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**Theoretical Investigation of Traffic Flow:
Inhomogeneity Induced Emergence**

A dissertation presented in partial fulfillment of the
requirements for the degree of

**Doctor of Philosophy
in
Computer Science**

at Massey University, Auckland, New Zealand

Mingzhe Liu

2010

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**I dedicate this thesis and my love to my wife Ping Wen,
my daughter Xiner Liu and my son Runxi Liu**

Abstract

This research work is focused on understanding the effects of inhomogeneity on traffic flow by theoretical analysis and computer simulations. Traffic has been observed at almost all levels of natural and manmade systems (e.g., from microscopic protein motors to macroscopic objects like cars). For these various traffic, basic and emergent phenomena, modelling methods, theoretical analysis and physical meanings are normally concerned.

Inhomogeneity like bottlenecks may cause traffic congestions or motor protein crowding. The crowded protein motors may lead to some human diseases. The congested traffic patterns have not been understood well so far.

The modelling method in this research is based on totally asymmetric simple exclusion process (TASEP). The following TASEP models are developed: TASEP with single inhomogeneity, TASEP with zoned inhomogeneity, TASEP with junction, TASEP with site sharing and different boundary conditions. These models are motivated by vehicular traffic, pedestrian traffic, ant traffic, protein motor traffic and/or Internet traffic.

Theoretical solutions for the proposed models are obtained and verified by Monte Carlo simulations. These theoretical results can be used as a base for further developments. The emergent properties such as phase transitions, phase separations and spontaneous symmetry breaking are observed and discussed. This study has contributed to a deeper understanding of generic traffic dynamics, particularly, in the presence of inhomogeneity, and has important implications for explanation or guidance of future traffic studies.

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Contents

Chapter 1 Introduction	1
1.1 Research description	1
1.2 Research motivation	4
1.3 Main contributions	5
1.4 Thesis outline	7
Chapter 2 Review on TASEP Models	9
2.1 Totally asymmetric simple exclusion process (TASEP)	9
2.1.1 Introduction	9
2.1.2 Updating procedures	10
2.1.3 Random update	12
2.1.4 Parallel update	14
2.2 TASEP with different particle properties	17
2.2.1 TASEP with large particles	17
2.2.2 TASEP with long-range hopping	18
2.2.3 TASEP with Langmuir kinetics	19
2.2.4 TASEP with two-species particles	21
2.3 TASEP with different lattice structures	25
2.3.1 TASEP with multiple parallel channels	25
2.3.2 TASEP with multiple-input multiple-output junctions	29
2.3.3 TASEP with local inhomogeneity	31
2.4 Research methods	34
2.5 Summary	36
Chapter 3 Local Inhomogeneity in a Single-channel System	38
3.1 Introduction	38
3.2 Model description	39
3.3 Mean-field approach	41
3.4 Domain wall approach	46
3.5 Theoretical calculations and computer simulations	49
3.6 Summary and conclusions	53
Chapter 4 Zoned Inhomogeneity on Asymmetric Exclusion	

Process	55
4.1 Introduction	55
4.2 Case V	56
4.2.1 Model description	56
4.2.2 Theoretical analysis	57
4.2.3 Results and discussion	63
4.3 Case W	65
4.4 Summary and conclusions	68
Chapter 5 Asymmetric Exclusion Process with Junction	72
5.1 Introduction	72
5.2 m -input 1-output junction	74
5.2.1 Model and mean-field analysis	74
5.2.2 Domain wall theory	82
5.2.3 Monte Carlo simulations and discussion	83
5.3 m -input n -output junction	86
5.3.1 Model and theoretical analysis	86
5.3.2 Results and discussion	90
5.4 Summary and conclusions	92
Chapter 6 Two-species TASEP with Site Sharing in a Single-channel System	102
6.1 Introduction	102
6.2 Model formation and theoretical analysis	104
6.3 Results and discussion	111
6.4 Summary and conclusions	115
Chapter 7 Spontaneous Symmetry Breaking in Asymmetric Exclusion Process with Site Sharing	118
7.1 Introduction	118
7.2 Model description	120
7.3 Monte Carlo simulations and discussion	122
7.4 Summary and conclusions	127
Chapter 8 Conclusions and Outlook	129
8.1 Research summary	130
8.1.1 Local inhomogeneity in a single-channel system	130
8.1.2 TASEP with m -input n -output junction	132
8.1.3 TASEP with site sharing and relaxed boundaries	133
8.1.4 TASEP with site sharing and constrained boundaries	134
8.2 Future Work	134
Bibliography	137
Appendix Publications	147

List of Figures

Figure 2.1	Illustration of TASEP in open boundary conditions	11
Figure 2.2	Phase diagrams of TASEP in open boundaries	14
Figure 2.3	Illustration of possible TASEP extensions	16
Figure 2.4	Sketch of the Bridge model	22
Figure 2.5	Sketch of two-channel TASEP with narrow entrances	24
Figure 2.6	Sketch of a four-channel model with narrow entrances	24
Figure 2.7	General two-channel TASEP models	25
Figure 2.8	Flow chart of Monte Carlo simulations	37
Figure 3.1	Illustration of TASEP with a local inhomogeneity	40
Figure 3.2	Phase diagrams of TASEP with a local inhomogeneity	44
Figure 3.3	Schematic diagram of the domain wall dynamics	46
Figure 3.4	Density profiles from theoretical analysis and MCS	50
Figure 3.5	Density profiles with different p	51
Figure 3.6	Dependence of current on entrance rate α	52
Figure 3.7	Density profiles near the phase boundaries	53
Figure 4.1	Illustration of TASEP with a zoned inhomogeneity	58
Figure 4.2	Diagrams of possible stationary-state phases	62
Figure 4.3	Currents with different hopping probability p	64
Figure 4.4	Density profiles from simulations with different p	66
Figure 4.5	Phase diagram in case W with p	68
Figure 4.6	Currents with fixed p	69
Figure 4.7	Density profiles with different p	70

Figure 5.1	Schematic diagram of TASEP with a MISO junction	75
Figure 5.2	Phase boundaries and phase diagram of the model	81
Figure 5.3	Density profiles from theoretical calculations and MCS	94
Figure 5.4	Density profiles from DW theory and MCS	95
Figure 5.5	Density profiles vs different m	96
Figure 5.6	Density profiles in random and parallel updates	97
Figure 5.7	TASEP with m -input n -output junction	98
Figure 5.8	Illustration of entering and exiting in two subsystems	98
Figure 5.9	Phase boundaries and phase diagram vs different λ	99
Figure 5.10	Density profiles vs different λ	100
Figure 5.11	Density profiles from DW theory and MCS	101
Figure 5.12	Currents from theoretical calculations and MCS	101
Figure 6.1	Illustration of the TASEP with site sharing	105
Figure 6.2	Four possible states on each site	105
Figure 6.3	Phase diagram of the model	112
Figure 6.4	Stationary currents vs sharing probability	113
Figure 6.5	Density profiles in the LD, HD and MC phases	114
Figure 6.6	Four possible states vs sharing probability	116
Figure 6.7	Currents in our model and the Bridge model	117
Figure 7.1	Illustration of the model	121
Figure 7.2	Symmetry breaking in the phase diagram	122
Figure 7.3	Histograms of densities in all phases	124
Figure 7.4	Flipping processes in the asymmetric phases	125
Figure 7.5	Finite-size effects with different system length	125
Figure 7.6	Stationary current with different β	126
Figure 7.7	Currents obtained from our model and the Bridge model	127

List of Tables

Table 2.1	Stationary properties of TASEP in random update	13
Table 2.2	Stationary properties of TASEP in parallel update	15
Table 2.3	Stationary properties of TASEP with large particles	18
Table 2.4	Stationary properties of TASEP with inhomogeneity	32
Table 3.1	Comparisons of TASEP with local inhomogeneity	46
Table 4.1	Details of Figure 4.5	69
Table 5.1	Possible phases and corresponding conditions	89