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Acceptance Sampling for Food Quality Assurance



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Dedicated to my mother, Carmen Fernández Ferrer
A ti madre querida, por ser ejemplo de dedicacion y amor.

“In God we trust, all others bring data.¹”

¹Attributed to W. Edwards Deming.

Declaration

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university. This dissertation is my own work and contains nothing which is the outcome of work done in collaboration with others, except as specified in the text and Acknowledgements. This dissertation contains fewer than 65,000 words including appendices, bibliography, footnotes, tables and equations and has fewer than 150 figures.

Edgar Santos-Fernández

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Abstract

Acceptance sampling plays a crucial role in food quality assurance. However, safety inspection represents a substantial economic burden due to the testing costs and the number of quality characteristics involved. This thesis presents six pieces of work on the design of attribute and variables sampling inspection plans for food safety and quality. Several sampling plans are introduced with the aims of providing a better protection for the consumers and reducing the sample sizes. The effect of factors such as the spatial distribution of microorganisms and the analytical unit amount is discussed. The quality in accepted batches has also been studied, which is relevant for assessing the impact of the product in the public health system. Optimum design of sampling plans for bulk materials is considered and different scenarios in terms of mixing efficiency are evaluated. Single and two-stage sampling plans based on compressed limits are introduced. Other issues such as the effect of imperfect testing and the robustness of the plan have been also discussed. The use of the techniques is illustrated with practical examples. We considered numerous probability models for fitting aerobic plate counts and presence-absence data from milk powder samples. The suggested techniques have been found to provide a substantial sampling economy, reducing the sample size by a factor between 20 and 80% (when compared to plans recommended by the International Commission on Microbiological Specification for Food (ICMSF) and the CODEX Alimentarius). Free software and apps have been published, allowing practitioners to design more stringent sampling plans.

Keywords:

Bulk material, Composite samples, Compressed limit, Consumer Protection, Double sampling plan, Food safety, Measurement errors, Microbiological testing, Sampling inspection plan.

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Declaration

This thesis complies with the ‘[Guidelines for Doctoral Thesis by Publications](#)’ and with the requirements from the Handbook for Doctoral Study by the Doctoral Research Committee (DRC), Massey University. January 2011. Version 7.

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The opinions, findings and conclusions in this thesis are solely those of the author(s). Under no circumstances will the author(s) be responsible for any loss or damage of any kind resulted from the use of these techniques. The software codes and the apps produced by this research are licensed under GPL $\geq$ 2.0 and it comes without warranty of any kind.

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