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**Enacting Challenging Tasks:
Maximising Opportunities for Students'
Mathematical Learning**

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requirements for the degree of
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Abstract

Three teachers of year 7 and 8 learners explored pedagogical approaches that exemplified current research on maximising opportunities for students to engage with and learn from challenging mathematics tasks. This study examined the learning opportunities afforded by the task enactments in the teachers' classrooms. The study also considered teachers' perspectives on a planning and lesson structure that exemplified explored approaches, and the challenges teachers experienced in implementing the tasks and approaches.

Reforms in mathematics education that have called for change in how teachers view mathematical knowledge, the value and purpose of social interaction in the classroom, and teachers' role as participants in classroom discourse, have influenced pedagogical approaches to the enactment of classroom tasks. Relevant literature was reviewed that illustrated the importance of tasks in affording opportunities for students to engage in meaningful mathematical practices and discourse, and construct conceptual mathematical understanding. Evidence was provided that teachers' pedagogical decisions and actions play a significant role in optimising opportunities for student learning from tasks, and that teachers' task implementations are mediated by their intentions, goals, knowledge, attitudes and beliefs.

The qualitative methodology chosen for this study aligned with case study and design-based research approaches. Multiple data sources were collected, and systematic analysis and triangulation of data alongside collaboration between the researcher and participant teachers strengthened the research findings.

The study revealed the influence of task selection on the type of mathematical activity afforded value in classrooms. The planning template and lesson structure prompted purposeful decision-making that strengthened teachers' task enactments, including explicit consideration of mathematical ideas inherent in tasks, students' prior understandings, and the role of task variations in supporting students' access to tasks. The study demonstrated that different enactments from the same planning resulted in contrasting opportunities for student learning. A noteworthy difference was the extent to which the mathematical ideas inherent in the task were explicitly addressed by teachers.

The results revealed the impact of teachers' decisions when selecting and implementing classroom tasks, and offered insights into purposeful pedagogical actions that teachers could incorporate into their practice to maximise opportunities for their students to learn mathematics.

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