GENERALISED KNOT GROUPS OF CONNECT SUMS OF TORUS KNOTS

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Abstract

Kelly (1990) and Wada (1992) independently identified and defined the generalised knot groups (G_n). The square (SK) and granny (GK) knots are two of the most well-known distinct knots with isomorphic knot groups. Tuffley (2007) confirmed Lin and Nelson's (2006) conjecture that $G_n(SK)$ and $G_n(GK)$ were non-isomorphic by showing that they have different numbers of homomorphisms to suitably chosen finite groups. He concluded that more information about K is carried by generalised knot groups than by fundamental knot groups. Soon after, Nelson and Neumann (2008) showed that the 2-generalised knot group distinguishes knots up to reflection. The goal of this study is to show that for certain square and granny knot analogues, the difference can be detected by counting homomorphisms into a suitable finite groups. This study extends Tuffley's work to analogues $SK_{a,b}$ and $GK_{a,b}$ of the square and granny knots formed from connect sums of (a, b)-torus knots. It gives further information about the generalised knot groups of the connect sum of two torus knots, which differ only in their orientation.

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