A MULTIVARIATE PLANNING MODEL - CITY STRUCTURE

A Thesis presented in partial fulfilment of the requirements for the
degree of Master of Science in Statistics

by

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Preface

The genesis of this study is post graduate research in Urban Geography at Canterbury University in 1966. At that time a crude multivariate Centroid model of 95 New Zealand towns and cities was constructed. Based upon 60 socio-economic variables two factors for each of the years 1951, 1956 and 1961 were extracted and compared. The present study, which is a considerable refinement upon the earlier research, incorporates not only tremendous advancement in multivariate design methodology and application, but also parallel advancements that have been made in computing facilities over the last five years.

The objective of this research is to construct a multivariate statistical planning model that is both statistically precise and meaningful in its application. Particular emphasis is placed upon the need to organise in a systematic and meaningful manner the increasingly greater variety of statistics that portray urban growth. Stress is placed upon the utility of the multivariate technique as a tool in the author's profession of Town Planning.

Acknowledgements

I would like to thank Professor B.I. Hayman not only as my supervisor but also as my tutor for three pleasant years at Massey University. My thanks must also go to the staff of the Computer Unit at Massey and in particular Miss Nola Gordon. The map and diagrams in the text testify to the artistic ability of Mrs. D. Harrod. Credit for the typing is due to Mrs. J. Cheer, while the quality of printing is a debt to the skill of 'Mac' McKenzie.

I am particularly grateful to the Palmerston North City Council, especially the Town Planning Department and the Town Planner, Mr. K. Nairn, for allowing me to indulge in my interest in Statistics.

Finally, a tribute must be made to the patience of my wife, Jane, who had to listen to so many of my little discoveries.

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I. MULTIVARIATE METHODOLOGY

1. Introduction:-

Multivariate methods are Statistical techniques concerned with relationships between variables. These relationships attain a particular level of significance in association with the volume of urban area statistics produced in New Zealand and the need to use such statistics in Town Planning. In particular, recent proposed legislation requiring the establishment of planning policy necessitates a more precise understanding of the nature of relationships between statistics used to delineate city development. This legislation is backed by precedence in decisions of the Town and Country Planning Appeals Board which has already stipulated that their determinations will be based upon planning policy where it exists. Few cities in New Zealand have established such policy. The Planner will therefore be required by statute to derive planning policy which will, on the whole, be obtained from a myriad of statistics all of varying degrees of importance. The problem is to develop a statistical technique which will incorporate and account for statistics used in Planning. The multivariate statistical technique of Factor Analysis appears to have the most potential for such an analysis.

2. Research Objectives:-

Research objectives in this study are two-fold - firstly to investigate the utility of a multivariate statistical technique in the delineation of urban relationships and hence the definition of planning policy, and secondly to assess problems of data distribution and mathematical meaningfulness inherent in multivariate modelling. Both the former and the latter objectives are analysed in terms of an examination of New Zealand's 18 cities over the 1951-71 period. The Multivariate Factor Analysis method is developed as the mathematical planning model.

3. Statistics:-

Multivariate statistical analysis is associated with a considerable body of statistical theory and knowledge which has developed since the 1940s from the work of Lawley. Earlier and simpler applications focussed upon univariate and bivariate relationships and the normal distribution. Much of the less systematic statistical methodology was developed by the early analytic psychologists; Charles Spearman, Cyril Burt, Karl Pearson, G.H. Thomson, J.C. Maxwell Garnett, Karl Holzinger, H. Hotelling, L.L. Thurstone, Galton and others. More recently, and in particular in the last decade, the development of computer science and more flexible numerical techniques has led to the relaxation of computational limitations upon applications of multivariate statistical theory. Work by Lawley, Howe, Anderson, Rao and Maxwell, Carroll, Ferguson, Neuhaus and Wrigley, Saunders and Kaiser on multivariate factor statistical methodology has been of considerable importance. At the same time refinement of the eigen-value problem by numerical analysts - Householder, Rutihauser, Francis and others - has greatly contributed to developments in multivariate analysis. The breakthrough by Joreskog in the establishment of a numerical method for the minimisation of a function of many variables in 1966 is of considerable importance. Methodological improvement by Joreskog in collaboration with others in the past few years has meant a simplification of the application of the technique's improved statistical base. Almost all of the improvements in the technique has meant an increase in ability to relate many variables in a statistically meaningful manner.

4. Planning:-

Town Planning involves the establishment of policy for city development goals, formulated from an interpretation of the patterns of urban growth. This interpretation involves prior knowledge from a determination of not only existing relationships and inter-relationships in cities, but also an understanding of the trends in such relationships and their relative degree of importance. Analysis of this kind, while implied under the Third Schedule of the Town and Country Planning Regulations 1960, is not stipulated.

Basic data used in the establishment of urban area inter-relationships are generally available from Census publications or from carefully designed sample surveys. Much of this material requires interpretation, particularly on complex issues where the outcome of a decision or policy implementation, may be consequential upon the complex interaction of a variety of variables. Indicative Planning in New Zealand has, until recently, been involved in the assessment of individual statistics or simple combinations of such statistics. More often than not, only univariate analysis is undertaken and frequently the population statistic was the sole index used in indicative planning.

Recent decisions of the New Zealand Town and Country Planning Appeals Board have emphasised the need for Planners to take cognizance of the more complex issues in establishing Town Planning policy[1]. The definition of the complex issues of planning require a more refined analysis in terms of the available statistics. The problem of the Planner is to arrange these statistics in a meaningful manner so that they may portray the complex issues and clarify the important aspects of city growth and development.

Outside of the classificatory work of the urban Geographers there has been little research undertaken in this area of statistical application. The American Ecological studies by Shevky-Bell, Haynes, Molotch and others have been concerned with spatial inter-relationships

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[1] An example is the Board's recent decision in the case G.U.S. Properties Ltd. and others v. Timaru City Council 1971 4 N.Z.T.C.P.A. 12.

and classification within urban areas. Most other studies including the above, criticised widely for lack of methodological framework, do not indicate an incorporation of a planning base.

5. Experimental Multivariate Planning Model:-

Multivariate statistical models involving determination of the simple and complex inter-relationships between many different statistics appear particularly suitable for an analysis of the characteristics of cities. Moreover, the multivariate Factor Analysis model has considerable potential as a planning model because it incorporates the principle of parsimony, i.e. the ability to precipitate a simple relationship from a complex combination of many variables. This study is an attempt to construct an experimental Factor Analysis planning model and to examine the relationship between the model and observable reality.

The study format is in three parts. In Chapter Two the mathematical and statistical framework for the model is established. Chapter Three consists of a detailed analysis of the application of the model to the New Zealand situation. In the final section, Chapter Four, the results and meaningfulness of the model are assessed in terms of the statistical accuracy and usefulness as a planning tool.

II. MATHEMATICAL MODEL - FACTOR ANALYSIS

1. Theoretical Framework:-

A multivariate mathematical model forms an ideal analytical base for demonstrating developments in inter-relationships. Initially the pattern of variable distributions can be portrayed in a univariate situation. Then may be considered the bivariate distributions which describe the relationships between pairs of variables. Multivariate patterns in turn may be portrayed in a Factor Analysis model. The relationships are essentially linear, but are systematic and the factor model is developed stage by stage (Figure 1).

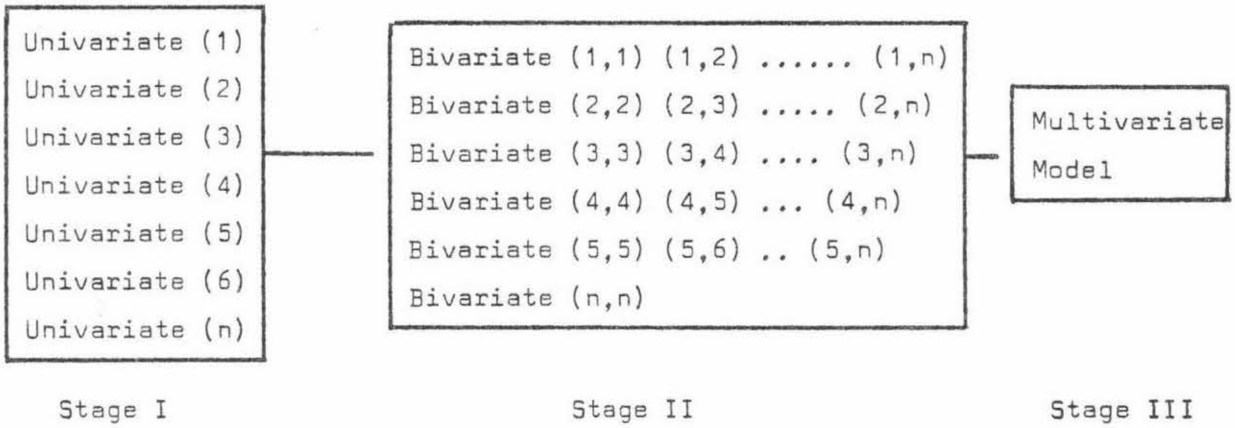

Figure 1. Staged Development of the Multivariate Factor Model

The advantage of the technique is that in any particular application an interpretation may be placed upon the various stages of the structuring of the model.

2. Data Cube:-

Consider a set of variables or characteristics, X_1, X_2 ....X_n describing particular entities over a set time period. A standard 'Data Cube' is formed. Such variables are selected on the basis of a particular hypothesis or research goal. In this instance the formation of a Data Cube which describes not only entities and their characteristics also allows for occasions, provides the basis for an analysis overtime. Such analysis forms a fundamental structure in the delineation of a planning model. The data cells of the three dimensional Data Cube form the basis for the model. In this particular research application occasions are combined in the final model with entities, and the standard data cube becomes two-dimensional. Separate time slices are used to study the structure of this model. The factoring matrix, therefore, conforms to the R-factor analysis. The significance of the datum cell is in the patterns of variation between characteristics over entities. Characteristics are the variables.

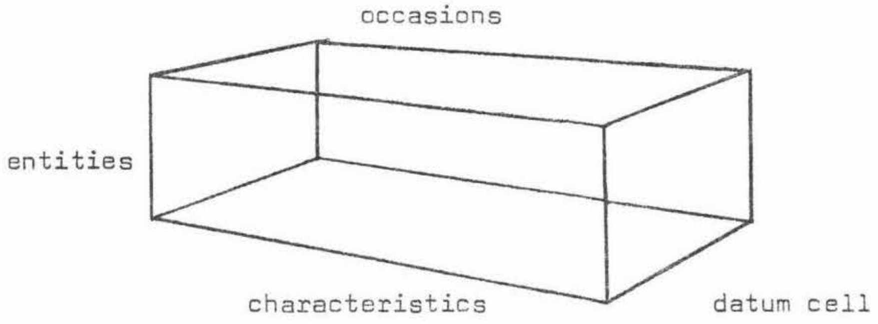

Figure 2. The Data Box

3. Means and Standard Deviations:-

The mean is a central value of a characteristic calculated as

$$\overline{X}_i = \sum_j X_{ij}/N$$

and indicating the general numerical location of the characteristic. Averaging of one characteristic for different entities at different points in time can reveal a simple pattern of change or a trend.

$$\overline{X}_{i,t_1} \quad ; \quad \overline{X}_{i,t_2} \quad ; \quad \overline{X}_{i,t_3} \quad \ldots\ldots\ldots\ldots$$

$$t = \text{time} \qquad i = 1, \text{———} n$$

The measure of location, however, may in particular instances, not take cognizance of the arrangement or spread of the individual values of the characteristics. Thus, in some situations the mean value may not provide enough information about the data distribution. Therefore, a measure of spread may be particularly useful in demonstrating patterns of change. Most commonly used is the variance value for the variable.

$$\sigma_i^2 = \frac{\sum (x_i - \bar{x}_i)^2}{N}$$

or its square root, the standard deviation,

$$\sigma_i \quad \sqrt{\frac{\sum(x_i - \bar{x}_i)^2}{N}}$$

which has the advantage of being on the same scale as the variable. For a normal variable, 68.26% of the sample lies within one standard deviation of the mean, etc. Thus, as a consequence a more precise description of data distribution is possible.

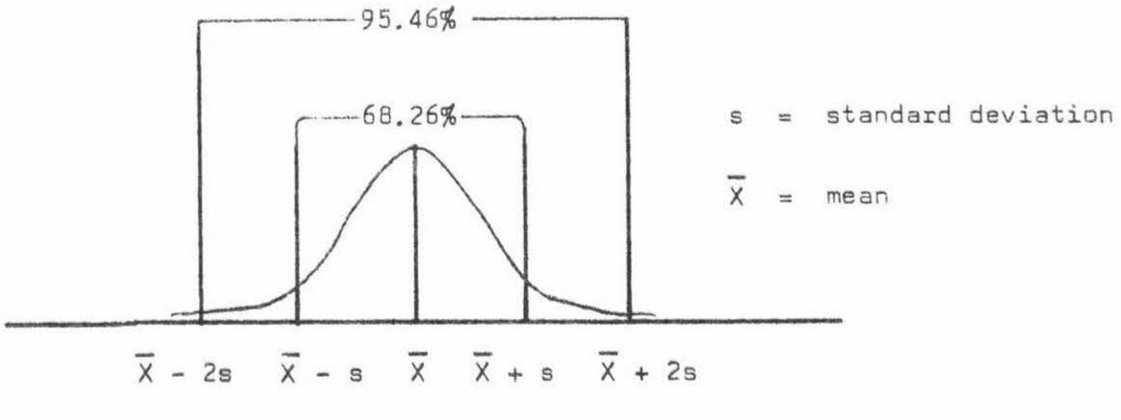

Figure 3. Areas Under a Normal Curve

Hence the standard deviation for a particular variable assessed for different entities at different points in time can reveal a pattern of change or a trend,

$$\sigma_{i,t_1} \quad ; \quad \sigma_{i,t_2} \quad ; \quad \sigma_{i,t_3} \quad ....$$

t = time, $i = 1,2, ... n.$

More so, if the changes in standard deviations are interpreted with the patterns of change associated with the development of the average values.

4. Correlation and Covariance:-

Until now the basic descriptive statistics have been associated with univariate situations. Fundamental in data analysis is the bivariate consideration - the pattern of relationships between two variables. The measures of covariance and correlation demonstrate the bivariate relationship. In the latter instance, however, the measure is a scaled quantity while the former retains the numerical data distribution.

$$\mathrm{cov}(x_i,x_j) = \sum (x_i - \overline{x}_i)(x_j - \overline{x}_j)$$

$$r_{i,j} = \frac{\sum (x_i - \overline{x}_i)(x_j - \overline{x}_j)}{\sqrt{\left[\sum (x_i - \overline{x}_i)^2\right]\left[\sum (x_j - \overline{x}_j)^2\right]}}$$

Further, $r_{i,j}^{2}$ represents the amount of variance which the two variables have in common. Either the covariance or the correlation coefficient can be used in an analysis of trends in a particular pair of characteristics. A relationship between two variables may intensify and therefore there will be greater inter-dependence. On the other hand, the converse situation may apply.

$$r_{i,j,t_1} \quad ; \quad r_{i,j,t_2} \quad ; \quad \ldots\ldots\ldots\ldots\ldots$$

$$i,j = 1,2,\ldots\ldots.n, \qquad t_1,t_2\ldots\ldots = \text{time}$$

$$\mathrm{cov}_{t_1}(x_i,x_j) \quad ; \quad \mathrm{cov}_{t_2}(x_i,x_j) \quad \ldots\ldots\ldots.$$

Both methods provide systematic measures which may be used to define changes in a bivariate relationship over a particular time period.

5. Principal Component Multivariate Model:-

Unlike partial, multiple and canonical correlations which are used to analyse the dependence structure of a multinormal population, the primary problem in correlation is the definition of dependent and independent variables. While the choice of dependent variable may be based upon response patterns and hence the research hypothesis, it is inevitable in a multivariate situation that the responses are symmetric or there are no a priori patterns of causality available.

Techniques developed to establish a dependence structure of observed responses based upon hypothetical independent variables come within the general category of Factor Analysis. Such statistical techniques attempt to define those hidden factors which have generated the dependence relation between, and the variation in, the responses. Observable variables are represented as functions of a smaller number of latent factor variables. These functions are such that they will generate the covariances or correlations amongst the responses. In this study we are concerned with generating the correlations amongst responses. The objective of the technique is to establish from amongst the responses of many variates a more simple or parsimonious description of dependence structure. It is assumed that the generating model is linear in form.

The principal component model developed by K. Pearson as a method of fitting planes by orthogonal least squares and extended by Hotelling for analysing correlation structures is the simplest of the Factor models and it is usual to use this model as the first step in estimating the structure of a factor model. The technique has widespread use in a variety of fields including human biology, cognitive psychology, mineralogy.

The model which merely partitions the variance amongst the computed components is derived from $X_1, \ldots\ldots\ldots\ldots X_p$ random variables with multivariate distribution mean vector $\underset{\sim}{\mu}$ and covariance matrix Σ Both the elements of $\underset{\sim}{\mu}$ and Σ are finite with the rank of Σ being $r \leq p$ and that the q largest characteristic roots of $\sum$ are distinct. Further, an N × p data matrix is established from a sample of N independent observation vectors.

$$X = \begin{bmatrix} x_{11} \ldots\ldots\ldots\ldots\ldots\ldots\ldots x_{1p} \\ \\ \\ x_{N1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots x_{Np} \end{bmatrix}$$

Note that neither $\sum$ nor X need be of full rank p, and further $\sum$ need not contain more than one characteristic root. Full rank, however, ensures simplicity in structure description and is generally assumed in practice.

An estimate of $\sum$ is either the variance-covariance matrix or the correlation matrix R. The latter is preferred instead of the former because of the scaling properties of the correlation coefficient. The first principal component of the observations X is the linear compound

$$Y_1 = a_{11}X_1 + \ldots\ldots\ldots\ldots + a_{p1}X_p$$

of the responses whose sample variance

$$s_{Y_1}^{\ 2} = \sum_{i=1} \sum_{j=1} a_{i1}a_{j1}s_{ij}$$

$$= \lambda_1 \quad \text{(The largest characteristic root)}$$

Continual factoring generates linear compounds of the original variates which account for a progressively smaller amount of the variance. The significant features of the model are that:-

a) the principal component analysis factorises R

b) principal component analysis factorization is unique

Because of the model's inherent characteristics it is therefore possible to construct principal component models portraying the relationships between many variables at different points in time. Moreover, a comparison between the models can be attempted on the basis of the changes in relationships and variation.

Principal Component = PC

PC_{1t_1} ; PC_{1t_2} ; PC_{1t_3} ....

PC_{2,t_1} ; PC_{2,t_2} ; PC_{2,t_3}..........

. . .

. . .

. . .

PC_{p,t_1} ; PC_{p,t_2} ; PC_{p,t_3}..........

PC_{i,t_j} = Principal Component
i = 1.....p, t = time j = 1, ... end of period.

Since the correlation between the original variables and the individual components can be obtained through the formula $a_{ij}\sqrt{\lambda_j}$ where a_{ij} are the estimated component loadings and λ_j the characteristic root of the jth component, it is possible to relate components and variables. Moreover, a simpler or parsimonious description is now possible in terms of a single linear component if it accounts for the greater part of the variance of the original variables.

6. Factor Analysis Multivariate Model:-

Despite its simplicity the Principal Component Multivariate model has shortcomings. While the model does factorise the covariance matrix the factorisation is more of a transformation rather than the consequence of a fundamental model for covariance structure. Further, the forms of components are not invariant under response scale changes and there is no strict criteria for deciding when sufficient variance has been accounted for. It is significant that no provision is made for error variance estimations.

This partition of the variance relates to the factor model in that "each response variate is represented as a linear function of a smaller number of unobservable common factor variates and a single latent specific variate. Common factors generate the covariances among the observable responses while the specific terms contribute only to the variances of their particular responses" (Morrison, 1967). This refinement in description over the Principal Component model is, however, gained at the expense of two assumptions:

a) the observations arose from a multinormal population of <u>full rank</u>.

b) the exact <u>number of common factors</u> can be <u>specified before analysis</u>.

Both these assumptions are an essential part of the Factor philosophy.

The mathematical model is based upon a multivariate system of p responses characterised by observed random variables $x_1 \ldots\ldots\ldots\ldots x_p . x_i$ having a nonsingular multinormal distribution. The model is of the form:

$$\begin{array}{lllll} x_1 & = & a_{11}y_1 & + \ldots\ldots + a_{1m}y_m & + \quad e_1 \\ . & & . & \quad\quad . & \quad . \\ . & & . & \quad\quad . & \quad . \\ . & & . & \quad\quad . & \quad . \\ x_p & = & a_{p1}y_1 & + \quad\quad + a_{pm}y_m & + \quad e_p \end{array}$$

y_j = jth common factor variate, $j = 1,2 \ldots\ldots m$

a_{ij} = parameter reflecting importance of jth factor in the composition of the ith response (loading of the ith reponse on the jth common factor)

e_i = ith specific factor variate

In matrix notation the factor model becomes $X = \Lambda Y + \mathcal{E}$

Now let the m common factor variates in y be distributed normally with zero means and unit variances i.e. $y \sim N(0,1)$. Further assume $\mathcal{E} \sim N(0, \Psi_i)$. Ψ_i being the specificity of the ith response.

$$\Psi = \begin{bmatrix} \Psi_1 & & & \\ & \Psi_2 & & \\ & & \ddots & \\ & & & \Psi_p \end{bmatrix}$$

Moreover, it is required that the variates y and $\mathcal{E}$ be independently distributed. Variance on the ith response from the properties of the latent variates are:

$$\sigma_i^2 = a_{i1}^2 + \dots\dots\dots\dots + a_{im}^2 + \Psi_i$$

and the covariance of the ith and jth response variate as

$$\sigma_{ij} = a_{i1}a_{j1} + \dots\dots\dots\dots + a_{im}a_{jm}$$

That is $\Sigma = \Lambda \Lambda' + \Psi$

Now $\sigma_i^2 - \Psi_i = \sum_{j=1} a_{ij}^2$

are the diagonal elements of $\Lambda\Lambda'$ and are called the communalities of the responses. a_{ij} is the covariance of the ith response with the jth common factor. However, when Σ is the population correlation matrix, R, the a_{ij} as in the case of the principal component model is the correlation of responses and common factors.

The basic problem in factor analysis is the determination of the a_{ij} with the elements of Ψ following as a constraint imposed upon the communalities. The fundamental aspect of the factor model, however, is that linearity becomes part of the research philosophy. Further, the research hypothesis is related directly to the number of factors. If there is not a fit between the hypothesised factors and the observed values then both the factor hypothesis and the linearity hypothesis may be rejected. Normally further factors may be hypothesised to test the fit. Some work by MacDonald has been undertaken on the problem of non-linearity rejection, but this work is in its early stages of development. Linearity is assumed throughout this research application.

Similar to the Principal Component technique, but at a more refined level of statistical analysis it is possible to relate not only variables to the factors, but also to construct factor models representing different analyses at different points in time. In addition, it is therefore possible to attempt a comparison between models on the basis of the changes in relationships and variations.

$$\begin{array}{lllll} F_{1,t_1} & ; & F_{1,t_2} & ; & F_{1,t_3} \dots\dots\dots\dots \\ F_{2,t_1} & ; & F_{2,t_2} & ; & F_{2,t_3} \dots\dots\dots\dots \\ & & & & \\ F_{m,t_1} & ; & F_{m,t_2} & ; & F_{m,t_3} \dots\dots\dots\dots \end{array}$$

F_{i,t_j} = Factor

$i = 1, ...m$ number of factors hypothesised

$t_j = 1, ...$ end of period under study

7. Varimax Rotation:-

As a corollary to factor production, maximisation of associations between factors and variables may be obtained by a rotation. The significant feature of the Principal Component model is not only the unique factorisation but also the orthogonal relationship between components. Thus Components are independent and theoretically unrelated. On the other hand, the Factor model does not have the condition that the sums of the squares become successively smaller as one passes from the first to the final factor. As a consequence orthogonal rotation of the loading matrix Λ does not affect the generation of covariances. In fact, it is to be appreciated that in factor analysis an infinity of loading matrices may be obtained from the correlation matrix.

As a result, a more "meaningful" application of the concept of simple structure may be applied to make a clearer definition of loadings. Further, it is reiterated by some that the "particular configuration of numbers obtained in an unrotated factor analysis loading matrix is largely a function of the method used to extract the eigenvalues and eigenvectors and therefore may have no empirical meaning". The concept of simple structure is a non-mathematical technique setting out several criteria for a rotation of factors:

a) existence of a positive manifold (i.e. a minimum number of negative values in the factor loading matrix)

b) a small number of high loadings and a large number of near zero loadings

c) each row of the factor loading matrix to have at least one near zero factor loading and at least one other large positive loading

d) it must account for the relative position of zeros and important high loadings

The principle is one of an application of Occam's Razor to the factor loading matrix and is felt by most to give a better or improved description to the factors.

Most commonly used, and the technique used in this application, is Kaiser's (1958) varimax rotation method which maximises the fourth power of the factor loadings and therefore maximises the scatter amongst the loadings. As the method retains the property of orthogonality which leaves the factors uncorrelated it is as a consequence widely used. In general a transformation matrix 'I' is developed over a cycle of rotations with the angle of each rotation chosen such that a function 'U' of the factor matrix is maximised.

$$U = m \sum_{i = 1}^{n} \sum_{j = 1}^{m} \left[\frac{g_{ij}}{h_i} \right]^4 - \sum_{j = i}^{m} \sum_{i = 1}^{n} \left[\frac{g_{ij}^{2}}{h^{2}_{j}} \right]^{2}$$

m = number of factors

n = number of variables

g_{ij} = element of factors matrix under rotation for ith variable jth factor

h_i^2 = communality

The rotated Factors assume particular importance because of this relationship, particularly in respect of the relationship between variables, factors and factor scores. Factor scores for particular entities are derived from the factors and demonstrate the relationship between individual entities in terms of the hypothesised factor constructed from many related variables. The degree of rotation and hence the dominance of a particular variable must be assessed in terms of the rotation. Different rotations applied for separate models representing different points in time tend to highlight differences between dominant variables.

8. Multivariate Statistical Factor Model:-

If occasions are combined with entities and the resultant two-dimensional data base is factor analysed a more general combined multivariate statistical factor model may be constructed. The model is still the simple format

$$X = \Lambda Y + \mathcal{E}$$

In this instance, however, the entities become entities for different occasions with the unique and distinct characteristics being associated with each particular point in time. Further, the descriptive basis allows the use of the factor hypothesis to delineate aspects of particular entities, and comparisons can be made between the different factor models

$F_{1,(t_1,t_2, \ldots\ldots t_k)}$; $F_{2,(t_1,t_2, \ldots\ldots t_k)}$; ....

.... $F_{m,(t_1,t_2, \ldots\ldots t_k)}$.

F = factor

F_i $i = 1,2,\ldots\ldots m$ number of hypothesised factors

t_j $j = 1,2,\ldots\ldots k$ time period under consideration

The particular significance of such a combined model is two-fold. Firstly, not only can the model be rotated with some consistency, but also the singular time scale models can be related in terms of specific variable - factor relationships. Secondly, hypothesised factor comparisons can be made. In this latter instance the factor scores, which relate entities and factors defining variation can map a particular trend in patterns of change as shown in the scores over a set time period. The former instance allows a check between the final factor model and observed reality. In fact the staged development from simple arithmetic means, variances, covariances, correlation, single time scale principal component and simple time scale factor models relates the cumulative model to the observable situation. A pattern of growth and inter-relationships may be defined in a complex multivariate situation through such a refinement of the application of the multivariate factor model.

9. Data Distribution:-

Basically the Factor Analysis model outlined focuses upon a delineation of similarities and differences, relationships and associations. Kendall (1957) stipulates that the application of Factor Analysis is a search for inter-relationships rather than dependency. The preciseness of the definition of such inter-relationships will be dependent upon a variety of factors. Not the least amongst these factors will be the data distribution.

It is only in recent years that a more sophisticated philosophy of factor analysis has been established from a multivariate normal hypothesis. Lawley's work in the early 1940s demonstrated the need for a sound data base while applications of the maximum likelihood philosophy has given a more formal theoretical statistical framework. Joreskog's breakthrough in 1967 has meant this theoretical framework can be applied to specific research applications. More specifically factors can be tested for significance in terms of a normal sampling situation.

Development of such a body of theory and techniques for application is a breakthrough of considerable importance, but is not undertaken in this study because the need to develop the technique, not yet available in New Zealand, was beyond the scope of the study. There was, however, a need to establish a reasonably consistent framework in which to develop the model. The normality of the data distribution and its effect on the fundamental model is the secondary objective of this piece of experimental research.

Data Distribution and normality considerations assume a particular degree of importance when it is considered that a bivariate normal distribution has the property that the regression relation between two variables is linear (Kendall and Stuart, 1958, vol. 1, p. 387). Further linearity in the bivariate inter-relationship of the data is a basic assumption of the model. Moreover, a sufficient condition for the correlation coefficients to be a true measure of statistical independence between two variables is that the bivariate distribution of the variables be normal. Thus, the importance of the normality of data distribution is a prime consideration not only in correlation, but in the final factor model.

In the development of the factor model it is proposed to examine this relationship between normality of data distribution and the model at its various stages of construction. Not only will the effect of normalising data be studied in derived correlation coefficients, but also the implications in terms of the fundamental factor model which is constructed from the correlation coefficients.

It is proposed to develop the model from basic data and repeat the application using the same data with a normal transformation. Both models will be assessed - the crude data model and the statistically exact model. Final examination will be the relationship between the model and its ability to portray the nature of the variation in relationships between variables.

III. MULTIVARIATE PLANNING MODEL - NEW ZEALAND CITIES

1. Experimental Design:-

The multivariate planning model is developed from an analysis of variables describing the characteristics of New Zealand cities, over the 1951-1971 post war development period. Research framework is structured on the basis of 22 variables and 18 cities (Tables 1 and 2 and Figure 4). Variables were selected on the basis of stipulation in The Third Schedule of the Town and Country Planning Regulations 1960, common characteristics, definitive identity and availability. Generally they embrace the diverse features of New Zealand cities in describing both social and economic characteristics as well as demographic aspects of change. A short title for descriptive purposes has been included (Table 3).

TABLE 1

22 Socio-economic and Demographic Variables Used in the Analysis of New Zealand Cities[†]

Description	Variable
I. Demographic Variables	
1. Population	X_1
2. Percentage of Population aged 0-14 years	X_2
3. Percentage of Population aged 15-64 years	X_3
4. Percentage of Population aged 65+ years	X_4
5. Females per 1000 males	X_5
II. Demographic Change Variables	
6. Per cent intercensal increase in total population	X_6
7. Per cent intercensal increase in population due to births and deaths	X_7
8. Per cent intercensal increase in population due to movement into the area	X_8
III. Political Variable	
9. Per cent of the voting population voting Labour in the last election	X_9
IV. Maori Population Variables	
10. Total Maori Population	X_{10}
11. Per cent intercensal increase in Maori population	X_{11}
V. Value Variable	
12. Per cent intercensal in gross capital value	X_{12}
VI. Industrial Activity Variables	
13. Per cent of Labour force women	X_{13}
14. Per cent of Labour force employed in Primary Industries	X_{14}
15. Per cent of Labour force employed in Primary Processing Industries	X_{15}
16. Per cent of Labour force employed in Construction Industries	X_{16}
17. Per cent of Labour force employed in Trade Industries	X_{17}
18. Per cent of Labour force employed in Service Industries	X_{18}
19. Per cent of Labour force employed in Seasonal	X

(Table 1 contd.)

Description	Variable
VII. Local Body Variable	
20. Rating in dollar averaged over intercensal period	X_{20}
VIII. Index of Economic Activity Variables	
21. Per cent of all building value new dwellings	X_{21}
22. Investment confidence index	X_{22}

[1] Source of variables and detailed description Appendix I

Static and dynamic aspects of the cities have been included within the model. The most significant feature about the above statistics is that they are commonly used in Town and Country Planning and can be readily obtained from the Government Statistician and Department of Labour. The number of variables selected was based upon obtaining a balanced description which would be reasonably comprehensible in terms of a meaningful modelling.

Eighteen New Zealand cities were selected as the basis for the model. The cities form the entities in the data cube. It is important to recognise that this is not a sample in the statistical sense and therefore the model does not incorporate an assessment of sampling error. In this study the choice of the urban universe was deliberate.

TABLE 2

Eighteen Largest New Zealand Cities

Whangarei
Tauranga
Napier
Wanganui
Wellington
Timaru
Auckland
Rotorua
Hastings
Palmerston North
Nelson
Dunedin
Hamilton
Gisborne
New Plymouth
Hutt
Christchurch
Invercargill

The time frame in which the variables and cities are to be assessed is from 1951 to 1971. Census quinquenniums 1951, 1956, 1961, 1966 and 1971 form the data slices for the initial simple models, while the combined

FIGURE 4

NEW ZEALAND CITIES

model incorporates the 1951-1971 twenty-year period. The period can be briefly described as one in which the New Zealand population has become increasingly more urbanised with the recent rapid growth and development taking place in the northern cities and in particular the Auckland metropolitan area. The time span is an ideal one to analyse as the patterns of change appear consistent with no major social, economic or political reversals occurring. The data cube used in this application is defined below (Figure 5).

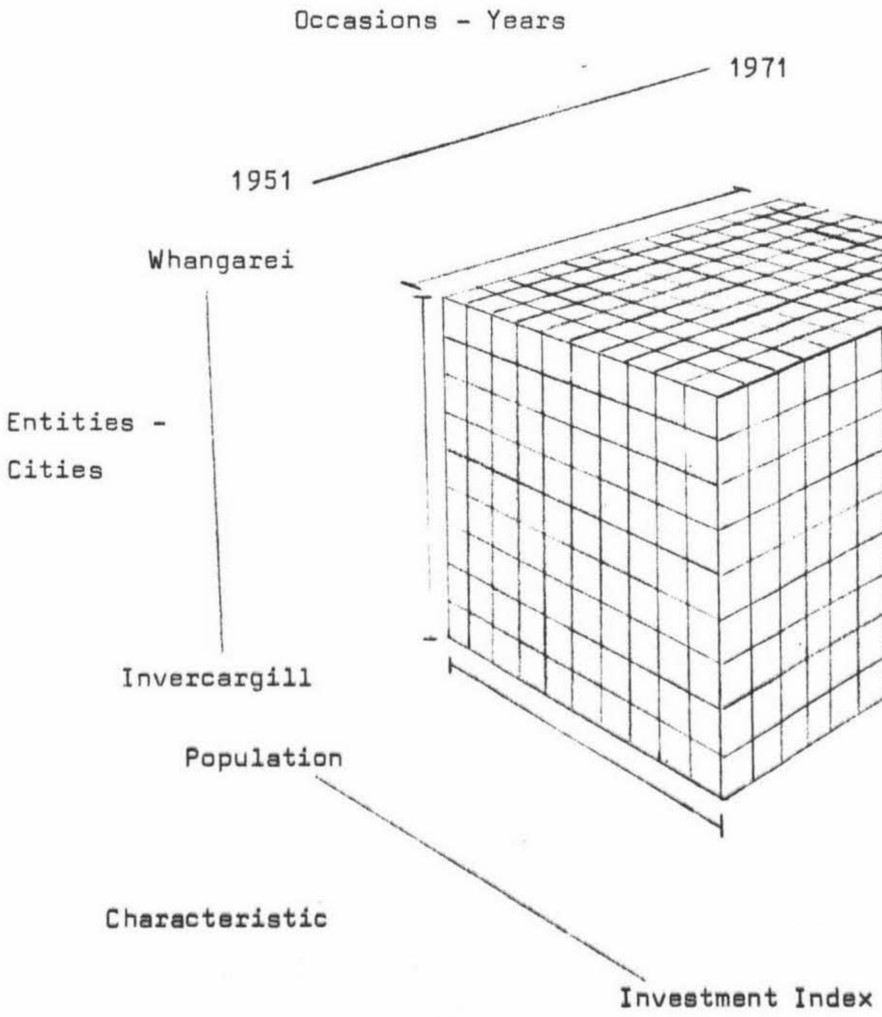

Figure 5. Study Data Cube

TABLE 3

Short Title of 22 Study Variables[1]

Variable	Long Title	Short Title
X_1	Total Population	Population
X_2	% total population aged 0-14 years	0-14 age group
X_3	% total population aged 15-64 years	15-64 age group
X_4	% total population aged 65+ years	65+ age group
X_5	Females per 1000 males	sex ratio
X_6	% intercensal increase in total population	population increase
X_7	% intercensal increase in population due to births and deaths	natural increase
X_8	% intercensal increase in population due to population migration into the area	movement
X_9	% of the voting population voting Labour in the last election	Labour vote
X_{10}	Total Maori population	Maori population
X_{11}	% intercensal increase in Maori population	Increase in Maoris
X_{12}	% intercensal increase in gross capital values	capital value increase
X_{13}	% of Labour force women	women in Labour force
X_{14}	% of Labour force employed in primary industries	primary industries
X_{15}	% of Labour force employed in primary processing industries	primary processing industries
X_{16}	% of Labour force employed in construction industries	construction industries
X_{17}	% of Labour force employed in trading industries	trading industries
X_{18}	% of Labour force employed in service industries	service industries
X_{19}	% of Labour force employed in seasonal industries	seasonal industries
X_{20}	Rating in the dollar averaged over the intercensal period	rating
X_{21}	% of all building value new dwellings	new dwellings
X_{22}	Investment confidence index	investment index

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[1] Short titles have been prepared to simplify descriptions of the variables and as an alternative to using mathematical substitutes (X_i). A complete description of each variable can be seen in the Appendix I.

Now it is proposed to construct 13 multivariate factor models although not all the models will necessarily be appearing in this study[1]. Comparative models will be developed for basic data and transformed normal distribution data for each of the years 1951, 1956, 1961, 1966 and 1971. Similarly, a combined model for both basic and normal transformation distribution data for 1951-1971 period will be constructed. An averaged model based on averaged correlation values is also to be constructed for comparative purposes.

The model construction is developed on the basis outlined in the mathematical model segment of the study. Univariate distributions are initially developed, bivariate situations are disclosed and both principal component and factor models are generated for discussion in terms of the patterns of city growth in New Zealand.

2. Univariate Patterns:-

The simplest statistical analyses not only define the means of the sample of variables, but also the standard deviations. Patterns of data distribution are highlighted in Table 4. The most basic of patterns is demonstrated in the analysis of the means. Population for instance, displays a steady pattern of growth, i.e.

$$\bar{X}_{1,1951} = 61,175$$

$$\bar{X}_{1,1956} = 68,467$$

$$\bar{X}_{1,1961} = 79,367$$

$$\bar{X}_{1,1966} = 92,897$$

$$\bar{X}_{1,1971} = 106,040$$

---

[1] The correlation matrices have been included in Appendix II.

Similarly, the steady decline in female dominance in New Zealand urban areas can be seen:-

$\overline{X}_{5,1951} = 1082$; $\overline{X}_{5,1956} = 1079$; $\overline{X}_{5,1961} = 1063$; $\overline{X}_{5,1966} = 1050$

$\overline{X}_{5,1971} = 1047$

Trends in other variables can be seen from the table.

Examination of the standard deviation patterns demonstrates the limitations of mean values in showing data distribution. An application of the standard deviation in conjunction with the arithmetic means does to some degree delimit the data distribution in the instance of each variable. Considerable variation can be seen amongst the variables and moreover, the patterns of variation change from time spectrum to time spectrum. Patterns demonstrated in Labour voting in the cities show the general extent of the variation.

$\overline{X}_{9,1951} = 47.7$; $\overline{X}_{9,1956} = 44.8$; $\overline{X}_{9,1961} = 45.0$; $\overline{X}_{9,1966} = 43.6$;

$\overline{X}_{9,1971} = 47.4$

$\sigma_{9,1951} = 6.8$; $\sigma_{9,1956} = 7.5$; $\sigma_{9,1961} = 9.1$; $\sigma_{9,1966} = 7.2$;

$\sigma_{9,1971} = 4.5$

Typically, the pattern of voting in the cities has varied over the period, but while the 1951 average value approximates to that of 1971 there is not the same degree of variation as is demonstrated by the fall in standard deviation. Hence, the analysis of deviations is of considerable value in determining data distribution.

TABLE 4
Univariate Data Distribution (Standard)
New Zealand Cities

Means

Variable	$\overline{X}_{i,1951}$	$\overline{X}_{i,1956}$	$\overline{X}_{i,1961}$	$\overline{X}_{i,1966}$	$\overline{X}_{i,1971}$	$\overline{X}_{i,1951-1971}$
X_1	61,175	68,467	79,367	92,897	106,040	81,591
X_2	25.3	26.4	28.1	31.6	31.5	28.6
X_3	65.1	63.9	63.0	59.6	59.0	62.0
X_4	9.5	9.9	9.0	9.3	9.4	9.4
X_5	1,082	1,079	1,063	1,050	1,047	1,064
X_6	20.8	14.8	16.2	15.9	9.7	15.5
X_7	10.0	7.3	7.6	7.7	5.8	7.7
X_8	11.0	7.7	8.5	8.1	3.9	7.8
X_9	47.7	44.8	45.0	43.6	47.4	45.7
X_{10}	1,079	1,606	2,633	4,592	6,159	3,214
X_{11}	32.7	87.2	64.9	77.6	27.9	58.1
X_{12}	87.5	109.1	48.7	46.3	36.3	65.6
X_{13}	28.3	28.0	29.0	30.2	30.5	29.2
X_{14}	0.4	0.3	0.2	0.2	0.2	0.3
X_{15}	8.0	7.4	7.8	7.0	6.2	7.3
X_{16}	29.4	30.6	29.8	30.5	24.3	28.9
X_{17}	33.3	32.9	32.5	32.1	37.7	33.7
X_{18}	26.5	25.3	25.8	26.2	26.3	26.0
X_{19}	2.8	3.4	3.9	3.7	5.2	3.8
X_{20}	6.0579	5.1387	3.8953	3.6484	3.7007	4.4882
X_{21}	67.9	51.1	51.2	47.6	43.5	52.3
X_{22}	88.8	85.4	58.1	43.0	26.2	60.3

Standard Deviations

Variable	$\sigma_{i,1951}$	$\sigma_{i,1956}$	$\sigma_{i,1961}$	$\sigma_{i,1966}$	$\sigma_{i,1971}$	$\sigma_{i,1951-1971}$
X_1	80,924	92,145	106,700	128,090	150,190	112,950
X_2	2.9	3.0	2.7	2.6	2.2	3.7
X_3	3.5	3.5	3.8	2.3	2.3	4.0
X_4	2.0	1.7	2.0	1.7	1.9	1.8
X_5	33.1	31.4	31.7	33.7	28.2	34.1
X_6	12.4	9.5	7.3	9.2	5.4	9.6
X_7	3.7	2.0	2.3	2.7	2.1	2.9
X_8	11.0	10.0	5.9	7.0	4.5	8.2
X_9	6.8	7.5	9.1	7.2	5.5	7.3
X_{10}	1,743	2,578	4,486	7,594	9,889	6,248
X_{11}	22.9	211.8	27.4	32.2	14.3	98.1
X_{12}	45.7	38.1	20.6	21.0	16.5	41.0
X_{13}	2.3	2.4	2.3	1.9	2.3	2.4
X_{14}	0.5	0.3	0.2	0.2	0.2	0.3
X_{15}	4.2	3.9	3.7	3.5	3.5	3.7
X_{16}	6.2	6.1	6.5	6.2	6.4	6.6
X_{17}	4.6	5.8	6.0	5.1	6.0	5.8
X_{18}	5.9	5.4	5.4	4.0	4.8	5.0
X_{19}	4.1	4.3	5.2	4.8	7.9	5.4
X_{20}	1.8749	1.4978	1.0349	0.6987	0.7561	1.5588
X_{21}	9.3	9.1	5.8	6.7	7.3	11.3
X_{22}	74.2	50.0	31.3	49.1	28.9	54.1

Combined, 1951-1971, values have been included and from these figures both means and standard deviations show that the combined value should be considered in conjunction with the individual values for particular years. On the whole, combined values tend to reflect averaged mean and standard deviation values for the period 1951 to 1971, while separate time phase analysis tends to reflect the dynamic elements of change not wholely demonstrated in the combined univariate results.

Close examination of mean and standard values, however, reflects a number of spurious results. Under population, for instance, the standard deviation in all instances is considerably larger than the mean value. Such results are generally indicative of non-normal distributions of data. Variable X_1, population, as a logarithmic distribution which is dominated by the City of Auckland. Other variables show similarly inconsistent patterns and as a result each of the variables was mapped by the means and standard deviations into distributions. While some retained an approximately normal distribution through the two-decade study period, some variables had not only different combined data distributions, but also distributions which varied considerably from year to year. In some instances, typical patterns consisted of left and right skew distributions as well as J and reverse J shaped distributions. Where applicable, standard transformations were applied to the raw data, normal or approximately normal distributions were established. The transformations were as follows:-

Variable	1951	1956	1961	1966	1971	1951-71
X_1	$\log X_1$	$\log X_1$	$\log X_1$	$\log X_1$	$\log X_1$	$\log X_1$
X_2	X_2	X_2	$\sqrt{X_2/100}$	X_2	X_2	X_2
X_3	$\sqrt{X_3}$	$\sqrt{X_3}$	$\sqrt{X_3}$	$\sqrt{X_3}$	X_3	X_3
X_4	X_4	X_4	X_4	X_4	X_4	X_4
X_5	X_5	X_5	X_5	X_5	X_5	X_5
X_6	$\sqrt{X_6}$	$\sqrt{X_6}$	$\sqrt{X_6}$	$\sqrt{X_6}$	X_6	$\sqrt{X_6}$
X_7	X_7	$\sqrt{X_7}$	$\sqrt{X_7}$	$\sqrt{X_7}$	$\sqrt{X_7}$	$\sqrt{X_7}$
X_8	$\log(X_8 +2.0)$	$\log(X_8+10)$	$\sqrt{10(X_8+10)}$	$\sqrt{(X_8+10)}$	X_8	$\log(X_8+10)$
X_9	X_9	X_9	X_9	X_9	X_9	X_9
X_{10}	$\log X_{10}$	$\log X_{10}$	$\log X_{10}$	$\log X_{10}$	$\log X_{10}$	$\log X_{10}$
X_{11}	$(X_{11}+9)^2$	$\sqrt{\log X_{11}}$	X_{11}	X_{11}	X_{11}	$\log(X_{11}+10)$
X_{12}	$\sqrt{X_{12}}$	X_{12}	$\sqrt{X_{12}}$	$\sqrt{X_{12}}$	X_{12}	$\sqrt{X_{12}}$
X_{13}	X_{13}	X_{13}	X_{13}	X_{13}	X_{13}	X_{13}
X_{14}	$\log[10(X_{14})+1]$	$\sqrt{X_{14}}$	$\sqrt{X_{14}}$	$\sqrt{X_{14}}$	$\sqrt{X_{14}}$	$\sqrt{X_{14}}$
X_{15}	$\sqrt{X_{15}}$	$\sqrt{X_{15}}$	$\sqrt{X_{15}}$	X_{15}	X_{15}	$\sqrt{X_{15}}$
X_{16}	$\sqrt{X_{16}}$	$\sqrt{X_{16}}$	$\sqrt{X_{16}}$	$\sqrt{X_{16}}$	$\sqrt{X_{16}}$	X_{16}
X_{17}	$(X_{17}-20)^2$	X_{17}	X_{17}	X_{17}	X_{17}	X_{17}
X_{18}	$\sqrt{X_{18}}$	$\sqrt{X_{18}}$	X_{18}	X_{18}	$\frac{1}{2}\log\left[\frac{1+X_{18}}{1-X_{18}}\right]$	X_{18}
X_{19}	$\sqrt{1/(X_{19} + 1)}$	$\sqrt{1/(X_{19}+1)}$	$\sqrt{1/(X_{19}+1)}$	$\sqrt{1/X_{19}}$	$\sqrt{1/X_{19}}$	X_{19}
X_{20}	$\sqrt{X_{20}}$	$\sqrt{X_{20}}$	X_{20}	X_{20}	X_{20}	$\sqrt{X_{20}}$
X_{21}	X_{21}	X_{21}	$\sqrt{X_{21}}$	X_{21}	X_{21}	X_{21}
X_{22}	$\sqrt{(X_{22}+20)}$	X_{22}	$\sqrt{X_{22}}$	X_{22}	X_{22}	X_{22}

Transformations generally fell within the following categories:-

Distribution	Variable	Transformation
Strongly right skew (logarithmic)	X_j	$X_j^* = \log X_j$
Slightly right skew	X_j	$X_j^* = (X_j)^{\frac{1}{2}}$
Left skew	X_j	$X_j^* = \frac{1}{2} \log \frac{1 + X_j}{1 - X_j}$

TABLE 5

Univariate Data Distribution (Normalised) New Zealand Cities

Means

Variable	$\overline{X}_{i,1951}$	$\overline{X}_{i,1956}$	$\overline{X}_{i,1961}$	$\overline{X}_{i,1966}$	$\overline{X}_{i,1971}$	$\overline{X}_{i,1951-1971}$
X_1	4.5484	4.6005	4.6845	4.7620	4.8198	4.6830
X_2	25.3	26.4	0.5	31.6	31.5	28.6
X_3	8.1	8.0	7.9	7.7	59.0	62.0
X_4	9.5	9.9	9.0	9.3	9.4	9.4
X_5	1082	1077	1063	1050	1047	1064
X_6	4.4	3.7	3.9	3.8	9.7	3.7
X_7	10.0	2.7	2.7	2.7	2.4	2.7
X_8	0.9914	1.2062	13.0	4.2	3.9	2.2159
X_9	47.6	45.1	45.0	43.7	47.7	45.7
X_{10}	2.7255	2.9083	3.1113	3.3632	3.4943	3.1206
X_{11}	223.4	1.2288	64.9	77.6	27.9	2.7087
X_{12}	91.0	109.1	6.7	6.8	34.8	7.8
X_{13}	28.3	28.0	29.0	30.2	30.5	29.2
X_{14}	0.5696	5.2	4.1	4.4	3.7	4.6
X_{15}	2.7	2.6	2.7	7.0	6.2	2.6
X_{16}	5.4	5.5	5.4	5.5	4.9	28.9
X_{17}	2.0	32.9	32.5	32.1	37.7	33.7
X_{18}	5.1	5.0	25.8	26.2	0.6425	26.0
X_{19}	0.6	0.6	0.6	0.8	0.7	3.8
X_{20}	2.4358	2.2467	3.8953	3.6484	3.7007	2.0915
X_{21}	67.9	51.1	7.1	47.6	43.5	52.3
X_{22}	7.7	85.4	7.4	43.0	26.2	60.3

Standard Deviations

Variable	$\sigma_{i,1951}$	$\sigma_{i,1956}$	$\sigma_{i,1961}$	$\sigma_{i,1966}$	$\sigma_{i,1971}$	$\sigma_{i,1951-1971}$
X_1	0.4346	0.4280	0.3967	0.3764	0.3726	0.4058
X_2	2.9	3.0	.03	2.6	2.2	3.7
X_3	0.2	0.2	0.2	0.5	2.3	4.0
X_4	2.0	1.7	2.0	1.7	1.9	1.8
X_5	33.1	31.6	31.7	33.7	28.2	34.1
X_6	1.4	1.1	0.9	1.1	5.4	1.2
X_7	3.7	0.3	0.4	0.5	0.4	0.5
X_8	0.3579	0.1863	3.2	0.8	0.4	0.1704
X_9	6.8	7.4	9.1	7.3	5.2	7.3
X_{10}	0.5310	0.5282	0.5204	0.5300	0.5349	0.5899
X_{11}	195.6	0.2377	27.4	32.2	14.3	0.3236
X_{12}	22.4	38.1	1.3	1.4	15.2	2.4
X_{13}	2.3	2.4	2.3	1.9	2.3	2.4
X_{14}	0.2731	2.9	2.7	2.5	2.4	2.7
X_{15}	0.7	0.7	0.7	3.5	3.5	0.7
X_{16}	0.5	0.5	0.5	0.5	0.7	6.6
X_{17}	1.3	5.8	5.9	5.1	6.0	5.8
X_{18}	0.5	0.5	5.4	4.0	0.5160	5.0
X_{19}	0.2	0.2	0.2	0.5	0.5	5.4
X_{20}	0.3599	0.3066	1.0349	0.6987	0.7561	0.3379
X_{21}	9.3	9.1	0.4	6.7	7.3	11.3
X_{22}	3.8	50.0	2.0	49.1	28.9	54.1

Using the transformed data,mean values and standard deviations were again calculated (Table 5). In almost all instances the distribution appeared to be more normal when calculated from the normalised transformed distribution. Unfortunately, because transformations were not necessarily consistent, it is however, impossible to relate all variables over the time scale 1951 to 1971. It must be recognised, however, that the combined model is much more useful in terms of data description - mean values are more normal and standard deviations more accurately describe the data distribution about the mean. The 1951-1971 values were obtained from a standard normalising transformation and here data distribution descriptions are more useful than in the basic data case.

3. Bivariate Patterns:-

Pearson's product moment correlation coefficients is the simplest method of demonstrating the relationships between any two variables. 22 x 22 product correlation matrices were developed as part of this analysis. Matrices were produced not only for the 1951, 1956, 1961, 1966 and 1971 data slices for both basic data and transformed normal data, but also for combined 1951-1971 data transformed and basic[1]. A matrix of averaged values from the basic data distribution time slice matrices was also constructed for comparison purposes.

While the full series of matrices has not been reproduced here, a number of examples are considered. The examples typify the characteristics of the matrices.

Example 1. Correlations between population (X_1) and percentage of the labour force engaged in primary processing industries (X_{15}).

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[1] Only the transformed correlation matrices of combined 1951-1971 results are reproduced here because of problems in reproduction. The remaining correlation matrices are in Appendix II.

	$r_{1,15,1951}$	$r_{1,15,1956}$	$r_{1,15,1961}$	$r_{1,15,1966}$	$r_{1,15,1971}$
1. Basic Data[1]	0.66	0.68	0.55	0.49	0.44
2. Normal Data[1]	0.80	0.79	0.63	0.55	0.44

	$r_{1,15,1951-71}$	$\bar{r}_{1,15,1951-1971}$[2]
1. Basic Data	0.49	0.56
2. Normal Data	0.61	

The table indicates a decrease in relationship between population and the proportion of labour force employed in primary processing industries over the survey period 1951-1971.

Example 2. Correlations between population (X_1) and the percentage of the population aged 15-64 years (X_3).

	$r_{1,3,1951}$	$r_{1,3,1956}$	$r_{1,3,1961}$	$r_{1,3,1966}$	$r_{1,3,1971}$
1. Basic Data	-0.21	-0.25	-0.22	0.49	0.45
2. Normal Data	-0.08	-0.16	-0.16	0.72	0.68

	$r_{1,3,1951-71}$	$\bar{r}_{1,3,1951-1971}$
1. Basic Data	-0.05	0.05
2. Normal Data	-0.08	

The table reflects a steadily improving relationship over the 1951-1971 period between total population and the proportion of population aged 14-64 years. In particular, the transformed data with the normal distribution indicates stronger coefficients than that portrayed by the basic data correlations.

Example 3. Correlations between population (X_1) and rating in the dollar (X_{20}).

	$r_{1,20,1951}$	$r_{1,20,1956}$	$r_{1,20,1961}$	$r_{1,20,1966}$	$r_{1,20,1971}$
1. Basic data	-0.06	-0.13	-0.20	0.08	-0.11
2. Normal data	0.10	-0.01	-0.09	0.11	-0.04

	$r_{1,20,1951-71}$	$\bar{r}_{1,20,1951-1971}$
1. Basic data	-0.14	-0.09
2. Normal data	-0.13	

---

[1] Basic data refers to the original data while normal data relates to the transformed data with a normal distribution.

[2] $\bar{r}$ is average correlation coefficient for 1951, 1956, 1961, 1966 and 1971.

The table typifies the correlation matrices - correlations are relatively low and only a small proportion of the variance is accounted for. Further coefficients, while low, also show considerable variation from year to year, i.e. $r_{1,20,1961}$ is slightly negatively correlated while $r_{1,20,1966}$ is the reverse in sign.

Example 4. Correlations between per cent increase in population (X_6) and total Maori population (X_{10}).

	$r_{6,10,1951}$	$r_{6,10,1956}$	$r_{6,10,1961}$	$r_{6,10,1966}$	$r_{6,10,1971}$
1. Basic data	-0.08	0.18	0.17	0.31	0.33
2. Normal data	-0.11	0.42	0.46	0.62	0.49

	$r_{6,10,1951-71}$	$\bar{r}_{6,10,1951-1971}$
1. Basic data	0.06	0.11
2. Normal data	0.12	

The changes in correlation over time show considerable degree of variation, also the distinct difference between basic and normal data distributions is clearly highlighted in the correlations between per cent increase in total population and Maori population.

Example 5. Correlations between per cent population increase due to migration (X_8) and the per cent of the voters voting Labour (X_9).

	$r_{8,9,1951}$	$r_{8,9,1956}$	$r_{8,9,1961}$	$r_{8,9,1966}$	$r_{8,9,1971}$
1. Basic data	-0.59	-0.47	-0.61	-0.67	-0.48
2. Normal data	-0.42	-0.37	-0.49	-0.69	-0.42

	$r_{8,9,1951-71}$	$\bar{r}_{8,9,1951-1971}$
1. Basic data	-0.51	-0.56
2. Normal data	-0.47	

While in the previous example variation between basic and normal data distributions were clearly apparent in an improvement in coefficients by the transformations, such improvement is not necessarily true for all instances as in this case. Correlation coefficients $r_{8,9,1951}$ and $r_{8,9,1956}$ and $r_{8,9,1961}$ demonstrate considerable differences between transformed and basic values.

Example 6. Correlations between percentage population increase due to migration (X_8) and percentage increase in gross capital values (X_{12}).

	$r_{8,12,1951}$	$r_{8,12,1956}$	$r_{8,12,1961}$	$r_{8,12,1966}$	$r_{8,12,1971}$
1. Basic data	0.40	0.69	0.48	0.84	0.75
2. Normal data	0.57	0.71	0.40	0.76	0.75

	$r_{8,12,1951-71}$	$\bar{r}_{8,12,1951-1971}$
1. Basic data	0.50	0.63
2. Normal data	0.53	

Transformations to normalise data distribution do not necessarily consistently improve correlations. Moreover, as the above example illustrates, in one instance an improvement may be incurred ($r_{8,12,1951}$) the converse may apply in a latter situation ($r_{8,12,1961}$). Such an outcome may only be a consequence of the small size of the population being used in the study.

Example 7. Correlations between total Maori population (X_{10}) and the percentage of the labour force who are women (X_{13}).

	$r_{10,13,1951}$	$r_{10,13,1956}$	$r_{10,13,1961}$	$r_{10,13,1966}$	$r_{10,13,1971}$
1. Basic data	0.35	0.16	0.28	0.29	0.33
2. Normal data	0.53	0.36	0.43	0.33	0.29

	$r_{10,13,1951-71}$	$\bar{r}_{10,13,1951-1971}$
1. Basic data	0.34	0.28
2. Normal data	0.50	

The relationship between the data slices and the combined matrix for 1951-71 can be seen in this example. Both the combined value, for the basic data, $r_{10,13,1951-71}$, and the averaged correlation coefficients, $\bar{r}_{10,13,1951-1971}$, demonstrate a degree of similarity with both each other and the basic data slices. On the other hand, however, despite relatively small coefficients in the transformed segment, the combined normal basic co-efficient, $r_{10,13,1951-71}$, is comparatively high and thus affords a better description of the relationships between variates.

Example 8. Correlations between the percentage of the labour force employed in seasonal industrial activities (X_{19}) and rating in the dollar (X_{20}).

	$r_{19,20,1951}$	$r_{19,20,1956}$	$r_{19,20,1961}$	$r_{19,20,1966}$	$r_{19,20,1971}$
1. Basic data	0.01	0.01	-0.19	-0.28	-0.09
2. Normal data	-0.08	-0.21	-0.08	-0.19	-0.25

	$r_{19,20,1951-71}$	$\bar{r}_{19,20,1951-1971}$
1. Basic data	-0.12	-0.11
2. Normal data	-0.14	

Correlations for combined data matrices, both basic and normal distributions, are not necessarily high as the above example illustrates.

Close examination of the differences between the matrices reveals the following points. Firstly, the normalised data distributions tend on the whole to give a more useful assessment of the relationships between the variables being correlated. Further, although not necessarily in every case, both high positive and negative correlations appear to be magnified. On the other hand, the lower values, of which there are a number in the matrices, tend to remain around zero. Secondly, a study of the relationships between both the combined basic data matrices and the averaged basic data distribution matrices reveals a relatively similar pattern in many instances[1]. Again, however, there are differences, but these are not always large. Finally, there is considerable variation between the combined normal data distribution matrix and the basic data distribution matrix. Infrequently combined correlations are higher and therefore demonstrate more clearly the relationships between the variables. Thus, it would appear that the combined correlation matrix derived from normalised data

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[1] Averaged correlation matrix for normalised data distributions was not possible. Firstly, because of the considerable variations in data distribution and secondly, the construction of a planning model necessitates a simple known transformation rather than a combined one.

distributions would be most useful in this analysis because of its more realistic portrayal of variable inter-relationships and its consistency with normal statistical theory. The full correlation matrix is reproduced in Table 6 with the complete set of correlation matrices given in Appendix II.

TABLE 6
Product Moment Correlation Matrix
New Zealand Cities 1951-1971
(Normalised Data Distribution)

X_1	1.00													
X_2	0.08	1.00												
X_3	-0.08	-0.87	1.00											
X_4	-0.04	-0.11	-0.36	1.00										
X_5	-0.47	-0.48	-0.24	0.48	1.00									
X_6	-0.33	0.04	0.11	-0.28	-0.07	1.00								
X_7	-0.25	0.05	0.19	-0.50	-0.16	0.70	1.00							
X_8	-0.33	0.04	0.07	-0.20	-0.03	0.96	0.53	1.00						
X_9	0.39	-0.08	0.03	0.12	-0.04	-0.42	-0.15	-0.47	1.00					
X_{10}	0.47	0.43	-0.25	-0.34	-0.55	0.12	0.10	0.14	-0.10	1.00				
X_{11}	0.07	0.20	-0.11	-0.17	-0.20	0.24	0.02	0.29	-0.23	0.25	1.00			
X_{12}	-0.38	-0.29	0.34	-0.11	0.17	0.55	0.45	0.53	-0.24	-0.20	0.09	1.00		
X_{13}	0.13	0.31	-0.30	-0.02	-0.12	0.02	0.00	0.04	-0.22	0.50	0.07	-0.24	1.00	
X_{14}	-0.05	-0.18	0.14	0.09	0.06	-0.04	0.04	-0.07	0.20	-0.13	-0.13	0.06	-0.11	1.00
X_{15}	0.61	-0.03	-0.13	0.34	-0.04	-0.38	-0.23	-0.39	0.62	-0.05	-0.07	-0.25	-0.11	0.01
X_{16}	0.20	0.16	-0.04	-0.22	-0.41	0.38	0.37	0.32	0.10	0.08	0.18	0.23	-0.38	0.12
X_{17}	-0.12	-0.11	-0.01	0.19	0.17	-0.24	-0.35	-0.17	-0.28	-0.11	-0.16	-0.04	0.23	-0.10
X_{18}	-0.48	-0.01	0.09	-0.16	0.13	0.20	0.28	0.18	-0.34	0.00	-0.08	0.10	0.58	0.00
X_{19}	-0.09	-0.03	0.06	-0.04	0.21	-0.14	-0.15	-0.10	0.10	0.06	0.04	-0.14	-0.23	-0.09
X_{20}	-0.13	-0.43	0.31	0.25	0.27	-0.13	0.12	-0.21	0.20	-0.43	-0.33	0.25	-0.34	0.11
X_{21}	-0.35	-0.40	0.40	0.00	0.40	0.54	0.47	0.50	-0.05	-0.30	-0.05	0.45	-0.30	0.07
X_{22}	-0.22	-0.26	0.22	0.02	0.13	0.36	0.20	0.35	-0.15	-0.19	0.03	0.38	-0.12	0.14

X_{15}	1.00							
X_{16}	0.27	1.00						
X_{17}	-0.33	-0.65	1.00					
X_{18}	-0.53	-0.41	0.23	1.00				
X_{19}	-0.10	-0.30	-0.24	-0.30	1.00			
X_{20}	0.22	0.00	0.10	-0.11	-0.14	1.00		
X_{21}	-0.11	0.13	-0.18	0.04	0.06	0.35	1.00	
X_{22}	-0.20	0.07	-0.04	0.14	-0.09	0.15	0.32	1.00

4. Multivariate Patterns:-

Principal Components provides the simplest method of determining multivariate structure. The technique, which is based upon the bivariate correlation patterns of a number of variables, provides a comparison of inter-relationships between patterns of phenomena. As explained earlier, the method involves no hypothesis and is merely a partitioning of variance.

To determine the most likely structural components and therefore a hypothesis, based upon the multivariate factor planning model the method of principal components provides a base on which to work. Principal components were extracted from both basic data and normal distributions as defined in the previous section. Components were obtained for the matrices established for 1951, 1956, 1961, 1966 and 1971 as well as the combined 1951-71 matrices and the averaged matrix for 1951-1971. The proportion of variance accounted for by each component is summarised in Table 7.

TABLE 7

Summary of Variance Accounted for by Principal Components (Percentage)

A. Basic Data Correlation Matrices

Components	1951	1956	1961	1966	1971	1951-71 combined	1951-1971 averaged
I	24.75	23.67	25.58	33.59	25.69	20.33	21.51
II	21.71	20.33	16.67	17.34	17.41	15.26	15.99
III	12.16	13.52	14.16	11.29	13.77	13.98	9.54
IV	10.22	12.11	9.33	9.39	9.32	8.26	9.10
V	8.86	8.16	8.37	7.44	7.34	7.61	8.10
VI	6.22	5.36	6.73	5.75	7.06	6.82	7.00
VII			4.99	4.76	4.09	5.54	5.59
Total variance accounted for with eigen-values $\geqslant 1.00$	83.92	83.15	85.83	89.66	85.59	77.80	76.83

B. Normal Data Correlation Matrices

Components	1951	1956	1961	1966	1971	1951-71 combined
I	25.86	24.69	27.19	35.91	26.91	21.34
II	23.61	20.88	16.87	17.61	17.82	17.39
III	11.21	14.23	13.66	11.19	12.26	13.95
IV	9.82	10.62	8.81	8.13	9.57	7.36
V	7.46	6.98	7.97	6.66	7.20	7.09
VI	6.48	5.45	6.25	5.91	6.33	5.34
VII			4.65		4.94	4.80
VIII						4.58
Total variance accounted for with eigenvalues $\geqslant$ 1.00	83.44	82.95	85.40	85.51	85.03	81.85

In accordance with general practice and the application of Guttman's lower bound theorem (1954) eigenvalues of < 1.00 are regarded as statistically insignificant, and therefore components with associated eigenvalues $\geqslant 1.00$ are extracted. Further, a measure of dependence may be considered in terms of 22 independent, and hence uncorrelated, variables would each account for 4.54% of the total variance. It can be seen from the table that the minor components approach this classification. All components would, however, need to be extracted in order to be able to reproduce the original correlation matrix. Extraction of additional components also includes the possibility of "noise" being generated as a consequence of numerical round-off error and subsequent spurious results.

A comparison between basic and normally distributed data matrices implies that there is possibly a better descriptive model being generated in the case of the latter. In particular the first few components account for a greater proportion of the variance in the case of the normally distributed data than in the basic data base. As a consequence, all future analysis will relate directly to the normally distributed data. In addition, variable description will be assumed to have a normal distribution either because one exists or a simple transformation has taken place. No distinction will be made in terms of variable description which will remain in the format X_1, ... X_{22}.

Table 8 contains a summary of the variance accounted for by the two and four components. In the instances of the data slices between 44% and 53% of the total variance is accounted for by two components constructed from the variation amongst 22 variables. Further, between 66% and 72% of the total variance is accounted for by four components in the same situation. Even the combined data has nearly 40% and 60% of its total variance accounted for by two and four components respectively. Such a description justifies the postulation of a two and a four factor hypothesis accounting for the variation amongst the variables that describe the cities of New Zealand. These hypotheses will form the focal issues for the remainder of the study.

TABLE 8

Summary of Variance Accounted for by 2 and 4 Components (Percentage)

Components	1951	1956	1961	1966	1971	1951-71 combined
I and II	48.47	45.57	44.06	53.52	44.73	38.73
I, II, III and IV	69.50	70.42	66.53	72.84	66.56	60.04

5. Factor Modelling:-

2- and 4-Factor multivariate models were constructed from the data matrices for 1951, 1956, 1961, 1966 and 1971. Initial communality estimates were derived from the squared multiple correlation coefficients. In most instances recycling of the matrix and iteration meant that the communalities rapidly converged and became stable after 4 iterations. Kaiser's Varimax rotation criteria was applied in an attempt to obtain the best description relating the hypothesis latent variate to the observed variables. This rotation, which is orthogonal, ensures the independence of hypothesised variates and thus enables them to be used for graphic display. Finally, factor scores were computed. Table 9 outlines the 5 basic 2-Factor multivariate models and their communality estimates.

TABLE 9

New Zealand Cities - 2 Factor Model[1]

Variable	1951 $F_1$1951	$F_2$1951	h^2	1956 $F_1$1956	$F_2$1956	h^2	1961 $F_1$1961	$F_2$1961	h^2	1966 $F_1$1966	$F_2$1966	h^2	1971 $F_1$1971	$F_2$1971	h^2
1	(-.81)	.27	.74	(.61)	.42	.55	(-.63)	-.04	.40	.15	(.73)	.56	-.40	(.67)	.61
2	-.32	(-.83)	.79	-.13	(.64)	.42	-.46	.05	.20	(-.70)	-.24	.55	(.73)	-.04	.53
3	.34	(.63)	.51	.01	-.48	.23	.41	.20	.21	.27	.48	.30	-.44	(.69)	.67
4	-.14	.01	.02	.09	-.18	.04	-.13	-.48	.25	(.74)	-.33	.65	-.35	(-.80)	.75
5	.43	.34	.30	-.13	(-.82)	.70	.39	-.43	.34	(.53)	(-.58)	.61	-.07	(-.80)	.65
6	.47	(-.86)	.97	(-.90)	.22	.87	.35	(.89)	.91	(-.99)	.09	.99	(.87)	.36	.89
7	.16	(-.69)	.51	-.43	.38	.33	.15	(.82)	.70	(-.91)	.07	.83	.48	(.64)	.63
8	.44	(-.84)	.90	(-.86)	.13	.76	.36	(.63)	.53	(-.95)	.07	.90	(.75)	.13	.58
9	(-.69)	.09	.49	(.50)	.44	.45	(-.55)	-.27	.37	(.77)	.27	.66	(-.53)	-.12	.30
10	-.11	-.01	.01	-.36	(.52)	.40	-.15	(.56)	.34	(-.55)	.41	.46	.26	(.70)	.56
11	.01	-.44	.19	(-.61)	.05	.38	-.04	-.29	.09	-.17	.39	.18	-.38	.25	.20
12	.13	(-.64)	.42	(-.72)	.03	.52	(.50)	.46	.46	(-.72)	-.08	.53	(.71)	-.03	.50
13	.37	.21	.18	-.43	.09	.19	.32	.38	.25	-.07	-.27	.08	.06	.29	.08
14	.24	-.24	.11	-.05	.12	.02	-.19	-.34	.15	.49	.18	.28	-.36	.09	.14
15	(-.88)	.07	.78	(.75)	.45	.76	(-.79)	-.24	.68	(.62)	.49	.62	(-.50)	.03	.25
16	-.42	(-.75)	.75	.03	(.67)	.45	(-.77)	.43	.77	-.32	(.69)	.58	.09	(.54)	.30
17	.11	(.51)	.27	.02	(-.62)	.38	(.68)	-.42	.65	-.02	(-.56)	.31	.14	-.15	.04
18	(.80)	.10	.65	(-.60)	.32	.46	(.78)	.35	.73	-.24	(-.74)	.61	.09	.04	.01
19	.28	.05	.08	-.34	.19	.15	.19	.45	.24	-.34	-.15	.14	.39	.29	.24
20	-.34	.02	.12	.35	-.19	.16	.02	-.36	.13	(.51)	.02	.27	(-.62)	-.35	.51
21	(.62)	-.40	.55	-.33	(-.61)	.48	-.44	-.18	.22	(-.64)	-.01	.41	.49	.10	.25
22	(.74)	.10	.56	-.02	(.57)	.33	.07	-.32	.11	(.63)	.04	.39	.00	-.43	.19

h^2 = communality ( ) = correlation $\geqslant \pm .50$

[1] Normalised Data Distribution N.B. Appendix III consists of 2-Factor Varimax Model for Basic Data

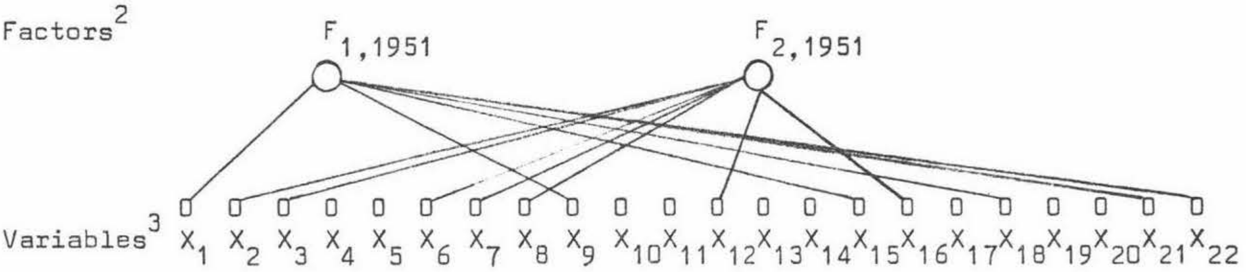
The significant feature demonstrated by the models is the shift in association of the population variable from the first factor to the second factor. The nature of the changes in association that have taken place over the 20-year period as demonstrated by the five 2 Factor models is displayed in Figure 6. In the diagrams, only variables with a correlation of ± 0.50 or greater are shown. The most encouraging feature of the factor descriptions is not only the relatively high communalities from such a diverse selection of variables, but also that at any one stage in the development of the models almost all the variables form part of the model.

To demonstrate the ability of the models to display a generalised description of New Zealand cities the factor scores have been mapped first on a lineal scale and secondly in vector terminology. The lineal mapping defines the type of description accounted for by the variation in the 22 variables and is shown for the 18 cities. Most significant, however, is the ability of the factor models to generate a pattern of association from amongst the variables for individual cities. Analysis of the variables in terms of highest correlation with each of the factors describes patterns of association consistent with known city characteristics. Factor scores for individual cities and derived from the computed multivariate factors, demonstrate both positive and negative associations between the variables. Interpreted in conjunction with high and low variable values and the patterns of associations between factors and variables a refined composite explanation of the relationships between city characteristics is possible. A close examination of both factor scores and the dominant variables in the factors reveals a correlation between patterns of association amongst the variables and extreme variable values. In particular, individual factor scores for each city reflects not only the patterns of association, but also specific variable value patterns of high positive associations in the

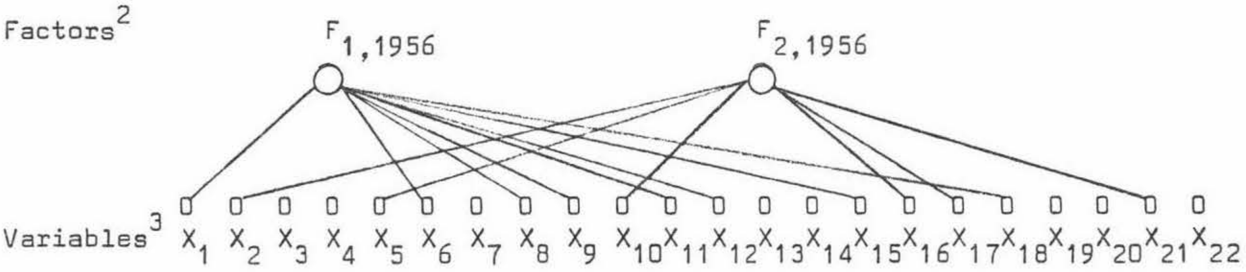
FIGURE 6

New Zealand Cities - 2-Factor Varimax Models[1]

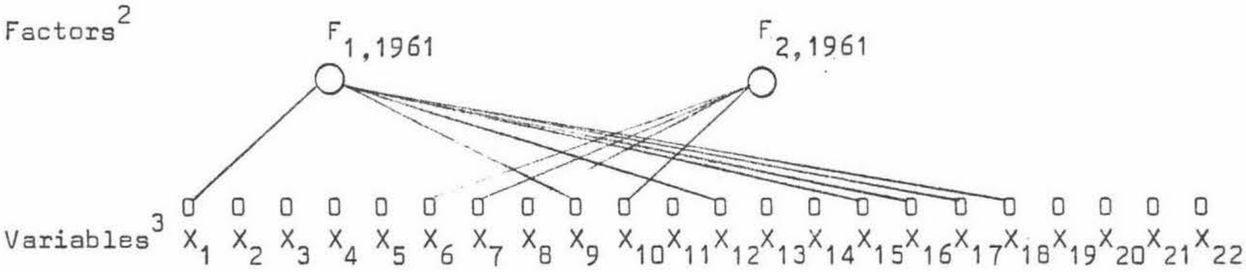
I 2-Factor Model 1951



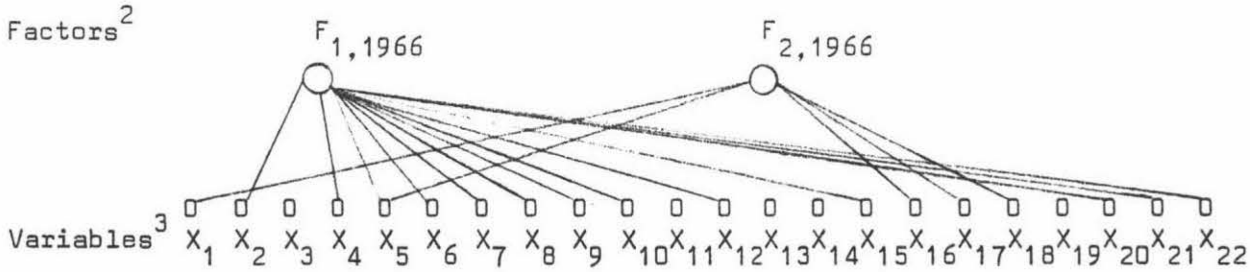
II 2-Factor Model 1956



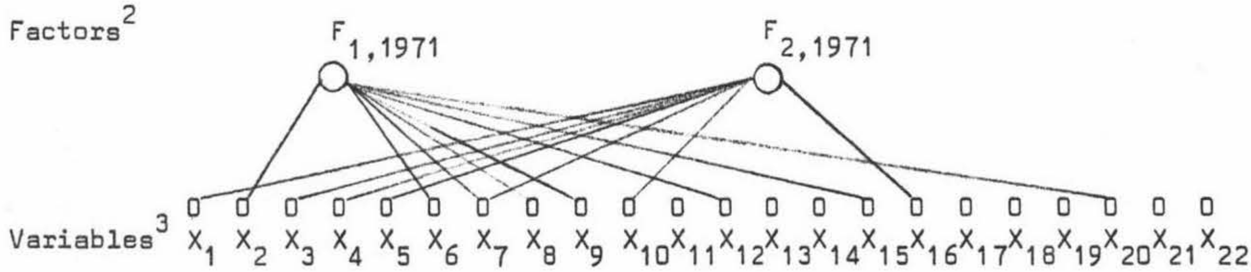
III 2-Factor Model 1961



IV 2-Factor Model 1966



V 2-Factor Model 1971



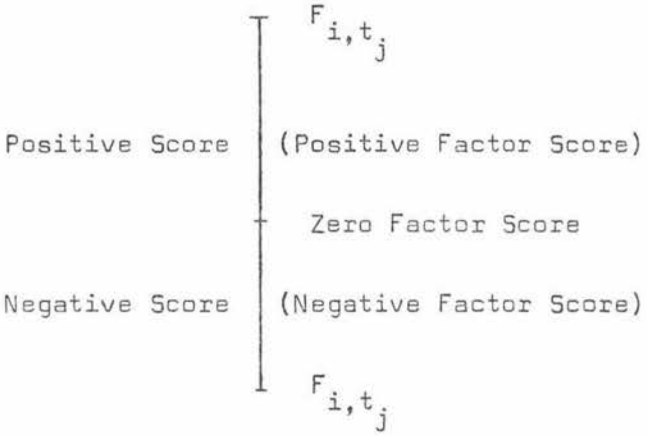
A Schematic representation of the relationships between the variables and the factors. Correlations of $\geqslant \pm 0.50$ are identified.

[1] Normal Data Distribution.

[2] Complete factor descriptions are given in Table 9.

[3] Variables may be identified using Table 3 as a reference.

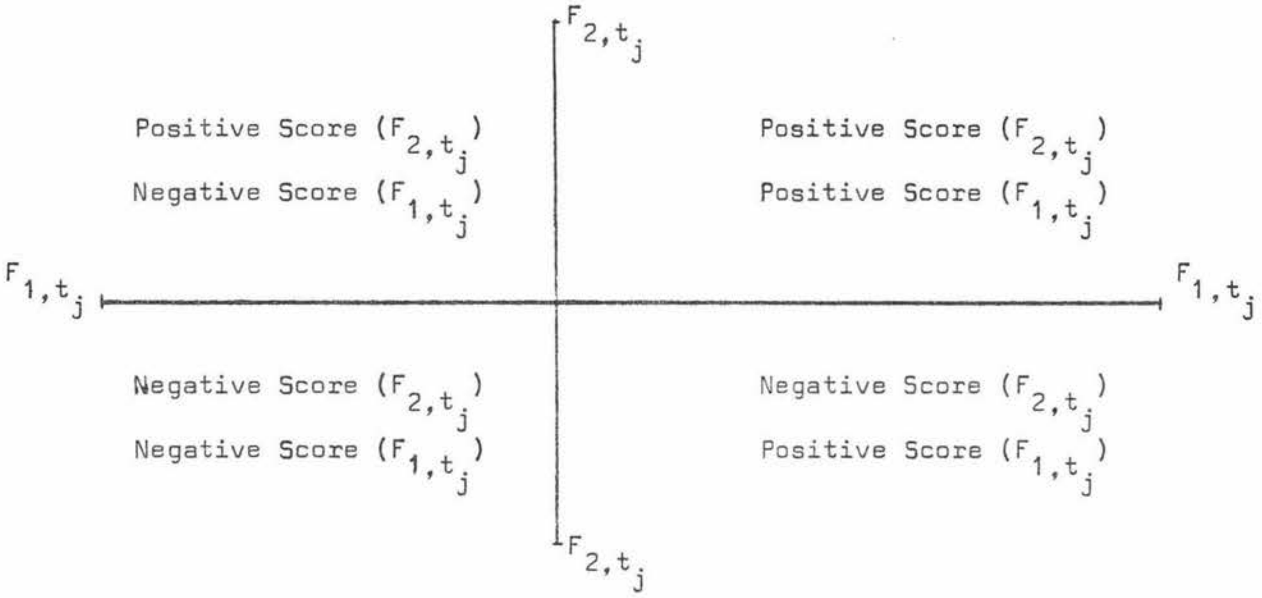
factors variables approximated to the highest variable values, while patterns of negative associations in the factors variables approximated to the smallest variable values. This relationship was clearly demonstrated in those cities with extreme factor scores.



Example of an Array of Factor Scores for Factor F_{i,t_j}
(Linear Diagram)

Mapped in a lineal format it can be demonstrated that the large positive Factor scores, described hereafter as Positive Scores, arise from a particular pattern of values of the variables. On the other hand, the large negative factor scores, described hereafter as Negative Scores, arise from the same pattern of values of the variables, but with reversed magnitudes. It is therefore now possible to describe two extreme groups of cities in terms of two main patterns of values of individual variables. Other cities, with near-zero factor scores, have various patterns of variable values.

Mapped in vector notation a much more comprehensive city description is obtained. Using the factors as reference axis, particular combinations of variable association can be seen to demonstrate a typology. Partitioned into quadrants, in the case of a 2-Factor analysis, Negative and Positive Scores, established from negative and positive factor scores for each city can be used in showing combinations of patterns of variable values in terms of the factors.



Example of an Array of Factor Scores for a
2-Factor Model F_{1,t_j} and F_{2,t_j}
(Vector Diagram)

Implicit advantage in the vector approach to city description is an increase in refinement in portrayal of the patterns of relationships that exist between variables. Factor 1 provides a major proportion of the description of relationships in accounting for much of the variation as is shown through the communalities. Factor 2 provides an additional refinement in accounting for more of the variation. Hence a more comprehensive description is provided in combining the factors in a relationship such as that provided by the vector notation. Incorporated within the relationship is the factor orthogonality quality and therefore independence of factors obtained through the Varimax rotation. Such a feature is important in that the vector portrayal provides a unique description for each city. Further, the qualities of this uniqueness may now be established in terms of the combinations of patterns of associations with particular variables and the values of these variables.

To clarify the argument, two examples are given from the 2-Factor models (Tables 10 and 11, 18 and 19). A complete description of each of the remaining three 2-Factor models must be made from the diagrams (Tables 12, 13, 14, 15, 16 and 17).

Example 1. 2-Factor 1951 Model: Table 10 and 11 contains both linear and vector models portraying factor scores and patterns of association with variables for the 2-Factor model of New Zealand cities in 1951, i.e. $F_{1,1951}$ and $F_{2,1951}$. Only variables with loadings greater than ± 0.50 are identified in the patterns of associations described by the factors. $F_{1,1951}$ has a pattern of highest positive association with the variables; service industries, expenditure on dwellings, investment confidence and highest negative association with variables; total population, labour voting and primary industries.[1] The cities of Hamilton, Tauranga, Rotorua, Hastings, New Plymouth and Nelson with their positive factor scores can be seen to have high values of positively associated variables and small values of negatively association variables. On the other hand, the cities of Auckland, Hutt, Christchurch, Dunedin and possibly Wellington with negative factor scores have a reversed pattern of values of the variables namely high values of total population, labour vote, primary processing industry and small values of service industries, new dwellings and investment confidence. Hence it appears that extreme factor values generate a reasonable description of the pattern of variable values of many of the cities.

Similarly, the values of $F_{2,1951}$ prescribes a pattern of values of the variables which give a logical description of New Zealand cities in 1951.

In both instances, factor 1 and factor 2 give a relationship that is not only logically meaningful,but one which is definitive in terms of

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[1] A shortened terminology for the variables will be used from now on to reduce the length of variable terminology. A complete description is shown on Table 3. Tables 10 to 14 and the remaining Figures use the variable terminology X_i and again Table 3 gives the most concise description of the variable.

variable value and the patterns of variable association. The factors generate what seems to be a useful portrayal of each particular city through the factor scores. It is, however, important to note that the better descriptions are provided by the more extreme factor score values.

Mapped in vector notation a more logical and comprehensive pattern of association becomes apparent. Not only does the mapping highlight patterns of associations between both factors, but also there is the added ability to refine the model through the incorporation of relative variable value into the patterns of positive and negative scores. In each of the four quadrants a combination of the scores can be seen, i.e. upper right quadrant contains Positive scores from both factors 1 and 2 with cities in that quadrant being identified in terms of small values of the variables; total population, young age group, increase in total population (both in migration and natural increase), labour vote, gross capital value, primary industry and construction industry,and high values of the variables; working population, service industry, commerce industry, building dwellings and investment index. Similar descriptions may be obtained for the other three quadrants from the figure. The vector scores show that the cluster of cities of Auckland, Christchurch and Dunedin demonstrate a common pattern of values of the variables - large for total population, Labour voting and primary processing and small for commerce industry, service industry, building dwellings and investment index, i.e. Negative score $F_{1,1951}$ and zero score $F_{2,1951}$. Wellington and Hutt, on the other hand, appear to have greater affinity with Negative score $F_{1,1951}$ and Negative score $F_{2,1951}$ with the actual patterns of values to be seen in the association of the variables in the two factors. For example, they have relatively high total population, a large proportion of 0-14 year olds, low proportion of persons in the 15-64 year old sector of the population, large increases in population, from both natural increase as well as migration into the cities, a high labour vote, high gross capital value increases, a high proportion of primary processing, transportation and commerce industries and a low proportion of service and seasonal industries as well as low proportion of building dwellings and a low investment index. Similar descriptions can be obtained from the diagram for each of the other cities. Again, the significant feature is the ability of the factors to generate and describe what appears to be a useful description of some of New Zealand's cities in 1951.

Example 2. 2-Factor 1971 Model: Figures 18 and 19 outline a model similar to that derived in the previous example.

TABLE 10

New Zealand Cities - 2-Factor Varimax Linear Model 1951

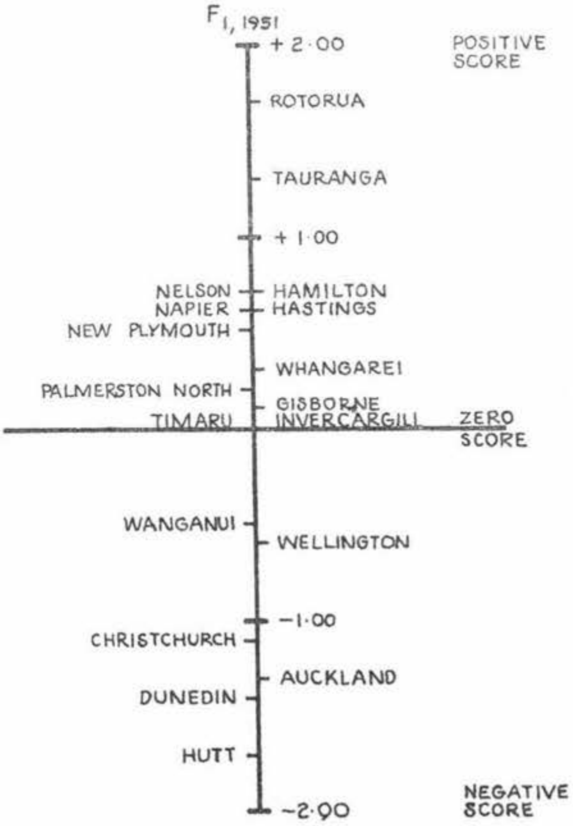
A. Linear Model*$F_{1,1951}$

a. Description

1. Positive Factor Score:-
    - i) High Value Variables: X_{18}, X_{21}, X_{22}
    - ii) Low Value Variables: X_1, X_9, X_{15}
2. Negative Factor Score:-
    - i) High Value Variables: X_1, X_9, X_{15}
    - ii) Low Value Variables: X_{18}, X_{21}, X_{22}

* Variables with loadings $\geqslant \pm 0.50$ in the factors are identified. Table 9 gives a complete description of the factors used to obtain the scores. Variable description is given in Table 3.

b. Factor Scores New Zealand Cities



B. Linear Model $F_{2,1951}$

a. Description

1. Positive Factor Score:
   - i) High Value Variables: X_3, X_{17}
   - ii) Low Value Variables: $X_2, X_6, X_7, X_8, X_{12}, X_{16}$
2. Negative Factor Score:
   - i) High Value Variables: $X_2, X_6, X_7, X_8, X_{12}, X_{16}$
   - ii) Low Value Variables: X_3, X_{17}

b. Factor Scores New Zealand Cities

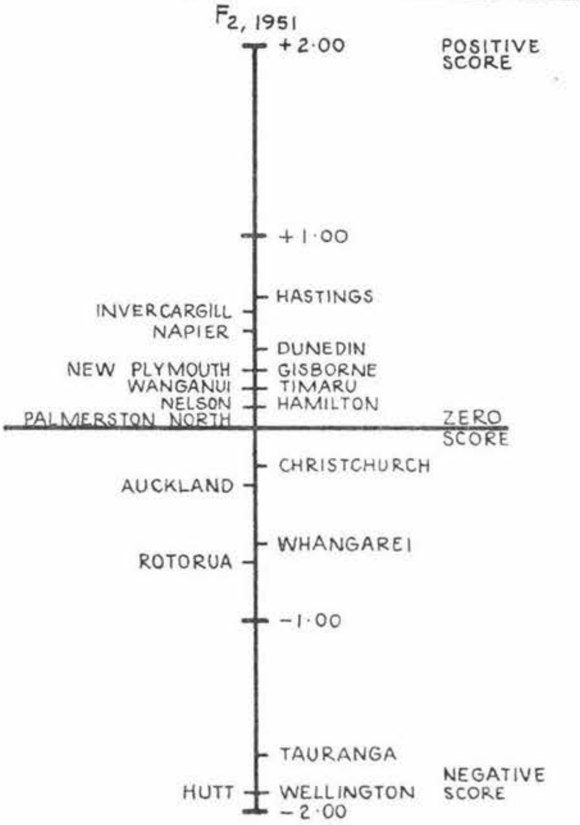


TABLE 11

New Zealand Cities - 2-Factor Varimax Vector Model* 1951

a. Description

$F_{2,1951}$
(Positive Factor Score)

Variable Value

High	Low
X_1	X_{18}
X_9	X_{21}
X_{15}	X_{22}
X_3	X_2X_8
X_{17}	X_6X_{12}
	X_7X_{16}

Variable Value

High	Low
X_{18}	X_1
X_{21}	X_9
X_{22}	X_{15}
X_3	X_2X_8
X_{17}	X_6X_{12}
	X_7X_{16}

$F_{1,1971}$ (Negative Factor Score) — (Positive Factor Score) $F_{1,1951}$

Variable Value

High	Low
X_1	X_{18}
X_9	X_{21}
X_{15}	X_{22}
X_2X_8	X_3
X_6X_{12}	X_{17}

Variable Value

High	Low
X_{18}	X_1
X_{21}	X_9
X_{22}	X_{15}
X_2X_8	X_3
X_6X_{12}	X_{17}

(Negative Factor Score)
$F_{2,1951}$

* Refer to Table 10 for further description.

b. Vector Model New Zealand Cities

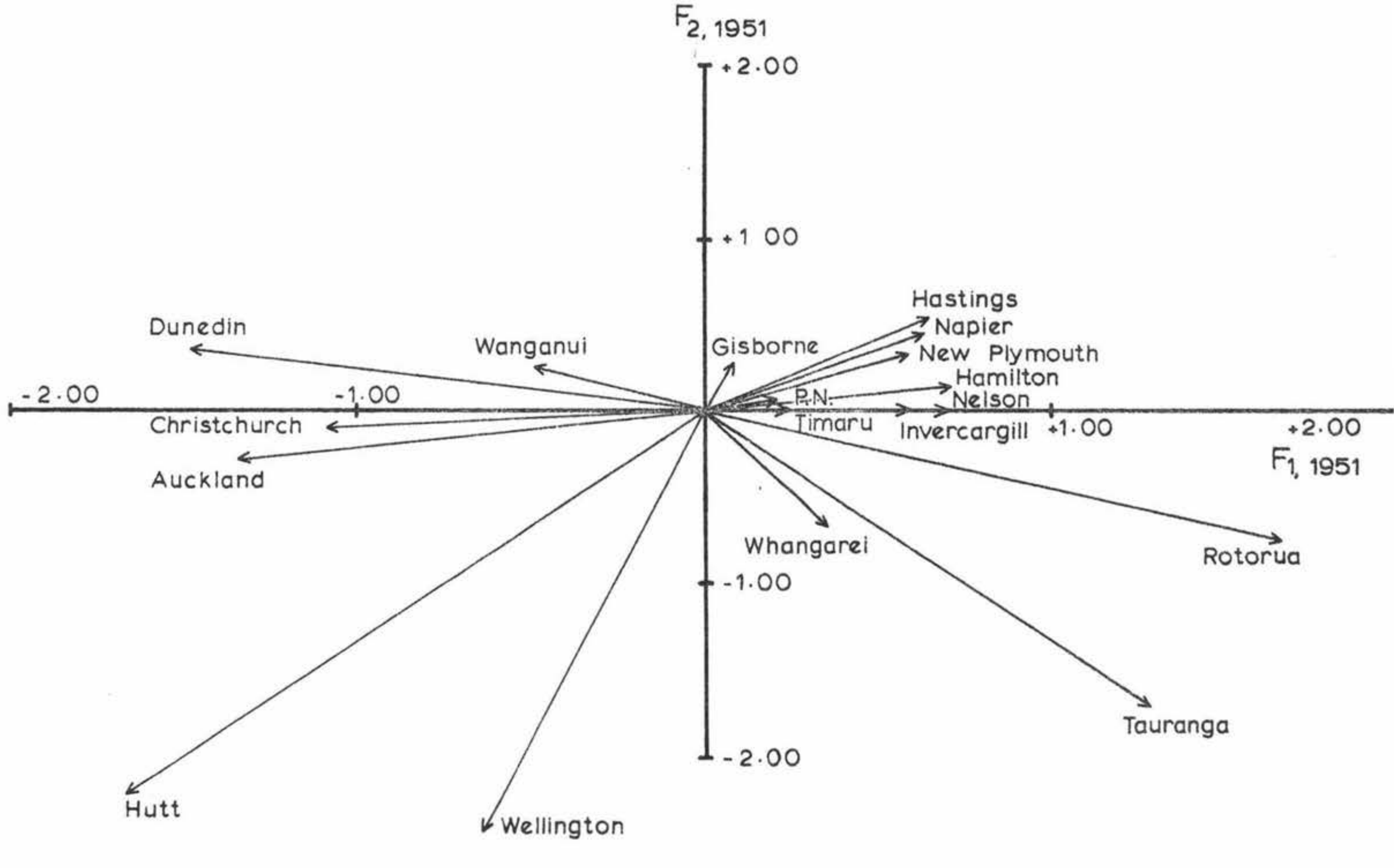


TABLE 12

New Zealand Cities - 2-Factor Varimax Linear Model 1956

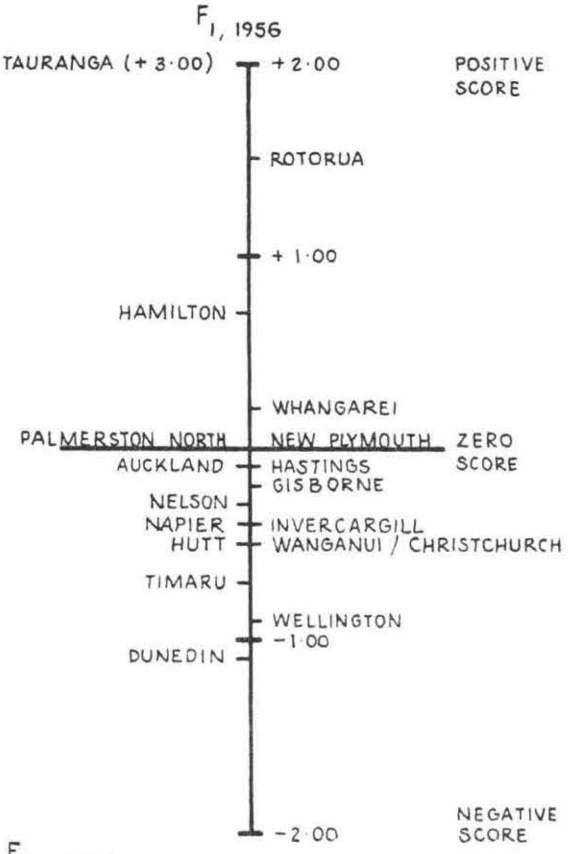
A. Linear Model* $F_{1,1956}$

a. Description

1. Positive Factor Score:-
   i) High Value Variables:
      X_1, X_9, X_{15}
   ii) Low Value Variables:
      $X_6, X_8, X_{11}, X_{12}, X_{18}$

2. Negative Factor Score:-
   i) High Value Variables:
      $X_6, X_8, X_{11}, X_{12}, X_{18}$
   ii) Low Value Variables:
      X_1, X_9, X_{15}

b. Factor Scores New Zealand Cities



B. Linear Model* $F_{2,1956}$

a. Description

1. Positive Factor Score:
   i) High Value Variables:
      $X_2, X_{10}, X_{16}, X_{22}$
   ii) Low Value Variables:
      X_5, X_{17}, X_{21}

2. Negative Factor Score:
   i) High Value Variables:
      X_5, X_{17}, X_{21}
   ii) Low Value Variables:
      $X_2, X_{10}, X_{16}, X_{22}$

* Variables with loadings $\geqslant \pm$ 0.50 in the factors are identified. Table 9 gives a complete description of the factors used to obtain the scores. Variable description is given in Table 3.

b. Factor Scores New Zealand Cities

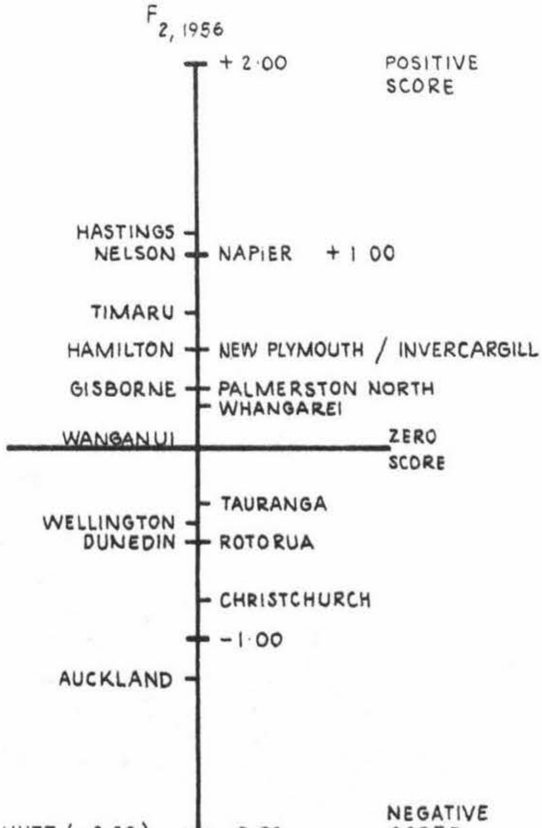


TABLE 13

New Zealand Cities - 2-Factor Varimax Vector Model 1956*

a. Description

$F_{2,1956}$
(Positive Factor Score)

Variable Value

High	Low
X_2X_{16}	X_5X_{21}
$X_{10}X_{22}$	X_{17}
X_6X_{12}	X_1X_{15}
X_8X_{18}	X_9
X_{11}	

Variable Value

High	Low
X_2X_{16}	X_5X_{21}
$X_{10}X_{22}$	X_{17}
X_1X_{15}	X_6X_{12}
X_9	X_8X_{18}
	X_{11}

$F_{1,1956}$ (Negative Factor Score) (Positive Factor Score) $F_{1,1956}$

Variable Value

High	Low
X_5X_{21}	X_2X_{16}
X_{17}	$X_{10}X_{22}$
X_6X_{12}	X_1X_{15}
X_8X_{18}	X_9
X_{11}	

Variable Value

High	Low
X_5X_{21}	X_2X_{16}
X_{17}	$X_{10}X_{22}$
X_1X_{15}	X_6X_{12}
X_9	X_8X_{18}
	X_{11}

(Negative Factor Score)
$F_{2,1956}$

* Refer to Table 12 for further description.

b. Vector Model: New Zealand Cities

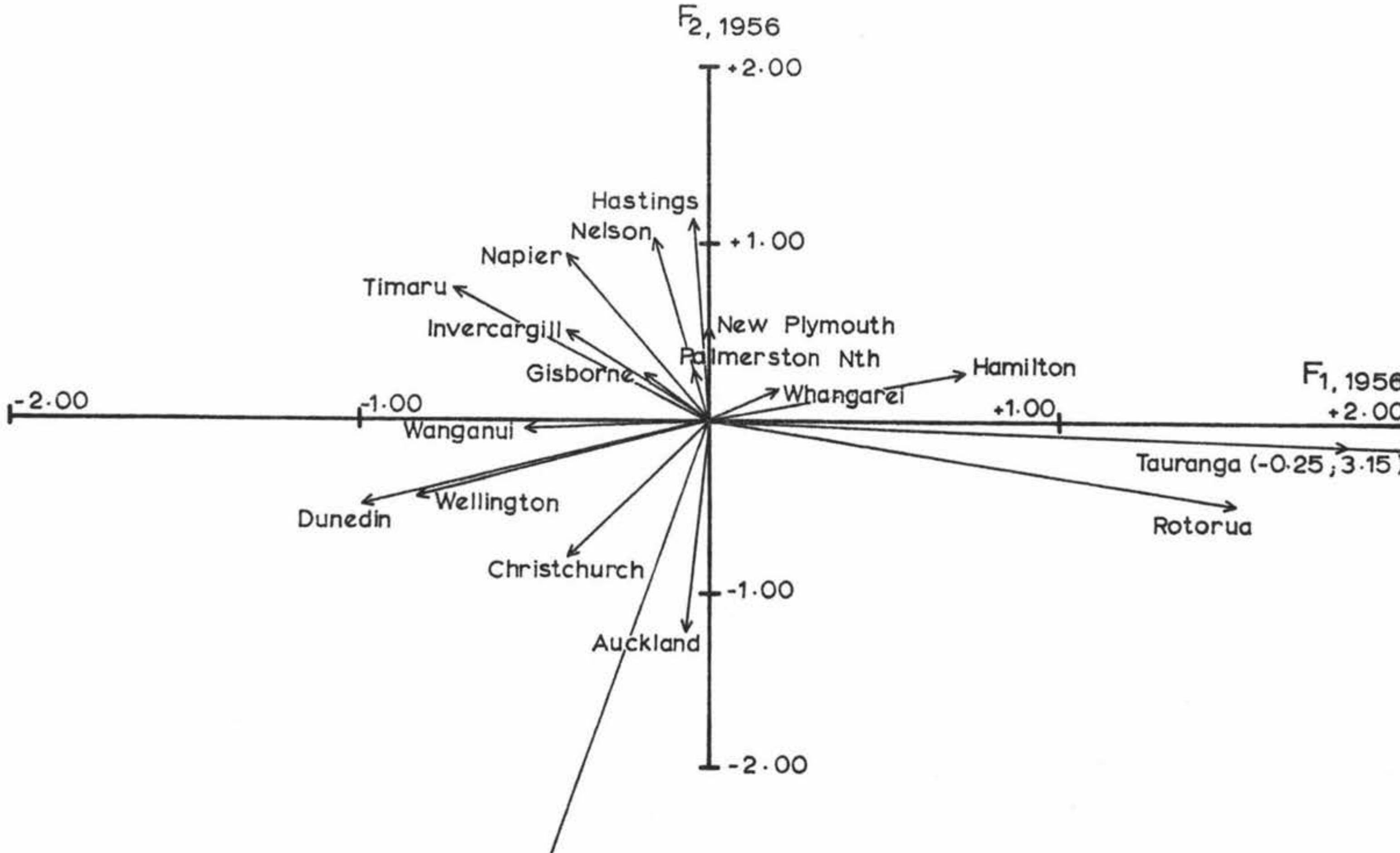


TABLE 14

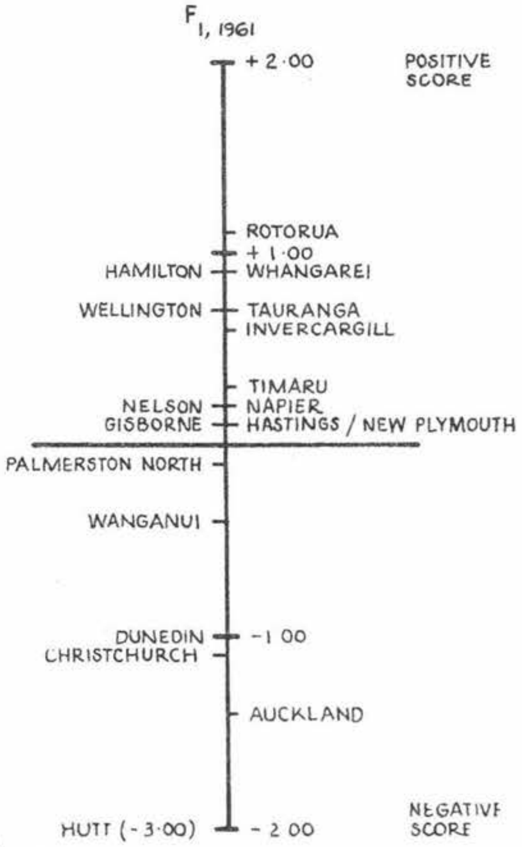
New Zealand Cities - 2-Factor Varimax Linear Model 1961

A. Linear Model $F^*_{1,1961}$

a. Description

1. Positive Factor Score:-
   i) High Value Variables:
      X_{12}, X_{17}, X_{18}
   ii) Low Value Variables:
      X_1, X_9, X_{15}, X_{16}
2. Negative Factor Score:-
   i) High Value Variables:
      X_1, X_9, X_{15}, X_{16}
   ii) Low Value Variables:
      X_{12}, X_{17}, X_{18}

b. Factor Scores New Zealand Cities



B. Linear Model $F^*_{2,1961}$

a. Description

1. Positive Factor Score:-
   i) High Value Variables:
      X_6, X_7, X_8, X_{10}
   ii) Low Value Variables:
      Nil
2. Negative Factor Score:-
   i) High Value Variables:
      Nil
   ii) Low Value Variables:
      X_6, X_7, X_8, X_{10}

* Variables with loadings $\geqslant \pm$ 0.50 in the factors are identified. Table 9 gives a complete description of the factors used to obtain the scores. Variable description is given in Table 3.

b. Factor Scores New Zealand Cities

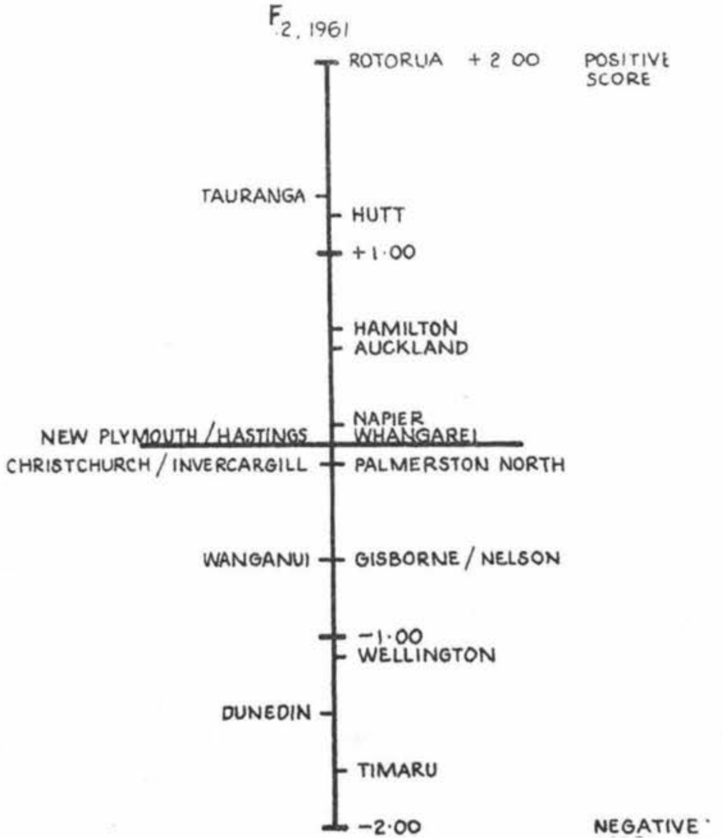
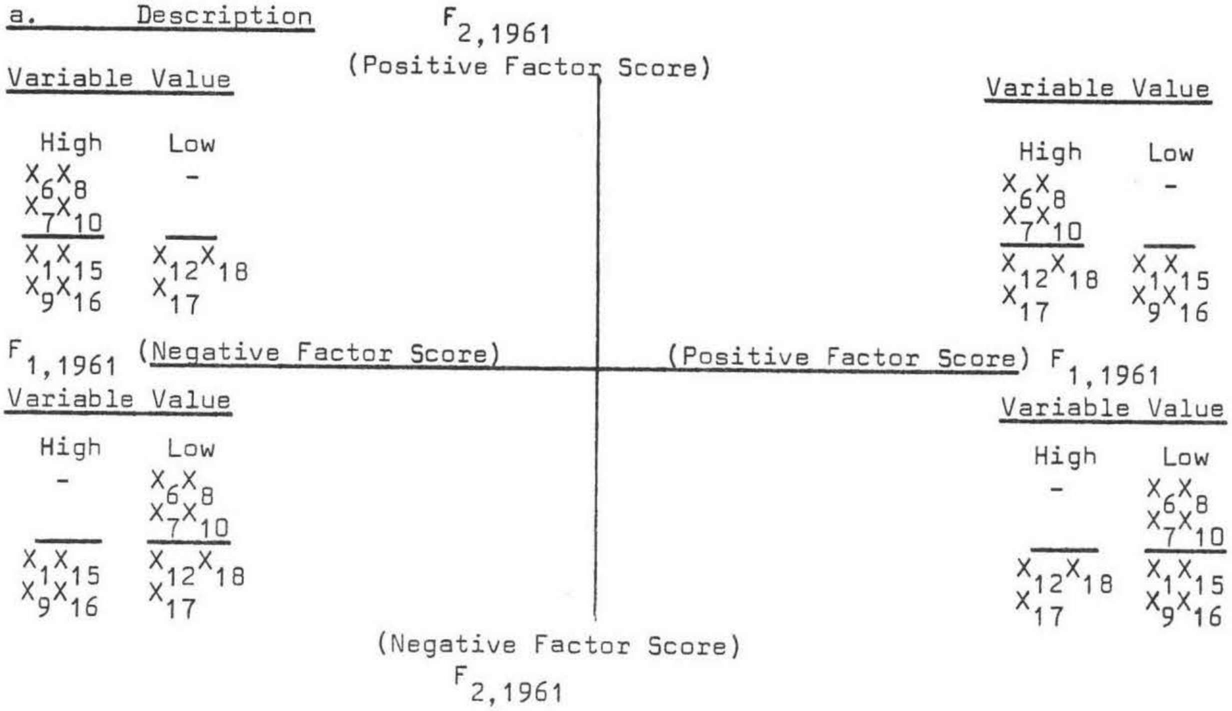


TABLE 15

New Zealand Cities - 2-Factor Varimax Vector Model 1961*

a. Description



* Refer to Table 14 for further description.

b. Vector Model: New Zealand Cities

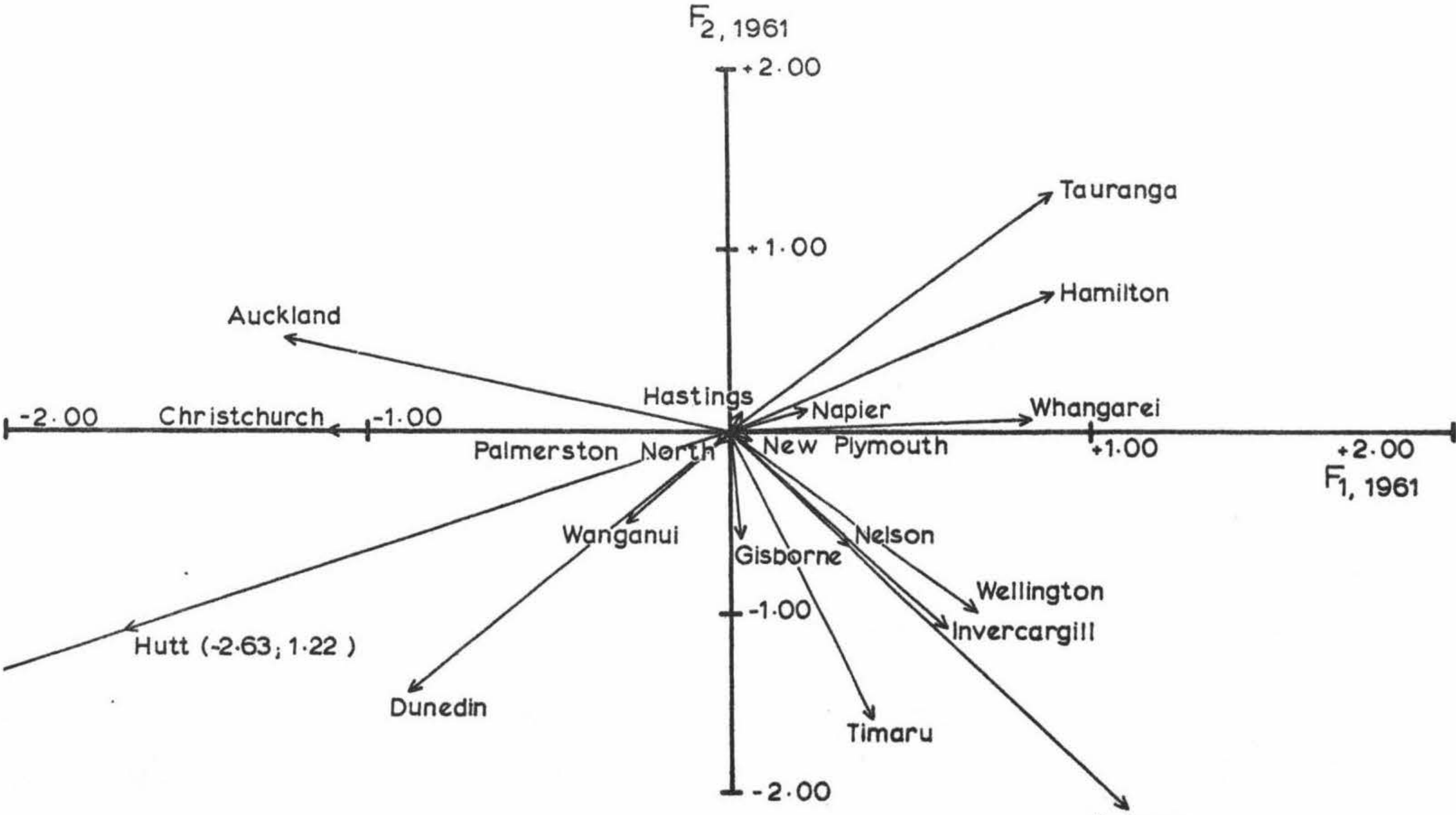


TABLE 16

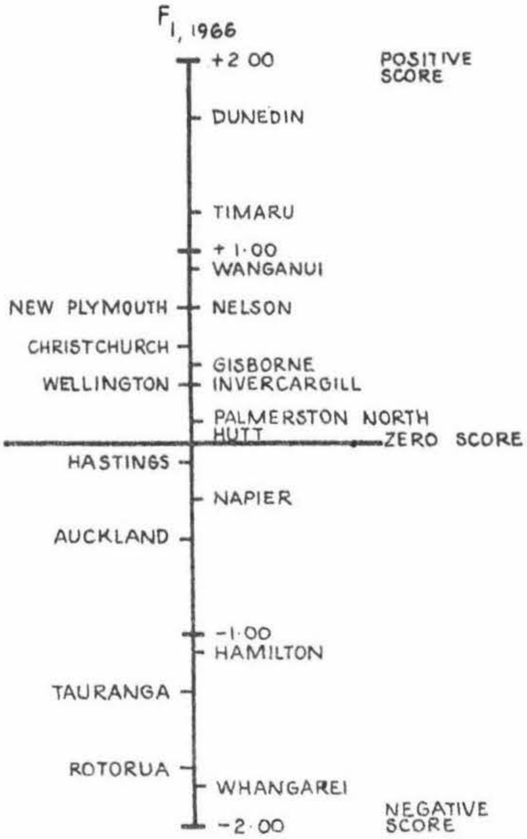
New Zealand Cities - 2-Factor Varimax Linear Model 1966

A. Linear Model $F^*_{1,1966}$

a. Description

1. Positive Factor Score:-
   - i) High Value Variables:
     $X_4, X_5, X_9, X_{15}, X_{20}, X_{22}$
   - ii) Low Value Variables:
     $X_2, X_6, X_7, X_8, X_{10}, X_{12}, X_{21}$
2. Negative Factor Score:-
   - i) High Value Variables:
     $X_2, X_6, X_7, X_8, X_{10}, X_{12}, X_{21}$
   - ii) Low Value Variables:
     $X_4, X_5, X_9, X_{15}, X_{20}, X_{22}$

b. Factor Scores New Zealand Cities



B. Linear Model $F^*_{2,1966}$

a. Description

1. Positive Factor Score:-
   - i) High Value Variables:
     X_1, X_{16}
   - ii) Low Value Variables:
     X_5, X_{18}, X_{19}
2. Negative Factor Score:-
   - i) High Value Variables:
     X_5, X_{18}, X_{19}
   - ii) Low Value Variables:
     X_1, X_{16}

* Variables with loadings > ± 0.50 in the factors are identified. Table 9 gives a complete description of the factors used to obtain the scores. Variable description is given in Table 3.

b. Factor Scores New Zealand Cities

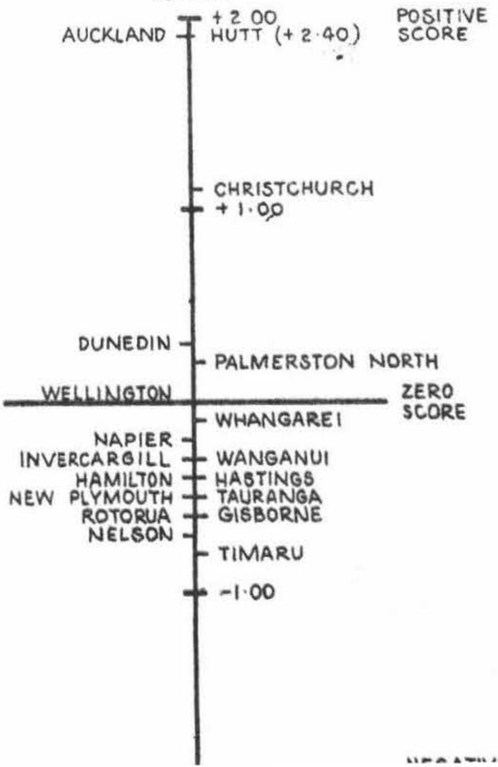


TABLE 17

New Zealand Cities - 2-Factor Varimax Vector Model 1966*

a. Description

$F_{2,1966}$

(Positive Factor Score)

Variable Value

High	Low
X_1	$X_5 X_{19}$
X_{16}	X_{18}
$X_2 X_{10}$	$X_4 X_{15}$
$X_6 X_{12}$	$X_5 X_{20}$
$X_7 X_{21}$	$X_9 X_{22}$
X_8	

Variable Value

High	Low
X_1	$X_5 X_{19}$
X_{16}	X_{18}
$X_4 X_{15}$	$X_2 X_{10}$
$X_5 X_{20}$	$X_6 X_{12}$
$X_9 X_{22}$	$X_7 X_{21}$
	X_8

$F_{1,1966}$ (Negative Factor Score) (Positive Factor Score) $F_{1,1966}$

Variable Value

High	Low
$X_5 X_{19}$	X_1
X_{18}	X_{16}
$X_2 X_{10}$	$X_4 X_{15}$
$X_6 X_{12}$	$X_5 X_{20}$
$X_7 X_{21}$	$X_9 X_{22}$
X_8	

Variable Value

High	Low
$X_5 X_{19}$	X_1
X_{18}	X_{16}
$X_4 X_{15}$	$X_2 X_{10}$
$X_5 X_{20}$	$X_6 X_{12}$
$X_9 X_{22}$	$X_7 X_{21}$
	X_8

(Negative Factor Score)

$F_{2,1966}$

* Refer to Table 14 for further description

b. Vector Model New Zealand Cities

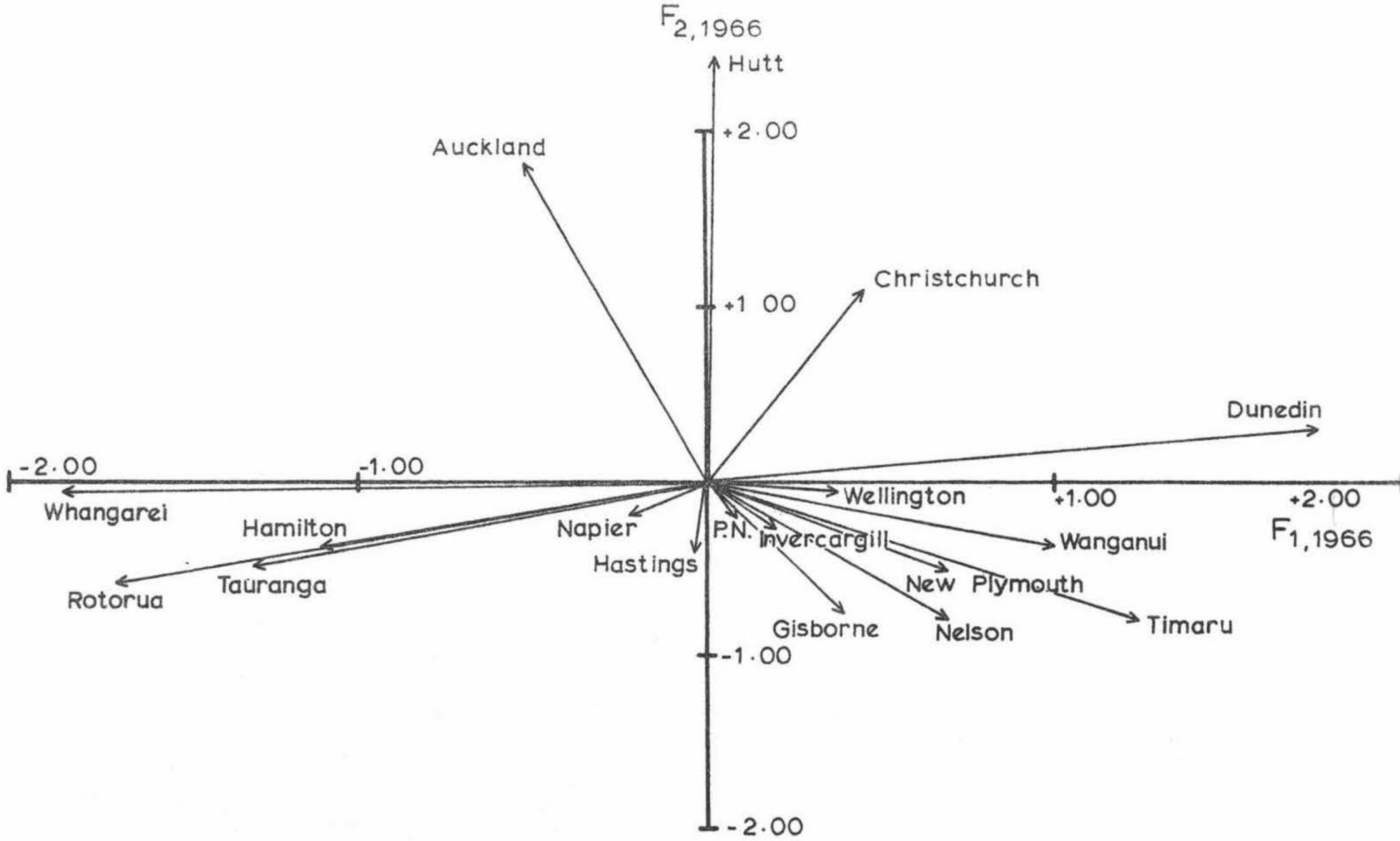


TABLE 18

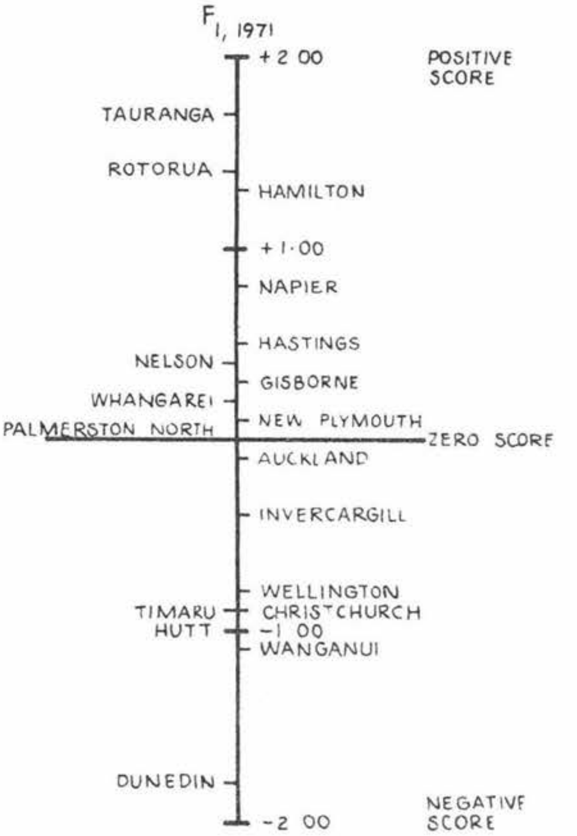
New Zealand Cities - 2-Factor Varimax Linear Model 1971

A. Linear Model*$F_{1,1971}$

a. Description

1. Positive Factor Score:-
   - i) High Value Variables: X_2, X_6, X_8, X_{12}
   - ii) Low Value Variables: X_9, X_{20}
2. Negative Factor Score:-
   - i) High Value Variables: X_9, X_{20}
   - ii) Low Value Variables: $X_2 . X_6 . X_8 , X_{12}$

b. Factor Scores New Zealand Cities



B. Linear Model*$F_{2,1971}$

a. Description

1. Positive Factor Score:-
   - i) High Value Variables: $X_1, X_3, X_7, X_{10}, X_{16}$
   - ii) Low Value Variables: X_4, X_5
2. Negative Factor Score:-
   - i) High Value Variables: X_4, X_5
   - ii) Low Value Variables: $X_1, X_3, X_7, X_{10}, X_{16}$

* Variables with loadings $\geq \pm$ 0.50 in the factors are identified. Table 9 gives a complete description of the factors used to obtain the scores. Variable description is given in Table 3.

b. Factor Scores New Zealand Cities

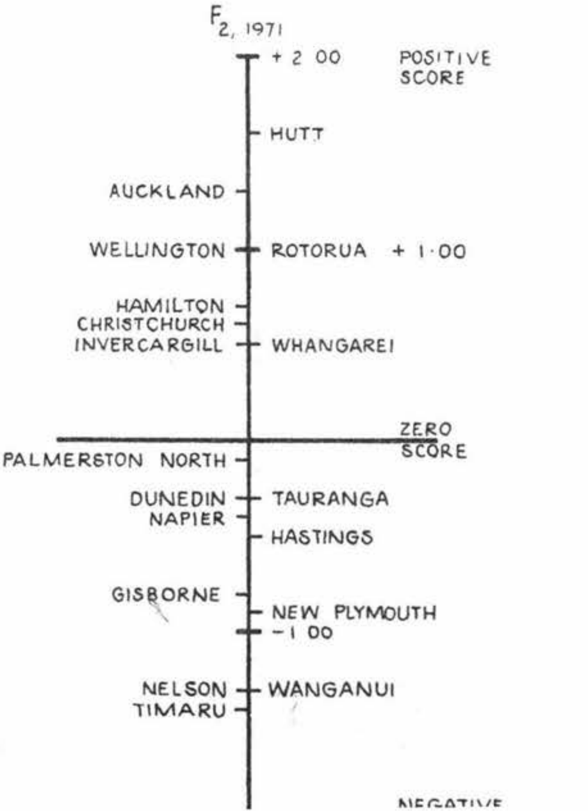


TABLE 19

New Zealand Cities - 2-Factor Varimax Vector Model 1971*

a. Description

$F_{2,1971}$
(Positive Factor Score)

Variable Value

High	Low
X_1X_{10} X_3X_{16} X_7	X_4 X_5
X_9 X_{20}	X_2X_8 X_6X_{12}

Variable Value

High	Low
X_1X_{10} X_3X_{16} X_7	X_4 X_5
X_2X_8 X_6X_{12}	X_9 X_{20}

$F_{1,1971}$ (Negative Factor Score) (Positive Factor Score) $F_{1,1971}$

Variable Value

High	Low
X_4 X_5	X_1X_{10} X_3X_{16} X_7
X_9 X_{20}	X_2X_8 X_6X_{12}

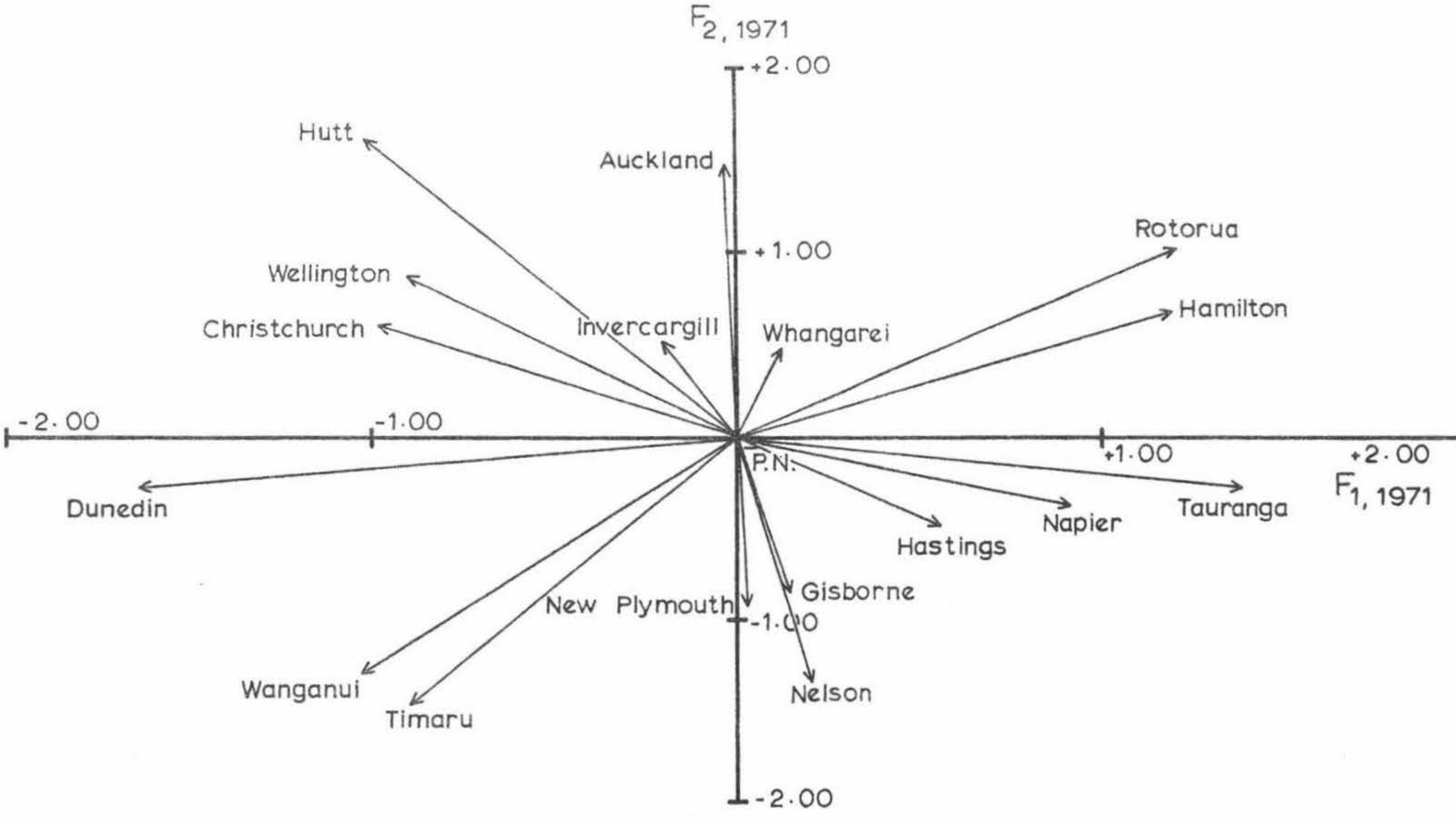
Variable Value

High	Low
X_4 X_5	X_1X_{10} X_3X_{16} X_7
X_2X_8 X_6X_{12}	X_9 X_{20}

(Negative Factor Score)
$F_{2,1971}$

* Refer to Table 18 for further description

b. Vector Model New Zealand Cities:



Factor scores are established for both the linear and the vector portrayals. $F_{1,1971}$ defines a positive pattern of association with 0-14 year olds, increase in population, particularly increases from in-migration, increase in gross capital value and negative pattern of association with the variables, labour vote and rating. The cities of Hamilton, Tauranga and Rotorua show high values of positively associated variables and low values of negatively associated variables. A reverse pattern of magnitudes of values realistically describes the cities of Wanganui, Hutt, Nelson, Christchurch, Timaru, Dunedin and Invercargill. $F_{2,1971}$ with a positive pattern of association of population, working age groups, natural increase in population, Maori population, construction industry and a negative pattern of association with the variables sex ratio, and the aged, defines in a similar way two groups of cities - Auckland, Hamilton, Rotorua, Hutt and Wellington with large positive factor scores, and Gisborne, Napier, Hastings, New Plymouth, Wanganui, Nelson and Timaru with large negative factor scores. Portrayed on a vector scale a more refined description is obtained. Rotorua, for instance, can be described in terms of large positive values of both factors and their respective variable values. Rotorua might generally be described to have a comparatively high population, a high increase in population from both natural increase and migration, many persons in the young dependent and working age groups, but few in the aged category, a low sex ratio, a non-Labour vote, a high Maori population and a high gross capital value increase, many persons employed in the construction industries, but a low rating. Such a description is consistent with actual variable values. Similar descriptions might be provided for each of the other 17 cities. The descriptions appear, in relationship to the basic values used to establish the model, to be realistic and logical.

From the analysis of the models it would appear that they are capable of generating a meaningful description of each of New Zealand's cities. While the models are static rather than dynamic, this description which combines many variables in a multivariate relationship does form from the pattern of association amongst the variable values a basis for planning. Such a description, while lacking a dynamic element in terms of assessment of direction or directions of change, does provide a description which can not only be used for descriptive policy, but also for comparative assessment of the relative values of variable in various cities. For example, the cities located in the lower right quadrant of the 2-Factor 1971 vector model all have labour force problems and problems associated with housing and the aged.

A set of 4-Factor models was also developed for each time slice. The attached table defines Kaiser Varimax orthogonal rotation solutions and the derived communalities (Table 20). A comparison between the 2-Factor and 4-Factor models shows that the 4-Factor model is merely a more refined 2-Factor description. Moreover, the most meaningful factors are to be seen in the first two factors rather than the latter two. The associations between each of the factors and individual variables in terms of factor scores for each city is schematically represented in Figure 7. Because of the complexity of demonstrating factor score representation and the need for the derivation of a simple factor model the 2-Factor model appears as the most acceptable form of factor analysis and the most suitable simple description of New Zealand's cities.

6. Multivariate Planning Model:-

A dynamic approach to factor modelling may be obtained through incorporating the time span within the construction. Normally such an inclusion means the loss of a dimension, either entities or characteristics. However, if entities are included with occasions a simple factor model can be reconstructed. This adjustment to the model is developed in relation to New Zealand's cities.

TABLE 20

New Zealand Cities - 4 Factor Varimax Model[1]

	1951					1956					1961				
Variable	$F_{1,1951}$	$F_{2,1951}$	$F_{3,1951}$	$F_{4,1951}$	h^2	$F_{1,1956}$	$F_{2,1956}$	$F_{3,1956}$	$F_{4,1956}$	h^2	$F_{1,1961}$	$F_{2,1961}$	$F_{3,1961}$	$F_{4,1961}$	h^2
1	(-.89)	.17	-.03	-.13	.85	.40	.47	.41	-.20	.59	-.23	(.52)	.33	-.17	.47
2	-.25	(-.86)	-.07	-.07	.81	-.13	.05	(.60)	.23	.43	(-.68)	.33	-.03	.06	.57
3	.22	(.69)	(.59)	-	.88	-	.18	(-.78)	.24	.70	(.95)	-.05	-.01	-.01	.90
4	-.03	.03	(-.98)	.11	.98	.05	-.24	.17	(-.74)	.65	(-.80)	-.39	.13	-.04	.81
5	(.52)	.44	(-.53)	.09	.75	.01	-.38	(-.71)	-.20	.69	-.08	(-.83)	-.13	-.27	.79
6	(.57)	(-.79)	-.07	-.02	.97	(-.97)	-.07	.07	.18	.98	.37	.15	(-.89)	.22	1.00
7	.18	(-.69)	.22	-.13	.58	-.26	-.15	.18	(.92)	.97	.34	.43	(-.51)	.37	.71
8	(.56)	(-.78)	-.13	.01	.93	(-1.00)	-.02	-	.02	1.00	.36	.01	(-.70)	.11	.63
9	(-.69)	.02	-.05	.05	.49	.38	(.52)	.22	.31	.57	-.10	.32	.45	-.37	.45
10	-.27	-.08	.14	(-.63)	.49	-.42	.07	.45	.12	.40	.08	.44	-.34	.12	.33
11	.05	-.43	.13	.08	.21	(-.73)	.02	-.06	-.02	.53	.16	-.03	.41	-.10	.20
12	.23	(-.63)	.39	.32	.71	(-.75)	-.20	.05	-.10	.62	.37	-.04	-.38	.41	.46
13	.22	.22	-.09	(-.96)	1.03	-.19	(-.71)	.43	-.06	.72	-.32	-.10	(-.54)	(.59)	.75
14	.18	-.25	.04	-.44	.29	.18	-.27	.14	.42	.30	-.03	.03	.44	-.11	.21
15	(-.85)	-.01	-.34	.10	.85	(.50)	(.53)	.48	-.29	.86	(-.65)	.33	.20	-.44	.76
16	-.31	(-.77)	-.01	.19	.73	-.23	(.63)	.41	.25	.67	-.21	(.77)	-.13	-.28	.74
17	.10	(.55)	.17	.27	.41	.20	(-.56)	-.29	-.48	.68	.02	(-.69)	.25	(.51)	.79
18	(.69)	.15	.18	(-.54)	.83	-.22	(-.87)	-.14	.23	.89	.30	-.32	-.39	(.63)	.75
19	.23	.06	.06	-.21	.10	-.07	(-.66)	.49	.12	.69	-.07	.28	-.18	(.77)	.70
20	-.28	-	-.25	.22	.19	.32	.08	-.14	-.17	.16	-.22	-.15	.35	.15	.22
21	(.67)	-.33	-.08	-.05	.57	-.43	-.01	(-.72)	-.24	.76	-.04	.09	.03	(-.73)	.55
22	(.75)	.18	-.13	-.07	.62	-.06	.02	(.68)	-.04	.47	.38	-.02	(.61)	.04	.51

h^2 = communality ( ) correlation coefficients ⩾ ± 0.50

[1] Normalised Data Distribution **N.B.** Appendix consists of 4-Factor Varimax Model for basic data

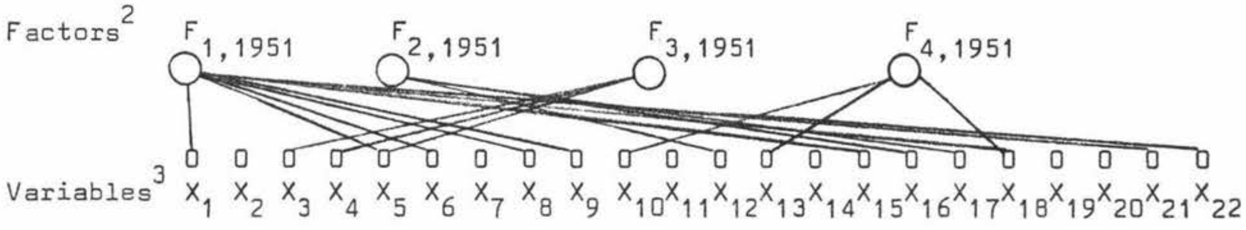
Table 20 (Contd.)

Variable	$F_{1,1966}$	$F_{2,1966}$	$F_{3,1966}$	$F_{4,1966}$	h^2	$F_{1,1971}$	$F_{2,1971}$	$F_{3,1971}$	$F_{4,1971}$	h^2
1	.35	.15	(.67)	(-.51)	.85	-.17	(.60)	.08	(-.56)	.71
2	-.31	-	.16	(.86)	.87	.68	.05	-.16	.57	.82
3	.08	.02	.37	(-.86)	.88	(-.50)	(.67)	.04	-.38	.85
4	.37	-.02	(-.76)	-.18	.76	-.19	(-.85)	.15	-.21	.82
5	.35	-.30	(-.67)	.14	.69	.06	(-.81)	.07	-.03	.67
6	(-.68)	.07	(.56)	.43	.97	.46	.36	(-.77)	.18	.97
7	-.49	-.11	(.68)	.47	.94	.38	(.72)	-.11	.34	.80
8	(-.71)	.12	.48	.39	.90	.37	.08	(-.82)	.04	.82
9	(.67)	.14	-.21	-.38	.66	.05	-.16	(.52)	-.39	.45
10	-.11	.03	(.78)	.18	.66	.20	(.67)	-.25	-.09	.56
11	-.05	.27	.31	-.06	.17	-.10	.16	.12	-.49	.29
12	(-.81)	.22	.09	.22	.76	.13	-.07	(-.88)	.14	.81
13	.16	(-.80)	.35	.11	.80	(-.53)	.37	-.22	.31	.57
14	.14	.24	-.30	(-.53)	.95	-.04	-.01	.08	(-.57)	.32
15	(.78)	.27	.05	-.21	.74	-.05	-.04	.31	(-.50)	.35
16	-.12	(.81)	.35	.08	.81	.32	(.54)	.08	-.09	.42
17	-.44	(-.61)	-.27	-.26	.70	(-.52)	-.20	(-.60)	.06	.68
18	-.38	(-.80)	-.12	.14	.83	-.40	.19	.09	(.69)	.68
19	-.41	-.10	.06	.05	.19	-.01	.29	-.47	.12	.32
20	.31	-.10	-.19	-.39	.30	(-.70)	-.35	.33	-.06	.72
21	(-.51)	.31	.10	.45	.57	(.55)	.09	-.28	.03	.39
22	(-.63)	.02	.33	.03	.50	.05	-.42	.05	.04	.18

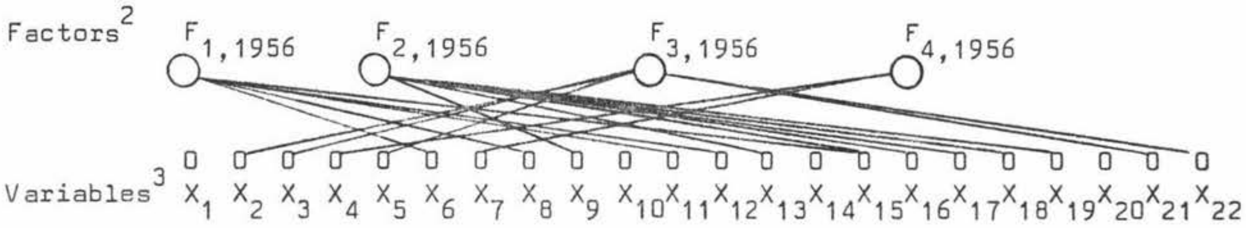
FIGURE 7

New Zealand Cities - 4-Factor Varimax Models[1]

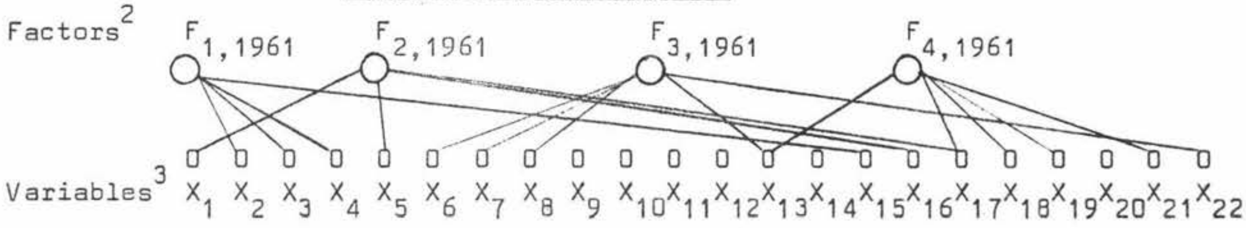
I 4-Factor Model 1951



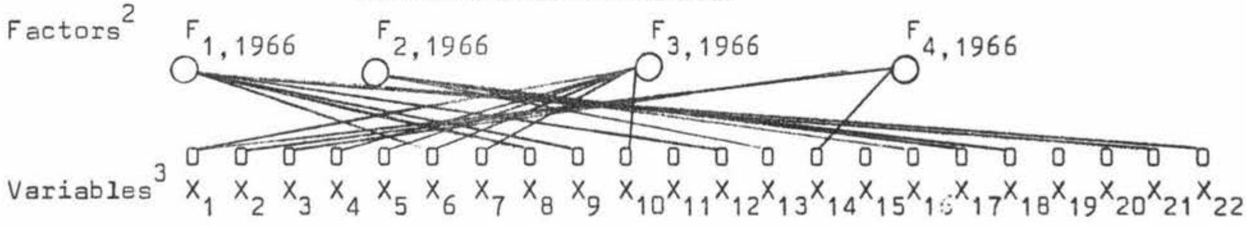
II 4-Factor Model 1956



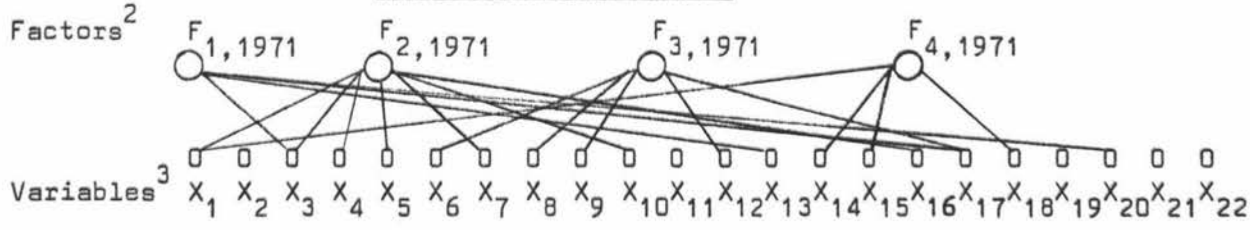
III 4-Factor Model 1961



IV 4-Factor Model 1966



V 4-Factor Model 1971



A Schematic representation of the relationships between the variables and the factors. Correlations of $\geqslant \pm 0.50$ are identified.

[1] Normal Data Distribution.

[2] Complete factor descriptions are given in Table 20.

[3] Variables may be identified using Table 3.

Consider instead of one city at one point in time, one city at different points in time, i.e. Auckland 1951, Auckland 1956, Auckland 1961, Auckland 1966 and Auckland 1971. Each city becomes a different entity at different points in time. From this mode of analysis it becomes possible to construct a factor model incorporating the elements of time. Such an incorporation is essential to establish trends and patterns of change. The correlation matrix for this model has already been described in a previous section under the nomenclature of a combined factor model based on transformed normally distributed data.

As in the last section a simple hypothesis of two and four linear factors was proposed. Similar models for the combined solution are outlined on the basis of a Varimax orthogonal rotation derivation, and are described in Tables 21 and 22 and Figures 8 and 9. $F_{1,1951-71}$ of the 2-Factor model has particularly high correlations with the variables associated with population change. There is also an association with changes in capital valuation and primary processing industrial activity, but this is to a much lesser degree than population change. It is significant to note that the variable combination highlighted by the factor is probably the most important aspect of urban change in New Zealand over the last 20 years. Post war migration of population began with movement from the countryside to the towns and cities, in the late fifties the shift in population distribution signified a movement from the small towns to the larger cities while in recent years the shift from the small cities to the large cities has been demonstrated in census studies.

$F_{2,1951-71}$ of the 2-Factor model does not display such a complete picture as shown by the earlier factor. There is, however, a linear relationship which has greatest affinity with Maori population and the dependent younger age group with some degree of association with the working segment of the population, the sex ratio rating and the building of dwellings.

Communality values for the 2-Factor model, however, indicate that there is a lack of completeness in factor description in association with particular variables. Hence the 4-Factor model was constructed and the communalities obtained demonstrate an improved assessment. This improvement is, however, gained at the expense of the simplicity of the 2-Factor model.

$F_{1,1951-71}$ of the 4-Factor model demonstrates a description relatively close to that of the first factor in the 2-Factor model. However, in this instance the variable associated with primary processing industries has diminished in importance and been replaced by variable X_{21}, dwellings. $F_{2,1951-71}$ in the 4-Factor model has, on the other hand, retained a description centralised upon the working section of the population and a negative association with the young dependent age group. Both variables X_2 and X_3 had relatively high relationships with factor II in the 2-Factor model. $F_{3,1951-71}$, on the other hand, is a little more complex and exhibits a pattern of high association with primary processing industry,construction industry, commerce and transport industries, service industries as well as a Labour vote. Not all associations are, however, positive, but the relationship appears to be a logical one. $F_{4',1951-71}$, the final factor in the 4-Factor model, has positive associations with the aged, the sex ratio and a negative association with the Maori population. Again this relation is logical and meaningful in terms of an expected relationship.

It is concluded that both factor models give a reasonable description of the variations in the 22 variables and can therefore be used with some confidence to portray patterns of urban change. The 4-Factor multivariate planning model has obviously the greater descriptive potential and from the communality estimates gives a much more refined description than that achieved in the 2-Factor model. However, because of the simplicity

TABLE 21

New Zealand Cities 1951-71
2-Factor Multivariate Planning Model (Varimax)[1]

		$F_{1,1951-71}$	$F_{2,1951-71}$	Communality (h^2)
Variable:-				
X_1	Population	-0.40	-0.41	0.33
X_2	0-14 age group	0.05	(-0.76)	0.59
X_3	15-64 age group	0.16	(0.57)	0.35
X_4	65+ age group	-0.40	0.25	0.22
X_5	sex ratio	-0.10	(0.66)	0.44
X_6	population increase	(0.97)	0.02	0.94
X_7	natural increase	(0.70)	0.05	0.49
X_8	movement	(0.90)	0.00	0.81
X_9	Labour vote	-0.49	0.08	0.24
X_{10}	Maori population	0.14	(-0.72)	0.54
X_{11}	increase in Maoris	0.25	-0.30	0.15
X_{12}	capital value increase	(0.57)	0.41	0.50
X_{13}	women in labour force	0.05	-0.44	0.20
X_{14}	primary industries	-0.04	0.20	0.04
X_{15}	primary processing industries	(-0.51)	-0.03	0.26
X_{16}	construction industries	0.27	-0.13	0.09
X_{17}	trading industries	-0.16	0.09	0.04
X_{18}	service industries	0.32	0.02	0.10
X_{19}	seasonal industries	-0.12	0.05	0.02
X_{20}	rating	-0.14	(0.59)	0.37
X_{21}	building dwellings	0.47	(0.58)	0.55
X_{22}	investment index	0.35	0.32	0.22

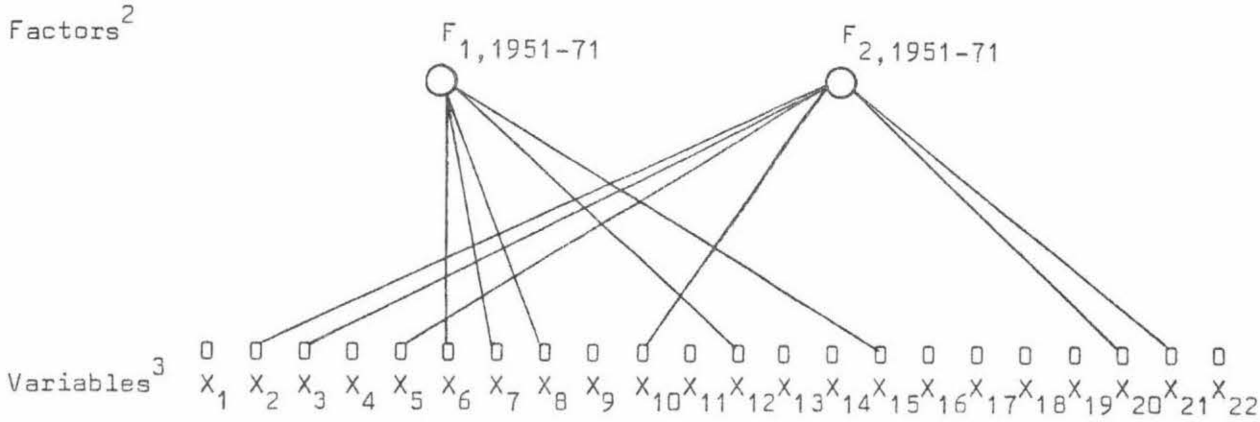
---

[1] Normal data distribution.

<u>N.B.</u> Appendix III consists of 2-Factor model for basic data.
A comparison may be made.

FIGURE 8

New Zealand Cities 1951-71 - 2-Factor Multivariate Planning Model (Varimax)[1]



A Schematic representation of the relationship between the variables and the factors. Correlations of $\geqslant \pm 0.50$ are identified.

---

[1] Normal Data Distribution.

[2] Complete factor descriptions are given in Table 21.

[3] Variables may be identified using Table 3 as a reference.

TABLE 22

New Zealand Cities 1951-71
4-Factor Multivariate Planning Model (Varimax)[1]

Variable:-		$F_{1,1951-71}$	$F_{2,1951-71}$	$F_{3,1951-71}$	$F_{4,1951-71}$	Communalities (h^2)
X_1	population	-0.49	-0.11	0.44	-0.42	0.62
X_2	0-14 age group	-0.02	(-0.86)	0.07	-0.16	0.77
X_3	15-64 age group	0.16	(0.95)	-0.10	-0.26	1.00
X_4	65+ age group	-0.26	-0.10	0.08	(0.74)	0.63
X_5	sex ratio	0.04	0.36	-0.24	(0.67)	0.64
X_6	population increase	(0.96)	-0.10	-0.01	-0.14	0.95
X_7	natural increase	(0.68)	0.09	0.06	-0.29	0.56
X_8	movement	(0.89)	-0.14	-0.06	-0.09	0.82
X_9	Labour vote	-0.42	0.20	(0.50)	0.03	0.47
X_{10}	Maori population	0.06	-0.44	-0.03	(-0.63)	0.59
X_{11}	increase in Maoris	0.20	-0.26	0.05	-0.20	0.15
X_{12}	capital value increase	(0.64)	0.29	0.00	-0.09	0.50
X_{13}	women in labour force	-0.12	-0.41	-0.48	-0.24	0.47
X_{14}	primary industries	0.01	0.19	0.11	0.07	0.05
X_{15}	primary processing industries	-0.42	0.02	(0.67)	0.10	0.64
X_{16}	construction industries	0.41	-0.09	(0.75)	-0.23	0.79
X_{17}	trading industries	-0.24	0.02	(-0.58)	0.19	0.44
X_{18}	service industries	0.22	-0.02	(-0.69)	-0.07	0.53
X_{19}	seasonal industries	-0.11	0.06	0.02	0.05	0.02
X_{20}	rating	0.00	0.48	0.15	0.34	0.38
X_{21}	building dwellings	(0.59)	0.41	0.09	0.24	0.59
X_{22}	investment index	0.40	0.21	-0.05	0.12	0.22

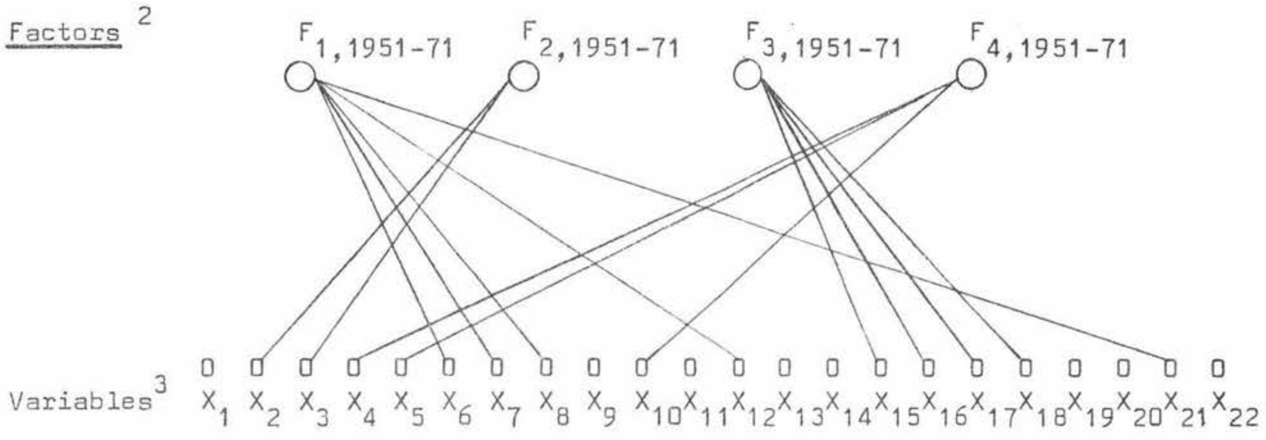
---

[1] Normal data distribution.

N.B. Appendix III consists of 4-Factor model for basic data.
A comparison may be made.

FIGURE 9

New Zealand Cities 1951-71 - 4-Factor Multivariate Planning Model (Varimax)[1]



A Schematic representation of the relationship between the variables and the factors. Correlations of ≥ ± 0.50 are identified.

---

[1] Normal Data Distribution.

[2] Complete factor descriptions are given in Table 22.

[3] Variables may be identified using Table 3 as a reference.

of the 2-Factor model and the fact that the first two factors of the 4-Factor model bear some similarity to the two factors of the 2-Factor model, it is developed further. Using factor scores derived from the 2-Factor model, simple vector models have been constructed for each city in New Zealand. Further, the vector models trace the pattern of changes that have taken place in each city over the 20-year study period in terms of intercensal quinquenniums.

Using the quadrant description derived from the factors and utilising the positive and negative scores combinations, the 2-Factor model becomes a powerful technique for analysing patterns of urban change. Each quadrant, described in Table 23, has a particular unique combination of Negative and Positive scores obtained from the variables. Quadrant II, for example, contains positive factor scores from Factor 1 and negative factor scores from Factor 2, i.e. high numbers of working age group population, a high sex ratio, primary processing industries and rating with low numbers of dependent youthful age group, low population increases in all categories as well as a small Maori population and small increase in capital value. The converse description is equally applicable to Quadrant IV with there being a high number of Maoris, many children, a large population increase from both natural increase and movement, few primary industries and low rating amongst other things. Similar complimentary descriptions apply to Quadrants I and III.

The following examples will illustrate the particular usefulness of the vector diagrams which have been constructed for the 18 cities of New Zealand (Figures 10, 11, 12, 13 and 14).

Example 1. Auckland The patterns of change as portrayed by the vector diagram which illustrates Auckland's growth, defines the considerable and consistent affinity the area has with the negative factor scores of

Factor 2. Almost all the vectors, particularly the most recent, 1971, tend towards the negative scores of Factor 2 with Factor 1 playing only a minor role in description, although some tendencies can be seen in this area in 1956 and 1966. Using the quadrant scheme it may be said that Auckland has the following characteristics - high number of young dependents, a low proportion or even deficiency in population in the working age group, a low urban sex ratio, a high Maori population and a low average rating. It might be reflected that percentage changes in both population and values would not necessarily show up in the Auckland area as a consequence of the very large basic population on which a per cent would be constructed. On the whole, this particular general description of Auckland might be considered as reasonable. Any changes that have taken place in the area are highlighted by the location of the vectors for 1956, 1961 and 1966 in Quadrant IV and the implicit associations with that sector's derived characteristics.

Example 2. Hamilton The model defines a clear pattern of change in Hamilton over the 1951-1971 period. The 1951 vector located in Quadrant I midway between the two positive factor areas defines a city with the following characteristics - low youthful population, many persons in the working age group, a high sex ratio, considerable population increase both natural and from movement, a low Maori population, few primary processing industries, but a high increase in capital value and a high rating. By 1966 a similar location midway in Quadrant IV was reached. As a result an implicit change in description had taken place with there now existing a high number of dependent young, a deficit in working age group, a low sex ratio, a high Maori population, but a low rating. A continued pattern of high increase in both population and in gross capital value can be seen. In 1971, however, the trend towards a negative score for Factor 2 continues and coupled with the reduced score for Factor 1, it might be postulated with some justification that Hamilton will by 1976 have assumed a factor description similar to that of Auckland.

TABLE 23

New Zealand Cities 1951-71

2-Factor Multivariate Planning Model (Varimax)[1]

Model[2] I $F_{1,1951-71}$ = 0.97(population increase) + 0.70(natural increase) + 0.90 (movement) + 0.57(capital value increase) - 0.51(primary processing industries).

Model[2] II $F_{2,1951-71}$ = 0.76(0-14 age group) + 0.57(15-64 age group) + 0.66(sex ratio) - 0.72(Maori population) + 0.59(rating).

Quadrant Vector Model

$F_{2,1951-71}$
(Positive Factor Score)

Quadrant II

Variable Value	Variable
Low	0-14 age group (X_2) Maori population (X_{10})
High	15-64 age group (X_3) sex ratio (X_5) rating (X_{20})
Low	population increase (X_6) natural increase (X_7) movement (X_8) capital value increase (X_{12})
High	primary processing industries (X_{15})

Quadrant I

Variable Value	Variable
Low	0-14 age group (X_2) Maori population(X_{10})
High	15-64 age group (X_3) sex ratio (X_5) rating (X_{20})
High	population increase (X_6) natural increase (X_7) movement (X_8) capital value increase (X_{12})
Low	primary processing industries (X_{15})

$F_{1,1951-71}$
(Negative Factor Score)

$F_{1,1951-71}$
(Positive Factor Score)

Quadrant III

Variable Value	Variable
High	0-14 age group (X_2) Maori population (X_{10})
Low	15-64 age group (X_3) sex ratio (X_5) rating (X_{20})
Low	population increase (X_6) natural increase (X_7) movement (X_8) capital value increase (X_{12})
High	primary processing industries (X_{15})

Quadrant IV

Variable Value	Variable
High	0-14 age group (X_2) Maori population(X_{10})
Low	15-64 age group (X_3) sex ratio (X_5) rating (X_{20})
High	population increase (X_6) natural increase (X_7) movement (X_8) capital value increase (X_{12})
Low	primary processing industries (X_{15})

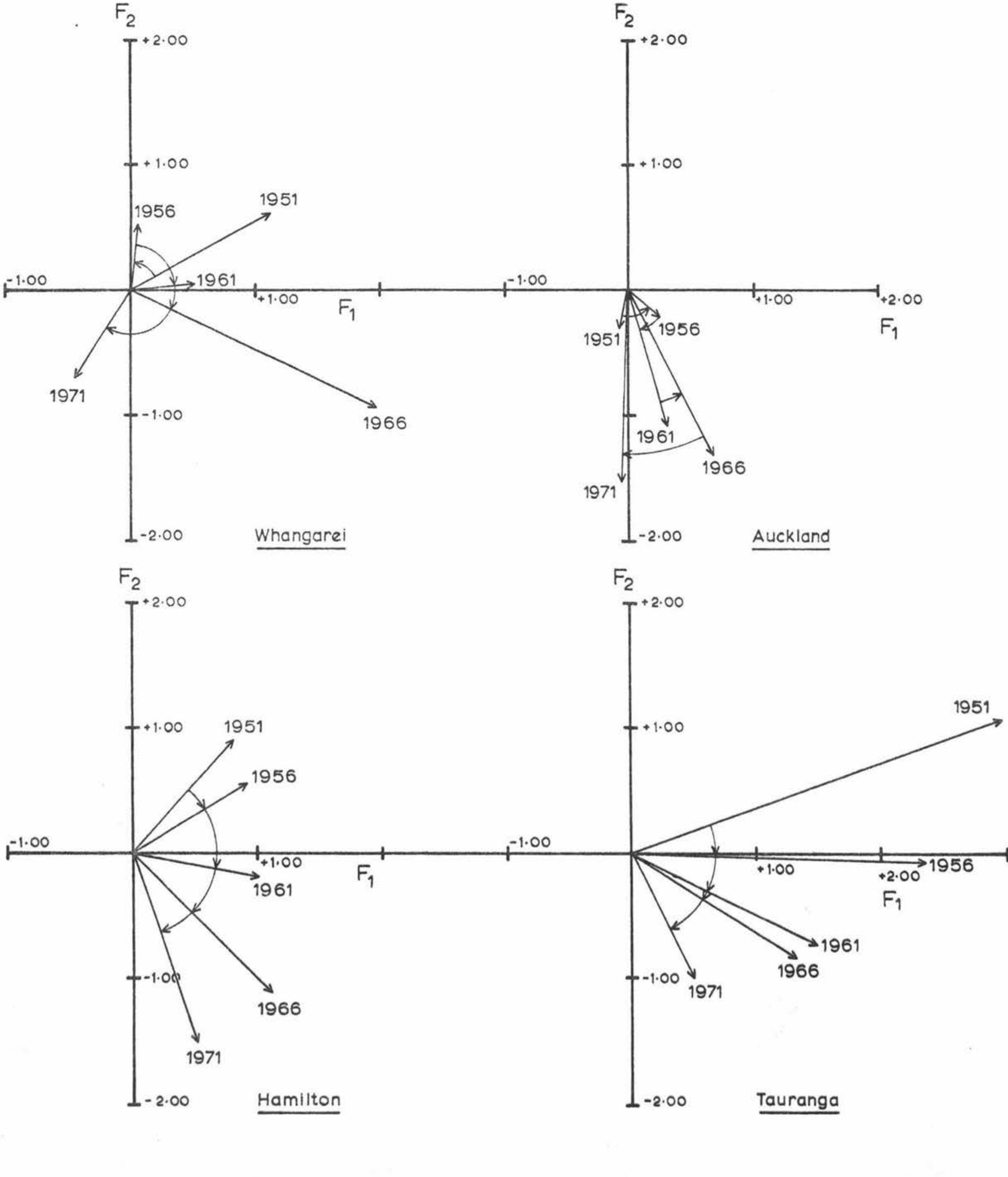
$F_{2,1951-71}$
(Negative Factor Score)

1 Normal data distribution.

2 Only variables with loadings $\geqslant \pm 0.50$ have been identified in the model. A more complete description with communalities can be seen on Table 21. The following figures are based on factor scores from the factors using all 22 var-

FIGURE 10

New Zealand Cities 1951-71 - 2-Factor Varimax Model



Example 3. Timaru While the 2-Factor model defines a pattern of change showing a general tendency towards a negative score from Factor 2 description in Hamilton, the trend in Timaru appears to show a systematic movement from a positive score Factor 2 in 1951 to a negative score Factor I description in 1971. Again, it could be postulated that by 1976 Timaru will be described solely by Factor I. In the intervening years of 1956, 1961 and 1966, while the vector movement swings through 90 degrees over the full period Timaru is described by the model as having few persons of youthful age, a high working age group population, a high sex ratio, but low population increase values, few Maoris, a low increase in capital value but a high rating and primary processing industries.

Similar descriptions might be obtained for the other 15 cities. It can be seen that by the introduction of a dynamic factor, the element of time, the model achieves more than a descriptive status. In some instances it is possible to postulate a future pattern of change and to provide a description of the likely future patterns of association of the variables with each city. While the description is crude, in that it is only a generalisation, it none-the-less is a description which provides a quantitative base for planning policy previously based on a consideration of population only. A more refined description might be obtained from a re-selection of variables on the basis of the experience of this investigation. Further investigation might relate vector length with degree of description, and the problems of near zero factor score descriptions (see Nelson City).

While the 2-Factor multivariate planning model may provide a crude description of the patterns of change, the 4-Factor multivariate planning model provides a very complex description. To illustrate the complexity of the description provided by the 4-Factor model, an example has been constructed from the factor scores (Table 24). The quadrant diagrams

FIGURE 11

<u>New Zealand Cities 1951-71 - 2-Factor Varimax Model</u>

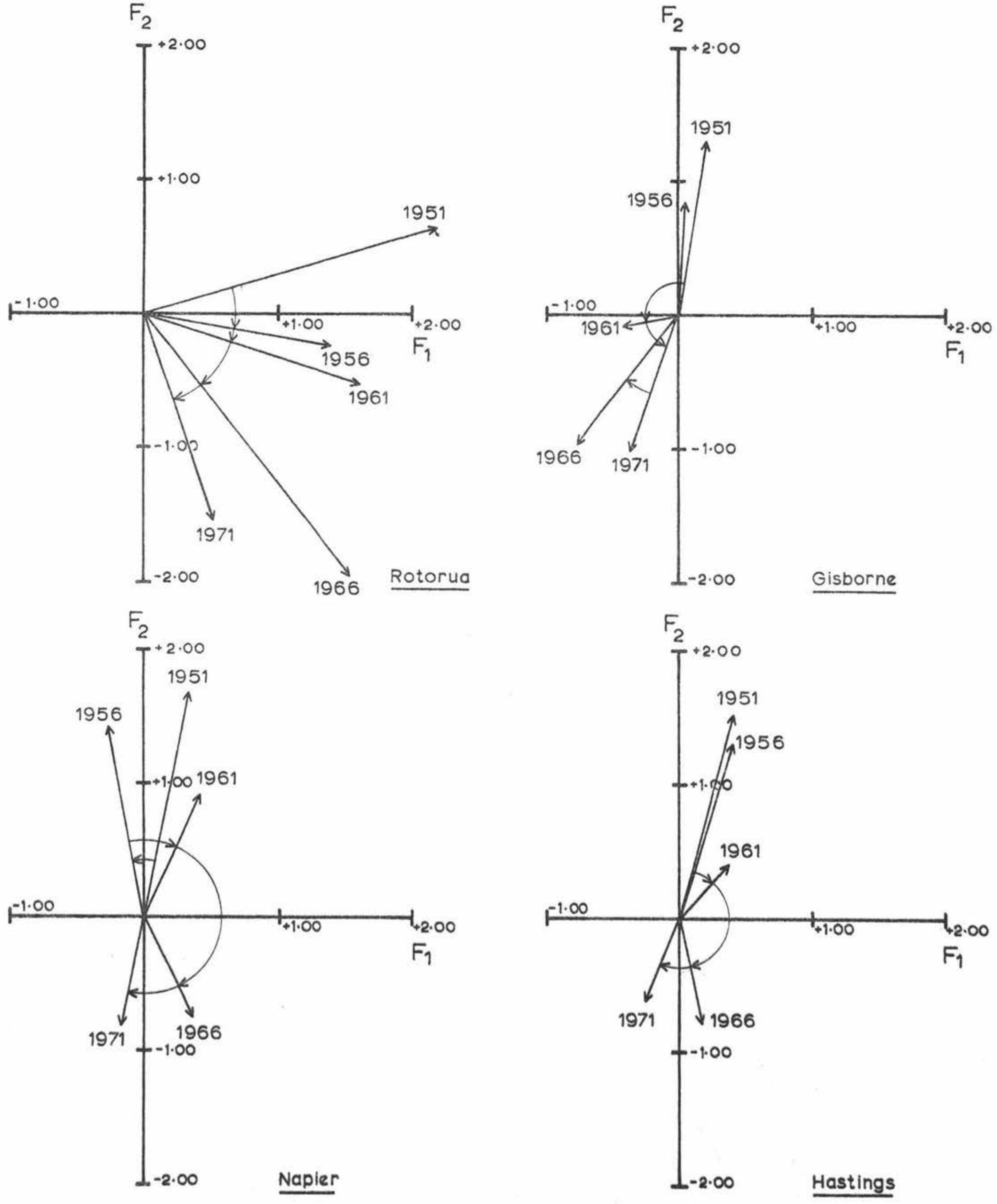


FIGURE 12

New Zealand Cities 1951-1971 - 2-Factor Varimax Model

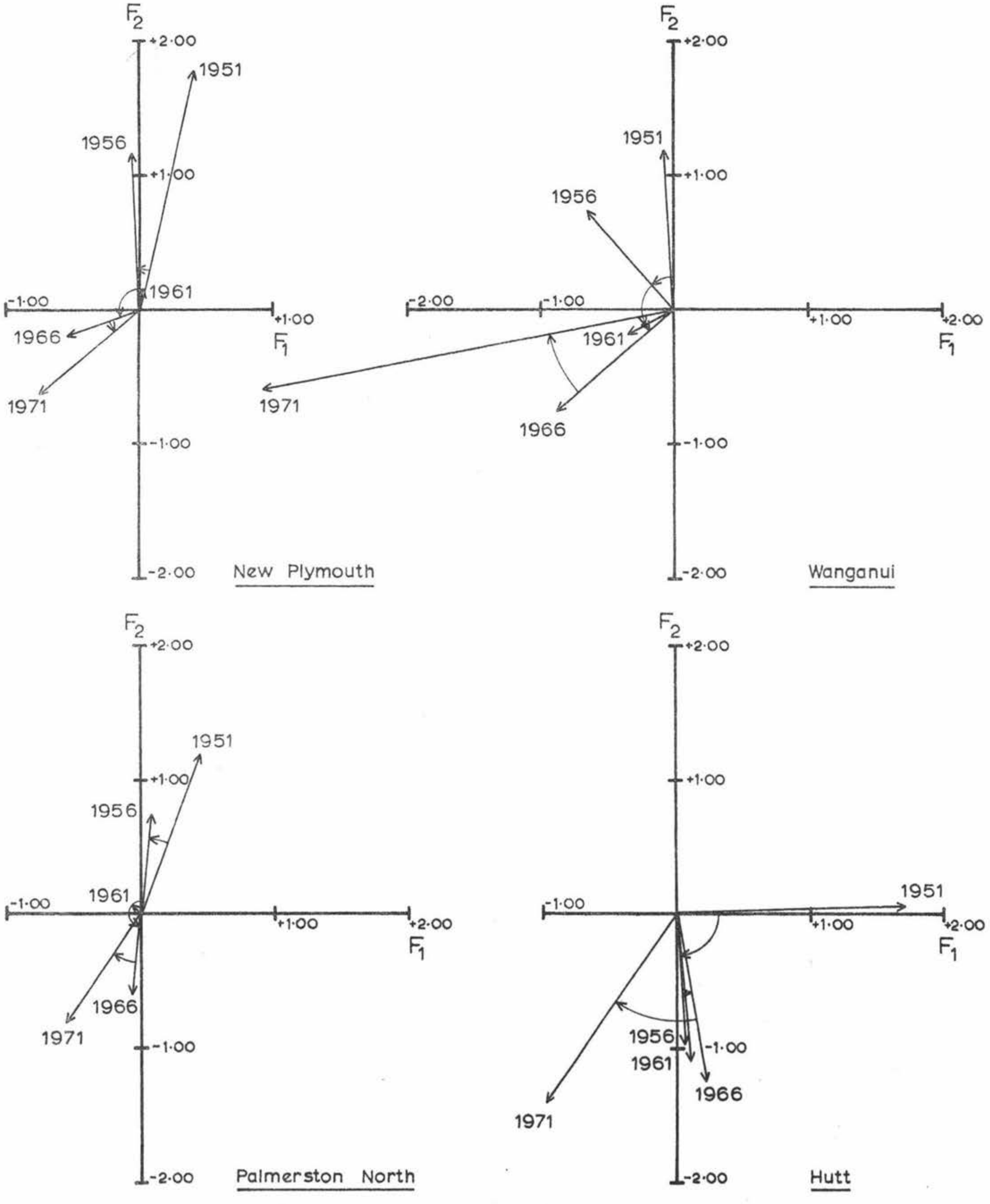


FIGURE 13

New Zealand Cities 1951-1971 - 2-Factor Varimax Model

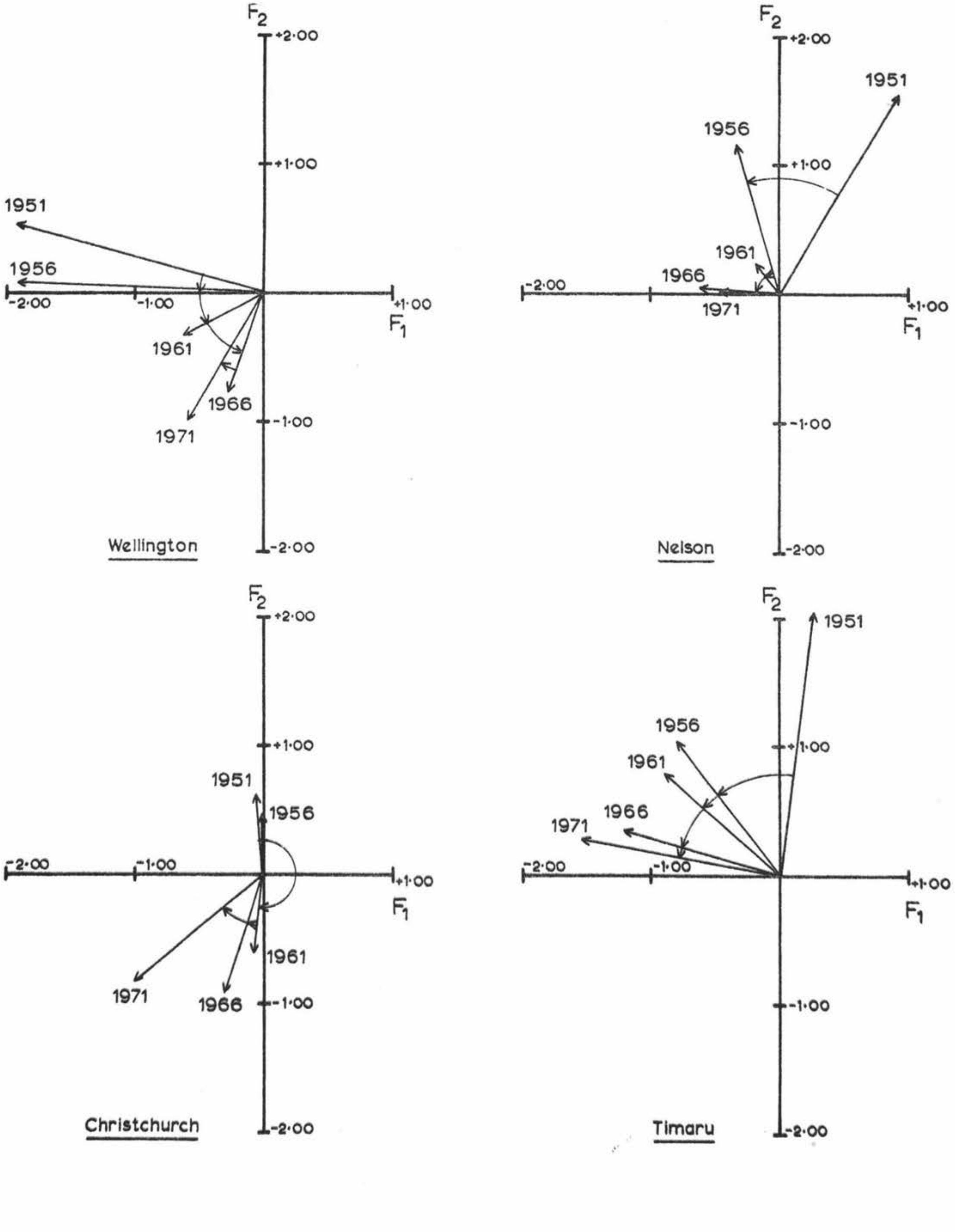
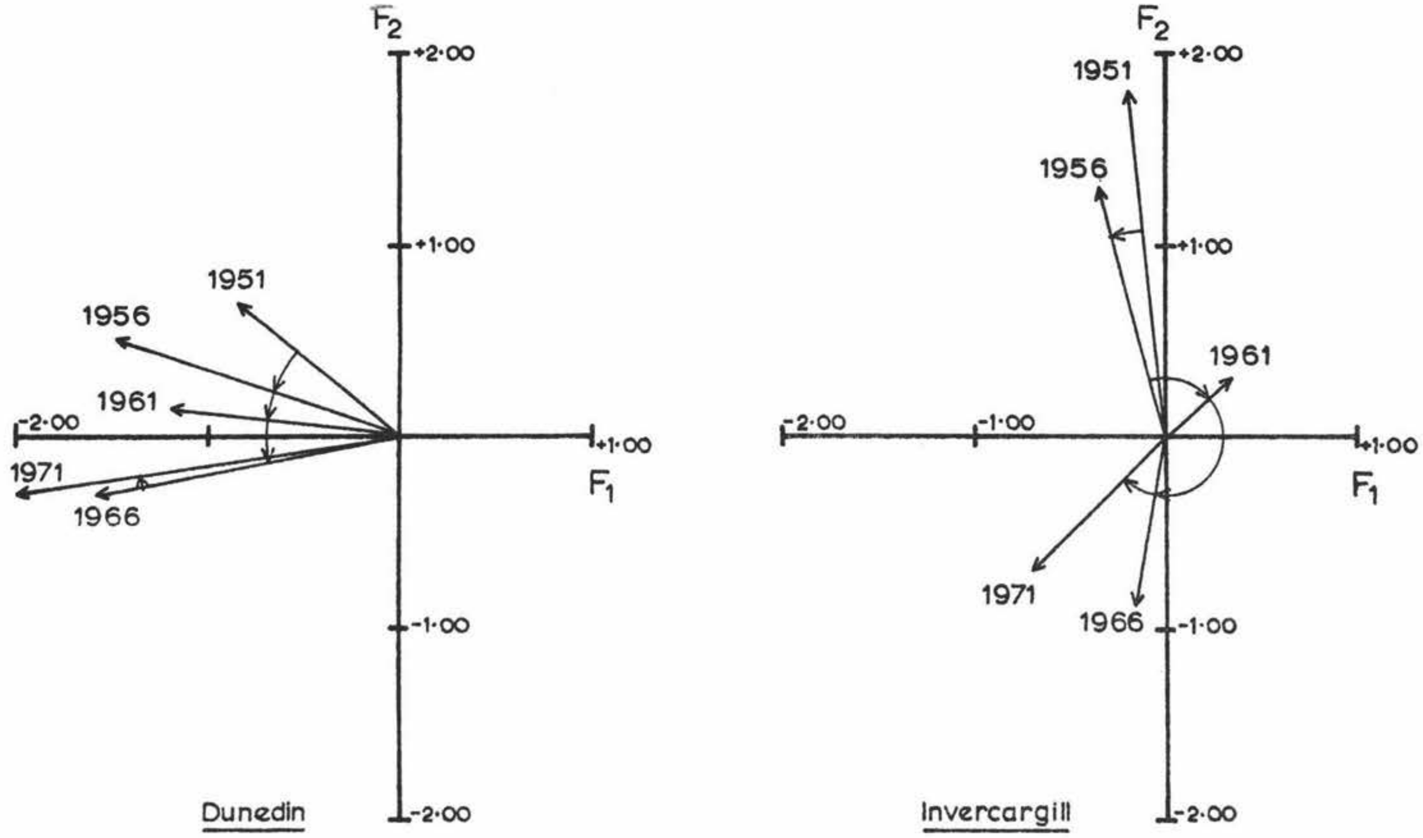


FIGURE 14

New Zealand Cities 1951-1971 - 2-Factor Varimax Model



Dunedin

Invercargill

TABLE 24

New Zealand Cities 1951-71
4-Factor Multivariate Planning Model (Varimax)[1]

Model^2 I $F_{1,1951-71}$ = 0.96(Population increase:X_6) + 0.68(Natural increase: X_7) + 0.89(Movement:X_8) + 0.64(Capital value increase: X_{12}) + 0.59(New dwellings:X_{21})

Model^2 II $F_{2,1951-71}$ = 0.86(0-14 age group:X_2) + 0.95(15-64 age group:X_3)

Model^2 III $F_{3,1951-71}$ = 0.50(Labour vote:X_9) + 0.67(Primary processing industries:X_{15}) + 0.75(Construction industries:X_{16}) - 0.58(Trading industries:X_{17}) - 0.69(Service industries:X_{18})

Model^2 IV $F_{4,1951-71}$ = 0.74(65+ age group:X_4) + 0.67(Sex ratio:X_5) - 0.63 (Maori population:X_{10})

Key to Quadrant Vector Models

Vector Diagram 1 $F_{2,1951-71}$ (Positive Factor Score)

High	Low	High	Low
$\underline{X_3}$ Value	$\underline{X_2}$ X_6 X_{12} X_7 X_{21} X_8	$\underline{X_3}$ X_6 X_{12} X_7 X_{21} X_8	$\underline{X_2}$
$F_{1,1951-71}$ (Negative Factor Score)		$F_{1,1951-71}$ (Positive Factor Score)	
High	Low	High	Low
$\underline{X_2}$	$\underline{X_3}$ X_6 X_{12} X_7 X_{21} X_8	$\underline{X_2}$ X_6 X_{12} X_7 X_{21} X_8	$\underline{X_3}$

(Negative Factor Score) $F_{2,1951-71}$

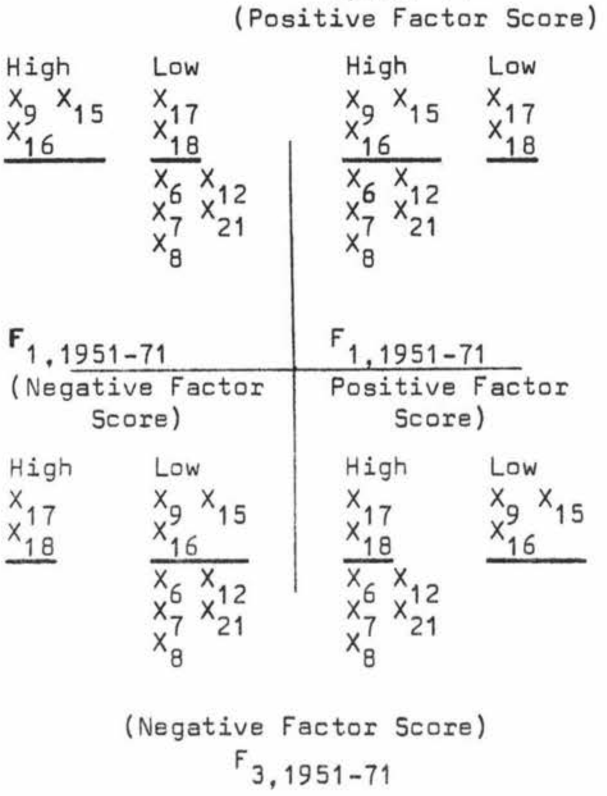
Vector Diagram 2 $F_{4,1951-71}$ (Positive Factor Score)

High	Low	High	Low
X_4 $\underline{X_5}$ X_{17} X_{18}	X_{10} —— X_9 X_{16} X_{15}	X_4 $\underline{X_5}$ X_9 X_{16} X_{15}	X_{10} —— X_{17} X_{18}
$F_{3,1951-71}$ (Negative Factor Score)		$F_{3,1951-71}$ (Positive Factor Score)	
High	Low	High	Low
X_{10} —— X_{17} X_{18}	X_4 $\underline{X_5}$ X_9 X_{16} X_{15}	X_{10} —— X_9 X_{16} X_{15}	X_4 $\underline{X_5}$ X_{17} X_{18}

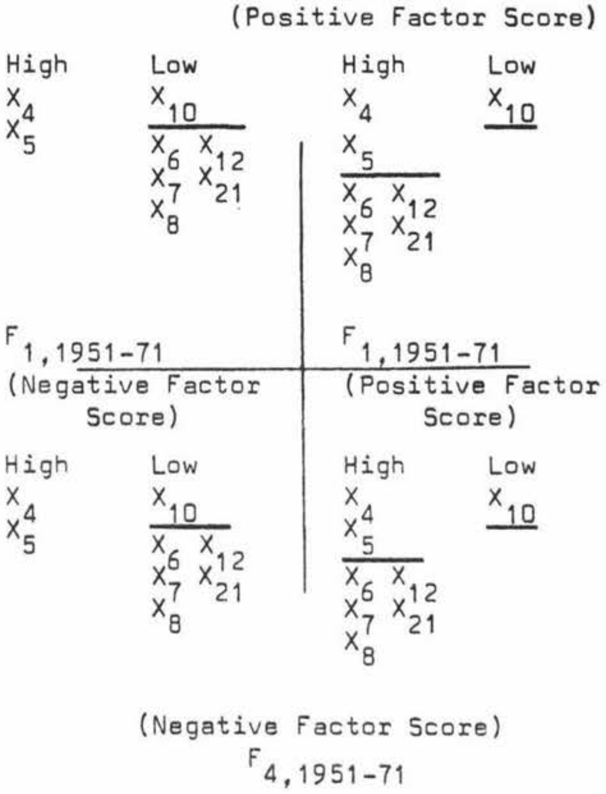
(Negative Factor Score) $F_{4,1951-71}$

TABLE 24 (Contd.)

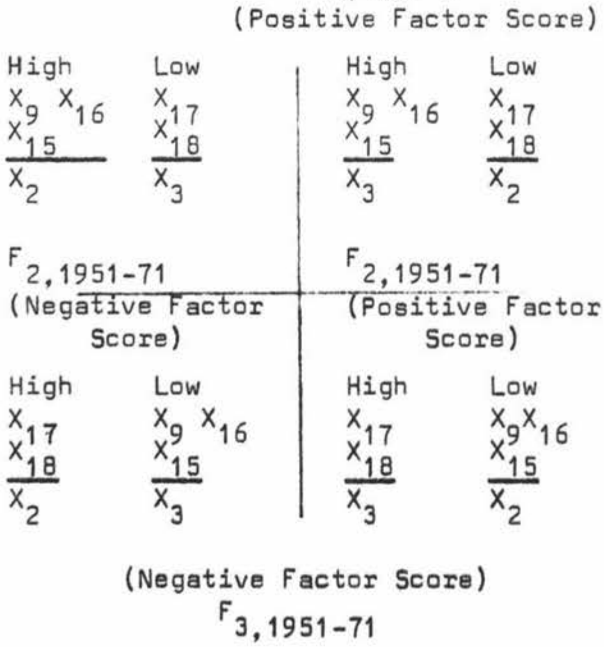
Vector Diagram 3



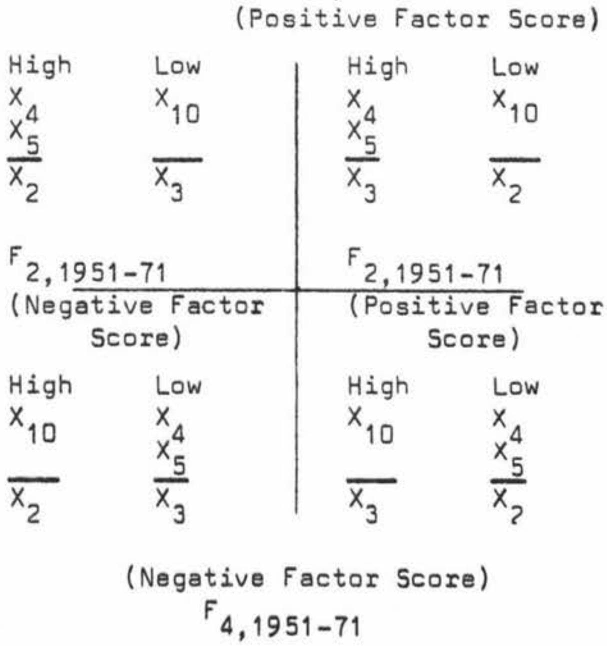
Vector Diagram 4



Vector Diagram 5



Vector Diagram 6



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[1] Normal Data Distribution.

[2] Only variables with loadings ≥ ± 0.50 have been identified in the models. A complete description of the factor loadings is given in Table 22.

define the relationships between each pair of the four factors and these can be seen to be very complex (Figures 15 and 16).

Example 1. Hamilton The relationship between Factor 1 and Factor 2 portrays a similar pattern to that described in the 2-Factor model. There is, however, some differences in the length of vector on each occasion and this might imply differing contributions of the variable values in the factor descriptions at each stage of the city's development. It is to be noted that there are some other differences in that the vector for 1961 is not located in Quadrant IV, but rather remains centrally located in Quadrant I. Such a change might be associated with the restructuring of Factor 2 in the 4-Factor model. This restructuring would be paralleled in a development in the city and most probably is associated with an increase in the youthful dependent population and a smaller number of persons in the working age group.

The relationship between Factor 1 and Factor 3 shows a much more complicated situation. The 1951 vector shifts from Quadrant IV to midway Quadrant I in 1961 with a reverse trend taking place from 1961 to 1966 and a shift to a near complete description by the converse of Factor 3. In this particular case, the magnitude of the changes imply considerable development within the city. Non-Labour voting and industrial activity especially seem to have played some role in this development. The factor descriptions in the relationship between Factor 1 and Factor 4 show a more general shift from Positive Factor 1 along to Negative Factor 4 alone. Age structuring of the population seems to be of some importance in this change rather than any changes in population. The shift in vectors, however,again defines situations of considerable complexity with both clockwise and anticlockwise swings taking place between the factors over Quadrant IV.

The relationship between Factor 2 and Factor 3 shows a pattern that seems to be dominated by changes in the city as defined by Factor 2. Again, these are changes that are associated with restructuring of the age of the city's population. In the case of Factors 2 and 4, both play a considerable part in describing a smooth pattern of change from Quadrant IV to Quadrant III - that is, shifts in industrial activity and in age structure. The relationship would be obvious in terms of obtaining manpower to develop such industries. Of all New Zealand cities Hamilton suffers from a lack of skilled manpower to man the city's rapid industrial expansion. Factors 3 and 4 with development taking place solely within Quadrant III indicate a steady pattern of change in association between large values of industrial activity, voting, Maori population, the aged and sex ratio. Changes in vector length demonstrate the amount of description increase and the change in orientation as well as the steady movement towards a description provided by Negative scores for Factor 4.

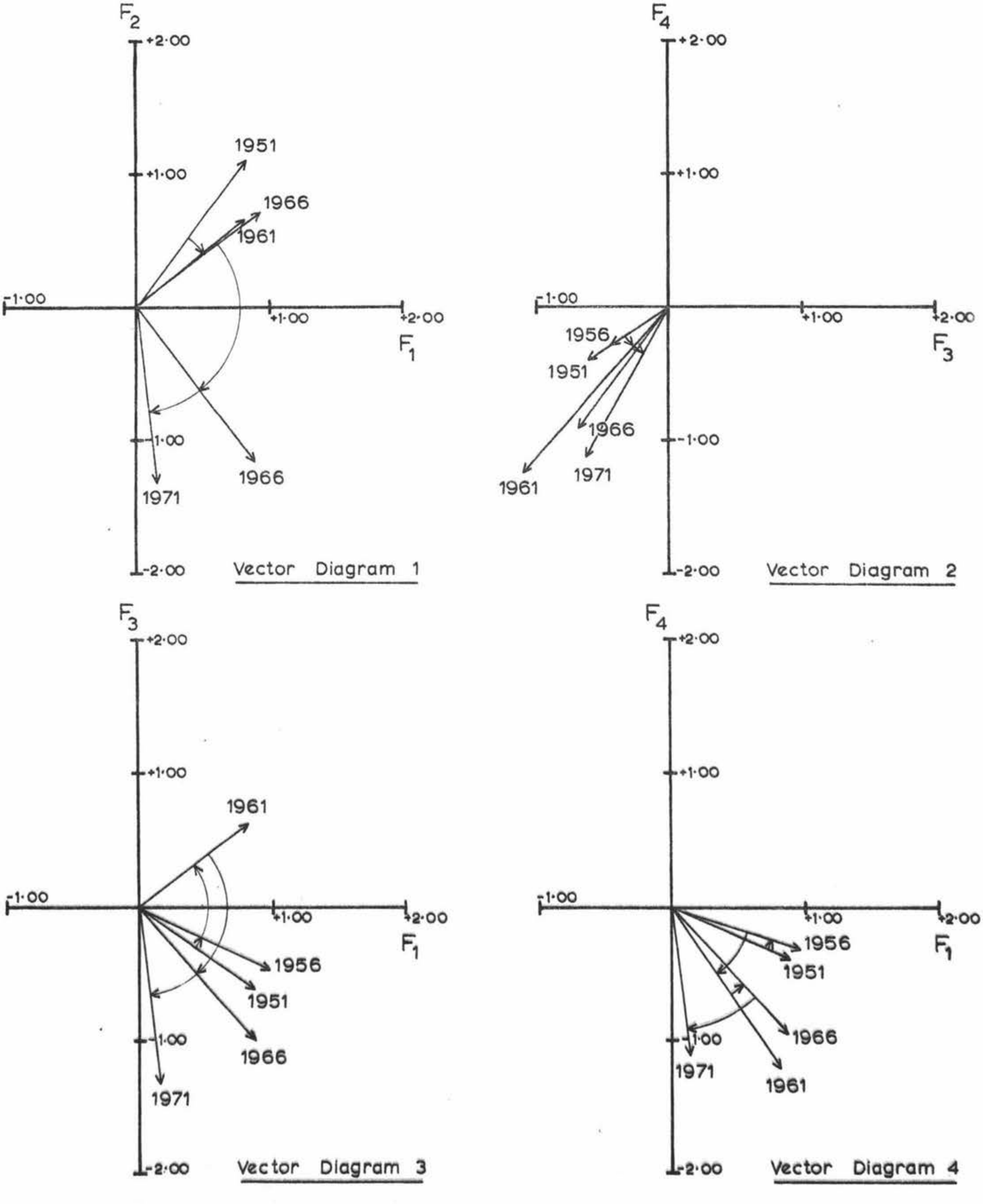
As in the 2-Factor multivariate planning model there are instances where a trend is so apparent that future development may be postulated with some degree of certainty. In the instance relating factors 3 and 4 a steady trend is apparent in Hamilton with a factor description relating to negative Factor 4 becoming increasingly more dominant. Hence it may be stated that within the next 5 years Hamilton will have problems associated with aged, further increases in Maori population would be expected and as a consequence of in-migration there would be an imbalance in the sex ratio in terms of less females.

Similar examples to that constructed for Hamilton City may be developed for the remaining 17 New Zealand cities. Again, as in the case of the 2-Factor multivariate planning model a powerful description of urban change is provided. In the instance of the 4-Factor model a far more

FIGURE 15

New Zealand Cities 1951-1971 - 4-Factor Varimax Model

HAMILTON



Vector Diagram 1

Vector Diagram 2

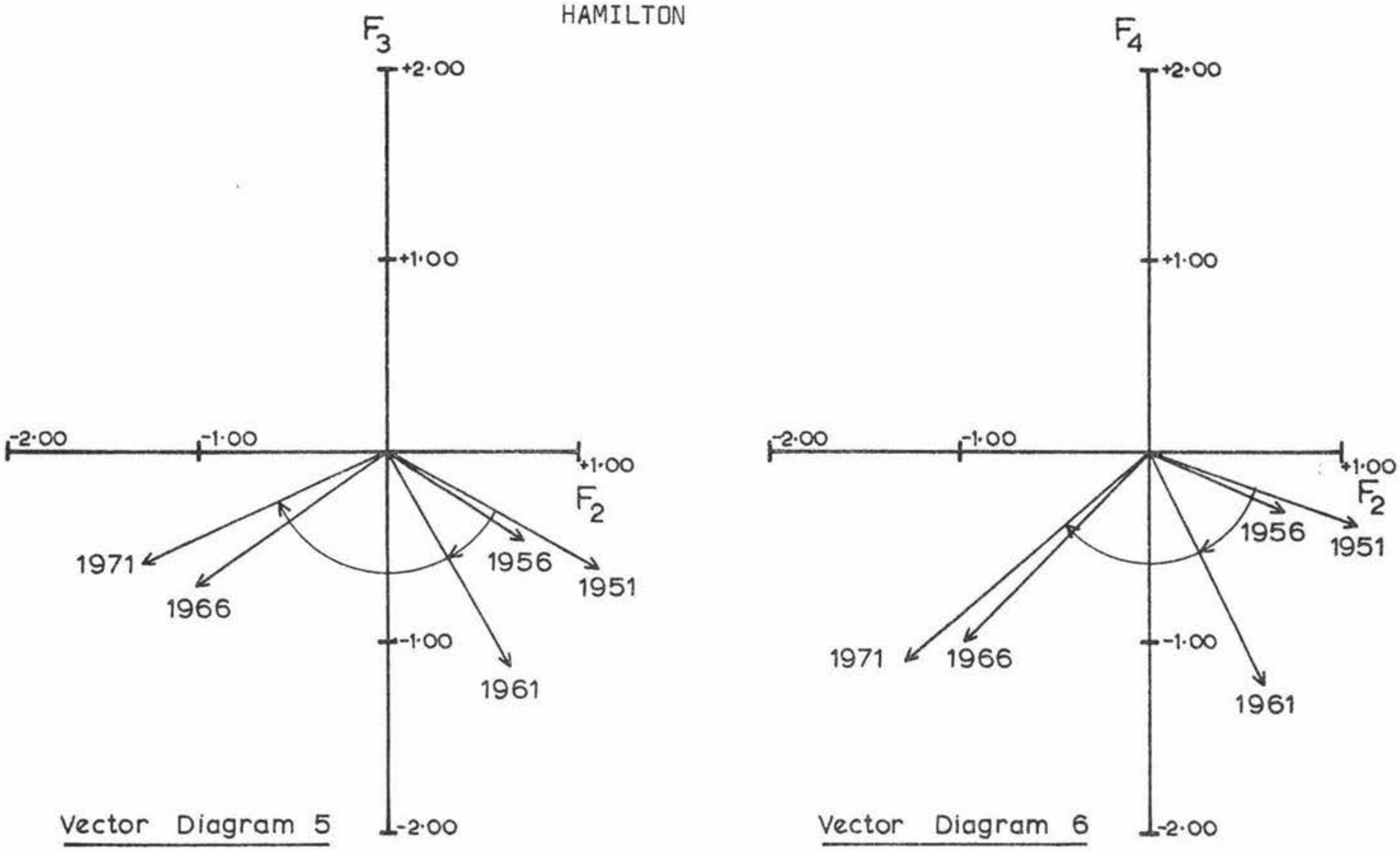
Vector Diagram 3

Vector Diagram 4

FIGURE 16

New Zealand Cities 1951-1971 - 4-Factor Varimax Model

HAMILTON



Vector Diagram 5

Vector Diagram 6

detailed description is given than that given in the 2-Factor model. Further, the models may provide a tool to planning when distinct trends are apparent. The power of the technique, however, is in the multivariate origin of the model.

IV. CONCLUSION - PLANNING POLICY AND STATISTICAL MODELS

1. Planning Model:-

One of the twin objectives of this study has been to construct a meaningful mathematical model capable of portraying city development patterns which could be used for the formulation of planning policy in New Zealand. In particular, the model was to incorporate the diverse elements of urban typology. While it was recognised that such incorporation must necessarily increase the complexity of the modelling, the fundamental goal was to formulate a simple model easily understandable and capable of relating many variables and their values.

Both the 2-Factor and 4-Factor multivariate models provide to some degree such a description. The 2-Factor model, because of its inherent simplicity, not only provides for meaningful relationship between many of the variables, but also generates patterns of city development in terms of the variables. The application however is successful, not in the preciseness of description, which is generally associated with statistical analysis, but in an ability to generalise from the models in a manner prescribed by statute.[1]

2. Statistical Models:

Second of the objectives of the study was the portrayal of city growth in New Zealand by a model that was consistent with established statistical theory. The models, which were constructed however, do not have the preciseness required for statistical testing. It would seem unlikely that such a model could be developed unless the Lawley formula was used. In lieu of the development simple transformations to normalise the distributions of the data base of the model highlighted the considerable difference between basic data and normalised data at the stage of correl-

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[1] Planning policies were to be expressed on a generalised basis, recognising general trends and developments in city areas rather than being specific in format. (The Role, Content and Form of the Scheme Statement: A Discussion Document, Town & Country Planning Div., Min. of Works, Wgton. April 1972).

ation. There seemed little distinction between the two approaches in terms of the eigenvalue solutions, but considerable difference in loadings were apparent once the factor models were established.

While the more complex 4-Factor model seems to have considerable potential in refining the model in terms of its description and trend delineation powers, the 2-Factor model was favoured in the study because of its inherent simplicity. Both models could, however, be improved upon through more precise experimental design. In the first instance the models used in this study suffered from the problems of arithmetic independence, i.e. using of individual variables and their summations also as variables - per cent population increase. The second instance was the related problem of singularity of the matrices. Almost all models used had cases of one or more zero eigenvalues. While this was not a problem in terms of the hypothesised factors, it may however have meant that there were high associations in areas that were neither arithmetically correct nor meaningful. Detection of variables with high association is, however, difficult when complex combinations may be present, but undetected until analysis is undertaken.

In conclusion, however, while the multivariate methodology provided a technique and a philosophy for planning, the model will need considerable refining through improved experimental design. The model developed here was an experimental one to explore the potential of the multivariate factor technique. The method has obvious potential in this area, but would require further refinement in both data base and research philosophy.

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APPENDIX I

THE VARIABLES AND THEIR SOURCES

Most of the 22 variables used in this analysis form part of Town Planning Scheme Statement requirements and were derived from material published by the Government Statistician. Actual sources of material are indicated. Unless otherwise indicated, all data includes Maoris. In most instances the time set applied to the census years 1951, 1956, 1961, 1966 and 1971. U.A. indicates Urban Area statistic, while C. indicates City statistic.

Demographic Variables:

1. (X_1) Enumerated population census. U.A.

Source: Population Census: Increase and Location of Population, 1951, 1956, 1961, 1966 and 1971 (Provisional)

2. (X_2) Percentage of population aged 0-14 years. U.A.

3. (X_3) Percentage of population aged 15-64 years. U.A.

4. (X_4) Percentage of population aged 65+ years. U.A.

Source: Population Census: Ages and Marital Status, 1951, 1956, 1961 and 1966.

Note: Values for 1971 were estimated on a pro rata basis as the results of the analysis of age structure have not yet been released by the Census Department.

5. (X_5) Females per 1000 males. U.A.

Source: Population Census: Increase and Location of Population, 1951, 1956, 1961, 1966 and 1971 (Provisional)

Total population has always been the most important statistic in town planning and as a consequence has been singularly used for this purpose. The 1960 Town and Country Planning regulations, while recognising the importance of population in the Third Scheme do, however, imply that the structure of the population may also be an essential indicator in planning and therefore stipulate an of population age structure in the format of variables X_2, X_3 and X_4 in the Scheme Statement. With each of these variables comes the associated problems of provision of schools, playgrounds, playcentres, employment opportunities, recreation facilities, senior citizens accommodation, etc. They are, therefore, of considerable importance in planning. Variable X_5, which indicates sex balance in an area, has been constructed as an index of sex imbalance and hence a sociological cause of change. This variable is also required under the Third Schedule.

Demographic Change Variables:

6. (X_6) Percentage intercensal increase in total population. U.A.
7. (X_7) Percentage intercensal increase in total population due to natural increase in population, i.e. excess births over deaths. U.A.
8. (X_8) Percentage intercensal increase in total population due to population movement into the area. U.A.

Source: Population Census: Increase and Location of Population, 1945, 1951, 1956, 1961, 1966 and 1971 (Provisional) and New Zealand Statistics: Vital 1945 to 1969.

Per cent population change along with total population has become a most important planning index. In particular, the former has become an index of the much maligned growth goal. It is, however, required to be included in the Scheme Statement of any Town Plan. All three variables used here are, it should be noted, adjusted for boundary changes and will therefore not in some cases tally directly with the stipulated population figures. While not noted in the regulations, Variables X_7 and X_8, natural increase and movement in population, are probably of real significance in the current planning environment. It is the latter variable which delimits population movement that is of considerable importance in maintaining population stability. This variable is derived from subtracting increases in population due to an excess of births over deaths from total population increase. While a crude index of population movement, it does signify shifts in population, particularly those shifts in recent years from the smaller cities to the larger metropolitan areas. The intercensal increases used to delimit population shifts refer to the years 1945-51, 1951-56, 1956-61, 1961-66 and 1966-71. In the case of the 1966-71 increases estimates on natural increases were extrapolated from the 1966, 1967, 1968 and 1969 values.

Political Variable:

9. (X_9) Percentage of the voting population voting Labour in the last general election. U.A.

Source: Appendix to the Journals of the House of Representatives of New Zealand, 1952, vol. III, general election 1951, paper H.33; 1955, vol. III, general election 1954, paper H.33; 1961, vol. IV, general election 1960, paper H.33; 1967, vol. III, general election 1966, paper H.33; 1970, vol. III, general election 1969, paper H.33.

Political decision making can play an important role in the development of an area. In particular political affiliation, and sometimes non-affiliation, can be of considerable social and economic importance. Marginal city seats often find specific advantages in their situation. General elections for 1951, 1954, 1960, 1966 and 1969 have been included in the analysis. Raw data was taken from the published polling places within the urban areas and therefore is taken as indicative of the political affiliation at the time of the election.

Maori Population Variable:

10. (X_{10}) Total Maori population. U.A.

11. (X_{11}) Percentage intercensal increase in Maori population. U.A.

Source: Population Census: Maori Population and Dwellings 1945, 1951, 1956, 1961 and 1966, and Population Census: Increase and Location of Population 1971 (Provisional)

Maori people are becoming an important component of New Zealand's urban scene. Shifts from the more distant rural areas of this country to the towns and cities has meant that the Maori has not only had to adjust his way of life, but he has at the same time had to contribute to the making of a new urban mosaic. For this reason alone, two indexes of Maori population have been included in the model.

Percentage increases in Maori population have been taken for the years 1945-51, 1951-56, 1956-61, 1961-66 and 1966-71, while adjustments have been made for boundary changes.

Value Variable:

12. (X_{12}) Percentage intercensal increase in gross capital value. C.

Source: Local Authority Statistics, 1945 to 1970

By using percentage increase in gross values, an index of development, particularly capital value, can be obtained. The variable defines what is basically the assets, that is buildings, within an area. One feature, however, needs to be borne in mind and it is that the actual valuation of an area is based upon 5-yearly valuations which, because of the volume of work involved, do not necessarily coincide with census. Intercensal increases have been based upon increases in the periods 1945-51, 1951-56, 1956-61 and 1961-66. Data for the remaining period was an extrapolation on the valuations from 1966, 1967, 1968, 1969 and 1970.

Industrial Activity Variables:

13. (X_{13}) Percentage of the total Labour force women. U.A.

14. (X_{14}) Percentage of the total Labour force employed in Primary Industries (forestry, logging, quarrying and mining). U.A.

15. (X_{15}) Percentage of the total Labour force employed in Primary Processing Industries (food, drink, tobacco - non-seasonal, textile, clothing, and leather). U.A.

16. (X_{16}) Percentage of the total Labour force employed in Construction Industries (building, construction, building material activities, engineering, metal work and miscellaneous manufacturing). U.A.

17. (X_{17}) Percentage of the total Labour force employed in Trading Industries (transport, commerce, insurance and finance). U.A.

18. (X_{18}) Percentage of the total Labour force employed in Service Industries (domestic, professional, personal, power, water and sanitary services). U.A.

19. (X_{19}) Percentage of the total Labour force employed in Seasonal Industries. U.A.

Source: Department of Labour, Wellington, unpublished and published returns relating to employment in all towns of over 1000 persons.

Clause 5, Part II of the Third Schedule of the Town and Country Planning Regulations 1960 specifies the inclusion of a descriptive occupational structure in the Scheme Statement. While the categories given are not those stipulated above, the specification in the Regulations does emphasise the importance of including an analysis of the Labour force in a planning model. The primary value of studying the Labour force is not only to put employment in perspective, but also to be able to make an assessment of the nature of industrial activity in any particular area.

All of the above data was obtained through the courtesy of the Department of Labour. In accordance with Section 13 of the Labour Department Act 1954 figures pertaining to individual firms have been combined. The actual data refers to the years 1953, 1956, 1961, 1966 and 1971. Year 1953 was the earliest that surveys of employment were undertaken in a consistent format and is given to represent 1951 values. It should be appreciated that the above data is an amalgam of 95 or more different labour codes used by the Department of Labour. In 1971, however, the Department conformed to an international labour code and as a consequence 1971 figures tend to differ from the previous definition. An attempt was made to achieve some degree of consistency in relating the changes to the earlier data by combining various results.

Local Body Variable:

20. (X_{20}) Rating in dollar averaged over the intercensal period. C.

Source: Local Authority Statistics, 1945 to 1970.

Capital works development programmes, loan repayments, subsidies and other areas of Local Authority expenditure are generally indicative of the wealth and foresight of a community. Consistent patterns of low rating in the dollar over 20 years generally denote a community with a considerable degree of planning ability. This variable has been included in the model on an experimental basis as an index of development decisions. The rate value obtained is an average of five years of rate payment - 1945-51, 1951-56, 1956-61, 1961-66 and 1966-71. There were two reasons for using an average. Firstly, to standardise the problem of revaluations and secondly, to depress the results of Local Authority election year ratings which are generally low. It should be noted too, that the 1970-71 rates values which are not yet published were obtained from an extrapolation on previous values.

Index of Economic Activity:

21. (X_{21}) Investment in new private dwellings as a per cent of total investment in all buildings averaged over the intercensal period. U.A.

Source: Statistics of Population, Migration and Buildings, 1946 to 1969.

22. (X_{22}) Investment Confidence index - per cent intercensal increase in the value of all building. U.A.

Source: Statistics of Population, Migration and Buildings, 1946 to 1969.

The building industry, an industry sensitive to economic change, provides an ideal index of economic activity. In particular, not only do increases in the value of building imply that a community is developing in a manner that makes such an investment worthwhile, but it also indicates that the builder envisages a return on his investment over the lifetime of the building. Building of new dwellings is, however, a complimentary characteristic in that a developing town generally requires homes for people who will participate in such development. Further, investment in homes is a more sensitive index of local economic potential than is an index of total investment. One problem not accounted for, however, is the problem of the effects of inflation. Variable X_{21} values have been averaged over each intercensal period in an attempt to reduce the considerable variations that take place in the building industry. Such averaging is an attempt at a more realistic portrayal of building. Variable X_{22} on the other hand displays all the vagarities of investment in total building in an area.

APPENDIX II

PEARSON PRODUCT MOMENT CORRELATION MATRICES: 22 VARIABLES AND 18 NEW ZEALAND CITIES

The following correlation matrices have been included in this Appendix for comparative purposes:-

(a)	1951 basic data and normalised data*	(Tables A and B)
(b)	1956 " " " " "	(Tables C and D)
(c)	1961 " " " " "	(Tables E and F)
(d)	1966 " " " " "	(Tables G and H)
(e)	1971 " " " " "	(Tables I and J)
(f)	1951-71 combined matrix based on basic data	(Table K)
(g)	1951-71 constructed from average correlations calculated for the basic data correlation matrices 1951, 1956, 1961, 1966 and 1971	(Table L)

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* Basic data correlation matrices were constructed from the original data while the normalised data correlation matrices were derived from data that had a normal distribution or had been transformed into a normal distribution.

TABLE A

Product Moment Correlation Matrix
New Zealand Cities 1951
(Basic Data Distribution)

Variables

X_1	1.00															
X_2	0.18	1.00														
X_3	-0.21	-0.82	1.00													
X_4	0.12	0.03	-0.59	1.00												
X_5	-0.33	-0.48	0.08	0.53	1.00											
X_6	-0.35	0.48	-0.38	-0.03	-0.13	1.00										
X_7	-0.35	0.49	-0.23	-0.29	-0.34	0.97	1.00									
X_8	-0.27	0.43	-0.37	0.04	-0.09	-0.55	0.31	1.00								
X_9	0.33	0.12	-0.10	0.02	-0.22	-0.08	0.03	-0.59	1.00							
X_{10}	0.78	0.24	-0.20	0.02	-0.25	0.37	-0.08	-0.06	0.01	1.00						
X_{11}	0.09	0.54	-0.33	-0.02	-0.26	0.48	0.02	0.44	-0.10	0.18	1.00					
X_{12}	-0.35	0.52	-0.20	-0.39	-0.34	-	0.57	0.40	-0.01	-0.26	0.39	1.00				
X_{13}	0.10	-0.13	0.12	-0.02	0.20	0.46	-0.04	-0.01	-0.17	0.35	0.03	-0.42	1.00			
X_{14}	-0.13	0.20	-0.09	-0.13	-0.09	-0.41	0.43	0.37	-0.24	0.21	-0.06	0.01	0.68	1.00		
X_{15}	0.66	0.26	-0.43	0.38	-0.29	0.45	-0.19	-0.37	0.75	0.26	-0.07	-0.19	-0.24	-0.26	1.00	
X_{16}	0.13	0.71	-0.59	0.01	-0.55	-0.37	-0.49	0.42	0.11	-	0.34	0.46	-0.47	-0.10	0.32	1.00
X_{17}	-0.11	-0.44	0.40	-0.07	0.34	0.27	-0.59	-0.25	-0.27	-0.22	-0.13	-0.18	-0.18	-0.40	-0.24	-0.35
X_{18}	-0.47	-0.23	0.34	-0.27	0.27	-0.16	0.30	0.17	-0.37	-0.06	-0.10	-0.04	0.70	0.67	-0.66	-0.49
X_{19}	-0.04	-0.23	0.25	-0.10	0.02	-0.32	-0.04	-0.17	0.14	0.02	-0.04	0.09	-0.20	-0.22	-0.06	-0.27
X_{20}	-0.06	0.09	-0.22	0.27	0.02	0.57	0.09	-0.37	0.39	-0.05	-0.27	-0.14	-0.29	-0.21	0.31	0.13
X_{21}	-0.58	0.05	-0.03	-0.03	0.37	0.51	0.50	0.47	-0.32	-0.19	0.16	0.24	0.13	0.30	-0.56	0.01
X_{22}	-0.41	-0.14	0.09	0.05	0.35	0.52	-0.01	0.55	-0.65	-0.15	0.22	-0.07	0.22	0.24	-0.55	-0.28

X_{17}	1.00					
X_{18}	-0.14	1.00				
X_{19}	-0.30	-0.15	1.00			
X_{20}	0.11	-0.34	0.01	1.00		
X_{21}	-0.27	0.43	0.24	0.07	1.00	
X_{22}	0.04	0.52	-0.07	-0.28	0.52	1.00

TABLE B

Product Moment Correlation Matrix
New Zealand Cities 1951
(Normalised Data Distribution)

Variables

X_{1}	1.00															
X_{2}	0.06	1.00														
X_{3}	-0.08	-0.83	1.00													
X_{4}	0.04	0.03	-0.59	1.00												
X_{5}	-0.38	-0.47	0.09	0.53	1.00											
X_{6}	-0.61	0.50	-0.43	0.05	-0.05	1.00										
X_{7}	-0.34	0.49	-0.22	-0.29	-0.34	0.65	1.00									
X_{8}	-0.57	0.49	-0.45	0.10	0.02	0.98	0.63	1.00								
X_{9}	0.59	0.12	-0.11	0.02	-0.22	-0.48	0.03	-0.42	1.00							
X_{10}	0.37	0.17	-0.01	-0.24	-0.30	-0.11	0.09	-0.09	0.08	1.00						
X_{11}	-0.05	0.54	-0.33	-0.20	-0.27	0.35	0.05	0.33	-0.17	0.02	1.00					
X_{12}	-0.43	0.47	-0.16	-0.39	-0.27	0.56	0.57	0.57	-0.06	-0.26	0.41	1.00				
X_{13}	-0.04	-0.13	0.12	-0.02	0.20	-0.05	-0.06	-0.06	-0.17	0.53	-0.07	-0.40	1.00			
X_{14}	-0.22	0.27	-0.17	-0.08	-0.16	0.36	0.29	0.30	-0.23	0.15	-0.07	0.10	0.46	1.00		
X_{15}	0.80	0.20	-0.37	0.36	-0.26	-0.43	-0.21	-0.39	0.78	0.01	-0.07	-0.28	-0.25	-0.14	1.00	
X_{16}	0.16	0.71	-0.62	0.06	-0.51	0.48	0.47	0.45	0.08	0.08	0.41	0.39	-0.45	0.02	0.26	1.00
X_{17}	-0.04	-0.44	0.45	-0.17	0.23	-0.43	-0.52	-0.44	-0.27	-0.29	-0.07	-0.19	-0.13	-0.10	-0.19	-0.33
X_{18}	-0.59	-0.27	0.37	-0.27	0.31	0.23	0.26	0.20	-0.38	0.13	-0.16	0.01	0.68	0.32	-0.68	-0.51
X_{19}	-0.16	-0.12	0.13	-0.05	0.14	0.12	0.14	0.07	-0.29	-0.01	0.02	-0.10	0.35	0.34	-0.20	0.08
X_{20}	0.10	0.08	-0.21	0.27	0.03	-0.22	0.12	-0.16	0.43	-0.04	-0.30	-0.16	-0.31	-0.18	0.39	0.12
X_{21}	-0.70	0.05	-0.03	-0.03	0.37	0.65	0.50	0.68	-0.32	-0.03	0.17	0.31	0.13	0.13	-0.54	0.01
X_{22}	-0.56	-0.30	0.21	0.07	0.47	0.31	-0.12	0.31	-0.64	-0.25	0.14	-0.12	0.30	0.02	-0.55	0.44

X_{17}	1.00					
X_{18}	-0.06	1.00				
X_{19}	0.43	0.33	1.00			
X_{20}	0.11	-0.35	-0.08	1.00		
X_{21}	-0.29	0.44	0.05	0.06	1.00	
X_{22}	0.10	0.58	0.08	-0.27	0.57	1.00

TABLE C

Product Moment Correlation Matrix
New Zealand Cities 1956
(Basic Data Distribution)

Variables

X_1	1.00															
X_2	0.06	1.00														
X_3	-0.25	-0.73	1.00													
X_4	0.20	-0.09	-0.44	1.00												
X_5	-0.45	-0.42	0.35	0.28	1.00											
X_6	-0.20	0.25	-0.15	0.01	0.04	1.00										
X_7	-0.25	0.45	0.01	-0.64	-0.21	0.40	1.00									
X_8	-0.16	0.17	-0.19	0.19	0.08	0.97	0.16	1.00								
X_9	0.46	0.15	-	-0.18	-0.17	-0.40	0.14	-0.47	1.00							
X_{10}	0.79	0.13	-0.19	0.11	-0.36	0.18	0.06	0.16	0.23	1.00						
X_{11}	-0.15	0.14	-0.30	0.36	0.02	0.79	-0.15	0.90	-0.51	0.01	1.00					
X_{12}	-	-0.04	0.07	0.07	0.11	0.69	0.13	0.69	-0.53	0.21	0.57	1.00				
X_{13}	0.01	0.28	-0.47	0.26	0.07	0.32	0.22	0.29	-0.21	0.16	0.25	0.34	1.00			
X_{14}	-0.14	0.18	-0.10	-0.12	-0.10	0.13	0.60	-0.03	0.01	0.14	-0.25	0.14	0.36	1.00		
X_{15}	0.68	0.31	-0.43	0.26	-0.48	-0.44	-0.33	-0.38	0.61	0.27	-0.23	-0.45	-0.16	-0.30	1.00	
X_{16}	0.13	0.49	-0.23	-0.27	-0.56	0.19	0.26	0.15	0.37	0.03	0.10	0.02	-0.41	-0.08	0.37	1.00
X_{17}	-0.11	-0.48	0.07	0.40	0.48	-0.24	-0.45	-0.14	-0.46	-0.15	0.03	0.04	0.18	-0.10	-0.35	-0.72
X_{18}	-0.45	0.03	-	-0.03	0.38	0.38	0.45	0.29	-0.34	-0.08	0.13	0.28	0.73	0.54	-0.61	-0.54
X_{19}	-0.06	-0.40	0.65	-0.33	0.12	-0.04	-0.08	-0.04	-	-0.01	-0.13	-0.34	-0.46	-0.22	-0.15	-0.09
X_{20}	-0.13	-0.02	0.09	0.11	-0.01	-0.45	-0.28	-0.41	-0.11	-0.19	-0.31	-0.18	-0.23	-0.08	0.12	-0.49
X_{21}	-0.22	-0.36	0.46	0.11	0.76	0.24	-0.20	0.28	-0.39	-0.01	0.12	0.30	-0.11	-0.17	-0.44	-0.32
X_{22}	0.32	0.29	-0.44	0.06	-0.49	0.06	0.11	0.05	0.17	0.17	0.09	0.21	0.35	0.02	0.21	0.31

X_{17}	1.00					
X_{18}	0.23	1.00				
X_{19}	-0.29	-0.28	1.00			
X_{20}	0.18	-0.24	0.01	1.00		
X_{21}	0.14	0.16	0.42	0.05	1.00	
X_{22}	-0.08	-0.19	-0.26	0.02	-0.53	1.00

TABLE D

Product Moment Correlation Matrix
New Zealand Cities 1956
(Normalised Data Distribution)

Variables

X_1	1,00															
X_2	-	1,00														
X_3	-0,16	-0,73	1,00													
X_4	0,08	-0,09	-0,44	1,00												
X_5	-0,44	-0,44	0,39	0,22	1,00											
X_6	-0,47	0,25	-0,06	-0,09	-0,04	1,00										
X_7	-0,27	0,44	0,04	-0,66	-0,18	0,46	1,00									
X_8	-0,45	0,15	-0,03	0,01	0,02	0,98	0,30	1,00								
X_9	0,61	0,07	0,11	-0,24	-0,37	-0,33	0,15	-0,37	1,00							
X_{10}	0,25	0,17	-0,13	-0,07	-0,46	0,42	0,28	0,41	0,20	1,00						
X_{11}	-0,30	0,02	0,08	-0,15	-0,15	0,68	0,09	0,73	-0,27	0,34	1,00					
X_{12}	-0,25	-0,04	0,06	0,07	0,03	0,69	0,13	0,71	-0,53	0,49	0,57	1,00				
X_{13}	-0,15	0,28	-0,47	0,26	0,03	0,23	0,17	0,19	-0,28	0,36	0,11	0,34	1,00			
X_{14}	-0,08	0,10	-	-0,15	-0,12	-0,11	0,42	-0,18	0,14	0,14	-0,16	-0,04	0,17	1,00		
X_{15}	0,79	0,28	-0,41	0,29	-0,43	-0,50	-0,38	-0,48	0,57	-0,03	-0,47	-0,48	-0,18	-0,14	1,00	
X_{16}	0,19	0,47	-0,22	-0,25	-0,53	0,28	0,28	0,24	0,32	0,21	0,07	0,07	-0,41	-0,08	0,33	1,00
X_{17}	-0.13	-0.48	0,07	0.40	0.47	-0.31	-0.45	-0.25	-0.51	-0.30	-	0.04	0.18	-0.10	-0.31	-0.71
X_{18}	-0.62	-0.01	0.02	0.01	0.41	0.32	0.36	0.27	-0.42	0.05	0.12	0.29	0.73	0.27	-0.63	-0.58
X_{19}	-0.21	0.20	-0.48	0.21	-0.14	0.21	0.32	0.13	-0.27	0.14	-0.02	0.20	0.57	0.37	-0.23	-0.04
X_{20}	-0.01	-0.04	0.11	0.11	0.05	-0.45	-0.29	-0.40	-0.10	-0.24	-0.19	-0.21	-0.25	0.09	0.17	-0.04
X_{21}	-0.40	-0.36	0.46	0.11	0.74	0.33	-0.20	0.45	-0.34	-0.11	0.36	0.30	-0.11	-0.23	-0.42	-0.28
X_{22}	0.36	0.29	-0.44	0.06	-0.50	0.02	0.11	-0.02	0.15	0.56	0.05	0.21	0.35	0.01	0.18	0.31

X_{17}	1,00					
X_{18}	0,29	1,00				
X_{19}	0,26	0,51	1,00			
X_{20}	0,17	-0,23	-0,21	1,00		
X_{21}	0,14	0,20	-0,46	0,05	1,00	
X_{22}	-0,03	-0,21	0,34	0,01	-0,53	1,00

TABLE E

Product Moment Correlation Matrix
New Zealand Cities 1961
(Basic Data Distribution)

Variables

X_1	1.00															
X_2	0.15	1.00														
X_3	-0.22	-0.87	1.00													
X_4	0.20	0.35	-0.76	1.00												
X_5	-0.39	-0.16	0.02	0.32	1.00											
X_6	-0.13	-0.13	0.28	-0.38	-0.15	1.00										
X_7	-0.14	0.01	0.30	-0.63	-0.40	0.69	1.00									
X_8	-0.10	-0.17	0.23	-0.23	-0.02	0.96	0.46	1.00								
X_9	0.29	0.18	-0.12	0.04	-0.18	-0.55	-0.18	-0.61	1.00							
X_{10}	0.85	0.07	-0.06	0.01	-0.31	0.17	0.11	0.17	0.11	1.00						
X_{11}	0.23	-0.17	0.16	-0.13	-0.08	-0.33	-0.16	-0.34	-0.06	0.07	1.00					
X_{12}	-0.36	-0.12	0.31	-0.43	-0.08	0.61	0.68	0.48	-0.18	-0.06	-0.17	1.00				
X_{13}	0.10	0.12	-0.21	0.22	0.06	0.51	0.39	0.47	-0.45	0.28	-0.28	0.31	1.00			
X_{14}	-0.06	0.07	-0.08	0.12	0.13	-0.38	-0.23	-0.37	0.26	-0.10	-0.28	-0.26	-0.21	1.00		
X_{15}	0.55	0.47	-0.55	0.41	-0.09	-0.49	-0.27	-0.49	0.48	0.24	0.12	-0.45	-0.07	-0.09	1.00	
X_{16}	0.27	0.54	-0.30	-0.16	-0.58	0.07	0.16	0.02	0.26	0.16	-0.13	-0.15	-0.29	0.09	0.42	1.00
X_{17}	-0.18	-0.31	0.02	0.38	0.37	-0.13	-0.28	-0.05	-0.43	-0.12	0.32	0.04	0.27	0.08	-0.41	-0.77
X_{18}	-0.47	-0.22	0.27	-0.20	0.17	0.56	0.55	0.48	-0.37	-0.18	-0.32	0.68	0.58	-0.06	-0.62	-0.52
X_{19}	-0.12	-0.43	0.49	-0.36	0.29	-0.13	-0.18	-0.09	0.16	-0.06	0.05	-0.21	-0.45	-0.07	-0.13	-0.20
X_{20}	-0.20	0.18	-0.26	0.24	-0.06	-0.43	-0.16	-0.46	0.03	-0.18	0.13	-0.10	-0.10	-0.15	-0.07	-0.15
X_{21}	0.24	-0.03	-0.05	0.12	0.12	-0.19	-0.16	-0.17	0.43	0.11	0.18	-0.25	-0.45	-0.25	0.52	0.24
X_{22}	-0.05	-0.13	0.18	-0.08	-0.19	-0.29	-0.24	-0.26	0.31	-0.12	0.22	-0.01	-0.45	0.22	-0.29	-0.17

X_{17}	1.00					
X_{18}	0.40	1.00				
X_{19}	-0.26	-0.29	1.00			
X_{20}	0.32	-0.05	-0.19	1.00		
X_{21}	-0.33	-0.42	0.23	-0.06	1.00	
X_{22}	0.29	0.14	-0.07	0.10	-0.01	1.00

TABLE F

Product Moment Correlation Matrix
New Zealand Cities 1961
(Normalised Data Distribution)

Variables																
X_1	1.00															
X_2	0.13	1.00														
X_3	-0.16	-0.88	1.00													
X_4	0.12	0.36	-0.76	1.00												
X_5	-0.50	-0.25	0.01	0.32	1.00											
X_6	-0.30	0.15	0.31	-0.42	-0.16	1.00										
X_7	-0.12	-	0.31	-0.63	-0.41	0.68	1.00									
X_8	-0.33	-0.18	0.27	-0.29	-0.07	0.87	0.44	1.00								
X_9	0.37	0.16	-0.12	0.04	-0.18	-0.54	-0.21	-0.49	1.00							
X_{10}	0.41	-0.06	0.17	-0.28	-0.42	0.46	0.44	0.41	-0.11	1.00						
X_{11}	0.34	-0.17	0.17	-0.13	-0.08	-0.33	-0.15	-0.25	-0.06	-0.08	1.00					
X_{12}	-0.52	-0.13	0.31	-0.41	-0.05	0.56	0.58	0.40	-0.22	-0.13	-0.15	1.00				
X_{13}	-0.06	0.14	-0.21	0.22	0.06	0.44	0.38	0.27	-0.45	0.43	-0.28	0.26	1.00			
X_{14}	0.13	0.03	-0.04	0.08	0.10	-0.48	-0.31	-0.35	0.46	-0.20	-0.05	-0.35	-0.38	1.00		
X_{15}	0.63	0.45	-0.56	0.43	-0.07	-0.51	-0.29	-0.53	0.49	-	0.09	-0.50	-0.06	0.06	1.00	
X_{16}	0.33	0.53	-0.31	-0.15	-0.56	0.13	0.18	0.05	0.25	0.23	-0.16	-0.14	-0.28	0.10	0.40	1.00
X_{17}	-0.22	-0.29	0.04	0.37	0.35	-0.13	0.26	-	-0.45	-0.22	0.29	0.05	0.29	0.03	-0.42	-0.76
X_{18}	-0.56	-0.21	0.18	-0.20	0.17	0.53	0.52	0.46	-0.37	0.03	-0.32	0.66	0.58	-0.26	-0.61	-0.51
X_{19}	-0.01	0.12	-0.04	-0.07	-0.37	0.33	0.47	0.14	-0.26	0.20	-0.25	0.46	0.56	-	-0.19	0.13
X_{20}	-0.09	0.18	-0.26	0.24	-0.06	-0.45	-0.18	-0.45	0.03	-0.21	0.13	-0.11	-0.10	-0.16	0.08	-0.16
X_{21}	0.24	-0.02	-0.07	0.12	0.12	-0.15	-0.15	-0.04	0.43	-0.10	0.18	-0.25	-0.46	-0.05	0.53	0.26
X_{22}	0.11	-0.18	0.25	-0.15	-0.21	-0.32	-0.24	-0.11	0.35	-0.27	0.34	-0.02	-0.52	0.31	-0.23	-0.17

X_{17}	1.00					
X_{18}	0.41	1.00				
X_{19}	0.13	0.46	1.00			
X_{20}	0.32	-0.05	-0.08	1.00		
X_{21}	-0.37	-0.43	-0.62	-0.06	1.00	
X_{22}	0.27	0.07	-0.14	0.14	0.06	1.00

TABLE G

Product Moment Correlation Matrix
New Zealand Cities 1961
(Basic Data Distribution)

Variables

X_1	1,00															
X_2	-0,39	1,00														
X_3	0,49	-0,78	1,00													
X_4	-0,07	-0,50	-0,15	1,00												
X_5	-0,24	-0,19	-0,33	0,78	1,00											
X_6	0,05	0,64	-0,23	-0,71	-0,51	1,00										
X_7	-0,05	0,72	-0,20	-0,87	-0,60	0,86	1,00									
X_8	0,06	0,58	-0,23	-0,60	-0,43	0,98	0,75	1,00								
X_9	0,30	-0,62	0,36	0,48	0,32	-0,68	-0,58	-0,67	1,00							
X_{10}	0,88	-0,03	0,18	-0,21	-0,26	0,31	0,23	0,25	0,08	1,00						
X_{11}	0,06	0,07	0,20	-0,40	-0,45	0,09	0,13	0,07	-0,39	-0,06	1,00					
X_{12}	-0,16	0,43	-0,15	-0,48	-0,39	0,80	0,57	0,84	-0,58	0,05	0,02	1,00				
X_{13}	0,18	0,04	0,07	-0,16	0,18	0,13	0,31	0,05	-0,08	0,29	-0,29	-0,18	1,00			
X_{14}	-0,02	-0,46	0,27	0,35	0,12	-0,41	-0,43	-0,35	0,19	-0,18	-0,14	-0,34	-0,25	1,00		
X_{15}	0,49	-0,48	0,27	0,38	0,18	-0,51	-0,49	-0,50	0,72	0,22	0,03	-0,52	-0,04	-0,01	1,00	
X_{16}	0,27	0,20	0,06	-0,41	-0,63	0,32	0,24	0,32	0,01	0,24	0,28	0,32	-0,56	0,15	0,17	1,00
X_{17}	-0,19	-0,16	0,05	0,18	0,17	-0,03	-0,06	0,07	-0,29	-0,15	-0,19	0,16	0,25	-0,05	-0,40	-0,65
X_{18}	-0,48	0,27	-0,20	-0,14	0,16	0,20	0,33	0,15	-0,44	-0,31	-0,22	0,13	0,56	-0,10	-0,61	-0,60
X_{19}	-0,10	0,05	-0,21	0,22	0,44	-0,22	-0,18	-0,22	0,08	-0,04	-0,03	-0,30	0,11	-0,10	-0,03	-0,33
X_{20}	0,08	-0,34	0,18	0,28	-0,03	-0,49	-0,41	-0,50	0,40	-0,08	-0,11	-0,33	-0,10	0,02	0,32	-0,12
X_{21}	-0,08	0,42	-0,35	-0,18	-0,12	0,69	0,39	0,75	-0,51	0,12	0,06	0,71	-0,19	-0,26	-0,27	0,27
X_{22}	0,20	0,26	0,02	-0,43	-0,38	0,72	0,49	0,75	-0,49	0,36	0,16	0,60	0,02	-0,11	-0,54	0,12

X_{17}	1,00					
X_{18}	0,61	1,00				
X_{19}	-0,34	-0,19	1,00			
X_{20}	0,23	-0,05	-0,28	1,00		
X_{21}	-0,05	-0,12	0,01	-0,68	1,00	
X_{22}	0,19	0,12	-0,03	0,25	0,41	1,00

TABLE H

Product Moment Correlation Matrix
New Zealand Cities 1966
(Normalised Data Distribution)

Variable																
X_1	1.00															
X_2	-0.52	1.00														
X_3	0.72	-0.78	1.00													
X_4	-0.18	-0.50	-0.15	1.00												
X_5	-0.36	-0.19	-0.33	0.78	1.00											
X_6	-0.03	0.63	-0.20	-0.71	-0.53	1.00										
X_7	-0.05	0.71	-0.18	-0.88	-0.63	0.88	1.00									
X_8	-0.04	0.58	-0.21	-0.62	-0.46	0.98	0.78	1.00								
X_9	0.31	-0.62	0.36	0.48	0.31	-0.68	-0.61	-0.69	1.00							
X_{10}	0.52	0.29	0.08	-0.58	-0.53	0.62	0.61	0.58	-0.36	1.00						
X_{11}	0.24	0.07	0.20	-0.40	-0.45	0.12	0.18	0.09	-0.38	0.12	1.00					
X_{12}	-0.27	0.40	-0.16	-0.40	-0.33	0.72	0.53	0.76	-0.53	0.20	-0.08	1.00				
X_{13}	0.16	0.04	0.06	-0.16	0.08	0.10	0.28	0.05	-0.08	0.33	-0.29	-0.22	1.00			
X_{14}	0.12	-0.57	0.36	0.41	0.09	-0.51	-0.60	-0.45	0.39	-0.31	-0.01	-0.33	-0.31	1.00		
X_{15}	0.55	-0.48	0.28	0.38	0.18	-0.54	-0.50	-0.52	0.72	-0.08	0.03	-0.56	-0.04	0.14	1.00	
X_{16}	0.27	0.23	0.03	-0.40	-0.60	0.35	0.26	0.37	-0.02	0.31	0.28	0.38	-0.56	0.10	0.15	1.00
X_{17}	-0.22	-0.16	0.05	0.18	0.17	0	-0.06	0.02	-0.29	-0.19	-0.19	0.18	0.25	-0.12	-0.40	-0.64
X_{18}	-0.47	0.27	-0.20	-0.14	0.16	0.17	0.29	0.14	-0.44	-0.14	-0.22	0.11	0.56	-0.26	-0.61	-0.57
X_{19}	-0.15	0.22	-0.08	-0.25	-0.23	0.31	0.34	0.31	-0.22	0.10	-0.25	0.50	0.13	0.03	-0.30	0.19
X_{20}	0.11	-0.34	0.18	0.28	-0.03	-0.56	-0.43	-0.55	0.40	-0.28	-0.11	-0.40	-0.10	0.20	0.32	-0.14
X_{21}	-0.22	0.42	-0.35	-0.17	-0.12	0.71	0.44	0.75	-0.51	0.29	0.06	0.71	-0.19	-0.36	-0.27	0.79
X_{22}	0.04	0.26	0.02	-0.43	-0.38	0.72	0.52	0.74	-0.49	0.37	0.16	0.52	0.02	-0.11	-0.54	0.13

X_{17}	1.00					
X_{18}	0.61	1.00				
X_{19}	0.27	0.34	1.00			
X_{20}	0.23	-0.05	-0.19	1.00		
X_{21}	-0.05	-0.12	0.15	-0.68	1.00	
X_{22}	0.18	0.12	0.10	-0.25	0.41	1.00

TABLE I

Product Moment Correlation Matrix
New Zealand Cities 1971
(Basic Data Distribution)

Variable																
X_1	1.00															
X_2	-0.38	1.00														
X_3	0.47	-0.64	1.00													
X_4	-0.12	-0.40	-0.45	1.00												
X_5	-0.27	0.03	-0.61	0.70	1.00											
X_6	0.12	0.55	-0.09	-0.53	-0.29	1.00										
X_7	0.06	0.62	0.11	-0.56	-0.53	0.56	1.00									
X_8	0.11	0.35	-0.16	-0.22	-0.10	0.92	0.19	1.00								
X_9	0.12	-0.34	0.02	0.38	0.28	-0.52	-0.19	-0.48	1.00							
X_{10}	0.90	-0.08	0.28	-0.25	-0.28	0.33	0.19	0.25	-0.07	1.00						
X_{11}	0.15	-0.26	0.41	-0.19	-0.16	-0.22	-0.02	-0.25	0.34	0.04	1.00					
X_{12}	-0.13	0.42	-0.21	-0.25	-0.07	0.78	0.36	0.75	-0.58	0.08	-0.32	1.00				
X_{13}	0.31	-0.16	0.36	-0.24	-0.23	0.19	0.24	0.11	-0.23	0.33	-0.17	0.27	1.00			
X_{14}	0.11	-0.20	0.06	0.17	-0.10	-0.16	-0.18	-0.11	0.19	-0.03	0.20	-0.20	-0.32	1.00		
X_{15}	0.44	-0.29	0.05	0.28	0.21	-0.32	-0.21	-0.28	0.63	0.25	0.28	-0.41	-0.14	-0.09	1.00	
X_{16}	0.38	0.08	0.27	-0.42	-0.56	0.28	0.35	0.17	-	0.31	-0.16	-0.06	-0.04	0.32	0.01	1.00
X_{17}	-0.23	-0.16	0.08	0.09	0.07	0.13	-0.14	0.21	-0.33	-0.17	0.09	0.50	0.31	0.13	-0.02	-0.51
X_{18}	-0.32	0.08	0.03	-0.14	-0.27	-0.09	0.14	-0.17	-0.29	-0.29	-0.34	0.02	0.50	-0.44	-0.33	0.01
X_{19}	-0.13	0.13	-0.32	0.23	0.47	-0.13	-0.16	-0.08	0.14	-0.06	0.14	-0.15	-0.43	-0.14	-0.09	-0.44
X_{20}	-0.11	-0.55	0.17	0.45	0.29	-0.69	-0.50	-0.58	0.34	-0.30	0.04	-0.45	0.11	-0.06	0.31	-0.52
X_{21}	0.06	0.34	-0.15	-0.21	-0.10	0.57	0.33	0.51	-0.06	0.19	-0.10	0.39	-0.29	-0.18	-0.23	0.18
X_{22}	-0.33	0.03	-0.35	0.39	0.32	-0.06	-0.44	0.14	0.10	-0.33	-0.14	-0.17	-0.07	0.17	-0.13	-0.13

X_{17}	1.00					
X_{18}	0.13	1.00				
X_{19}	-0.34	-0.56	1.00			
X_{20}	0.31	0.22	-0.09	1.00		
X_{21}	-0.19	-0.23	0.25	-0.51	1.00	
X_{22}	-0.07	-0.04	0.25	-0.02	-0.30	1.00

TABLE J

Product Moment Correlation Matrix
New Zealand Cities 1971
(Normalised Data Distribution)

Variable																
X_1	1.00															
X_2	-0.50	1.00														
X_3	0.68	-0.64	1.00													
X_4	-0.23	-0.40	-0.45	1.00												
X_5	-0.41	0.03	-0.61	0.70	1.00											
X_6	0.03	0.55	-0.09	-0.53	-0.29	1.00										
X_7	0.04	0.63	0.12	-0.87	-0.53	0.56	1.00									
X_8	0.03	0.35	-0.16	-0.22	-0.10	0.92	0.19	1.00								
X_9	0.17	-0.26	-	0.30	0.29	-0.45	-0.23	-0.42	1.00							
X_{10}	0.55	0.26	0.27	-0.62	-0.51	0.49	0.61	0.31	-0.33	1.00						
X_{11}	0.27	-0.26	0.41	-0.19	-0.16	-0.22	0.01	-0.25	0.37	0.10	1.00					
X_{12}	-0.21	0.30	-0.22	-0.08	0.04	0.68	0.12	0.75	-0.52	0.23	-0.24	1.00				
X_{13}	0.33	-0.16	0.36	-0.24	-0.23	0.19	0.21	0.12	-0.21	0.29	-0.17	0.11	1.00			
X_{14}	0.21	-0.38	0.24	0.15	-0.10	-0.26	-0.21	-0.19	0.11	-0.05	0.33	-0.15	-0.36	1.00		
X_{15}	0.44	-0.29	0.05	0.28	0.21	-0.32	-0.20	-0.28	0.61	0.03	0.28	-0.39	-0.14	0.21	1.00	
X_{16}	0.44	0.09	0.27	-0.42	-0.55	0.31	0.38	0.20	0.05	0.30	-0.19	-0.04	-0.03	0.26	-0.01	1.00
X_{17}	-0.28	-0.16	0.08	0.09	0.07	0.13	-0.16	0.22	-0.38	-0.16	0.09	0.51	0.31	0.12	-0.20	-0.49
X_{18}	-0.16	0.05	0.06	-0.13	-0.30	-0.06	0.11	-0.14	-0.25	-0.18	-0.39	-0.06	0.57	-0.46	-0.33	0.11
X_{19}	-0.06	0.27	0.05	-0.37	-0.32	0.55	0.46	0.41	-0.14	0.24	0.07	0.43	0.28	-0.11	0.12	0.08
X_{20}	-0.04	-0.56	0.17	0.45	0.29	-0.69	-0.52	-0.58	0.28	-0.49	0.04	-0.41	0.11	-	0.31	-0.52
X_{21}	-0.12	0.34	-0.15	-0.21	-0.10	0.37	0.33	0.52	-0.07	0.06	-0.10	0.35	-0.29	-0.21	-0.23	0.18
X_{22}	-0.24	0.03	-0.35	0.38	0.32	-0.06	-0.46	0.14	0.11	-0.33	-0.16	-0.15	-0.07	0.03	-0.13	-0.13

X_{17}	1.00					
X_{18}	0.11	1.00				
X_{19}	0.43	0.15	1.00			
X_{20}	0.31	0.15	-0.25	1.00		
X_{21}	-0.19	-0.23	0.07	-0.51	1.00	
X_{22}	-0.07	-0.06	-0.21	-0.02	-0.30	1.00

TABLE K

Product Moment Correlation Matrix
New Zealand Cities 1951-71
(Basic Data Distribution)

Variable																
X_1	1.00															
X_2	0.03	1.00														
X_3	-0.05	-0.87	1.00													
X_4	0.03	-0.11	-0.36	1.00												
X_5	-0.34	-0.48	0.23	0.49	1.00											
X_6	-0.13	0.06	-0.07	-0.24	-0.05	1.00										
X_7	-0.16	0.06	0.19	-0.52	-0.19	0.66	1.00									
X_8	-0.10	0.05	0.01	-0.10	-	0.96	0.43	1.00								
X_9	0.27	-0.08	-0.03	0.12	-0.05	-0.47	-0.13	-0.51	1.00							
X_{10}	0.83	0.21	-0.14	-0.13	-0.33	0.06	0.02	0.06	0.04	1.00						
X_{11}	-0.03	0.05	-0.09	0.10	-0.03	0.33	-0.05	0.47	-0.29	-0.02	1.00					
X_{12}	-0.20	-0.28	0.32	-0.10	0.16	0.53	0.44	0.50	-0.21	-0.16	0.29	1.00				
X_{13}	0.20	0.31	-0.30	-0.02	-0.12	0.07	0.03	0.06	-0.22	0.34	0.04	-0.25	1.00			
X_{14}	-0.07	-0.10	0.08	0.03	0.06	0.17	0.22	0.11	0.04	-0.06	-0.12	0.11	0.10	1.00		
X_{15}	0.49	-0.03	-0.13	0.03	-0.04	-0.34	-0.18	-0.33	0.60	0.14	-0.08	-0.19	-0.18	-0.13	1.00	
X_{16}	0.20	0.16	-0.03	-0.22	-0.41	0.33	0.35	0.28	0.10	0.09	0.12	0.24	-0.38	0.07	0.29	1.00
X_{17}	-0.17	-0.11	-0.01	0.19	0.18	-0.22	-0.34	-0.13	-0.28	-0.05	-0.04	-0.06	0.23	-0.13	-0.34	-0.65
X_{18}	-0.40	-0.01	0.09	-0.16	0.13	0.27	0.31	0.20	-0.34	-0.17	-	0.10	0.58	0.28	-0.57	-0.41
X_{19}	-0.08	-0.03	0.06	-0.04	0.20	-0.16	-0.16	-0.14	0.10	-	-0.05	-0.13	-0.23	-0.16	-0.11	-0.30
X_{20}	-0.14	-0.41	0.29	0.24	0.26	-0.14	0.13	-0.20	0.18	-0.24	-0.16	0.25	-0.33	0.01	0.23	0.02
X_{21}	-0.16	-0.40	0.40	-	0.40	0.50	0.46	0.43	-0.05	-0.13	-	0.42	-0.30	0.12	-0.07	0.13
X_{22}	-0.11	-0.26	0.22	0.02	0.14	0.40	0.19	0.40	-0.15	-0.12	0.09	0.34	-0.12	0.21	-0.19	0.07

X_{17}	1.00					
X_{18}	0.22	1.00				
X_{19}	-0.24	-0.30	1.00			
X_{20}	0.09	-0.12	-0.12	1.00		
X_{21}	-0.17	0.04	0.06	0.36	1.00	
X_{22}	-0.04	0.14	-0.09	0.15	0.32	1.00

TABLE L

Product Moment Correlation Matrix
Averaged: New Zealand Cities 1951-1971
(Basic Data Distribution)

Variable																
X_1	1.00															
X_2	-0.07	1.00														
X_3	0.05	-0.77	1.00													
X_4	0.06	-0.12	-0.48	1.00												
X_5	-0.34	-0.27	-0.10	0.52	1.00											
X_6	-0.10	0.36	-0.11	-0.33	-0.21	1.00										
X_7	-0.14	0.46	-	-0.66	-0.42	0.61	1.00									
X_8	-0.07	0.27	-0.14	-0.16	-0.11	0.96	0.37	1.00								
X_9	0.30	-0.10	0.03	0.15	-0.06	-0.54	-0.18	-0.56	1.00							
X_{10}	0.84	0.07	-	-0.06	-0.29	0.11	0.12	0.16	0.07	1.00						
X_{11}	0.07	0.06	0.03	-0.11	-0.18	0.14	-0.04	0.16	-0.14	0.05	1.00					
X_{12}	-0.20	0.24	-0.04	-0.30	-0.15	0.67	0.46	0.63	-0.38	-	0.10	1.00				
X_{13}	0.14	0.03	-0.03	0.01	0.06	0.23	0.22	0.18	-0.23	0.28	-0.09	0.06	1.00			
X_{14}	-0.05	-0.04	0.01	0.08	-0.01	-0.07	0.03	-0.10	0.10	0.02	-0.11	-0.13	0.05	1.00		
X_{15}	0.56	0.05	-0.13	0.34	-0.09	-0.44	-0.30	-0.41	0.64	0.25	0.04	-0.40	-0.13	-0.11	1.00	
X_{16}	0.24	0.04	-0.16	-0.25	-0.51	0.26	0.30	0.27	0.15	0.15	0.08	0.12	-0.36	0.07	0.26	1.00
X_{17}	-0.16	-0.31	0.12	0.20	0.29	-0.12	-0.30	-0.03	-0.36	-0.16	0.02	0.11	0.17	-0.07	-0.32	-0.60
X_{18}	-0.44	-0.01	0.09	-0.16	0.14	0.27	0.35	0.19	-0.36	-0.18	-0.17	0.21	0.61	0.12	-0.57	-0.43
X_{19}	-0.16	-0.17	0.17	-0.07	0.27	-0.14	-0.13	-0.12	0.10	-0.03	-	-0.12	-0.29	-0.15	-0.09	-0.27
X_{20}	-0.09	-0.13	-0.01	0.27	0.04	0.48	-0.25	-0.46	0.21	-0.15	-0.10	-0.24	-0.12	-0.10	0.22	-0.14
X_{21}	-0.12	0.09	-0.02	-0.03	0.20	0.37	0.17	0.37	-0.17	0.04	0.09	0.28	-0.18	-0.11	-0.19	-0.08
X_{22}	-0.05	0.06	-0.10	-	-0.08	0.19	-0.02	0.24	-0.15	-0.01	0.11	0.11	0.01	0.11	-0.26	-0.03

X_{17}	1.00					
X_{18}	0.13	1.00				
X_{19}	-0.34	-0.29	1.00			
X_{20}	0.31	-0.09	-0.11	1.00		
X_{21}	-0.19	-0.04	0.23	-0.22	1.00	
X_{22}	-0.07	0.11	-0.04	-0.09	0.19	1.00

APPENDIX III

2- AND 4-FACTOR VARIMAX MODELS

The 2- and 4-Factor varimax models given in this appendix demonstrate differences in factor models constructed from data with difference base distributions. Two types of data distribution have been used in the study:-

a) Basic data - computed from data that has not been subject to any form of transformation.

b) Transformed data - computed from data that has been subject to a normal transformation, i.e. converted to a normal distribution.

A comparison of differences between basic data and the transformed data used in the text is clearly demonstrated in the final solutions given in the 2- and 4-factor models. Both types of model show differences in the associations of the variables that constitute each hypothesised factor. The differences are further demonstrated in the models combining the 1951 to 1971 results. Not only can the contrast be seen between the basic data models and the normalised data but the averaged model, derived from averaging each of the matrices based on basic data for each of the years 1951, 1956, 1961, 1966 and 1971, also displays unique characteristics when compared with the other two types of models. This difference in data base and the differences in computed models is of major importance in terms of the considerable variety of interpretations that may be placed upon the role of the factor model and what it demonstrates. Moreover, it emphasised the need for care when interpreting factor models.

The following information is provided in this appendix for comparison purposes:-

1. 2-Factor models derived from basic data and for the years 1951, 1956, 1961, 1966 and 1971 (Table A).
2. 4-Factor models derived from basic data and for the years 1951, 1956, 1961, 1966 and 1971 (Table B).
3. 2-Factor models derived from basic data and averaged correlations for 1951-1971 (Table C).
4. 4-Factor models derived from basic data and averaged correlations for 1951-1971 (Table D).

TABLE A

New Zealand Cities - 2 Factor Varimax Model (Basic Data)

	1951			1956			1961			1966			1971		
Variable	F_1 1951	F_2 1951	h^2	F_1 1956	F_2 1956	h^2	F_1 1961	F_2 1961	h^2	F_1 1966	F_2 1966	h^2	F_1 1971	F_2 1971	h^2
1	(-.63)	-	.40	-.24	(-.50)	.31	.09	(.59)	.36	-.10	(.67)	.45	-.23	(.66)	.49
2	-.22	(-.90)	.86	.27	(-.67)	.52	.05	(.55)	.30	(.70)	-.22	.54	(.73)	-.16	.56
3	.28	(.65)	.50	-.28	(.62)	.46	-.26	-.44	.26	-.27	.42	.25	-.35	(.72)	.64
4	-.19	.04	.04	.11	.05	.01	.44	.10	.20	(-.72)	-.28	.60	-.44	(-.67)	.65
5	.34	.46	.32	.13	(.82)	.69	.26	-.46	.27	(-.52)	(-.55)	.57	-.16	(-.76)	.60
6	(.61)	(-.76)	.95	(.93)	-.07	.87	(-.99)	-.10	1.00	(.99)	.10	.98	(.89)	.30	.89
7	.24	(-.60)	.42	.33	-.24	.16	(-.81)	.04	.65	(.87)	.04	.76	(.54)	(.52)	.57
8	(.54)	(-.67)	.74	(.89)	-.01	.79	(-.83)	-.14	.71	(.94)	.09	.90	(.74)	.09	.55
9	(-.68)	.04	.47	(-.52)	(-.52)	.55	-.40	.42	.34)	(-.78)	.30	.70	(-.62)	-.09	.39
10	-.23	-.09	.06	.09	-.37	.14	-.18	.41	.20	.16	(.50)	.28	.05	(.61)	.37
11	.06	-.47	.22	(.71)	-.01	.51	.29	-.04	.08	.19	.29	.12	-.35	.20	.17
12	.12	(-.59)	.36	(.69)	.10	.48	(-.65)	-.29	.51	(.79)	.02	.62	(.79)	.01	.63
13	.36	.18	.16	(.55)	-.10	.31	-.49	-.11	.25	.06	-.28	.08	.08	.33	.12
14	.46	-.23	.27	.24	-.09	.07	.29	-.04	.08	-.38	.05	.15	-.23	.06	.06
15	(-.89)	-.11	.80	(-.51)	(-.65)	.69	.39	(.75)	.72	(-.62)	(.50)	.64	(-.53)	.10	.29
16	-.33	(-.82)	.78	-.03	(-.62)	.39	-.15	(.81)	.69	.29	(.76)	.66	.13	(.57)	.35
17	.01	(.51)	.26	-.01	(.51)	.26	.22	(-.69)	.52	.05	(-.52)	.28	.10	-.13	.02
18	(.78)	.14	.63	(.57)	.31	.42	(-.56)	(-.63)	.71	.26	(-.76)	.65	.10	.03	.01
19	-.03	.13	.62	-.23	.31	.15	.16	-.12	.04	-.16	-.16	.05	-.01	-.41	.16
20	-.35	.05	.13	-.36	.13	.15	.34	-.10	.12	-.47	.04	.22	(-.65)	-.26	.50
21	(.64)	-.25	.47	.16	(.66)	.46	.26	.36	.20	(.63)	.05	.40	(.50)	.10	.26
22	(.72)	-.01	.52	.12	(-.54)	.30	.27	-.23	.13	(.65)	.10	.43	-.02	-.47	.22

h^2 = communality

( ) = correlations ⩾ ± 0.50

TABLE B

New Zealand Cities - 4 Factor Varimax Model (Basic Data)

		1951						1956					1961		
Variable	F_1 1951	F_2 1951	F_3 1951	F_4 1951	h^2	F_1 1956	F_2 1956	F_3 1956	F_4 1956	h^2	F_1 1961	F_2 1961	F_3 1961	F_4 1961	h^2
1	(-.80)	-.03	-.21	.26	.76	-.15	(-.51)	.04	-.38	.43	.48	.15	.12	(-.74)	.82
2	-.16	(-.88)	-.01	.23	.86	.14	(-.62)	.30	.23	.54	.08	.43	(.78)	.06	.81
3	.14	(.62)	-.10	(-.70)	.91	-.16	(.86)	.20	-.09	.81	-.24	-.08	(-.96)	-	.98
4	-.02	.17	.18	(.86)	.80	.17	-.29	(-.69)	-.29	.68	.37	-.43	(.70)	-.04	.81
5	.44	(.59)	.05	.39	.69	.12	(.63)	(-.51)	.09	.68	.05	(-.51)	-.05	.17	.30
6	(.64)	(-.70)	-.18	.16	.96	(.96)	-.03	.18	.21	1.01	(-.85)	.15	-.13	-.45	.97
7	.21	(-.65)	-.11	-.28	.55	.12	-.05	(.62)	(.76)	.98	(-.71)	.39	-.13	-.18	.70
8	(.61)	(-.60)	-.14	.25	.81	(1.00)	-.03	.02	.02	1.02	(-.73)	.02	-.11	-.44	.74
9	(-.66)	-.02	.22	-.11	.50	(-.51)	-.34	.44	-.13	.58	(.53)	.37	-.02	.36	.44
10	-.46	-.12	-.48	.21	.50	.11	-.36	.08	-.10	.16	.14	.11	.04	(-.73)	.57
11	.07	-.46	-.05	.07	.22	(.88)	-.13	-.18	-.16	.86	.33	-.17	-.22	-.06	.19
12	.24	(-.65)	.28	-.37	.70	(.71)	.06	-.07	.12	.51	(-.72)	.07	-.13	.02	.55
13	.05	.21	(-.90)	.11	.86	.28	-.39	-.42	(.59)	.75	(-.57)	-.31	.45	-.48	.85
14	.22	-.26	(-.74)	-.01	.66	-.03	-.10	.09	(.66)	.46	.16	-.01	.09	.32	.14
15	(-.82)	-.12	.22	.32	.84	-.40	(-.65)	.12	(-.51)	.85	(.64)	.31	.40	-.26	.74
16	-.15	(-.81)	.33	.11	.80	.13	-.37	(.70)	-.26	.70	.14	(.88)	.22	-.09	.84
17	.11	(.53)	.29	-.04	.38	-.10	.21	(-.80)	.06	.70	-.10	(-.90)	.10	.12	.85
18	(.54)	.14	(-.69)	-.23	.84	.26	.16	-.31	(.85)	.92	(-.82)	-.32	-.02	.11	.79
19	-.02	.11	.12	-.21	.07	-.02	(.57)	.32	-.31	.52	.25	.01	(-.61)	.06	.44
20	-.21	.06	.34	.12	.18	-.34	.08	-.15	-.15	.16	.16	-.18	.25	.31	.21
21	(.67)	-.20	-.11	-.01	.50	.30	(.65)	-.20	-.13	.57	.46	.22	-.13	-.13	.30
22	(.71)	.06	-.25	.18	.61	.06	(-.56)	.07	.02	.33	.14	-.11	-.18	.38	.17

h^2 = communality

( ) = correlations ⩾ ± 0.50

Table B (Contd.)

		1966						1971		
Variable	F_1 1966	F_2 1966	F_3 1966	F_4 1966	h^2	F_1 1971	F_2 1971	F_3 1971	F_4 1971	h^2
1	.11	.15	(.96)	.15	.98	.06	.30	.17	(.87)	.89
2	(.77)	.06	-.29	-.09	.69	(.50)	.22	-.44	(-.51)	.75
3	.45	-.05	.39	(.53)	.64	-.27	.48	.49	.39	.69
4	(-.58)	-.02	-.10	(-.60)	.71	-.25	(-.85)	-.08	.12	.80
5	-.30	-.24	-.13	(-.87)	.92	-.03	(-.80)	-.27	-.04	.72
6	(.96)	-.04	.14	.26	1.00	(.92)	.31	-	-.01	.97
7	(.81)	-.12	.07	.33	.78	.34	(.75)	-.06	-.19	.72
8	(.93)	-	.12	.22	.92	(.90)	.01	.03	.07	.82
9	(-.73)	.22	.29	-.21	.70	(-.54)	-.17	-.30	.35	.54
10	.22	.06	(.88)	.04	.82	.29	.34	.10	(.67)	.66
11	.09	.26	-.07	.36	.21	-.28	.09	-.05	.30	.18
12	(.73)	.03	-.11	.28	.63	(.87)	-.02	.25	-.21	.86
13	.11	(-.60)	.33	-.16	.51	.13	.18	(.70)	-	.54
14	-.42	.11	-.12	.05	.21	-.14	-.03	-.11	.26	.10
15	(-.56)	.39	.41	-.13	.67	-.35	-.11	-.10	(.52)	.42
16	.21	(.83)	.08	.44	.93	.06	(.64)	-.15	.20	.48
17	-.02	(-.74)	-.14	.11	.58	.22	-.33	(.60)	-.14	.55
18	.22	(-.86)	-.33	.05	.89	-.16	.22	(.51)	(-.62)	.71
19	-	.07	-	(-.52)	.28	.03	-.34	(-.57)	.05	.44
20	(-.56)	-.13	.02	.18	.37	(-.62)	-.43	.42	-.02	.75
21	(.73)	.25	-.04	-.13	.62	(.51)	.19	.36	.06	.43
22	(.60)	-.12	.19	.28	.49	-.02	-.40	-.17	-.16	.21

TABLE C

New Zealand Cities - 2 Factor Varimax Models (Basic Data and Averaged)

	1951-71 (Basic Data)[1]			1951-1971 (Averaged)[2]		
Variable	F_1 1951-71	F_2 1951-71	h^2	F_1 1951-1971	F_2 1951-1971	h^2
1	.78	-.38	.18	-.14	(.56)	.34
2	-.14	(-.79)	.64	.43	.29	.27
3	-.06	(.64)	.41	-.10	-.12	.02
4	.34	.13	.13	-.47	-.11	.23
5	.15	(.59)	.38	-.32	(-.58)	.45
6	(-.99)	.09	.98	(.96)	-.06	.93
7	(-.64)	.12	.42	(.69)	.11	.49
8	(-.87)	.06	.76	(.83)	-.08	.70
9	(.50)	.02	.25	(-.52)	.47	.49
10	-.03	-.47	.22	.11	.39	.16
11	-.31	-.05	.10	.14	.11	.03
12	(-.51)	.45	.45	(.67)	-.14	.47
13	-.14	-.44	.21	.14	-.36	.15
14	-.19	.12	.05	-.03	-.01	-
15	.44	-.08	.20	(-.50)	(.65)	.67
16	-.27	-.10	.08	.35	(.75)	.68
17	.15	.05	.02	-.18	(-.56)	.35
18	-.37	.05	.14	.31	(-.60)	.45
19	.19	.08	.04	-.14	-.02	.02
20	.19	(.54)	.32	-.46	-.06	.21
21	-.36	(.60)	.50	.31	-.03	.09
22	-.35	.34	.24	.19	-.08	.04

[1] Basic raw data combined 1951-71.

[2] Averaged correlation matrices 1951 to 1971.

h^2 = communality

( ) = correlations ⩾ ± 0.50

TABLE D

New Zealand Cities - 4 Factor Varimax Models (Basic Data and Averaged)

	1951-71 (Basic Data)[1]					1951-1971 (Averaged)[2]				
Variable	$F_1$1951-71	$F_2$1951-71	$F_3$1951-71	$F_4$1951-71	h^2	$F_1$1951-1971	$F_1$1951-1971	$F_3$1951-1971	$F_4$1951-1971	h^2
1	.16	-.33	.46	-	.35	-.32	.23	(-.92)	.05	.99
2	-.10	(-.79)	.05	-.04	.64	.33	.38	-.02	(-.63)	.65
3	.09	(.70)	-.05	.43	.69	-.05	-.04	-.01	(.86)	.74
4	-	.14	.02	(-.84)	.72	-.43	-.40	-	(-.52)	.62
5	-.04	(.59)	-.29	-.42	.61	-.11	(-.70)	.16	-.19	.57
6	(-.94)	.01	-.03	.31	.98	(.93)	.19	-.14	-.12	.93
7	-.44	.11	.06	(.66)	.64	(.62)	.34	-.01	.09	.52
8	(-.96)	-.02	-.05	.09	.92	(.81)	.12	-.15	-.17	.72
9	.47	.12	.47	-.04	.46	(-.64)	.25	-.09	-.01	.48
10	.03	-.44	.24	.11	.26	.01	.16	(-.83)	.05	.72
11	-.44	-.10	-	-.17	.23	.09	.15	-.04	.02	.03
12	(-.53)	.41	.02	.17	.48	(.67)	.11	.03	-.02	.46
13	-.01	(-.52)	-.44	.05	.47	.32	(-.60)	-.45	-.08	.67
14	-.12	.10	-.09	.16	.06	-.02	-.03	-.01	-	-
15	.28	.01	(.69)	-.27	.63	(-.68)	.28	-.35	-.25	.73
16	-.35	-.07	(.72)	.28	.73	.08	(.81)	-.13	-.19	.71
17	.16	-	(-.57)	-.25	.41	.02	(-.59)	.08	.10	.36
18	-.17	-.04	(-.74)	.29	.67	(.50)	-.47	.07	.12	.49
19	.16	.10	.03	-.08	.04	-.15	.07	.24	.17	.11
20	.09	(.55)	.12	-.08	.34	-.41	-.17	.14	-.03	.22
21	-.41	(.59)	.08	.14	.54	.29	.12	.10	-.06	.11
22	-.38	.30	-.06	.07	.24	.20	-.01	.02	-.06	.05

[1] Basic raw data combined 1951-71.

[2] Averaged correlation matrices 1951 to 1971.

h^2 = communality

( ) = correlation $\geqq \pm$ 0.50

APPENDIX IV

COMPUTATION METHODOLOGY

All Factor models computed in this study were based upon the 1130 Statistical System. The package was used on Massey University Computer Unit's 1130 for which the system was designed. An outline of the package is given below but for a full description the reader is referred to the "User's Manual" which contains an outline of the type of analysis, a description of the computational algorithms used, the form and content of the control, cards, operating instructions and sample problems.

One important modification to the system was made to the package. The writer is indebted to Mr. Chris Freyberg, Junior Research Officer at the Computer Unit, for the incorporation of an automatic iterative technique into the package. As the computed factor solution needs to be recycled to obtain a convergence in communalities the addition of an automatic recycling procedure, hitherto done by hand, was of considerable help in speeding up the computation of the factors.

The Factor Analysis programme is based upon the observations X_i and the output is centred upon a considerable array of options. A correlation matrix is computed along with means and standard deviations. A given factor matrix can be constructed while correlations between variables and the computed factors are also established. The factor matrix may be rotated to an approximate simple structure as desired. The rotation by an analytic criterion was in this instance orthogonal and the communality estimates for factoring were based upon the squared multiple correlation for the ith variable.

The characteristic roots for the above matrix with the squared multiple correlation coefficients in the doagonal were computed by a Householder tridiagonalisation followed by the use of the QR algorithm. The characteristic vectors were computed by Wilkinson's method. Loadings were calculated as requested while the rotations were made in accordance with simple structure procedure in an orthogonal reference frame by the Normal Varimax method of Kaiser. Factor scores were computed by Harman's short regression method.