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Title

A thesis presented in partial fulfillment of the requirements for the degree of Master of Philosophy in Biotechnology at Massey University

Full name : Aroon A. Parshotam
Year : 1988

Topic - Mathematical Analysis of the reaction diffusion processes occuring within a biofilm

by

Aroon A. Parshotam

Submitted as a 3 paper thesis for the degree of Masters of Philosophy in Biotechnology at Massey University in New Zealand.

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Abstract

This thesis investigates the behaviour of the solutions to mass transfer type reaction-diffusion equations within a biological film around a spherical particle, around a rotating cylinder and on a slab. In general these biological films may be found in any biological system and as examples of systems with spherical, cylindrical and slab geometry we examine biofilms in a fluidised bed biofilm reactor, biological growth around a rotating cylinder and on a rotating biological disk contactor.

These equations model concentrations of substrates within a single biofilm as a function of position and time.

In dimensionless coordinates , these equations have the form

$$\frac{\partial y}{\partial t} = \frac{1}{x^{a-1}} \frac{\partial}{\partial x} \left(x^{a-1} \frac{\partial y}{\partial x}\right) - \phi^2 F(y) \qquad (P)$$

for a = 1,2,3 being geometries of a slab, cylinder and sphere respectively and where

F(y) may correspond to F_0, F_1, F_n or F_{mm} and where

$F_0 = 1$ corresponds to zero order kinetics

$F_1 = y$ corresponds to first order kinetics

$F_n = y^n$ corresponds to nth order kinetics where n is an integer and

$F_{mm} = \frac{y}{1+\beta y}$ corresponds to Michaelis-Menten reaction kinetics

The dependent variable y corresponds to concentration and the independent variables x and t correspond to distance and time respectively.

The model parameters are saturation parameter β, which describes the concentration and Thiele modulus ϕ, which is the ratio of reaction to diffusion coefficients.

The boundary conditions are

$$\frac{\partial y}{\partial x}(\alpha,t)=0 \quad \text{and} \quad y(1,t)=1 \text{ for all time } >0$$

Here α is the internal boundary, a parameter that corresponds in spherical geometries to the ratio of the radius of a support media to the total radius of the bioparticle, in cylindrical geometries to the radius of a cylinder without biofilm to the total radius of the cylinder with biofilm and in slab geometry to be the ratio of the inactive region of diffusion to the thickness of biofilm measured from the centre of the slab. This may vary but for our purposes we shall take it to be a constant.

The major part of this thesis is concerned with the solution to the steady state associated with problem (P).

Using the maximum principle and, methods of upper and lower solutions and standard topologocal results from non-linear analysis, existence , uniqueness and monotonicity results are obtained.

In particular it is shown that the steady state problem has a unique solution for all values of the parameters when $\alpha \geq 0$, $\beta \geq 0$, $\phi^2 \geq 0$ and F(y) of the form above in the geometries slab, cylinder and sphere.

It is also shown that if $F(0) = 0$, $F(y) \geq 0$, the unique solution of the steady state problem associated with (P) is strictly greater than zero. This is indeed true for nth order and Michaelis-Menten kinetics.

For the zero order case $F(0) \neq 0$ implies that our solution to the steady state problem associated with (P) could become negative. Having a negative concentration is not a physical reality and we impose a third boundary condition that redefines α in terms of Thiele modulus, ϕ.

We also show that our solution to the steady state problem associated with problem (P) is monotonically decreasing in ϕ and monotonically increasing in β for all geometries and all orders. This may be generalised to all time for the unsteady state case.

The final part of this thesis considers the equation of the existence of solutions to problem (P) which are defined for all time . Such existence results are derived and approximate analytical solutions are obtained for some special cases ; but for general regions , only partial answers are obtained.

Dedication

Towards the last few weeks of typing up this thesis I came to the realisation that it may be far more important to study the creator of biofilms than to study these measely biofilms themselves. These biofilms seem to me now so insignificant in spite of the bother they have been causing me. I therefore humbly and sincerely dedicate this thesis to God.

Declaration

The work embodied in this thesis is to the best of my knowledge and belief, original, except as acknowledged in the text, and has not been submitted for a degree at any university either in whole or in part.

Aroon A. Parshotam 22/11/88

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Chapter 1 -Introduction, the model and the problem

1.1 Biofilms -an introduction

Definition (Biofilms)
Biofilms are multiple layers of microorganisms on a solid surface of any geometry in an aqueous environment.

Adhesion of micro-organisms to submerged or intermittently wetted surfaces is a virtually universal phenomenon which has been studied extensively by the engineering and medical fraternities, largely from the standpoint of the deleterious effects of biofilm formation (biofouling). These are manifest in engineering technology as heat transfer reduction, fluid flow resistance, corrosion and limitations imposed on mass transfer and chemical transformations - medical applications include fouling of prosthetic implants, dental decay and pathogen activity. The significance of microbial films in fermentation processes has also not been overlooked.

In natural environments, frequently nutrient deficient, observed increases in the concentration of nutrients at interfaces (gas/liquid, gas/solid, liquid/solid) bestows obvious advantages on microbes colonizing the phase boundary. In contrast, the objective of controlled application of biofilms in bioengineering is expedited through the provision of optimal conditions for substrate transfer at the interface and within the biofilm.This has been discussed by Anderson,G.K. and Sanderson,J.A. [1].

Biofilms will form at almost any interface where there is a microbial presence, indeed, it is more than many operators can do to prevent biofilm formation. Development of a biofilm is not simply the outcome of biological activity but the net result of a complex combination of physical, chemical and biological interactions at the phase boundary.
A sequential series of events occurs during initial biofilm formation leading ultimately to an unstable equilibrium condition:

(i) transport and adsorption of organic molecules to the wetted surface forming an organic conditioning layer. This is virtually spontaneous and the conditioning layer is comprised mainly of glyco-proteins, proteoglycans or their humic residues

(ii) transport of microbial cells to the 'conditioned' surface

(iii) microbial adhesion

(iv) biofilm proliferation and metabolism

(v) detachment (sloughing) of the microbial film

The dynamic equilibrium is maintained when (iv) and (v) are occuring simultaneously.

As examples of biofilm growth in various geometries, we shall examine biological growth around spheres, cylinders and on slabs.

Examples of these geometries in various biological systems are also given .

In particular, the fluidised bed biofilm reactor (FBBR) is an example of where spherical bioparticles may be found, a rotating cylinder is an example of a cylinder and a Rotating Biological Contactor Plant , an example of many slabs close together to support biological film growth.

We shall analyse these systems in more detail in the following examples.