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# **Process Mnemonics and Mathematics Learning Disabilities**

**Emmanuel Manalo**

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# Abstract

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This study investigated the effects of process mnemonic instruction on the computational skills performance of 13- to 14-year-old students with mathematics learning disabilities (LD). Two experiments were carried out. In Experiment 1, 29 students with mathematics LD were assigned to either a process mnemonic instruction group, a demonstration-imitation instruction group (which served as a comparison instruction group), a study skills group (which served as a placebo instruction group), or a no instruction group. Those in the process mnemonic and the demonstration-imitation groups were provided with instructions in computational skills. The present author acted as instructor. Assessments of performance were undertaken at pre-instruction, immediate post-instruction, 1 week later, and 6 to 8 weeks later. The results showed that those in the process mnemonic group made significant improvements following the instructions provided. During the earlier stages of post-instruction, the magnitude of improvements they made were generally equivalent to that made by students in the demonstration-imitation group. However, in the longer term, the improvements made by the students in the process mnemonic group maintained better. No significant changes in performance were observed in the study skills and no instruction groups.

In Experiment 2, 28 students with mathematics LD were assigned to groups similar to those in Experiment 1, but without the study skills group. Two research assistants acted as instructors to control for any possible unintentional bias, and to investigate the effectiveness of the process mnemonic method when used by other instructors. Assessments of performance were undertaken at pre-instruction, immediate post-instruction, 1 week later, 4 weeks later, and 8 weeks later. The results

obtained were similar to those in Experiment 1. Furthermore, the students in the process mnemonic group generally made greater performance improvements compared to those in the demonstration-imitation group. No significant changes in performance were observed in the no instruction group.

Areas focused on in the discussion include the possible reasons why the process mnemonic method of instruction proved to be effective, the method's potential applications in mathematics LD remedial instruction, and the implications of the findings about the mathematics LD condition.

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*This thesis is dedicated to the memory of my late grandmother, Emilia O. Olivares, who walked with me and provided me with a lot of encouragement when I first started school.*

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# Introduction

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There exist a considerable number of papers in the psychological and educational literature about the use of mnemonic techniques to teach students with learning disabilities (LD). In some ways, the present study can be considered as belonging to this genre which began with the rekindling of interest in mnemonic usage in the late 1970s and early 1980s. However, in many ways, the present study is also different.

Even though mnemonic rhymes such as “an i before an e except after c or when sounded like an a as in neighbour and weigh,” which serve as reminders of rules, have been around for some time, until recently mnemonics have almost exclusively been considered only as tools to aid memorisation of facts. As Morris (1979) pointed out, mnemonic techniques can effectively be used to remember facts, particularly when the material being dealt with is disconnected and devoid of meaning.

The published research on mnemonic usage to teach students with LD has almost exclusively been on fact mnemonics. Studies have employed the pegword method, the keyword method, and various other mnemonic techniques to help students with LD learn various assortments of facts. They have shown mnemonics to be very effective in facilitating this learning.

With so many studies that have been carried out on the educational use of mnemonic strategies, it appeared that by the late 1980s most avenues of research on this topic had been exhausted. The viable fact mnemonic strategies had been employed. A wide range of subject topics, from foreign vocabulary to science facts, had been investigated. And the

possible populations, from normal individuals to ones with LD, had been examined.

However, particularly where mnemonic use with students who have LD was concerned, there was something unsatisfactory which was unacknowledged in the literature. Fact mnemonic instruction was being described, for example, as “an effective learning and memory strategy for handicapped learners” (Mastropieri, Scruggs, & Levin, 1987, p. 369). But, as will be discussed in later chapters, handicapped learners such as those with LD are generally not characterised in terms of their inability to learn facts. Some of them may have problems in learning facts, but they are considered LD because of their inability to learn specific skills such as reading, spelling, and mathematics. Fact mnemonic usage was therefore not addressing the central issues of concern in LD.

There is, however, another type of mnemonics, process mnemonics. Some process mnemonics are used on a daily basis by many people to remember rules and procedures in spelling, trigonometry, mathematics, and science. For many years, one type of process mnemonics has been used successfully to teach students in a school in Japan anything from how to solve quadratic equations to how to speak the English language. Furthermore, a derivative of this Japanese process mnemonic method had been successfully employed to teach primary school children in California how to solve problems involving fractions. However, very little empirical research has been carried out on process mnemonic usage, let alone its applications in special education.

The present study investigated the usefulness of process mnemonics in teaching students with mathematics LD. The primary question it addressed was whether process mnemonic instruction can demonstrably improve the performance of students with LD who manifest significant deficits in executing basic computational operations in addition, subtraction, multiplication, and division. Thus, the present study is

different from other earlier studies in that it used a new kind of mnemonic, process mnemonics, to teach students with LD rules and procedures rather than facts. It addressed a central issue of concern in mathematics LD – deficits in computational skills.

The first three chapters of this thesis review relevant literature in the areas of LD, mathematics LD, and mnemonics, as a basis for the study which follows. A rationale and hypotheses chapter explains more explicitly the reasons behind the study and the questions it endeavours to address. The rest of this introduction provides a preview and a rationale for the materials and issues covered in the three literature review chapters.

The first chapter on LD briefly explains what LD is, outlines the many types of LD that there are, and describes one way of organising and classifying these types. It shows how manifestations of significant deficits in computational skills is a subtype of mathematics LD, which in turn is a form of academic LD.

The next section in this chapter looks into some of the factors that may be considered when addressing the question of whether individuals with LD are truly capable of learning. These include the reported cases of success in the remedial intervention literature which are incongruent with reported cases of persistence of LD problems into adulthood. It is suggested that perhaps persistence of problems need not be taken in many cases as evidence for an inability to learn, but of a problem in remembering the skills that have been learned in the long term.

Finally, the first chapter looks into the question of what the best way is of identifying individuals with LD, and examines some of the arguments for and against the use of IQ test scores in the selection of experimental participants with LD. It concludes that it is unnecessary to exclude individuals with below average IQs from LD research studies and

remedial intervention programmes. The position taken is that individuals who score below average in IQ tests should still be considered as LD as long as they are not seriously impaired intellectually.

The second chapter on mathematics LD provides a concise coverage of some of the important factors and issues relating specifically to this form of LD. It begins with a brief description of mathematics LD and then it describes some of the many skills necessary in mathematics learning. It points out that the acquisition of computational skills competence relies on the development of rapid fact retrieval, procedural knowledge, and monitoring of the sequence of steps being executed.

The second chapter then outlines the typical pattern of mathematics skills acquisition in normal children, and then describes the error patterns of children who have mathematics LD. The point is made that there appears to be a regular, predictable progression of computational and other mathematical skills acquisition in children. Some common types of errors can also be identified among individuals with mathematics LD who manifest computational skills deficits.

The next section in the second chapter examines some of the reasons that have been put forward to explain the deficits exhibited by individuals with mathematics LD. Because of their ability to explain the types of computational errors frequently exhibited by individuals with mathematics LD, some of these explanations are considered more convincing. These include explanations that propose inadequate functioning of subcomponents of the calculation system, the developmental delay theories, and the explanations that focus on memory deficits.

The final section of the second chapter then looks at the remedial interventions that have been used with individuals who have mathematics LD, and their respective effects. One of the main purposes of

this section is to identify some of the important features that have been found effective in remedial intervention efforts that have been undertaken thus far. The features identified include motivating students to use new and more successful strategies, reducing the load demand on the memory of students through the use of simple rules, and teaching at a more concrete level.

The third chapter on mnemonics, first very briefly defines the term and mentions the two main types: fact and process mnemonics. It then provides a concise historical background to the uses of mnemonics, and describes two more recent reports of mnemonic feats. The main purpose of these sections is to show how the need to remember various forms of information has an almost predisposing effect on people to employ mnemonic techniques in natural, as well as unnatural, settings.

The third chapter then describes the types of fact mnemonics that there are, and their uses that have been reported in various research studies: first in teaching normal students, and second in teaching students with LD. The purpose of these sections is to stress the extensive uses and the effectiveness of fact mnemonic instruction in educational settings – noting particularly the special education area.

The next section deals with the question of why mnemonics work. Explanations put forward include: that mnemonics facilitate recoding, relating, and retrieval of target information; that mnemonic use facilitates the qualities that distinguish students who are effective in their learning; that mnemonics function in the same way as memory schemas; and that mnemonics enhance the successful execution of knowledge acquisition and performance during information processing.

The following section describes process mnemonics, as exemplified by the yodai method used at the Ryoyo Institute in Japan. A brief historical account of the development and use of yodai mnemonics is provided, as

well as examples of its applications in learning. The pool mnemonic method for teaching and learning operations with fractions, developed for American primary school-aged children, is outlined with the aim of noting the cross-cultural applicability of some of the notions behind yodai mnemonics.

A section that reports on comments about yodai and process mnemonics follows. It is pointed out that despite some important criticisms that need to be considered, process mnemonics nevertheless present potentially useful tools for teaching students with special needs, such as those with LD. This section also stresses that mnemonic use does not obstruct understanding of materials being learned.

The third chapter concludes with a brief description of the only published study so far on the use of process mnemonics in LD remedial instruction. The need for further research is highlighted: particularly to find out if process mnemonic instruction can produce performance improvements in other individuals with LD; if process mnemonic instruction can effectively be used to teach small groups; and if the performance improvements that result from process mnemonic instruction maintain in the long term.

## Chapter 1

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# Learning Disabilities

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### *Introduction*

Individuals with learning disabilities (LD) perform inadequately in areas of academic achievement despite seemingly adequate intellectual skills and educational opportunity. Mathematics is one of the areas of academic achievement that can be affected; mathematics LD is sometimes referred to as dyscalculia. Other common forms of LD are in reading and in writing/spelling, sometimes referred to as dyslexia and dysgraphia, respectively.

While the brief description above of LD appears simple and straightforward, adequately defining LD is far from being simple and straightforward. The most recent definition provided by the National Joint Committee on Learning Disabilities (NJCLD) in the United States of America, given in a letter to member organisations and cited in Hammill (1990), is as follows:

Learning disabilities is a general term that refers to a heterogeneous group of disorders manifested by significant difficulties in the acquisition and use of listening, speaking, reading, writing, reasoning, or mathematical abilities. These disorders are intrinsic to the individual, presumed to be due to central nervous system dysfunction, and may occur across the life span. Problems in self-regulatory behaviours, social perception, and social interaction may exist with learning disabilities but do not by themselves constitute a learning disability. Although learning disabilities may occur concomitantly with other handicapping conditions (for example, sensory impairment, mental retardation, serious emotional disturbance) or with extrinsic influences (such as cultural differences, insufficient or inappropriate instruction), they are not the result of those conditions or influences. (NJCLD, 1988, p. 1: cited in Hammill, 1990, p. 77)

The above NJCLD definition is in fact the second definition that the committee has issued. The first, which is the one often cited in literature dealing with the topic of LD definition, was issued in 1981 (see Hammill, Leigh, McNutt & Larsen, 1981). The 1988 definition contains a few changes to the 1981 version: it spells out more clearly that LD occurs across the life span, and it points out that nonverbal disabilities such as those relating to social interaction and self-regulatory behaviour are different from LD. Also, according to Hammill (1990), for the purposes of precision the second version made wording changes to several points: for example, the word "direct" was dropped from the original so that it can be stated more precisely that "... [LD] is not the result of those [other] conditions or influences."

Hammill (1990) considered the 1988 NJCLD definition to be the best of those currently available because it best describes "the nature of learning disabilities" (p. 82). He was also of the opinion that consensus was near regarding the issue of defining LD, as indicated by the popularity of certain definitions and fundamental agreement on most issues among those definitions. Other authors, however, are not as optimistic. Kavale and Forness (1992), for example, while conceding that the field appears to have reached a status quo with regard to the issue of definition, believed that the problem of definition can only be resolved by ceasing "... efforts at modifying all available definitions" (p. 21).

A discussion of the detailed arguments concerning the definition of LD is outside the scope of this study. For the present purposes, the previously mentioned simple and generally accepted notion of LD as manifestations of inadequate performance in areas of academic achievement despite seemingly adequate intellectual skills and educational opportunity, will be used as the basic guideline for what LD means. The specifications of the 1988 NJCLD definition will generally be adhered to in considerations of the more detailed aspects of the LD condition.

The rest of this chapter will look at the types of LD that there are, particularly in view of how mathematics LD fits in existing schemes; the question of whether individuals with LD are capable of learning and overcoming their learning deficits; and the identification of LD and the reasons why IQ test scores may be considered irrelevant to this process.

### *Types of LD*

Apart from difficulties associated with acquiring skills that are commonly and explicitly taught in schools – namely reading, writing, and mathematics skills – the NJCLD definition (cited in Hammill, 1990, p. 77) includes difficulties in listening, speaking and reasoning. Elsewhere, Kirk and Gallagher (1989, p. 186) noted that “... there are so many learning disabilities that it’s impossible to classify them or even to draw up a specific list of the different types.” Their observation is not far from the truth. Hazel and Schumaker (1988), for example, suggested that deficits in social skills constitute a learning disability, and more recently Stanovich (1993) proposed a new specific learning disability called *dysrationalia*. According to Stanovich, *dysrationalia* is “the inability to think and behave rationally despite adequate intelligence” (p. 503). Some resistance was shown to accepting social skills deficits and *dysrationalia* as constituting forms of LD (e.g., Forness & Kavale, 1991; Kavale, 1993; Sternberg, 1993), but their proposals highlight the fact that because learning is a very complex matter there are many disabilities that can be associated with it. Whether or not all these disabilities ought to be included in the LD category is, of course, another matter.

With the existence of so many possible forms of LD, perhaps as a starting point it is useful to distinguish between developmental and academic forms of LD, as Kirk and Gallagher (1989) suggested. They described *developmental learning disabilities* as “deviations from normal development in psychological or linguistic functions” and noted that

“often these disabilities are related to information processing, the way the individual receives, interprets, and responds to sensory input” (p. 187). They included in this developmental category disabilities that are manifested as disorders in attention and perception, memory, perceptual-motor skills, thinking, and language. In contrast, they described *academic learning disabilities* as “conditions that significantly inhibit the process of learning to read, spell, write, or compute arithmetically” (p. 190). They pointed out that these disabilities are often manifested and identified when individuals are in academic settings and performing well below their potential.

Kirk and Gallagher’s (1989) distinction between developmental and academic LD should not be confused with the distinction between ‘developmental’ and ‘acquired’ forms of LD. Acquired LD, in this latter context, refers to LD following brain injury in an individual who previously experienced no specific learning problems. For example, *acquired dyslexics* were previously competent readers who suffered an impairment of that ability due to brain injury. Developmental LD in this context refers to learning disorders in individuals who have never sustained any brain damage. For example, *developmental dyslexics* have not lost their ability to read: for some reason they never attained an adequate ability to read in the first place. Since Kirk and Gallagher were not discussing individuals who have sustained various forms of brain injury, the LD categories they were referring to are both ‘developmental’ in nature rather than ‘acquired.’

There are many different ways that have been proposed for organising and classifying the diverse forms of disorders that are subsumed under the label of LD. For example, Barkley (1981) came up with disorders of reading, disorders of arithmetic, and disorders of written expression. Siegel and Heaven (1986) suggested the categories of reading disability, written work/arithmetic learning disability, and attention deficit disorder. Another example is Bender and Golden’s (1990) study which, after

subjecting the test scores of 57 children with LD to cluster analysis, came up with a group of language deficit children, a group of visual deficit children, a group with no notable deficits, a group with very poor reading achievement paired with very high self-concept, and a group of behaviourally disordered children. When it comes to subtyping specific forms of disabilities (e.g., reading disabilities) the situation becomes even more complicated.

With this complicated and confusing state of affairs in mind, Kirk and Gallagher's (1989) suggested differentiation between academic and developmental forms of LD appears most sensible at this initial level. This would enable distinction of deficits in skills that are taught in schools from those that are primarily a function of maturation. Academic learning disabilities – those that manifest themselves as disabilities in acquiring and using basic skills that are explicitly taught in schools (i.e., reading, writing and mathematics) – are closely tied in with the LD individual's reception of, and response to, instruction. In contrast, developmental learning disabilities – such as disabilities in listening, speaking, and reasoning – are closely tied in with the LD individual's natural process of development. These skills, while they may be implicitly cultivated in schools, are not usually explicitly taught. Research on these forms of LD assumes that, other relevant factors being normal, at certain stages of chronological development certain levels of skills acquisition could be expected, independent of instruction. For example, it could be expected that in terms of listening skills a twelve-year-old would be better than an eight-year-old, simply because the former is older and has presumably developed this skill to a more advanced level. Thus a significant negative deviation from the expected skill level could constitute a 'developmental' learning disability.

Making this initial distinction between academic and developmental forms of LD could provide some order to the many and diverse classification systems that have been put forward. In the examples

mentioned earlier, all of Barkley's (1981) categories, Siegel and Heaven's (1986) disorders of reading and written work/arithmetical learning disability, and Bender and Golden's (1990) group with very poor reading achievement, would be types of academic LD. Siegel and Heaven's attention deficit disorder, and the rest of Bender and Golden's categories, on the other hand, would be types of developmental LD. While even after making this distinction it is obvious that agreement about what ought to be included in each form of LD is far from reached (e.g., Bender and Golden do not mention anything about writing or mathematics skills deficits, while Siegel and Heaven lump the two together), it at least makes it a little easier to understand the diversity of initial classification systems that exist in the LD literature.

The overwhelming majority of research that has been carried out in the LD field is on the academic forms of LD. This is perhaps quite understandable since academic forms of LD are in many ways more salient: in literate cultures formal education takes up a considerable proportion of time in childhood and young adulthood, and much of the ability to succeed in (or cope with) education hinges on learning how to read, write, and solve mathematical problems. Failing to acquire and use reading, writing and mathematical skills, therefore, is a highly noticeable problem – both to the individual concerned, and to educators. Reading, writing and mathematics also lend themselves more easily to empirical research when it comes to questions about acquisition and use of skills: for example, tests to directly assess competency levels in literacy and mathematics skills are available, as well as normative data relating to expected levels of attainment at different chronological ages or school grades. Furthermore, the majority of individuals with LD experience difficulties of the academic kind: Lyon (1985), for example, noted that the deficits in 60–80% of the LD population are primarily in reading decoding or comprehension skills.

Having made the distinction between academic and developmental forms of LD at the initial level, the situation remains complicated at the next level down because there exist so many different proposals for subtyping specific forms of disabilities (e.g., reading LD, writing LD, or mathematics LD). Essentially, these subtyping systems fall into two categories: *extrinsic* and *intrinsic*. Extrinsic subtyping systems are based on concomitant disabilities in oral language, motor skill, perceptual capacity, short-term memory, and so on. An example of an extrinsic system for subtyping dyslexia is the one that Mattis (1978) proposed: the subtypes under this system were language disorder, articulatory and graphomotor incoordination, and visuospatial disorder. Intrinsic subtyping systems, on the other hand, are based on actual learning performance and strategies per se. An example of an intrinsic system for dyslexia is Boder's (1973) categories of reading and spelling patterns: these were dysphonetic, dyseidetic, and mixed dysphonetic-dyseidetic. Not all subtyping systems fit cleanly into one or the other of these two categories though. Barkley's (1981) subtypes of dyslexia, for example, appear to utilise an integration of the intrinsic and extrinsic classification systems. He proposed the subtypes of disorders of language skills (extrinsic), disorders of sequencing and verbal expression (intrinsic), and disorders of visuospatial skills (extrinsic).

A question that could be asked is: Is either of these subtyping systems better than the other? Ellis (1985) argued in favour of intrinsic systems. He pointed out that the main problem with extrinsic systems is that there would be overlaps between individuals with LD and those without LD where the classifying disorders (e.g., oral language, motor skill, etc.) are concerned. It is likely, for example, that there are as many non-dyslexic individuals with visuospatial disorders as there are dyslexics. It was found, for example, in Feagans and Merriwether's (1990) study that 34% of their subjects who manifested high error rates in visual discrimination tasks *did not* have LD – not quite as high a percentage as those with LD but still a considerable proportion. With such overlaps, the causal role

that the extrinsic symptoms play in bringing about LD comes into question and, consequently, their usefulness as bases for subtyping also becomes questionable.

In contrast, where intrinsic subtyping systems are concerned, Ellis (1985) noted that there are often parallels between the proposed categories and findings about how the particular skills (e.g., reading, writing) are actually executed. He was referring in particular to Boder's (1973) intrinsic classification system for dyslexia and findings about reading routes based on studies about acquired dyslexia (e.g., Ellis, 1984; Morton & Patterson, 1980). The reading patterns that distinguish Boder's classifications of dysphonetics and dyseidetics closely parallel the distinctions between the direct and analytical reading routes.

If the intrinsic subtyping system is employed, reading LD can further be subdivided into deficits in whole-word reading skills (direct), and phonic word-analysis skills (analytical). Recent research focusing on performance characteristics of individuals with reading LD (e.g., Castles & Coltheart, 1993) support the existence of at least these two varieties. In the other forms of academic LD, writing and mathematics, intrinsic subtypes can also be identified. Writing and spelling, of course, are closely linked to reading (Barkley, 1981). Most individuals with writing and spelling LD (dysgraphia) display spelling performance patterns like those described by Boder and Jarrico (1982). There are two basic types: those who spell by sight alone and can therefore only spell words that are in their sight vocabulary, and those who spell words phonetically and therefore often make spelling errors that are good phonetic equivalents.

With regard to mathematics LD or dyscalculia, Sutaria (1985) observed that disabilities of this kind can be subsumed under two broad groups: those which primarily involve computational skills, and those which deal with reasoning skills. Like individuals with reading and writing LD,

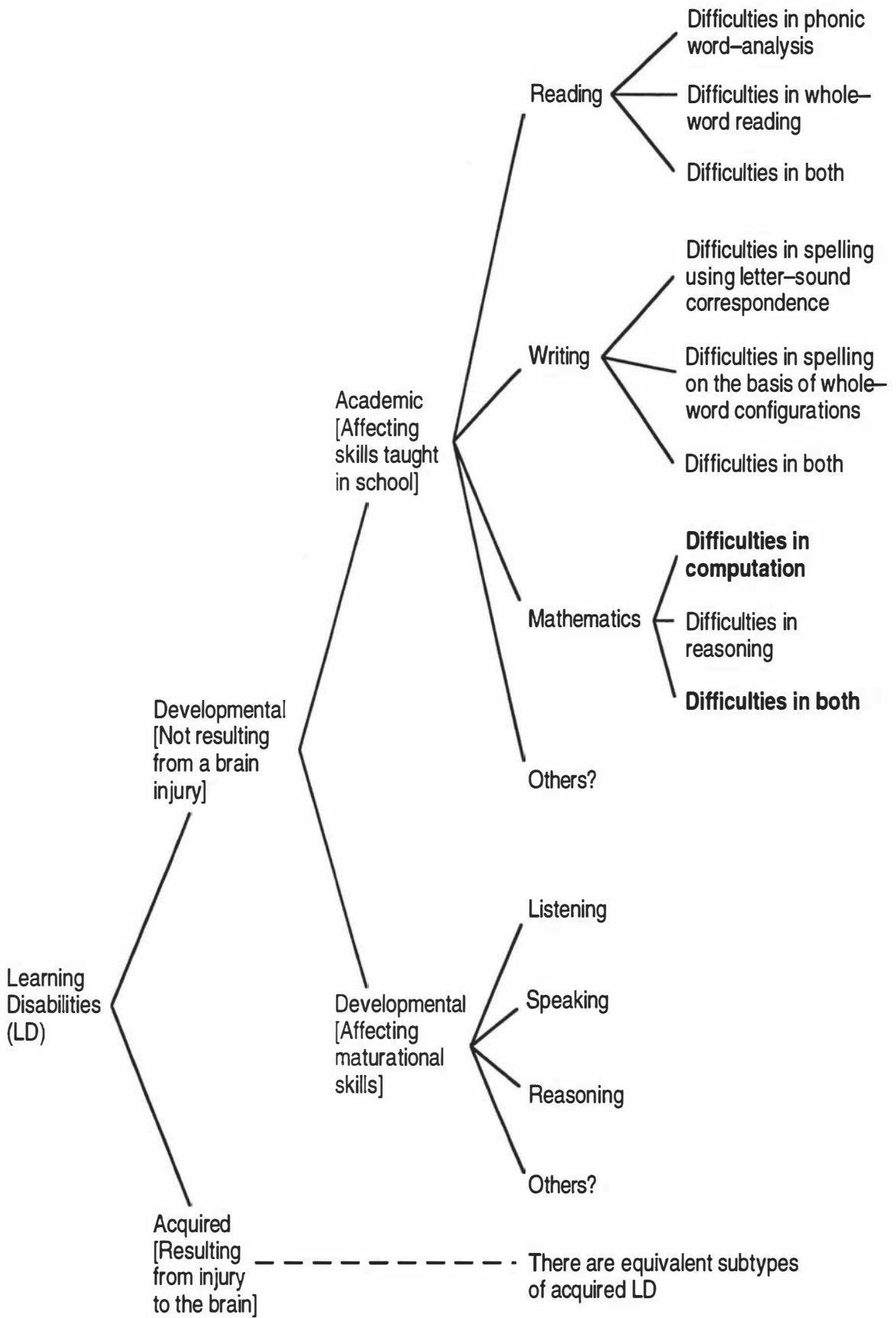
some individuals with mathematics LD manifest both kinds of deficits in their performance.

Figure 1 outlines one possible way of organising the many different types of LD that exist. This organisation system first distinguishes between forms of LD that are caused by injury to the brain and those that are not. The academic–developmental distinction is then made to sort those that do not result from any known injury to the brain. Of the academic forms of LD, an intrinsic approach is used to describe the different subtypes.

The subtype of mathematics LD that is of concern in the present study – that which is characterised by difficulties in computational skills – is in bold print. Mathematics LD involving difficulties in both computation and reasoning is also in bold print since, presumably, many of those who have deficits in computational skills also experience considerable difficulties in mathematical reasoning.

### *Are individuals with LD capable of learning?*

The use of the term LD to describe individuals with specific problems in learning suggests that these individuals are not capable of learning. The term *disabilities* implies impairment of abilities, a state that has some degree of permanence associated with it. The overarching assumption in most remedial intervention studies, however, runs contrary to this. These studies are based on the notion that individuals with various forms of LD *are* capable of learning and at least partially overcoming the specific difficulties they experience. Instructional methods and procedures used in these studies are often innovative and/or quite different from regular classroom instruction. Inherent in these studies is the belief that what experimental participants initially cannot do because of their LD can be learned by these very same individuals, provided the



**Figure 1. Types of learning disabilities.**

modifications in methods and procedures of instruction undertaken in these studies are employed.

Results from remedial instruction studies largely support the assumption that individuals with LD are capable of learning (e.g., see reviews by Mastropieri, Scruggs, & Shiah, 1991; Pereira & Winton, 1991; Scruggs & Mastropieri, 1990; Singh, Deitz, & Singh, 1992). These results need to be reconciled with the earlier mentioned implication of the term LD, as well as the finding in some studies that LD problems persist into adulthood (e.g., Gerber, Schnieders, Paradise, Reiff, Ginsberg, & Popp, 1990; Horn, O'Donnell, & Vitulano, 1983; Schonhaut & Satz, 1983; Spreen, 1982). It is necessary to examine the normal acquisition of the skills affected by LD; and the similarities between the skills capabilities at different stages of acquisition and the performance profiles of different subtypes of individuals with LD. Although the focus of the present study is on mathematics LD, the majority of the studies cited in this section pertain to reading acquisition and reading LD because no parallel studies have been undertaken on the topic of mathematics LD. The studies cited here are important because they convey the possibility that subtypes of LD fit in a learning continuum similar to that which normal individuals go through. These studies suggest that individuals with LD are capable of progressing through the stages of skills acquisition represented in the continuum.

Ellis (1985) made an interesting observation that there are strong similarities between the skills acquired at different stages of normal reading development proposed in Marsh, Friedman, Welch, and Desberg's (1981) cognitive-developmental model and the symptoms manifested in different kinds of reading LD. Ellis only referred to the Marsh et al. model, but Frith (1985) and Seymour and MacGregor (1984) later proposed similar models. And almost 20 years earlier, both Luria (1966) and Gibson (1968) proposed models similarly suggesting that the sequence of stages in learning to read is orderly and developmental.

In a simplified form, these models of reading acquisition propose that beginning readers acquire a small sight vocabulary of words they can read as wholes. Later, they acquire some phonic knowledge which enables them to 'sound out' regular words. Still later, they learn to use other higher order rules of decoding, and analogy, which enable them to read even words that are unfamiliar and irregular. These models suggest that the progression in the development of these skills is largely independent of the instructions the individual receives.

Ellis (1985) pointed out that the errors and skills manifested by beginning readers are very similar to those of one of the subtypes of reading LD, the dysphonetic or phonological dyslexic who lacks phonic word analysis skills. Readers at the later stage of development, on the other hand, show errors and skills similar to another subtype of reading LD, the dyseidetic or surface dyslexic who tends to over-rely on the sounding out approach in reading and who lacks adequate whole-word reading strategies.

Ellis (1985) further suggested that perhaps developmental dyslexics pass through various metamorphoses from one reading profile to another, in a way similar to normal readers. He noted that Coltheart, Masterson, Byng, Prior, and Riddoch's (1983) research participant CD, who they described as a developmental surface dyslexic, could have previously been a developmental phonological dyslexic. Coltheart et al. considered CD a surface dyslexic because she was more successful at reading aloud regular than irregular words and was reasonably competent in reading nonwords. At 16 years of age, CD had a reading age equivalent of only 9 to 10 years. Ellis pointed out, however, that since Coltheart et al. described her as someone who was "always slow at reading" it is possible that she progressed through the normal stages of reading development before becoming fixed (if she was fixed) at the reading age of a 9- to 10-year-old. Thus his point was that surface dyslexics, being similar to normal readers at a later stage of development in the skills and errors they manifest, are very likely to have been classifiable as phonological dyslexics earlier on

and manifested skills and errors similar to beginning readers. They could simply have progressed through the normal stages of reading acquisition at an abnormally slow rate.

Additional evidence that dyslexics pass through various reading profiles like normal readers, only slower, is found in Manalo (1986). He described a 20-year-old student, SA, who was a developmental surface/dyseidetic dyslexic with a reading age of only 13 years. Although in 1986, SA had good phonic skills, she had serious problems in this area prior to receiving additional reading and spelling instruction from a qualified tutor. There was documented evidence not only that her reading age was previously lower than 13 years, but also of shifts in her reading profile and the gains in skills she made over specific periods of time.

As previously noted, there are very few studies on mathematics LD that specifically deal with this topic. Piaget (1952) proposed that the acquisition of mathematical skills is orderly and developmental in nature. Nesher (1986), in reviewing research that has been undertaken on various mathematical skills such as counting, also arrived at the conclusion that there is a sequence and hierarchy involved in acquisition, largely governed by maturation and cognitive development. Shalev, Manor, Amir, and Gross-Tsur (1993) investigated the development of arithmetic skills in normal school-children on the basis of a cognitive model of acquired dyscalculia, and their findings also support the notion that there is an orderly and developmental sequence in mathematics skills acquisition. These studies will be discussed further in the next chapter which specifically examines issues pertinent to mathematics LD.

Kulak (1993) suggested that the majority of children with mathematics LD manifest the condition because of a developmental delay in moving through the normal sequence of strategy acquisition. Putnam, deBattencourt, and Leinhardt (1990) found support for this developmental delay notion, and argued that children with mathematics

LD are capable of moving through the various stages of skills acquisition. Putnam et al. investigated students' justifications and evaluations of 'derived-fact strategies' which are strategies for solving unknown combinations in computational problems by using known combinations. For example to solve  $6 + 7$ , an individual might use this strategy: "I know that 6 plus 6 is 12 so 6 plus 7 must be 13, because 7 is 1 more than 6, and 13 is 1 more than 12." They found that students with LD not only tended to use previously learned or invented methods which were unsuccessful, but also that their performance very closely resembled those of younger normally achieving students. For example, in Addition, the fifth-grade students with LD provided approximately the same number of correct strategy justifications as normal achieving third-graders.

Although more comprehensive research is required to substantiate the claim that individuals with various forms of LD pass through various metamorphoses from one skills profile to another, similar to individuals without LD only slower, it appears difficult to counter the argument that individuals with LD *are* capable of learning. The more contentious aspects of this issue concern questions like: How much can they learn? Does the learning they manifest produce metamorphic profile shifts, or simple enhancement of the very same skills they already have? Do they reach a plateau, where no more skills improvement appears possible?

The last question in particular appears very pertinent because, although the overwhelming majority of intervention studies reported in the literature suggest that individuals with LD are by and large receptive to remedial instruction, as previously pointed out some research studies have also found that for many individuals with LD problems persist into adulthood (e.g., Gerber et al., 1990; Horn et al., 1983; Schonhaut & Satz, 1983; Spreen, 1982). This could be taken to mean that individuals with LD have fixed limits in their learning capabilities: they reach but cannot go beyond sub-optimal levels.

However, it is still possible that at least in some cases the persistence of problems is the result of non-provision of appropriate remedial instruction and assistance. Naidoo's (1981) review, for example, noted that the most widely used teaching schemes for children with dyslexia all employ a phonic approach. Assuming that instruction should be directed at an individual's weaknesses rather than his/her strengths, the phonic approach would be appropriate for teaching individuals with reading and/or writing LD who experience phonological skills difficulties. Such an approach, however, would not be appropriate for individuals with reading and/or writing LD who are dyseidetics or developmental surface dyslexics. The phonic approach would not address the latter group's need to overcome difficulties in perceiving whole-word configurations, making use of conditional rule patterns and other higher order rules of orthographic structure, and so on. It is highly likely that a similar situation exists where mathematics LD is concerned, where a predominant focus in or method of remedial instruction tends to neglect needs or fails to address deficits of particular subtypes.

The point here is that possible gains in skills proficiency or shifts in skills profile depend on the focus and method of remedial instruction provided to individuals with LD. This being the case, it is very important that performance characteristics directly related to the skill of concern (e.g., reading, writing, or mathematics) are adequately assessed. Areas of capabilities and deficiencies have to be determined, as well as strengths and weaknesses. And, as pointed out above, it is very important that the instruction provided is directly appropriate to the profile of the remedial group of individuals with LD, so that it addresses the skills deficits they have.

Perhaps one way to reconcile the apparent incongruence between the responsiveness to remedial instruction of individuals with LD on the one hand, and the persistence of problems into their adulthood on the other, is to view both as being true but that the former need not always imply

permanence. Thus, individuals with LD may largely be responsive to remedial instruction as most research studies suggest, but the effects of instruction and consequent improvements in performance may not always be lasting, and so some adults with LD are found still suffering from the same learning problems some years afterwards. Montague (1992), for example, found that students with LD learned the cognitive and metacognitive components of a strategy for solving mathematics problems with relative ease, but the students did not maintain the strategy over time. It is possible that in many cases the problem lies not so much in actual learning but in retaining or remembering what is learnt. After all, reading, writing, and mathematics all involve not just learning and retention of facts but also of quite complicated rules and procedures. This matter will be examined further in the next two chapters.

### *Identification of LD and the use of IQ*

Theorists, researchers, and practitioners in the field of LD are not in agreement about many issues. One of these issues concern the identification of individuals with LD and the use of IQ tests in that process. Just what is the best way of identifying individuals with LD? How necessary is it to use IQ test scores?

A frequent requirement in the process of identifying individuals with LD is to *exclude* those whose difficulties can be attributed to causes other than LD. Salvia and Ysseldyke (1988), for example, pointed out that the areas that have to be excluded from the LD category include mental retardation, blindness or visual impairment, deafness or hearing impairment, physical handicaps, emotional disturbance that is severe enough to require therapy, and environmental and/or cultural factors. Basically the point is that individuals with LD are those whose failure to acquire literacy and/or numeracy skills cannot be attributed to any obvious extrinsic factor such as lack of intelligence, inadequate opportunity to

learn, or sensory impairment. Vellutino's (1979) list of characteristics that developmental dyslexics must (or must not) have is typical of the sort of criteria commonly used to identify LD individuals by exclusion:

- 1 An IQ of 90 or above on either the Verbal or the Performance scale of the Wechsler Intelligence Scale for Children;
- 2 Adequate peripheral sensory functions (sight and hearing);
- 3 Absence of severe neurological or physical disability;
- 4 Absence of 'significant social or emotional problems';
- 5 Not socioeconomically disadvantaged; and
- 6 Adequate opportunity to learn to read.

A criterion that is related to exclusion and also almost always required in the identification of individuals with LD is discrepancy. For individuals to be identified as LD, it is necessary for there to be a significant difference or discrepancy between their expected and actual achievement. For example, for an individual to be identified as having mathematics LD, a criterion that could be used is that their mathematics achievement score must be one standard deviation lower than their *expected* mathematics achievement score, according to the norms of a standardised and acceptable mathematics test.

Both the exclusion and the discrepancy criteria, however, are problematic in many ways. Kavale and Forness (1992), for example, argued that the exclusion criterion does not adequately differentiate between individuals with LD and those with similar conditions such as mental retardation (MR), brain dysfunction (BD) and environmental, cultural or economic disadvantage (CD).

Intelligence, as measured by IQ tests such as the Wechsler Intelligence Scale for Children – Revised (WISC–R), is a very important component of both the exclusion and the discrepancy criteria. As mentioned earlier, individuals who have below average IQs are excluded from the LD

category because it is assumed that their inability to acquire literacy and/or numeracy skills is attributable to lack of intelligence. Thus it is often considered critical that a discrepancy exists between the achievement test score and the intelligence test score of an individual with LD.

However, the requirement that individuals must have at least average IQs before they can be identified as being LD has been argued against by numerous authors in the field. As early as 1972, for example, Cruickshank had expressed the opinion that average or above IQ is not a prerequisite for LD. He did not agree with school psychologists' use of IQ and other mental age tests in the identification of LD because they "produce sterile information which assists no one, teacher or child or parent" (Cruickshank, 1972, p. 388). He favoured instead the use of qualitative diagnostic tests of the child's capacities.

More recently, Siegel (1989) expressed her dissatisfaction with what she referred to as the 'discrepancy definition' of LD and argued that IQ is irrelevant to the definition of LD. She explained that there are four basic assumptions in the IQ-achievement discrepancy. The first of these is that IQ tests measure intelligence. The second is that intelligence and achievement are independent, and the presence of LD will not affect IQ test scores. The third assumption is that IQ scores predict reading and/or arithmetic scores, meaning that, for example, children with low IQ scores should be poor readers and children with high IQ scores should be good readers. The fourth assumption is that individuals with reading disabilities (RD) of different IQ levels have different cognitive processes and information processing skills (Siegel, 1989, p. 469). In examining each of these assumptions, Siegel arrived at the conclusion that each was invalid.

Regarding the first assumption, Siegel (1989) looked at the subtests in the common 'measure of intelligence' in the field of LD, the Wechsler Intelligence Scale for Children-Revised (WISC-R), and noted that they do

not really measure the skills that are usually implied by the term *intelligence* - namely problem-solving skills, logical reasoning, and adaptation to the environment. Instead, the scale measures expressive language skills, memory, fine motor abilities, and specific factual knowledge, and it is biased against children with LD. For example, children with reading disabilities (RD) are likely to be exposed to significantly less text than normal children and would therefore, in comparison, have more limited opportunities for vocabulary growth (supported in a study carried out by Biemiller, 1977-78). Many of the subtests in the Verbal scale of the WISC-R tap word knowledge and, consequently, the child with RD who reads less and is likely to know fewer words will have a lower IQ score. Similarly, in the Performance scale subtests of the WISC-R, Siegel noted that children with LD are likely to perform poorer (as a consequence of what the tests measure rather than what they are *supposed* to measure) and thus achieve lower IQ scores. For example, in many of the subtests, additional points are awarded if the child completes the tasks under the time limits given. Siegel found in a study carried out with Barsky (Barsky & Siegel, 1987) that children with LD are capable of solving as many problems as non-LD children within the allowed time limits - but they do so more slowly. Thus children with LD are not likely to gain 'extra IQ points' that are contingent on speed of task completion.

In addressing the second assumption, Siegel (1989) argued that it is wrong to assume that the presence of LD will not affect IQ test scores. She again stressed that standard IQ tests measure abilities such as expressive language skills, short-term memory abilities, information processing speed, speed of responding, and knowledge of specific facts - each of which is noted in many studies as being deficient in individuals with LD. Thus IQ tests that use some combination of these functions as their basis for measuring *intelligence* cannot be a true measure of an LD individual's intelligence. Siegel further pointed out that LD children's IQ scores may decrease over time (as found in a number of studies, e.g., Arnold,

Barneby, McManus, Smeltzer, Conrad, Winer & Desranges, 1977; Bishop & Butterworth, 1980; and Share & Silva, 1987), that their lower IQ scores may be a consequence of their LD, and that IQ scores may underestimate the real intelligence of individuals with LD. She cited a study by Stanovich (1986), in which he coined the description *Matthew effects*, to partially substantiate these claims about IQ test scores and the presence of LD. Stanovich found evidence that children experiencing problems in learning to read have less experience with print and lower self-esteem and motivation, and these very results of reading acquisition failure lead to further problems in the development of reading skills.

Siegel (1989) also did not agree with the third assumption of the discrepancy definition – which suggests that children with low IQ scores should be poor readers, and those with higher IQ scores should be better readers. She pointed out that according to this formulation, it should not be possible for a child with low IQ to be a good reader, but a significant number of such cases exist. The existence of such cases implies that children with low IQ scores can learn to read, in many cases as well as normal children with higher IQ scores. This means, according to Siegel, that children with low IQ scores who fail to read should be considered as genuinely RD, and their reading failure should not be regarded as being the result of their low IQ scores.

Finally, regarding the fourth assumption, Siegel (1989) argued that the cognitive processes of poor readers with high and low IQ scores do not differ significantly, and therefore it is unnecessary to separate readers with LD using IQ scores just to understand cognitive processes. Siegel pointed out that a number of other researchers have obtained evidence suggesting that certain cognitive processes may not be significantly different in LD children with lower IQ scores compared to those with higher IQ scores. Hall, Wilson, Humphreys, Tinzmann, and Boyer (1983), for example, found this to be the case for short-term memory tasks; and Taylor, Satz and Friel (1979) found this to be the case for cognitive processes such as

reversal of letters or letter sequences within a word. Siegel also cited an earlier study she carried out (Siegel, 1988) to determine whether LD children with different IQ levels differed in performance on a variety of reading, spelling, language, memory, and arithmetic tests. The findings of the study cited were that differences in performance across the IQ levels were not significant. Siegel therefore concluded that it is unnecessary to differentiate children with LD on the basis of IQ if the purpose is just to study the cognitive processes that underlie learning.

Siegel further pointed out that children from different cultural or minority backgrounds with LD are likely to miss out on being given appropriate remedial instruction when IQ test scores are used to identify the presence of LD. Children from minority backgrounds, lower social class families, and/or different ethnic groups may score lower on IQ tests because of the specific knowledge and experienced that is required for at least some of the questions. Children in these groups who also have LD (e.g., they may be reading disabled) may instead be labelled as “slow learners” and not be considered intelligent enough to benefit from remedial programs for LD children, thus depriving them of needed assistance.

In the same issue of the *Journal of Learning Disabilities* where Siegel (1989) put forward her arguments for why IQ is irrelevant to the definition of LD, there was a mixed response from the other contributing authors. Some strongly supported Siegel’s case (e.g., Bryan, 1989; Lyon, 1989; Swanson, 1989), some disagreed with her (e.g., Graham & Harris, 1989; Torgesen, 1989), and others partially agreed and partially disagreed with her assertions. More recently, a study by Glez and Lopez (1994) found that the differences between LD and NLD (non-learning disabled) groups in word recognition cannot be explained by IQ – a finding similar to the results obtained by Siegel (1988), mentioned earlier. On the basis of the results they obtained, Glez and Lopez concluded that cognitive processes of individuals with LD who have different levels of IQ are

similar when the ability tests used are not related to reading activities. They therefore supported Siegel's earlier claims that it is not necessary to segregate LD experimental participants as a function of IQ to understand their cognitive processes.

It seems clear that there are challenges to the legitimacy of the place that IQ test scores have for so long held in the process of identifying individuals with LD. Arguments that have been put forward against the necessity for a discrepancy between IQ and achievement in LD identification are often quite convincing and – at the very least – thought-provoking, even though in most cases there are corresponding counter-arguments.

Does the LD field then have to simply hold out hope for a better intelligence test? Even though positive developments are constantly being made with regard to how intelligence is conceptualised (e.g., see works of Baron, 1985; Gardner, 1983; and Sternberg, 1986), the likelihood of the ideal intelligence test emerging in the near future seems remote. As Siegel (1989, p. 477) put it, “the multifaceted nature of mental processes and the possible influence of educational and cultural factors on test performance make it unlikely that a suitable intelligence test could be developed.” As noted earlier, individuals from different cultural or minority backgrounds, and those from lower social class families, are generally disadvantaged when taking intelligence tests because of the influence that educational and cultural factors have on test performance. In New Zealand, for example, some researchers have found significant differences between Maori and Pakeha/European children in their intelligence test performance (e.g., see Chapman & Nicholls, 1976; Flynn, 1988; Harker, 1978; Lovegrove, 1966), and most of these authors have attributed the differences largely to environmental factors.

Is there a solution to the problem of using IQ in identifying the existence of LD? Siegel (1989) considered the ideal solution to be the total

abandonment of the IQ test in the definition of LD. However, the compromise solution she offered is perhaps more acceptable and viable. She suggested lowering the cut-off IQ score to 80 instead of the usual requirement of '90 or above,' thus including individuals in the "Low Average" categories. Siegel suggested this solution because she believed that many investigators would probably not want to ignore IQ scores, and that the LD field in general was perhaps not ready to abandon the use of IQ. The thrust of her argument for the lower IQ score cut-off is that children with LD who obtain lower IQ scores (for whatever reason) should also receive remedial help and should not simply be dismissed or assumed to be hopeless.

Concerning the broader issue of LD identification, and the place of the exclusion and discrepancy criteria in it, Hammill (1990, p. 78) noted that the NJCLD made a distinct effort in formulating its definition to make it clear that "the 'exclusion clause' did not rule out the coexistence of learning disabilities and other handicapping conditions." The NJCLD definition was based on the 1977 U.S. Office of Education (USOE) definition which, in part, stated that "the term [specific learning disability] does not include children who have learning disabilities which are primarily the result of visual, hearing, or motor handicaps, or mental retardation, or emotional disturbance, or of environmental, cultural, or economic disadvantage" (USOE, 1977, p. 65083: cited in Hammill, 1990). In contrast, the NJCLD definition was expressed in a less exclusive fashion when it pointed out that although learning disabilities are not the result of other handicapping conditions or extrinsic influences, they "may occur concomitantly" with those conditions or influences (NJCLD, 1988, p. 1: cited in Hammill, 1990). Thus, in essence, the arguments against the exclusion from the LD category of individuals with less than average IQ test scores can find some support in the NJCLD view of what constitutes learning disabilities.

In practice, as reflected in reported research, the methods used to identify and select LD research participants vary considerably. Just to give some examples, Fox (1994, p. 43) described her sample of ten dyslexic readers as attending special reading classes; having reading ages that were at least two years behind their chronological ages; having a mean IQ of 111.7 (SD = 16.9), which was comparable with the mean IQ score obtained by the control readers; and coming from a similar socio-economic background as the control readers. In contrast, Dagnan, Dennis, and Wood (1994, p. 39) described their research participants simply as:

Eleven people with learning disabilities ... These were all clients who have been seen by three members (2 men and 1 woman) of the psychology department in the previous year for more than 5 sessions on a face to face basis. All were clients referred through the community teams for people with learning disabilities ... The group had a mean raw score on the British Picture Vocabulary Scale ... of 76 (SD = 27) which indicates a reasonable level of verbal ability. One man has a severe physical disability such that he needs to use a communication board.

Dagnan et al. did not give details about their participants' IQ levels, and apart from the one man's severe physical disability, they did not provide any other information about the presence (which seemed to be implied) of other handicapping conditions.

In the *Journal of Learning Disabilities*, authors usually mention their research participants' IQ scores and how their disabilities were identified. Lyytinen, Rasku-Puttonen, Poikkeus, Laakso, and Ahonen's (1994, p. 187) description is typical:

Each subject was administered the Wechsler Intelligence Scale for Children-Revised ... Only children who had normal IQ (WISC-R total IQ above 80) and whose learning problems met the diagnostic criteria were included in the LD group ... Their difficulties in reading, spelling or language were identified by teachers and confirmed by achievement testing ...

Note the use of the IQ cut-off score suggested as a compromise by Siegel (1989). Another author, Brier (1994), included in his diagnostic criteria for LD “the presence of either a Verbal IQ or a Performance IQ in the average range (80 or greater)” (p. 217). There now seems to be a greater acceptance of the concept that LD can occur concomitantly with other handicapping conditions such as lower intellectual functioning. Another example of the inclusion of individuals with lower IQs in LD studies and research is to be found in Roffman, Herzog, and Wershba-Gershon’s (1994) paper in which they described “young adults with learning disabilities who also function in the low-average range intellectually” (p. 414).

It is perhaps fitting to close this section by mentioning some interesting comments made by Baldwin and Vaughn (1989). They believed the real issue to be whether or not *intelligence* (not IQ test score) has any bearing at all on LD. They pointed out that if we assume LD is physiologically based rather than the result of bad teaching or political agendas, then it becomes very difficult to conceive how intelligence would have anything to do with it. As they put it, they did not know of “any disease organisms, birth defects, or neurological disorders that selectively attack humans with above-average intelligence” (p. 520). Thus the chances that an individual with below-average intelligence would be affected by LD would be no greater or lesser than the chances for an individual with average or above-average intelligence.

### *Summary*

This chapter briefly looked at the definition of LD, and then examined three crucial issues: the types of LD that exist and how these might be systematically and comprehensively organised; whether or not individuals with LD can learn and overcome their learning deficits; and how LD is identified and the arguments that IQ test scores are irrelevant to this process.

It was pointed out that the varieties of LD are many, and one way to organise these is to first distinguish between the kinds that are caused by a known brain injury (acquired) and those that are not (developmental). Of the developmental kind, a distinction can be made between those that are deviations from normal development in psychological or linguistic functions (also denoted as 'developmental') and those that significantly inhibit the processes of learning to read, write, spell, or carry out mathematical tasks (academic). The main categories of academic LD are in reading, writing/spelling, and mathematics. In subtyping these categories, it was argued that an intrinsic system is more useful than an extrinsic system because the subtypes that result relate to how the skills are actually executed, and they correspond to theories and findings about the processes underlying those skills. Under the system of organisation proposed here, difficulties in computation, which is the disability of concern in the present study, is a subtype of mathematics LD. Mathematics LD, in turn, is a form of academic LD, which in turn belongs to the developmental category of LD.

In the next section, it was pointed out that there are strong similarities between skills acquired at different stages of normal academic skills development, and the symptoms manifested by different subtypes of LD. It was proposed that individuals with LD progress through the same stages of skills acquisition as normal-achieving individuals, only slower. It was then argued that, despite the implications of the term LD and the finding in some studies that individuals with LD retain their learning deficits through to adulthood, individuals with LD are capable of learning and of overcoming their learning deficits. A couple of studies suggesting profile shifts in individuals with LD were cited, and reference was made to the overwhelming majority of research studies attesting to the responsiveness of individuals with LD to remedial instruction. In an effort to reconcile the findings of these studies with the previously mentioned finding in some studies that LD problems persist into adulthood, it was suggested that perhaps in some cases the problem is not

so much in learning but in retaining or remembering the skills that have been learned in the long term.

The final section of this chapter pointed out that a common requirement in the process of identifying individuals with LD is to exclude those whose learning deficits can be attributed to causes other than LD. Included in these causes is mental retardation, and one frequent requirement in selection is that individuals with LD have at least average IQ test scores. Siegel's (1989) arguments concerning the irrelevance of IQ to the definition of LD were then outlined. Basically, her arguments were that: existing IQ tests do not truly measure intelligence; the presence of LD affects IQ test scores and thus IQ tests underestimate the real intelligence of individuals with LD; it is wrong to assume that below average IQ is necessarily the cause of LD; and the cognitive processes of individuals with LD who have high and low IQs do not differ. The fact that individuals from different cultural or minority backgrounds tend to score lower in IQ tests because of various socio-economic and environmental factors was also mentioned. In addition, Siegel's suggested compromise of lowering the IQ cut-off score to 80 (thus including those in the 'low average' categories), and its apparent acceptance by many current researchers in the LD field, were noted. The position taken in this study is that it is unnecessary to exclude individuals with below average IQs from LD research studies and remedial intervention programmes: individuals with below average IQ test scores should also be considered as LD as long as they do not exhibit serious intellectual impairment.

## Chapter 2

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# Mathematics LD

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### *Introduction*

The National Joint Committee on Learning Disabilities in the U.S. included “significant difficulties in the acquisition and use of ... mathematical abilities” in its definition of LD (NJCLD, 1988, p. 1: cited in Hammill, 1990, p. 77). The NJCLD definition is quite extensively based on the U.S. Public Law 94-142 definition of LD. PL 94-142, however, goes into more detail by specifying two areas in mathematics that could be affected by LD: mathematics calculation and mathematics reasoning (Hammill, 1990).

As noted in the previous chapter, mathematics LD is sometimes referred to as *dyscalculia*. Badian (1983) and Bush and Andrews (1973), pointed out that the term came from *acalculia* which Herschen used in 1925 to describe an acquired loss in the ability to perform mathematical functions in adults. Kosci (1974) used the term *developmental dyscalculia* to describe significant difficulties in mathematics that are not due to direct damage of the brain, and out of the whole sample of 375 children he studied he found that 6.4% of them had the disorder. This led him to speculate that nearly 6% of children who are seemingly ‘normal’ are likely to have the symptoms of developmental dyscalculia. In New Zealand, Chapman, St. George, and van Kraayenoord’s (1984) data shows that 47% of their LD ‘pool group’ were underachieving in mathematics. These children comprised 5% of their cohort of 1,220 pupils at five intermediate schools in Palmerston North and Feilding. Assuming that Chapman et al.’s and Kosci’s samples of children reasonably represent the performance

characteristics of school children in literate countries, then mathematics LD clearly affects a considerable proportion of these children.

This chapter will first briefly look at the basic mathematics skills that must be acquired. It will then discuss the typical acquisition pattern of these skills in normal children, and how children with mathematics LD deviate from this pattern. Next, theories and models that have been proposed to account for the differences between children with mathematics LD and normal children will be examined. Finally, some of the intervention strategies that have been tried on students with mathematics LD and the effects that these have had will be outlined.

### *The basic skills required in mathematics*

According to Sutaria (1985), to be able to carry out mathematical tasks, it is necessary to understand the number system that mathematics uses as well as the rules that apply to the operations used in mathematics. The number system used is referred to as the Hindu–Arabic system which uses algorithms. In contrast, earlier systems – the Egyptian system and the Roman system – used a repetitive principle and a subtractive principle respectively. In the Hindu-Arabic system, the significance of each numerical symbol that appears depends on its position in the number system. For example, the numerical symbol 2 appears in 12, 29, and 208, but its actual value differs in each because of the different positions it takes.

The number system used in mathematics has certain properties that Jensen (1973) described. These properties include the fact that it is a decimal system – that is, ten is used as a base. Thus, among other things, groupings are done in tens and there are ten symbols (i.e., 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9) that are called digits. These digits are also referred to as whole numbers, positive integers, or natural numbers. In this system, zero (0) is

used to denote the lack of object existence at a given place. As with the other digits, the placement of zero determines its value. Thus zero by itself simply means there is nothing there, but in 60 it means there are no 'ones,' while in 704 it means there are no 'tens.' With the use of zero and the value attached to the placement of digits, numbers can be written to infinity.

While most mathematically educated adults probably take the properties of the number system for granted and never really think about the rules that govern arithmetic computations, children have to gradually understand and learn these properties and rules. They are by no means acquired automatically.

Reisman and Kauffman (1980) provided a good overview of the cognitive factors that influence the ability to acquire mathematical skills. These factors, of course, influence the acquisition of many other skills apart from mathematics, but they still need to be considered to obtain a reasonable understanding of the process of mathematics learning. One of the factors that Reisman and Kauffman mentioned is the child's rate of learning relative to that of others of the same age. They considered this important because abstract reasoning and use of a symbolic language of relationships are involved in mathematics. Thus, the cognitive functioning of a child would greatly influence the kind and amount of mathematics he or she could learn at any stage of development. Problems could arise if, as Reisman and Kauffman believed, the development of quantitative concepts in children with mathematics LD are in the same order and stages as normal children but considerably later. These children with mathematics LD would therefore not be able to keep up with the normal children. Later in this chapter, theories and research concerning the notion that children with mathematics LD are experiencing a developmental delay will be discussed.

Memory is also an important factor, according to Reisman and Kauffman (1980). They pointed out that unless prerequisite ideas are remembered, it will be difficult to learn anything new. They gave the example of the need for the child to be able to remember the configuration of the digits 0 to 9 if the child is to be able to write those digits from recall. They also pointed out that in mathematics the child needs to be able to remember the correct sequence when performing operations, and suggested that failure to do so is one important reason why some children make errors when solving computational problems.

Batchelor, Gray, and Dean (1990), in examining neuropsychological functions that underlie arithmetic problem solving, also found memory to be a central component. The results they obtained from administering psychometric tests to 989 students with LD suggest that irrespective of modality used to present arithmetic problems – visual or aural – memory processing was crucial. More specifically, the canonical correlation analysis they carried out indicates that mental flexibility and the ability to effectively access general information from memory facilitate problem solving on arithmetic tasks. They referred to access of general information from memory as “remote memory” and used the Information subtest of the Wechsler Intelligence Scale for Children–Revised to measure it. They found attention, concentration, verbal-auditory discrimination, and visual tracking to also be necessary for arithmetic performance.

Returning to Reisman and Kauffman (1980), these authors pointed out that a child also needs to be able to learn arbitrary associations and symbol systems. They explained how mathematics is communicated with the use of various symbols that are arbitrarily associated with the ideas they represent. In particular, mathematics incorporates logographs in which the symbol unit represents one or two words which in turn represent a complete thought (e.g., digit, subtraction sign, division sign, square root sign, etc.). They stressed the fact that a mathematics symbol directly

represents an abstraction (e.g., the digit 2 represents the abstraction “twoness”) which sometimes does not even have a concrete counterpart (e.g.,  $\pi$ , the symbol for *pi* and what it represents). Reisman and Kauffman noted that although symbol or graphic systems have now become more systematised, some children still find it extremely difficult to acquire language in graphic form.

Another factor that Reisman and Kauffman (1980) considered important is the ability to attend to salient aspects of a situation, which simply means being able to consciously choose what attribute or aspects of a situation are necessary to the task being dealt with. For the child to be able to abstract the number property of equivalent sets he or she must be able to focus just on the number of elements in the sets, and disregard other available (and, in this case, extraneous) information such as colour, shape, size, texture, or position of objects in space. In contrast, if colour becomes the salient feature under consideration, then all other features – including the number of elements – must be disregarded.

Concerning the successful development of computational skills in particular, there are a number of cognitive skills that researchers and theorists working in the area suggest as necessary. Woodward (1991) pointed out that the development of the ability to rapidly retrieve fact knowledge is critical. He stressed the need in computation for improved organisation to produce a unified sequence of actions. Thus, an effective procedural integration of the many subcomponents of a skill (e.g., the many steps that need to be taken to divide a 3-digit, 2-place decimal number by a 2-digit, 1-place decimal number) is required.

Kulak (1993) suggested very similar requirements. She was of the opinion that one general cognitive process that is important in arithmetic skill development (which she also believed important in reading skill development) is the assembly of small units of information into larger units. The resulting larger units can therefore be retrieved in a relatively

quick fashion and with high levels of accuracy. Woodward (1991) discussed this process in terms of moving toward automaticity. He argued that automaticity is required before an initially declarative skill or knowledge (i.e., one that contains basic factual information that can be verbalised) can be embedded into a newer operation, which in turn may be learned at a declarative level. The example he used was the skills involved in the early phases of subtraction – initially declarative in their representation – which need to be automated before being embedded in a newer operation like long division.

Nesher (1986) noted that we do not fully understand all that is involved in executing procedures. She pointed out that more than just the knowledge of the sequence of steps to be taken is required. In particular, she stressed the necessity of being able to monitor the entire process so that when non-viable results are reached, alarm signals would be triggered to prompt one to seek where the error or the problem occurred. To reach this level of competence in executing a procedure, a reasonable understanding of the entire set of relationships that appears in the task is required. And this, she believed, is what distinguishes an expert from a naive problem-solver. She cited an example from Fischbein, Deri, Sainati, and Marino (1985: cited in Nesher, 1986) of  $5 \div 25$ , which many naive problem-solvers answer as “5.” In this case, the naive problem-solvers do not possess a *control rule* of some kind saying that dividing a small number by a larger one produces a fraction. The expert problem-solvers presumably possess this kind of knowledge since it is neither stated in the problem itself nor explicitly conveyed by the specific numbers in the problem.

### *The development of mathematical skills*

In the previous chapter on learning disabilities, it was noted that numerous psychologists and educators (e.g., Frith, 1985; Marsh et al., 1981;

Seymour & MacGregor, 1984) believe that there is a natural progression in the development of reading and writing skills. These researchers have gathered evidence suggesting that the acquisition of reading and writing skills is largely determined by maturation. Thus, as children get older they naturally develop more and more skills that enable them to more successfully decode and encode words. This process of development, according to these theorists, is largely independent of instructions the children receive. Similar beliefs are held about the development of mathematical skills and concepts.

The most prominent of the cognitive developmental theorists was Jean Piaget, a Swiss psychologist who devoted most of his work to the study of the acquisition of understanding in children, including the understanding of mathematical concepts. Piaget's view was that children more or less follow a timetable established by nature in their development of the ability to think and execute various skills. The publication of his book, *The child's conception of number* (1952), has had a tremendous and lasting impact on mathematics education. Of course, there have been criticisms of Piaget's ideas and claims (e.g., Bryant & Trabasso, 1971; Groen & Kieran, 1983). However, as Sutherland (1993) noted, Piaget's original work nevertheless continues to greatly influence the study and practice of mathematics education.

Piaget referred to the first two years of the child's life as the *sensorimotor stage*. Among many other things, it is at this time that the child starts to develop the concept of numbers: the basis of counting is formed as soon as the child starts to undertake such activities as holding objects one after another, or shifting objects from one place to another. Another theorist, Jensen (1973), suggested that during this time the child learns about numbers because of hearing and imitating his or her parents count in relation to objects that are manipulated. The second or *preoperational stage* occurs when the child is between the ages of 2 and 7 years, approximately. Language development is a prominent feature of this

stage, and with this development the child begins to form a basic understanding of mathematics in his or her use of words and concepts such as “more,” “all gone,” “no more,” etc.

According to Piaget (1952), the third stage – which he called the *concrete operational stage* – occurs when the child is approximately between the ages of 7 and 12 years. This stage marks the beginnings of the child being able to think in a mathematically logical way (e.g., he or she shows a grasp of the principles of conserving and reversing). Between the ages of 12 and 15 years, the child goes through the final stage which Piaget called the *formal operations stage*. This final stage is when the child is able to use logic to solve problems – to make use of combinations, permutations, proportions and other concepts in solving problems.

Piaget (1952, 1953) discussed numerous areas of mathematical concepts that he considered important in the development of mathematical skills. Four areas will briefly be outlined here because of their direct bearing on the topic of the present investigation. These areas are: numeration; addition and subtraction; multiplication and division; and fractions and proportions. Piaget (1953) believed that the child does not acquire a real understanding of number concepts, or *numeration*, until he or she has achieved mental maturity – which does not occur until the child is about six-and-a-half to seven years of age. In Piaget’s view, understanding the concept of numbers develops spontaneously and independently of teaching. If the child is not mentally ready when given instruction about numbers, he or she may display learning that is purely verbal. For example, the child may recite numbers by rote, but not really understand their significance. To really understand numeration, the child must first come to terms with the principles of one-to-one correspondence (e.g., 9 red chips are matched one by one to 9 green chips) and conservation of quantity (e.g., when 7 blue chips are spread wider apart, they do not increase in quantity in comparison to when they were placed close together).

With regard to *addition and subtraction* skills, Piaget (1952) believed that although many first graders (children who are about 6 years of age) can verbally recite information relating to these operations (e.g., “two plus two equals four,” “two plus three equals five,” ... “seven minus seven equals zero,” “seven minus six equals one,” etc.) many of them are probably able to do so only by rote. Many do not really understand what these operations mean until they reach the age of 7 or 8 years. By this age, the child would have acquired the necessary grasp of two facts: first, that the sum of two sub-classes will always be larger than each sub-class; and second, that the difference between them will always be smaller than the sub-class being subtracted from.

Piaget (1952) was of the opinion that *addition* and *multiplication* are parallel operations that are dependent on each other. He considered it a mistake to think that the concept of addition is acquired earlier and then generalised to multiplication. According to him, both operations are learnt by the child at about the same time, which is approximately when the child reaches 7 or 8 years of age. *Division*, of course, is the inverse of multiplication and is learnt around the same time. In fact, Piaget believed that because of their inverse relationship, understanding either multiplication or division is essentially dependent on understanding both operations.

Regarding the concepts of *fractions and proportions*, Piaget (1952) noted that even when the child is only around 3 years of age, he or she can usually understand that some things can be cut and shared. However, the child at this age does not really grasp the true relationship of the parts to the original (which is necessary to understand conservation of the whole), and this does not happen until the child is about 7 years of age.

As the child gets older, he or she develops and displays a greater and more complex repertoire of mathematical skills and knowledge. Gesell and Ilg (1946) charted the normal developmental progression of mathematical

skills in children from the age of one through to the age of nine. Rather than outlining all of Gesell and Ilg's observations and findings about what the child could do at each of these different ages, just those directly relating to computational skills will be mentioned here, since that is the primary area in mathematics that is of concern to this investigation. The present author acknowledges that Gesell and Ilg's observations were made many years ago, but there appear to be no similarly detailed observations that have been undertaken more recently. It also needs to be noted that Gesell and Ilg based their observations on American children, and it cannot automatically be assumed that these observations apply universally. However, these limitations aside, Gesell and Ilg's observations are still useful in depicting the regular, predictable progression of computational and other mathematical skills in children.

According to Gesell and Ilg (1946), at one-and-a-half years of age, the child uses the word "more," and by two he or she grasps the idea of "one more." By four-and-a-half years of age the child is able to provide an answer to "how many" type questions, and by five can add and subtract up to 5 using his or her fingers. By the age of five the child can also write some numbers down on paper as the numbers are dictated to him or her, but he or she may reverse the numbers. Also the child may copy a number but not recognise it.

By five-and-a-half years of age the child can correctly add up to 5 while counting on fingers and starting with the smaller number. The child is also able to subtract correctly up to 5. By six years of age the child can write in 'large hand' – usually horizontally across the page and, when writing numbers, may reverse some of them. He or she can also, by this stage, correctly add up to 10 starting with the larger number. The child is also likely to show a liking for adding doubles (e.g., "two plus two," "three plus three," etc.).

By the time the child gets to seven years of age he or she likes to write a long number of several digits, and can write 1 to 20 or higher horizontally or vertically, with occasional reversals. The child adds correctly up to 20, subtracts correctly up to 10, and knows doubles facts. He or she makes errors of plus or minus 1 in computation, and is likely to be confused when both additions and subtractions are on the same page.

At eight years of age the child writes in smaller, more uniform handwriting with correct spacing between numbers. In writing numbers up to 20, errors are now rare. The child also knows many addition and subtraction facts; learns to carry or borrow in up to 3-digit problems; becomes familiar with some multiplication facts, especially doubles; and is now able to carry out simple division. Furthermore, by this stage the child is able to make process shifts, such as using addition in multiplication problems. In solving computational problems, the child may make occasional errors of plus or minus 1.

When the child reaches the age of nine, he or she can write numbers fairly accurately but may occasionally make mistakes especially when writing to dictation or when copying. The child may also show a preference by this time for writing numbers vertically rather than horizontally. He or she knows all basic addition, subtraction, and multiplication facts; and can use the long division method with 2- to 5-digit dividends and 1-digit divisor. The child may also use multiplication instead of addition to arrive at an answer to a computational problem more quickly. Furthermore, by this stage the child is able to tell what process he or she is using when solving mathematical problems.

More recently, Shalev, Manor, Amir, and Gross-Tsur (1993) investigated the acquisition of arithmetic in normal children using a model of arithmetic processing and calculation put forward by McCloskey, Caramazza, and Basili (1985) for individuals with dyscalculia. This model divides arithmetic function into three basic subskills: number

comprehension, number production, and calculation processing. Number comprehension includes comprehension of quantities, the symbolic nature of numbers, and digit order. Number production includes skills like counting, reading and writing numbers. Calculation processing includes comprehension of operation symbols, execution of arithmetic exercises, and memorisation of numerical facts.

Shalev et al. (1993) found normal children to have mastered counting, number comprehension, and number production by the third grade (in Israel; these children's mean age was 9.6 years). They noted that by this age the children had good concepts of number magnitude and quantities, as well as number order, and could read and write multidigit numbers. Citing Piaget (1952), Shalev et al. pointed out that mathematical knowledge is known to appear quite early in normal children, and mentioned evidence indicating that such knowledge develops independently of cultural and educational background – in fact even in the absence of usual education (Ginsburg, Posner, & Russell, 1981; Ginsburg & Russell, 1981: both cited in Shalev et al., 1993).

Shalev et al. (1993) also found, where number facts were concerned, that children who had achieved good levels of accuracy also worked faster. They found that when children had to revert to counting techniques and other immature strategies – thus taking longer amounts of time to complete tasks – they generally did not improve their scores. Shalev et al. also found evidence clearly indicating that skills in addition, subtraction, multiplication, and division improved with age and experience. Unlike older children, for example, those in the third grade had not mastered the procedure for complex multiplication exercises, even when they knew their multiplication tables well.

As a final point in this section, it is worth noting Piaget's (1953) observation that children can usually perform many mechanical tasks in mathematics because they have been adequately drilled to do so.

However, it is not until they can clearly show an understanding of the underlying logical relationships and apply what they understand to real life situations that they can really be said to have developed mathematical skills. This suggests that many of the mechanical aspects of mathematics such as counting, recitation of math facts, and computational skills are generally acquired and developed prior to true mathematical reasoning skills.

### *The error patterns of children with mathematics LD*

Sutaria (1985) observed that individuals with mathematics LD fall into two basic categories with respect to the difficulties they experience. There are those whose primary difficulties relate to computational skills, and those who display deficits in their reasoning skills. Sutaria identified and described twelve areas where individuals with mathematics LD commonly experience problems. Since the focus of the present study is on computational skills deficits, only two of these areas that directly relate to this focus will be discussed here: difficulties associated with comprehension of place value, and problems manifested in solving computational tasks.

Comprehension of place value is essential in most mathematical operations, and it is an area where many individuals with mathematics LD experience problems. Sutaria (1985) pointed out that many of them, for example, may not understand how or why 760, 607, and 706 are different from each other. If they cannot understand the principle of place value, many tasks involved in solving computational problems will be very difficult to learn and perform. Adding and subtracting by columns, regrouping, carrying and borrowing, the steps involved in long multiplication, and the use of decimals simply will not make much sense.

Sutaria (1985) gave examples of some common errors that emanate from a lack of understanding of place value. An individual may add all the numbers together instead of by columns, as in

$$\begin{array}{r} 41 \\ + 43 \\ \hline 12 \end{array}$$

Carrying may not be used, as in

$$\begin{array}{r} 73 \\ + 9 \\ \hline 712 \end{array}$$

And borrowing may not be applied when required, as in

$$\begin{array}{r} 68 \\ - 19 \\ \hline 51 \end{array}$$

From the present author's own observations, the last one in particular is a very common type of error that children with mathematics LD make when subtracting. Apart from a failure to borrow, it also shows a reversal such that the top digit (the 8) has been subtracted from the bottom digit (the 9).

According to Sutaria (1985), there are many difficulties that children with mathematics LD encounter when performing computations involving the four fundamental operations of addition, subtraction, multiplication, and division. Many stem from a lack of appreciation of the fact that subtraction and division are the inverse operations, or the reverse, of addition and multiplication, respectively. Without this understanding,

errors such as that in the following example could go undetected and unchecked:

$$\begin{array}{r} 59 \\ - 27 \\ \hline 31 \end{array}$$

In subtraction, not knowing that the difference (the solution) cannot be greater than the original number that was subtracted from can produce errors like:

$$\begin{array}{r} 71 \\ - 7 \\ \hline 88 \end{array}$$

Sutaria (1985) noted that similar types of errors can be seen in efforts at solving addition and multiplication problems when it is not understood that the sum and the product ought to be greater than the sub-classes. Examples he gave were:

$$\begin{array}{r} 25 \\ + 11 \\ \hline 9 \end{array}$$

$$\begin{array}{r} 82 \\ \times 3 \\ \hline 6 \\ + 24 \\ \hline 30 \end{array}$$

Sutaria (1985) also thought that the right-to-left direction utilised in addition, subtraction, and multiplication may cause confusion and frustration because it goes counter to the left-to-right progression required

in reading and writing. The fact that many arithmetic problems are presented in vertical columns could also add further confusion. Furthermore, when the child with mathematics LD has learnt the right-to-left direction used in addition, subtraction, and multiplication, he or she may then be confused by the left-to-right direction required in long division!

Another observation that Sutaria (1985) made was that some individuals with mathematics LD tend to leave problems half complete (without realising that they are not quite finished) despite starting them correctly. For example:

$$\begin{array}{r} 2 \\ 53 \\ \times \underline{27} \\ 371 \end{array}$$

They may also copy problems incorrectly, or write numbers too closely to each other (with inadequate spacing between them) and therefore make confusion about alignment almost inevitable.

Manalo (1991) described numerous types of errors in computation made by an individual who had mathematics LD. The errors he described were those of only one person, but they are similar to those described by McCloskey et al. (1985) as typical errors of individuals with acquired dyscalculia, which in turn Shalev et al. (1993) appraised as being similar to the errors manifested by children with developmental mathematics LD. Thus, to some extent, the errors manifested by Manalo's experimental participant can be considered as typical of individuals with mathematics LD who have computational difficulties. Many of the computational errors described by Manalo are also similar to those described by Sutaria (1985), and noted earlier. Hence, only some of those that are different will be detailed here.

Manalo (1991) noted the problem of not lining up the numbers properly using the decimal point (when decimals are involved). For example,  $28 + 2.1$  could be erroneously lined up and solved as:

$$\begin{array}{r} 28 \\ + 2.1 \\ \hline 4.9 \end{array}$$

Lining up the numbers inappropriately as shown above does not readily allow the addition of units to units, tens to tens, and so on. The source of the problem is probably a lack of comprehension of place value (as described by Sutaria, 1985). This problem of not lining up the numbers properly was also noticed by Manalo in subtraction. An example he provided was:

$$\begin{array}{r} 80.5 \\ - 52 \\ \hline 85.3 \end{array}$$

In multiplication, Manalo (1991) noted the problem of seeming not to know that multiplying by the next digit up (e.g., by the 'tens' after multiplying by the 'units') would result in a product ten times greater than the previous product obtained, and which should be placed one space along to the left below the previous product. Sometimes his experimental participant carried out the necessary shift to the left, and sometimes not. The participant also did not seem to know where to place the decimal point in the solutions he arrived at. An example Manalo provided was:

$$\begin{array}{r}
 29.81 \\
 \times \underline{31.6} \\
 \hline
 16886 \\
 2981.0 \\
 \underline{87730.6} \\
 924096
 \end{array}$$

In division, like many of the children with computational difficulties the present author has observed, Manalo's (1991) experimental participant could solve basic, one-step division problems (e.g.,  $35 \div 7$ , or  $68 \div 2$ ) but shied away from ones requiring a sequence of steps to solve. When the participant did attempt more complex division problems, his efforts strongly suggested a lack of familiarity with the procedures involved in long division, despite instructions he had received at school.

Careless mistakes (e.g., adding 6 to 7, and getting 12) were also mentioned by Manalo (1991). Although children with normal ability also make these kinds of mistakes, children with computational skills difficulties appear to commit such mistakes more often. The present author has frequently observed that whilst many children with computational skills difficulties can usually correctly answer simple math fact questions (e.g.,  $8 - 7$ ;  $13 + 2$ ), when these become embedded in problems that require more steps to solve (e.g.,  $183 - 71$ ;  $213 + 22$ ) careless mistakes start to appear in their work.

### *Possible reasons for the deficits in mathematical skills*

McCloskey et al. (1985) distinguished between the ability to understand and produce numbers, and the ability to calculate. They proposed a cognitive systems model that has a number processing system and a calculation system. The number processing system includes mechanisms for comprehending and producing numbers. The calculation system has

the facts and procedures required for carrying out calculations. Their interpretation of dyscalculia was in terms of impairment or damage to either, both, or parts of these components.

The calculation system included in the McCloskey et al. (1985) proposal has three major subcomponents: (i) processing of operational symbols (e.g., +) or words (e.g., "plus"); (ii) retrieval of basic arithmetic facts; and (iii) execution of the required procedures. Results of their investigation of patients with acquired dyscalculia support the notion that these are distinct subcomponents that can be impaired selectively (as well as in combinations) following injury to the brain of adults who previously had no known deficits in mathematics. As just noted in the previous section, the error manifestations of individuals with acquired mathematics LD are very similar to, and basically indistinguishable from, the errors of individuals with developmental mathematics LD. Presumably, therefore, the same impairments in components or subcomponents are present in the latter – except that theirs did not result from any known injury to the brain. Instead, the difficulties evidenced by individuals with developmental mathematics LD appear to stem from inadequate development of these components or subcomponents.

The developmental delay interpretation of mathematics LD has already been mentioned previously. In reviewing various relevant strands of evidence, Kulak (1993) concluded that the conditions of the majority of children with mathematics LD can best be explained in terms of a delay in moving through the normal sequence of strategy acquisition. She noted that the performance characteristics of children with LD differ from that of their normal peers in a *quantitative* way rather than a qualitative way. This means that they are only *delayed* in the acquisition of otherwise normal processing skills. She conceded, however, that the reasons for the delay are not clear.

The findings of studies carried out by Geary and Brown (1991), Geary, Brown, and Samaranayake (1991), Goldman, Mertz, and Pellegrino (1989), and Goldman, Pellegrino, and Mertz (1988) support the delay argument for mathematics LD that Kulak (1993) put forward. Basically, these studies have found that children with mathematics LD use combinations of strategies that are less mature than those used by their peers without LD. For example, they manifest a delay in moving from less efficient to more efficient counting strategies. They would count all the items in an addition problem, or count on from the first number given irrespective of its magnitude relative to the second number given, whilst children without LD would choose the larger of the two numbers and count the number of times indicated by the smaller number. Thus, given  $3 + 5$ , children with mathematics LD would count "1, 2, 3 ... 4, 5, 6, 7, 8" or "3 ... 4, 5, 6, 7, 8" to arrive at the answer. Children without LD of the same-age, who are more advanced in their skills acquisition, would choose the larger of the two numbers (i.e., 5) and count on from there – thus, "5 ... 6, 7, 8." These studies have also found children with mathematics LD to be delayed in developing direct retrieval of computational facts (e.g., multiplication facts) compared to their same-age peers.

Putnam et al. (1990) also obtained results in their study that support the developmental delay notion. The performance of students with LD in their study lagged considerably behind that of their normally achieving peers. Where addition strategies were concerned, for example, the students with LD in the fifth grade provided a similar proportion of adequate explanations as normally achieving third graders. Putnam et al. were of the opinion that children with mathematics LD lag behind their peers but are quite capable of developing the knowledge needed to reason mathematically, given adequate time and support. Again, there was not a lot in terms of explanations for the delay, but Putnam et al. did find students with LD to be more inflexible in their mathematical thinking. They found these students to be more likely to use their own well-learned strategies even when these were inappropriate, than to use new strategies

that had been modelled for them. An inflexibility in mathematical thinking would certainly contribute to delays in skills acquisition since, as a consequence of reluctance to try out new strategies, less experience and practice in using more advanced strategies and procedures would ensue.

Siegler and Shrager (1983: cited in Nesher, 1986) provided a model that suggests an explanation for the use of immature strategies by students with mathematics LD. Their proposed model suggests that usually an individual will first attempt to retrieve an answer when presented with a computational problem to solve. If this fails, an elaboration of the representation (i.e., of the problem to be solved) will be carried out and the individual will try to retrieve an answer again. If this still fails, the individual is likely to resort to counting (i.e., of the objects in the representation). Thus counting – a strategy viewed as being less mature in the sequence of strategy development – is used when retrieval from memory fails.

The real source of the problem could therefore be in memory. It was mentioned in an earlier section that memory is an important factor in being able to undertake mathematical tasks (cf. Batchelor et al., 1990; Reisman & Kauffman, 1980). Swanson, Cochran, and Ewers (1990) found in their study that subgroups of students with LD manifested moderate to severe deficiencies in working memory. For example, they found students with LD to be inferior to other students in being able to access resources for processing and retrieval of information from their memory. They concluded that working memory processes underlie individual differences in learning ability. This conclusion was partly based on their finding of significant performance differences between individuals belonging to the different memory subtypes they identified on all the basic academic skills measures they examined, including mathematics.

Batchelor et al. (1990) argued that the causes of poor arithmetic performance include not just working memory but also long-term

memory. This argument would be in line with Neshet's (1986) assertion that accessibility in memory is explained by trace strength, which in turn is determined by frequency of presentation and recentness of use. Trace strength of computational procedures in long-term memory would therefore likely be weak in individuals with mathematics LD if presentation and recent use of those procedures are incomplete or affected in some detrimental way by problems in working memory. These problems could be in areas such as selection of relevant stimuli, activation of appropriate processes, verbal representation and storage, and imagery and spatial representation (cf. Swanson et al., 1990). With difficulties such as these affecting their performance, individuals with mathematics LD would not be able to easily attain what Neshet described as "constructing and improving procedural strategies by lumping subprocedures ... into one chunk" (p. 1119) – a process which is considered to aid efficiency in terms of speed of access to the required solution.

In discussing procedural knowledge in mathematics, Neshet (1986) emphasised the slow, constructive process involved. She explained how the elements that make up a procedure are gradually put together and integrated to form the mastered skill. It is clear that an impairment in procedural memory would hamper this process: difficulties could be experienced in retaining the individual elements (the sub-procedures) that make up the procedure, or in integrating those elements into a complete, fluent whole.

Many studies have discussed procedural memory as a distinct component of human memory, and one that can be selectively impaired (e.g., Cohen, 1984; Cohen & Bacdayan, 1994; Cohen & Squire, 1980; Craik, 1991; DiGuilio, Seidenberg, O'Leary, & Raz, 1994; Mishkin, Malamut, & Bachevalier, 1984; Mitchell, 1989; Mitchell, Brown, & Murphy, 1990; Morris, 1984; Ostergaard & Squire, 1990; Roediger, 1990; Tulving, 1985). However, as far as the present author is aware, the relationship between procedural memory and LD has not been thoroughly investigated. A

study by Lorschach, Sodoro, and Brown (1992) found that children with LD who also had language disabilities (i.e., language/learning disabled, or L/LD) were impaired in episodic but *not* procedural memory. In reviewing literature on hyperlexia (a condition characterised by a precocious ability to read words, significant difficulty in understanding verbal language, and abnormal social skills), Goldberg (1987) argued that idiot savants have intact declarative memories but dysfunctional procedural memory systems. Apart from these two studies, which strictly speaking were not on conventional forms of LD, the possible link between forms of LD and procedural memory has not been examined. It is conceivable, for example, that general impairments (or underdevelopment of capabilities) in procedural memory are the cause of failure to learn arithmetic procedures in individuals with mathematics LD.

There are, of course, other views about what the causes of mathematics LD are. For example, Sutaria (1985) noted deficits in perception, symbolisation, cognition, and behaviour – along with memory – as the possible psychological roots of disabilities in mathematics. Lerner (1985) pointed out the following as possible causes of mathematics LD: associated disturbances of abilities in motor and visual perception, language and reading, and direction and time; deficits in some basic prerequisite skills (relevant to mathematics), body image, and social maturity; and impairments in memory and concepts of spatial relationships. Hughes, Kolstad, and Briggs (1992) argued that calculation and reasoning deficits of students with mathematics LD stem from failure to comprehend some of the fundamental spatial aspects of mathematics. Montague and Applegate (1993) linked the deficiencies in mathematics of students with LD to limitations in their cognitive and metacognitive strategy use. And Waldron and Saphire (1992) found that students with LD have significant weaknesses in auditory and visual discrimination, memory, sequencing, and spatial abilities.

It is outside the scope of the present study to try to explain in any conclusive manner the reasons for the deficits that individuals with mathematics LD manifest. However, even though there is a wide range of explanations that have been put forward by other authors, where computational skills deficits are concerned, inadequate functioning of one or more of the subcomponents described by McCloskey et al. (1985) seems the most straightforward and feasible. It satisfactorily explains the error features of the computational skills performance of individuals with mathematics LD. The inadequate functioning of these subcomponents, in turn, could be due to a developmental delay and/or impairment in some aspect of memory. The developmental delay and memory explanations of mathematics LD are congruent with two important premises behind the present study: that individuals with mathematics LD are capable of learning and thus of overcoming the performance deficits they manifest, and that an instruction strategy specifically aimed at aiding memory would be effective in teaching such individuals.

### *Interventions for mathematics LD, and their effects*

In comparison to reading and writing LD, there are considerably fewer studies on remedial interventions for mathematics LD. In fact, there are simply fewer studies that focus on mathematics LD. Kulak (1993) considered the study of mathematics LD to be equally worthy as that of reading LD, but noted the relative neglect of mathematics LD until recently when more investigations into the incidence, causes, and correlates of this form of LD have begun to appear in the published literature. Batchelor et al. (1990) pointed out that one reason for this lack of effort in the mathematics LD area is the school personnel's consideration of mathematic problems as not being as serious or debilitating as reading and writing problems. This opinion about the relative seriousness of problems manifested by students with LD translates to practice. As Shalev et al. (1993) pointed out, while remedial

intervention for dyslexia is usually commenced during the very early years of schooling, intervention for arithmetic is often postponed or neglected at that age.

Most of the research studies that have been done on mathematics instruction for individuals with LD have focused on computational abilities (Cawley, Fitzmaurice, Shaw, Kahn, & Bates, 1978), rather than reasoning abilities. This predominant focus on computation may be justifiable on two counts. First, both Kauffman (1981) and Mercer (1987) have observed that the majority of individuals diagnosed as having mathematics LD manifest deficits in computational skills. Even in a group of gifted students with LD (with a mean WAIS-R IQ of 132), Waldron and Saphire (1992) found the weakest area to be in written computational skills. Second, as Piaget (1953) and later Sutaria (1985) suggested, the development of computational skills is likely to be prior and essential to the development of true mathematical reasoning skills. Thus, assuming this is correct, it would be logical and more beneficial to address computational needs first.

Pereira and Winton (1991) reviewed studies that employed applied behavioural analysis in the teaching and remediation of mathematics. In their search of the available literature, they found 55 such studies published between 1968 and 1989. Twenty-six of the 55 studies they reviewed used research participants who were LD. The rest used participants who were brain damaged, low achievers, developmentally retarded, educable mentally retarded, hyperactive, or who had attention disorders. With a few exceptions, the studies that Pereira and Winton reviewed focused on computational skills.

The applied behavioural analysis approach to remedial instruction of individuals with mathematics LD has been one of the more common approaches used in this area. For example, since the Pereira and Winton (1991) review, quite a number of intervention studies utilising this

approach have appeared in the literature. These studies have focused on a range of mathematics difficulties, employing a variety of behavioural strategies in efforts to produce performance improvements. For example, Koscinski and Gast (1993) and Williams and Collins (1994) both used the constant time delay procedure in teaching multiplication facts to students with LD. This procedure provides for the systematic introduction of teacher assistance when the student cannot provide the correct answer. Case, Harris, and Graham (1992) taught their students with LD a self-regulated strategy for comprehending simple addition and subtraction word problems and devising appropriate solutions. Maag, Reid, and DiGangi (1993) found that self-monitoring of productivity and self-monitoring of accuracy improved their research participants' arithmetic performance. Ross and Braden (1991) found that the use of token reinforcement and cognitive behaviour modification techniques were equally effective, and superior to direct instruction, in improving the mathematics skills performance of students with LD. Seabaugh and Schumaker (1994) taught their research participants four self-regulation skills (behaviour contracting, self-recording, self-monitoring, and self-reinforcement), and found considerable increases in these participants' rate of lesson completion as a result. And Fuchs, Fuchs, Hamlett, and Whinnery (1991) found that greater performance stability was achieved when goal line feedback (i.e., feedback that incorporates goal lines superimposed on graphs of the students' actual performance rates) was used with students who had LD.

It appears, however, that where the applied behavioural approach to remedial instruction of students with mathematics LD is concerned, demonstration/modeling strategies are one of the most effective – particularly for teaching basic computational skills. Pereira and Winton (1991) conveyed such an opinion in their review and, in a later study (Laird & Winton, 1993), found that the performance accuracy of students with LD in solving long multiplication and division problems improved

following interventions that involved modeling and imitation to verbalise self-instructions of strategies while solving problems.

Earlier demonstration/modeling research includes studies undertaken by Blankenship (1978), Rivera and Smith (1987, 1988), and Smith and Lovitt (1975). The usual method was to demonstrate to the student how to correctly solve a problem. A solved problem could then be made available to the student while he or she attempted other problems. That is, the solved problem acted as the “permanent model” that the student could consult. Variations included requiring the student to imitate the demonstrated skills before attempting to independently solve other problems, and the incorporation of key words that the student recited. The incorporation of key words was aimed at helping the student remember better the sequence of steps involved in solving particular types of problems.

Rivera and Smith (1988) observed that the demonstration and imitation strategy, by itself, was not very effective in teaching long division because students became lost in the division process and were unable to use the permanent model to help guide them through the many steps involved. Thus they extended the strategy by incorporating the use of key guide words. This meant that during the imitation phase, the students with LD who were their experimental participants were also required to repeat key words which were: (does a) *go into* (b), *place dot* (to indicate which digit or digits are being divided), *divide*, *multiply*, *subtract*, *check* (to see if the subtraction answer is smaller than the divisor), *bring down*, *repeat*, and *put up remainder*. They referred to this as the *demonstration-imitation-key words strategy*, and they found it effective in teaching eight students with LD how to compute long division.

Apart from intervention studies that employed demonstration/modeling strategies, there are also those that used self-instruction and/or self-monitoring strategies, as mentioned earlier (e.g., Case et al., 1992; Maag et

al., 1993; Seabaugh & Schumaker, 1994). One of the most noteworthy of these is Wood, Rosenberg, and Carran's (1993) study, in which it was found that the use of taped cues made a positive difference to students' successful use of self-instruction. But perhaps more importantly, they found that the students with LD were able to use the self-instruction procedures they were taught, even after the taped cues had been faded. This finding suggests that cues could play a very useful role in the remedial instruction of students with mathematics LD, and that the positive effect of cues on performance could go beyond the immediate or short term – even after they are no longer presented.

Mastropieri, Scruggs, and Shiah (1991), like Pereira and Winton (1991), reviewed the area of remedial intervention for mathematics LD, but looked at other types of interventions apart from those employing the behavioural approach. These other forms of intervention included those that employed goal setting, verbalisation and feedback, word problem solving strategies, strategies to facilitate computation, and strategies using alternative instruction delivery systems (e.g., computer assisted instruction or CAI; peer mediation). Mastropieri et al. found 30 studies published between 1975 and 1989, all of which reported successful outcomes.

One study of particular interest that Mastropieri et al. (1991) reported was carried out by Willott (1982: cited in Mastropieri et al., 1991). In this study, mnemonic imagery was used to facilitate recall of basic multiplication facts among 15 students with LD. Willott obtained the best results when the students were presented with "concrete-concrete" pictures in which both multiplication factors and product were represented by images of associated mnemonic pegwords. For example, for  $8 \times 7 = 56$ , a picture was presented of a gate (which stands for 8 in the pegword rhyme "one is a bun, two is a shoe, ...") in heaven (which stands for 7 in the same rhyme) and a bunch of "gifted (i.e., gift-wrapped) sticks" representing 56. As noted above, the students with LD remembered more

of the multiplication facts they were taught when provided also with the mnemonic pictures, compared to no pictures or the provision of mnemonic pictures only for the factors or the products but not both.

Published research in the area suggests that other forms of intervention, in addition to those that employ the behavioural approach, are also effective in producing performance improvements in individuals with mathematics LD. For example, recent studies include Miller and Mercer's (1993a) report of a strategy they referred to as a "graduated word problem sequence." Using this strategy, word problem solving is approached by dealing initially with simple words, progressing onto phrases, sentences, and paragraphs, and later including extraneous information. Miller and Mercer found this strategy effective in teaching subtraction and multiplication to elementary students with LD. Zaiwaza and Gerber (1993) also dealt with word problem solving, and found that methods of translating word problems that do not clearly convey the required arithmetic operation(s) to students with LD, coupled with the use of diagrams, produced the best performance results. Beirne-Smith (1991) found peer tutoring to be an effective method for facilitating the acquisition of single-digit addition facts among primary-aged students with LD. Bryan and Bryan (1991) found that positive mood induction improved the accurate problem-solving output of students with LD. Kamann and Wong (1993) found that teaching students with LD a coping strategy based on cognitive behaviour modification increased these students' use of positive self-talk to a level comparable to that of normally achieving students. And Scott (1993) successfully used a multisensory approach to teach addition and subtraction skills to children with mild disabilities.

There are some studies, though, that stand out, not just because of the results they obtained, but also because of what their findings suggest about the required ingredients of effective remedial interventions for mathematics LD. Okolo (1992a) is one such study. She found that an

instructional computer game was effective in significantly improving the arithmetic facts proficiency of students with LD. More importantly, she found that the way the instructions were delivered – in the form of a computer game – had a positive effect on the continuing motivation of students who initially scored low on a measure of attitudes toward mathematics as a subject. In a second study, Okolo (1992b) looked at the effects of incorporating attribution retraining in a mathematics CAI programme. Students with LD received either ability- and effort-focused attribution retraining feedback from the computer (e.g., “You really know these,” “You are really trying hard,” “You can get it if you keep trying,” etc.), or neutral feedback (e.g., “You are meeting your goal,” “You met your goal,” “You are not meeting your goal,” etc.). Although the attribution retraining did not appear to have a significant impact on the students’ attributions, Okolo found that students who were in the attribution retraining condition completed significantly more levels of the mathematics programme. These students also scored significantly higher than control (neutral feedback) participants on a multiplication computation posttest. Because there was a significant correlation between posttest score and number of programme levels completed, Okolo suggested that the attribution retraining may have affected the students’ performance scores primarily because of its effects on their persistence.

Van Houten (1993) conducted experiments that examined the effectiveness of teaching subtraction fact rules to children with LD who had demonstrated difficulty in learning these facts by rote. He found that children learned the subtraction facts more quickly when a rule teaching and correction strategy was used. For example, in teaching subtraction of 7 from a teen number, the children were told, “There is an easy trick you can use when subtracting seven from a teen number. You simply add three to the number above the seven.” Furthermore, Van Houten found that children with LD were able to generalise the use of the rule. Children taught the rule for the subtraction of 9 and 8 were found to use the equivalent rule for the subtraction of 7, which was not taught.

Van Houten's (1993) study emphasised the importance of knowing and applying rules where the performance in computational skills of children with LD is concerned. Rules may be helpful because they serve an organising and guiding role in executing tasks. Furthermore, as Van Houten noted, rules may lighten the load on memory. For example, with the use of the rules employed in the Van Houten study, there would be considerably fewer subtraction facts to memorise.

Miller and Mercer (1993b) investigated the effects of using the concrete–semiconcrete–abstract (CSA) instruction on students with mathematics LD. This method of instruction is based on the belief that students first learn mathematics through the manipulation of concrete objects; they then move onto learning through pictures or other pictorial representations of objects; and finally, they learn using abstract symbols (e.g., Suydam & Higgins, 1977: cited in Miller & Mercer, 1993b). Thus, CSA instruction would begin with concrete level instruction involving the use of three-dimensional objects (e.g., coins, sticks, buttons, etc.) to solve mathematics problems. After mastery at the concrete level, instruction moves to the semiconcrete level where drawings are used to solve the mathematics problems. Finally, after mastery at the semiconcrete level, instruction moves to the abstract level where students are taught to look at the mathematics problem and try to solve it without using objects or drawings.

Miller and Mercer (1993b) found that students with mathematics LD were able to transfer learning to the abstract level after they had received only concrete or semiconcrete levels of instruction. In other words, they did not need abstract-level instruction to be able to employ the skills they had learned at the concrete and semiconcrete levels of instruction to solve mathematics problems presented at the abstract level. This finding has important implications for the presentation of skills instruction to students with mathematics LD. It emphasises the value of concrete and semiconcrete forms of delivery to facilitate skills acquisition, and provides

reassuring evidence of transfer occurring to the abstract level. Miller and Mercer also found that most students needed both concrete and semiconcrete lessons before the transfer of skills to the abstract-level problems was evidenced.

The overwhelming majority of studies report favourable outcomes as a result of providing various forms of instruction to students with mathematics LD. As noted earlier, for example, all of the 30 studies published between 1975 and 1989 that Mastropieri et al. (1991) reviewed reported successful outcomes consequent to the interventions used. This is a good sign: it very strongly suggests that, although they might manifest serious deficits in mathematical skills, individuals with mathematics LD are generally receptive to remedial instruction and can learn.

The question about the ability of individuals with LD to learn was raised in the previous chapter. It was noted that despite reports of successful outcomes as a result of various remedial instructions that have been employed, there is evidence indicating that problems persist into adulthood (e.g., Gerber et al., 1990; Horn et al., 1983; Schonhaut & Satz, 1983; Spreen, 1982). Thus a pertinent new question is: What is required of instructional interventions to avoid this apparent persistence of problems?

Assuming that individuals with LD are in fact generally capable of learning, it is possible that the reason why problems appear to persist is because these very same individuals have problems in long term retention of facts and/or skills they learn. Thus it could be important that the method of instruction used assists in the long-term retention of the information taught. Very few of the studies reviewed above examined the extent of maintenance or long term retention of the skills dealt with. When they did, follow-up or maintenance measures taken were within days or a few weeks at the most, which does not really give a clear

indication of the likelihood of lasting beneficial effects from the intervention.

It is also important that the strategies taught are generalisable to real situations. If the methods employed to teach individuals with mathematics LD require that they be equipped with materials they are unlikely to have handy in natural settings (e.g., multiplication tables, solved problems as models, etc.), they may find it difficult to execute the strategies outside of the artificial laboratory or classroom situation. They may get very little practice in making natural use of those strategies. Hence, after the instructional interventions have ceased, skills learned using such methods may soon be forgotten.

Other factors may also be important, such as getting the student motivated enough to use the strategies taught to solve mathematics problems (e.g., see Okolo, 1992a), reducing the load demand on the memory of the students (e.g., through the use of rules, as suggested in the Van Houten, 1993, study), and teaching them at a level that they would respond well to (e.g., at the concrete or semiconcrete level, as in the Miller & Mercer, 1993b, study). These are but a few: there are many other factors that could be important when considering the issue of maintaining improvements in performance of students with mathematics LD.

Finally, explanations as to why certain intervention procedures work are much needed. As Mastropieri et al. (1991) noted, none of the research studies they reviewed made any attempt to link the intervention procedures used to specific theories or even to characteristics of individuals with LD, other than their general deficit in executing mathematical tasks. These intervention procedures do not just work: there must be reasons why they work, and those reasons must be somehow related to the particular characteristics of individuals with mathematics LD. Considering these reasons would therefore contribute

to a better understanding of mathematics LD and the needs of individuals affected by it.

### *Summary*

Mathematics is a complex skill: there are many rules, for example, that govern arithmetic computation. Many cognitive factors have been suggested as being essential for the successful acquisition of mathematical skills. These include the child's rate of learning, memory, the ability to learn arbitrary associations and symbols, and the capacity to attend to salient aspects of a situation. Where computational skills are concerned, the development of the ability to rapidly retrieve fact and procedural knowledge is also important, together with the capacity to monitor the sequence of steps being taken so that, for example, non-viable results can be detected.

Although they were formulated many years ago, Piaget's views about how mathematical skills and concepts develop are still influential today. These, and other views expressed by cognitive-developmental theorists, suggest that the acquisition of mathematical skills is developmental and orderly, and largely independent of instructions provided. A regular, predictable progression of computational and other mathematical skills can be observed in children. The acquisition of the mechanical aspects of mathematics, which include computational skills, is also indicated as being prior to the acquisition of true mathematical reasoning skills. More recent studies, while few in number, largely support these views.

Studies and observations of individuals with mathematics LD who manifest computational skills deficits reveal some common types of errors that are made. These include errors that appear to result because of the following reasons: a lack of understanding of place value and, associated with this, a failure to align numbers properly when decimals

are involved; not borrowing in subtraction when required; a lack of appreciation of the fact that subtraction and division are the inverse operations of addition and multiplication, respectively; not knowing that in subtraction the difference cannot be greater than the original number that was subtracted from; a lack of understanding that in addition and multiplication the sum and the product ought to be greater than the sub-classes; leaving the solution to problems half complete; and not knowing where to place the decimal point in solutions arrived at. In long division, a lack of familiarity with the procedures involved is common. It has also been observed that children with computational skills difficulties tend to make careless mistakes more often than children with normal ability.

Many possible reasons for deficits in mathematical skills have been proposed. In this study, the explanation of inadequate functioning of one or more of the subcomponents of the calculation system (i.e., processing of operational symbols, retrieval of basic arithmetic facts, and execution of the required procedures) is subscribed to because it most satisfactorily explains the typical types of computational errors manifested by individuals with mathematics LD. It was proposed that inadequate functioning of these subcomponents could be due to a developmental delay and/or impairment in memory. The developmental delay explanation suggests that children with mathematics LD move through the same sequence of strategy acquisition as children of normal ability, except in a slower and significantly delayed fashion. This explanation is congruent with the notion that individuals with mathematics LD are capable of learning and of overcoming the skills deficits they manifest. Where memory is concerned, there are many aspects that have been proposed as the source of problems in mathematics learning, and research evidence confirms the centrality of memory in the execution of mathematical tasks. The memory deficit explanation of mathematics LD suggests the need for an instruction strategy that would specifically address long term retention of skills learnt.

It was noted that the majority of studies that have been carried out on mathematics instruction for individuals with LD have focused on computational skills. Such a focus may be warranted because the majority of individuals affected manifest computational skills deficits, and it appears that the development of computational skills is prior and essential to the development of true mathematical reasoning skills. The demonstration/modeling method of instruction has been shown to be effective in teaching computational skills to students with mathematics LD. Other methods of instruction, however, have also been shown to be effective, and some of these are particularly noteworthy because they suggest instructional features that may be necessary for the successful remediation of mathematics LD. These features include motivating the students to use the strategies taught for solving mathematics problems, reducing the load demand on the memory of the students, and teaching the students at a more concrete level. The importance of teaching strategies that are generalisable to real situations was also pointed out, as well as the desirability of finding satisfactory explanations for intervention procedures that are found to be effective.

## Chapter 3

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# Mnemonics

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### *Introduction*

Mnemonics can be broadly defined as schemes for assisting memory. There are basically two types of mnemonics: *fact* and *process* mnemonics. Fact mnemonics – the more commonly known form – are used on a one-to-one basis to remember facts. Typically, one mnemonic association is constructed for each item to be remembered. In contrast, process mnemonics – the lesser known form – are used to help remember rules, principles, and procedures. In essence, process mnemonics assist in remembering the processes that underlie problem solving (see Higbee, 1987).

Among other names, fact mnemonics have also been referred to as *artificial* and *fantastical* memory (see D'Assigny, 1985: originally published in 1697; Herrmann & Chaffin, 1988; Yates, 1966). Their use can be traced a long way back in history.

### *Historical background*

Marcus Tullius Cicero, who lived from 106 B.C. to 43 B.C., described in his *De oratore* how Simonides of Ceos invented the art of memory: the mnemonic of places and images, or *loci* and *imagines* (Herrmann & Chaffin, 1988; Yates, 1966). Apparently, while Simonides was dining at the house of a nobleman named Scopas, he composed and recited a poem in honour of his host. However, in the poem he also included a passage

honouring Castor and Pollux, and so Scopas nastily informed him that he was paying for only half the agreed fee – that Simonides could obtain the balance from the two gods to whom he decided to devote half of the poem. Later in the evening, Simonides received a message that two young men were waiting outside wanting to talk to him, but when he went outside there was no one there. While he was outside, the roof of the dining room collapsed, crushing Scopas and all his other guests to death. Thus, as Cicero interpreted it, the invisible callers Castor and Pollux paid for their share of the poem by saving Simonides' life! Apparently, the bodies in the dining room were so mangled that their relatives could not identify them. However, Simonides remembered the places where they were sitting and therefore was able to show the relatives which of the dead belonged to them. The whole experience suggested to Simonides the importance of an orderly arrangement for a clear memory. He inferred that if a person wants to train their memory they must select localities, form mental images of the facts they wish to remember, and store those images in the localities. Later they can retrieve those facts by mentally going through the arranged localities with the associated images representing the facts.

Apart from the description given by Cicero, the method of loci was also discussed by two other 'ancient' authors: one was an unknown teacher who wrote *Ad Herennium* around 86 to 82 B.C., and the other was Quintillian (c. 40 A.D. – c. 96 A.D.) who wrote *Institutio oratoria*. Essentially, the method requires committing to memory a series of loci or places. The most common type used was architectural structure. In fact, Quintillian advised on a spacious and many-featured building that includes the forecourt, the living room, the bedroom, the parlours, the statues and other ornaments decorating the rooms, and so on. Images are then created of the speech to be remembered, and these images are placed in imagination on the places that have been sequentially committed to memory. After doing this, and when the facts/speech details have to be

recalled, one has to mentally visit the places in the right sequence and retrieve the images that have been placed in each of them.

Yates (1966) noted that nowadays it may be rather difficult to appreciate the value and utility of this mnemonic technique, but she put in a reminder that in the ancient world there was no printing, no paper for notetaking or typing of lectures, and thus a trained memory was extremely important for some. She described an ancient orator – for whom essentially the method of loci was devised – as moving in imagination through the building he has memorised whilst reciting his speech. From the memorised places he extracts the images he had placed on them, thus making sure that the parts of the speech are remembered in the right order, since the places in the building are revisited in a fixed sequence.

The anonymous author of *Ad Herennium* described an example of how a concept or idea can be represented by a multi-faceted image which can then be associated with a location, which he referred to as the “background”:

Often we encompass the record of an entire matter by one notation, a single image. For example, the prosecutor has said that the defendant killed a man by poison, has charged that the motive for the crime was an inheritance, and declared that there are many witnesses and accessories to this act. If in order to facilitate our defence we wish to remember this first point, we shall in our first background form an image of the whole matter. We shall picture the man in question as lying ill in bed, if we know his person. If we do not know him, we shall yet take someone to be our invalid, but not a man of the lowest class, so that he may come to mind at once. And we shall place the defendant at the bedside, holding in his right hand a cup, and in his left tablets, and on the fourth finger a ram’s testicles. In this way we can record the man who was poisoned, the inheritance, and the witnesses. In like fashion we shall set the other counts of the charge in background successively, following their order, and whenever we wish to remember a point, by properly arranging the patterns of the backgrounds and carefully imprinting the images, we shall easily succeed in calling back to mind what we wish. (Anonymous, c. 86 B.C.: in Herrmann & Chaffin, 1988, p. 89)

Although noting that this mnemonic device certainly has its uses, Quintillian had reservations about its usefulness in remembering connected speech (cited in Herrmann & Chaffin, 1988). He pointed out that certain things, such as conjectures, are impossible to represent by symbols. He also believed that the flow of speech will inevitably be hampered by the double task imposed on memory – of having to utter words in a flowing manner and at the same time look back at separate symbols for each individual word. He advised instead that if a long speech needs to be remembered, one should learn it piecemeal since memory does not respond very well to being overburdened.

D'Assigny (1985, originally published in 1697) also did not have absolute faith in the method of loci (which he described in the final chapter of his book), primarily because he believed that it depends too much on the strength of an individual's imagination. He did think that it could sometimes be useful, but the main focus of his book was on teaching ways to improve the *natural* powers of memory. Like many during his time, he believed that animal spirits formed the basis of memory, and that it was necessary to ensure the maintenance of bodily equilibrium through proper regimen and medical treatment if one were to have good memory. For example, he recommended moderation in intake of food and drink, getting sufficient exercise, ensuring that the head and feet are not exposed to too much cold, and avoiding sexual excess – especially on a full stomach or out of wedlock!

Yates (1966) was more positive towards the method of loci, and paid particular attention to the one described in *Ad Herennium*, where the anonymous author gave a detailed description of the method (in contrast to Cicero's and Quintillian's writings which both assumed that the reader already knew of the technique and the associated terminology). While acknowledging that a tremendous amount of effort is required to carry out these "mnemonic gymnastics," she had no doubt that they work. In her book, *The art of memory*, she pointed out the great importance

through the ages of *Ad Herennium*: how the practice of memory as an art in the Western tradition was so much dependent on the memory section of this document. *Ad Herennium* was well known and much used during the Middle Ages and during the Renaissance. Every mnemonic technique devised in the Western world, even the mysterious and occult systems of the Renaissance, subscribed in one way or another to the basic plan of *Ad Herennium*: its rules for places, rules for images, memory for things, and memory for words.

The long history of fact mnemonic use can be considered a testimony to its capacity to address an important need in humans: the need to remember various facts. Whether the facts that need to be remembered concern the sequence of an oratory or the details of a court proceeding, the use of mnemonics is very understandable prior to the availability of paper. It would have been impractical to construct stone tablets to record the details of every speech, let alone to lug the heavy tablets around for the simple purpose of consulting them. However, that their use survived the advent of paper suggests that there is a little more to them than just an alternative to reminder notes.

### *More recent reports of mnemonic feats*

Luria's (1968) patient S, whose memory capacity appeared to have "no distinct limits" (p. 11), independently devised and used a form of the method of loci to remember seemingly endless amounts of information. Luria described:

When S read through a long series of words, each word would elicit a graphic image. And since the series was fairly long, he had to find some way of distributing these images of his in a mental row or sequence. Most often (and this habit persisted throughout his life), he would 'distribute' them along some roadway or street he visualised in his mind. Sometimes this was a street in his home town, which would also include the yard attached to the house he had lived in as a child and which he recalled vividly. On the

other hand, he might also select a street in Moscow. Frequently he would take a mental walk along that street – Gorky Street in Moscow – beginning at Mayakovsky Square, and slowly make his way down, ‘distributing’ his images at houses, gates, and store windows ... This technique of converting a series of words into a series of graphic images explains why S could so readily reproduce a series from start to finish or in reverse order; how he could rapidly name the word that preceded or followed one I’d selected from the series. To do this, he would simply begin his walk, either from the beginning or from the end of the street, find the image of the object I had named, and ‘take a look at’ whatever happened to be situated on either side of it. (pp. 31–33)

In addition, S appeared to have an extremely highly developed ability to form images (as well as taste and tactile impressions!) of just about any piece of information given to him. For example, Luria (1968, p. 23) reported that when S was presented with a tone pitched at 50 cycles per second and an amplitude of 100 decibels, S “saw a brown strip against a dark background that had red, tongue-like edges. The sense of taste he experienced was like that of sweet and sour borscht, a sensation that gripped his entire tongue.”

Of numbers, Luria (1968) reported S as saying:

Even numbers remind me of images. Take the number 1. This is a proud, well-built man; 2 is a high-spirited woman; 3 is a gloomy person (why, I don’t know); 6 a man with a swollen foot; 7 a man with a moustache; 8 a very stout woman – a sack within a sack. As for the number 87, what I see is a fat woman and a man twirling his moustache. (p. 31)

Unlike the deliberate and purposeful use of mnemonics described by Herrmann and Chaffin (1988) and Yates (1966), the use of mnemonic strategies by Luria’s (1968) patient S was more natural – and initially, largely unintentional. Although he later became a professional mnemonist, S (who worked as a newspaper reporter when he was first referred to Luria) was baffled that other people did not operate the way he did – how they had to take notes, and could not remember everything they were told. Assuming that the information about himself that he provided Luria was accurate, his extraordinary abilities seem to have been

something he had even as a child. S reported that when he was only two or three years of age, he was taught the words to a Hebrew prayer that he did not understand. The words of the prayer settled in his mind as puffs of steam or splashes, and as an adult he could still hear the sounds of the prayer because he could recall to mind the puffs and splashes.

S's amazing ability to 'remember' also produced its problems. For instance, he often could not understand the meaning of passages he read because the images they conjured kept on interfering with his thoughts and comprehension. Thus he could remember the exact words of passages without having a clear appreciation of what they meant. S had many other problems, but this particular problem he experienced highlights an important point: that through the use of mnemonic strategies, recall is possible even without thorough comprehension.

Another contemporary use of mnemonics (outside of instruction and learning) was reported by Ericsson, Chase, and Faloon (1980). They conducted an experiment where they took an undergraduate (referred to as SF), with average memory abilities and average intelligence for a college student, and asked him to practise a memory span task for about 1 hour per day, 3 to 5 days a week, for 20 months. In the resulting 230 hours of practice in the laboratory, SF was able to increase his memory span from 7 to 79 digits. He devised a method for recoding the digits he was given as running times, ages, and dates. For example, 3492 became "3 minutes and 49 point 2 seconds, near world-record mile time," and 893 became "89 point 3, very old man" (p. 1181). He also incorporated organisation into his retrieval structure by segmenting his groups of digits into subgroups.

Ericsson et al. (1980) argued that although SF dramatically increased his digit span, he did not really increase his short-term memory capacity. There was, for example, no transfer found when a switch was made from digits to letters of the alphabet. Ericsson et al. believed instead that

increases in memory span, as demonstrated by SF, were due to the use of mnemonic associations in long-term memory.

The findings of the Ericsson et al. (1980) study highlight a couple of important points. First, that mnemonic devices are not necessarily unnatural and artificial: they need not be taught, and they can be developed 'naturally' in response to a need to remember information. Second, that because mnemonic devices are linked to long-term memory information, it appears important to their effectiveness that they consist of salient material that is at least partially interesting to the user. In SF's case, for example, his development and use of running times as a mnemonic tool to recode digits is congruent with the fact that he was a good long-distance runner who competed in races throughout the eastern United States. Thus, as Ericsson et al. reported, he was able to classify running times into at least 11 major categories, from half-mile to marathon, with several subcategories within each.

Therefore, even though mnemonics have been referred to as 'artificial' memory strategies, there is evidence to suggest that there are aspects to them that are not artificial. Luria's (1968) patient S's case is an exceptional one, but he spontaneously developed his own mnemonic techniques and even independently devised a method of loci. Similarly, Ericsson et al.'s (1980) experimental participant SF – a much more 'average' person with no exceptional abilities – came up with his own mnemonic-based method. Thus it seems that although there are mnemonic features that can be considered artificial, there are also aspects to them that make them a normal human response to having to remember facts.

### *Types of fact mnemonics*

Morris (1979) provided a review of fact mnemonic systems. It is interesting to note that three of the systems he described (including the

method of loci previously described here) require explicit, prior 'construction' of structures in long-term memory for information to be remembered. Like SF's recoding of numerical digits into running times and ages, these methods make explicit use of associations in long-term memory.

The phonetic translation system is used to remember numbers. It works by linking particular consonant sounds with each digit from 0 to 9. Thus 1=t or d, 2=n, 3=m, 4=r, 5=l, 6=j or sh or ch, 7=k or g, 8=f or v, 9=b or p, and 0=s or c or z. Numbers that need to be remembered are translated into meaningful words and/or phrases by first converting the digits into their appropriate consonant sounds, and then inserting vowel sounds that would enable the construction of meaningful words. The example Morris (1979) gave was that of having to remember someone's extension number: 4180 could be translated to "ReD FaCe."

As Morris (1979) pointed out, in most cases there are structural similarities or phonetic/semantic associations between the digits and their respective sounds. For example, t which corresponds to 1 has one downstroke, n(2) has two, and m(3) has three; s which corresponds to 0 represents the s/z sound of the word zero; and l which corresponds to 5 is the Roman numeral for 50 (L). However, the important point here is that the digit-sound associations have to be learnt and memorised first (committed into long-term memory) before a person can use the translation system and translate numbers into words and/or phrases that they would remember better.

Similarly, an ordered sequence of peg words have to be constructed and memorised prior to using peg mnemonics. Peg mnemonics are mnemonic devices that can enable recall – in the right sequence if desired – of unrelated concrete items, such as a shopping list. The most commonly known and used peg mnemonic is the rhyme "one is a bun, two is a shoe, three is a tree, four is a door, five is a hive, six is sticks,

seven is heaven, eight is a gate, nine is wine, and ten is a hen." The words bun, shoe, tree, and so on constitute the peg words. If the rhyme has been memorised, it can be used to retain a list of ten items by creating images that link the items to the peg words. For example, if a list starting with onion, carrot, and bread has to be remembered, a link would first be made between onion and bun (the first peg word) by creating a mental image such as that of an onion on top of a bun. Similarly, mental images would be created that would link carrot to shoe (e.g., a carrot inside a shoe), and bread to tree (e.g., slices of bread hanging from a tree). Later retrieval of the items on the list would require recitation of the rhyme to remember the peg words *and* the images that have been created that incorporates the items on the list. Morris (1979) pointed out that obviously the rhyme, the way it is, is limited to retention of only ten items since there are only ten peg words. However, if more than ten items need to be remembered, a virtually unlimited number of peg words can be created using the phonetic translation system. For example, for 20, 21, and 22, "nose," "net," and "nun" could be used.

Other fact mnemonic devices, described by Morris (1979), do not require prior construction and memorisation of loci, translation systems, or peg words. However, they still draw from, or tap into, information stored in long-term memory to enable the retention of the target items or information. For example, there is the link method, where images are formed to link each item with the preceding and subsequent item. And there is the use of stories and associations, where stories or sentences are composed to link the items together. In both, the images and stories constructed are based on one's prior knowledge, information contained in long-term memory, and/or interest. Thus, just as SF (described by Ericsson et al., 1980) could conceivably have used weights or any other system instead of running times to recode digits, there would be extensive variations between individuals with regard to images and sentences constructed when using the link method or stories and associations. For example, if the items to be remembered were car, table, tower, and

window, an image could be composed about a car carrying a table on its roof rack that parks outside a tower with stained-glass windows. Or, if the mnemonist had a slightly more violent and morbid imagination, it could be about a car that is driving so fast it smashes into a table that is located outside a gothic-looking, run-down tower with broken windows.

There are two features of mnemonic use that are important to note here. First, mnemonics enable the memorisation of semantic-based information even if the information is not clearly understood. And second, they make possible the retention of much more information than otherwise possible using more 'natural' methods (such as rehearsal). The Hebrew prayer and much of what S read did not make much sense to him; the numbers that SF retained were not in themselves meaningful; and numbers recoded using the phonetic translation system and lists encoded using peg mnemonics are usually devoid of inherent meaning. Rehearsal alone would not have enabled SF to remember up to 79 digits; many of S's memory feats would have been impossible using normal techniques for remembering; and similarly, the amount of information that often can be retained using various mnemonic techniques is certainly greater than can be expected by employing 'natural' methods.

Another important point (noted earlier) is that mnemonics tap into information, knowledge, and even personal preferences stored in long-term memory. This linkage to stored information can be explicit as when 'structures' are deliberately constructed in long-term memory and used for the specific purpose of remembering certain facts (e.g., when using the method of loci, the phonetic translation system, and peg mnemonics). Or it can be incidental (e.g., when using the link method, or stories and associations). In essence, mnemonics provide a means for ordering, enhancing salience, and – to some extent – personalising target information. When viewed this way, they can be seen to function in much the same way as effective study techniques, such as paraphrasing and elaboration (e.g., see Buzan, 1995; Hookham, 1995; Novak & Gowin,

1984). They enable the individual to 'own' the target information that he or she is trying to memorise, hence making the information more likely to be retained and accessible in memory. The obvious difference lies in the kinds of target information they normally deal with: while study techniques are usually applied to meaningful material, the sequence of facts memorised using mnemonics are often devoid of inherent meaning.

### *Fact mnemonics and school learning*

Levin (1983) discussed the use of pictorial strategies for school learning. Most of the strategies he referred to can be considered forms of mnemonics as they essentially make use of the same procedures as mnemonics do in improving retention and learning. These procedures include the use of imagery, elaboration, recoding, and/or association with previously learned or previously known material. He referred, for example, to an earlier study that he and his colleagues carried out (Levin, McCormick, Miller, Berry, & Pressley, 1982) to show the difference between memory-facilitative and non-facilitative pictures. The tasks involved having to remember the meanings of words, such as that *surplus* means "having some left over, having more than was needed." In the memory-facilitative picture, stimulus recoding was involved: the students were shown a picture of someone pouring lots of syrup over pancakes and saying that there was a surplus of it in the cupboard. Apart from illustrating a situation where surplus was involved, the word *syrup* was also used as a keyword to help remember the meaning of the word *surplus*. Thus, as noted earlier, the strategy – as with most of the others mentioned in Levin's paper – although designated "pictorial" was clearly also mnemonic in nature.

Levin's (1983) article, while pointing out that not all pictorial strategies are beneficial in all learning situations for all people, was essentially

positive towards the use of this form of strategy for remembering and learning. He argued, for example, that pictures should be used as school learning aids, and he addressed common criticisms leveled at the use of pictures. These criticisms include claims that pictorial strategies are incongruent with the value society places on abstract conceptual thinking, and that they are artificial in their approach because most verbal information encountered outside the classroom is not accompanied by pictures. Levin pointed out that the use of pictures has particular value in special education where individuals involved have previously not responded sufficiently to traditional methods of teaching, and that even for high achieving students pictures (like physical representations, demonstrations, and concrete analogies) can provide valuable clarifications and insights into many abstract concepts that are difficult to grasp. He also argued that pictures are not there simply to serve as a 'crutch' for learners, but that the eventual aim is for learners to acquire the skill of generating their own pictures to help in their learning.

Levin (1983) referred to studies showing that pictorial mnemonic strategies are useful teaching tools in the language arts area, particularly in word recognition and prose learning. In a study by Shriberg, Levin, McCormick, and Pressley (1982), for example, the keyword method and pictures were used to teach students the supposed accomplishments of various fictitious individuals as written in prose. Those in the experimental group were given each individual's name phonetically recoded into a concrete referent, and a picture that semantically related the accomplishment to the concrete referent. For example, with Charles McKune who was famous for having a cat who could count, McKune was recoded as *raccoon* and a picture was provided of a cat counting raccoons. Shriberg et al. found that these students recalled more name-accomplishment information than the no-keyword and no-picture control students. They also found that students were equally if not more successful in recalling the required information when they had to generate the images/pictures themselves.

The keyword method has also been shown to be effective in vocabulary learning. McDaniel, Pressley, and Dunay (1987) found that college students learned the definitions of obscure English words faster under a condition that involved the use of keywords than under a condition in which semantic contexts for the words were provided. However, they reported no difference in retention between the groups one week after the trials. McDaniel and Pressley (1989) compared a keyword, a semantic context, and a no-strategy control group on speed of text comprehension (measured by reading time), actual comprehension (measured by the number of correct responses on a true-false questionnaire), and recall of vocabulary word definitions. Again, college students were used as experimental participants, and they were required to learn the meanings of unfamiliar Old English words using one of the three strategies. For speed of comprehending texts in which the words were used, no significant differences were found between the three groups. However, those in the keyword condition performed significantly better than the other two groups in correctly answering a true-false test designed to measure actual comprehension of the texts, and in recalling the definitions of the Old English words.

The effectiveness of the mnemonic keyword method has also been shown in facilitating the acquisition of German nouns and their grammatical gender (Desrochers, Gelinas, & Wieland, 1989), and botany concepts (Rosenheck, Levin, & Levin, 1989). In the former study, a modified version of the keyword method was used so that gender tags were recoded to make them more concrete. The intention was to enable French-speaking students to form a semantic link between the referent of the German noun and the recoded gender tag. Desrochers et al. examined the effect of this method on the recall of French translations and German genders, with the German nouns provided as retrieval cues. Compared to a free-strategy control group, those in the keyword method group showed better acquisition of the meaning of German nouns and their grammatical gender.

In the Rosenheck et al. (1989) study, college students who were either in a mnemonic, a taxonomic, or a free study condition, had to learn a plant classification system and the distinguishing characteristics of the plant groups within that system. Those in the mnemonic condition were provided with keywords and illustrations. For example, the keyword for *angiosperms* was 'angel' and the illustration was that of a happy angel with a bouquet of flowers (distinguishing characteristic) in her hands. Those in the taxonomic condition were provided with a chart (boxes connected by lines) to illustrate the plant classification system. And those in the free study condition were asked to just use their own best method for learning, and were also given blank paper for notetaking. The students were then tested on a variety of learning measures (taxonomy production, taxonomy use, definitions, and problem solving) immediately after reading, two days later, and two months later. Rosenheck et al. reported that the students in the mnemonic group performed significantly better than the free study students on every measure, and better than the taxonomy students on almost all of the measures. Rosenheck et al. pointed out that the results of their study clearly show that the use of mnemonic strategies can be beneficial, not just in simple recall of facts, but also in tasks that require the manipulation of information learned and problem solving. For example, on the problem solving task, the students had to identify the order of a specimen and list the subsumed orders when provided with only one or two distinguishing clues. Rosenheck et al. also highlighted the durability of mnemonic strategy effects: their study found mnemonic superiority after a 2-month delay. In fact, as they noted, their results suggest an increase in mnemonic superiority over time.

Ehri, Deffner, and Wilce (1984) used pictorial mnemonics in teaching letter-sound associations to prereaders. In their study they compared the effectiveness of using integrated pictures (picture mnemonics representing letter-sound relationships) with disassociated pictures in teaching phonic skills to pre-school, kindergarten, and first grade

children. Children in the integrated-picture group were taught letter-sound associations using pictures in which the name of the object shown began with the appropriate letter (e.g., wings for *w*), and the letter itself was a part of the picture (*w* forming the outline of the lower part of the butterfly's wings depicted in the picture). Children in the disassociated-picture group were taught the letter-sound associations using pictures in which the name of the object shown also began with the appropriate letter (e.g., wings for *w*) but they were drawn differently so that the letter did not form any part of the picture. The results they obtained showed that children taught with the integrated pictures learned more letter-sound associations and letter-picture associations than those taught with the disassociated pictures. In Ehri et al.'s second experiment, they found that children in the integrated-picture group again performed better, and that the performance of children in the disassociated-picture group did not differ from that of a control group of children who were shown no pictures at all.

### ***Fact mnemonic instruction and students with LD***

Mastropieri, Scruggs, and Levin (1987) reviewed research on the use of mnemonic instruction in teaching students with LD. Their review focused mainly on the areas of vocabulary learning and content-area information learning. One study they discussed (Mastropieri, Scruggs, Levin, Gaffney, and McLoone, 1985) focused on vocabulary learning and used junior high students with LD as experimental participants. Those in a mnemonic condition were given strategy instruction and provided with interactive pictures showing recoded stimulus and response items. For example, if they had to learn that *barrister* means "lawyer," they were provided with a keyword (e.g., bear) that was a concrete familiar word resembling a salient part of the vocabulary word. They were then shown an interactive picture relating the keyword to the definition - for example, that of a bear acting like a lawyer. Compared to a control group of

students who were given direct instruction and non-interactive pictures (pictures depicting only the vocabulary words), those in the mnemonic group performed significantly better (80 vs. 31% correct) on a subsequent test. In a second experiment, they found that students placed in a mnemonic condition still outperformed those in a similar control condition (69 vs. 47% correct) even when the former were only provided keywords and had to generate their own interactive images.

Other studies have also found the keyword method, combined with interactive pictures, effective in teaching new vocabulary to children with LD. Conduis, Marshall, and Miller (1986), for example, compared the performance on vocabulary learning of students with reading LD in a keyword–image condition with those in a picture–context condition, a sentence–context condition, and a control condition. Those in the picture–context condition were given illustrations representing the definitions of the vocabulary words; those in the sentence–context condition were given sentences that explained the definitions of the words, and asked to relate the meaning of the words to personal experiences; and those in the control condition were asked to use their own study methods to learn the vocabulary word meanings. Conduis et al. found that those in the keyword–image condition significantly outperformed those in all the other conditions in remembering word meanings. They found this not just immediately after instruction but also one week, two weeks, and eight weeks afterwards.

Mastropieri, Scruggs, and Fulk (1990) used adolescents with LD as experimental participants, and assigned them to either a keyword mnemonic condition or an experimenter-directed rehearsal condition. These adolescents had to learn 16 difficult vocabulary words – half of them concrete (e.g., *oxalis*, meaning clover-like plant), and the other half abstract (e.g., *vituperation*, meaning abusive speech). They found that on both recall and comprehension tests, those in the mnemonic condition performed significantly better than those in the rehearsal condition.

The pegword mnemonic described earlier (“one is a bun, two is a shoe, ... ten is a hen”) has also been found to be an effective strategy in teaching students with LD. Elliott and Gentile (1986) used both LD and non-LD students as experimental participants. They were assigned to either a peg-mnemonic condition or a control condition in which they used their own strategies for learning. The students had to learn lists of words. Elliott and Gentile found that those in the mnemonic condition, for both LD and non-LD groups, recalled more words than those in the control condition.

In a study on content-area information learning, Scruggs, Mastropieri, Levin, and Gaffney (1985) used a combination of keyword, imagery, and pegword strategies in the experimental mnemonic condition to teach three specific attributes of eight North American minerals to adolescents with LD. For example, these students had to learn that (i) pyrite is sixth on the hardness scale of these minerals, (ii) it is yellow in colour, and (iii) it is used in the manufacture of acid. Those in a mnemonic condition were given strategy instruction and provided with interactive pictures incorporating a keyword for the mineral in question, the colour of the mineral, a peg word representing its position on the hardness scale, and something representing its use. For example, for pyrite, a picture of a yellow (colour) pie (keyword) supported by sticks (peg word for six in the previously mentioned peg mnemonic technique that begins “one is a bun, two is a shoe ...”) with acid (use) being poured over it was shown. Three control groups were used: a free-study, and two direct-instruction groups. Students in the free-study group were provided with the necessary materials and told to study independently; those in one of the direct-instruction groups were taught using a teacher-directed question and answer format; and those in the other direct-instruction group were taught using a similar teacher-directed format but were given only half of the minerals list to learn. Scruggs et al. found that students in the mnemonic group performed significantly better than those in control groups on a test that was subsequently administered. They also found

that mastery rate (number of attributes learned over a given amount of time) was higher for those in the mnemonic group compared to those in the control group that only had to learn half of the list.

Scruggs, Mastropieri, McLoone, Levin, and Morrison (1987) found that using mnemonic pictures was also effective even in the absence of teacher-directed instruction. In their Experiment 1, students with LD were required to remember attribute dichotomies of North American minerals. Students in both the mnemonic and the control conditions were provided with study booklets containing instruction and expository prose. Those in the mnemonic condition, however, had mnemonic pictures in their booklets. For example, to remember that the mineral crocoite is soft, dark, and used in the home, a picture was provided of a crocodile (keyword for crocoite) that was dark (its colour), next to a baby (symbolising soft – instead of hard, which was symbolised by an old man), and in a home setting (to represent home use instead of industrial use). Those in the control condition were also provided with pictures in their booklets, but the pictures were non-mnemonic. The main difference between this experiment and previous similar experiments was that the students with LD had to independently read and learn from the expository prose passages. In a test following the students' independent reading of the passages, Scruggs et al. found that those in the mnemonic condition remembered significantly more.

In Scruggs et al.'s (1987) Experiment 2, the students with LD were required to remember specific attributes of North American minerals (e.g., that crocoite is level 2 on the hardness scale, it is dark orange in colour, and is used for display cases in the home) rather than dichotomous attribute classifications. The procedures were much the same as in Experiment 1, and students in the mnemonic condition were provided with mnemonic pictures (e.g., for crocoite: of a crocodile that was orange in colour, wearing shoes, and inside a display case) while those in the control condition were provided with non-mnemonic pictures. In a test on

specific attributes that immediately followed independent studying of the passages, those in the mnemonic condition scored significantly better than those in the control condition. The students were also tested on dichotomous attribute classifications of the minerals, which were not explicitly taught and had to be inferred from the material covered in the passages. The results showed those in the mnemonic condition again performing better, indicating that even in an LD population, mnemonic-based learning can facilitate information comprehension and usage. That is, information that has been explicitly learnt through mnemonics can be transformed for other purposes. Scruggs et al. also assessed longer term retention (one week after studying the passages) and found that students in the mnemonic condition retained significantly more information than those in the control condition. This was found in both specific-attribute recall and dichotomous-attribute identification.

In two experiments carried out by Peters and Levin (1986), the effectiveness of mnemonic imagery in facilitating prose information recall by good and poor readers was investigated. In their first experiment, the students had to learn the names and accomplishments of fictitious characters (similar to the Shriberg et al., 1982, study described earlier). They found that mnemonic imagery helped the recall of not just the good readers but also the poor readers: both scored better than their control counterparts both immediately and after one week. In their second experiment, Peters and Levin examined whether the mnemonic strategy would still be effective in ecologically valid instructional passages. Thus they used nonfictitious passages (again, about famous people and their accomplishments) taken from actual school reading materials. For example, the students had to learn that a violinist named Joseph E. Maddy (the keyword 'mad' was used for those in the mnemonic condition) started the National Music Camp in Interlochen in 1928. Again, as in their first experiment, they found that the mnemonic strategy was effective in facilitating recall of both the good and the poor readers. Both good and poor readers in the mnemonic condition

performed better than their control counterparts in immediate and delayed tests.

### *Why mnemonics work*

The mnemonic techniques that have been employed in teaching students with LD include: imagery or pictorial depictions, the keyword method, and the pegword method. These techniques all make use of what Levin (1983) referred to as the three R's of mnemonic techniques: recoding, relating, and retrieving. They recode unfamiliar stimuli by transforming them into more familiar and more memorable representations. By relating in an elaborative fashion the recoded stimuli to previously known information, a semantic context is created so that the stimuli can be learned in a more meaningful manner. And retrieval is a systematic process, thus reducing uncertainty when the desired response (remembering the stimuli) is called for.

Regarding mnemonic use in the more general sense, Rohwer and Thomas (1987) discussed the role that mnemonic strategies play in study effectiveness. They pointed out the importance of remembering information (facts, concepts, principles, as well as procedures) to academic success. They also stressed the value of adeptness. They pointed out that while effectiveness in the ability to recall information one needs is important, it is similarly important to be effective in what one does in preparing to remember and in using recalled information to perform well. According to Rohwer and Thomas, mnemonics play an important part here: they improve the adeptness with which students remember information.

Rohwer and Thomas (1987) pointed out that three factors distinguish students who are good at strategy use from those who are not: generativity, executive monitoring, and personal efficacy. Generativity

refers to the reformulation of information that has been given, and to the generation of further information over and above what has been explicitly given. This has been found to enhance performance and, as Rohwer and Thomas pointed out, mnemonic use facilitates generativity. Executive monitoring involves the appraisal of strategy needs, the deployment of the appropriate strategy, and monitoring of the strategy's effectiveness. When applied to mnemonic use, it means that it is important to know when mnemonic strategies are called for, to carefully consider which mnemonic strategies are best to use given the features of a task, and to periodically monitor the effectiveness of their use. Finally, Rohwer and Thomas considered it important that students believe in their ability to control the outcomes of their learning – that is, they should have a good sense of personal efficacy. Without this, they would be unlikely to try to make use of different strategies (such as mnemonic or mnemonic-related strategies) to improve their ability to learn and perform well in academic settings.

Bellezza (1987) did not subscribe to the commonly held belief that mnemonics are unnatural forms of remembering. He viewed mnemonics as having many features in common with memory schemas. Memory schemas are organised knowledge structures that can be thought of as generic concepts about persons, objects, situations, events, actions, and so on. A schema gets activated when information that is similar to it in content gets presented. For example, if a person walks into a room where the other people present are talking to each other, drinking, and eating nibbles, that person's schema for a 'party' might get activated. Bellezza argued that mnemonic devices, like memory schemas, possess the properties of useful mental cues which are constructibility, associability, discriminability, and invertibility. Both mnemonic devices and memory schemas can be recalled to mind during the learning of information and during the recall of that information (constructibility); both can be associated with new information (associability); both are difficult to confuse with others of their kind (discriminability); and in

both, there is a two-way direction between the cue (the schema or mnemonic) and the learned information (invertibility). On this last point, during the learning stage the direction is from the new information to the cue, and during the retrieval stage this is reversed – that is, the direction becomes from cue to the newly learned information. To illustrate the similarity between memory schemas and mnemonic devices, Bellezza used the example of a neighbourhood schema: if organised and memorised systematically, features of it can serve as locations for the method of loci.

Bellezza (1987) suggested that mnemonic devices may be more constructible and associable than memory schemas. Schemas tend to change with experience and therefore they could easily become less constructible. For example, our schemas for different types of people are likely to gradually change as we meet more and more people, and schemas we related to people we met a while back may no longer come to mind so easily. Schemas are also not always associable – the effectiveness of their associability depends on the material to be learned. Thus, a restaurant schema might be useful for remembering items usually found in restaurants, but possibly not for a randomly selected set of objects. In contrast to this, mnemonic devices are constant, and therefore – as long as they are well-learned devices – they always remain constructible. The associability of the majority of mnemonic devices also appear almost limitless: peg mnemonic items, for example, can be linked to any object that can be visualised.

There are of course some important differences between memory schemas and mnemonic devices, as Bellezza (1987) acknowledged. For example, while memory schemas tend to be activated automatically by incoming sensory information, mnemonic devices are usually activated only through strategic decisions. Bellezza also pointed out that while memory schemas can often be used as guides for behaviour, mnemonic devices cannot be used as such. (Here he was referring to fact

mnemonics: as will be discussed later in this chapter, *process* mnemonics can be considered guides for behaviour in that they help in remembering procedures and rules). Bellezza made the observation that mnemonic devices are like simple memory schemas and as such they tend to simplify acquisition when applied to the learning process.

It is perhaps this quality of simplifying acquisition that makes mnemonic strategies particularly useful tools in special education. Turnure and Lane (1987), in writing about the special education applications of mnemonics, pointed out that the effectiveness of mnemonic techniques can be understood better when these techniques are considered in terms of Sternberg's (1985) componential model of knowledge acquisition and performance. They pointed out that steps taken in the use of mnemonic techniques directly correspond to components in knowledge acquisition and performance. For example, one of the steps facilitated by the use of mnemonics is the discrimination of stimulus parts or features: this corresponds to a 'selective-encoding' component, sifting out relevant from irrelevant information. Another step is the interactive combination of images: this corresponds to a 'selective combination' component which combines selectively encoded information in such a way that the parts are cohesive and make sense. When viewed this way, it is clear that mnemonic techniques make explicit the requirements of successful knowledge acquisition and performance. The techniques make it more likely that those requirements would be met during information processing, which may be very important especially when dealing with some special education groups such as those with LD or those with attention-deficit disorders.

### ***Process mnemonics: The Yodai method***

The techniques used for teaching that have been described thus far in this chapter are all *fact* mnemonic techniques. They are used on a one-to-one

basis to remember facts and, typically, one mnemonic association is made for each item to be remembered. As mentioned at the beginning of this chapter, however, there is another type of mnemonic called *process* mnemonics. These are used to help remember rules, principles, and procedures.

A Japanese educator, Masachika Nakane, was probably the first to develop and use forms of process mnemonics for instructional purposes (see Higbee, 1987; Higbee & Kunihiro, 1985a). Prior to his death in 1984 at the age of 95, he spent some 70 years developing and using what he called *yodai* mnemonics to teach students at the Ryoyo Institute in Kyoto where he worked as a teacher and principal. He used the term *yodai*, which means "the essence of structure," because the primary aim of these mnemonics is to summarise the organisation and the process of problem solving. Nakane claimed, for example, to have condensed the essence of trigonometry into a few rhymes that can be read within 60 seconds (Nakane, 1981: cited in Higbee & Kunihiro, 1985a, who also reported that these rhymes cannot be meaningfully translated into English).

Higbee and Kunihiro (1985a) noted that *yodai* is used in several Japanese schools (including the Ryoyo Institute) to teach a range of subjects including arithmetic, algebra, geometry, trigonometry, calculus, inorganic and organic chemistry, physics, biology, spelling and grammar, and the English language. Phrases, sentences, rhymes, and even songs are used in the *yodai* system to help in learning and remembering the orderly steps involved in solving problems. Some even include verbal commands for gestures and/or activities requiring the use of blackboard, or paper and coloured pens. The Ryoyo Institute where Nakane worked is a demonstration and experimental school which has students from kindergarten through to high school. Higbee and Kunihiro pointed out that the use of *yodai* mnemonics basically goes contrary to common approaches in Japanese and Western education (where comprehension precedes doing) in that, typically, *yodai* use involves doing before

understanding. They pointed out, however, that most mnemonics are similar in this sense, and that there is evidence to show that doing can effectively lead to learning. For example, this method is used in the Suzuki School of Music in Japan with successful outcomes, and Kunihiro and Asher (1965) showed that a foreign language can be taught without mediation of the first language – and therefore initially without understanding.

Some specific examples would probably help explain better what yodai mnemonics are all about and how they work. Higbee (1987) and Higbee and Kunihiro (1985a) provided several examples of yodai use, but most were in mathematics – perhaps because the others were too difficult to translate and explain in a coherent form in English. They pointed out that yodai mnemonics use familiar metaphors expressed in familiar words to teach mathematical operations in a simple and uncomplicated way to children. For example, because Japanese children apparently like playing with bugs, fractions have been represented by bugs. Thus a fraction – described as a bug to the children – has a head (the numerator) and a wing (the denominator). Terms like *fraction*, *numerator*, and *denominator* are dispensed with. To add fractions, children are taught a rule to “count the heads when the wings are the same.” To multiply, they have to “put the heads together and put the wings together.” The multiplication sign ( $\times$ ) signifies the crossed horns or feelers of the bugs, and cues the children to carry out the instruction just mentioned. In dividing, they have to turn one bug upside down and then multiply. Neither Higbee, nor Higbee and Kunihiro, said anything about how they subtract fractions, but presumably the procedure/rule would be much the same as in addition – except children would probably have to take ‘heads’ away.

Wrestling is involved when multiplying binomials of the form  $(a+b)(c+d)$ . In this case, each term in parenthesis represents a wrestler of either the east team or the west team. Each member of the east team has

to meet and wrestle with each member of the west team, thus  $(a+b)(c+d) = ac + ad + bc + bd$ . Passengers and luggage on a train are used to describe multiplication of binomials of the form  $(a+b)^2$  or  $(a+b)(a-b)$ . Even the Japanese folk story of a boy named Momotaro gets utilised – when multiplying a monomial by a polynomial of the form  $a(b+c+d)$ . The boy (a) is on a journey and he meets a dog (b), a pheasant (c), and a monkey (d), and takes each of them with him. Thus  $a(b+c+d)$  can be solved as  $ab + ac + ad$ . Parenthesis can also be described as baskets, and + and – signs can be represented by male and female respectively. Thus  $(a-b)$  could be a male bug and a female bug inside a basket (Higbee, 1987; Higbee and Kunihiro, 1985a).

Perhaps the most impressive example of yodai use described by Higbee (1987) was in solving quadratic equations of the form  $ax^2 + bx + c = 0$ . At the Ryoyo Institute, Higbee observed and videotaped kindergarten children, who are about 5 years of age, solving such equations.

To solve for  $x$  in such an equation, the formula given below needs to be used.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The children at the Ryoyo Institute remember this formula and how to use it through a yodai mnemonic called the “flute song.” The song goes as follows: *fu-e-no* (“flute’s”), *hi-bi-ki* (“sound”), *wa* (topic marker which means “as for the previous”), *mi-mi* (“ear”), *hi* (which means to hold the previous sound when singing), *yo-a-shi* (“good”). In English, the song means “As for the flute’s sound, it is good to the ear.”

The syllables of the song correspond to components of the formula. Thus a child singing the flute song with others in the class could write on the board the solution for  $x$ :

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-\text{bi ki} \sqrt{\text{mi mi} - \text{yo a su}}}{\text{fu e}}$$

Higbee (1987) pointed out that every component of the formula derives from the song except for *su* which replaces *shi*. Each syllable not only has a meaning in the song but in the formula as well. For example, the denominator contains the word *flute* (fu-e): but “fu” is also the first syllable of a Japanese word for 2 (*futatsu*), and “e” sounds like the letter *a* as usually pronounced in Japan.

Higbee (1987) and Higbee and Kunihiro (1985a) also briefly described uses of yodai mnemonics in organic chemistry. They pointed out that various molecules from the chemical elements C (carbon), H (hydrogen), and O (oxygen) are portrayed as family members including mother, father, brother, sister, grandmother, grandfather, aunt, uncle, and so on. Yodai mnemonics are used to identify various compounds, and to combine them and figure out the results. Children also draw “kites” when working out the configurations of ring compounds like benzene.

Apart from anecdotal reports that a high percentage of Nakane’s graduates from the Ryoyo Institute have successfully entered various professions such as medicine and engineering, and that many of them have attributed their interest in science and mathematics to how they were taught at the school, there is also some empirical research evidence showing that yodai mnemonics can be very effective teaching tools. Takizawa, Hatori, Kunihiro, and Machida (1980: cited in Higbee & Kunihiro, 1985a) found that children instructed using the bug mnemonics were able to compute with fractions as well as children who were in the grade above them and who had an advantage of a year of regular instruction. In another report, they found yodai mnemonics highly effective in improving the ability of high school students to solve complex computational problems (Hatori, Takizawa, Kunihiro, & Machida, 1980: cited in Higbee & Kunihiro, 1985a). Yodai mnemonics

were also shown to be more effective than traditional instruction methods in helping Japanese junior high school students learn how to construct trigonometric ratios and functions and to compute with them (Kunihira & Machida, 1981: cited in Higbee & Kunihira, 1985a). And Machida and Carlson (1984) found that yodai mnemonics were more effective than traditional methods of instruction in teaching Japanese students how to construct useful equations to solve algebra sentence problems using monomials and polynomials.

### *The Pool mnemonics method*

To address the question of whether yodai-type mnemonic instruction can be as useful in other countries as it is in Japan, Kunihira developed similar mnemonic techniques for use with American children (Higbee, 1987; Higbee & Kunihira, 1985a). Since American children are generally not interested in playing with bugs, Kunihira looked for representations he could use that would be more familiar and more interesting to them. He used swimming pools and joggers, and came up with *pool mnemonics* for computing with fractions.

Pool mnemonics are similar to yodai in many ways: their instructional components include numbers being represented by more familiar objects, rhymes, and visual illustrations. In Kunihira's pool mnemonics, fractions are joggers: they have a number on their jersey (the numerator) and patches on their pants (the denominator). Thus a fraction like  $\frac{2}{3}$  would be a jogger with the number 2 on their jersey, and 3 patches on their pants. When multiplying fractions, the  $\times$  between the 'two joggers' is described as representing an x-shaped swimming pool. The presence of the x-shaped swimming pool cues children to play the multi-POOL game. The rhyme they have to remember for this game is "Pool shirts to shirts, patches to patches," meaning multiply the numerators together and the denominators together (Higbee & Kunihira, 1985a).

With regard to division, children are taught that they can imagine the division sign ( $\div$ ) between the two joggers as a diving board with two beach balls, one above it and one below it. They are told that when they see this diving board, that the jogger on the right is actually fooling around and is standing on his or her head – and that they have to play the DIVE-ide game. Thus they have to first “Flip the fool into the pool” to ensure that this jogger is the right side up. After flipping the fool, the rest of the rhyme (“... into the pool”) cues the children to then play the multi-POOL game with the joggers (i.e., “Pool shirts to shirts, patches to patches”) (Higbee & Kunihiro, 1985a).

For addition and subtraction of fractions the children are taught the MATCH-PATCH game. In describing this game to children, the  $t$  in each of the two words represents the addition sign (+) while the hyphen between the two words represents the minus sign (-); when presented to children, these (the  $t$ 's and the hyphen) are written in a colour different from the other letters. When the joggers have the same number of patches on their pants (i.e., equal denominators) the rhyme to remember what to do is simple: “Match the patches, don't take a chance; count the shirts and leave the pants.” This instructs children to simply add the numerators and to leave the denominators alone, the answer will have a denominator identical to the two joggers'. If the number of patches on the joggers' pants are not the same (i.e., different denominators) then the rhyme is: “If the patches do not match, pool the other person's patch.” This gives the children instruction to first play the multi-POOL game with each jogger using the number of patches on the other jogger's pants. For example, if the problem was  $\frac{2}{3} + \frac{1}{5}$ , they have to first 'multiply each jogger' by the other jogger's patches – hence  $\frac{2}{3} \times \frac{5}{5}$  and  $\frac{1}{5} \times \frac{3}{3}$ . Thus they would get  $\frac{10}{15} + \frac{3}{15}$  which would now be conducive to the MATCH-PATCH game with equal denominators (Higbee, 1987).

Apart from anecdotal reports that both Higbee and Kunihiro have found pool mnemonics very effective in teaching fractions to children (see

Higbee & Kunihiro, 1985a, p. 60), Kunihiro, Kuzma, Meadows, and Lotz, in a paper they presented at the 61st annual meeting of the Western Psychological Association in 1981 (cited in Higbee & Kunihiro, 1985a), reported convincing results obtained in an empirical study they carried out. They taught 8-year-old third graders, who had not started studying fractions yet, how to add, subtract, multiply, and divide fractions using pool mnemonics. They gave the instructions in three 1-hour class sessions. They pointed out that usually the material they covered on operations with fractions is spread over three years from grades 4 to 6. On immediate posttest, the third graders performed calculations as well as sixth graders, who had already been given the traditional form of instruction for 3 years, and better than fourth and fifth graders. On a test two weeks after instruction, the third graders no longer performed as well as the sixth graders, but their performance was still better than fourth graders and equal to fifth graders.

### *Comments on yodai and process mnemonics*

Pressley (1985) and Levin (1985) wrote positive commentaries on the Higbee and Kunihiro (1985a) paper on yodai mnemonics. Pressley believed that the paper would be of interest to a considerable number of educational researchers, and that it may motivate research on new and more complex forms of mnemonics. However, he expressed three concerns. First, that it was not made clear enough exactly what constitutes a yodai mnemonic. Second, Pressley felt that the available data supporting yodai use as learning aids were inadequate. And third, he felt that further investigation was required on precisely how yodai mnemonics work. Despite being very positive and enthusiastic about yodai mnemonics, Levin (1985) expressed similar concerns to those noted by Pressley. He stressed the necessity for a more comprehensive definition and description of yodai, and pointed out the lack of controlled research data on their use.

In response to the questions that Pressley (1985) and Levin (1985) raised, Higbee and Kunihiro (1985b) pointed out that they were still in the process of learning more about yodai mnemonics themselves, which was the reason behind the sub-optimal clarity in their definition and descriptions. They agreed that yodai mnemonics (and process mnemonics, generally) require more experimental investigation, not just on whether they work, but also on how they work. They wrote that they hoped their paper would stimulate such investigation.

Kilpatrick (1985) was more negative towards yodai mnemonic use. One of the few values he could see was in its utilisation of recitation, a feature he felt was lacking in American classrooms. He also believed that mnemonics could be useful in remembering mathematical conventions such as remembering to multiply and divide first before adding and subtracting in expressions without grouping symbols (e.g.,  $3 + 4 \times 5 - 2$ ). But he did not think mnemonics were useful where the associations involved are not arbitrary – that is, where there are good reasons for carrying out the procedures involved. Essentially, he disagreed with yodai mnemonics' reliance on 'doing' at the expense (he believed) of 'understanding,' because one does not naturally follow on from the other.

Kilpatrick (1985) questioned, first, the value of yodai's use of familiar terms and metaphors, in that it just delays what ultimately need to be learned (i.e., the technical terms). Second, he questioned the usefulness of operations that yodai can help in learning, such as multiplication and division of fractions, when the necessity for such operations hardly ever falls outside the domain of contrived situations. And third, he raised questions about the actual cross-cultural applicability of yodai mnemonics: he believed they work well in Japan primarily because of the language and the culture which support regularity, repetition, rhyming, and so on. Kilpatrick also noted the confusion that could arise from all the different metaphors that yodai mnemonics use:

To be sure, the Yodai mnemonic of wrestlers pairing off captures well the combinations one wants in  $(a + b)(c + d)$ . But when students have learnt  $(a + b)^2$  represents passengers on a train and that  $(a + b)$  represents two male bugs in a basket, how do they come to see that  $(a + b)$  is a factor of  $(a + b)^2$  and that  $(a + b)^2$  is a special case of  $(a + b)(c + d)$ ? Are these male bugs wrestling in baskets on a train? (p. 66)

In response to Kilpatrick's (1985) comments, Higbee and Kunihiro (1985b) stated that they agreed understanding is important in learning. They pointed out that yodai mnemonic use is not at the expense of understanding – rather, they believed that their use might precede, accompany, or follow understanding. They might help in the retention of procedures that have already been understood. They might also help in giving students confidence and successful experiences which would stimulate interest and on-task behaviour, which in turn could lead to understanding and further learning. They noted that students with special needs or problems have already been shown to benefit from traditional types of mnemonics, and thus yodai-type mnemonics might equally be beneficial in their learning.

Higbee and Kunihiro (1985b) also argued that, contrary to Kilpatrick's (1985) suggestions, yodai mnemonics help in remembering and applying the processes required for problem solving. By problems here, they meant not just what Kilpatrick considered "real" problems which were word problems or story problems, but also equivalent mathematical expressions which occur in typical mathematics textbooks. They stressed the importance of learning procedures, and noted that even in traditional forms of instruction a considerable amount of emphasis is placed on this. The main difference is that yodai mnemonics – such as Kunihiro's pool mnemonics – teach procedures in a more meaningful, more interesting, and more memorable way to students. As an example, they pointed out the relative ease of remembering "Flip the fool into the pool" compared to "To divide fractions, multiply by a reciprocal" (from a 5th grade text) and "A quotient can be expressed as the product of the dividend and the reciprocal of the divisor" (from an 8th grade text).

According to Higbee (1987), the effectiveness of mnemonics derives from their use of the principles of learning and memory which include meaningfulness, organisation, association, attention, and visualisation. Yodai mnemonics, and process mnemonics in general, are no exceptions. The language they use is familiar and meaningful to children: for example, fractions are “bugs” with “heads” and “wings” in Japan, and “joggers” with “numbers on their jerseys and patches on their pants” in California. They provide a structure to the material to be learnt, in the form of rhymes (which create a condition where only a limited number of alternatives will fit), categorisation (the grouping of bugs as “boys” [+] and girls [-]), and chunking (e.g., apparently in yodai, one rhyme can generate hundreds of chemical formulas). Links are made between abstract symbols and concrete associations: for example, the division sign ( $\div$ ) is linked to a more concrete “diving board with two beach balls – one on top and one below it.” They use metaphors that children are interested in, and in Japan, attention is further facilitated by group recitation of rhymes and songs, as well as engagement in gestures and motor activities. Higbee also pointed out that although process mnemonics are largely verbal or linguistic in nature, visual aids are sometimes used, as in teaching Kunihiro’s pool mnemonics. But even in the absence of these visual aids, visual imagery is still fostered as a result of the concreteness of the rhymes, phrases, descriptions, and stories used. For example, it is difficult to avoid forming mental images of the boy Momotaro and his meetings with a dog, a pheasant, and a monkey when one thinks of the yodai mnemonic for multiplying a monomial by a polynomial of the form  $a(b + c + d)$ .

Higbee (1987) used yodai mnemonics to illustrate process mnemonics, but he explained that not all yodai mnemonics are process mnemonics. Some are used to remember facts rather than processes, and can be considered as more like traditional encoding and organisational mnemonics. Similarly, there are process mnemonics that are not part of the yodai system.

Process mnemonics, as Higbee (1987) explained, provide the processes for getting the required information – as opposed to giving the information directly which is what fact mnemonics do. The rhyme “Thirty days has September, April, June, and November ...,” for example, is a fact mnemonic because all the information that could be required is contained in the rhyme itself. In contrast, a mnemonic like *soh cah toa* (for solving trigonometric problems to do with right-angled triangles) is a process mnemonic because, although it provides the factual information about how to solve trigonometric problems, all the possible solutions to such problems are not contained in the mnemonic itself. Thus, as Higbee pointed out, fact mnemonics can be considered *duplicative* in that they duplicate the information from the mnemonic itself, while process mnemonics are *generative* in that they allow the generation of the required information from the mnemonic.

### *Process mnemonic instruction and LD*

Earlier sections of this chapter have not only discussed research findings that attest to the useful instructional applications of fact mnemonics to a reasonably wide range of topics, but also those that show the responsiveness of individuals with LD to fact mnemonic instruction. Higbee (1987) included children with LD in the group he suggested could very well benefit from instruction utilising process mnemonics, and Higbee and Kunihiro (1985b) mentioned that Kunihiro had helped in developing a mathematics course in Japan that included yodai mnemonic components, aimed at junior high students who had not responded well to the regular classroom instruction. Use of process mnemonics in teaching students with LD appears very promising. Since they help in learning and remembering processes and rules, it is conceivable that they could directly address some of the difficulties in reading, spelling, and arithmetic that students with LD experience. However, since the Higbee and Kunihiro paper in 1985, and the Higbee chapter in McDaniel and

Pressley's book in 1987, the only paper that has been published on the topic of process mnemonic use with LD populations is Manalo's (1991) case report.

Manalo (1991) used a single subject, multiple baseline design across the four arithmetic operations of addition, subtraction, multiplication, and division. The experimental participant was a 15-year-old boy with difficulties in reading, spelling, and mathematics, although Manalo dealt only with his mathematics LD. The study looked at the relative effectiveness of the demonstration-imitation strategy and a strategy that involved process mnemonics, demonstration and imitation, in teaching the participant computational skills that he had previously failed to master in school. Process mnemonic instruction was implemented by presenting numbers as characters (warriors) and the operations as stories. The details of this mnemonic instruction method will be described later, as they are used in the present research.

In Manalo's (1991) case report, the demonstration-imitation (DI) method alone was first used in addition. The participant improved sufficiently and maintained this improvement over the subsequent sessions, so that it was deemed unnecessary to provide further instruction that incorporated a process mnemonic component. However, in multiplication, the participant did not make significant progress despite the DI intervention initially provided. Thus, several sessions later, a demonstration-imitation-and-mnemonics (DI&M) intervention was given. A significant increase in his performance accuracy was observed after the DI&M intervention, both in the immediate post-instruction assessment and in subsequent assessments when no instruction was given.

In both subtraction and division, the DI&M intervention was given at the onset and, in both cases, clear improvements were observed in the

participant's assessment performance both in immediate post-instruction and in subsequent sessions (Manalo, 1991).

Manalo's (1991) case report indicated that process mnemonics could be usefully incorporated in instruction for students with mathematics LD. While the demonstration-imitation strategy was effective (as shown by the participant's response to the DI instruction in addition, and as claimed by other researchers like Rivera & Smith, 1987), the inclusion of a process mnemonic component appeared to produce better results. This was evidenced by the participant's response to the DI&M instruction in subtraction and division, and in multiplication where he initially failed to make adequate improvement after the DI intervention alone.

Because Manalo (1991) used only one experimental participant, generalisability of the findings is limited and quite a few questions were left unanswered. Would the same 'warrior' mnemonics be effective in teaching other children with similar mathematics LD? Manalo did not consult the experimental participant about his interests prior to formulating the warrior mnemonics instruction used, but the instruction did prove effective in improving that participant's computational skills performance. Manalo was of the opinion that the instruction would be effective with other children as well because themes from computer games influenced him in formulating the mnemonics' characterisations and stories, which interest many children.

The comparison between instruction incorporating process mnemonics and instruction using only the demonstration-imitation approach was not thoroughly carried out in the Manalo (1991) study. For example, DI instruction (without mnemonics) was never provided in subtraction and division. Therefore, while the DI&M intervention proved effective, there were no data to show how well or how poorly the participant would have responded to just a DI intervention in these two operations. It is important to find out whether instruction incorporating process

mnemonics is superior to the demonstration-imitation approach, since the latter approximates the form of instruction currently given in most LD remedial settings.

Another question is whether other instructors would get similarly good results using the process mnemonics Manalo (1991) devised in teaching students with mathematics LD. Manalo formulated, and was very familiar with, the characterisations and stories involved in his warrior mnemonics. Would they be easy enough for other instructors or researchers to learn and use as effective and practical teaching tools? A different but related question is whether warrior mnemonics can be used for small group instruction, as opposed to the one-to-one teaching carried out in Manalo's case report. Small group instruction would be more efficient and cost-effective when considering the realities of remedial instruction classrooms.

Finally, there is the question of maintenance of improvements in performance following process mnemonic instruction. Many of the research studies that have been carried out using fact mnemonics to teach students with LD have found that experimental participants who were given mnemonic instruction retained information better in the long term (up to ten weeks) than those given other forms of instruction (see earlier section on this topic). However, Higbee and Kunihiro's (1985a) report on using pool mnemonics to teach fractions to non-LD students show that after two weeks the students' performance had deteriorated quite considerably. At immediate posttest, the third graders who were given instruction using pool mnemonics performed calculations as well as sixth graders. Two weeks later, however, they no longer performed as well as the sixth graders: their performance by then had gone down to equal the fifth graders. If we crudely extrapolate from this reported data, in 4 weeks their performance would have probably deteriorated to fourth grade level, and in 6 weeks they would have been back to where they started – at third grade level.

Manalo (1991) did not collect adequate follow-up data to show whether or not his experimental participant's improvements were maintained in the longer term. This is therefore an important question that needs to be addressed: Assuming other children with mathematics LD show improvements in their computational skills performance following warrior mnemonics instruction, would such improvements maintain in the longer term? The answer to this question, and to some of the others posed earlier, have important implications concerning the usefulness of Manalo's warrior mnemonics – and process mnemonics in general – as tools for teaching students with LD.

### *Summary*

There are two basic types of mnemonics: fact mnemonics and process mnemonics. The former are used on a one-to-one basis to remember facts, while the latter are used to help remember rules, principles, and procedures. The use of mnemonics has a long history chronicled by ancient authors such as Cicero, the anonymous author of *Ad Herennium*, and Quintillian. The enduring use of mnemonics suggests that they address a central need in humans to remember various forms of information often exceeding the capacity of natural methods.

Mnemonics have been referred to as artificial memory strategies. However, more recent reports of mnemonic feats – involving people both of unusual and normal memory capabilities – suggest that mnemonic use can be natural and largely unintentional; it can be 'induced' as a response to having to remember various facts.

Fact mnemonics are the better known and more commonly used type of mnemonics. Many forms of fact mnemonics require explicit, prior construction of structures in long-term memory for the retention of target information. However, even those that do not, still tap into information

stored in long-term memory. Two features of mnemonic use were noted. First, that mnemonics enable the memorisation of semantic-based information even when the information is not clearly understood. Second, that mnemonics enable the memorisation of much more information than otherwise possible using more natural methods such as rehearsal. Research on the use of fact mnemonics in school learning has shown that they are useful teaching tools in tasks that include word recognition and prose learning; vocabulary learning; the acquisition of foreign language nouns and their grammatical gender; the learning of science concepts; and the acquisition of letter-sound associations among prereaders.

Research studies that have been reported indicate that mnemonics are effective in teaching students with LD. The usefulness of mnemonic-based instruction has been shown in teaching these students vocabulary words and their meanings, word lists, and subject-related factual information and descriptive details. All the studies indicate that those given mnemonic-based instruction remember significantly more information taught than those given various other forms of instruction. Quite a number of the studies show that this effect is not limited to the immediate post-instruction period: the superiority of mnemonic instruction is maintained even in delayed tests. Other findings worth noting include those that show mnemonic pictures can be effective even without teacher-directed instruction, and that mnemonic-based learning can facilitate information comprehension and usage.

The mnemonic techniques that have been used in teaching students with LD recode unfamiliar stimuli into more familiar, memorable representations; they relate the recoded stimuli to previously known information; and they establish mechanisms that make retrieval a systematic process. Mnemonic strategies have been described as facilitating generativity, executive monitoring, and personal efficacy – the possession of which qualities distinguish students who are good at

strategy use from those who are not. The effectiveness of mnemonics has been explained in terms of features they share with memory schemas: constructibility, associability, discriminability, and invertibility. The effectiveness of mnemonic strategies as tools in special education has also been explained in terms of their making it more likely that the requirements of successful knowledge acquisition and performance are met during information processing.

Yodai mnemonics, most of which are forms of process mnemonics, are used in several Japanese schools to teach a range of subjects including mathematics, science, spelling, grammar, and the English language. Phrases, sentences, rhymes, songs, stories, and gestures are used in yodai to help in learning and remembering the orderly steps involved in solving problems. Examples of the method's effectiveness include that of enabling kindergarten children to solve quadratic equations. Various researchers in Japan have also found yodai more effective in teaching mathematics to children than traditional methods.

The pool mnemonic method was developed in the U.S. to test whether yodai-type mnemonic instruction can effectively be used with American children. The method teaches addition, subtraction, multiplication, and division of fractions. Reported findings suggest that pool mnemonics, compared to traditional forms of instruction, are more effective in teaching children computations with fractions.

The main criticism that has been levelled at yodai and process mnemonics is that they rely on doing at the expense of understanding. However, counterarguments in support of these methods stress that understanding is not neglected at all: mnemonic use might precede, accompany, or follow understanding. The potential use of process mnemonics in teaching students with special needs has also been emphasised, based on findings that fact mnemonics are effective in teaching such student populations. It has also been pointed out that like

traditional forms of mnemonics, process mnemonics make use of principles of learning and memory which include meaningfulness, organisation, association, attention, and visualisation. However, unlike fact mnemonics, process mnemonics are generative (rather than duplicative), meaning that they allow the generation of the required information from the mnemonic.

Only one paper has been published on the use of process mnemonics to teach individuals with LD. The study employed stories and metaphors to teach basic computational skills to an individual with significant deficits in mathematics. Positive results were obtained. However, the study used a single-subject design which limited the conclusions that could be drawn and the generalisability of its findings. It was pointed out that further research is required to find out whether process mnemonic instruction would be effective in teaching other individuals with similar deficits in mathematics. Investigations are also required to find out, among other things, whether process mnemonic instruction can effectively be used to teach small groups; and whether improvements in performance that result from process mnemonic instruction would maintain beyond the short term.

## Chapter 4

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# Rationale & Experimental Questions

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### *Introduction*

The primary purpose of the present research was to investigate the usefulness of process mnemonics as a tool for teaching individuals with mathematics LD. The experimental participants in this study were students who had deficits in the basic computational skills of Addition, Subtraction, Multiplication, and Division. Thus the process mnemonics instruction was aimed at remediating skills deficits in executing these basic operations.

Historically, among educated people, mnemonics have been useful (see Yates, 1966). While there may have been debates over the extent of their application, their actual usefulness was never in question. Even critics of mnemonic use, like Quintillian (cited in Herrmann & Chaffin, 1988) and D'Assigny (1985: originally published in 1697), conceded that mnemonics have useful applications.

With the advent of pen and paper, many of the original uses of mnemonics have become obsolete: there is little need for mnemonics when one can have notes to consult, for example. However, reports of exceptional feats as a result of mnemonic use have continued (e.g., see Luria, 1968; Ericsson et al., 1980) and since the 1970's there has been renewed interest in their usefulness as educational tools.

### *Questions about mnemonics*

As described in the previous chapter, various uses of mnemonics in education have been shown to be effective. However, in order for the use of mnemonics to be justifiable, it is necessary to demonstrate that they are more effective than other methods, or that they address a need not met by these other methods.

Much of the research on mnemonic use has focused on remembering facts, such as the meanings of new, foreign, or difficult words; selected story or prose details; letter-sound associations; or science facts. There are obviously some practical limitations to the kinds of materials that mnemonics can help in remembering. However, the fact remains that mnemonic learning has in many cases been demonstrated to be more effective than learning with the use of other alternative methods. It is understandable, for example, that using a keyword like “angel” and a pictorial representation of a happy angel holding flowers could facilitate remembering of information about *angiosperms* far more effectively than using a chart representation of plant features (as in the Rosenheck et al., 1989, study): the pictorial representation is a salient, single but complete item that has a distinct image, while the chart has several associated parts and can be difficult to mentally distinguish from other chart representations. Thus, the number of chunks to remember (i.e., one versus several) and distinctiveness are likely to impact on memorability in favour of the mnemonic approach.

However, given the arguments that mnemonics are artificial and not particularly facilitative of comprehension, perhaps it is easier to justify mnemonic use where it can address learning needs not adequately addressed by other techniques. Thus, where individuals with LD are concerned, it is easier to justify mnemonic use because other methods of instruction have presumably failed.

A substantial amount of research has been carried out on the use of mnemonics to teach students with LD. Especially in the 1980's, collaborative efforts between Mastropieri, Scruggs, Levin, and their colleagues have produced a considerable number of papers on the topic. In these studies, various forms of fact mnemonics (mainly the keyword method, peg mnemonics, and imagery) were used to teach students with LD a wide range of factual information. These studies have shown that mnemonics are more effective than other methods (such as recitation, direct instruction, or rehearsal) in teaching students with LD information such as new vocabulary words, attributes of minerals, and accomplishments of fictitious and actual famous people. Thus it can be said that, for teaching students with LD, mnemonics have useful applications in vocabulary learning, science, and social studies.

There is, however, a point of concern here. Individuals with LD are not usually characterised as experiencing difficulties in learning that *oxalis* is a clover-like plant, that *crocoite* is orange and used for display cases, or that Joseph E. Maddy started the National Music Camp in Interlochen in 1928. Instead the focus tends to be on the difficulties they experience in learning to read, write, spell, and solve arithmetic problems. Thus, although they may very well experience problems in remembering who Joseph E. Maddy was or what *crocoite* and *oxalis* are, these really are incidental problems. Hence, fact mnemonics primarily address incidental problems that individuals with LD experience.

There are two likely reasons why researchers have used fact mnemonics to teach students with LD, despite this type of mnemonics not addressing the central problems noted above. First, students with LD often do manifest difficulties in memory tasks, including retention of facts (e.g., Batchelor et al., 1990; Reisman & Kauffman, 1980). Therefore, research examining the usefulness of fact mnemonics to teach students with LD can provide valuable contributions to the remedial instruction area. The second likely reason is related to researcher curiosity, and to convenience.

Up until recently, only fact mnemonics have been discussed in the widely available literature. They have been shown to be effective in teaching normal students. Thus, an examination of their usefulness with special populations was inevitable. Furthermore, because research studies utilising fact mnemonics do not directly address the process deficits of students with LD, they are usually relatively easy and not very time consuming to conduct, thus making them quite attractive projects to undertake. For example, most of the studies that have been conducted have simply involved instruction of students with LD on a limited set of facts, followed by assessments of how much the students remembered of those facts.

Because process mnemonics are used to remember the processes, the rules, and the procedures that underlie problem solving, they hold a lot of promise. Conceivably, they can be used to teach various aspects of reading, writing, spelling, and mathematics. In Japan, for example, yodai mnemonics are used to teach a wide range of subjects including arithmetic, geometry, organic and inorganic chemistry, spelling and grammar, and the English language (Higbee & Kunihiro, 1985a). Although much of the evidence is anecdotal, reports about the use of yodai mnemonics for teaching in Japan largely attest to their effectiveness (Higbee, 1987). Similarly Kunihiro's pool mnemonics, another form of process mnemonics, have been shown to be effective in teaching fractions to children in California in the U.S. (Higbee & Kunihiro, 1985a).

Mathematics, being a subject that has relatively constant rules and procedures, can be considered the logical place to start investigations into the usefulness of process mnemonics to teach students with LD. It is more difficult to imagine process mnemonic applications in teaching reading and spelling. However, as just noted, process mnemonics are used in teaching spelling, grammar, and the English language to students in some schools in Japan. Furthermore, in the previously mentioned study by Ehri et al. (1984), mnemonic images were used to represent

consonant sounds. Thus, there certainly is the possibility that process mnemonics can be used to teach some reading and spelling skills to students with LD.

Admittedly, very little empirical research has been carried out on process mnemonic use with normal populations, let alone LD populations. For normal populations, there are just Machida and Carlson's (1984) study and those reported by Higbee and Kunihiro (1985a). For LD populations, there is just Manalo's (1991) case report. However, the facts remain that: (i) the effectiveness of fact mnemonics as teaching tools has been demonstrated with both normal and LD populations; (ii) there is evidence to show that process mnemonics have useful applications for teaching normal students; and (iii) an individual with LD had responded very well to arithmetic instruction incorporating process mnemonics. Thus, the question of whether other students with mathematics LD will respond similarly to process mnemonic instruction needs to be addressed. Addressing this question will contribute information necessary to answer the wider question of whether process mnemonics can effectively be used as tools for teaching individuals with LD.

Mnemonics are memory aids, and one important question that is inherent in all mnemonic research is whether or not they truly help in remembering information, both in the immediate and in the longer term. Where immediate remembering is concerned (usually straight after learning or instruction), published research on fact mnemonics almost invariably reports significant improvements or differences in performance in favour of mnemonic learning. Where longer term remembering is concerned (one week or longer), some have reported no advantage in favour of mnemonics (e.g., McDaniel, Pressley, & Dunay, 1987). However, the majority of studies have found that fact mnemonic instruction maintains better in the long term than other forms of instruction, for both experimental participants with LD and those with

normal ability (e.g., Condit et al., 1986; Peters & Levin, 1986; Rosenheck et al., 1989).

Where process mnemonics are concerned, Higbee and Kunihiro's (1985a) reported findings on pool mnemonic instruction showed significant gains on immediate post-instruction assessment. However, in the longer term, after two weeks, the gains made had decreased considerably. These findings raise the question of whether or not process mnemonic instruction is effective in helping remember information in the long term, as most studies have found with fact mnemonics. Thus one of the aspects important for the present study to examine was the maintenance of learning that results from process mnemonic instruction. This was particularly important since the experimental participants with LD in the study had previously failed to retain procedures for computational operations in the long term.

### *Questions about mathematics LD*

Most studies that have looked at remedial interventions for individuals with mathematics LD have focused on computational skills deficits. This may very well be warranted for two reasons. First, it appears that the majority of individuals with mathematics LD have deficits in computational skills (Kauffman, 1981; Mercer, 1987; Waldron & Saphire, 1992); and second, the development of computational skills is likely to be prior and essential to the development of true mathematical reasoning skills (Piaget, 1953; Sutaria, 1985).

It is assumed that students with mathematics LD have failed to learn mathematical skills despite adequate educational opportunities. Educational opportunities here basically mean the learning opportunities that are provided in schools, the primary ingredient of which are the instructions students are given in classrooms. Classroom instruction in

mathematics usually takes the form of direct instruction where students are presented with problems, shown how to solve them, and then asked to solve similar problems. Although this is the exact form of instruction that students with mathematics LD tend not to respond to, some researchers (e.g., Blankenship, 1978; Rivera & Smith, 1987, 1988) have found that formalising and systematising the usual steps involved in direct instruction transforms the procedure into an effective remedial strategy for teaching basic computational skills to these students. Hence the demonstration-imitation (DI) strategy, where the steps that lead to the correct solution of a problem are first shown to students and then they are required to correctly imitate and execute those steps on similar problems, has been shown to be an effective method of remedial instructional in a number of research studies.

In reviewing studies that employed applied behavioural analysis in the remedial instruction of mathematics, Pereira and Winton (1991) considered demonstration/modeling strategies, such as the DI strategy, to be one of the more effective methods. Among other things, the systematic approach used by this method in skills demonstration, the use of imitation, the provision of feedback, and the implementation of correction must help in keeping students focused, and in identifying and addressing specific areas of weaknesses.

There are many other factors that researchers have found relevant and useful in teaching students with mathematics LD. A number of these factors – namely concreteness, interest, and rules – are particularly pertinent in the present study both because of the process mnemonic instruction employed, and the nature of the experimental participants who received the instruction. The latter issue and its relationship to the factors named will briefly be explained here.

As mentioned in an earlier chapter, the results of Miller and Mercer's (1993b) study on concrete–semiconcrete–abstract (CSA) instruction

emphasised the value of concrete delivery when teaching students with mathematics LD. They found concrete instruction facilitated skills acquisition, which then generalised to the abstract level. Supporting evidence for the importance of concrete delivery can also be found in Willott's (1982: cited in Mastropieri et al., 1991) study where concrete-concrete pictures produced the best results in facilitating recall of basic Multiplication facts among students with LD.

Okolo's (1992a) findings suggest that the way the instructional material is packaged – so that it is relevant to the interests of participants – has an important bearing on motivation to learn. Okolo used an instructional computer game which she found had a positive effect on the continuing motivation of students with LD, who initially had low motivation to learn mathematics.

Van Houten (1993) found rule teaching to be effective (e.g., provision of “a trick to use when subtracting 7 from a teen number”), and facilitative of generalisation of skills acquired to similar, but different, problems. Van Houten stressed the organising and guiding role of rules when executing tasks. Rules may also lighten the load on memory by reducing the number of single isolated chunks of information to remember (e.g.,  $19 - 7 = 12$ ;  $18 - 7 = 11$ ; etc.) and shifting the focus to the one rule to remember and apply. If, as noted several times before, individuals with LD have deficits of some form in memory, then the provision of rules and the consequent lightening of the load on memory are very likely to help in their learning.

That individuals with mathematics LD have deficits in memory is one of the most commonly proposed explanations for why they manifest the learning problems that they do. And it is difficult to argue against assertions made, for example by Batchelor et al. (1990) and Reisman and Kauffman (1980), that memory is essential in being able to undertake mathematical tasks. Both working memory (e.g., Swanson et al., 1990)

and long-term memory (e.g., Batchelor et al., 1990; Nesher, 1986) have been implicated. And if the constructive processes of procedural knowledge acquisition described by Nesher (1986) are in any way detrimentally affected, then the computational skills deficits experienced by the majority of individuals with mathematics LD would be understandable. While the present study does not directly investigate the possible causes of mathematics LD, the results of the instructional interventions used would have some implications relating to these causes.

In attempting to explain deficits manifested by individuals with mathematics LD, some researchers subscribe to the developmental delay interpretation of this condition. For example, Kulak (1993) reviewed research relevant to this issue and concluded that the majority of children with mathematics LD are simply delayed in moving through the normal sequence of strategy acquisition, and Putnam et al. (1990) provided some empirical evidence in support of this developmental delay theory. Inherent in the developmental delay explanation is the notion that individuals with mathematics LD have the *potential* to move through the normal sequence of stages. This suggests the capability to maintain improvements in performance, that may result from instructional interventions, beyond the short term.

The question about the permanence of deficits that individuals with mathematics LD manifest is an important and contentious one. It is closely linked to the question of how truly responsive these individuals are to remedial instruction. On the one hand, there are research studies (e.g., Gerber et al., 1990; Horn et al., 1983; Schonhaut & Satz, 1983; Spreen, 1982) suggesting that problems persist into adulthood. And on the other, there is the fact that the overwhelming majority of interventional research studies in the area report favourable outcomes; for example, all 30 studies found and reviewed by Mastropieri et al., 1991, reported successful outcomes as a result of the instructional interventions used.

The present study, which looks at performance improvement beyond the immediate post-instruction phase, attempts to address this question.

Finally, a question that is relevant not just to studies in mathematics LD but also to other forms of LD, is whether these students differentially respond to treatment as a function of their IQs. This question spawns another important question: whether students with lower IQs ought to be excluded from LD remediation studies or programmes because they may not benefit from the instructions given in the same way that students with average or above average IQs could. The position held by a considerable number of researchers and professionals in the field is that students with lower IQs benefit equally from such instructions, and hence differentiating between experimental participants with LD who have average or above average IQs and those with lower IQs is not necessary where remedial intervention studies are concerned. Such a position is consistent with Siegel's (1989) assertion that individuals who show the same specific learning deficits, even if they score below average on IQ tests, are still LD and should not be excluded from remedial instruction programmes.

For practical reasons, an IQ test was not used in the present study. Instead, a group-administered pen-and-paper test providing scores that correlate well with IQ test scores was used. Both those who scored average or better, and those who scored below average, were included in the study. The aim was to examine the extent to which the students' scores in the test accounted for differences in their responses to the instructions provided. Thus, the present study also addressed the questions concerning the relevance of IQ to LD intervention research.

### *Questions about instructional strategies*

As previously noted, remedial intervention studies in mathematics LD, and in LD in general, usually report successful outcomes. The present author found 15 remedial intervention studies in LD across a six-year period, and of those 13 reported performance improvements in the experimental participants as a result of the intervention methods used. In the more specific area of mathematics LD, Mastropieri et al. (1991) found 30 such studies, all of which reported successful outcomes subsequent to the provision of various instructional interventions.

Therefore, it can be concluded that there are many effective remedial instruction methods available to address the learning needs of students with LD. However, question of whether these instructional methods produce lasting improvements in the skills performance of individuals with LD is unresolved. As noted previously, there are studies reporting that the problems of individuals with LD persist into adulthood.

One possible explanation is that most remedial intervention methods do not directly address the psychological process deficits that individuals with LD may have. The basic psychological processes that Beale and Tippett (1992) identified as possible sources of LD included imagery, attention, memory, concept formation, and perception. However, in reviewing research on remediation of psychological process deficits, Beale and Tippett concluded that there are many vital shortcomings in the majority of research studies that have been carried out so far, making it very difficult to accurately assess the viability of process remediation. Mastropieri et al. (1987) were even less favourable towards such intervention approaches. They described an example where students with visual figure-ground orientation deficiencies were provided with guided practice in finding objects embedded within larger pictures. Thus as part of the training given to these students with LD, they were required to find all the fish figures within the pictures presented to them.

Mastropieri et al. pointed out that such efforts rarely produced improvements in the reading or other academic skills performance of students with LD, who are often referred precisely because of their deficits in academic skills. According to Mastropieri et al., it was partly because of the failure of these process deficits remediation methods that direct instruction, and other methods dealing directly with the academic skills or subskills concerned (e.g., reading, spelling, arithmetic), began to gain popularity.

However, the use of methods that directly teach the required skills raises numerous other questions. For example, what aspects of these methods are different from, and render them substantially more effective than, previously unsuccessful classroom instruction? Is it the systematic presentation of skills? Or, alternatively, is it the use of computers or other forms of delivery mechanisms? There are many possibilities. Without linking methods and findings to processes involved and deficits that may be present, as Mastropieri et al. (1991) observed of the 30 studies they reviewed, it is difficult to ascertain why particular methods of instruction work. Without such understanding, it is also difficult to accurately predict their likely long-term effects on skills performance. Maintenance measures can be taken, but there are usually practical limitations to the length of time after cessation of instructions that these measures can be taken.

Basically, the line of reasoning being pursued here is that if remedial interventions do not in some way address the underlying process deficits, then those deficits will remain a problem and can eventually cause a reversion to previous states of incompetence in the skills concerned.

A considerable number of remedial intervention studies in LD do undertake maintenance measures of performance improvements, and the majority report results in favour of the instruction methods employed (e.g., Ross & Braden, 1991; Scott, 1993; Wood et al., 1993).

However, there are also reports of non-maintenance (e.g., Kershner, Cummings, Clarke, & Hadfield, 1990) and, even if the effects of an intervention method are shown to maintain for a few weeks or a few months, that is no guarantee of its maintenance in the long term. A rationale for expecting maintenance is therefore desirable: one that links features of the intervention method employed to process deficits either identified or suspected. In the present study, the use of mnemonic instruction was essentially based on the issue of skills retention of individuals with LD. Efforts were made to link features of this intervention method to known characteristics of these individuals.

Another question pertinent to instructional strategies concerns the extent to which skills taught can be generalised to 'real world' settings. In some studies, the methods of instruction used and shown to be effective contain elements that are helpful but not commonly found in environments that experimental participants would usually find themselves in. For example, a permanent model that students could consult while solving arithmetic problems was used by Blankenship (1978), Rivera and Smith (1987), and Smith and Lovitt (1975). And Rivera and Smith (1988) prompted their students to check their answers when computational errors were made – during the stages when data were being collected to assess improvements made as a result of instructions provided. In the present study, computational skills instructions were provided with the aim of achieving generalisation of skills acquired to natural settings, and assessments of the students' post-instruction performance were carried out in settings as similar as possible to the ones the students would normally encounter in the educational environment.

Two features of instructional strategies are also important for practical reasons: instructor user-friendliness, and cost-effectiveness. It is important that the programme or strategy is easily learned and used by instructors, and that it can produce similarly good results when used by instructors other than its originator(s). The provision of individual

instruction is also costly in terms of human resourcing and thus, most schools, if they do provide remedial programmes for students with LD, are usually restricted in being able to offer this form of instruction. Small group instruction is more cost-effective and is the form of remedial instruction used in most schools. The present study investigated whether process mnemonic instruction can successfully be used by other instructors, and whether it can be effective when used in a small group mode of teaching. Affirmative answers to these questions would indicate more convincingly that there are 'real world' practical applications of process mnemonic instruction for students with LD.

### *Summary*

The logic behind the present study can be summarised as follows:

- 1 In educational settings, fact mnemonics have been shown to be effective tools in helping normal, non-LD students remember various forms of factual information.
- 2 Similar results have been obtained with students who have LD. Indeed, it has repeatedly been shown that students with LD respond to fact mnemonic instruction better than to other forms of instruction.
- 3 Reports have indicated that process mnemonics are helpful in teaching processes, rules, and procedures in the mathematics domain to students of normal ability.
- 4 Therefore, it is conceivable that process mnemonics, like fact mnemonics, would also be useful and effective in teaching students who have LD.

In essence, the present research investigated whether or not process mnemonic instruction would be effective in teaching the computational operations of Addition, Subtraction, Multiplication, and Division to students who have mathematics LD. In investigating this, the following questions in particular were addressed:

- 1 Can process mnemonic instruction demonstrably improve the performance of students with LD who initially display significant deficits in the performance of basic computational operations in Addition, Subtraction, Multiplication, and Division?
- 2 Would improvements in performance resulting from process mnemonic instruction be better than improvements that could arise using placebo instruction or no instruction?
- 3 Would improvements resulting from process mnemonic instruction be significantly better compared to improvements that could be produced using the demonstration-imitation (DI) strategy? As noted previously, the DI strategy draws extensively from, but is more systematic than, regular classroom instruction; and it has been shown to be effective in teaching students with mathematics LD.
- 4 If improvements result from process mnemonic instruction, would these improvements maintain over time, and how would this maintenance compare to that resulting from other forms of instruction (particularly the DI instruction)?

Two other questions, pertaining more to the practicality of using process mnemonic instruction, were implicitly posed in this investigation:

- 1 Would the process mnemonic form of instruction originally developed by Manalo (1991) produce equally good results when

used by other instructors in teaching students with mathematics LD? This issue is important since the real usefulness of an instruction strategy hinges to a considerable extent on its ability to be used by instructors other than its originator(s).

- 2 Would the same process mnemonic instruction be effective in teaching small groups? The importance of this lies in the economic reality that it is more cost-effective to provide small group remedial instruction than it is to provide individual remedial instruction to children with LD in schools.

Another issue touched on by the present investigation concerns the relevance of IQ to LD remedial instruction research. The extent to which the response of students with LD to remedial instruction varied in relation to their approximate IQs was considered. The position taken was that, barring serious intellectual or sensory deficits, most children who exhibit specific deficits in learning could benefit from effective remedial instruction and show improvements in their performance even if they do not meet the usual criterion of having an IQ that is average or above. Thus, assuming support for this position is found, it would suggest that exclusion on the basis of IQ is not necessary where remedial instruction programmes and research are concerned.

The chapters that follow will describe the methods used in this study to address the issues specified in this chapter. The results of the two main experiments conducted will be described and discussed. A general discussion chapter will then examine more closely a number of pertinent issues that include the reasons why process mnemonics work, the implications of the study's findings about the mathematics LD condition, the wider implications of the results obtained, the limitations of the investigations carried out, and the possibilities for future research.

## Chapter 5

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# General Method

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This chapter will first describe the construction of, and collection of normative data for, a computational skills test used in the study. It will then describe the different instructional interventions used in the study: the process mnemonic method, the demonstration-imitation method, study skills instruction, and no instruction. The results of a pilot study are then reported. Finally the chapter will describe the profile of participants sought for the study, and the steps taken in the selection and instruction of students who fitted the profile.

### *The need for a computational skills test*

As noted in previous chapters, most intervention studies in mathematics LD have focused on computational skills problems. The present study was no different. It looked at the four basic computational skills of Addition, Subtraction, Multiplication, and Division.

Most studies that have sought to provide computational skills remediation have simply stated that their experimental participants were LD in mathematics or LD in general (e.g., Rivera & Smith, 1988). They established their participants' deficits in carrying out the skills dealt with through the collection of baseline data prior to the provision of instruction.

There are some problems associated with this approach. First, the precise type of deficit is not identified – whether it is computational, reasoning,

or both. Thus, remedial instruction in computational skills could be provided to participants whose primary difficulties lie in mathematical reasoning. The second problem is that, despite the baseline data collected, the participants' computational skills abilities/disabilities in comparison to their peers are not clearly established. That is, they could be half a year behind, two years behind, or even not behind at all, in comparison to their same-age peers in carrying out particular computational operations. It is also conceivable that they are not performing well in particular computational operations during baseline simply because the baseline problems provided are too difficult and their same-age peers (without LD) would similarly not perform very well. Baseline problems used in such studies are usually generated specifically for data-collection purposes by the experimenters, and are not standardised.

Thus, in this study, it was decided that some form of an assessment was required to establish that deficits in the computational skills operations to be dealt with were manifested by the participants selected for the study, and to have a reasonable gauge of the extent of their disabilities or deficits in relation to their peers. However, there was a problem, as a suitable assessment tool could not be found to meet these needs.

One instrument considered was the Progressive Achievement Test in Mathematics (PAT Maths Test, for short), which is widely used in schools in New Zealand. This test, however, does not allow a sufficiently detailed analysis of the computational skills which the present study was specifically concerned with. It contains items such as:

147. Which of these operations will never change the value of a number?
- I      Multiplying it by 1
  - II     Dividing it by 1
  - III    Multiplying it by 0

- (A) I only
- (B) II only
- (C) III only
- (D) I and II only
- (E) I and III only

Most of the items do require the utilisation of Addition, Subtraction, Multiplication, and Division skills, and thus could be described as assessing the sorts of abilities in computation which were of interest in the present study. However, these assessments of computational skills are indirect and/or incidental.

The PAT Maths Test also contains many items similar to the following:

178. If  $576 \div q = q$ , what is the value of  $q$ ?

- (A) 24
- (B) 26
- (C) 34
- (D) 36
- (E) None of these

Even with good computational skills, these sorts of items would be difficult to answer correctly without sound mathematical reasoning. The PAT Maths is therefore not really a computational skills test, but more a test of *general* mathematical skills with a bias towards mathematical reasoning.

A particular strength of the PAT Maths Test is that there are New Zealand norms for particular age groups of children. As such, it had utility in the present study as an indicator of overall mathematics LD, and was used as an additional tool for selecting children whose mathematics

achievements were lower than the average for children in the same form as they were.

In terms of tests that specifically assess computational skills, the only ones that could be found were subtests within France's Profile of Mathematical Skills (France, 1979). This test has nine subtests which include Addition, Subtraction, Multiplication, and Division. It was the test used to establish computational skills disability in Manalo's (1991) case report.

There were, however, some problems in using the Profile of Mathematical Skills in the present study. First, its age-based norms were gathered from British children. The performance of New Zealand children could differ in some important ways, but this cannot be ascertained as there are no New Zealand norms. Secondly, and probably more importantly, the items in the subtests do not appear to represent or gauge the computational skills that New Zealand children are meant to master through instruction based on the national syllabus.

None of the tests described was considered adequate for the purposes of this study. Thus, it was decided to design an appropriate computational skills test.

### *The rationale for the Computational Skills Test*

For the purposes of the present study, a computational skills test was required that would

- (i) effectively assess ability to carry out the correct procedures in Addition, Subtraction, Multiplication, and Division;
- (ii) have items based on what children should have learned in computation through instruction following the national syllabus;

- (iii) have norms that would show average scores (and standard deviations of those scores) of children at different levels of schooling.

With respect to the first requirement, it was important to find out with the test whether or not children knew *how* to solve computational skills problems – that is, whether or not they could correctly execute the appropriate procedures. A distinction was made between this ability and knowing computational facts (e.g.,  $8 \times 1 = 8$ ;  $8 \times 2 = 16$ ;  $8 \times 3 = 24$ ; etc.) because the study was investigating the effectiveness of a strategy (process mnemonics) that teaches the procedures, or the “how to” aspects – not the facts. To properly and accurately assess learning resulting from the use of the strategy, it was necessary to minimise ‘contamination’ of the children’s performance from possible limitations in their knowledge of computational facts.

There are three ways of minimising this possible ‘contamination’ of performance. One is to exclude students who have not yet mastered the computational facts, but this might have left too few students with mathematics LD in the study to be feasible. A second method is to provide tables from which they can look up the facts. And a third method is to avoid the inclusion of more difficult facts in the assessment measure used. In some of the earlier studies (e.g., Manalo, 1991; Rivera & Smith, 1988) the second method was employed. In this study, the third method was chosen for two reasons. First, the condition under which the students would have to execute the procedural steps to solve the problems would then be closer to natural settings (i.e., settings that students with LD, as well as students of normal ability, would normally find themselves in). They would not have tables to consult, and in the regular classroom environment students usually do not have tables to consult during tests and other forms of assessment. The second reason is that it was deemed possible to assess the children’s skills in computational procedures without the use of difficult computational facts.

With respect to the second requirement mentioned above, it was considered important for the test to have not just an 'intuitive' mix of items, but for it to have items that assess whether or not procedures covered in classroom instruction following the national syllabus have been learned. This way, it can more accurately be claimed that the students selected for the study have failed to master relevant computational skills procedures despite regular classroom instruction.

With respect to the third requirement also mentioned above, it was important for the test to be able to show how a student was performing relative to peers in the same form. Of particular interest in this study were students performing significantly below average. The criterion of obtaining a score that is one standard deviation below average, or a score that is at a level two years below their age or school-group affiliation (e.g., the standard or form<sup>1</sup> that the student is in), is usually employed in identifying children who have significant deficits or who are LD in particular areas of learning.

The school syllabus describes objectives for what students should be able to do in Addition, Subtraction, Multiplication, and Division (as well as other topics like algebra and geometry) by particular years at school, up to Form 4. In Addition and Subtraction, by Form 3, students should be able to do the following:

- show that the order of numbers is important in Subtraction
- know the effect of adding and subtracting zero
- subtract by decomposition
- solve Addition and Subtraction problems to 999 999
- add and subtract four-digit numbers
- add and subtract to 9999.99

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<sup>1</sup> The ages of children attending standards 1–4 are approximately 7 to 10 years, at the beginning of the year. Children attending forms 1–7 are approximately 11 to 17 years of age, also at the beginning of the year. Standards 1–4 are equivalent to grades 2–5 in the U.S. system, and forms 1–7 are equivalent to grades 6–12.

- add and subtract two-place decimals (e.g.,  $3 - 0.4$ ;  $125 - 3.67$ ;  $23.6 - 8.74$ )
- add and subtract three-place decimals (e.g.,  $14.2 - 1.256$ ;  $2 - 0.984$ ).

These skills are included as objectives in preceding years, up to and including Form 2, and therefore should be mastered by the time students get to Form 3.

In Multiplication and Division, by Form 3, students are meant to be able to do the following:

- show the effect of multiplying by 0 and 1
- solve problems involving multiplying a two-digit factor by a single-digit factor, and a three-digit factor by a two-digit factor
- solve Division problems, with and without remainders, involving a single-digit divisor and a product less than 1000
- solve Multiplication problems of the type  $27.64 \times 5 =$
- solve Division problems of the type  $7.32 \div 6 =$
- multiply two-place decimals by single-digit whole numbers
- multiply two-digit, one-place decimals (e.g.,  $0.2 \times 0.6$ ;  $1.4 \times 2.8$ )
- multiply with a two-digit multiplier (e.g.,  $2.34 \times 5.6$ )
- divide two-place decimals by single-digit whole numbers
- divide whole numbers by one-place decimals
- divide three-significant-figure decimals by one-significant-figure decimals.

Again, these skills are included as objectives in preceding years, up to and including Form 2, and therefore should be well-learnt by the time students get to Form 3.

### *Construction of the Computational Skills Test*

A decision was made to have ten numerical problems (i.e., no words or stories involved) for each of the four computational skills in the test being devised. Words and stories in computational problems were deemed to require additional skills outside of those of concern in the present study, and thus they were not included. The 40 numerical test items were intended to assess the range and multifaceted nature of procedural skills in computation that children, by Form 3, ought to be capable of.

Form 3 was chosen because that is the form by which children would have already been given all the instruction in the basic skills for executing computational procedures. Prior to Form 3, various skills are still being introduced and taught. For example, in Form 2, multiplication using a two-digit multiplier, one of which could have two-place decimals (e.g.,  $3.17 \times 2.5$ ), is taught. This is a progression from what is taught in Form 1 which is to be able to multiply two-digit, one-place decimals (e.g.,  $2.6 \times 0.4$ ). Essentially, by Form 2 all procedural skills in computation have been covered. Therefore, by the time they reach Form 3, children ought to have learned most of these skills adequately. There are of course more complicated procedures beyond those covered in instruction to Form 2 but these are not explicitly taught. It seems that children are expected to utilise and/or extend their knowledge in solving simpler problems to solve more complicated ones, or to use a calculator in solving more laborious computational tasks.

After drafting the contents of the computational skills test, consultation was carried out with a number of University staff members and teachers in schools. They were asked their opinions on the appropriateness of the items included, the range of the skills assessed, the layout/presentation of the test, and – of the school teachers – whether or not the test items would be appropriate in terms of what they observed as the actual

capabilities of children in the different forms. Modifications were made on the basis of feedback received.

A copy of the final version of the computational skills test, the one used in the study, is included in Appendix A. In the Addition section, the first item requires one instance of carrying over, but the two numbers to be added are already properly lined up. From item 2 onwards, lining up is required, which tests the students' knowledge of place value. Items 2 to 7 require the lining up and addition of two numbers. These numbers range from those with one-place decimals (items 2 and 3) to those with three-place decimals (items 6 and 7), the addition of which type of numbers is taught in Form 2. Items 8 and 9 require the addition of three numbers (including ones with three-place decimals), and item 10 requires the addition of four numbers (one of which has four decimal places). Not all the items require carrying over: items 2, 4, 6, and 8 do not.

In the Subtraction section, the first two items require 'borrowing' (also known as "number decomposition") but the numbers in them are presented already lined up. Items 3 to 10 require lining up, which, as noted earlier, tests the students' knowledge of place value. In the case of Subtraction, items 3 to 10 also test their knowledge that number order is important (i.e., which number goes at the top and which at the bottom – in other words, which one is being subtracted from which). Item 3 does not require borrowing, but the rest from item 4 onward do. (Item 3 was placed in the test purely to gauge the students' ability to line numbers up properly). Items 3 and 4 include numbers with one decimal place, items 5 and 6 have numbers with two decimal places (covered in Form 1), and items 7 to 9 have numbers with three decimal places (covered in Form 2). One of the numbers in item 10 has four decimal places.

In both the Addition and Subtraction sections, some of the numbers have zero in them to enable the assessment of whether or not the students know the effect of adding and subtracting zero. In both sections, the

inclusion in the last items of numbers with four decimal places (not explicitly covered in school instruction) serves two purposes. The first is to examine whether students can generalise what they have learned about addition and subtraction of numbers with three-place decimals to numbers with four-place decimals. The second purpose is to include a slightly more difficult item in both sections to counteract possible ceiling effects.

In the Multiplication section, the items ranged in difficulty from two-digit numbers multiplied by one-digit numbers (both without decimals), to four-digit, two-decimal place numbers multiplied by two-digit, one- or two-decimal place numbers. These cover procedural skills in computation introduced and taught up to the Form 2 level. All the items have computational problems that require carrying over, except for items 1 and 4. The actual digits to be multiplied were kept as simple as possible. The aim was to have a test where if one knew how to carry out the computational procedures required then one should be able to compute the right answers in most of the items, even if one's basic facts proficiency was relatively poor. The use of higher digits was kept to a minimum, so that 9 is not used at all as a multiplier, and 7 and 8 are each used only once. In seven of the items, the other multiplier digit in any single instance of multiplication that needs to be carried out is always a 1, 2, or 3. Two of the other items (6 and 8) require a response to  $4 \times 5$ , and the last item requires a response to  $7 \times 5$ . Thus, the basic facts component of the Multiplication section can be considered relatively simple.

In the Division section, the first item is a simple check of whether the student knows that the digits of the dividend (the number being divided) are dealt with from left to right, one at a time, in long division. The second item ( $205 \div 5$ ) checks whether or not it is known that if a digit in the dividend is too small to be divided by the divisor, then that digit is combined with the next digit of the dividend to the right. The third item ( $429 \div 3$ ) checks on knowing that the remainder from a digit of the

dividend being divided by the divisor is combined with the next digit of the dividend to the right. Items 4 and 5 involve dividing numbers with two decimal places by one-digit whole numbers (covered in Standard 4). Items 6 and 7 involve dividing whole numbers by numbers with one decimal place (covered in Form 1). They check whether it is known that the decimal point on the divisor needs to be shifted to the right, and that a zero needs to be added to the end of the dividend – hence making both divisor and dividend ten times greater prior to commencing the actual division. Items 8 to 10 involve dividing three-significant-figure decimals by one-significant-figure decimals (covered in Form 2). They check whether it is known that the decimal points in both the divisor and the dividend need to be shifted the same number of places to the right to ensure that the number being divided is always a whole number.

As in the Multiplication section, an effort was made to keep the basic facts required in the Division section as simple as possible. All but one of the divisors (in some instances, after transformation) are in the range of 2 to 6. Item 10 has a divisor of 11 (after transformation), but the numbers it divides are 23, 14, and 33 – thus it is relatively simple if the correct procedural steps are known.

### *Collection of normative data for the Computational Skills Test*

The norms for the Computational Skills Test were obtained by administering the test to 411 students in three schools in the Palmerston North area. Of these students, 118 were in Form 1, 126 in Form 2, and 167 in Form 3. The gender distributions were: in Form 1, 53 females and 65 males; in Form 2, 63 females and 65 males; and in Form 3, 90 females and 77 males.

The appropriate procedures were taken in seeking permission to test the children in the schools. The teachers concerned were informed that

children with a range of abilities were needed to sit the test. Thus in each school, the test was administered to either mixed ability classes or equivalent numbers of high-, average-, and low-ability classes.

Teachers administering the test were given brief instructions in written form. They were told that students should work on their own in doing the problem sheets, and that the students were not allowed to use calculators, Multiplication tables, or Division tables. The teachers were also told that if the students were not familiar with the division format of  $y\overline{x}$ , then they should tell them that it is the same as  $x \div y$  or  $\frac{x}{y}$ , whichever one the children were familiar with. The teachers were told that no strict time limit in working on the problem sheets needed to be applied, but it was estimated that students should be able to comfortably finish all four sections within the usual single class period of 45 to 50 minutes.

The computational skills test was scored out of 40 (10 in each of the basic computational operations). Table 1 shows the overall means, standard deviations ( $\sigma_{n-1}$ ), and ranges of the scores of the students in Forms 1, 2, and 3. Table 2 shows the means and standard deviations ( $\sigma_{n-1}$ ) of the scores in the Addition, Subtraction, Multiplication, and Division sections of the test.

**Table 1. Form 1, 2, and 3 Means, Standard Deviations and Ranges of Scores on the Computational Skills Test**

Form	N	Mean	Standard deviation	Range
1	118	17.87	8.49	3 – 40
2	126	20.83	8.62	3 – 39
3	167	25.99	9.28	3 – 40

**Table 2. Means and Standard Deviations of Scores in Addition, Subtraction, Multiplication, and Division for Form 1, 2, and 3 Students**

Form	Means				Standard Deviations			
	Add	Sub	Mul	Div	Add	Sub	Mul	Div
1	5.24	3.41	4.93	4.30	2.95	2.66	2.38	2.19
2	6.10	4.24	5.67	4.81	3.25	2.96	2.63	2.04
3	7.49	6.05	6.44	5.95	2.66	3.41	2.77	2.45

The students' overall performance in the computational skills test was poorer than expected. For example, children in Form 3 were expected to score an average of around 8 or 9 in each of the four sections of the test. Instead their averages were around 6 or 7. This finding suggests that there are considerable numbers of children in intermediate and early high school levels who have not mastered computational skills procedures that are part of the school syllabus.

The children's poorer performance in the test, however, did not affect the usefulness of the test for assessment purposes in the present study. The test still clearly differentiates between the performance capabilities of children in the different forms. Form 3 children's overall performance was better than those of Form 2 children, who in turn performed better than their Form 1 counterparts (see Table 1). This pattern of performance is also evident in each of the sections of the test (see Table 2).

A one-way analysis of variance (ANOVA) on the children's total scores, grouped according to the Forms they were in and with correction for different group sizes, was undertaken. The group effect was statistically significant,  $F(2, 418) = 71.93, p < 0.0001$ . The least squares means (LS means) of the children's scores in each of the Forms were also all significantly different from each other at the  $p = 0.0001$  level.

It was hoped that the Form 3 children's overall mean score minus its standard deviation would be approximately equal to the Form 1 children's overall mean score. This way, a deficit of one standard deviation from the mean would approximate a deficit of two school years. From the performance data obtained, the Form 3 mean score minus one standard deviation equals 16.71, while the Form 1 mean score is 17.87. They are not equal, but reasonably close. For the purposes of the present study, the more conservative and stricter of these was adhered to as a cut-off point score, so that Form 3 children considered to have computational skills LD and selected for the study had to score 16 or lower in the test.

### *The Basic Facts Test*

It was noted earlier that the computational skills test devised was intended to assess students' ability to carry out basic computational procedures. Difficult computational facts were avoided so that students' scores would not be confounded by possible limitations in their basic facts proficiency. As an added measure to ensure that children selected for the study did not score poorly in the computational skills test largely because of poor basic facts knowledge, a basic facts test was also devised.

A copy of the Basic Facts Test is included in Appendix A. It consists of forty items, ten in each of Addition, Subtraction, Multiplication, and Division. The items assess ability to solve basic facts components of items in the Computational Skills Test. For example, item 3 in Addition is  $4 + 3$ . This corresponds to a component of item 2 on the computational skills test,  $67.4 + 1.3$ , and checks whether or not the answer to that component is known. In the Multiplication section, items 6 and 8 ask  $5 \times 4$  and  $5 \times 3$  respectively. These assess whether or not components of item 8 ( $2.43 \times 5.1$ ) in the Computational Skills Test are known.

The Basic Facts Test was used to demonstrate that poor scores in the Computational Skills Test could not be attributed to inadequate mastery of basic mathematical facts. An examination of the items in the Basic Facts Test will show that there is a reasonably comprehensive coverage of the basic facts components of the Computational Skills Test, especially the more demanding ones.

Norms were not required for the Basic Facts Test since it was used in the study to establish a required standard of performance, and comparison with peers or other groups was irrelevant. It was acknowledged that among other things careless mistakes would be made, and that to expect perfect scores would be unreasonable. A score of 75% (or 30/40) correct on the Basic Facts Test was therefore established as the minimum score for students to be included in the study. This way, a fairly confident claim could be made that the students selected could solve at least *most* of the basic facts components of the Computational Skills Test, indicating that their main difficulties were in computational procedures.

### *The process mnemonic and DI interventions*

The two main instructional components of the present study were the process mnemonic (PM) and the demonstration-imitation (DI) interventions. The PM instruction used in this study was based on, and was a further development of, Manalo's (1991) warrior mnemonics. These mnemonics were in turn inspired by yodai mnemonics, Kunihira's pool mnemonics, and themes from arcade computer games. DI instruction, which essentially uses components of normal classroom instruction, was developed as a remedial instruction method by researchers such as Smith and Lovitt (1975), Blankenship (1978), and Rivera and Smith (1987, 1988). The main comparison made in the present study was between these two forms of instruction in terms of

their effectiveness in teaching computational skills to children with mathematics LD.

The procedures for the PM instruction and the DI instruction were written down (see Appendices B & C). The written instructions were used as guidelines to ensure consistency in the instructions provided. The actual instructions delivered in the classrooms would have differed in some minor ways since instructors did not directly read from the written procedures, but only referred to them. However, the content of the written instructions were adhered to as much as possible to ensure that phrasing, sequencing of steps, and what was included remained consistent. The exercises used during the instruction sessions were also written down so that the same ones could be used throughout the study (copies of these are also included in Appendix D).

Consistency in the delivery of instructions and in the exercises used was considered necessary in the present study. Essentially, this meant making sure that the intended and claimed instructions were delivered, and that no major deviations occurred in different sessions or with different groups. These were also important in terms of ensuring that procedures carried out were replicable.

### *The process mnemonic instruction*

As noted in earlier chapters, process mnemonics often incorporate salient stories and characterisations that are meant to facilitate better memorisation of particular sets of rules or procedures. In the present study, the rules and procedures concerned were those used in carrying out numerical Addition, Subtraction, Multiplication, and Division. PM instruction was implemented by presenting numbers as characters (warriors), and operations involved as situational stories. The characters and stories used were similar to those used in Manalo (1991) for teaching

Subtraction, Multiplication, and Division. The PM instruction employed in this study also utilised the basic DI components of demonstrating the steps that lead to the correct solution of a mathematical problem, and then requiring students to imitate the teacher-demonstrated steps in solving problems prior to being allowed to work on other problems independently.

### *Addition*

In Addition, students were told to imagine the numbers they were adding as sets of warriors. These warriors had a number on them which indicated their size, weight, and strength. Thus, for example, a warrior with the number 7 would be bigger, heavier, and stronger than one with the number 4. All these were illustrated on the board using simple drawings. Simple drawings, to illustrate concepts the students were being asked to imagine, were an integral part of the PM instruction provided (see written instructions in Appendix B).

For each set of warriors, the decimal point (referred to simply as “the dot”) indicated which warriors were ranked (“the officers”) and which were not. The digits to the left of the dot were described as ranked warriors, while those to the right were unranked warriors.

The students were told to imagine Addition as the warriors getting onto a boat. The Addition sign (+) was described as the mast of the boat, and the line beneath it was the boat itself where the warriors all hop on. The students were told that when the warriors boarded the boat, they had to line up properly so that ranked warriors were lined up with warriors from the other set(s) who were also ranked. Similarly, those not ranked had to line up with others also not ranked. The students were shown how to use the dots on the sets of warriors to correctly do this. A couple

of examples were shown to them, and then they were asked to complete exercises that required them to properly line numbers up for addition.

As part of the usual procedure, while the students were completing the exercises given, the instructor circulated among them to check that they were carrying out the tasks correctly. The instructor provided individual assistance when necessary, and encouraged the students to try again when mistakes were made. Each of the students had to correctly solve at least one of the problems in each set of exercises given before the instructor proceeded onto the next segment of instruction.

The next segment of instruction described to the students how numbers without decimal points could be imagined as ranked warriors, and the dot – although not shown on them – was located on their right. The students were also informed that zeros could be placed to the right of the dot, and that these zeros could be imagined as very weak and useless warriors who were not ranked. The students were then asked to complete a set of exercises in which they had to place the dot on numbers without decimal places, and add some zeros if they wanted.

After these exercises, the students were shown how to line up numbers that contained decimal points (e.g., 38.4) with numbers without decimal points (e.g., 4). The students were asked to imagine them as warriors, and were shown the correct and the erroneous ways of lining the numbers up. After this demonstration, they were asked to complete another set of exercises in which they had to line up numbers similar to those they had just been shown.

The students were then told that once the numbers had been lined up properly, they were ready to be added up. They were told that the adding up process could be imagined as trying to find out how heavy the lined up warriors were, and that the warriors (or digits) furthest to the right were the ones they had to start on. With the use of an example, this process

was illustrated to them on the board. The students were shown how the Addition answer – the unit weight of all the warriors on the boat – could be derived. Their attention was also drawn to placing the dot on the final answer: this had to line up with the other dots on the sets of warriors.

Included in these demonstrations of the Addition process were instructions about how to carry over. The importance of recording these carry overs at the appropriate places was stressed, and the examples used in the demonstrations incorporated the need to carry over. The students were also reminded of the consequences of adding zero to numbers. Following this last segment, the students were again asked to work on a set of problems; this time they were required not only to line up the numbers but also to add them up.

### *Subtraction*

In Subtraction, the students were similarly told to imagine the numbers they were dealing with as warriors. The first two segments of instruction (see Appendix B) dealt with the necessity of properly lining numbers up before subtraction is carried out. Thus, like in Addition, the students were told which of the warriors were ranked and which were not, depending on which side of the dot they were located. They were told about numbers without decimal points and how to line them up with numbers that have decimal points – although this time, they were told that it was quite important to add the “zero” warriors when some warriors had no one lined up against them. For example, lining up 275 – 3.16 would result in

$$\begin{array}{r} 275. \\ - \quad 3.16 \\ \hline \end{array}$$

where the 1 and the 6 of the bottom number would have no digits (warriors) lined up against them from the top number. Thus the students were told that, in this case, it would be necessary to include two zero warriors – to the right of the dot of the top number, opposite the 1 and the 6 of the bottom number. The Subtraction problem would then look like

$$\begin{array}{r} 275.00 \\ - \quad 3.16 \\ \hline \end{array}$$

Exercises in lining numbers up properly, and putting in the zeros where necessary, followed the first two segments in Subtraction.

The students were asked to imagine the two numbers as two sets of warriors fighting each other. Referring to numbers that had been properly lined up, the top group were described as the “attackers” while the bottom group were described as the “defenders.” For example,  $75.6 - 43$  would be lined up as

$$\begin{array}{r} 75.6 \\ - \quad 43.0 \\ \hline \end{array}$$

Thus, 75.6 would be the attackers, and 43.0 the defenders. The students were told that the defenders were easily recognisable because they had their knives drawn: the Subtraction sign, –.

The students were asked to imagine that the attackers were trying to get past the defenders so they could pass through the gate behind the defenders (the line below the bottom set of numbers). However, as they battled their way past the defenders, they became weaker. Thus by the time each of them had gone through their opposite defender, and past the gate, their strength (the number on them) would have gone down by the

same amount as the strength of the defender they had to go through. In the example given above, attacker 6 would retain his strength after easily getting past defender 0, but attacker 5 would only have a strength of 2 ( $5 - 3$ ) after going past defender 3, and attacker 7 would be reduced to 3 ( $7 - 4$ ) by the time he got past defender 4. Thus to find out an attacker's strength after going past a defender and through the gate, required the subtraction from the attacker's strength of the opposite defender's strength. The students were instructed to always start with the rightmost pair of warriors, and it was stressed to them that in Subtraction "we take away the bottom number from the top number, not the other way round." After this segment, the students were asked to work on a set of exercises requiring them to properly line numbers up and then carry out subtraction as they had been shown.

In the next segment of instruction in Subtraction, the students were informed that when the attacking warrior's strength was lower than the opposite defending warrior, then it was necessary to first increase that attacking warrior's strength by 10 – otherwise he would not be able to get past the stronger defending warrior. However, there was a consequence to this: the next defending warrior's strength had to be increased by 1. For example, in

$$\begin{array}{r} 8.0 \\ - \underline{3.2} \end{array}$$

attacker 0 would not be able to get past defender 2. Thus, that warrior's strength would have to be increased by 10, making his new strength level 10 ( $0 + 10 = 10$ ). The consequence would be that defender 3 (the next defending warrior) would have 1 added to his strength – thus becoming 4 ( $3 + 1 = 4$ ). The students were told to make sure they marked these changes, and they were shown how an answer of 4.8 could be derived from the example given. The placement of the dot, in line with the other

dots, was also stressed to them. They were asked to work on the next set of problems on their exercise sheet, in which “borrowing” was necessary.

The next segment of instruction involved demonstrating to the students the application of the same principles on a more difficult problem,  $40.2 - 0.3587$ . The importance of working systematically through the problem, adding 10 strength points to attacking warriors and 1 strength point to the next defending warriors when necessary, was stressed. Any questions that the students might have had were dealt with, and then they were asked to work on the last set of more difficult problems on their Subtraction exercise sheet.

### *Multiplication*

In Multiplication, the students were asked to imagine the numbers being multiplied as warriors at a meeting to “x-change” battle techniques. The bottom set of warriors, next to the Multiplication sign (x), were described as the “x-perts.” These warriors were at the meeting to teach the top group of warriors their special battle techniques. To do so, each member of the bottom group of x-perts needed to meet with each member of the top group of warriors, and the products of their meetings (“what they got out of them”) needed to be obtained and recorded. For example, in

$$\begin{array}{r} 34 \\ \times \underline{2} \end{array}$$

the 2 would be the x-pert warrior needing to meet with, firstly the warrior 4, and then the warrior 3, from the top group. To obtain the products of their meetings, the numbers on the warriors had to be multiplied. Thus,

in the example,  $2 \times 4 = 8$  and then  $2 \times 3 = 6$  had to be figured out, providing an answer of 68 as the overall product of the warriors' meetings.

An example ( $27 \times 3$ ), which required carrying over, was also given. Then the students were asked to work on the first set of problems on their Multiplication exercise sheet.

In the next segment of instruction, the students were told that quite often more than one x-pert warrior is involved – there could be two or more. In those situations, each of the x-pert warriors in turn (starting with the one furthest to the right) would have to meet with each of the warriors in the top group. For example, in

$$\begin{array}{r} 314 \\ \times \quad 25 \\ \hline \end{array}$$

the x-pert warrior 5 would need to meet with each of warriors 4, 1, and 3 from the top group. Then x-pert warrior 2 would have to do the same. However, before moving onto x-pert warrior 2, the students were told that members of the x-pert group increased in expertise the farther to the left they were positioned in relation to the others. Hence, x-pert warrior 2 would be more of an expert than x-pert warrior 5. To mark this shift in expertise, the students were told they had to first place a zero: thus, in the example, before multiplying by the 2, a zero would have to be placed like so:

$$\begin{array}{r} 314 \\ \times \quad 25 \\ \hline 1570 \\ 0 \end{array}$$

The students were then told that they had to add the products of the warriors' meetings (in the example above, 1570 and 6280), to obtain the overall product (the answer to the Multiplication problem). After this segment, the students were asked to work on the next set of problems on their exercise sheet.

The next segment of instruction dealt with multiplication when decimals are involved. The students were asked to imagine the decimal point ("the dot") as separating the ranked from the unranked warriors. However, the students were told not to worry about the dot and the ranks when they start solving Multiplication problems – they could worry about those later. Thus the students were informed that initially they simply had to follow the same procedures as described in the previous segment, and those were demonstrated again to them using a Multiplication problem ( $30.42 \times 2.3$ ).

Once the products of the x-pert warriors' meetings with the top group of warriors had been obtained (9126 and 60840) and added, the number 69966 was arrived at. The students were informed that the next step would be to figure out where to place the dot in this number. This they could figure out by counting the number of unranked warriors in both the x-pert group (in the example, only one) and the top group (in the example, two). These had to be added to get the total number of unranked warriors from both groups (in the example, three). The students were then told they had to count that many number of digits from the right of the Multiplication answer they previously obtained. Thus in 69966, "one" would be the first 6 on the right, "two" would be the second 6 from the right, and "three" would be the first 9 from the right. Thus the dot would have to be placed after this 9, that is, between the 9's. The final answer would therefore be 69.966.

The next set of problems the students were asked to work on involved placing the dot correctly on Multiplication problems that had already been

solved, but without the decimal points included in the final answers. After they had done these, the students were given a quick revision of the Multiplication process taught – from imagining the two numbers as two groups of warriors x-changing battle techniques, to placing the dot on the final answer. They were told that being careful was important, and reminded about the results of multiplying by 1 and by 0. The students were then asked to work on the final section of their Multiplication problem sheet.

### *Division*

The students were asked to imagine numbers involved in Division as consisting of a warrior and a rack of armour pieces. They were given the example of

$$2 \overline{)462}$$

where 2 was described as the warrior, and the 4, 6, and 2 as the pieces of armour. The students were told that the warrior, whose armour size was the number on him (2, in this case), was standing next to the rack trying on the armour pieces one by one, starting with the piece closest to him (4, in this case). To find out how well the armour pieces fitted the warrior, their sizes were divided in turn by the size of the warrior. Thus, in the example, there were the following divisions to carry out:  $4 \div 2$ ,  $6 \div 2$ , and  $2 \div 2$ , yielding 2, 3, and 1 respectively. The students were told that they had to write these answers (how well each armour piece fitted the warrior) directly above the armour pieces concerned. They were shown how each digit of 231 (the answer to the problem) was obtained and written above “the rack” directly over 462.

The students were told that sometimes some of the armour pieces hanging on the rack would be too small (i.e., some of the digits of the

dividend would be too small by themselves to be divided by the divisor). For these armour pieces to be useful to the warrior, they would need to be combined with other armour pieces. They were given the example of

$$5 \overline{)305}$$

where the first armour piece, the 3, was too small and had to be combined with the 0 to make an armour piece of size 30 before it could be divided by the warrior's size of 5. However, before combining an armour piece that was too small with the next armour piece, the students were told that they had to first place a zero above the armour piece deemed to be too small. Once this had been done, and the small piece had been combined with the next armour piece, then division could proceed. Another example was demonstrated to the students ( $416 \div 4$ , where 1 was too small and had to be combined with the 6) before they were asked to work on the next set of exercises on their Division problem sheet.

The next segment of instruction dealt with Division remainders. This was described to the students as a situation where an armour piece the warrior tried on did not quite fit "an exact number of times." They were shown an example of

$$3 \overline{)456}$$

where the first armour piece tried out (the 4) fitted the warrior once, but there was a remainder of 1 ( $4 \div 3 = 1$ , remainder 1). The students were told to put the 1 above the 4, and the remainder, 1, beside the next armour piece (the 5). Combining the remainder 1 with the 5 would give 15, and this would be the size of the next armour piece the warrior would try on. The students were shown how an answer of 152 could be derived from the problem.

Next they were shown another example ( $652 \div 2$ ), in which the armour piece of size 5 did not fit the warrior of size 2 an exact number of times: it fitted twice (2), but with a remainder of 1. The students were shown how the 1 would be combined with the 2 to make 12, which would then become the size of the next armour piece for the warrior to try out. After demonstrating to them the derivation of the answer of 326, the students were asked to work on the next set of Division problems on their sheet.

The next segment of instruction dealt with decimals. The students were told that a decimal point could be a part of either or both the number being divided and/or the number dividing it. They were first shown how to deal with decimals in the dividend. An example was given of

$$4 \overline{)4.92}$$

The armour pieces were described to the students as being separated by a dot: those to the left of the dot were referred to as the expensive pieces (in this case, only the 4), and those to the right as the bargain pieces (the 9 and the 2). The students were told that if the dot was only on the armour pieces and not on the warrior, then they had to also put a dot in the answer derived in line with this dot. Thus a dot had to be placed in 123 between the 1 and the 2, giving a final answer of 1.23. After this, the students were asked to work on the next set of exercises on their problem sheet.

The next segment of instruction dealt with having a decimal point in the divisor. The students were again told that a dot in the warriors separated those ranked from those not ranked. They were also told that only ranked warriors were allowed to purchase armour, and so if the warrior given was not ranked he either had to get ranking or pretend he was ranked. The students were given an example of

$$0.3 \overline{)3.99}$$

In this, the 0.3 was an unranked warrior. The students were told that they had to shift the dot in this warrior to the right (making it a 3) so that he could pretend to be a ranked warrior.

There was, however, a consequence to this. The shopkeeper (not shown) would know what was happening – that the warrior was only pretending to be ranked – and so the shopkeeper would also shift the dot in the rack of armour one space along to the right. This shift would result in one more piece of expensive armour and one less piece of bargain armour. Thus in the example given, the first 9 on the left which used to be bargain-priced would end up becoming priced expensively. The students were told they had to re-write the problem given. The example re-written would look like

$$3 \overline{)39.9}$$

The students were told that even though this looked different from the original  $3.99 \div 0.3$ , it would give the same answer and would be a lot easier to solve.

The students were also informed that sometimes they might have to add a zero to the number being divided. They were given an example of

$$0.4 \overline{)56}$$

The dot in the warrior would have to be shifted to the right. But no dot existed in the armour pieces to be shifted – they were all expensive pieces already. The students were told that in such a case, the shopkeeper would instead include at the end of the rack a poor quality piece of armour of

size 0. Thus the 56 would become 560 and the divisor problem would look like

$$4 \overline{)560}$$

After this demonstration, the students were asked to work on the next section of their problem sheet which required them to re-write the Division problems after shifting the decimal points and, in some cases, adding zeros to the dividend as well. However, the students were not required to actually solve the Division problems given.

In the next instruction segment, the students were given a demonstration of how to solve  $46.31 \div 1.1$ , asking them to imagine the numbers and the procedures as previously described. Any questions that the students had were dealt with, and then they were asked to work on the final set of problems on their Division exercise sheet.

### *The demonstration-imitation instruction*

The DI instruction, as noted a number of times before, involves demonstrating to students the steps to follow in solving a mathematical problem. After this, students are asked to imitate those same steps in solving similar problems. It was pointed out earlier that this method simply constitutes a systematisation of mathematics instruction normally carried out in school classrooms. Researchers, however, have found this to be an effective way of teaching students with mathematics LD.

There are a number of variations to the DI instruction such as providing a permanent model of a solved problem for students to consult while working on other problems, and incorporating key guide words. In the present study, however, the basic form of the DI instruction was used, for two main reasons. First, the intention was to employ a comparison

method of instruction that was as similar to normal classroom instruction as possible, but one that has been shown to be effective in teaching students with LD. This way PM instruction would essentially be compared with a systematised and effective, but familiar, form of instruction in mathematics. The second reason was so that the DI instruction and the PM instruction would differ on a single feature: the inclusion of the characterisations and stories in the PM instruction. The PM instruction provided utilised the principles of the DI instruction, plus those characterisations and stories which make up the process mnemonic component. Thus, if the PM instruction proved more effective in teaching students with mathematics LD, the difference could be attributed to this one critical feature.

The exercise sheets used in the DI instruction were exactly the same as those used in the PM instruction. The demonstrated numerical examples, as well as the sequence in which materials and procedural steps were presented, were also exactly the same. In fact, everything was exactly the same except that the process mnemonic components of imagining the numbers as warriors and the computational procedures as situational stories were omitted. This being the case, the DI instructions in Addition, Subtraction, Multiplication, and Division will be described only briefly here.

### *Addition*

The first instruction segment in Addition dealt with properly lining numbers up using their decimal points. After this, the students were asked to work on the first set of problems on their Addition exercise sheet. All instruction segments were followed by appropriate exercises given on the sheet handed out to the students. As in the PM instruction, the instructor went round checking the work of the students and providing help to those who needed it.

The next segment dealt with numbers that did not have decimal points, and how the decimal points in those actually went on their right. In the third segment, the students were shown how to line these numbers up with numbers that had decimal points.

In the fourth instruction segment, the addition of properly lined up numbers was demonstrated to the students. It was stressed to them that they needed to start adding from the rightmost digits, and that recording carry overs correctly was important. The importance of placing the decimal point in the final answer, in line with the decimal points of the numbers that were added, was emphasised to the students. Finally, the students were reminded of the result of adding zero to a number, before they were asked to work on the final set of problems on their exercise sheet.

### ***Subtraction***

In Subtraction, the first two segments of instruction involved demonstrating to the students how to properly line numbers up, including numbers without decimal points. They were also told that placing zeros, in line with digits not lined up with any other digits, was important in Subtraction. Examples of these were given and demonstrated.

In the third segment, the students were given a demonstration of actually subtracting one number from another ( $75.6 - 43$ ). It was stressed to them that once properly lined up, the number at the bottom would be the one they would be taking away from the number at the top, and *not* the other way round. They were also shown how to correctly place the decimal point in the final answer, in line with the other two decimal points.

In the fourth segment, the students were shown how to deal with situations where a digit of the top number was smaller than the digit being subtracted from it. They were told that in those cases they needed to “borrow.” However, they were told that the borrowing method demonstrated to them would probably be different from the one they had been taught. In the method shown to them, 10 was added to the small top digit and 1 to the next digit (to the left) being subtracted. For example, in

$$\begin{array}{r} 8.0 \\ - 3.2 \\ \hline \end{array}$$

ten would be added to the 0 of the top number, making it a 10, and 1 to the 3 of the bottom number, making it a 4. The students were told to mark the digits affected so that they would remember. They were shown how to derive the correct answer of 4.8 in the example given.

In the final segment of instruction, the steps involved in solving a more complicated Subtraction problem, requiring a number of instances of “borrowing” ( $40.2 - 0.3587$ ), were demonstrated to the students. Working systematically through the problem was emphasised, and the students were given an opportunity to ask any questions they had before they were asked to work on the last set of problems on their Subtraction exercise sheet.

### ***Multiplication***

In the first segment of instruction in Multiplication, the students were told that each digit of the top number needed to be multiplied by the bottom number. They were given an example of

$$\begin{array}{r} 34 \\ \times \underline{2} \end{array}$$

where both the 4, and then the 3, had to be multiplied by the 2 in turn. That they had to start with the digit furthest to the right (4, in the example) was explained.

They were also told about the need to carry over when the answer to the multiplication of two particular digits exceeded 9. This process was demonstrated to them using an example of  $27 \times 3$ .

Segment two of instruction dealt with having to multiply by more than one digit. An example of

$$\begin{array}{r} 314 \\ \times \underline{25} \end{array}$$

was given, where they were told that each of the digits of 314 had to be multiplied by 5 (the 4, the 1, and then the 3 in turn), and then by the 2. But before multiplying by the 2, the students were told that they had to place a zero as shown below

$$\begin{array}{r} 314 \\ \times \underline{25} \\ 1570 \\ 0 \end{array}$$

This was explained to them in terms of the 2 in the 25 really being a 20 (since  $25 = 20 + 5$ ). Therefore, before multiplying by the 2, the zero had to be placed to remind them and acknowledge the fact that the 2 really was a 20.

Once each of the digits of 314 had been multiplied by the 5 and then the 2, it was demonstrated to the students how the two products obtained (1570 and 6280) had to be added together to get the final answer of 7850.

After the students had completed the appropriate problems on their Multiplication exercise sheet, instruction moved on to multiplication with decimals. They were told that procedures were the same as shown to them in the previous instruction segment, and that initially they did not need to worry about the decimals. The steps involved were demonstrated to them using an example of

$$\begin{array}{r} 30.42 \\ \times \quad 2.3 \\ \hline \end{array}$$

Once the products of multiplying 30.42 by the 3 and then by the 2 had been obtained and then added, 69966 was arrived at. The students were told that they had to place the decimal point in this, and to do so they had to first count the number of digits to the right of the decimal point in the numbers they multiplied. In the example, there were three of these digits: two in 30.42 (the 4 and the 2), and one in 2.3 (the 3). The students were told they had to count that many digits from the right of the number previously obtained (the 69966) to place the decimal point. It was shown to them how counting three from the right end of 69966 would place the decimal point between the 9's, thus giving an answer of 69.966.

The next set of exercises on their sheet required them only to place the decimal points in Multiplication problems where the answers had already been worked out – minus those decimal points. The next segment of instruction that followed this was only a revision of the steps in Multiplication that had been covered. The importance of being careful to avoid unnecessary mistakes was emphasised, and the students were reminded of the results of multiplying by 1 and by 0. After this, they were

asked to work on the final set of exercises on their Multiplication problem sheet.

### *Division*

Demonstration-imitation instruction in Division started with explaining to the students which number was being divided by which. In an example given to them of

$$2 \overline{)462}$$

they were told that the 462 was being divided by the 2. Hence it was pointed out to them that the task would be to try to find out the number of 2's in 462, or how many times 2 would go into 462. To find the answer to this, each of the digits of 462 – starting with the digit furthest to the left (the 4 in this case) – would be divided by the 2 in turn. The process of arriving at the answer of 231 was demonstrated to the students.

They were also shown how to deal with situations where some of the digits of the number being divided were too small to be divided on their own by the divisor. The example of  $305 \div 5$  (in which the 3 was too small) and  $416 \div 4$  (in which the 1 was too small) were given. They were told that in these cases they had to combine those small digits with the next digit of the dividend: thus, in the examples, 3 would be combined with 0 to make 30, and 1 with the 6 to make 16. However, they were told that before doing they had to first place a zero above the number too small to be divided by the divisor. The steps required to solve the two examples were demonstrated to the students.

The second segment of instruction dealt with remainders, and the students were shown how to combine remainders with the following digit of the dividend. An example of

$$3 \overline{)456}$$

was used where the remainder of 1 from  $4 \div 3$  was combined with the 5 to make 15, thus making the next step  $15 \div 3$ . Obtaining the correct solution to this problem was demonstrated to the students. These steps were shown to them again using  $652 \div 2$  before they were asked to work on the next set of exercises on their Division problem sheet.

The third segment of instruction dealt with decimal points in the dividends of Division problems. The students were told to ensure that they place a decimal point in their answer in line with the decimal point in the number divided (the dividend). This was demonstrated to them using  $4.92 \div 4$  as an example.

The students were then shown what to do when both the dividend and the divisor had decimal points in them. Using an example of

$$0.3 \overline{)3.99}$$

they were told that the decimal point in the divisor had to be shifted one space along to the right making the 0.3 a 3. They were told that this had to be done because it was quite complicated to divide by a number with a decimal point. However, the same thing had to be done with the dividend: the decimal point had to be shifted one space along to the right, making the 3.99 a 39.9. They were told that re-writing the problem was important to avoid confusion. In the example used, they were shown how the problem could be re-written as

$$3 \overline{)39.9}$$

The students were told, and shown, that sometimes a zero would need to be added to the number being divided. They were informed that this

applied to cases where there was no decimal point to shift in the dividend. They were given an example of

$$0.4 \overline{)56}$$

where shifting the decimal point in the 0.4, and adding a zero to the 56 would transform the problem to

$$4 \overline{)560}$$

On the next set of problems on their sheet, the students only had to re-write the Division problems given after shifting decimal points and, in some cases, adding zeros to the dividends as well. They did not have to actually solve the Division problems given.

In the final segment of instruction, the students were shown the steps to solving a particular example:  $46.31 \div 1.1$ . They were then given an opportunity to ask any questions they had, and then they were asked to work on the final set of problems on their Division exercise sheet.

### *The control groups*

In the present study, two types of control groups were used: a study skills instruction group, and a no instruction group. Students placed in the "No Instruction" groups simply received no instructions, but were assessed at various intervals corresponding to assessment of students in the instruction groups.

Study skills instruction was used in the present study as a placebo instruction, to control for the possibility that improvements in computational skills could occur simply because of providing additional

attention and special treatment to the students (sometimes referred to as the *Hawthorn effect* – see Brown, 1954; Tuckman, 1978). Students in the study skills groups received the same number of instruction sessions, of the same duration, as students in the PM and DI groups. They were also provided these instruction sessions around the same time as the students in the PM and DI groups. Skills covered in the study skills instruction sessions were: better reading, notetaking, mindmapping, and concentration. These skills are useful in themselves but they were not directly relevant to the computational skills assessed in the present study.

### *The pilot study*

This section briefly describes a pilot study that was carried out before the main experiments were conducted. The pilot study was carried out after the PM and the DI methods of instruction had been devised, but prior to practical procedures being finalised. The construction of the Computational Skills Test and the Basic Facts Test had not been completed at that time, and these tests were not used in the pilot study.

The primary purposes of conducting the pilot study were to get an indication of children's response to the instruction methods planned for use, to fine-tune the methods of instruction and their forms of delivery, and to help decide on the design for the main experiments of the study.

### *Method*

#### *Participants*

The experimental participants were two female and two male Form 1 students at a school in Palmerston North that incorporated both

intermediate and high school. These students are referred to by the initials of their first names: D, L, B, and M.

These students were selected by the Head of Mathematics of the school in consultation with their teachers. The criteria for selection were that the students should be at least average in general ability, but performing considerably poorer than their peers in mathematics. When individually administered a short form of the WISC-R, all except one of the students obtained a prorated IQ score in the average range. L scored three points below the lower parameter of the average range. The students were also administered the Addition, Subtraction, Multiplication, and Division sections of France's Profile of Mathematical Skills. All of them scored at least two years below the average scores for their ages.

### *Procedure*

Two students, D and L, were assigned to Group 1, and received DI instruction. The remaining two students, B and M, were assigned to Group 2, and received PM instruction. The students were instructed in these groups. For both groups, the first stage of instruction dealt with Addition and Subtraction, while the second stage dealt with Multiplication and Division. The students were instructed and/or assessed three to four times a week during their mathematics class period. For each group, each session lasted 15 to 20 minutes.

Demonstration of the various computational skills was done using pen and paper, rather than the board. The children were also provided with pen and paper to work on computational problems when required to imitate the steps demonstrated to them. In addition to the pen and paper demonstrations, simple cardboard cut-outs of warriors with numbers on them were used during the PM instructions. However, other than the process mnemonic component in the PM instruction, and the use of the cardboard cut-outs, the DI and the PM instructions were the same in

terms of skills covered, the sequence in which these were covered, the numerical content of examples demonstrated, and the problems given for the children to work on during the imitation segments.

In each session, the students' computational skills were assessed: Addition and Subtraction in the first stage, and Multiplication and Division during the second stage. The assessments were carried out using problem sheets each containing 10 problems. There were three versions of these sheets for each of Addition, Subtraction, Multiplication, and Division. The different versions were equivalent to each other in terms of difficulty: the skills assessed were the same, only the numbers used varied. These different versions were used in rotation from session to session.

An extended ABA design was used for both the Addition and Subtraction stage, and the Multiplication and Division stage. In a standard ABA design, no treatment is provided during the A phases, and treatment is provided during the B phase. During the first A phase, baseline data points are collected to establish the experimental participant's usual performance prior to treatment. During the B phase, data points are collected on the participant's performance subsequent to each treatment session. During the second A phase, treatment is removed; the reason for doing so is either to check maintenance of improvements made as a consequence of treatment, or to establish that the participant's improvements in performance are dependent on the provision of treatment used. In the pilot study being described here, treatment consisted of the provision of instructions, and several treatment and maintenance phases were implemented.

#### *Addition and Subtraction stage*

Two baseline points were collected prior to any instruction being given. Then students were provided with two sessions of either DI or PM

instruction in Addition, each session being followed by an assessment of their skills level. Maintenance of any skills acquired in Addition was checked in the fifth session with no instruction being provided prior to assessment. This was followed by two more instruction sessions in Subtraction, then another assessment of maintenance. For D in Group 1, and B and M in Group 2, who demonstrated clear improvements in their Addition and Subtraction skills following these instruction sessions, a follow-up assessment was administered two weeks after their second maintenance assessment. For L, who did not show much improvement following the DI instruction, four additional instruction sessions were provided utilising the PM method to see whether her performance would improve.

#### *Multiplication and Division stage*

The Multiplication and Division stage was basically the same in structure as the previously described Addition and Subtraction stage. Three baseline points were collected prior to any instruction being provided. These three baseline points were followed by two instruction sessions in Multiplication, followed by two instruction sessions in Division, and then by no instruction maintenance sessions. L, who again did not show as much improvement as the other three students, was not provided with additional PM instruction sessions during this stage. The reasons were, first, that the students' teachers were expressing concern about the amount of time the students were spending away from their regular mathematics classes. They were worried that this might set the students back further in terms of the syllabus topics being covered in their classes. The second reason was that L did show noticeable improvements after the DI instruction. Thus, in consultation with her teachers and the Head of Mathematics, it was decided that the improvements she had shown were adequate, and that it would be better for her to return to her regular mathematics classes than to be provided with further instruction sessions.

All the students were administered a follow-up assessment two weeks after the last maintenance assessment.

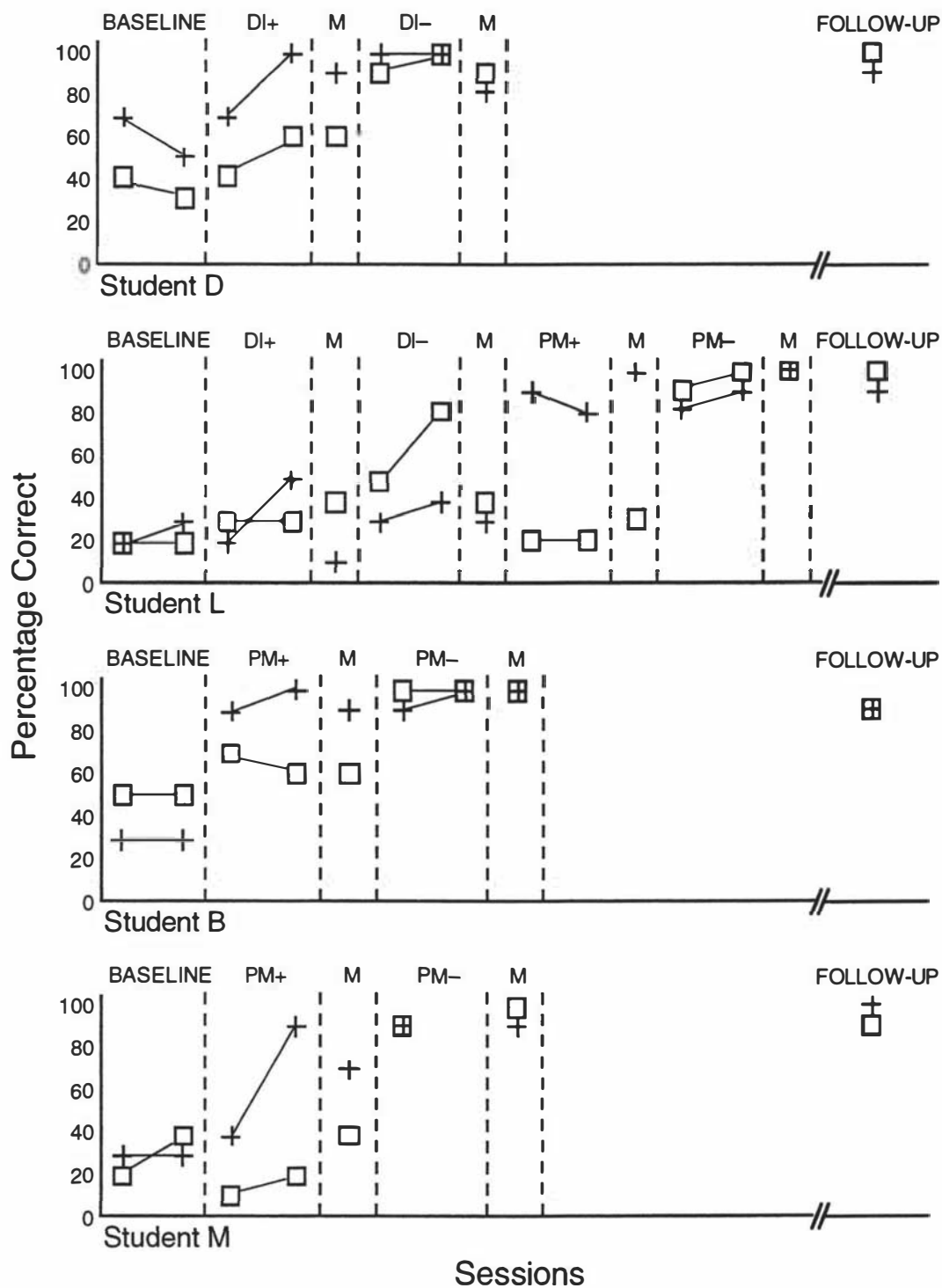
### *Results*

The results from the Addition and Subtraction stage are depicted in Figure 2. Figure 3 shows the results from the Multiplication and Division stage.

Group 1 students, D and L, responded to the DI instruction in different ways. After four DI instruction sessions in Addition and Subtraction, D's assessment scores improved considerably, such that it was deemed unnecessary to provide him with further instruction using the alternative PM method. The improvements in his performance showed good maintenance at the two-week follow-up.

The other Group 1 student, L, showed noticeable improvements during the DI instruction sessions, but these improvements did not maintain during the assessment sessions when no instruction was provided. By the second maintenance assessment session, her score in Subtraction was only marginally better than at baseline, and her score in Addition overlapped with baseline levels.

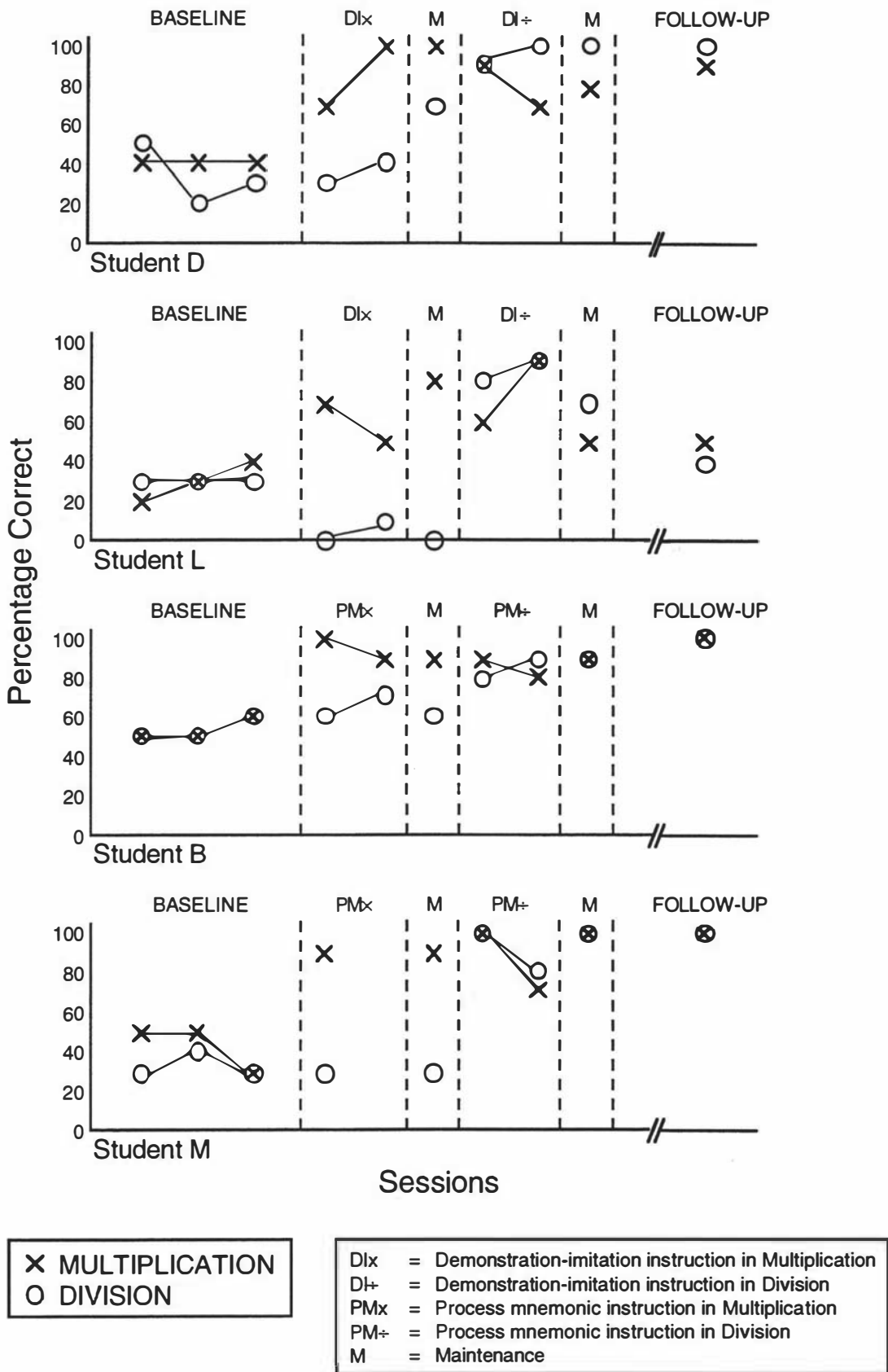
As noted earlier, following this second maintenance assessment session, it was decided to provide L with additional instruction using the PM method. She immediately showed improvements following instructions – first in Addition (first two sessions), then in Subtraction (second two sessions). By the second maintenance assessment session following the PM instructions, the improvements in L's Addition and Subtraction scores were evident, and these were carried through to the follow-up two weeks later.



+ ADDITION  
□ SUBTRACTION

DI+ = Demonstration-imitation instruction in Addition  
 DI- = Demonstration-imitation instruction in Subtraction  
 PM+ = Process mnemonic instruction in Addition  
 PM- = Process mnemonic instruction in Subtraction  
 M = Maintenance

**Figure 2. Student participants' progress in addition and subtraction across baseline, intervention, maintenance, and follow-up conditions.**



**Figure 3. Student participants' progress in multiplication and division across baseline, intervention, maintenance, and follow-up conditions.**

Both Group 2 students, B and M, achieved noticeable improvements in their Addition and Subtraction performance after a total of four PM instruction sessions. It was deemed unnecessary to give these students additional instruction sessions using the DI method alone. The improvements in their computational skills performance showed good maintenance at follow-up, carried out two weeks later.

The students' pattern of responses to the DI and the PM instructions in Multiplication and Division were very similar to the patterns during the Addition and Subtraction stage. Again, D from Group 1, and B and M from Group 2, exhibited considerable improvements, and it was deemed unnecessary to provide them with further instruction sessions using the alternative methods. L's pattern of responding was better than in Addition and Subtraction. By the second maintenance assessment session, her scores were moderately better and did not overlap with her baseline levels. As noted earlier, because of her teachers' concern about the amount of time she was spending away from regular classes, it was decided not to provide her with additional PM instruction sessions, even though her performance had not reached a level comparable to the other three students.

### *Discussion*

The results of the pilot study provided some useful answers to questions relating to the intervention methods and their implementation in the main experiments of the present research. First, although there were only four experimental participants, the usefulness of the DI and PM methods for teaching computational skills to students with mathematics LD was clearly demonstrated. Improvements in the students' performance followed the provision of both the DI and the PM instructions. The instruction methods were also used in small group settings (two children in each group), and this did not appear to reduce their effectiveness. Of

particular interest were the Group 2 students' responses to the PM instruction because it had previously only been used in a one-to-one setting.

There was some indication that where the DI instruction failed, the PM instruction proved effective. This is in view of L's performance during the Addition and Subtraction stage. However, a clear-cut conclusion cannot really be derived from this. The question arises, for example, of whether providing further DI instruction sessions would have produced similar improvements in L's performance. Another question that comes up is whether providing her with the PM instruction from the beginning (i.e., if she was in Group 2) would have avoided the necessity of providing additional instruction sessions. Most importantly, it is very difficult to draw any firm conclusions from the performance of only one student, who was also atypical in comparison to the other three students where her below average WISC-R score was concerned.

It is worth noting, however, that L did make considerable gains in her performance following the instruction sessions provided, despite having a prorated IQ score that was three points below the lower parameter of the normal range in the WISC-R. Thus, even though she did not, strictly speaking, meet the sometimes required criterion of having an IQ that is at least in the normal range, she quite clearly benefited from the remedial instruction sessions provided. L's positive response to remedial instruction lends some support to earlier mentioned arguments about the irrelevance of IQ to LD (e.g., Siegel, 1989), especially where provision of instruction is concerned. Although caution is again necessary in drawing any conclusions based on the performance of only one student, L's performance in this pilot study suggests that individuals with LD who have IQs below the normal range need not be excluded from remedial programmes because they could similarly benefit from being provided effective instruction.

Having found instruction that incorporates process mnemonics to be effective in teaching computational skills on a one-to-one basis (Manalo, 1991) and in a small, tutorial type setting involving only two students, an examination of its effectiveness in slightly larger groups of four or six students was decided upon for the main experiments. This would enable an investigation into its usefulness and applicability to real remedial classroom type settings where teaching small groups of students (rather than individuals) is more cost-effective and viable.

With respect to the procedures for teaching computational skills using the DI and PM methods, those described and used in the pilot study were deemed satisfactory. However, a decision was made to use blackboard and chalk (or whiteboard and marker pens) for the main experiments of the study, rather than pen and paper, since the former would be more practical when slightly larger groups of students are being taught. Consequently, to make the DI and PM instructions equivalent to each other, the cardboard cut-outs of warriors in the PM method were discarded. All demonstrations and illustrations of processes and procedures would instead be carried out on the board.

A final issue that was considered important to investigate after the completion of the pilot study was the longer-term maintenance of improvements following the instructions provided. In the pilot study, it was found that improvements resulting from both the DI and PM instructions maintained over a two week period. It would be important to find out whether they would show maintenance over longer periods, and whether either method would prove better than the other in the longer term.

## *Selection of participants for the main experiments*

### *Instruments and criteria used*

Selection of experimental participants for the present study was based largely on scores that students obtained in four tests: the Computational Skills Test, the Basic Facts Test, the PAT Maths Test, and the Test of Scholastic Abilities (TOSCA). Although teachers in schools initially made recommendations of students whom they believed would be suitable for the study, all recommended students were tested and it was on the basis of their performance in these tests that decisions were made about inclusion in the study.

The rationale, construction, and collection of normative data for the Computational Skills Test were described earlier in this chapter. The test consists of a total of 40 computational problems: ten each of Addition, Subtraction, Multiplication, and Division. The Basic Facts Test, also described earlier in this chapter, consists of basic facts questions, of which there are a total of 40, ten in each of the four basic computational operations.

The Computational Skills Test and the Basic Facts Test were the first screening tests administered to the students whom their teachers considered to be performing at a level significantly lower than their peers in mathematics. It was indicated to the teachers that students with serious intellectual, sensory, or physical handicaps would probably not benefit from the interventions to be provided in the study. Other than that, all students considered to be performing below average in mathematics could be put forward by their teachers and tested. There was no requirement that they be of at least average IQ, intelligence, or general ability.

The Computational Skills Test and the Basic Facts Test were administered together. On the Computational Skills Test, students had to obtain a total score of less than 17 (out of 40) to be selected after this first round of testing. As noted earlier, a score of 16 is more than one standard deviation lower than the mean score of students in Form 3, based on the normative data collected. A score of 16 is also lower than the mean for Form 1 students. Thus, students in Form 3 who score 16 or less in the Computational Skills Test can be considered to be performing significantly below the level expected of their age group, and performing at a level that is approximately equal to that of students two years their juniors in school.

Apart from scoring less than 17 in total, students also had to score no higher than the mean of the Form 3 normative data in any of the Addition, Subtraction, Multiplication, and Division sections. These means were 7.49, 6.05, 6.44, and 5.95 respectively. Thus, all students selected scored 7 or under in Addition, 6 or under in Subtraction and Multiplication, and 5 or under in Division.

On the Basic Facts Test, students had to score at least 30 out of 40. As noted earlier in this chapter, the Basic Facts Test was used to establish a required standard of performance in the students' basic facts knowledge. It was used as a means of showing that the poor scores in the Computational Skills Test of the students selected were due to errors in executing computational procedures – and *not* lack of knowledge about the basic facts required in the computational problems given. As noted previously, since careless and other unintended mistakes were likely to be made, a score of 75% correct in the Basic Facts Test was decided on as the criterion for selection.

The PAT Maths Test has two versions, A and B; Form B was used in this study. For Form 3 students, the test consists of 50 multiple-choice questions. Based on the normative data collected by the New Zealand

Council for Educational Research (N. A. Reid, 17 June 1992: personal communication), the raw score mean for Form 3 children is 23.87 out of 50, and the standard deviation is 10.06.

The PAT Maths Test was used in the present study as a means of providing additional confirmation (independent of tests designed by the researcher) that the selected students had difficulties in mathematics. As such, a more relaxed criterion for selection was employed in using this test: students who obtained a raw score of at least one-half the standard deviation below the mean (18 or lower) were selected, instead of just those who scored at least one standard deviation below the mean. By the time the PAT Maths Test was administered, two characteristics of the students had already been established: (1) that they were considered by their teachers to be underachieving in mathematics, and (2) that they were significantly below their peers in their levels of ability to carry out computational procedures. Thus the criterion employed was considered adequate for the purpose of confirming the selected students' mathematics difficulty and underachievement.

Another reason that influenced the decision to employ a more relaxed criterion in using the PAT Maths Test was that the test appeared to have a high probability of a 'miss' or 'false negative' outcome because of the way it has been structured. The PAT Maths Test is a multiple-choice test with five choices (A to E) in most of the 50 questions (Questions 155, 158, 164, and 170 have only four choices). Therefore, by chance alone, the likely score would be about 10 out of 50. If someone picks B for all their answers, they would in fact score 16 out of 50, which is higher than the Form 3 mean minus one standard deviation (13.81). This suggests that someone only guessing – and presumably experiencing serious difficulties in mathematics – has a chance of exceeding the mean minus one standard deviation criterion, and therefore would not be detected as significantly underachieving in mathematics.

Students who met the criterion that was set for the PAT Maths Test were administered the TOSCA. The TOSCA is a pen and paper group test that is intended to assess the scholastic abilities of New Zealand children in schools. It can be administered to a group of students within the regular school class period of around 50 minutes, since students have a time limit of only 30 minutes to work on the questions of the test. Furthermore, scores in the TOSCA correlate well with scores in WISC-R subtests (Reid & Gilmore, 1988), and the TOSCA has been used as a tool for estimating intelligence (Chapman, St. George, & van Kraayenoord, 1984).

The TOSCA was considered to be an adequate indicator of students' general ability to learn and achieve in the school setting and, for practical reasons, it was used in the present study instead of an IQ test such as the WISC-R. Administration of even a short form of the WISC-R to potential participants in the study would have been too time consuming and impractical in terms of scheduling and resource utility. The delays that would inevitably result, the disruption to regular classes, and the difficulties in scheduling mutually acceptable times and rooms to use in the schools were just some of the factors that precluded administration of the WISC-R.

As noted in earlier chapters, it is questionable whether children have to be of at least average intelligence to be considered LD or to benefit from remedial instruction. The TOSCA was used in the present study as a measure of the students' general ability, but only those who fell in the bottom category (the bottom 4%) were excluded. Decisions were based on the stanine scores that the students obtained: these were 9 = "superior," 7-8 = "above average," 4-6 = "average," 2-3 = "below average," and 1 = "low." Those who obtained a stanine score of 1 were excluded, but those with 2's and 3's, in the "below average" category, were included. This was a compromise similar to that suggested by Siegel (1989) of lowering the IQ cut-off point to 80 (instead of 90) in the selection of students with LD. In effect, a stance was taken that students with below average abilities as

indicated in tests should also be considered as LD, and as equally likely to benefit from remedial instruction as those with average abilities or better.

To summarise, the students selected for the experiments of this research study were those who:

- 1 Scored a total of less than 17 out of 40 in the Computational Skills Test, and no more than the respective normative data means in each of the subtests of Addition, Subtraction, Multiplication, and Division;
- 2 Scored a total of at least 30 out of 40 in the Basic Facts Test;
- 3 Obtained a raw score of 18 or lower in the PAT Maths Test; and
- 4 Obtained a stanine score of 2 or better in the TOSCA.

#### *Procedures in participant selection*

Prior to approaches being made to schools for conducting the experiments of the study, ethical approval was sought from the Massey University Human Ethics Committee. One of the important requirements passed by the Committee was that students placed in control groups, and students not selected because of failure to meet other selection criteria, should be given an opportunity to receive equivalent computational skills instruction if they so wished. This requirement was adhered to in conducting both experiments of the present study, so that after experimental group instructions and assessments had been completed, students in the control groups were given the opportunity to attend instruction sessions in computational skills that were provided. Teachers were also asked to inform students who met the criteria of the Computational Skills Test and the Basic Facts Test, but not the PAT Maths

Test and/or the TOSCA, that they could also attend the instruction sessions provided later if they wished<sup>2</sup>.

In approaching and working in the schools, the first step made was to contact the Principal of the school by telephone. If the Principal was receptive to the possibility of conducting research in the school, a letter was sent to them providing more details about what would be involved in the research, the number of students and sessions required, the amount of teacher involvement, the amount of time it would take, and so on. After this, if the Principal agreed to allow the research to be carried out, the Principal or the Head of Department (HOD) of Mathematics at the school contacted the present author (the researcher) to arrange a meeting to further discuss details.

The ensuing meetings usually involved only the HOD of Mathematics and appropriate class teachers. The requirements of the study, the procedural and administrative steps involved, and scheduling were discussed during these initial meetings.

Suitable times to administer the tests to the children were arranged. In all schools, the teachers informed the author that the students' consent to take the tests was at this stage adequate, and that parental/guardian consent would only be required when the students selected had to participate in the intervention programmes of the study. The cover sheet of the booklet containing the Basic Facts Test and the Computational Skills Test, the first tests to be administered to the students, required them to provide information about their date of birth and age, gender, and school. The cover sheet also provided instructions and information about the research to the students, and asked for their consent to take the tests. The students had to write their name and sign if they agreed to participate.

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<sup>2</sup> Only a few of the students in the control groups, and none of those not selected for the experiments, attended the additional instruction sessions provided.

Students whose scores met the criteria required for the Computational Skills Test and the Basic Facts Test were administered the PAT Maths Test at the next scheduled testing session. Those who met the PAT Maths criterion were next administered the TOSCA.

The students who met the criteria of all the tests administered were then given, through their teachers, a letter and a consent form to take home to one of their parents or guardians. The letter provided information on the study and what would be involved should they permit their child to participate. The consent form required both the parent's or guardian's name and signature, as well as the student's. Copies of the letter and the consent form are included in Appendix E.

Students whose parent or guardian consented to their participation in the study were randomly assigned to the experimental instruction or control groups. The random number generator of a calculator (a Casio fx-100) was used for these assignments.

Whenever they asked for it, students participating in the study were provided information about their individual performance and progress (i.e., their scores in the tests administered and/or their assessment scores in computational skills gathered at various points during the course of the study). The HOD of Mathematics and the students' mathematics teachers were also provided with these performance and progress scores. A brief report of findings was also provided at the conclusion of the experiments to the HOD of Mathematics, and a letter expressing gratitude was sent to the Principal.

## Chapter 6

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# Experiment 1

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### *Introduction*

The primary purpose of Experiment 1 of the present study was to examine the effectiveness of using process mnemonic instruction in teaching computational skills to students with mathematics LD. In doing this, two main hypotheses were pertinent. The first of these was that compared to no instruction and placebo instruction, process mnemonic (PM) instruction would produce improvements in the computational skills performance of students identified as having significant deficits in this area. The second main hypothesis was that compared to the DI instruction, PM instruction would produce greater improvements in performance of such students.

Apart from these main hypotheses, a number of other important questions were raised. First, there was the question of how well any improvements produced by the instructional interventions (PM and DI) would maintain over time. Thus, the study was designed to explore whether improvements shown at immediate post-instruction would still be present one week later and six to eight weeks later at follow-up.

Second, there was the question of whether students' levels of scholastic ability (as measured by the TOSCA) would influence their responses to the instructional interventions. As noted in earlier chapters, some theorists and researchers in the LD area argue that individuals with LD need to have an IQ in the average range or better to be considered LD and/or to be included in intervention programmes. While the TOSCA is not an IQ test, it correlates well with subtests of the WISC-R (Reid &

Gilmore, 1988), and in this study students who scored in the “below average” range of the TOSCA were included. Therefore, an important question that needed to be addressed was whether these students would perform (i.e., show improvements in computational skills) any worse than students with average or better scores in the TOSCA. This question could also be posed as: Could any resulting variance in computational skills scores after the interventions had been administered be attributed to students’ TOSCA scores?

Another relevant question was whether any resulting variance in computational skills scores after the interventions could be attributed to students’ PAT Maths Test scores. The PAT Maths Test was used as a measure of the students’ general difficulty and underachievement in mathematics. Again a more relaxed criterion was used: students with PAT Maths Test scores that were one-half of a standard deviation or more below the mean were included in the study, rather than just those who scored at least a whole standard deviation below the mean. (The reasons for this were discussed in the previous chapter). Thus it would be relevant to investigate whether any resulting variance in computational skills scores after the intervention had been administered could be attributed to students’ PAT Maths Test scores.

Finally, whether those in the placebo instruction group would show a pattern of performance different from the no instruction group was examined in Experiment 1. As noted in the previous chapter, study skills instruction (the placebo instruction) should be of some benefit to the students. However, the skills dealt with in these instruction sessions would be irrelevant to, and theoretically should not affect, computational skills performance. If students in the placebo instruction group improved in their performance in comparison to those in the no instruction group, and if that improvement was of comparable magnitude to that shown by students in the instruction groups, then doubts could be cast on the real

effectiveness of the computational skills instruction methods used in the study.

## *Method*

### **Participants**

The experimental participants were 29 Form 3 students from two schools in the Palmerston North area. Fourteen of the students came from one school, and fifteen came from the other. The experimenter carried out the study in one of the schools first and then, when work there had been completed, moved onto the second school. Thus, each school contributed students to all four groups of the study.

Twenty-three of the students were females, and six were males. The predominance of females was due to one of the schools being a single-sex girls' school. The students' ages ranged from 13 years to 14 years and 6 months at the time of initial testing.

All student participants had been put forward by their teachers as low achieving in mathematics in comparison to their peers, but without any serious intellectual, physical, or sensory impairments to which the underachievement could be attributed. The 29 students met the criteria, discussed in the previous chapter, for the Computational Skills Test, the Basic Facts Test, the PAT Mathematics Test–Revised, and the TOSCA.

In phase 1, the Addition and Subtraction phase, the mean scores of the student participants in the selection tests are depicted in Table 3, according to the groups to which they were randomly assigned.

**Table 3. Mean Test Scores of the Student Participants in Each of the Four Groups, During the Addition and Subtraction Phase, Experiment 1**

Test	Groups			
	PM (N = 9)	DI (N = 7)	Study Skills (N = 7)	No Instruct. (N = 6)
Comp. Skills Test (/40)	10.44	9.43	11.29	11.50
Basic Facts Test (/40)	38.11	35.57	38.43	38.33
PAT Maths Test (/50)	13.33	14.00	11.57	16.17
TOSCA (stanine)	3.33	3.71	4.14	3.33

An alpha level of .05 was used for all statistical tests, except where stated otherwise.

Using an analysis of variance, it was found that the groups did not differ significantly from each other in the four tests used for selection. In the Computational Skills Test,  $F(3,25) = .75, p = .5353$ ; in the Basic Facts Test,  $F(3,25) = 2.60, p = .0746$ ; in the PAT Maths Test,  $F(3,25) = .21, p = .8881$ ; and in the TOSCA,  $F(3,25) = .47, p = .7044$ . This finding meant that, where the tests used for selection were concerned, the student participants in the four groups could be considered equivalent to each other as they entered into the Addition and Subtraction phase of the experiment.

In phase 2, the Multiplication and Division phase, the students were randomly re-assigned to the groups. For each of the groups, the mean scores in the tests used for selection are depicted in Table 4 below.

**Table 4. Mean Test Scores of the Student Participants in Each of the Four Groups, During the Multiplication and Division Phase, Experiment 1**

Test	Groups			
	PM (N = 7)	DI (N = 8)	Study Skills (N = 6)	No Instruct. (N = 8)
Comp. Skills Test (/40)	13.00	10.63	9.50	9.38
Basic Facts Test (/40)	39.14	38.13	37.00	36.00
PAT Maths Test (/50)	16.43	12.13	13.33	13.00
TOSCA (stanine)	3.71	4.00	3.83	3.00

Again, an analysis of variance was carried out on these scores. No significant differences were found across the groups in the Basic Facts Test,  $F(3,25) = 2.42, p = .0898$ ; the PAT Maths Test,  $F(3,25) = .37, p = .7763$ ; and the TOSCA,  $F(3,25) = .60, p = .6187$ . This finding meant that, where these three test were concerned, the student participants in the four groups could be considered equivalent to each other when they began phase 2 of the experiment.

However, a significant difference between the groups was found in the Computational Skills Test,  $F(3,25) = 3.12, p = .0440$ . Tukey's Studentized Range (HSD) Test, with an alpha level at .05, was employed to determine the exact nature of the significant difference found. It showed that the PM, DI, and study skills groups did not significantly differ from each other; nor did the DI, study skills, and no instruction groups significantly differ from each other. The significant difference was between the PM group and the no instruction group. Looking at the mean scores in Table 4, the PM group had a higher mean score (13.00/40) than the no instruction group (9.38/40). This difference between groups necessitated

an examination of the extent to which the Computational Skills Test scores of the students contributed to variance observed during the different stages of phase 2.

### **Procedure**

The students were randomly assigned to one of the four groups of the study, first for phase 1 dealing with Addition and Subtraction, and then for phase 2 which covered Multiplication and Division. In phase 1, a total of 9 students were assigned to the PM group, 7 to the DI group, 7 to the study skills group, and 6 to the no instruction group. In phase 2, students were randomly re-assigned to the groups. A total of 7 students were assigned to the PM group, 8 to the DI group, 6 to the study skills group, and 8 to the no instruction group.

In each school, during each phase, there was one each of the PM, DI, study skills, and no instruction groups. Group sizes ranged from three to five students. Instruction sessions were carried out either in regular classrooms not in use during the period concerned, or in slightly smaller remedial classrooms, depending on what was available. In all cases, the classrooms had either a whiteboard or a blackboard for instructional use.

The different methods of teaching the groups were described in the previous chapter. In each phase, the PM, DI, and study skills groups were each provided with five instruction sessions. For the PM and DI groups, sessions 1 and 3 dealt with Addition or Multiplication (depending on the phase), sessions 2 and 4 with either Subtraction or Division, and session 5 provided a review of the computational skills taught followed by the immediate post-instruction assessment. The study skills groups were provided with instruction on a different topic in each of sessions 1 to 4, and then in session 5 a quick review of those topics was given followed by the immediate post-instruction assessment of computational skills. The

no instruction groups were also administered the assessment around the time the other three groups were assessed in session 5.

Each instruction session was approximately 25 minutes in duration, and usually each of the instruction groups were seen twice a week. Students in all groups were again administered the relevant computational skills assessment one week after session 5. Another assessment was then carried out at follow-up which was six to eight weeks after the cessation of instruction in session 5. There was some variance on the time this last follow-up assessment session was carried out because of a number of factors outside the experimenter's control. These included student absence due to illness and other reasons, clashes with various important school functions or activities, and school holidays. The schedules for instruction and assessment are depicted in the Table 5 below.

**Table 5. Schedules for Instruction and Assessment of the Four Groups in Experiment 1**

Sessions	Groups			
	PM	DI	Study Skills	No Instruct.
1	Addition or Multiplication	Addition or Multiplication	reading skills	n/a
2	Subtraction or Division	Subtraction or Division	notetaking skills	n/a
3	Addition or Multiplication	Addition or Multiplication	mindmapping	n/a
4	Subtraction or Division	Subtraction or Division	concentration skills	n/a
5	review then assessment	review then assessment	review then assessment	assessment
1 week later	assessment	assessment	assessment	assessment
6–8 weeks later	assessment	assessment	assessment	assessment

## *Results*

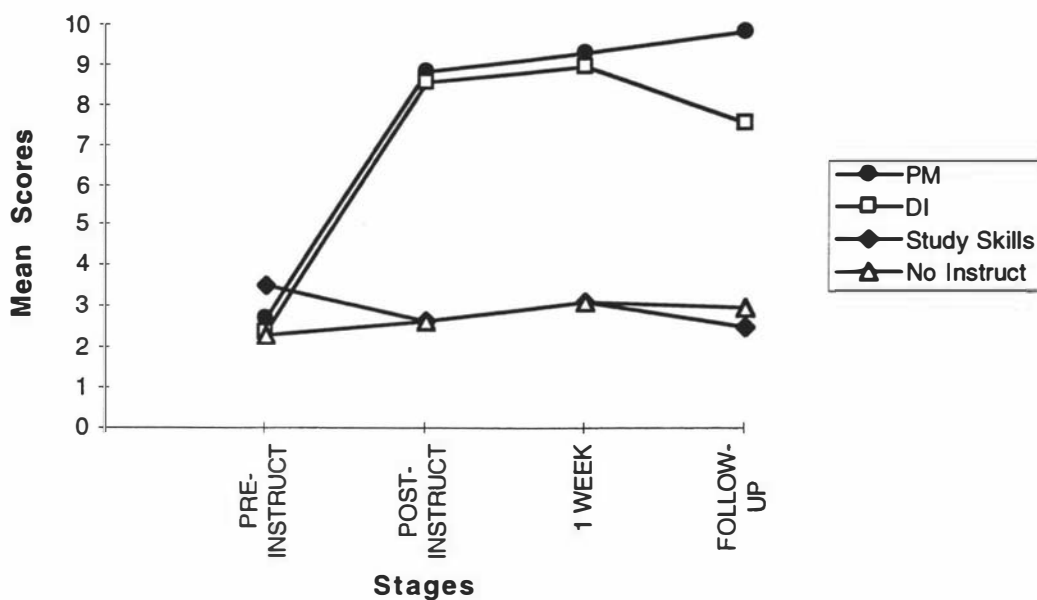
### **Addition and Subtraction Phase**

The mean scores in Addition at pre-instruction, immediate post-instruction, one week later, and at follow-up for each of the four groups are shown in Figure 4. Similarly, the mean scores in Subtraction are shown in Figure 5. From these figures, it appears that for both Addition and Subtraction, the four groups were approximately equal at pre-instruction. Then, at immediate post-instruction, the PM and DI groups show noticeable improvements in their scores while the study skills and no instruction groups' scores remain at a level similar to the level at pre-instruction. These latter two groups remain more or less the same in their mean scores 1 week later and at follow-up, some 6 to 8 weeks later. One week later, the PM and DI groups appear to maintain the improvements shown immediately after instruction. The improvements in scores also appear to be present at follow-up, although by this stage the mean scores of these two groups start to diverge, with the PM group score seeming to improve slightly while the DI group score appears to deteriorate a little.

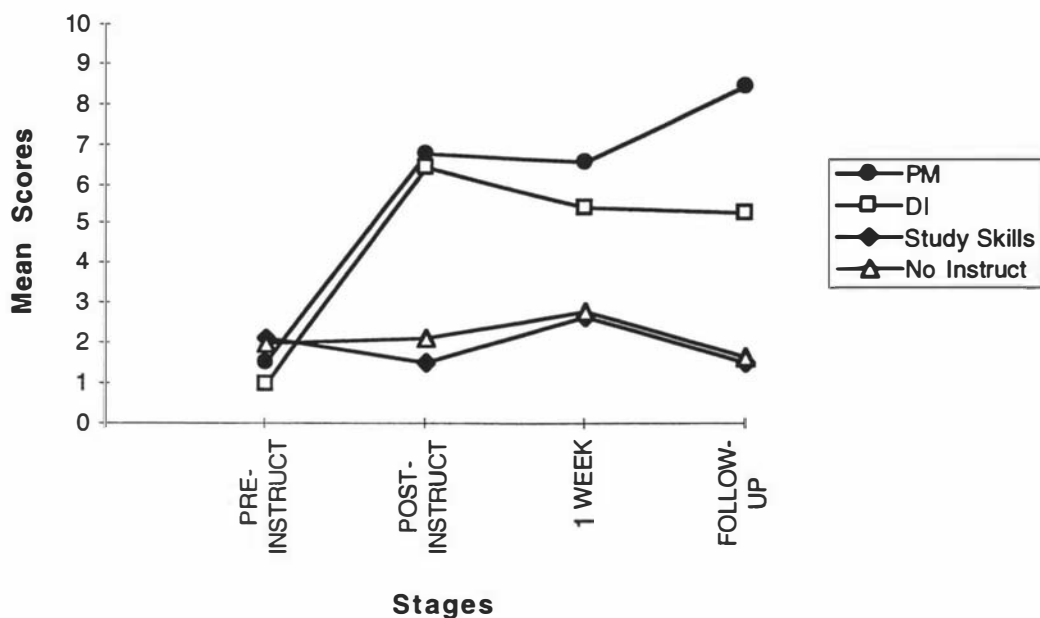
### *Addition*

An analysis of variance using a repeated measures design was employed to analyse the Addition data. The students' TOSCA scores and PAT Maths Test scores were used as covariates in analysing the data to investigate whether these contributed significantly to the variance observed in the experiment.

A significant group effect was found,  $F(3, 23) = 71.08, p = .0001$ . However, no significant effects due to the covariates were found: for the TOSCA,  $F(1, 23) = .09, p = .7624$ ; and for the PAT Maths Test,  $F(1, 23) = .46, p = .5022$ .



**Figure 4.** Addition mean scores of the groups across the different stages of Experiment 1



**Figure 5.** Subtraction mean scores of the groups across the different stages of Experiment 1

The effect due to the stages of the experiment (pre-instruction, immediate post-instruction, 1 week, and follow-up) was not significant,  $F(3, 69) = 1.36$ ,  $p = .2621$ . However, a significant interaction was found between group and stages of the experiment,  $F(9, 69) = 14.00$ ,  $p = .0001$ . The nature of this interaction is clearly seen in Figure 4.

At the pre-instruction stage, there was no significant effect found due to group,  $F(3, 23) = .82$ ,  $p = .4983$ . However, significant effects due to group were found at the immediate post-instruction stage,  $F(3, 23) = 36.69$ ,  $p = .0001$ ; the 1 week stage,  $F(3,23) = 75.57$ ,  $p = .0001$ ; and the follow-up stage,  $F(3, 23) = 32.05$ ,  $p = .0001$ .

The means of the four groups are shown in Table 6.

**Table 6. Means of the Four Groups in Addition at Different Stages of Experiment 1**

Groups	Stages			
	Pre-Instruct	Post-Instruct	1 Week	Follow-Up
PM	2.78	8.89	9.33	9.89
DI	2.43	8.57	9.00	7.57
Study Skills	3.57	2.71	3.14	2.57
No Instruct	2.33	2.67	3.17	3.00

The simple main effects of the interaction between group and stages of the experiment were analysed in two ways: first, by examining the effect of group at each stage, and second, by examining the effect of the stages for each group. Tukey's Studentized Range (HSD) Test was used to analyse the simple main effects because this test controls the Type I experimentwise error rate without increasing the likelihood of a Type II error (SAS/STAT User's Guide, 1990). In statistical analyses, Type I error is committed when a conclusion is drawn that a difference exists in a

comparison when in reality there is none. Type II error is committed when it is concluded that a difference does not exist when in reality it does.

Using an alpha level of .05, the comparisons of groups at each of the stages revealed the following:

- 1 As noted earlier, no significant group effect was found at the pre-instruction stage of Addition, so use of the Tukey's Test was not necessary.
- 2 At immediate post-instruction, the means of the PM and DI groups had increased, so that they were significantly higher than the means of the study skills and no instruction groups. However, the PM and DI groups did not significantly differ from each other. The study skills and no instruction groups also did not significantly differ from each other.
- 3 The results were much the same 1 week after the final instruction session, with the PM and DI groups not being significantly different from each other, but being significantly higher in their mean scores than the study skills and no instruction groups. These latter two groups were in turn not significantly different from each other.
- 4 At follow-up, six to eight weeks later, the PM and DI groups still had mean scores significantly higher than the study skills and no instruction groups. These latter two groups did not significantly differ from each other. The PM and DI groups also remained not significantly different from each other.

To investigate the effect of the experimental stages for each of the groups, a one-way ANOVA was carried out on the data obtained for each of the groups. Tukey's Test was then employed to further examine any

significant effects found. An alpha level of .05 was used. The findings were:

- 1 For the PM group, the effect of stages was significant,  $F(3, 32) = 57.42$ ,  $p = .0001$ . Tukey's Test showed that the means at the immediate post-instruction, 1 week, and follow-up stages were all significantly different from the mean at the pre-instruction stage. However, all pairwise comparisons of means from the post-instruction stages revealed no statistically significant differences.
- 2 For the DI group, the effect of stages was significant,  $F(3, 24) = 19.08$ ,  $p = .0001$ . Tukey's Test showed that the means at the immediate post-instruction, 1 week, and follow-up stages were all significantly different from the mean at the pre-instruction stage. However, all pairwise comparisons between post-instruction stages revealed no statistically significant differences.
- 3 For the study skills group, the effect of stages was not significant,  $F(3, 24) = 1.24$ ,  $p = .3181$ .
- 4 Similarly, for the no instruction group, the effect of the stages was not significant,  $F(3, 20) = .46$ ,  $p = .7122$ .

### *Subtraction*

An analysis of variance using a repeated measures design was employed to analyse the Subtraction data. The students' TOSCA scores and PAT Maths Test scores were again used as covariates in analysing the data to investigate whether these contributed significantly to the variance observed in the experiment.

A significant group effect was found,  $F(3, 23) = 15.09, p = .0001$ . However, no significant effects due to the covariates were found: for the TOSCA,  $F(1, 23) = .10, p = .7534$ ; and for the PAT Maths Test,  $F(1, 23) = .40, p = .5315$ .

The effect due to the stages of the experiment was not significant,  $F(3, 69) = .95, p = .4236$ . However, a significant interaction was found between group and stages of the experiment,  $F(9, 69) = 5.38, p = .0001$ . The nature of this interaction is clearly seen in Figure 5.

At the pre-instruction stage, there was no significant effect found due to group,  $F(3, 23) = 2.48, p = .0864$ . However, significant effects due to group were found at the immediate post-instruction stage,  $F(3, 23) = 12.16, p = .0001$ ; the 1 week stage,  $F(3,23) = 3.41, p = .0346$ ; and the follow-up stage,  $F(3, 23) = 16.45, p = .0001$ .

The means of the groups are shown in Table 7.

**Table 7. Means of the Four Groups in Subtraction at Different Stages of Experiment 1**

Groups	Stages			
	Pre-Instruct	Post-Instruct	1 Week	Follow-Up
PM	1.56	6.78	6.56	8.44
DI	1.00	6.43	5.43	5.29
Study Skills	2.14	1.57	2.71	1.57
No Instruct	2.00	2.17	2.83	1.67

As in Addition, the simple main effects of the interaction between group and stages of the experiment were analysed in two ways: first, by examining the effect of group at each stage, and second, by examining the effect of the stages for each group. Tukey's Studentized Range (HSD) Test

was again used to analyse these simple main effects. An alpha level of .05 was used.

The comparisons of groups at each of the stages revealed the following:

- 1 As noted earlier, no significant group effect was found at the pre-instruction stage of Subtraction, so use of the Tukey's Test was not necessary.
- 2 At immediate post-instruction, the means of the PM and DI groups had increased, so that they were significantly higher than the means of the study skills and no instruction groups. However, the PM and DI groups did not significantly differ from each other. The study skills and no instruction groups also did not significantly differ from each other.
- 3 One week after the final instruction, none of the groups were significantly different from each other.
- 4 However, at follow-up, six to eight weeks later, the PM group had a mean score that was significantly higher than the mean scores of the study skills group, the no instruction group, and the DI group. The mean of the DI group was also significantly higher than those of the study skills and no instruction groups. These latter two groups did not significantly differ from each other.

As in Addition, a one-way ANOVA was carried out on the Subtraction data obtained for each of the groups to investigate the effect of the experimental stages on each of these groups. Tukey's Test was then employed to further examine any significant effects. An alpha level of .05 was used. The findings were:

- 1 For the PM group, the effect of the stages was significant,  $F(3, 32) = 10.78, p = .0001$ . Tukey's Test showed that the means at the immediate post-instruction, 1 week, and follow-up stages were all significantly different from the mean at the pre-instruction stage. However, all pairwise comparisons of means from the post-instruction stages revealed no statistically significant differences.
- 2 For the DI group, the effect of the stages was significant,  $F(3, 24) = 7.58, p = .0010$ . Tukey's Test showed that the means at the immediate post-instruction, 1 week, and follow-up stages were also all significantly different from the mean at the pre-instruction stage. However, all pairwise comparisons of means from the post-instruction stages revealed no statistically significant differences.
- 3 For the study skills group, the effect of stages was not significant,  $F(3, 24) = 1.59, p = .2189$ .
- 4 Similarly, for the no instruction group, the effect of stages was not significant,  $F(3, 20) = .67, p = .5793$ .

### **Multiplication and Division Phase**

The mean scores in Multiplication at pre-instruction, immediate post-instruction, 1 week later, and at follow-up for each of the four groups are plotted in Figure 6. The mean scores in Division are plotted in Figure 7. From these plots it appears that the levels of performance of the study skills and no instruction groups did not change much during the post-instruction stages as compared to their levels at the pre-instruction stage. In contrast, both the PM and DI groups show improvements in their performance at the immediate post-instruction stage. In Multiplication, these improvements appear approximately equivalent in magnitude, but in Division, the improvement made by the PM group appears greater than that made by the DI group. These improvements in performance

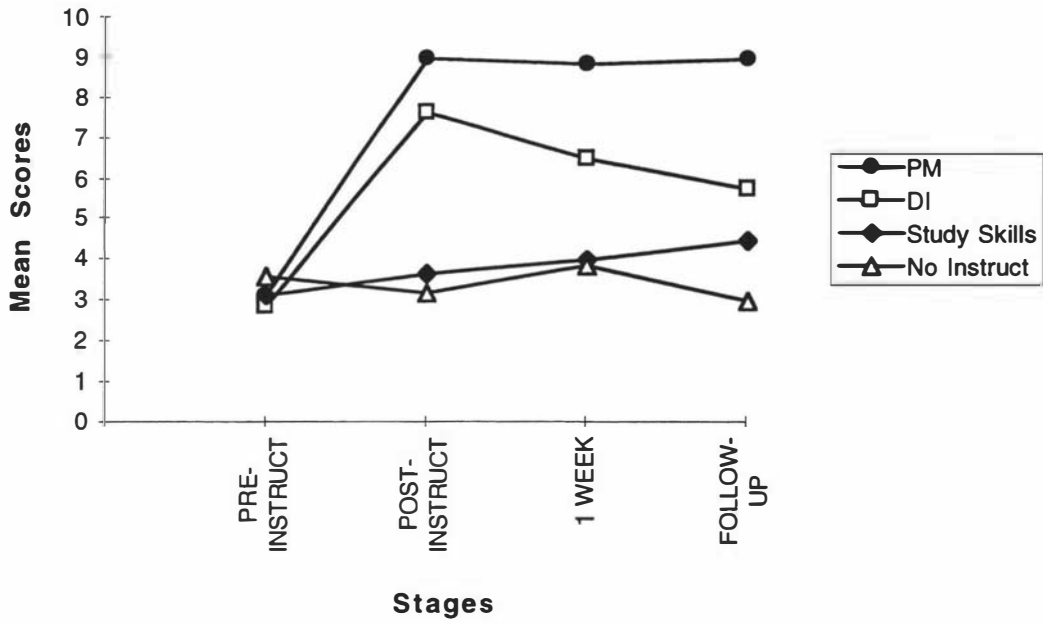


Figure 6. Multiplication mean scores of the groups across the different stages of Experiment 1

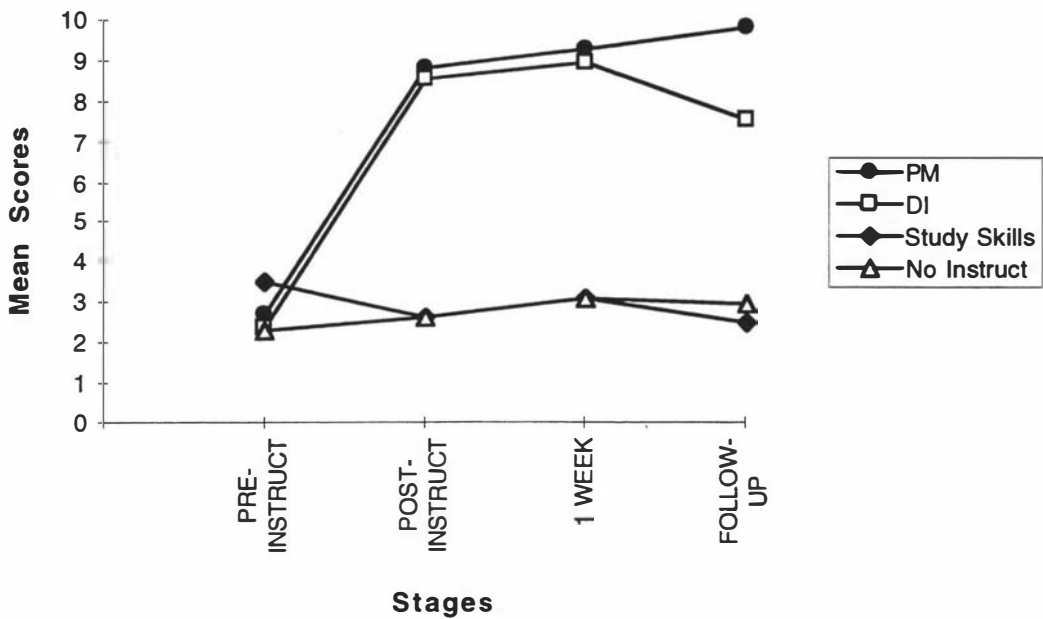


Figure 7. Division mean scores of the groups across the different stages of Experiment 1

appear to have been at least maintained 1 week later. At follow-up, some 6 to 8 weeks later, the performance of the PM and DI groups are at levels still above those at pre-instruction. However, the trends seem to be for the PM group to improve slightly in performance, and for the DI group to deteriorate – slightly in Multiplication, and quite markedly in Division.

### *Multiplication*

An analysis of variance using a repeated measures design was employed to analyse the Multiplication data. The students' TOSCA scores and PAT Maths Test scores were used as covariates in analysing the data to investigate whether these contributed significantly to the variance observed in the experiment. The students' Computational Skills Test scores were also used as a covariate because, as pointed out earlier in the Participants subsection of the Method section, a significant difference was found between the groups in this test during the Multiplication and Division phase. Thus it was important to find out whether scores in the Computational Skills Test could significantly account for the variance observed during the different stages of the experiment.

However, there was a problem in using the students' Computational Skills Test scores as a covariate in the analysis because the students' scores in this test were composed in part of the pre-instruction scores in Multiplication and Division. Thus including the Computational Skills Test scores as a covariate, and keeping the students' pre-instruction scores in the analysis as the first level of the 'stages of the experiment' variable, results in partly counting those scores twice. To investigate whether this would in any important way affect the results of the analysis, two analyses were first conducted: one which included all four levels of the stages of the experiment, and one which excluded the pre-instruction level from the stages of the experiment. Both analyses led to the same conclusions.

For the sake of consistency, the analysis including all four levels of the stages of the experiment – parallel to the Addition and Subtraction analyses – is reported here.

In Multiplication, a significant group effect was found,  $F(3, 22) = 6.70$ ,  $p = .0022$ . A significant effect was also found for the stages of the experiment,  $F(3, 66) = 5.63$ ,  $p = .0017$ . And a significant interaction effect was found between group and stages of the experiment,  $F(9, 66) = 7.49$ ,  $p = .0001$ .

No significant effects were found due to the TOSCA,  $F(1, 22) = .16$ ,  $p = .6904$ ; or to the PAT Maths Test,  $F(1, 22) = .13$ ,  $p = .7246$ .

No significant effect was found due to the Computational Skills Test,  $F(1, 22) = .14$ ,  $p = .7159$ . This meant that only a negligible amount of the variance found could be explained in terms of the Computational Skills Test scores of the participants. However, a significant interaction effect was found between the Computational Skills Test and stages of the experiment in Multiplication,  $F(3, 66) = 2.97$ ,  $p = .0380$ . This meant that the patterns of responding (reflected in scores obtained in Multiplication) of the participants, grouped according to their Computational Skills Test scores, were not consistent across the stages of the experiment. An examination of the data concerning this interaction suggests that students who had higher Computational Skills Test scores made generally larger gains at the immediate post-instruction stage than those who had lower Computational Skills Test scores. However, the students' scores in this test were not predictive of performance at the 1 week or follow-up stages.

At the pre-instruction stage, no significant effect due to group was found,  $F(3, 22) = 1.30$ ,  $p = .3000$ . However, significant group effects were found at the immediate post-instruction stage,  $F(3, 22) = 8.22$ ,  $p = .0007$ ; the 1 week stage,  $F(3, 22) = 3.59$ ,  $p = .0299$ ; and the follow-up stage,  $F(3, 22) = 12.10$ ,

$p = .0001$ . No significant effects due to the covariates were found at any of these stages, all  $p > .05$ .

The means of the groups are shown in Table 8.

**Table 8. Means of the Four Groups in Multiplication at Different Stages of Experiment 1**

Groups	Stages			
	Pre-Instruct	Post-Instruct	1 Week	Follow-Up
PM	3.14	9.00	8.86	9.00
DI	2.88	7.63	6.50	5.75
Study Skills	3.17	3.67	4.00	4.50
No Instruct	3.63	3.25	3.88	3.00

The simple main effects of the interaction between group and stages of the experiment were analysed in two ways: first, by examining the effect of group at each stage, and second, by examining the effect of the stages for each group. Tukey's Studentized Range (HSD) Test was used to analyse these simple main effects. As noted in the Addition section, this test was employed because it controls the Type I experimentwise error rate without increasing the likelihood of a Type II error (SAS/STAT User's Guide, 1990). An alpha level of .05 was used.

The comparisons of groups at each of the Multiplication stages revealed the following:

- 1 As noted earlier, no significant group effect was found at the pre-instruction stage of Multiplication, so use of the Tukey's Test was not necessary.

- 2 At immediate post-instruction, the means of the PM and DI groups had increased, so that they were significantly higher than the means of the study skills and no instruction groups. However, the PM and DI groups did not significantly differ from each other. The study skills and no instruction groups also did not significantly differ from each other.
- 3 At the 1 week stage, the mean of the PM group remained significantly higher than the means of the study skills and no instruction groups. However, the mean of the PM group did not significantly differ from the mean of the DI group. The mean score of the DI group had also decreased to the point that it was no longer significantly different from the mean scores of the study skills and no instruction groups, both of which increased slightly at this stage. The means of these latter two groups did not significantly differ from each other.
- 4 At the follow-up stage, six to eight weeks later, the PM group had a mean score that was significantly higher than the mean scores of the study skills and no instruction groups, as well as the DI group. The means of the DI, study skills, and no instruction groups did not significantly differ from each other.

To investigate the effect of the experimental stages for each of the groups, a one-way ANOVA was carried out on the data obtained for each of the groups. Tukey's Test was then employed to further examine any significant effects found. An alpha level of .05 was used. The findings were:

- 1 For the PM group, the effect of stages was significant,  $F(3, 24) = 13.42$ ,  $p = .0001$ . Tukey's Test showed that the means at the immediate post-instruction, 1 week, and follow-up stages were all significantly different from the mean at the pre-instruction stage.

However, all pairwise comparisons of means from the post-instruction stages revealed no statistically significant differences.

- 2 For the DI group, the effect of stages was significant,  $F(3, 28) = 6.47, p = .0018$ . Tukey's Test showed that the means at the immediate post-instruction and 1 week stages were significantly different from the mean at the pre-instruction stage. However, the means at the pre-instruction and follow-up stages did not significantly differ from each other, indicating that six to eight weeks after the provision of instruction the mean of this group was no longer significantly different from its mean at pre-instruction. All pairwise comparisons of means from the post-instruction stages revealed no statistically significant differences.
- 3 For the study skills group, the effect of stages was not significant,  $F(3, 20) = .42, p = .7397$ .
- 4 Similarly, for the no instruction group, the effect of stages was not significant,  $F(3, 28) = .65, p = .5911$ .

### *Division*

An analysis of variance using a repeated measures design was employed to analyse the Division data. The students' TOSCA scores and PAT Maths Test scores were used as covariates in analysing the data to investigate whether these contributed significantly to the variance observed in the experiment. The students' Computational Skills Test scores were also used as a covariate for reasons explained in the previous Multiplication section.

A significant group effect was found,  $F(3, 22) = 11.16, p = .0001$ . No significant effect was found for the stages of the experiment,  $F(3, 66) = 2.28$ ,

$p = .0872$ . However, a significant interaction effect was found between group and stages of the experiment,  $F(9, 66) = 7.04, p = .0001$ .

No significant effects were found due to the TOSCA,  $F(1, 22) = .07, p = .7951$ ; or to the PAT Maths Test,  $F(1, 22) = 1.59, p = .2205$ .

No significant effect was found due to the Computational Skills Test,  $F(1, 22) = 1.92, p = .1795$ . This meant that only a negligible amount of the variance found could be explained in terms of the Computational Skills Test scores of the students. Unlike in Multiplication, no significant interaction effect was found between the Computational Skills Test and stages of the experiment in Division,  $F(3, 66) = 2.04, p = .1165$ .

At the pre-instruction stage, no significant effect due to group was found,  $F(3, 22) = 2.58, p = .0795$ . However, significant group effects were found at the immediate post-instruction stage,  $F(3, 22) = 8.95, p = .0005$ ; the 1 week stage,  $F(3, 22) = 9.20, p = .0004$ ; and the follow-up stage,  $F(3, 22) = 14.97, p = .0001$ . A significant effect due to the Computational Skills Test was also found at the pre-instruction stage,  $F(1, 22) = 18.09, p = .0003$ . This meant that the students' scores in this test significantly contributed to variance observed at the pre-instruction stage of Division. This is not surprising since a significant difference was found between the groups in the Computational Skills Test which was composed in part of the pre-instruction score in Division, as noted earlier. However, because no significant effects due to the Computational Skills Test were found in the post-instruction stages of Division, the significant effect at the pre-instruction stage was considered as having no important bearing on the results of this experiment. No significant effects due to the other covariates, the TOSCA and the PAT Maths Test, were found at any of the Division stages, all  $p > .05$ .

The means of the four groups in Division are shown in Table 9.

**Table 9. Means of the Four Groups in Division at Different Stages of Experiment 1**

Groups	Stages			
	Pre-Instruct	Post-Instruct	1 Week	Follow-Up
PM	4.86	9.00	8.86	9.00
DI	2.50	4.75	5.88	3.63
Study Skills	2.83	3.00	2.50	2.50
No Instruct	1.88	1.25	2.00	1.88

Again, the simple main effects of the interaction between group and stages of the experiment were analysed in two ways: first, by examining the effect of group at each stage, and second, by examining the effect of the stages for each group. Tukey's Studentized Range (HSD) Test was employed to analyse these simple main effects, and an alpha level of .05 was used.

The comparisons of groups at each of the Division stages revealed the following:

- 1 As noted earlier, no significant group effect was found at the pre-instruction stage of Division, so use of the Tukey's Test was not necessary.
- 2 At immediate post-instruction, the mean of the PM group had increased and was significantly higher than the means of the DI, study skills, and no instruction groups. The mean of the DI group had also increased and was significantly higher than the mean of the no instruction group. No significant differences were found between the means of the DI group and the study skills group, or between the means of the study skills group and the no instruction group.

- 3 At the 1 week stage, the mean of the DI group had increased so that it was no longer significantly different from the mean of the PM group. The means of the PM and the DI groups were significantly higher than the means of the study skills and no instruction groups. These latter two groups, in turn, had means that were not significantly different from each other.
- 4 At the follow-up stage, six to eight weeks later, the PM group had a mean score that was significantly higher than the mean scores of the DI, study skills and no instruction groups. The means of the DI, study skills, and no instruction groups did not significantly differ from each other.

In investigating the effects of the experimental stages on each of the groups, the one-way ANOVA and Tukey's Test, with alpha level set at .05, revealed the following:

- 1 For the PM group, the effect of stages was significant,  $F(3, 24) = 10.05$ ,  $p = .0002$ . Tukey's Test showed that the means at the immediate post-instruction, 1 week, and follow-up stages were all significantly different from the mean at the pre-instruction stage. However, all pairwise comparisons of means from the post-instruction stages revealed no statistically significant differences.
- 2 For the DI group, the effect of stages was significant,  $F(3, 28) = 3.19$ ,  $p = .0390$ . Tukey's Test showed that the means at the pre-instruction stage and at the 1 week stage were significantly different from each other. All pairwise comparisons of means from the post-instruction stages revealed no statistically significant differences. However, the means at the immediate post-instruction and follow-up stages were not significantly different from the mean at the pre-instruction stage.

- 3 For the study skills group, the effect of stages was not significant,  $F(3, 20) = .10, p = .9597$ .
- 4 Similarly, for the no instruction group, the effect of stages was not significant,  $F(3, 28) = .32, p = .8137$ .

## *Discussion*

The results of the present study clearly indicate that PM instruction produced improvements in computational skills performance in students identified as having significant deficits in this area. Compared to the study skills and no instruction groups, the PM group performed significantly better during all the post-instruction stages in Addition, Multiplication, and Division. The PM group also performed better during the immediate and follow-up stages in Subtraction. Considering that the PM group did not significantly differ from the other three groups at the pre-instruction stages, the capacity of PM instruction to produce improvements in computational skills performance – as compared to providing no instruction or providing study skills instruction – has clearly been demonstrated.

The 1 week stage in Subtraction was odd since a significant effect due to group was found at this stage, with  $p = .0346$ , but an analysis of the simple main effects using Tukey's Test revealed no significant pairwise differences between the four groups. This finding may be due to both the study skills and no instruction groups' mean scores increasing slightly at this stage, while both the PM and DI groups' mean scores decreased slightly. Although none of these slight increases and decreases were significant, as confirmed by the analysis of the effects of the stages on the groups, they could have been enough to bring the groups' scores closer together, rendering the pairwise differences between the groups not significant under the Tukey's Test. Despite this single exception, it needs

to be kept in mind that the PM group performed significantly better than the study skills and no instruction groups in eleven of the twelve post-instruction stages in this experiment.

The results of the present study also indicate that the DI method of instruction is, in general, effective in teaching computational skills to students who have significant deficits in this area of learning. In at least one post-instruction stage in all of Addition, Subtraction, Multiplication, and Division, students in the DI group scored significantly higher than students in the no instruction group and/or those in the study skills group. Since the groups did not significantly differ at pre-instruction, this finding attests to the effectiveness of the DI method compared to the provision of irrelevant instruction or no instruction at all, as other researchers have previously found (e.g., Blankenship, 1978; Rivera & Smith, 1987, 1988; Smith & Lovitt, 1975). In Addition and Subtraction, the DI group consistently performed better than the study skills and no instruction groups across the post-instruction stages (again, with the exception of the 1 week stage in Subtraction). However, in Multiplication and Division the DI group's results were not as consistent: while the DI group performed better than the study skills and no instruction groups at the immediate post-instruction and/or 1 week stages, the significant differences were no longer present at the follow-up stages. This suggests that although the DI method may be useful in producing immediate to short-term improvements in Multiplication and Division skills, it does not adequately facilitate longer-term retention of those skills.

The analyses of the results showed that the study skills and the no instruction groups did not significantly differ from each other during any stages of the experiment in Addition, Subtraction, Multiplication, or Division. It is worth noting also that, in examining the effects of the experimental stages on the study skills and no instruction groups, no significant differences were found in these groups across the stages of all four computational skills areas. This clearly indicates that no significant

changes in computational skills performance were manifested by students in the study skills and no instruction groups during the duration of the study.

The study skills group was used as a placebo instruction group in the present study to provide a means of checking that any improvements shown in the other instruction groups were not simply due to the provision of additional attention and special treatment to the participants (sometimes referred to as the *Hawthorne effect*: see Brown, 1954; Tuckman, 1978). The Hawthorne effect was not observed in the present study since the study skills group did not significantly differ from the no instruction group at any of the post-instruction stages. It is indicated, therefore, that improvements made by the PM and DI groups were not likely to have been due to the attention given to the participants, their perception of the attention given to them, and the expectations associated with it.

As noted, both the PM and DI groups showed improvements in their performance after the instructions provided. A comparison of these two groups in Addition shows that the effects of the PM and DI methods of instruction were very similar. Although at the follow-up stage the mean of the PM group increased slightly while the mean of the DI group decreased slightly, the resulting difference was not significant. An examination of the effects of the experimental stages on these two groups revealed very similar patterns of performance: both groups significantly improved on their pre-instruction scores following the provision of instructions, and these improvements were maintained right through to the follow-up stage. Thus, it can be concluded that where Addition is concerned, the PM and DI methods of instruction produced equivalent improvements in the performance of students with mathematics LD.

A comparison of the effects of the PM and DI methods of instruction in Subtraction reveals response patterns very similar to those observed in

Addition. Both groups improved subsequent to the provision of instructions, and maintained these improvements through to the follow-up stage. However, while both groups had mean scores significantly higher than the mean scores of the study skills and no instruction groups at the follow-up stage, the PM and DI groups also significantly differed in their mean scores at this stage – with the PM group being higher. An examination of the effects of the experimental stages on the PM and DI groups in Subtraction reveals that both groups significantly improved on their pre-instruction scores following provision of the instructions, with these improvements being maintained through to the follow-up stage. Thus it can be concluded that in Subtraction both the PM and DI methods produced improvements in the performance of students with mathematics LD. However, because the PM group's score was significantly higher than the DI group's score at the follow-up stage, there are indications that the effects of the PM instruction may be longer lasting.

Comparing the PM and DI methods of instruction in Multiplication shows that these methods produced improvements that were not significantly different from each other during the immediate post-instruction and 1 week stages. Their mean scores, however, were significantly different at the follow-up stage. An examination of the effects of the experimental stages on these groups shows that the PM group significantly improved following instruction and maintained that improvement right through to the follow-up stage. The DI group also improved following instruction, but by the follow-up stage its score was no longer significantly different from that at pre-instruction. These findings in Multiplication indicate that while both the PM and DI methods of instruction produced improvements lasting up to a week, only the PM method produced improvements that maintained in the longer-term of six to eight weeks.

In Division, the post-instruction means of the PM and DI groups differed at the immediate post-instruction and follow-up stages, but not at the

1 week stage. An examination of the effects of the experimental stages shows that while the PM group improved in performance following the instructions provided, the DI group did not: the DI group's immediate post-instruction mean did not significantly differ from its pre-instruction mean. And while the DI group's mean score at the 1 week stage was significantly higher than its mean score at the pre-instruction stage, its mean at the follow-up stage no longer differed significantly from its mean at pre-instruction. In contrast, the improvement in performance manifested by the PM group at the immediate post-instruction stage was maintained right through to the follow-up stage.

The results of the present study attest to the effectiveness of PM instruction in producing improvements in the computational skills performance of students with mathematics LD. In contrast, the results where the DI method is concerned were not as consistent: although no significant differences were observed between the PM and DI groups during the post-instruction stages in Addition, in the other three computational skills the PM group's scores were significantly higher than the DI group's scores by the follow-up stage. Furthermore, in Multiplication and Division, the DI group no longer differed significantly from the study skills and no instruction groups by the follow-up stage.

There are a number of factors that could contribute to the differences in performance observed following the provision of the PM and DI methods of instruction. Memory is likely to be an important factor since learning always implies some form of memory use, and the failure to learn suggests memory failure and/or ineffective use. As discussed in an earlier chapter, many theorists and researchers in the LD field believe that individuals with mathematics LD have deficits in memory (e.g., Batchelor et al., 1990; Reisman & Kauffman, 1980; Swanson et al., 1990). Process mnemonics are strategies that are designed particularly to aid in remembering rules and execution of certain procedures (Higbee, 1987; Higbee & Kunihira, 1985). In the present study, the process mnemonic

strategies employed in the PM instruction were designed particularly to aid in remembering the rules and procedures involved in the computational skills of Addition, Subtraction, Multiplication, and Division. Thus they addressed a likely deficit in the learning process of the students with mathematics LD, and viewed this way the resulting improvements in performance which maintained in the longer-term are quite understandable.

Another important factor is the DI method's ability to address particular issues in learning failure that students with mathematics LD have. The DI method of instruction employed in the present study is simply a more structured and systematic approach to teaching students with LD what they had previously been taught in the classroom and failed to retain. Therefore, unless it is believed that all that is required to facilitate better retention is better structuring and systematisation of instruction, one might suspect that in the longer term the students with LD would forget the procedures taught – much in the same way they could have done following regular classroom instructions. Taking these points into account, the results obtained in the present study make sense: the method of instruction that was specifically designed to aid retention resulted in better long term maintenance of the skills acquired.

It was noted earlier that in Multiplication and Division, the groups differed significantly from each other in the Computational Skills Test prior to instruction being provided: in particular, the PM group was significantly higher than the no instruction group. The Computational Skills Test scores were used as covariates in the statistical analysis carried out on the Multiplication and Division data. This showed that the test did not significantly account for the variance observed during the different stages of the experiment. It therefore appears likely that the students' initial Computational Skills Test performance did not have a significant influence on their performance (and hence their responses to the different instructions) during the Multiplication and Division phase

of the experiment. The variance observed could better be explained in terms of group assignment and instructions provided. Hence, the significant difference between the groups in the Computational Skills Test did not have an important impact on the outcomes of the experiment and the conclusions that can be drawn.

The students' TOSCA scores, used as a covariate, did not significantly influence the students' responses to the instructional interventions. Subscribing to the premise that the TOSCA scores were approximations of students' IQs would suggest that students who were below average in 'intelligence' (but did not have serious intellectual, physical, or sensory impairments) responded to the instructions provided in a similar manner to the students who were average or above in 'intelligence.' This finding lends support to Siegel's (1989) contention that many individuals who are below average in IQ should also be considered as LD, and that they respond to remedial instruction as much as those with normal or above average IQs do.

The students' PAT Maths Test scores, also used as covariates, did not significantly influence the students' responses to the instructional interventions. This indicates that even though a more lax criterion was used where the PAT Maths Test was concerned, this did not affect the outcomes in any important way. In turn, this suggests that the aspect of mathematics LD focused on in the study – computational skills – was quite specific, and perhaps distinct from the aspects assessed by the PAT Maths Test. Therefore, the decision to use the PAT Maths Test scores only as indicators of *general* difficulties in mathematics rather than as *the* indicator of mathematics LD can be considered a sound one: it did not compromise the outcomes of, and the conclusions that can be drawn from, the study. And it enabled the inclusion in the study of students who really had significant difficulties in computation (and therefore arguably LD in mathematics) but who did not meet the strict criterion for

the PAT Maths Test perhaps simply because they did not have significant difficulties in the skills assessed by the test.

In summary, the findings of this experiment were:

- 1 PM instruction in the computational skills of Addition, Subtraction, Multiplication, and Division produced significant improvements in the performance of students with mathematics LD who were deficient in these skills areas.
- 2 In general, during the earlier stages of post-instruction, the magnitude of improvements in performance resulting from PM instruction was equivalent to that resulting from DI instruction.
- 3 Improvements in performance resulting from PM instruction maintained better in the longer term, as assessed during the follow-up stages. In contrast, the initial improvements in the performance of students provided with DI instruction diminished by the follow-up stage, significantly so in Multiplication and Division.
- 4 The Hawthorne effect was not observed in students who were provided with the study skills instruction.
- 5 The students' levels of scholastic ability, as measured by the TOSCA, did not influence their responses to the instructional interventions. If the TOSCA is accepted as providing a good approximation of IQ, this indicates that students with LD who are below average in general intelligence can equally benefit from receiving remedial instruction, at least where computational skills are concerned.

- 6 The contribution of the students' PAT Maths Test scores to the variance observed during the post-instruction stages of the experiment was insignificant. This suggests that the mathematical skills focused on in this study were largely separate from the skills assessed by the PAT Maths Test, and that the correct decision had been taken in using the PAT Maths Test only as an indicator of general mathematics difficulties, rather than as the determinant of mathematics LD.

## Chapter 7

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# Experiment 2

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### *Introduction*

The primary purpose of Experiment 2 was to replicate the results of Experiment 1, but with the use of instructors other than the present author. This use of other instructors was done primarily to examine the effectiveness and usefulness of the process mnemonic (PM) method when used by other instructors to teach computational skills to students with mathematics LD. In effect, this would investigate the possibility that unintended bias in teaching could partially explain the results of Experiment 1, since the present author (who was the instructor in Experiment 1) had a vested interest in the outcomes of the study.

Even though the written instructions for the PM and DI groups were objectively equivalent (except for the process mnemonic components of the PM instruction), it was still possible that the present author (as instructor) inadvertently provided the instructions in a manner which in some way advantaged the PM group over the other groups. For example, he could have been unintentionally more enthusiastic in teaching the PM group, or more encouraging of the students in that group. Thus, by using other instructors without a vested interest in the outcomes of the study, this potential confounding variable could be eliminated in Experiment 2.

It was also important to investigate the ease with which other instructors could use the PM method described in this study, as well as the effectiveness of the method in teaching computational skills to students with mathematics LD when used by these instructors. If the PM method of teaching computational skills is to have a genuine practical application

in remedial classrooms, it needs to be reasonably easy for teachers to learn and use. Its effectiveness needs to be demonstrated, not just when used by its originator, but also when used by other instructors. In essence, it is necessary to examine the question of whether the effectiveness of the instructions used depends in any important way on the instructors.

Since the group that was provided the placebo instruction in Experiment 1, the study skills group, did not perform differently from the no instruction group at any of the stages of either the Addition and Subtraction phase or the Multiplication and Division phase, it was decided to discard this group in Experiment 2. It had served its purpose in Experiment 1, confirming that the changes in performance levels of the instruction groups could not be attributed to the Hawthorne effect (see Brown, 1954; Tuckman, 1978). As there was no reason to suspect otherwise in conducting Experiment 2, the no instruction group was the only control group in this second experiment.

The results of Experiment 1 indicated that the PM method of instruction produced improvements in computational skills performance that maintained better in the longer term compared to the DI method. These results are important and warranted re-examination in Experiment 2.

In Experiment 2, with different instructors, the intention was to find out whether results would indicate approximately equivalent magnitudes of improvement resulting from the PM and DI methods of instruction, but with better maintenance resulting from the PM method. It would have also been ideal to conduct longer term follow-up assessments on the students, but scheduling constraints and considerations about disruptions caused to the students' regular classroom attendance effectively prohibited these. In order to investigate levels of maintenance in Experiment 2, four follow-up assessments were scheduled. These occurred immediately after instruction, one week after instruction, four weeks after, and eight weeks after. The reason for including an assessment four weeks after instruction

was to obtain a clearer and more continuous picture of maintenance patterns following the instructions provided. An additional improvement in Experiment 2 was the administration of the final follow-up at a standard time interval.

In Experiment 1, the results indicated that neither the students' TOSCA scores nor their PAT Maths Test scores had significant effects on their post-instruction performance. It was important to confirm this result in Experiment 2 because of wider issues relating to participants' TOSCA and PAT Maths scores. The first of these issues concerns the relevance of IQ in LD research involving the provision of remedial instruction, and the second concerns the relative independence of the computational skills assessed in the present study from the general mathematics skills assessed by the PAT Maths Test. As noted earlier, two arguments put forward in the present study were that: (i) participants who are "below average" in their IQ or other 'ability' test score could similarly benefit from remedial instruction, and therefore need not be excluded from such programmes or research; and (ii) the PAT Maths Test, while arguably a good measure of mathematics reasoning ability, is not a good measure of computational skills ability. Evidence supporting these arguments was found in Experiment 1, and it was important to investigate the extent to which these arguments were supported in Experiment 2.

However, putting these other issues aside, the main hypotheses of Experiment 2 were basically the same as those of Experiment 1. The first hypothesis was that compared to no instruction, PM instruction would produce improvements in the computational skills performance of students identified as having significant deficits in this area. The second hypothesis was that compared to DI instruction, PM instruction would produce greater improvements in the computational skills performance of these same students. The third hypothesis was that compared to DI instruction again, improvements in students' performance resulting from PM instruction would maintain better over time.

## *Method*

### **Instructors**

The instructors in this experiment were two women, aged 26 years (instructor 1) and 21 years (instructor 2). One had completed a Masters degree in Psychology and was working as a part-time tutor in the Psychology Department of the University of Auckland (instructor 1), and the other was in her first year of a Masters degree in Psychology (instructor 2).

Both instructors had some knowledge about learning disabilities through courses they had taken. One had had a few years' experience teaching tutorials at the undergraduate level (instructor 1), while the other had only taught students on an individual basis (instructor 2). Both were recommended by the present author's colleagues in the Psychology Department of the University of Auckland. Through a research grant that the present author was awarded by the Dyslexia Learning Foundation, both instructors were paid on an hourly basis for the work they carried out. The instructors were each provided with a total of seven hours of training in the use of both the PM and the DI methods of instruction. The training sessions involved the present author demonstrating the use of the instruction methods, and the instructors getting practice in the use of those methods. Each instructor was assigned to a single school.

### **Participants**

The experimental participants were 28 Form 3 students from two schools in Auckland. One school was located on the North Shore of Auckland. It had a student population of approximately 1200 students. The majority of students in this school were of European descent, although there were significant numbers of students from Polynesian and other ethnic

minority groups. The other school was located in West Auckland, with a student population of approximately 700 students. The majority of students in this school were Polynesians (Maori and Pacific Island students), although there were significant numbers of students of European and other ethnic minority descent. Both schools were public schools, and both were co-educational (i.e., attended by both male and female students).

Eighteen of the students came from one school, and ten came from the other. Each school had students in all three groups of the study: PM, DI, and no instruction groups. The experiment was commenced, and consequently completed, a number of weeks earlier in one of the schools.

Eleven of the student participants were females, and seventeen were males. The students' ages ranged from 13 years and 1 month to 14 years and 6 months at the time of initial testing.

All students of Experiment 2 had been put forward by their teachers as low achieving in mathematics in comparison to their peers, but without any serious intellectual, physical, or sensory impairments to which the underachievement could be attributed. The initial pool of recommended students were first administered the Computational Skills Test and the Basic Facts Test. Those who met the criteria required for these tests were administered the PAT Mathematics Test-Revised (PAT Maths Test) on the next testing sessions scheduled. Those who met the PAT Maths Test criterion were then administered the TOSCA.

Thirty students met the criteria, previously discussed, for the Computational Skills Test, the Basic Facts Test, the PAT Maths Test, and the TOSCA. Ten of these students were assigned to the PM instruction group, nine to the DI instruction group, and eleven to the no instruction group. However, prior to completion of instructions during the Addition and Subtraction phase, one student withdrew from the PM group, and

another withdrew from the DI group. Thus, the PM group consisted of nine students, and the DI group of eight students. Since incomplete data sets were collected from the two students who withdrew from the study, they were not included in the ensuing analyses of data: only the results of the 28 students who remained were analysed and reported here.

The student participants remained in the groups they were assigned to for both phase 1, the Addition and Subtraction phase, and phase 2, the Multiplication and Division phase. That is, they were not re-assigned to different groups for phase 2, as was done in Experiment 1. The mean scores of the students in the tests used for selection are depicted in Table 10 below, according to the groups they were randomly assigned to.

**Table 10. Mean Scores in the Selection Tests for the Three Instructional Groups, Experiment 2**

Test	Groups		
	PM (N=9)	DI (N=8)	No Instruction (N=11)
Computational Skills Test (/40)	7.89	5.63	7.27
Basic Facts Test (/40)	36.33	37.38	36.00
PAT Maths Test (/50)	13.67	12.50	10.64
TOSCA (stanine)	3.11	3.38	2.82

An alpha level of .05 was used for all statistical tests, except where stated otherwise.

Using an analysis of variance, it was found that the groups did not differ significantly from each other in three of the four tests used for selection.

No significant differences were found across the groups in the Computational Skills Test,  $F(2, 25) = 1.05$ ,  $p = .3655$ ; the Basic Facts Test,  $F(2, 25) = .46$ ,  $p = .6364$ ; or the TOSCA,  $F(2, 25) = 1.11$ ,  $p = .3451$ . This meant that where these three tests were concerned, the students in the three groups of the experiment could be considered equivalent to each other.

However, a significant difference between the groups was found in the PAT Maths Test,  $F(2, 25) = 3.79$ ,  $p = .0364$ . Tukey's Studentized Range (HSD) Test with alpha level at .05, was employed to determine the exact nature of the difference found. It showed that while the PM and DI groups did not significantly differ from each other and the DI and no instruction groups did not significantly differ from each other, the PM group was significantly higher than the no instruction group. This difference between groups necessitated an examination of the extent to which the PAT Maths Test scores of the students contributed to variance observed during the different stages of the experiment.

## Procedure

Subsequent to the administration of the tests used for selection (described in the previous subsection), the students selected were randomly assigned to one of the three groups of the study. As previously noted, the students remained in the groups they were assigned to for both phases of the experiment (Addition and Subtraction, and Multiplication and Division).

In each school there was only one of each of the PM, the DI, and the no instruction groups. Group sizes ranged from two to seven students (there were fewer participants in one of the schools which resulted in group sizes being smaller). As in Experiment 1, instruction sessions were carried out either in regular classrooms not in use during the period concerned, or in slightly smaller remedial classrooms, depending on what was

available. In all cases, the classrooms had blackboards for instructional use.

The PM and DI methods of teaching the groups were described in the General Methods chapter. In each phase of Experiment 2, the PM and DI groups were each provided with five instruction sessions. Sessions 1 and 3 dealt with Addition in phase 1, and Multiplication in phase 2. Sessions 2 and 4 dealt with Subtraction in phase 1, and Division in phase 2. Session 5 provided a review of the computational skills taught followed by the immediate post-instruction assessment. The no instruction groups were also administered the immediate post-instruction assessment around the time the PM and DI groups were assessed in session 5.

Each instruction session was approximately 25 minutes in duration, and usually each of the instruction groups were seen twice a week.

All three groups were again administered the relevant computational skills assessments one week after session 5, at four weeks, and at eight weeks after the cessation of instruction in session 5. The schedules for instruction and assessment are depicted in Table 11 below.

**Table 11. Schedules for Instruction and Assessment of the Three Groups in Experiment 2.**

Sessions	Groups		
	PM	DI	No Instruction
1	Addition or Multiplication	Addition or Multiplication	n/a
2	Subtraction or Division	Subtraction or Division	n/a
3	Addition or Multiplication	Addition or Multiplication	n/a
4	Subtraction or Division	Subtraction or Division	n/a
5	review, then assessment	review, then assessment	assessment
1 week later	assessment	assessment	assessment
4 weeks later	assessment	assessment	assessment
8 weeks later	assessment	assessment	assessment

## *Results*

### **Addition and Subtraction Phase**

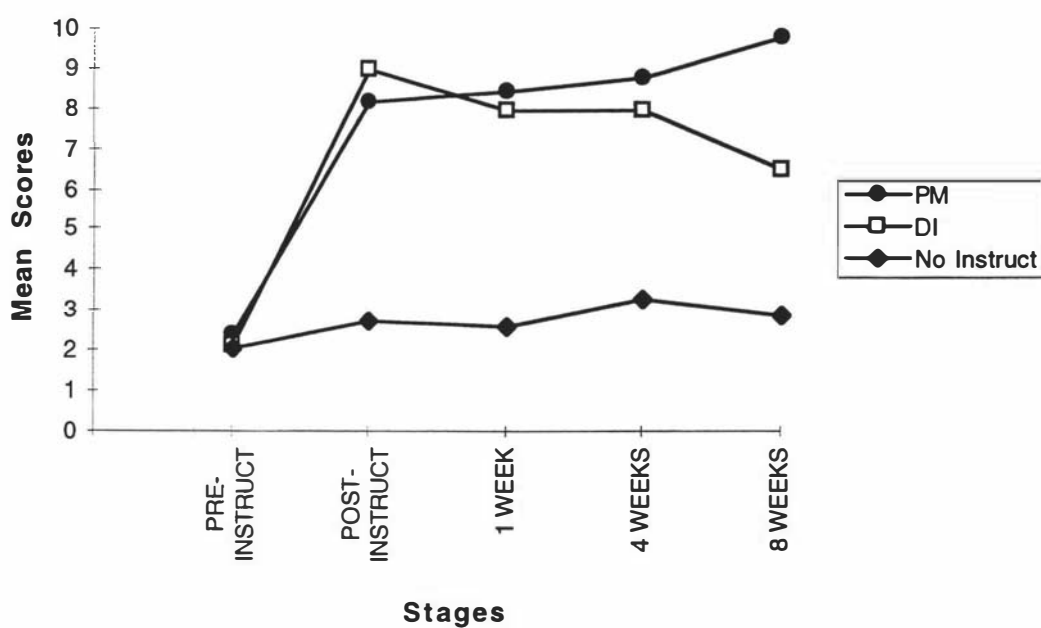
The mean scores in Addition at the pre-instruction, immediate post-instruction, 1 week, 4 weeks, and follow-up (8 weeks) stages for each of the PM, DI, and no instruction groups are plotted in Figure 8. Similarly, the

mean scores in Subtraction are plotted in Figure 9. From these plots, it appears that for both Addition and Subtraction, the three groups were approximately equivalent at pre-instruction. At immediate post-instruction, the PM and DI groups show noticeable improvements in their scores, the magnitudes of which appear approximately equivalent. In Addition, the mean scores of the PM and DI groups appear to progressively diverge from the 1 week stage to the 8 week stage, with the PM group increasing slightly and the DI group decreasing. In Subtraction, both the PM and DI groups' mean scores increased at the 1 week stage. However, while the PM group's mean score continued to increase at the 4 week stage, the DI group decreased very slightly. And at 8 weeks, the PM group decreased very slightly in its mean score, while the DI group had a noticeable drop. The no instruction group's mean scores in both Addition and Subtraction remained approximately stable across the stages of the experiment.

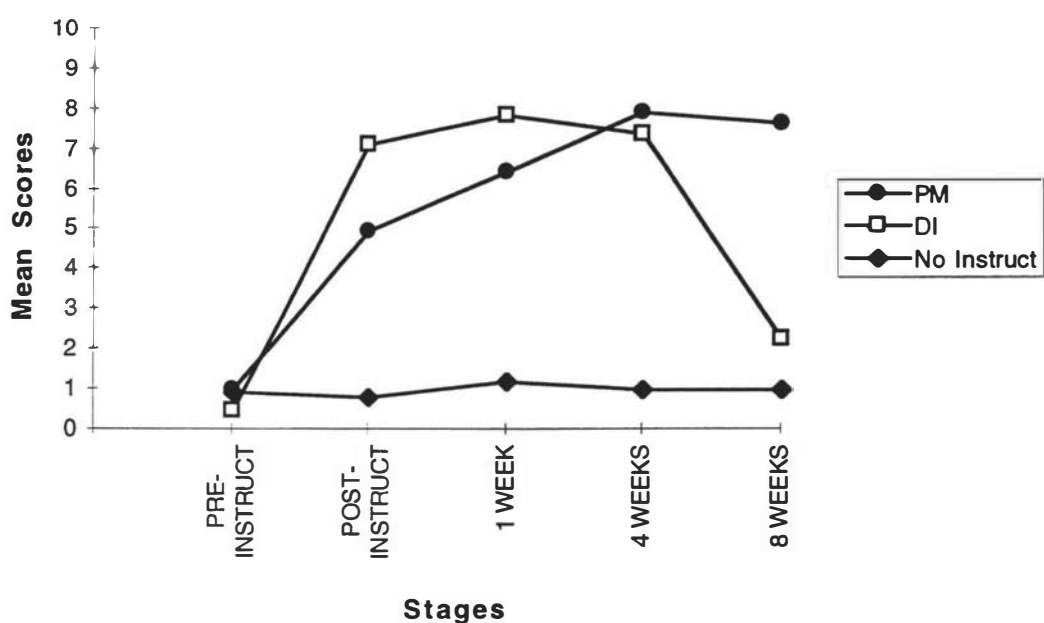
### *Addition*

An analysis of variance using a repeated measures design was employed to analyse the Addition data. The students' TOSCA scores and PAT Maths Test scores were used as covariates in analysing the data to investigate whether these contributed significantly to the variance observed in the experiment. Where the PAT Maths Test scores were concerned, it was particularly important to investigate this contribution to the variance observed because, as noted in the Participants subsection of the Methods section, the groups significantly differed on this test.

A significant group effect was found in Addition,  $F(2, 23) = 31.55, p = .0001$ . However, no significant effects due to the covariates were found: for the TOSCA,  $F(1, 23) = 2.21, p = .1509$ ; and for the PAT Maths Test,  $F(1, 23) = 1.21, p = .2829$ .



**Figure 8.** Addition mean scores of the groups across the different stages of Experiment 2



**Figure 9.** Subtraction mean scores of the groups across the different stages of Experiment 2

The effect due to the stages of the experiment was significant,  $F(4, 92) = 4.12, p = .0041$ . The interaction between groups and stages of the experiment was also significant,  $F(8, 92) = 9.79, p = .0001$ . The nature of this interaction is clearly seen in Figure 8.

At the pre-instruction stage in Addition, no significant effect due to group was found,  $F(2, 23) = .17, p = .8484$ . However, significant group effects were found at the immediate post-instruction stage,  $F(2, 23) = 30.51, p = .0001$ ; the 1 week stage,  $F(2, 23) = 28.68, p = .0001$ ; the 4 week stage,  $F(2, 23) = 15.92, p = .0001$ ; and the 8 week stage,  $F(2, 23) = 17.55, p = .0001$ .

The means of the three groups are shown in Table 12.

**Table 12. Means of the Three Groups in Addition at the Different Stages of Experiment 2**

Groups	Stages				
	Pre-Instr	Post-Instr	1 Week	4 Weeks	8 Weeks
PM	2.44	8.22	8.44	8.78	9.78
DI	2.13	9.00	8.00	8.00	6.50
No Instr	2.09	2.73	2.64	3.27	2.91

As in Experiment 1, the simple main effects of the interaction between groups and stages of the experiment were analysed in two ways: first, by examining the effect of group at each stage, and second, by examining the effect of the stages for each group. Tukey's Studentized Range (HSD) Test was used to analyse the simple main effects because this test controls the Type I experimentwise error rate without increasing the likelihood of a Type II error (SAS/STAT User's Guide, 1990).

Using an alpha level of .05, the comparisons of groups at each of the stages revealed the following:

- 1 As noted earlier, no significant group effect was found at the pre-instruction stage of Addition, so use of the Tukey's Test was not necessary.
- 2 At immediate post-instruction, the means of the PM and DI groups had increased, so that they were significantly higher than the mean of the no instruction group. However, the PM and DI groups did not significantly differ from each other.
- 3 The results were much the same 1 week later, with the PM and DI groups not being significantly different from each other, but being significantly higher in their mean scores than the no instruction group.
- 4 At 4 weeks, the PM and DI groups remained not significantly different from each other, but with mean scores significantly higher than the mean score of the no instruction group.
- 5 At the 8 week stage, the mean of the PM group had slightly increased, while the mean of the DI group had slightly decreased. This rendered their mean scores significantly different from each other. Both remained significantly higher in their means compared to the no instruction group.

To investigate the effect of the experimental stages for each of the groups, a one-way analysis of variance was carried out on the data obtained for each of the groups. Tukey's Test was then employed to further examine any significant effects found. An alpha level of .05 was used. The findings were:

- 1 For the PM group, the effect of stages was significant,  $F(4, 40) = 21.86$ ,  $p = .0001$ . Tukey's Test showed that the means at the immediate post-instruction, 1 week, 4 week, and 8 week stages were

all significantly higher compared to the mean at the pre-instruction stage. However, all pairwise comparisons of means from the post-instruction stages revealed no statistically significant differences.

- 2 For the DI group, the effect of stages was significant,  $F(4, 35) = 21.87$ ,  $p = .0001$ . Tukey's Test showed that the means at the immediate post-instruction, 1 week, 4 week, and 8 week stages were all significantly higher compared to the mean at the pre-instruction stage. However, it is interesting to note that while the mean at the 8 week stage did not significantly differ from the means at the 1 week and 4 week stages, it was nevertheless significantly lower than the mean at the immediate post-instruction stage. The means at the 1 week and 4 week stages did not significantly differ from the mean at the immediate post-instruction stage. Thus, the only significant difference among the post-instruction means was between the immediate post-instruction stage and the 8 week stage.
- 3 For the no instruction group, the effect of the stages was not significant,  $F(4, 50) = .54$ ,  $p = .7057$ .

### *Subtraction*

An analysis of variance using a repeated measures design was employed to analyse the Subtraction data. The students' TOSCA scores and PAT Maths Test scores were used as covariates in analysing the data to investigate whether these contributed significantly to the variance observed in the experiment.

A significant group effect was found,  $F(2, 23) = 24.64$ ,  $p = .0001$ . However, no significant effects due to the covariates were found: for the TOSCA,  $F(1, 23) = .02$ ,  $p = .9021$ ; and for the PAT Maths Test,  $F(1, 23) = .10$ ,  $p = .7517$ .

The effect due to the stages of the experiment was not significant,  $F(4, 92) = 1.94, p = .1104$ . However, a significant interaction was found between group and stages of the experiment,  $F(8, 92) = 17.66, p = .0001$ . The nature of this interaction is clearly seen in Figure 9.

At the pre-instruction stage in Subtraction, no significant effect due to group was found,  $F(2, 23) = .94, p = .4041$ . However, significant group effects were found at the immediate post-instruction stage,  $F(2, 23) = 17.20, p = .0001$ ; the 1 week stage,  $F(2, 23) = 17.86, p = .0001$ ; the 4 week stage,  $F(2, 23) = 37.75, p = .0001$ ; and the 8 week stage,  $F(2, 23) = 22.92, p = .0001$ .

The means of the three groups are shown in Table 13.

**Table 13. Means of the Three Groups in Subtraction at the Different Stages of Experiment 2**

Groups	Stages				
	Pre-Instr	Post-Instr	1 Week	4 Weeks	8 Weeks
PM	1.00	5.00	6.44	7.89	7.67
DI	.50	7.13	7.88	7.38	2.25
No Instr	.91	.82	1.18	1.00	1.00

As in Addition, the simple main effects of the interaction between groups and stages of the experiment were analysed by examining the effect of group at each stage, and by examining the effect of the stages for each group. Tukey's Studentized Range (HSD) Test was used to analyse the simple main effects, with the alpha level set at .05.

The comparisons of groups at each of the stages revealed the following:

- 1 As noted earlier, no significant group effect was found at the pre-instruction stage of Subtraction, so use of the Tukey's Test was not necessary.

- 2 At immediate post-instruction, the means of the PM and DI groups had increased, so that they were significantly higher than the mean of the no instruction group. However, the PM and DI groups did not significantly differ from each other.
- 3 Similar results were obtained 1 week later, with the PM and DI groups not being significantly different from each other, but being significantly higher in their mean scores than the no instruction group.
- 4 At 4 weeks, the PM and DI groups remained not significantly different from each other, and both remained significantly higher than the no instruction group.
- 5 At the 8 week stage, the mean of the PM group remained approximately the same as at the 4 week stage, while the mean of the DI group decreased considerably. This rendered the mean score of the PM group significantly higher than the mean scores of the DI and no instruction groups. The mean score of the DI group was no longer significantly different from the mean score of the no instruction group at this stage.

A one-way analysis of variance was carried out on the Subtraction data obtained for each of the groups. The purpose of this was to investigate the effect of the experimental stages on each of the groups. Tukey's Test was employed to further examine any significant effects found. An alpha level of .05 was used. The findings were:

- 1 For the PM group, the effect of stages was significant,  $F(4, 40) = 14.79$ ,  $p = .0001$ . Tukey's Test showed that the means at the immediate post-instruction, 1 week, 4 week, and 8 week stages were all significantly higher compared to the mean at the pre-instruction

stage. However, all pairwise comparisons of means from the post-instruction stages revealed no statistically significant differences.

- 2 For the DI group, the effect of stages was significant,  $F(4, 35) = 18.27$ ,  $p = .0001$ . Tukey's Test showed that the means at the immediate post-instruction, 1 week, and 4 week stages were all significantly higher compared to the mean at the pre-instruction stage. However, the mean at the 8 week stage did not significantly differ from the mean at the pre-instruction stage. Pairwise comparisons of the DI group's Subtraction means at the immediate post-instruction, 1 week, and 4 week stages revealed no significant differences.
- 3 For the no instruction group, the effect of the stages was not significant,  $F(4, 50) = .19$ ,  $p = .9417$ .

### **Multiplication and Division Phase**

The mean scores in Multiplication at pre-instruction, immediate post-instruction, 1 week later, 4 weeks later, and 8 weeks later for each of the PM, DI, and no instruction groups are plotted in Figure 10. The respective mean scores in Division are plotted in Figure 11. From these plots, it appears that for both Multiplication and Division, the three groups were approximately equivalent at the pre-instruction stage. The PM and DI groups' scores improve at immediate post-instruction, although the magnitude of the PM group's improvements appear greater than the DI group's. At the 1 week, 4 week, and 8 week stages, scores for the PM group appear to improve slightly. In contrast, for the same stages, the DI group displays quite erratic trends, with alternating increases and decreases in its scores. The no instruction group displays slight but noticeable improvements in its scores at the immediate post-instruction stages, in both Multiplication and Division. These slight improvements, however,

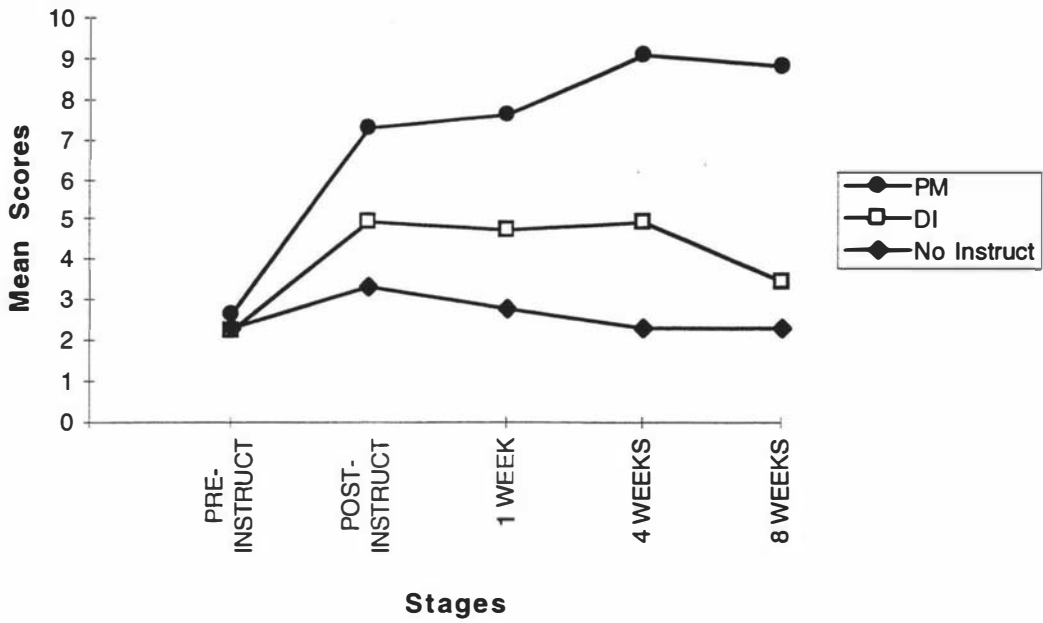


Figure 10. Multiplication mean scores of the groups across the different stages of Experiment 2

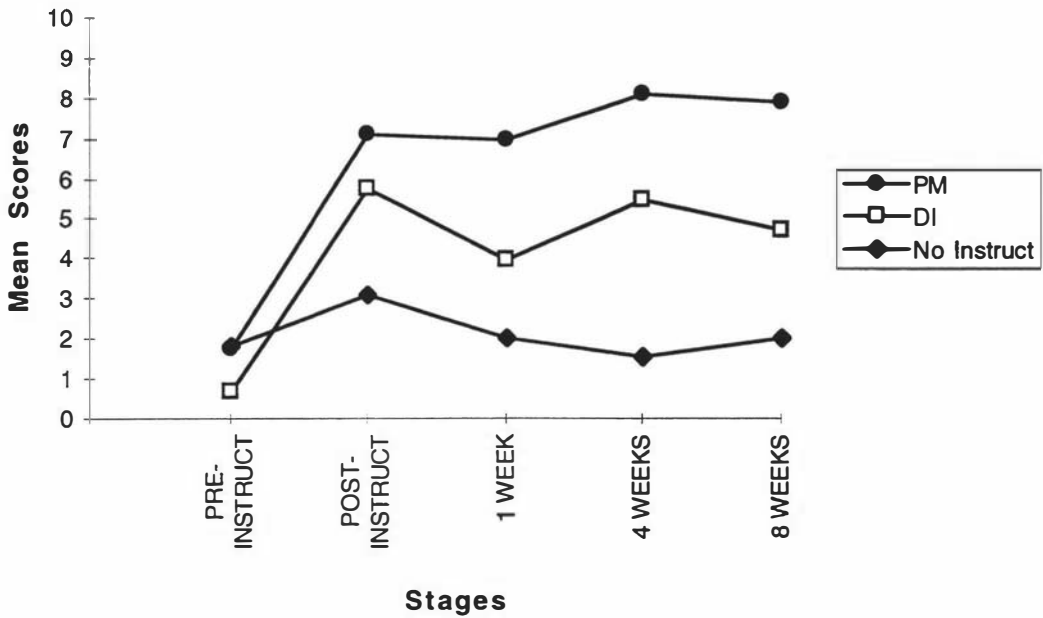


Figure 11. Division mean scores of the groups across the different stages of Experiment 2

were not maintained and were no longer evident in the subsequent stages.

### *Multiplication*

An analysis of variance using a repeated measures design was employed to analyse the Multiplication data. The students' TOSCA scores and PAT Maths Test scores were used as covariates in analysing the data to investigate whether these contributed significantly to the variance observed in the experiment.

A significant group effect was found,  $F(2, 23) = 21.01, p = .0001$ . No significant effects were found due to the TOSCA,  $F(1, 23) = .63, p = .4340$ ; and to the PAT Maths Test,  $F(1, 23) = .35, p = .5582$ .

The effect due to the stages of the experiment was not significant,  $F(4, 92) = 2.03, p = .0961$ . However, a significant interaction was found between group and stages of the experiment,  $F(8, 92) = 6.73, p = .0001$ . The nature of this interaction can be seen in Figure 10.

At the pre-instruction stage in Multiplication, no significant effect due to group was found,  $F(2, 23) = .20, p = .8198$ . However, significant group effects were found at the immediate post-instruction stage,  $F(2, 23) = 6.12, p = .0074$ ; the 1 week stage,  $F(2, 23) = 5.96, p = .0082$ ; the 4 week stage,  $F(2, 23) = 34.44, p = .0001$ ; and the 8 week stage,  $F(2, 23) = 35.85, p = .0001$ .

The means of the three groups are shown in Table 14.

**Table 14. Means of the Three Groups in Multiplication at the Different Stages of Experiment 2**

Groups	Stages				
	Pre-Instr	Post-Instr	1 Week	4 Weeks	8 Weeks
PM	2.67	7.33	7.67	9.11	8.89
DI	2.25	5.00	4.75	5.00	3.50
No Instr	2.36	3.36	2.82	2.36	2.36

Again, the simple main effects of the interaction between groups and stages of the experiment were analysed by examining the effect of group at each stage, and by examining the effect of the stages for each group. Tukey's Studentized Range (HSD) Test was used to analyse the simple main effects, with the alpha level set at .05.

The comparisons of groups at each of the stages revealed the following:

- 1 No significant group effect was found at the pre-instruction stage of Multiplication, so use of the Tukey's Test was not necessary.
- 2 At immediate post-instruction, the mean of the PM group had increased so that it was significantly higher than the mean of the no instruction group. However, the PM group did not differ significantly from the DI group; nor did the DI group differ significantly from the no instruction group.
- 3 At the 1 week stage, the mean of the PM group was significantly higher than the means of the DI and no instruction groups. These latter two groups did not significantly differ from each other where their means were concerned.
- 4 At 4 weeks, the PM group's mean score remained significantly higher than the mean scores of the DI and no instruction groups.

The DI group's mean score had increased slightly, while the no instruction group's mean score had decreased slightly: these changes rendered the DI group mean significantly higher than the no instruction group mean at this stage.

- 5 At the 8 week stage, the mean of the PM group was significantly higher than the means of the DI and no instruction groups. The latter two groups did not differ significantly from each other.

A one-way analysis of variance was carried out on the data obtained for each of the groups to investigate the effect of the experimental stages on these. Tukey's Test was employed to further examine any significant effects found. An alpha level of .05 was used. The findings were:

- 1 For the PM group, the effect of stages was significant,  $F(4, 40) = 19.51, p = .0001$ . Tukey's Test showed that the means at the immediate post-instruction, 1 week, 4 week, and 8 week stages were all significantly higher compared to the mean at the pre-instruction stage. However, all pairwise comparisons of means from the post-instruction stages revealed no statistically significant differences.
- 2 For the DI group, the effect of the stages was not significant,  $F(4, 35) = 1.91, p = .1299$ .
- 3 For the no instruction group, the effect of the stages was also not significant,  $F(4, 50) = 1.08, p = .3768$ .

### *Division*

An analysis of variance using a repeated measures design was employed to analyse the Division data. The students' TOSCA scores and PAT Maths Test scores were again used as covariates in analysing the data to

investigate whether these contributed significantly to the variance observed in the experiment.

A significant group effect was found,  $F(2, 23) = 10.41, p = .0006$ . No significant effects were found due to the TOSCA,  $F(1, 23) = .13, p = .7255$ ; or to the PAT Maths Test,  $F(1, 23) = 1.41, p = .2479$ .

The effect due to the stages of the experiment was not significant,  $F(4, 92) = .67, p = .6157$ . However, a significant interaction effect was found between group and stages of the experiment,  $F(8, 92) = 5.02, p = .0001$ . The nature of this interaction can be seen in Figure 11.

At the pre-instruction stage in Division, no significant effect due to group was found,  $F(2, 23) = 2.09, p = .1465$ . The effect due to group was also not significant at the 1 week stage,  $F(2, 23) = 3.33, p = .0539$ . However, significant group effects were found at the immediate post-instruction stage,  $F(2, 23) = 4.04, p = .0314$ ; the 4 week stage,  $F(2, 23) = 22.86, p = .0001$ ; and the 8 week stage,  $F(2, 23) = 13.93, p = .0001$ .

The means of the three groups are shown in Table 15.

**Table 15. Means of the Three Groups in Division at the Different Stages of Experiment 2**

Groups	Stages				
	Pre-Instr	Post-Instr	1 Week	4 Weeks	8 Weeks
PM	1.78	7.11	7.00	8.11	7.89
DI	.75	5.75	4.00	5.50	4.75
No Instr	1.91	3.18	2.09	1.64	2.09

As with the other computational skills, the simple main effects of the interaction between groups and stages of the experiment in Division were analysed by examining the effect of group at each stage, and by examining

the effect of the stages for each group. Tukey's Studentized Range (HSD) Test was used to analyse the simple main effects, with the alpha level set at .05.

The comparisons of groups at each of the stages revealed the following:

- 1 As noted earlier, no significant group effect was found at the pre-instruction stage of Division, so use of the Tukey's Test was not necessary.
- 2 At immediate post-instruction, the mean of the PM group had increased so that it was significantly higher compared to the mean of the no instruction group. However, the means of the PM and DI groups did not significantly differ from each other, nor did the means of the DI and no instruction groups.
- 3 No significant group effect was found at the 1 week stage of Division, so use of the Tukey's Test was not necessary.
- 4 At the 4 week stage, the mean score of the PM group was significantly higher than the mean scores of the DI and no instruction groups. The mean score of the DI group was also significantly higher than the mean score of the no instruction group.
- 5 The findings were the same at the 8 week stage: the mean score of the PM group was significantly higher than the mean scores of the DI and no instruction groups, and the mean score of the DI group was significantly higher than the mean score of the no instruction group.

A one-way analysis of variance was carried out on the Division data obtained for each of the groups, so as to investigate the effect of the

experimental stages on these. Tukey's Test, with the alpha level set at .05, was employed to examine further any significant effects found. The findings were:

- 1 For the PM group, the effect of stages was significant,  $F(4, 40) = 10.61$ ,  $p = .0001$ . Tukey's Test showed that the means at the immediate post-instruction, 1 week, 4 week, and 8 week stages were all significantly higher compared to the mean at the pre-instruction stage. However, all pairwise comparisons of means from the post-instruction stages revealed no statistically significant differences.
- 2 For the DI group, the effect of stages was significant,  $F(4, 35) = 4.92$ ,  $p = .0030$ . Tukey's Test showed that the means at the immediate post-instruction, 4 week, and 8 week stages were all significantly higher compared to the mean at the pre-instruction stage. The mean at the 1 week stage did not differ significantly from the means at the immediate post-instruction, 4 week, and 8 week stages; however, neither did it differ significantly from the mean at the pre-instruction stage.
- 3 For the no instruction group, the effect of the stages was not significant,  $F(4, 50) = 1.53$ ,  $p = .2080$ .

## *Discussion*

The results of the present study were very similar to those of Experiment 1. They clearly showed the effectiveness of PM instruction in teaching computational skills to students with significant deficits in this area. The PM group did not significantly differ from the no instruction group at pre-instruction in any of the four computational skills. However, following instruction, the PM group performed significantly better than the no instruction group in all of the post-instruction stages in all of the

computational skills, except at the 1 week stage in Division when no significant effect due to group was found.

The 1 week stage in Division deserves comment. At this stage, compared to the immediate post-instruction stage, the scores of all three groups decreased, and the standard deviations of the PM and DI groups increased. These factors would have been the reasons that the  $p$  value obtained for group effect at the 1 week stage just failed to meet the alpha level of .05 that was used ( $p = .0539$ ).<sup>1</sup>

The results of this experiment also indicate that the DI method of instruction is effective, in a somewhat inconsistent way, in teaching computational skills to students who have significant deficits in this area of mathematics. In at least one post-instruction stage in all of Addition, Subtraction, Multiplication, and Division, students in the DI group scored significantly higher than students in the no instruction group. The DI group and the no instruction group did not differ significantly at the pre-instruction stages, so these results clearly indicate improvements as a result of the DI method of instruction provided.

The results obtained in this experiment for the DI method of instruction are not as clear-cut as those obtained in Experiment 1. The pattern of immediate improvement, but poorer long term maintenance, that was observed in Experiment 1 was not found in Experiment 2. For example, while in Subtraction the DI method produced improvements in performance that did not maintain to the 8 week stage, in Division the improvements in performance it produced were only evident (relative to the no instruction group) in the longer term. Nor did the DI method of instruction always produce immediate post-instruction improvements: it failed to do so in Multiplication and in Division.

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<sup>1</sup> Because the  $p$  value obtained was so close to the critical alpha level employed, the Tukey's Test result generated by the statistical programme was examined: according to this, the PM group was significantly higher than the no instruction group at this stage.

The no instruction group in this experiment did not manifest any significant changes in computational skills performance. This indicates that any changes in computational skills performance shown by the PM and DI groups were likely to have been the result of the instructional interventions provided to these groups.

#### *Comparison of the PM and DI groups*

The results showed that both the PM and DI groups manifested improvements in their performance following the instructions provided. Comparisons between these two groups are therefore useful. In Addition, during the post-instruction stages, the two groups did not significantly differ from each other until the 8 week stage, when the PM group's mean score became significantly higher than the DI group's mean score. Examining the effect of stages of the experiment on the two groups showed that the PM group made a significant improvement in performance at the immediate post-instruction stage and maintained that level of performance right through to the 8 week stage. The DI group's pattern of performance was similar, except by the 8 week stage its level of performance – while significantly higher than at pre-instruction – was nevertheless also significantly lower than at immediate post-instruction. This suggests a tendency towards poorer maintenance in the longer term following DI instruction.

Comparing the effects of the PM and DI methods of instruction in Subtraction shows that, compared to the provision of no instruction, these methods produced improvements in performance that were not significantly different from each other at the immediate post-instruction, 1 week, and 4 week stages. However, at the 8 week stage, while the improvement resulting from the PM instruction maintained, the improvement resulting from the DI instruction did not: by the 8 week stage, the mean score of the DI group was no longer significantly different from the mean score of the no instruction group. Examining the effects of

the experimental stages on the PM and DI groups in Subtraction reveals that the PM group made a significant improvement in performance at immediate post-instruction and maintained that improvement right through to the 8 week stage. In contrast, the DI group also made a significant improvement in its performance at the immediate post-instruction stage but maintained that level of performance only through to the 4 week stage. By the 8 week stage, the DI group's mean score had adequately deteriorated that it was no longer significantly different from its mean score at pre-instruction. These results suggest that while the PM method is capable of producing improvements in Subtraction performance that maintain in the longer term, improvements in performance resulting from the DI method appear not to maintain in the longer term.

In Multiplication, the PM method of instruction produced an improvement in performance at the immediate post-instruction stage that rendered the PM group's score significantly higher than the no instruction group's score but not the DI group's score. The PM group's scores were significantly higher than the no instruction group's scores during all post-instruction stages, and higher than the DI group's scores at the 1 week, 4 week, and 8 week stages. In contrast, the DI group's score was only significantly higher than the no instruction group's score at the 4 week stage. Looking at the effects of the experimental stages on the PM and DI groups reveals that the PM group improved at immediate post-instruction and maintained that improvement through to the 8 week stage. In contrast, no significant effect due to stages of the experiment was found in the DI group: its scores during the post-instruction stages did not significantly differ from its score at the pre-instruction stage. These results suggest that while the PM method is effective in producing improvements in Multiplication performance that maintain in the longer term, the DI method is not.

In the Division post-instruction stages, the means of the PM and DI groups did not significantly differ at immediate post-instruction and at 1 week. However, the PM group's mean scores were significantly higher than the DI group's mean scores at the 4 week and 8 week stages. An examination of the effects of the experimental stages on these two groups reveals that, as in the other computational skills, the PM group improved at immediate post-instruction and maintained that improvement right through to the 8 week stage. In contrast, the DI group's scores in all post-instruction stages except at 1 week, were significantly higher than its pre-instruction score, indicating some improvement that maintained. In general, these results suggest that both the PM and DI methods of instruction are effective in producing performance improvements in Division. However, there are indications that the magnitude of improvement resulting from the PM method is greater than that resulting from the DI method.

In general, the results of this experiment suggest that the PM method of instruction is more effective than the DI method in teaching computational skills to students with mathematics LD. Improvements resulting from the PM intervention were not only significant but also maintained through to the 8 week stage in all computational skills dealt with. In contrast, the DI method produced an improvement in performance that did not maintain to the 8 week stage in Subtraction, and no significant improvements in Multiplication.

Figures 8 to 11 show that in thirteen of the sixteen combined post-instruction stages (four in each of Addition, Subtraction, Multiplication, and Division), the mean score obtained by the PM group was higher than the mean score obtained by the DI group. The differences between the PM and DI groups only reached statistical significance in seven of these instances, but perhaps the most important point to note is that by the 8 week stage in all of Addition, Subtraction, Multiplication, and Division the PM group was significantly higher in its mean score compared to the

DI group. Clearly, therefore, the PM method of instruction was more effective where longer term retention of computational skills was concerned.

As noted in Experiment 1, a likely reason for the better retention of computational skills taught using the PM method lies in the very nature of the strategy employed: process mnemonics are intended to aid in remembering rules and procedures (Higbee, 1987; Higbee & Kunihiro, 1985a). The PM method of instruction employed in the present study was designed particularly to aid in remembering the rules and procedures involved in computation. By incorporating characterisation, stories, and imagery in the numerical rules and procedures of computation, the intention was to make those rules and procedures more interesting, salient, and memorable to the students.

Note that features of the DI method of instruction were incorporated in the PM method used: the correct procedures were demonstrated in a systematic way to the students, and imitation of those procedures was required. Those features were considered useful and important in the delivery of instructions. The DI method by itself, however, is simply a better structured and more systematic version of regular classroom instruction. And students with mathematics LD who have deficits in computational skills have previously failed to acquire those computational skills despite the provision of regular classroom instruction. Hence, as the results of this experiment suggest, although students do tend to learn various computational skills following DI instruction, they also tend to forget much of what they learned in the longer term.

It was noted earlier that the groups differed significantly from each other where the PAT Maths Test was concerned: in particular, the PM group had a significantly higher PAT Maths Test score compared to the no

instruction group. The students' PAT Maths Test scores were therefore used as covariates in the statistical analyses carried out. This revealed that in all of Addition, Subtraction, Multiplication, and Division, the test did not significantly account for the variance observed in the experiment. This finding indicates that the students' PAT Maths Test scores did not have a significant influence on how they performed and how they responded to the different instructions provided. The variance observed can be explained better in terms of group assignment and instructions provided. Thus, the significant difference between the groups in the PAT Maths Test did not have an important impact on the outcomes of the experiment or the conclusions that can be drawn.

The finding that the students' PAT Maths Test scores did not significantly account for variance observed also indicates that, although a more relaxed criterion than that commonly employed in LD research was used in participant selection, this did not affect the outcomes in any important way. This finding lends support to the conclusion drawn in Experiment 1 that the aspect of mathematics LD focused on in the study – computational skills – was quite specific and perhaps distinct from the aspects assessed by the PAT Maths Test. Thus, it can strongly be argued that the decision to include students who clearly manifested significant difficulties in computation, but who did not meet the strict criterion of the PAT Maths Test, was justified.

As in Experiment 1, the students' TOSCA scores were used as covariates, and it was found that these did not significantly influence the students' responses to the instructional interventions. TOSCA scores have been reported as good approximations of IQ scores (Reid & Gilmore, 1988), and they have previously been used to estimate research participants' intelligence (e.g., Chapman et al., 1984). Thus, the findings in this experiment suggest that students who are below average in 'intelligence,' but without serious intellectual, physical, or sensory impairments, do not differ from students who are average or better in 'intelligence' in their

responses to instructions such as those provided in the present study. This lends support to Siegel's (1989) claim that many individuals with below average IQs should also be considered as LD and ought not to be excluded from remedial instruction programmes.

There was a confound in the present experiment that needs to be acknowledged: that of instructors and schools. As noted in the Method section, there were two instructors and two schools. One instructor was assigned per school because it was sensible and expedient to do so, as explained earlier. It is possible that the two instructors differed in their teaching skills, enthusiasm, and so on. It is also possible that the schools differed in ways that could have affected their students' capacity to respond to the remedial instructions provided (e.g., economic status of the students and their families, cultural/ethnic factors, general quality of educational environment, teachers' attitudes, etc.). The groups in the experiment, however, were considered too small to warrant a comparison of instructors/schools. Besides, differences between the schools and/or the instructors per se were not really issues of concern in the present study: the different instructors were employed to investigate whether the results of Experiment 1 could be replicated with the use of instructors who did not have a vested interest in the outcomes of the study.

The findings of this experiment demonstrated the effectiveness and usefulness of the PM method when used by other instructors. Of course, results from just two instructors do not establish conclusively that the method would be effective and useful when used by all instructors who might want to use it, but the indications from this experiment are promising.

Neither of the instructors reported any difficulties in using either the PM or the DI methods of instruction. The DI method, of course, has been used by other instructors in other studies. However, the PM method described in this study had previously only been used by the present

author. Thus the findings can be considered encouraging where practical application of the PM method for teaching computational skills in remedial classrooms is concerned.

In summary, the findings of Experiment 2 were:

- 1 PM instruction was effective in teaching Addition, Subtraction, Multiplication, and Division to students with significant deficits in computational skills and were considered to be mathematics LD.
- 2 Compared to DI instruction, PM instruction in most instances produced greater improvements in the computational skills performance of the students. PM instructions also produced improvements that maintained better in the longer term.
- 3 In Multiplication, PM instruction appeared to be more effective overall than DI instruction. The magnitude of improvement of students provided with PM instruction was greater than that of students provided with DI instruction. Maintenance was also better for the PM group.
- 4 In Division, results following PM instruction also appeared better than those following DI instruction. The post-instruction scores of the PM group were consistently greater than those of the DI group, although differences between the groups did not always reach significant levels. The PM group and the DI group both generally maintained the improvements they made in Division.
- 5 In Addition and Subtraction, the results indicate that while the magnitude of initial improvements resulting from PM instruction and DI instruction were equivalent, the improvements in performance that followed PM instruction maintained better in the longer term.

- 6 The lack of significant effects due to the PAT Maths Test suggests that the mathematical skills focused on in this study, computational skills, are quite distinct from the skills assessed by this test. Thus, the correct decision had been taken in using the PAT Maths Test only as an indicator of general mathematics difficulties, and not as *the* determinant of mathematics LD. Furthermore, the significant difference between the groups in the PAT Maths Test does not appear to have affected in any important way the students' responses to the instructions provided or the outcomes of the experiment.
- 7 The lack of significant effects due to the TOSCA suggests that, if the test is considered a reasonable approximation of IQ, then students who are below average in general intelligence can equally benefit from remedial instruction in computational skills.
- 8 The findings of Experiment 1 were supported: even with other instructors who had no vested interest in the outcomes of the study, the effectiveness and general superiority of the PM method of instruction were demonstrated.

## Chapter 8

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# General Discussion

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### *Summary of findings*

It was found in both Experiment 1 and Experiment 2 that process mnemonic (PM) instruction produced improvements in the performance of students with LD who initially manifested significant deficits in the basic computational skills of Addition, Subtraction, Multiplication, and Division. The improvements in performance following PM instruction were, in all instances, significantly better than any changes in performance that followed the provision of placebo instruction (Experiment 1) or no instruction (Experiments 1 and 2). In most cases, PM instruction also produced significantly better longer-term maintenance than that produced by demonstration-imitation (DI) instruction. In the majority of cases, the magnitude of improvements resulting from PM instruction was also greater than the magnitude of those resulting from DI instruction, although differences in the performance of these two groups did not always reach significant levels.

Combined, Experiments 1 and 2 of the present study had three main hypotheses. The first of these was that compared to no instruction and placebo instruction, PM instruction would produce improvements in the computational skills performance of students identified as having significant deficits in this area. The second hypothesis was that compared to DI instruction, PM instruction would produce greater improvements in the performance of such students. And the third hypothesis was that compared to DI instruction, improvements in students' performance resulting from PM instruction would maintain better over time.

In direct response to these experimental hypotheses, first, PM instruction did produce improvements in the computational skills performance of students identified as manifesting significant deficits in this area. The post-instruction scores obtained, and improvements made by those in the PM groups were significantly greater in all cases than those who received no instruction. This study, therefore, has clearly demonstrated the effectiveness of PM instruction in teaching basic computational skills to students with mathematics LD.

Second, PM instruction in most cases produced greater improvements in the computational skills performance of the students than DI instruction did. However, the differences between these two groups – in terms of magnitude of immediate to medium-term improvements in scores attained – did not, in many cases, reach significant levels. In Experiment 1, the overall conclusion reached was that during the earlier stages of post-instruction, the magnitude of improvements in performance resulting from PM instruction was equivalent to that resulting from DI instruction. However, in Experiment 2, the overall conclusion was that compared to DI instruction, PM instruction in most instances produced greater improvements in the computational skills performance of the students. This was particularly the case with reference to the significantly greater improvements made by the PM group, in comparison to the DI group, in Multiplication and Division.

Third, in most cases, improvements resulting from PM instruction maintained better in the longer-term than those resulting from DI instruction. In all but one of the computational skills, the PM group scored significantly higher than the DI group at the follow-up stage (6–8 weeks in Experiment 1, and 8 weeks in Experiment 2). The only exception was Addition in Experiment 1 when the PM and DI groups did not differ at the follow-up stage. Improvements made by the PM group, in all cases, statistically maintained to the follow-up stage. In contrast, improvements following DI instruction in most cases deteriorated by the

follow-up stage – in many cases, significantly so and to the point where levels of scores were no longer different from the pre-instruction stage. This finding strongly indicates the effectiveness of PM instruction in facilitating the long-term retention of procedures involved in basic computational skills in students with mathematics LD.

It is also important to note the finding in Experiment 1 that those in the placebo instruction group did not show a pattern of performance different from the no instruction group. Thus the Hawthorne effect (Brown, 1954; Tuckman, 1978) was not observed in the present study. Therefore, improvements made by the instruction groups (PM and DI) can legitimately be attributed to the skills provided through the instructions, and not to other incidental factors such as participants' expectations.

The results of the present study also indicate that the students' TOSCA and PAT Maths Test performance did not significantly contribute to the variance in scores observed during the post-instruction stages of the experiments. These findings suggest that the students' levels of scholastic ability (an approximation of their IQs) did not influence their responses to the instructional interventions in any significant way. They also suggest that the particular computational skills focused on in this study may be quite different to and largely independent of the skills assessed by the PAT Maths Test. The findings lend support to the decision to use more lax criteria, where the TOSCA and the PAT Maths Test were concerned, in selecting participants for the study.

Finally, in Experiment 2, the results obtained by the two instructors without any vested interest in the outcomes of the study were similar to those obtained by the present author who acted as instructor in Experiment 1. These findings indicate that the instruction methods used were accessible to other instructors, and their effectiveness is intrinsic (i.e., directly related to the methods themselves) rather than instructor-specific. It was particularly important to investigate and establish this with respect

to the PM method of instruction, because it was a new method and it had not previously been used by instructors other than the present author. The findings also alleviate any possible concerns that the favourable results relating to the PM method could have been due to unintended bias in the way the author provided the instructions in Experiment 1.

The present study primarily concerned two areas: process mnemonics and mathematics LD. Its findings have important implications for these areas which will be discussed in the following sections. The findings also have wider implications about the necessity for students to understand what they are taught in mathematics, how computational skills may be taught more effectively, and the selection of participants in LD research and remediation programmes. These, as well as consideration of the present study's limitations and the possibilities for future research, will be discussed in subsequent sections.

### ***Process mnemonics***

The central and most important finding of the present study is that process mnemonics can effectively be used to teach computational skills to students with mathematics LD. This finding raises a number of pertinent questions regarding the reasons why process mnemonics worked, how criticisms about mnemonic use in remedial education might be addressed, and the overall relevance of process mnemonics to the area of mathematics LD remedial instruction.

### **Why process mnemonics worked**

In the earlier chapter on mnemonics, it was noted that although mnemonics are often viewed as unnatural or artificial forms of learning, it can be argued that they are actually a natural response to the human

need to remember information. The long history of mnemonic use (Herrmann & Chaffin, 1988; Yates, 1966) supports this argument, as do findings by researchers such as Ericsson et al. (1980) and Luria (1968).

Mnemonics make it possible for individuals to retain much more information in memory than otherwise possible (e.g., Ericsson et al., 1980). They enable the memorisation of semantically-based information even when the information is not clearly understood (e.g., Luria, 1968) and, in fact, mnemonics have been prescribed as a means for remembering information that is devoid of inherent meaning (Morris, 1979). If individuals with mathematics LD have deficits in memory as many researchers in the field believe (e.g., Batchelor et al., 1990; Nesher, 1986; Swanson et al., 1990), then process mnemonics could work by facilitating the retention of much greater amounts of information than that dictated by the limitations in memory these individuals might have. And if individuals with mathematics LD experience developmental delays that render them deficient in understanding the necessary concepts and reasons in mathematics (e.g., Kulak, 1993; Putnam et al., 1990), then process mnemonics could work by providing a means for these individuals to remember procedures that they might not yet fully comprehend. Thus, superficially, the effectiveness of process mnemonics in teaching computational skills to students with mathematics LD, found in the present study, is understandable. However, a more thorough analysis of such reasons is required to better understand the mechanisms by which process mnemonics exert their influence within the context of LD remediation.

#### *The three Rs of mnemonics, and the principles of learning and memory*

In discussing why fact mnemonics work, Levin (1983) referred to the three Rs of mnemonic techniques: recoding, relating, and retrieving. Mnemonics recode unfamiliar information by transforming it into a more memorable representation. In an elaborative fashion, they relate

the recoded information to previously known information, thus creating a semantic context which makes it possible to learn the information in a more meaningful manner. Mnemonics also make retrieval a more systematic process, so that uncertainty is reduced when recall of the information is required.

The properties of fact mnemonics described by Levin (1983) also apply to the process mnemonics used in the present study, and this could partly explain their effectiveness as tools of instruction. The numerical, symbolic, and procedural components of computation were recoded into more familiar and more memorable characters and stories about warriors. During this process, the components of computation were effectively elaborated on and related to information and concepts that the students were probably more familiar with through the media (e.g., that warriors lose strength when they fight, that rank and order is important in a military environment, etc.). Retrieval was also likely to have been made more systematic as a consequence: instead of trying to remember the correct procedures through numbers and symbols they had previously dealt with unsuccessfully, the students had the metaphors incorporating the warrior stories and actions.

Of course, it is difficult to state with absolute certainty that the process mnemonics used in the present study were more familiar and memorable, that they consisted of information previously known to the students, and that they made retrieval more systematic. However, considering the students' previous failure to retain computational skills procedures, and the apparently lasting improvements in performance that followed PM instruction, it is likely that the process mnemonic components of the instruction provided did make the procedures more memorable. Also, as noted in an earlier chapter, the metaphors employed in the process mnemonic components were derived largely from arcade computer games. Many of these games have themes about battles, acquisition and loss of strength, donning of armour and other

implements, sequential and systematic confrontation with adversaries, and so on. These themes are also employed in home computer games, videos, and movies catering largely for the younger age group market. Thus, most young people would be quite familiar with the concepts and ideas used in the PM instruction of this study.

According to Higbee (1987), process mnemonics work because they utilise five principles of learning and memory. These principles are meaningfulness, organisation, association, attention, and visualisation. It is easy to see that if students clearly understand the language being used to explain procedures, it would be much easier for them to grasp which parts of a particular procedure are relevant and need to be remembered, as opposed to those that are irrelevant or incidental. In Subtraction, for example, taking the bottom digit away from the top digit is relevant information, whilst the exact values of the digits being used can be considered incidental as they would change from problem to problem. The language used in the PM method employed in the present study makes this distinction. The attacker's strength is always reduced by the defender's strength, but the strength of those warriors would vary with different sets of attackers and defenders.

The metaphors employed in PM instruction provide new, organising structures to the important elements of the procedure. These structures render those elements more cohesive and sensible. For example, in Multiplication, use of the process mnemonic metaphor avoids the ambiguity of "putting the decimal point in the answer *somewhere*." Instead, the structure of the metaphor specifies the need to count the total number of unranked warriors in the problem, and to count the same from the right of the final answer before placing the decimal point. Thus, because process mnemonics provide a structure to the material to be learnt, the relevant parts of previously confusing information are selected and then combined together in a way that would make better sense.

According to Higbee (1987), process mnemonics also make effective use of association. The linking of abstract symbols with concrete associations facilitates the cohesive combination of the relevant parts of a procedure. In the present study, the Addition sign was likened to the mast of a boat where the warriors had to hop on but in an organised manner. The Subtraction sign was likened to a knife that was drawn, and marking which of the two sets of warriors were the defenders. The Multiplication sign marked the "x-pert warriors." And the sign for long Division was likened to a rack on which the armour pieces of varying sizes hung in an ordered fashion. The symbols, and other abstract aspects of computation, were associated with concrete elements that were integral and cohesive parts of the respective computational procedures.

Because process mnemonics use metaphors that children are interested in, they make it more likely that students would attend to the relevant procedural steps being taught (Higbee, 1987). To have strategies for capturing attention is very important especially where special education groups are concerned. Students with mathematics LD who experience difficulties in computation, for example, would have previously experienced repeated failures in acquiring the correct procedural steps involved. Many are likely, therefore, to 'switch off' when instructed again in more or less the same way as they had been instructed numerous times in the past. In contrast, attention may not only be captured but also held for longer periods of time if appropriate metaphors are incorporated in the instructions provided. This use of appropriate metaphors is an integral part of most process mnemonic methods of instruction: for example, bugs in Nakane's yodai mnemonics, joggers and swimming pools in Kunihiro's pool mnemonics, and computer game themes (warriors and their activities) in the PM method used in the present study. These metaphors focus the students' attention on the most important parts of the procedures being taught. In the PM method for teaching Subtraction in the present study, for example, the main thrust of the metaphor used was on the reduction of the strength levels of the

“attackers” as they encountered the corresponding “defenders.” This captures the essence of the Subtraction process: taking away (reducing) particular digits by the appropriate corresponding digits.

According to Higbee (1987), process mnemonics often foster visualisation even if they are largely verbal in nature. Like Kunihiro’s pool mnemonics, the process mnemonics used in the present study were largely verbal in nature but accompanied by some visual aids: illustrations and demonstrations were provided on the board. But even without the visual aids, the process mnemonic characters, stories, and descriptions used were quite concrete and drawn largely from easily imagined scenarios so that visual imagery was likely to have been fostered. With visual imagery, discrimination of the relevant features of the procedure being covered is facilitated. In Multiplication, for example, the visual image likely to result from the descriptions and board illustrations distinguish quite clearly between the “top group of warriors” and the “x-pert warriors” – the latter group being the ones who need to individually meet with each and every member of the former group to impart their knowledge and expertise.

#### *Features shared with other successful intervention methods*

The effectiveness of the PM method found in the present study can partly be explained in terms of features that it shares with other intervention methods that have previously been found successful in teaching students with LD. As described in the general methods chapter, the PM method of instruction used in this study incorporated the demonstration-imitation (DI) method. Components of the DI method, on their own, have previously been found effective in teaching computational skills to students with mathematics LD (Blankenship, 1978; Rivera & Smith, 1987; Smith & Lovitt, 1975), except where long Division is concerned (Rivera & Smith, 1988). According to Rivera and Smith (1988), students with LD tended to get confused and lost in the many steps involved in long

Division, despite the provision of DI instruction. Hence, they included the use of key guide words in the method to make it effective in teaching long Division.

The DI method is effective because it is systematic, it provides opportunities for practice, and it gives feedback on performance to both instructors and students so that any correction necessary can be implemented. The steps to correctly solving a problem are demonstrated to the student in an incremental and systematic fashion. Thus, every necessary component of the procedure concerned is adequately covered, and the method lessens the likelihood that the student will get overwhelmed as a consequence of being presented with too much information all at once. Supporting evidence for the effectiveness of sequential, systematic, and piecemeal delivery of skills instruction to students with mathematics LD is found in Miller and Mercer (1993a). Miller and Mercer dealt with word problems, but their results are nevertheless relevant here in that they found positive improvements in performance as a consequence of the piecemeal approach, starting from the very basic simple components and moving gradually onto the more complex.

Using the DI method, the student is required to correctly imitate the demonstrated skills before attempting to independently solve other similar problems. The student, therefore, gets opportunities to practise the skills necessary prior to attempting the ones which would be scored. Such practice is important to the retention of performance skills (Morris, 1979). And, checks are made on the student's problem solving performance so that the student can be directed to correcting any errors being made, and further instruction in identified areas can be provided as required. This ensures that the correct steps are emphasised to the student, and faulty execution of procedures are corrected during the practice sessions.

The PM instruction used in this study benefited from all these qualities since, as noted previously, it was essentially the DI instruction with the process mnemonic components incorporated into it.

In discussing the functions and mechanisms of fact mnemonics, Morris (1979) pointed out that they essentially provide cues for recall. Process mnemonics are no exception. And it is important to note that process mnemonics provide cues for recall that are generalisable to natural settings. Thus, the metaphors used in the instructions provided would still serve as cues for remembering the procedural steps in solving computational problems, even when the instructions are no longer being provided and the numerical features of the problems given have changed. This generalisability of process mnemonic cues to natural settings can be considered as stemming from their being 'internal' rather than 'external' cues: process mnemonic cues are built in as inherent parts of the procedural components themselves, rather than being external to them. Thus, features of the problems to be solved act as cues for the appropriate metaphors, which in turn act as cues for the appropriate procedures to be executed. For example, the multiplication sign cues the metaphors of "x-changing battle techniques" and "x-pert warriors," which in turn cue the necessity to multiply each of the top digits by each of the bottom digits in the correct sequence.

The PM instruction used in this study provided a more concrete dimension to the concepts and procedures involved in solving computational problems. Miller and Mercer (1993b) found that, for most of the students with LD in their study, it was necessary to teach at the concrete and semiconcrete levels before a transfer of mathematical problem-solving skills to the abstract level occurred. They explained this finding in terms of a belief that students first learn mathematics through the manipulation of concrete objects; then they move onto learning through pictures or other pictorial representations of objects; and finally they learn using abstract symbols. Students with mathematics LD who are

not coping at the abstract level therefore need to be taken back to the more concrete levels before they can progress any further. Manipulation of concrete objects was not provided in the present study, but pictorial representations of the metaphors used in the PM instruction were provided on the board. Thus, not only were metaphors provided to associate the computational procedures with more concrete parallel 'procedures' (i.e., the warriors, their activities, and what happens to them), but pictorial representations were also given which would have facilitated in the students a better appreciation of the information being delivered to them. If Miller and Mercer's belief about the necessity of more concrete delivery to students with mathematics LD is correct, then it would partly explain the effectiveness of PM instruction found in this study.

A contributing factor to the effectiveness of process mnemonic instruction could be that it enhances the motivation of the students being taught. Okolo (1992a) found that a computer game format of instructions delivery to students with mathematics LD had a positive effect on the continuing motivation of those who initially scored low on a measure of attitudes towards mathematics as a subject. No measure of attitudes was carried out in the present study so such shifts in attitude cannot be ascertained. However, the findings of the Okolo study suggest that features of computer games assist in motivating students with mathematics LD. As pointed out previously, the metaphors used in the PM instruction utilised themes from computer games. Thus, this may have facilitated positive shifts in the motivation of the students provided with this form of instruction.

The teaching of simple and specific rules, which was part of the PM instruction employed, may have also contributed to its effectiveness. Van Houten's (1993) study found teaching of Subtraction fact rules to children with mathematics LD to be effective. According to Van Houten (1993), rules may be helpful because they serve an organising and guiding role in

executing tasks, and they may lighten the load on memory. In the PM instruction used in the present study, there were many simple and specific rules employed, as well as “easy tricks” – although they were not referred to as such. For example, for correctly lining numbers up in Addition and Subtraction, the students were instructed to line up the “warriors” according to whether they were “ranked or unranked.” Another example, is in subtracting a larger digit from a smaller digit: the students were instructed to “increase the attacking warrior’s strength by 10 points” and to “add 1 strength point to the next defending warrior.”

If students with mathematics LD have deficits in memory (e.g., Batchelor et al., 1990; Nesher, 1986; Swanson et al., 1990), then rules may help because they lighten the load on memory, as Van Houten (1993) suggested. In the PM method used in this study, the many component parts and considerations involved in executing computational skills procedures were often organised and encapsulated in simple rules relating to the metaphors employed. Thus, there would have been less for the students to hold, retrieve, and process in memory where the procedures needed to be executed were concerned.

*Strategy use, knowledge acquisition and performance components, and memory schemas*

The effectiveness of process mnemonic instruction found in the present study could also be explained in terms of memory- and performance-related factors that have previously been used to explain why fact mnemonics work. Rohwer and Thomas (1987) believed that fact mnemonics facilitate generativity, executive monitoring, and personal efficacy. They considered these factors to distinguish students who are good at strategy use from those who are not. Turnure and Lane (1987) explained the usefulness of fact mnemonics in special education in terms of similarities to knowledge acquisition and performance components, particularly selective encoding and selective combination. In addition,

Bellezza (1987) suggested that fact mnemonics are useful memory aids because they share many common features with memory schemas.

'Generativity' refers to the capacity to generate further information over and above what has explicitly been given (Rohwer & Thomas, 1987). The PM strategy used in the present study added the characters, stories, and scenarios to the standard information provided about basic computation. The operations and the numbers involved were in effect reformulated, with the inclusion of further information that was elaborative but comprehensible and thematically consistent (e.g., the reduction of warrior strength theme was consistent with the process of Subtraction). Rohwer and Thomas considered this act of generating further information to be performance enhancing. This is understandable since, when the elaborative material used is cohesive with the actual material being learnt, deeper processing is facilitated. The individual concerned needs to think about and make sense of the linkages between the component parts that make up the combination of the original and the elaborative materials. More pathways for retrieving the material being learnt are also created. For example, not only would there be the pathways of the sequential steps involved in computation, but also the metaphors now associated with those steps, to make it more likely that the target information (what to do in computation) is not forgotten.

An aspect of executive monitoring, the 'deployment of appropriate strategy' (Rohwer & Thomas, 1987), was likely to have been facilitated by the PM method employed in this study. The students with mathematics LD were provided with mnemonic methods to use for each of Addition, Subtraction, Multiplication, and Division. These methods included ways to help correctly identify operational cues, such as the computational symbols themselves, and distinguish the respective procedures from each other. Thus, for example, there were the "boat's mast" in Addition, the "drawn knife" in Subtraction, the "x-change of battle techniques" in Multiplication, and the "rack with armour pieces hanging" in Division.

These metaphors would have helped in developing recognition of different features of a task, and the consequent deployment of the appropriate responses required. For example, the metaphors used in Subtraction were likely to have been helpful in addressing not only questions like "What sort of a problem is this?," but also "Which of these digits do I subtract from which?"

Rohwer and Thomas (1987) considered it important that students believe in their ability to control the outcomes of their learning. They referred to this factor as 'personal efficacy.' Through the use of process mnemonic techniques, students in the present study were able to observe themselves solve, and a considerable amount of time later still solve, problems that for a number of years they had failed to come to grips with. Rohwer and Thomas would consider this facilitation of personal efficacy as a factor that would have contributed to the effectiveness of process mnemonics.

The effectiveness of process mnemonics when used with students, such as those with mathematics LD in the present study, may also be explainable in terms of their simplification of acquisition. Turnure and Lane (1987) argued this to be the case for fact mnemonics. An examination of the steps taken in the use of process mnemonics reveals that they also correspond to components of knowledge acquisition and performance, constructs that Sternberg (1985) proposed. Process mnemonics, for example, facilitate discrimination of stimulus features and the sifting out of relevant from irrelevant information: the important procedural steps in computation are associated with distinctive metaphors that focus the student on those steps and reduce the likelihood of confusion with other procedural steps. Process mnemonics also facilitate the task of the selective combination component: the selectively encoded information is combined in such a way that the parts are cohesive and make sense. For example, the procedural steps in solving Multiplication problems, combined with the process mnemonic

metaphors describing those procedures, are cohesive and they 'make sense.'

Bellezza (1987) argued that fact mnemonics are effective in facilitating learning and retention because they share many features with memory schemas. While Bellezza is the only one who has previously explicitly drawn this parallel between mnemonics and memory schemas, most other authors' views about memory schemas and their relationship to learning are congruent with Bellezza's opinions. A consideration of these other views lends support to the comparison that Bellezza made. Brewer (1987, p. 188), for example, referred to schemas as "mental structures that underlie the molar aspect of human knowledge and skill." Hart (1983) went as far as explaining learning in terms of acquiring useful schemas. He referred to schemas as sequences used for attaining goals; these sequences are triggered when the learner recognises a pattern.

Bellezza's (1987) opinions about how fact mnemonics work could also be applied to process mnemonics. For example, memory schemas are discriminable: they have distinctive features that distinguish between similar but different kinds. In an individual's schema, a cat's meow may distinguish this kind of a four-legged animal from a dog which barks instead. Likewise, process mnemonics, such as those used in the present study, are discriminable: they are not easily confused with others of their kind. They have distinctive, purposeful features which not only help prevent confusion, but also assist in making them memorable. For example, the use of  $x$  in Multiplication: it not only stands for the Multiplication sign but also for the theme of "x-changing battle techniques," and marks the "x-pert warriors."

Perhaps more importantly, process mnemonics, like memory schemas, can be used as guides for behaviour. Process mnemonics can be considered as providing the sequences used for attaining goals that Hart (1983) described. Bellezza (1987) actually pointed out that fact mnemonics

cannot be used as guides for behaviour. However, where process mnemonics are concerned, that is in fact their purpose: to help in remembering procedures and rules to follow.

This similarity to memory schemas in serving as guides for behaviour may largely account for the effectiveness of process mnemonics found in the present study. When using memory schemas in deciding the appropriate course of action, individuals draw from previous similar experiences, observations, and information provided by other people – all linked to, and forming composite parts of, the relevant schema. When using process mnemonics, the individual would consciously or subconsciously be asking what the particular behaviour required had been likened to (e.g., “What had the procedural steps required in Division been likened to?”). In this way, the effectiveness of process mnemonics may be understood in terms of adding potent, distinctive and accessible reference features in relevant memory schemas. Thus, in the memory schema for solving long Division problems, the reference feature that the procedural steps are like the actions and consequences of a warrior trying on armour pieces, would have been added.

### **The question of understanding and cohesion**

The results of the present study provide evidence that the PM method of instruction is an effective tool for teaching students with mathematics LD. However, sometimes, mnemonics are not considered ideal methods of instruction because they do not foster understanding. In the case of process mnemonics, Kilpatrick (1985) expressed such concerns quite strongly.

In response to such concerns, first it needs to be noted that the population of individuals with LD is really a special case. These individuals have previously failed to learn or retain various academic skills when

instructed using other methods. Thus, where individuals with LD are concerned, an important question that needs to be asked is: Would it be better for them to learn these skills even without total (or good) understanding of the underlying processes, or to not learn them at all if these processes cannot be readily understood? Where basic computational skills are concerned, it would be very hard to argue against learning-even-without-total-understanding, considering the value of such skills in society.

Nor is it the case that process mnemonic instruction obliterates or totally ignores comprehension of the processes underlying the skills being learnt. The PM instruction used in the present study, for example, simply informed students with LD that the numbers and procedures involved in basic computation could be imagined as characters and stories. They were shown on the board, and asked to imagine, how these characters and stories might look like and how they might interact. However, the students were not informed that the numbers and procedures were actually the characters and stories themselves. Telling them so could have confused meanings and purposes of the computational procedures they were learning. Instead, the students were presented with an analogy that could be applied to computational tasks they deal with: they were told that they could “imagine” or that the procedures were “like” something else.

Similarly, Nakane’s (cited in Higbee, 1987) and Kunihiro’s (Higbee & Kunihiro, 1985a) yodai mnemonic methods of instruction did not tell students that the numbers used in computation were the joggers, the bugs, the wrestlers, or whatever characters were used in constructing stories and scenarios to facilitate retention of the procedural steps involved. The students were told, likewise, to “imagine” or that the procedures were “like” something else. If this were not the case, confusion could ensue, for example, about the underlying rationale for multiplying numbers on jerseys and patches on pants of joggers (not a

common real life undertaking), or for 'multiplying wrestlers' from the east and west teams. These would (and could) only make sense if it is kept in mind and understood that those joggers, wrestlers, and other metaphors are really numbers, and that the process or yodai mnemonic techniques are simply there to aid in remembering what to do. No doubt even younger students can make these distinctions between real and imaginary, between numbers and wrestlers or other characters.

Kilpatrick (1985, p. 66) raised the question of: "... how do they [the students] come to see that  $(a + b)$  is a factor of  $(a + b)^2$  and that  $(a + b)^2$  is a special case of  $(a + b)(c + d)$ ? Are these male bugs wrestling in baskets on a train?" His point about the confusion that could be generated is acknowledged. It prompts the consideration of two important issues. First is the need to clarify the purposes of the mnemonic or any other instructional method being used. And second is the desirability of maintaining some consistency in the use of characters, stories, and/or scenarios in such instructional methods to avoid the possibility of confusion as noted.

Where the first issue is concerned, perhaps it is simply beyond the purpose of process mnemonics to facilitate the comprehension of the deeper relationships that underlie the procedures taught. This, of course, does not mean that mnemonics would consequently hinder such comprehension. Earlier instruction on any mathematical topic provided in the regular classroom does not go into underlying relationships either, and such instruction is not expected to do so because that is outside of its purpose. Comprehension of more in-depth relationships is generally expected to come later, after the mechanisms and procedures of the more basic components have been mastered. Higbee and Kunihiro (1985b) were of the opinion that mnemonics could help in giving students confidence and successful experiences which would stimulate interest and on-task behaviour. Understanding and further learning could then follow. The important point here is that in serving its particular purposes (the main

one being retention of procedures), process mnemonics appear to be very successful and it is inappropriate to criticise them for not facilitating what is beyond their intended purposes. Hindrance of understanding, and the possibility of confusion that Kilpatrick suggested, could only be expected if process mnemonics taught, and students really believed, that the numbers and procedures being dealt with were indistinguishable or inseparable from the characters, stories, and scenarios given.

Where the second issue is concerned, using different metaphors as Nakane did (reported in Higbee, 1987; Higbee & Kunihiro, 1985a) when dealing with closely related procedures with binomials, admittedly makes the whole method rather non-cohesive and untidy: with wrestlers, bugs, and passengers in teams, baskets, and trains, it is not hard to imagine that confusion could ensue. In the present study, every effort was made to ensure cohesion across the computational skills in the stories, characters, and scenarios used in the PM instructions provided. For example, all digits represented warriors, except for the ones "hanging off the rack" (the 'armour') in Division. The numerical values of the digits always represented personal attributes of the warriors (weight, strength, expertise, armour size). And the scenarios and stories were consistent with warrior-like activities (getting on boats, fighting, exchanging battle techniques, trying on armour). Such consistency is important even if it is just to ensure cohesion of the components of the method, and to avoid any possible confusion. The method used in the present study still does not provide any direct facilitation of in-depth understanding (e.g., that Multiplication is a special case of Addition), but that is beyond its intended purposes.

It is important to stress also that the mnemonic instruction used in the present study contained no less material to facilitate comprehension of processes underlying computational procedures than the DI instruction did. Essentially, the mnemonic method of instruction used was the DI method, but with the stories and characterisations added. There was

nothing that was taken away from the DI method in formulating the mnemonic method that could have deprived the latter of any ability to convey meaning and facilitate understanding.

Finally, the indications from the present study are that the students with mathematics LD did not experience any difficulty in transferring to the abstract level what they learned through the semiconcrete form of instruction employed by the PM method. Drawings of warriors on the board were used to illustrate how to solve computational problems during the provision of the PM instruction. This was done to concretise the numbers, concepts and procedures involved, and can be considered as semiconcrete-level instruction (Miller & Mercer, 1993b). However, during data collection when the students were required to solve computational problems independently, such drawings were not included which meant that the students had to operate at the abstract level. In general, and as the results indicate, students provided with the PM instruction did not have any difficulty with this. This is consistent with Miller and Mercer's finding that students did not need abstract-level instruction to be able to employ skills learned at the concrete and semiconcrete levels of instruction to solve problems presented at the abstract level. It alleviates concern particularly about the PM method employed in this study, and the transferability of skills acquired under this method to the abstract level where students must operate under normal circumstances.

### **Relevance of process mnemonics to mathematics LD remedial instruction**

The findings of the present study demonstrate that process mnemonics have a place in the remedial instruction of students with mathematics LD. The results suggest that the mnemonic method of instruction addresses a need not otherwise adequately addressed by other common methods of remedial instruction.

*Comparison with demonstration/modeling strategies: Facilitation of long term retention*

It is not exactly the case that other methods are ineffective in teaching basic computational skills to students with mathematics LD. The DI method, used as a comparison in this study, has many times been shown to be effective in other studies (e.g., Blankenship, 1978; Rivera & Smith, 1987, 1988; Smith & Lovitt, 1975). However, although the DI method of instruction did produce improvements in the computational skills performance of students in the present study, those improvements did not maintain as well in the longer term as improvements resulting from mnemonic instruction. In fact, in a number of instances, the performance of the DI group had sufficiently deteriorated by the 8 week follow-up stage that it was no longer significantly different from the no instruction group's performance. The magnitude of the DI group's improvements in performance was also in many cases lower than the magnitude of the mnemonic group's improvements.

Demonstration/modeling strategies, such as the DI method, draw largely from regular classroom instruction practices. Certain components of regular classroom instruction appear to be essential in remedial teaching. For example, it would be difficult to teach students the procedures involved in basic computation without the opportunity to demonstrate to them what exactly those procedures are and what sequential order needs to be followed in executing them. Imitation seems to be an element that serves not only the facilitation of learning, but also the provision of feedback to both instructor and student. An important question, however, is whether simple systematisation of these components is adequate.

A closer examination of demonstration/imitation-derived methods shows that certain components are often introduced which render the resulting 'environment' somewhat artificial and different from that

usually encountered by students. For example, the provision of a permanent model that students could consult while solving arithmetic problems (e.g., Blankenship, 1978; Rivera & Smith, 1987; Smith & Lovitt, 1975) deviates from conditions that students would normally encounter when their ability to solve such problems is being tested. In fact, the possession of a permanent model is like having 'crib notes' – which, under a different set of circumstances, could be considered as cheating. These additional forms of assistance make it difficult to assess the real capability of the students to execute the target computational skills independently, without the additional help during data collection. Their inclusion also makes it difficult to accurately assess whether or not the skills have really been learnt and retained by the students.

It seems that where students with LD and computational skills are concerned, there are two factors that need to be addressed: learning what the correct procedural steps are, and remembering/retaining those procedural steps. The demonstration and imitation components of instruction appear to be aimed at the first of these factors. The additional help provided (i.e., the permanent model, the various prompting methods) appears intended to address the problem with remembering/retaining the procedural steps. However, it could be argued that provision of the permanent model and prompts do not appropriately solve the retention problem because outside of the remedial classroom environment those are unlikely to be available. Thus, their provision limits the students' capability to solve the problems to just the artificial environment created. (Furthermore, there seems to be an inherent assumption behind this approach that, unlike normal students, students with LD cannot retain the procedural steps in question – which is why the artificial environment needs to be supplied. This assumption may or may not be correct, but it should be tested.). A real challenge therefore would be to address the students' problems in remembering/retaining the correct procedural steps *and* do so in a way that adheres more closely to real life environments (classrooms or

otherwise), so that the students with mathematics LD would not need to have special conditions provided to them just to be able to correctly add, subtract, multiply, and divide.

Strong evidence has been found in the present study that process mnemonics address problems in remembering/retaining computational procedures that students with mathematics LD have. The comparison DI method used was without the additional forms of assistance (e.g., permanent model, prompting) and in most instances the improvements made by students taught using the mnemonic method maintained significantly better in the longer-term (up to 8 weeks). The important point to note here is that during the post-instruction assessments of performance, no additional forms of assistance were provided to the students. Thus, in solving the problems given, they had to rely solely on their prior learning and their ability to recall the procedures they had been taught.

As noted previously, both mnemonic and DI groups showed improvements immediately following instruction. Although the mnemonic group often made greater improvements than the DI group, the differences between the groups did not always reach significant levels. Thus, the results indicate that generally both mnemonic and DI methods of instruction addressed the need that the students with mathematics LD had for “learning what to do in computation.” Judging by the results obtained, both methods of instruction also generally addressed the problem of “remembering/retaining” the procedural steps in the short to medium term. The most consistent significant differences between the two groups occurred in the longer term follow-up assessments when, in the majority of cases, the mnemonic group manifested good maintenance of improvements made while the DI group markedly deteriorated in its performance. The clear findings here are that, first, the DI method of instruction appears not to address the problem of long-term retention. Second, using process mnemonic strategies (i.e., stories, characterisations,

scenarios, and other metaphors), but keeping the demonstration and imitation components, proved effective in facilitating not only “learning what to do” in computation but also “remembering what to do” in the long term. Thus, it can be said that in teaching computational skills to students with mathematics LD, process mnemonics complement the effective components of the DI method of instruction.

*Small group teaching, and use of the method by other instructors*

It was found in the present study that the process mnemonic method used in a single-case design in Manalo (1991), with some modifications and adjustments, could successfully be used in small group remedial instruction. This clearly indicates that the effectiveness of the method is not limited to the one particular individual for whom the original version was designed. It also indicates that the method has practical applications in the remedial classroom, where small group teaching rather than individual instruction is the more viable option because of its cost-effectiveness and adherence to usual limitations in resources.

Ideally, remedial instruction methods should be accessible to instructors coming from a reasonably wide range of prior training and experience. The methods should be effective and reasonably consistent in the results they produce, and they should not be overly demanding in terms of resource requirements. All evidence collected in the present study indicates that the PM method possesses these desired qualities. Results indicating the method’s effectiveness in teaching computational skills to students with mathematics LD have been described previously. These results were of reasonable consistency across computational operations (Addition, Subtraction, Multiplication, and Division) and instructors. The instructors, who experienced no difficulties in learning and using the method, did not possess a great deal of prior teaching experience and/or training. And the only resource requirements for the PM method of

instruction were a teacher, a room with a board, and some photocopied exercise sheets.

### *Mathematics LD*

The present study found that students with mathematics LD responded positively to the instructions provided, and that improvements in their computational skills performance resulting from PM instruction maintained well in the long term. These findings have important implications about mathematics LD. More specifically, the findings suggest that mathematics LD need not be viewed as a permanent condition; that the provision of direct instruction addressing intrinsic difficulties is effective; and that some of the learning failure manifested by students with mathematics LD is likely to stem from deficits in memory.

#### **Should mathematics LD be considered a permanent condition?**

It was noted in earlier chapters that the skills performance of children with various forms of LD resemble the performance of children who are younger in chronological age. With regard to reading, this observation led to Ellis (1985) suggesting that perhaps dyslexics progress through various metamorphic profiles in reading acquisition, and that these profiles may follow the cognitive-developmental stages proposed by Marsh et al. (1981) for normal children, except at a delayed and much slower rate. Such a developmental delay explanation of deficient performance may apply not just to individuals with dyslexia but also those with other forms of LD.

Kulak (1993) suggested that, like children with reading LD, children with mathematics LD acquire skills in the same way except at a slower rate compared to normal children. She observed that the performance

characteristics of children with LD differ from those of their normal peers in a quantitative way rather than a qualitative way, suggesting that the former are only delayed in acquiring otherwise normal processing skills. Kulak also cited results from a number of studies indicating that children with mathematics LD use combinations of strategies that are less mature than those used by their peers without LD. She therefore concluded that the conditions of the majority of children with mathematics LD can best be explained in terms of a delay in moving through the normal sequence of strategy acquisition. Putnam et al. (1990) obtained results supporting this developmental delay notion in their study that examined the performance of students with mathematics LD.

There is evidence to show that individuals with LD do make metamorphic shifts in their skills profiles. Korhonen (1991) found such shifts when he examined the stability of neuropsychological profiles of children with LD. The majority of changes he observed over the three year period of the study were positive, with most shifts in performance occurring so that severe deficiencies were reduced to mild deficiencies or even average ("normal") performance over that period. For example, in two of the groups of children he tested, approximately 70% had severe reading disabilities initially, but three years later the proportion with severe reading disabilities had decreased to approximately 25%. Data obtained by Feagans and Merriwether (1990) also show shifts in the skills performance profiles of students with LD. During the three year duration of their study, the performance scores of the students with LD closely approximated the previous year's scores of the students without LD in the Reading Recognition, Reading Comprehension, and Mathematics subtests of the Peabody Individualised Achievement Test.

The implication of the claims, observations, and results of these prior studies is that individuals with LD can learn and progress through the same stages of skills development as normal individuals. They support the premise that individuals with LD are not stuck at any particular stage

of skills development, manifesting a particular insurmountable kind of deficit.

The finding in some research studies that, for many individuals with LD, problems persist into adulthood (e.g., Gerber et al., 1990; Horn et al., 1983; Schonhaut & Satz, 1983; Spreen, 1982) has previously been noted. The possibility that the persistence of problems results from non-provision of appropriate remedial instruction was suggested earlier. It is important that remedial instruction provided to individuals with LD directly addresses the difficulties and performance problems they manifest. In the present study, for example, computational skills instruction was provided to students who were established as being significantly deficient in this particular area of mathematics.

Another possible reason suggested earlier is that individuals with LD may largely be responsive to remedial instruction, but that the effects of instruction and consequent improvements in performance may not always be lasting. Thus the problem may lie not so much in actual learning but in retaining or remembering what is learned. There is evidence in the present study to support this contention. Students in both the PM and DI groups showed improvements in their performance following instructions, but in the long term only those in the PM group maintained the improvements they had made. Those in the DI group generally did not retain the computational skills they had initially acquired. The implication of this is that remedial instruction methods need to directly address the issue of retention: they need to incorporate means to facilitate beneficial effects that maintain in the long term.

### **A case for the provision of direct remedial instruction**

No support was found in the present study for Lerner's (1985) suggestion that psychological process deficits may need to be addressed prior to the

actual mathematical difficulties. The skills contained in the instruction methods used focused directly on computational difficulties the student participants had manifested rather than any learning process deficits they may have had. And significant improvements in performance were obtained not just in the immediate and short term, but also in the long term.

This finding is not surprising: as Mastropieri et al. (1987) pointed out, many efforts at providing remedial instruction to students with LD have utilised the direct instruction approach. This approach, under which the DI method fits, incorporates teacher-led instruction usually at a brisk pace, cumulative review, specific correction procedures, and teaching that allows generalisation of problem-solving skills to other similar problems. It has proven to be more effective than psychological process approaches, since the latter often do not show any beneficial effects on the actual LD problems (e.g., reading). An example that Mastropieri et al. gave of psychological process training was for correcting 'visual figure-ground' orientations: the experimental participants would be given guided practice in picking out particular objects embedded within larger pictures (e.g., they were asked questions like, "How many fish can you find in this picture?"). As Mastropieri et al. pointed out, whether such methods of training are beneficial in some ways or not, they very rarely show any direct influence on experimental participants' actual reading, spelling, or mathematics performance.

It was noted in earlier chapters that there are many possible cognitive deficits that could produce the difficulties in performance associated with mathematics LD. However, where actual performance problems are concerned, as Sutaria (1985) pointed out, there are only two basic types: those involving computational difficulties, and those involving reasoning difficulties. Irrespective of the other possible cognitive deficits occurring concomitantly with or even causing the mathematics LD, it appears possible to address just the actual performance difficulties

themselves. This study focused on computational difficulties and was able to demonstrate improvements in the student participants' performance following remedial instruction. This suggests that individuals with mathematics LD experiencing difficulties with computation respond positively to particular components of the instruction methods used.

Direct instruction methods, such as those used in this study, may not directly address the cognitive deficits that individuals with mathematics LD presumably have. In effect, the instruction methods may only be treating the symptoms rather than the causes. Thus the underlying cognitive deficits may remain, and the remedial instruction – if it is effective – may simply provide a way of circumventing the difficulties resulting from those deficits. Providing direct remedial instruction this way, rather than focusing on the process deficits, may be useful since, first, the role that cognitive deficits have in causing various forms of LD is often questioned. Korhonen (1991), for example, found equal numbers of children with LD as children without LD who exhibited the same visuo-motor-spatial problems in the schools where he conducted his study. Second, this may be acceptable since, as pointed out earlier, remediation of process deficits usually does not show any direct beneficial effects on the actual skills of concern (e.g., mathematics). Thus, at least where individuals with mathematics LD experiencing problems in computation are concerned, the direct instruction approach that incorporates process mnemonic components appears to be a very effective approach to use if it is desired that these individuals learn – and retain in the longer term – the procedural skills involved.

### **The possible deficit in memory**

The experimental components of the present study did not focus on the causes of mathematics LD. However, the results obtained do imply some

of these. In particular, the finding that improvements in performance following PM instruction maintained better in the longer term implies a number of things about the nature of the problems experienced by those with mathematics LD.

A deficit or problem in memory, for example, is strongly indicated. Since it was established that the student participants of the study did not have difficulties in remembering basic facts, the significant deficits they manifested in computational skills must stem from problems in remembering procedures to be followed. The main finding of the study that the method of instruction specifically designed to aid in the retention of processes, procedures, and rules adds further and very important support to this. As previously pointed out, many other researchers believe that individuals with mathematics LD have some form of a deficit in memory (e.g., Batchelor et al., 1990; Neshet, 1986; Swanson et al., 1990).

An explanation for the problem of remembering procedures was suggested by Neshet (1986). She explained that procedural knowledge in mathematics involves a slow constructive process whereby elements that make up a procedure need to gradually be put together and integrated to form the mastered skill. Individuals with mathematics LD could experience difficulties in retaining the individual elements (the sub-procedures) that make up the procedure, or in integrating those elements into a complete, fluent whole. Thus, the trace strength of those procedures, which determines accessibility in memory, would be weak.

Many studies in the literature have discussed procedural memory as a distinct component of human memory, and one that can be selectively impaired (e.g., Cohen, 1984; Cohen & Bacdayan, 1994; Cohen & Squire, 1980; Craik, 1991; DiGuilio et al., 1994; Mishkin et al., 1984; Mitchell, 1989; Mitchell et al., 1990; Morris, 1984; Ostergaard & Squire, 1990; Roediger, 1990; Tulving, 1985). However, many of these studies tend to view procedural memory (memory of how to do something) as synonymous

with implicit memory (memory for how to do tasks without conscious recollection of previous experiences). Therefore, many of the ways by which 'procedural memory' is examined and assessed in these studies are not appropriate for the kind of procedural memory that needs to be examined in relation to computational procedures. For example, Cohen and Corkin (1981: cited in Cohen, 1984) used the Tower of Hanoi puzzle which involves learning through practice the optimal solution to shifting wooden blocks from one peg to another, while adhering to certain constraints. The kind of memory assessed by this procedure, however, would be different from that utilised in learning computational procedures. The latter would be more deliberate and conscious, and may be similar to memory utilised in playing board games, breaking codes, and perhaps even reading and writing. Thus to effectively investigate the procedural memory capabilities of individuals with mathematics LD would require not only the construction of more appropriate strategies for assessing this kind of memory, but also a delineation of it as distinct from the kind of procedural memory often described in the literature. To do these were considered outside the scope of the present experimental investigation.

It is also possible that instead of an actual deficit in procedural memory, individuals with mathematics LD may simply employ an immature and ineffective approach to remembering procedures. Often associated with the developmental delay explanations discussed earlier (e.g., Kulak, 1993; Putnam et al., 1990) are observations that individuals with LD use immature strategies in undertaking various learning-related tasks. Thus, the PM instruction provided in this study could have simply taught the students a more effective approach to remembering computational procedures. This is a viable alternative explanation to the one suggested earlier in this section that the PM instruction provided a means for circumventing the possible deficits in procedural memory. The relative merits of these explanations can only be established through further

research into the memory-related characteristics of individuals with mathematics LD.

### *Wider implications*

#### **Is understanding always necessary in mathematics?**

Even without explicit emphasis on facilitating understanding of the reasons underlying the procedures used in computation, mnemonic instruction need not be considered deficient in any important way if it is effective in helping students learn the 'how to' aspects of instructions given. Some theorists, such as Piaget (1953), have suggested that children's repeated execution of mathematical tasks contribute to their eventual understanding of the mathematical reasoning behind those tasks. Higbee (1987) noted that the 'doing helps understanding' premise is subscribed to in many schools in Japan, not just in teaching mathematics but also in other subjects such as music. But even if the notion that 'doing leads to understanding' is not always true, it is not difficult to argue that being able to do something even without complete understanding is preferable to not being able to do it at all. Many situations that present themselves in day-to-day living require some ability to carry out at least the basic components of computational procedures.

There are also many aspects of computational skills that even individuals *without LD* also do not completely comprehend. For example, many people do not fully understand the reasons relating to the shifting of decimals in long Division (i.e., so that the divisor is a whole number). Nor do some people understand the reasons behind counting of the digits to the right of the decimal points (of the numbers multiplied) to determine where to place the decimal point in the answer to a Multiplication problem.

The point here is that it may not always be crucial to understand particular aspects of mathematics, especially where computational operations are concerned. Perhaps it is wrong to assume that understanding is always the ultimate goal, and perhaps it would be better to consider the actual functional reasons one might have for learning something. Where students with mathematics LD are concerned, an important reason for teaching them computational skills would be to equip them with skills considered useful and of practical advantage to have in both school and out-of-school settings. Having already experienced serious difficulties in learning those skills, to also achieve good understanding could be considered desirable but not absolutely necessary for these students, since it lies outside the basic functional reason for teaching them the skills.

### **Some ideas for teaching computational skills more effectively**

The results of the present study, showing that improvements in performance followed both PM and DI instructions but that PM instruction produced better maintenance in the longer term, suggest that the primary problem experienced by students with mathematics LD who have computational skills difficulties is in long term retention of those skills. When the students were provided only with the DI instruction, they improved in performance in the immediate term but generally failed to retain the skills through to the 8 week stage. Thus, it appears that the students were able to learn the procedures involved in mathematics computation but had difficulties in remembering those procedures for periods longer than several weeks. Although no proper evaluation of the components of the apparently more effective PM instruction was undertaken, it is nevertheless possible to make some inferences from the results and consider some of the ways that computational skills may be more effectively taught – particularly to students with mathematics LD.

The common features of remedial instruction, such as smaller group teaching and focus on specific topics of difficulty, would certainly have contributed to the success of the instruction methods employed in this study. However, these features alone cannot significantly account for the improvements in performance of the students since most of them had previously been placed in remedial classes without making any lasting improvements in their computational skills performance. The students' improvements in performance also cannot be attributed to other extraneous factors and conditions, such as the presence of the researcher or the absence of grades, since students in the placebo group (also referred to as the Study Skills group) in Experiment 1 did not manifest any significant improvements despite sharing the same factors and conditions. The important factors that facilitated improvements in the students' computational skills performance must therefore lie in features of the PM and DI methods of instruction.

The results of the present study suggest that a systematic approach to procedural instruction is important. As noted previously, the PM instruction used incorporated features of the DI method of instruction. The DI method is systematic in its approach and presents procedural steps in a highly ordered manner: they are presented bit by bit, gradually building on the simpler steps with more complicated steps, and ensuring that they are largely learned through imitation (which in itself provides practice) prior to moving on (Blankenship, 1978; Rivera & Smith, 1987, 1988; Smith & Lovitt, 1975).

Using a similar approach in teaching computational skills in the classroom may avoid a lot of the difficulties and problems that many students encounter. The distinctive presentation of important parts and the clear linkages drawn between related steps probably help in avoiding confusion and in enabling students to follow and subsequently imitate the necessary steps. The imitation of the steps gives practice in executing those steps (making it more likely that they can be executed again in the

future) and provides opportunities for correcting any initial errors. With the DI method drawing most of its features from regular classroom instruction methods, it should not be very difficult to adapt or modify classroom instruction itself to incorporate more of the useful features of the DI method.

The results of the present study also lend support to Reisman and Kauffman's (1980) suggestion, and Miller and Mercer's (1993b) finding, that it would pay for teachers to concretise concepts and ideas for children if they are found to be experiencing difficulties in dealing with the abstract and complex aspects of the materials being taught. Topics in mathematics – including computational skills – are full of abstract and complex aspects. It may therefore help generally if much of what is covered in mathematics instruction is concretised and made more salient to the learning child. In this study, this was done using process mnemonics. It is likely to prove effective, and unlikely to be very difficult, to incorporate some of this 'concretising' of ideas, enhancement of salience, and linking of component parts in regular classroom instruction of mathematical skills. Process mnemonics – or aspects of it – could be used. And it may contribute to making mathematics learning less daunting, and more fun and enjoyable for students.

Making instructions more interesting through their delivery may also be very important, and one that ought to be considered and implemented as much as possible in classroom teaching. As noted previously, the metaphors used in the PM instruction of the present study were likely to have generated greater interest in the computational procedures taught among the student participants. Generating interest in the instructions being provided is important, particularly with students who have LD, for two main reasons: it motivates the students (Okolo, 1992a), and it may encourage them to try out and keep using new and more successful procedural methods.

Another factor that could help in teaching computational skills more effectively to students with mathematics LD is the inclusion of simple and specific rules. Van Houten's (1993) findings about the effectiveness of teaching rules in the form of "easy tricks" to follow was mentioned earlier. The findings of this study lend further support to the usefulness of rules: the metaphors in the PM instruction used, such as having to line up warriors according to whether they were ranked or not ranked, provided the students with simple and salient rules to follow which may have greatly assisted in their longer term retention of the procedures. The use of rules like these is reasonably easy to incorporate in regular classroom teaching. In talking to teachers in the schools where the experiments of the present study were conducted, the present author discovered that some of the teachers already occasionally employed similar strategies for teaching. Perhaps it would be beneficial for teachers to more often include simple and specific rules in the instructions they provide – particularly when it appears necessary to do so, such as when students manifest difficulties in learning procedures.

Finally, the results of the present study highlight the need to focus on long term retention when teaching computational skills to students with mathematics LD. This suggests that teachers should find ways to facilitate not just learning in the immediate or short term, but also remembering of the procedures they teach to students in the long term. This study looked at the use of process mnemonics, which proved effective. However, the method of describing computational procedures in terms of warrior metaphors is just one way to incorporate process mnemonics in teaching. There must be a great number of other possibilities. Process mnemonics may also be but one method to facilitate long term retention of procedural skills: if it is kept in mind that facilitation of long term retention is a vital facet of providing instruction to students with mathematics LD, other methods could be developed and investigated.

### **Selection of participants in mathematics LD intervention programmes**

When the students' TOSCA scores were used as a covariate in the statistical analyses of results, they had no significant influence on the observed variance, and hence the students' responses to the instructional interventions. The TOSCA is not an IQ test: it is a test of scholastic achievement. Nevertheless, as explained in the General Methods chapter, the TOSCA correlates well with various subtests of IQ tests (such as the WISC-R) and is generally considered a good estimate of IQ. If the premise that TOSCA scores are reasonable approximations of IQs is accepted, the results suggest that students who were below average in 'intelligence' (but did not have serious intellectual, physical, or sensory impairments) responded to the instructions no differently than students who were average or above in 'intelligence.' This finding supports Siegel's (1989) contentions that many individuals who are below average in IQ should also be considered as LD and that they respond to remedial instruction as well as individuals with average or better IQs do.

As explained in the first chapter on learning disabilities, Siegel's (1989) claim about the irrelevance of IQ to the definition of LD was based on arguments against four basic assumptions in the IQ-achievement discrepancy. Basically, the assumptions that Siegel rejected are that: (1) IQ tests measure intelligence; (2) intelligence and achievement are independent, and the presence of LD will not affect IQ test scores; (3) IQ scores predict reading and/or arithmetic scores; and (4) individuals with LD of different IQ levels have different cognitive processes and information processing skills.

According to Siegel (1989), if a test truly measures intelligence then it ought to tap into problem-solving skills, logical reasoning, and adaptation to the environment. This view is congruent with Sternberg, Wagner, Williams, and Horvath's (1995) observation that researchers are now looking into new constructs (e.g., practical intelligence) to find measures

to supplement existing psychometric intelligence tests, which have been found to account for only up to 25% of variance in people's real-world performance. Siegel pointed out that most supposed intelligence or IQ tests instead measure other attributes such as expressive language skills, memory, specific factual knowledge, and so on. This was her argument against the first assumption. The TOSCA would be no different: despite good correlations with subtests of IQ tests and being a good approximation to IQ scores, it is nonetheless a test of scholastic achievement. It would be limited in its ability to assess the attributes Siegel considered desirable for intelligence tests. But using proper IQ tests would arguably not have been much better: this argument might sound circular but the very fact that the TOSCA correlates so well with IQ tests suggests that those IQ tests are tapping markedly into achievement rather than intellectual potential. Siegel's argument against the second assumption is related to this: because of what IQ tests actually measure, they tend to bias against individuals with LD. Thus because LD affects IQ test scores, such scores cannot be considered true measures of the intelligence of individuals with LD. Likewise, because of what it measures, the TOSCA would almost certainly have underestimated the 'real' intelligence of the present study's student participants with LD.

However, Siegel's (1984) argument against the third assumption is probably more pertinent in explaining the good post-instruction performance of the present study's students, including those who were in the "below average" category of the TOSCA. Siegel argued against the IQ-discrepancy definition of LD and pointed out that children with low IQ scores can learn to read, in many cases, as well as those with high IQ scores. Thus children with low IQ scores who fail to learn to read should be considered as reading LD; their reading failure should not be considered as resulting from their low IQ scores. What Siegel said about reading LD applies also to mathematics LD: it cannot be assumed that low IQ scores would automatically cause symptoms of mathematics LD such as computational disabilities or deficits in mathematical reasoning. Idiot

savants would be extreme examples, but they illustrate the fact that even with extremely low levels of intellectual functioning some selected skills and abilities (e.g., in mathematics computation) could be well-developed. The point is that even if the students of the present study in the “below average” category of the TOSCA were really below average in their IQs, it would have been an inadequate explanation for their poor mathematics skills performance and certainly cannot be assumed as *the* cause for it. There would have been other students, in the same classes as these students, who would have similarly been in the “below average” category of the TOSCA (and presumably below average in IQ) but not have manifested similar difficulties in mathematics, particularly computation.

Against the fourth assumption, Siegel (1989) argued that the cognitive processes of poor readers with high and low IQ scores do not differ significantly, and thus that separation of readers with LD using IQ scores to understand cognitive processes is unnecessary. Cognitive processes were not directly examined in this study. However, computational skills performance and response to instructions were examined, and there were no indication that those presumably of low IQ (because they were in the “below average” category of the TOSCA) performed any differently to those presumably of average IQ (because they were in the “average” category of the TOSCA). With similar responses to the same instructions, this implies that the cognitive processes causing the problems in computation were similar for both categories of students. Thus, if the cognitive processes underlying difficulties in computation were the focus of interest, the results of this study suggest that differentiating between those with low and those with average IQs would probably not be necessary.

It is important to stress that the study demonstrated that the students, irrespective of their approximate IQs, were generally able to learn computational skills. Thus it would be wrong to assume that the previous inability of any of the students was due to inadequate

intelligence. There are many possible reasons (including perhaps memory deficiency) that could have been at the root of their inability to learn computational procedures.

Siegel (1989) also pointed out that, for various reasons, individuals from different ethnic/cultural and minority backgrounds and those from lower socio-economic groups are likely to obtain lower IQ scores. This possibility is also relevant to the present study. Although no formal procedures were carried out to establish the ethnic affiliation or background of the student participants of the present study, a considerable proportion of them were from ethnic/cultural minority groups (Maori and Pacific Island groups, in particular). In fact, in one of the Auckland schools, the students were almost exclusively Maori and Pacific Island students. Various research studies conducted in New Zealand have found that students from these groups score significantly lower in IQ tests than Pakeha/European students (e.g., Chapman & Nicholls, 1976; Flynn, 1988; Harker, 1978; Lovegrove, 1966), and the authors attribute this largely to environmental factors. More Maori and Pacific Island children tend, for example, to come from lower socio-economic group families and their home environments may be comparatively impoverished in the educational sense, with regard to availability of books, computers, and so forth. But whatever the reasons, the fact remains that individuals from different minority, cultural or ethnic backgrounds obtain significantly lower IQ scores. With this in mind, it cannot be considered fair to base decisions on whether or not students from ethnic minority backgrounds are LD on their IQ scores. Some will fall undeservedly into "below average" categories and therefore be excluded. The consequences of exclusion, could be a different, less favourable attitude to these students, or even disqualification from receiving appropriate remedial instruction.

The present study included students who scored in the "below average" category of the TOSCA, but not those in the very bottom "low" category. This methodological decision was in line with Siegel's (1989) suggested

compromise of lowering the IQ cut-off score from 90 to 80, which also appears to be the practice adopted in a number of recently published studies (e.g., Brier, 1994; Lyytinen et al., 1994). Whether or not it is necessary to exclude those in the lowest IQ category from such intervention programmes is a matter that may need to be addressed in future research. Perhaps responses to similar instruction in computational skills (or other mathematical skills) could be examined in relation to the different IQ (or approximate IQ) score categories of students with LD, including those in the lower categories.

Another issue related to selection of student participants that was touched on in the present study concerns the use of the PAT Maths Test: students who were one-half of a standard deviation (rather than one standard deviation) or more below the mean were included in the study. This raised the question of whether those included in the study only through the more lax criterion adopted here performed differently in response to instructions, than those who would have met the stricter criterion. Statistical analyses showed that the students' PAT Maths Test scores used as a covariate exerted no significant influence on the variance observed in the study.

Consideration of the PAT Maths Test scores raised a wider question of whether individuals need to demonstrate a *general* mathematics disability in a psychometric test (such as the PAT Maths Test) before they can be considered as mathematics LD, or whether it would be sufficient to simply demonstrate that they have a significant deficit in a particular area of mathematics, such as basic computation. In the present study, the latter approach was followed.

Aside from some of the problems associated with the PAT Maths Test discussed in the previous chapters (e.g., its bias towards mathematical reasoning), and the finding that the students' PAT Maths test scores did not have a significant influence on their post-instruction performance,

the basic position taken here is that individuals need not be experiencing problems in all aspects of mathematics to be considered mathematics LD. Their area of difficulty could be quite specific, such as the computational skills focused on in this study. But despite being specific, the area of difficulty needs to be broad enough to produce significant difficulties in mathematics and to render the individual incapacitated in many common mathematical tasks and/or activities. For example, it would not be appropriate to consider a person as having mathematics LD if he or she experiences significant problems only in trigonometry and absolutely no other areas in mathematics. Experiences, observations and findings from the present study support the position taken here: a considerable number of the students included in the study would not have qualified as having mathematics LD if the decision was based on their PAT Maths Test performance. However, taking into account these students' performance and significant deficiencies in basic computation, it would seem unreasonable to consider them as normal achieving in mathematics.

### *Where to from here*

#### **Limitations of the present study and potential improvements**

The present study had a number of limitations relating to: (1) the length of time subsequent to provision of instructions when maintenance assessments were taken; (2) the generalisability of findings to other age groups of individuals with mathematics LD; (3) the amount of instruction time provided; and (4) the number of student participants in the study. These limitations should be considered when interpreting the results, and when considering improvements to future investigations on process mnemonics and mathematics LD.

The first limitation concerns the lengths of time subsequent to provision of instruction when maintenance assessments were carried out. As noted

in this chapter and in the previous two chapters, the PM method of instruction produced performance improvements that maintained better in the *longer term* than those produced by the DI method of instruction. The ability of the students with mathematics LD to overcome their difficulties in basic computation has also been noted suggesting that the students provided with PM instruction could have *permanently* learnt the computational procedures, and that maintenance would continue over and beyond the time it was actually observed.

In actual fact, maintenance assessments were taken a maximum of only eight weeks subsequent to provision of instruction. Conducting maintenance assessments up to eight weeks after instruction is approximately on a par with studies that have examined the longer term effects of fact mnemonic instruction. Longer term assessments are required to establish whether or not the effects produced by PM instruction maintain beyond the 8-week period. After all, the comparison method of instruction (the DI method) often produced comparable improvements in student performance which maintained reasonably well up to four weeks. If maintenance assessments were taken only up to four weeks after instruction, then different conclusions about the relative merits of the instruction methods would have been drawn. It is possible, where the effects of the PM instruction are concerned, that student performance would have changed after eight weeks.

The main practical limitations to undertaking even longer term maintenance assessments concern the students and the schools they attend. Most teachers from the schools who were involved in the present study were keen to have the instructions and assessments undertaken efficiently and in as short a time span as possible so as to minimise the disturbances and disruptions to the students' regular classroom instructions. In two of the schools, class schedules also changed from term to term: thus it became necessary in the latter stages of the experiments to take students out of regular classes other than

mathematics. Some of the teachers from the other subject classes were not very happy with this, since the initial arrangement was that the students were going to participate in the experiments only during their mathematics classes.

Therefore, with the students' best interests in mind, and the schools' schedules and resources to consider, undertaking longer term maintenance assessments is not likely to be easy or to be viewed favourably by the schools. However, if these difficulties could be overcome, it would be helpful to carry out further maintenance assessments six months and twelve months after the provision of instruction. It would then be possible to establish whether or not (1) the beneficial effects of PM instruction are truly long-lasting, and (2) students with mathematics LD experiencing difficulties in basic computation are able to overcome those difficulties permanently.

The present study also only looked at the effects of the PM and the DI methods of instruction on students in Form 3, approximately 13- to 14-year-olds. An important reason for this was that the computational skills covered in the instructions are meant to have been mastered by students by the time they reach Form 3. However, this means that the generalisability of the finding that process mnemonics are useful instruction tools for teaching students with mathematics LD is somewhat limited. This generalisability can be extended if the PM method of instruction (particularly, where the content is concerned) can be modified to suit other age groups (e.g., those who are younger) and be trialed on those groups.

The present study provided PM and DI instructions in each of the computational operations in two instruction sessions and one review session prior to the post-instruction assessments. It would be worth examining whether this amount of instruction time is optimal. It is, for example, possible that the same results can be achieved with lesser

amounts of instruction time. Or perhaps the students' skills in computational operations will be better consolidated with more instruction time, and consequently longer term maintenance will be improved both for those receiving PM instruction and those receiving DI instruction. The methods of PM instruction for each of the computational operations were used in small group remedial teaching of students with mathematics LD for the first time in this study, and there would certainly be room to make adjustments, modifications, and improvements on them.

Finally, it is possible that the results of the study could have been improved with more student participants. Some of the differences between the performance of students in the PM group and those in the DI group might then have been a little clearer. For example, quite often after immediate post-instruction the performance of the DI students deteriorated while those of the PM students generally maintained well. However, usually it was not until the 8-week post-instruction assessment that significant differences between these two groups of students manifested. Perhaps with larger numbers of student participants significant differences would have been evidenced at the 4-week or even the 1-week stages. However, while larger numbers of student participants would be desirable, the reality was that there were not very many students who met the criteria, and who were willing and allowed to participate in the study.

### **Future directions**

With regard to further examination of the effectiveness and usefulness of process mnemonics in teaching computational skills to students with mathematics LD, it would be useful to take the method to schools and request actual remedial teachers in those schools to trial the method themselves. To get results and feedback from such teachers would

contribute not only to further scrutiny of the method but also to fine tuning it. It may also stimulate the development of further similar methods of instruction (i.e., using process mnemonics) among the people who, on a day-to-day basis, actually provide remedial instruction to students with mathematics and other forms of LD.

It may be worth examining whether the PM method of instruction described in this study can effectively be carried out using computers. After all, the inspiration and basic ideas used in it came from themes commonly employed in computer games. The last decade or so has seen tremendous developments in computer technology and the use of such technology in schools. It would certainly save a lot of teacher time, and may prove more effective, to provide the PM instruction described here through self-pacing computer programmes incorporating good, interesting graphics and animation. The only restriction is that measures need to be taken to ensure that skills learnt through the computer can be generalised to pen and paper tasks (the usual form tests and examinations take) and other situations that may require computational skills.

A necessary future direction to take is the investigation of some cognitive processes of individuals with mathematics LD who experience significant difficulties in computational procedures, as the students of the present study did. Mastropieri et al. (1991) pointed out that none of the studies on mathematics LD remediation which they reviewed made any attempt to link the intervention procedures used to specific theories or even to characteristics of individuals with LD, other than their general deficit in executing mathematical tasks. Intervention procedures do not just work: there must be reasons why they work, and those reasons must be somehow related to the particular characteristics of individuals with mathematics LD. In this and the previous two chapters, the possible reasons for the success of the PM method of instruction have been discussed. The reasons include ones that suggest various difficulties, problems, or deficits that individuals with mathematics LD may have.

For example, individuals with mathematics LD may have deficits in any of a number of aspects of memory, or in the strategies they employ to remember information, particularly procedures. However, experimental investigations are required to actually determine these characteristics and to satisfactorily explain why process mnemonics were found to be an effective tool for teaching students experiencing difficulties in mathematics computation. Future studies will need to specifically address these issues.

Finally, future directions in research ought to investigate the possibility of developing process mnemonic methods of instruction for other areas of mathematics LD (other than basic computational procedures), and perhaps other areas of LD. As noted previously, process mnemonics are intended to aid the learning and retention of rules and procedures, which are the processes underlying problem solving. As such, they potentially could address many of the problems in learning encountered by individuals with LD. While reading and writing are not as rule-based as mathematics there are still significant areas in these that are rule-based, and which often present difficulties to individuals with LD. Perhaps as a starting point a closer examination needs to be made of how yodai mnemonics are used at the Ryoyo Institute in Kyoto, Japan to teach other subjects/topics (including learning of the English language) to their students. Since teachers at the school have been using yodai mnemonics to teach their students for quite some time, there may be a lot in terms of strategies and development of methods that could be learnt from them.

### *Summary*

This discussion chapter focused on the relevance of the present study's findings to the topics of process mnemonics and mathematics LD. It also looked at the wider implications of the findings, the limitations of the study, and directions for future research.

The possible reasons suggested for the success of the PM instruction include its facilitation of recoding, relating, and retrieving of information to be learned – hence the use of the three Rs of mnemonics. Process mnemonics also utilise five principles of learning and memory: meaningfulness, organisation, association, attention, and visualisation.

Another reason which may help explain why the PM method of instruction worked is that it shared many features with other successful intervention methods. It was essentially the DI method with the process mnemonic metaphors added on so, like the DI method, it was systematic, it provided opportunities for practice, and it gave useful feedback to both instructors and students. The PM instruction used also provided cues for recall which were generalisable to natural settings; it provided a more concrete dimension to the concepts and procedures involved in solving computational problems; it made the computational skills procedures more interesting; and it employed many simple and specific rules which could have helped to lighten the load on the memory of the student participants.

The effectiveness of the PM instruction was also explained in terms of memory- and performance-related factors that have previously been used to explain why fact mnemonics work. Process mnemonic use facilitates generativity, executive monitoring, and personal efficacy. Process mnemonics simplify acquisition of target information by facilitating discrimination of features, sifting out what is relevant from what is irrelevant, and combination of selectively encoded information in such a way that the parts are cohesive and make sense. Process mnemonics also share many features with memory schemas. It was pointed out that, like memory schemas and unlike fact mnemonics, process mnemonics can serve as guides for behaviour. This may account in large measure for the effectiveness of the PM method of instruction in teaching the students *what to do* when solving computational problems.

Some of the concerns previously expressed by other researchers about the use of process mnemonics in teaching were addressed. It was stressed that the PM instruction used in this study did not obliterate or hinder comprehension of the processes underlying the skills taught. Possible confusion that could arise from using many incongruent metaphors was also avoided in the present study.

The primary relevance of process mnemonics to mathematics LD remedial instruction is that they address a need not otherwise addressed by other common methods of remedial instruction: that of long term retention of the skills learnt. There were also clear indications from the present study's results that the PM method has practical applications in the remedial classroom since it can effectively be used in small group teaching, and other instructors were able to use it successfully.

From the findings of the present study, there were clear indications that mathematics LD need not be viewed as a permanent condition. Supporting evidence was found that students with mathematics LD are capable of making significant improvements in their computational skills performance and of maintaining the improvements made in the long term. The improvements in performance were also achieved without resorting to psychological process deficits remediation: the provision of direct remedial instruction proved adequate. Hence, it was proposed that direct instruction methods may provide a way to successfully circumvent any process deficits that exist.

Although the experimental components of the present study did not focus on the causes of mathematics LD, the results obtained suggested some of these. These were discussed, particularly the possible deficit in memory that students with mathematics LD may have. Deficits in memory may arise because of difficulties in retaining the individual elements (the subprocedures) that make up a procedure, or in integrating those elements into a complete fluent whole.

The question of whether understanding is always necessary in mathematics was raised. Arguments were put forward that it is better to acquire skills such as basic computation even in the absence of complete understanding because such skills are important in day-to-day living.

Some ideas for teaching computational skills more effectively were also discussed. These included the use of a systematic approach to procedural instruction; concretising of concepts and ideas involved, particularly if children manifest difficulties in grasping those; making instructions more interesting to facilitate motivation and the willingness to try out new and more effective strategies; the incorporation of simple and specific rules to follow; and the need to focus on facilitating long term retention particularly where students with mathematics LD are concerned.

The selection of experimental participants with mathematics LD was discussed with particular reference to the TOSCA and the PAT Maths Test used in this study. It was argued that neither the presence of an IQ (or approximate IQ) that is average or better, nor the presence of deficiencies in a wide range of aspects of mathematics, are needed in the selection of participants with mathematics LD in research or in remedial programmes.

The limitations of the present study and how they might be addressed in future studies were examined. The limitations identified related to the extent to which maintenance assessments were taken; the narrow age range of the student participants used; the lack of investigation into the optimal length and amount of time for providing instructions; and the desirability of using greater numbers of student participants in investigating the effects of instructional intervention methods.

Finally, directions for future research were considered. It was deemed useful to take the PM method to schools and allow regular classroom teachers to use it and provide feedback on its effectiveness. The possibility

of developing and providing PM instruction using computers was noted. The necessity to undertake experimental investigations into the characteristics of students with mathematics LD that might explain why they respond positively to PM instruction was stressed. And it was pointed out that future research should investigate the possibility of developing process mnemonic methods of instruction for other areas of mathematics LD, as well as other forms of LD.

The present study was the first of its kind to investigate in any substantial way the usefulness of process mnemonics in teaching students with LD. It clearly showed that process mnemonics can be used as an effective tool for teaching computational skills to those with mathematics LD. It addressed many questions pertaining to mnemonic use, mathematics LD, and the practice of providing remedial instruction. The study also raised many questions and possibilities that should help stimulate further research into these important areas.

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# Appendices

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*Appendix A. Instruction to Students, The Basic Facts Test,  
and The Computational Skills Test*

## TO THE STUDENT

The mathematics problems on the following pages have been designed as part of a research project I am doing. The research focuses on how children learn computational skills (addition, subtraction, multiplication and division).

Depending on the scores you get in solving the problems, you may be asked to take part in an instructional programme. In the programme I will be teaching different ways of learning computational skills.

It does not matter whether you are good or poor in mathematics for you to have a go at the problems on these pages.

The scores you get will not be available to anyone else apart from myself, my two supervisors at university, and your maths teachers. You can ask me what scores you get once I have marked the sheets.

If you understand what I have written above, and you agree to do the problems, please write your name and put your signature in the appropriate spaces below:

*Student's name* .....

*Student's signature* .....

## PLEASE ANSWER THESE QUESTIONS

Your date of birth: \_\_\_\_\_

Your age: \_\_\_\_\_ years and \_\_\_\_\_ months

BOY / GIRL (Circle the one that applies)

Your school: \_\_\_\_\_

## INSTRUCTIONS

Please attempt all the problems on all of the sheets. If you are having difficulty with a particular problem, you can leave that problem in the meantime, and go on to the next one. You can do the same if you are having difficulty with a particular page. Later on, you may if you wish go back to previous problems or previous pages.

Some of the computational problems may be rather difficult to do. Do not let this bother you too much. Just do the best that you can.

## BASIC COMPUTATIONAL FACTS

### ADDITION

1	$3 + 8 =$
2	$5 + 0 =$
3	$4 + 3 =$
4	$6 + 5 =$
5	$7 + 2 =$
6	$6 + 6 =$
7	$6 + 2 =$
8	$9 + 3 =$
9	$9 + 4 + 2 =$
10	$4 + 7 + 1 =$

### SUBTRACTION

1	$6 - 3 =$
2	$12 - 6 =$
3	$10 - 4 =$
4	$5 - 3 =$
5	$16 - 7 =$
6	$13 - 9 =$
7	$10 - 7 =$
8	$12 - 3 =$
9	$17 - 9 =$
10	$11 - 3 =$

### MULTIPLICATION

1	$2 \times 4 =$
2	$6 \times 2 =$
3	$4 \times 3 =$
4	$3 \times 1 =$
5	$0 \times 6 =$
6	$5 \times 4 =$
7	$8 \times 2 =$
8	$5 \times 3 =$
9	$7 \times 2 =$
10	$7 \times 5 =$

### DIVISION

1	$5 \div 5 = \underline{\quad} r \underline{\quad}$
2	$20 \div 5 = \underline{\quad} r \underline{\quad}$
3	$4 \div 3 = \underline{\quad} r \underline{\quad}$
4	$16 \div 4 = \underline{\quad} r \underline{\quad}$
5	$7 \div 3 = \underline{\quad} r \underline{\quad}$
6	$13 \div 3 = \underline{\quad} r \underline{\quad}$
7	$36 \div 6 = \underline{\quad} r \underline{\quad}$
8	$15 \div 2 = \underline{\quad} r \underline{\quad}$
9	$23 \div 11 = \underline{\quad} r \underline{\quad}$
10	$33 \div 11 = \underline{\quad} r \underline{\quad}$

EM.91b.A.1

Name: \_\_\_\_\_

**ADDITION**

1  
$$\begin{array}{r} 253 \\ + 108 \\ \hline \end{array}$$

2  
$$67.4 + 1.3$$

3  
$$28.3 + 3$$

4  
$$11 + 6.92$$

5  
$$23.6 + 8.54$$

6  
$$152 + 7.831$$

7  
$$17.06 + 3.965$$

8  
$$172.4 + 6 + 0.391$$

9  
$$829.3 + 4.926 + 2$$

10  
$$624.6 + 57.031 + 1 + 0.6295$$

**SUBTRACTION**

1

$$\begin{array}{r} 623 \\ - 360 \\ \hline \end{array}$$

2

$$\begin{array}{r} 510 \\ - 334 \\ \hline \end{array}$$

3

$$55.8 - 31$$

4

$$3 - 0.4$$

5

$$23.6 - 8.74$$

6

$$125 - 3.67$$

7

$$14.2 - 1.256$$

8

$$2 - 0.984$$

9

$$510.07 - 3.384$$

10

$$30.1 - 0.2658$$

EM.91b.M.1b

Name: \_\_\_\_\_

**MULTIPLICATION**

1

$$\begin{array}{r} 22 \\ \times 4 \\ \hline \end{array}$$

2

$$\begin{array}{r} 16 \\ \times 2 \\ \hline \end{array}$$

3

$$\begin{array}{r} 404 \\ \times 3 \\ \hline \end{array}$$

4

$$\begin{array}{r} 213 \\ \times 12 \\ \hline \end{array}$$

5

$$\begin{array}{r} 0.2 \\ \times 0.6 \\ \hline \end{array}$$

6

$$\begin{array}{r} 1.4 \\ \times 2.5 \\ \hline \end{array}$$

7

$$\begin{array}{r} 23.8 \\ \times 1.2 \\ \hline \end{array}$$

8

$$\begin{array}{r} 2.43 \\ \times 5.1 \\ \hline \end{array}$$

9

$$\begin{array}{r} 20.34 \\ \times 1.3 \\ \hline \end{array}$$

10

$$\begin{array}{r} 13.71 \\ \times 0.25 \\ \hline \end{array}$$

**DIVISION**

1

$$2 \overline{)826}$$

2

$$5 \overline{)205}$$

3

$$3 \overline{)429}$$

4

$$4 \overline{)8.56}$$

5

$$3 \overline{)7.32}$$

6

$$0.2 \overline{)46}$$

7

$$0.6 \overline{)636}$$

8

$$0.4 \overline{)0.812}$$

9

$$0.2 \overline{)1.524}$$

10

$$1.1 \overline{)23.43}$$

*Appendix B. PM Instructions in Addition, Subtraction,  
Multiplication, and Division*

## PM INSTRUCTION: ADDITION

When we are adding numbers, we can imagine that they are warriors with a number on them. The number shows how heavy, how strong and how big the warrior is.

For example, a warrior who has the number 7 is heavier, stronger and bigger than a warrior with the number 4. (Illustrate this on the board).

Some warriors are “ranked” – they are the officers. Some warriors are not ranked – they are just ordinary warriors. When they are in groups, they have to line up properly. The dot – or decimal point – shows which warriors are ranked and which are not. Those to the left of the dot are ranked, while those to the right are not ranked.

For example, when we have 27.49, the warriors with 2 and 7 are ranked while those with 4 and 9 are not ranked. (Illustrate this on the board).

We can imagine Addition as the warriors getting onto a boat. The plus sign (+) is the mast of the boat, and the line beneath it (\_\_\_\_\_) is the boat itself where they all hop on. (Illustrate this on the board).

When they get on the boat, the warriors must line up properly so that the ones who are ranked are lined up with others who are also ranked, while those who are not ranked are lined up with others also not ranked. We can use the dot (or decimal point) to do this correctly because the dot in one group of warriors must always line up with those in other groups.

For example, if we were adding  $57.2 + 2.6$ , as I said we can imagine them to be two groups of warriors getting onto a boat. To get on the boat they have to line up properly. So we'll get

$$\begin{array}{r} 57.2 \\ + 2.6 \\ \hline \end{array}$$

(Illustrate this process of ‘getting on the boat’ on the board by drawing the numbers as warriors, etc.)

Notice that the dots or decimal points are lined up with each other. As a result, the warrior with the 2 from the top group is lined up with the warrior with the 6 from the bottom group. Both these warriors are next on the right of the dots. Likewise, the warrior with the 7 from the top group is lined up with the warrior with the 2 from the bottom

group. Both these warriors are next on the left of the dot. (Point to the appropriate 'warriors' on the board.)

Another example is if we were adding  $32.4 + 7.59$ , we can imagine them to be warriors getting on a boat. Using the dots or decimal points we can line them up as

$$\begin{array}{r} 32.4 \\ + 7.59 \\ \hline \end{array}$$

(Illustrate this process of 'getting on the boat' on the board by drawing the numbers as warriors, etc.)

Notice again that the dots are properly lined up with each other. Notice also that the warrior with the 4 from the top group is lined up with the warrior with the 5 from the bottom group – which is correct. It would have been wrong if we lined up the warrior with the 4 (point to it) with the warrior with the 9 (point to it, and then illustrate how doing it that way is wrong – i.e., some ranked and some unranked warriors would end up lined up with each other, e.g., the 2 and the 5). Both the warrior with the 4 and the warrior with the 5 are next to the right of the dots, so they must be lined up with each other (point to the appropriate warriors). Similarly the warrior with the 2 from the top group and the warrior with the 7 from the bottom group are correctly lined up with each other (point to the appropriate warriors). They are both the closest to the left of the dots.

On the sheet I have given you, I want you to have a look at the numbers given in part 1 and imagine that they are groups of warriors – just like I have shown you on the board. Then have a go at lining up these numbers that we're imagining to be groups of warriors. Don't worry about adding up the numbers, just have a go at lining them up properly.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

Some numbers are given to us without decimal points, for example: 6, 2, 35, 817. We can imagine these numbers also to be warriors – but they are all ranked. That means they're officers, as I said before. So the dot or decimal point goes on their right.

For example, in these numbers the dot or decimal point goes as follows

6.      2.      35.      817.

(Illustrate where the dot goes on drawings of warriors with numbers on them.)

What goes on the right of the dot in these numbers is just 0 or nothing. We can imagine the 0 to be a very weak and basically useless warrior who is not ranked. So if we want we can put

6.0                  2.00                  35.0                  817.000

(Illustrate this using warrior drawings.)

Notice that we can put one, two, three or even more 0 (zero) warriors if we want. Or we can put none. It doesn't really matter in this case because the 0 (zero) warrior is pretty weak, light, and small anyway to make too much difference.

On the sheet I have given you, I want you to again imagine that the numbers given in part 2 are warriors. They do not have the dots or decimal points because they are ranked officers. I want you to quickly have a go at putting the dots or decimal points, and 0's if you want, on these numbers.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

Now that we know where the dot or decimal point goes on these numbers, we can line them up properly with numbers with decimal points. We can simply imagine that these numbers are ranked warriors and that the dot goes on their right. Then we can use that dot to line it up properly with other numbers already with a dot.

For example, if we were adding  $38.4 + 4$ , we can imagine them to be a group of warriors (the 38.4) and a ranked warrior (the 4). The ranked warrior will have a dot to the right. So we'd be able to line them up properly as

$$\begin{array}{r} 38.4 \\ + 4. \\ \hline \end{array}$$

(Illustrate this process using drawings of warriors, on the board.)

Notice that the ranked warrior on the bottom (the 4) is lined up with the warrior with the 8 from the top group who is also ranked. Both the warrior with the 4 and the warrior with the 8 are next to the left of the dots – so they are properly lined up with each other. It would have been wrong if we lined up the warrior with the 4 from the top group, who is not ranked, with the bottom warrior with the 4 who is ranked. (Point to the appropriate warriors, and illustrate with drawings when explaining erroneous way of lining up).

On the sheet I have given you, I want you to again imagine that the numbers in part 3 are warriors. Then have a go at lining up these numbers. Again, don't worry about adding them up, just line them up properly.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

Now that we have gone over how to line up the numbers correctly, the next thing to do is to look at how to correctly add up the numbers. As I said before, we can imagine that the numbers are warriors getting on a boat. When they get on the boat they have to line up properly – which is just what we've been going over. Once they are lined up properly, we can start adding. We can imagine that we are trying to find out how heavy the lined-up warriors are.

When we are doing this, we always start from the rightmost numbers – or, as we're imagining them, the warriors who are furthest to the right.

For example, if we were adding  $19.02 + 5.297$ , we can imagine them to be two groups of warriors getting on a boat. As they get on the boat, they have to line up properly like this

$$\begin{array}{r} 19.02 \\ + 5.297 \\ \hline \end{array}$$

(Illustrate this process using warrior drawings with the numbers.)

Once lined up properly, we can start adding their weights – starting with the warriors furthest to the right. In this case, we'd first add the 7 to nothing or 0 (zero), and so we still get 7. Then we'd add the 2 from the top group to the 9 from the bottom group, which gives us 11. We therefore write 1 next to the 7 (show this on the board), and carry the other 1 over. So we write this carried over 1 above the 0 (show this on the board).

Make sure that you are careful and that you record any carry overs correctly.

So we add the carried over 1 to the 0 from the top group and the 2 from the bottom group. This gives us 3. Then we add the 9 from the top group to the 5 from the bottom group, which gives us 14. Therefore, we write the 4 next to the 3, and carry over the 1. Then we add the carried over 1 to the 1 of the top group, which gives us 2. (Illustrate all of this on the board).

So we can imagine that the weight of these warriors once they've all hopped on the boat is 24.317 units. This is our addition answer.

Notice that the dot, or decimal point, also goes in the answer. This is very important. The dot in this case is between the 4 and the 3. It is lined up with the other dots. When we're adding, the dot or decimal point in the answer must always line up with the other dots or decimal points. (Point to the appropriate numbers and decimal points on the board.)

Sometimes the warrior who is furthest to the right does not line up with anyone at all. When that happens, we don't have to add the weight of that warrior to any other warrior's. So we just find out how heavy that warrior is and write that number down.

An example is in the problem I have shown you just now. Or if we were adding  $858.4 + 3.664$ . Again we can imagine that these are two groups of warriors getting on a boat. And so using the dots or decimal points, we line them up as

$$\begin{array}{r} 858.4 \\ + 3.664 \\ \hline \end{array}$$

In this case, two of the non-ranked warriors in the bottom group do not line up with any of the warriors from the top group. These warriors from the bottom group are the one with the number 4 and the one with the number 6 (point to the appropriate warriors). So we don't add their weights to anyone's. We just write them down here as 4 (show) and 6 (show again). Then we continue adding the rest: the 4 from the top group to the 6 from the bottom group, and so on.

Always remember also that when you add 0 to a number, or a number to 0, you still get that number. We can imagine that warriors with 0's (zeros) on them are so light that they don't make any difference.

For example,

$$4 + 0 = 4 \quad 0 + 9 = 9 \quad 26 + 0 = 26 \quad 0 + 431 = 431$$

Now I want you to have a look at the problems in part 4 of the sheet I have given you. I want you to imagine that the numbers there are groups of warriors who are about to hop onto a boat. So first of all, line up these warriors properly using the dots. Then add up their weights properly. Remember to start with the warriors furthest to the right. Also make sure you record and use carry overs. And work carefully.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

## PM INSTRUCTION: SUBTRACTION

When we are solving Subtraction problems, like in Addition, we can imagine the numbers as warriors. The numbers on these warriors show how strong, how heavy and how big they are. A warrior with the number 8, for example, is going to be stronger, heavier and bigger than a warrior with the number 5.

Like in Addition, some warriors are ranked, and some are not ranked. The ranked warriors, who are to the left of the dot or decimal point, are the officers. The other warriors, to the right of the dot, are not ranked – they are just ordinary warriors.

For example, if we have 36.28, the warriors with the 3 and the 6 are ranked, while the warriors with the 2 and the 8 are not ranked. (Illustrate this on the board).

We can imagine Subtraction as being two groups of warriors fighting each other. The first thing we have to do is line up the two groups, and we can do this properly by using the dots or decimal points. They must always line up with each other so that ranked warriors face other ranked warriors, while those who are not ranked face warriors who are also not ranked.

For example, if we have  $34.2 - 6.51$ , we can imagine them to be two groups of warriors and line them up as

$$\begin{array}{r} 34.2 \\ - 6.51 \\ \hline \end{array}$$

Notice that the dots or decimal points are properly lined up with each other. Notice also that the nonranked warrior with the 2 from the top group is lined up with the nonranked warrior with the 5 from the bottom group. This is correct since both the warrior with the 2 and the warrior with the 5 are immediately to the right of their group's dot or decimal point. So they line up correctly with each other. (Illustrate and point to the appropriate numbers on the board).

It would have been a mistake if we had lined up the warrior with the 2 from the top group with the warrior with the 1 from the bottom group (illustrate this on the board). This way the dots or decimal points are not lined up with each other. The warriors from the two groups are also not lined up properly with each other. For example, the ranked warrior with the 4 from the top group will be wrongly lined up with the nonranked warrior with the 5 from the bottom group. (Illustrate these on the board).

On the sheet I have given you, I want you to have a look at the numbers given in part 1 and imagine that they are groups of warriors – just like I’ve shown you on the board. Then have a go at properly lining up these numbers that we’re imagining to be groups of warriors. Don’t worry about subtracting the numbers from each other, just have a go at lining them up properly.

(Circulate among the children and make sure that they are doing the task correctly. If not, tell them “That is not quite right,” show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

As we saw when we had the class on Addition, some numbers are given to us without decimal points. For example: 5, 3, 42, 619 (write these on the board). We can imagine these numbers also to be warriors – but they are all ranked. That means they are officers, as I said before. So their dot or decimal point will go on their right. On these particular numbers the dots or decimal points will go as follows:

5.      3.      42.      619.

(Illustrate where the dot goes on drawings of warriors with numbers on them.)

Now that we know where the decimal point or dot goes on these numbers, we can line them up with other numbers which have decimal points. So, for example, if we have  $56.9 - 23$ , we now know that the dot on the 23 goes to its right. Remember that we can imagine the 23 to be two ranked warriors – one with a 2 on it, and the other with a 3 on it (illustrate this on the board). So putting the dot on it will look like this

23.

Now we can line it up properly with the 56.9, like this

$$\begin{array}{r} 56.9 \\ - 23. \end{array}$$

As I said before, we can imagine that these are two groups of warriors, and using the dots or decimal points on them we have properly lined them up with each other. Notice that the ranked warrior with the 6 from the top group – which is next on the left of the top group’s dot (show this on the board) – is properly lined up with the ranked warrior with the 3

from the bottom group – which is also next on the left of the bottom group’s dot (show this on the board).

It is also a good idea to put 0’s (zeros) in where some of the warriors after the dot in one of the groups are not lined up with any other warriors from the other group. For example, in this case the nonranked warrior with the 9 from the top group is not lined up with any other nonranked warrior from the bottom group. So we can put in a warrior with a 0 in the bottom group to line up with it. We can imagine the 0 to be a very weak and basically useless warrior who is not ranked. So it will look like this

$$\begin{array}{r} 56.9 \\ - 23.0 \\ \hline \end{array}$$

Another example of this is if say we had  $275 - 3.16$ . We can imagine the 275 to be a group of ranked warriors and that the dot or decimal point on it goes on its right (show this on the board). So we can properly line it up with the 3.16 – which we can also imagine to be a group of warriors – like this

$$\begin{array}{r} 275. \\ - 3.16 \\ \hline \end{array}$$

Now we can see that the warrior with the 1 and the warrior with the 6 of the bottom group line up with nothing from the top group. So what we do is put in two warriors with 0’s after the dot of the top group. So the top group now becomes 275.00. As I said before this is not really going to be much different from just 275 since the warriors with the 0’s are pretty weak and useless anyway. So now our Subtraction problem will look like this

$$\begin{array}{r} 275.00 \\ - 3.16 \\ \hline \end{array}$$

In part 2 of the sheet I have given you, I want you to imagine that the two numbers given are two groups of warriors. Using the dots or decimal points, have a go at properly lining up the two groups. Also put in the 0’s where they are needed. Don’t worry about actually doing the subtraction, just line up the numbers properly and put in the 0’s where they are needed.

(Circulate among the children and make sure that they are doing the task correctly. If not, tell them “That is not quite right,” show them individually how it is correctly done, and

then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

Now that we know how to line up the numbers properly, and also where to put in the necessary 0's, we can get on with doing the actual subtraction.

We can imagine Subtraction to be a situation where there are two groups of warriors. One group consists of the "attackers," and the other group the "defenders." For example, if we have  $75.6 - 43$ , we line them up properly and put in the 0 after the decimal point on the 43. So we get

$$\begin{array}{r} 75.6 \\ - 43.0 \\ \hline \end{array}$$

The top group, the 75.6, are the attackers. The bottom group, the 43.0, are the defenders. The defenders have their knives drawn, which we can imagine the minus sign (the  $-$ ) to be. The defenders are always next to this minus sign.

Now the attackers are trying to get past the defenders so that they can pass through the gate behind the defenders – which is shown here by the line below them (the         , illustrate and explain this on the board). But as the attackers go past the defenders they become weaker. (As I said before, we can imagine these numbers to be warriors with numbers on them – and the numbers show how strong they are.) By the time the attackers go through the gate, their strength is usually less. We find out what their strength is by subtracting the defending warriors' strength from their strength. And we always start with the rightmost pair of warriors.

So in the example that we have, we start with the attacking warrior with the 6 trying to get past the defending warrior with the 0. These are the two warriors furthest to the right (show this on the board). Now as I said before, warriors with a 0 on them are pretty weak and useless. So in this case, the attacking warrior with the 6 can quite easily get past the defending warrior with the 0. The attacking warrior's strength (the number on him) by the time he gets through the gate is his original strength (which is 6) minus the defending warrior's strength (which is 0). The answer to 6 minus 0 is 6. So the attacking warrior's strength is still 6. We write that down here. (Illustrate all of these on the board).

The important thing to remember in Subtraction too is that we are taking away the bottom number from the top number, not the other way round. By imagining that our numbers are warriors, we can say that the attacking warriors' strength (the top number) goes down or

becomes weaker by the defending warriors' strength (the bottom number). So in this case we are taking 43.0 (the number which shows the defending warriors' strength) away from 75.6 (the number which shows the attacking warriors' strength). We can also say this as 75.6 take away 43.0.

Back to our subtraction, so far we have seen the attacking warrior furthest to the right go past the defending warrior he is lined up against. The attacking warrior had a strength of 6, we subtracted the defending warrior's strength of 0 from this, and we therefore still got 6 – which is the strength of the attacking warrior by the time he goes through the gate here. (Point to the appropriate warriors/numbers on the board).

Next we put the dot or decimal point here, making sure that it is properly lined up with the other two dots above. Remember that it is important to put the dot in – it let's us know which of our warriors are ranked, and which are not (those to the left of it are ranked, and those to the right are not).

Then we have this ranked attacking warrior with a strength of 5 trying to get past this ranked defending warrior with a strength of 3. So we have 5 minus 3 which is 2. What this means is that this attacking warrior's strength is only 2 by the time he gets past this defending warrior and through the gate. So we write the 2 here. (Illustrate these on the board).

Finally we have this ranked attacking warrior with a strength of 7 trying to get past this ranked defending warrior with a strength of 4. So we have 7 minus 4 which is 3. What this means is that this attacking warrior's strength is only 3 by the time he gets past this defending warrior and through this gate here. So we write the 3 here. (Illustrate these on the board).

Our answer then is 32.6 (point to this answer on the board).

In part 3 of the sheet I have given you, I want you to quickly have a go at solving the Subtraction problems given there. Imagine that the numbers are groups of warriors, line them up properly first, then put in the 0's where they are needed. As I said we can imagine the top number to be the attacking group of warriors, and the bottom number to be the defending group of warriors. Start with the attacking and defending warriors furthest to the right. Remember that as the attacking warrior goes past the defending warrior, his strength goes down. We take away the defending warrior's strength from the attacking warrior's strength. Also remember to put in the dot or decimal point in your answers.

(Circulate among the children and make sure that they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

When sometimes the attacking warrior's number or strength is less than the defending warrior's, we first have to increase that attacking warrior's strength by 10. Otherwise that attacking warrior will not get past the defending warrior who is stronger than him.

As an example, let's say we have 8 – 3.2 which we line up as

$$\begin{array}{r} 8.0 \\ - 3.2 \\ \hline \end{array}$$

The attacking warrior with the strength of 0 has to get past the defending warrior with the strength of 2. Obviously the attacking warrior with the 0 won't make it. (Illustrate these on the board).

So what we do is add 10 strength points to the warrior with the 0. So now that warrior has 0 plus 10 strength points – which equals 10 (show this on the board). But whenever we do this we must remember that something else happens – and that is that the next defending warrior's strength increases by 1. So in this case the ranked defending warrior with the 3 has 1 strength point added to it. We can remind ourselves of this by putting a slash mark here below the 3 (show this on the board). So now that defending warrior's strength is 3 plus 1 which equals 4.

So now when the attacking warrior with the 10 (which used to be the 0 – remember?) goes past the defending warrior with the 2, his strength will get lower. It will be 10 minus 2 which equals 8. What that means is that by the time he gets past the defending warrior and the gate, his strength will only be 8. So we write down the 8 here.

Next we put the dot or decimal point in. Remember that it has to be in line with the other dots (show this on the board).

Then we have the ranked attacking warrior with the 8 trying to get past this defending warrior with the 4 (3 plus 1 – which is what the slash stands for). By the time this attacking warrior gets past this defending warrior his strength will be 8 minus 4, which is 4. So we write the 4 here. (Illustrate these on the board.)

As you can see, the answer to our Subtraction problem is 4.8 (point to this answer on the board).

In part 4 of the sheet I have given you, I want you to have a go at the Subtraction problems given there. Imagine that they are two groups of warriors, the first group are the attacking warriors and the second group are the defending warriors. Line them up properly and put in the 0's where they are needed. Then if an attacking warrior's strength is lower than the defending warrior's he is lined up against, increase that attacking warrior's strength by 10. But remember also to increase the next (to the left) defending warrior's strength by 1.

Do you have any questions? (Answer any questions posed, explaining things on the board where necessary.) Also remember to put the dot or decimal point in your answers!

(Circulate among the children and make sure that they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

We've seen how if an attacking warrior's strength is lower than the defending warrior's we have to add 10 strength points to that warrior, and then add 1 to the next defending warrior's strength. Sometimes we have to do this several times in a Subtraction problem. For example, if we have  $40.2 - 0.3587$ , we can line them up and put in the necessary 0's to get

$$\begin{array}{r} 40.2000 \\ - \underline{0.3587} \end{array}$$

We can imagine the 40.2000 to be the group of attacking warriors, and the 0.3587 to be the group of defending warriors (illustrate this on the board). So first the attacking warrior with the 0 – which is furthest to the right – has to get past the defending warrior with the 7. Now 0 is lower strength than 7, so the attacking warrior won't be able to get through. We therefore add 10 strength points to it, making it's strength 10 (0 plus 10 equals 10). But we also have to put a slash just under this defending warrior with the 8 to remind ourselves to add 1 to its strength of 8. That defending warrior's strength therefore becomes 9.

Back to our subtraction, when this attacking warrior which now has 10 strength points goes past this defending warrior with the 7, his strength goes down to 3 (10 minus 7 equals 3). So we write down the 3 here (illustrate this on the board).

The next attacking warrior also has a strength of 0, which is less than the strength of the defending warrior he is lined up against – which is 9 (remember, this used to be the 8 that we added the 1 to). So we add 10 strength points to this attacking warrior, making its strength 10. But we have to add 1 to the strength of the next defending warrior – this one which has a 5. We put a slash below it to remind ourselves that it is now a 6 (5 plus 1 equals 6).

Back to the subtraction, we have an attacking warrior with a 10 and a defending warrior with a 9. By the time the attacking warrior has gone past the defending warrior and onto the other side of the gate, his strength will be 10 minus 9 which equals 1. So we write the 1 here (illustrate this on the board).

Next we have another attacking warrior with a 0 and a defending warrior with a 6 (5 plus 1, remember?). The strength of 0 is clearly lower than the 6, so we add 10 strength points to this attacking warrior like we've done with the others. So now he has a strength of 10. And we add 1 strength point to the next defending warrior – which is the one with the 3, making that a 4. So by the time this attacking warrior with the 10 gets past this defending warrior with the 6, his strength will be down to 4 (10 minus 6 equals 4). So we write down the 4 here (illustrate this on the board).

Next we have this attacking warrior with the 2, and this defending warrior with the 4 (3 plus 1). Again the attacking warrior has a lower strength, so we add 10 strength points to him, making his total strength 12 (2 plus 10 equals 12). And we add 1 strength point to the next defending warrior which has a 0, making his total strength 1 (0 plus 1 equals 1). Back to the subtraction, we have an attacking warrior with a 12 and a defending warrior with a 4. So by the time the attacking warrior gets past the defending warrior and to the other side of the gate his strength will have gone down to 8 (12 minus 4 equals 8). So we write down the 8 here (show this on the board).

Next we put the dot or decimal point here (show this on the board). As we can see, it lines up properly with the other two dots. It is very important to remember to put the dot or decimal point in our answer – otherwise our answer will not be correct.

Back to the Subtraction problem, we next have a ranked attacking warrior with a strength of 0 trying to get past a ranked defending warrior with a strength of 1 (which used to be a 0, but we added 1 strength point to him – remember?). Again the attacking warrior is too weak so we add 10 strength points to him, making his strength 10 (0 plus 10 equals 10). We also have to add 1 strength point to the next defending warrior on the left. But there isn't any warrior there, so we just imagine that there is a 0 there under which we put a slash to

remind ourselves of the 1 that we add to it. Back to the subtraction, we have an attacking warrior with a 10 trying to get past a defending warrior with a 1. By the time the attacking warrior gets past the defending warrior and onto the other side of the gate, his strength will be down to 9 (10 minus 1 equals 9). So we write down the 9 here. (Illustrate all of these on the board.)

Next we have this ranked attacking warrior with a strength of 4 and this defending warrior with a strength of 1 (remember there used to be nothing there – ?). By the time the attacking warrior gets past the defending warrior, his strength will be down to 3 (4 minus 1 equals 3). We write down the 3 here. (Illustrate these on the board.)

Therefore the answer to our Subtraction problem is 39.8413 (point to this on the board).

In this Subtraction problem that we've done we have seen what to do when we have to subtract a digit which is bigger than the digit we are subtracting it from. Sometimes this happens quite a number of times in a Subtraction problem – as in this problem. If we imagine our numbers as attacking and defending warriors, when a particular attacking warrior is weaker than his opposite defending warrior, we just add 10 strength points to that attacking warrior. But we must also remember to add 1 strength point to the next defending warrior. It is a fairly easy rule to remember. (Illustrate this on the board using the problem previously solved).

We must also always remember to put in the dot or decimal point in our answers. And work very carefully.

Do you have any questions? (Answer any questions posed, explaining things on the board where necessary.)

In part 5 of the sheet I have given you, I want you to have a go at the Subtraction problems given there. Imagine that the numbers are warriors: the first group being the attacking warriors, and the second group the defending warriors. Line them up properly using the dots or decimal points. Then put in any necessary 0's. Then do your subtractions carefully. Remember that the attacking warriors are trying to get past the defending warriors and onto the other side of the gate. As they do this their strength goes down according to the strength of the defending warriors. So by the time they get to the other side of the gate, their strength will be their original strength minus the strength of the defending warrior.

Remember that if the attacking warrior's strength is lower than the strength of the defending warrior he is lined up against, we have to add 10 strength points to that

attacking warrior first. So his strength will be his original strength plus 10. But we must also add 1 strength point to the next defending warrior along to the left. We mark this 1 additional strength point with a slash.

Be careful in working through the problems. And remember to put the dots or decimal points in your answers.

(Circulate among the children and make sure that they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

## PM INSTRUCTION: MULTIPLICATION

We can imagine Multiplication to be a meeting of two groups of warriors – there is a top group and a bottom group. For example, if we have

$$\begin{array}{r} 34 \\ \times 2 \\ \hline \end{array}$$

the situation is that in the top group there are two warriors, one with a 4 on him and one with a 3, and in the bottom group there is only one warrior with a 2 (illustrate these on the board using warrior drawings).

These two groups of warriors have come to this meeting to exchange battle techniques. We can imagine the 'x' or Multiplication sign to stand for "X-change" of battle techniques. To exchange battle techniques, each member of the bottom group, of which in this case there is only one, must meet with each member of the top group – starting with the one furthest to the right. The warriors in the bottom group – lined up with the x (point to this on the board) – are the "X-perts" (experts) and they must teach each member of the top group their special battle techniques (point to and illustrate these on the board).

So first we have the X-pert warrior with the 2 meeting up with the warrior with the 4 from the top group – who is furthest to the right in that group. We get the 'product of their meeting' – which means what sort of things did they get out of it – by multiplying the two numbers. So we have 4 times 2 which is 8. We write that 8 down here. (Illustrate these on the board.)

Next we have the X-pert warrior with the 2 meeting with the warrior with the 3 from the top group. Again we multiply the two numbers; 3 times 2 is 6. And we write down the 6 here. So the answer we get is 68. (Illustrate these on the board.)

Sometimes when we are doing multiplication, or this X-change of battle techniques as we can imagine it to be, we have to do 'carrying over.' We do this when the answer to the two numbers we multiplied is larger than 9. For example, if we have

$$\begin{array}{r} 27 \\ \times 3 \\ \hline \end{array}$$

imagining that these are two groups of warriors, we first have the X-pert warrior with the 3 meeting up with the warrior with the 7 from the top group. We multiply these numbers;

the answer we get to 7 times 3 is 21. But we only write down the 1 of this answer here (show on the board). The 2 of the 21 gets carried over. We write that 2 on top of this warrior with the 2 of the top group. We write it there to remind ourselves to add it to our next answer.

Next we have the X-pert warrior with the 3 meeting up with the warrior with the 2 of the top group. So we multiply these numbers – 2 times 3 is 6. But we don't just write the 6 down here. First we add the carried over 2 to it. So it's 6 plus 2 which is 8. It is this 8 which we write down here next to the 1. So the answer we get is 81. (Illustrate these steps on the board).

On the sheet I have given you, I want you to have a go at the Multiplication problems given in part 1. You can imagine the two numbers to be two groups of warriors at a meeting to X-change battle techniques. The bottom group has the X-pert warrior. The X-pert warrior must meet up with each of the warriors of the top group starting with the one furthest to the right. You get the product of their meeting by multiplying the number on the X-pert warrior by the number on the warrior from the top group he is meeting with. When you're doing the multiplication, don't forget to carry over when necessary.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

Quite often there are two or more X-pert warriors in the bottom group. For example

$$\begin{array}{r} 314 \\ \times 25 \\ \hline \end{array}$$

In this case there are two X-pert warriors – the one with the 5, and the one with the 2 (illustrate these on the board). Each of these X-pert warriors must meet up with each of the warriors from the top group to teach them their special battle techniques. We always begin with the warriors furthest to the right, so in this case we begin with the X-pert warrior with the 5. This warrior has to meet up first with the warrior with the 4 from the top group (again starting with the one furthest to the right), then the warrior with the 1, then the warrior with the 3. Similarly, the X-pert warrior with the 2 has to meet up with the warrior with the 4 from the top group, then the one with the 1, and finally with the warrior with the 3. (Illustrate these on the board.)

So we first have 4 times 5, which is 20. We write down the 0 here and carry over the 2. Remember how we carry over? We write down the 2 above this 1, to remind ourselves to add it on later. Then we multiply 1 by 5, which is still 5. We add the 2 to this 5, which gives us 7. And we write that 7 down here. Next we have 3 times 5, which is 15. We write that 15 down here. So this tells us that the product of the meeting between the X-pert warrior with the 5 and the warriors of the top group is 1570. (Illustrate the steps taken on the board.)

[We can, if we want, carry over the 1 of the 15 and just write the 5 down here first. We write the carried over 1 here, and since there is nothing to add it to, we just bring it down and write it next to the 5. So our answer remains the same as what I showed you before. (Show and explain this on the board.)]

Next, as I mentioned earlier, the X-pert warrior with the 2 has to meet up with each of the warriors from the top group. One thing which we have to know about these X-pert warriors is that as you move along to the left they become more of an expert. What this means is that the warrior with the 2 has more expertise than the warrior with the 5. The product of the X-pert warrior with the 2's meeting with the top group will be 10 times greater than that of the X-pert warrior with the 5 and the top group.

What we have to remember to do then is to place a 0 (zero) first, to mark the fact that we have now moved on to a more X-pert warrior. If we have more than two X-pert warriors, we put down two zeros next time, then three, and so on. In the situation we presently have, we place that 0 like this:

$$\begin{array}{r} 314 \\ \times \underline{25} \\ 1570 \\ 0 \end{array}$$

Now that we have done this, we can go ahead with the X-pert warrior with the 2 meeting up with each of the warriors from the top group. We start with the warrior with the 4 since he is furthest to the right. So 4 times 2 gives us 8. We write that 8 down here next to the 0. Then we have 1 times 2, which is still 2. We write that 2 down here next to the 8. Finally, we have 3 times 2, which is 6. And we write that 6 down here next to the 2. So the product of the X-pert warrior with the 2 meeting with the top group of warriors is 6280. (Illustrate these on the board.)

The next thing we do is add together these two products. We add the 1570 to the 6280. So we draw a line here below the 6280, and start adding. Zero plus 0 is still 0, which we write down here. Seven plus 8 is 15, so we write down the 5 here and carry the 1 over on top of the 5. Five plus 2 is 7, plus the carried over 1 gives us 8, which we write down here. Finally, 1 plus 6 is 7, which we write down here. So, after the two X-pert warriors have met up with the warriors from the top group, the overall product – and the answer to our Multiplication problem – is 7850. (Show the addition steps on the board.)

On the sheet I have given you, I want you to have a go at the Multiplication problems given in part 2. Remember that, if you imagine that these numbers are groups of warriors, you start with the X-pert warrior of the bottom group who is furthest to the right. That warrior has to meet up with each of the top group warriors, beginning again with the one furthest to the right. You multiply the numbers on these warriors together to find out the products of their meeting. When you move on to the next, more X-pert warrior don't forget to place that 0 first. Then when you've got the products of the two X-pert warriors' meetings with the top group of warriors, add them together. This will give you the overall product and the answer to the Multiplication problem. Also don't forget to carry over when necessary.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

Quite often the numbers we have to multiply have decimals. An example of this is if we get

$$\begin{array}{r} 30.42 \\ \times \underline{2.3} \end{array}$$

Both the top number and the bottom number have decimals. We can still imagine these numbers to be two groups of warriors – the top group has four warriors, and the bottom group has two X-pert warriors. The decimal point or dot, in this case, shows us which warriors are ranked and which ones are not. The ranked warriors are the officers, and they are to the left of the dot. So in the top group there are two, the warrior with the 0 and the warrior with the 3. And in the bottom group there is one X-pert warrior who is ranked – the one with the 2 (illustrate and point these out on the board). There are two warriors in the top group who are not ranked, the one with the 2 and the one with the 4. And there is the X-pert warrior with the 3 in the bottom group who is not ranked (again, illustrate and point these out on the board).

When we start our multiplication we don't worry about which warriors are ranked and which are not. We worry about that later on. In the meantime we just go ahead with the multiplication or, as we can imagine it to be, the X-change of battle techniques. So the X-pert warrior from the bottom group with the 3 meets up with the warriors from the top group with the 2, 4, 0, and 3. Likewise, the X-pert warrior with the 2 meets up with each of these warriors from the top group (point to the appropriate 'warriors' on the board).

Starting with the X-pert warrior with the 3 from the bottom group, he first meets up with the warrior furthest to the right in the top group – that's the one with the 2 (point to these numbers on the board). So 2 times 3 is 6, which we write down here. Next he meets up with the warrior with the 4, so we get 4 times 3 which is 12. We write down the 2 here and carry over the 1. Then meeting up with the warrior with the 0: 0 times 3 is still 0. We add to this the 1 we carried over – so we have 0 plus 1 – which gives us 1, which we write down here. Finally, meeting with the top group warrior with the 3: 3 times 3 is 9, which we write down here. So the answer we get, which we can imagine to be the product of the meetings between the X-pert warrior with the 3 and the top group warriors, is 9126. (Show these steps on the board.)

Next the X-pert warrior with the 2 from the bottom group meets with each of the warriors from the top group. But as I showed you before, when we move onto the next X-pert warrior, who has more expertise than the previous one, we must first write a 0 to mark what's happening. We do that like this

$$\begin{array}{r} 30.42 \\ \times \quad 2.3 \\ \hline 9126 \\ 0 \end{array}$$

Then we can go ahead with X-pert warrior with the 2 meeting with the warriors from the top group, starting with the 2 which is furthest to the right. So, 2 times 2 is 4, which we write down here. Then 4 times 2 is 8, which we write down here. Zero times 2 is still 0, so we write down the 0 here. Finally, 3 times 2 is 6, which we write down here. So we get 60840 as the product of the X-pert warrior with the 2 meeting with the warriors of the top group. (Show these steps on the board.)

Next, as I showed you before, we add the 9126 and the 60840 together, so we draw a line under the 60840. Six plus 0 is 6, which we write down here. Two plus 4 is 6, which we write down here. One plus 8 is 9, which we write down here. Nine plus 0 is 9, which we write

down here. And finally 6 plus nothing is still 6, so we write that 6 down here. So adding these two products gives us 69966.

The next step is figuring out where to put the decimal point or dot in this 69966. What we do first is count the number of warriors who are not ranked in both the top group and the bottom group. Remember that the warriors to the right of the decimal point or dot are not ranked. In the top group there are the warriors with the 2 and the 4 who are not ranked. And in the bottom group there is the warrior with the 3 who is not ranked. So there are three warriors altogether who are not ranked: two from the top group and one from the bottom group.

We use this number – which we got from counting the warriors who are not ranked – to place the decimal point or dot in our answer. We must count this number – the three – of digits from the right of our answer. So we go, “one” the 6, “two” the second 6, and “three” the 9. We put the dot or decimal point in after this third digit. So the decimal point goes here between the two 9’s. And our answer is 69.966. We must always remember to put the dot or decimal point in our answers when necessary, otherwise the answers we give will not be correct.

On the sheet I have given you, I want you to have a go at placing the decimal points in the answers to the problems given in part 3. The Multiplication steps have already been done for you. All you have to do is put the decimal points in the correct positions in the answers given. So imagining that the numbers which have been multiplied together are groups of warriors, count the number of warriors who are not ranked in both the top and the bottom groups. Remember that the warriors who are to the right of the dot or decimal point are the ones who are not ranked. Using the number that you get of warriors who are not ranked in both top and bottom groups, count that many digits from the right of the final answer before placing the decimal point. Do you have any questions? (Answer any questions posed, explaining things on the board where appropriate.) Go ahead and work on part 3 of your sheets.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them “That is not quite right,” show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

Next I want you to have a go at the Multiplication problems given in part 4 of the sheet I have given you. You can, as I suggested, imagine the numbers to be two groups of warriors. You start with the X-pert warrior of the bottom group who is furthest to the right. That

warrior has to meet up with each of the top group warriors, beginning again with the one furthest to the right. You multiply the numbers on these warriors together to find out the products of their meeting. When you move on to the next, more X-pert warrior don't forget to place that 0 first. Then when you've got the products of the two X-pert warriors' meetings with the top group of warriors, add them together.

Once you've added the two products together, the next step is to put the decimal point or dot in your answer. You can do this easily by first counting the number of warriors who are not ranked in both the top and the bottom groups. Remember that the warriors who are to the right of the dot or decimal point are the ones who are not ranked. Using the number that you get of warriors who are not ranked in both top and bottom groups, count that many digits from the right of the final answer before placing the decimal point. That should then give you the correct answer.

Are there any questions? (Answer any questions posed, explaining things on the board where necessary.)

Now be careful not to make any careless mistakes. And remember that when you multiply a number by 1, you still get that number. For example, 4 times 1 is still 4; 9 times 1 is still 9; and so on. Also remember that if you multiply any number by 0 you get 0. For example, 3 times 0 is 0; 7 times 0 is 0; and so on.

Go ahead and start on the questions in part 4.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

## PM INSTRUCTION: DIVISION

We can imagine Division to be a situation where a warrior is standing next to a rack of armour pieces. For example, if we have

$$2 \overline{)462}$$

the warrior is the 2, and the 4, 6 and 2 are pieces of armour. The warrior is trying on these armour pieces. He tries on each piece – starting with the piece closest to him – to see how many times he fits into it. The armour size of the warrior is the number on him. For example, in this case it is 2. (Illustrate these on the board using warrior drawings, etc.).

So what happens is that the warrior first tries on the armour of size 4. He will fit that 2 times. We can also say this as 4 divided by 2, which is 2. It is basically the same thing. We write that 2 above the 4.

Next the warrior tries on the armour piece of size 6. So it's 6 divided by 2 – or how many times does the warrior of size 2 fit into the armour of size 6? The answer is 3, and we write that 3 above the 6.

Next is the armour piece of size 2. So it's 2 divided by 2 – or how many times does the warrior fit into this armour piece of size 2? The answer is 1, since the armour piece is exactly the same size as the warrior. We write that 1 above the 2. (Show these steps on the board).

Our Division answer therefore is 231.

Sometimes some of the digits we are trying to divide are smaller than the number we are dividing by. We can imagine this to be where some of the armour pieces hanging on the rack are too small. For them to be useful to the warrior they have to be combined with other armour pieces. For example, if we have

$$5 \overline{)305}$$

the warrior whose armour size is the number on him – which is 5 – is first trying on the armour piece of size 3. Obviously, it is too small. What we do is we first write a 0 above that 3 to remind ourselves that it was too small to fit. And then we combine this armour piece of size 3 with the next one of size 0, and now we have an armour piece of size 30. So

we have 30 divided by 5 – or how many times does the warrior of size 5 fit into the armour of size 30? The answer is 6, and we write that 6 above the 0. (Illustrate these steps on the board).

Next the warrior tries on the armour of size 5 – which is exactly his size. Five divided by 5 gives us 1. We write this 1 above the 5. (Again illustrate these steps on the board).

The answer to our Division problem therefore is 61.

Another example of this happening is if say we had

$$4 \overline{)416}$$

The warrior of size 4 first tries on the armour of size 4 also – it is exactly his size. Four divided by 4 is 1, and we write this 1 above the 4. Next the warrior tries on the armour of size 1. It is obviously too small so we put a 0 above the 1 to remind us of this. And we combine this armour of size 1 with the next armour piece of size 6. We now have an armour of size 16, and the warrior tries this out. Sixteen divided by 4 is 4 – the warrior fits 4 times into the size 16 armour. We write that 4 above the 6.

So the Division answer we get is 104.

On the sheet I have given you, I want you to have a go at the problems given in part 1. As I told you, you can imagine the Division problems to be situations where a warrior is trying on armour pieces on a rack. The warrior's armour size is the number on him. He starts with the armour piece closest to him, and finds out how many times he fits into that piece. So it's the size of the armour divided by the size of the warrior. The answer we put above the armour piece in question. And then we move on to the next armour piece.

If the armour piece is too small, put a 0 above it and then combine it with the next armour piece – as I've just shown you.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

Quite often when we divide we also get remainders. We can imagine this to be situations where the armour pieces that the warrior tries on don't quite fit an exact number of times. For example, if we have

$$3 \overline{)456}$$

we can imagine that the warrior – whose armour size is 3 – first tries on the armour of size 4. So it's 4 divided by 3 – or how many times does the warrior of size 3 fit into that armour of size 4? The answer is once – or 1 – but there is also a remainder of 1. So it's 1 remainder 1. We put the first 1 above the 4, and the second 1 – the remainder – we put next to the armour of size 5 here, to combine it with it (illustrate these steps on the board).

So now that armour is of size 15, and that's what the warrior tries on next. Fifteen divided by 3 is 5, and we put that 5 above the 5. Next it's the armour piece of size 6. Six divided by 3 is 2, and we put that 2 above the 6. (Show these steps on the board).

Therefore our Division answer is 152.

Another example of this is if say we had

$$2 \overline{)652}$$

The warrior of armour size 2 first tries on the armour piece of size 6. Six divided by 2 is 3, and we put that 3 above the 6. Next the warrior tries on the armour piece of size 5. This time, the warrior does not fit the armour piece an exact number of times. Five divided by 2 is 2 plus a remainder of 1 – so it's 2 remainder 1. We put the 2 above the 5, and the remainder 1 goes next to this 2 to remind us to combine it with it, so that now it is a 12. Next therefore are these armour pieces of size 12. Twelve divided by 2 is 6, and we put that 6 above the 2. (Illustrate these steps on the board).

The answer therefore is 326.

On the sheet I have given you, I want you to have a go at the problems given in part 2. As I showed you, you can imagine the Division process to be a situation where a warrior (the number we are dividing by) is trying on armour pieces hanging on a rack (the digits we are dividing). Sometimes the warrior does not fit the armour pieces an exact number of times and we get remainders. What we do is combine the remainder with the armour piece next to it – as I showed you just now. Go ahead and try this out on the problems given in part 2.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them “That is not quite right,” show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

Quite often the number we have to divide, the number we are dividing by, or both, will have a decimal point. If only the number we are dividing has a decimal point, we only have to remember to put a decimal point in our answer – which is in line with the decimal point of the number we divided. For example, if we have

$$4 \overline{)4.92}$$

we can imagine that hanging on the rack are three pieces of armour. Between the first two – the one sized 4 and the one sized 9 – is a dot which separates the expensive armour from the bargain priced armour. To the left of the dot is the expensive piece – the armour sized 4. To the right of the dot are the bargain pieces – the pieces sized 9 and 2. We only have to remember at the end to put a dot also in our answer which is in line with this dot. (Point these things out on the board).

We go ahead with our normal division. So imagining still that it’s a warrior trying on armour pieces, the warrior of size 4 tries on the armour of size 4 first. That’s 4 divided by 4, which is 1. We write this 1 above the 4. Next it’s the armour piece of size 9. This piece does not fit the warrior an exact number of times – 9 divided by 4 is 2 remainder 1. So we write the 2 above the 9, and the remainder 1 next to this 2 here to remind us that it is now a size 12 instead of just a 2. Twelve divided by 4 is 3, so we write that 3 above the 2.

As you can see the ‘answer’ we have is 123. But we still have to put the dot in our answer – and that dot must be in line with the dot separating the expensive armour from the bargain armour hanging on the rack. So it goes between the 1 and the 2, making 1.23 our final answer.

Now I’d like you to have a go at solving the problems in part 3 of the sheet I have given you. Work carefully and remember to put the decimal point in the answer that you get.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them “That is not quite right,” show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

As I said earlier, sometimes the number we are dividing by also has a decimal point. When this happens, we have to shift the decimal point both in that number and in the number we are dividing. For example, if we have

$$0.3 \overline{)3.99}$$

we can imagine the situation again to be a warrior trying on pieces of armour on a rack. The armour pieces are separated by a dot. The piece to the left – the one sized 3 – is expensive, and those to the right – the pieces both sized 9 – are bargains.

There is one rule we have to remember. Only ranked warriors – those who are officers – are normally allowed to purchase armour. The others can try them on but usually are not allowed to buy them. Our warrior in this case, the 0.3 is not ranked. When there is a dot involved, those to the left of it are ranked while those to the right are not. So the 0.3 is a nonranked warrior. If that warrior is to buy armour he has to get ranking or pretend that he is ranked. So we have to shift the dot to the right and the 0.3 is now a 3. (Illustrate these steps on the board).

There is a consequence to this though. The shopkeeper knows what's happening – that the warrior is only pretending to be ranked – so she also shifts the dot on the rack of armour. She shifts it also one space along to the right. So now there are more expensive pieces of armour than there are bargain priced ones. The first armour which is sized 9 is no longer bargain priced, but given an expensive price. (Illustrate these on the board). Now the 3.99 has become 39.9. Our Division problem therefore can now be written instead as

$$3 \overline{)39.9}$$

Even though this looks different, it will still give us the same answer as 3.99 divided by 0.3 – but it is much easier to do.

Sometimes also when we shift decimal points, we have to add a 0 to the number we are dividing. For example, if we have

$$0.4 \overline{)56}$$

we can imagine that the nonranked warrior 0.4 wants to purchase armour and so pretends to be ranked. Thus by shifting the dot, the 0.4 becomes 4. The shopkeeper also has to shift the dot in the armour rack – but there is no dot to shift. All the pieces of armour are already priced expensively. So what the shopkeeper does is put in another piece of

armour – one which is not very good and is sized 0 – at the end of the rack, next to the armour sized 6. Thus the 56 now becomes 560.

We can re-write our Division problem is now as

$$4 \overline{)560}$$

Even though it looks different, it should still give us the same answer as 56 divided by 0.4.

The thing to remember here is that if there is nothing after the decimal point in the number we are dividing, and we have to shift the decimal point, we just add a 0 to that number. Just like the shopkeeper adding a no good piece of armour at the end of the rack. In the example, we just added a 0 to the 56 to make it 560.

In the sheet that I have given you, I want you to now have a go at the problems in part 4. Remember that if the number we are dividing by has a decimal point in it, we have to shift the decimal point to the right – both in it and the number we are dividing. We can imagine a nonranked warrior pretending to be ranked, and the shopkeeper shifting the dot in the rack of armour so that there are more expensive pieces. Or adding a 0 piece which is a useless piece, if there is no dot in the rack and all the pieces of armour are already expensive.

All I want you to do is shift decimal points where necessary in the problems given in part 4. And then re-write the problems to what they become after you have shifted the decimal points. Don't worry about actually solving the problems.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

Once we have shifted decimal points as necessary, we're ready do solve the Division problem as we normally do. For example, if we have

$$1.1 \overline{)46.31}$$

the first thing we have to do shift decimal places in both the number we are dividing by and the number we are dividing. Again we can imagine that the number we are dividing by

– the 1.1 – are warriors, in this case there are two warriors – one ranked and the other not. Both are sized 1. The one who is not ranked – the one to the right of the dot (point to the one on the board) – has to pretend that he is, so we have to shift the dot or decimal point to the right (illustrate on the board). The 1.1 is therefore now an 11.

Next we have to shift the decimal point in the number we are dividing. Again we can imagine these to be pieces of armour hanging on a rack. The shopkeeper finds out that one of the warriors is just pretending to be ranked, so she decides to shift the dot separating expensive from bargain pieces of armour on the rack. She shifts it also one space to the right so that now there are more expensive pieces than there are bargain pieces (illustrate this on the board). Therefore, what used to be 46.31 is now 463.1. So we can re-write the problem as

$$11 \overline{)463.1}$$

Now we can just go ahead with the division. Imagining that our two warriors are both trying on the armour pieces, they first have a go at the piece sized 4 to see how many times they fit into it. Obviously it is too small, and we put a zero above it, and combine it with the next piece sized 6. Now we have armour pieces of size 46 and we want to know how many times the two warriors – combined size 11 – fit into them. So it's 46 divided by 11 which gives us 4 remainder 2 (four 11's are 44, plus the remainder 2 make up 46). We put the 4 above the 6, and the remainder 2 next to this 3 (show on the board) to combine it with it so that now that is a 23.

So the next thing we have are pieces of armour sized 23 and the warriors are trying them out. Twenty-three divided by 11 is 2 remainder 1. So we put the 2 above the 3, and the remainder 1 next to this 1 here to combine it with it – making 11 (show this on the board). At this stage we should put in the dot or decimal point in our answer so that we don't forget to put it on later. It goes in line with the dot or decimal point of the number we are dividing, so it goes next to this 2 (illustrate on the board).

Finally we have these armour pieces sized 11. The warriors – also sized 11 – tries them on. So it's 11 divided by 11 which gives us exactly 1. We put this 1 above this 1 (show on the board).

Our answer therefore is 42.1.

Do you have any questions about what I did? (Answer any questions posed, explaining things on the board as necessary).

Now I want you to have a go at solving the problems in part 5 of the sheet I have given you. Remember to shift the dots or decimal points and re-write the problems first if necessary. Remember also to correctly place the dot or decimal point in your answer when necessary. Work carefully.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

*Appendix C. DI Instruction in Addition, Subtraction,  
Multiplication, and Division*

## DI INSTRUCTION: ADDITION

In Addition it is very important that we properly line up the **numbers** we are adding together.

We can do this using the decimal points of the numbers we have.

For example, if we were adding  $57.2 + 2.6$ , we have to line them up first as

$$\begin{array}{r} 57.2 \\ + 2.6 \\ \hline \end{array}$$

Notice that the decimal points are lined up with each other. As a result, the 2 which is the digit to the right of the decimal point of the top number is lined up with the 6 which is the digit to the right of the decimal point of the bottom number. The same goes with the 7 of the top number and the 2 of the bottom number. (Point to the appropriate **numbers** on the board).

Another example is if we were adding  $32.4 + 7.59$ , we have to line them up first as

$$\begin{array}{r} 32.4 \\ + 7.59 \\ \hline \end{array}$$

Notice that the decimal points are again properly lined up. Notice also that the 4 of the top number is lined up with the 5 of the bottom number – and not the 9, which is what could have happened if we had lined up the numbers wrongly (illustrate this on board). The 4 of the top number is correctly lined up with the 5 of the bottom number because they are both immediately to the right of their number's decimal point. In other words, they are closest on the right of the decimal points. Similarly the 2 of the top number and the 7 of the bottom number are correctly lined up. The 2 and the 7 are both closest on the left of the decimal points. (Again, point to the appropriate numbers on the board).

On the sheet I have given you, I want you to have a go at lining up the **numbers** given in part 1. Don't worry about adding up the numbers, just have a go at lining them up properly.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

Some numbers are given to us without decimal points, for example: 6, 2, 35, 817. Sometimes lining these numbers up with numbers with decimal points becomes confusing. Actually, the decimal point in these numbers goes on their right.

For example, in these numbers the decimal point goes as follows

6.      2.      35.      817.

What goes on the right of the decimal point in these numbers is just 0 or nothing. So if we want we can put

6.0                  2.00                  35.0                  817.000

On the sheet I have given you, I want you to quickly have a go at putting the decimal points, and 0's if you want, on the numbers given in part 2.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

Now that we know where the decimal point goes on these numbers, we can line them up properly with numbers with decimal points.

For example, if we were adding  $38.4 + 4$ , we'd line them up as

$$\begin{array}{r} 38.4 \\ + 4. \\ \hline \end{array}$$

Notice that the 4 of the bottom number lines up with the 8 of the top number – and not with the other 4, which is what is likely to have happened had the numbers been lined up wrongly (illustrate this on the board). The 8 and the 4 are lined up correctly because they are both the next closest on the left of their decimal point.

On the sheet I have given you, I want you to have a go at lining up the numbers in part 3. Again, don't worry about adding them up, just line them up properly.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then

ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

Now that we have gone over how to line up numbers correctly for addition, we are basically ready to start actually adding up the numbers.

After we have lined up the numbers correctly, we always start adding from the rightmost numbers – that is, the numbers furthest to the right.

For example, if we were adding  $19.02 + 5.297$ , we'd line them up correctly first, and then start adding from the right.

$$\begin{array}{r} 19.02 \\ + 5.297 \\ \hline \end{array}$$

In this case, we'd first add the 7 to nothing or 0 (zero), and so we get 7. Then we'd add the 2 to the 9, which gives us 11. We therefore write 1 next to the 7 (show this on the board), and carry the other 1 over. So we write this carried over 1 above the 0 (show this on the board).

Make sure that you are careful and that you record any carry overs correctly.

So we add the carried over 1 to the 0 and the 2, which gives us 3. Then we add the 9 to the 5, which gives us 14. Therefore, we write the 4 next to the 3, and carry over the 1. Then we add the carried over 1 to the 1 of the top number, which gives us 2. (Illustrate all of this on the board).

So the answer is 24.317. Notice that we must put the decimal point in our answer as well. In this case between the 4 and the 3. This makes it lined up with the other decimal points. When we're adding, we must always line up the decimal point in our answer with the other decimal points. (Point to the appropriate numbers and decimal points on the board).

Notice also that in some cases, the rightmost number does not have any other numbers lined up with it. In those cases, just bring that number down or write it down as the sum for that line of numbers.

An example is in the problem I have shown you just now. Or if we were adding  $858.4 + 3.664$ :

$$\begin{array}{r} 858.4 \\ + \underline{3.664} \end{array}$$

In this case the 4 and the first 6 on the right of the bottom number are added to nothing or 0's since there is nothing lined up with them in the top number. (Show this on the board.)

Always remember also that when you add 0 to a number, or a number to 0, you still get that number.

For example,

$$4 + 0 = 4 \quad 0 + 9 = 9 \quad 26 + 0 = 26 \quad 0 + 431 = 431$$

Now I want you to have a go at the problems in part 4 of the sheet I have given you. First of all, correctly line up the numbers given. Then add them up. Remember to start with the rightmost numbers. Make sure you record and use carry overs when appropriate. And work carefully.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

## DI INSTRUCTION: SUBTRACTION

In Subtraction it is also very important that we properly line up the numbers we are using. Like in Addition, we can do this by using the decimal points of the numbers we have.

For example, if we have  $34.2 - 6.51$ , we have to line them up as

$$\begin{array}{r} 34.2 \\ - 6.51 \\ \hline \end{array}$$

Notice that the decimal points are properly lined up with each other. Notice also that the 2 of the top number is lined up with the 5 of the bottom number – which is correct. Both this 2 of the top number and this 5 of the bottom number are immediately to the right of their number's decimal point. So they line up correctly with each other. (Point to the appropriate numbers on the board).

A mistake would have been if we had lined up the 2 of the top number with the 1 of the bottom number (illustrate this on the board). This way, the decimal points are not properly lined up with each other. The digits of the numbers are also not properly lined up with each other. The 4 of the top number, for example, which is in the units or 1's column, will be wrongly lined up with the 5 of the bottom number which is in the 1/10's column. (Point to the appropriate numbers on the board).

On the sheet I have given you, I want you to have a go at lining up the numbers given in part 1. Don't worry about subtracting the numbers from each other, just have a go at lining them up properly.

(Circulate among the children and make sure that they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

As we saw when we had the class on Addition, some numbers are given to us without a decimal point. For example: 5, 3, 42, 619 (write these on the board). So that we can line these numbers up properly with other numbers with a decimal point, we have to know where their decimal point goes. The decimal points in these numbers always go on their right. Thus in these numbers the decimal points go as follows:

5.      3.      42.      619.

Now that we know where their decimal points go, we can line these sorts of numbers up with other numbers with decimal points. So, for example, if we have  $56.9 - 23$ , we now know that the decimal point on the 23 goes to its right like this

23.

What this means is that we can line it up properly with the 56.9, like this

$$\begin{array}{r} 56.9 \\ - 23. \\ \hline \end{array}$$

It is also a good idea to put 0's (zeros) in where some of the digits after the decimal point in one of the numbers are not lined up with any other digits from the other number. So in this example that we are looking at, we can put in a 0 after the decimal point of the bottom number to line up with the 9 of the top number. So it will look like this

$$\begin{array}{r} 56.9 \\ - 23.0 \\ \hline \end{array}$$

Another example of this is if say we had  $275 - 3.16$ . We know that the decimal point on the 275 goes to its right (show this on the board), and so we can properly line it up with the 3.16, like this

$$\begin{array}{r} 275. \\ - 3.16 \\ \hline \end{array}$$

Now we can see that the 1 and the 6 of the bottom number line up with nothing from the top number, so what we do is put in two 0's after the decimal point of the top number. So the top number becomes 275.00 – which is not really different from just 275 since putting 0's in is just a way of showing that there is nothing there. So now our Subtraction problem will look like this

$$\begin{array}{r} 275.00 \\ - 3.16 \\ \hline \end{array}$$

In part 2 of the sheet I have given you, I want you to have a go at properly lining up the numbers given. Also put in the 0's where they are needed. Don't worry about actually doing the subtraction, just line up the numbers properly and put in the 0's where they are needed.

(Circulate among the children and make sure that they are doing the task correctly. If not, tell them “That is not quite right,” show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

Now that we know how to line up the numbers properly, and also where to put in the necessary 0’s, we can now get on with doing the actual subtraction.

We always start with the digits furthest to the right. For example, if we have  $75.6 - 43$ , we line them up properly and put in the 0 after the decimal point on the 43. So we get

$$\begin{array}{r} 75.6 \\ - 43.0 \\ \hline \end{array}$$

We then start our subtraction by subtracting 0 from 6 – these are the two furthest digits to the right (show this on the board). Subtracting 0 from 6 means 6 take away 0. Now whenever we take 0 away from any number we still get that number. So in this case, 6 take away 0 is equal to 6. If we take nothing or 0 away from 6, we still get 6.

The important thing to remember in Subtraction too is that we are taking away the bottom number from the top number, not the other way round. So in this case we are taking 43.0 away from 75.6 (which we can also say as 75.6 take away 43.0).

Back to our subtraction, so far we have 6 take away 0 equals 6, and so we write down the 6 here (show this on the board). Then we put the decimal point in our answer, making sure that it is properly lined up with the other two decimal points (show this on the board). Then we have 5 take away 3 which equals 2, so we write down the 2 here (show this on the board). And finally we have 7 take away 4 which equals 3, so we write down the 3 here (show this on the board). Therefore our answer is 32.6 (point to this answer on the board).

In part 3 of the sheet I have given you, I want you to quickly have a go at solving the Subtraction problems given there. Remember to line up the numbers properly first, and put in the necessary 0’s. Then start with the digits furthest to the right. Remember that you are subtracting or taking away the bottom digits from the top digits – not the other way round. Remember also to put in the decimal point in your answers.

(Circulate among the children and make sure that they are doing the task correctly. If not, tell them “That is not quite right,” show them individually how it is correctly done, and

then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

When in some cases the top digit we are dealing with is smaller than the bottom digit we are subtracting, then we have to 'borrow.' Borrowing is actually quite simple and what I'm going to do is show you a method which will probably be quite different from the one you've been taught.

As an example, let's say we have  $8 - 3.2$  which we line up as

$$\begin{array}{r} 8.0 \\ - 3.2 \\ \hline \end{array}$$

The first thing we have to do is take away 2 from 0. But 0 is smaller than 2, so we can't really do this just like that. First we have to add 10 to the 0, so now that 0 is a 10 (show this on the board). But as a consequence, we increase the next digit we are subtracting, the 3, by 1. To remind ourselves of this we put a slash underneath it like this (show this on the board). So we know that that 3 is now a 4.

We can now do our subtraction, so 10 (which used to be the 0) take away 2 is 8 – we write this down; then we put our decimal point in; then 8 take away 4 (which used to be the 3) equals 4 – and we write this down (show all of this on the board). So our answer is 4.8 (point to this answer on the board).

In part 4 of the sheet I have given you, I want you to have a go at using this method in solving the Subtraction problems given there. First, line up the numbers properly and put in the 0's where needed. Then if the top digit you are dealing with is smaller than the bottom digit you are subtracting, increase the top digit by 10, and increase the next bottom digit (to the left) by 1. Do you have any questions before you start? (Answer any questions posed, explaining things on the board where necessary.) Remember also to put the decimal point in your answers!

(Circulate among the children and make sure that they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

Sometimes we have to borrow several times when we are solving a Subtraction problem. For example, if we have  $40.2 - 0.3587$ , we can line them up and put in the necessary 0's to get

$$\begin{array}{r} 40.2000 \\ - \underline{0.3587} \end{array}$$

Now when we have to do our subtraction, we see that first we have 0 take away 7. The 0 is smaller than the 7, so as I showed you how to do before, we add 10 to the 0 making it 10, and add 1 to the next bottom digit, which is 8, to make it 9. To do this we just put a slash to remind us to add 1, underneath the 8 (show this on the board). So we have 10 take away 7 which is 3, and we write down the 3 here (show this on the board).

Next we have 0 take away 9 (which used to be the 8, remember?). Again the 0 is smaller than the 9, so we add 10 to the 0 making it 10, and add 1 to the next bottom digit along, which is the 5, making it a 6. We put a slash in to mark this (show on the board). So now we have 0 take away 9 which is 1, and we write down the 1 here (show this on the board).

Next we have 0 take away 6 (which used to be the 5, remember?). The 0 is smaller than the 6, so we add 10 to the 0 to get 10. And we add 1 to the next bottom digit, which is 3, to make it a 4. We put in a slash to remind us of this (show this on the board). So we have 10 take away 6 which is 4, so we write down the 4 here (show this on the board).

Next we have 2 take away 4 (which used to be the 3). Again the 2 is smaller than the 4 so we add 10 to it. Two plus 10 is 12. We also have to add 1 to the next bottom digit along, which is this 0, which now becomes 1. We put a slash here beneath the 0 to remind us of this (show this on the board). So now we have 12 take away 4, which is 8. So we write the 8 here (show this on the board).

Next we put the decimal point in our answer. As we can see, it lines up properly with the other two decimal points. It is very important to remember to put the decimal point in our answer – otherwise our answer will not be correct.

Back to our Subtraction problem, we next have 0 take away 1 (which used to be a 0, remember?). Like before, the 0 is smaller than the 1, so we add 10 to it, making it a 10. But we must also add a 1 to the next digit along in the bottom number. In this case there is no next digit along (point to this on the board). But we can imagine that there is a 0 there, so we add the 1 to the 0, and we get 1. We put a slash here to remind us of this (show this on the board). So now we have 10 take away 1, which is 9. We write down the 9 here (show on the board).

Next we have 4 take away 1 (which used to be nothing). The answer to this is 3. We write down the 3 here (show on the board). So the answer to our Subtraction problem is 39.8413 (point to this on the board).

In this Subtraction problem that we've just done we have seen that we can 'borrow' quite a number of times. The thing to remember is that whenever a digit is smaller than the one we're subtracting from it, what we must do is add 10 to it. But we must always remember to also add 1 to the next digit along to the left of the bottom number (illustrate this on the board using the problem previously solved).

We must also always remember to put in the decimal point in our answers. And be careful not to make careless mistakes.

Do you have any questions? (Answer any questions posed, explaining things on the board where necessary.)

In part 5 of the sheet I have given you, I want you to have a go at the Subtraction problems given there. So first make sure that you line up the numbers properly using the decimal points. Then put in any 0's necessary. Then do your subtractions carefully. Remember that if the top digit is smaller than the bottom digit you are subtracting from it, you can add 10 to the top digit. But you must also add 1 to the next digit along (to the left) of the bottom digit. Be careful in working through the problems. And remember to put your decimal points in.

(Circulate among the children and make sure that they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

## DI INSTRUCTION: MULTIPLICATION

In Multiplication, each of the digits in the top number gets multiplied by the bottom number. We start with the digit furthest to the right in the top number. For example, if we have

$$\begin{array}{r} 34 \\ \times 2 \\ \hline \end{array}$$

we first multiply the 4 – which is the digit furthest to the right in the top number – by the 2. The answer we get is 8, which we write down here (show on the board). Next we multiply the 3 by the 2. We get 6, which we write down here (show on the board). So our answer is 68.

Sometimes we have to ‘carry over’ when the answer to two digits we multiplied is larger than 9. For example, if we have

$$\begin{array}{r} 27 \\ \times 3 \\ \hline \end{array}$$

we first multiply the 7 by the 3. The answer we get for 7 times 3 is 21. We only write the 1 down here (show on the board). The 2 of the 21 gets carried over, we write it here on top of the 2 of the top number. We write it there to remind ourselves to add that 2 to our next answer.

Next we multiply the 2 by the 3. The answer we get is 6. And we add the 2 we carried over to the 6, to get 8. And we write that 8 down here. So the answer to our Multiplication problem is 81. (Show steps on the board.)

On the sheet I have given you, I want you to have a go at the Multiplication problems given in part 1. Remember to multiply all the digits of the top number by the bottom number. Start with the digit furthest to the right. And don’t forget to carry over when necessary.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them “That is not quite right,” show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

Quite often the number we multiply by has more than one digit. An example is if we have

$$\begin{array}{r} 314 \\ \times 25 \\ \hline \end{array}$$

In this case we are multiplying 314 by 25, and 25 has two digits – the 2 and the 5. The 5 is the furthest to the right so we start with it. We multiply each of the digits of 314 by the 5, starting with the 4 which is furthest to the right of that number (show and explain these on the board). So 4 times 5 is 20. We write the 0 down here, and carry the 2 of the 20 over – as I showed you earlier. We write that 2 above the 1. (Show these on the board).

Next we multiply the 1 by the 5, which gives us 5. We add the 2, which we carried over, to the 5 which gives us 7. And we write this 7 here. Next we have 3 times 5, which is 15. Since the 3 is the last digit of the top number that we have to multiply by 5, we don't need to carry over anything even if our Multiplication answer is greater than 9. So we just write down the 15 here, giving us an answer of 1570 after we have multiplied the digits of the top number by the first digit of the bottom number. (Show these steps on the board.)

[We can, if we want, carry over the 1 of the 15 and just write the 5 down here first. We write the carried over 1 here, and since there is nothing to add it to, we just bring it down and write it next to the 5. So our answer remains the same as what I showed you before. (Show and explain this on the board.)]

Next we have to multiply each of the digits of 314 by the 2. The 2 of the 25 is in the 10's column of that number. What that means is that the 25 really is 20 plus 5 – the 5 is in the 1's column, while the 2 is in the 10's column. What this also means is that the 2 really is a 20. So before we start multiplying by the 2, we first must write down a 0 here to remind us of the fact that that 2 really is a 20. (Show this on the board.)

$$\begin{array}{r} 314 \\ \times 25 \\ \hline 1570 \\ 6280 \\ \hline 0 \end{array}$$

Now that we have done this, we can start multiplying by the 2. We have 4 times 2, which is 8. Then 1 time 2, which is 2. And finally, 3 times 2, which is 6. So we get 6280 after multiplying the digits of the top number by the 2 of the bottom number.

What we have to do next is add the 1570 and the 6280 together. So we draw a line here below the 6280, and start adding. Zero plus 0 is still 0, which we write down here. Seven plus 8 is 15, so we write down the 5 here and carry the 1 over on top of the 5. Five plus 2 is 7, plus the carried over 1 gives us 8, which we write down here. Finally, 1 plus 6 is 7, which we write down here. So our Multiplication answer is 7850. (Show the Addition steps on the board.)

On the sheet I have given you, I want you to have a go at the Multiplication problems given in part 2. Remember to first multiply all the digits of the top number by the digit furthest to the right of the bottom number. Then put in a 0 first when you start multiplying by the second digit (on the 10's column) of the bottom number. Once you have multiplied by both of the digits of the bottom number, add together the two answers you get. Don't forget to carry over when necessary.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

Quite often the numbers we have to multiply have decimals. An example of this is if we get

$$\begin{array}{r} 30.42 \\ \times 2.3 \\ \hline \end{array}$$

Both the top number and the bottom number have decimals. But we don't worry about these decimals at first, we just go ahead and do the multiplication like I showed you just before.

We start with the 3 from the bottom number. We multiply each of the digits of the top number by this 3, starting with the digit furthest to the right which is 2 (point to these numbers on the board). So 2 times 3 is 6, which we write down here. Then 4 times 3 is 12, so we write down the 2 here and carry over the 1. Then 0 times 3 is still 0, plus the 1 we carried over – so we have 0 plus 1 – gives us 1, which we write down here. Then 3 times 3 is 9, which we write down here. So the answer we get after multiplying the top number digits by the 3 of the bottom number is 9126. (Show these steps on the board.)

Next we multiply the top number digits by the 2 of the bottom number. But first, as I showed you before, when we move on to multiply by the next digit of the bottom number, we must first put a 0 in our answer like this

$$\begin{array}{r}
 30.42 \\
 \times \underline{2.3} \\
 \hline
 9126 \\
 0 \\
 \hline
 \end{array}$$

Then we can start multiplying by the 2. So, 2 times 2 is 4, which we write down here. Then 4 times 2 is 8, which we write down here. Zero times 2 is still 0, so we write down the 0 here. Finally, 3 times 2 is 6, which we write down here. So we get 60840 after multiplying the top number digits by the 2 of the bottom number. (Show these on the board.)

Next we add the 9126 and the 60840 together, so we draw a line under the 60840. Six plus 0 is 6, which we write down here. Two plus 4 is 6, which we write down here. One plus 8 is 9, which we write down here. Nine plus 0 is 9, which we write down here. And finally 6 plus nothing is still 6, so we write that 6 down here. So our addition gives us 69966.

This is not where we stop. We still have to put the decimal point in our answer. To do this, we first have to count the number of digits in the top number which are to the right of the decimal point. There are 2 – the 4 and the 2 (point to these on the board). Then we count the number of digits in the bottom number which are to the right of the decimal point. There is only 1 – the 3 (point to this on the board). Then we add the 2 and the 1 together which gives us 3.

To place the decimal point in our answer, we must count this number – the 3 – of digits from the right of our answer. So we go, “1” the 6, “2” the second 6, and “3” the 9. We put the decimal point in after this third digit. So the decimal point goes here between the two 9’s. And our answer is 69.966. We must always remember to put the decimal point in our answers when necessary, otherwise the answers we give will not be right.

On the sheet I have given you, I want you to have a go at placing the decimal points in the answers to the problems given in part 3. The Multiplication steps have already been done for you. All you have to do is put the decimal points in the correct positions in the answers given. So count the number of digits after the decimal point of the top number, and add this to the number of digits after the decimal point of the bottom number. Then whatever number you get, count that many digits from the right of the final answer before placing the decimal point. Do you have any questions? (Answer any questions posed, explaining things on the board where appropriate.) Go ahead and work on part 3 of your sheets.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them “That is not quite right,” show them individually how it is correctly done, and then

ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

Next I want you to have a go at the Multiplication problems given in part 4 of the sheet I have given you. Remember to first multiply all the digits of the top number by the digit furthest to the right of the bottom number. Then put in a 0 first when you start multiplying by the second digit (on the 10's column) of the bottom number. Once you have multiplied by both of the digits of the bottom number, add together the two answers you get. Then, as I showed you just now, don't forget to put the decimal point in your final answer. Count the number of digits after the decimal point – to the right – of the top number and the bottom number, then add them up. What you get is the number of times you count the digits – from the right – of your final answer before you place the decimal point.

Are there any questions? (Answer any questions posed, explaining things on the board where necessary.)

Now be careful not to make any careless mistakes. And remember that when you multiply a number by 1, you still get that number. For example, 4 times 1 is still 4; 9 times 1 is still 9; and so on. Also remember that if you multiply any number by 0 you get 0. For example, 3 times 0 is 0; 7 times 0 is 0; and so on.

Go ahead and start on the questions in part 4.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

## DI INSTRUCTION: DIVISION

In Division, we are the dividing the **number** inside the Division mark by the **number** outside. For example, if we have

$$2 \overline{)462}$$

we are dividing the 462 by the 2. In other words, we are trying to find out how many 2's there are in 462. We can also say that we are trying to find out how many times 2 goes into 462.

To get our answer, we just take each of the digits of the **number** inside the Division mark and divide them by 2. We always start with the number furthest to the left – in this case the 4. So 4 divided by 2 is 2. This means there are 2 (two) 2's in 4. We write this 2 directly above the 4. Next we have 6 divided by 2, which is 3. This means there are 3 (three) 2's in 6. We write this 2 directly above the 6. Finally we have 2 divided by 2, which is 1. This means there is only 1 (one) 2 in 2. And we write this 1 directly above the 2. (Show all these steps on the board.)

Our answer therefore is 231.

Sometimes some of the digits inside the Division mark are too small and we have to combine digits together before we can divide. Let me explain. For example, if we have

$$5 \overline{)305}$$

the first division we have to do is 3 divided by 5. But 3 is too small for the 5 to divide. So we first write a 0 above the 3 which indicates that the 3 on its own cannot be divided by the 5. And then what we do is we combine the 3 with the next digit which is the 0. Now we have 30 divided by 5, which gives us 6. This means there are 6 (six) 5's in 30. We write that 6 above the 0. Next is 5 divided by 5, which is 1. This means there is 1 (one) 5 in 5. We write that 1 above the 5. (Show these steps on the board.)

The answer we get is 61.

Another example of this happening is if say we had

$$4 \overline{)416}$$

We first have 4 divided by 4, which is 1. We write that 1 above the 4. Next we have 1 divided by 4, which doesn't go since 1 is too small for the 4 to divide. So we write a 0 above the 1 which indicates that it cannot be divided by the 4. And then what we do is combine the 1 with the next digit which is the 6 – giving us 16. Now we have 16 divided by 4, which is 4. And we write that 4 above the 6. (Show these steps on the board.)

So the answer we get is 104.

On the sheet I have given you, I want you to have a go at the problems given in part 1. Remember that you are dividing the number inside the Division mark by the number outside it. To do this, you are taking each of the digits of the inside number one by one – starting with the one furthest to the left – and dividing them by the outside number (point to appropriate digits of worked examples on the board to clarify what is meant). If a digit you are dividing is too small, write a 0 above it. Then combine it with the next digit, and divide the combined digits next – as I have just shown you.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them “That is not quite right,” show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

Quite often when we divide we also get remainders. We have to combine these remainders with the next digit we get. For example, if we have

$$3 \overline{)456}$$

we first have to divide 4 by 3. So 4 divided by 3 gives us 1 – meaning that there is 1 (one) 3 in 4 – plus a remainder of 1. So the answer to 4 divided by 3 is 1 remainder 1. We write the first 1 above the 4, and the remainder 1 we write next to the 5 to remind ourselves that that 5 is now a 15. (Show this on the board.) So our next Division task is 15 divided by 3, which gives us 5 – meaning that there are 5 (five) 3's in 15. We write that 5 above the 5. Finally we have 6 divided by 3, which gives us 2 – meaning that there are 2 (two) 3's in 6. We write that 2 above the 6. (Show these steps on the board.)

Therefore our answer is 152.

Another example of this is if say we had

$$2 \overline{)652}$$

We'd first have 6 divided by 2, which is 3. We write that 3 above the 6. Then we have 5 divided by 2, which is 2 remainder 1. This means that there are 2 (two) 2's in 5 – which is 4 – plus 1 remainder. We write the 2 above the 5, and the remainder 1 next to the 2 to remind ourselves that that is now a 12. So next we have 12 divided by 2, which is 6. We write that 6 above the 2. (Show these steps on the board.)

The answer therefore is 326.

On the sheet I have given you, I want you to have a go at the problems given in part 2. Remember what to do with the remainder. If you divide a digit and there is a remainder, you combine that remainder with the next digit you're supposed to divide. For example, in the problem I've just shown you, 5 divided by 2 gives you 2 and a remainder of 1. So you combine that 1 with the next digit you're supposed to divide – which is the 2. And so you get 12 – which you next divide by 2. (Point to the appropriate digits on the board.)

(Circulate among the children and make sure they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

Quite often the number we have to divide, the number we are dividing by, or both, will have a decimal point. If only the number we are dividing has a decimal point, we only have to remember to put a decimal point in our answer – which is in line with the decimal point of the number we divided. For example, if we have

$$4 \overline{)4.92}$$

we do our normal division, so first we have 4 divided by 4, which is 1. Then 9 divided by 4, which is 2 remainder 1. As I showed you before, we write the 2 above the 9, and the remainder 1 goes next to the 2 of the number we are dividing – to remind us that that is now a 12. So 12 divided by 4 is 3. (Show these steps on the board.)

As you can see the 'answer' we have is 123. But we still have to put the decimal point in this answer. It goes between the 1 and the 2 – which is in line with the decimal point in the number we divided. (Show this on the board.) Our answer therefore is 1.23.

Next I'd like you to have a go at solving the problems in part 3 of the sheet I have given you. Work carefully and remember to put the decimal point in the answer that you get.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

As I said earlier, sometimes the number we are dividing by also has a decimal point. When this happens, we have to shift the decimal point both in that number and in the number we are dividing. For example, if we have

$$0.3 \overline{)3.99}$$

we have to shift the decimal point in the 0.3 to make it a whole number – a number without a decimal point. We do this because it is quite complicated to divide by a number with a decimal point.

So what we do is we move the decimal point one space along to its right. (Illustrate this on the board.) This turns the 0.3 into just a 3. But we must do the same thing to the number we are dividing. We must also shift its decimal point one space along to the right. (Illustrate this on the board.) So now the 3.99 becomes 39.9. Our Division problem therefore has now become

$$3 \overline{)39.9}$$

Even though it looks different, it will still give us the same answer as 3.99 divided by 0.3 – but it is much easier to do.

Sometimes also when we shift decimal points, we have to add a 0 to the number we are dividing. For example, if we have

$$0.4 \overline{)56}$$

we first shift the decimal point in the 0.4 to the right, so it now becomes a 4. Next we have to shift the decimal point in the 56. At the moment it is to the right of the 6. After the 6 and the decimal point, there is nothing – or 0 (zero). So if we shift the decimal point in the 56 one space along to the right, we end up shifting it after the 0. And the 56 is now 560. So our Division problem is now

$$4 \overline{)560}$$

Even though it looks different, it should still give us the same answer as 56 divided by 0.4.

The thing to remember here is that if there is nothing after the decimal point in the number we are dividing, and we have to shift the decimal point, we just add a 0 to that number. In the example, the 56 had nothing after where the decimal point goes in it, so we just added a 0 to it to make it 560.

In the sheet that I have given you, I want you to now have a go at the problems in part 4. Remember that if the number we are dividing by has a decimal point in it, we have to shift the decimal point to the right – both in it and the number we are dividing. All I want you to do is shift decimal points where necessary, and re-write the problems to what they become after you have shifted the decimal points. Don't worry about actually solving the problems.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

Once we have shifted decimal points as necessary, we're ready to solve the Division problem as we normally do. For example, if we had

$$1.1 \overline{)46.31}$$

we shift decimal points so that we get 11 instead of 1.1 as the number we are dividing by, and 463.1 instead of 46.31 as the number we are dividing (show the decimal point shifts on the board). We can re-write the problem as

$$11 \overline{)463.1}$$

Now we can just go ahead with the division. Four divided by 11 does not go – the 4 is too small for the 11 to divide. So we put a 0 (zero) above the 4, and combine it with the next number along which is the 6. Now we have 46 divided by 11. How many 11's are there in 46? The answer is 4 remainder 2. (The 4 (four) 11's give us 44, plus the remainder 2 to make up the 46.) So we write the 4 above the 6, and the remainder 2 next to the 3 to remind ourselves that that is now a 23. Therefore, next we have 23 divided by 11, which is 2 remainder 1. (The 2 (two) 11's give us 22, plus the remainder 1 to make up the 23.) We write the 2 above the 3, and the remainder 1 next to the 1 to remind ourselves that that is now an 11. At this stage also we should put in the decimal point after the 2 in our answer, to make sure that we don't forget it later (show this on the board). Finally, we have 11 divided by 11, which is just 1. And we write this 1 above the 1.

Our answer therefore is 42.1.

Do you have any questions about what I did? (Answer any questions posed, explaining things on the board as necessary).

Now I want you to have a go at solving the problems in part 5 of the sheet I have given you. Remember to shift decimal points and re-write the problems first if necessary. Remember also to correctly place the decimal point in your answer when necessary. Work carefully.

(Circulate among the children and make sure they are doing the task correctly. If not, tell them "That is not quite right," show them individually how it is correctly done, and then ask them to try again. Keep doing this until they do at least one of the problems or items correctly.)

*Appendix D. Exercises Used During the Instruction Sessions*

**ADDITION EXERCISES****PART ONE**

$76.1 + 3.8$	$40.2 + 6.15$	$28.5 + 0.293$	$16.07 + 2.569$
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**PART TWO**

3	8	91	476
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**PART THREE**

$69.2 + 2$	$33 + 4.55$	$261 + 9.238$	$124.7 + 5 + 0.119$
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**PART FOUR**

$67.6 + 6$	$12.8 + 9.32$	$18.06 + 2.961$	$538.2 + 3.953 + 3$
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**SUBTRACTION EXERCISES****PART ONE**

$25.2 + 6.34$	$12.4 - 2.319$	$320.02 - 6.571$	$40.2 - 0.3769$
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**PART TWO**

$88.4 - 26$	$5 - 0.2$	$217 - 4.83$	$9 - 0.615$
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**PART THREE**

$69.1 - 52.1$	$7.36 - 2.2$	$85.4 - 12$	$310.86 - 200.7$
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**PART FOUR**

$5 - 0.8$	$3 - 1.6$	$7.2 - 4.9$	$13.1 - 0.7$
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**PART FIVE**

$224 - 2.59$	$5 - 0.928$	$710.03 - 4.241$	$90.1 - 0.2733$
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## MULTIPLICATION EXERCISES

### PART ONE

$\begin{array}{r} 71 \\ \times 2 \\ \hline \end{array}$	$\begin{array}{r} 36 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 303 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 243 \\ \times 4 \\ \hline \end{array}$
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### PART TWO

$\begin{array}{r} 322 \\ \times 21 \\ \hline \end{array}$	$\begin{array}{r} 125 \\ \times 42 \\ \hline \end{array}$	$\begin{array}{r} 237 \\ \times 13 \\ \hline \end{array}$	$\begin{array}{r} 3102 \\ \times 16 \\ \hline \end{array}$
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### PART THREE

$\begin{array}{r} 1 \\ 0.7 \\ \times 0.2 \\ \hline 14 \\ 000 \\ \hline 014 \end{array}$	$\begin{array}{r} 5.1 \\ \times 6.4 \\ \hline 204 \\ 3060 \\ \hline 3264 \end{array}$	$\begin{array}{r} 2 \\ 8.41 \\ \times 2.6 \\ \hline 15046 \\ 16820 \\ \hline 21866 \end{array}$	$\begin{array}{r} 2 \\ 20.29 \\ \times 0.31 \\ \hline 2029 \\ 60870 \\ \hline 000000 \\ \hline 62899 \end{array}$
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### PART FOUR

$\begin{array}{r} 0.5 \\ \times 0.3 \\ \hline \end{array}$	$\begin{array}{r} 3.5 \\ \times 1.2 \\ \hline \end{array}$	$\begin{array}{r} 1.44 \\ \times 3.1 \\ \hline \end{array}$	$\begin{array}{r} 21.36 \\ \times 0.15 \\ \hline \end{array}$
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<b>DIVISION EXERCISES</b>
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**PART ONE**

$2 \overline{)108}$	$5 \overline{)405}$	$4 \overline{)812}$	$3 \overline{)915}$
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**PART TWO**

$3 \overline{)723}$	$4 \overline{)528}$	$5 \overline{)575}$	$2 \overline{)314}$
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**PART THREE**

$3 \overline{)3.96}$	$2 \overline{)3.08}$	$5 \overline{)30.5}$	$4 \overline{)9.32}$
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**PART FOUR**

$0.5 \overline{)8.05}$	$0.2 \overline{)88}$	$0.6 \overline{)966}$	$1.2 \overline{)38.52}$
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**PART FIVE**

$0.4 \overline{)8.16}$	$0.6 \overline{)0.366}$	$0.2 \overline{)1.746}$	$1.1 \overline{)34.21}$
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*Appendix E. Introduction Letter and Consent Form*

## INTRODUCTION LETTER

Dear Parent/Guardian

I am carrying out a research project on the effectiveness of alternative ways of teaching arithmetic (addition, subtraction, multiplication and division) to children. The project is part of a PhD study that I am doing through Massey University.

This letter is a formal request for you to give permission for your child to take part in the project. If you agree for your child to take part, he/she – together with other children – will take part in extra lessons in arithmetic. In these lessons, the children will be taught new ways of solving arithmetic problems. These new ways may make it easier for some children to remember the many steps involved in arithmetic.

Your child does not have to be good in maths to take part in the project. In fact it is better if he/she is at present finding arithmetic a little difficult. This way your child may get some benefit out of the extra arithmetic lessons described above.

At any stage, if you wish to do so, you can withdraw your child from taking part in the project.

Any information about your child will be treated with strict confidence. Within [name of school], information about your child will be made available to your child's teachers (unless you request otherwise). Outside of [name of school], the information will only be available to myself, my research assistant, and my two supervisors at Massey University.

Your child's performance in the project will not affect his/her other marks in school.

Enclosed with this letter is a Consent Form. Please explain the project to your child. I need both of you to fill out and sign the Consent Form. After filling it out and signing it, please ask your child to return it to his/her maths teacher.

If you or your child have any questions you would like to ask before completing the Consent Form, please do not hesitate to contact me. I work as the Director of the Student Learning Centre at the University of Auckland and you can contact me there during the day at this number: 373 7599 extension 7896. You can also contact me at home in the evening at this number: 444 1433.

Thank you for your co-operation.

Yours sincerely

Emmanuel Manalo  
UNIVERSITY OF AUCKLAND

**CONSENT FORM**PARENT/GUARDIAN, PLEASE FILL OUT THIS PART

I agree to allow my child to take part in the project.

I do not want my child to take part in the project.

(Please tick one of the boxes above.)

Parent's or Guardian's signature:

Parent's or Guardian's name:

Home address:

Telephone number:

STUDENT, PLEASE FILL OUT THIS PART

I agree to take part in the project.

I do not want to take part in the project.

(Please tick one of the boxes above.)

Student's signature:

Student's name:

Date of birth:

Age: \_\_\_\_ years and \_\_\_\_ months

Boy / Girl (Circle the one that applies.)

PLEASE RETURN THIS CONSENT FORM TO YOUR MATHS  
TEACHER.

*Appendix F. Participants' Scores in the Selection Tests and the Different Stages of Addition, Subtraction, Multiplication, and Division Phases of Experiment 1*

### Experiment 1 Participants' Scores in the Selection Tests and the Different Stages of Addition and Subtraction

	STUDENT	BASIC FACTS (/40)	COMP SKILLS (/40)	PAT MATHS	TOSCA	ADDITION				SUBTRACTION			
						PRE-INSTR.	POST-INSTR.	1 WEEK	FOLLOW-UP	PRE-INSTR.	POST-INSTR.	1 WEEK	FOLLOW-UP
MNEMONICS	1	39	8	15	3	1	10	10	10	1	6	9	9
	2	35	10	10	3	3	10	9	9	1	9	2	10
	3	36	10	15	2	2	10	10	10	2	9	7	10
	4	40	15	16	3	3	10	9	10	2	9	2	9
	5	39	13	18	4	2	10	10	10	3	8	10	10
	6	37	9	9	5	4	10	8	10	1	7	8	6
	7	37	5	12	2	1	4	9	10	0	1	2	9
	8	40	13	18	6	4	9	10	10	2	10	10	10
	9	40	11	7	3	5	7	9	10	2	2	9	3
DEMO-IMITATION	10	30	9	10	5	4	10	10	6	2	9	4	5
	11	39	11	14	5	2	5	6	9	2	4	3	2
	12	38	12	18	5	3	10	9	9	1	7	2	3
	13	33	6	13	2	2	9	10	8	0	9	7	8
	14	37	8	17	3	2	9	9	2	0	5	5	2
	15	39	11	10	3	1	9	10	10	1	5	9	10
	16	33	9	16	3	3	8	9	9	1	6	8	7
STUDY SKILLS	17	38	9	11	3	5	2	4	2	3	1	7	1
	18	38	8	8	6	1	2	2	2	2	2	2	2
	19	40	12	19	5	4	3	4	2	2	2	1	2
	20	40	11	9	2	3	3	3	4	2	1	1	2
	21	37	14	18	6	4	4	3	3	3	2	3	1
	22	37	13	9	2	5	2	2	1	2	2	2	1
	23	39	12	16	5	3	3	4	4	1	1	3	2
NO INSTRUCTION	24	36	6	12	5	2	2	2	1	1	2	1	1
	25	39	8	13	2	1	2	1	4	2	0	1	0
	26	37	12	19	3	1	2	4	4	2	2	3	3
	27	40	12	19	3	2	3	4	1	1	2	6	0
	28	39	16	8	4	4	3	4	3	3	4	4	3
	29	39	15	16	3	4	4	4	5	3	3	2	3

### Experiment 1 Participants' Scores in the Selection Tests and the Different Stages of Multiplication and Division

	STUDENT	BASIC FACTS (/40)	COMP SKILLS (/40)	PAT MATHS	TOSCA	MULTIPLICATION				DIVISION			
						PRE-INSTR.	POST-INSTR.	1 WEEK	FOLLOW-UP	PRE-INSTR.	POST-INSTR.	1 WEEK	FOLLOW-UP
MNEMONICS	1	37	12	19	3	4	10	10	10	5	10	10	10
	2	40	12	19	3	4	10	9	10	5	10	10	10
	3	40	11	9	2	2	10	10	10	4	8	8	10
	4	39	16	8	4	3	10	10	10	6	10	10	10
	5	40	13	18	6	2	10	10	10	5	10	10	10
	6	39	15	16	3	3	10	9	9	5	10	7	9
	7	39	12	16	5	4	3	4	4	4	5	5	5
DEMO-IMITATION	8	38	9	11	3	1	8	5	7	0	4	6	3
	9	36	6	12	5	2	10	9	10	1	9	8	6
	10	39	8	13	2	4	8	7	9	1	0	2	1
	11	38	8	8	6	4	6	6	5	1	5	4	5
	12	40	12	19	5	3	9	10	4	3	5	8	4
	13	37	14	18	6	2	6	5	5	5	5	4	3
	14	40	15	16	3	5	10	8	4	5	8	9	5
	15	37	13	9	2	2	4	2	2	4	2	6	2
STUDY SKILLS	16	30	9	10	5	2	2	3	3	1	0	1	1
	17	39	11	14	5	5	4	3	3	2	0	1	2
	18	38	12	18	5	3	2	4	4	5	5	4	4
	19	39	11	10	3	4	9	9	6	5	5	5	5
	20	33	9	16	3	3	3	3	4	2	5	2	3
	21	37	5	12	2	2	2	2	7	2	3	2	0
NO INSTRUCTION	22	39	8	15	3	3	4	4	3	3	5	6	5
	23	35	10	10	3	4	3	3	4	2	0	1	0
	24	36	10	15	2	3	4	4	3	3	3	2	2
	25	33	6	13	2	2	2	2	1	2	0	0	1
	26	37	8	17	3	4	2	3	3	2	0	0	2
	27	39	13	18	4	5	4	9	4	3	2	3	3
	28	37	9	9	5	4	4	3	2	0	0	2	0
	29	40	11	7	3	4	3	3	4	0	0	2	2

*Appendix G. Participants' Scores in the Selection Tests and the Different Stages of Addition, Subtraction, Multiplication, and Division Phases of Experiment 2*

### Experiment 2 Participants' Scores in the Selection Tests and the Different Stages of Addition and Subtraction

	STUDENT	BASIC FACTS (/40)	COMP SKILLS (/40)	PAT MATHS (RAW)	TOSCA (STANINE)	ADDITION					SUBTRACTION				
						PRE-INSTR.	POST-INSTR.	1 WEEK	4 WEEKS	8 WEEKS	PRE-INSTR.	POST-INSTR.	1 WEEK	4 WEEKS	8 WEEKS
MNEMONICS	1	40	12	11	3	1	10	10	9	10	1	8	10	10	7
	2	40	10	17	3	2	10	10	10	10	2	6	10	10	10
	3	36	8	11	2	3	10	9	10	10	0	5	9	9	8
	4	40	9	17	4	3	4	5	4	10	2	7	7	6	5
	5	32	5	14	4	3	9	10	10	10	1	3	1	7	8
	6	40	7	10	3	1	10	10	10	10	2	5	8	8	8
	7	32	4	17	3	2	9	8	10	8	0	7	6	8	6
	8	33	10	16	3	4	8	10	9	10	1	2	6	9	10
	9	34	6	10	3	3	4	4	7	10	0	2	1	4	7
DEMO-IMITATION	10	38	4	13	5	2	8	6	6	4	0	10	10	10	4
	11	36	3	13	4	2	10	9	10	10	0	5	8	8	1
	12	39	4	12	3	1	9	6	6	6	0	9	8	8	8
	13	34	9	12	2	3	10	10	10	7	2	9	10	9	4
	14	40	6	10	3	2	9	8	8	6	0	10	8	8	0
	15	39	8	13	3	2	8	8	8	2	1	6	6	5	1
	16	37	7	13	4	2	10	9	10	8	1	7	8	7	0
	17	36	4	14	3	3	8	8	6	9	0	1	5	4	0
NO INSTRUCTION	18	32	2	11	3	1	0	3	2	1	0	1	2	0	0
	19	32	6	10	2	0	3	3	3	1	1	0	1	1	1
	20	38	13	14	3	2	4	5	7	6	3	2	4	3	3
	21	39	1	6	2	0	1	2	1	1	0	2	1	1	0
	22	33	5	8	2	2	2	2	8	9	1	0	2	1	0
	23	31	4	12	2	1	5	3	1	2	0	0	0	0	0
	24	38	10	11	4	2	2	1	1	1	0	1	1	1	2
	25	38	8	9	4	2	2	2	2	2	1	2	1	2	2
	26	36	6	9	2	4	4	4	4	4	1	0	0	0	1
	27	40	11	14	4	4	3	3	3	4	1	1	1	2	2
	28	39	14	13	3	5	4	1	4	1	2	0	0	0	0

## Experiment 2 Participants' Scores in the Selection Tests and the Different Stages of Multiplication and Division

	STUDENT	BASIC FACTS (/40)	COMP SKILLS (/40)	PAT MATHS (RAW)	TOSCA (STANINE)	MULTIPLICATION					DIVISION				
						PRE-INSTR.	POST-INSTR.	1 WEEK	4 WEEKS	8 WEEKS	PRE-INSTR.	POST-INSTR.	1 WEEK	4 WEEKS	8 WEEKS
MNEMONICS	1	40	12	11	3	6	7	10	10	10	4	8	10	10	10
	2	40	10	17	3	3	10	10	10	10	3	10	10	10	10
	3	36	8	11	2	2	7	5	9	9	3	6	5	9	7
	4	40	9	17	4	2	8	10	8	7	2	10	10	8	7
	5	32	5	14	4	1	4	4	7	10	0	7	2	5	5
	6	40	7	10	3	2	8	9	10	10	2	9	9	9	8
	7	32	4	17	3	2	7	9	9	7	0	8	9	8	8
	8	33	10	16	3	3	9	9	9	9	2	3	8	8	9
	9	34	6	10	3	3	6	3	10	8	0	3	0	6	7
DEMO-IMITATION	10	38	4	13	5	2	2	0	3	3	0	9	0	7	5
	11	36	3	13	4	1	6	7	5	2	0	5	1	4	2
	12	39	4	12	3	1	2	2	5	7	2	2	1	5	10
	13	34	9	12	2	4	8	9	9	5	0	7	5	6	5
	14	40	6	10	3	3	6	7	7	3	1	8	9	8	3
	15	39	8	13	3	3	4	3	3	2	2	2	4	4	2
	16	37	7	13	4	3	10	7	7	4	1	9	8	8	8
	17	36	4	14	3	1	2	3	1	2	0	4	4	2	3
NO INSTRUCTION	18	32	2	11	3	1	4	3	3	3	0	2	2	1	2
	19	32	6	10	2	3	2	3	2	0	2	3	2	0	1
	20	38	13	14	3	3	3	4	4	4	5	5	5	5	4
	21	39	1	6	2	1	8	2	1	1	0	7	1	1	1
	22	33	5	8	2	0	4	4	4	4	2	2	0	0	2
	23	31	4	12	2	2	1	2	2	2	1	0	1	1	1
	24	38	10	11	4	5	4	2	2	2	3	3	2	3	2
	25	38	8	9	4	3	3	2	1	1	2	2	2	1	2
	26	36	6	9	2	1	1	2	2	1	0	1	1	1	1
	27	40	11	14	4	3	4	4	3	4	3	5	4	3	4
	28	39	14	13	3	4	3	3	2	4	3	5	3	2	3

*Appendix H. ANOVA Summary Tables for Experiment 1*

**Summary of ANOVA in Addition, Experiment 1**

<i>Source</i>	<i>DF</i>	<i>Type III SS</i>	<i>F Value</i>	<i>Pr &gt; F</i>
GROUP	3	579.87547128	71.08	0.0001
TOSCA	1	0.25457955	0.09	0.7624
PATMATH	1	1.26419662	0.46	0.5022
Error	23	62.54548913		

<i>Source</i>	<i>DF</i>	<i>Type III SS</i>	<i>Mean Square</i>	<i>F Value</i>	<i>Pr &gt; F</i>
STAGES	3	7.5184851	2.5061617	1.36	0.2621
STAGES*GROUP	9	232.1155105	25.7906123	14.00	0.0001
STAGES*TOSCA	3	2.7248564	0.9082855	0.49	0.6882
STAGES*PATMATH	3	6.0682879	2.0227626	1.10	0.3558
Error (STAGES)	69	127.0792893	1.8417288		

**Summary of ANOVA in Subtraction, Experiment 1**

<i>Source</i>	<i>DF</i>	<i>Type III SS</i>	<i>F Value</i>	<i>Pr &gt; F</i>
GROUP	3	319.26112050	15.09	0.0001
TOSCA	1	0.71305336	0.10	0.7534
PATMATH	1	2.84700482	0.40	0.5315
Error	23	162.25733762		

<i>Source</i>	<i>DF</i>	<i>Type III SS</i>	<i>Mean Square</i>	<i>F Value</i>	<i>Pr &gt; F</i>
STAGES	3	10.7097468	3.5699156	0.95	0.4236
STAGES*GROUP	9	182.8094438	20.3121604	5.38	0.0001
STAGES*TOSCA	3	10.4437435	3.4812478	0.92	0.4350
STAGES*PATMATH	3	1.8013225	0.6004408	0.16	0.9235
Error (STAGES)	69	260.5668759	3.7763315		

### Summary of ANOVA in Multiplication, Experiment 1

<i>Source</i>	<i>DF</i>	<i>Type III SS</i>	<i>F Value</i>	<i>Pr &gt; F</i>
GROUP	3	203.36748876	6.70	0.0022
CS	1	1.37526634	0.14	0.7159
TOSCA	1	1.64764872	0.16	0.6904
PATMATH	1	1.28837406	0.13	0.7246
Error	22	222.56991651		

<i>Source</i>	<i>DF</i>	<i>Type III SS</i>	<i>Mean Square</i>	<i>F Value</i>	<i>Pr &gt; F</i>
STAGES	3	34.1536233	11.3845411	5.63	0.0017
STAGES*GROUP	9	136.1619842	15.1291094	7.49	0.0001
STAGES*CS	3	18.0224497	6.0074832	2.97	0.0380
STAGES*TOSCA	3	3.3455059	1.1151686	0.55	0.6487
STAGES*PATMATH	3	2.7012269	0.9004090	0.45	0.7213
Error (STAGES)	66	133.3799666	2.0209086		

### Summary of ANOVA in Division, Experiment 1

<i>Source</i>	<i>DF</i>	<i>Type III SS</i>	<i>F Value</i>	<i>Pr &gt; F</i>
GROUP	3	338.59415535	11.16	0.0001
CS	1	19.43813391	1.92	0.1795
TOSCA	1	0.69849154	0.07	0.7951
PATMATH	1	16.08440051	1.59	0.2205
Error	22	222.49004227		

<i>Source</i>	<i>DF</i>	<i>Type III SS</i>	<i>Mean Square</i>	<i>F Value</i>	<i>Pr &gt; F</i>
STAGES	3	9.95049904	3.31683301	2.28	0.0872
STAGES*GROUP	9	92.09020860	10.23224540	7.04	0.0001
STAGES*CS	3	8.90327021	2.96775674	2.04	0.1165
STAGES*TOSCA	3	2.35678520	0.78559507	0.54	0.6563
STAGES*PATMATH	3	3.04207298	1.01402433	0.70	0.5568
Error (STAGES)	66	95.93340115	1.45353638		

*Appendix I. ANOVA Summary Tables for Experiment 2*

### Summary of ANOVA in Addition, Experiment 2

<i>Source</i>	<i>DF</i>	<i>Type III SS</i>	<i>F Value</i>	<i>Pr &gt; F</i>
GROUP	2	494.24122874	31.55	0.0001
TOSCA	1	17.29096694	2.21	0.1509
PATMATH	1	9.46980779	1.21	0.2829
Error	23	180.16723895		

<i>Source</i>	<i>DF</i>	<i>Type III SS</i>	<i>Mean Square</i>	<i>F Value</i>	<i>Pr &gt; F</i>
STAGES	4	37.7504403	9.4376101	4.12	0.0041
STAGES*GROUP	8	179.5355007	22.4419376	9.79	0.0001
STAGES*TOSCA	4	7.4212016	1.8553004	0.81	0.5223
STAGES*PATMATH	4	2.3007082	0.5751771	0.25	0.9084
Error (STAGES)	92	210.8932559	2.2923180		

### Summary of ANOVA in Subtraction, Experiment 2

<i>Source</i>	<i>DF</i>	<i>Type III SS</i>	<i>F Value</i>	<i>Pr &gt; F</i>
GROUP	2	471.19037301	24.64	0.0001
TOSCA	1	0.14799122	0.02	0.9021
PATMATH	1	0.98040428	0.10	0.7517
Error	23	219.90080290		

<i>Source</i>	<i>DF</i>	<i>Type III SS</i>	<i>Mean Square</i>	<i>F Value</i>	<i>Pr &gt; F</i>
STAGES	4	16.3133700	4.0783425	1.94	0.1104
STAGES*GROUP	8	297.0612631	37.1326579	17.66	0.0001
STAGES*TOSCA	4	5.1341891	1.2835473	0.61	0.6561
STAGES*PATMATH	4	4.4485215	1.1121304	0.53	0.7147
Error (STAGES)	92	193.4350723	2.1025551		

**Summary of ANOVA in Multiplication, Experiment 2**

<i>Source</i>	<i>DF</i>	<i>Type III SS</i>	<i>F Value</i>	<i>Pr &gt; F</i>
GROUP	2	371.87604879	21.01	0.0001
TOSCA	1	5.61173851	0.63	0.4340
PATMATH	1	3.12366064	0.35	0.5582
Error	23	203.54367326		

<i>Source</i>	<i>DF</i>	<i>Type III SS</i>	<i>Mean Square</i>	<i>F Value</i>	<i>Pr &gt; F</i>
STAGES	4	18.5158052	4.6289513	2.03	0.0961
STAGES*GROUP	8	122.5442051	15.3180256	6.73	0.0001
STAGES*TOSCA	4	9.6185789	2.4046447	1.06	0.3827
STAGES*PATMATH	4	11.9548173	2.9887043	1.31	0.2710
Error (STAGES)	92	209.4201923	2.2763064		

**Summary of ANOVA in Division, Experiment 2**

<i>Source</i>	<i>DF</i>	<i>Type III SS</i>	<i>F Value</i>	<i>Pr &gt; F</i>
GROUP	2	256.77772225	10.41	0.0006
TOSCA	1	1.55805660	0.13	0.7255
PATMATH	1	17.32777551	1.41	0.2479
Error	23	283.58875386		

<i>Source</i>	<i>DF</i>	<i>Type III SS</i>	<i>Mean Square</i>	<i>F Value</i>	<i>Pr &gt; F</i>
STAGES	4	7.3995629	1.8498907	0.67	0.6157
STAGES*GROUP	8	111.1248142	13.8906018	5.02	0.0001
STAGES*TOSCA	4	16.3600662	4.0900166	1.48	0.2155
STAGES*PATMATH	4	16.0392961	4.0098240	1.45	0.2245
Error (STAGES)	92	254.7334501	2.7688418		