Unconscious Processing in Children:

Developing Mathematical Concepts Through Mathematical Puzzles

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Abstract

The degree of occurrence of unconscious versus conscious processing of information in children is unclear. Also unclear is knowledge of mechanisms and reasons for selective transference of unconsciously processed sensory and perceptual information into conceptual, conscious and verbal awareness. Examples of unconscious processing were evidenced in the difficulty some children experienced verbalising processes and naming classifications after solving some mathematical puzzles. Correct solutions indicated children had clear conceptual understanding of the structure of the task. A series of personally designed junior mathematical puzzles utilising environmental materials to aid development of pre-mathematical concepts of classification, patterning, seriation, ideas of conservation and one to one correspondence in five and six year old New Zealand primary school children were used, and extended to incorporate mathematical concepts taught to twelve year old children at New Zealand intermediate school Form 2 level. These senior puzzles incorporated concepts of set theory, probability, matrices, tessellations, and rotational patterning and ordering, with some puzzles developed to adult difficulty levels. Some adults had difficulty with some junior puzzles, and found senior puzzles as difficult as the twelve year olds they were designed for. Mathematically able six year old children solved some senior puzzles successfully. A hypothesis developed that children could master mathematical concepts considered beyond their age ability defined by the current school curriculum, provided concepts were presented in manipulable and visual form. This was supported in the present study in 1985 - 1986 where 92 six to ten year old junior and senior children with the highest and lowest mathematical ability or special learning difficulties in three primary schools researched the puzzles. Schools selected senior children from national age normed Progressive Achievement Tests (P.A.T.) mathematics results. Junior children were teacher assessed. Unexpected findings included unconscious processing of some unfamiliar concepts with difficulties verbalising unfamiliar and familiar concepts, contrasting

conscious deliberation required for multiple concepts, transfer of learning and use of strategy, indicating ability, especially in conjunction with speed, novel approach, use of symmetry and a younger age of child. Puzzles were diagnostic in detecting and in remediating mathematical understanding of single concepts. The formal mathematics of some remedial and extension children improved, suggesting unconscious transfer of concepts, and some children who previously disliked mathematics or school in general developed a liking for both. No gender performance differences emerged. P.A.T. performance did not correlate with puzzle performance, emphasising differences between P.A.T. formal verbal mathematics and nonverbal visual spatial logic puzzle mathematics, or predominantly left versus right brain mathematical processing respectively, possibly explaining difficulties children had verbalising nonverbal actions. Two P.A.T. average children included with extension children performed above the highest P.A.T. children. Lack of P.A.T. correlation indicates formal mathematics alone may be inadequate to identify all mathematically able children, to remediate all having difficulties, or to extend those needing lateral enrichment. As pre-mathematical concepts incorporated into junior puzzles are prerequisites for formal mathematics, mathematical concepts incorporated into senior puzzles may aid unconscious transfer into formal mathematics through conscious awareness from verbal introspection, providing useful remediation and enrichment if embedded within future curricula.

For my children,

Andrea, David, Catherine, Steven and Timothy,

and the creativity

of all the world's children.

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Preface and Overview

'Unconscious Processing in Children,' presents experiences indicating information transmits both unconsciously and consciously, intrapsychically within the person, interpsychically between people, and between the person and the environment, with information processed, interpreted, integrated and applied both unconsciously and consciously. Little is known of how sensations translate into perceptions, and perceptions move into concepts. Little is also known of how concepts are available to operable consciousness and action awareness, or of how action awareness transfers into consciously expressible language. The operation of multiple concepts, transfer of learning and strategic application indicate conscious awareness of structure, and applied conscious choice.

The differentiation between what is consciously learnt and what is unconsciously absorbed and processed is unclear. Active awareness and focus of attention may express through sensory and motor systems as action, or verbally through verbal thought and speech. Individuals 'know' more than conscious content. Mechanisms selecting incoming and processed information into conscious awareness are not established, nor is there a unified definition of consciousness or unconsciousness. Why do some items and not others become conscious? Is consciousness to be defined in terms of what can be verbally described, when actions describe knowledge awareness where language can not? Why some items come to conscious awareness and some do not is unknown, but consciousness manifestations must be selected by unconscious mechanisms via sensory awareness into perceptual awareness, and have central meaning to the individual, showing exercise of free will operates strongly at both unconscious and conscious levels.

The degree of conscious scanning in which a person participates is dependent on the degree of sensory and selective perceptual and conceptual awareness coming into conscious attention. This focus of attention is also dependent on individual consciousness states which may alter the nature of the potential for conscious processing at any given time. Variables such as sleeping, waking, degree of consciousness, alertness, tiredness, physique, health, emotions, aloneness, togetherness, age, sensory and perceptual acuity, cognitive awareness and integration, motivation, and cultural biases and language will affect what comes into conscious awareness. Unconscious scanning and processing appear fully present and constantly aware through all consciousness variables, with overall awareness of experiences and learnings.

The present study looks at how specific intrapsychic nonverbal and possibly unconscious information transmission processes contrast with specific intrapsychic verbal and conscious information transmission processes involved in the solving of mathematical puzzles designed for children and adults, and the interacting effect external judicious questioning has in extending these processes. The focus on unconscious processing emerged from the present study, in the part unconscious processes played in supporting the original hypothesis that children are able to understand mathematical concepts at much higher levels than is generally believed or taught within the educational system, and from unexpected findings that emerged. Unconscious processing is the theme underlying each major finding in the present study, for which there are no definitive interpretations. They include:

- Performing a correct answer without conscious awareness of the process, and difficulty verbalising classifications.

- The ability of children to perform above what is considered usual for their age.

- The breaking of Piaget's linear steps of conceptual development, with mixed conceptual performance in children.

- High ability untapped and unknown in two children considered average. What are the P.A.T.'s and the curriculum missing? What is the unidentified ability?

- The incidence of upper and lower ranges of children's formal mathematics improving using the puzzles, particularly in remedial junior and senior children.

- The biggest question - how manipulating things is translated into, and the forerunner of formal mathematical work.

The present study was not intended as a scientific experiment in the traditional sense, but was an exercise in teaching in progress. Its purpose was an exercise in discovery and extension for individual children, with puzzles and questioning techniques aimed to facilitate maximun self discovery of learning and minimum content teaching. Individual work in small groups facilitated 'catching the moment' with judicious questioning helping children take the discovery as far as possible. This encompassed diagnostic, remediation and lateral extension and enrichment at every level for every age of child. Children worked at their own pace and level. Emphasis was always on building onto what children could already do and achieve by themselves, and how far this could be extended. It was essential to work with children in this manner as good teaching practice, and for the purposes of the present study to find as full a range of possibilities and benefits and uses for the puzzles as possible.

The non experimental approach was selected as non intrusive, non disruptive and low key, emotionally and educationally appropriate, and to simultaneously maximise individual educational effects. It was also necessary to simulate the way the puzzles might be used in normal classroom use to determine their impact as classroom materials. No attempt was made to experimentally have each child attempt the same puzzles, or to attempt them in the same order as any other child, for reasons of varying age groups and ability levels within mathematical groups in addition to individual differences and needs. Questioning was always responsive to the actions of individual children reflecting these differences. Maximising educational advantage meant tailoring to individually assessed needs.

The present study also sits in limbo between a quantitative and a qualitative approach. One quantitative aspect is the use of national age normed P.A.T. mathematics results of quantitative formal mathematical understanding used to group senior children for the present study. Although many puzzles had correct solutions, and some puzzles had more than one arrangement or creative interpretation for a correct answer, all puzzles allowed for individual perceptual and conceptual awareness to manifest in levels of performance. Performance levels typically ranged from too difficult through to exceptional or novel performance, with various questioning and non questioning speed and strategic performance levels exhibited within these extremes. Performance levels provided ordinal data in the present study for the qualitative nature of the puzzles themselves. Non parametric analyses were used, with Spearman's Rho correlations measuring levels of puzzle performance of children against their formal school mathematical groups, ages and class levels respectively, and Mann-Whitney U-Tests measuring gender differences in levels of performance with the puzzles.

There are three parts to the present study:

Part 1 - Introduction
Part 2 - The Puzzles
Part 3 - Implications

Part 1
Introduction

Chapter 1
Unconscious Processing

Chapter 1 looks at current possible theoretical explanations and explorations of unconscious cognitive processing, presenting some definitions of consciousness indicating lack of consensus amongst theorists as to what consciousness is. Conscious versus unconscious processes are discussed. The degree to which these occur and can be introspected also divide the theorists. Theories on modes of learning continue the debate over whether people process information consciously or unconsciously, and in which order. Issues of introspection and protocol procedures with children look at advantages and disadvantages of the talking aloud method and the clinical interview, and the part language plays in this introspection. Consciousness in the form of unconscious awareness of concepts through actions, through to conscious awareness of concepts through language expression is discussed, along with the acquisition of language as a means of bringing the concept into conscious awareness. Nonverbal versus verbal conscious awareness is discussed. Consciousness is presented as processed integrated awareness, with cerebral evidence of unconscious processing and left and right hemispheric processing leading into a discussion of possible mechanisms for the movement of sensations into perceptions and concepts, and into conscious awareness.

Chapter 2

The Present Study

Chapter 2 introduces the present study, firstly looking at aims of the present study with prior assumptions and hypotheses underlying these aims. A rationale for the use of the mathematical puzzles in the teaching of mathematics and pre-mathematics to school children incorporates Piaget's stages of cognitive development. Pre-mathematical concepts embedded in the puzzles, mathematical readiness in children, and senior puzzle concepts are described. A method section describes the participants and their selection, followed by the rationale and use of the New Zealand Progressive Achievement Tests in mathematical group selection as the quantitative measure incorporated in the present study. Materials used in the puzzles and the nature of the puzzles themselves and how children process them are the qualitative measure. Procedures and types of questioning used with the children are presented. The qualitative aspects of the data collection and analysis follow.

Part 2

The Puzzles

Chapters 3 - 8 highlight and integrate some facets of unconscious and conscious information processing observed in children attempting the mathematical puzzles.

Chapter 3

Children Unable to Verbalise Classifications

Chapter 3 discusses unconscious information transmission process events where children were able to solve some puzzles correctly which could not be solved without conceptual understanding, yet these children were unable to verbalise or name their chosen classifications and processes without extensive questioning. Puzzles incorporating process of elimination, rote performance, and confabulation follow, showing how none of these elements were involved in this unconscious processing. Chance factors are unlikely.

Chapter 4

Perceptual Priorities

Chapter 4 looks at perceptual priorities children exhibited while solving the puzzles, where unexpected concepts were likely to be processed unconsciously or to create perceptual and conceptual difficulties. Puzzles involving expected or familiar elements versus unexpected or unfamiliar elements within perceptual dimensions such as colour, shape, size, pattern, reversals and rotations are presented. Child profiles showing how children progressed through perceptual and conceptual difficulties are presented.

Chapter 5

Multiple Concepts

Chapter 5 elaborates on perceptual difficulties occurring more frequently in puzzles where in addition to perceptual dimensions, multiple concepts such as ordering, alternating or patterning required for the solution of a puzzle necessitated a conscious focus of attention.

Chapter 6

Transfer of Learning and Use of Strategy

Chapter 6 shows how some children manipulating multiple concepts exhibited a conscious transfer of learning, while other children applied a conscious use of strategy, increasing the speed of the solution of the puzzle, and indicating ability in the child concerned, especially where these approaches manifested in a younger child.

Chapter 7

Diagnostic Progressions

Chapter 7 looks more closely at several puzzles giving progressive diagnostic information on a child's level of perceptual and conceptual awareness as indicated by the levels of performance and specific difficulties encountered. They encompass all of the previously presented difficulties in previous chapters. These fall into two groups, of puzzles not involving enumeration and puzzles involving enumeration.

Chapter 8

Range in Levels of Performance

Chapter 8 presents some puzzles where children exhibited a mixed range of levels of performance within and across mathematical group categories, emphasising the general lack of correlation between mathematical group categories and levels of performance. Some puzzles present children exhibiting a mixed range of levels of performance within a particular mathematical group, similarly showing a lack of correlation between mathematical group category and levels of performance. Also presented are two children from an average mathematical group category exhibiting a high level of performance, supporting the general lack of correlation between mathematical group categories and levels of performance in this study.

Part 3

Implications

Chapter 9

Correlation Results

Chapter 9 presents correlation results from comparisons of levels of performance across mathematical group categories, school classes, and ages of children using nonparametric analyses of Spearman's Rho correlations, and Mann-Whitney U-Tests for measuring gender performance differences. These are looked at in conjunction with puzzle difficulty level.

Chapter 10

Implications in the Teaching of Mathematics

Chapter 10 looks at ways which support the hypothesis that children are able to understand mathematical concepts at much higher levels than is generally believed or taught within the educational system, and provides possible explanations for unconscious processing, with implications for the teaching of mathematics. Previously presented findings in the present study, previously unpresented findings, and findings relating specifically to unconscious processing are listed. Some possible reasons for the general lack of correlation between P.A.T. mathematics scores and puzzle performance are examined in the discussion, with an exploration of P.A.T. mathematics and the mathematical puzzles and what they each measure, and the nature and degree of these different mathematical assessments. It includes discussion on verbal and nonverbal processing and the validity of both P.A.T. mathematics and the mathematical puzzles in determining the acquisition of mathematical concepts in children. Left and right brain function, consciousness and the brain, and aspects of quantum consciousness lead into the movement of percepts to concepts and the function of language in concept formation. Thoughts on Piaget's stages of cognitive development, and the place of intuitive, creative and divergent thinking are followed by definitions of mathematical ability and able children. Acceleration and enrichment programmes are discussed, with concluding implications for the teaching of mathematics, limitations of the present study and possibilities for future research followed by a conclusion.

Appendices include junior mathematical puzzles, the Eileen Churchill Number Readiness Test, three manufactured adult puzzles, and puzzles in approximate order of difficulty.

Part 1

Introduction

Unconscious Processing

The Present Study

Each part of me acts its own part
receiving information it is designed to receive.
Each sensory system its actions.
Each cell a sensory system
built to receive and respond
to and for my whole self.
My conscious mind only one such sensory system
of my body mind soul
and verbal language one of its actions.

I know what I do when I do it.
My actions know the thoughts
that my thoughts do not yet know.
My thoughts know those thoughts
when they know where my actions come from.

Marion Moxham

Chapter 1

Unconscious Processing

The processes of transfer from sensory information into perceptions leading into concepts, and from pre-mathematical concepts leading into formal mathematical concepts have long been considered to involve some unconscious information transmission processes of an intrapsychic nature. The degree and utilisation of conscious versus unconscious, and verbal versus nonverbal processing has been largely elusive. The gradual movement from sensation to perception and from perception into conceptual understanding, and the movement of conceptual understanding that is expressed through action to conceptual understanding that can be expressed through language is a process that asks why and how do conscious and unconscious processes work.

Chapter 1 looks at current theoretical explanations and explorations of possible unconscious cognitive processing, presenting some definitions of consciousness indicating the lack of consensus amongst theorists as to what consciousness is. Conscious versus unconscious processes are discussed. The degree to which conscious and unconscious processes occur and can be introspected also divide theorists. Theories on modes of learning continue the debate over whether people process information consciously or unconsciously, and in which order and whether these processes can be introspected. Issues of introspection and protocol procedures with children look at advantages and disadvantages of the talking aloud method and the clinical interview, and the part language plays in these types of introspection. Consciousness in the form of unconscious awareness and indication of conceptual development through actions, through to conscious awareness of concepts through language expression is discussed, along with the acquisition of language as a means of aiding the bringing of a concept into conscious awareness. Nonverbal versus verbal conscious awareness is also discussed. Consciousness is presented as processed integrated awareness, with cerebral evidence of unconscious processing, and left and right hemisphere processing leading into a discussion of possible mechanisms for the movement of sensations into perceptions and concepts, and into conscious awareness and language. Physicist definitions of consciousness and its selections as to what emerges into conscious awareness through sensoy, perceptual and conceptual information transmission processes are considered.

Definitions of Consciousness

'Consciousness is primary. Its subdivisions are merely expressions of its primacy. Saying hello to itself through the different media of awareness, body action, thought, language and sensory registration. Consciousness is the only reality, yet we are able to glimpse it only through the action that gives rise to the material and mental aspects of our observational processes' (Goswami, Reed & Goswami 1993). This is one definition of consciousness. Goswami et al consider all or any expressions of consciousness are different manifestations of one overall source. Consciousness is everything there is.

There is no universally agreed upon definition of what constitutes consciousness or unconsciousness, creating interpretive difficulties in deciding on the consciousness or otherwise of many cognitive, awareness and behavioural states and activities common in people. The Reader's Digest Universal Dictionary (1988) lists some definitions:

conscious - refers to mental processes such as thoughts or emotional reactions of which a person is aware.

subconscious - pertains to thoughts or feelings outside immediate awareness either wholly or partly.

preconscious - refers to mental processes that are outside the consciousness but are easily brought into the conscious mind.

unconscious - alludes to all mental processes that a person is not aware of, including thoughts or feelings that have been forgotten or repressed, and also images, instincts, desires, and the like. It is often used interchangeably with subconscious.

Vagueness remains. Sutherland (1989) summarises the confusion over a definition of consciousness in the Macmillan Dictionary of Psychology:

consciousness - The having of perceptions, thoughts, and feelings; awareness. The term is impossible to define except in terms that are unintelligible without a grasp of what consciousness means. Many fall into the trap of equating consciousness with self-consciousness. To be conscious it is only necessary to be aware of the external world. Consciousness is a fascinating but elusive phenomenon: it is impossible to specify what it is, what it does, or why it has evolved. Nothing worth reading has been written on it.

In the absence of adequate theories of consciousness, theories focus more on aspects of what consciousness may do, to help establish what consciousness is. Examples include representation (Lloyd 1989), intentional state (Searle (1983), working memory (Anderson (1983), focal-attention processing, awareness and introspection (Velmans (1991), the hidden observer (Hilgard 1986), setting and selecting goals (Shallice 1972), and multiple awareness (Dennett 1991). Klein (1991) finds consciousness a representational workshop supporting activities of decision making, imagination, planning, problem solving, hypothesis testing and the novel use of habitual routines, but distinguishes between awareness of ideas, sensations, images, movements and actions referred to as consciousness, and awareness of oneself or self-awareness also considered as consciousness.

Dretske (1991) distinguishes further, believing consciousness is not what we are conscious of. It is not an object of awareness. It is the act of awareness, a process where we are made aware of whatever objects we are aware of. People can be conscious of the external thing they identify without being conscious of the internal processes that make them aware of it. Navon (1991) considers consciousness is the awareness of a person, but draws attention to a validity issue, where in the absence of an external observer, the mind may not readily and objectively examine itself by itself, and theories of consciousness should instead deal with issues of states, structures, representations, processes and transmission of information between processes. Consciousness is the medium where individuals obtain information on their mental events, and is both the universe to be explained by a first person account and the source of data for such an account.

Further difficulties arise in addition to what consciousness may be and what it may not be and what it may do. These concern evidence of knowledge being present in an individual without the individual having conscious awareness of such knowledge. Tulving (1985) finds evidence of subliminal perception demonstrating semantic processing can take place without awareness, and information can be available without knowledge of its source or awareness of its having been a perception at the time of stimulus exposure, as an unconscious perception. On the basis of unconsciously perceived information being accessible through some form of behavioural response, the state of being conscious may manifest as consciously known or unconsciously known awareness from the person's point of view, removing any clear boundaries between potential definitions of conscious and unconscious events.

Furthermore, the timing of the consciousness of the events adds to the complications of the definition of conscious versus unconscious processing. Perception comes from sensory information, a process which is unconscious, and concepts come from perceptions and actions, creating a time lag between events and awareness. Concepts may be defined as general ideas or understandings. They may come from finding similarities between things, or from abstracting a pattern from events or objects, and may be abstract or concrete. Wagstaff (1991) finds subjective reactions to consciousness necessarily follow the events themselves, causing actual awareness, reaction time to awareness, and memory for awareness to be easily confounded. As sensations, perceptions and conceptions are individual interpretations from events entering the sensory system rather than the events themselves, issues of reality validity are added to any questions on a definition of consciousness.

We may also possess a limited and inadequate repertoire of verbal and nonverbal responses for describing the contents of our consciousness. Consequently there is no guarantee that our descriptions of awareness will accurately reflect the actual events in consciousness or the times of their occurrence Wagstsff (1991). Lloyd (1989) finds introspective awareness shares the stage of consciousness with primary awareness or unreflective consciousness which is the awareness of the world as opposed to awareness of one's own mental states. This distinction within consciousness gives rise to the possibility of states of awareness that are neither introspective nor introspected, and there must be such states. If every state of awareness had to be the subject of a secondary state of introspective awareness, then the secondary states, themselves states of awareness, would need tertiary states of awareness directed at them, and so on in an infinite regress. The assumptions that consciousness is exclusively introspective and operationalised through verbal reports results in a narrowing of the concept of consciousness. A broader view would include non introspective awareness and would consider experience to be signalled by any discriminative response. This amounts to a broadly reductive strategy, identifying states of consciousness with causally active cognitive states.

Introspection: Conscious Versus Unconscious Processing

There are current hypotheses in the field of cognitive psychology regarding conscious versus unconscious processing and the degree to which these processes are available through introspection. Introspection is the procedures of thought in mental processes. Whereas Nisbett & Wilson (1977) claim mental processes in concept formation are unconscious and therefore not available to introspection, Ericsson & Simon (1980) find introspection a valuable aid to understanding cognitive processes in task performance as a process of conscious thought. These views are opposing. Kellogg (1982) conducted experiments showing both views are valid. Depending on task demand, unconscious or conscious thought processes or both may be used with associated degrees of reliability of introspection.

Kellogg terms this a dual-factor view of cognition, referring to unconscious processing as feature-frequency processing, and conscious processing as hypothesis testing procedures.

Kellogg's first set of experiments found a positive correlation between the degree of introspection subjects claimed to have undertaken when learning was conscious, and no correlation when it was unconscious. Subjects were given a dual task situation involving difficult oral multiplication, at the same time being shown a series of faces on a secondary channel. This was the dual task condition. Single task conditions were given just the multiplication or the faces. Accuracy of multiplication and recognition of faces was tested. Single task conditions showed both conscious and unconscious processing, whereas dual task conditions showed the recognition of faces to be entirely unconscious. Subjects were correct in their estimation of their degree of introspection when using conscious processes, but not when their mental processes were unconscious.

In Kellogg's second series of experiments the aim was to find which events were consciously attended to and which were unconsciously attended to, the hypothesis being that if trial n was consciously attended to, then it would be stored in short term memory on trial n + 1. It was found with random questioning probes for the existence of introspection, hypothesis recognition was inaccurate, as most learning occurred unconsciously, although under conditions where all learning was probed, hypothesis recognition was much nearer 100%. The conclusion was that if the subject realised the emphasis was on hypothesis testing, then conscious attention was given to this. Also, the processes underlying task performance may be conscious or unconscious, depending on the nature of the task.

Lewicki, Hill, & Bizot (1988) illustrated a process of unconsciously acquired information about a pattern of stimuli, which facilitated subjects' subsequent performance. Their hypothesis was that the cognitive system memorises more information about encountered stimuli than can be processed through consciously controlled channels, with concept formation occurring largely unconsciously. Some of this involuntarily processed and memorised information is believed to influence subsequent cognitive processes, even when perceivers are unable to articulate the knowledge they use. Lewicki et al (1988) asked subjects to find a target in a sequence of frames where target location followed a complex pattern. Although subjects' responses became increasingly accurate following pattern repetition, no subject reported conscious awareness of a pattern, nor could any subject verbalise one.

Perruchet, Gallego, & Savy (1990) dispute the Lewicki et al (1988) findings. In replicating the pattern of stimuli, they found the frequency of sequencing the reason for subsequent performance facilitation, not unconscious acquisition of this information. They found subjects reacted more slowly to infrequent events located in the early phase of the Lewicki et al experiments, possibly explaining subsequent improvement in task performance. They do not believe people unconsciously abstract rules that can be discovered through deliberate logical analysis. Nor do they consider inability to articulate the manipulated pattern implies operation on an unconscious level, although they do acknowledge processes underlying improvement in performance may still be unconscious. They discuss traditional concept learning and problem solving experiments, where the adaptive first mode of learning is activated in simple learning situations where people perform conscious operations on identified isolated variables of a task. Conscious operations become inefficient with large numbers of variables or when the structure of the situation is not obvious, requiring the second mode of learning to operate, where the processing of representations of events is both unavailable to conscious awareness and beyond attentional control. Both conscious and unconscious, or explicit and implicit modes of learning are thought to occur during the process of learning the rules underlying any situation subjects are exposed to.

In a study of artificial grammar learning, Reber, Kassin, Lewis, & Cantor (1980) found that when the underlying grammar was evident, subjects categorized better, using the first mode of learning when given explicit instructions stressing the need to pay attention to rules. When structure was difficult to discover with implicit instructions to pay attention to items, subjects always performed better than chance. Reber (1989 a & b) finds knowledge acquired from implicit learning procedures is always ahead of subjects' explicit knowledge, but as subjects are unable to verbalise implicit rules, performance is thought to be the result of an unconscious abstraction process. Dulany, Carlson, & Dewey (1984, 1985) dispute the Reber et al (1980) claim of unconscious abstraction processing, providing an alternate view of conscious attention to a large number of simple and approximate rules instead.

Unconscious processing does occur along with conscious processes, although no existing definition of consciousness or unconsciousness is universally accepted. It is believed unconscious sensory information is unconsciously selected to become perceptual information. Perceptual information is abstracted into concepts, perhaps both consciously and unconsciously. There are differences amongst theorists as to whether conscious or unconscious processing comes first or second during task performance, or whether this depends on the nature of the task.

Modes of Learning

Conflict remains between cognitive theorists who believe conscious processing occurs before unconscious processing, and cognitive theorists who believe unconscious processing occurs before conscious processing. In mathematics, Dienes (1959) distinguishes between analytic and constructive thinking, acknowledging these are not the only ways of thinking, nor are they mutually exclusive. Whereas analytical thought involves logic with concepts formed before they are used, constructive thinking involves creative intuitive perception not based on reasoning, of things not fully understood. These distinctions resemble the first and second modes of learning discussed by Perruchet et al (1990). Dienes believes constructive thinking develops before analytic, with both types required in mathematics. He notes Meredith's (1956) observation that mathematicians use intuition and insight to discuss their theorems, and logic to prove them. He defines mathematical intuition as insight gained into mathematical phenomena without conscious reasoning. Reasoning verifies intuitions, but is dependent on past experience and knowledge. He finds children more likely at first to build concepts intuitively without awareness of relationships between these and other concepts.

Bruner (1986) describes a narrative mode of thinking and a paradigmatic mode of thought. Similarly, Skemp (1971) distinguishes between intuitive and reflective thinking. He considers mathematics is learnt at the intuitive level long before the reflective level is functional, with learners at the intuitive level largely dependent on the way material is presented to them. The degree of accommodation possible at the intuitive level is less than that achieved by reflection, with intuitive thinking leading into Piaget's defined progressive stages of concrete reasoning and formal thinking (Wadsworth 1971). Dienes, in responding to Becker & Rogers (1969) finds these stages are not necessarily linear. Children can and do make intuitive leaps. Depending on the familiarity and usage of the particular concept involved, children may currently possess any mixture of intuitive, concrete operational, and formal operational thinking, and these levels of conceptual understanding may vary from one application of a concept to another.

Modes of learning are an important consideration to the present study where children show evidence of both conscious and unconscious processing. More important than which mode comes first or second is why these modes exist, and what kinds of tasks will access which mode. Children may use unconscious modes first as they develop concepts, but the development of concepts may not be as linear as Piaget suggests. Children may show mixed levels of conceptual awareness, and conscious or unconscious awareness and use of them.

Protocols for Eliciting Introspection in Children

Questioning children in classroom practice is designed to not only monitor conceptual understanding and thinking processes, but to aid conceptual development through the translation of intuitively understood concepts into verbally expressed language. Children may find verbal naming of classifications and processes difficult because some classification procedures may be made largely unconsciously, or consciously but nonverbally. Bringing unconscious or conscious nonverbal processes to articulate conscious awareness through the verbal naming of classifications appears much more difficult than making original unconscious or conscious nonverbal selections.

What does it mean if children do not naturally or continuously think out aloud as they complete a problem solving task? Observation indicates children perform many problem solving tasks silently, or silently with only occasional verbal comment. This suggests some options for the natural ways children may think during task performance:

- verbally and consciously
- silently but verbally and consciously
- nonverbally and consciously
- nonverbally and unconsciously

Perruchet et al (1990) reinforce that any demonstrations of unconscious complex learning will be limited by the researcher's ability to ask subjects the right questions when assessing conscious knowledge. Protocol procedures for eliciting introspection processes used in research on mathematical thinking are described by Ginsburg, Kossan, Schwartz, & Swanson (1983). These include the talking aloud method used by Newell & Simon (1972), and the clinical interview developed by Piaget (Piaget 1929; Piaget & Szeminska 1952).

The talking aloud method was traditionally used with children, who were considered to lack verbal skills and have poor or minimal comprehension. The talking aloud method encourages subjects to verbalise every thought as it appears as a means of eliciting complex forms of problem solving, sequential activity, formulation and testing of hypotheses, and the execution of multistep algorithms while solving challenging problems. There is minimum investigator intervention during this process. Ginsburg et al (1983) believe that requesting verbalisation of a process during problem solving may affect both the course and completion of the task. When the child is asked to talk aloud as a problem is solved, the necessary verbal process may slow task completion and confuse it. The child may not be able to do both tasks at once, or may become self-conscious and use different strategies or means that might not be used when left alone during task completion. The possibility is for talking aloud procedures to alter the very phenomena under examination (Brentano 1973).

The clinical interview originally used with adults involves asking the child after completion of a task what the child did, or what the child did in the preceding moment. Verbalisation after the event may be more difficult with the child unable to retrace original thoughts and intentions. When a child speaks spontaneously, it is with intention. This may have an enabling part to play in whatever process is being undertaken. When the moment of intention has passed, what cue will access memory of a now past intention? For a six year old girl who could not say what was the same about some transparent buttons she had correctly classified and placed, the cue to the memory of her original conceptual processing was a replication of her original conceptual processing combined with a focus of attention on naming the concept. She had to see through a button again to consciously remember the button was 'see through'. It involves both access to memory, and conscious observation of an event to talk after the event. Difficulties of the clinical interview include memory decay, interference from current thinking while reporting, and confabulation, where a rationale, or one originally unconnected with the production of an answer is invented. The way a problem is worded also affects the way the child is able to solve the problem asked (Riley, Greeno, & Heller 1983). Similarly, how a question is asked will help or hinder the child's ability to retrieve accurate memory of processing, facilitating verbal access to conceptual understanding (Donaldson 1978). The wrong question may elicit an answer suggesting erroneously that the child does not possess adequate conceptual knowledge involved in the task.

Donaldson (1978) finds several reasons why the child may have difficulty verbalising problem solving. The child has less knowledge of language and may be less confident about using it and may rely more on non-linguistic cues. The child may also not have learnt to distinguish between situations where primacy to the language is important, and situations where it is not. The child may find paying scrupulous attention to language very difficult, and where interpretation of words differs with some other expectation, words may take second place. In Donaldson's view, the child makes sense of situations first, then uses this kind of understanding to make sense of what is said.

Protocols for eliciting introspection in children are an important consideration in the present study. With task processes possibly being altered through asking the child to talk aloud during task completion, and memory decay occurring when asking children questions after the event, verbal introspection protocols may not be the most accurate or adequate means for assessing the understanding of concepts in children. Whereas verbal descriptions demand conscious awareness of process as well as the abstraction of applying vocabulary to this description either during or after the event, action awareness may bypass conscious pathways altogether through nonverbal means. Furthermore, conscious awareness of an unconscious process may be available only through simulation of the original perceptual and conceptual stimulus. For the child who is struggling with a task, an additional verbal one may not be possible either simultaneously or at task completion. The child's level of conceptual understanding may be more apparent through the strategies and solutions the child exhibits.

Unconscious Awareness of Concepts Through Actions

Lloyd (1989) presents a broader view of consciousness to include introspective awareness along with experience signalled by any discriminate response. Adults acting with conceptual understanding, who are unable to define the concept in verbal terms have been observed by Lovell (1961). Lovell finds this very frequent, and not necessarily due to lack of vocabulary. Ginsburg & Opper (1969) cite Piaget's deep insight suggesting the child's unconscious logic may be a more efficient way of describing thought than the child's own imprecise natural language. Brody (1989) similarly finds a recognition-like procedure provides a better assessment of consciousness than simple introspective reports. If a child's actions show the presence of thought, the absence of a verbal means of expressing this thought does not mean the thought does not exist. The solution already shows that it does. Riley et al (1983) believe the solution of a problem indicates the child already has both the conceptual knowledge and the procedure. Evaluating the success in verbal terms is to see what the benefit of solving the problem has been. In mathematical thinking, the benefit has already been significant. Donaldson (1978) believes that what encourages awareness of language may also encourage awareness of one's own thinking. For Piaget, mathematical thinking can develop only if the child can become aware of unconscious processes (Piaget 1974).

Conscious Awareness of Concepts Through Language.

Jung (1954) conceives growth of consciousness as closely related to intellectual growth, with some control over thinking required for intellectual growth to take place. He believes there is no control over thinking where there is no awareness of thinking processes. Also, many processes a child uses in solving a problem are unconscious, and the child is far more able to act and understand than to express verbally. These ideas appear contradictory until Jung's concept of control over thinking is defined in verbal terms. The attainment of control over verbal thinking means bringing thought out of its primitive unconscious embeddedness and interacting with others (Jung 1954; Vygotsky 1962; Donaldson 1978). Skemp (1971) relates making an idea conscious to associating it with a symbol, as in the acquisition of words in the development of language. Once the association has been made, the symbol seems to act as a combined handle and label for retrieval from memory. Verbal thinking is the use of pronounceable symbols used for communication, with voluntary control over thoughts achieved largely by use of symbols. Naming classifications is a separate and additional task to classifying them, requiring transfer of conceptual processes into verbal symbols.

Lovell (1961) defines mathematics as a mental activity studying order abstracted from the particular objects and phenomena which exhibit it, and in a generalized form. Skemp (1961) considers that if an individual does badly in mathematics, it may not be through lack of concepts, but through lack of reflective ability. To help the child develop mathematical concepts, language and symbols of mathematics must be taught. Individuals must have the concepts, although they may not be able to formulate a clear definition of the concept in verbal terms. Skemp finds the transition from elementary number work to mathematics involves the use of reflection on the relevant concepts and operations. Lovell emphasises that getting children to appreciate the significance of their own actions so they become aware of what they are doing is crucial.

Piaget (1976) defines consciousness as basically a conceptualisation process, with emergent consciousness of an action the means of transferring it into a concept, with conceptualisation being a process, as it is not immediate. Because it is a process, the degree of consciousness varies, depending on the degree of integration. For Piaget the issue is more the degree of integration than one of making the unconscious conscious. He considers that where a child is aware of goals and end results, but not of process, there is lack of conceptualisation and integration, although he acknowledges action in itself constitutes autonomous and already powerful knowledge. There is conflict in Piaget defining consciousness as the acquisition of concepts. Can a correct answer derived unconsciously and nonverbally emerge from undeveloped conceptual awareness? This question remains unanswered. If consciousness develops through the acquisition of concepts, what comes first? As language is itself a symbolic conceptual abstraction and application, verbalisation in Skemp's terms provides a label and a handle for the concept to be expressed, remembered, and available for transfer to further situations. On this basis verbalisation of concepts may be the means of integrating and adding to concepts rather than of forming them. Verbalising conceptual processes emerges as one of the functions of language itself, as a tool for bringing to consciousness conceptual processes for classification purposes to make them more readily useful and applicable for the individual.

Conscious awareness is not necessarily able to be verbalised. Conceptual knowledge may also be outside of conscious awareness. The question remains unanswered as to whether concepts arise through language or before language, but there must be a circular effect, where the verbal expression of a concept enhances conscious awareness and enables a higher level of integration of the concept.

Nonverbal Versus Verbal Conscious Awareness

The question remains. What can be known and expressed through language is minuscule in comparison with what can be known. To verbalise every life process would not be useful. Schooler & Engstler-Schooler (1990) find some components of visual memories can not be put into words, such as face recognition, and that verbalising previous visual stimuli may impair subsequent recognition performance by producing a verbally biased memory representation that can interfere with the application of the original visual memory. Furthermore, memory verbalisation impairs memory for stimuli that are difficult to put into words. Whereas it has been found that verbal processing generally improves memory performance, especially of verbal stimuli, the benefits of verbalising are limited to what it is useful to remember. Gardiner & Java (1990) found 'know' responses were more accurate for non words than for words, whereas 'remember' responses were more accurate for words. Lloyd (1989) also questions an assumption that consciousness is best operationalised as accurate memory reporting. Language and memory are inadequate to capture moment to moment experience, with some aspects of experience unable to be captured by language.

Consciousness as Processesd Integrated Awareness

Velmans (1991) presents an extreme view that consciousness may be outside the area of cognitive psychology altogether, believing information entering consciousness is already integrated, with consciousness of an input occurring relatively late in the information processing sequence. Awareness of a stimulus follows analysis and selection, and awareness of a response follows its execution, as does awareness of inner processes such as creativity. Physicist Penrose (1989) believes the actual thought that surfaces in the brain is the solution of some problem, with the action of conscious thinking resolving alternatives. Mandler (1985) finds conscious contents are made available for further processing involving choice and decision.

Velmans (1991) believes a process is conscious to the degree there is consciousness of the process, or consciousness accompanying the process, or consciousness entering into or causally influencing the process, with problem solving, thinking, and planning processes becoming conscious only to the degree they are accessible to introspection. As Velmans finds human information processing required for analysis, selection of stimuli, memory encoding and responding is unconscious and not generally available to introspection, he considers consciousness can not enter into or causally influence the process. Processes creating awareness are involved in perception, emotion and thinking, causing awareness, or being correlates of it, but are not acts of awareness. Awareness does not play an active part in the processes creating awareness. Events, information encoding, storage, retrieval, and transformation to output enter consciousness only if they are the focus of attention. We may be conscious of the stimuli we analyse and select for more detailed attention, conscious of what we learn and commit to memory, conscious of responses we make to such stimuli, and aware of effort given to planning and monitoring of thoughts, emotions and images, but the dissociation of awareness from information processing invariably occurs.

This extreme view suggests all conscious awareness is the result of prior unconscious processing. Velmans has been criticized for taking an epiphenomenalist view, with epiphenomenalism defined by Thomas Huxley claiming consciousness to be a form of output caused by brain processing, which in turn has no causal influence on that processing. Velmans does not support epiphenomenalism. He takes a monist view, considering functioning and awareness different perspectives of one process. Velmans refers to consciousness as primarily a state of awareness, and focal-attentive processing as a functional subdivision in an information processing model of the brain.

The perceptual process is unconsciously selected from sensory information, and integration of perceptual information into functional concepts must at first also be unconscious. In the present study, the movement of percepts into concepts is of relevant interest as to how children acquire mathematical concepts from manipulating materials in the environment.

Cerebral Evidence For Unconscious Processing

Libet (1991) finds a stimulus to the human somatosensory thalamus can be detected correctly in a forced choice response even when subjects are unaware of a stimulus induced sensation, showing cerebral information processing can be dissociated from awareness of the processing, and focal attention and cognitive responses can be separated from conscious features of a signal. Libet (1985) demonstrated EEG analyses show shifts of negative potentials long before subjects respond or are aware of their decision. Recent studies of hypnosis indicate information may also be able to enter long term memory and be recalled, without first entering consciousness. (Hilgard 1986). Similarly, Weiskrantz (1987) discusses how in the amnesic syndrome a person can demonstrate verbal, perceptual, and skill learning, but not recognise or have any consciousness that anything has been learnt. The dissociation between the capacity to learn and to retain is clear, along with the person's knowledge of this fact. Here, the monitoring of the process is what is missing from the process. Penrose (1989) does not believe language is necessary for thought or for consciousness. From split half brain experiments, it is known the two sides of the brain can be independently conscious. When a person has blindsight caused by damage to the visual cortex, information may not be consciously perceived, but is revealed by correctness of the person's guesses as to what is there.

Again, evidence that information enters the brain before conscious awareness of that information occurs, indicates the likelihood of unconscious processing preceding actions of awareness. Conscious knowledge seems to follow unconsciously known knowledge.

Left and Right Hemisphere Processing

Blakeslee (1980) attempts to explain the phenomenon of unconscious and nonverbal thinking by translating nonverbal, unconscious, and intuitive processes into right brain activity, versus verbal, conscious, and logical processes of the left brain. The right brain thinks in images, dealing with spatial processes. Control normally passes to the hemisphere with the fastest answer, but hemispheres can be trained to perform better with encouragement. Whereas the left brain examines issues sequentially and logically, the right brain is holistic, viewing the whole situation at once. For verbal explanation of right brain activity, the left brain must take an observers role, frequently confabulating or making up its interpretations and explanations rather than accessing them. Yet most creative breakthroughs, even in mathematics, result from intuitive leaps made in the right brain, whereas logic and language are so rigidly structured they are not suitable for the kind of thinking needed for creative exploration. Penrose (1989) comments that Einstein did not think verbally, nor did the geneticist Galton. Most mathematical thinking for Penrose is visual, not verbal, supporting the idea that much conscious mathematical activity takes place in the right hemisphere.

This relates to the possibility that the mathematics puzzles are processed in the right hemisphere more than in the left. Puzzle solutions that are arrived at without conscious awareness are likely to involve this creative process, and may not involve verbal processing.

Sensation into Perception

Insufficient knowledge regarding specific locations of conscious and unconscious thinking in the brain leaves unanswered the question of how much each process belongs to any one hemisphere. It is thought likely both hemispheres undertake both conscious and unconscious processing to some extent. Pribram (1987) considers memory traces to be distributed throughout the brain. When patients suffer damage to their forebrains they do not lose particular memory traces. The patient may not be able to speak, identify objects visually or tactilely, or recall a whole mnemonic category, but individual specific memories seem to be sufficiently distributed to be able to be recalled despite extensive damage. Pribram (1971; 1976) adds that with all information enfolded over the whole brain, as in a hologram, the response when the holographic record in the brain is suitably activated is to create a pattern of nervous energy constituting a partial experience similar to that which produced the initial hologram. It is different, less detailed, and with memories from different times possibly merging together and becoming connected by association and logical thought to give further order to the whole pattern. If sensory data is simultaneously being attended to, the overall experience merges to a given new whole. Nelson (1990) describes data stored in the brain as intermingling wave patterns of diverse frequencies and amplitudes distributing each bit of information evenly within the whole so that anything that alters a relationship in any part simultaneously alters all relationships everywhere.

Skarda & Freeman (1987) also consider learning and memory are functions which may be distributed throughout the neural network of the brain, and outline a neurologically based approach to cognition as an alternative to current cognitivism. They found a spatial pattern of chaotic activity covering the entire olfactory bulb in the rabbit, involving equally all the neurons in it, and existing as a carrier wave for a few tens of milliseconds with each stimulus of a new or familiar odour. Freeman & van Dijjk (1988) also found the same basic neural dynamics in the visual cortex of the rhesus monkey. Skarda & Freeman hypothesize chaotic behaviour is the basic form of collective neural activity for all perceptual processes, and functions as a controlled source of noise, generating chaos as an essential precursor to ordered states, as a means to ensure continual access to previously learned sensory patterns, and to learn new sensory patterns. By chaos, they mean activity that appears to be random but instead is deterministic. The brain organises its own time-space patterns of function and its own structure and should be viewed as a self-organised process of adaptive interaction with the environment. Garfinkel (1987) describes chaos as essential for optimal performance of systems in search of their own goals or states of minimal energy. In terms of the present study, some ideas are suggested as to how the actual process of perception may take place. Children processed familiar concepts in a more conscious manner, suggesting previously learned patterns are available, but that there is ample opportunity for new ideas to be incorporated, and the chaos theory may be the fastest route to access old or acquire new information.

This model of brain function resembles the distributed, self-organised processes of connectionist models rather than the rule driven symbol manipulating processes characteristic of the digital computer. Connectionist models are explicitly cognitive, dealing with mental processes of pattern recognition and completion, generalisation, discrimination, associative memory and mechanisms responsible for cognition. With connectionism, a distributed system of interacting elements is able to produce behaviour previously thought to require rules and symbols. The Skarda & Freeman model resembles connectionist models in being a system that exhibits regularities and processes information without being rule driven and manipulating symbols. Werner (1987) considers internal states are the neuronal system's own symbols. The chaotic background state provides the system with continued open-endedness and readiness to respond to completely novel as well as familiar input, without requirement for an exhaustive memory search. The nervous system thrives on an enormous degree of sloppiness in its design, as chaos. Chaos makes the difference in survival between a creature with a brain in the real world and a robot that can not function outside a controlled environment, although Grossberg (1987) disputes that chaos is necessary for the processes Skarda & Freeman claim it is necessary for. Skarda & Freeman emphasise that their model is competent to simulate only preattentive cognition and the instantaneous apperception of a stimulus, and not attentive inspection or sequential analysis. This suggests mechanisms for children to take a creative approach to puzzle solutions, in a situation where they do not have previous rules and rote learning already laid down, but simultaneously are able to access what is already familiar to them and incorporate it in a creative manner.

Roberts (1991) concludes that perceptual experience, whether visual, auditory, tactile, or other, is created within an independent stratum of consciousness, by consciousness itself. The creation and control of such images is determined by what the consciousness believes is the situation, irrespective of the sensory input to the brain and the resulting cerebral activity, which it is able to disregard. One example is rotating advertising signs that can be seen to reverse simply by willing them to do so. The brain believes what it wants to believe. Visual images are not necessarily created by a pattern in the visual cortex, but consciousness creates them within itself. The image we actually see depends on whether volition or sensory input has the greater control. Similarly, physicist Bohm (1980) finds the process of thought is not merely a representation of the manifest world, but it makes an important contribution to how we experience the world, for this experience is a fusion between sensory information and memory, which consists of thought built into its very form and order. This indicates that perceptions are already interpretations made by the individual, and that perceived reality may be different for each person. It may indicate why some children have different priority modes such as observing colours before shapes, or sizes before patterns, where another child may reverse these. The conscious mind may be more inclined to see what it knows, and not see what it does not already know consciously.

Concepts into Consciousness

The debate between conscious versus unconscious processing continues. The nature of consciousness may come to be seen as the central problem of research on the brain (Blakemore & Greenfield, 1987). Rumelhart (1977) could find no complete and widely accepted theory of problem solving, and Skarda & Freeman (1987) recognise that all models are abstractions. Bohm (1980) emphasises a theory is primarily a form of insight and a way of looking at the world, not a form of knowledge of how the world is. Integration of thought would be one further fragmented point of view. He believes different ways of thinking are better considered as different ways of looking at one reality, each with some domain in which it is clear and adequate. As all experiments are performed in a limited domain, to a limited degree of approximation, results from experiments performed in new domains to new degrees of approximation may not fit current theories, giving no proof that basic concepts of given theories have completely universal validity. Penrose (1989) claims algorithms, in themselves, never ascertain truth. External insights are needed to decide the validity of an algorithm, and consciousness is needed for awareness of the right algorithm to use. To Penrose, seeing the truth of a mathematical argument and being convinced of its validity is the essence of consciousness. Bohm sees fact and theory as different aspects of one whole in which analysis into separate but interacting parts is irrelevant. Theories are not always forced to fit facts in currently accepted orders, but changes in what is meant by fact may be required for assimilation into new theoretical notions of order. Perceptions do not begin with abstract preconceptions as to what the order has to be, with adaptation of perceptions to that order, but begin by observing the whole fact, and then assimilating it by degrees into an order, and developing it into further order, form and structure, with theoretical concepts.

Thought, and visual and auditory and other perceptions involve active transformation as they enfold into the brain. Images are too fast to be regarded as separate, and Bohm therefore sees these expressed as movement, as part of the implicate order binding the sharp break between concrete immediate experience and abstract logical thought. Each moment of consciousness has an explicit content as a foreground, and an implicit content as a background. Consciousness is therefore a series of moments. Attention shows a given moment can not be fixed exactly in relation to time, but covers a vaguely defined and variable extended period of duration. Each moment is experienced directly in the implicate order with one moment giving rise to the next. Content previously implicate is now explicate while previously explicate content becomes implicate. This continuation accounts for moment to moment change. There is much occurrence and stability in thought. Memory allows content to be held in fairly constant form. It is necessary the content be organised into basic categories such as space, time, causality and universality, through relatively fixed associations and rules of logic for concepts and mental images to be developed. This describes the movement of perception into some sort of integration in the mind, and as a continuous process. Attention is focused on this movement of moments. What is let go as a past moment may be incorporated into an existing or new concept, but must be an ever adaptive and unfixed process of the formation of ideas from memory.

Penrose (1989) asks what we can do with conscious thought that can not be done unconsciously. He considers the problem more elusive when considering that anything we seem originally to require consciousness for appears able to be learnt and later carried out unconsciously. Consciousness seems necessary to handle situations requiring new judgements, where rules have not been previously laid down. Piaget (1956) describes this consciousness of the familiar as operating only to a small extent in early life where the implicate order is more immediate and direct. Infants learn in sensori-motor experience, later connecting this with its expression in language and logic. Penrose discusses Plato's view of mathematics where ideas have an existence of their own, and mathematical discovery is a form of remembering. Penrose believes because mathematical knowledge is already known in some form, no information in the technical sense passes to the discoverer. The information was there all the time. It requires putting things together and 'seeing' the answer. Globus (1987) sees mind as the action of unfolding. In perception, mind is a process of selecting from autonomous processes.

Summary

Chapter 1 has looked at current theoretical explanations and explorations of possible unconscious cognitive processing, and some tentative definitions of consciousness. Conscious versus unconscious processes have been discussed, and theories on modes of learning continued the debate over whether people process information consciously or unconsciously, to what degree, in which order, and whether or not these can be introspected. Issues of introspection and protocol procedures with children looked at advantages and disadvantages of the talking aloud method and the clinical interview, and the part language plays in these types of introspection. Consciousness in the form of unconscious awareness and indication of conceptual development through actions, through to conscious awareness of concepts through language expression was discussed, along with the acquisition of language as a means of aiding the bringing of a concept into conscious awareness, bringing the issue of nonverbal versus verbal conscious awareness to attention. A child's inability to articulate a classification was considered to indicate unconsciously aware processing does occur, taking the form of consciousness manifested in action rather than introspective conscious awareness of that consciousness. Here, language was considered to play no part, being an applied secondary mode after the event, and on request. When requests for classification in language terms are made, intellectual growth is aided through the application of verbal terms, adding terminology and strategic skills to the conscious memory bank of what is already known, but not known it is known. Unconscious conscious knowledge becomes conscious conscious knowledge, providing the individual with future learning resources. Consciousness was presented as processed integrated awareness, with cerebral evidence of unconscious processing, and left and right hemispheric processing leading into a discussion of possible mechanisms for the movement of sensations into perceptions and concepts, and into conscious awareness and language. Ideas from physicists as to the nature of this process were discussed, and questions regarding the nature of consciousness and its conscious selections were viewed.

Chapter 1 discusses the importance of children bringing task processing concepts into conscious awareness, firstly through actions and secondly through verbalising. Lack of verbalising ability does not mean the idea is not there, but that the child may not be consciously aware of the idea or know how to explain it. As concepts are built from perceptions, integration of perceptions into concepts is crucial in education. Unconscious processes allow for speedy access to previously learned information, or to deal with novel situations with a creative approach. The more modes through which conscious awareness can exhibit, the more likely the concept is to be integrated and become fully conscious. It can then be actively applied in new and consciously chosen endeavours.

Chapter 2 looks at the present study, its methodology, and how conscious and unconscious processes might relate to the teaching of mathematics to children.

Chapter 2

The Present Study

Chapter 2 looks at aims, prior assumptions and hypotheses in the present study, with a rationale for the use of the mathematical puzzles. It involves Piaget's stages of cognitive development inherent in the structure of the puzzles, for developing pre-mathematical and mathematical concepts as taught in New Zealand primary school mathematics. A method section describes the present study research carried out in 1985 and 1986, including sections on participants in the study, measures used, and the nature and materials and mathematical concepts used in the puzzles. The procedure describes how the research was carried out, including the setting of the mathematics lessons, selection of puzzles, structure of the programme and indications as to questioning methods used with participating children. A section on data analysis discusses the quantitative and qualitative aspects of the present study, and analyses used.

Aims of The Present Study

Aims of the 1985 study:

1. To discover the upper and lower ranges of ability and ages at which the puzzles can be solved.
2. To find if the same children who score high or low in P.A.T. mathematics exhibit the same ability levels in solving the puzzles.
3. To discover the degree to which the puzzles are conceptually diagnostic for mathematical concepts and perceptual difficulties.
4. To discover the degree to which the puzzles are effective in meeting the needs of children within certain mathematical group categories: extension, remedial, and special learning difficulties.
5. To determine any gender differences in working with the puzzles.
6. To establish the puzzles in approximate order of difficulty.

Additional aims of the 1986 study:

1. To extend the time and scope of the 1985 aims and results.
2. To incorporate puzzles newly designed in 1986.
3. To enlarge the number of children in the specialised mathematical groups studied to decrease chance results.
4. To determine if the results were consistent across schools.
5. To evaluate and compare two average P.A.T. children who appeared able with the puzzles, with top P.A.T. mathematics children.

Prior Assumptions

Hypotheses

1. Children can master mathematical concepts considered beyond their expected age ability as defined by the current school curriculum, provided concepts are presented in manipulable and visual form.

Traditional beliefs held that children learnt mathematics in a gradual step by step process. More difficult concepts could not be taught before simpler concepts had been mastered. Piaget's stages of cognitive development meant children could not carry out formal operational adult thinking processes before moving through various stages of concrete and intuitive and trial and error practical processing to find out how the world and things in it work. In this sense Bruner (1960) believes that any subject can be taught effectively in some intellectually honest form to any child at any stage of development. Dienes believes children may possess mixed levels of conceptual understanding within the rigid stages Piaget proposes, depending on previous experiences and the nature of the task in hand (Becker & Rogers 1969). Finding the right tasks for children to develop and exhibit higher levels of conceptual understanding depends on the kinds of structured materials embedding complex and simple concepts within them and the nature of the problem solving tasks children are given to manipulate.

2. No gender differences would be found in ability with the mathematical puzzles in primary school children.

Traditional ideas of females being better at verbal tasks and males at mathematical and scientific tasks were accentuated by customary adult career roles and maturational factors regarding child development. Adult roles have seen more males in mathematical and scientific employment, and females in people related and verbal roles. Dweck & Licht (1980) discuss the popular finding that males exhibit superior mathematical ability and females exhibit superior verbal skills. With girls reaching puberty before boys, girls have appeared maturationally advantaged in language areas at primary school. Boys caught up during adolescence, frequently surpassing girls in mathematics, especially visual-spatial mathematics, and in scientific endeavours.

Rationale for the Puzzles

The rationale for the extensive use of environmental materials in the teaching of mathematics to young children is multiple. Mathematical concepts are more easily absorbed and developed when perceptual models are as varied as possible and mathematical ideas penetrate everyday thinking (Dienes 1959; Lovell 1961). In addition, children must handle real things in free undirected activity, with the opportunity to discuss developing concepts to acquire the language of mathematics (Churchill 1961). Mathematical concepts can not be brought about by using symbols, verbalisations, mechanical processes, or perceptual reconstructions (Lovell 1961). Piaget finds concepts develop from the schemata of action and thought arising from handling materials, but not directly from handling materials themselves where perception plays only a part. The development of mathematical concepts is based on the formation and systematisation of two operations: classification and ordering (Piaget & Inhelder 1959). Experiences of grouping, comparing, sharing, ordering, numbering and finding the relationship between things aids the development of mathematical concepts (Dienes 1959; Churchill 1961).

The mathematical puzzles provide a wide range of materials for children to classify and order. The structuring of materials into puzzles to incorporate desired concepts to be developed, means children attend to concepts inherent in puzzles simply by handling them. The greater the variety of materials structured to repetitively encompass the same concepts allows for a greater chance for the concepts to become integrated. The level of conceptual understanding gained will depend on previous experience and what the child discovers can be done with the elements of the puzzle. A child might not normally think to place objects in order of size, or to match the same coloured buttons during normal play. When the structure of the puzzles is focused on possibilities for grouping and arranging elements, it is more likely the child will absorb the concepts, consciously or unconsciously, and verbally or nonverbally.

Life is full of patterns and sequences. Knowledge is acquired long before there is language for the child to describe it. The child learns about the surrounding world through the senses and the proprioceptive experience of movement. These experiences become the basis for understanding how the world operates and how to operate on the world. With increasing awareness and significance of these variables and their uses, the stage of naming is reached, translating perceptual awareness into conceptual understanding. Operational knowledge becomes more functional as it is conceptualised and verbalised (Churchill 1961). Churchill finds verbal patterns need linking with understanding gained through experience. The transformation of direct experience into symbolism such as naming is made possible by language. Naming things brings what is already known into consciousness.

The primary function of language is to enable man to deal with experience symbolically, and communication with others is a later development. Langer (1951) finds that through language the child finds a way of patterning and ordering experience. Language itself influences modes of learning. It determines to a large extent what aspects of experience are reflected on and what is ignored. Churchill (1961) adds that the language habits of a community predispose certain choices of interpretation. If words are acquired in the context of real concrete experiences they can become symbols for ordering experiences at the mental as well as the concrete. Unless the language of mathematics is communicated to the child in such a way that it forces the child to think relationally about experienced events it will not provide a firm basis for later more advanced mathematical thinking. Children have to experience repeatedly to develop concepts defined as systems of learned responses and to organise and interpret data provided by sensory perception. It becomes a means of applying past learning to present situations to develop more complex systems.

The child can only develop analytical thinking through constructive activities with structured materials that precede analytical insights (Dienes 1959; 1960). The eventual attainment of mathematical concepts depends on the opportunity the child is given to handle real things. In time children come to realise the sameness in the tasks performed with different materials. It is important to provide for mathematical variability. The structure of the concept is preserved while the variables which make up the concept are varied. Dienes has in mind the needs of children at Piaget's stage two of concrete operational group structures, for any concept. This is the intermediate stage when imagery plays such an important part in reconstituting schema and building new connections. The child can imagine before he can structure, and through play with images can intuitively evolve new operational structures. Images are representations of real experiences and depend on initial concrete experience and capacity to retain and recall. The child deprived of experience at the concrete level is deprived of the sources of the images on which capacity to learn depends. These images are not ideas but the means for imagination to create ideas from the activity.

McKellar (1957) finds all image making depends on perceptual experience. Implicit or explicit analogy from perceived objects and events is the core around which the thinker models thoughts and communications. Original thought always depends on past perceptions both for its form and its content, and quite often on past perceptions of a very concrete kind. McKellar uses the term mental models to denote the more or less explicit figures of speech which are used to assist thought, understanding and exposition, considering the models we choose are selected from our own personal experience and limited by it.

The child builds a store of operational knowledge used as a frame of reference as a result of sensori-motor and affective responses to the world. This map of expectancies guides the approach to present experiences which can be called on at will because of the power to image. Operational knowledge is converted into conceptualised understanding through the process of abstraction. Activity and experience comes before knowledge to be acquired and facts to be stored (Churchill 1961).

Bruner (1960) believes a person must have the general nature of the phenomenon being dealt with clearly in mind to enable recognition of the applicability or otherwise of an idea to a new situation, and to broaden learning. Mastery of the fundamental ideas of a field also involves the development of an attitude towards learning and inquiry, including the use of guessing and hunches and the possibility of solving problems alone. Unless detail is placed into a structured pattern it is rapidly forgotten. Detailed material is conserved in memory by the use of simplified ways of representing it. An understanding of fundamental principles and ideas also appears necessary to adequate transfer of learning.

Bruner (1962) finds it is only through the exercise of problem solving and the effort of discovery that the working heuristics of discovery are learned. The more one has practice, the more likely one is to generalise what one has learned into a style of problem solving or inquiry that serves for any kind of task encountered. Bruner asks the question, 'How can I know what I think until I represent what I do?' Manipulation and representation are necessary conditions for discovery and may be a process of intuition. In mathematics it involves the embodiment or concretisation of an idea, not yet stated, in the form of some sort of operation. Bruner (1960) differentiates an operation from a simple action in that it is internalised and reversible. Internalised means the child does not have to go about problem solving any longer by overt trial and error, but can carry this out mentally. Piaget (1955) emphasises that every perception lasts and every perception extends. Mathematics is always abstract and must be thought into the concrete situation (Churchill 1961).

Handling many different materials in the world allows a child to develop ideas of properties of those materials, what they do, and what can be done with them. It is necessary for the child to relate together categories of materials and the ways they behave or can be manipulated. Mathematical learning encompasses ideas such as quantity and order, time and space and groups of things that belong together. Pre-mathematical experiences occur from birth onwards, from the discriminations of colours, shapes, sizes, of object permanence, reversibility, and multiples. Everything is mathematics in some form. Formal mathematical numerical concepts can not be learnt before ideas of larger than or smaller than or more than or less than or the same as or different from are experienced and observed. The puzzles provide these experiences in structured form, focusing attention on specific concepts to be acquired. Handling the materials allows multi-sensory experience to provide as much information concerning the concept as possible, and learning the language of mathematics inherent in the puzzles helps integrate it.

Piaget's Stages of Cognitive Development

Piaget (1955) believes all thought originates in the interiorisation of actions. Some of the thought activities include composition, where any two unit operations can be combined to produce a new unit; reversibility, where things can be separated again; and associativity where the same result may occur from combining units in different ways. These unit operations refer to actions capable of being internalised, reversed and coordinated into systems of thought.

There are three stages in this development.

1. Sensori-motor 0-2 years.

Children can only perform actions. There are no operations because the child can not yet internalise activities. The child forms notions of objects and learns to construct the notion of a permanent object lying beyond the field of vision. This is the beginning of intelligence.

2. Concrete operation group structures.

Pre-operational thought - 2-4 years. Actions become internalised because the child can use imagery to imitate an action. The beginning of symbolic behaviour and the use of language follows. Internalised actions are not yet reversible, and conservation is only understood at sensori-motor level. Schema are concepts or categories. They never stop changing or becoming more refined. The cognitive schemata of the adult are derived from the sensori-motor schema of the child. The processes responsible for the change are assimilation and accomodation. Assimilation is the cognitive process where new perceptual, motor or conceptual information is integrated into existing schema or patterns of behaviour. Assimilation accounts for the growth of schema but accomodation accounts for the change or modification of schema. A balance between the two allows for equilibrium to ensure the child's efficient interaction with the environment (Wadsworth 1971).

Intuitive thought - 4-7 years. Concrete operations are becoming organised, but thinking is still tied to perceptual factors, and structures are still rigid and irreversible.

Logical thought - 7-11 years. The development of logical operations where reasoning processes become logical. An intellectual operation is an internalised system of actions that is fully reversible. The child applies logical operations to concrete problems. The child is no longer perception bound to deal with reversible processes.

Concrete operational children can not deal with complex verbal problems involving propositions, hypothetical problems or those involving the future. The reasoning of concrete operational children is content bound and tied to available experience. To this extent a concrete operational child is not free of past and present perceptions.

3. Formal operations

Formal operational thought - 11-15 years. A child develops the reasoning logic to solve all classes of problems. There is a freeing of thought from direct experience. Not all people fully develop all the possibilities of formal operations (Elkind 1962). Aspects of formal operational thinking include hypothetical deductive reasoning, scientific inductive reasoning, reflective abstraction, propositional or combinatorial operations, formal operational schemes as less abstract than propositional schemes, and probability.

Piaget finds these stages linear, with one stage following the establishment of a previous one. Children need experiences to help them develop as many concepts of the world as possible within the developmental mental framework they are capable of. Clearly, mathematical puzzle work fits into these stages at the different levels of thinking. Concepts of classification and order, of reversibility and conservation, of quantity and space are essential before abstract mathematics manipulating symbols of quantity and reversibility can be understood and operated.

Pre-mathematical Concepts

In order for children to experience mathematics at the concrete manipulative level as a prerequisite to formal mathematical learning, basic concepts of classifying and ordering are needed to acquire the language and experience of mathematics. Pre-mathematical concepts included in the junior school mathematics programme include concepts of classification, one to one correspondence, ideas of conservation and seriation.

Classification

Children's number concepts result from a synthesis of the logical operations of order (seriation) and group membership (classification) (Piaget & Inhelder 1969). Four or five year old children typically select objects that go together based on similarities, but may not be consistent about what the similarities are throughout a whole set of matched objects, or be aware of differences. From 5-7 years like objects are placed together but any idea of subsets is missing. Children do not understand relationships between subsets of items as class inclusion, a concept which develops around eight years of age.

Traditional classroom classification experiences utilizing mainly visual and tactile sensory modes focus attention primarily on discrimination of colour, shape, size and pattern to develop concepts of quantity, equality and inequality. Classification experiences also aid the development of pre-mathematical language through questioning children and encouraging them to verbalize their classifications. Examples of pre-mathematical language include: colour names such as red, yellow, white, purple, blue; shape names such as circle, triangle, square, rectangle, hexagon; and size names such as big, bigger than, biggest, small, smaller than, smallest, many, few, wide, narrow, long, short, tall, short, thin, thick, high, low, more than and less than. Other visual, tactile, and auditory examples include heavy, light, fast, slow, loud, soft, high, low, cold, warm, smooth, rough, shiny, dull, sharp, blunt, the same as and different from.

One to One Correspondence

One to one correspondence is essential to ideas of equality and counting. In discussing pre-mathematical concepts with adults who did not know what a one to one correspondence was, two volunteers were asked to sit opposite each other. A large pile of small blocks was placed between them. They were asked to pretend they were from a society with no developed counting system and to give each other an equal share of the blocks. One volunteer made two equal looking piles by rough visual division. The other volunteer was asked if she thought she had the same number of blocks as her partner had. She said they looked the same but she couldn't be sure. She was asked how she could find if they were equal. She placed one block from her pile and one block from her partner's pile away from their original piles, matching block for block until she was able to accurately establish the equality or inequality of the original piles without counting them. This equality or inequality was determined by use of one to one correspondence.

The ability to make a one to one correspondence is essential to any child before reading or enumeration can take place. When a small child is asked to count, the child may rote count numbers up to 10. Place in front of the child the number of objects the child can rote count to and ask how many there are. The child may even enumerate using a one to one correspondence matching a spoken numeral with the touching of an object for up to about 3 objects before losing the one to one correspondence and continuing random rote counting and random object touching, but the child is more likely to randomly touch objects and rote count numbers with neither a one to one correspondence nor enumeration. That the child may enumerate by a one to one correspondence matching a spoken numeral with the touching of an object for up to about 3 objects before losing the one to one correspondence and continuing random rote counting and random object touching, indicates that concepts of one to one correspondence and enumeration are present and operational for a limited range of numbers. The child is quite satisfied with either of the above described performances in the absence of constancy or range of these concepts. Similarly, a pre-reading child may also recite a memorised story in a book, making no visual one to one correspondence between written and spoken words.

Ideas of Conservation

A number is understandable only if it remains unchanged irrespective of the way in which its units may be arranged. The idea of conservation or permanence of a quantity is a necessary condition for the understanding of numbers (Lee 1963). The potential for one to one correspondence matching in the absence of enumerative ability is available to a child when tested for the development of ideas of conservation or invariance of quantity, as defined by Piaget. Ideas of conservation or invariance of quantity means that in concepts such as numerical value or volume, a whole persists unchanged no matter how its parts may be rearranged. A child will verify two small equal piles of blocks are the same by visual appearance or through making a one to one correspondence or by enumeration. Condense one pile into as small a space as possible and spread the other pile widely. Ask the child which pile has the most blocks. If you ask which pile is the biggest the child is right in saying the spread pile is the biggest. It takes the most space. The child with developed ideas of conservation will see both piles have the same number of blocks, irrespective of how they are arranged. The piles can be reversed with the spread pile condensed and the condensed pile spread and the same question asked. The child with undeveloped ideas of conservation will say the spread pile has more blocks, and will reverse this to the other pile when the positions are reversed.

After making a one to one correspondence by placing a number of plastic eggs in the same number of egg cups (1 egg in 1 egg cup) a child will verify there are enough eggs for the egg cups and vice versa. The number of eggs and egg cups is the same. Spread the eggs in their egg cups out and the child will still say there are enough eggs for the egg cups. Remove the eggs from the egg cups and put them into as tight a group as possible, leaving the egg cups spread out. Ask the child if there are still enough eggs for the egg cups. A child lacking ideas of conservation will say there are too many egg cups for the eggs now because they look more. Reverse this by grouping the egg cups close together and spreading the eggs. The same child will now say there are too many eggs for the egg cups because they look more. A child with developed ideas of conservation will tell you there are still enough because none have been added or taken away.

The development of ideas of conservation begins in babyhood and continues throughout mathematical conceptual formation. A baby learns mother exists whether or not she is visible at this moment, and later that self is separate from mother. A personal memory of being over my mother's shoulder in Queen Street, Auckland at two years of age was of the view looking backwards being different from the view looking forwards. Two year old thinking thought there were two different places via the turn of the head backwards or forwards. There was no concept of each view being a panoramic extension of the other. The views looked like different places. They were different places to two year old thinking. The magical sense of power imprinted the memory.

A child with pe-mathematical ideas of conservation of quantity needs to develop difficult formal mathematical ideas of reversibility:

$3 + 4 = 7$; $4 + 3 = 7$; $7 - 4 = 3$; $7 - 3 = 4$.

More difficult is:

$2 \times 3 = 6$; $3 \times 2 = 6$; $6 \div 3 = 2$; $6 \div 2 = 3$.

$(3 + 3 = 6)$ $(2 + 2 + 2 = 6)$

Even more difficult reversibility: $4 - 7 = -3$; $3 - 7 = -4$.

Ideas of conservation and reversibility extend into algebra and geometry. Both algebraic equations and geometric theorems operate on hypotheses of reversibility. Understanding of geometry requires an ability to mentally and physically rotate spatial features.

Seriation

Ability to seriate objects is necessary to develop concepts of the cardinal and ordinal meanings of numbers. Seriation is the putting of a group of objects in order of size (or weight or volume). This prepares for enumeration when a child learns each number represents one more than the one before it and one less than the one after it. Language such as before, after, in between, smaller than, bigger than, more than and less than are important when questioning children about their seriations. Once seriation transfers into enumeration, the cardinal and ordinal meanings of numbers become important. Cardinal meanings of numbers are the quantities they represent. How many? The cardinal meaning of 2 is its two-ness. Ordinal meanings of numbers are their ordinal positions or places between other numbers. Which one? The ordinal position of 4 is 4th. Other ordinal numbers include 1st, 5th and 10th.

Mathematical Readiness

Lee (1963) lists the following considerations to decide when a child is ready to move on to a higher level of learning in junior mathematics:

1. Can the child make discoveries with the materials provided?
2. Does the child discover many relationships between quantities and see many possibilities in the materials, or is the child's thinking relatively limited?
3. Is the child able to express ideas well?
4. Is the child showing a lively interest in work?
5. Does the child respond well to the type of guidance that stimulates the ability to see new relationships and make new discoveries with materials?
6. To what extent does progress depend on teacher guidance? Is there too much dependence on teacher help?
7. Does the child show by successive discoveries with materials what is remembered and understood from past experiences?
8. To what extent can the child apply what has been learned to simple everyday problems that arise in the classroom?

Children are ready to move on to elementary formal mathematical concepts involving enumeration and symbols when pre-mathematical concepts of classification, matching and one to one correspondence, some ideas of conservation and reversibility, seriation, and rudimentary understanding of the language of mathematics is understood. Concepts of more than and less than and before and after transfer gradually into a cardinal and ordinal understanding of numbers. Tests such as 'A Number Readiness Test' (Churchill 1961) - (Appendix 2) will assess the degree to which this transition has taken place in children. Wertheimer (1938) studied numerical concepts in primitive peoples, finding that a one to one method of evaluating groups by matching does not contribute to the development of abstract numbers. Wherever there is no natural relationship, no vividly concrete and relevant connection amongst things themselves, there is also no logical connection, nor is any logical manipulation of these things possible. Dantzig (1947) points out that it is counting that consolidated the concrete notion of plurality into the abstract number concept which made mathematics possible. This reinforces the importance of concrete manipulations in combination with the language of mathematics preceding the transition into formal mathematical work, again a task of the mathematical puzzles.

Mathematical Concepts of the Senior Puzzles

A considerable part of mathematics is concerned with the study of numbers which do not possess the same concrete existence as the objects that are counted. They are properties or attributes, just as colours, shapes and textures are properties. There is no such object as 'a large' but there are large objects. Nor is there an object which is 'a blue' but there are blue objects. Number is a property which refers to sets of objects. No one object can have the property of six. A set of objects can have a property of six. Objects described in numerical terms are therefore sets of objects, and sets are already abstractions. There are universal sets of objects with defined attributes. There are relationships between sets such as one set being included in another, or one set having no common member with another, or one set having the same members as another in which case it is not really 'another' (Dienes 1966).

The senior puzzles in the present study incorporate mathematical concepts expanded to increasingly difficult levels from those inherent in the junior puzzles. Mathematical concepts include those of set theory as the study of the mathematical properties of sets; and elements of probability, a concept based on understanding chance and proportion and dealing with the range of possible combinations within defined sets. Matrices or arrays with a rectangular arrangement of ordered sets; and tessellations forming an inlay or mosaic of shapes, as taught at New Zealand Intermediate school Form 2 level; and geometric forms and classifications requiring increasing degrees of logic, use of strategy, ordering, patterning, and rotation are further concepts.

The rationale for puzzles of this type of mathematical activity superimposed on formal mathematical teaching is the same for senior and junior puzzles. Manipulative operations encourage creative solutions requiring understanding of the mathematical structure of the task rather than one of rote learning where a method is applied which has been taught but not understood. Lack of structural understanding results in rote learning which will not be sufficiently flexible to be applied in unfamiliar situations. Dienes challenges Becker & Rogers (1969) in the previously held Piagetian view that mathematics must be taught in steady steps as children do not make sudden difficulty leaps in their understanding. Dienes finds that when children are simply taught certain procedures, the actual structures of what they are learning do not become any clearer for the children in the sense a mathematician understands them. Taught in this manner there is very little difference between the teaching of new mathematics, old mathematics or any other kind of mathematics children have learned in schools. They just make different noises from what they were making before, and children are taught to do different kinds of problems than they were taught before. This is in reference to older styles of teaching where rote learning of method rather than an inside understanding of the problem through first hand experience was the norm.

Dienes (Becker & Rogers 1969) looks for behavioural criteria where it can be said a child has or has not mastered a certain structure. He believes the kinds of tasks given children should be suitable for children and adults. The tasks must be unfamiliar and not too difficult for children or too easy for adults. They should be challenging for the very able and possible for the less able. They should be suitable for standardising so comparisons can be made, but should allow sufficient freedom for different strategies to be applied and observed. Dienes found children coped with multiple embeddedness of concepts better than adults, as opposed to generalisation which children were not so able to do. It was easier for children to transfer learning from the complex concept that had simpler concepts embedded within the structure of the complex task, than it was for children to move forwards in graded steps from the simple concept task to the complex.

The senior mathematical puzzles incorporate structure, embedding the simple within the complex, and have sufficient unfamiliarity to enable true problem solving strategies to emerge. The puzzles are challenging for any age group, and many puzzles encompass multiple layers of solution embeddedness to be mastered at varying levels, and in different ways observable to the onlooker. For this reason they have an inherent diagnostic as well as learning function.

Method

The method section examines participant selection processes and demographic features of the children concerned; measures used in the selection processes of the children, and measures used in the form of the materials used - the puzzles; the procedure for the mathematics lessons, and methods of data analysis used in the present study.

Participants

A total of 92 children took part in the present study across 1985-1986.

1985

Five groups of children with approximately 6 children per group participated in the 1985 study, with three junior and three senior groups in one primary school. The three groups in each age range consisted of the highest (extension group) and lowest (remedial group) of mathematical performers, and children with special learning difficulties respectively. No junior mathematics readiness tests were used to select the junior groups who were selected by the Senior Teacher of Junior Classes on the basis of mathematical performance. Senior groups were selected from Progressive Achievement Test (P.A.T.) mathematics scores (Reid & Hughes 1974), this identification considered the most reliable by the school. Children with special learning difficulties such as verified brain injury, attention deficit disorder, dyslexia, perceptual difficulties, colour blindness, emotional or behavioural difficulties, and unspecified other difficulties were selected by class teachers, not from mathematical test scores. The children the puzzles quickly diagnosed as having problems with colour discriminations were placed in the group because they were experiencing a general range of perceptual difficulties. Children ranged in age from 6 years to 10 years of age. Some children were put into or out of the programme during the course of work, accommodating ongoing assessment by their respective teachers using a 'revolving door' system approach, where children are moved in and out of a specified instruction group according to their ongoing needs (Renzulli 1984).

1986

Groups from 1985 were continued in this school in 1986 with some changes of children from 1985. Some children had gone on to Intermediate school, and some had come up through the junior department. Some children were no longer considered to be in these most extreme categories of highest or lowest mathematical groups, or of having special learning difficulties, and some other children now fitted into these criteria more than previous children had done. An additional group of the second highest P.A.T. mathematics children was added, as teachers were interested to find out how well these children coped with the puzzles in comparison with the highest ability children, and felt there was little distinction, if any between these two groups of children. Five groups from a second school were also worked with, using the same grouping system and selection processes as for the 1985 study in the first school. Children ranged in age from 6 years to 10 years, with one 11 year old. One group of the four highest P.A.T. mathematics children and two P.A.T. average children in a third school were also worked with in 1986. Their ages ranged from 8 to 10 years old. The two P.A.T. mathematics average children were brought to me privately initially. Their ability with the puzzles was immediately apparent, and permission was given for these two children to be worked with in their own school with that school's highest P.A.T. mathematics children. Parental consent was obtained for all children in all schools.
Table 1 lists mathematical group categories, school class levels, ages, and numbers of children in the 1985-1986 study:

Table 1: Mathematical Group Categories, School Class Levels, Ages, and Numbers of Children. (N = 92).

Mathematical Group Categories		Class	Age	N
Junior Special Learning Difficulties -	J-SLD	J1-Std 1	6-8	9
Junior Remedial -	JR	J1-Std 1	6-8	18
Junior Extension -	JE	J1-Std 1	6-8	18
Senior Special Learning Difficulties -	S-SLD	Std2 - Std4	8-10	2
Senior Remedial Children with lowest P.A.T. mathematics scores -	SR	Std2 - Std4	8-10	15
Senior Extension Children with average P.A.T. mathematics scores -	AV	Std2 - Std4	8-10	2
Senior Extension Children with second highest P.A.T. mathematics scores -	SE 2	Std2 - Std4	8-10	10
Senior Extension Children with highest P.A.T. mathematics scores -	SE 1	Std2 - Std4	8-11	18

Measures

Progressive Achievement Tests (P.A.T.) Mathematics

The P.A.T. tests were developed in 1972 by mathematics personnel in New Zealand. Objectives and emphases reflected current aims, practices and expertise in New Zealand and overseas. Samples of children in all types of New Zealand schools with the exception of private schools were used to gain the norms for the tests. Children were aged from 8 years in Standard 2, to 15 years in Form 4, the ages and levels the tests are designed for. Age and class norms are available in percentile rankings for each sex.

There were conceptual assumptions in the development of the tests:

1. Children are at different levels of cognitive development and mastery.
2. Progress is more rapid in these skills when instruction is suited to their present stage of development and understanding.
3. Those who are successful through mastery of suitable material (to their needs) are more likely to develop desirable attitudes to mathematics and continue with confidence and comprehension.
4. Teachers need assistance in gauging pupil attainment for planning.
5. Reliable standardized tests as a supplement to sensitive teacher observation can provide a useful and objective estimate of pupil achievement. Teachers use these to make decisions about instruction and materials most appropriate.

P.A.T. tests are used early in the New Zealand school year in March when the norms were carried out, to test achievement levels from the preceding school year. The main functions of the tests are:

1. To provide a broad estimate of children's mathematical achievement. (for grouping purposes)
2. To provide information to set realistic goals and for planning effective work programmes.
3. To identify children making unsatisfactory progress, leading to special diagnostic and remedial attention.
4. To indicate able pupils who require enriched programmes.
5. To find weaknesses and strengths in the class and to help assess the levels of cognitive objectives of the curriculum, and assess whether or not these are being achieved.

The P.A.T. tests are of the omnibus type emphasizing power rather than speed. They cover four mathematical areas: recall, computation, understanding, and application. Two aspects are examined: the content or subject matter of mathematics, and the process or ability being tested to see what is done with it. Content is determined by the syllabus of the New Zealand Department of Education. Concepts of recall and computation are straightforward, but for understanding, it is hoped the pupil comprehends mathematical terms, symbols, formulae, processes, concepts and principles, and can explain and illustrate these, identifying relationships, comparing and contrasting between related mathematical concepts, processes, terms, etc. For application, the pupil is able to apply knowledge of mathematics to unfamiliar situations, showing versatility, ability to make generalizations, make estimates, and draw inferences and predictions. At junior to senior levels, emphasis on understanding and computation remains the same. At senior level, emphasis on recall is reduced and emphasis on application is increased. At junior level there are no questions involving logic or geometry. At senior level questions involving spatial relations are phased out. There are two equivalent forms of the tests, although Reid & Hughes warn the equivalent forms reliability coefficients reported in the P.A.T. tests are low estimates of the reliability of the tests because the norms were taken from children who sat one test straight after the other.

Reid & Hughes caution the use of the tests in several ways:

1. No test is perfectly reliable. Children must not be labeled as a result of test scores.
2. Not all levels of attainment are to be reached by all children.
3. The tests should not be taken at short intervals.
4. The results should not be used to classify children into classes, only into groups within classes. P.A.T. tests have been designed specifically as an aid to the classroom teacher to enable better assessment of the skills and abilities of pupils, with a view to providing individualised instruction adapted to their present stages of development (Reid & Hughes 1974).

Materials

Specific Types of Junior Puzzles

In 1967, during the development of some of the beginning junior puzzles in the present study, the New Zealand junior school mathematical programme (Lee 1963) required the use of structured apparatus such as Cuisenaire rods, Stern blocks and Adsum blocks, as well as environmental materials to provide children with experiences in classifying and ordering to aid the development of pre-mathematical concepts in five and six year old children. Environmental materials included natural and manmade small objects such as shells or buttons that were readily available within the environment for use in the development of pre-mathematical concepts. As environmental materials were not provided by schools it was necessary for teachers of junior children to provide their own range of environmental materials for classroom use. The junior puzzles designed from environmental materials and used in the present study were displayed at a national teachers' mathematics inservice course held in Blenheim, New Zealand, in January 1977, and are shown in Appendix 1 (Plates 44 - 77). Some of these puzzles and puzzle types are also documented in Moxham (1978). Permission was given by the present writer for some of these pre-mathematical puzzle design concepts to be incorporated into the subsequent and current New Zealand junior mathematical programme, 'Beginning School Mathematics,' (B.S.M.) compiled by Diane Barriball and others (Department of Education 1985; Ministry of Education 1992).

Specific types of junior puzzles include colour matching sets, shape matching sets, pattern matching sets, one to one correspondence sets, cardinal and ordinal sets, and sets involving size matching and ordering. Some sets involve preliminary fractions such as halves and quarters, and bilateral symmetry as in matching gloves or faces cut into two halves. Others include symmetrical patterning, geometric shapes, tessellations where a mosaic pattern of shapes is repeated with no left over spaces between the shapes, and elementary arrays or matrices, where matched, classified and ordered components are arranged with horizontal, vertical, and diagonal relationships.

Specific Types of Senior Puzzles

Earlier junior puzzle concepts and materials were extended to senior and adult difficulty levels to include set theory using such items as Venn Diagrams as sets that have members belonging to more than one set, and one difference sets adapted from the attribute blocks designed by Dienes

(1966) used in school mathematics, where each adjacent member has only one attribute difference from its surrounding neighbours, or must have a one attribute relationship to its surrounding members. Probability involving all possible combinations of items or elements provided; matrices or arrays as structured classified and consecutive ordering; tessellations using shapes; geometric forms using shapes; and classifications requiring increasing degrees of logic, use of strategy, ordering, patterning, and rotation of elements were concepts inherent in other senior puzzles.

It is important to remember the difficulty level of 'Beginning Senior' puzzles was based on set theory as taught at 12 year old level in New Zealand Intermediate Schools in 1985 (Adamson & Turner 1967). It is noteworthy some 6-8 year old children in the present study were able to master some puzzles at this level, and some 8-10 year old children were able to master puzzles extending to adult difficulty levels. All puzzle diagrams represented in the present study are reduced in size from the original puzzles used by participating children. Most puzzles are presented to the reader firstly in an incomplete form to simulate the original presentation to the child as nearly as possible, and are followed by a solution diagram. Appendix 4 lists very advanced junior puzzles through to adult puzzles in approximate order of difficulty.

Environmental Materials

All levels of the puzzles utilised miscellaneous environmental materials that were easily and cheaply available. Ideas for structuring environmental materials into puzzle form came mostly from the materials themselves, or from observations of what children did with them. Occasionally materials were looked for that were appropriate for a specific idea. Examples of materials used included buttons, buckles, gift wrapping papers, wallpaper sample books, fabric and fabric samples, shells, stones, magazine pictures or any pictures, doyleys, vinyl samples, metal washers, coloured plastic tubing, sandpaper, leaves, seeds, shoe toe and heel plates, corks, packaging materials, coloured clothes pegs, magnetic shape memos, magnetic tape, foam sponge, wool, postage stamps, cotton reels, paint charts, toy catalogues, embroidery tapes and laces, sheepskin offcuts, coloured cardboard, coloured transparent plastic envelopes and folders, small plastic toys, and lids and canisters. These environmental materials could be classified according to shape, pattern, size or colour, or matched if cut into halves or quarters. They could make tessellations, be ordered according to size, and matched in a one to one correspondence. Utilising existing materials in the environment for the purposes of teaching mathematics enables children to gain wide experience in handling objects, and to absorb the mathematical elements inherent in the world around them.

Procedure

The present study evolved over two years and across three different schools, adding new puzzles and new children between the 1985 and 1986 studies.

1985

The research continued for 9 weeks. Lessons were for 30 minutes once per week. Children were taken to a spare room away from their normal classrooms. Children worked on the floor with the puzzles.

1986

The research continued for 15 weeks in the same school using the same procedure, but with an additional group of the second highest P.A.T. mathematics children as well as original groups. I concurrently worked in a second school for 6 weeks using the same original five mathematical groups to compare children in the mathematical group categories. Class lessons were also for 30 minutes, and children were also taken to a spare room away from their normal classrooms, and worked on the floor with the puzzles.

The last group of children in the third school were given 45 minutes per week with the puzzles for 15 weeks, working in a spare room and on the floor as in the other schools.

Methods of Working with Children.

The particular mathematical groups the children were placed into encompassed diagnostic, remediation and lateral extension and enrichment at every level for every age of child. Children worked at their own pace and level. Emphasis was always on building onto what children could already do and achieve by themselves, and how far this could be extended. It was essential to work with children in this manner as good teaching practice, and for the purposes of this study to find as full a range of possibilities and benefits and uses for the puzzles as possible. Every child considered capable of managing any particular puzzle was not necessarily given the opportunity to attempt it because of time constraints. A child given a particular puzzle was considered capable of mastering the level required, or needing to develop embedded concepts, or in need of a challenge. Sample groups for each puzzle were different in the numbers of children who attempted them and in the particular children who attempted them, with some puzzles too easy or too difficult to be given to any particular individual child. The selection of puzzles for each child was based on progressive assessment, considering age, mathematical grouping, interest level and performance observation, to maximise educational value and prevent frustration or boredom. With the exception of one puzzle, the Birthday Candle Array, all other puzzles were unsuitable for all levels of capability.

Questioning Children

It is important to think about comments and questions made to a child before a puzzle is attempted, during its completion, and at completion. When choosing or presented with a puzzle a child may ask, 'What do I have to do?'

Answer, 'Look very hard and see if you can find any things that belong together.'

When the child has completed the puzzle do not say, 'That's right,' or, 'That's wrong. 'Say, 'I see you have looked and thought very hard about what you were doing. How did you find which things belong together?'

In this way children are helped to think about what they have done and to acquire pre-mathematical language. Many puzzles can be achieved successfully at progressive levels. A child's performance level and the degree to which this can be extended through questioning is diagnostic in determining pre-mathematical concept formation.

A child may peg coloured material squares onto a clothes line by matching one material square to one peg. A successful one to one correspondence has been made. At this level the colours of the pegs and the colours of the squares are not colour matched but placed at random. Sometimes a child has randomly matched a red peg with a red square. To find if this was accidental or deliberate and if the child is able to see the colour relationship, say, 'I see you have pegged all of the squares on the clothes line.' Point to the colour matched peg and square and ask, 'Can you tell me why you put this peg with this square?'
Almost always the child has done this accidentally, and sometimes is unable to see the 'redness' of the matched pair being pointed at. If the child is unable to see this relationship or to see another way to match the pegs to the squares, diagnostically, in this situation, this child can make a one to one correspondence but can not colour match. Give this child another puzzle that focuses on either colour matching alone or another combining one to one correspondence with colour matching.

The child may notice the redness of the square and the peg together when attention is drawn to it through being asked, 'Can you tell me why you put this peg with this square?'
A confabulated reply, 'Because they are the same,' or, 'Because they are red,' are common answers.
If the child answers, 'Because they are the same,'
ask, 'What is the same about them?'
If the child answers, 'They are the same colour,' but doesn't know the colour name, teach the colour 'red'.

If the child does not now automatically match all of the squares and pegs by their colours, ask, 'Can you see another way you can match the pegs and squares?'
The child may now change the pegs around and match red squares with

red pegs, and yellow squares with yellow pegs and so on. If this is completed accurately, ask, 'Can you tell me the names of all the colours?' Teach colour names not known to the child. Then ask, 'Can you find another way to match your pegs and squares?'
An advanced pre-mathematical level child might now place all of the red pegs with red squares beside each other, and do this with each of the colours.

When a child has made a mistake, make no comment until the work is completed to the child's satisfaction. Frequently the child will spontaneously discover and correct the mistake during the completion process. If the mistake is still not noticed by the child on completion of the puzzle, say, 'Can you show me which things belong together?' Agree with the child on all correct items. During this process the child is likely to discover and correct the mistake. If the child insists mismatched items are correct do not continue questioning but give the child a simpler puzzle or another involving the same concepts. In this way the development of pre-mathematical concepts occurs at the child's own pace.

Data Analysis

The present study uses elements of both a quantitative and a qualitative approach. One quantitative aspect is the use of national age normed P.A.T. mathematics scores as an index of formal mathematical understanding used to group senior children for the present study. The P.A.T. mathematics tests were undertaken by the schools concerned in March of each year as is the custom in New Zealand schools, to determine achievement levels of mathematical understanding carrying over from the previous school year. As the present writer was not in general classroom teaching practice at the time of the present study, specific P.A.T. mathematics results of participating children were not available to the present writer, but only to class teachers of the children concerned. It is therefore not possible to report any differences in upper or lower levels in P.A.T. results from individual schools, but to accept that the children provided were in those categories for those particular schools. It is for this reason that larger samples of children were sought to reduce the effects of chance deviations from the normal distribution of results that might occur in any individual school because of chance innate capability levels of the particular children in the sample at the time.

The present study was not a scientific experiment with scientific controls, but teaching in progress. Puzzles and questioning techniques were aimed to 'catch the moment' with judicious questioning helping children take their conceptual understanding as far as possible. Although many puzzles had correct solutions, and some puzzles had more than one arrangement or creative interpretation for a correct answer, all puzzles allowed for individual perceptual and conceptual awareness to manifest in levels of performance. Performance levels typically ranged from too difficult through to exceptional or novel performance, with various questioning and non questioning speed and strategic levels exhibited within these extremes. All of the senior puzzles manifested the following levels of performance, with occasional variations to accomodate specific features of the puzzles concerned. They included:

0 - Too difficult
1 - Difficult and inaccurate with a lot of questioning
2 - Difficult accurate completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

A numerical assessment was applied to these performance levels, and was expanded where necessary to accomodate finer grades of performance than presented here. These performance levels provided ordinal data giving an additional quantitative aspect to the present study besides P.A.T results. Mathematical group categories were also given ordinal status, and scatterplots were drawn to observe correlation trends between levels of performance and mathematical group categories. As the ordinal data from both of these measures was non interval, non parametric analyses were used, with Spearman's Rho correlations measuring levels of performance of children against mathematical groups, ages and class levels respectively in puzzles with larger 'n' samples, and Mann-Whitney U-Tests measuring gender differences in performance levels of the puzzles already tested with Spearman's Rho analyses. The ordinal data for mathematical groups was as follows:

1 - Junior Special Learning Difficulties - J-SLD
2 - Junior Remedial - JR
3 - Junior Extension - JE
4 - Senior Special Learning Difficulties - S-SLD
5 - Senior Remedial - SR
6 - Average - AV
7 - Second Highest Senior Extension - SE 2
8 - Senior Extension - SE 1

The puzzles provided the qualitative aspect of the measures in the present study. Their order of difficulty can only be guessed from observing numerous children using the puzzles. Puzzle lists in Appendix 4 are only approximate, and group categories of difficulty levels are arbitary divisions in the sliding scale from very advanced junior puzzles to adult puzzles. The difficulty level of the very advanced junior puzzles equates with 6-9 year old difficulty level approximately. The beginning senior puzzles are around 12 year old difficulty level. From this it can be seen that all following puzzles come very close to an adult difficulty range. The many junior puzzles used in the present study are pictured in Appendix 1.

Summary

The aims, hypotheses, prior assumptions, rationale for the puzzles, mathematical concepts, participants, measures, the nature of the puzzles, procedure and data analysis have been presented in this chapter on the methodology used in the present study. Part 2 looks at puzzle processing chapters, the first of which is Chapter 3 examining puzzles showing children exhibiting unconscious processing of concepts.

Part 2

The Puzzles

Children Unable to Verbalise Classifications

Perceptual Priorities

Multiple Concepts

Transfer of Learning and the Use of Strategy

Diagnostic Progressions

Range in Levels of Performance

Chapter 3

Children Unable To Verbalise Classifications

The event drawing my attention to the possibility of intrapsychic unconscious information transmission and conceptual processing came from the following beginning senior puzzle represented in Figure 1:

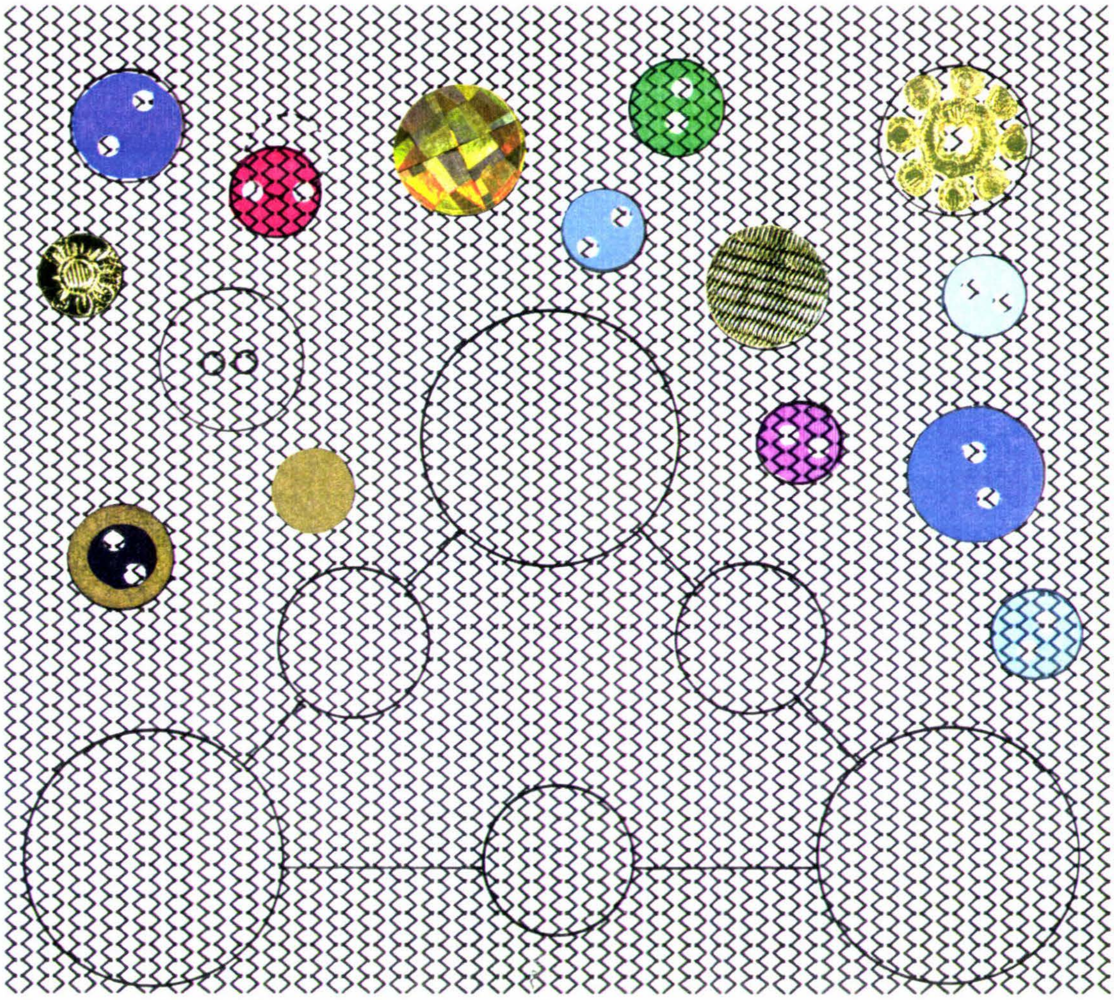


Figure 1: Three Intersecting Button Sets:

Verbal Instructions:

- Find which buttons belong together.
- Put each set of buttons that belong together in a big circle on the
- diagram.
- One button goes in each little circle. The button you put in each little circle must belong to [intersect] the buttons in each big circle it is connected to.

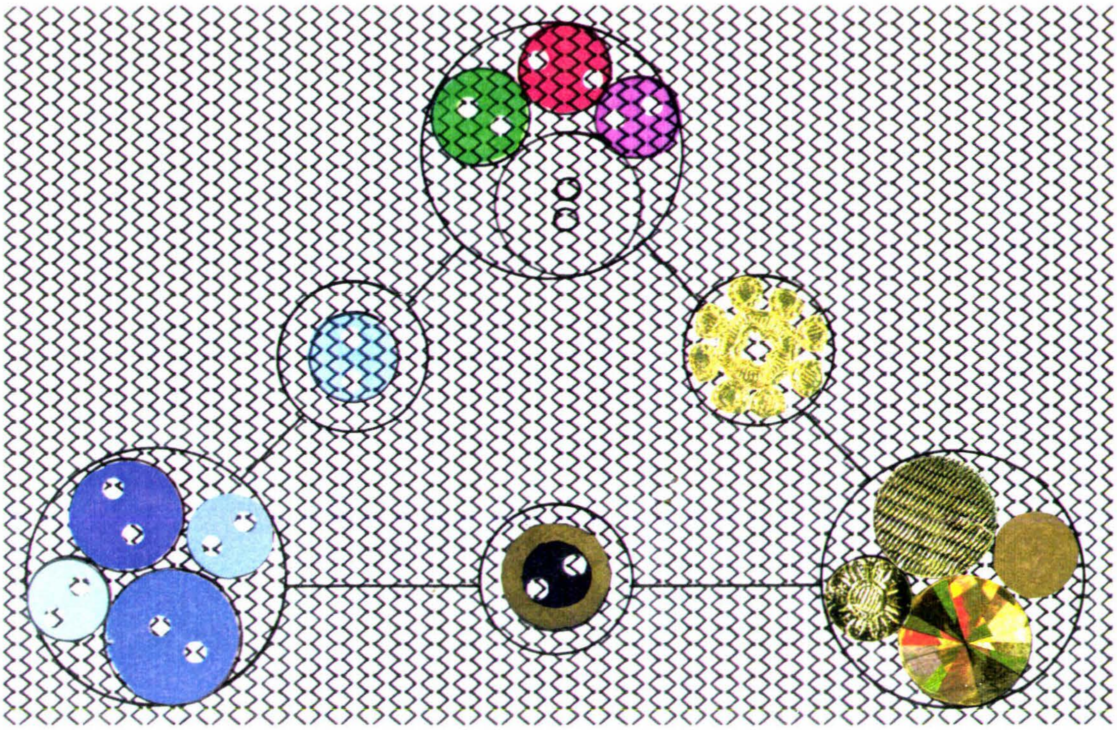


Figure 1a: Three Intersecting Button Sets.

Verbal instructions:

- Can you name your sets [big circles]?

  If the child has finished the puzzle or is stuck, I ask the child to name the sets, irrespective of whether or not intersecting buttons are placed.

  If the child can not name the sets, I point to the buttons the child has placed in one of the big circles:

- Can you give your set [these buttons] a name?

  I do this for each big circle. If the child is still unable to name the sets, I point to each big circle again:

- What is the same about the buttons you have put in this circle?

  The puzzle is too hard if the child can not verbalise any reasons.

  If intersecting buttons have not been placed but sets are named:

- Can you find a button to put in this little circle that belongs in this big circle and this one? I point to the two connecting big circles.

  When the child has done this, or has previously placed intersecting buttons into the little circles and has named the big circle sets:

- Tell me about the buttons you have put into the little circles.

Children Unable To Verbalise Classifications

Figure 1: Three Intersecting Button Sets: Blue / Transparent / Gold.

Because of the difficulty of representing transparency in a photocopied diagram of the buttons for this thesis, I placed the representation of the diagram and buttons over a patterned background. This as well as the elimination of the tactile and three dimensional features of the buttons may make the transparency of the buttons easier for the reader to detect than it is for a child manipulating the real puzzle with the diagram and buttons placed on a plain cardboard background.

A large proportion of children had the same difficulty with Figure 1. Although able to classify and place sets and intersecting buttons correctly, verbally naming any set was difficult, and many children were at first unable to name their transparent set. It could be argued children who classified and placed sets and intersecting buttons correctly but who had difficulty naming the transparent set, used a process of elimination to find the remaining transparent set following the placement of the gold and blue sets. Some children did use a process of elimination, but intersections were not correct when this occurred, unless some conceptual awareness followed, consciously or unconsciously. Correctly placed intersecting buttons supports a conceptual rather than an elimination or perceptual process alone. The separation of the blue transparent button from the blue opaque ones necessitates an awareness of a conception of transparency. Perceptions and conceptions appear to have been processed unconsciously and nonverbally as indicated by the difficulty retrieval for verbal reporting of classifications shows, also suggesting unconscious and nonverbal processing may be easier for the child than conscious verbal processing.

Inadequate vocabulary was also not supported as a reason for a child classifying and intersecting the sets correctly with difficulty naming the transparent set. With the exception of one 6 year old Junior Extension Group girl with a reading age of 10, every child was eventually able to call the transparent set, 'see through', 'clear', or 'transparent', when asked what is the same about the buttons in this set, despite only a few children being familiar with the word 'transparent'. Although the above mentioned 6 year old child completed the placement of her sets and intersections correctly and named her gold and blue sets, she was unable to name her transparent set or to tell me what was the same about the transparent buttons she had placed together. I took one colourless transparent button and one opaque blue button from her diagram and placed them side by side on the multi-coloured striped carpet we were sitting on. I asked her to tell me about these two buttons. She said one was 'see through', and immediately named her set 'see through' without further questioning, suggesting she already had available both the concept of transparency and an adequate vocabulary to complete her task. This problem was well represented at senior level also. One 10 year old Senior Extension Group girl placed her button sets and intersecting buttons correctly, but was only able to name her transparent set when I pointed to it and asked her to tell me what was the same about these buttons. Figure 1b shows Fig. 1 levels of performance across mathematical groups:

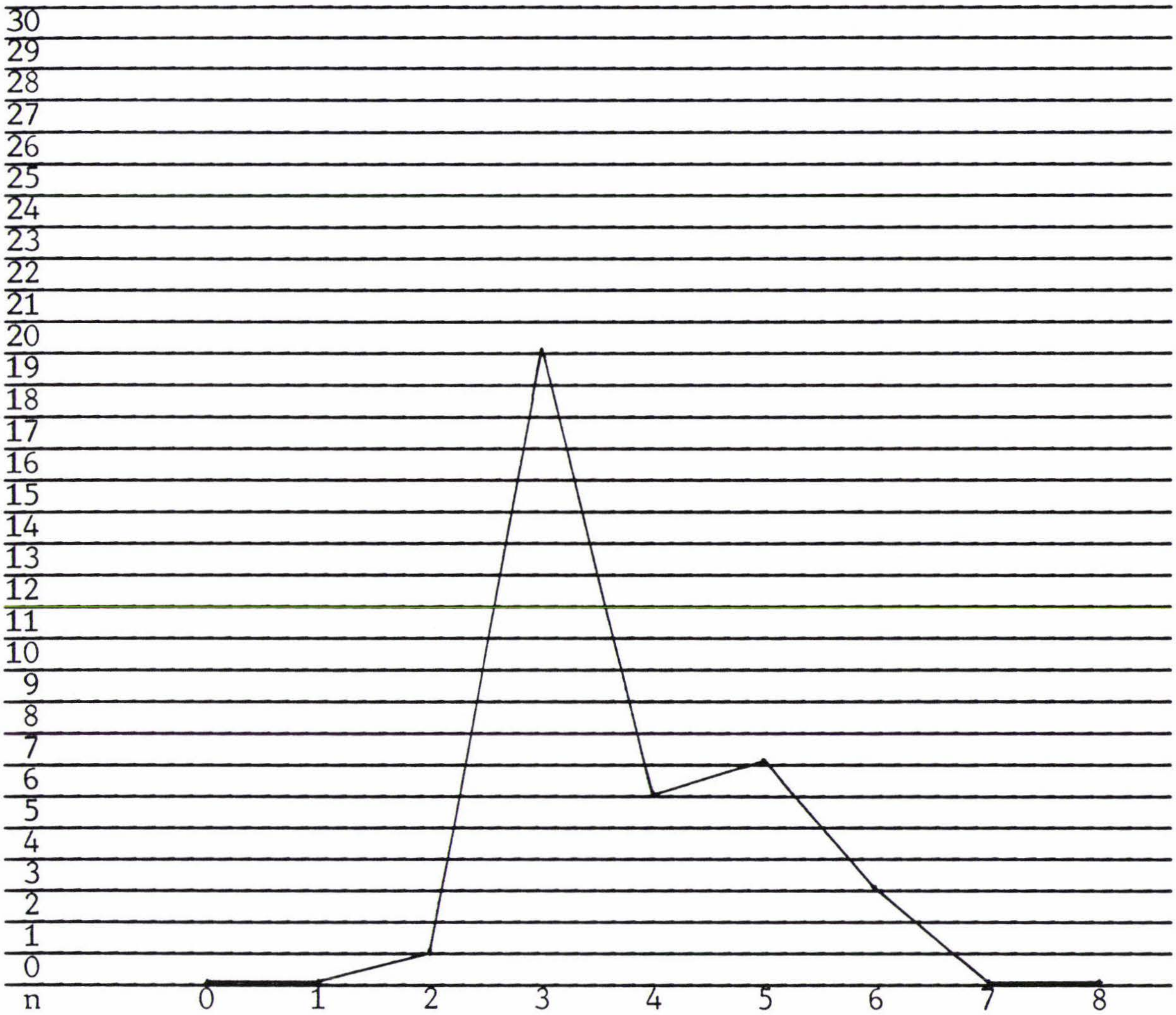
8								
7								
6					1	1		1
5			1	1	1		3	1
4			4					2
3			8		3	1	3	5
2			1					
1								
0								
Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 1b: Three Intersecting Button Sets: Blue / Transparent / Gold: Levels of Performance Across Mathematical Groups. n = 37.

Figure 1c shows levels of performance across all children who attempted Figure 1:



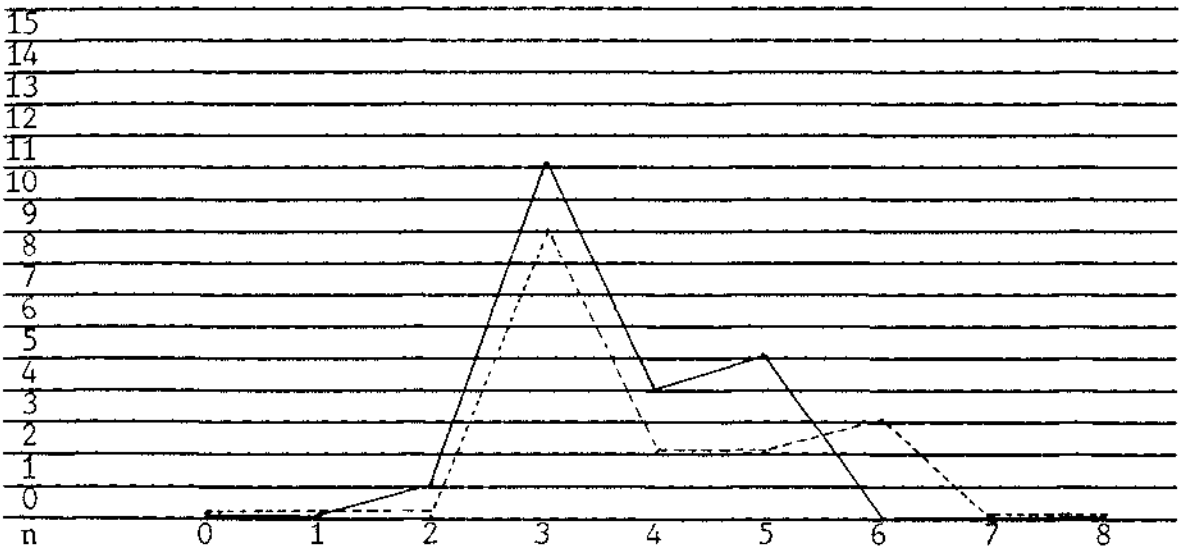
Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 1c: Three Intersecting Button Sets: Blue / Transparent / Gold: Levels of Performance. n = 37.

Children requiring questioning numbered 27 (73%). Most questioning related to difficulties with the transparent concept. The absence of children finding Figure 1 too difficult to complete indicates a screening process as this puzzle is considered to be a beginning senior puzzle and was only offered to children from Junior Extension level upwards.

Of all children attempting Figure 1, 16 of 21 males (76.1%) and 11 of 16 females (68.75%) required questioning to complete the puzzle. Subject numbers are too small for these results to be significant or reliable in showing gender differences in levels of performance.
Figure 1d examines gender performance across levels of performance:



Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

n = 21 ―――――――― Males
n = 16 ---------- Females

Figure 1d: Three Intersecting Button Sets: Blue / Transparent / Gold: Gender Performance Across Levels of Performance. n = 37.

Figure 1e shows children at the highest level of performance for Figure 1 across mathematical groups and age. Higher levels of performance were expected in children from higher mathematical group categories and older age levels. The child achieving this highest level of performance from the lowest mathematical group category combined with the youngest age indicates the most unexpected performance. The most unexpected performance for Figure 1 is an 8 year old with an average P.A.T. mathematics score.

expected

Age	Maths Groups	J SLD	JR	JE	S SLD	SR	AV	SE 2	SE 1
11									
10						1			
9									
8							1		1
7									
6									

unexpected

Figure 1e: Three Intersecting Button Sets: Blue / Transparent / Gold: Highest Level of Performance: Very fast completion with no questioning: Mathematical Groups Across Ages. n = 3.

Table 1A shows Spearman's Rho correlation coefficients between levels of performance and mathematical groups, school class levels, and ages, for Figure 1. Correlation levels are not significant.

Table 1A: Spearman's Rho Correlation Coefficients: Figure 1: Three Intersecting Button Sets: Blue / Transparent / Gold: Levels of Performance: n = 37.

Across mathematical groups	$r_s = 0.263$	$p = >0.05$
Across school class levels	$r_s = 0.238$	$p = >0.05$
Across ages	$r_s = 0.198$	$p = >0.05$

Process of Elimination, Rote Performance, Confabulation.

There were several other ways besides unconscious nonverbal conceptual processing where a child's performance resulted in a correct solution to a puzzle or part of a puzzle, with the child being unable to give an adequate verbal report showing conscious conceptual understanding of the process or product. The child may not have had understanding, and may or may not have gained it later following questioning. These include using a process of elimination, rote performance, and random correct performance with later confabulated reasoning.

Process of Elimination

Figure 2, representing an average senior puzzle, gives an example of a child using a process of elimination:

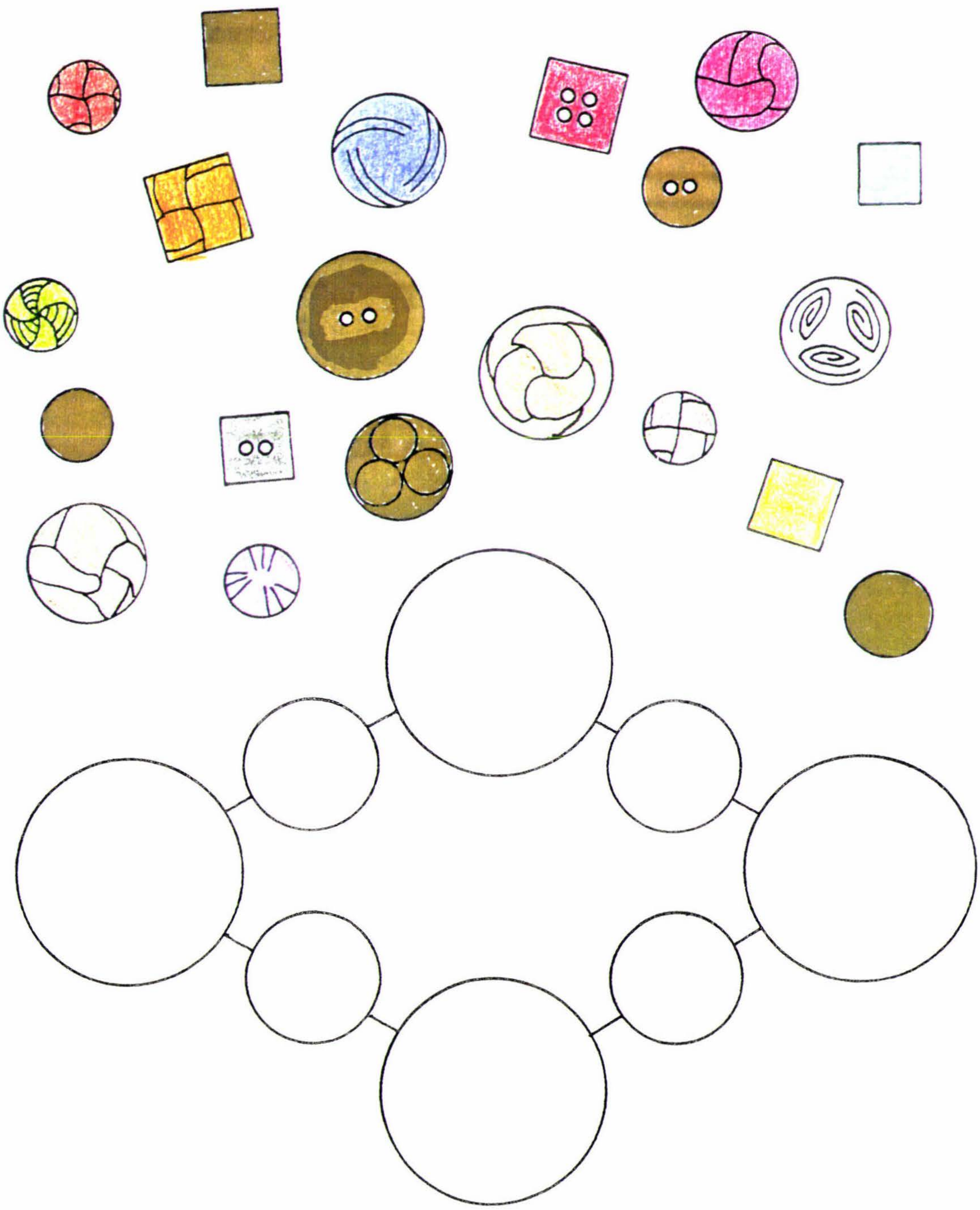


Figure 2: Four Intersecting Button Sets.
Instructions and questioning as for Figure 1:

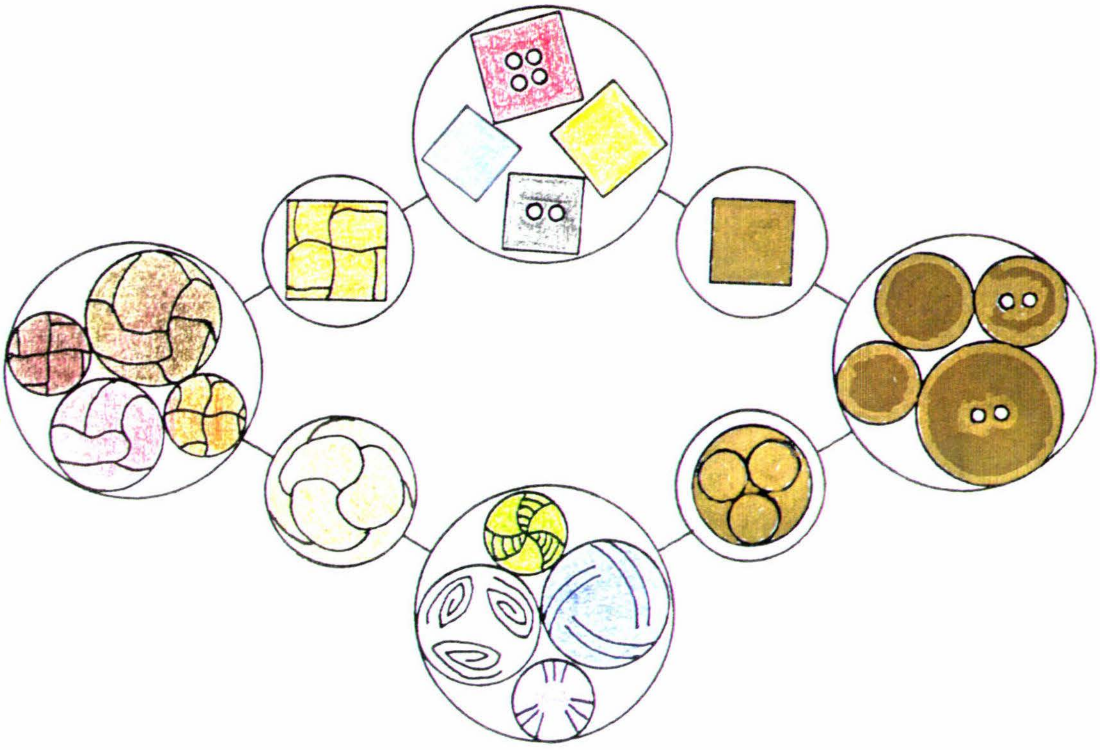


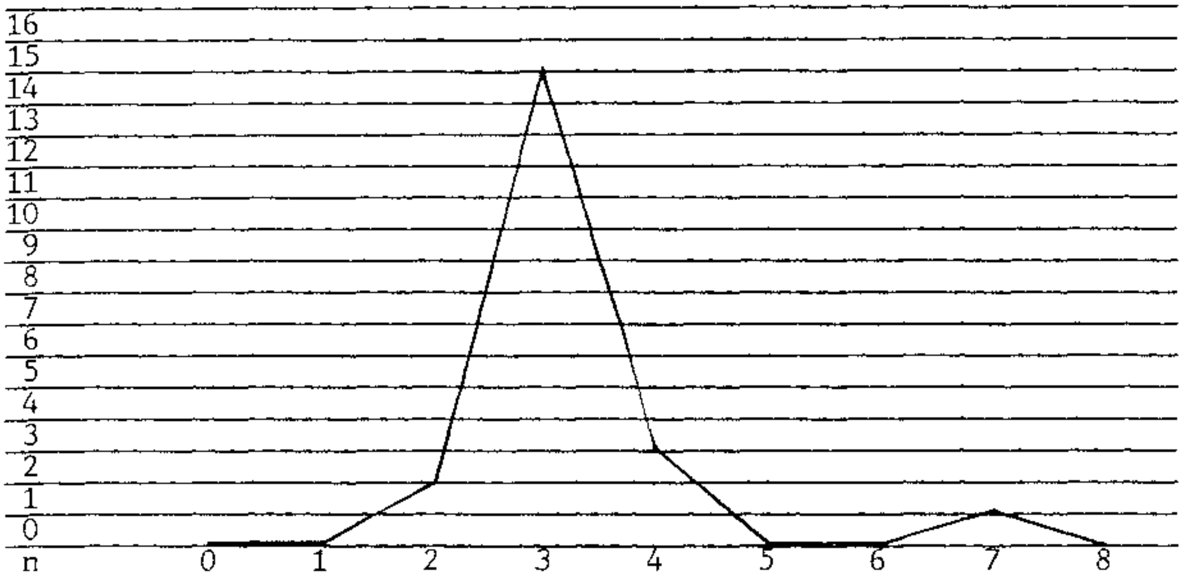
Figure 2a: Four Intersecting Button Sets.
Instructions and questioning as for Figure 1a:

One 10 year old SE 1 girl classified and correctly placed her set of patterns of 3 through a process of elimination following the placement of her leather, square, and gold sets. Some questioning was required before she could perceive or name her patterns of 3 classification and place her intersecting buttons. In contrast, as in Figure 1, one 9 year old SE 1 boy had difficulty naming his correctly classified and placed patterns of 3 set and intersections, and he had not used a process of elimination. Table 2 shows Figure 2 correlation coefficients between levels of performance and mathematical groups, school class levels, and ages.

Table 2: Spearman's Rho Correlation Coefficients: Figure 2: Four Intersecting Button Sets: Leather /Square /Gold /Patterns of 3: Levels of Performance: n = 21.

Across mathematical groups	$r_s = 0.363$	$p = >0.05$
Across school class levels	$r_s = 0.459$	$p = <0.05$
Across ages	$r_s = 0.411$	$p = <0.05$

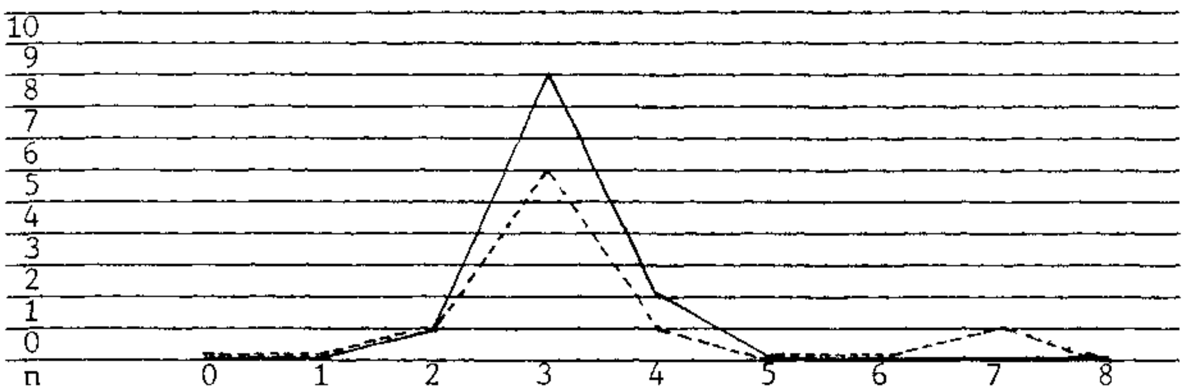
Figure 2b shows levels of performance for Figure 2:



Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 2b: Four Intersecting Button Sets: Leather / Square / Gold / Patterns of 3: Levels of Performance. n = 21.



n = 12 ―――――――― Males
n = 9 ---------- Females

Figure 2c: Four Intersecting Button Sets: Leather / Square / Gold / Patterns of 3: Gender Performance Across Levels of Performance. n = 21.

Figure 2d shows levels of performance across mathematical groups.

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7						1		
6								
5								
4						1		2
3			4				3	8
2			1				1	
1								
0								

<u>Levels of Performance</u>

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

<u>Figure 2d: Four Intersecting Button Sets: Leather / Square / Gold / Patterns of 3: Levels of Performance Across Mathematical Groups. n = 21.</u>

The highest level of performance was achieved by a 10 year old girl with an average P.A.T. mathematics score. Her 2 minutes speed of performance was achieved through the use of her own strategy. Most other children first classified all sets, and then worked out their juxtaposition from the intersecting members. This child classified one of her sets first, and placed it on her diagram, then found an intersecting member which told her the classification of her next set. She worked around her diagram in this orderly fashion until she met her starting point. This strategy contrasts very differently with the neutral type of strategy used by the girl using a process of elimination.

Rote Performance

Rote performance was a second way besides unconscious nonverbal conceptual processing where a child's performance resulted in a correct solution to a puzzle or part of a puzzle with the child being unable to give an adequate verbal report showing conscious conceptual understanding of the process or product. The child may not have had understanding, and may or may not have gained it later following questioning. In rote performance the child is taught a procedure for doing a puzzle. The child is able to complete the puzzle but may have no idea of the related concepts involved during completion, or later through questioning. Figure 3, a very advanced junior puzzle, provided examples of rote and correct performance occurring with instances of no conceptual understanding of the relationships involved:

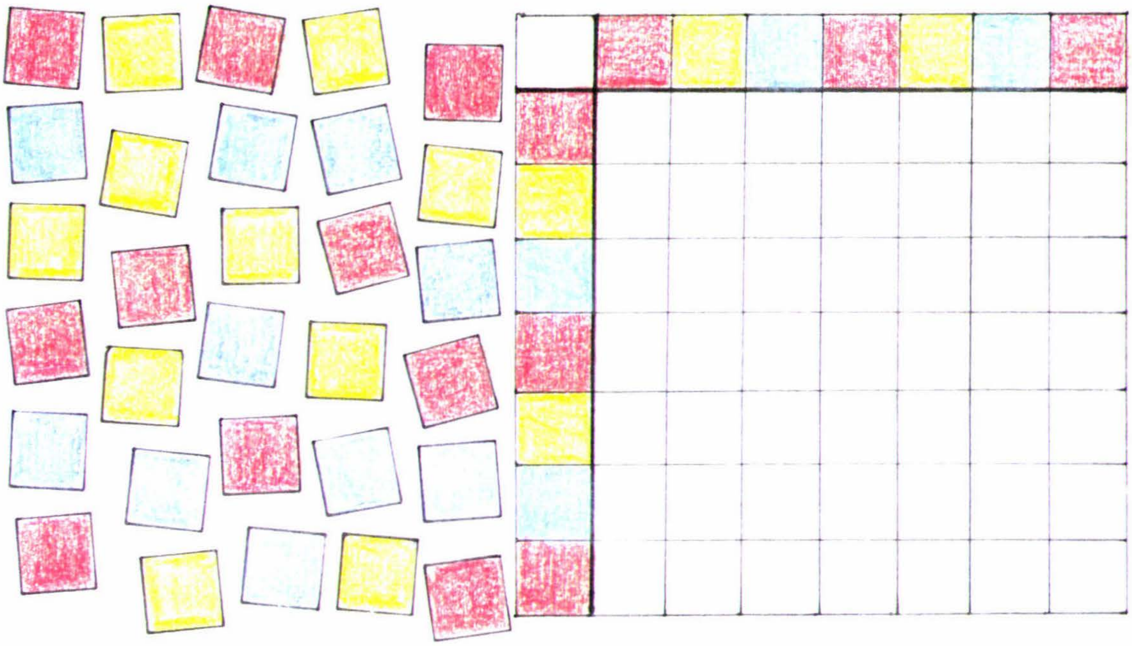


Figure 3: Coloured Transparent Plastic Film - Addition Array.

Verbal instructions

[I set a pattern for a junior child.] Senior children set their own:

- Make a pattern of colours that repeats itself across the top row. eg. red/yellow/blue.
- Make the same pattern of repeating colours down the left column. r/y/b.
- Make an addition array following the patterns arranged in your top row and left column. Fill in all of the squares.- Place two colour squares on each square in your diagram. [for any child who does not understand these instructions, I point to any square on the diagram and ask which colours need to go in this square. From this square the child checks the top row and the left column to find which colours to place here].

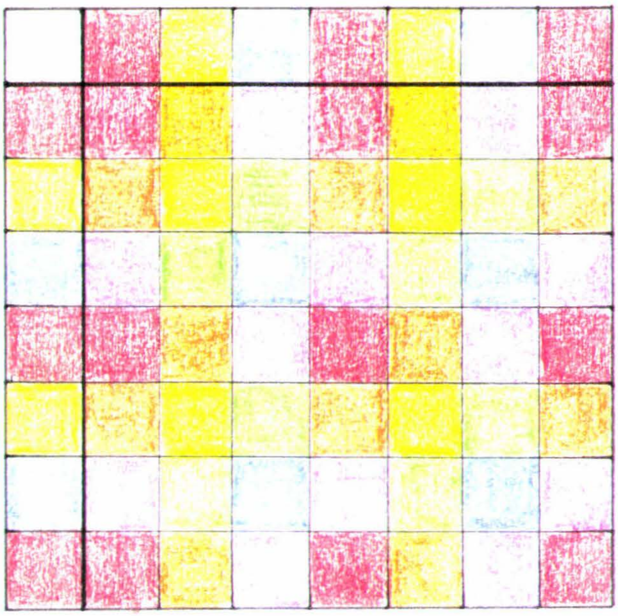
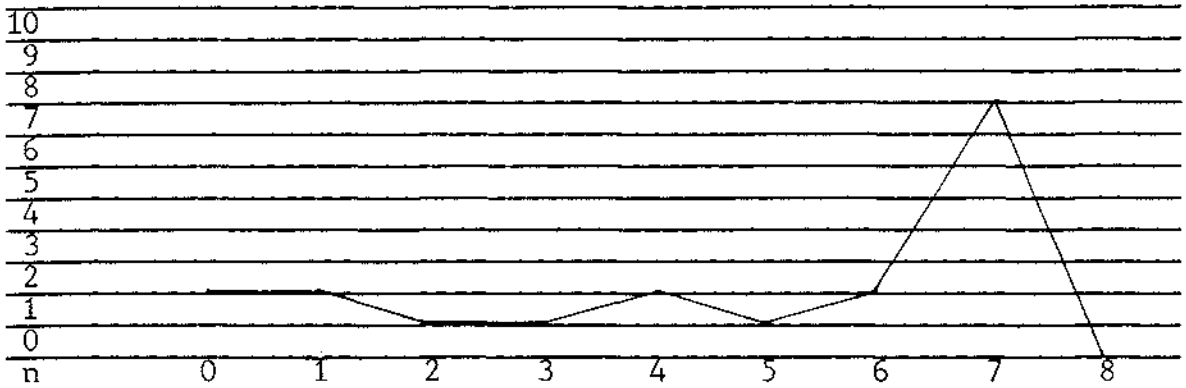


Figure 3a: Coloured Transparent Plastic Film - Addition Array.

Verbal instructions

- If the child has not completed the puzzle, questioning regarding original instructions is repeated until understanding is achieved or the puzzle is considered too difficult for that child and exchanged for another.
- If the puzzle is mostly correct with some mistakes, I start by pointing to a correct example and asking the child to explain how the colours were selected for this square. I then point to a square where a mistake has been made and ask the same question. Usually the child will see and correct the mistake.
- When the diagram has been completed accurately, I ask the child to tell me about the colours that have been made. I attempt to get the child to observe the horizontal, vertical, and diagonal patterning obtained.
- I attempt to assist the acquisition of the concept of addition through the use of the three primary colours, and ask the child how the three additional colours of green, orange, and purple have occurred.

Figure 3b shows levels of performance across Figure 3:

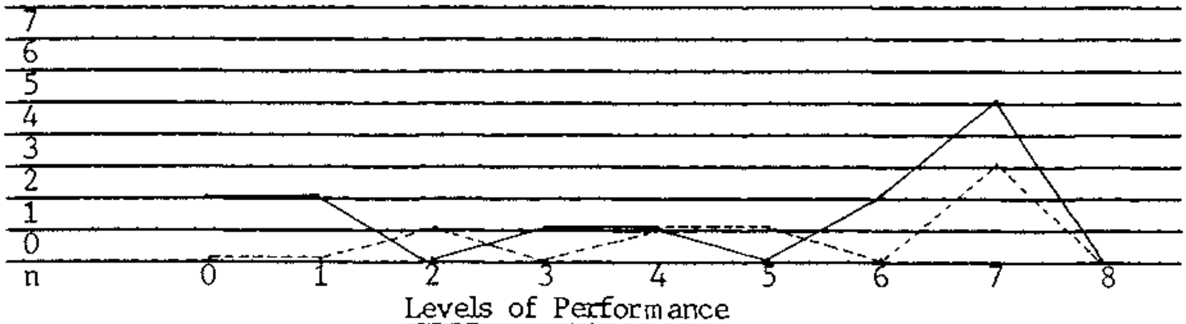


Levels of Performance

0 - Too difficult.
1 - Incomplete; unable to see array was incomplete.
2 - Slow and difficult completion; unable to observe horizontal, vertical, and diagonal colour relationships.
3 - Slow and difficult completion; horizontal, vertical, and diagonal colour relationships observed but not understood.
4 - Slow and difficult completion; horizontal, vertical, and diagonal colour relationships observed and understood.
5 - Slow and difficult completion; strategy using colour patterns to check accuracy; horizontal, vertical, and diagonal colour relationships observed and understood.
6 - Complete with some questioning; horizontal, vertical, and diagonal colour relationships observed and understood.
7 - Accurate without difficulty; horizontal, vertical, and diagonal colour relationships observed and understood.
8 - Fast completion; horizontal, vertical, and diagonal colour relationships observed and understood.

Figure 3b: Figure 3; Coloured Transparent Plastic Film - Addition Array: Levels of Performance. n = 19.

Figure 3c shows gender performance across levels of performance:



Levels of Performance

n = 13 ________ Males
n = 6 ---------- Females

Figure 3c: Coloured Transparent Plastic Film - Addition Array: Gender Performance Across Levels of Performance. n = 19.

Figure 3d shows levels of performance across mathematical groups.

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7			4	1	1	2		
6		1						1
5					1			
4		1	1					
3					1			
2					1			
1			1		1			
0		2						

<u>Levels of Performance</u>

0 - Too difficult.
1 - Incomplete; unable to see array was incomplete.
2 - Slow and difficult completion; unable to observe horizontal, vertical, and diagonal colour relationships.
3 - Slow and difficult completion; horizontal, vertical, and diagonal colour relationships observed but not understood.
4 - Slow and difficult completion; horizontal, vertical, and diagonal colour relationships observed and understood.
5 - Slow and difficult completion; strategy using colour patterns to check accuracy; horizontal, vertical, and diagonal colour relationships observed and understood.
6 - Complete with some questioning; horizontal, vertical, and diagonal colour relationships observed and understood.
7 - Accurate without difficulty; horizontal, vertical, and diagonal colour relationships observed and understood.
8 - Fast completion; horizontal, vertical, and diagonal colour relationships observed and understood.

<u>Figure 3d: Coloured Transparent Plastic Film - Addition Array: Levels of Performance Across Mathematical Groups. n = 19.</u>

Table 3 shows correlation coefficients between levels of performance and mathematical groups, school class levels, and ages.

<u>Table 3: Spearman's Rho Correlation Coefficients: Figure 3: Coloured Transparent Plastic Film - Addition Array: Levels of Performance: n = 19.</u>

Across mathematical groups	$r_s = 0.345$	$p = >0.05$
Across school class levels	$r_s = 0.424$	$p = <0.05$
Across ages	$r_s = 0.492$	$p = <0.05$

Figure 3e shows children at the highest level of performance for Figure 3 across mathematical groups and age. The most unexpected performance was from a 7 year old Junior Extension boy.

expected

11									
10					1	1	1		
9									
8				3			1		
7				1					
6									
	unexpected								
Age	Maths Groups	J SLD	JR	JE	S SLD	SR	AV	SE 2	SE 1

Figure 3e: Coloured Transparent Plastic Film - Addition Array: Highest Level of Performance: Accurate Without Difficulty; Horizontal, Vertical, and Diagonal Relationships Observed and Understood. Mathematical Groups Across Ages. n = 8.

Random Correct Performance with Confabulated Reasoning.

Random correct performance with confabulated reasoning is the third way besides unconscious nonverbal conceptual processing where a child's performance results in a correct solution to a part of a puzzle with the child being unable to give an adequate verbal report showing conscious conceptual understanding of the process or product. As the correct performance is accidental, the child does not have understanding, and may or may not gain this later following questioning. Piaget (1929) noted children without proper conceptual understanding may invent an answer that does not reflect a well thought out belief, or invent a rationale originally not connected with the production of the answer. Sometimes the child observes the correct rationale when questioned after the event.

An example of random correct performance with confabulated reasoning was given in the introduction where a child might randomly match a red peg with a red material square and later be able to perceive this colour relationship through questioning. The child now confabulates a reason for making the accidental match, perhaps verbalising the conceptual understanding that occurred during the later questioning process, but which was not present at the time of the random placement of the matched piece. Random correct performance with confabulated reasoning of this type occurred with a number of children during the performance of one puzzle, the Birthday Candle Array, and will be discussed in a later section on diagnostic progressions.

Unconscious Processing

There were other puzzles in addition to Figures 1 and 2 where examples of unconscious conceptual processing with difficulty verbalising classifications occurred. Figure 4, a beginning senior puzzle, gives one example:

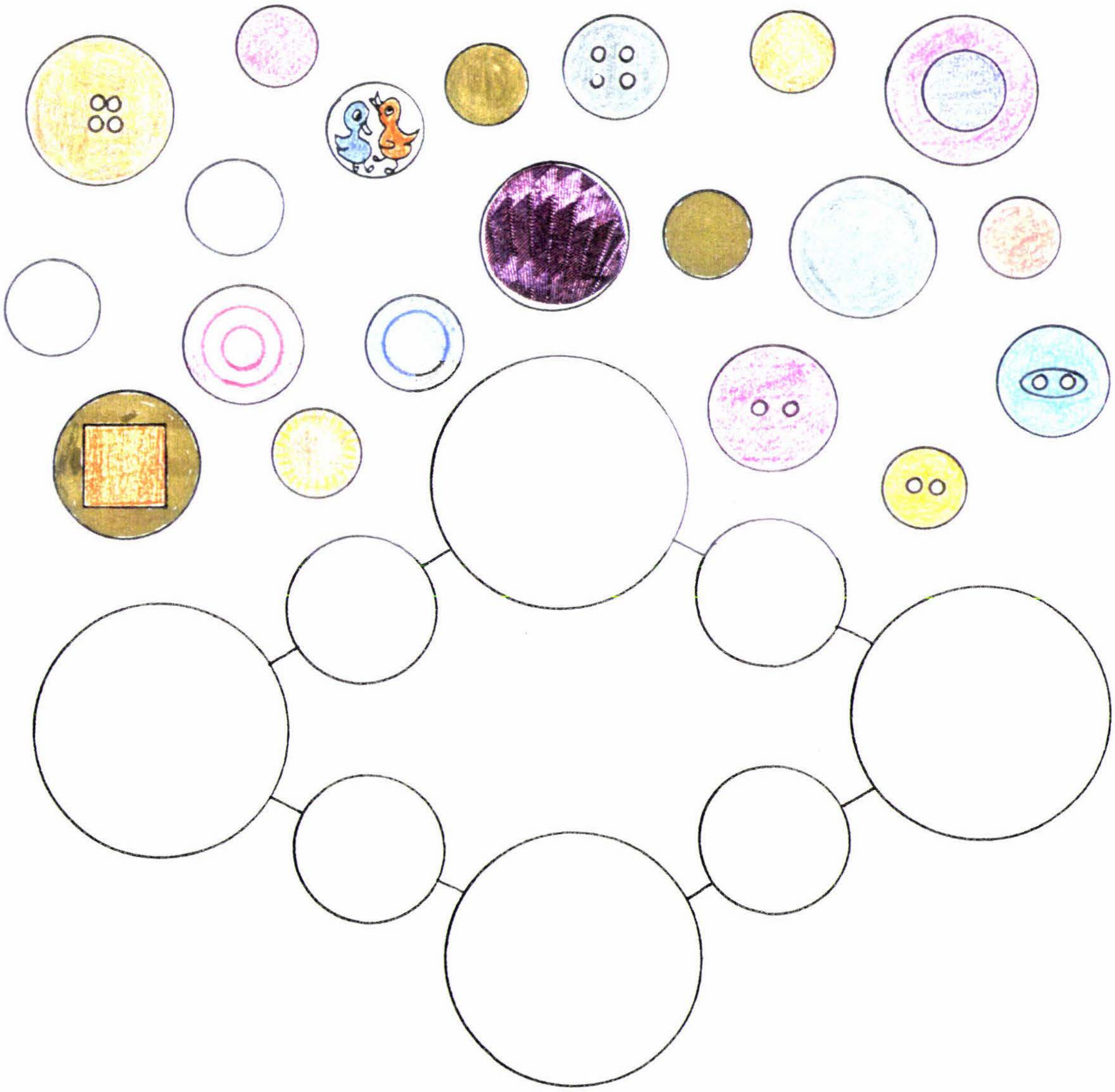


Figure 4: Four Intersecting Button Sets:
Instructions and questioning as for Figure 1.

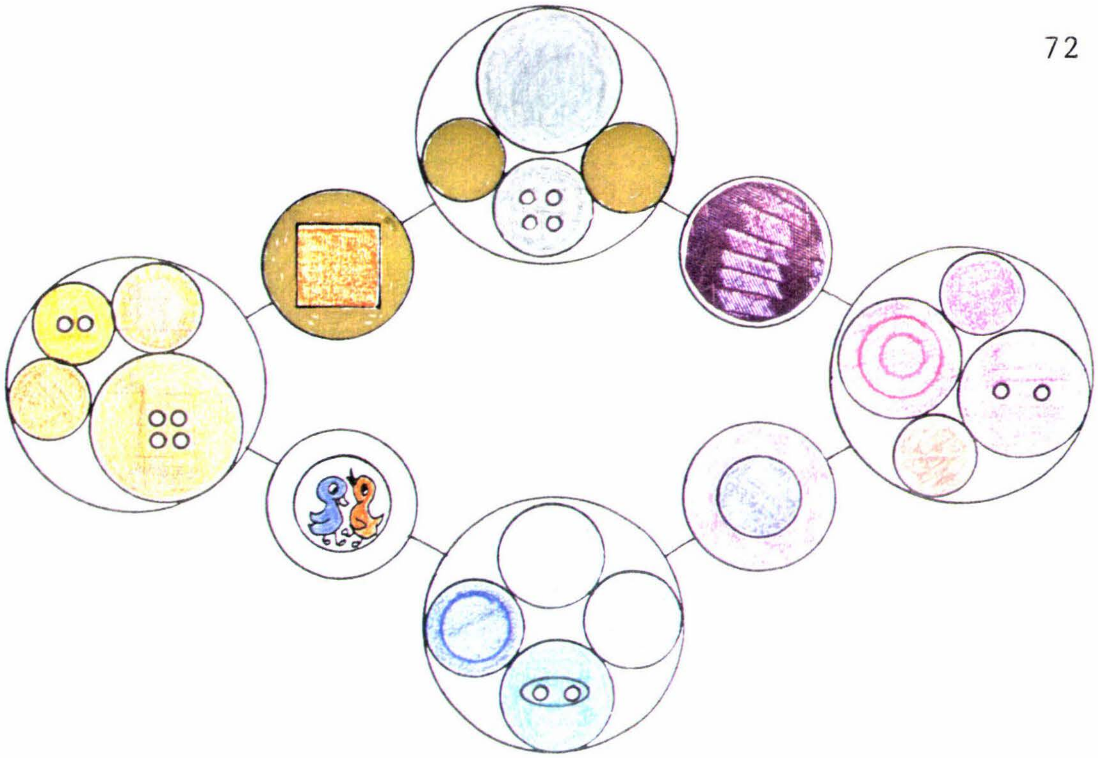


Figure 4a: Four Intersecting Button Sets.
Instructions and questioning as for Figure 1a.

Figure 4b shows levels of performance across mathematical groups.

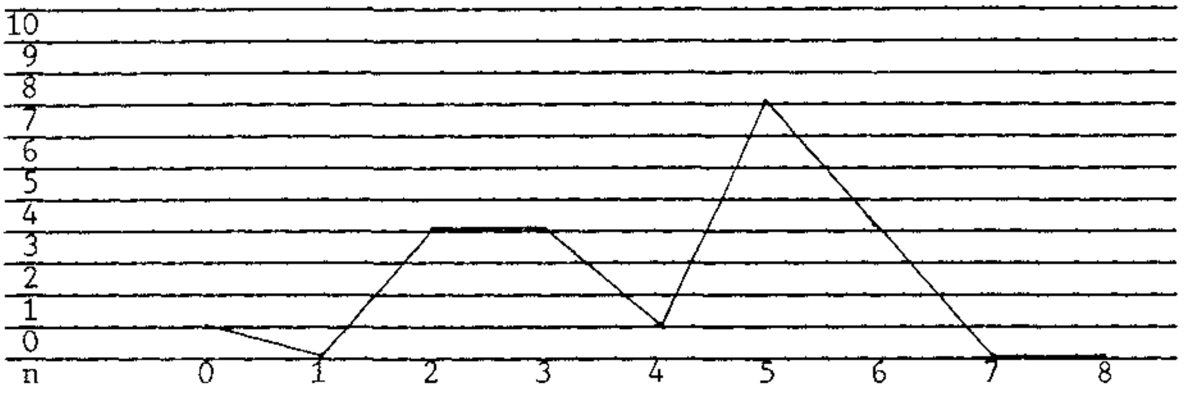
Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6			1			2	1	
5			2		2		3	1
4			1					
3			1		3			
2			1		1		1	1
1								
0					1			

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 4b: Four Intersecting Button Sets: Blue / Pink / Orange / Metal.
Levels of Performance Across Mathematical Groups. n = 22.

Figure 4c shows levels of performance across Figure 4:

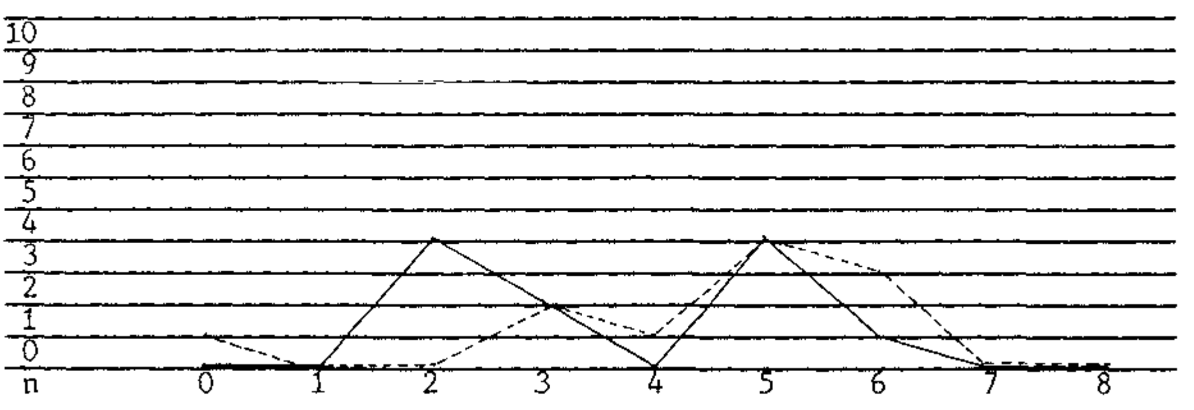


Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 4c: Four Intersecting Button Sets: Blue / Pink / Orange / Metal. Levels of Performance. n = 22.

Figure 4d examines gender performance across levels of performance: [Levels of performance as for Figure 4c]



Levels of Performance

n = 11 ________ Males
n = 11 ---------- Females

Figure 4d: Four Intersecting Button Sets: Blue / Pink / Orange / Metal. Gender Performance Across Levels of Performance. n = 22.

An example of unconscious conceptual processing and difficulty verbalising classifications occurred in Figure 4 with a 7 year old JE girl who couldn't at first name her metal set, although with questioning she knew what metal was, and that these buttons were made of metal, with the word 'metal' already in her vocabulary. This was the same child who recognised the transparency of the button on the carpet in Figure 1 as 'see through'. This child had not used a process of elimination, and had correctly placed her intersecting buttons.

Table 4 shows correlation coefficients between levels of performance and mathematical groups, school class levels, and ages, for Figure 4.

Table 4: Spearman's Rho Correlation Coefficients for Figure 4: Four Intersecting Button Sets: Blue / Pink / Orange / Metal. Levels of Performance: n = 22.

Across mathematical groups	$r_s = 0.169$	$p = >0.05$
Across school class levels	$r_s = 0.094$	$p = >0.05$
Across ages	$r_s = -0.039$	$p = >0.05$

Figure 4e shows children at the highest level of performance for Figure 4, across mathematical groups and age. The most unexpected performance for Figure 4 was from a 6 year old Junior Extension boy.

expected

Age	J SLD	JR	JE	S SLD	SR	AV	SE 2	SE 1
11								
10						1		
9							1	
8						1		
7								
6			1					

unexpected

Age / Maths Groups

Figure 4e: Four Intersecting Button Sets: Blue / Pink / Orange / Metal. Highest Level of Performance: Very fast completion with no questioning: Mathematical Groups Across Ages. n = 4.

A further puzzle in addition to Figures 1, 2, and 4 where examples of unconscious conceptual processing with difficulty verbalising classifications occurred was Figure 5, an above average senior puzzle:

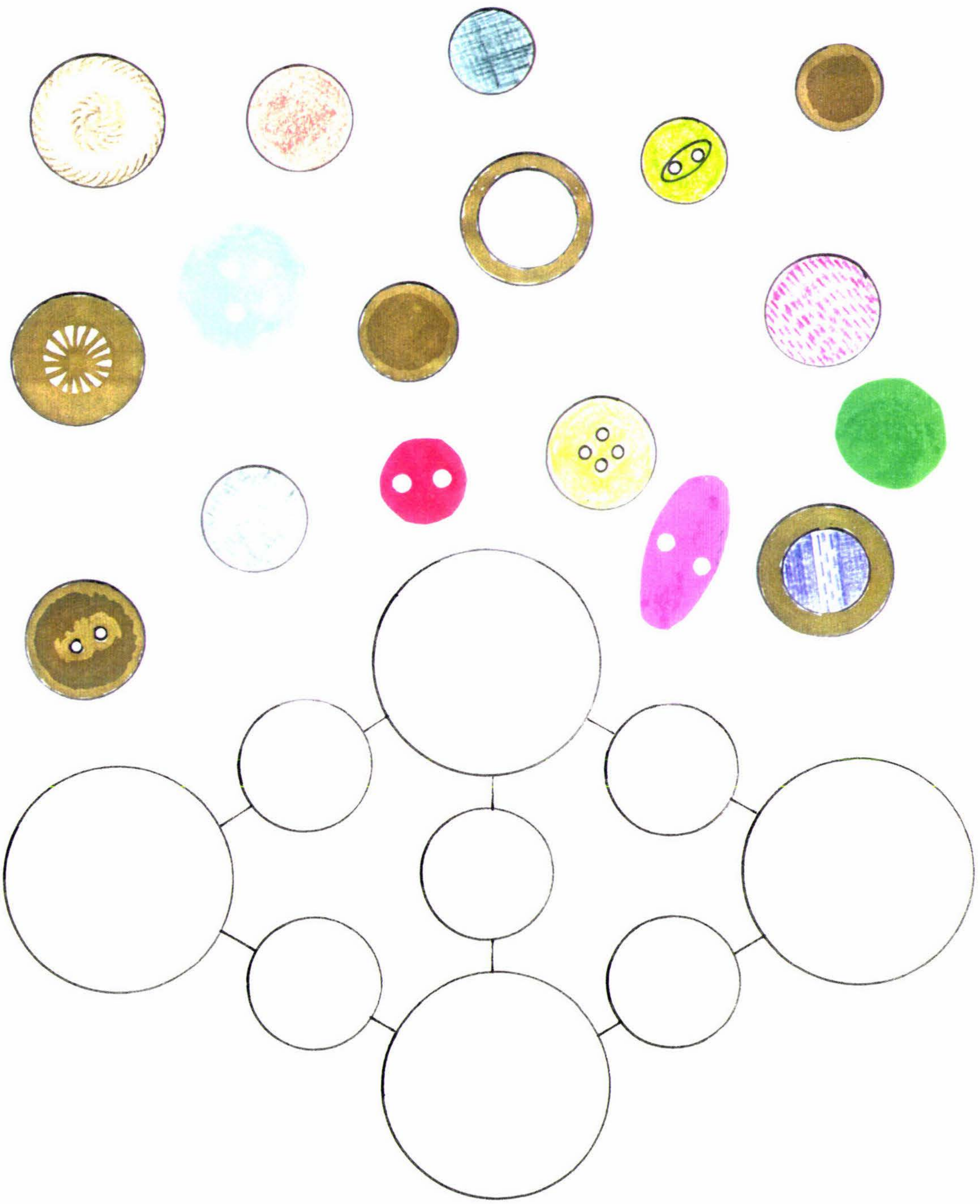


Figure 5: Four Intersecting Button Sets.
Instructions and questioning as for Figure 1.

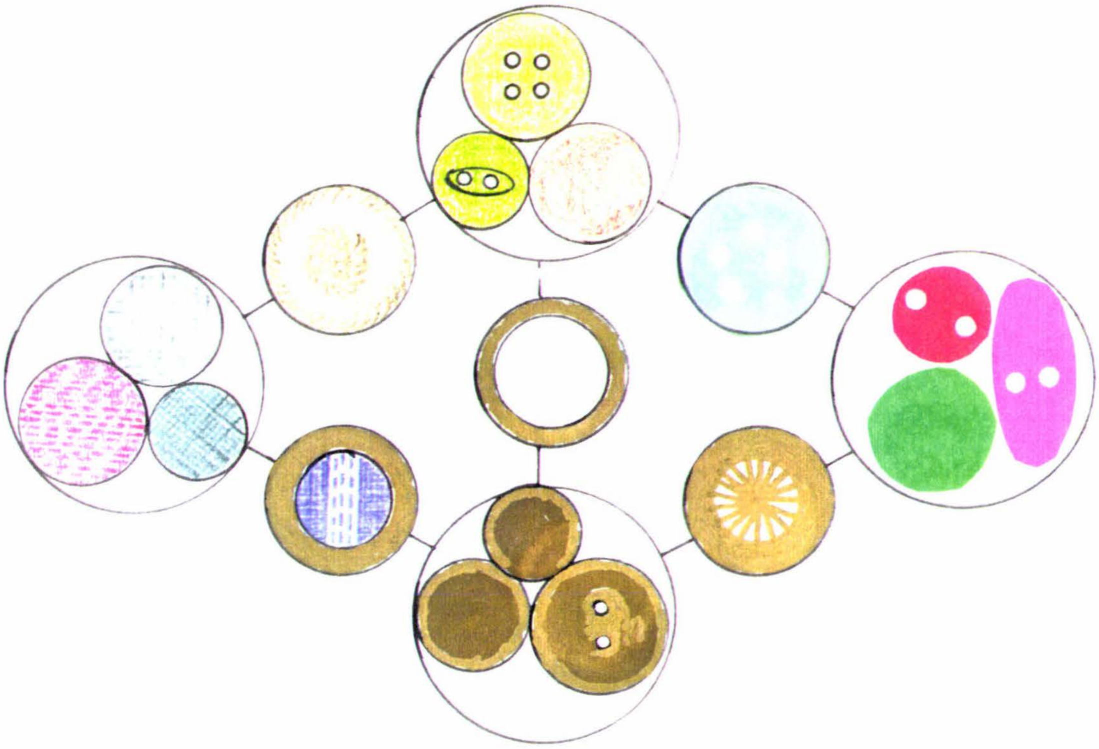


Figure 5a: Four Intersecting Button Sets.
Instructions and questioning as for Figure 1a.

Figure 5b shows levels of performance across mathematical groups.

Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7						1		
6								1
5								
4								2
3						1	1	1
2							1	2
1								
0								

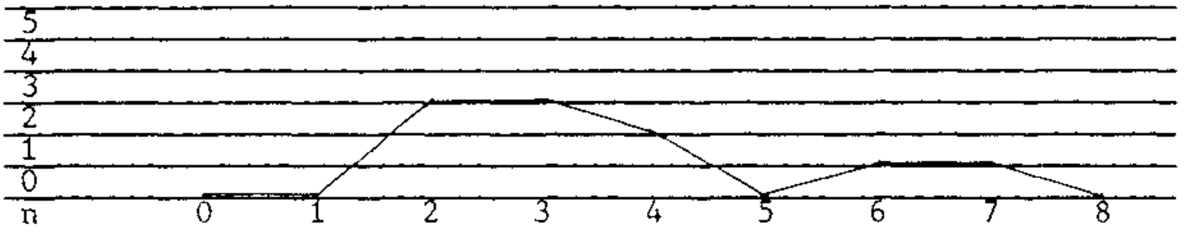
Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion. Difficulty naming sets
7 - Very fast completion. Clear naming of sets
8 - Exceptional or novel performance

Figure 5b: Four Intersecting Button Sets: Transparent / Plastic / Gold / Fabric: Levels of Performance Across Mathematical Groups. n = 10.

The most unexpected performance for Figure 5 was from an 8 year old girl with an average P.A.T. mathematics score and the highest level of performance: very fast completion with no help, and clear naming of sets.

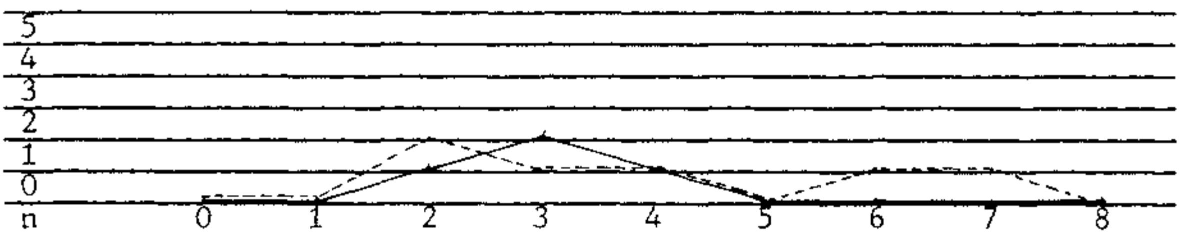
Figure 5c shows levels of performance across Figure 5:



0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion. Difficulty naming sets
7 - Very fast completion. Clear naming of sets
8 - Exceptional or novel performance

Figure 5c: Four Intersecting Button Sets: Transparent / Plastic / Gold / Fabric: Levels of Performance. n = 10.

Figure 5d examines gender performance across levels of performance:
[Levels of performance as for Figure 5c]



n = 4 ________ Males
n = 6 ---------- Females

Figure 5d: Four Intersecting Button Sets: Transparent / Plastic / Gold / Fabric: Gender Performance Across Levels of Performance. n = 10.

Table 5 shows correlation coefficients between levels of performance and mathematical groups, school class levels, and ages, for Figure 5.

Table 5: Spearman's Rho Correlation Coefficients for Figure 5: Four Intersecting Button Sets: Transparent / Plastic / Gold / Fabric: Levels of Performance: n = 10.

Across mathematical groups	$r_s = 0.042$	$p = >0.05$
Across school class levels	$r_s = 0.127$	$p = >0.05$
Across ages	$r_s = 0.127$	$p = >0.05$

Another puzzle in addition to Figures 1, 2, 4, and 5, where an example of unconscious conceptual processing with difficulty verbalising classifications occurred was Figure 6, an average senior puzzle:

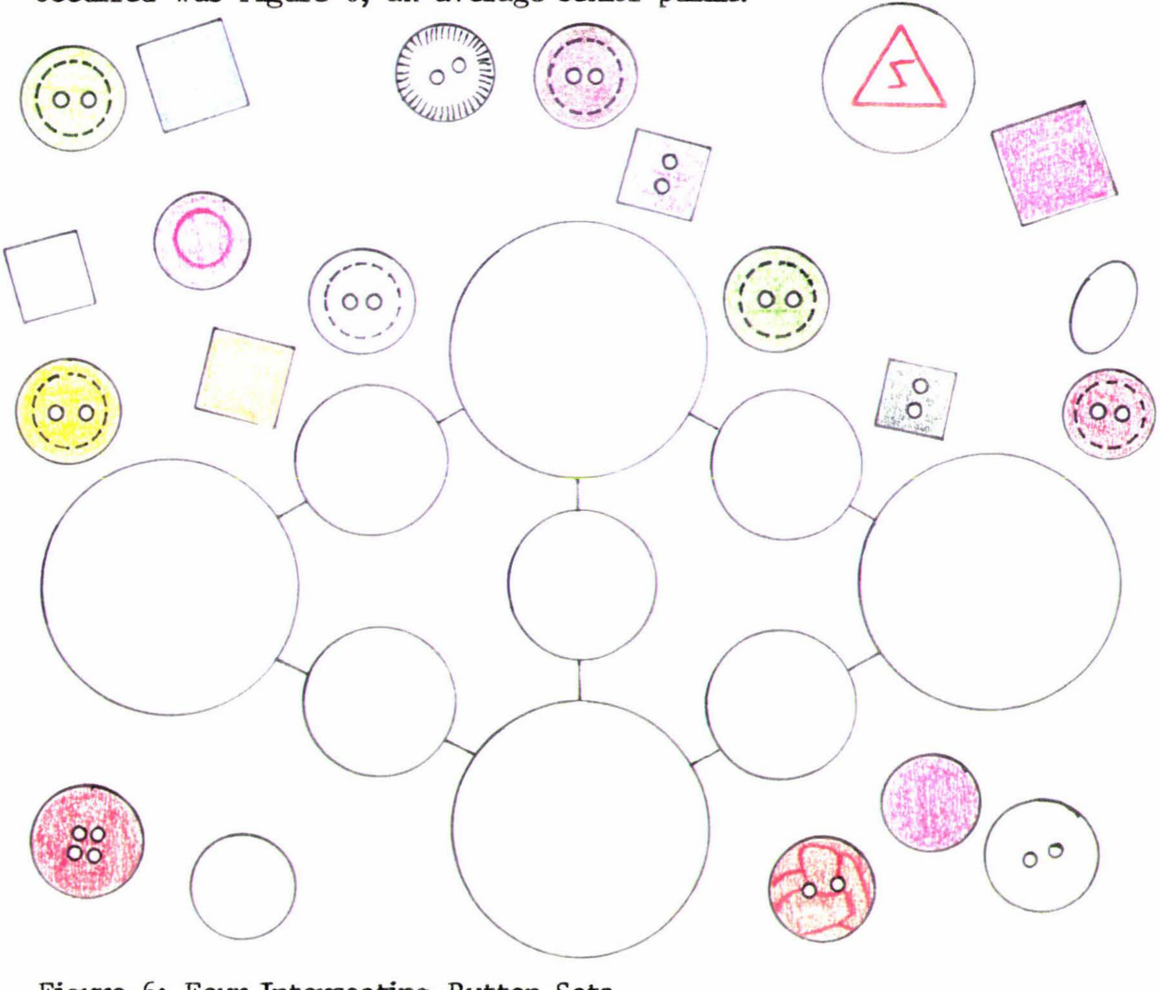


Figure 6: Four Intersecting Button Sets.
Instructions and questioning as for Figure 1.

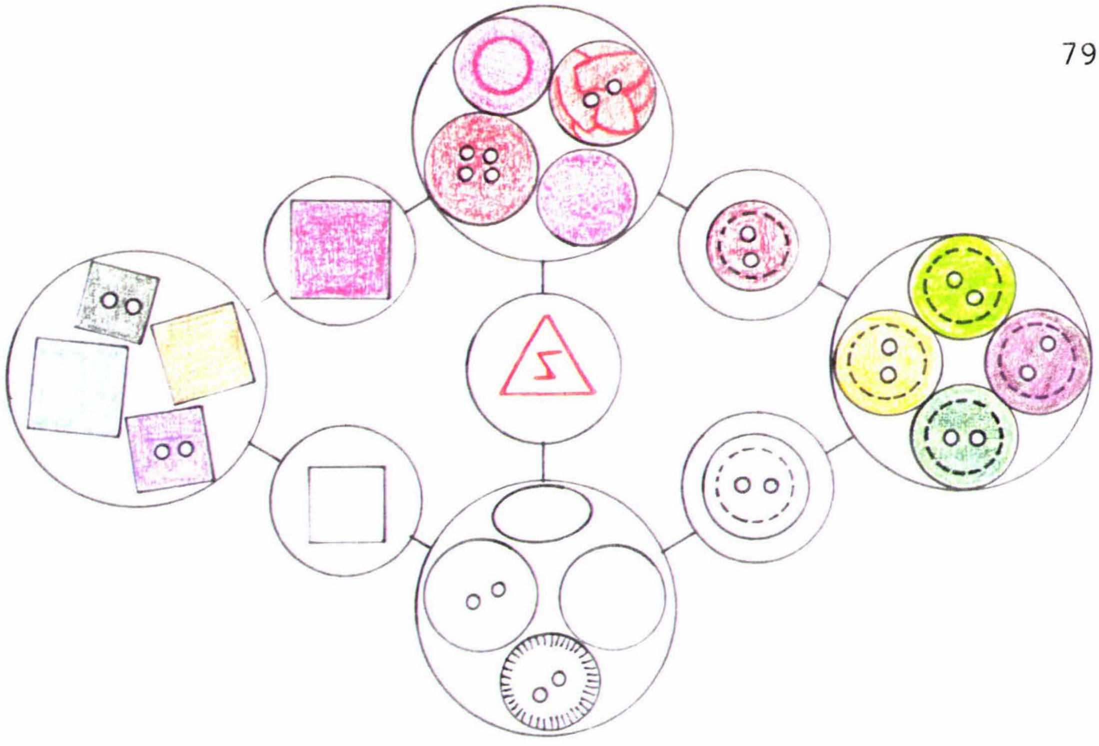


Figure 6a: Four Intersecting Button Sets.

Instructions and questioning as for Figure 1a.

Figure 6b shows levels of performance across mathematical groups.

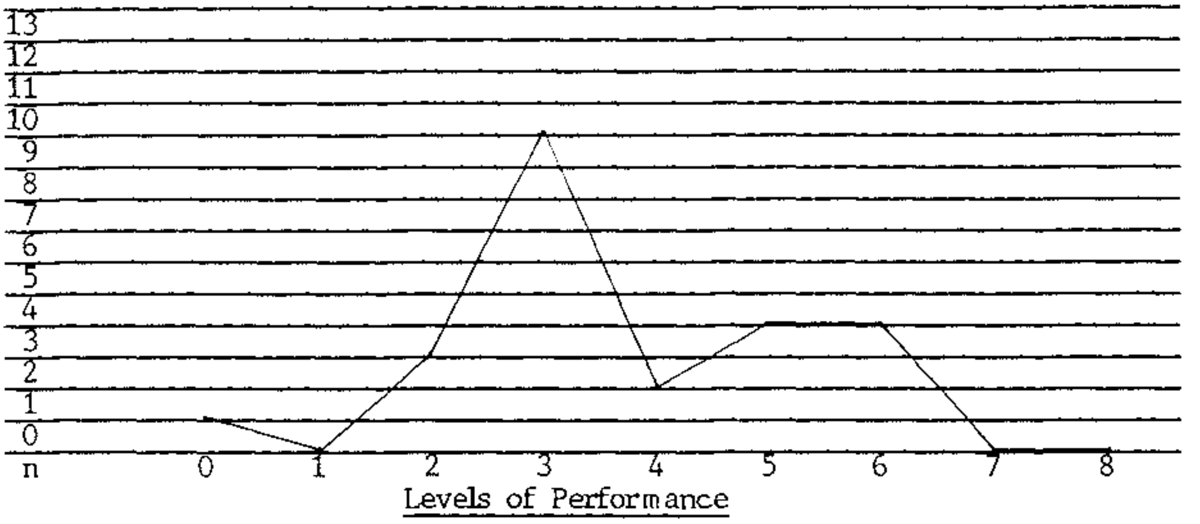
Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6					1	1		1
5			1	1	1		3	1
4			4					2
3			8		3	1	3	5
2			1					
1								
0								

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 6b: Four Intersecting Button Sets: Pattern / Red / White / Square: Levels of Performance Across Mathematical Groups. n = 24.

Figure 6c shows levels of performance across Figure 6:



0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 6c: Four Intersecting Button Sets: Pattern / Red / White / Square: Levels of Performance. n = 24.

Figure 6d examines gender performance across levels of performance:
[Levels of performance as for Figure 6c]

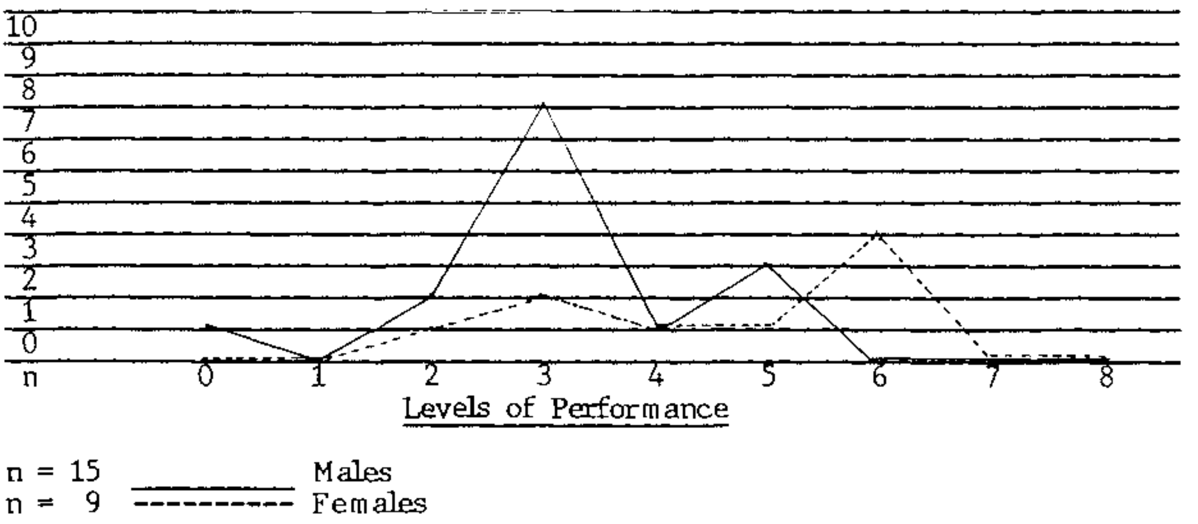


Figure 6d: Four Intersecting Button Sets: Pattern / Red / White / Square: Gender Performance Across Levels of Performance. n = 24.

Table 6 shows correlation coefficients between levels of performance and mathematical groups, school class levels, and ages, for Figure 6.

Table 6: Spearman's Rho Correlation Coefficients for Figure 6: Four Intersecting Button Sets: Pattern / Red / White / Square: Levels of Performance: n = 24.

Across mathematical groups	$r_s = 0.333$	$p = >0.05$
Across school class levels	$r_s = 0.158$	$p = >0.05$
Across ages	$r_s = 0.214$	$p = >0.05$

The child who used an unconscious conceptual process but who had difficulty verbalising classifications in Figure 6 was a 10 year old girl with an average P.A.T. mathematics score who arranged all of her buttons correctly, but who couldn't name her square set without further questioning.

Figure 6e shows children at the highest level of performance for Figure 6, across mathematical groups and age. The most unexpected performance for Figure 6 was an 8 year old SE 1 girl.

expected

Age	Maths Groups: J SLD	JR	JE	S SLD	SR	AV	SE 2	SE 1
11								
10								2
9							1	
8								1
7								
6								

unexpected

Figure 6e: Four Intersecting Button Sets: Pattern / Red / White / Square: Highest Level of Performance: Very fast completion with no questioning: Mathematical Groups Across Ages. n = 4.

Levels of performance across mathematical groups, classes and ages for puzzles demonstrating unconscious processing did not correlate with any significance, with the exception of slight significant correlation at 0.05 level for classes and ages but not mathematical groups for Figure 2. One reason for this general lack of correlaton may be that P.A.T. tests testing formal mathematical learning, and the puzzles extending pre-mathematical activity of classification and ordering into sets, measure different abilities in children, with the puzzles requiring perceptual, conceptual and creative flexibility utilising logical reasoning and problem solving skills.

Table 6A shows two tailed Mann-Whitney U-Tests testing gender differences in levels of performance across mathematical groups at 0.05, where U (M) = Males, U1 (F) = Females, and z shows use of normal approximation for larger samples. Figure 6 is significant in favour of performance of females at 0.05. As this result was one of very few puzzles showing gender differences, and larger numbers of tests are more likely to encounter chance significance, this result is more likely to be due to chance.

Table 6A: Mann-Whitney U-Tests For Chapter 3: Gender Differences: Levels of Performance Across Mathematical Groups:

Figure	U (M)	U1 (F)	z	Males	Females
Puzzles demonstrating unconscious processing					
1	149.5	186.5	+ 0.56	-	-
2	51.5	56.5		-	-
4	41.5	79.5		-	-
5	9.5	14.5		-	-
6	34	101		-	*
Puzzle demonstrating rote performance					
3	31	47		-	-

* Significant at 0.05

Summary

Children showed evidence of unconscious processing of less familiar concepts, where a process of elimination, rote performance or confabulated reasoning were not shown to have occurred. Further examples in addition to Figures 1, 2, 4, 5, and 6, of examples of unconscious conceptual processing with difficulty verbalising classifications occurred, and will be shown in later chapters. There were no gender differences of significance which may not have been attributed to chance.

Chapter 4 looks at possible reasons for the selections of sets processed unconsciously, where children may process familiar concepts in a more consciously aware manner than unfamiliar concepts. This leads to a discussion of perceptual priority modes where colour, size, shape and pattern classifications are more familiar to children in junior school mathematics. Puzzles show variations in perceptual priority modes children used, and perceptual discrimination difficulties within modes, such as the differences between colours or shapes. Puzzles with unfamiliar concepts are presented, and puzzles showing further perceptual difficulties children had, such as in discriminating reversals, manipulating rotations, and leading on to puzzles with multiple concepts embedded within them.

Chapter 4

Perceptual Priorities

The phenomenon of unconscious nonverbal processing leads to questions regarding perceptual priorities. Which perceptions take prominence? Do people first perceive attributes with which they are familiar, or do differences take priority? A sense of difference must emerge from a sense of what is familiar first. A difference isn't a difference until it is different from what is already known. Children unable to verbalise their classified sets had more difficulty verbalising sets with unexpected attributes. Actions, as accurate classifications and intersections of button sets, were evidence the children perceived the differences, but their conscious expectations must have focused on familiar pathways rather than on the novel experience they were confronted with. Actions showed unconscious adaptation to the unexpected, also showing clear conceptual understanding of the situation. Conscious verbal interpretation of these correct actions was lagging, only brought to consciousness on request and with further questioning to elicit the concepts involved. Once the concept of 'transparent' was brought to the conscious awareness of a child, this concept was consciously remembered if encountered in a subsequent puzzle.

The classification a child was unable to verbalise in the five previously mentioned puzzles is underlined:

Figure 1: blue / <u>transparent</u> / gold
Figure 2: leather / square / gold / <u>patterns of three</u>
Figure 4: blue / pink / orange / <u>metal</u>
Figure 5: transparent / <u>plastic</u> / gold / fabric
Figure 6: pattern / red / white / <u>square</u>

The surprise element in Figure 5, was that all of the transparent buttons except one were made of glass. The one plastic transparent button was the intersecting button between the transparent and plastic sets. The difference between the glass transparent buttons and the plastic transparent one was not in the child's conscious awareness, even though all transparent and plastic buttons were correctly placed and intersected, including the transparent/plastic, transparent/gold, gold/plastic, gold/fabric, and fabric/plastic intersections. Clear conceptual awareness of the 'plastic' concept was shown through three correctly placed intersecting buttons made or partly made with plastic, but the child could not verbalise 'plastic' until asked her to hold all of her transparent buttons and tell me what they were made of. She felt them all, felt the weight and coldness of the glass buttons and the way they clinked together in her hand. She told me they were made of glass, but that one of them wasn't made of glass. When asked what it was made of she said 'plastic'. Only when asked her why she had intersected her transparent set and her next set with this particular button, and pointed to her plastic set, asking her what these buttons were made of, did she name her set 'plastic'.

There is support for the idea it is easier to find similarities than it is to find differences. Velmans (1991) finds semantic analysis of familiar stimuli in the attended channel can occur without reportable consciousness, with consciousness being unnecessary for such analysis. He also considers processing accompanied by awareness of the stimulus is different from processing that is not. Libet, Wright, Feinstein, & Pearl, (1979), and Neeley (1977) consider conscious focus of attention during processing cannot be necessary for the analysis and identification of such stimuli, even when occurring in novel, complex combinations. The analysis of familiar stimuli proceeds involuntarily, irrespective of whether the stimuli are in the attended channel or not. Even during conscious attention to a given stimulus, it may be difficult to prevent certain analyses from occurring. The analysis is automatic in this sense.

Velmans further considers consciousness may be essential when novel stimuli or skills are being learned. Consciousness of the input stimulus and of associations being formed result from focal-attentive processing, to which we have no introspective access, and which are normally explained in entirely neurophysiological terms. Subjects cannot prevent perceptual analysis of irrelevant attributes of an attended object. Even if a stimulus is consciously attended to, what is analysed may not be under conscious voluntary control. Input selection is preconscious. Task selection is under conscious, voluntary control. Velmans believes much learning and memory is to do with recurring stimulus patterns. The stimuli may be present to consciousness, but that they form a recurring pattern may not. Schacter (1987), and Reber (1989) find exposure to successive recurring patterns may produce learning of those patterns, whether or not there is any intention to learn, and in the absence of any explicit knowledge of the pattern being learned. Perruchet et al (1990) find availability in memory of a particular item is highly dependent on the number of repetitions of this item during the learning session. Greater frequency yields greater retention.

This may explain why children more easily name classifications with greater familiarity to them. Skemp (1961) believes when generalization takes place the mind searches for all points of similarity between the ideas in the data before it, as discrimination. Points of agreement must be consistent with observed differences. This conflicts with another idea that likenesses and differences automatically separate out, with likenesses being considered. Skemp (1971) adds that mathematics comes from perceptions leading to classifications. These lead to invariant properties and common properties, or as Dienes (1959) says, mathematics patterns and regularities found at successive levels, with similarities easier to find than differences. We see what we know. This is a process of abstraction. Classification may be by function or by naming objects. Neeley (1977) finds strong support for Posner & Synder's 'two-process' theory: an initial preconscious identification process accesses memory traces of the input stimulus and those of semantically related stimuli. This is followed by a conscious identification process that not only facilitates the recognition of expected stimuli but also inhibits the recognition of unexpected stimuli.

Classifications as concepts may develop as a holographic process of repeated sensory information, consisting of many perceptions over time, of many different images, dimensions, variations, positions, moods, and applications for each separate idea. The child learns from memory the essential elements holding constant each time a particular experience is met. Items not holding constant are discarded as not part of the concept.

met. Items not holding constant are discarded as not part of the concept. This is similar to pre-mathematical ideas of conservation. Similarly, children show awareness of sequences related to events. Their need for rhythm and order is strong (Churchill 1961). Later, the child can work out how to do something before doing it, and can anticipate results before they occur. This is one beginning of true thought, arising out of actions that have become internalised (Piaget & Inhelder 1959).

Familiar Concepts

Gagné (1964; 1977; & Gagné & Briggs 1979) orders learned behaviour from discrimination learning, concrete concept learning, defined concept learning, rule learning, higher order rule learning and cognitive strategy learning. During the early stage of discrimination learning it is interesting to observe perceptual priorities children demonstrate. Many children see colour likenesses before observing pattern similarities. Other children discriminate shapes more readily than colours. In junior classes children are regularly exposed to materials enabling awareness of colour, shape, size and pattern to develop. Colour names; shape names; size differentiation involving length, area, volume, weight, thickness, ordering and time; and pattern discrimination and creation of patterns are included. Figure 7, an advanced junior puzzle shows one child's perceptual priority mode between these dimensions:

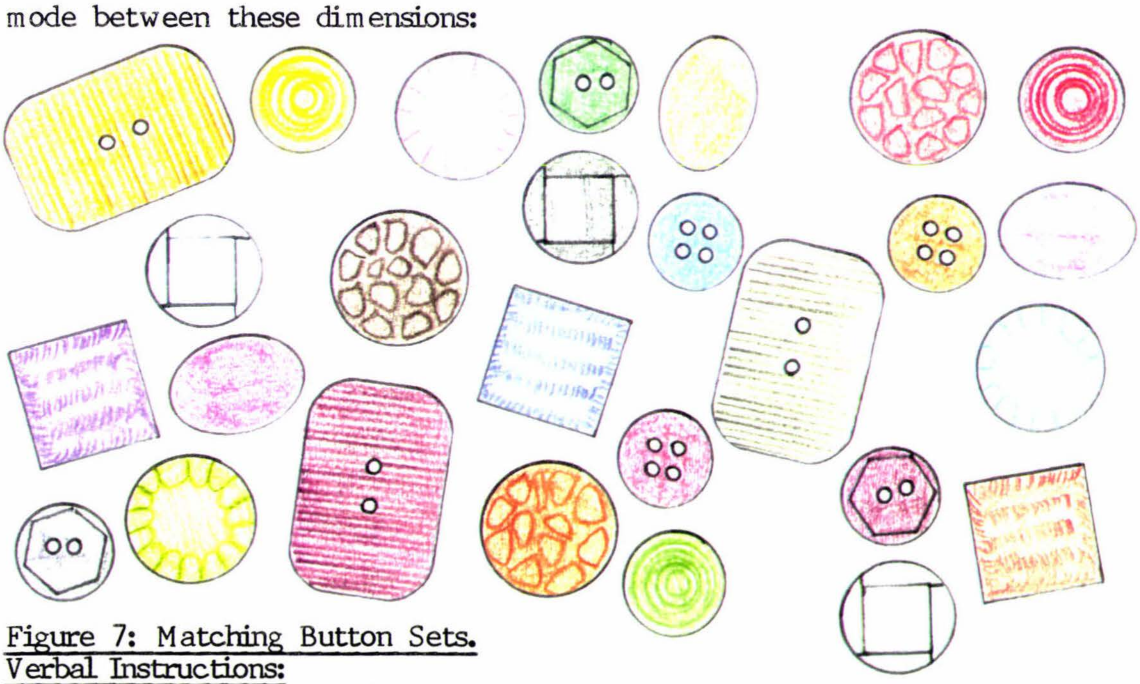


Figure 7: Matching Button Sets.
Verbal Instructions:
- Find which buttons belong together.

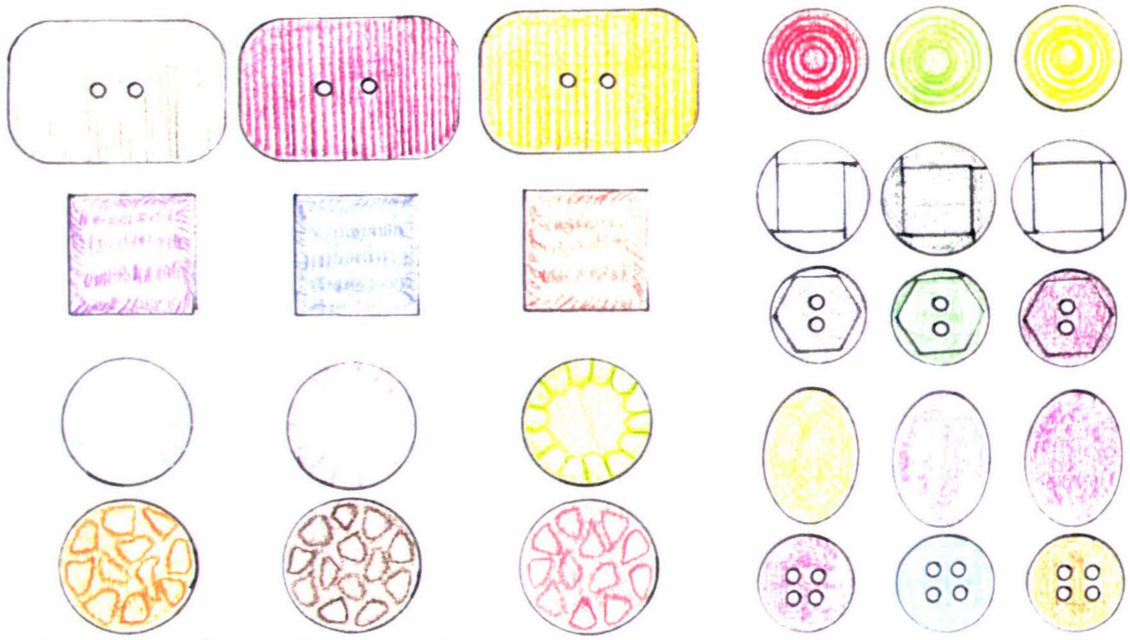


Figure 7a: Form Matching Button Sets.

A difficulty many children have is knowing which way up buttons face. I teach all children to put buttons up the right way first. One 8 year old JE child matched his buttons into three size groups, only becoming aware of and classifying different patterns when I took one of his size groups, helped him turn all of his buttons up the right way, and asked him if he could find any buttons that were the same in another way besides their size. I gave him a similar puzzle, Figure 8, an advanced junior puzzle, using different fabrics and textures, which was completed easily by him:

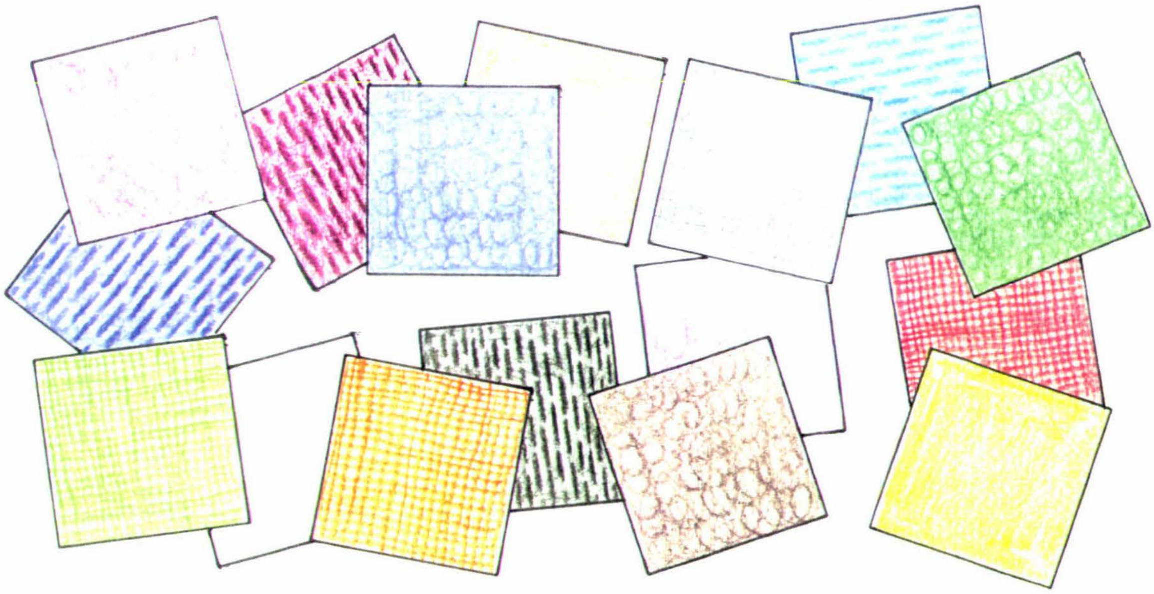


Figure 8: Fabric Texture Match: Sets of Four.

Figure 9, a beginning senior puzzle was another example of a child's perceptual priority mode between dimensions:

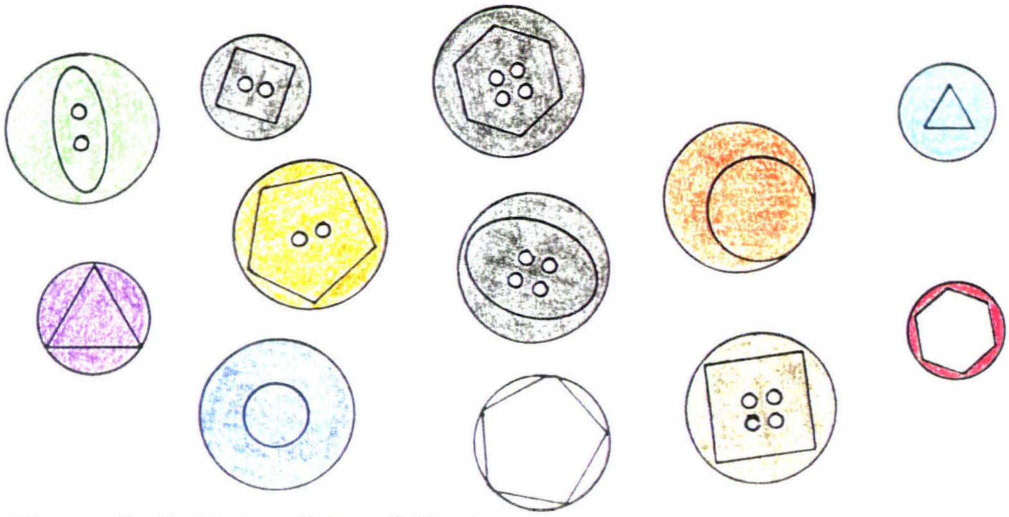


Figure 9: Buttons: Sets of Two:
Verbal Instructions:
Find sets of two buttons that belong together.

Figure 9a shows levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6								
5						1		
4								
3						1	1	1
2								
1								
0								1

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 9a: Circular Buttons: Sets of Two with Matching Internal Shapes: Levels of Performance Across Mathematical Groups. n = 5.

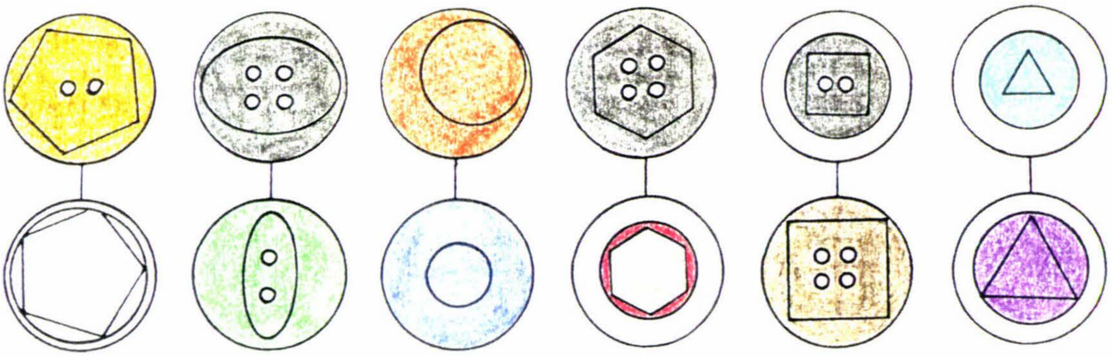


Figure 9b: Circular Buttons: Sets of Two with Matching Internal Shapes:

One 10 year old SE 1 boy put his buttons into colour groups. He did not notice the shapes until I asked him if there was another way he could match his buttons. An 8 year old AV girl had difficulty discriminating within the shapes in Figure 9, at first mixing pentagons with hexagons until I asked her how many sides each had. She also had difficulty recognising shapes in Figure 10, a beginning senior puzzle. Her first attempt at Figure 10 showed no awareness of shape, despite all buttons being black to eliminate the colour dimension. I had to hold one shape button and ask her to tell me about it before she noticed it incorporated shape. She completed the puzzle quickly with no further help. Her 10 year old AV sister completed Figure 10 in two minutes, with no questioning:

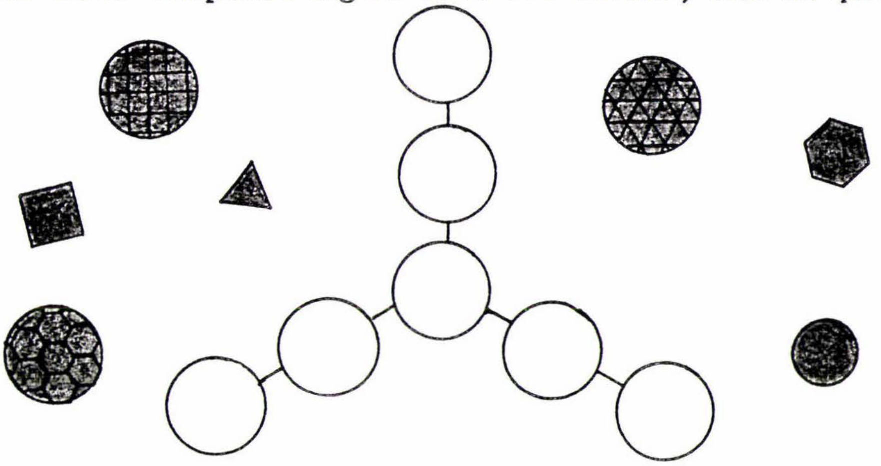


Figure 10: One Difference Button Arms [Black]:

Verbal Instructions:

- Find which buttons belong together.
- Arrange the buttons with no more than one thing different between any buttons you place in connecting circles.

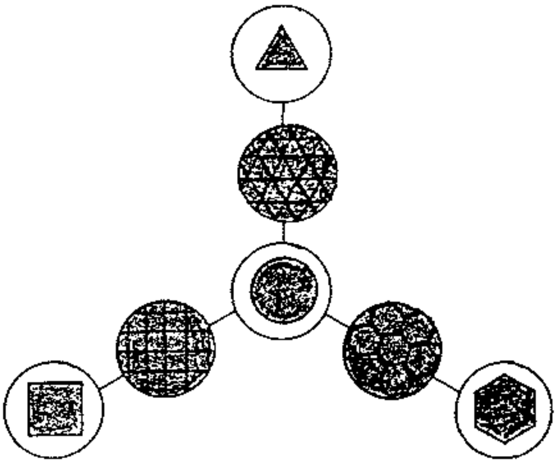


Figure 10a: One Shape Difference Button Arms [Black]:

Some puzzles eliminate one dimension to help children develop another. Figure 11, an advanced junior puzzle uses embossed white wallpaper samples cut into squares the same size, to focus attention on pattern by eliminating colour, size and shape of pieces. [Plate 64].

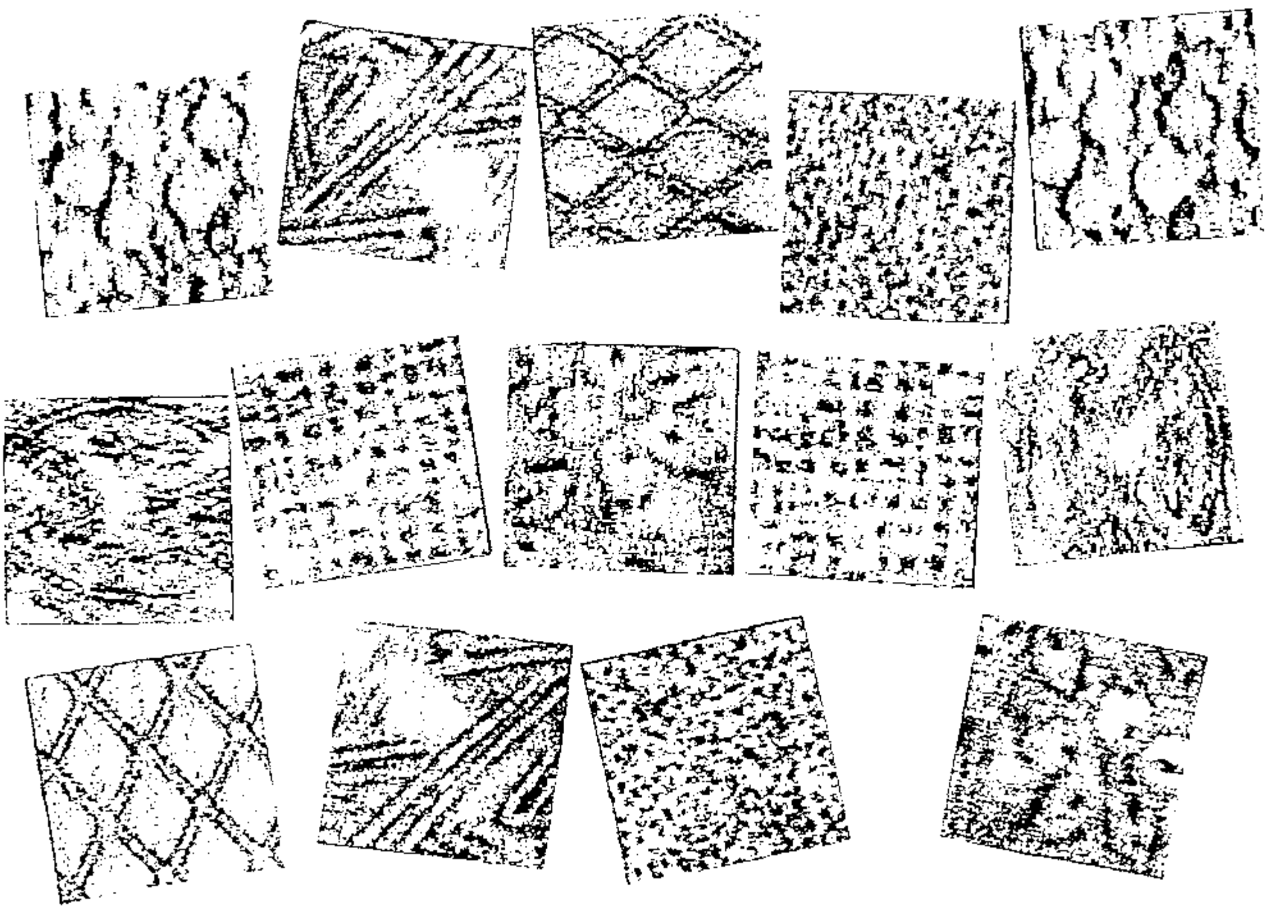


Figure 11: Embossed White Wallpaper Patterns: Sets of Two.

Samples of brown sandpaper can be used for grading as well as matching. [Plate 65]. Figure 12, a very advanced junior puzzle directs attention to size of pieces, with potential for ordering. Nine identically coloured, patterned and sized circles are cut into a buildup fraction set. One is whole and can be used as a base if a child wants to build circles up rather than spreading them separately. Other circles are cut into halves, thirds, quarters, fifths, sixths, eighths, tenths and twentieths. One 9 year old SR girl had difficulty discriminating between sizes. She put the two halves onto the whole circle first, then attempted another circle using two quarters, a fifth and a third, finding these wouldn't fit. I asked her how she made her first circle. She said she measured the sizes of the pieces together. I asked her if she could measure the sizes of all of her other pieces together to find any the same. She made size piles, completing each size pile into a circle, and told me which circles were made with quarters, tenths, etc. This contrasts with an 8 year old AV girl who, with no questioning, graded her circles in order from the whole one through to the one containing twentieths.

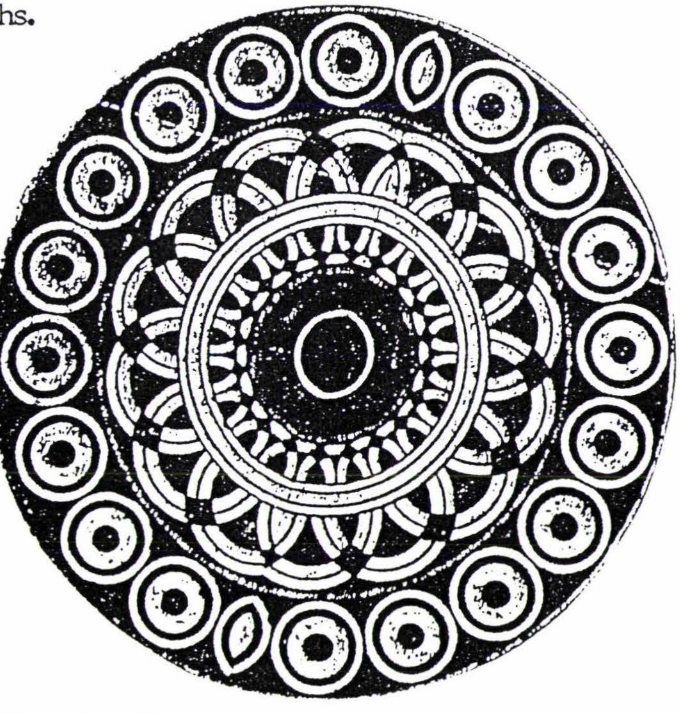


Figure 12: Buildup Circular Fraction Set.

Some children have difficulty with subtleties within colours. A 10 year old AV girl needed help to sort between three shades of red and three shades of orange in Figure 13, an above average senior puzzle. Sometimes a child can be taught a subtle colour name such as vermilion red:

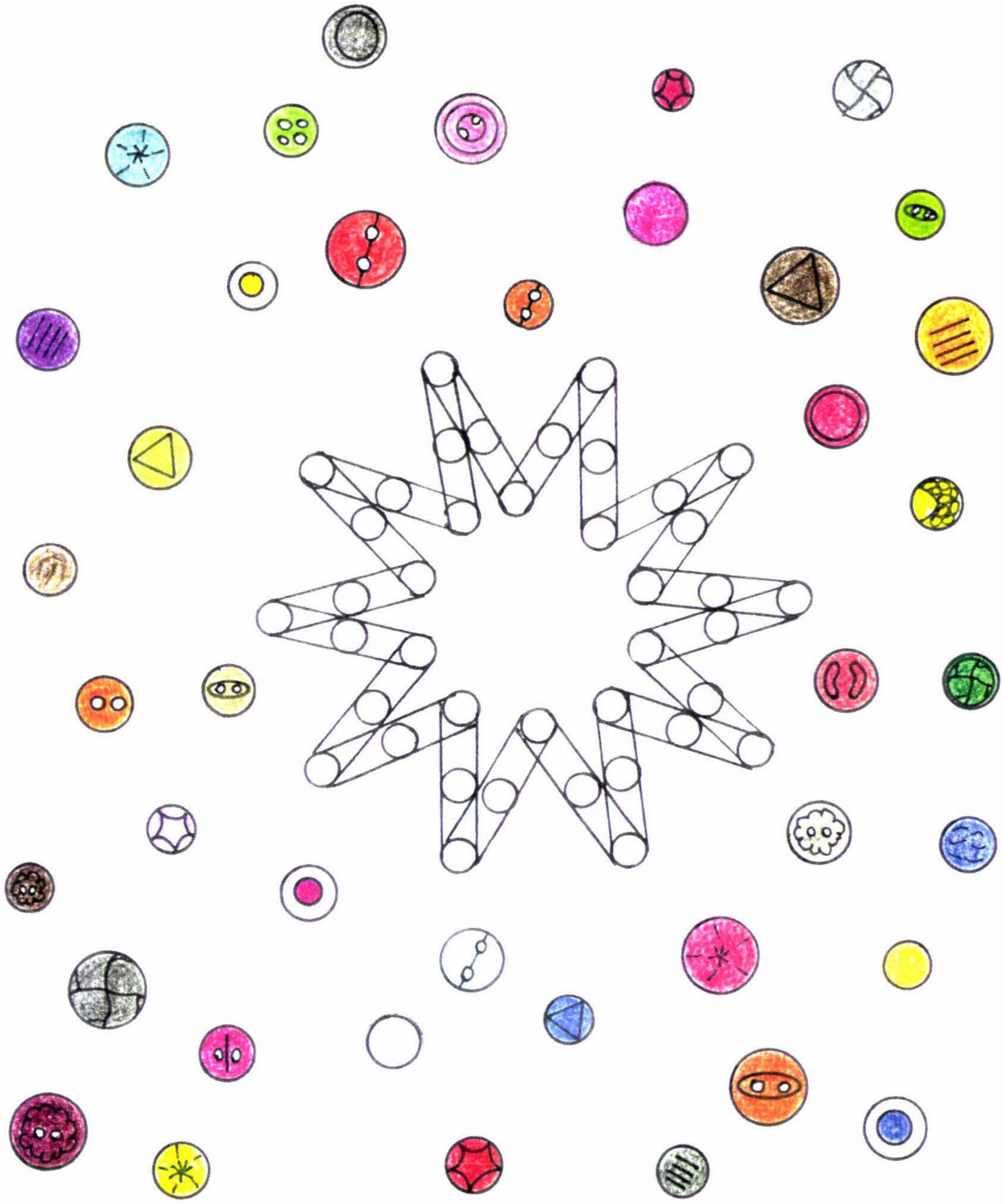


Figure 13: Intersecting Seriated Sets of Three Button Star:

Verbal Instructions:

- Find sets of three buttons.
- Place any one set on the diagram.
- Find another set to connect with this set.
- Intersect all of the sets as you place the button sets around the diagram.
- Your last set must intersect your first set.

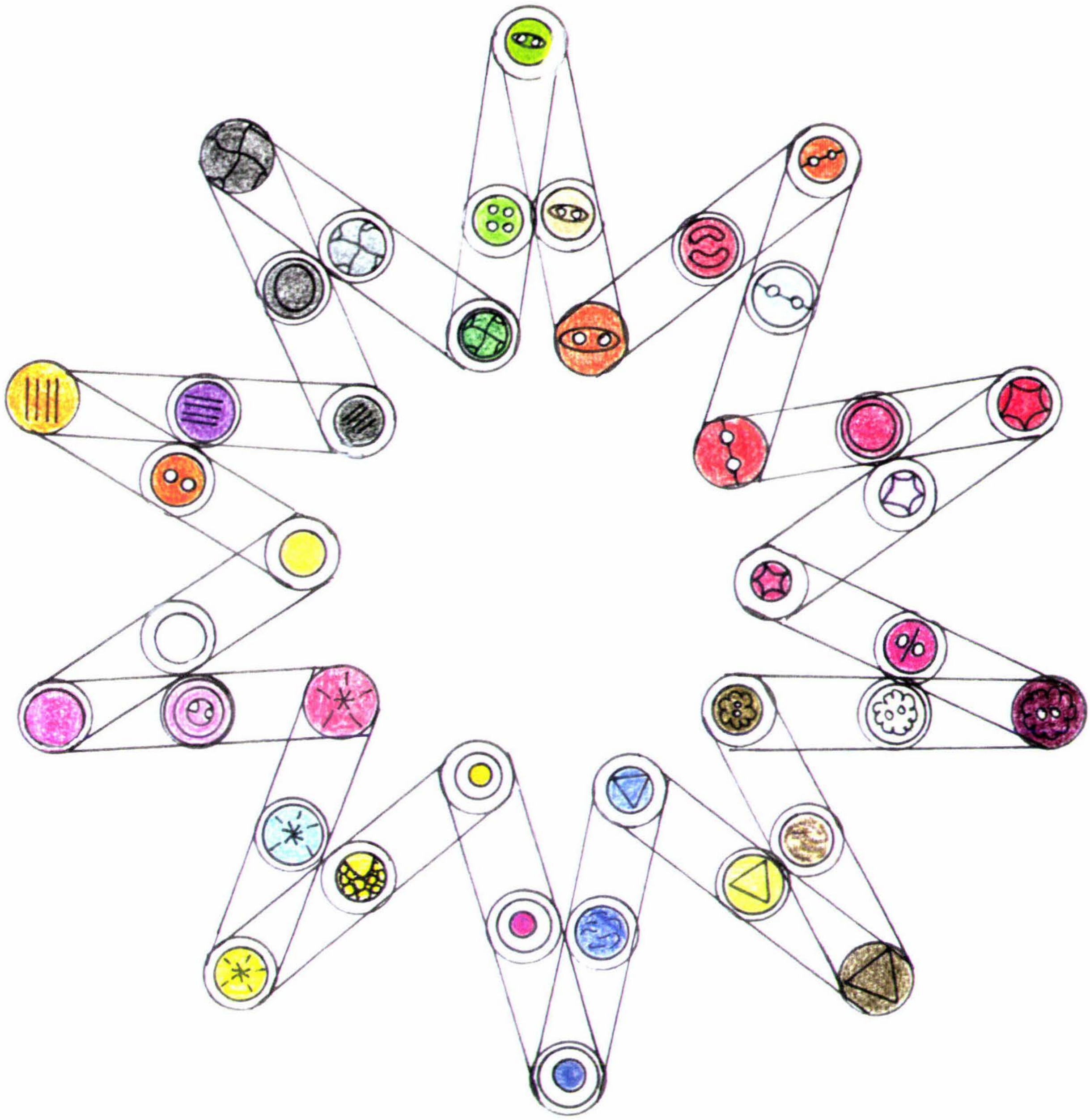


Figure 13a: Intersecting Seriated Sets of Three Button Star: Three Colour / Three Pattern: Verbal Instructions:

- Name your sets. [colour / pattern].
- Tell me something else about your sets. [seriated].

This puzzle was completed quickly once colour groups were distinguished.

An 8 year old SE 1 girl did not know light brown belonged to the brown family. She did not know the name 'fawn', which I taught her. This occurred in Figure 14, a beginning senior puzzle:

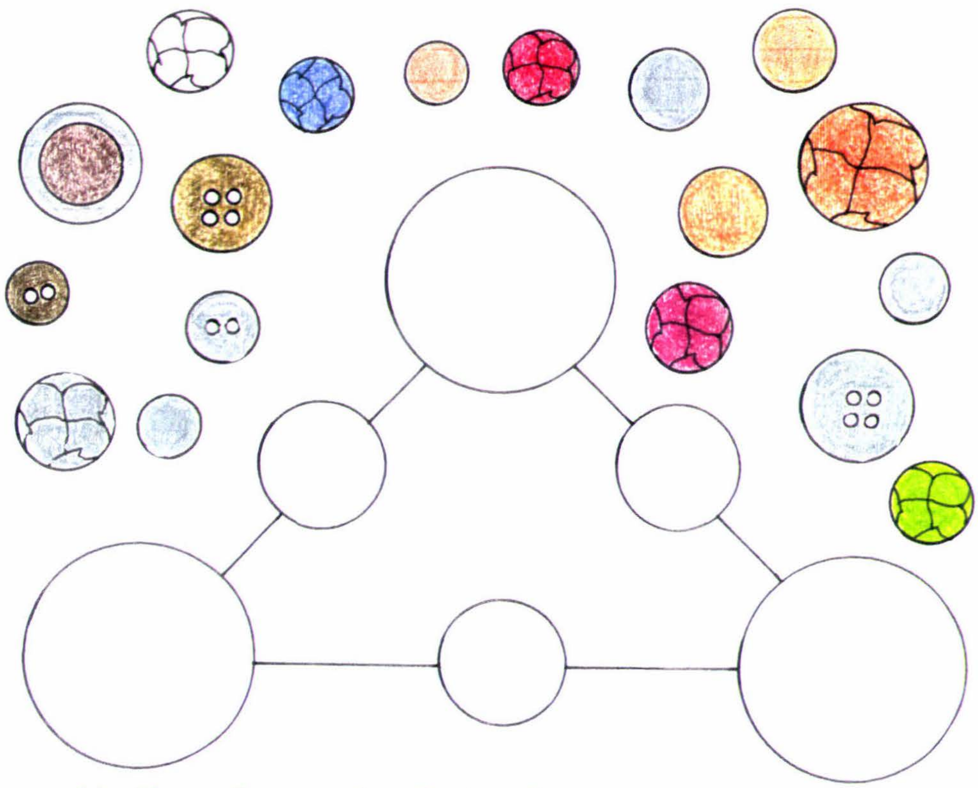


Figure 14: Three Intersecting Button Sets.
Instructions and questioning as for Figure 1.

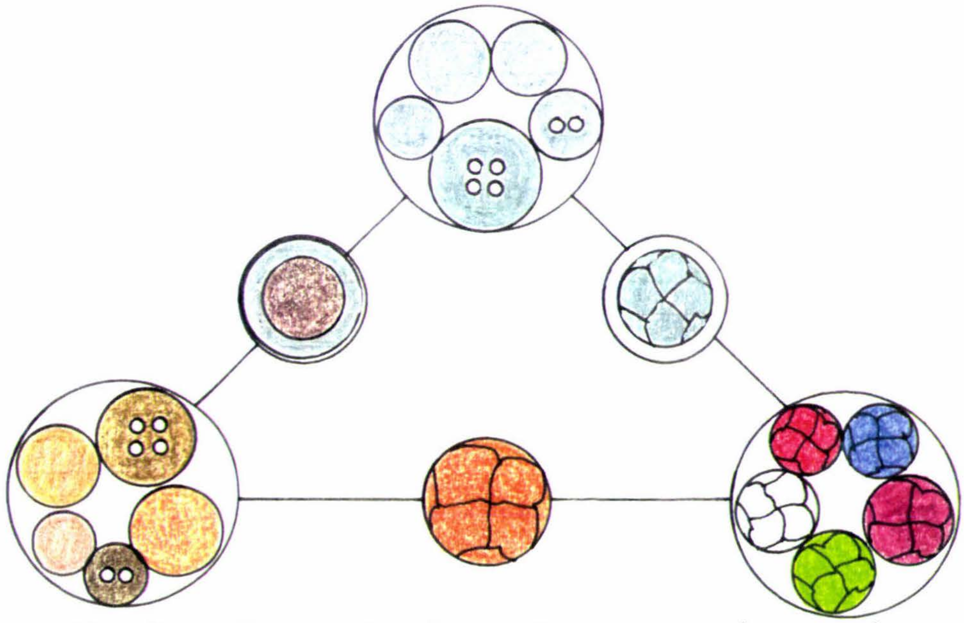


Figure 14a: Three Intersecting Button Sets: Pattern / Brown / Silver.
Instructions and questioning as for Figure 1a.

Figure 14b shows Figure 14 levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6					1			2
5			1	1			3	1
4			1					
3	1		8		3	2	4	6
2					1			
1					1			
0								1

<u>Levels of Performance</u>

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

<u>Figure 14b: Three Intersecting Button Sets: Pattern / Brown / Silver: Levels of Performance Across Mathematical Groups. n = 37.</u>

Figure 14c shows children at the highest level of performance for Figure 14 across mathematical groups and age:

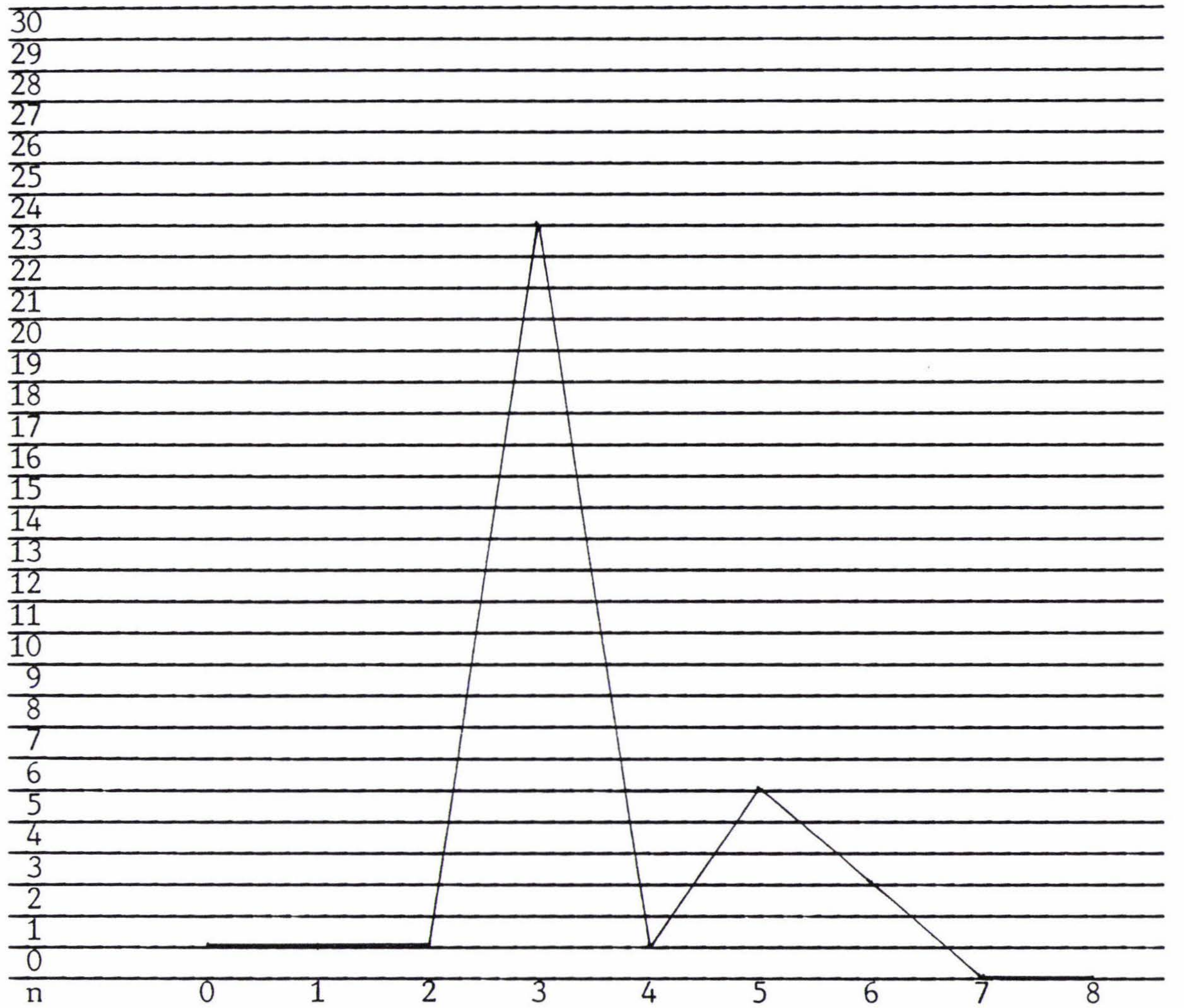
expected

Age	Maths Groups J SLD	JR	JE	S SLD	SR	AV	SE 2	SE 1
11								
10					1			1
9								
8								1
7								
6								

unexpected

<u>Figure 14c: Three Intersecting Button Sets: Pattern / Brown / Silver: Highest Level of Performance: Very fast completion with no questioning: Mathematical Groups Across Ages. n = 3.</u>

Figure 14d shows levels of performance across all children who attempted Figure 14:

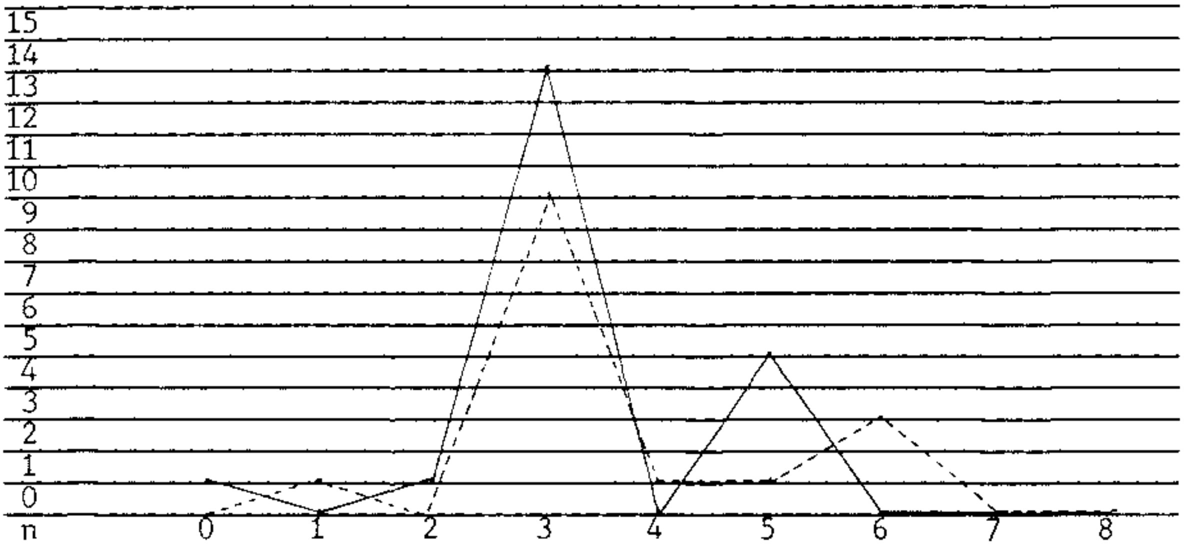


Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 14d: Three Intersecting Button Sets: Pattern / Brown / Silver: Levels of Performance. n = 37.

Figure 14e examines gender performance across levels of performance:



Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

n = 21 ________ Males
n = 16 ---------- Females

Figure 14e: Three Intersecting Button Sets: Pattern / Brown / Silver: Gender Performance Across Levels of Performance. n = 37.

Table 1B shows correlation coefficients between levels of performance and mathematical groups, school class levels, and ages.

Table 7: Spearman's Rho Correlation Coefficients: Figure 14: Three Intersecting Button Sets: Pattern / Brown / Silver: Levels of Performance: n = 37.

Across mathematical groups	$r_s = 0.250$	$p = >0.05$
Across school class levels	$r_s = 0.314$	$p = <0.05$
Across ages	$r_s = 0.345$	$p = <0.05$

School classes and ages are slightly significant for Figure 14, but not for mathematical group categories. The reason for this is unclear, and may be due to chance factors, but is more likely to be due to perceptual difficulties more prevalent in younger children. As these types of perceptions are not so involved in P.A.T. mathematical tests, it is not surprising mathematical group categories do not correlate with levels of performance in this puzzle, nor that levels of significance for mathematical groups do not equate with levels of significance for school classes and ages of children. Ages equate to school class levels much more than mathematical group categories do. Most perceptual difficulties and questioning in Figure 14 related to children not noticing a pattern set in association with two colour sets. Mathematical group categories were represented significantly at the level of performance of requiring some questioning to enable the child to become aware of this pattern set when their expectations were to find a third colour set. A Mann-Whitney U-Test finds no gender differences in Figure 14 levels of performance.

Unfamiliar Concepts

Perceptual mode priorities and discrimination difficulties are individual for different children, reflecting habitual preferences of awareness such as colour before shape, size before pattern, or unfamiliarity with subtle discriminations between colours or shapes etc. A child may find by process of elimination that the element to be classified is different from their expectations or automatic first or second choice for grouping. Some children find difficulty manipulating more than one perceptual dimension within the same puzzle. This difficulty with multiple concepts is discussed in Chapter 5. In Figures 15, 16 and 17, perceptual difficulties arose with unexpected elements. Significance in Figure 15 levels of correlation between mathematical group categories, school class levels, and ages across levels of performance was the highest in each case, of all of the puzzles tested in this study. The multiple concepts and perceptual dimensional range and higher difficulty level may have been responsible for this result, although subject numbers are too small to draw conclusions from. In contrast, Spearman's Rho correlations were not significant in Figure 16 where a single dimension of material types was involved. A Mann-Whitney U-Test finds no gender differences in Figure 15 levels of performance, but significance favouring females in Figure 16.

Asking children to name their classifications helps focus their attention on what is unexpected as well as on what is familiar. Figure 15, an average senior puzzle contained an unexpected element giving difficulty:

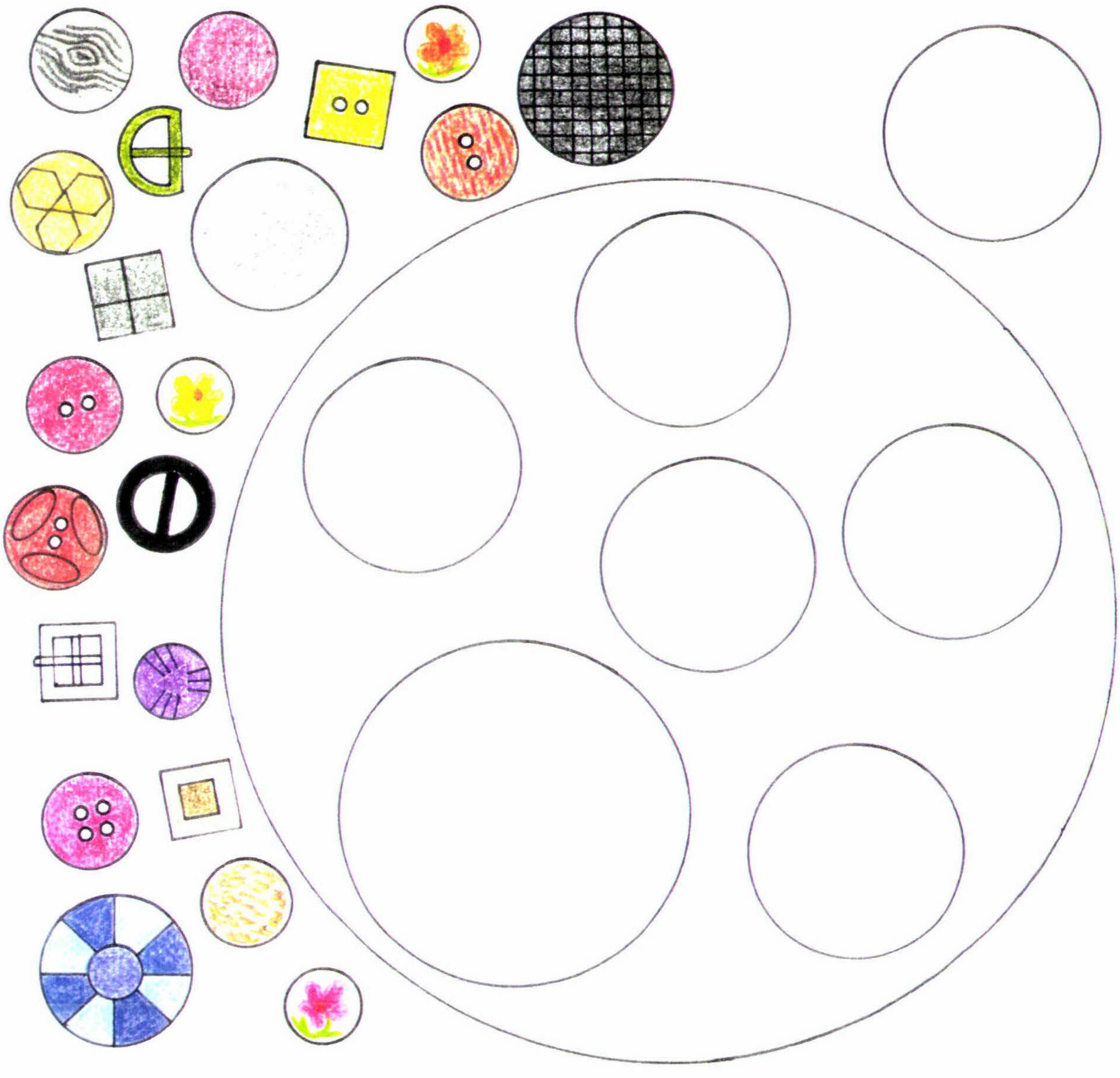


Figure 15: Button Subsets and Classifications:
Verbal Instructions:

- The diagram has one big universal set containing smaller subsets.
- One set belongs outside the universal set.
- Find sets that belong together.
- Name the sets.
- Place your sets in the subset circles provided.
- Place the set that does not belong, outside the universal set.

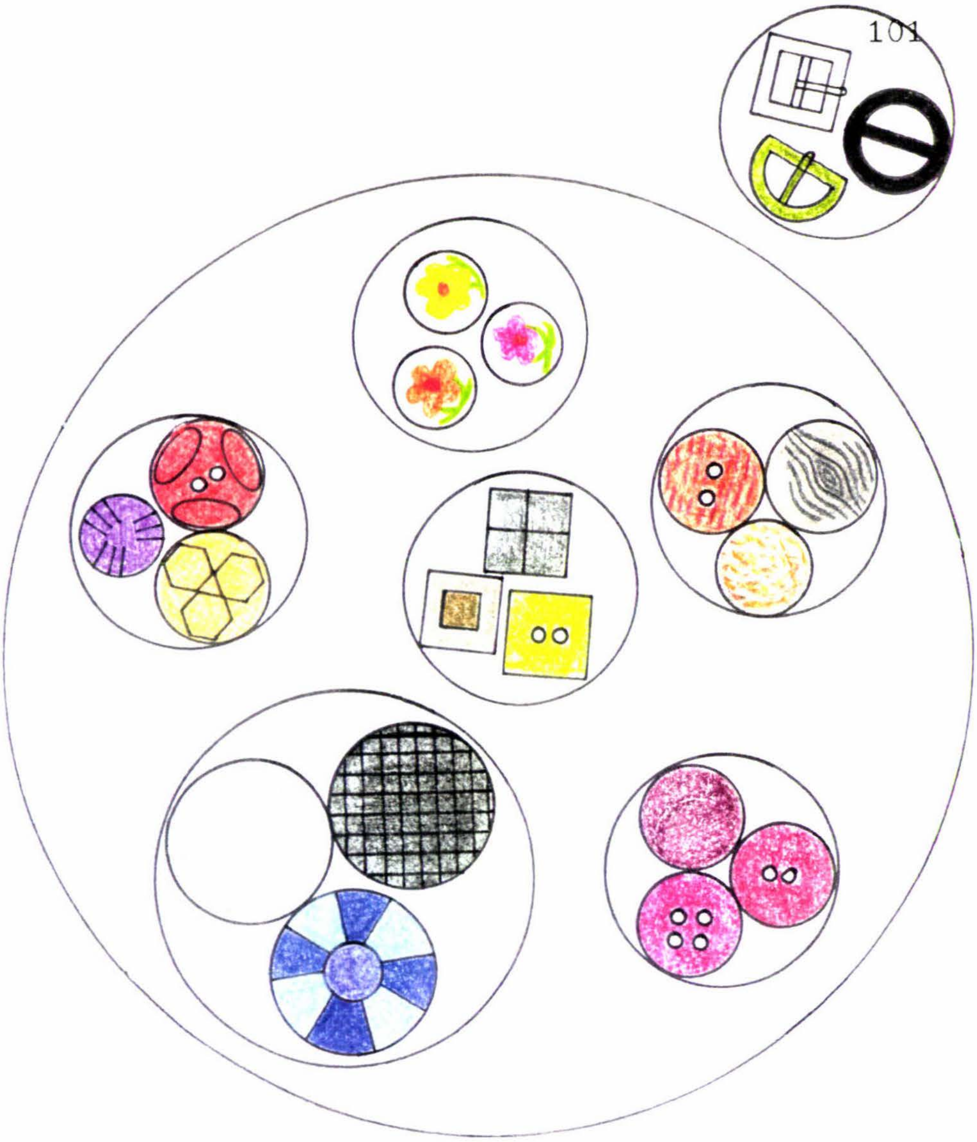


Figure 15a: Button Subsets and Classifications:

[Big; Maroon; Square; Wood; Pattern; Repeated Patterns of Three; Buckles]. This is one of the few puzzles clearly correlating levels of performance with mathematical group categories. Table 8 shows Figure 15 correlation coefficients between levels of performance and mathematical groups, school class levels, and ages:

Table 8: Spearman's Rho Correlation Coefficients: Figure 15: Figure 15: Button Subsets and Buckle Classifications: Levels of Performance: n = 14.

Across mathematical groups	$r_s = 0.885$	$p = <0.05$
Across school class levels	$r_s = 0.664$	$p = <0.05$
Across ages	$r_s = 0.617$	$p = <0.05$

Figure 15b shows levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6								
5								2
4								
3					2	2		3
2			2	1				
1								
0			2					

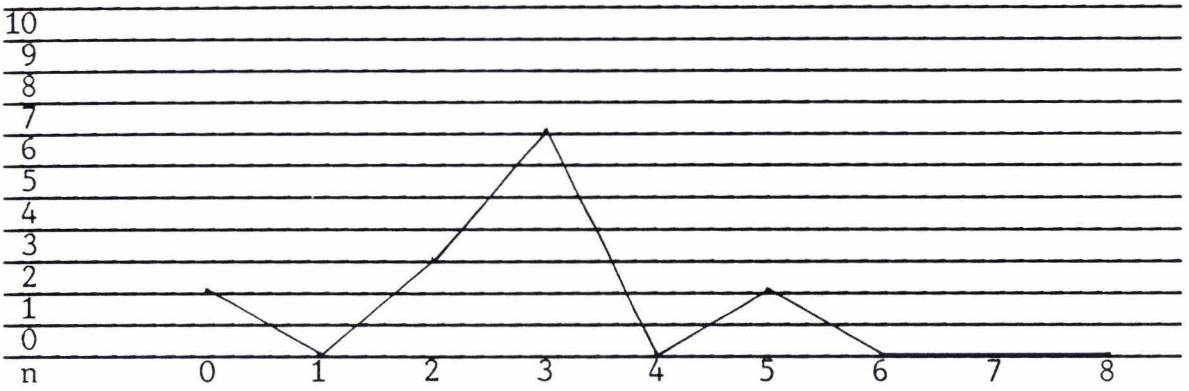
Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 15b: Button Subsets and Buckle Classifications:
Levels of Performance Across Mathematical Groups. n = 14.

An 8 year old AV girl placed uneven numbers of buttons in her subsets, and the wooden buttons outside her universal set. I asked her to count how many items she had in each subset. As there were three items in most subsets, she closely examined subsets she had given more or less than three items. She was unable to make the number of subset items even, so I asked her to name her subsets. She corrected mistakes in her classification groups as she named her subsets. More difficult was naming her universal set. Only after naming her universal set could she see that the wooden buttons belonged inside the universal set, and the buckles belonged outside it. Many children required questioning to separate buttons from buckles. Many did not know what a buckle was. All children needing questioning to separate buttons from buckles needed to name their universal set first. Creating further difficulty in this puzzle was utilisation of multi-perceptual dimensions: subsets incorporating size, shape, colour, pattern, repeated patterns, material type and object type.

Figure 15c shows levels of performance across all children who attempted Figure 15:

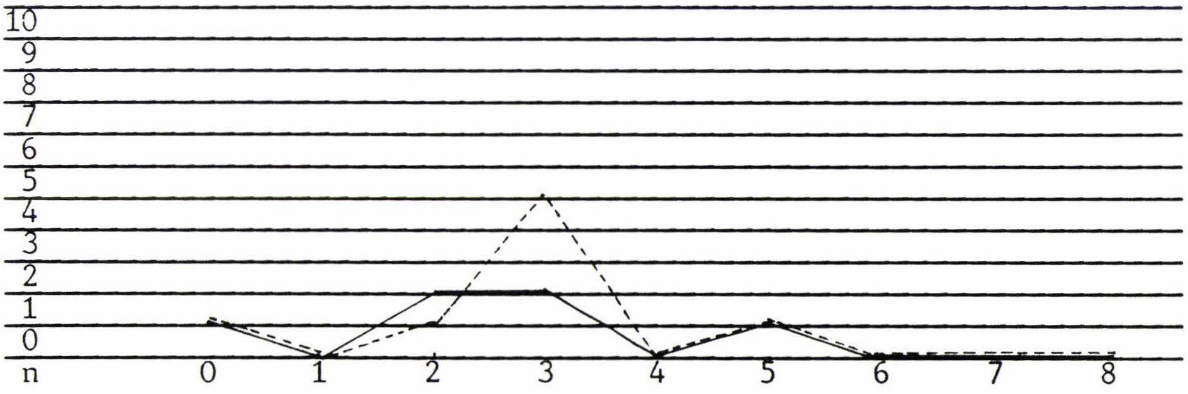


Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 15c: Button Subsets and Buckle Classifications: Levels of Performance. n = 14.

Figure 15d examines gender performance across levels of performance: [Levels of performance as for Figure 15c].



n = 6 ________ Males
n = 8 ---------- Females

Figure 15d: Button Subsets and Buckle Classifications: Gender Performance Across Levels of Performance. n = 14.

Another example of the unexpected creating perceptual difficulties for children was an average senior puzzle, Figure 16:

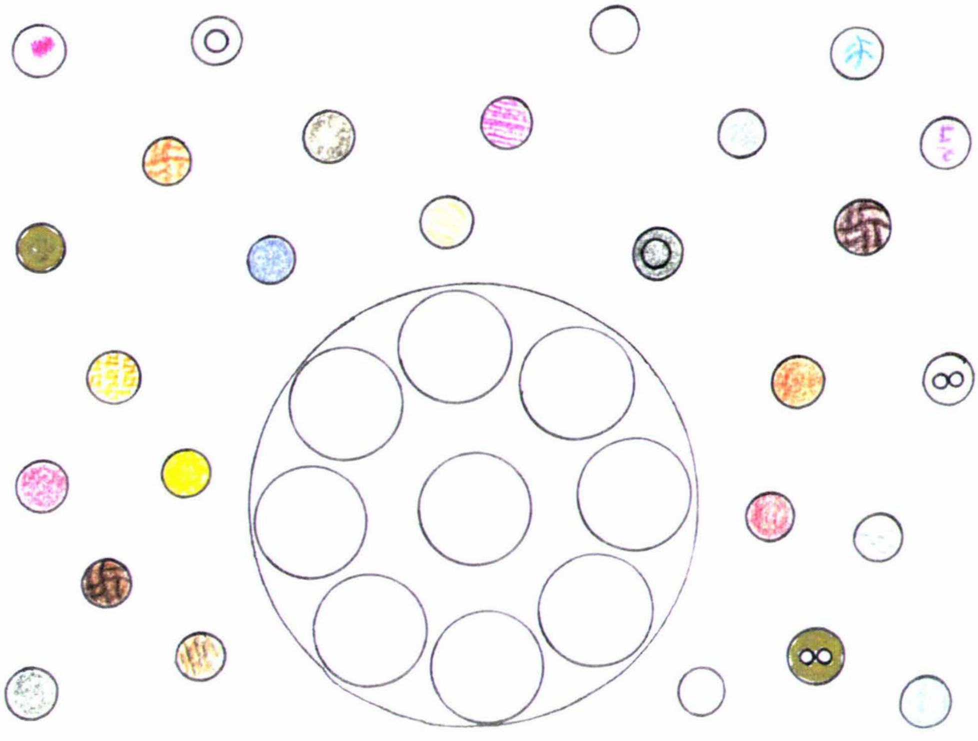


Figure 16: Button Subsets of Material Types:

Verbal Instructions:

- The diagram has one big universal set containing smaller subsets.
- Find sets that belong together.
- Name the sets.
- Place your sets in the subset circles provided.

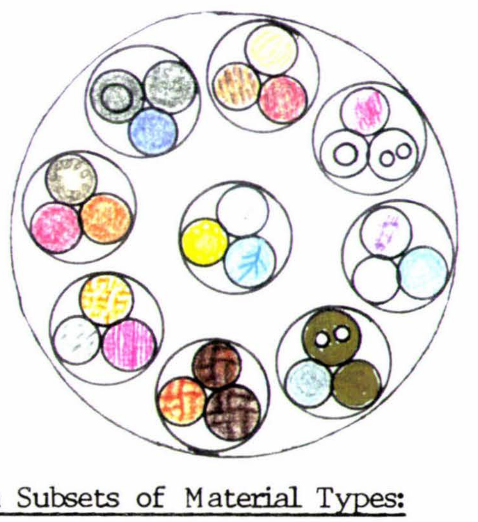


Figure 16a: Button Subsets of Material Types:

[Plastic; Wood; Glass; Shell; Vinyl; Fabric; Metal; Leather; China].

Figure 16b shows levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6						1		
5								1
4			1					
3			2		2	1		2
2			2					1
1								
0								

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

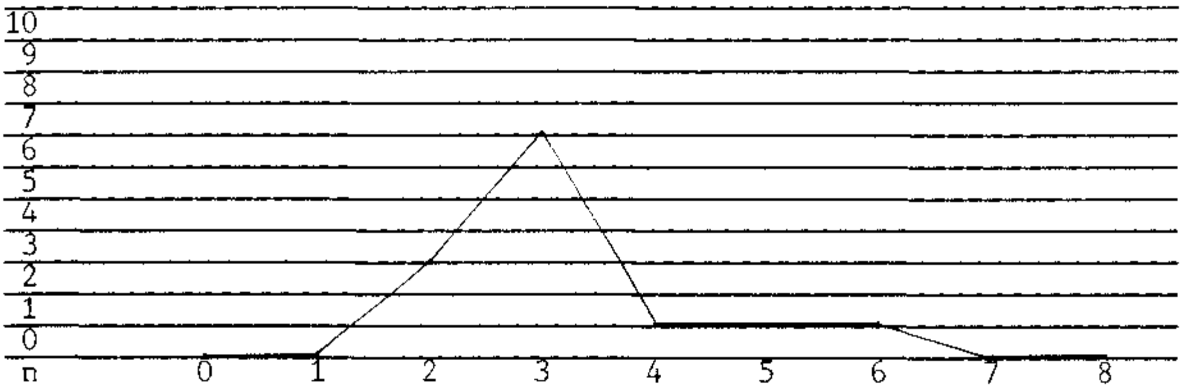
Figure 16b: Button Subsets of Material Types:
Levels of Performance Across Mathematical Groups. n = 13.

Table 9 shows Figure 16 correlation coefficients between levels of performance and mathematical groups, school class levels, and ages:

Table 9: Spearman's Rho Correlation Coefficients: Figure 16:
Figure 16: Button Subsets of Material Types:
Levels of Performance: n = 13.

Across mathematical groups	$r_s = 0.307$	$p = >0.05$
Across school class levels	$r_s = 0.322$	$p = >0.05$
Across ages	$r_s = 0.278$	$p = >0.05$

Figure 16c shows levels of performance across all children who attempted Figure 16:

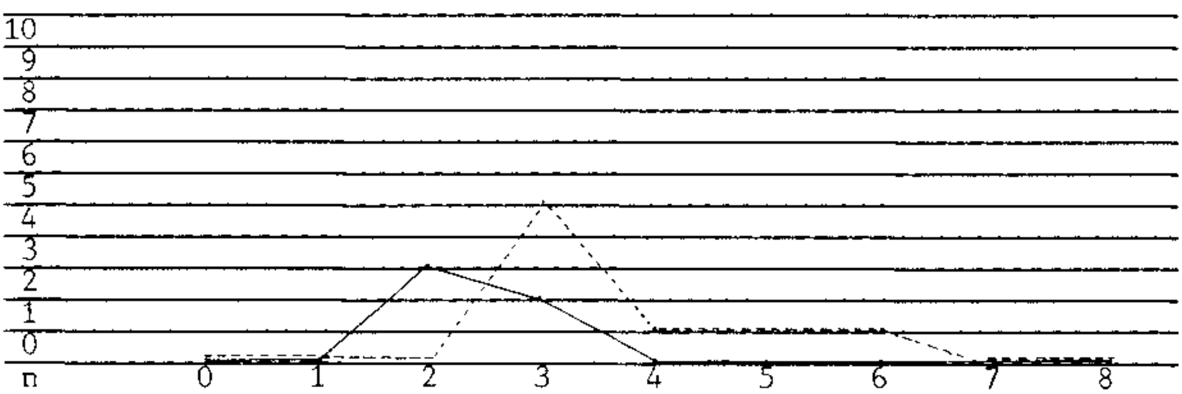


Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 16c: Button Subsets of Material Types:
Levels of Performance. n = 13.

Figure 16d examines gender performance across levels of performance: [Levels of performance as for Figure 16c].



n = 5 ________ Males
n = 8 ---------- Females

Figure 16d: Button Subsets of Material Types:
Gender Performance Across Levels of Performance. n = 13.

Many children were not aware of the different material types, nor of their names.

Another example of the unexpected occurred in a beginning senior puzzle, Figure 17:

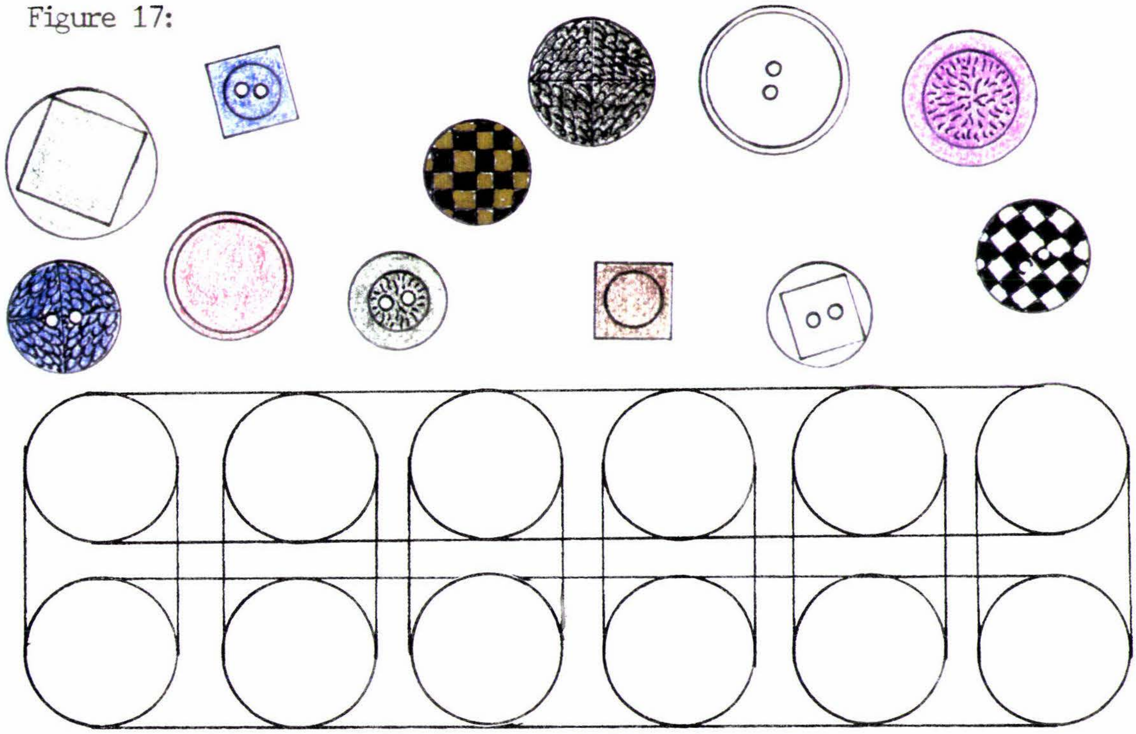


Figure 17: Button Array: Subsets of Two:
Verbal Instructions:

- Find sets of two and place on the diagram.
- Arrange subsets into an array with buttons forming two big sets.
- Name the big set in the top row. Name the big set in the bottom row.

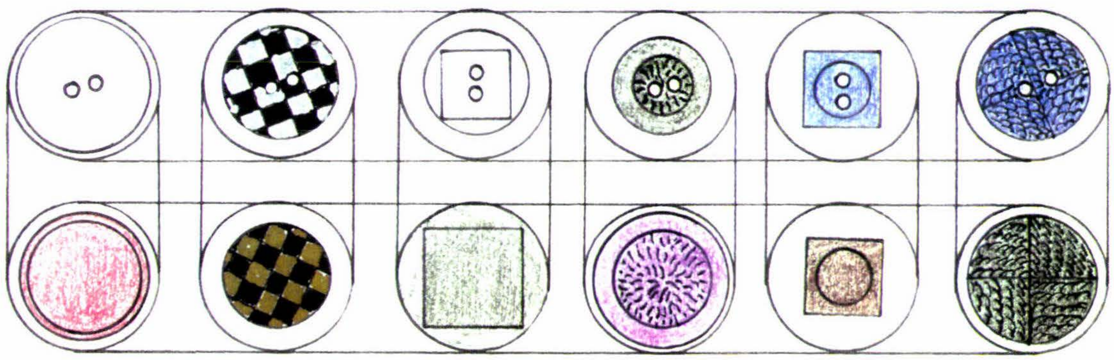


Figure 17a: Holes / No Holes Button Array: Subsets of Two:

A 10 year old AV girl classified her subsets, but required questioning to observe the holes difference. This compares with her 8 year old AV sister who completed this puzzle quickly and without questioning.

Reversals

Some children had difficulty perceiving rotations, reversals, and even lines. This included an inability to match reversed colours as in the junior sets Felt Flowers [Plate 51], and Hessian Houses [Plate 50]. Reversed shapes such as Reversed Buildings [Plate 70], and Reversed Cars [Plate 74], also created difficulty for some. Figure 18, a very advanced junior puzzle shows how difficult reversals are for some children. This puzzle consists of a large coloured base board containing reversed sailing ships. The loose pieces shown are for the child to match onto the base board: [Plate 71].

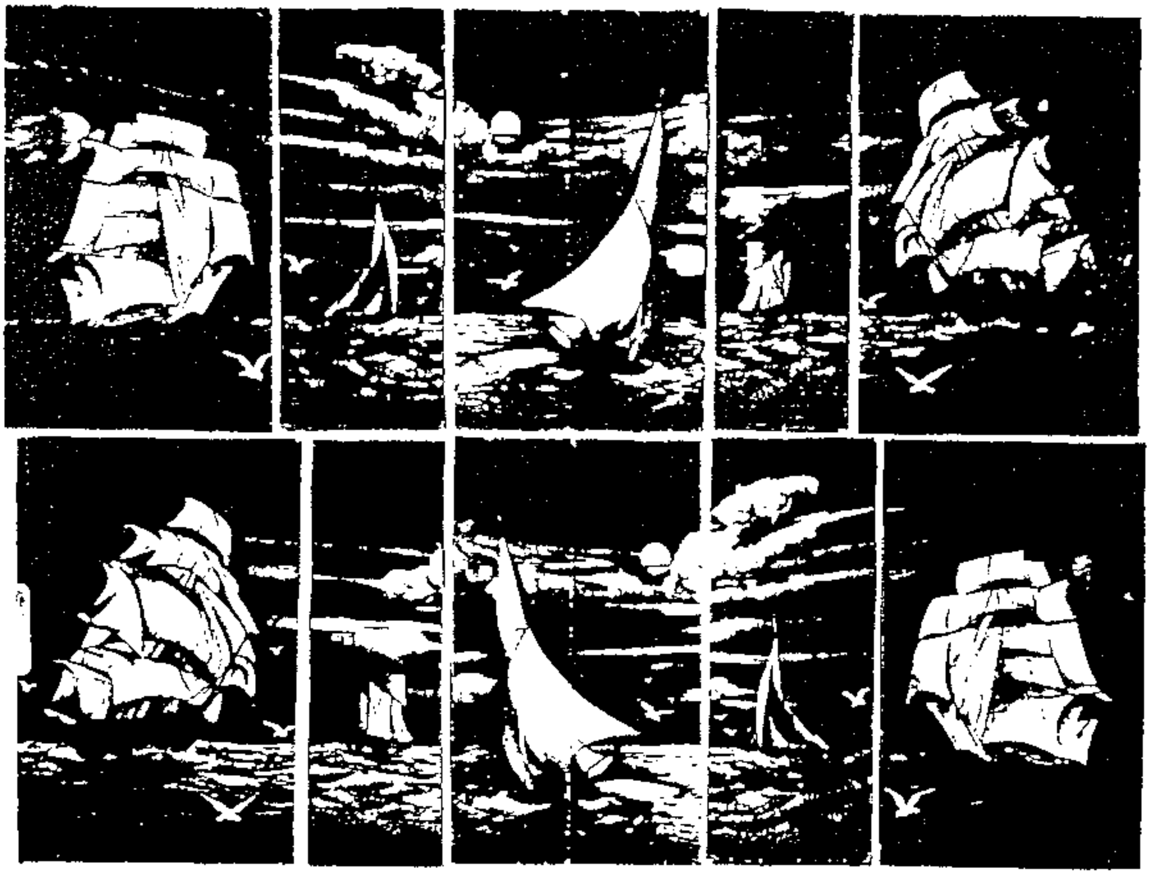


Figure 18: Reversed Sailing Ships

Figure 18a shows Figure 18 levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6	2	1	1		2			
5		1				2		
4		1	1					
3	1	1						
2		1						
1		1						
0								

Levels of Performance

0 - Too difficult
1 - Mismatched all but one pair of ships
2 - Three mismatches: Uncorrected
3 - One mismatch: Uncorrected
4 - Two mismatches: Corrected
5 - One mismatch: Corrected
6 - Complete with no questioning
7 - Very fast completion with no questioning
8 - Exceptional or novel performance

Figure 18a: Reversed Sailing Ships:
Levels of Performance Across Mathematical Groups. n = 15.

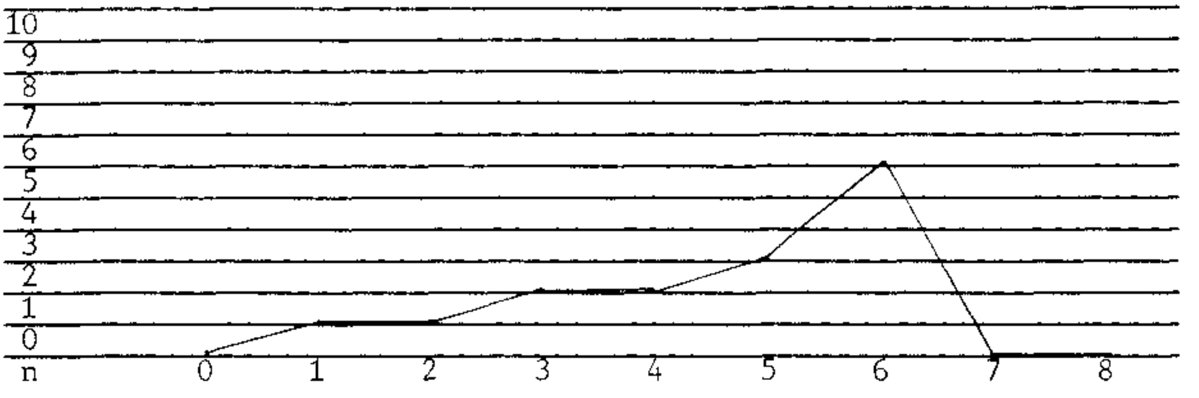
Table 10 shows Figure 18 correlation coefficients between levels of performance and mathematical groups, school class levels, and ages.

Table 10: Spearman's Rho Correlation Coefficients: Figure 18:
Figure 18: Reversed Sailing Ships:
Levels of Performance: n = 15.

Across mathematical groups	$r_s = 0.223$	$p = >0.05$
Across school class levels	$r_s = 0.445$	$p = <0.05$
Across ages	$r_s = 0.458$	$p = <0.05$

There is a correlation between levels of performance and school class levels and ages, but not for mathematical groups.

Figure 18b shows levels of performance across all children who attempted Figure 18:

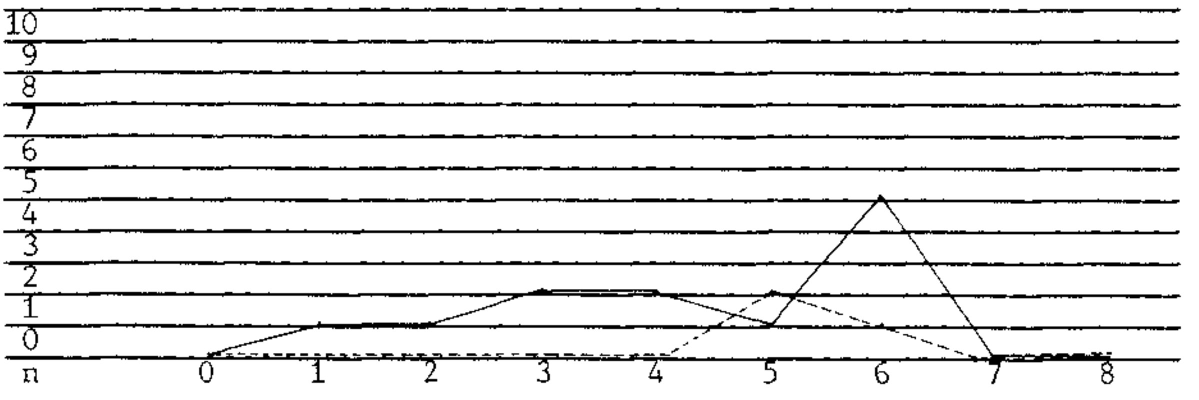


Levels of Performance

0 - Too difficult
1 - Mismatched all but one pair of ships
2 - Three mismatches: Uncorrected
3 - One mismatch: Uncorrected
4 - Two mismatches: Corrected
5 - One mismatch: Corrected
6 - Complete with no questioning
7 - Very fast completion with no questioning
8 - Exceptional or novel performance

Figure 18b: Reversed Sailing Ships:
Levels of Performance. n = 15.

Figure 18c examines gender performance across levels of performance: [Levels of performance as for Figure 18b].



n = 12 ________ Males
n = 3 ---------- Females

Figure 18c: Reversed Sailing Ships:
Gender Performance Across Levels of Performance. n = 15.

Subject numbers are too small to rule out chance, and senior extension groups of children were not given this junior puzzle. It can still be seen that there is no correlation between mathematical group categories across levels of performance when the highest level of performance was reached by children from J-SLD, JR, JE, and SR groups, but not reached by the two AV girls who represented the highest mathematical group level attempting this puzzle. Although school classes closely follow children's ages, school class and age correlations were able to be significant when mathematical groups were not, because each mathematical group category incorporates a three year age range, and three school class levels. A Mann-Whitney U-Test finds no gender differences in Figure 18 levels of performance.

In puzzles incorporating reversals, Figures 18, 19, 20, 21, 22 and 23, awareness of the reversals, especially where bilateral symmetry as in Figure 20 was involved, was not necessarily automatic over and above the matching of the individual units within each puzzle. This kind of awareness was progressive according to the child's level of conceptual understanding, and therefore diagnostic. For Figure 19, subject numbers were small, but in Figure 20, there was no significance in correlations between mathematical group categories, school classes, and ages across levels of performance, where children from JE, SR and AV mathematical group categories shared the highest level of performance, and children from JR, JE and SR shared the lowest. A Mann-Whitney U-Test finds gender differences in Figure 20 levels of performance in favour of males. Again, because the number of gender tests showing significant difference is so small across a large number of tests, the reason is likely to be chance factors alone.

Subject numbers for Figure 21 are also small, giving a greater likelihood of chance performance of one JE child reaching the highest level of performance for this puzzle, and a SE 1 child achieving the lowest. The three children attempting Figure 22 found this puzzle difficult, needing to progress through stages from initial classification of subsets into named groups of subsets, and finally into reversed named sets across groups.

A further puzzle creating difficulty with reversals was an average senior puzzle, Figure 19:

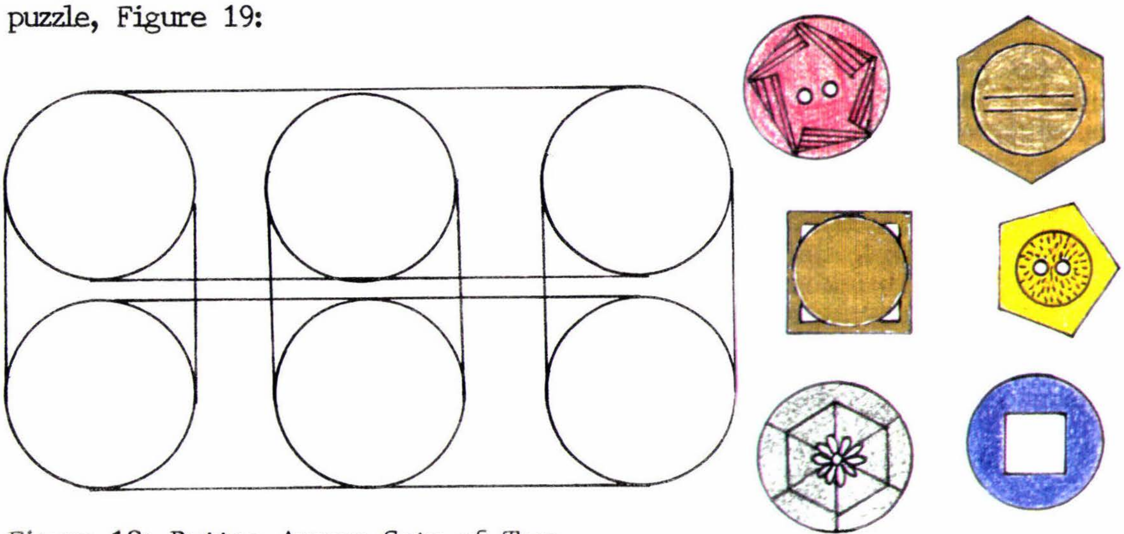


Figure 19: Button Array: Sets of Two.

Verbal Instructions:

- Find subsets of two and place them on the diagram.
- Arrange subsets into an array so all of the buttons make two big sets.
- Name your big set in the top row.
- Name your big set in the bottom row.
- Place your subsets in order from left to right.

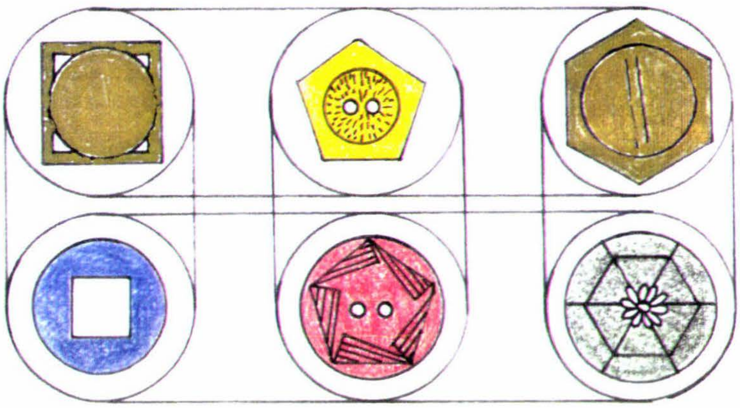


Figure 19a: Button Array: Reversed Internal and External Shapes: Sets of Two. [Top: Circles Inside Shapes; Bottom: Shapes Inside Circles; 4,5,6].

Two AV children needed questioning to form arrays and ordinal sequencing.

Symmetry in reversals can make perceptual differentiation especially difficult. Figure 20, a very advanced junior puzzle utilising bilateral symmetry not only reverses shape and pattern, but colour as well. The two bottom corner pieces are glued down on a base board for Figure 20:

Figure 20: Reversed Tree: Reversed Shape / Pattern / Colour:

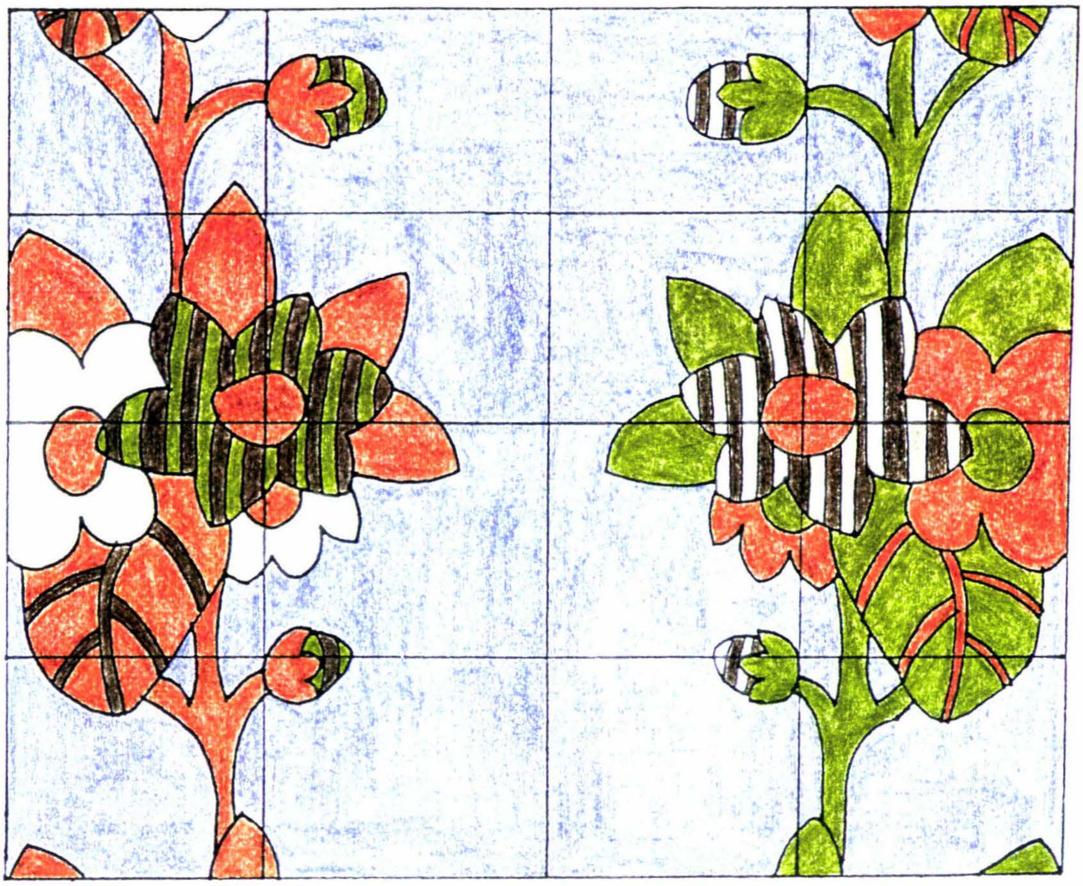


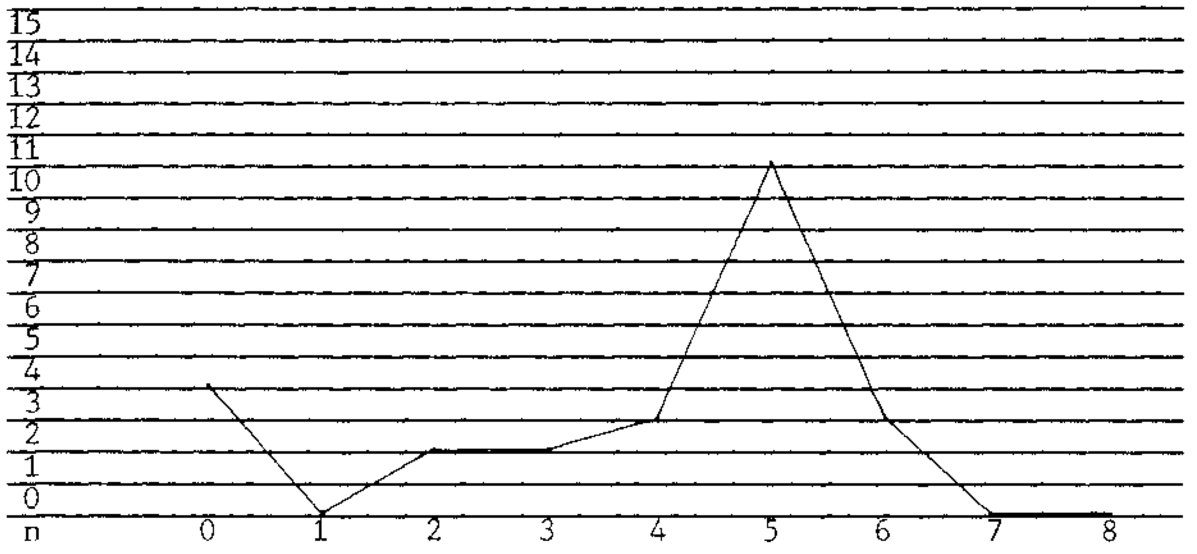
Figure 20a: Reversed Tree: Reversed Shape / Pattern / Colour:

Table 11 shows Figure 20 correlation coefficients between levels of performance and mathematical groups, school class levels, and ages.

Table 11: Spearman's Rho Correlation Coefficients: Figure 20: Figure 20: Reversed Tree: Reversed Shape / Pattern / Colour: Levels of Performance: n = 25.

Across mathematical groups	$r_s = 0.283$	$p = >0.05$
Across school class levels	$r_s = 0.056$	$p = >0.05$
Across ages	$r_s = 0.014$	$p = >0.05$

Figure 20b shows levels of performance across all children who attempted Figure 20:

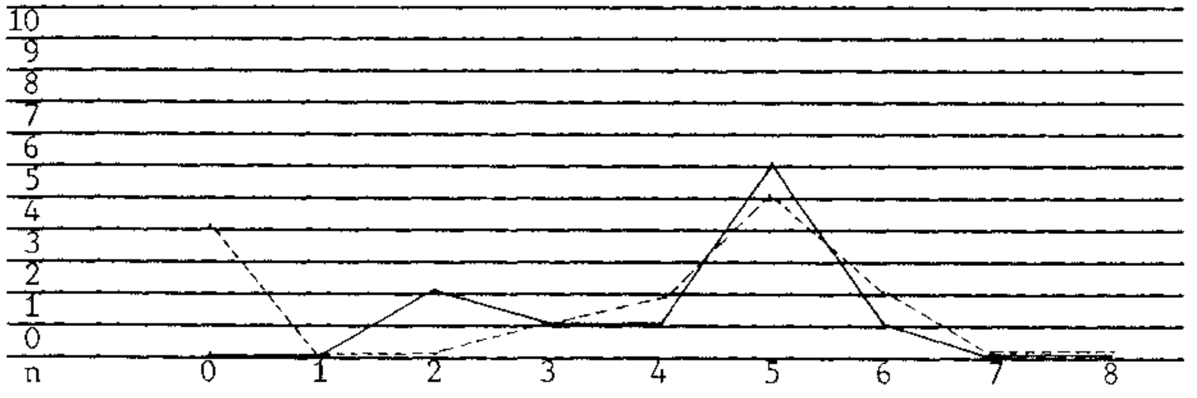


Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 20b: Reversed Tree: Reversed Shape / Pattern / Colour: Levels of Performance. n = 25.

Figure 20c examines gender performance across levels of performance: [Levels of performance as for Figure 20b].



n = 14 ———— Males
n = 11 ----------- Females

Figure 20c: Reversed Tree: Reversed Shape / Pattern / Colour: Gender Performance Across Levels of Performance. n = 25.

Figure 20d shows levels of performance across mathematical groups:

Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6			1		1	1		
5		3	2		5	1		
4		2	1					
3				1	1			
2		1			1			
1								
0		1	1		2			

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 20d: Reversed Tree: Reversed Shape / Pattern / Colour: Levels of Performance Across Mathematical Groups. n = 25.

Figure 21, a beginning senior puzzle, added rotation to the reversals:

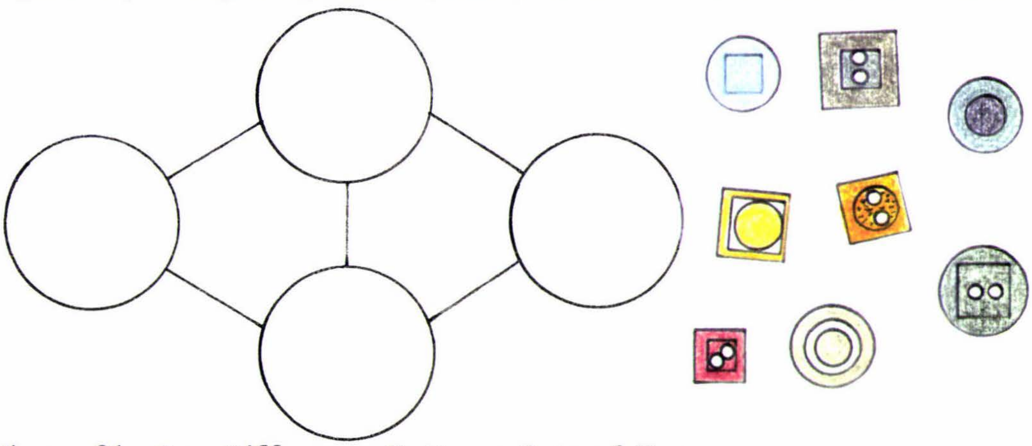


Figure 21: One Difference Buttons: Sets of Two.

Verbal Instructions:

- Find which buttons belong together.
- Name your sets.
- Arrange your sets into the circles, with no more than one thing different between any sets you place in connecting circles.

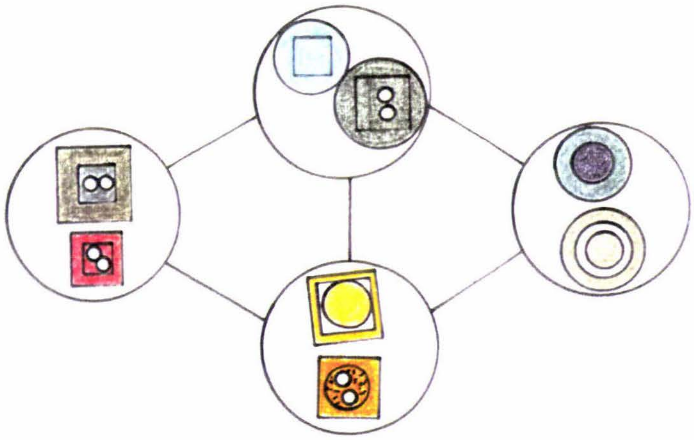


Figure 21a: One Difference Buttons: Sets of Two:
Internal and External Shapes.

Finding sets of shapes was easy for an 8 year old AV girl, but she had difficulty placing them in a one difference order.

Figure 21b shows levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6			1					
5								
4								
3							1	
2						2		1
1								
0								

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 21: One Difference Buttons: Sets of Two:
Internal and External Shapes.
Levels of Performance Across Mathematical Groups. n = 5.

The highest level of performance was from a 7 year old JE girl.

The same 8 year old AV girl having difficulty with Figure 21 had no difficulty with Figure 22, a beginning senior puzzle, despite two further dimensions being added to the concept of reversal:

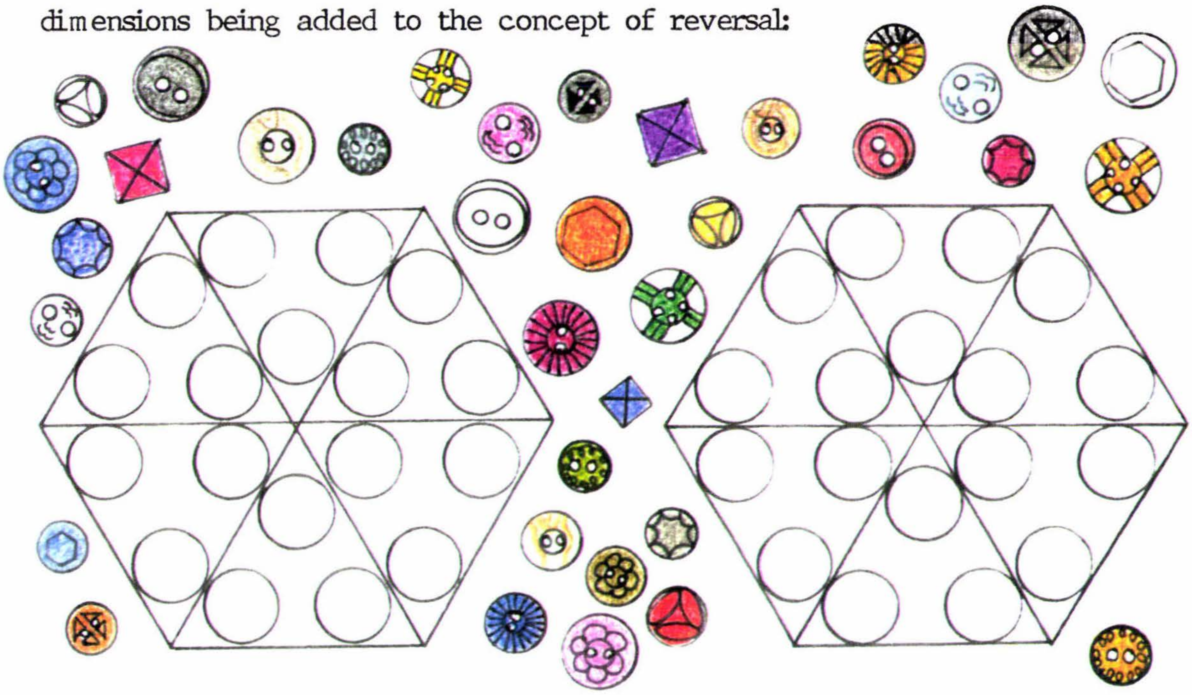


Figure 22: Buttons: Two Sets: Subsets in Triangles Forming Two Hexagons: Verbal Instructions:

- Find buttons that belong together.
- Arrange sets of buttons belonging together in the triangles so that:
- Buttons forming the outer ring in each hexagon belong together.
- Name these outer sets.
- Buttons forming the inner ring on each hexagon belong together.
- Name these inner sets.

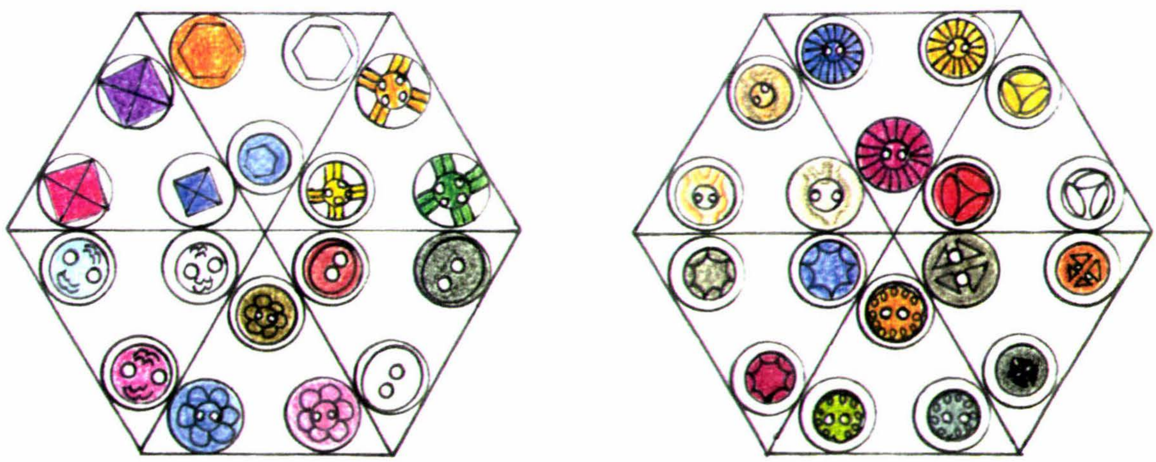


Figure 22a: Buttons: Two Sets Reversed Sizes: Pattern Matched Subsets in Triangles Forming Two Hexagons: [Left Hexagon: Outer Set Big, Inner Set Small; Right Hexagon: Outer Set Small, Inner Set Big].

The two added dimensions to reversal in Figure 22 were pattern and size matching. Figure 23, an average senior puzzle, is a similar design, but with mobile triangles forming a single hexagon, adding a rotational element:

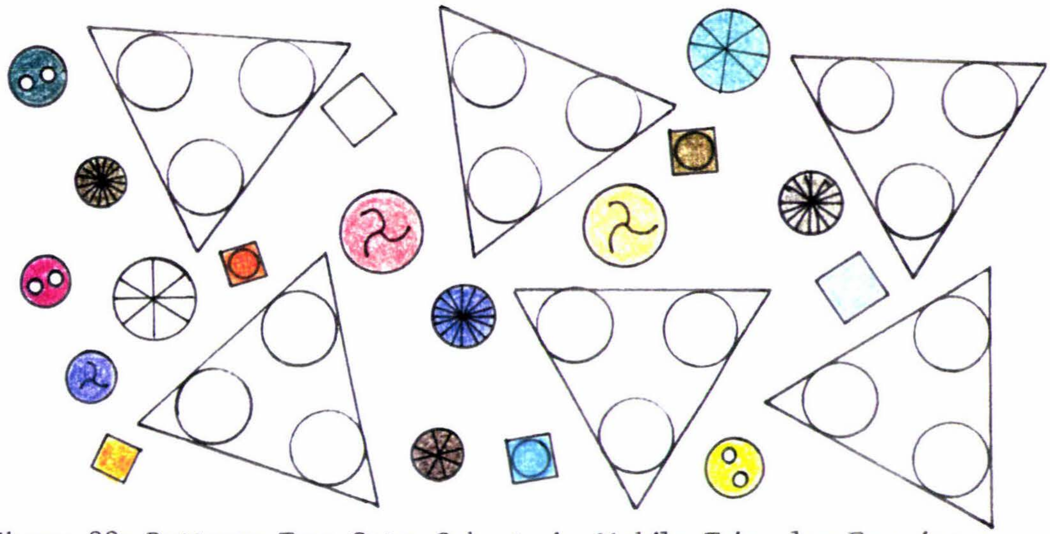


Figure 23: Buttons: Two Sets: Subsets in Mobile Triangles Forming a Hexagon:

Verbal Instructions:

- Find buttons that belong together.
- Arrange sets of buttons belonging together onto the triangles.
- Arrange triangles into a hexagon so that:
- Buttons forming the outer ring in the hexagon belong together.
- Name this outer set.
- Buttons forming the inner ring in the hexagon belong together.
- Name this inner set.

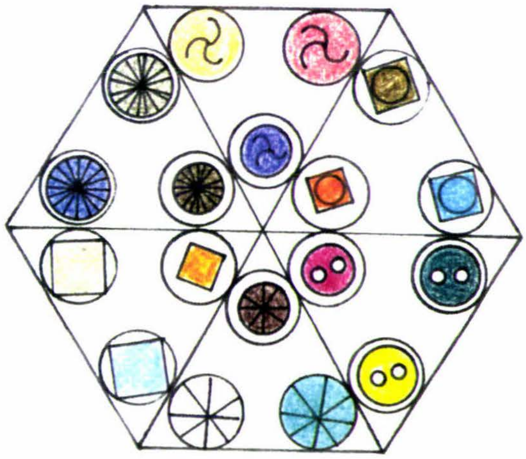


Figure 23a: Buttons: Two Reversed Sizes Sets: Pattern Matched Subsets in Mobile Triangles Forming a Hexagon: [Outer Set Big, Inner Set Small].

One 10 year old AV girl made two different arrangements of her subsets before observing the size differences of the buttons. Her first attempt was to place all 'pie' fraction patterned buttons into the centre and name her sets 'divided' and 'undivided.' When there weren't enough of these for this to work, she placed all buttons with a square in their centre in the inner set. Finally she named her external set 'big' and her internal set 'little.' Her 8 year old AV sister had difficulties rotating the triangles into a hexagonal shape, but quickly arranged and named her sets once she had accomplished this.

Rotation

Rotations can be formidable for children to perceive. One 6 year old JR boy had considerable visual difficulties. He was unable to see the straight edges of pieces. In one junior puzzle with a base board of colourful pirates, and individual pirates cut to match onto this board, [Plate 72], he was unable to decide which pieces were the external pieces with the straight edges. Instead of matching these onto the base board, he placed them right off the board around the outside of it. He also had difficulties knowing the difference between the right way up and upside down.
Figure 24, an average senior puzzle, required the four corner pieces to be glued down to provide a boundary and starting point:

Figure 24: Fabric Pattern Match Domino Set.

Figures 25a,b,c, very advanced junior puzzles, [Plate 64], all have loose pieces:

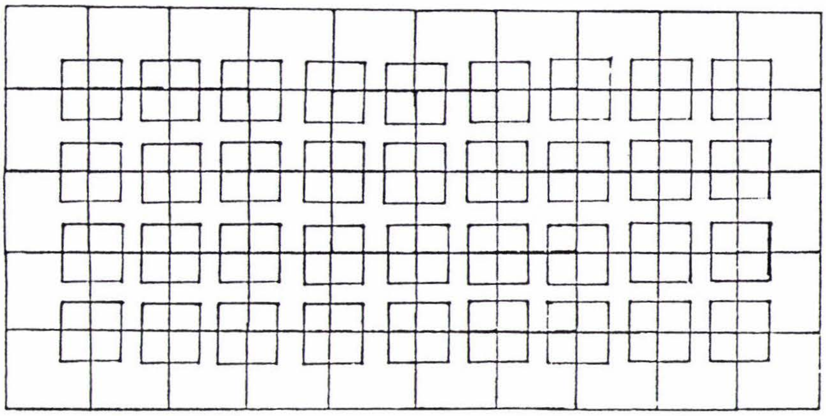


Figure 25a: Paint Chart Puzzle:

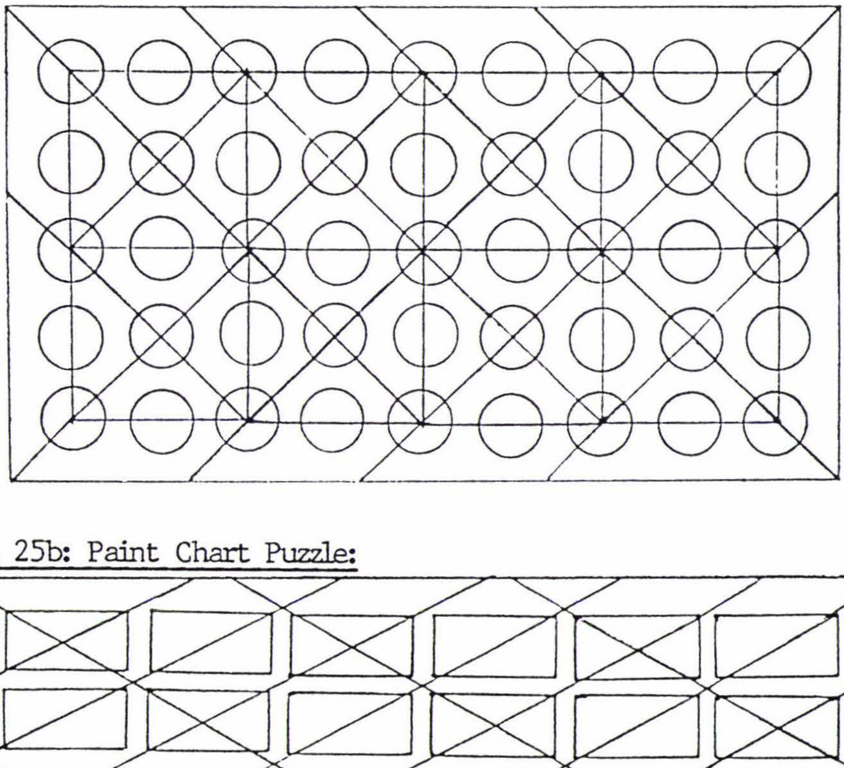


Figure 25b: Paint Chart Puzzle:

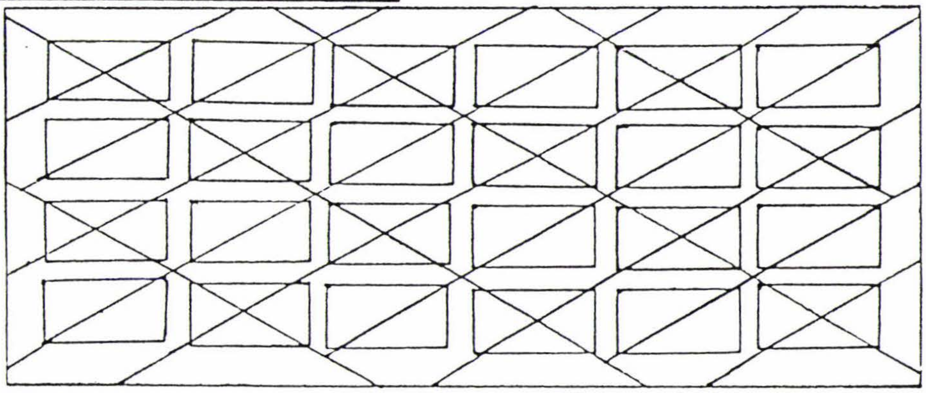


Figure 25c: Paint Chart Puzzle:

Figure 25d shows Figure 25a,b,c levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6								
5						2		
4								
3								
2								
1								1
0					2			

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 25d: Paint Chart Puzzles: Figures 25a,b,c.
Levels of Performance Across Mathematical Groups. n = 5.

Figure 26, an average senior puzzle, can be done in several ways. One 10 year old AV girl made a 3 x 6 rectangle, whereas her 8 year old AV sister made a 2 x 9 one.

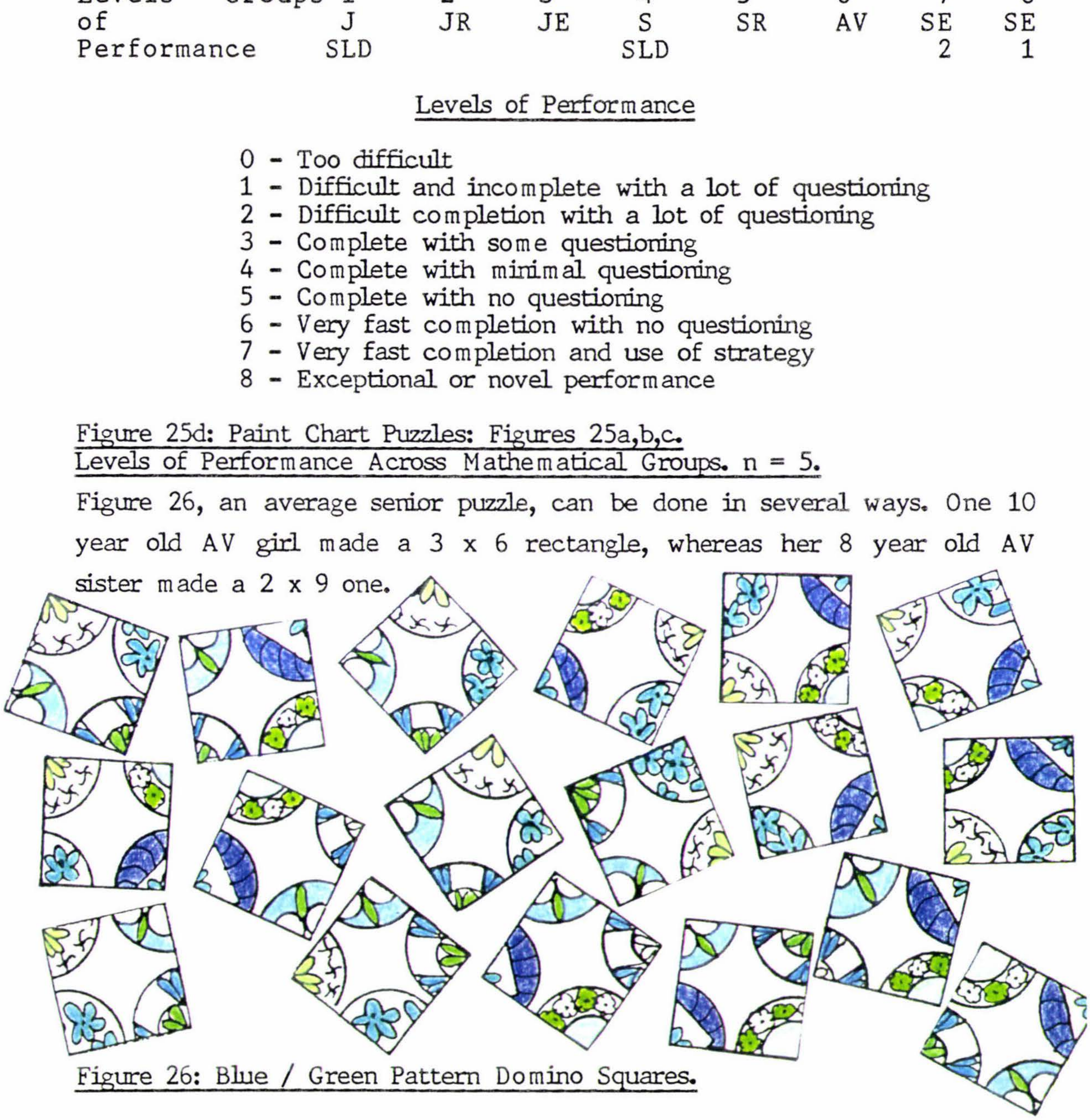


Figure 26: Blue / Green Pattern Domino Squares.

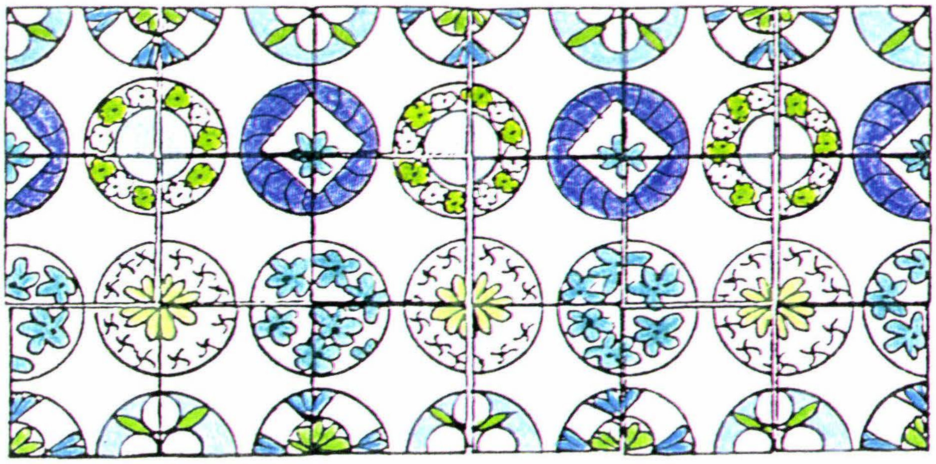


Figure 26a: Blue / Green Pattern Domino Squares.

Figure 27 shows rotation combined with pattern differentiation:

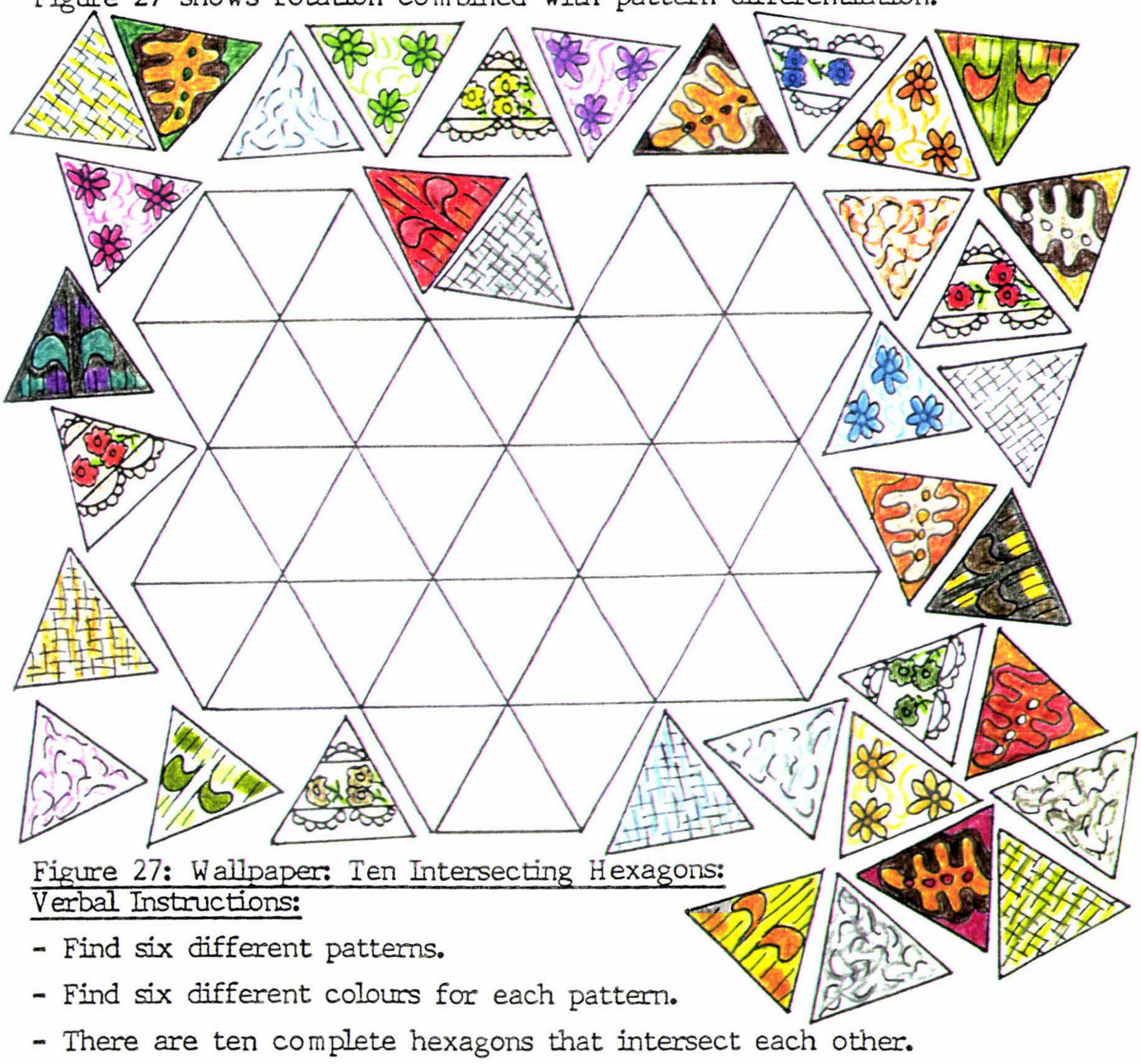


Figure 27: Wallpaper: Ten Intersecting Hexagons:

Verbal Instructions:

- Find six different patterns.
- Find six different colours for each pattern.
- There are ten complete hexagons that intersect each other.
- Place only one piece from each pattern in each complete hexagon.
- Place only one piece from any pattern in each incomplete hexagon.

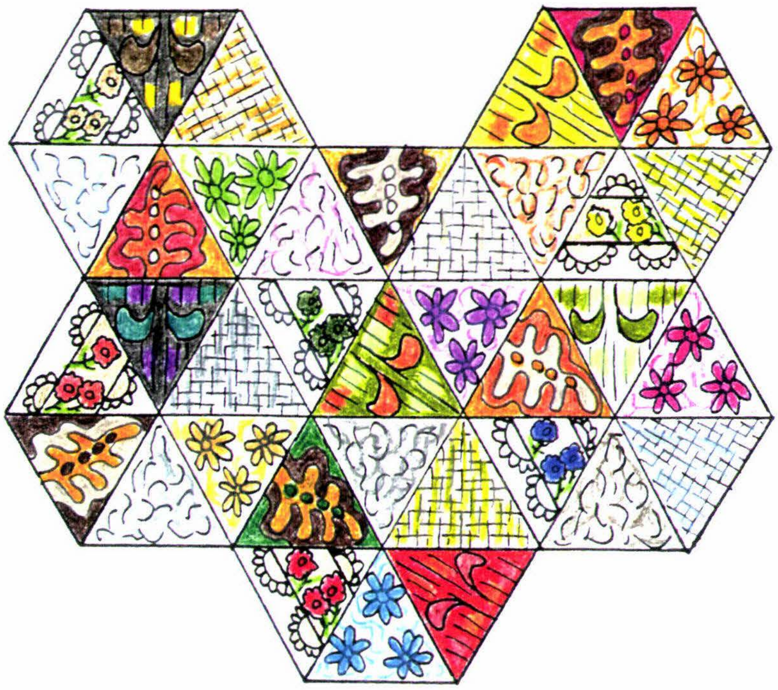


Figure 27a: Wallpaper: Ten Intersecting Hexagons.

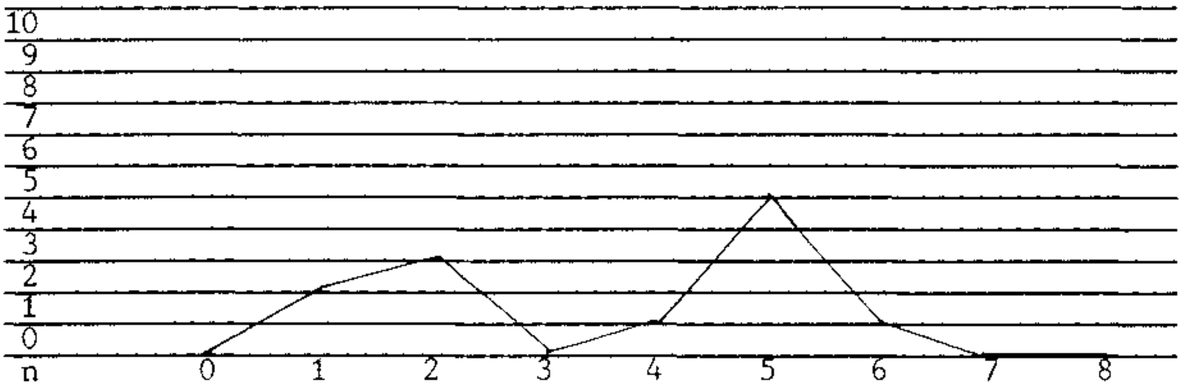
This puzzle required conscious concentration on more than one concept at once. Being aware of hexagonal boundaries and rotating patterns to allow only one of each pattern per hexagon, including external hexagons, was difficult. Being aware of incomplete hexagons was the most difficult part, and some children were unable to manage these external incomplete hexagons, even when their internal hexagons were correctly patterned.

Table 12 shows correlation coefficients between levels of performance and mathematical groups, school class levels, and ages for Figure 27:

Table 12: Spearman's Rho Correlation Coefficients: Figure 27: Wallpaper: Ten Intersecting Hexagons.
Levels of Performance: n = 12.

Across mathematical groups	$r_s = 0.141$	$p = >0.05$
Across school class levels	$r_s = 0.005$	$p = >0.05$
Across ages	$r_s = 0.003$	$p = >0.05$

Figure 27b shows levels of performance across all children who attempted Figure 27:

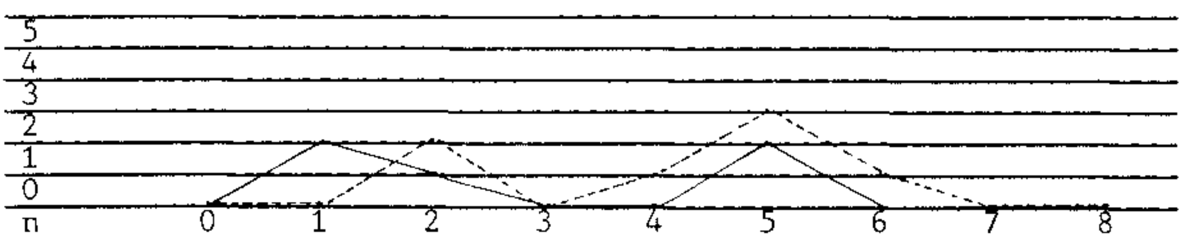


Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 27b: Wallpaper: Ten Intersecting Hexagons:
Levels of Performance. n = 12.

Figure 27c examines gender performance across levels of performance: [Levels of performance as for Figure 27b].



n = 5 ________ Males
n = 7 ---------- Females

Figure 27c: Wallpaper: Ten Intersecting Hexagons:
Gender Performance Across Levels of Performance. n = 12.

The highest level of performance was an 8 year old AV girl.

Figure 27d shows Figure 27 levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6						1		
5							1	4
4						1		
3								
2								3
1								2
0								

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 27d: Wallpaper: Ten Intersecting Hexagons:
Levels of Performance Across Mathematical Groups. n = 12.

Figure 28, a very advanced junior puzzle, [Plate 64], gives clear examples of the difficulties children may have when faced with multiple concepts:

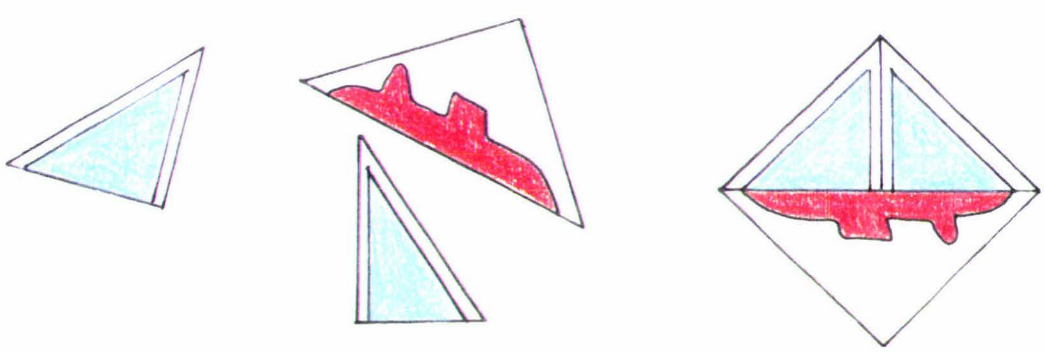
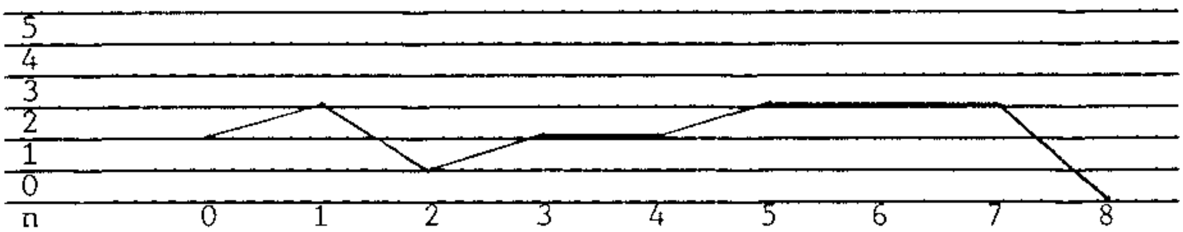


Figure 28: Yachts.

Many children had difficulty with rotations. This lay in the formation of squares made from half and quarter triangles. Some children lay sails flat or back to front. Whereas some children found rotations difficult with no colour problems, others were unable to focus on colour, with no rotational difficulties. For these children one concept was enough at a time.

Figure 28a shows levels of performance across Figure 28:

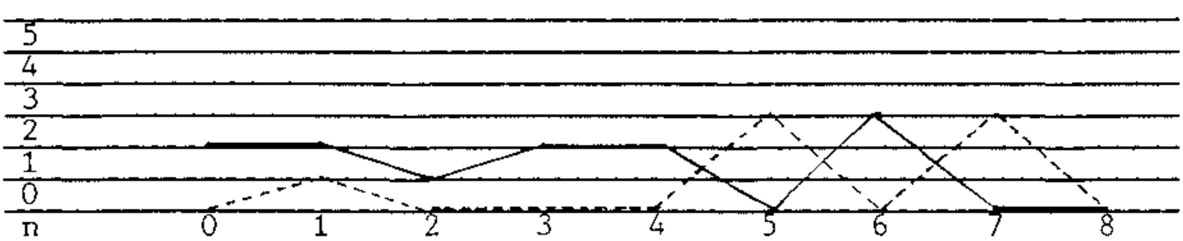


Levels of Performance

0 - No rotations. No colour matching.
1 - Rotations with questioning. No colour matching.
2 - Rotations with questioning. Colours with questioning.
3 - Rotations with questioning. Colour matching.
4 - Good rotations. No colour matching.
5 - Good rotations. Colour matching incomplete..
6 - Accurate but incomplete.
7 - Accurate completion.
8 - Exceptional, fast or novel performance.

Figure 28a: Yachts:
Levels of Performance. n = 19.

Figure 28b examines gender performance across levels of performance: (Levels of performance as for Figure 28a).



n = 12 ———— Males
n = 7 - - - - - - Females

Figure 28b: Yachts:
Gender Performance Across Levels of Performance. n = 19.

Table 13 shows correlation coefficients between levels of performance and mathematical groups, school class levels, and ages.

Table 13: Spearman's Rho Correlation Coefficients: Figure 28: Yachts: Levels of Performance: n = 19.

Across mathematical groups	$r_s = 0.483$	$p = <0.05$
Across school class levels	$r_s = 0.374$	$p = >0.05$
Across ages	$r_s = 0.512$	$p = <0.05$

Figure 28c shows Figure 28 levels of performance across mathematical groups:

Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7			1			2		
6	1	1			1			
5			2		1			
4	1	1						
3			1		1			
2		1						
1	1	1	1					
0		2						

Levels of Performance

0 - No rotations. No colour matching.
1 - Rotations with questioning. No colour matching.
2 - Rotations with questioning. Colours with questioning.
3 - Rotations with questioning. Colour matching.
4 - Good rotations. No colour matching.
5 - Good rotations. Colour matching incomplete.
6 - Accurate but incomplete.
7 - Accurate completion.
8 - Exceptional, fast or novel performance.

Figure 28c: Yachts:
Levels of Performance Across Mathematical Groups. n = 19.

It was unexpected to find a 7 year old J-SLD boy, and a 6 year old JR boy at a high level 6 level of performance, only surpassed by two 8 and 10 year old AV children and one 7 year old JE child. The significance in correlation levels found in Figure 28 between mathematical groups across levels of performance may well be attributed to the high performance of the two AV girls, combined with the two JR children finding this puzzle too difficult. Ages across levels of performance were also significant, but school classes were not. Gender performance for Figure 28 is tested with a Mann-Whitney U-Test, showing significance in favour of females. This is most likely chance.

In contrast, Figure 27, Wallpaper: Ten Intersecting Hexagons, Spearman's Rho correlations for mathematical groups, school classes, and ages across levels of performance were not significant. These very low correlations can be attributed to the highest level of performance coming from a child in the lowest mathematical group category attempting this puzzle, an AV girl, combined with the lowest level of performance for this puzzle coming from two children in the highest mathematical group categpory of SE 1 children. Perceptual difficulties such as pattern matching with colour distractions, combined with awareness of shape, and rotations within these shapes, do not correlate well with P.A.T. mathematics, but more to the ability of a child to perceive and conceive multiple dimensions and concepts simultaneously. A Mann-Whitney U-Test shows no gender differences for Figure 27.

Table 13A shows two tailed Mann-Whitney U-Tests testing gender differences in levels of performance across mathematical groups for Chapter 4, at 0.05, where U (M) = Males, U1 (F) = Females, and z shows use of normal approximation for larger samples.

Table 13A: Mann-Whitney U-Tests For Chapter 4: Gender Differences: Levels of Performance Across Mathematical Groups:

Figure	U (M)	U1 (F)	z	Males	Females
Puzzles demonstrating unexpected dimensions					
14	147.5	188.5	+ 0.62	-	-
15	20	28		-	-
16	5	35		-	*
Puzzle demonstrating reversals					
18	13.5	22.5		-	-
20	127.5	26.5		*	-
Puzzles demonstrating rotations					
27	10	25		-	-
28	18	66		-	*

* significant at 0.05.

Subject numbers for the other puzzles involving rotation that have been mentioned in this section were too small for valid comments to be made, but it is interesting in the paint chart puzzles, that the two AV girls had no problems, whereas both SR children found these puzzles too difficult, and one SE 1 child completed them with difficulty.

Individual Child Profiles

Several remedial children improved their normal classroom mathematics following work with the puzzles. The progressive movement was evident in their ability to process puzzles and concepts that were previously difficult for them. Following work with the puzzles one senior remedial 9 year old boy's formal mathematical work improved, and one junior remedial girl moved into an average mathematical group in her school class. Two further children show explicit difficulties and movement of progress:

Junior Remedial boy - age 6 years

The first child had major problems recognising and rotating shapes to match them, gradually learning to rotate and match, although not name the shapes, and finally managing some quite advanced puzzles. This boy firstly matched sailing ship halves with two sails together instead of a ship base and sail, with rotational problems. He was unable to distinguish between flower and pentagonal shaped buttons. When asked what shape a white felt circle was, he said it was white. On his fourth session he was finally able to match halves and quarters cut horizontzlly, vertically and diagonally, but called a triangle 'rectangle.' On his sixth session he attempted Figure 89, the birthday candle array, managing a cardinal and ordinal array with vertical colour sequences, and later matched adsum blocks onto shaped cards successfully (Figure 86), although he had no idea of conservation of area. He made a large tessellated rectangle with the adsum vinyl shapes, and performed another cardinal and ordinal array with the button holes array (Figure 88), achieving the most unexpected performance for this puzzle and the flower pattern matching strips (Figures 92, 93). He was also able to classify, rotate and match the aeroplane array (Figure 53), but still called a triangular aeroplane nose a rectangle, then a square.

Junior Remedial girl - age 6 years

This girl had problems with shapes, and with bilateral symmetry. In attempting to match some vertically cut halves with Red Indians on them,

(Plate 77), some figures were mismatched to have faces with one or three eyes each. She could not see anything wrong with this arrangement. She could not seriate a set of diamond shapes, and although she could match colours and shapes for vertically halved diamonds, could not match horizontally cut halved diamonds. The following week she was able to both seriate, and to match faces correctly. At a later session she completed the button holes array (Figure 88) successfully with questioning, and made tessellated patterns with some semicircles on rectangular shapes. She had difficulty rotating pieces to make yachts (Figure 28), and made no colour matches. The following year she achieved the second to highest level of performance for the adsum shape conservation set (figure 86) and completed the reversed tree puzzle (Figure 20) with minimun questioning. Her teacher said her school mathematical work developed suddenly following work with the puzzles.

Summary

Some children do have perceptual mode preferences such as colour or shape. Some children may also have difficulty with variations within these dimensional modes, such as differentiation between colours or shapes, and further difficulties with the names of such differentiations. Other children may find difficulty with reversals, rotations, and in perceiving lines. In addition to these perceptual difficulties, the combination of multiple concepts may mean a child is able to concentrate on one concept or another, but not both at the same time.

Chapter 5 looks at children's ability to process multiple concepts. It elaborates on perceptual difficulties occurring more frequently in puzzles where in addition to perceptual dimensions, multiple concepts such as ordering - alternating, rotational or consecutive, and consecutive in array form, or patterning required for the solution of a puzzle necessitated a conscious focus of attention.

Chapter 5

Multiple Concepts

Distraction

When a child is attending to multiple concepts simultaneously as previously discussed, difficulty may occur from an unexpected element or the difficulty of focusing on several separate ideas such as colour, pattern, shape, reversal or rotation at the same time. Additionally, difficulty may manifest as a distraction element, as evidenced in the following three puzzles, Figures 29, 30, and 31. Figure 29, an average senior puzzle, involved all of these types of difficulty:

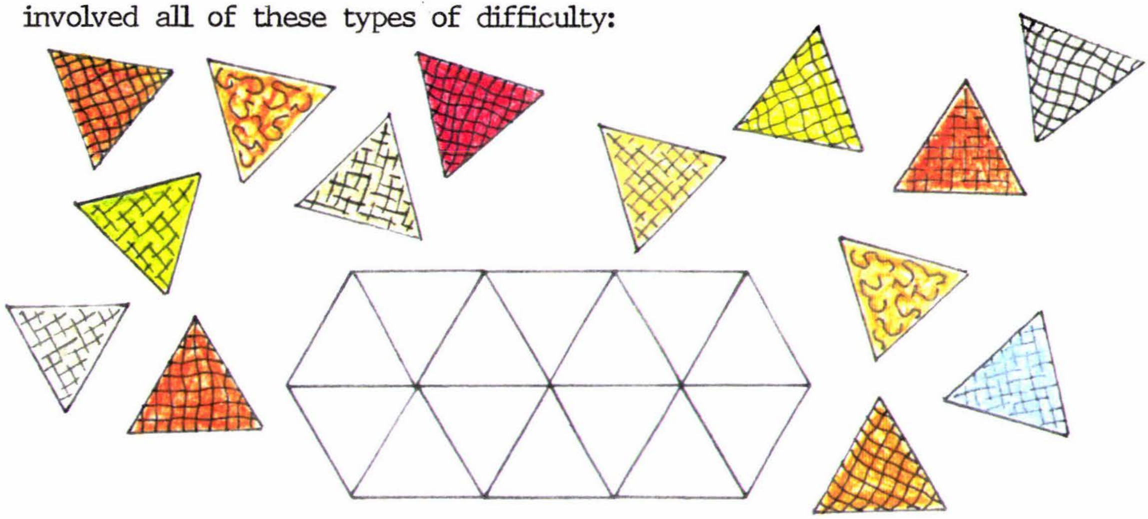


Figure 29: Wallpaper: Three Intersecting Hexagons:
Verbal Instructions:

- Find three sets.
- Find three hexagons.
- Place each set in its own hexagon.

Table 14 shows Figure 29 correlation coefficients between levels of performance and mathematical groups, school class levels, and ages.

Table 14: Spearman's Rho Correlation Coefficients:
Figure 29: Wallpaper: Three Intersecting Hexagons:
Levels of Performance: n = 15.

Across mathematical groups	$r_s = 0.247$	p = >0.05
Across school class levels	$r_s = 0.243$	p = >0.05
Across ages	$r_s = 0.325$	p = >0.05

Figure 29a shows Figure 29 levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6						1	1	1
5							1	2
4						1		1
3			1					1
2			1					3
1								
0							1	

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 29a: Wallpaper: Three Intersecting Hexagons: Levels of Performance Across Mathematical Groups. n = 15.

The distraction element in Figure 29 was the concept of pattern matching where colour was irrelevant, and an unexpected reversal with an orange colour set intersecting the two pattern sets. Rotation was also involved.

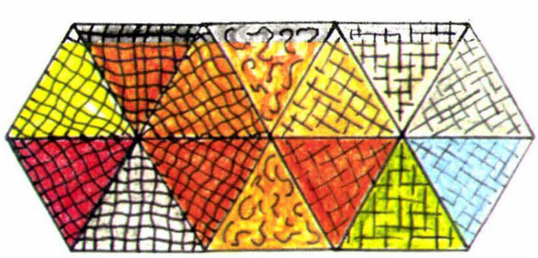
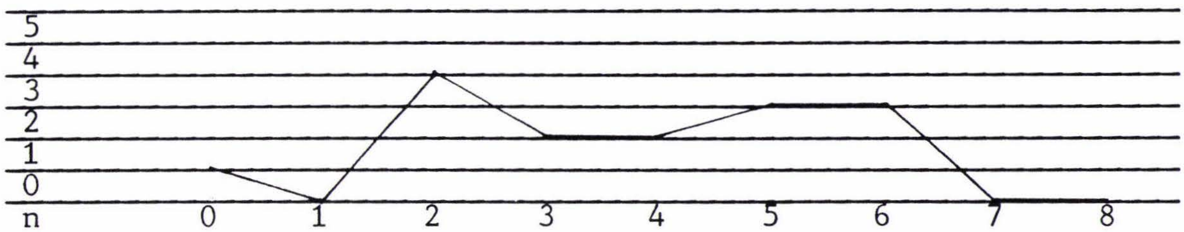


Figure 29b: Wallpaper: Three Intersecting Hexagons.

Figure 29c shows levels of performance across all children who attempted Figure 29:

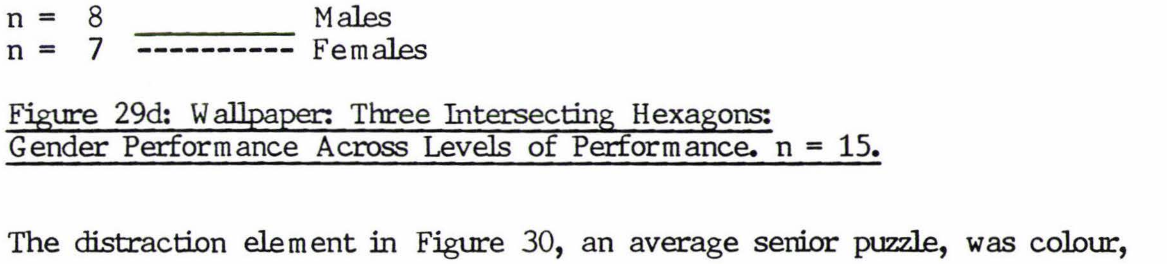


Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 29c: Wallpaper: Three Intersecting Hexagons: Levels of Performance. n = 15.

Figure 29d examines gender performance across levels of performance: [Levels of performance as for Figure 29c].



n = 8 __________ Males
n = 7 ---------- Females

Figure 29d: Wallpaper: Three Intersecting Hexagons: Gender Performance Across Levels of Performance. n = 15.

The distraction element in Figure 30, an average senior puzzle, was colour, making it difficult for children to attend to the pattern continuation and ignore colour. With only one piece glued into place, all other pieces created rotational difficulties. Pieces of pattern needed to be rotated to find their connections.

Figure 30: Wallpaper: Red / Brown Pattern Continuation Squares.

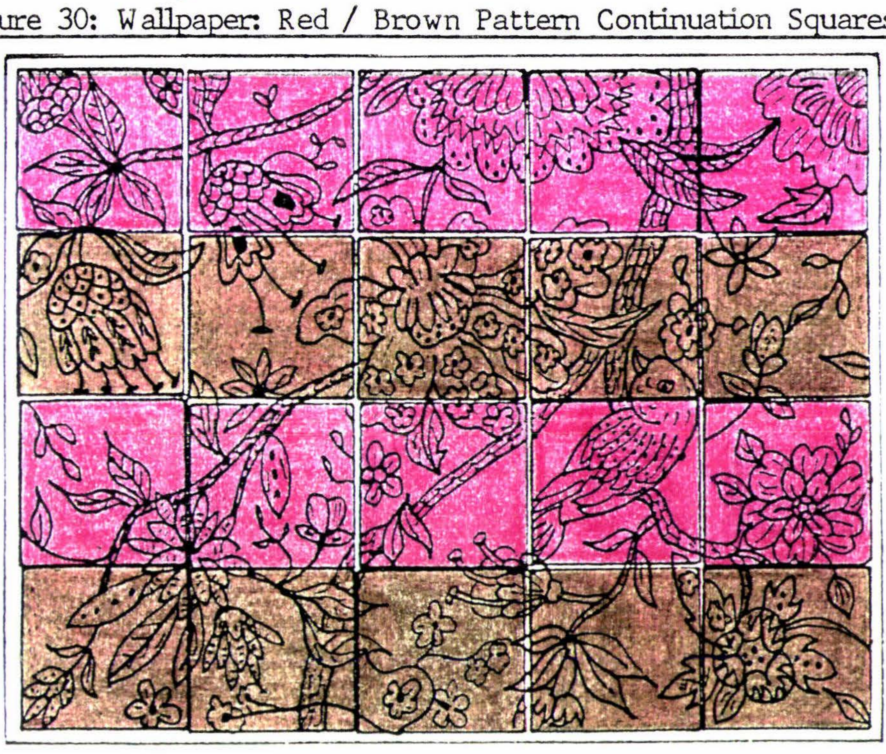


Figure 30a: Wallpaper: Red / Brown Pattern Continuation Squares.

Only three children attempted Figure 30, Wallpaper: Red / Brown Pattern Continuation Squares. It was too difficult for one 10 year old SR girl. The 8 year old AV girl found this difficult, but completed it, and her 10 year old AV sister completed it with no problems.

As well as showing elements of distraction, Figures 29, 30 and 31 also incorporate concepts of rotational ordering, discussed later in this chapter. Neither Figure 29 nor Figure 31 show significance at 0.05 levels of significance for Spearman's Rho correlations between mathematical groups, school classes or ages across levels of performance. Subject numbers are small, requiring a higher r_s for significance between correlations.

In Figure 29, Wallpaper: Three Intersecting Hexagons, one AV girl was able to achieve the highest very fast completion with no questioning level of performance 6, exhibited in this puzzle, along with SE 1 and SE 2 children. Curiously, the lowest level of performance was also exhibited by a SE 2 child who found this puzzle too difficult. Two JE children completed the puzzle with difficulty and questioning. Three of the eight SE 1 children attempting Figure 29 also completed it with difficulty and questioning, with more children achieving at this low level 2 than at any other level. This was one puzzle where boys seemed to have more difficulty than girls, with boys peaking at difficult completion with a lot of questioning level 2, and girls peaking at level 6 very fast completion with no questioning. A Mann-Whitney U-Test testing gender differences in levels of performance for Figure 29 shows no significant gender differences.

Gender performance for Figure 31 included three boys and two girls who found this puzzle too difficult. This is more than half of the eight children attempting Figure 31. Two boys and one girl completed this puzzle successfully. Gender differences are not apparent. This difficulty centred around rotation, where each block of repeated pattern was cut differently, with pieces not interchangeable with pieces in other pattern blocks. Colour distraction also created significant difficulty.

Figure 31, an advanced senior puzzle, had a distraction element of colour, where the real task was of repetitive matched patterns with rotation:

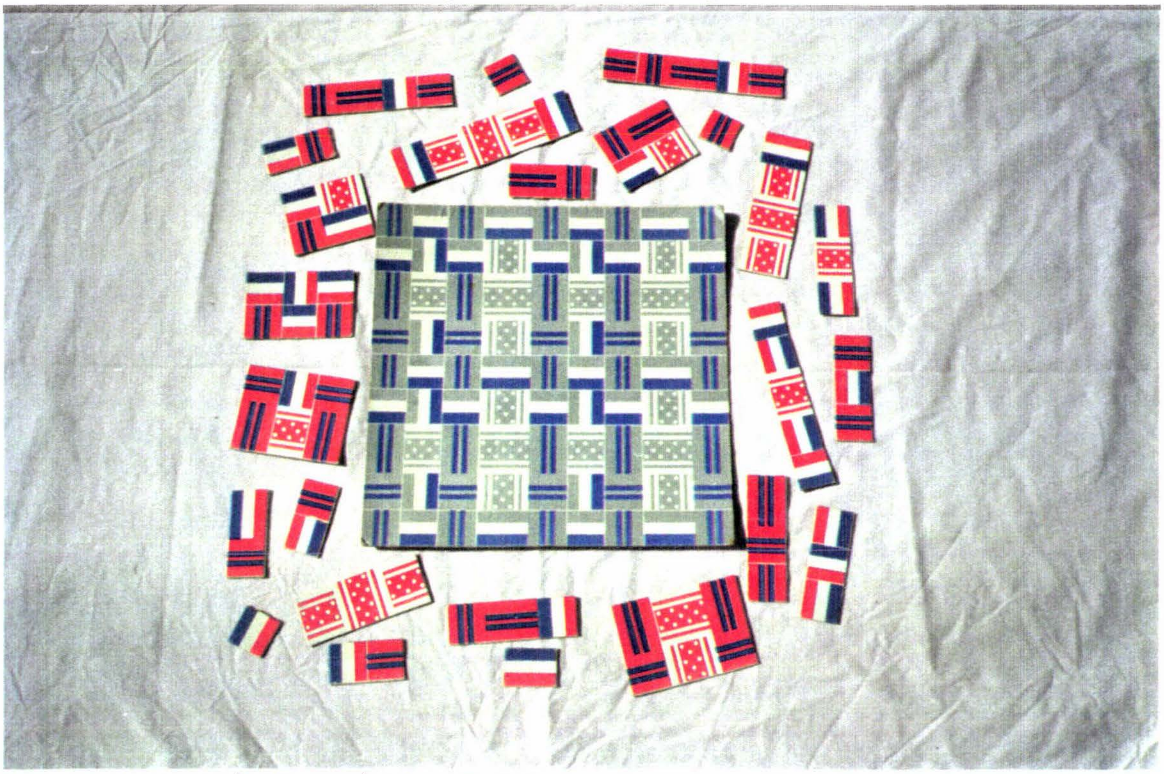


Figure 1: Red / Silver / Blue Buildup Pattern Match Jigsaw.

Plate 2 : Figure 31a: Red / Silver / Blue Buildup Pattern Match Jigsaw.

Figure 31b shows Figure 31 levels of performance across mathematical groups:

Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6								
5								1
4						1		
3								
2								1
1								
0					1	1	2	1

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of rearranging
2 - Difficult completion with a lot of rearranging
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 31b: Red / Silver / Blue Buildup Pattern Match Jigsaw.
Levels of Performance Across Mathematical Groups. n = 8.

Table 15 shows Figure 31 correlation coefficients between levels of performance and mathematical groups, school class levels, and ages.

Table 15: Spearman's Rho Correlation Coefficients:
Figure 31: Red / Silver / Blue Buildup Pattern Match Jigsaw.
Levels of Performance: n = 8.

Across mathematical groups	$r_s = 0.440$	$p = >0.05$
Across school class levels	$r_s = 0.434$	$p = >0.05$
Across ages	$r_s = 0.434$	$p = >0.05$

Alternating Concepts

A further difficulty with multiple concepts concerned alternating concepts. [Figures 32, 33, 34, 35, 36, 37]. Alternating concepts form an ordered pattern in themselves, so the child's task is to classify, order, and connect, requiring much conscious attention. Figure 32, an average senior puzzle, shows an example of alternating concepts:

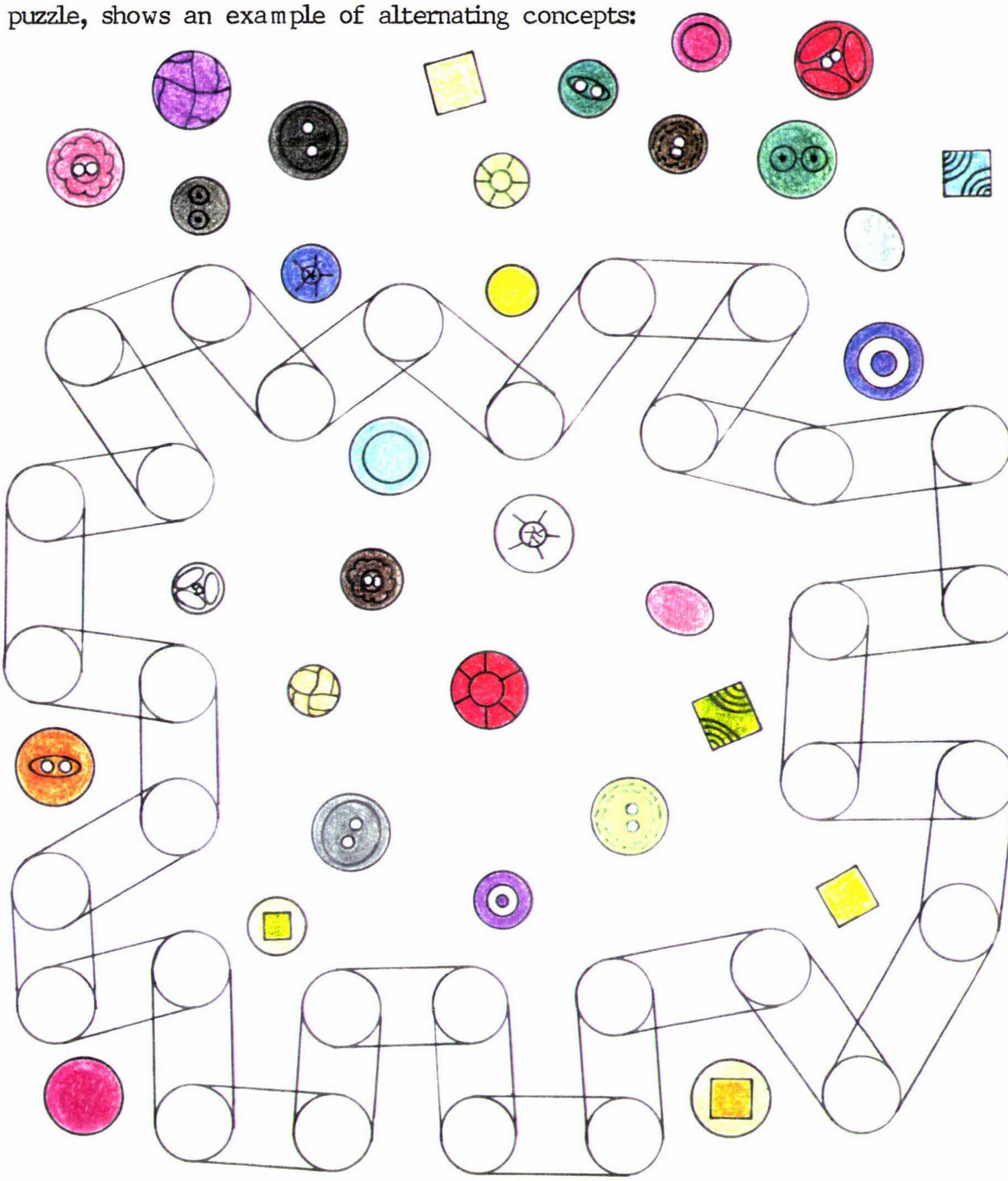


Figure 32: Chain Link: Intersecting Sets of Two Buttons:
Verbal Instructions:

- Find sets of two buttons.
- Each chain link is a set of two.
- Place sets of two buttons in the chain links.
- The last button you place must make a set with your first button.

One 10 year old SE 1 girl had difficulty distinguishing green from turquoise, and in establishing an alternating colour / pattern sequence. Colour differentiation and alternating pattern difficulties were the major difficulties encountered in this puzzle.

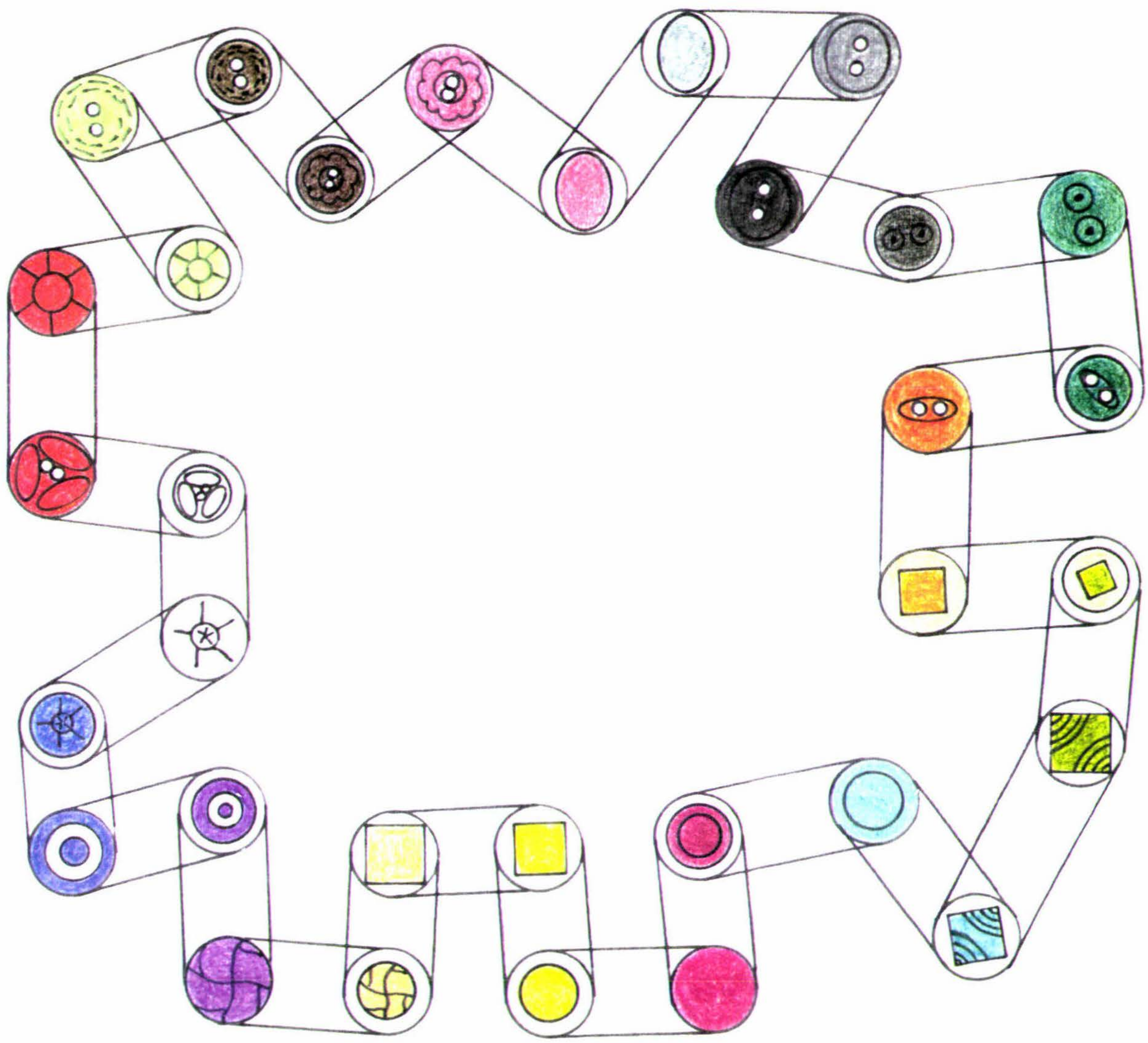


Figure 32a: Chain Link: Intersecting Colour / Pattern Sets of Two Buttons: Verbal Instructions:

- If a child has difficulty placing buttons into sets of two because each button belongs to a colour set and a form / pattern set, I ask the child to find one set of two. The child will select either a colour or a pattern set of two. The set is placed on the diagram. I ask the child to find another button to match one of these two buttons.

Figure 32b shows Figure 32 levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6								
5								
4			1			2	1	
3								2
2		1					1	2
1								1
0		1						

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

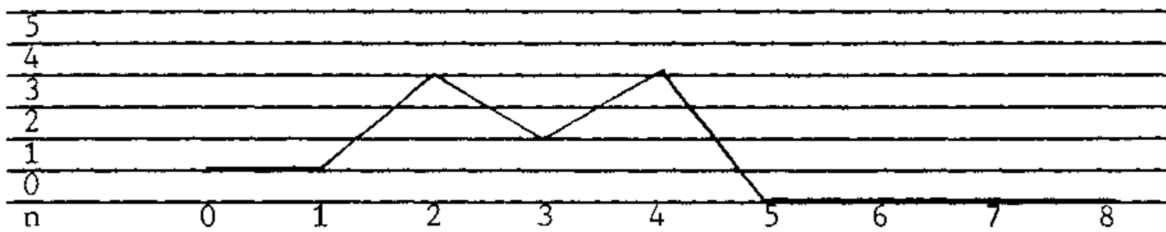
Figure 32b: Chain Link: Intersecting Colour / Pattern Sets of Two Buttons: Levels of Performance Across Mathematical Groups. n = 12.

Table 16 shows correlation coefficients between levels of performance and mathematical groups, school class levels, and ages.

Table 16: Spearman's Rho Correlation Coefficients:
Figure 32: Chain Link: Intersecting Colour / Pattern Sets of Two Buttons: Levels of Performance: n = 12.

Across mathematical groups	$r_s = -0.029$	$p = >0.05$
Across school class levels	$r_s = 0.054$	$p = >0.05$
Across ages	$r_s = 0.208$	$p = >0.05$

Figure 32c shows levels of performance across all children who attempted Figure 32:

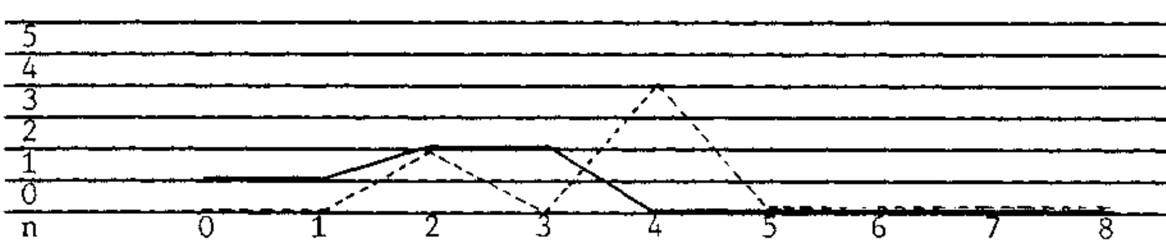


Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 32c: Chain Link: Intersecting Colour / Pattern Sets of Two Buttons: Levels of Performance. n = 12.

Figure 32d examines gender performance across levels of performance: (Levels of performance as for Figure 32c).



n = 6 ________ Males
n = 6 ---------- Females

Figure 32d: Chain Link: Intersecting Colour / Pattern Sets of Two Buttons: Gender Performance Across Levels of Performance. n = 12.

It is not surprising there is no correlation between mathematical groups and levels of performance for Figure 32. No SE 1 child reaches as high a level of performance of 4 of complete with minimal questioning, as the two AV girls and one JE and one SE 2 child. As well, the second to lowest level of performance of 1 is included in the SE 1 group, with a difficult incompletion. There were equal numbers of children achieving a difficult completion with a lot of questioning, as there were who completed the puzzle with minimal questioning. Gender performance appears to show girls achieving the higher level of performance. A Mann-Whitney U-Test testing gender differences in levels of performance in Figure 32 shows no significant gender differences.

Colour discrimination was not more difficult for boys than girls. The diffficulty seemed more related to the necessity to classify according to colour and pattern, and to alternate these in pattern form. Conscious effort to keep track of these alternations seemed very difficult for many of the children.

Figure 33, an above average senior puzzle, is an extension of the concepts in Figure 32. The same 10 year old SE 1 girl experienced similar difficulty with alternating concepts in Figure 33 as she did in Figure 32. One SE 1 boy had difficulty finding pattern matched sets of two buttons, being distracted by their differing colours. A further distraction element was the inclusion of a third colour matched button that was not related to any particular pattern set, creating the need for rotational ordering over and above the alternating colour / pattern circular sequence.

Figures 33, 34, 35, 36 and 37 are extensions of the basic concept in Figure 32, including variations on a similar theme. Each puzzle adds an additional surprise element. Subject numbers are too small across all of these puzzles to examine trends in levels of performance, with mixed AV and SE 1 children only attempting these puzzles.

Of note is speed of performance which may indicate transfer of learning for some of these children. Transfer of learning will be discussed in Chapter 6. In Figure 36, one SE 1 girl completed this puzzle accurately in five minutes, with no questioning. The same girl completed Figure 37 in ten minutes, as did the 8 year old AV girl, both performances following minimal questioning. The 10 year old AV girl completed Figure 37 accurately with no questioning in just a few minutes.

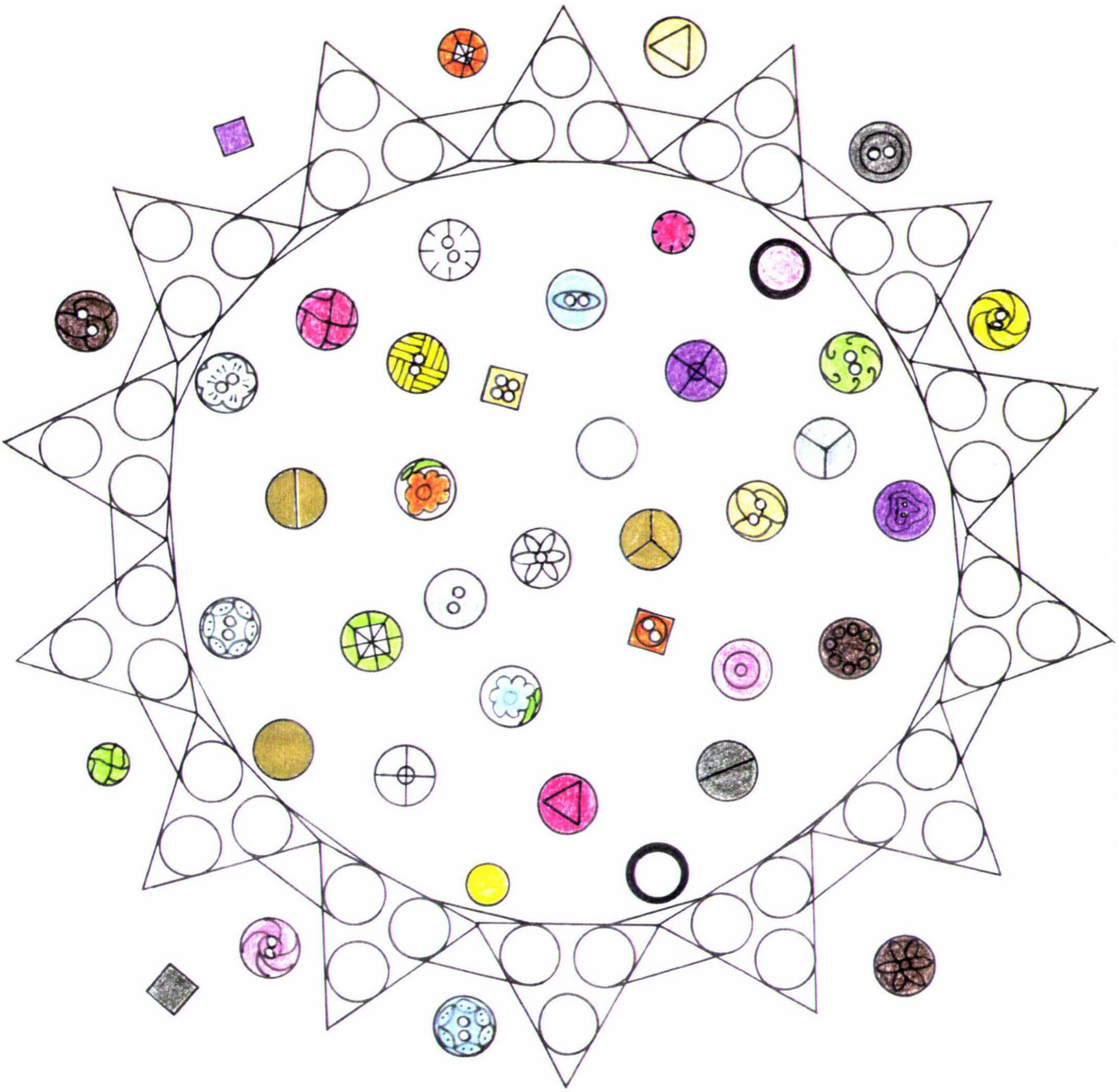


Figure 33: Intersecting Sets of Three / Sets of Two Button Star: Verbal Instructions:

- Each triangle is a set of three. Each chain link is a set of two.
- Find buttons that belong together.
- Place sets of two buttons in the chain links.
- Place sets of three buttons in the triangles.
- The last set you place must link with your first set.

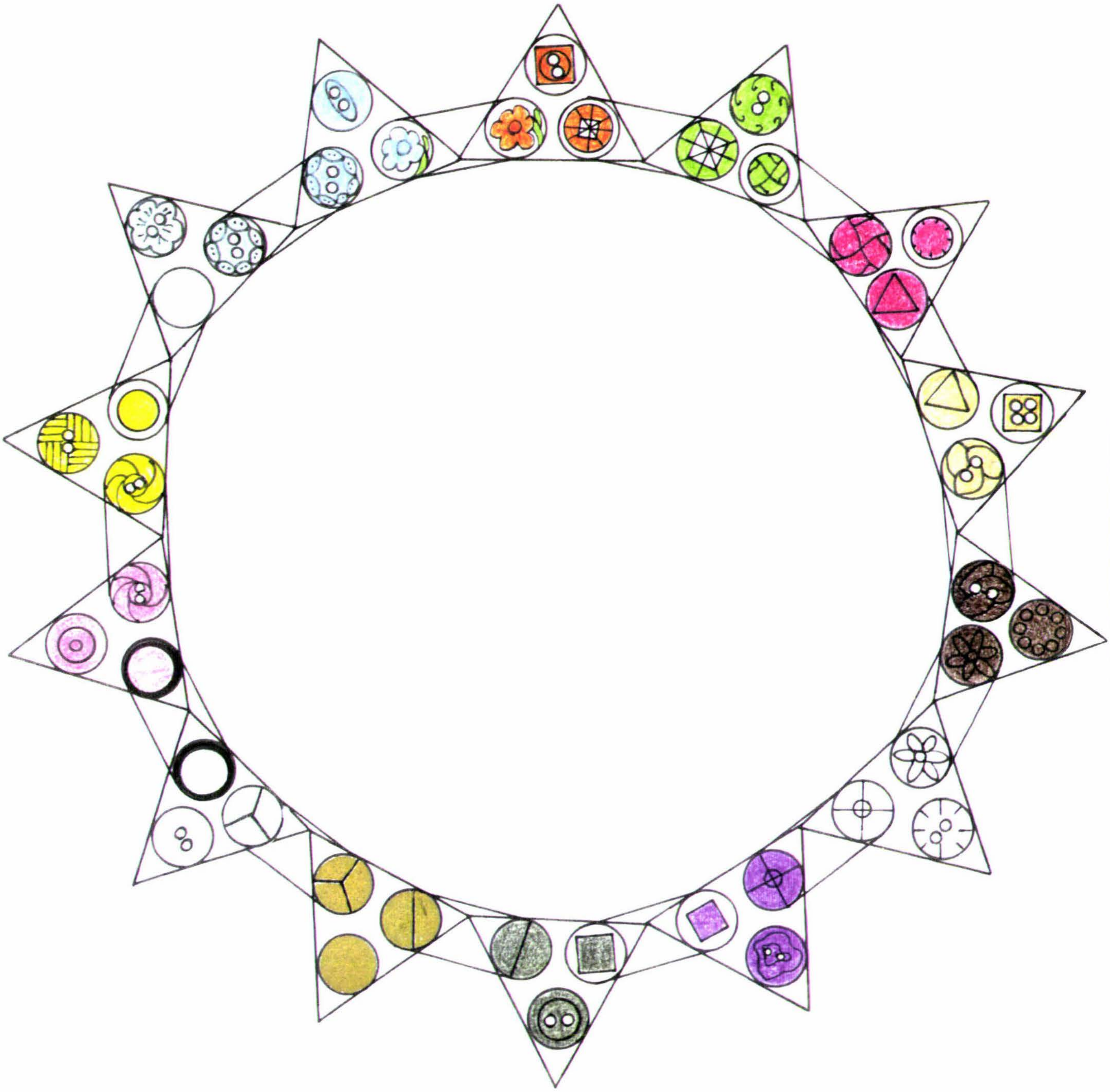


Figure 33a: Intersecting Colour Sets of Three / Pattern Sets of Two Button Star:

Figure 33b shows levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6								1
5						1		
4						1		1
3								1
2								1
1								
0								

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Very fast completion with minimal questioning
6 - Complete with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 33b: Intersecting Colour Sets of Three / Pattern Sets of Two Button Star: Levels of Performance Across Mathematical Groups. n = 6.

Figure 34, an above average senior puzzle, shows Figure 34 levels of performance across mathematical groups: [Levels of performance as for Figure 33b].

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6								
5						1		3
4						1		
3								
2								1
1								
0								

Figure 34: Intersecting Sets of Three Button Star: Levels of Performance Across Mathematical Groups. n = 6.

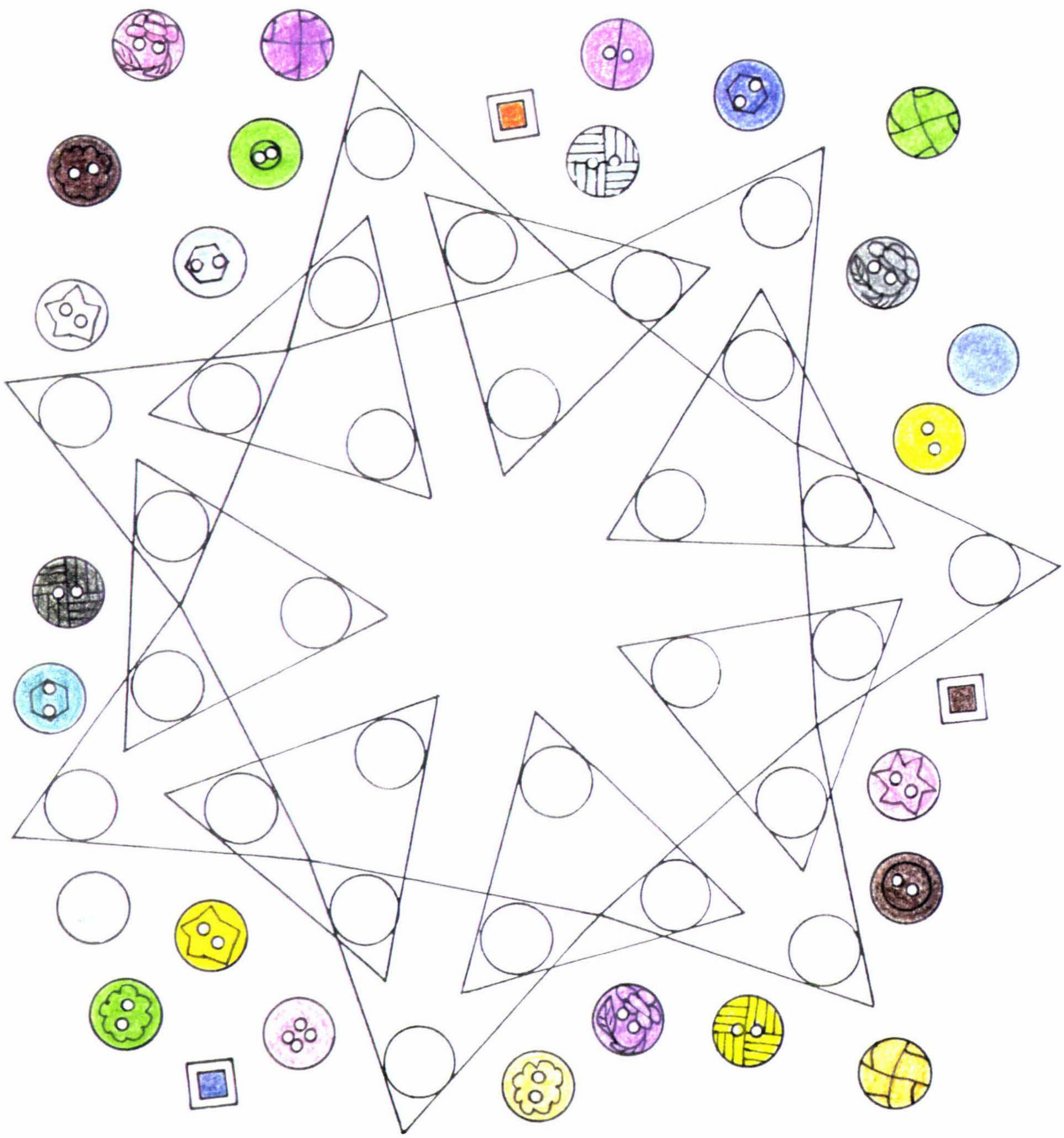


Figure 34a: Intersecting Sets of Three Button Star:
Verbal Instructions:

- Each triangle is a set of three.
- Find buttons that belong together.
- Place sets of three buttons in the triangles.
- The last set you place must link with your first set.

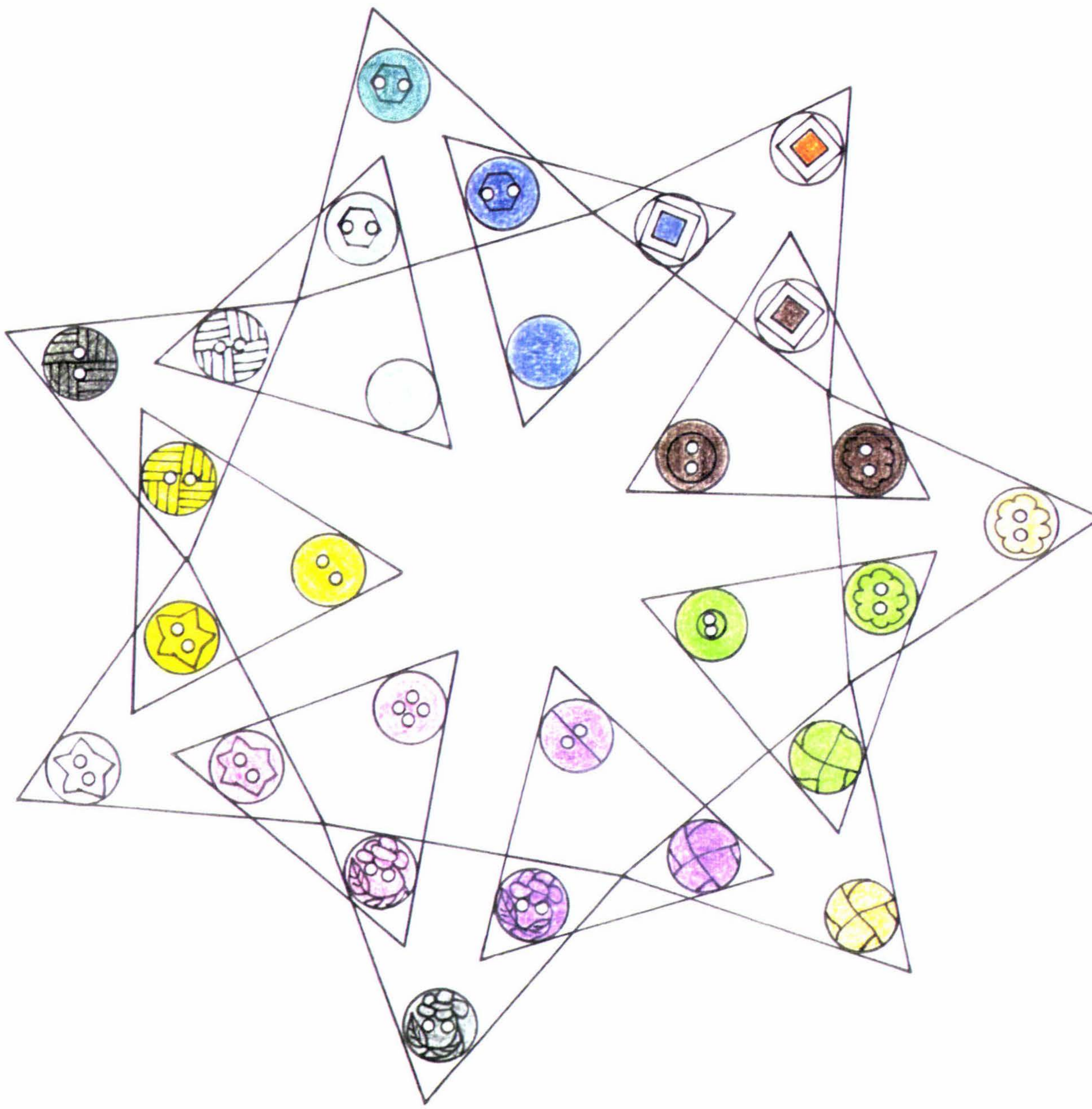


Figure 34b: Intersecting Colour Sets of Three / Pattern Sets of Three Button Star:

Figure 35, an above average senior puzzle, is a further example of alternation:

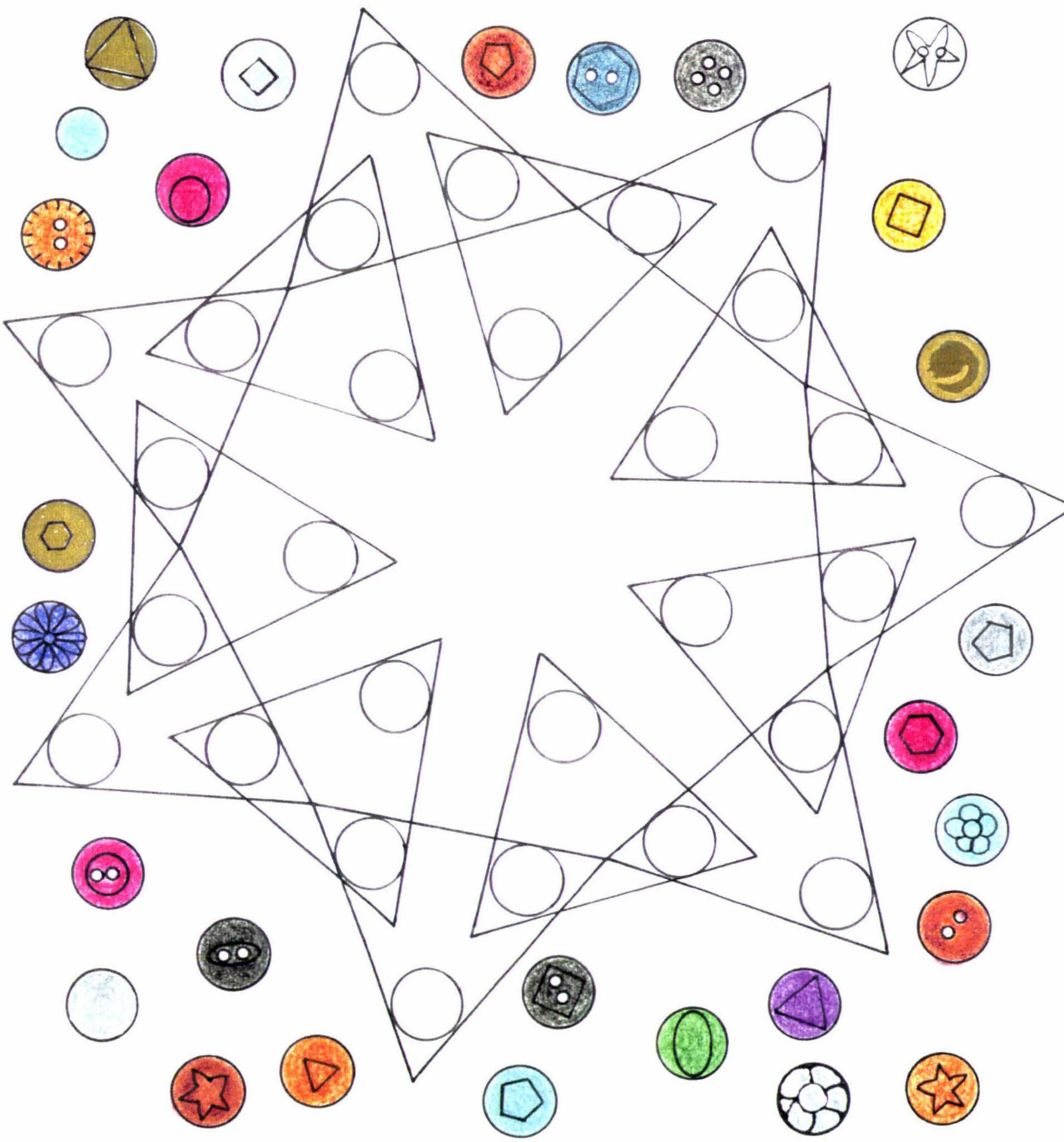


Figure 35: Intersecting Sets of Three Button Star:
Verbal Instructions:

- Each triangle is a set of three.
- Find buttons that belong together.
- Place sets of three buttons in the triangles.
- The last set you place must link with your first set.

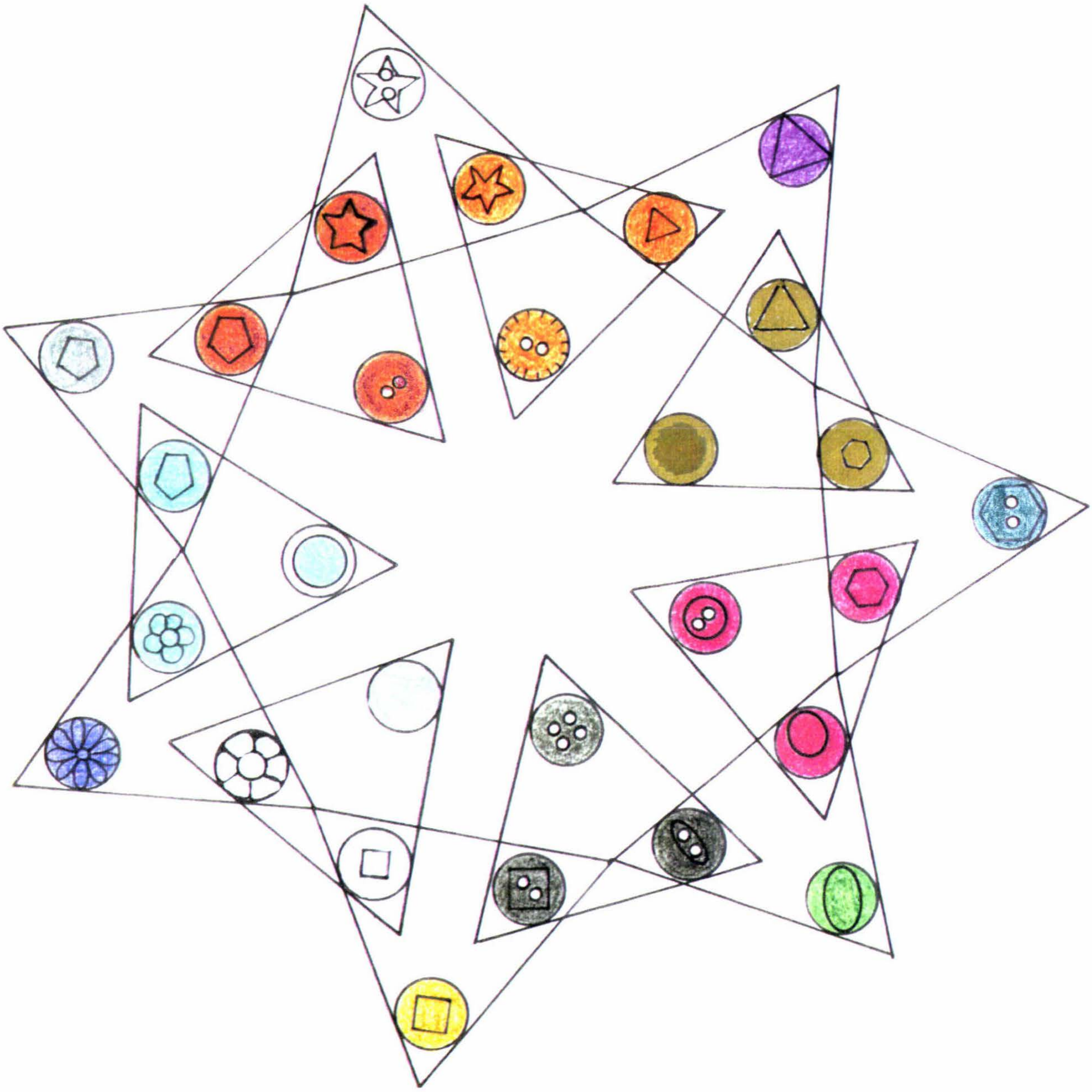


Figure 35a: Intersecting Colour Sets of Three / Shape Sets of Three Button Star.

Figure 35b shows levels of performance across mathematical groups:

Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6								
5								
4						1		
3						1		1
2								
1								
0								

<u>Levels of Performance</u>

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Very fast completion with minimal questioning
6 - Complete with no questioning
7 - Very fast completion with no questioning
8 - Exceptional or novel performance

<u>Figure 35b: Intersecting Colour Sets of Three / Shape Sets of Three Button Star: Levels of Performance Across Mathematical Groups. n = 3.</u>

Figure 36, an average senior puzzle, shows Figure 36 levels of performance across mathematical groups. [Levels of performance as for Figure 35b].

Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								1
6								
5						1		
4								
3								2
2						1		
1								
0								

<u>Figure 36: Intersecting Sets of Two Circular Chain with Intersected Triangular Points: Levels of Performance Across Mathematical Groups. n = 5.</u>

The child with the highest level of performance was the same 10 year old SE 1 girl who earlier had difficulty with alternating concepts in Figures 32 and 33.

Figure 36a: Intersecting Sets of Two Circular Chain with Intersected Triangular Points:
Verbal Instructions:

- Find sets of two buttons.
- Each chain link is a set of two.
- Place sets of two buttons in the chain links.
- The last button you place must make a set with your first button.
- Find one button to complete each triangle. Each button completing a triangle will intersect a chain linked set.

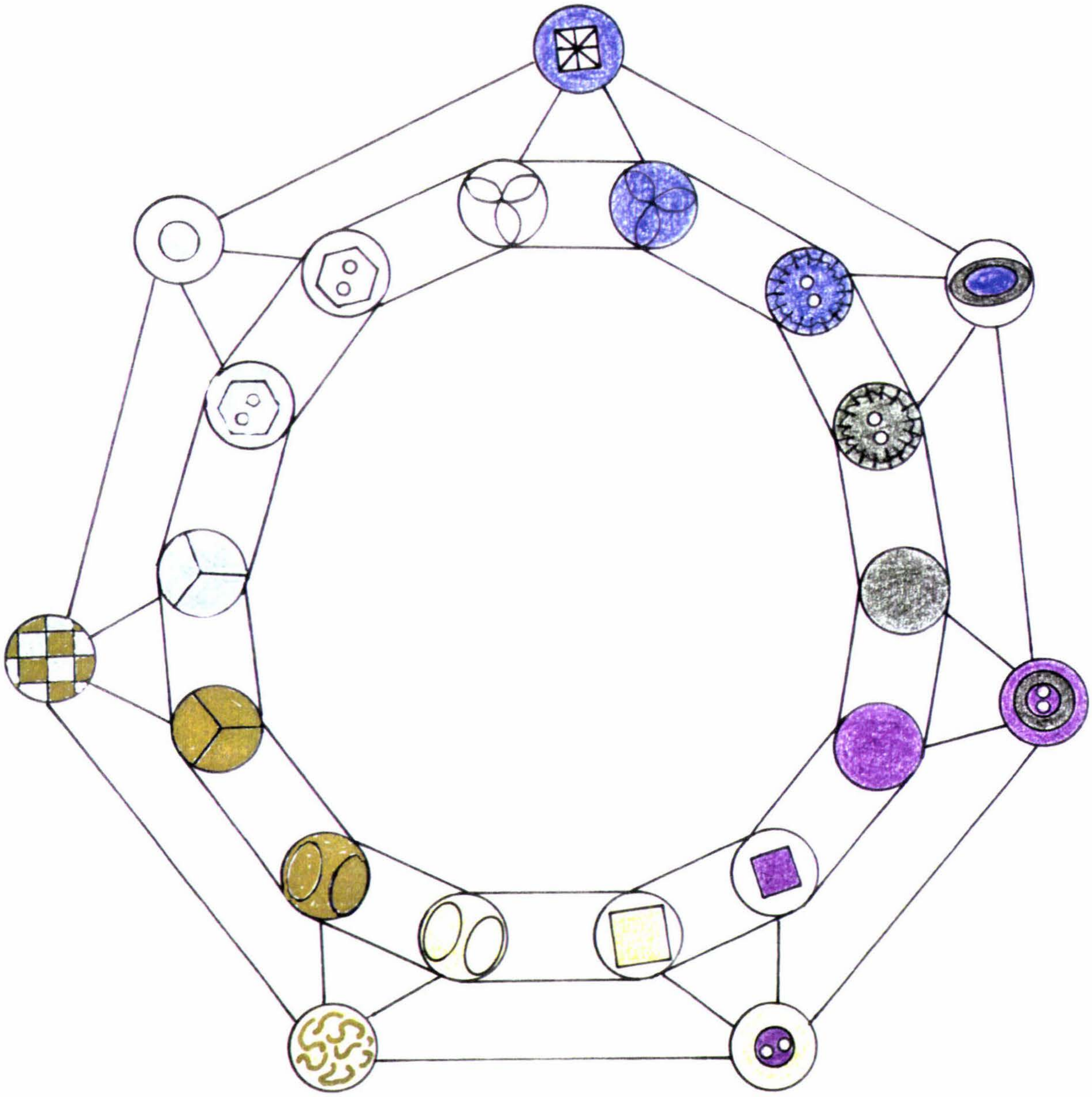


Figure 36b: Intersecting Colour / Pattern Sets of Two Circular Chain with Colour Intersected Triangular Points.

Figure 37, an above average senior puzzle, exhibited multiple alternating concepts:

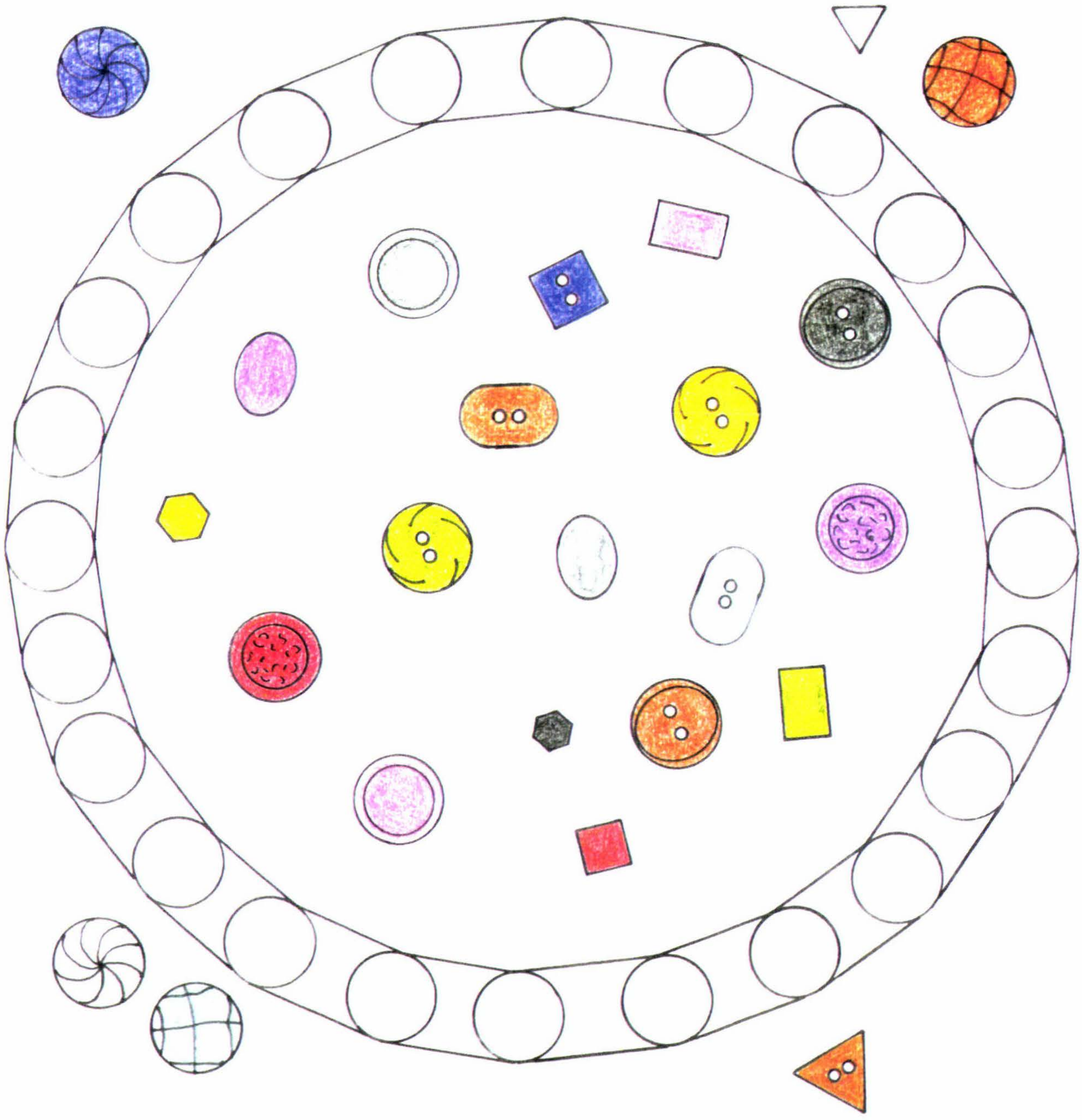


Figure 37: Intersecting Sets of Two Circular Chain:
Verbal Instructions:

- Find sets of two buttons.
- Each chain link is a set of two.
- Place sets of two buttons in the chain links.
- The last button you place must make a set with your first button.

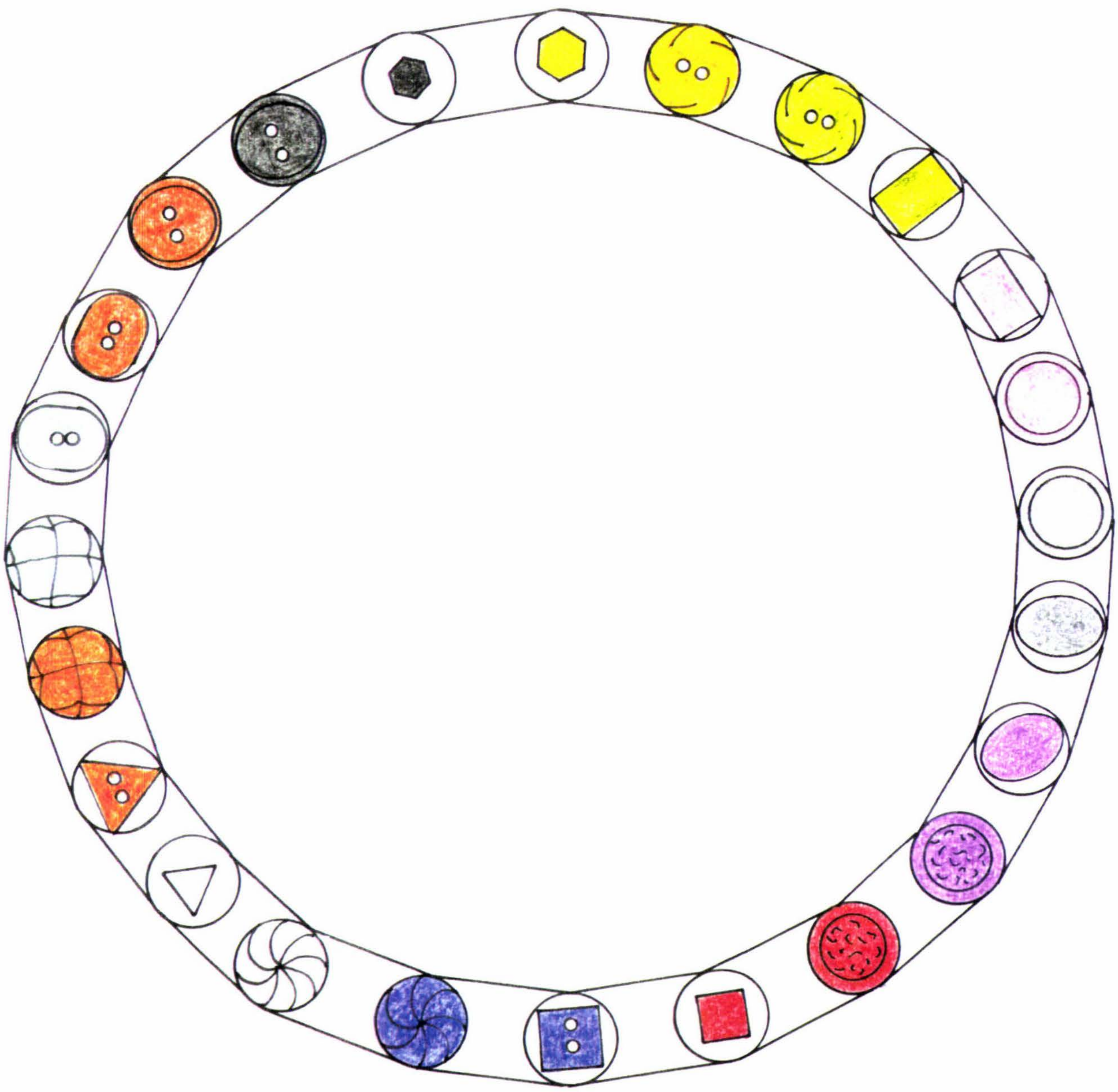


Figure 37a: Intersecting Shape / Colour / Pattern / Colour Sets of Two Circular Chain.

Figure 37b shows levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7						1		
6								
5						1		1
4								
3								
2								
1								
0								

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Very fast completion with minimal questioning
6 - Complete with no questioning
7 - Very fast completion with no questioning
8 - Exceptional or novel performance

Figure 37b: Intersecting Shape / Colour / Pattern / Colour Sets of Two Circular Chain: Levels of Performance Across Mathematical Groups. n = 3.

Rotational Ordering

A further difficulty with multiple concepts occurred with rotational ordering. [Figures 38, 39, 40, 41, 42, 43]. Figure 38 is a very advanced junior puzzle:

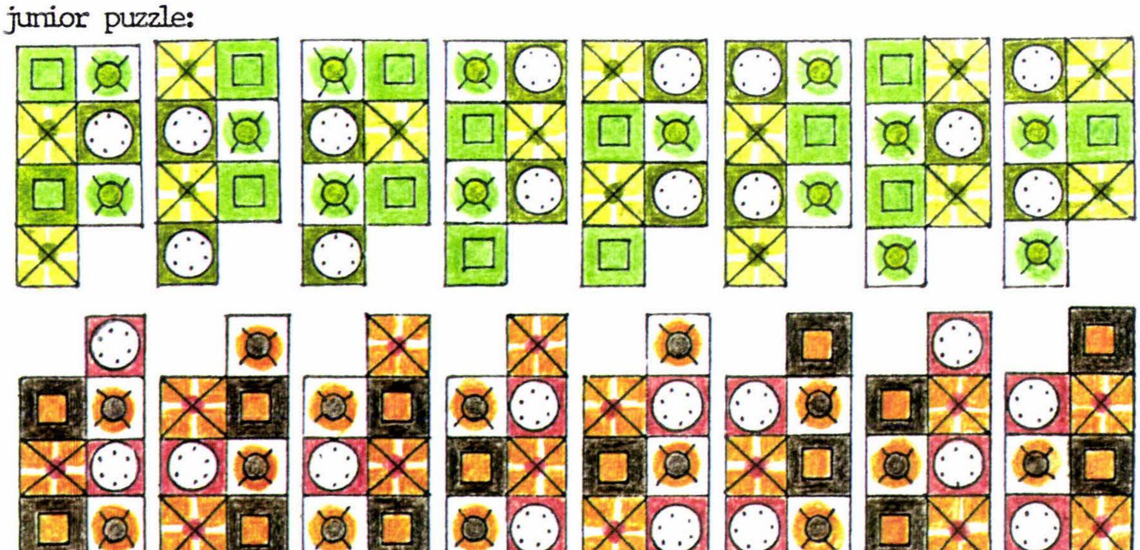


Figure 38: Green / Brown Pattern Continuation Rectangles.

Children attempted to colour match rather than pattern match, and found rotating pieces difficult. Children also had rotational ordering difficulties with Figure 39, a beginning senior puzzle:

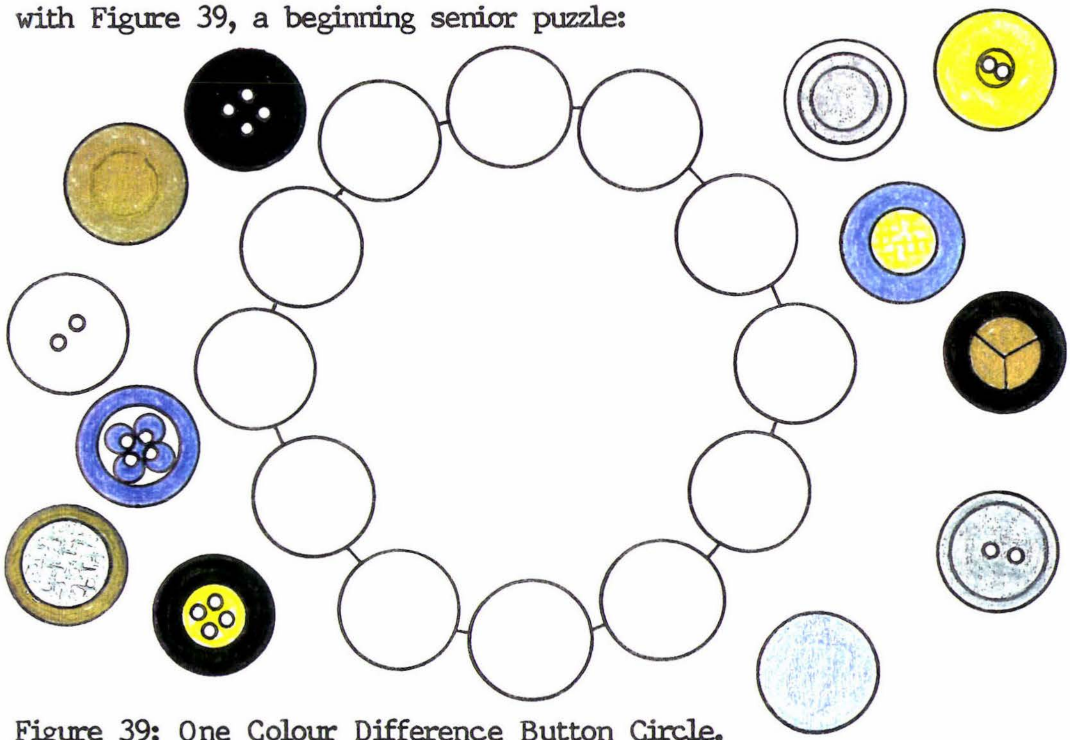


Figure 39: One Colour Difference Button Circle.

Verbal Instructions:

- Arrange the buttons with one colour connection and no more than one colour difference between any buttons you place in connecting circles.

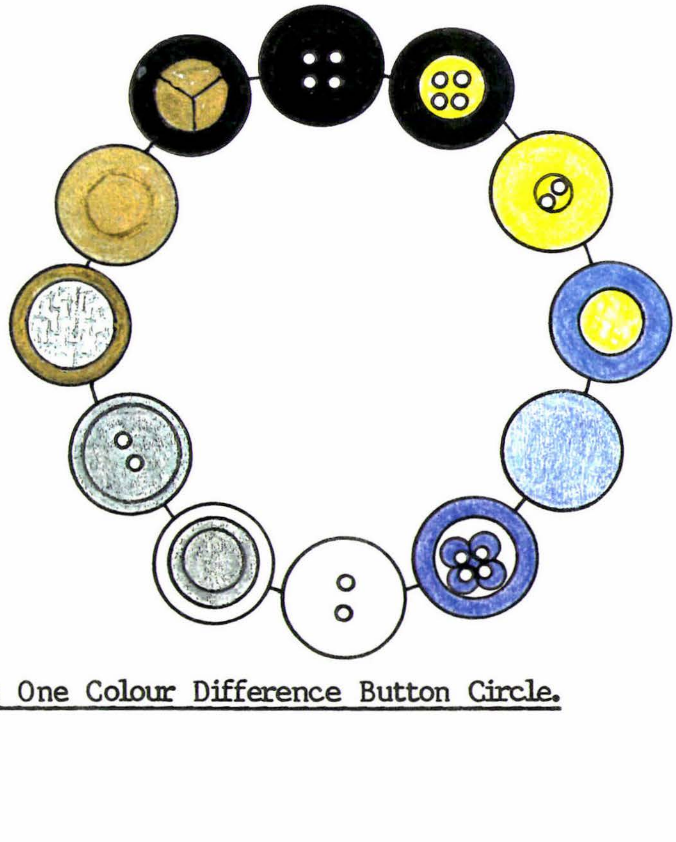


Figure 39a: One Colour Difference Button Circle.

Figure 39b shows Figure 39 levels of performance across mathematical groups:

Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7						1		
6					1	1	1	3
5			1		2		1	2
4							1	
3		1	5		1		1	
2							1	2
1								
0		1			1			

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 39b: One Colour Difference Button Circle.
Levels of Performance Across Mathematical Groups. n = 27.

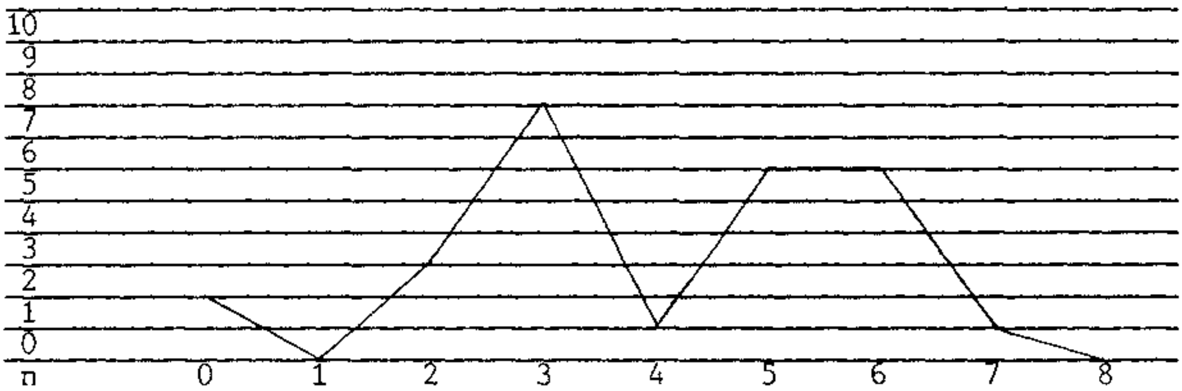
The highest level of performance of exceptional speed [30 seconds] was from an 8 year old AV girl. Many children did not notice the alternating pattern of one colour / two colours, nor apply an ordering strategy.

Table 17 shows Figure 39 correlation coefficients between levels of performance and mathematical groups, school class levels, and ages.

Table 17: Spearman's Rho Correlation Coefficients:
Figure 39: One Colour Difference Button Circle.
Levels of Performance: n = 27.

Across mathematical groups	$r_s = 0.404$	$p = <0.05$
Across school class levels	$r_s = 0.346$	$p = <0.05$
Across ages	$r_s = 0.389$	$p = <0.05$

Figure 39c shows levels of performance across all children who attempted Figure 39:



Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 39c: One Colour Difference Button Circle.
Levels of Performance. n = 27.

Figure 39d examines gender performance across levels of performance: [Levels of performance as for Figure 39c].

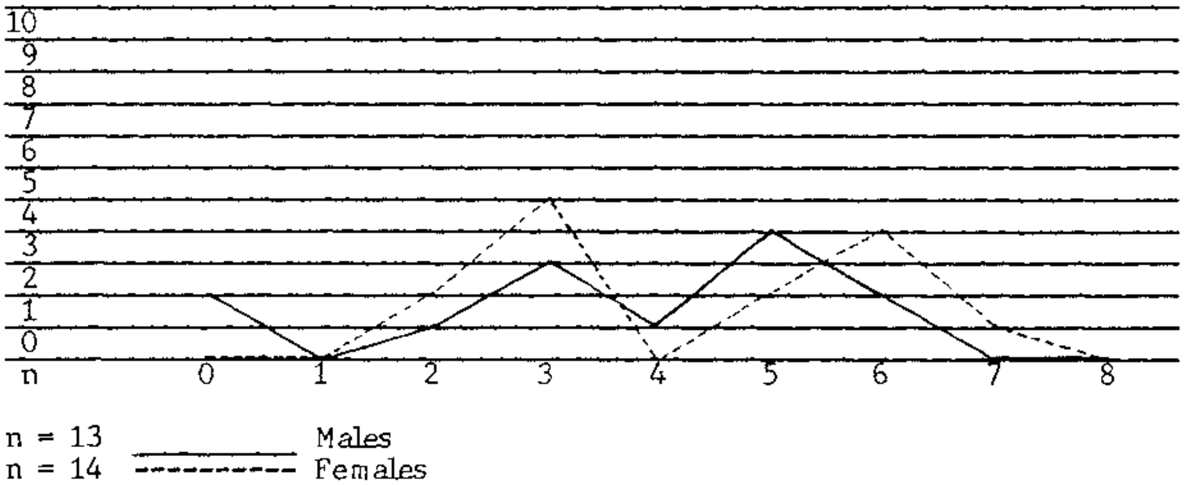


Figure 39d: One Colour Difference Button Circle.
Gender Performance Across Levels of Performance. n = 27.

Spearman's Rho correlations for Figure 39 between mathematical groups, school classes and ages across levels of performance were all significant at a 0.05 level of significance, despite the highest level of performance being from an 8 year old AV Std. 2 girl, and three SE children finding this puzzle very difficult. In contrast, no junior child reached the higher levels of performance recorded here, and one JR child and one SR child found this puzzle too difficult. Significance levels are not high and may be due to chance alone. A Mann-Whitney U-Test testing gender differences in levels of performance shows no gender differences.

Figures 40 and 41 as examples of puzzles incorporating rotational ordering are increasingly difficult variations on the one difference circuit theme. The position of items in sets is important for the correct solution in all rotational ordering puzzles. Classifying sets is a necessary pre-requisite before the ordering positions of these sets can be linked.

In contrast to Figure 39, Figure 40 Spearman's Rho correlations between mathematical groups, school classes and ages across levels of performance were not significant at a 0.05 level of significance. This is not surprising when SE 1 children ranged in levels of performance from too difficult to very fast completion and use of strategy. The too difficult category was also represented by children in the SE 2 mathematical group. Additionally, both AV children achieved the highest level of performance. There appears to be a difference in gender performance, with boys achieving at lower levels of performance than girls. A Mann-Whitney U-Test testing gender differences in levels of performance shows no significant differences.

Two further examples of rotational ordering are demonstrated in Figures 42 and 43, both puzzles requiring fixed positional ordering of sets. In Figure 42, Spearman's Rho correlations between mathematical groups, school classes and ages across levels of performance were also not significant at a 0.05 level of significance. This is also not surprising when two SE 1 children found this puzzle too difficult, and junior levels of performance were similar to senior levels of performance. A Mann-Whitney U-Test testing gender differences in levels of performance shows no difference.

Figure 40, an average senior puzzle, was different from Figure 39 in that a strategy was useful to determine which button needed to take the central position in the diagram, although a time consuming visual trial and error approach was successful for some children:

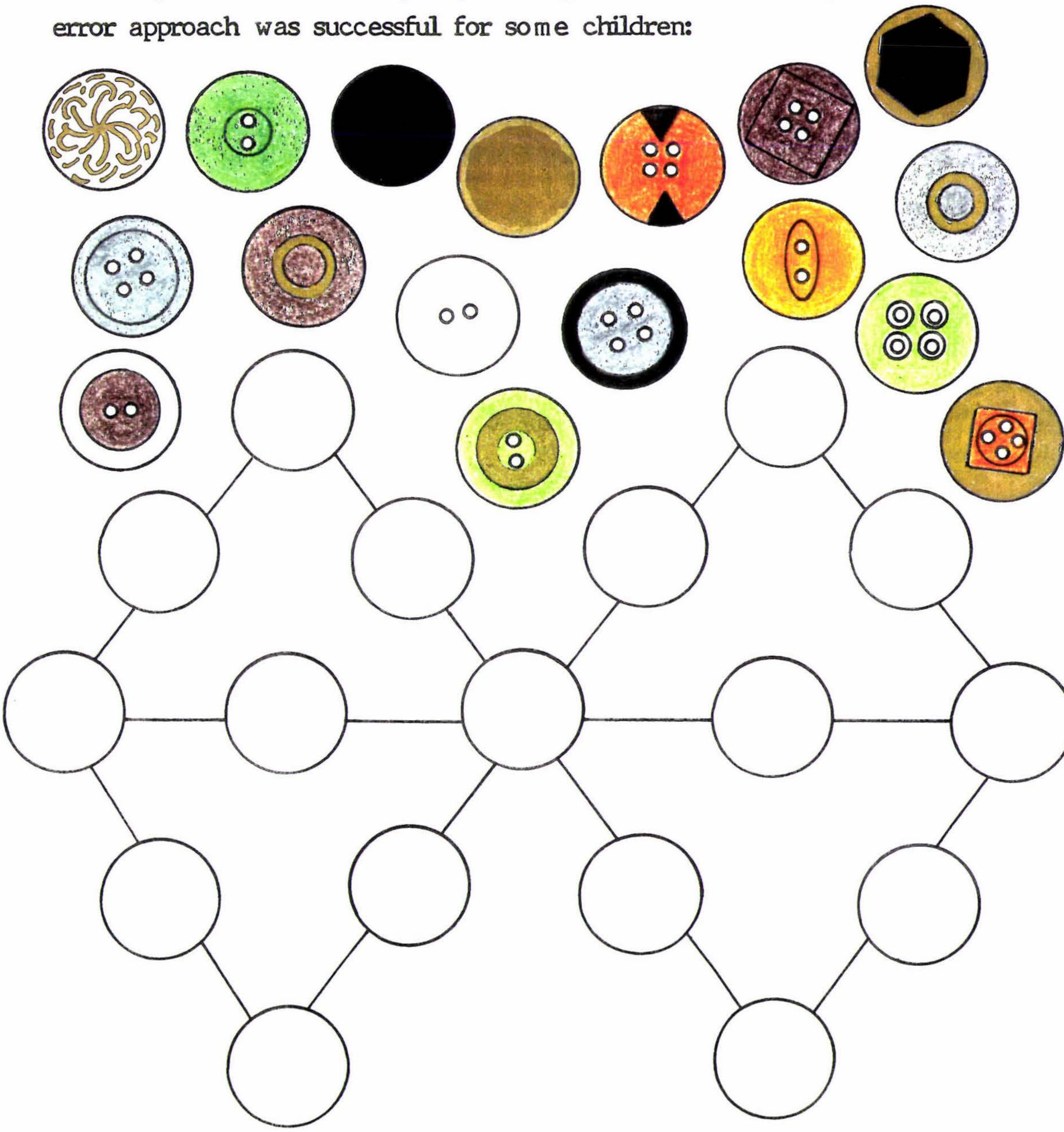


Figure 40: One Colour Difference Button Figure Eight:
Verbal Instructions:

- Arrange the buttons with one colour connection and no more than one colour difference between any buttons you place in connecting circles.
- The button you place in the centre must have a colour relationship with every button circle it is connected to.

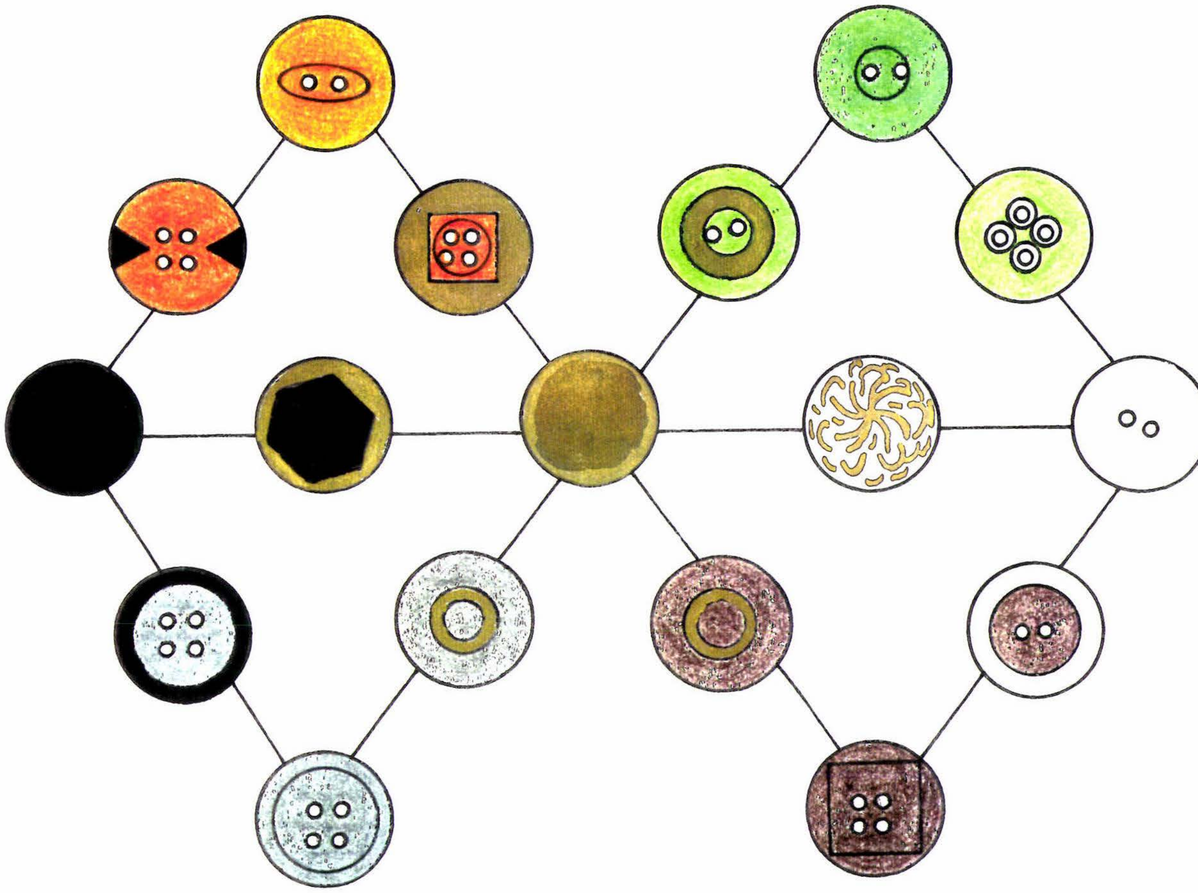


Figure 40a: One Colour Difference Button Figure Eight:

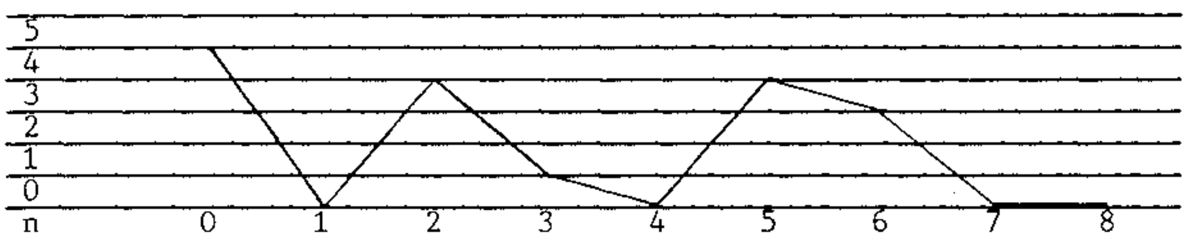
Table 18 shows correlation coefficients between levels of performance and mathematical groups, school class levels, and ages.

Table 18: Spearman's Rho Correlation Coefficients:
Figure 40: One Colour Difference Button Figure Eight:
Levels of Performance: n = 17.

Across mathematical groups	$r_s = 0.034$	$p = >0.05$
Across school class levels	$r_s = 0.237$	$p = >0.05$
Across ages	$r_s = 0.237$	$p = >0.05$

The time it took children to complete this puzzle varied enormously. Two SE 2 children completed this puzzle in 30 minutes each, and the two AV children and one SE 1 child completed it in 5 minutes each. The faster children worked out a strategy of counting the number of circles connecting with the central circle, and then counting the number of buttons any particular colour appeared on. Finding this central button was essential to the ordered rotation in this puzzle.

Figure 40b shows levels of performance across all children who attempted Figure 40:

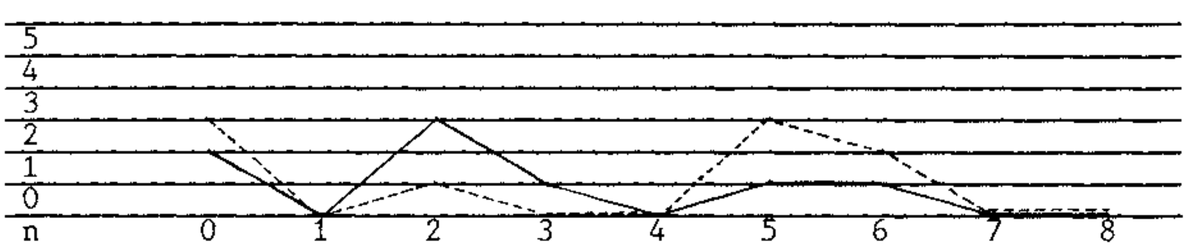


Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 40b: One Colour Difference Button Figure Eight: Levels of Performance. n = 17.

Figure 40c examines gender performance across levels of performance: [Levels of performance as for Figure 40b].



n = 8 ———— Males
n = 9 - - - - - Females

Figure 40c: One Colour Difference Button Figure Eight: Gender Performance Across Levels of Performance. n = 17.

Figure 40d shows levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7						2		1
6								
5					1		1	2
4								
3							1	
2							2	2
1								
0					1		2	2

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 40d: One Colour Difference Button Figure Eight: Levels of Performance Across Mathematical Groups. n = 17.

Figure 41, an above average senior puzzle was only attempted by one 10 year old AV girl who completed it after a lot of rearranging:

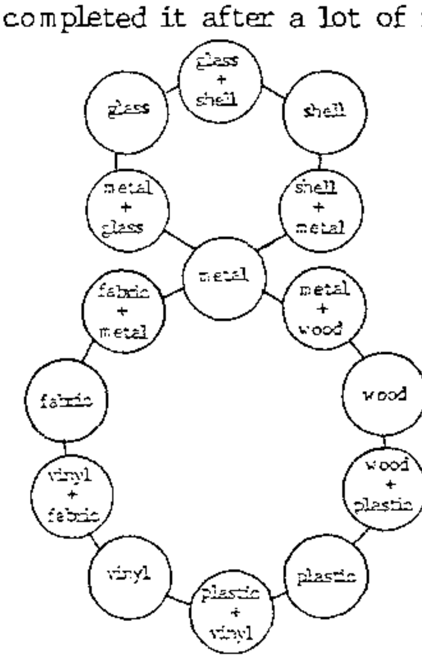


Figure 41: One Material Difference Button Figure Eight:

Figure 42, an above average senior puzzle, also required a particular order, often involving children in much rotational rearranging before the correct intersections could be found. The clear naming of sets was essential to the effective ordering of elements in this puzzle.

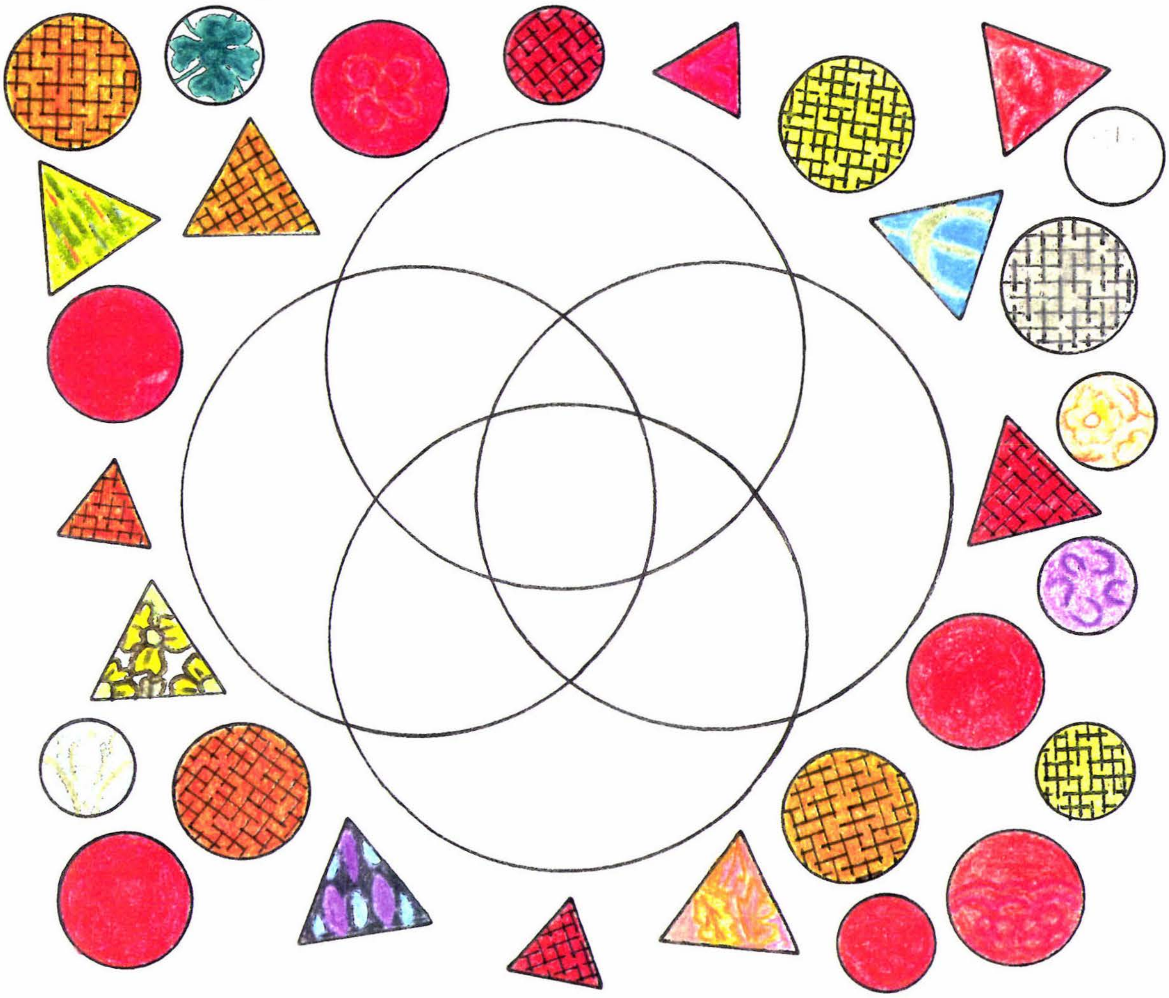


Figure 42: Venn Diagram: Four Intersecting Wallpaper Sets:
Verbal Instructions:

- Find four sets.
- One set is a colour set.
- One set is a size set.
- One set is a pattern set.
- One set is a shape set.
- Name each set.
- Place each set in its own circle.
- Some pieces belong to more than one set. Place these pieces where the sets intersect each other in the intersecting circles.

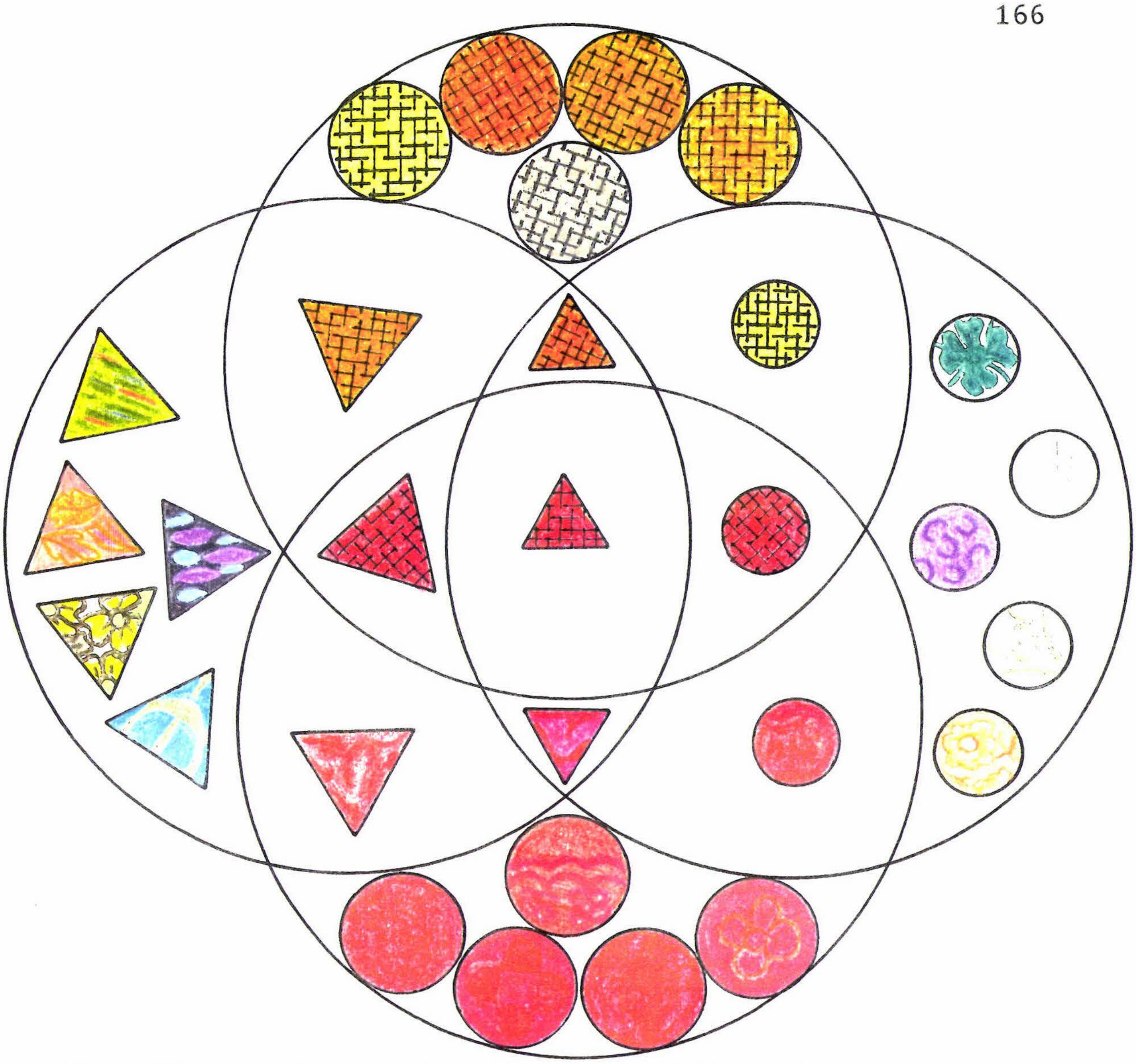


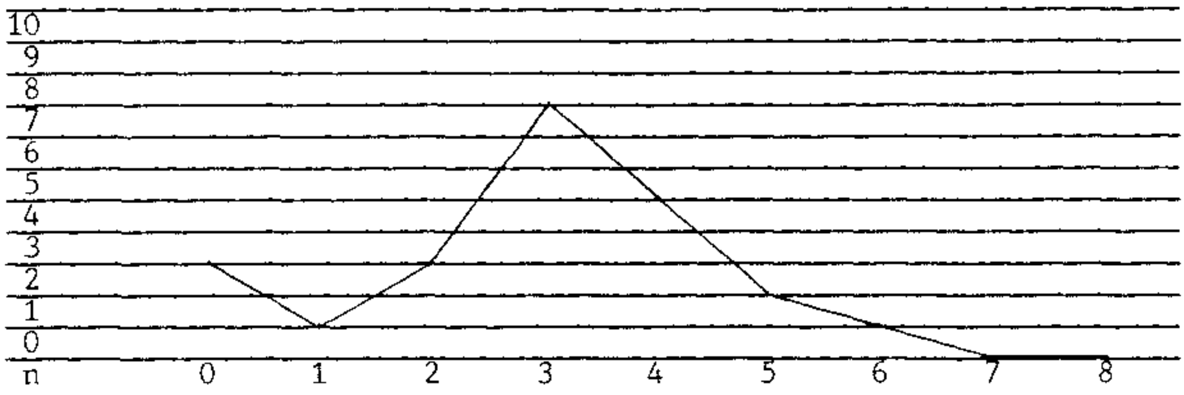
Figure 42a: Venn Diagram: Four Intersecting Wallpaper Sets: Red / Small / Pattern / Triangle.

Table 19 shows Figure 42 correlation coefficients between levels of performance and mathematical groups, school class levels, and ages.

Table 19: Spearman's Rho Correlation Coefficients: Figure 42: Venn Diagram: Four Intersecting Wallpaper Sets: Red / Small / Pattern / Triangle. Levels of Performance: n = 23.

Across mathematical groups	$r_s = 0.140$	$p = >0.05$
Across school class levels	$r_s = 0.147$	$p = >0.05$
Across ages	$r_s = 0.204$	$p = >0.05$

Figure 42b shows levels of performance across all children who attempted Figure 42:



Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 42b: Venn Diagram: Four Intersecting Wallpaper Sets:
Red / Small / Pattern / Triangle.
Levels of Performance. n = 23.

Figure 42c examines gender performance across levels of performance:
[Levels of performance as for Figure 42b].

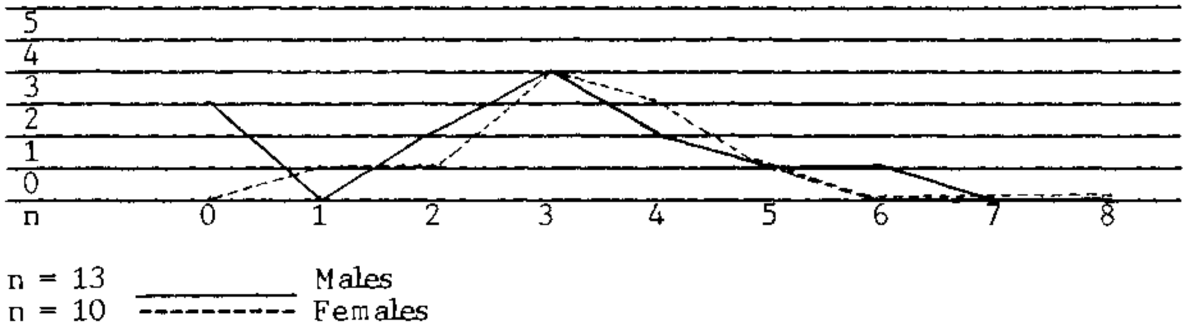


Figure 42c: Venn Diagram: Four Intersecting Wallpaper Sets:
Red / Small / Pattern / Triangle.
Gender Performance Across Levels of Performance. n = 23.

Figure 42d shows Figure 42 levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6							1	
5								2
4			1		1	1		2
3			1				1	6
2						1	1	1
1							1	
0			1					2

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 42d: Venn Diagram: Four Intersecting Wallpaper Sets: Red / Small / Pattern / Triangle.
Levels of Performance Across Mathematical Groups. n = 23.

The 9 year old SE 2 boy with the highest level of performance completed Figure 42 in ten minutes. He did this by clearly naming his sets and by attending only to these named elements without being distracted and confused by other elements such as 'big' and 'circle'.

Figure 43, a very advanced senior puzzle, is a fixed position domino set with movable buttons. Much rotational ordering to intersect sets is involved in this puzzle, made more difficult by the multiple classification concepts incorporated. A 10 year old AV girl completed this puzzle, but found it difficult and required a lot of questioning.

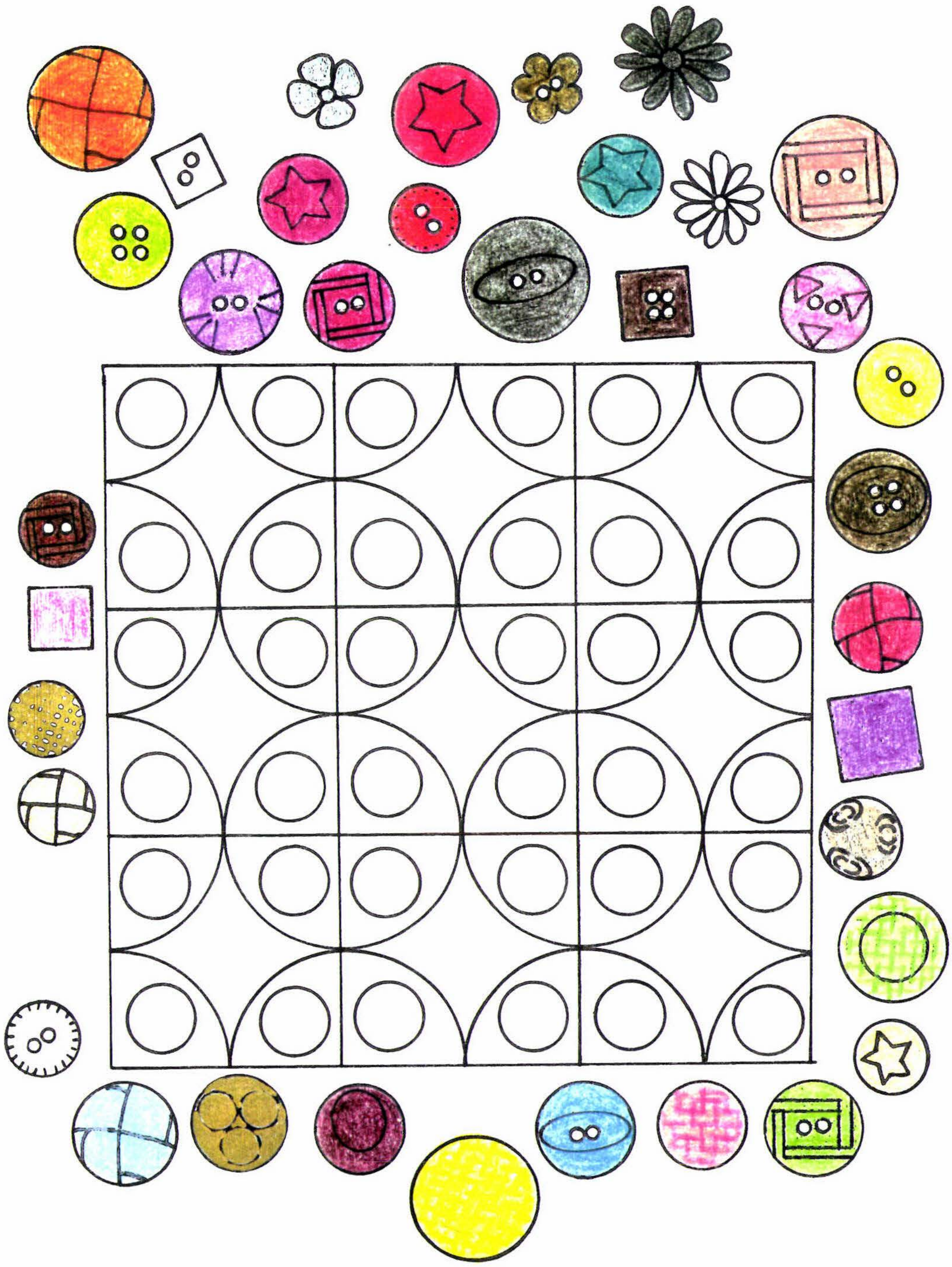


Figure 43: Domino Multiple Concept Button Sets:
Verbal Instructions:

- Find sets of buttons that belong together.
- Each square on the diagram is a set.
- Each circle on the diagram is a set.
- Each half circle [semicircle] and quarter circle on the diagram is part of a set.
- Put each set of buttons in its own circle or square.

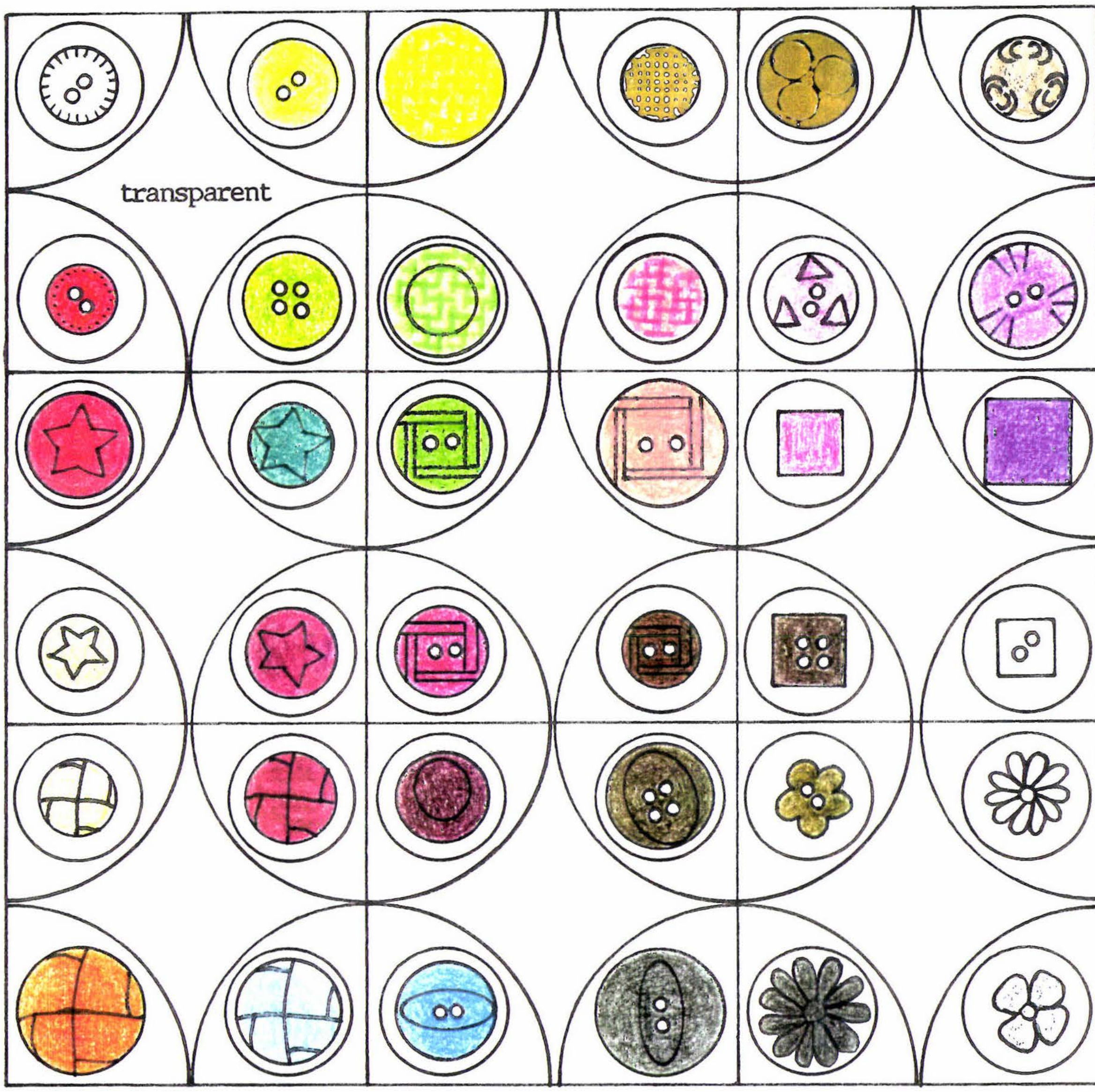


Figure 43a: Domino Multiple Concept Button Sets.

As all square sets are complete, with some circular sets incomplete, it is necessary for the child to count the number of sets of four and the number of incomplete sets to determine which sets belong in the squares and which sets belong in the circles on the diagram.

Increased conceptual performance from a child indicates learning, whereas decreased conceptual performance may indicate tiredness from the conscious focus of attention on multiple concepts, causing one or another concept to progressively slip from attention. Figure 44, a very advanced junior puzzle, is composed of four differently patterned sets of twelve triangles of equal shape and size. Children create their own matching, symmetrical, geometric or tessellated patterns from these. [Plate 36].

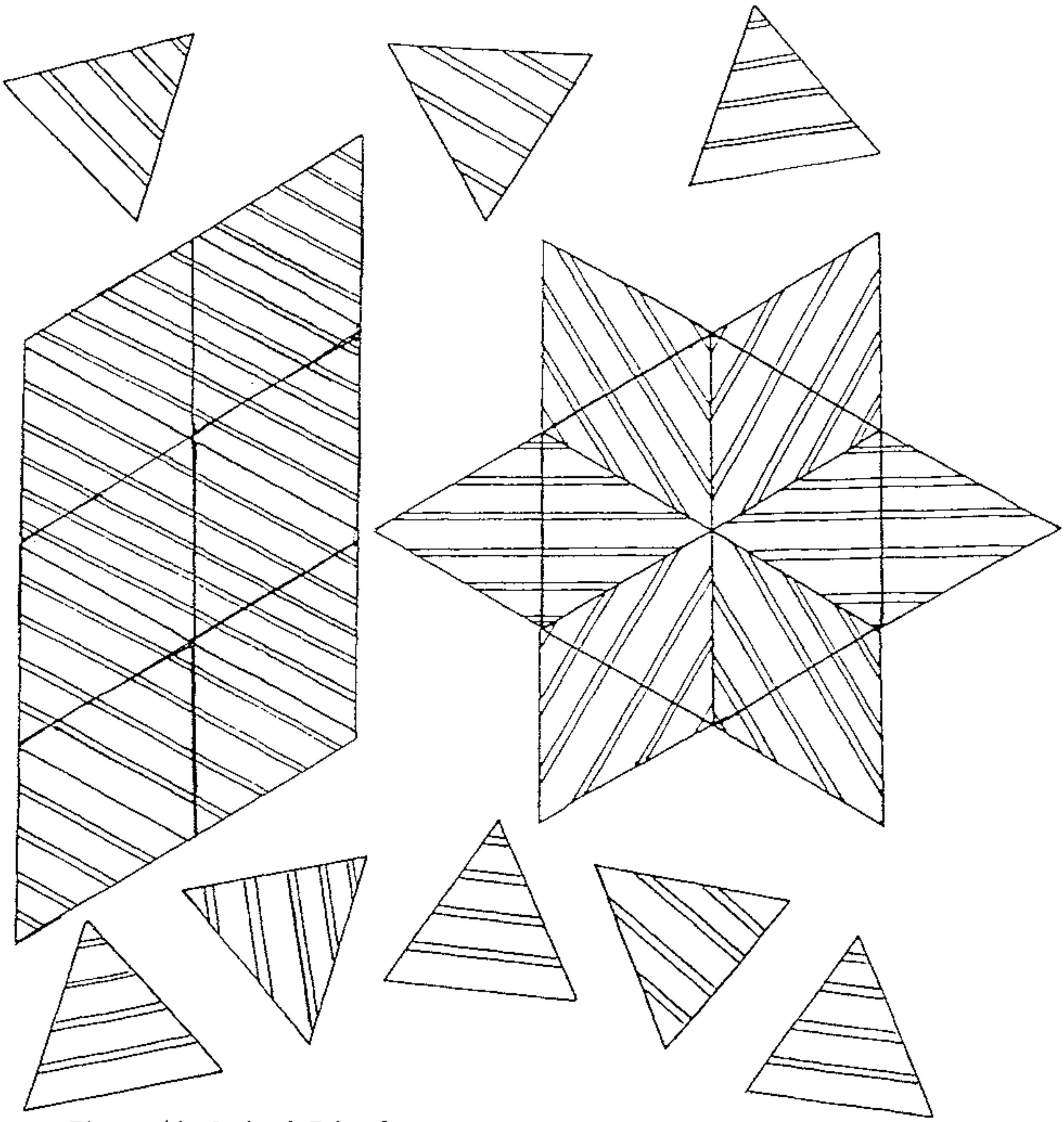


Figure 44: Striped Triangles:
Verbal Instructions:

- Make patterns with the triangles.

Figure 44a shows Figure 44 levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8						1		
7	1				1			
6					1			
5			1					
4						1		
3					1			
2			1					
1					1			
0								

Levels of Performance

0 - No patterns or pattern matching
1 - Hexagons with no pattern matching
2 - Symmetrical pattern matched patterns
3 - Pattern matched hexagon, mismatched hexagons, incomplete hexagons
4 - Pattern matched hexagons
5 - Very fast pattern matched hexagons
6 - Pattern matched hexagons and some stars
7 - Pattern matched hexagons and stars
8 - Pattern matched hexagons, stars and creative symmetrical patterns

Figure 44a: Striped Triangles:
Levels of Performance Across Mathematical Groups. n = 9.

One 9 year old SR boy formed six pattern matched triangles into a hexagon. He followed this with several more mismatched patterned hexagons, and finally with incomplete hexagons. This contrasts with the unexpectedly high level of performance from an 8 year old J-SLD boy who made a haphazard design at first, followed by pattern matched hexagons which he finally turned into stars by adding matched triangles to the outsides of his hexagons.

Consecutive Ordering

Consecutive ordering was another form of multiple concept where children might experience difficulty and exhibit progressive levels of conceptual understanding. Figures 45, 46, 47, 48, 49, 50, 51, 52 and 53 show examples of consecutive ordering in addition to classification. Some of these puzzles form an array or matrix with an ordered pattern or progression both horizontally and vertically in rows and columns. Figure 45, a beginning senior puzzle, may be completed horizontally or vertically: top to bottom, or left to right:

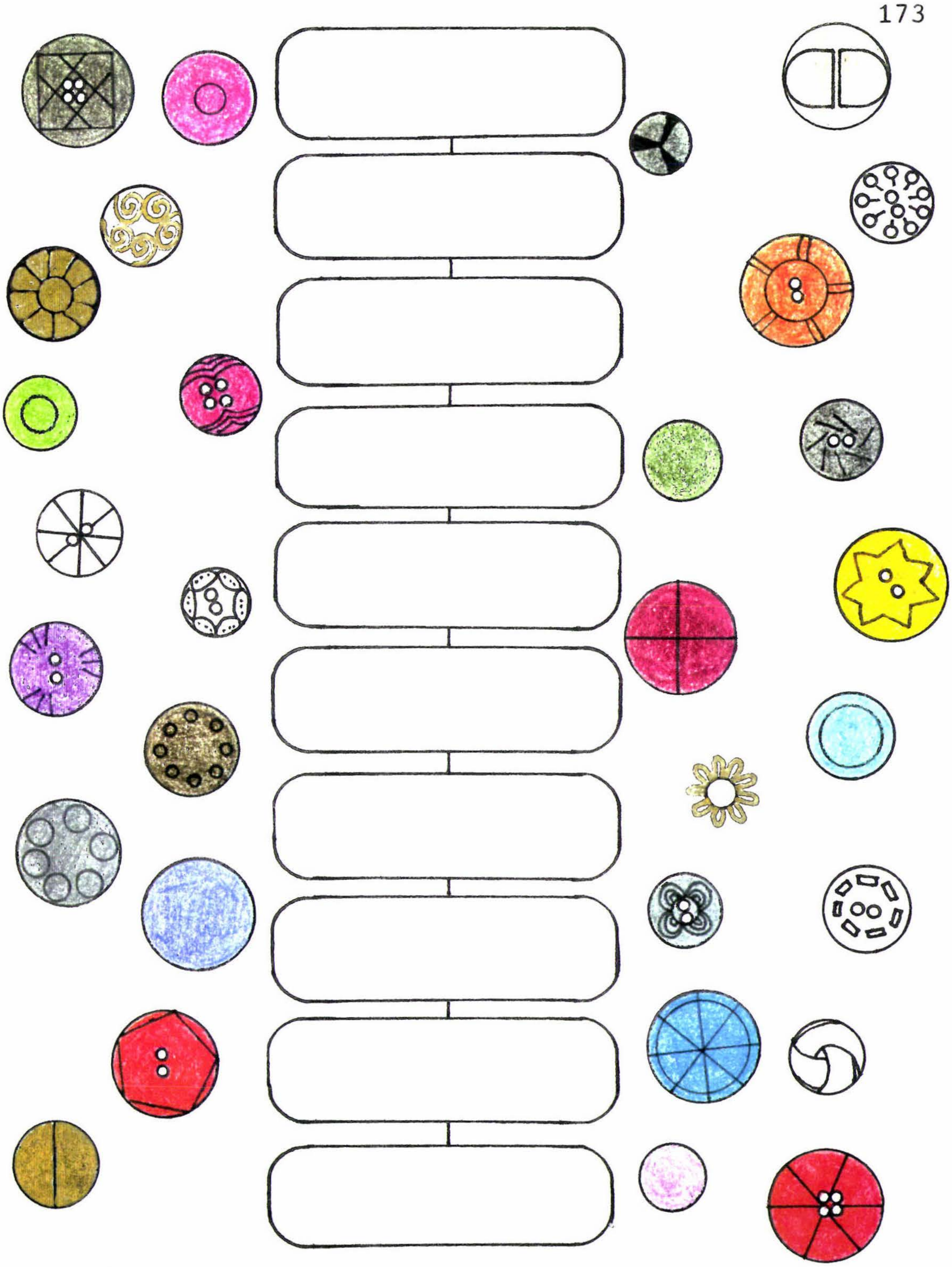


Figure 45: Button Sets:
Verbal Instructions:

- Find buttons that belong together.
- Arrange button sets on the diagram.
- There is a horizontal relationship between buttons across rows.
- There is a vertical relationship between buttons down columns.

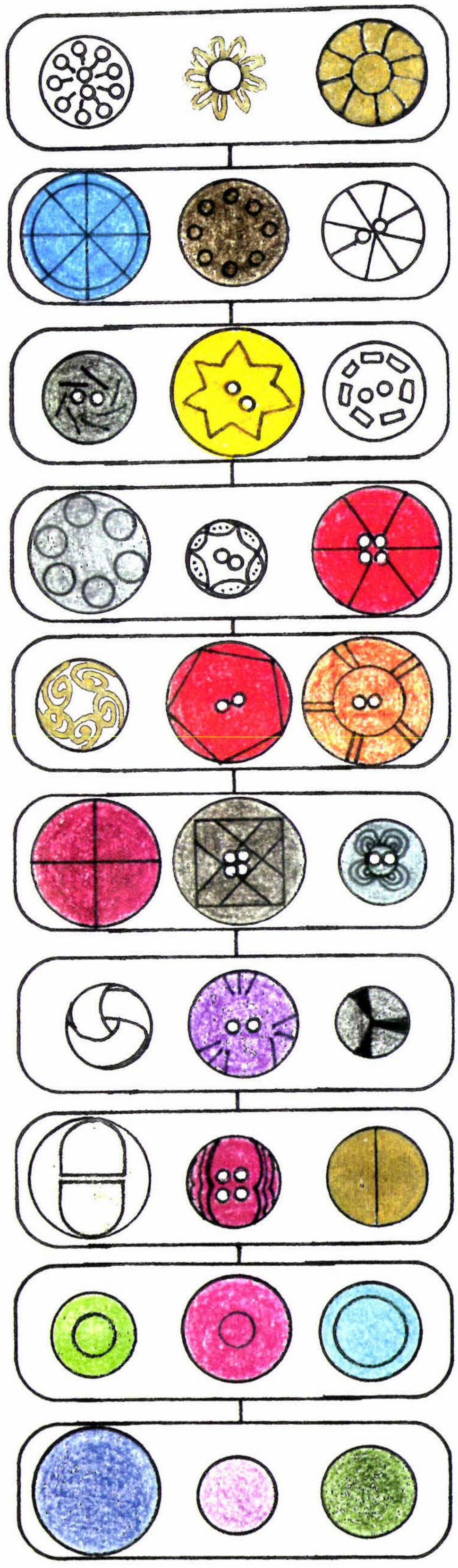


Figure 45a: Repeated Patterns Cardinal and Ordinal Button Sets.

Figure 45b shows Figure 45 levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6								
5						1		
4								
3					1	1		
2								2
1								
0								

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 45b: Repeated Patterns Cardinal and Ordinal Button Sets: Levels of Performance Across Mathematical Groups. n = 5.

One 9 year old SE 1 boy used a strategy by first counting his buttons and dividing this number by the number of rows in the diagram to find how many buttons he could expect there to be in each set. He then tried to classify the buttons according to colour. When this did not work he tried matching the buttons according to their pattern types. He finally classified them according to the number of repeated patterns the buttons contained, and after a lot of rearranging, ordered the sets on the diagram correctly.

Figure 46, an average senior puzzle, contrasts with Figure 45. In Figure 45 there are no colour relationships, and ordering occurs between rows with cardinal horizontal relationships within rows, and ordinal vertical relationships between rows. In Figure 46 there are colour relationships, and ordering occurs within rows with ordinal horizontal relationships within rows, and cardinal vertical relationships between rows.

Figure 46: Repeated Patterns Coloured Cardinal and Ordinal Button Sets

Verbal Instructions:

- Find buttons that belong together.
- Arrange button sets on the diagram.
- There is a horizontal relationship between buttons across rows.
- There is a vertical relationship between buttons down columns.

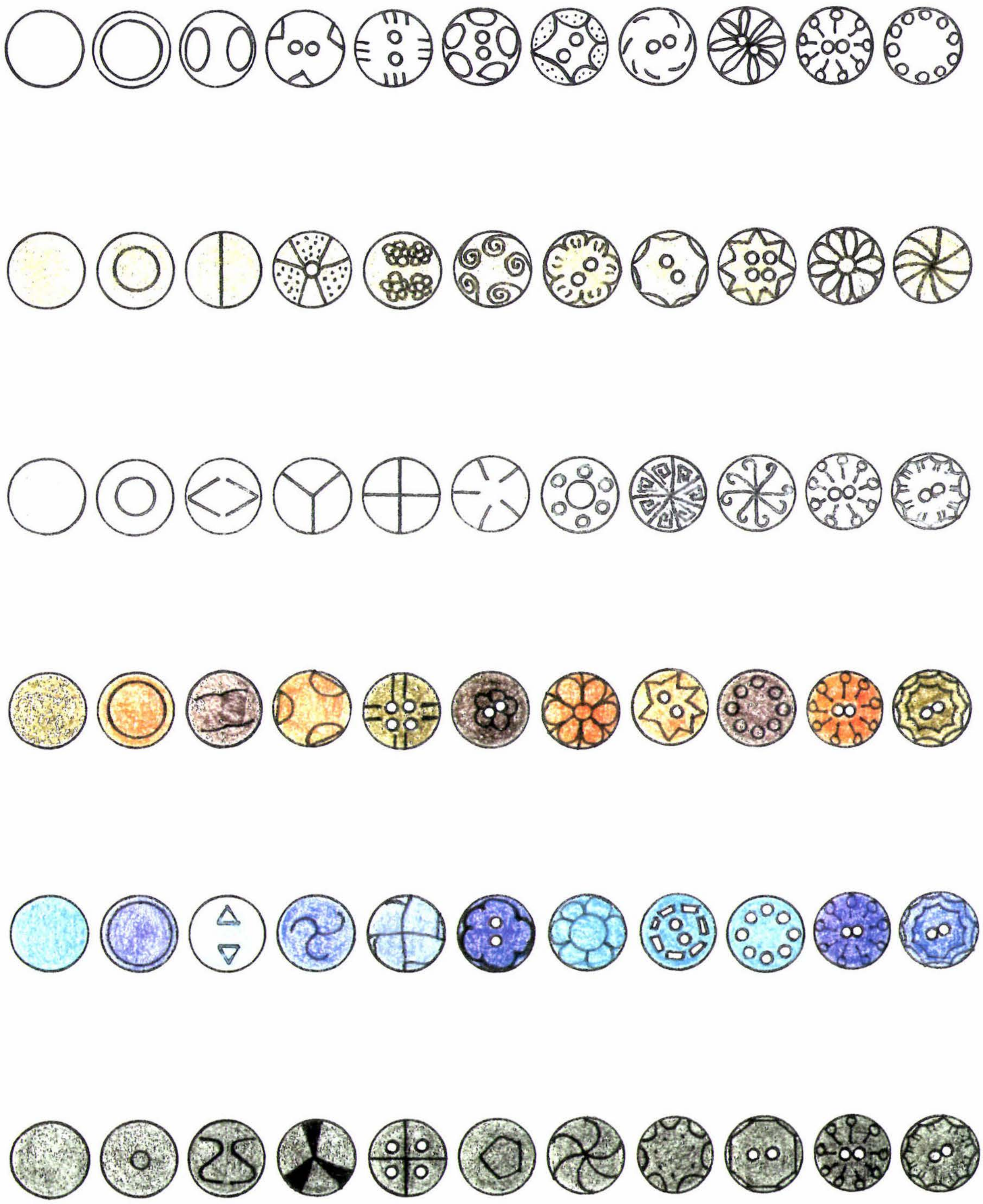


Figure 46a: Repeated Patterns Coloured Cardinal and Ordinal Button Sets.

An 8 year old AV girl formed her colour rows, but because she had difficulty counting the numbers of repeated patterns on some of the buttons, some buttons were placed in order and some were not. Her 10 year old AV sister classified the colours and observed the repeated patterns, but needed questioning to initiate ordinal sequencing.

Three further multiple conceptual puzzles involving consecutive ordering included hierarchy triangles. Figure 47, an advanced senior puzzle, involved one perceptual dimension:

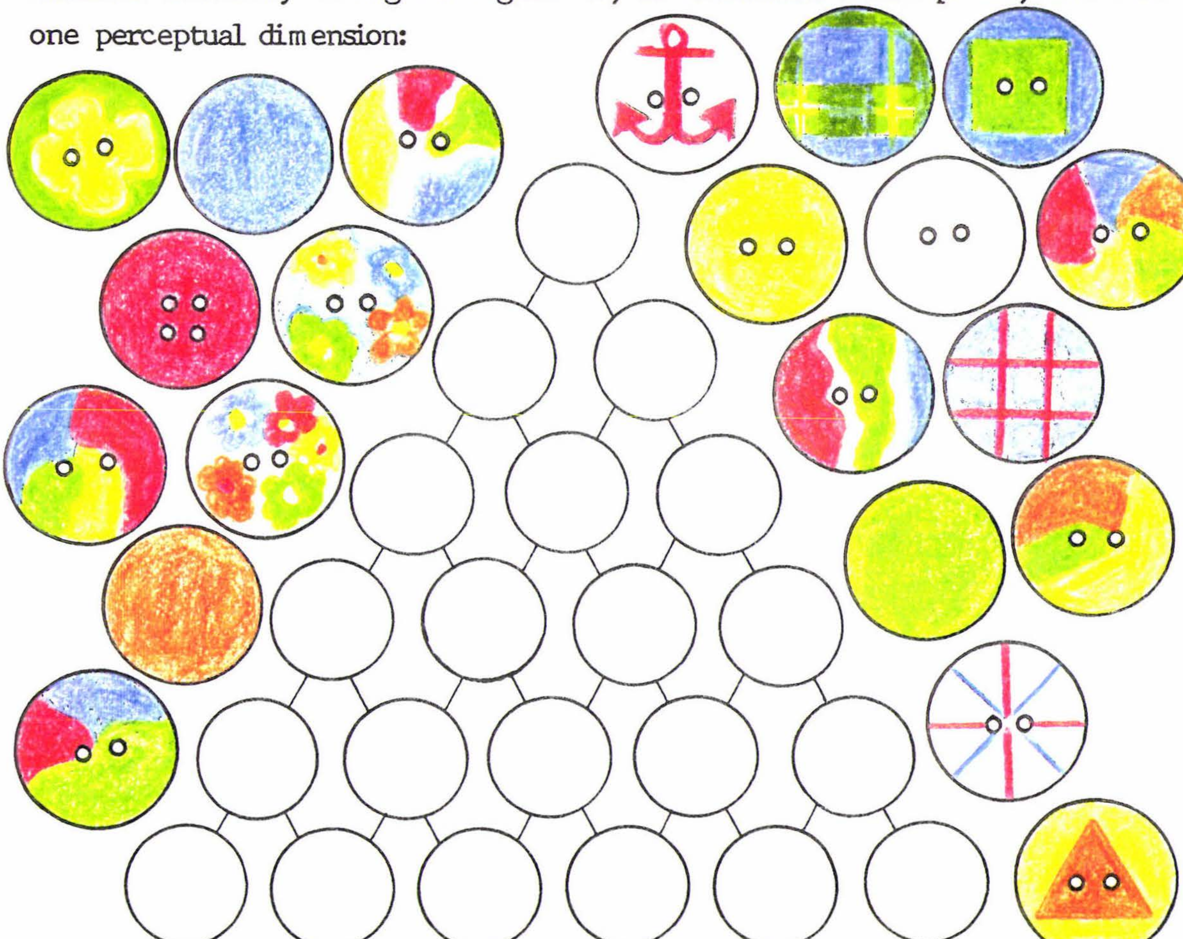


Figure 47: Six Button Sets One Difference Hierarchy Triangle:
Verbal Instructions:

- Find 6 sets. Name your sets.
- Find the 6 buttons that name your sets.
- Place the 6 buttons that name your sets at the bottom of your hierarchy triangle.
- Arrange the remainder of the buttons with no more than one thing different between any buttons you place in connecting circles.

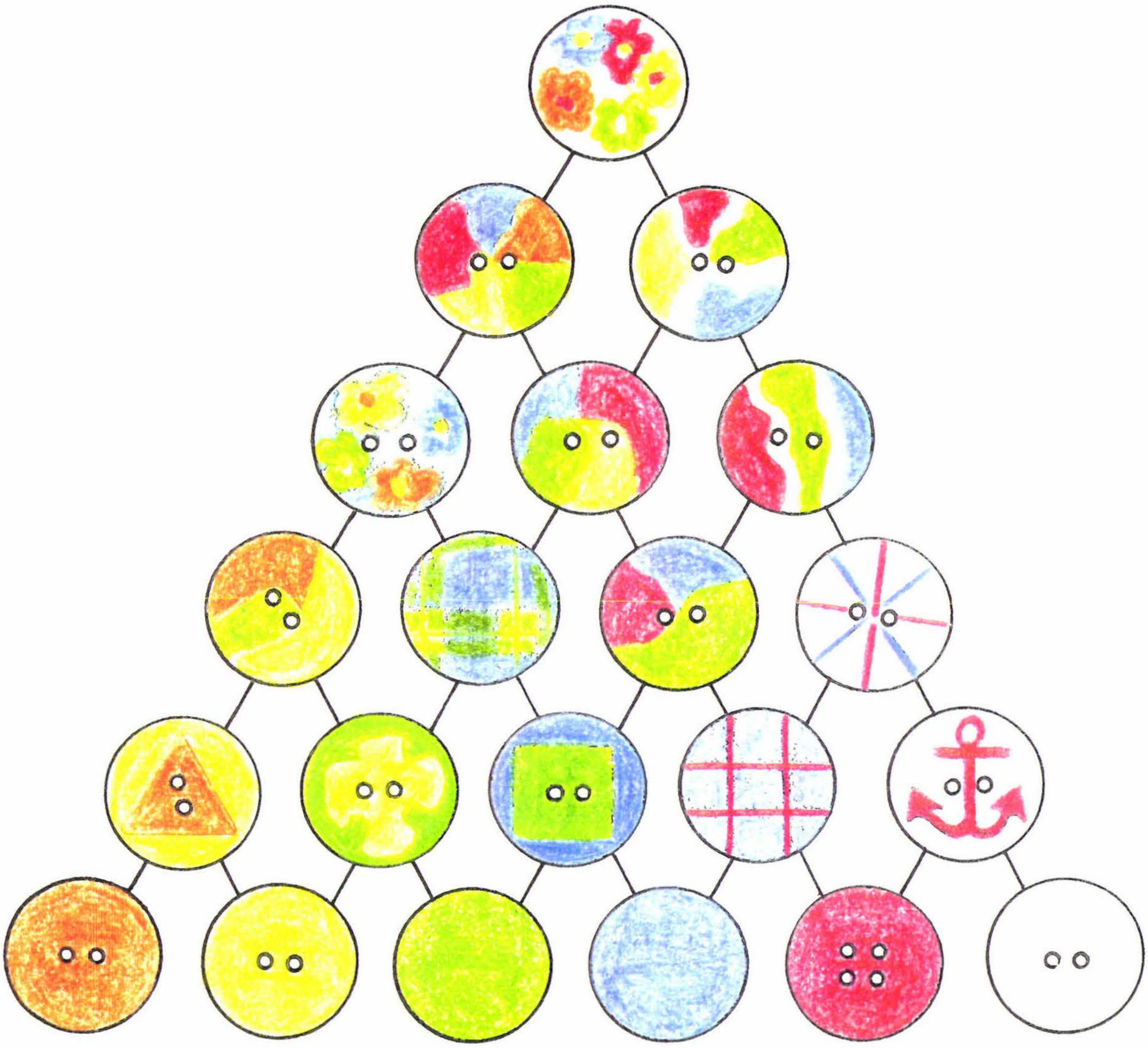


Figure 47a: Six Colour Button Sets One Difference Hierarchy Triangle:

Both AV girls needed questioning to complete Figures 47, 48, and 49. Figure 48, an advanced senior puzzle was another example of a hierarchy triangle involving an additional perceptual dimension:

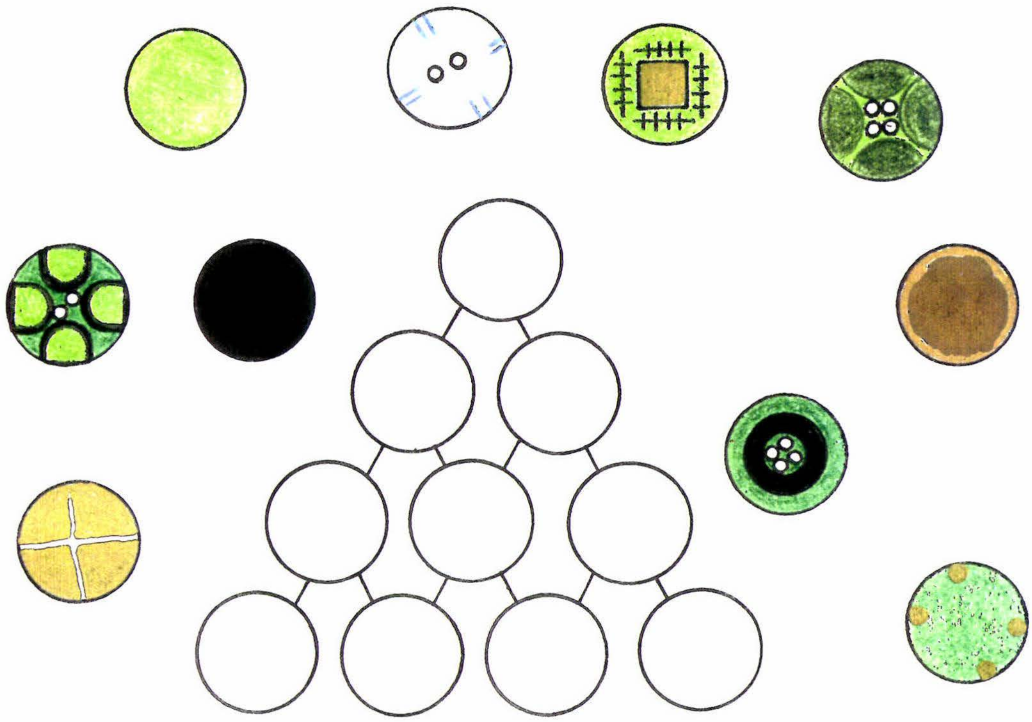


Figure 48: Four Button Sets One Difference Hierarchy Triangle:
Verbal Instructions:

- Find 4 sets. Name your sets.
- Find the 4 buttons that name your sets.
- Place the 4 buttons that name your sets at the bottom of your hierarchy triangle.
- Arrange the remainder of the buttons with no more than one thing different between any buttons you place in connecting circles.

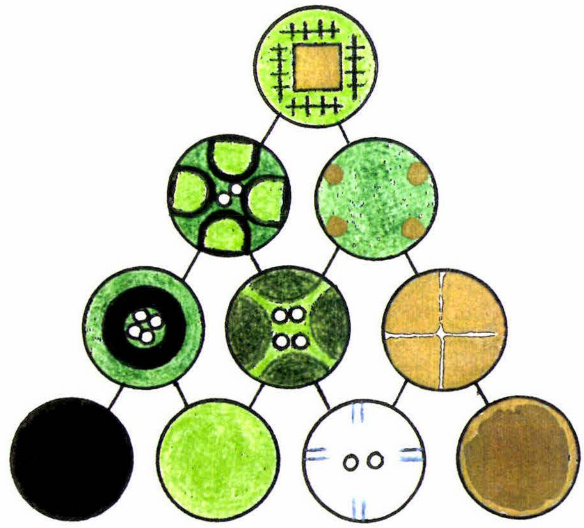


Figure 48a: Four Colour / Repeated Patterns of Four Button Sets One Difference Hierarchy Triangle.

Figure 49, an average senior puzzle, was the third hierarchy triangle:

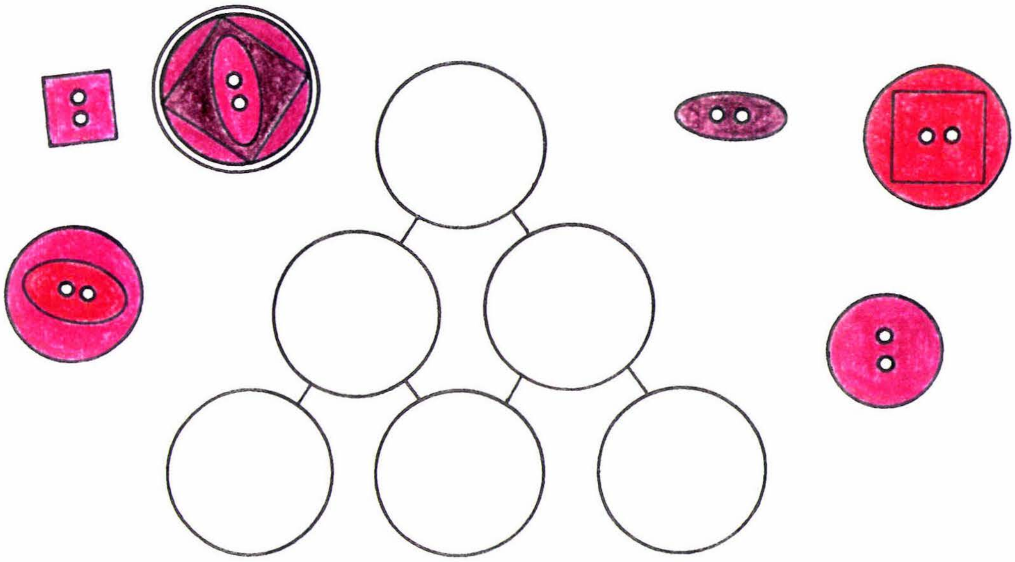


Figure 49: Three Button Sets One Difference Hierarchy Triangle:
Verbal Instructions:

- Find 3 sets. Name your sets.
- Find the 3 buttons that name your sets.
- Place the 3 buttons that name your sets at the bottom of your hierarchy triangle.
- Arrange the remainder of the buttons with no more than one thing different between any buttons you place in connecting circles.

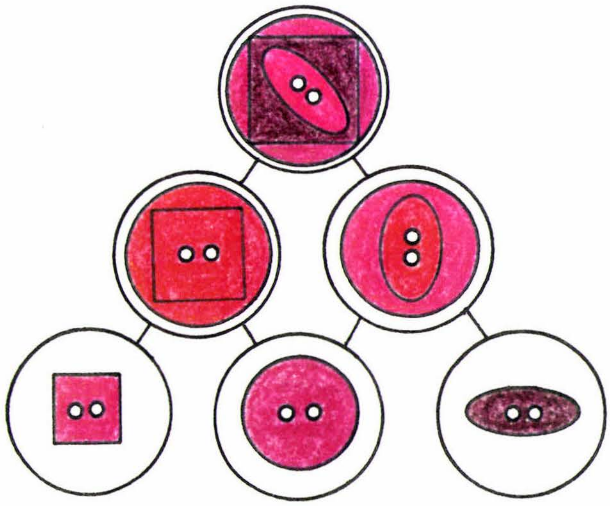


Figure 49a: Three Shape Button Sets One Difference Hierarchy Triangle.

Ordering appears to be a more difficult task for children than classifying, when ordering of sets is an additional task to the initial classification of sets. It requires a conceptual leap for a child to simultaneously and consciously attend to and link these two functions. For cardinal and ordinal sets involving repeated patterns of specific numbers, some children found consecutive ordinal ordering of these classifications was not an automatic progression, despite awareness of the differing numbers of repeated patterns as evidenced by the child's classifications, and the visual assistance towards layout from the design of the diagrams. Counting the numbers of repeated patterns in the higher numbers was difficult for all children. Successful children found a strategy essential for accuracy, of marking a starting point for counting each button. In Figure 45 there were insufficient numbers of subjects to make any correlational interpretations between mathematical group categories and levels of performance, but the two lowest performing children were from the highest mathematical group category, and only one 8 year old AV girl required no questioning at all. Some children became confused by the large number of buttons in Figure 46, finding this puzzle too difficult, despite conceptual understanding.

Consecutive ordering in the hierarchy triangle series of puzzles involved concepts of progressive accumulative addition of elements. The only children to attempt these puzzles were the two AV girls, both of whom required questioning to complete each puzzle. The array concept seemed to be the most difficult aspect of these puzzles.

Arrays

Most of the remaining puzzles in Chapter 5 incorporating consecutive ordering are arrays of various types. These include Figures 50, 52, 53, 55 and 56. The issue of an ordering process following a classification one is evident in performance levels in these puzzles, with the ordering aspect again not being a natural progression for the child, but a separate and distinctive function. For Figure 50, Spearman's Rho correlations between mathematical groups across levels of performance were significant at a 0.05 level of significance, but not for school classes and ages. It may be the high performance of the two AV girls that create this significance. A Mann-Whitney U-Test testing gender differences in levels of performance for Figure 50 shows no significant differences.

Figure 50, a very advanced junior puzzle with multiple concepts requiring consecutive ordering, showed various perceptual difficulties and levels of conceptual understanding:

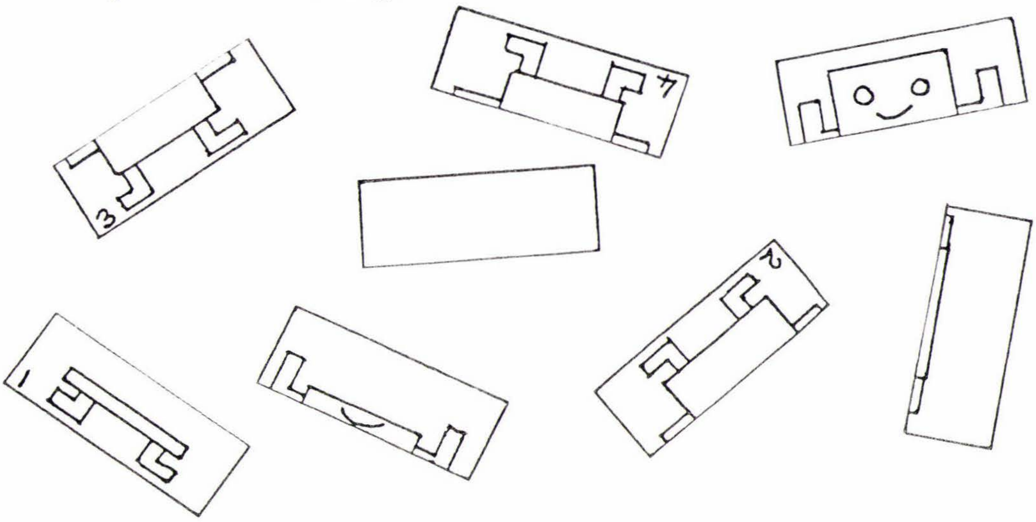
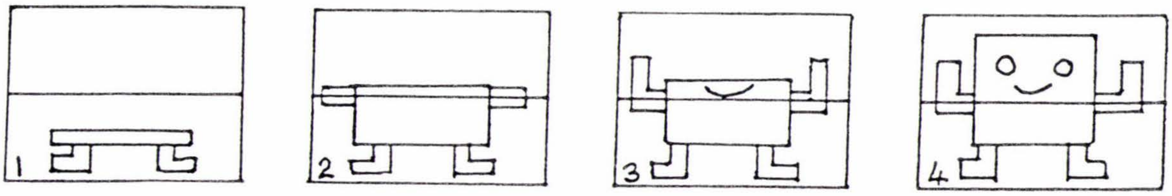


Plate 3 : Figure 50: Lego Ordinal Array:
Verbal Instructions:

- Make Lego models.
- Arrange your Lego models together to make sets.



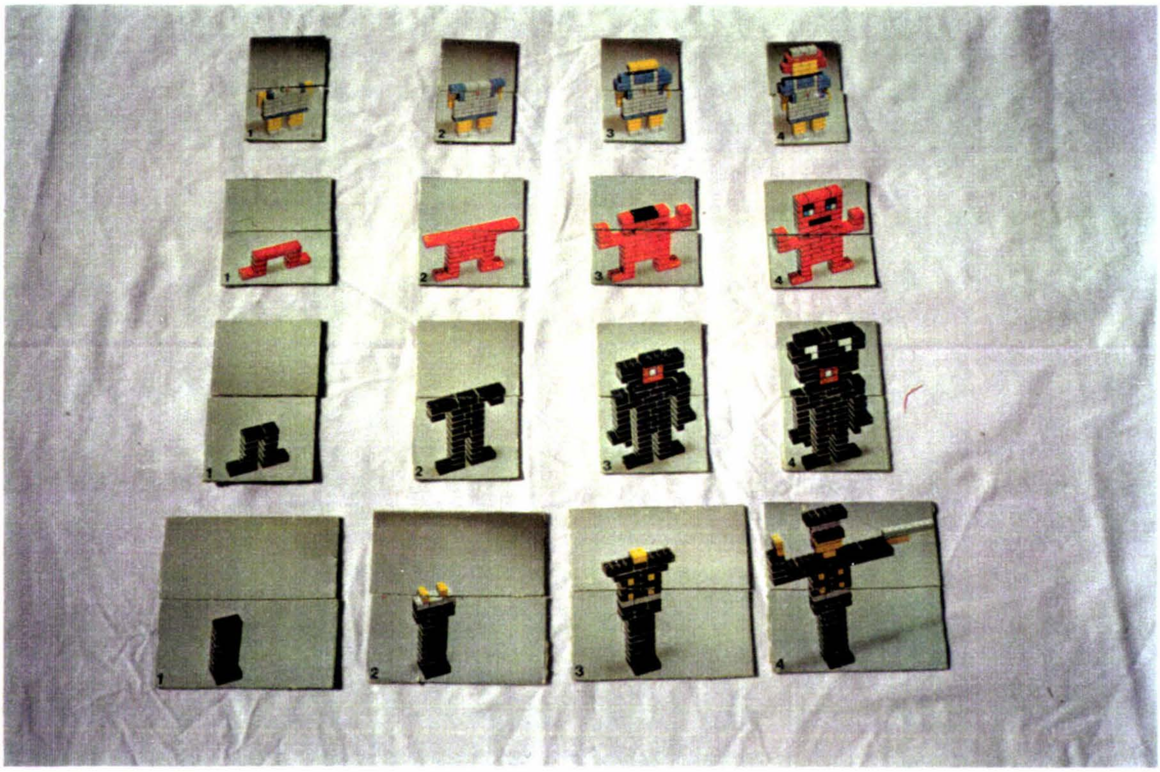


Plate 4 : Figure 50a: Lego Ordinal Array:

A 7 year old JR boy had difficulty classifying the size of the pieces in one model which used the same colour combinations of another model with differently sized pieces. He then had great difficulty ordering these sets. Another 7 year old JR boy mixed models and their sequences. Some children needed questioning to arrange their first model, and were then able to complete other models alone. Some children made arrays with their models, and some children could not find a way of arranging their separately placed models together.

Figure 50b shows Figure 50 levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7						1		
6			1					
5						1		
4			1					1
3		1			1			
2	1	1	1					
1		1						
0					1			

Levels of Performance

0 - Too difficult.
1 - Difficult and incomplete with a lot of questioning.
2 - Difficult completion of models with a lot of questioning.
3 - Completion of models with initial questioning. No array.
4 - Completion of models with no questioning. No array.
5 - Array with some questioning
6 - Array with no questioning
7 - Array with very fast completion
8 - Exceptional or novel performance

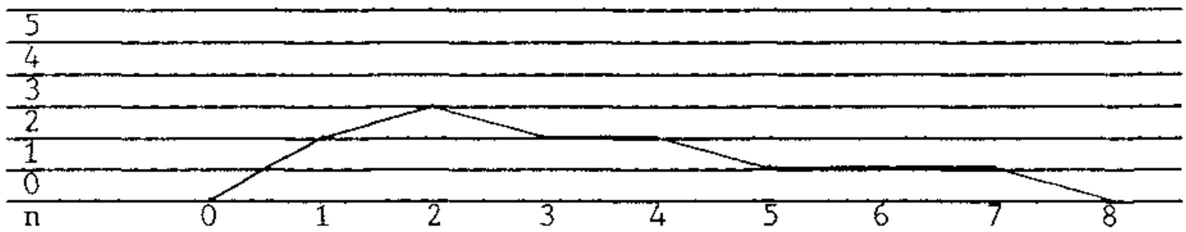
Figure 50b: Lego Ordinal Array:
Levels of Performance Across Mathematical Groups. n = 12.

Table 20 shows Figure 50 correlation coefficients between levels of performance and mathematical groups, school class levels, and ages. There is a correlation between levels of performance and mathematical groups, but not between levels of performance and school class levels or ages.

Table 20: Spearman's Rho Correlation Coefficients:
Figure 50: Lego Ordinal Array:
Levels of Performance: n = 12.

Across mathematical groups	$r_s = 0.554$	$p = <0.05$
Across school class levels	$r_s = 0.337$	$p = >0.05$
Across ages	$r_s = 0.339$	$p = >0.05$

Figure 50c shows levels of performance across all children who attempted Figure 50:



Levels of Performance

0 - Too difficult.
1 - Difficult and incomplete with a lot of questioning.
2 - Difficult completion of models with a lot of questioning.
3 - Completion of models with initial questioning. No array.
4 - Completion of models with no questioning. No array.
5 - Array with some questioning
6 - Array with no questioning
7 - Array with very fast completion
8 - Exceptional or novel performance

Figure 50c: Lego Ordinal Array:
Levels of Performance. n = 12.

Figure 50d examines gender performance across levels of performance:

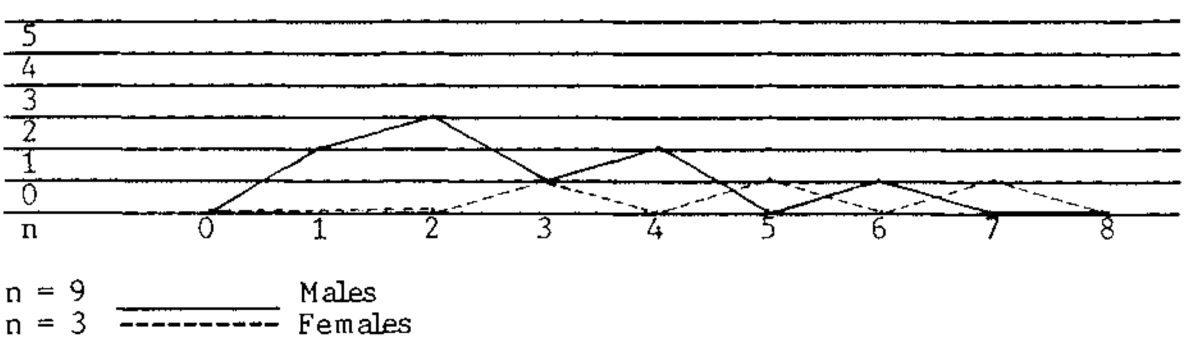


Figure 50d: Lego Ordinal Array:
Gender Performance Across Levels of Performance. n = 12.

Figure 51, a beginning senior puzzle, was another example of a multiple conceptual puzzle involving consecutive ordering causing difficulty. Although these button classifications needed consecutive ordering, there was more than one way of arranging them, and more than one name for one of the sets with a multiple classification.

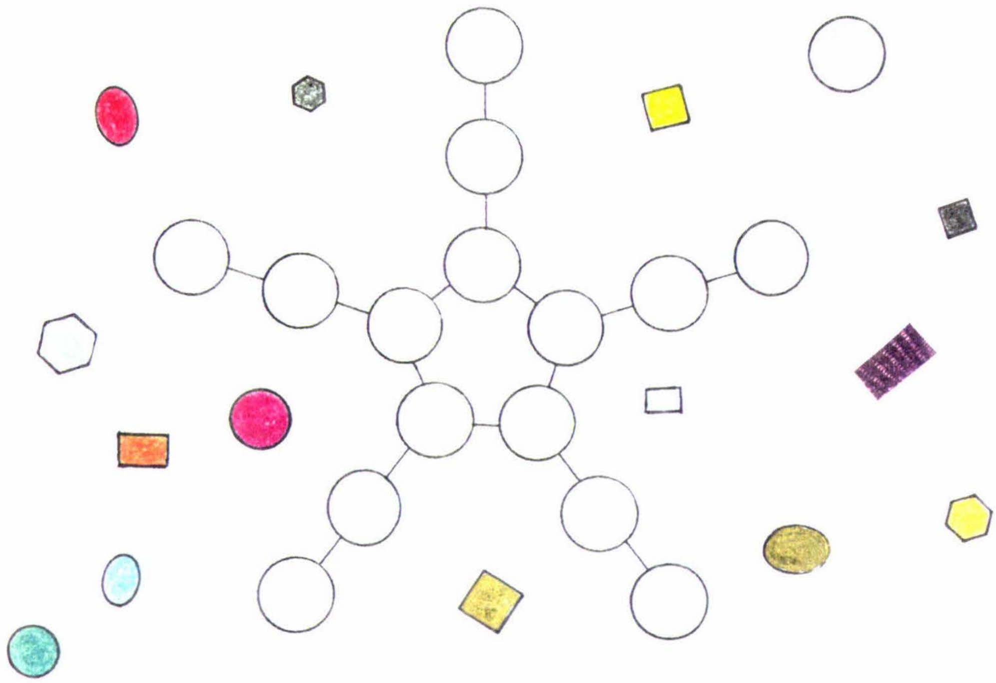


Figure 51: One Difference Multiple Concept Button Arm Set:
Verbal Instructions:

- Find buttons that belong together.
- Name your sets.
- Arrange your sets on the diagram with no more than one thing different between any buttons you place in connecting circles.
- Name each button arm set.
- Name your central set.

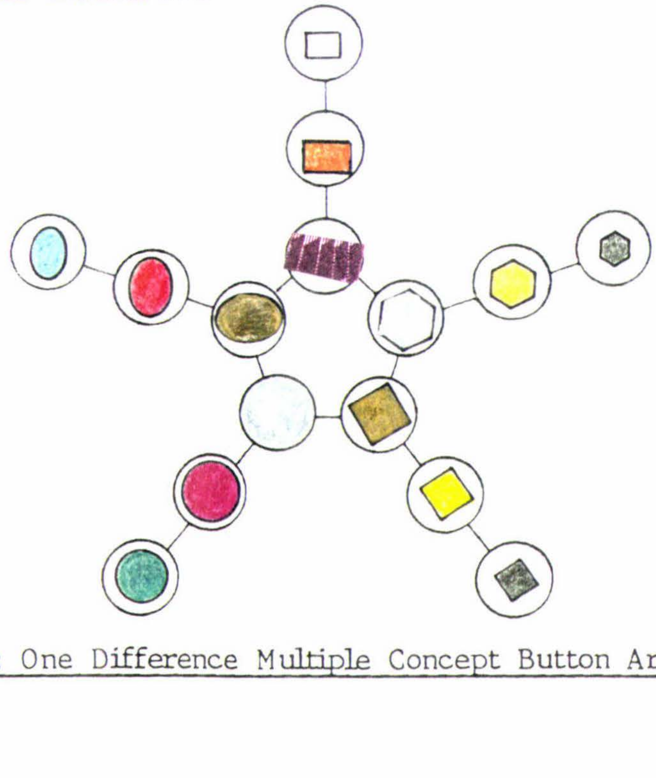


Figure 51a: One Difference Multiple Concept Button Arm Set:

A 10 year old AV girl arranged her shape sets in seriated form with the biggest buttons in the centre, but did not notice her metal set. Her 8 year old AV sister reversed this, placing the smallest buttons in the centre. When I asked her if she could find another central set, she reversed the seriations and called her central set 'big.' I asked if she could find another name for her big set. Only then did she notice it was metal.

Figures 52 and 53, very advanced junior puzzles, were multiple conceptual puzzles requiring consecutive ordering in the form of an array: [Plate 14]

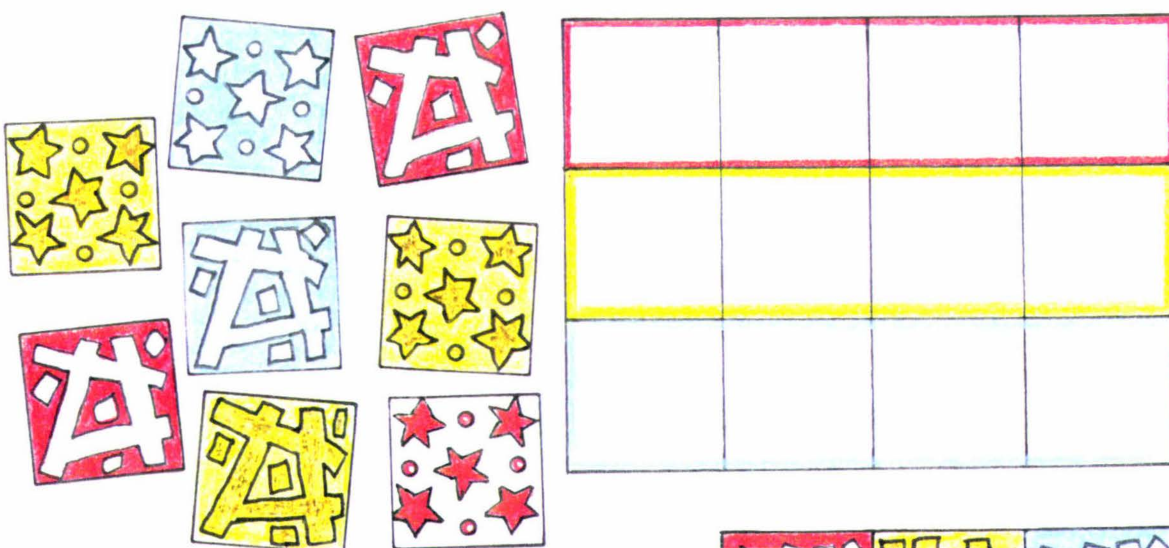
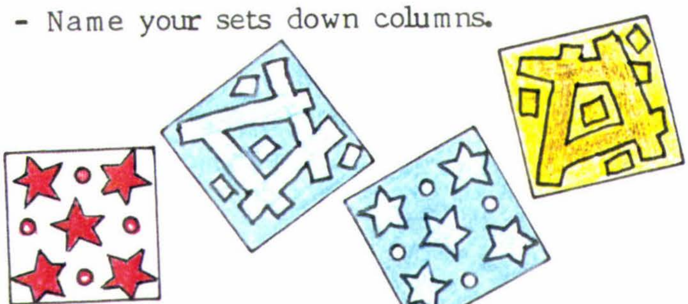


Figure 52: Contact: Colour / Pattern Array: Verbal Instructions:

- Find pieces that belong together.
- Match them together onto the diagram.
- Place sets of pieces together.
- Name your sets along rows.
- Name your sets down columns.



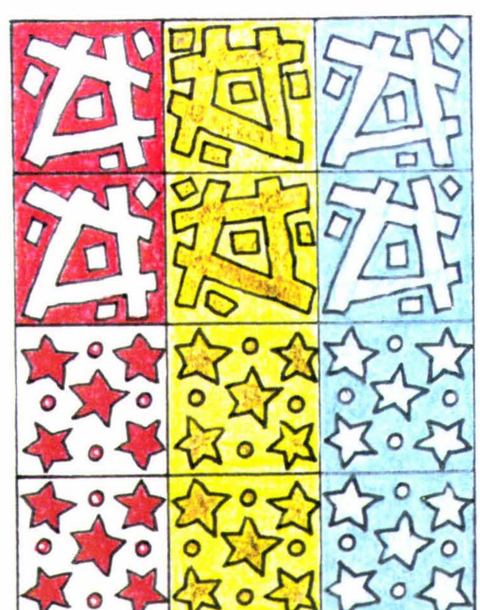


Figure 52a: Contact: Colour / Pattern Array:

Figure 52b shows Figure 52 levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6						2		
5		2	1					
4								
3		1	1					
2								
1								
0		1						

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 52b: Contact: Colour / Pattern Array:
Levels of Performance Across Mathematical Groups. n = 8.

An 8 year old JE boy made a correct array, but had difficulty naming his sets, especially his colour ones. He imaginatively called his star pattern set 'milky way.' Another array was Figure 53: [Plate 19].

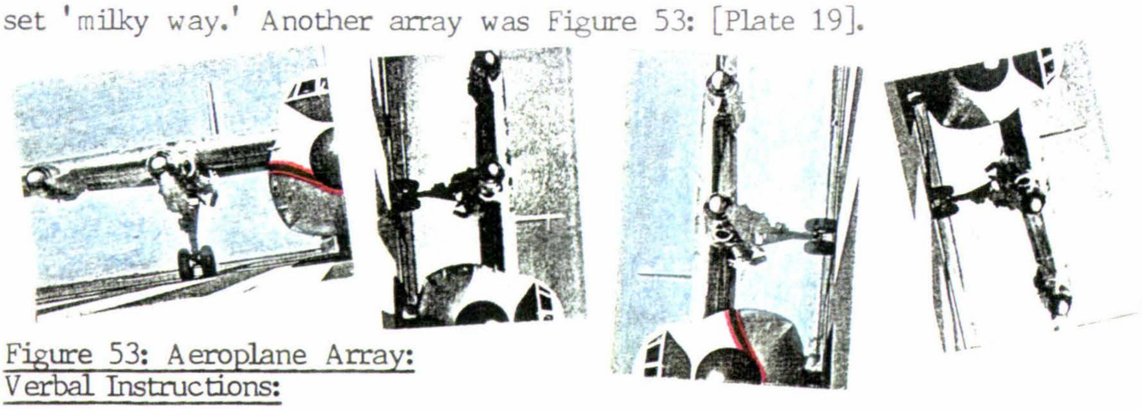


Figure 53: Aeroplane Array:
Verbal Instructions:

- Match your aeroplane halves into aeroplane pictures.
- Place aeroplanes that belong together on the diagram.
- Name each row.
- Name each column.

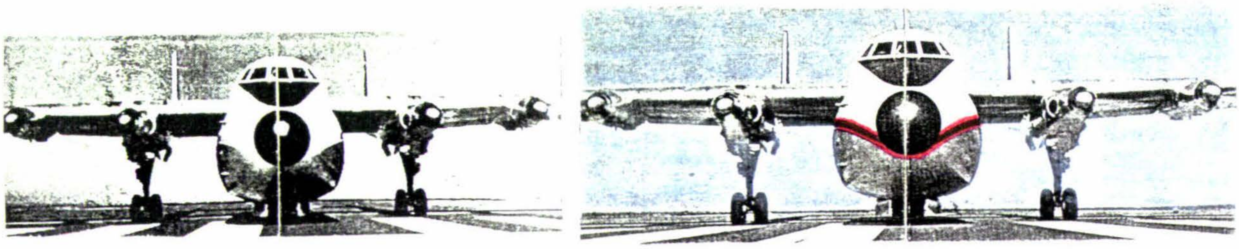


Figure 53a: Aeroplane Array:
Figure 53b shows Figure 53 levels of performance across mathematical groups:

Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8						1		
7			2		1			
6								
5	1	2	1		1			
4		1						
3			1					
2	1							
1		1						
0		3						

Levels of Performance

0 - Too difficult.
1 - Mismatched planes. Mismatched colours.
2 - Correct plane matches. Mismatched colours. No sets of two.
3 - Correctly matched planes. No array.
4 - Difficult completion with extensive questioning.
5 - Complete with some questioning.
6 - Complete with minimal questioning.
7 - Complete with no questioning.
8 - Very fast completion with no questioning.

Figure 53b: Aeroplane Array:
Levels of Performance Across Mathematical Groups. n = 16.

Sometimes dealing with multiple concepts or difficult perceptual dimensions may create frustration for a child. This happened in Figure 54, a very advanced junior puzzle, where one 9 year old SE 1 boy could not at first see the patterns he was trying to match. He exhibited frustration with several other puzzles as well. He had not been in New Zealand long, and had not come from a school system where he had been given many applied problems. He was accustomed to being outstanding in all of his school work, and became upset when he could not do something new.

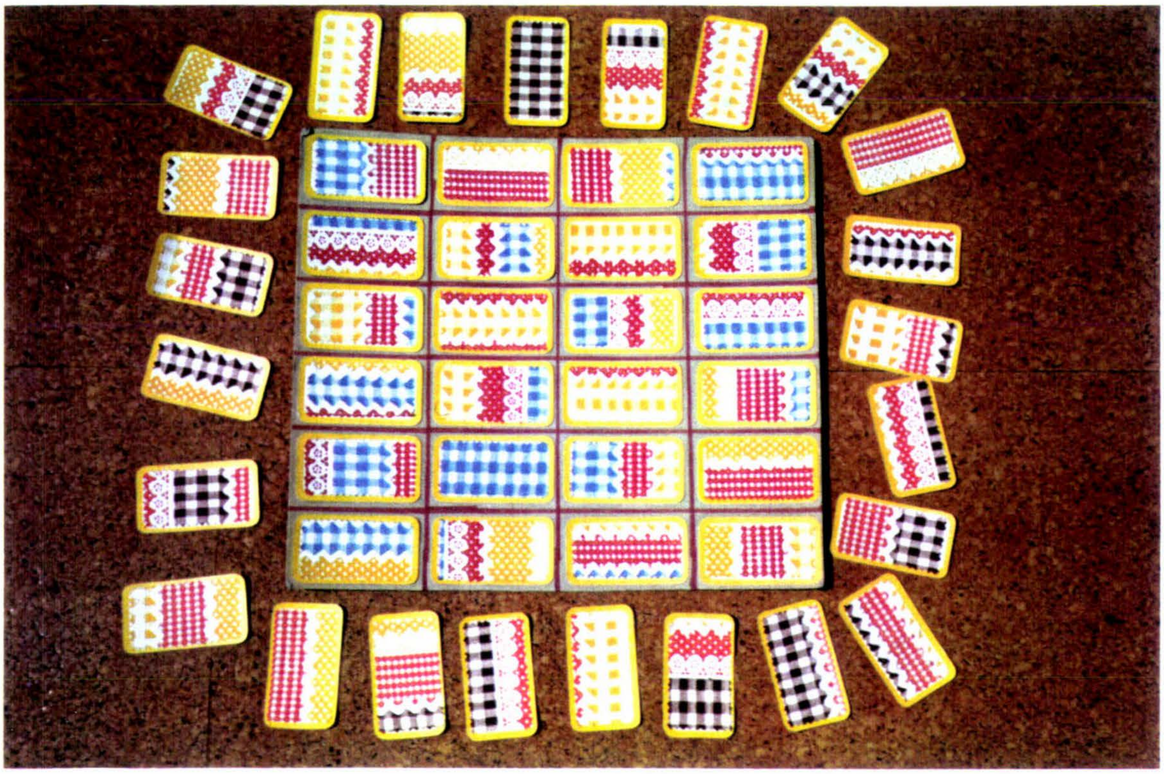
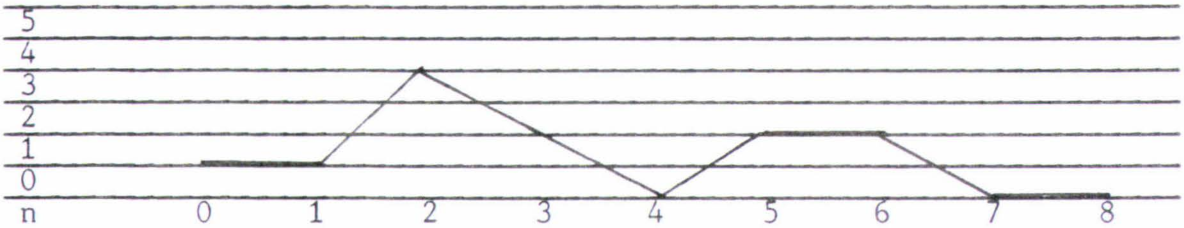


Plate 5: Figure 54: Fabric Pattern Match: Red / Blue - Orange / Brown:. Verbal Instructions:

- Match the patterns.

Figure 54a shows levels of performance for Figure 54:

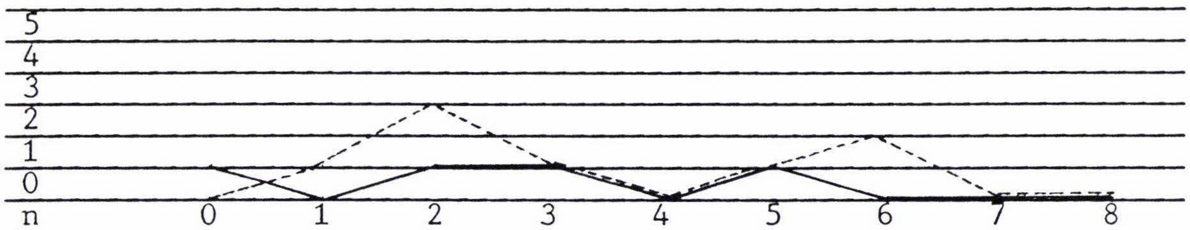


Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 54a: Fabric Pattern Match: Red / Blue - Orange / Brown: Levels of Performance. n = 12.

Figure 54b examines gender performance across levels of performance: (Levels of performance as for Figure 54c). A Mann-Whitney U-Test shows no gender differences in levels of performance.



n = 4 ________ Males
n = 8 ---------- Females

Figure 54b: Fabric Pattern Match: Red / Blue - Yellow / Brown: Gender Performance Across Levels of Performance. n = 12.

Figure 54c shows Figure 54 levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6			1			1		
5			1			1		
4								
3					1			1
2			1		2		1	
1					1			
0		1						

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 54c: Fabric Pattern Match: Red / Blue - Yellow / Brown: Levels of Performance Across Mathematical Groups. n = 12.

Table 21 shows Figure 54 correlation coefficients between levels of performance and mathematical groups, school class levels, and ages.

Table 21: Spearman's Rho Correlation Coefficients:
Figure 54: Fabric Pattern Match: Red / Blue - Yellow / Brown:
Levels of Performance: n = 12.

Across mathematical groups	$r_s = 0.256$	$p = >0.05$
Across school class levels	$r_s = 0.118$	$p = >0.05$
Across ages	$r_s = 0.215$	$p = >0.05$

Incubation of multiple concepts sometimes occurs when a child is unable to solve a puzzle on the first attempt, but has no difficulty on a subsequent occasion. This occurred to the same child in Figures 55 and 56, above average senior puzzles, adapted from a puzzle using flowers and leaves by Z.P. Dienes:

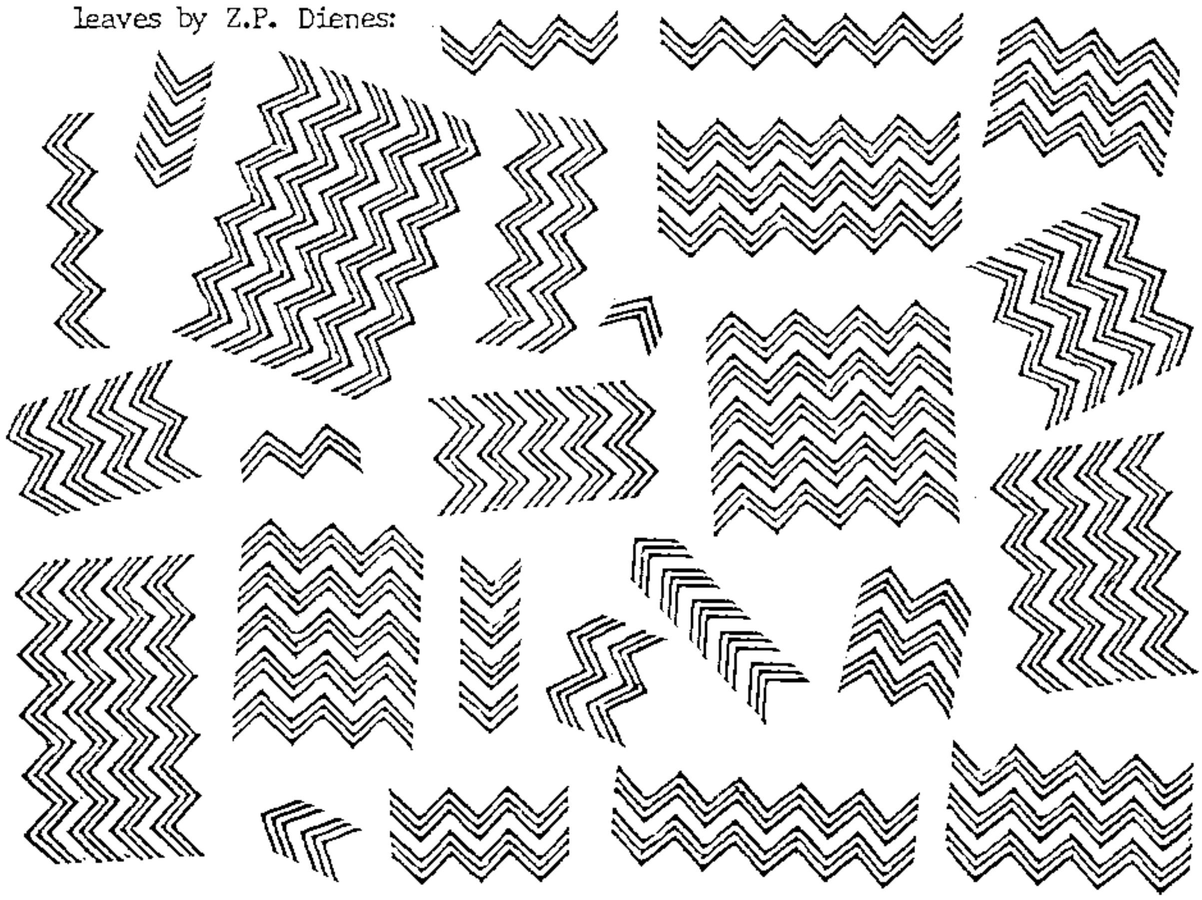


Figure 55: One Difference Zigzag Array:
Verbal Instructions:

- Find pieces that belong together. Name your sets.
- Arrange pieces with no more than one thing different between any pieces you place next to each other.
- Each row is a set. Name it. Each column is a set. Name it.

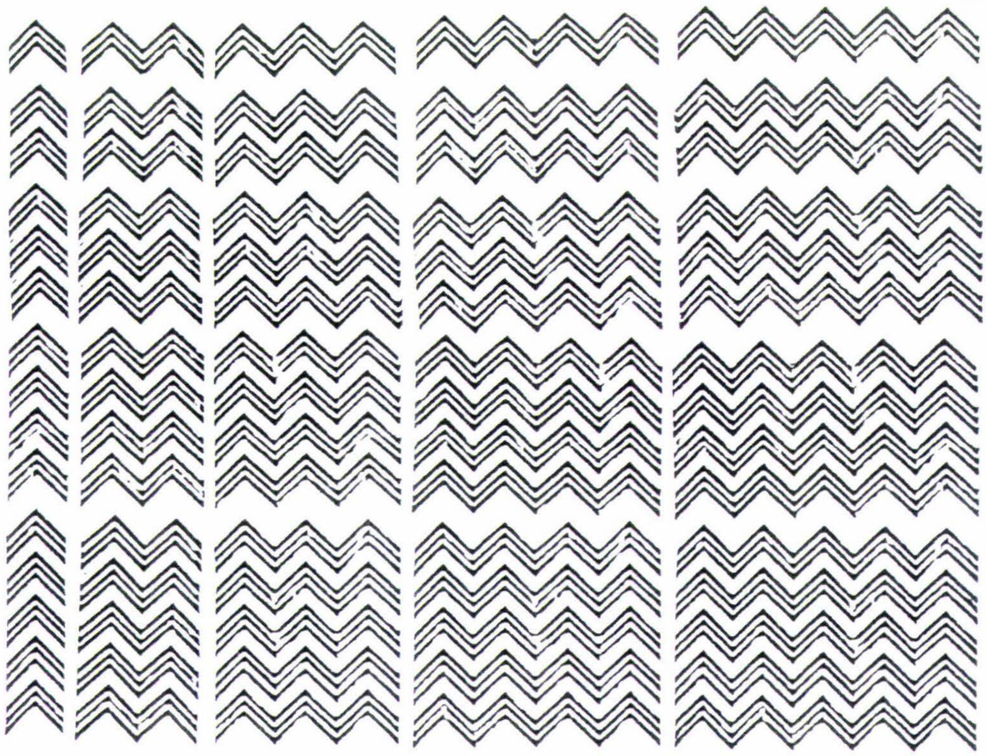


Figure 55a: One Difference Rectangular Cardinal and Ordinal Zigzag Array: Three SE 1 children made a rectangular or irregular shape fitting the pieces together. When questioned about how many items there were on each piece, they were able to start with the smallest piece and order an array. A 10 year old AV girl could not do this on her first attempt, but on her second, completed the array quickly and accurately. She could not solve Figure 56 on her first attempt either: [Instructions as for Figure 55].

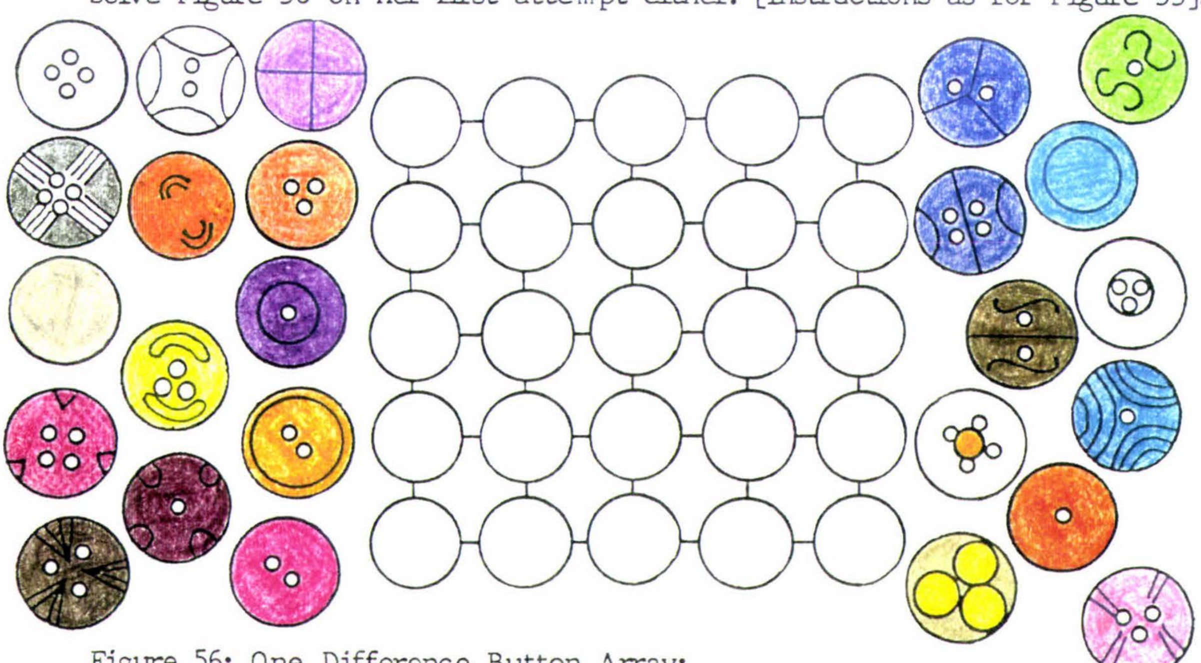


Figure 56: One Difference Button Array:

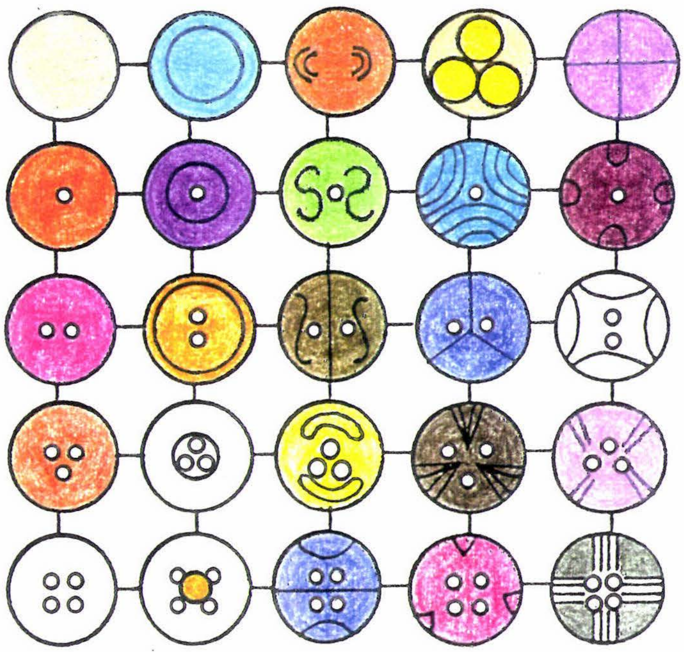


Figure 56a: One Difference Holes / Repeated Patterns Cardinal and Ordinal Button Array:

The 10 year old AV girl classified some buttons according to pattern but not repeated patterns, and she did not notice the holes. I gave her Figure 45 [Repeated Patterns Cardinal and Ordinal Button Sets] and a button holes puzzle which will be discussed in the chapter on diagnostic progressions. She then attempted Figure 56 again, completing it with no questioning or problems. Not only was incubation involved, but also transfer of conscious learning. Her 8 year old AV sister made a holes array, but did not make a holes / repeated patterns array until I asked her to tell me about the pattern on one button.

Multiple concepts may create various difficulties, such as distraction from and by particular perceptual dimensions, inability to maintain alternating patterns, rotational ordering problems, consecutive ordering problems, frustration, incubation, and the need for experience at the single concept level where unexpected unfamiliar concepts are involved, to facilitate conscious transfer of learning.

Table 21A shows two tailed Mann-Whitney U-Tests testing gender differences in levels of performance across mathematical groups for Chapter 5, at 0.05, where U (M) = Males, U1 (F) = Females, and z shows use of normal approximation for larger samples.

Table 21A: Mann-Whitney U-Tests For Chapter 5: Gender Differences: Levels of Performance Across Mathematical Groups:

Figure	U (M)	U1 (F)	z	Males	Females
Puzzles demonstrating distraction elements					
29	12.5	43.5		-	-
Puzzles demonstrating alternating concepts					
32	6	30		-	-
Puzzles demonstrating rotational ordering					
39	75.5	106.5	+ 0.75	-	-
40	31	41		-	-
42	53.5	72.5		-	-
Puzzles demonstrating consecutive ordering					
50	13.5	13.5		-	-
Puzzles demonstrating frustration with multiple concepts					
54	24.5	7.5		-	-

* Significant at 0.05

Difficulties and performance levels did not differ with any significance between genders for any of the Chapter 5 puzzles tested. Even when single concepts within a puzzle are understood, any child may have difficulty sequencing them in progressive conceptual steps. This was particularly noticeable in the multi level arrays such as the aeroplane, lego, and contact arrays. Simple matching within units did not mean a child was able to see the possibility of matching between units. For example, being able to match individual models in the lego array did not mean a child would automatically put the matched models in ordinal sequence, or the ordinal sequences into a horizontal and vertical array. In the aeroplane array, individual aeroplane matching did not necessarily mean a child had an awareness of colour versus non colour, and if this awareness was there, it was not necessarily automatic to arrange matched pairs putting all coloured aeroplanes together and all non coloured aeroplanes together. Similarly, questioning was required for some children attempting the contact array, to place individual colour and pattern matched pairs first into pattern, and finally into colour groups.

The final two cardinal and ordinal arrays, Figures 55 and 56 had a progressive sequence from start to finish. If the basic concept was missed in the beginning stages, the puzzle could not be successfully completed on any level. Sometimes a particular concept needs to be experienced in several different forms before the pattern is consciously recognised by a child and conscious transfer of learning may take place between one puzzle and another.

Summary

There were many difficulties with multiple concepts embedded within puzzles. Much conscious focus of attention was required to pay attention to the patterning of puzzles with alternating concepts, and consecutive ordering in particular. Children found this very difficult, even when the patterning was very regular. Observing the patterning in the first place was the hardest part for most children. This observation acted as a form of strategy for completing the puzzle more quickly. Unless this patterning was firmly understood, children were easily distracted by the multiple concepts they were processing.

Graded puzzle series involving transfer of learning, and puzzles which can and can not be done without the use of a strategy are presented in Chapter 6.

puzzles. This caused distraction and perceptual difficulties, but levels of performance were balanced by the learned and transferred conceptual understanding and processing of the puzzle type. Smaller numbers of children increased their levels of performance, or decreased them, showing respective ability or inability to think independently beyond these variables. Where Venn Diagrams incorporating multiple concepts or major changes of materials were added, levels of performance tended to decrease or undulate for most children, unless the conceptual grasp was consolidated to facilitate improved performance.

Series of puzzles of the same or related types with progressive difficulty levels showed the same mixed trends of increase, decrease, equality and undulation of levels of performance as the Venn Diagram series, and for the same reasons. There was more than one example of a child with an exceptionally high level of performance at a lower difficulty level being unable to transfer obviously understood concepts to a successively difficult puzzle when there were too many multiple conceptual variables to manipulate simultaneously. This was more a feature of younger children, indicating that the quantity of difficult variables as well as the quality of difficult variables was involved in the transfer of learning problem.

Some puzzles required the use of strategy as a thought out in advance step by step method to reduce the process of puzzle solution from a time consuming visual manipulation trial and error approach to a speedy, structured, ordered process requiring less visual checking. Use of strategy, often involving rotation and symmetry, always indicates both advanced conceptual understanding of the nature of a problem, and exceptional ability in the child using it, especially where this occurs with a younger child.

Chapter 7 focuses on puzzles where all of the previous kinds of difficulties were exhibited, from all of the previous chapters. Their broad range of embedded concepts allowed for the observation of much diagnostic information from children.

Chapter 6

Transfer of Learning and the Use of Strategy

Transfer of Learning

Issues of transfer of learning and use of strategy are discussed in this chapter. Whereas transfer of learning may occur unconsciously or consciously, the use of strategy needs conscious application. It is difficult to know how much transfer of learning occurs unconsciously and how much needs conscious attention. Children have shown evidence of preferred perceptual dimensional modes and the processing of familiar concepts both unconsciously and consciously with the ability to verbalise classifications, and of unconscious classification of unfamiliar and unexpected concepts with an inability to verbalise these classifications. It is confusing to consider when the familiar becomes processed unconsciously yet is able to be in the child's conscious awareness, and when the unfamiliar becomes familiar and no longer needs conscious processing, when children are already able to process the unfamiliar unconsciously. Did the 10 year old AV girl use both incubation and transfer of learning, or either of these on their own? A series of three beginning senior Venn Diagram puzzles given to children in a specific order reflects these questions. Figure 57 is given to children first because all of the pieces hold size, shape and shades of colour constant, whereas the Figure 58 and 59 button sets do not hold these elements constant.

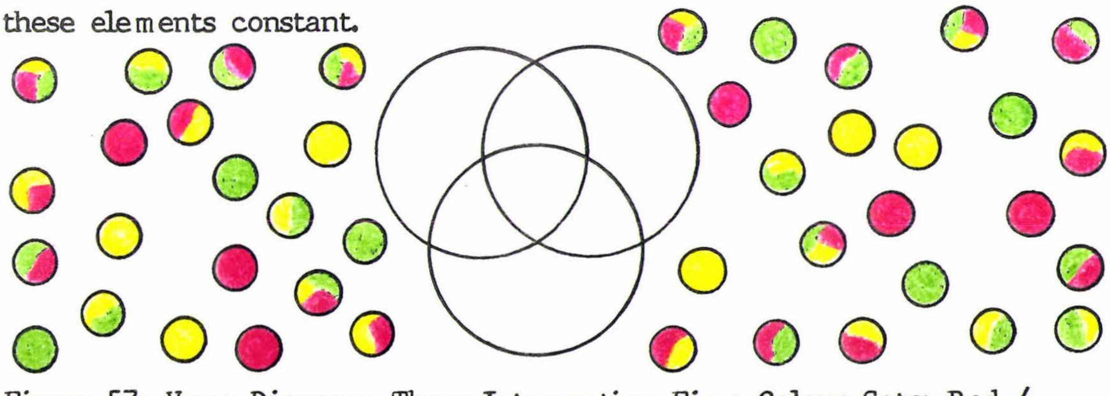


Figure 57: Venn Diagram: Three Intersecting Fimo Colour Sets: Red / Yellow / Green: Verbal Instructions:

- Find three sets. Name each set.
- Place each set in its own circle.
- Some buttons belong to more than one set. Place these buttons where the sets intersect each other in the intersecting circles.

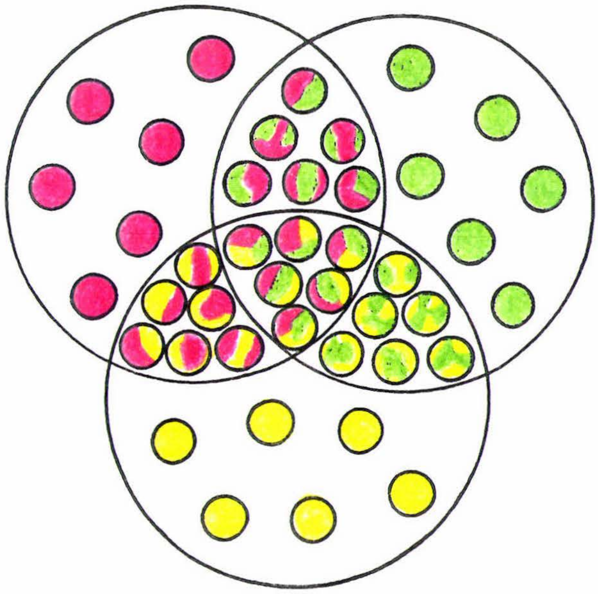


Figure 57a: Venn Diagram: Three Intersecting Fimo Colour Sets: Red / Yellow / Green.

Figure 57b shows Figure 57 levels of performance across mathematical groups:

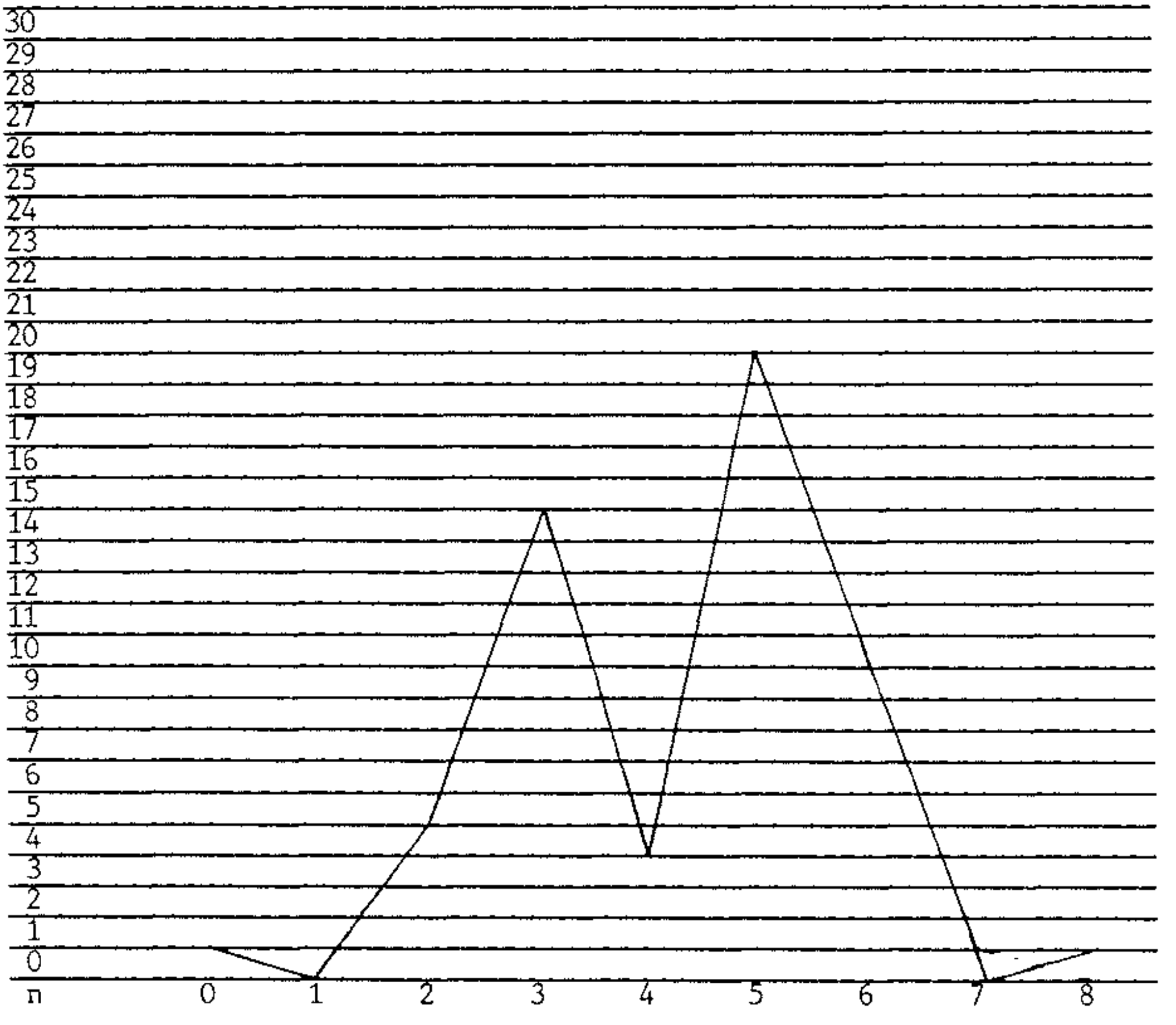
Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								1
7								
6			3		1	2	2	2
5			5	1			7	7
4			2		1			1
3		1	5		4		1	4
2		1	1		2			1
1								
0							1	

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Exceptional speed
8 - Exceptional use of strategy

Figure 57b: Venn Diagram: Three Intersecting Fimo Colour Sets: Red / Yellow / Green:
Levels of Performance Across Mathematical Groups. n = 56.

Figure 57c shows levels of performance across all children who attempted Figure 57:

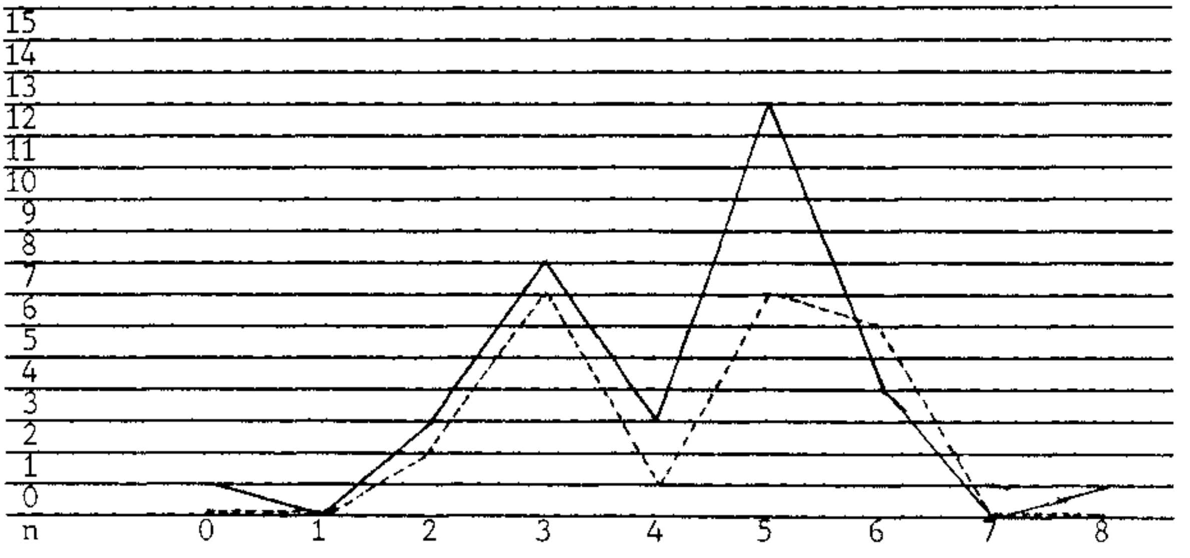


Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Exceptional speed
8 - Exceptional use of strategy

Figure 57c: Venn Diagram: Three Intersecting Fimo Colour Sets: Red / Yellow / Green: Levels of Performance. n = 56.

Figure 57d examines gender performance across levels of performance:



Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Exceptional speed
8 - Exceptional use of strategy

n = 33 ———— Males
n = 23 - - - - - - Females

Figure 57d: Venn Diagram: Three Intersecting Fimo Colour Sets: Red / Yellow / Green: Gender Performance Across Levels of Performance. n = 56.

Table 22 shows Figure 57 correlation coefficients between levels of performance and mathematical groups, school class levels, and ages:

Table 22: Spearman's Rho Correlation Coefficients:
Figure 57: Venn Diagram: Three Intersecting Fimo Colour Sets: Red / Yellow / Green: Levels of Performance: n = 56.

Across mathematical groups	$r_s = 0.237$	$p = <0.05$
Across school class levels	$r_s = 0.135$	$p = >0.05$
Across ages	$r_s = 0.152$	$p = >0.05$

Of note is a 9 year old SE 1 boy with the highest level of performance exhibiting exceptional use of strategy. He accurately visualised and planned the placements of all of his sets mentally before placing the pieces, verbally explaining their positions to me before putting them on the diagram. Most children attempted Figures 58 and 59 following Figure 57, Venn Diagrams of three colours as in the fimo set, but with buttons instead of fimo. Some children found this transition difficult with the introduction of differently sized buttons and various shades of the colour sets they represented, with different numbers of set members from the fimo set. This difficulty represents elements of both distraction and perceptual difficulty. Other children solved these two puzzles at the same level of performance as Figure 57, indicating this was a consolidating exercise, or the level of puzzle was still either too easy or too difficult. A third group of children improved their level of performance, perhaps indicating conscious transfer of learning, and a fourth group undulated performance levels. Of note is the exceptional speed of performance for Figures 58 and 59 from a 10 year old AV girl who completed these puzzles in 10 seconds each. Exceptional speed, along with the use of strategy as mentioned in Figure 57, is an indication of exceptional ability in a child.

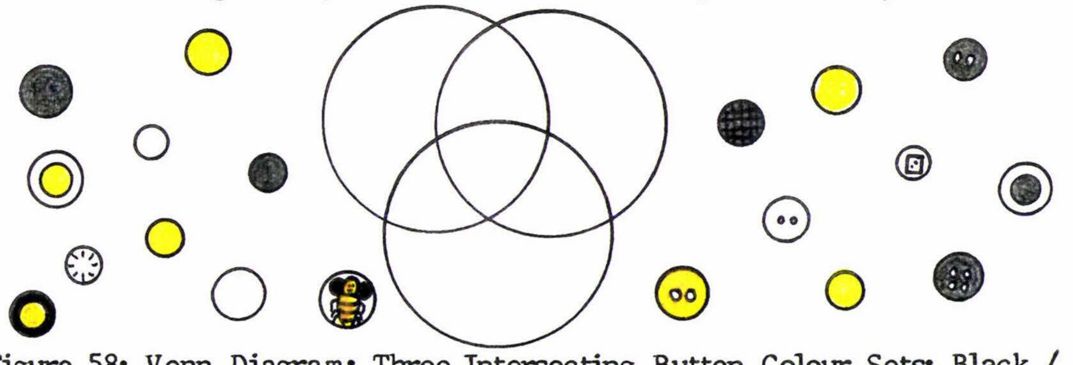


Figure 58: Venn Diagram: Three Intersecting Button Colour Sets: Black / Yellow / White: Verbal instructions as for Figure 57.

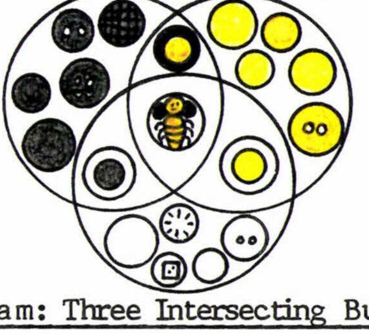


Figure 58a: Venn Diagram: Three Intersecting Button Colour Sets: Black / Yellow / White.

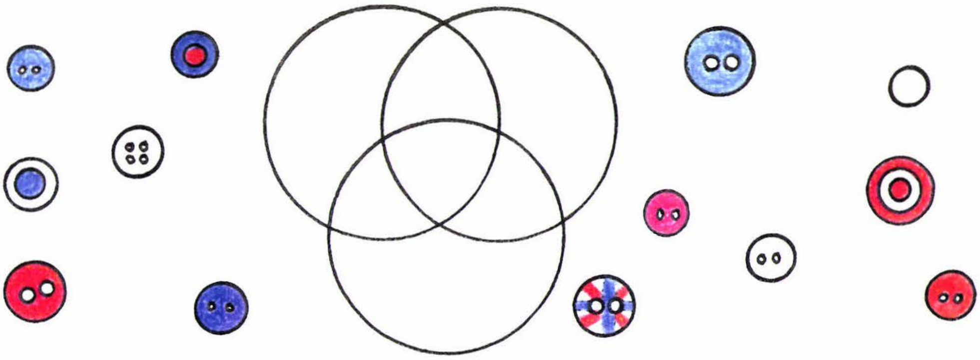


Figure 59: Venn Diagram: Three Intersecting Button Colour Sets: Red / White / Blue: Verbal instructions as for Figure 57.

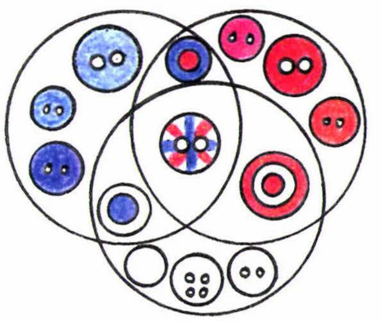


Figure 59a: Venn Diagram: Three Intersecting Button Colour Sets: Red / White / Blue.

Tables 23, 24, 25 and 26 show Figures 57, 58, and 59 levels of performance represented by decrease (-); equality (=); improved (+); and mixed performance (x).

- in Table 23 for children whose level of performance decreased, indicating distraction elements and perceptual difficulties;

= in Table 24 for children whose levels of performance stayed the same, indicating they were either consolidating this conceptual level, or that this level of puzzle was still too easy or too difficult for them; and

+ in Table 25 for children whose levels of performance increased, indicating transfer of learning.

x in Table 26 where children exhibited undulating levels of performance across Figures 57, 58 and 59:

Table 23: - Decreased Levels of Performance: Figures 57, to 58, 59: Venn Diagrams: Three Intersecting Colour Sets: n = 3.

Mathematical Groups	Figure 57	Figure 58	Figure 59
JE	4	3	3
JE	5	3	3
JE	6	5	

Table 24: = Equal Levels of Performance: Figures 57, to 58, 59:
Venn Diagrams: Three Intersecting Colour Sets: n = 28.

Mathematical Groups	Figure 57	Figure 58	Figure 59
JE	2	2	2
JE	3	3	3
JE	5	5	5
S-SLD	5	5	5
SR	3	3	3
SR	2	2	2
SR	2	2	2
SR	6	6	6
AV	6	6	6
SE 2	0	0	0
SE 2	5	5	5
SE 2	5	5	5
SE 2	5	5	5
SE 2	5	5	5
SE 2	5	5	
SE 2	5	5	
SE 1	2	2	2
SE 1	3	3	3
SE 1	3	3	3
SE 1	3	3	3
SE 1	3	3	3
SE 1	4	4	4
SE 1	5	5	5
SE 1	5	5	5
SE 1	5	5	5
SE 1	5	5	
SE 1	6	6	6
SE 1	6	6	6

Table 25: + Increased Levels of Performance: Figures 57, to 58, 59:
Venn Diagrams: Three Intersecting Colour Sets: n = 10.

Mathematical Groups	Figure 57	Figure 58	Figure 59
JR	3	3	5
JE	3	5	
JE	3	4	4
JE	4	4	5
JE	3	6	6
SR	3	5	5
SR	3	5	5
SR	4	6	6
AV	6	7	7
SE 1	5	6	6

Table 26: x Mixed Levels of Performance: Figures 57, to 58, 59: Venn Diagrams: Three Intersecting Colour Sets: N = 1.

Mathematical Groups	Figure 57	Figure 58	Figure 59
JE	6	3	5

It could be expected that children decreasing their performance would be junior children exhibiting more perceptual and distraction difficulties and less experience in transfer of training, but surprisingly, the only JR child to attempt these puzzles increased his performance. To negate the expectation that junior children might decrease their levels of performance, some JE children increased their levels of performance and decreased it, as well as keeping it equal. Also surprising was that all children except one from the two senior extension groups maintained their levels of performance without any apparent transfer of learning. The change of perceptual levels and distraction may have balanced the effect of transfer of learning, keeping the levels of performance equal. Some children attempted further Venn Diagrams with multiple concepts involved. These included average senior puzzles Figures 60 and 61, and beginning senior puzzles Figures 62, 63 and 64:

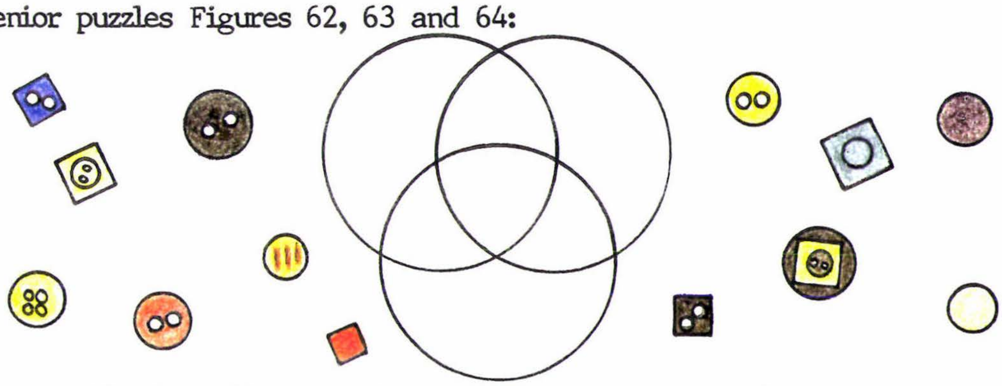


Figure 60: Venn Diagram: Three Intersecting Button Sets: Verbal instructions as for Figure 57.

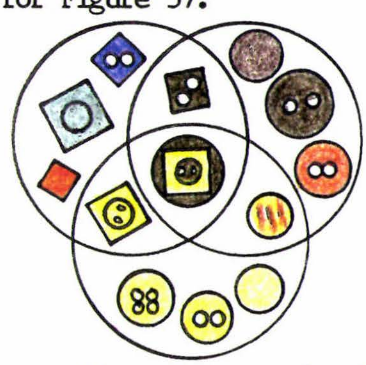


Figure 60a: Venn Diagram: Three Intersecting Button Sets: Cream / Brown / Square.

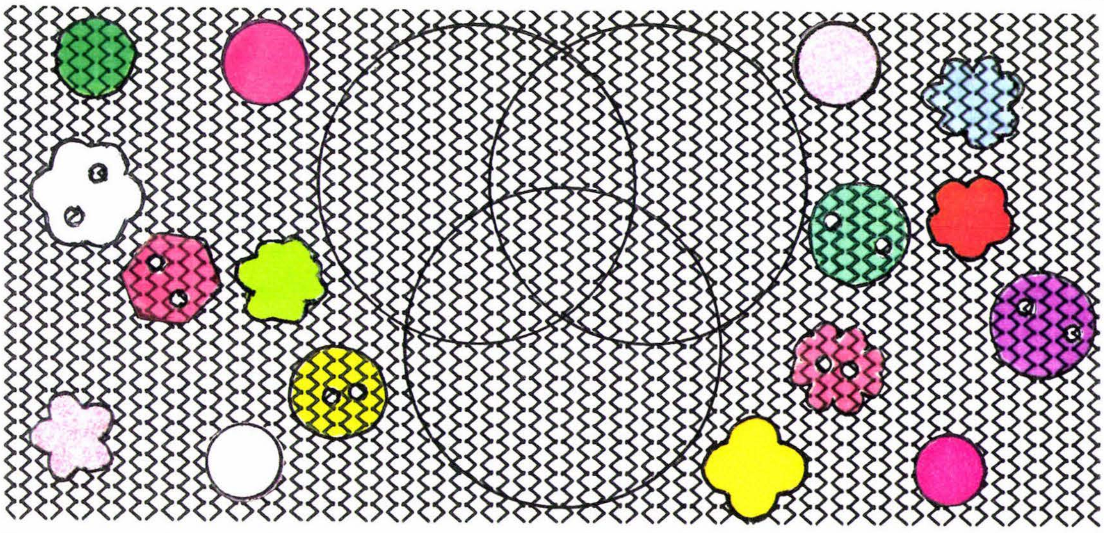


Figure 61: Venn Diagram: Three Intersecting Button Sets:
Verbal instructions as for Figure 57.

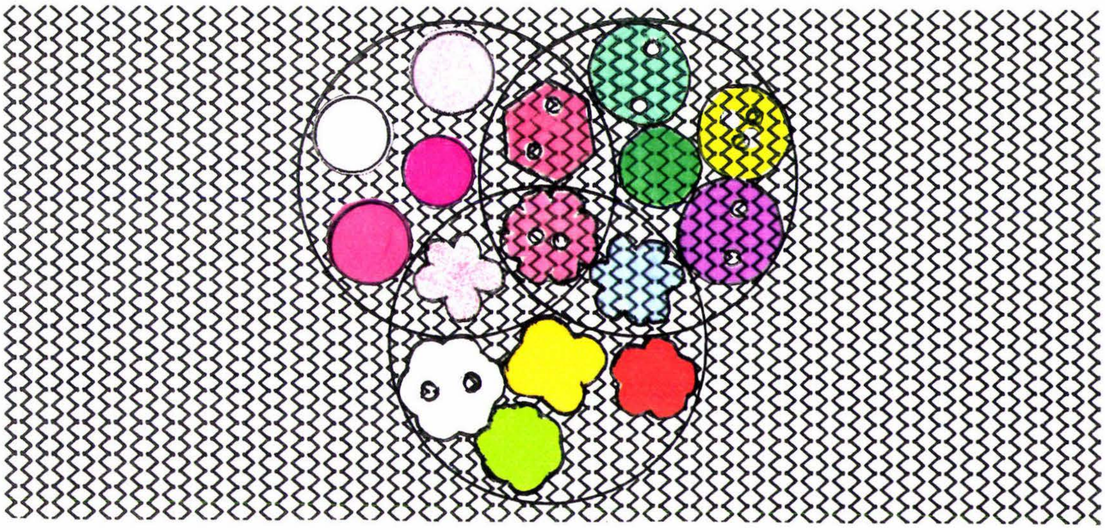


Figure 61a: Venn Diagram: Three Intersecting Button Sets:
Pink / Flower / Transparent.

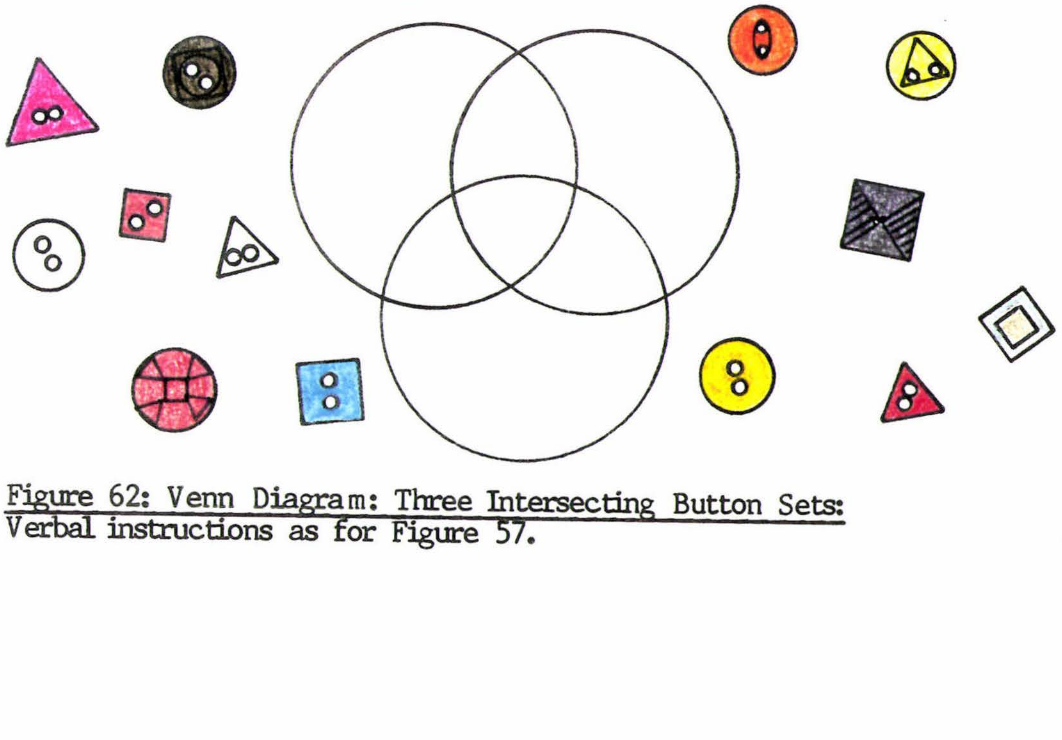


Figure 62: Venn Diagram: Three Intersecting Button Sets:
Verbal instructions as for Figure 57.

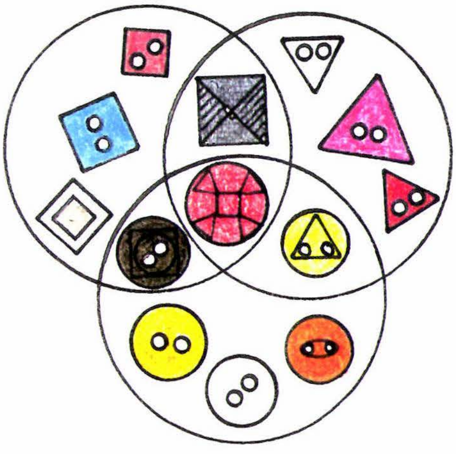


Figure 62a: Venn Diagram: Three Intersecting Button Sets: Triangle / Square / Circle.

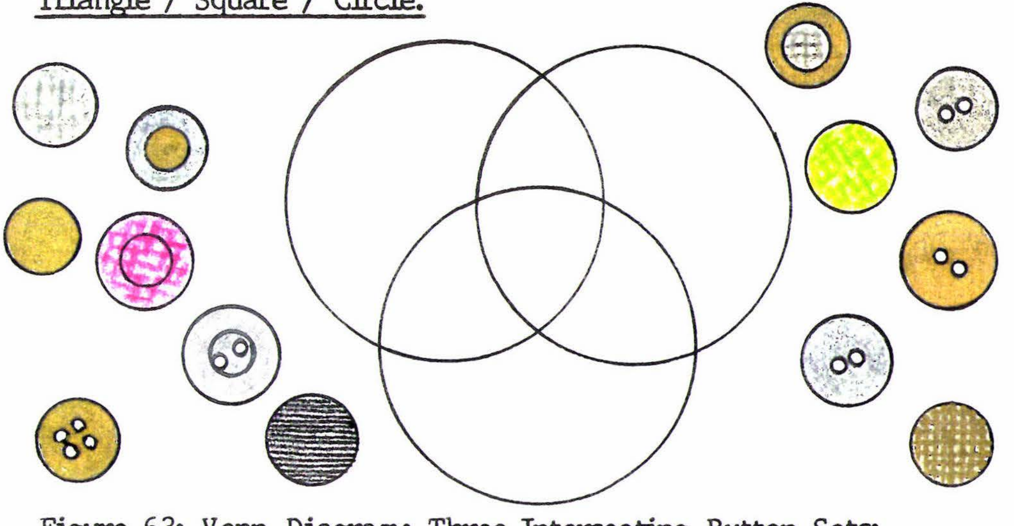


Figure 63: Venn Diagram: Three Intersecting Button Sets: Verbal instructions as for Figure 57.

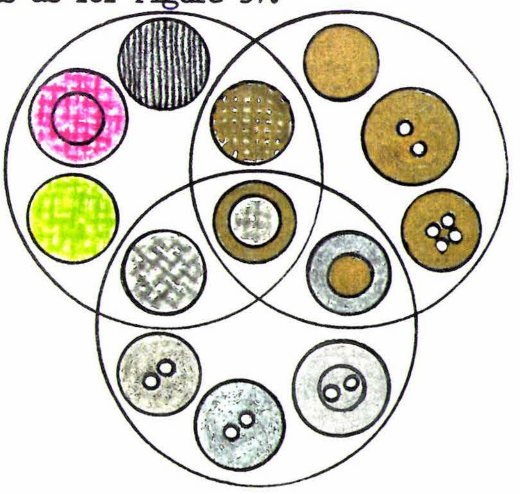


Figure 63a: Venn Diagram: Three Intersecting Button Sets: Gold / Grey / Fabric.

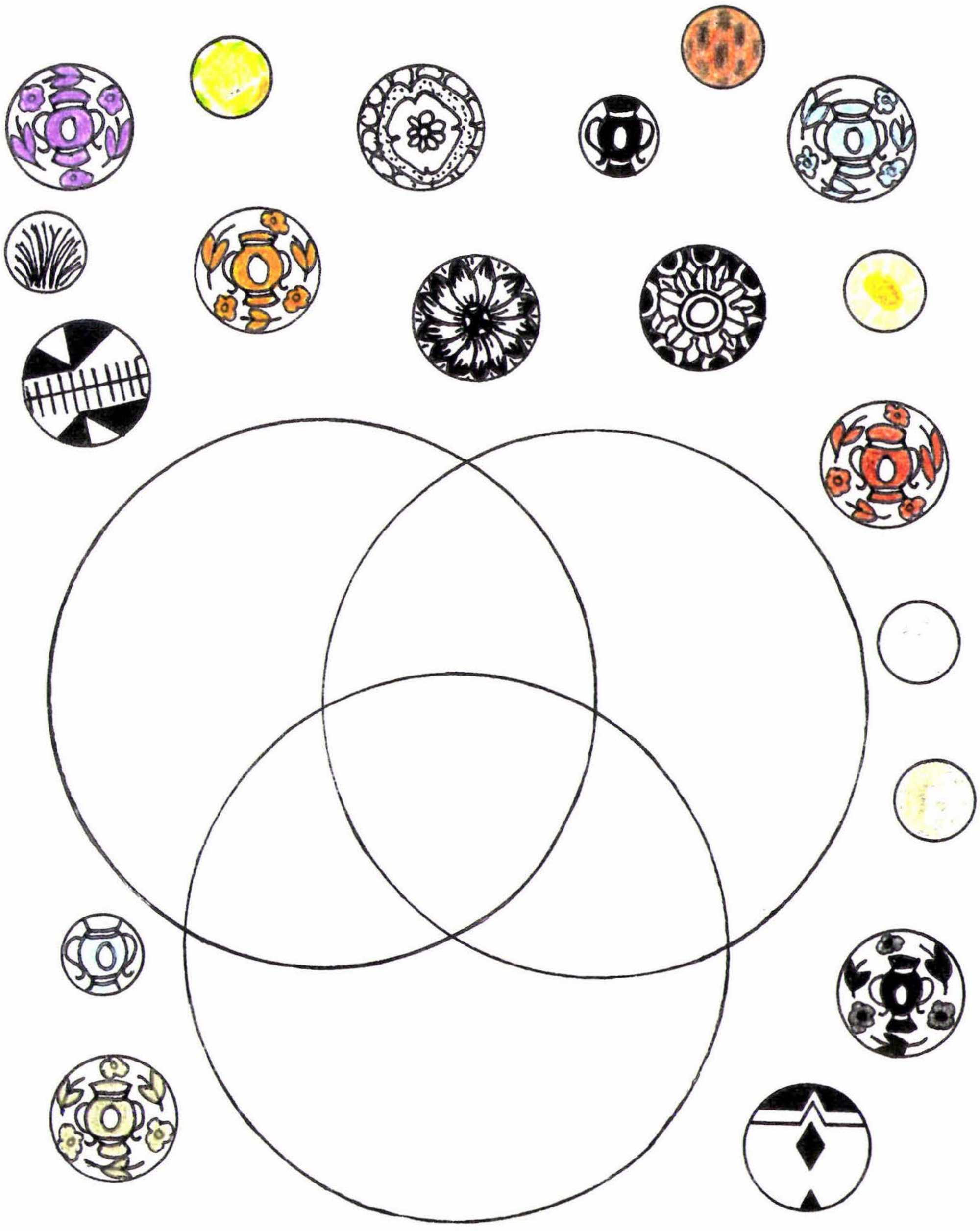


Figure 64: Venn Diagram: Three Intersecting Wallpaper Sets: Verbal instructions:

- Find three sets.
- One set is a colour set.
- One set is a size set.
- One set is a pattern set.
- Name each set.
- Place each set in its own circle.
- Some pieces belong to more than one set. Place these pieces where the sets intersect each other in the intersecting circles.

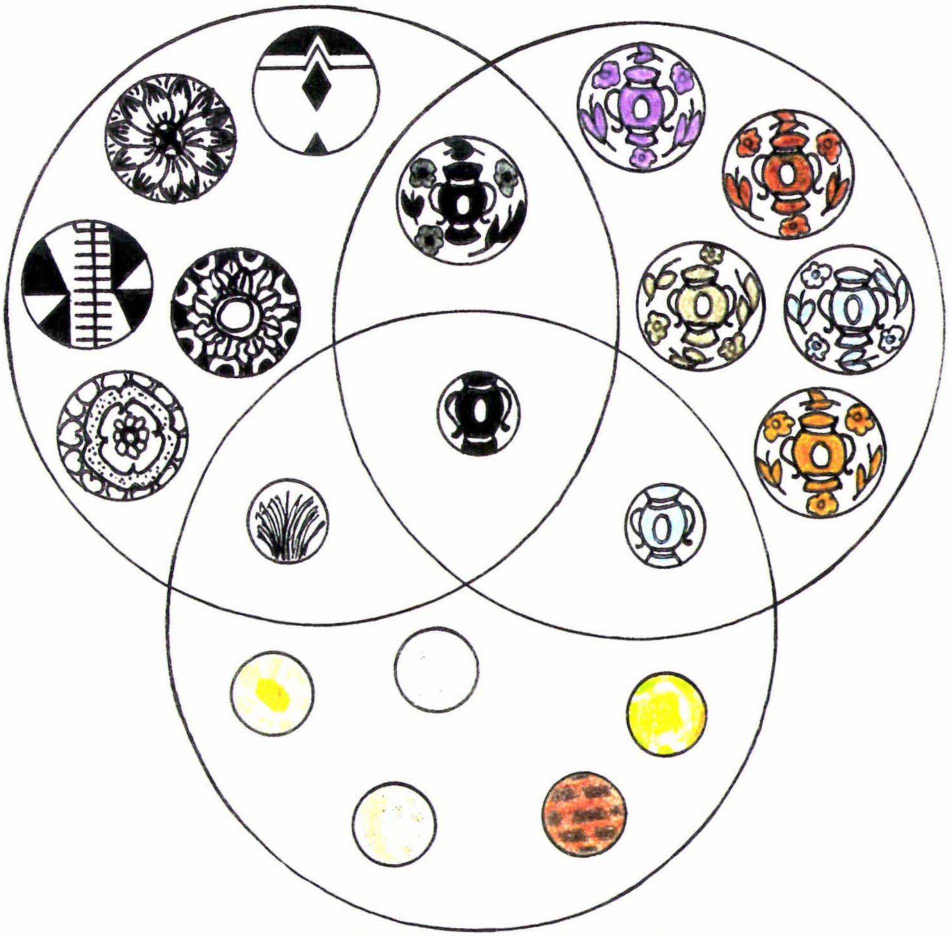


Figure 64a: Venn Diagram: Three Intersecting Wallpaper Sets: Black and White / Small / Jug Pattern.

Table 27 shows Figure 64 correlation coefficients between levels of performance and mathematical groups, school class levels, and ages:

Table 27: Spearman's Rho Correlation Coefficients:
Figure 64: Venn Diagram: Three Intersecting Wallpaper Sets:
Black and White / Small / Jug Pattern:
Levels of Performance: n = 35.

Across mathematical groups	$r_s = 0.490$	$p = <0.05$
Across school class levels	$r_s = 0.307$	$p = <0.05$
Across ages	$r_s = 0.239$	$p = >0.05$

Figure 64b shows Figure 64 levels of performance across mathematical groups:

Levels of Performance	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6			1			1	3	4
5			1				1	6
4						1	2	
3					1		3	4
2			5	1	1			
1								
0								

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 64b: Venn Diagram: Three Intersecting Wallpaper Sets: Black and White / Small / Jug Pattern: n = 35.

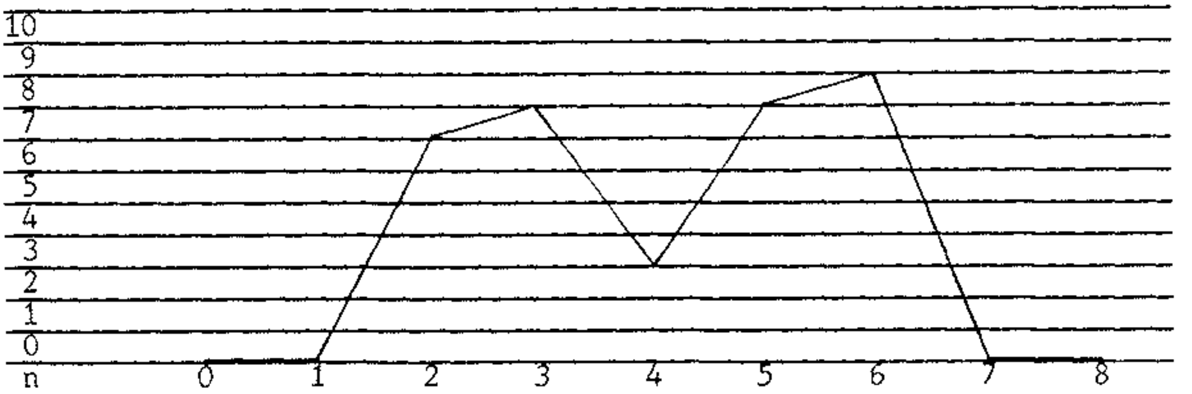
Figure 64c shows children at the highest level of performance for Figure 64 across mathematical groups and age. Of note is a 6 year old JE boy.

Age	Maths Groups J SLD	JR	JE	S SLD	SR	AV	SE 2	SE 1
								expected
11								
10								2
9							2	1
8						1	1	1
7								
6			1					
unexpected								

Figure 64c: Venn Diagram: Three Intersecting Wallpaper Sets: Black and White / Small / Jug Pattern:.

Highest Level of Performance: Very fast completion with no questioning: Mathematical Groups Across Ages. n = 9.

Figure 64d shows levels of performance across all children who attempted Figure 64:

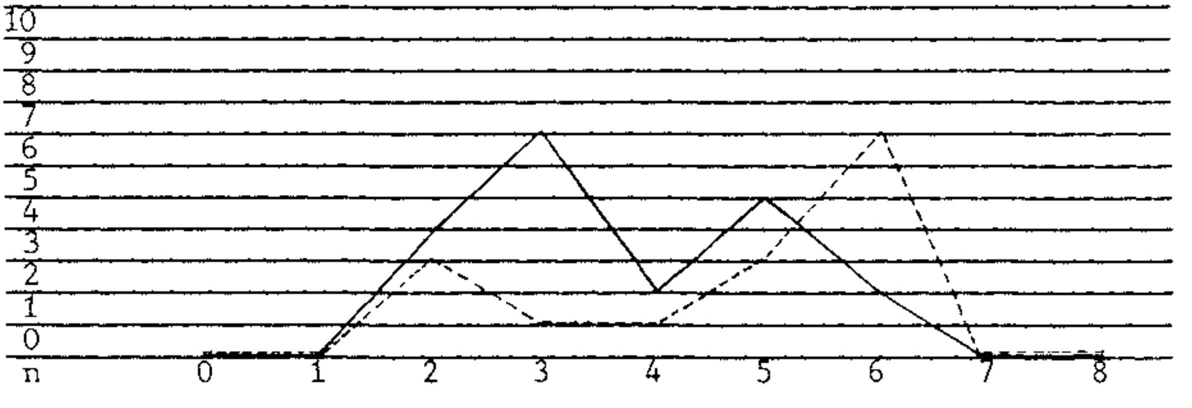


Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 64d: Venn Diagram: Three Intersecting Wallpaper Sets:
Black and White / Small / Jug Pattern:
Levels of Performance. n = 35.

Figure 64e examines gender performance across levels of performance: [Levels of performance as for Figure 64d].



n = 20 ________ Males
n = 15 ---------- Females

Figure 64e: Venn Diagram: Three Intersecting Wallpaper Sets:
Black and White / Small / Jug Pattern:
Gender Performance Across Levels of Performance. n = 35.

Both Figure 57 and Figure 64 were significant at 0.05 level for levels of performance correlated with mathematical group categories. Figure 64 was also significant at school class level correlated with levels of performance. It is difficult to find a possible reason for these correlations occuring with two puzzles of identical type. It is also difficult to determine a reason why correlations are significant when most of the puzzles did not show significant correlations between mathematical groups and levels of performance, and also to determine whether these results occurred from chance alone or for some specific reason relating to the puzzles.

One reason for significance in Figure 57 could be the exceptional score of one SE 1 child using an exceptional level of strategy. Otherwise score ranges were the same for JE and SE 1 children. This high score was able to override the effect of one SE 2 child with the lowest level of performance who found this puzzle too difficult. A further factor could be the small number of JR children attempting this puzzle, eliminating a greater number of potential levels of performance across the range.

For Figure 64, levels of performance ranges were spread from JE to SE 1 children, with the numbers of children at higher levels being greater for senior extension children.

Table 27A shows two tailed Mann-Whitney U-Tests showing no gender differences in levels of performance across mathematical groups for Figures 57 and 64 at 0.05, where U (M) = Males, U1 (F) = Females, and z shows use of normal approximation for larger samples.

Table 27A: Mann-Whitney U-Tests For Chapter 6: Figures 57 and 64: Gender Differences: Levels of Performance Across Mathematical Groups:

Figure	U (M)	U1 (F)	z	Males	Females
Puzzles demonstrating transfer of learning					
57	353.5	405.5	+ 0.43	-	-
64	108.5	191.5		-	-

* Significant at 0.05

Tables 28, 29, 30 and 31 show the same children as Tables 23, 24, 25 and 26 who attempted some of the multiple concept Venn Diagrams (Figures 60, 61, 62, 63 and 64). Trends are mixed as to which children stay at decreased, equal, increased or mixed levels of performance:

Table 28: - Decreased Figure 57-59 Levels of Performance/ Fig 60-64: Venn Diagrams: Three Intersecting Sets: N = 2.

Mathematical Groups / Figures	57	58	59	60	61	62	63	64
JE	4	3	3					2
JE	6	5						2

Table 29: = Equal Figure 57-59 Levels of Performance/ Fig 60-64: Venn Diagrams: Three Intersecting Sets: N = 22.

Mathematical Groups / Figures	57	58	59	60	61	62	63	64
JE	2	2	2		2			2
JE	5	5	5					5
S-SLD	5	5	5					2
SR	3	3	3					2
SR	2	2	2					3
AV	6	6	6	3	4		6	6
SE 2	5	5	5					4
SE 2	5	5	5					5
SE 2	5	5	5					3
SE 2	5	5	5					6
SE 2	5	5						3
SE 2	5	5		6	3			6
SE 1	3	3	3					5
SE 1	3	3	3					5
SE 1	3	3	3					5
SE 1	4	4	4	2				
SE 1	5	5	5	3				5
SE 1	5	5	5					3
SE 1	5	5	5	3				6
SE 1	5	5						2
SE 1	6	6	6					3
SE 1	6	6	6					3

Table 30: + Increased Figure 57-59 Levels of Performance/ Fig 60-64: Venn Diagrams: Three Intersecting Sets: n = 4.

Mathematical Groups / Figures	57	58	59	60	61	62	63	64
JE	3	5						6
SR	4	6	6	3	3			
AV	6	7	7	6	6	6	6	4
SE 1	5	6	6					6

Table 31: x Mixed Figure 57-59 Levels of Performance/ Fig 60-64: Venn Diagrams: Three Intersecting Sets: n = 1.

Mathematical Groups / Figures	57	58	59	60	61	62	63	64
JE	6	3	5					2

The majority of children equalled their levels of performance across Figures 57-59, but with the addition of multiple concepts to the same type of puzzle in Figures 60-64, the main trend was a lowering of levels of performance. Figure 64 added an extra dimension of a different material type which created transfer of learning difficulties for some children. A smaller group of children did make a positive transfer of learning. Table 32 lists trends of the movement of levels of performance for each of the -, =, + and x groups:

Table 32: Movement in Levels of Performance: Figures 57-59 to 60-64: Venn Diagrams: Three Intersecting Sets:

Table No.	N	-	=	+	x
Table 28 -	2	2			
Table 29 =	20	9	3	6	2
Table 30 +	4	2	1	1	
Table 31 x	1	1			
Total	27	14	4	7	2

The issues are illustrated again by a special case not included in the previous tables. The SE 1 boy concerned started with Figure 58 (red / white / blue buttons) instead of the usual Figure 57 (fimo). His level of performance for Figure 58 was 5, showing he had a good conceptual grasp and was not too distracted with the perceptual elements of the buttons. He followed with Figure 62 (triangle / square / circle buttons), showing his difficulties with the inclusion of multiple concepts and elimination of colour by a level of performance of 3. He finally completed Figure 64 (Wallpaper: black & white / small / jug pattern) with a level of performance of 6, showing a definite transfer of learning.

Tables 33 and 34 show a further special case category of children not included in previous tables, who give evidence of the need for consolidating experience for adequate transfer of learning, and reinforce the difficulties created by the addition of perceptual variables and multiple concepts to a puzzle. These children each attempted two Venn Diagrams only, moving straight from Figure 57 (fimo) to Figure 64 (Wallpaper: black & white / small / jug pattern). In Table 33, levels of performance decreased from the first puzzle to the second, despite most of these children belonging to one of the two highest SE mathematical group categories. The child previously achieving a high 7 level of performance through exceptional use of strategy was unable to transfer his visualising ability when perceptual and conceptual variables were not held constant:

Table 33: - Decreased Levels of Performance
From Figure 57 to 64:
Venn Diagrams: Three Intersecting Sets: N = 6.

Mathematical Groups	Figure 57	Figure 64
JE	5	2
SE 2	5	3
SE 2	5	4
SE 2	6	3
SE 1	5	3
SE 1	7	3

Table 34 shows children attempting only Figures 57 and 64 in the Venn Diagram group, who maintained equal levels of performance, indicating significant transfer of learning despite the fact that no child increased performance level when attempting these two Venn Diagrams alone:

Table 34: = Equal Levels of Performance
From Figure 57 to 64:
Venn Diagrams: Three Intersecting Sets: N = 3.

Mathematical Groups	Figure 57	Figure 64
JE	5	5
SE 2	5	5
SE 1	6	6

Figures 65, 66, 67 and 68, very advanced senior puzzles, required transfer of learning between each of these related ordered progressions of puzzles. All children attempting any of these four puzzles were 10 years old. Table 35 shows levels of performance and movement in levels of performance between these progressively difficult puzzles. Performance level changes indicate ability to transfer concepts, with a mixture of children maintaining, increasing and decreasing performance levels across puzzles.

Table 35: Movement in Performance Levels: Figure 65-66-67-68: N = 9.
Figure 65: Two Intersecting Button Sets with Subsets of Two.
Figure 66: Two Intersecting Button Sets with Subsets of Three.
Figure 67: Four Button Classification Sets with Subsets of Two.
Figure 68: Four Button Classification Sets with Subsets of Three.

Mathematical Groups	Figure 65	Figure 66	Figure 67	Figure 68
SE 1	2			
SE 1	2	2	3	5
SE 1	2	2		
SE 1	2	6	4	6
AV	3	5	6	6
SE 1	3	5	5	5
SE 1	4			2
SE 1	4			
SE 1			4	5

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

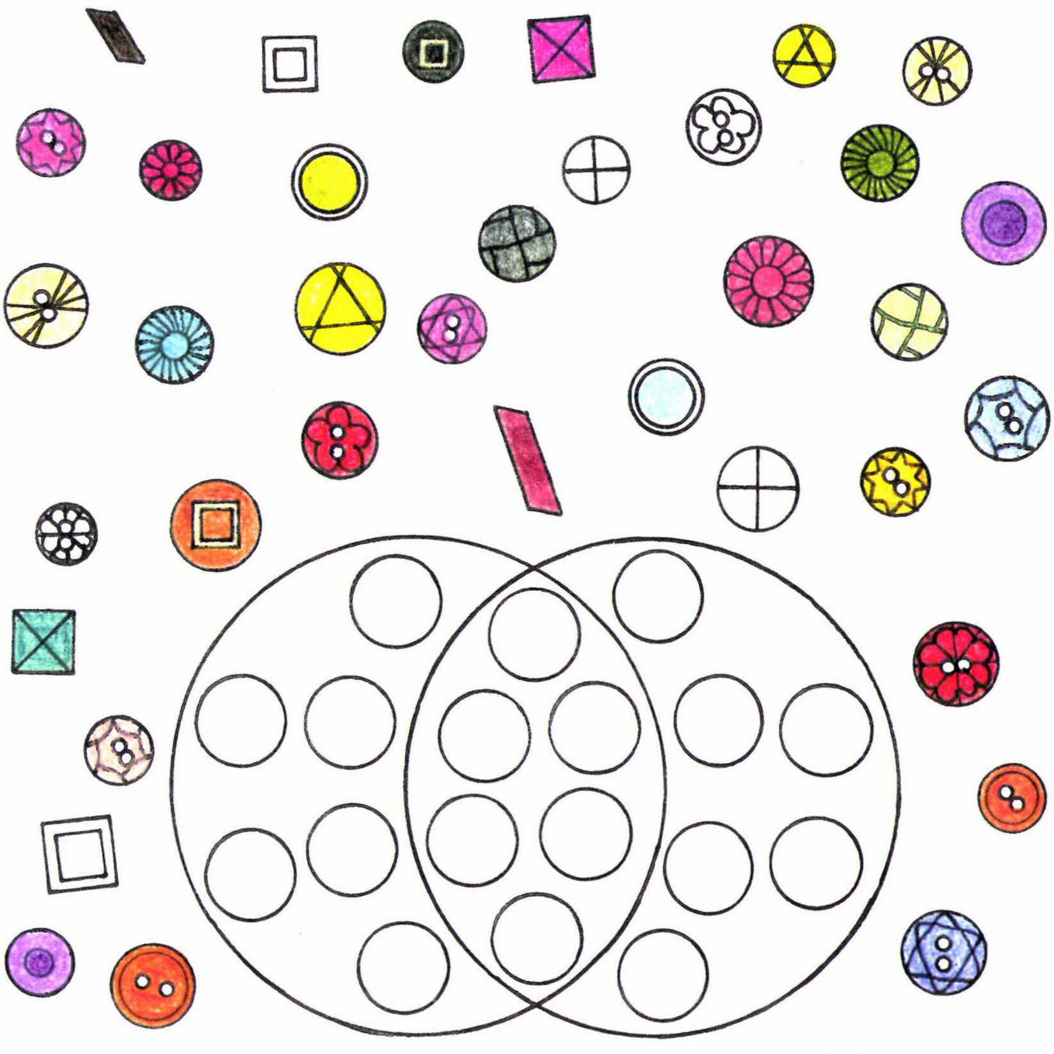


Figure 65: Two Intersecting Button Sets with Subsets of Two:
Verbal Instructions:

- Find buttons that belong together.
- Take one subset of two that you have found. What is the same about these two buttons? What is different about them? Give this subset a name that states what is the same or what is different.
- Find other subsets of two with the same name.
- Group other subsets with different names.
- You now have three different groups of subsets.
- Place the named groups of subsets on the diagram to make two sets. Each subset of two has its own little circle.
- The group of named subsets you place in the intersecting space between the two big sets must belong to both sets.
- Name the two sets.

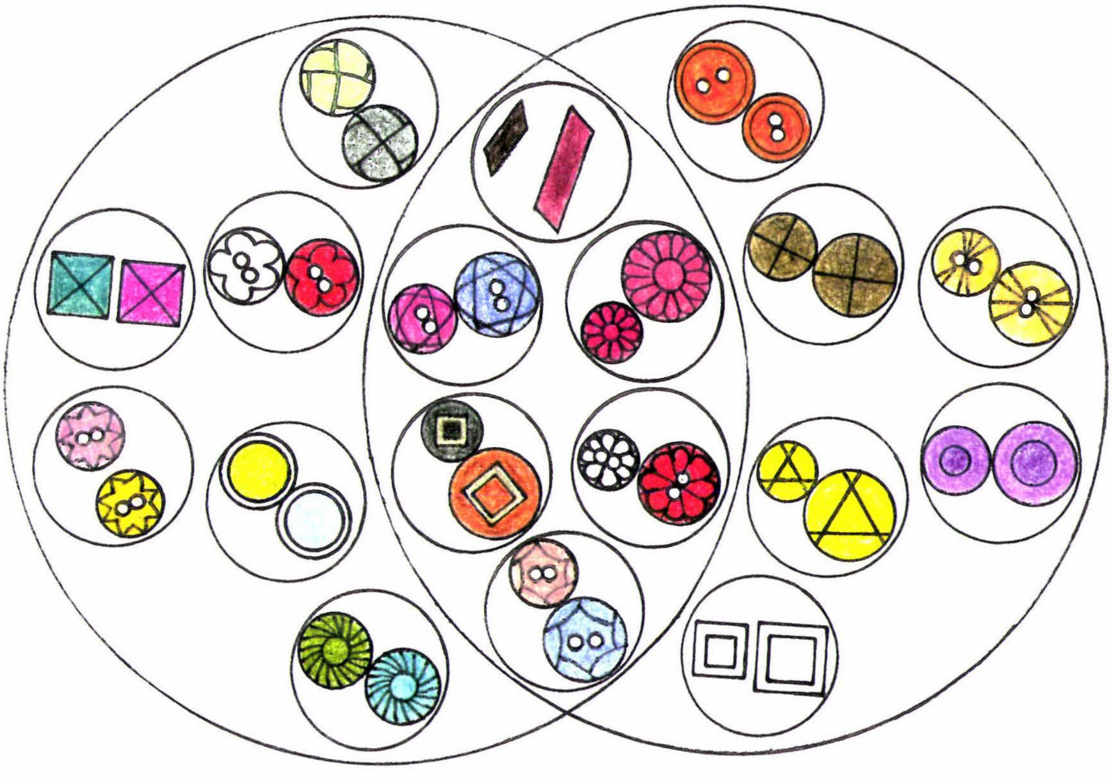


Figure 65a: Two Intersecting Button Sets with Subsets of Two:
Verbal Instructions:

- What are the names of the three groups of subsets of two?
    - Same colour / different size
    - Different colour / same size
    - Different colour / different size
- What are the names of the two big sets?
    - Different colour
    - Different size

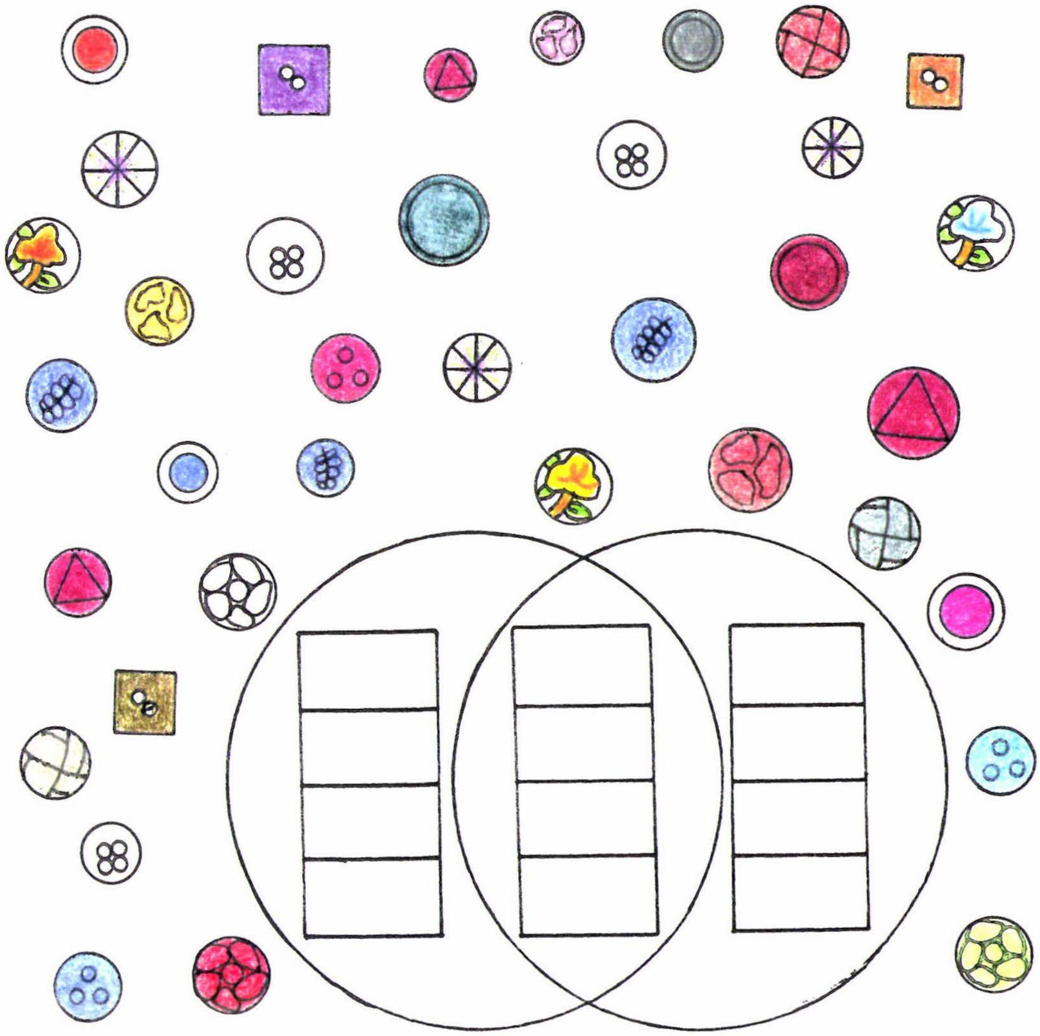


Figure 66: Two Intersecting Button Sets with Subsets of Three: Verbal Instructions:

- Find buttons that belong together.
- Take one subset of three that you have found. What is the same about these three buttons? What is different about them? Give this subset a name that states what is the same or what is different.
- Find other subsets of three with the same name.
- Group other subsets with different names.
- You now have three different groups of subsets.
- Place the named groups of subsets on the diagram to make two sets. Each subset of three has its own rectangle.
- The group of named subsets you place in the intersecting space between the two big sets must belong to both sets.
- Name the two sets.

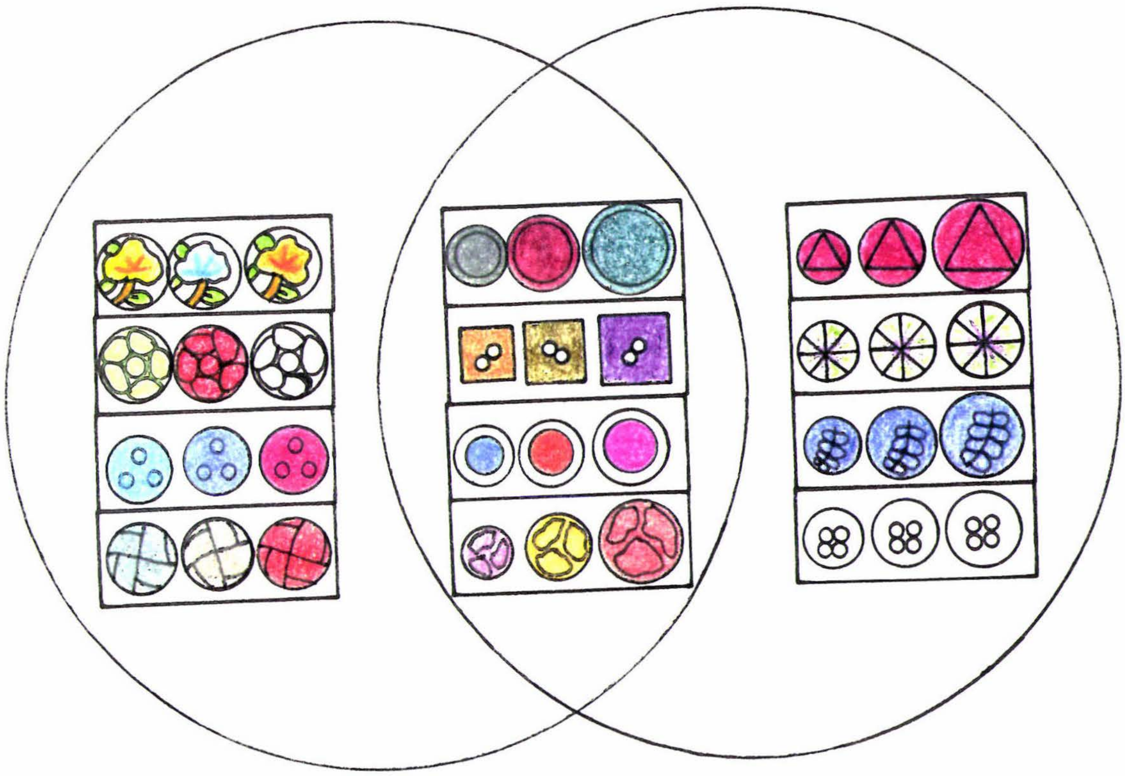


Figure 66a: Two Intersecting Button Sets with Subsets of Three: Verbal Instructions:

- What are the names of the groups of subsets of three?
    - Same colour / different size
    - Different colour / same size
    - Different colour / different size
- What are the names of the two big sets?
    - Different colour
    - Different size

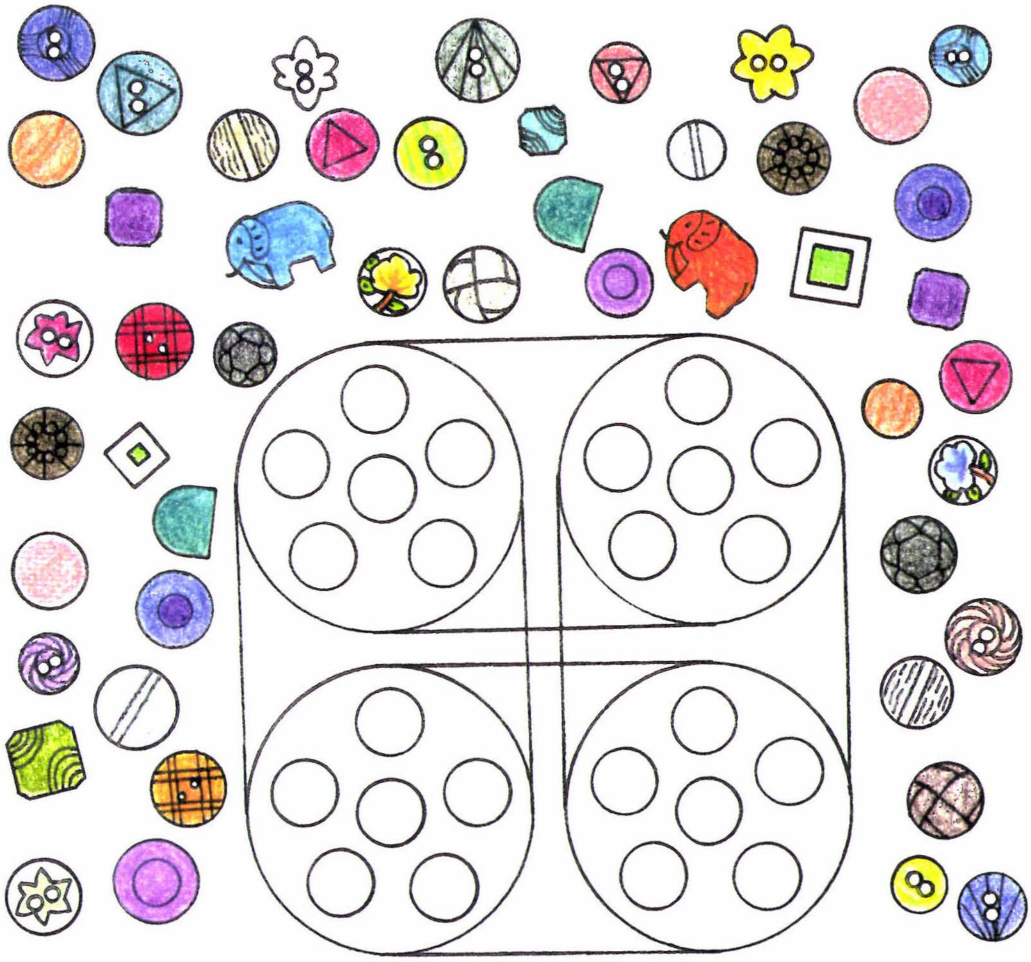


Figure 67: Four Button Classification Sets with Subsets of Two:
Verbal Instructions:

- Find buttons that belong together.
- Take one subset of two that you have found. What is the same about these two buttons? What is different about them? Give this subset a name that states what is the same or what is different.
- Find other subsets of two with the same name.
- Group other subsets with different names.
- You now have four different groups of subsets.
- Place the four named groups of subsets in the four big circles on the diagram, with each subset in its own little circle.
- Each of the four big circles is connected vertically and horizontally with another big circle.
- There are four sets. Each set is made up of two connecting big circles.
- Name the four sets.

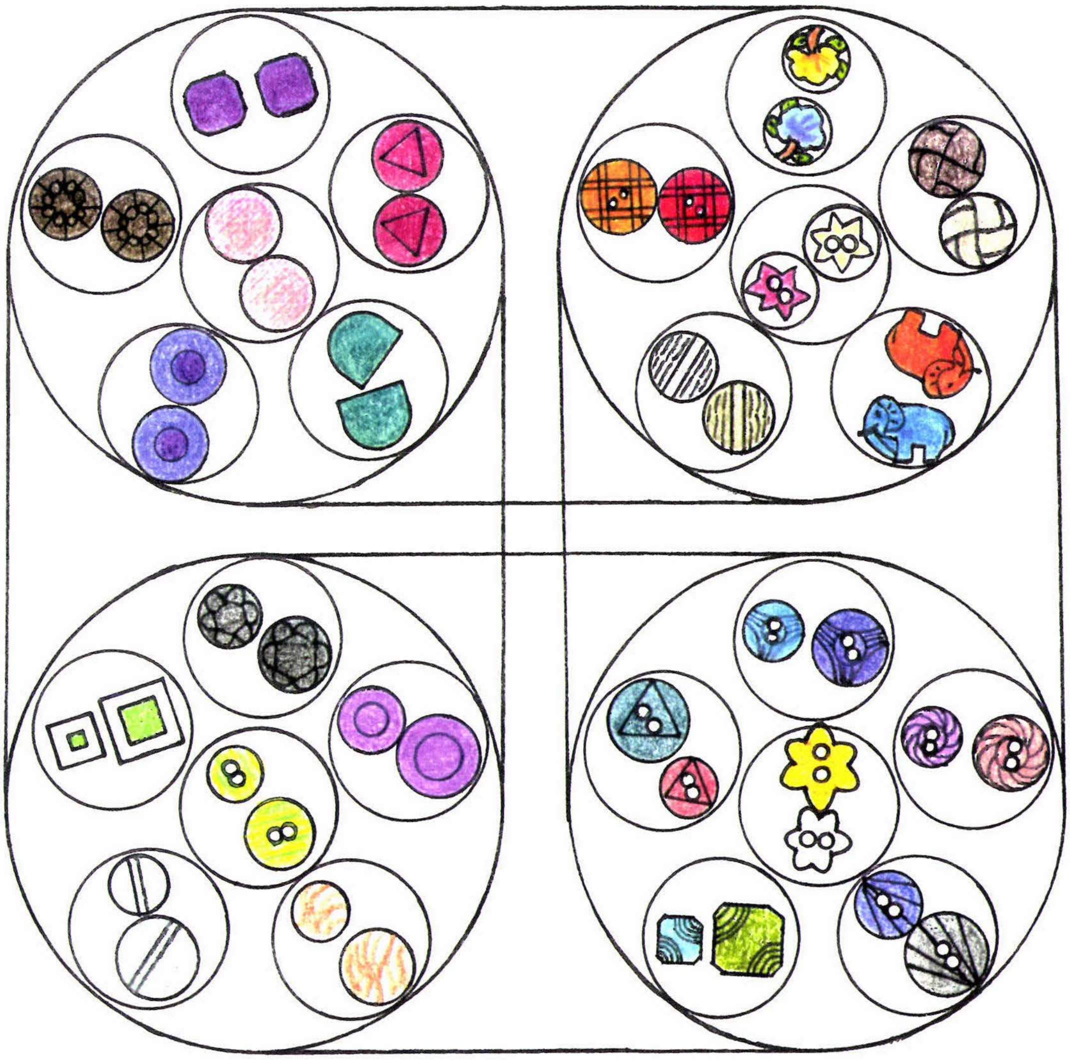


Figure 67a: Four Button Classification Sets with Subsets of Two:
Verbal Instructions:

- What are the names of the groups of subsets of two?
  - Same colour / different size
  - Different colour / same size
  - Different colour / different size
  - Same colour / same size
- What are the names of the four connected sets?
  - Same size
  - Different colour
  - Different size
  - Same colour

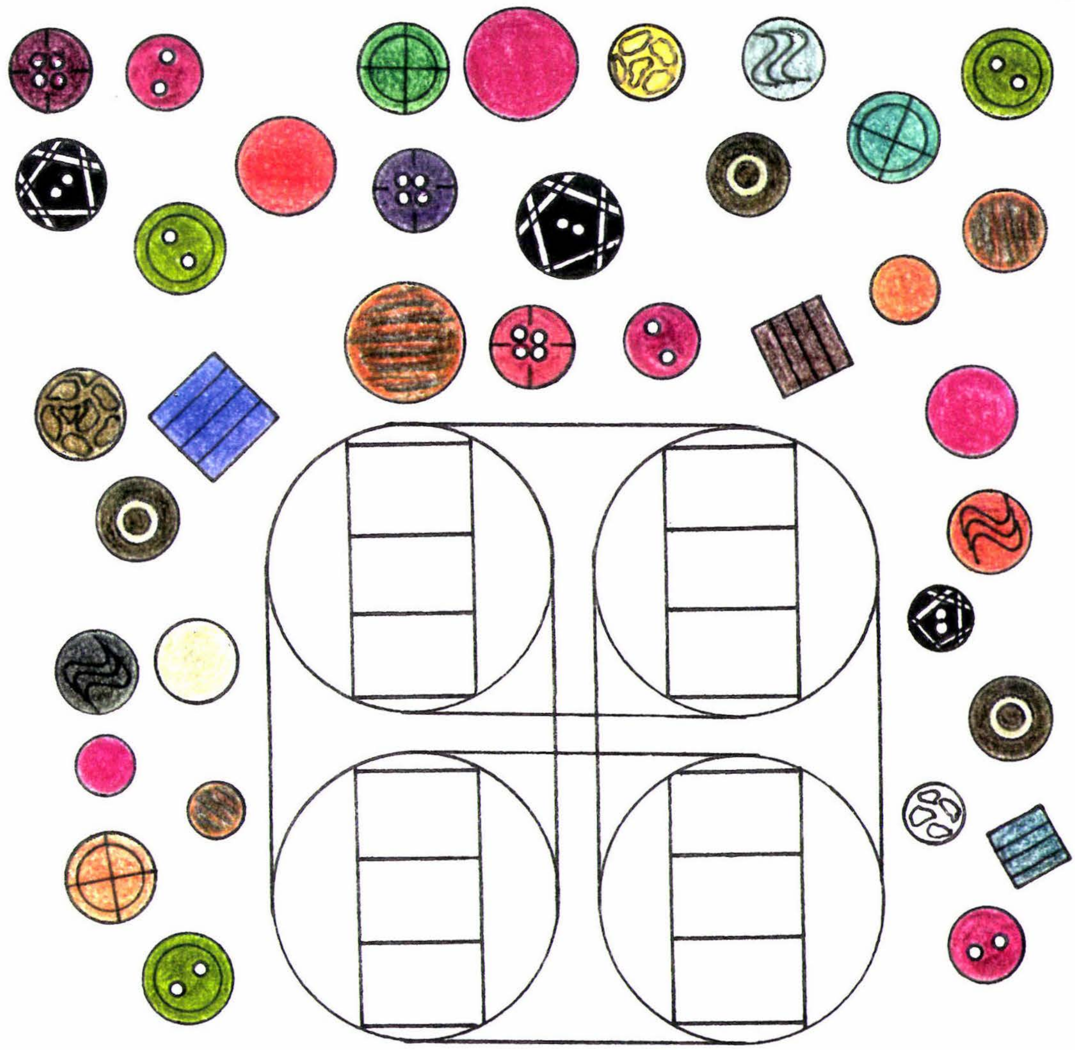


Figure 68: Four Button Classification Sets with Subsets of Three: Verbal Instructions:

- Find buttons that belong together.
- Take one subset of three that you have found. What is the same about these three buttons? What is different about them? Give this subset a name that states what is the same or what is different.
- Find other subsets of three with the same name.
- Group other subsets with different names.
- You now have four different groups of subsets.
- Place the four named groups of subsets in the four big circles on the diagram, with each subset in its own rectangle.
- Each of the four big circles is connected vertically and horizontally with another big circle.
- There are four sets. Each set is made up of two connecting big circles.
- Name the four sets.

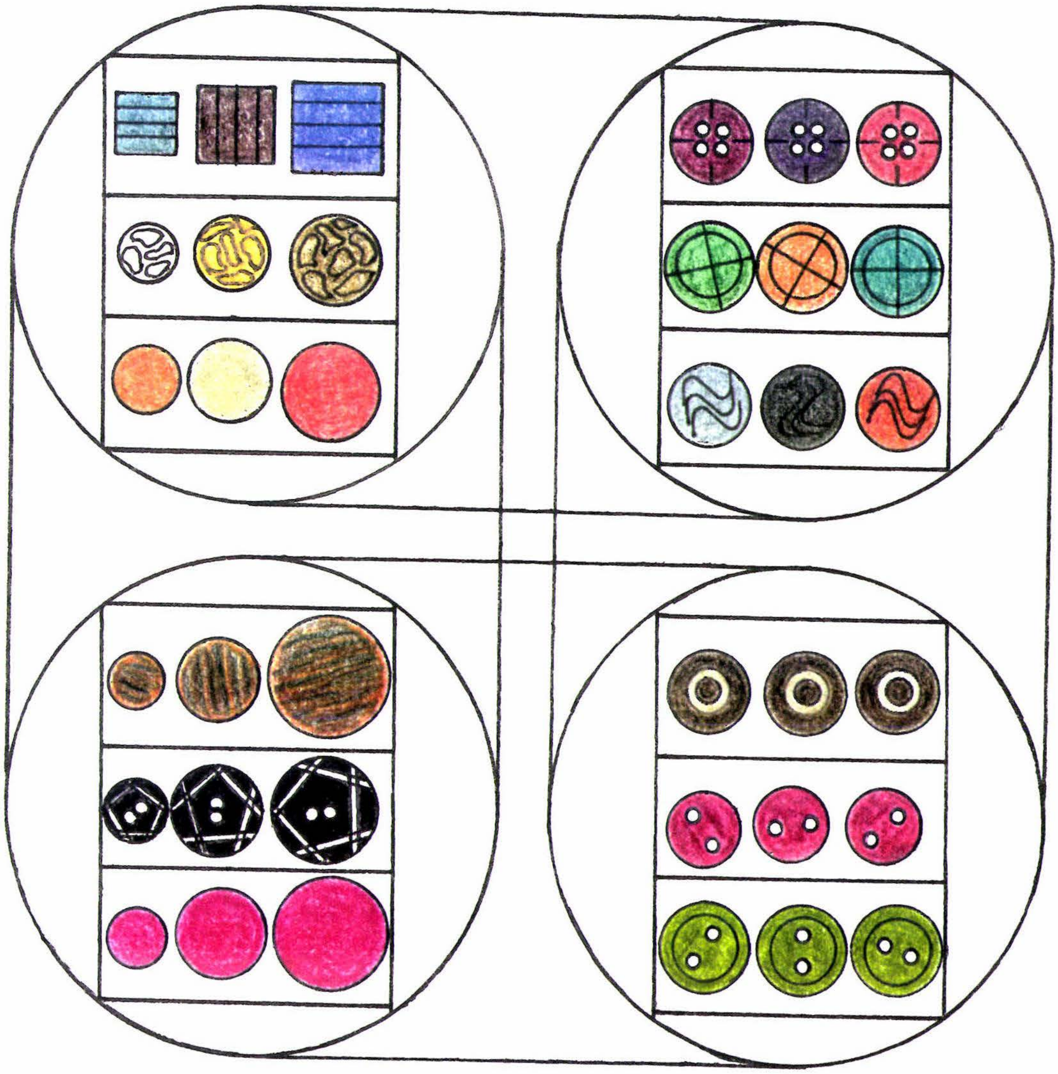


Figure 68a: Four Button Classification Sets with Subsets of Three:
Verbal Instructions:

- What are the names of the groups of subsets of three?
    - Same colour / different size
    - Different colour / same size
    - Different colour / different size
    - Same colour / same size
- What are the names of the four connected sets?
    - Same size
    - Different colour
    - Different size
    - Same colour

Figures 69, 70 and 71, beginning senior puzzles, Figure 72, an advanced senior puzzle, and Figure 73, a very advanced senior puzzle, also required transfer of learning between each of these related and progressively difficult puzzles. Table 36 shows levels of performance and movement in levels of performance, including conscious use of strategy. Performance level changes indicate ability or inability to transfer concepts, with a mixture of children maintaining, increasing, decreasing and undulating performance levels across puzzles. Also included is Figure 10 (Chapter 4).

Table 36: Performance Level Movement: Figure 69-70-10-71-72-73: N = 17.

Figure 69: One Colour Difference 5 Button Arms: White Centre.

Figure 70: One Colour Difference 7 Button Arms: Gold Centre.

Figure 10: One Shape Difference 3 Button Arms: Circle Centre.

Figure 71: One Shape Difference 8 Button Arms: Circle Centre.

Figure 72: One Colour Difference 6 Intersecting Button Arms: Black.

Figure 73: One Colour Difference 9 Intersecting Button Arms: Transparent.

Mathematical Groups	Figure 69	70	10	71	72	73
SR	2					
SE 1	2	5			7	
SE 1	3	3				
SR	3	0				
SR	4	5				
SR	5	5				
SR	5	5				
SR	5					
SE 1	6	6			5	4
AV	6	6	6	3	7	8
AV	6	6	4	4	8	0
SE 1	6				2	
SE 2		2				
SE 1					2	
SE 1					3	
SE 2					3	
SE 2					4	

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Use of strategy with minimal questioning
8 - Use of strategy with no questioning
9 - Very fast completion with use of strategy

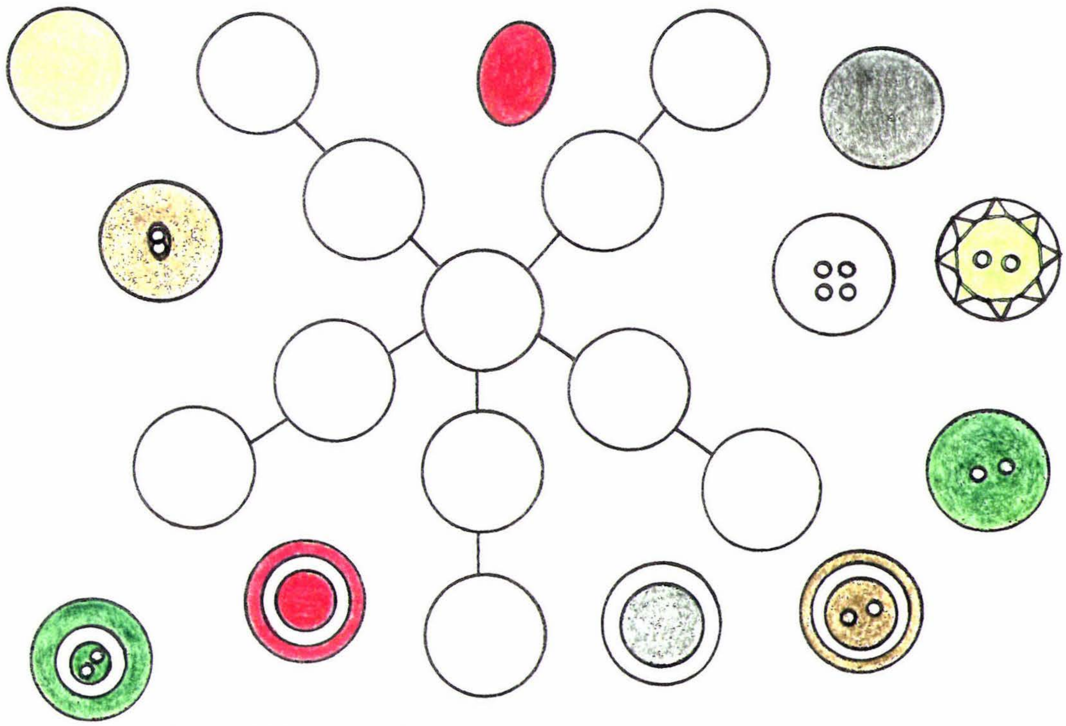


Figure 69: One Difference 5 Button Arms:
Verbal Instructions:

- Find which buttons belong together.
- Arrange the buttons with no more than one thing different between any buttons you place in connecting circles.

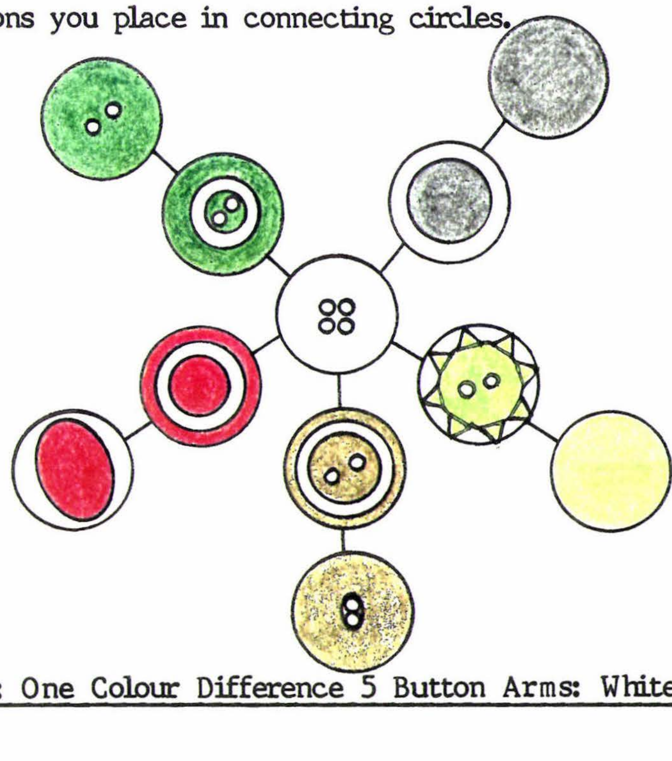


Figure 69a: One Colour Difference 5 Button Arms: White Centre.

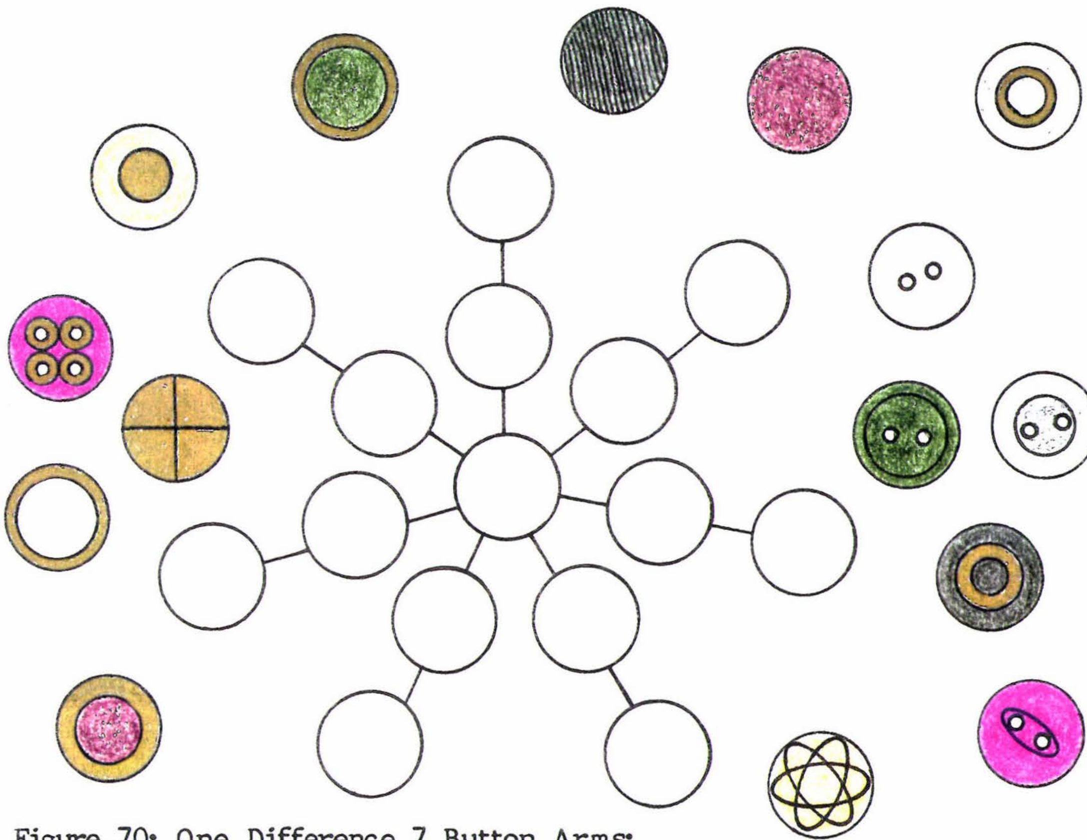


Figure 70: One Difference 7 Button Arms:
Verbal Instructions:

- Find which buttons belong together.
- Arrange the buttons with no more than one thing different between any buttons you place in connecting circles.

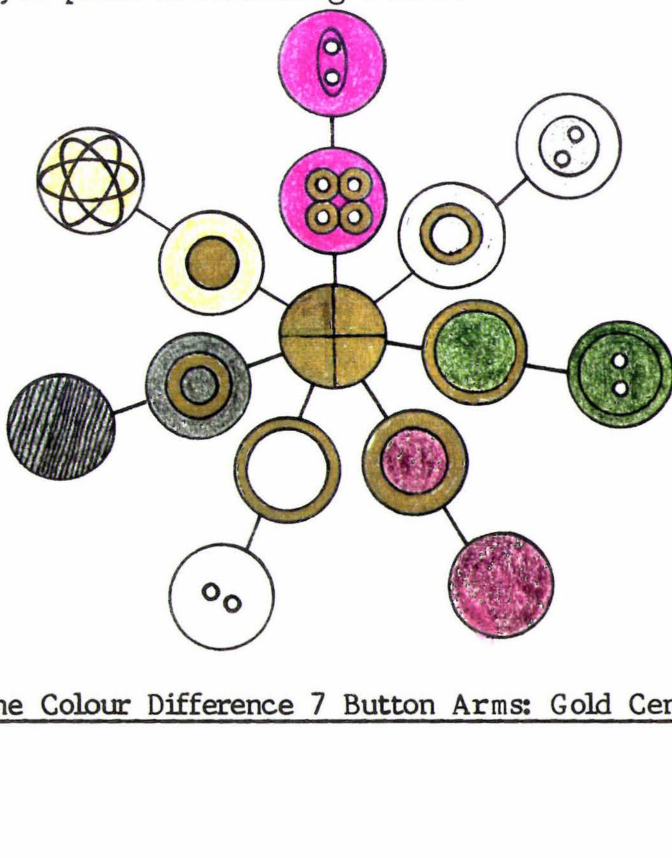


Figure 70a: One Colour Difference 7 Button Arms: Gold Centre.

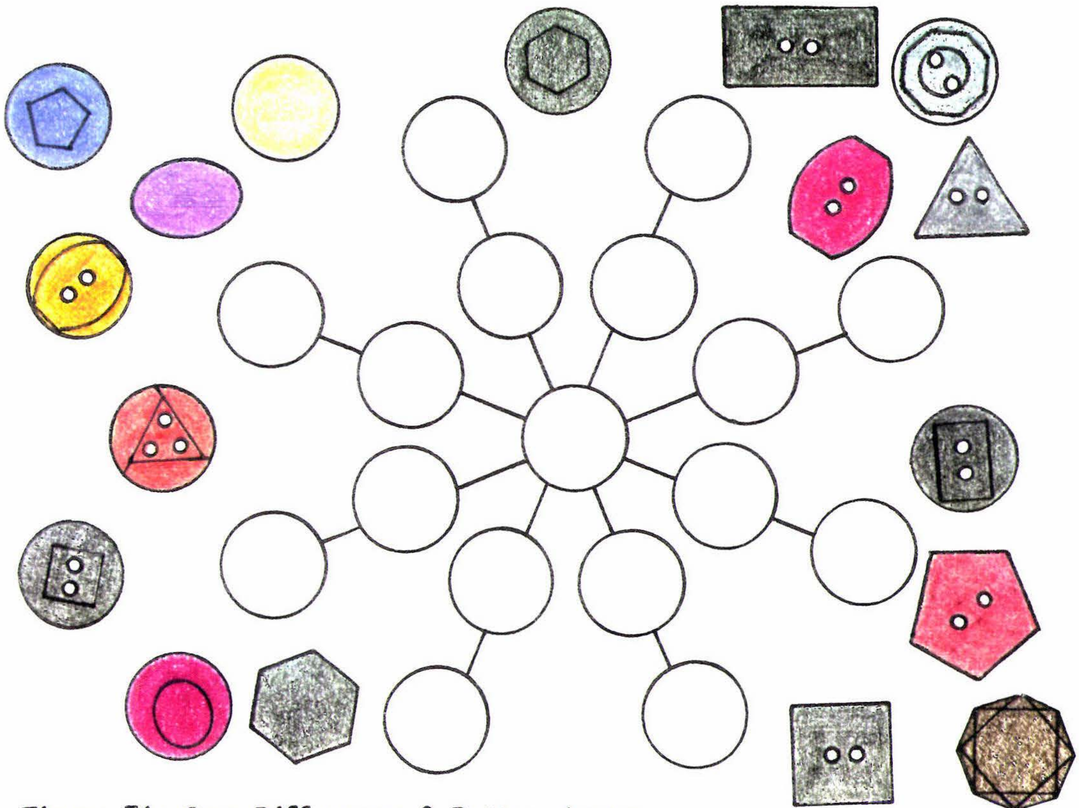


Figure 71: One Difference 8 Button Arms:
Verbal Instructions:

- Find which buttons belong together.
- Arrange the buttons with no more than one thing different between any buttons you place in connecting circles.

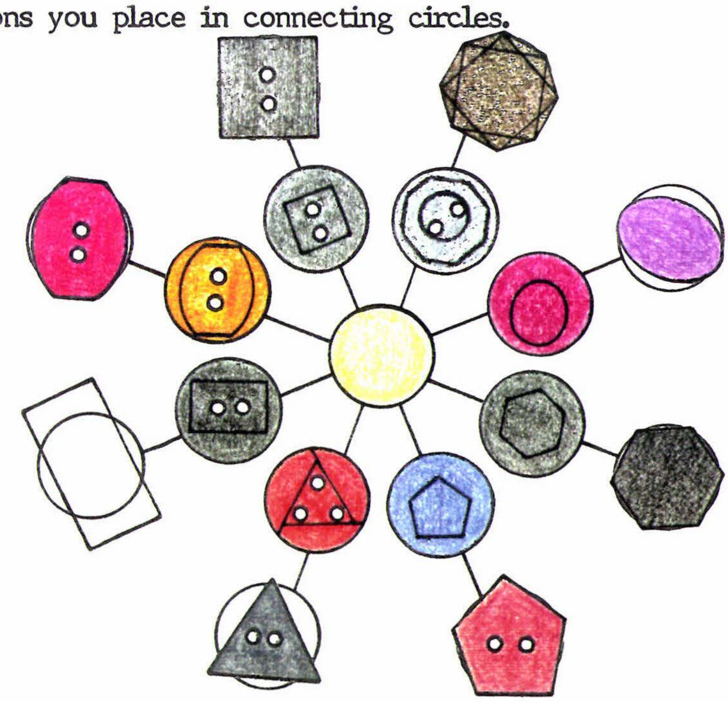


Figure 71a: One Shape Difference 8 Button Arms: Circle Centre.

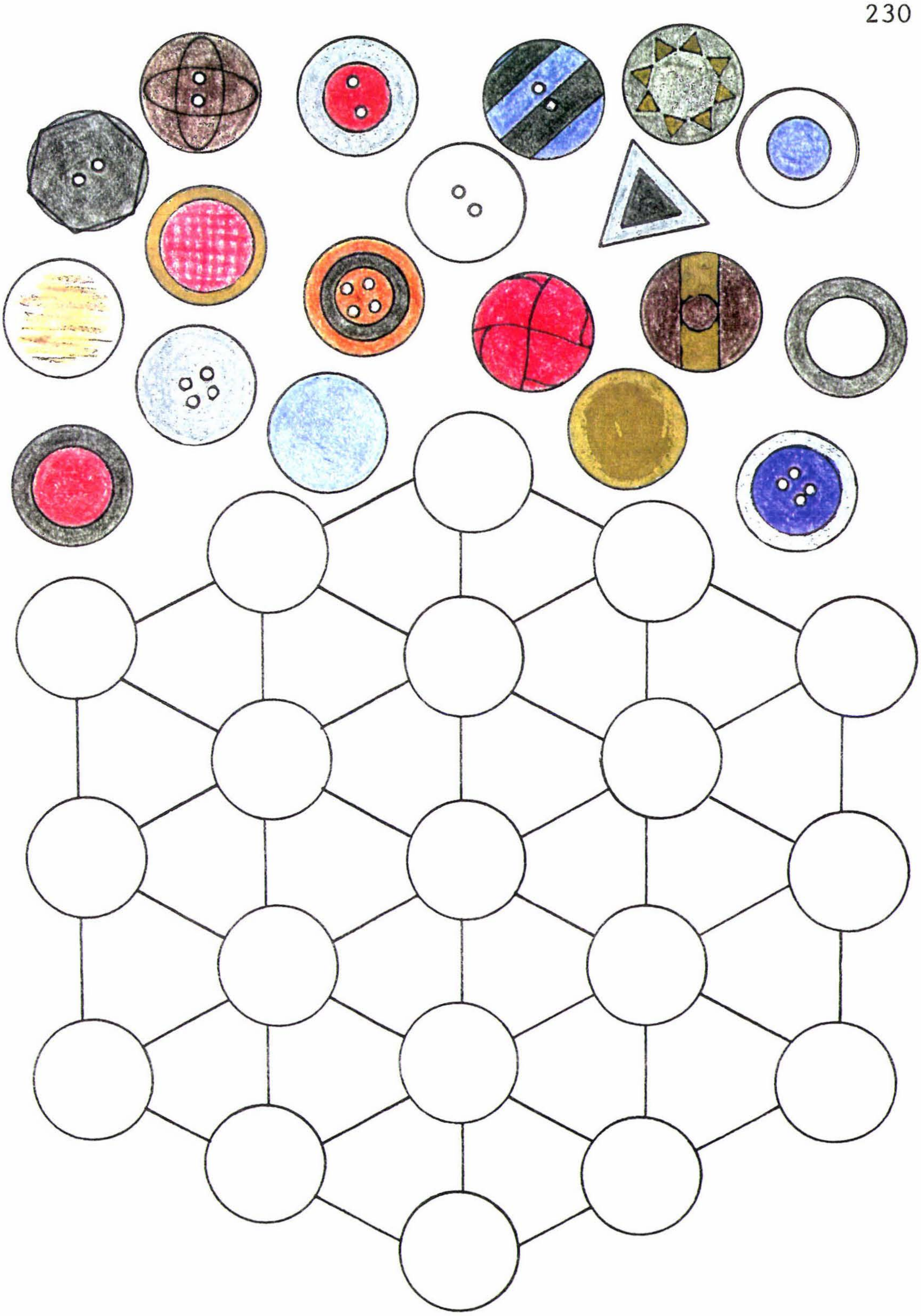


Figure 72: One Difference 6 Intersecting Button Arms:
Verbal Instructions:

- Find which buttons belong together.
- Arrange the buttons with no more than one thing different between any buttons you place in connecting circles.

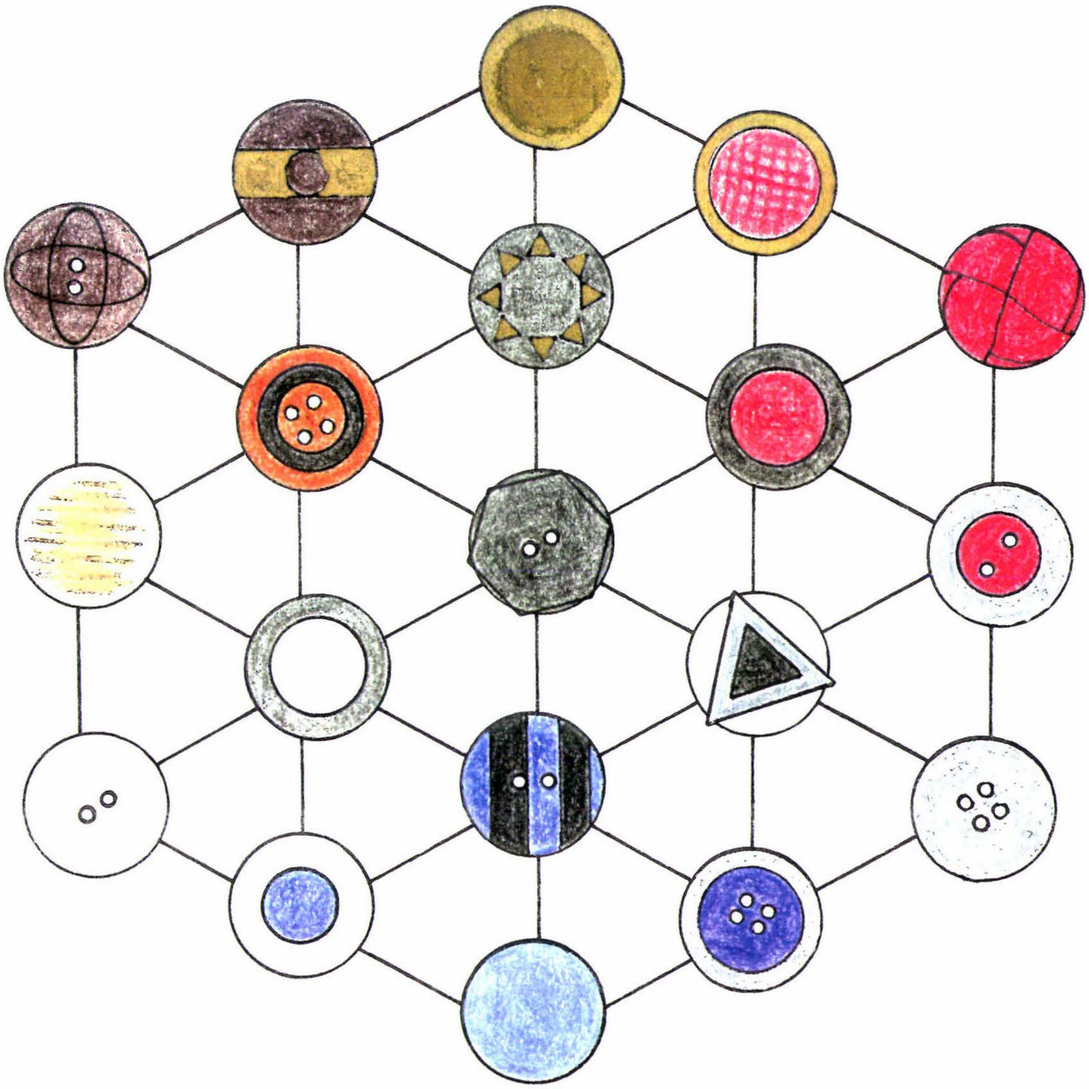


Figure 72a: One Colour Difference 6 Intersecting Button Arms: Black.

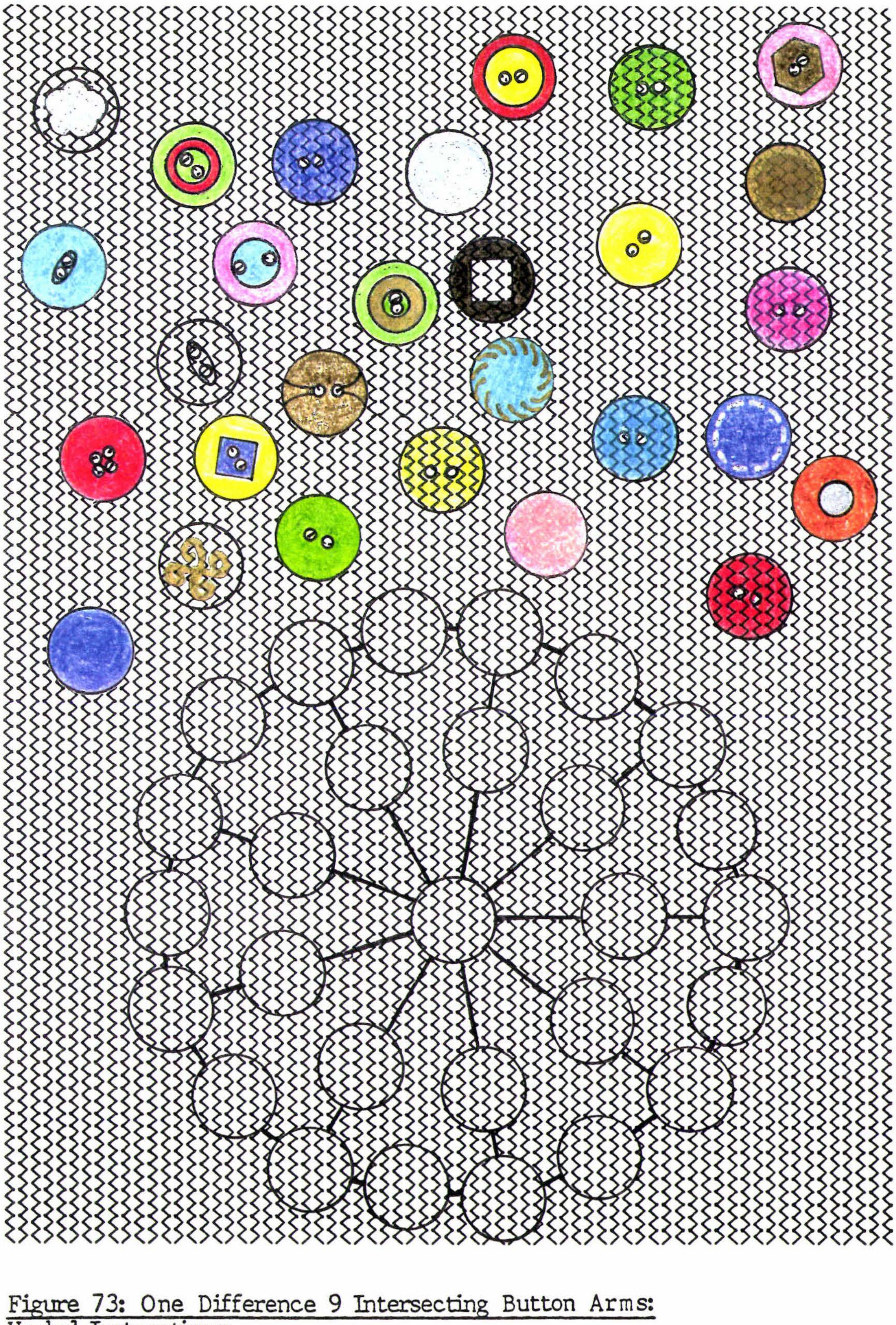


Figure 73: One Difference 9 Intersecting Button Arms:
Verbal Instructions:

- Find which buttons belong together.
- Arrange the buttons with no more than one thing different between any buttons you place in connecting circles.

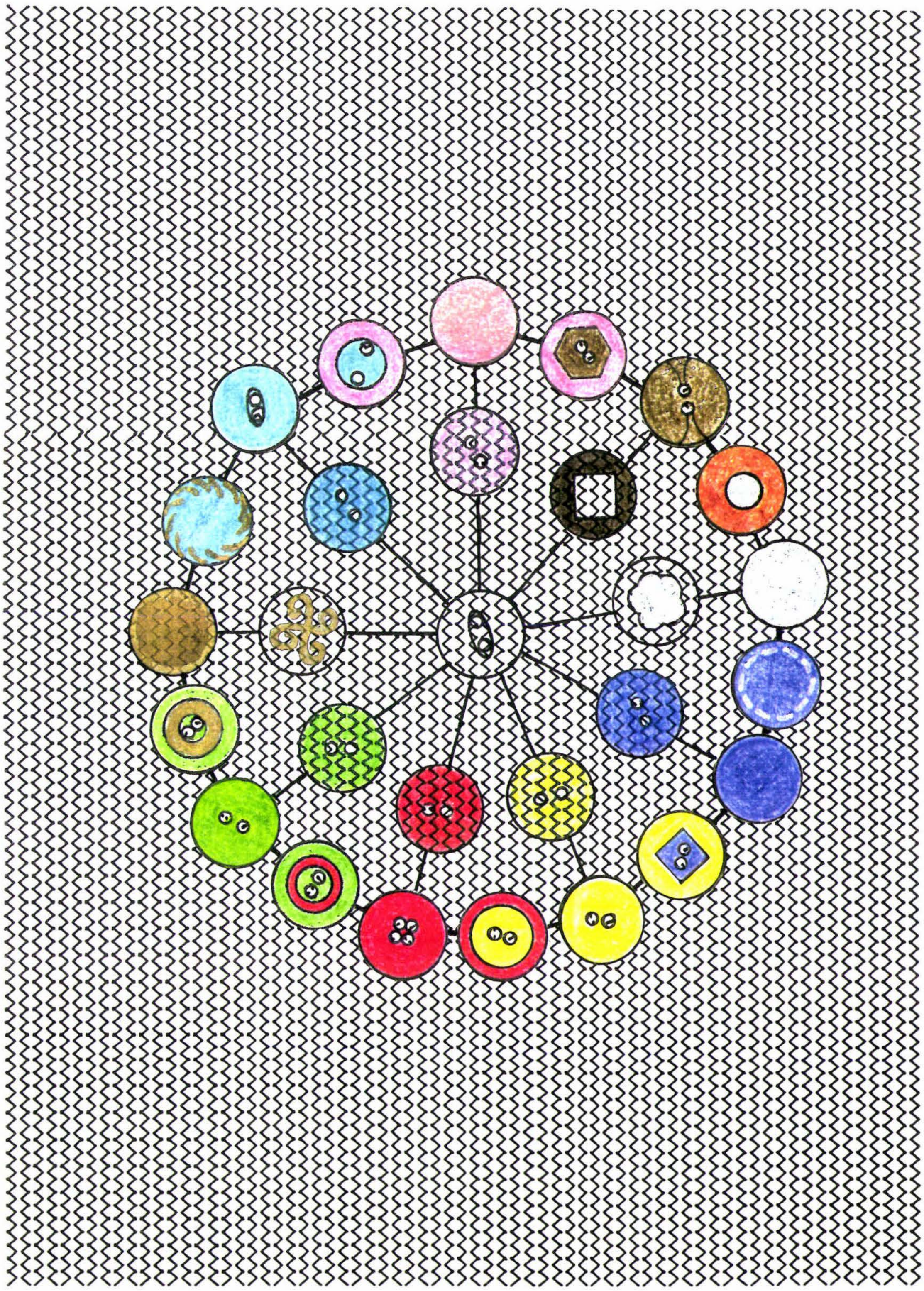
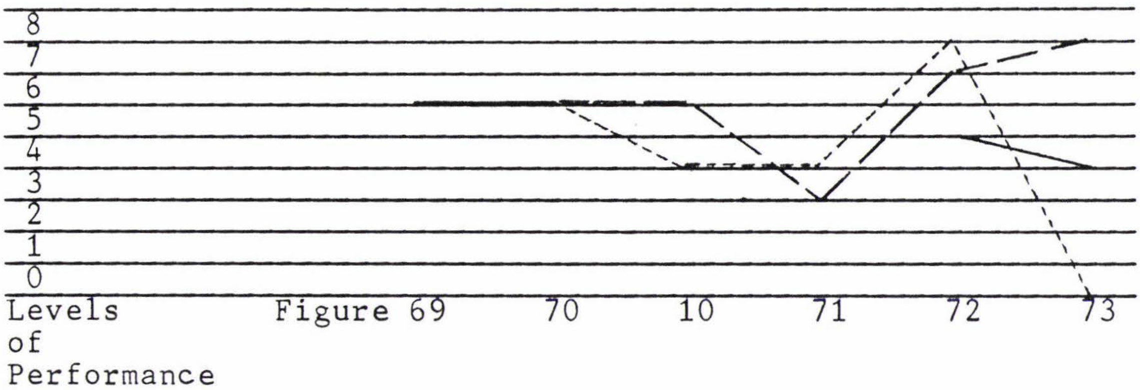


Figure 73a: One Colour Difference 9 Intersecting Button Arms: Transparent.

Figure 73b shows in graphic form the upward and downward movements of performance levels of the three children who attempted four or more puzzles across difficulty levels from Figures 69-70-10-71-72-73. Each of the three children obtained a performance level of 6 for Figures 69 and 70. Maintained and upward movements indicate clear transfer of learning. Decreased movements indicate difficulty in transfer of learning with the addition of multiple concepts to simultaneously attend to. It is interesting that the younger 8 year old AV girl who achieved the highest level of performance of 8 for Figure 72, was unable to solve Figure 73 at all:



Key	Mathematical Group	Age
————————	SE 1	10
— — — —	AV	10
- - - - - - - - - -	AV	8

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 73b: Performance Level Movement: Figure 69-70-10-71-72-73: N=3.

Figure 69: One Colour Difference 5 Button Arms: White Centre.
Figure 70: One Colour Difference 7 Button Arms: Gold Centre.
Figure 10: One Shape Difference 3 Button Arms: Circle Centre.
Figure 71: One Shape Difference 8 Button Arms: Circle Centre.
Figure 72: One Colour Difference 6 Intersecting Button Arms: Black.
Figure 73: One Colour Difference 9 Intersecting Button Arms: Transparent.

For children to solve these puzzles at speed, the use of strategy is necessary, rather than the slow trial and error method used by some children. Counting the number of times a colour or element is repeated across buttons determines the central button with the most connections. The puzzle can then be completed quickly. Figures 72 and 73 required the addition of a particular order of the button arms determined by the intersecting buttons. For Figure 73, a greater number of button arms and a transparency concept meant the difference between the 8 year old AV girl being able to solve one puzzle with intersecting button arms, but not the other, despite clear evidence of her conceptual understanding and previous use of strategy.

Table 37 shows transfer of training between Figure 74, a very advanced senior puzzle, and Figure 75, an adult puzzle, each puzzle using similar conceptual processing to the other, but having different diagrams. (Figure 75 is one of a trio of my adult puzzles marketed in New Zealand and Australia in 1993, with the commercial name 'Arrange-A-Way'). It is interesting that the 10 year old SE 1 girl found both puzzles too difficult, whereas one AV girl decreased and one AV girl increased her level of performance. The younger 8 year old AV girl was unable to complete the more difficult puzzle, despite showing conceptual understanding of the basic structure of the puzzle:

Table 37: Transfer of Learning: Figure 74-75: N = 3.

Figure 74: Intersecting Colour / Pattern Button Sets of Three: Steps.

Figure 75: Intersecting Colour / Pattern Button Sets: Diamond Array.

Mathematical Groups	Figure 74	Figure 75
SE 1	0	0
AV	3	1
AV	4	6

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

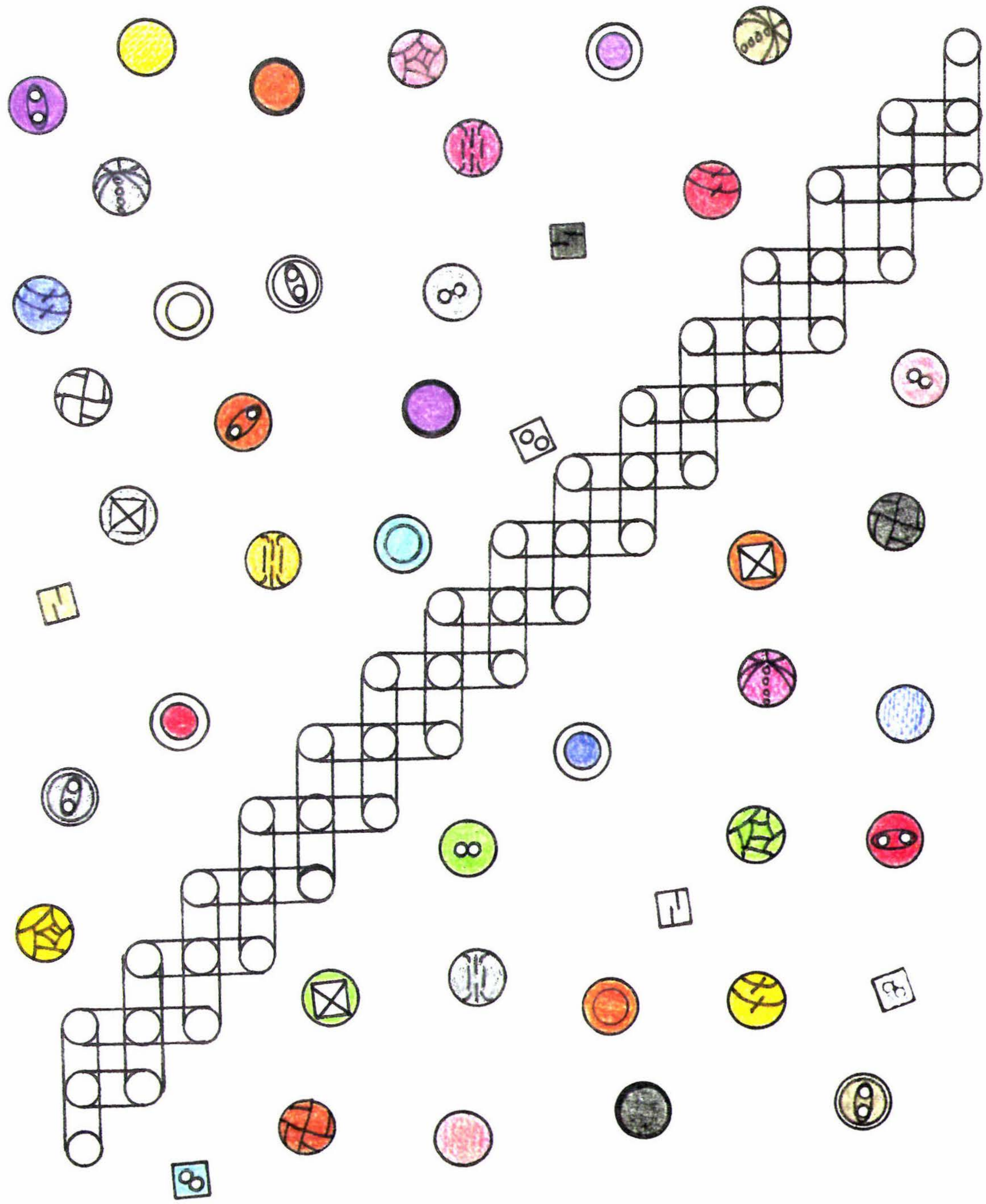


Figure 74: Intersecting Colour / Pattern Button Sets of Three: Steps: Verbal Instructions:

- Each row and column of three connecting circles is a set.
- Find sets of three buttons.
- Choose one set to start. Find another set to connect with it.
- Put each member of each button set together.

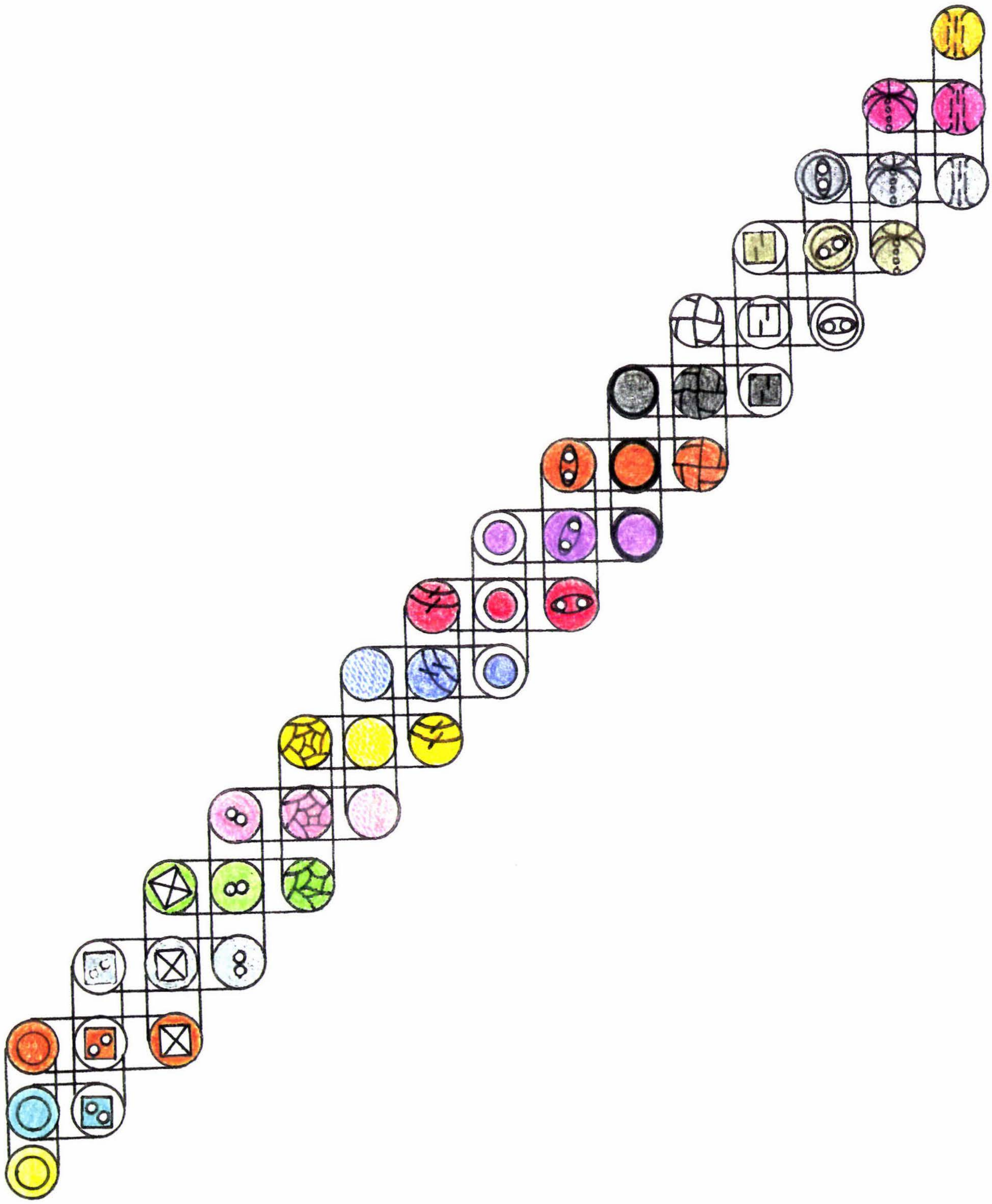
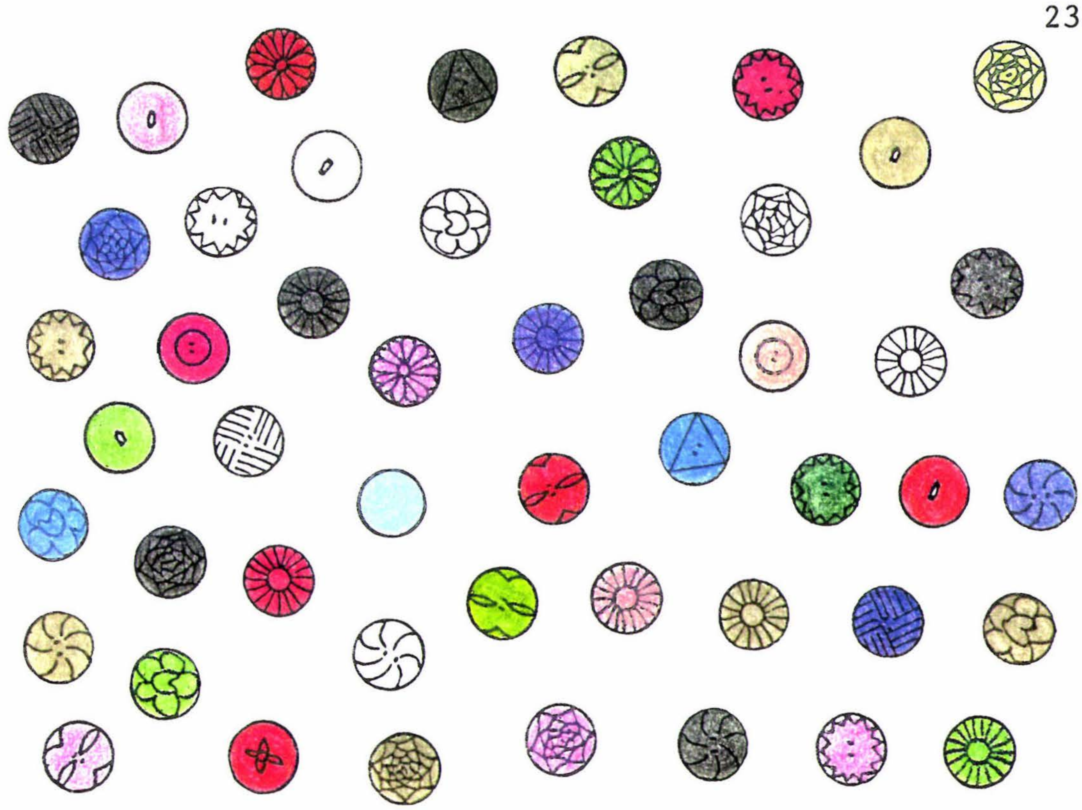


Figure 74a: Intersecting Colour / Pattern Button Sets of Three: Steps:

Verbal Instructions:

- Find colour and pattern sets.



Put each member of each set together.

Figure 75: Intersecting Colour / Pattern Button Sets: Array.
Verbal Instructions:

- Find sets.
- How many buttons in each set?
- Choose one set to start. Find another set to connect with it.
- Put each member of each set together.
- Do not leave any gaps.

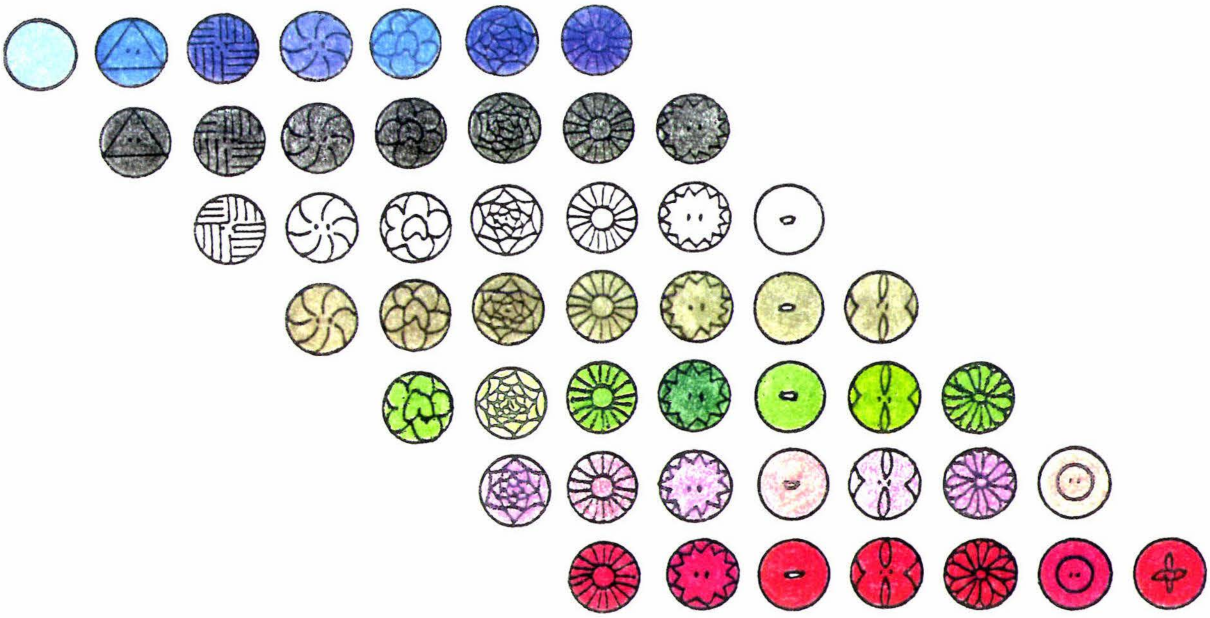


Figure 75a: Intersecting Colour / Pattern Button Sets: Diamond Array.

Strategy:

- Find colour and pattern sets.
- Use columns and rows to connect all of the sets.

Transfer of learning may include transfer of consciously or unconsciously learned concepts, or a method of processing used in one puzzle and transferred to another. So far Chapter 6 shows how children may consciously or unconsciously transfer conceptual learning from one puzzle to another, and consciously transfer a method they have discovered to solve one puzzle to solve another of the same type. It would be expected that children would increase their speed and level of performance between such puzzles. Where levels of performance remained equal, there has been found to be an increase in perceptual and conceptual difficulties created by such variables as elements of colour and size not being held constant, or the introduction of multiple concepts or of a different material type or diagram. These effects create a learning jump, balancing out the positive effect of the transfer of the learning by maintaining an equal level of performance. Children who increase their levels of performance despite these perceptual distractions and conceptual additions show a consolidated and conscious conceptual understanding of the processes they are manipulating. Children who decrease their level of performance, despite conceptual understanding, may be unable to retain and mentally manipulate the number of variables involved simultaneously. This occurs especially with younger children.

Two further related puzzles involving transfer of learning and use of strategy were above average senior puzzles Figures 76 and 77:

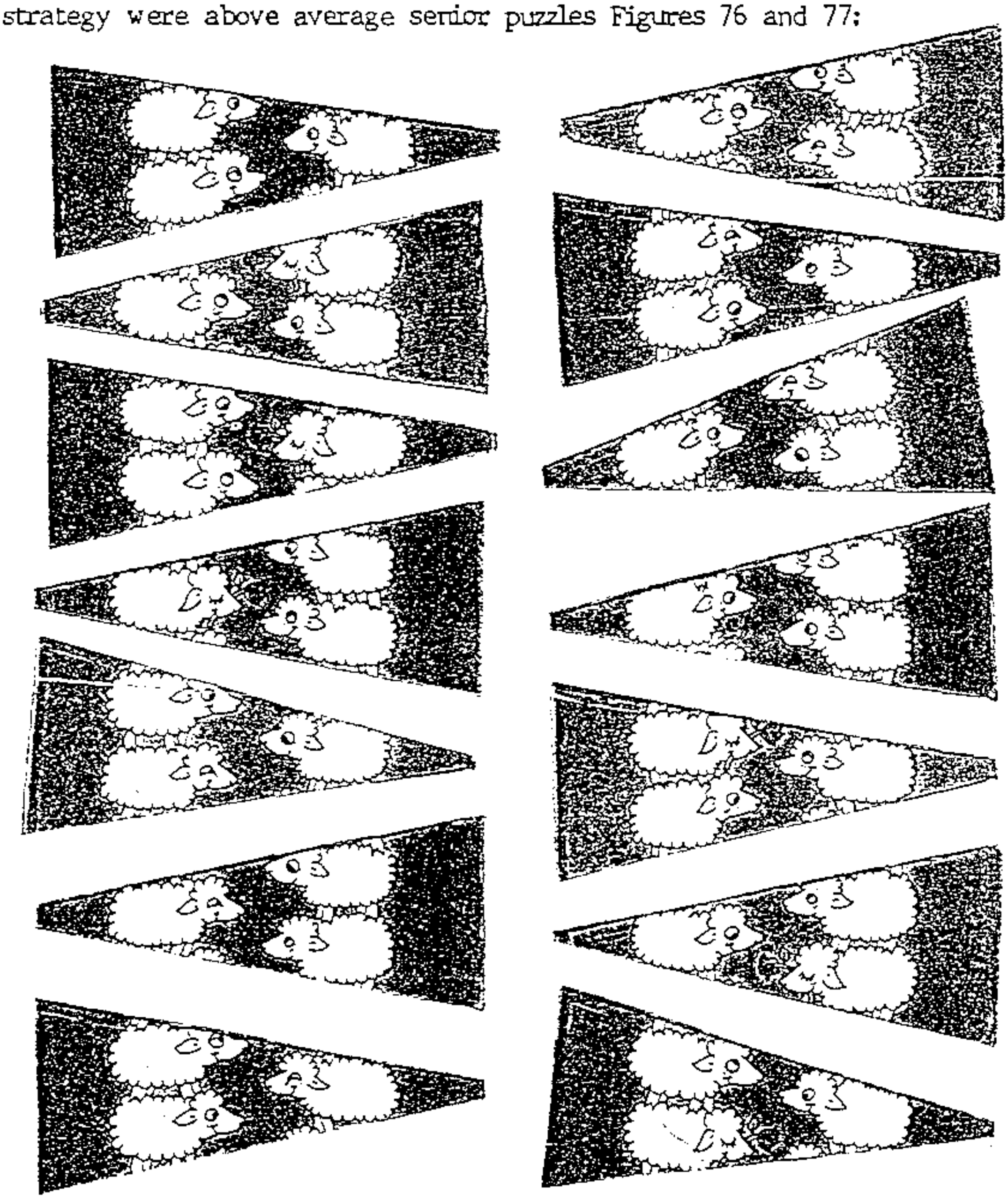


Figure 76: Ordered Sheep Triangles (3's):

Verbal Instructions:

- Put the sheep in order.

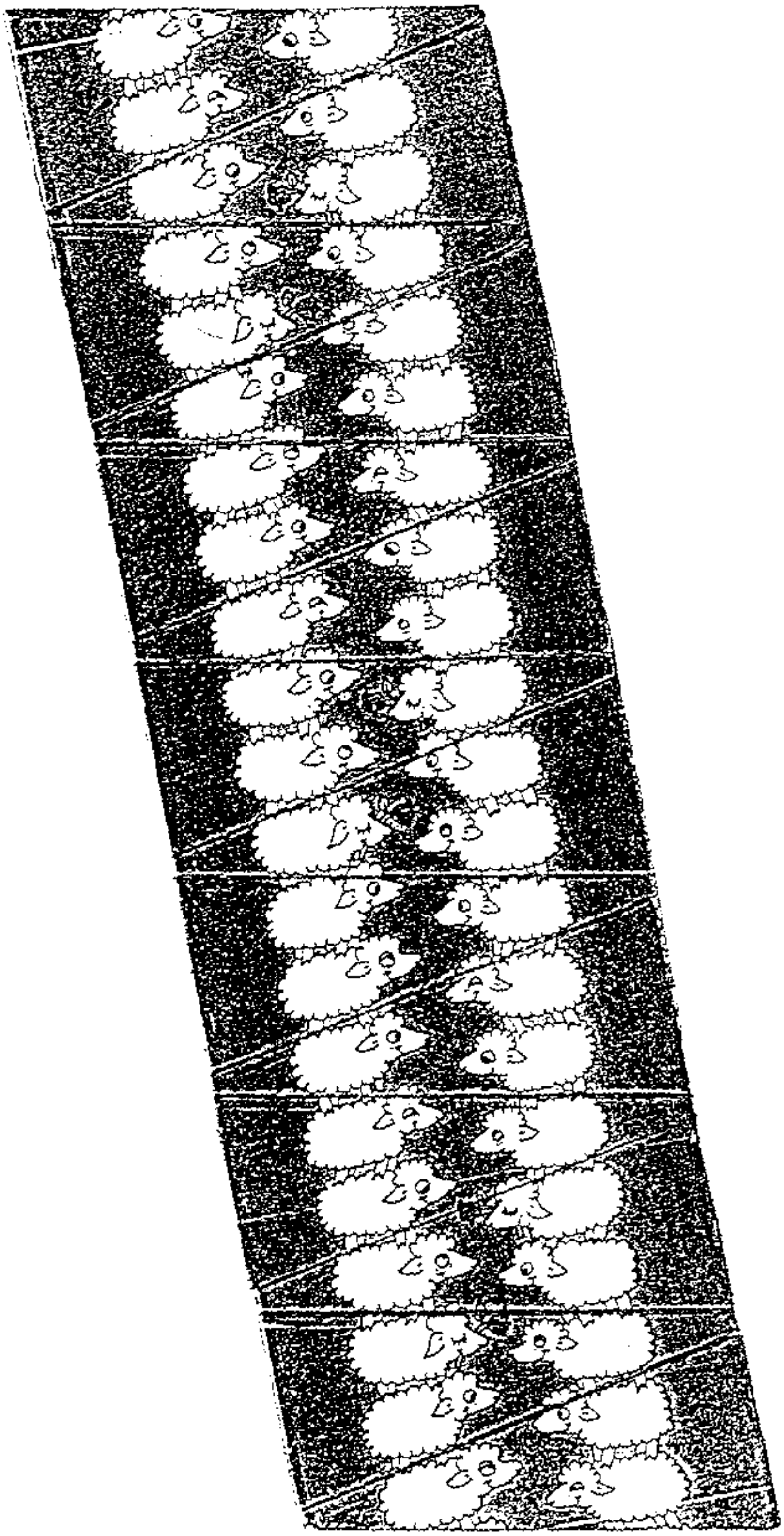


Figure 76a: Ordered Sheep Triangles (3's):

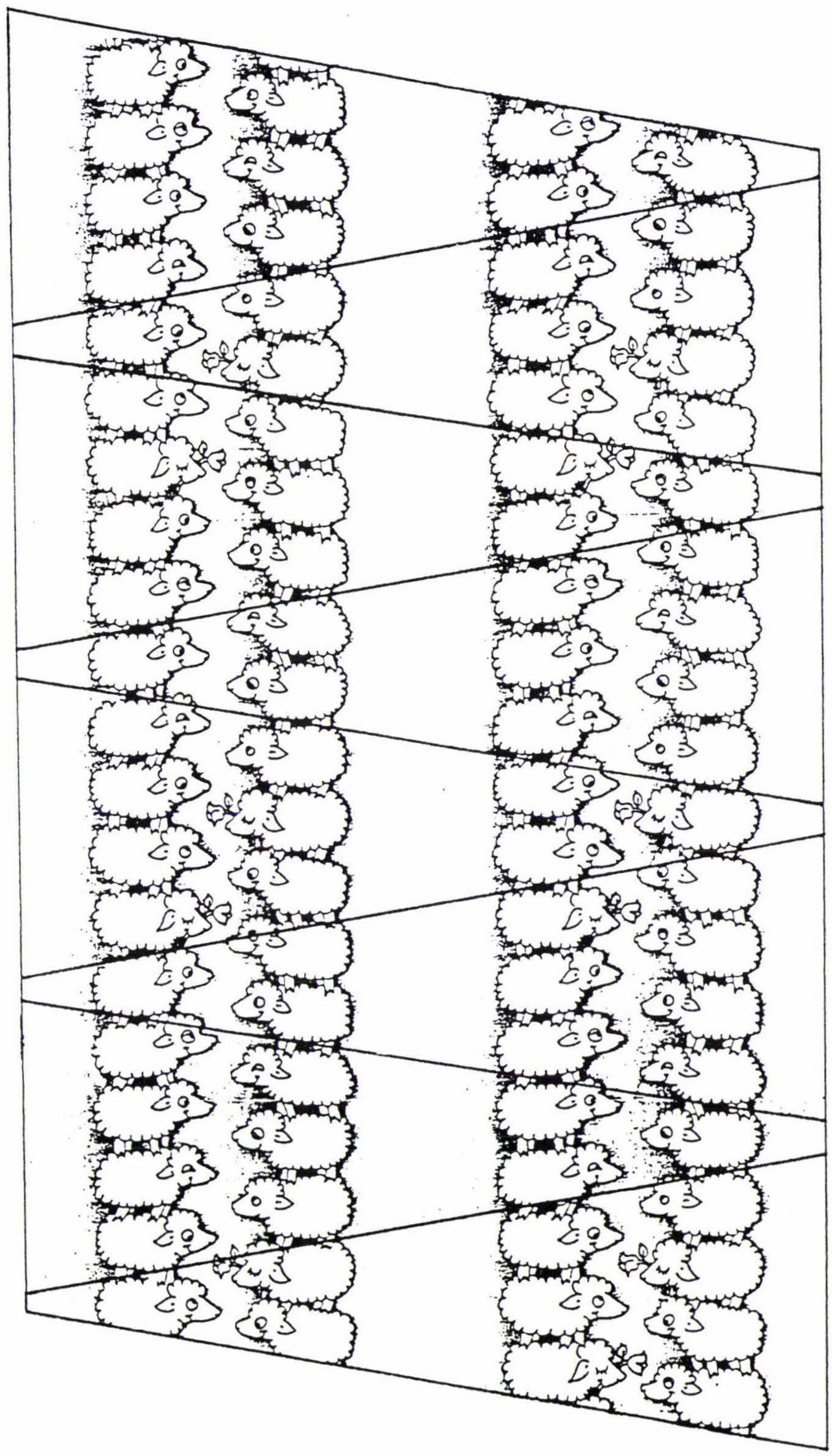


Figure 77: Ordered Sheep Triangles (10's). (Part of Completed Set):
Verbal Instructions:

- Put the sheep in order.

The two AV sisters attempted these two related puzzles, Figure 76 and Figure 77. The 8 year old AV girl ordered some sheep correctly by observing repeated patterns, and incorrectly visually mismatched other individual sheep. With no consistent use of strategy she was unable to transfer any learning to Figure 77 which was too difficult for her. She became confused by the extra number of sheep. Her 10 year old sister attempted Figure 77 only, achieving a level of performance of 7, very fast completion and use of strategy, through applying an ordering strategy examining repeats of particular sheep rather than trying to visually match each individual sheep. Neither child noticed that the last sheep triangle linked with the first.

Use of Strategy

Some puzzles were difficult to solve without the application of a strategy. A strategy is mentally thought out in advance, and a step by step method used to solve a puzzle, indicating advanced conceptual grasp and ability in a child. The use of strategy speeds the process from the time consuming trial and error approach of visual manipulation of objects, reducing the amount of visual attention and manipulation required, to a process executing logic, structure, order or probability. The following six puzzles, advanced senior puzzle Figure 78, and adult puzzles Figures 79, 80, 81, 82 and 83 were difficult to solve without the application of a strategy. Figure 78 is an adaptation of an original puzzle involving probability, using houses, with a differently coloured roof, window and door, by Z.P. Dienes:

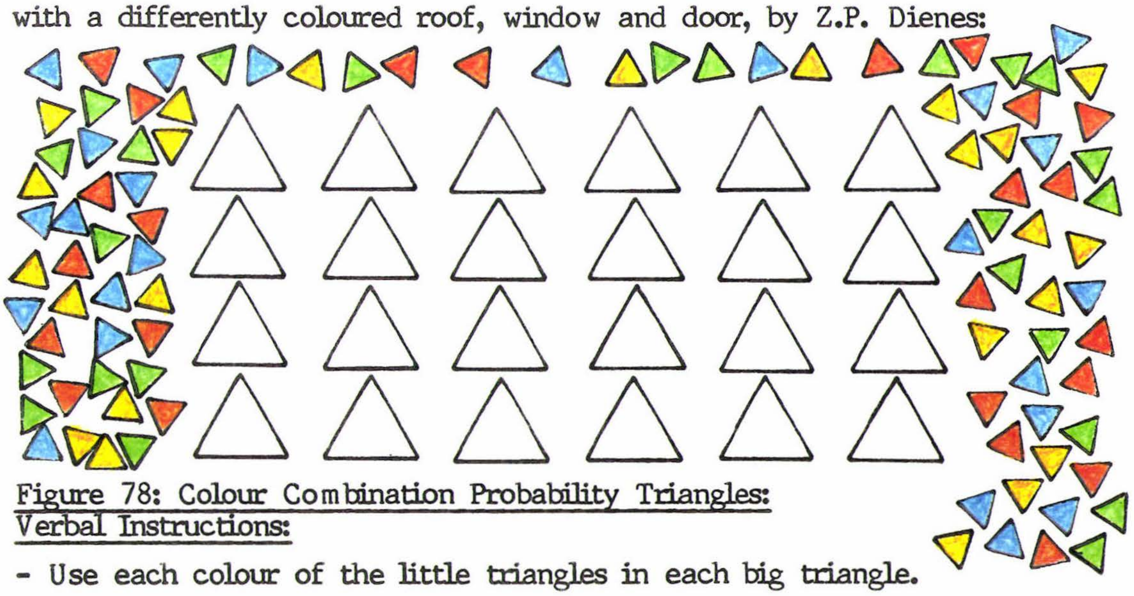


Figure 78: Colour Combination Probability Triangles:
Verbal Instructions:

- Use each colour of the little triangles in each big triangle.
- Make each big triangle different.

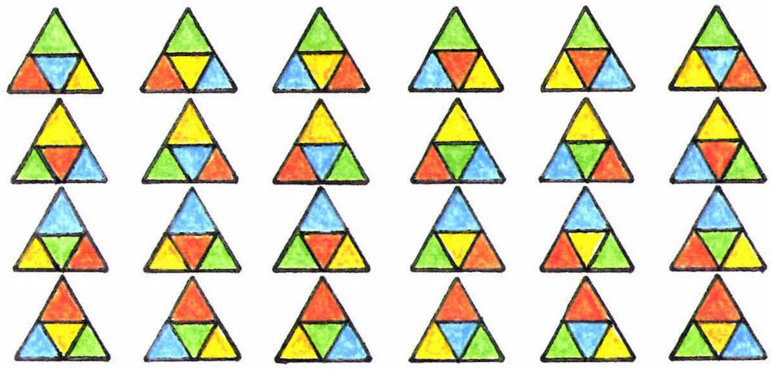


Figure 78a: Colour Combination Probability Triangles:

The 8 year old AV girl completed Figure 78 accurately using a visual checking approach with no strategy. Six other children attempting Figure 78 applied a strategy which they worked out for themselves, giving a different colour to the top points of the big triangles in each row. Five of the six children visually manipulated the other colours. Of these children, two 9 year old SE 1 children completed only half of the puzzle, finding the remainder too difficult. Three 9 and 10 year old SE 1 children using this strategy accurately completed the puzzle. One 10 year old AV girl completed this puzzle very quickly using the above strategy, but extending it to rotate the other three colours within each big triangle, eliminating any visual checking to ensure the big triangles were arranged differently.

Figure 79 is also one of the trio of adult puzzles marketed in 1993, with the commercial name of 'Scottish Nitemare'.

Plate 6:

Figure 79: Checked Fabric Pattern Strips: Two Identical Square Arrays:

Verbal Instructions:

- Make two squares with an identical pattern.

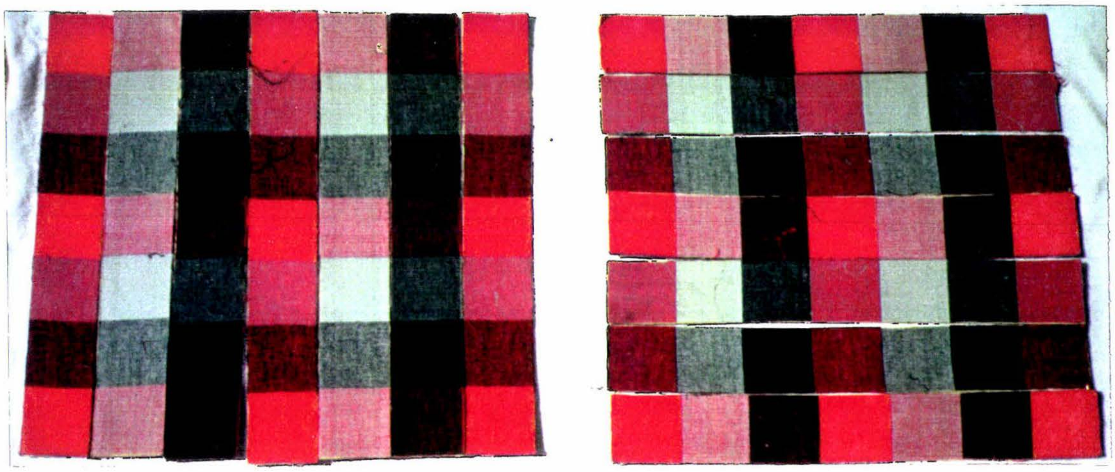


Plate 7:
Figure 79a: Checked Fabric Pattern Strips: Two Identical Square Arrays:
Strategy:

- One square is cut horizontally and one vertically.
- Use any piece to order the pieces in the other square.

Only one SE 1 child was able to solve Figure 79. After much trial and error he discovered the above strategy required to solve this puzzle.

Figure 80 is the third of the trio of adult puzzles marketed in 1993, with the commercial name 'Compare Square'.

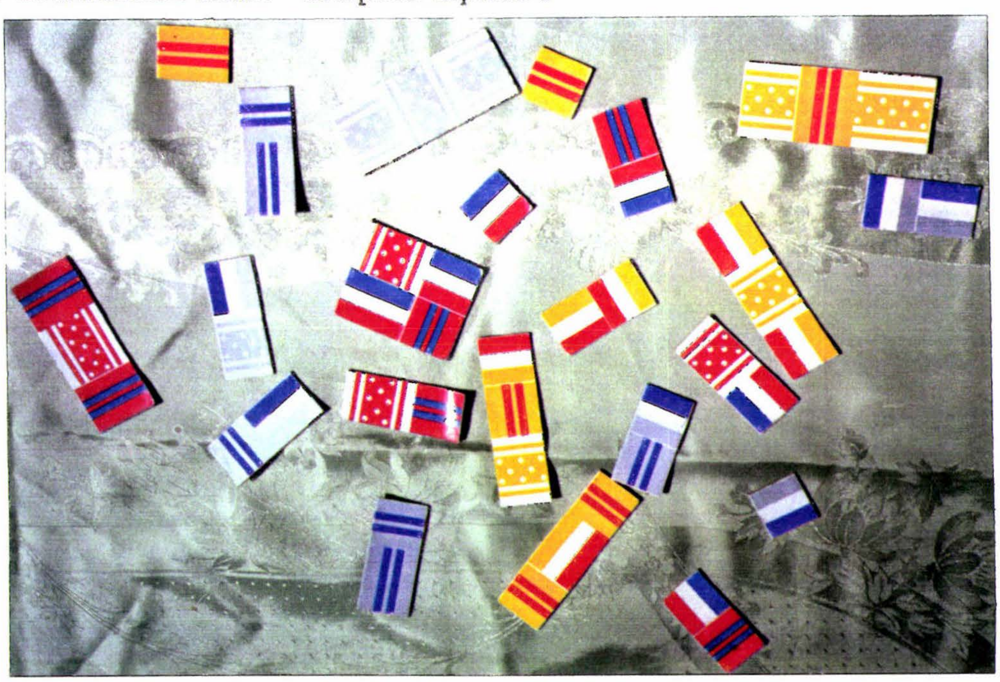


Plate 8 : Figure 80: Three Identically Patterned Squares:
Verbal Instructions:

- Make three squares with an identical pattern.

Plate 9 : Figure 80a: Three Identically Patterned Squares:
Strategy:

- Each colour square is cut differently.
- Find one piece from each colour square that contains the same pattern.
- Arrange each piece to make the pattern in each colour square identical.
- Find other pattern pieces to connect with the first three pieces.
- Make all three colour squares at the same time.

Three children attempted Figure 80. One of the two 10 year old SE 1 children found this too difficult, and the other completed only half of the puzzle, but did use a strategy. The 10 year old AV girl completed Figure 80 in 35 minutes, using a strategy, and with no additional questioning.

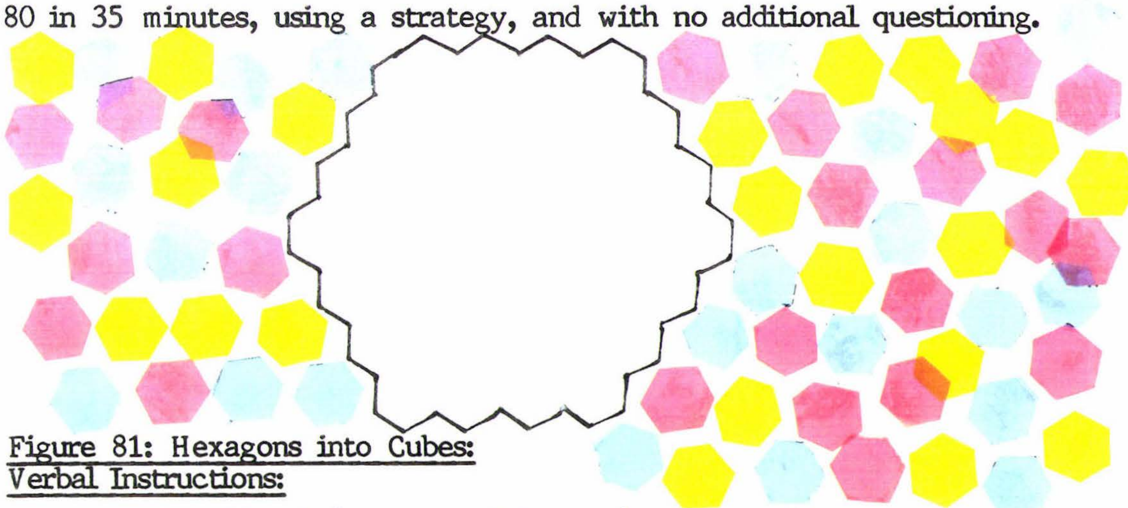


Figure 81: Hexagons into Cubes:
Verbal Instructions:

- Make a tessellated (no spaces left over) pattern of hexagons.
- Intersect the pattern of hexagons with further hexagons overlaid to change the pattern of hexagons into a pattern of cubes.
- Arrange the colours so each adjacent side of any completed cube is distinguished by a different colour.

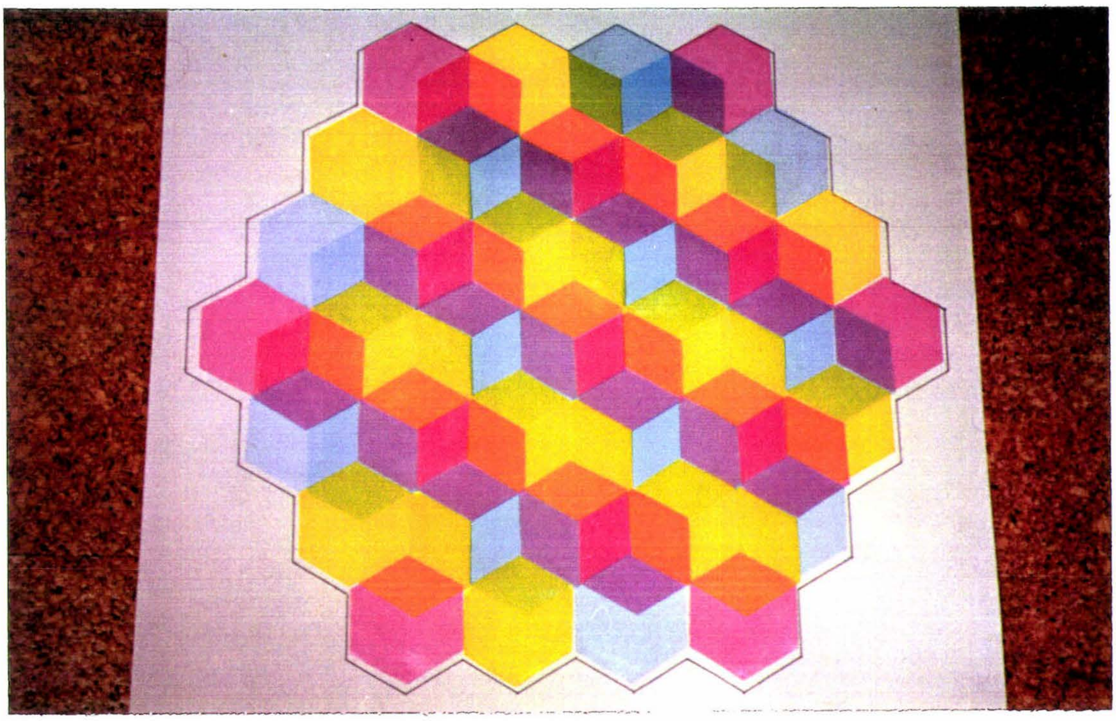


Plate 10: Figure 81a: Hexagons into Cubes:
Strategy:

- Pattern the base pattern of hexagons in an order, such as red / yellow / blue.
- Colour pattern the overlaid hexagons to enable the sides of the completed cubes to be distibguished with different colours.

Four children attempted Figure 81. One 10 year old SE 1 child found this too difficult, and another completed the puzzle accurately after a lot of difficulty intersecting the hexagons to make cubes, although with no problem making the initial colour patterning. The two AV 8 and 10 year old girls completed Figure 81 with no difficulties or questioning.

Figure 82, another adult puzzle, required written instructions:

- There are 7 sets of buttons.
- There are 7 members in each set.
- All buttons except 7 belong to more than one set. These 7 buttons name the sets. Name the sets.
- Find the 7 buttons that name the sets, and place them on the diagram.
- Arrange the remaining intersecting buttons into an array, so each member of each set is in continuous linkage with its other members.
- Notice the relationships at triangle points.

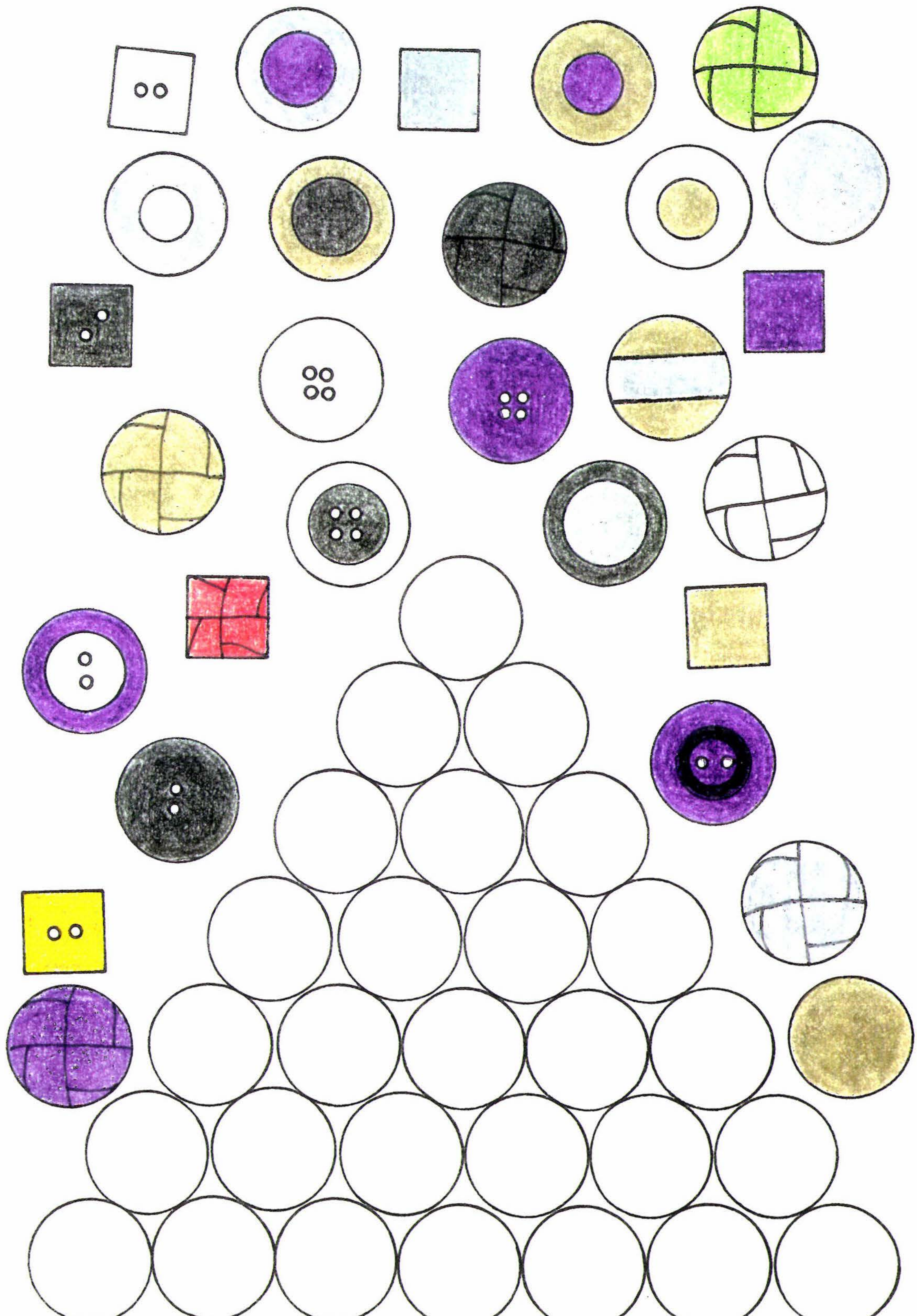


Figure 82: Seven Set Triangular Button Array.

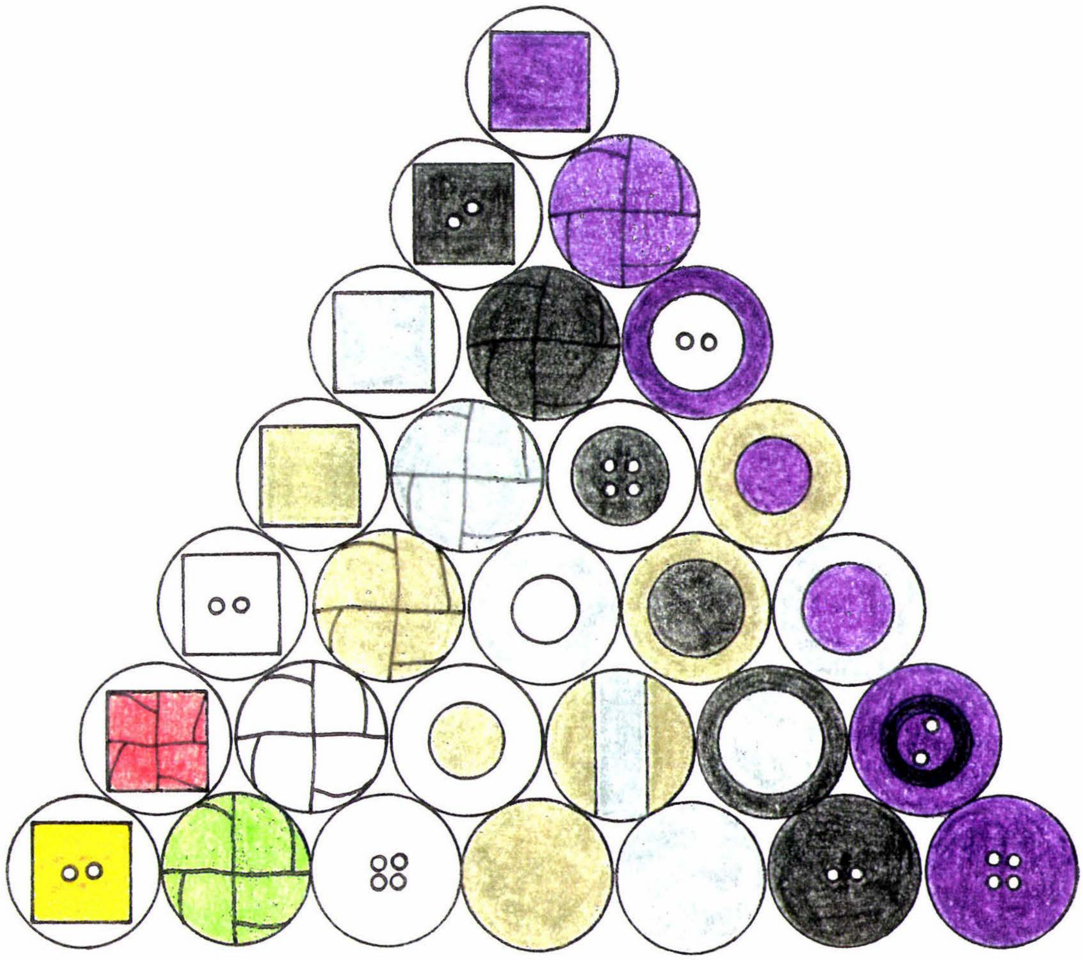


Figure 82a: Seven Set Triangular Button Array:
Strategy:

- Place the 7 set naming buttons along one side of the outside of the big triangle, such as along the base.
- A fast way to find the correct intersecting buttons is to add the elements from the two bottom points of any triangle within the diagram, and place the button containing this addition of elements at the top point of the triangle concerned.
- Observe the pattern of ticks the continuous linkage of sets creates.

Figure 82b shows Figure 82 levels of performance across mathematical groups. The 10 year old SE 1 girl with a level of performance of 2 needed extensive questioning to find the set naming buttons. She placed these along the base of the triangle with no further questioning, but needed questioning to intersect the remaining buttons. The 10 year old SE 1 boy on level 3 followed the written instructions unaided, made a baseline of set naming buttons and began intersecting them up the triangle. Because there were some mistakes in the baseline, he was unable to complete the intersections without questioning to find the correct set naming buttons. The 9 year old SE 2 boy on level 3 needed questioning to find the baseline set, but completed the remainder of the puzzle unaided. The two AV sisters completed Figure 82 quickly and unaided, with the 10 year old completing this puzzle in 15 minutes.

7								
6						2		
5								
4								
3							1	1
2								1
1								
0								
Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1

<u>Levels of Performance</u>

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with strategy and extensive questioning
3 - Complete with strategy and some questioning
4 - Complete with strategy and minimal questioning
5 - Complete with strategy and no questioning
6 - Very fast completion with strategy and no questioning
7 - Exceptional or novel performance

<u>Figure 82b: Seven Set Triangular Button Array: Levels of Performance Across Mathematical Groups. n = 5.</u>

Figure 83 is the final adult puzzle presented in Chapter 6. The partly completed designs are glued onto the diagram:

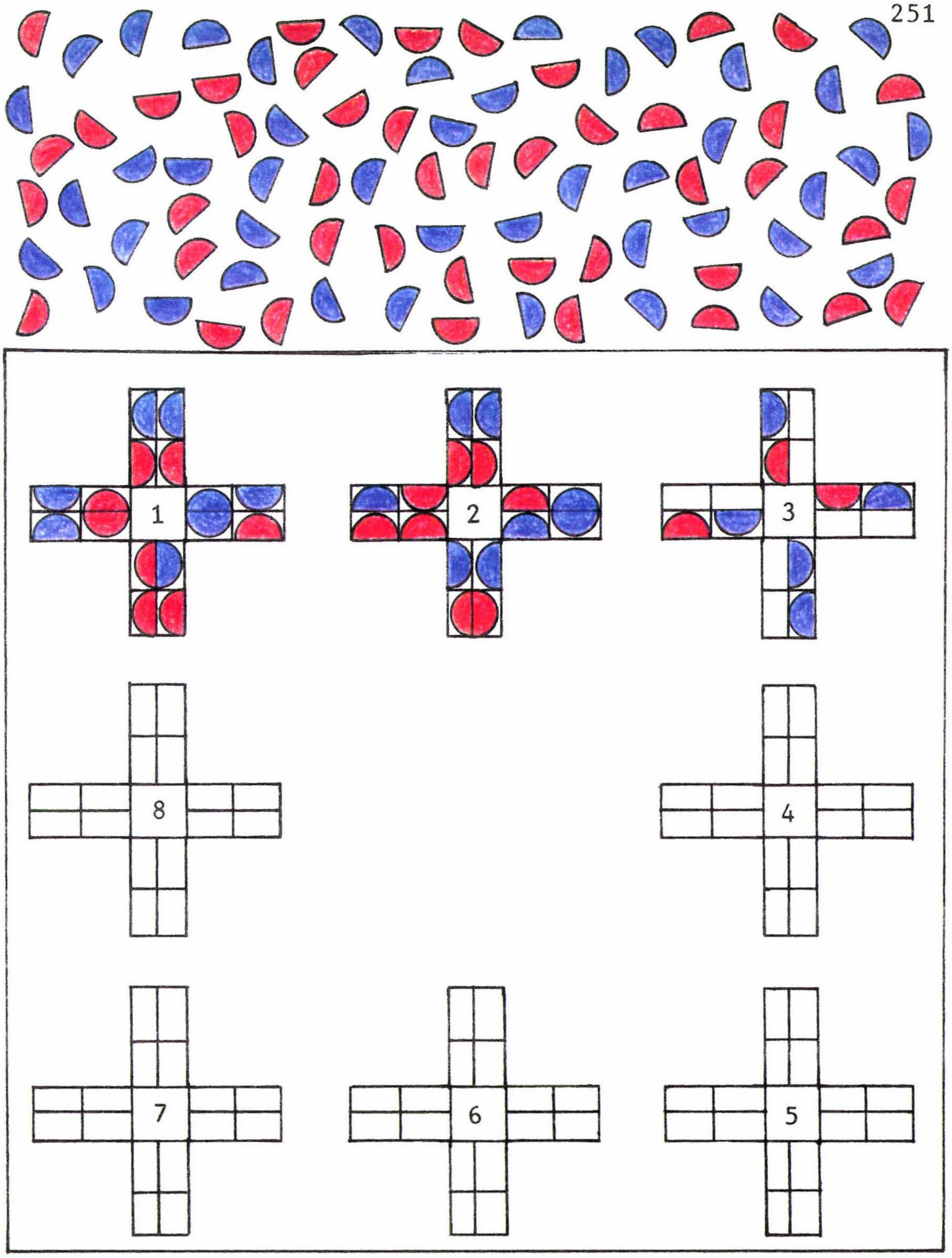


Figure 83: Rotational Circuit:
Verbal Instructions:

- Design 1 is the starting pattern.
- Design 2 is a rotation of Design 1.
- Design 3 is a half completed rotation of Design 2.
- Complete the rotations so Design 8 rotates to link with Design 1.

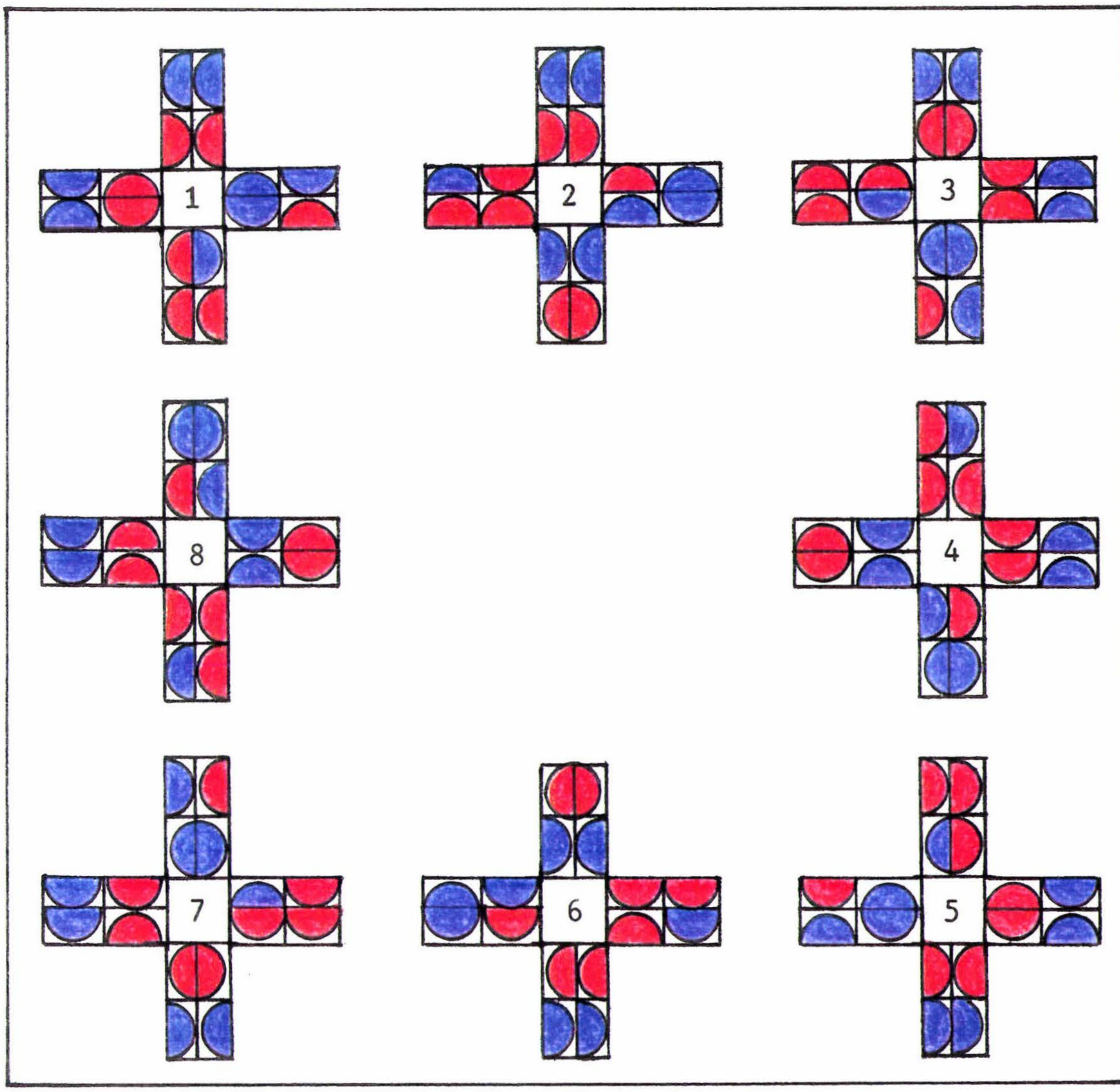


Figure 83a: Rotational Circuit:
Strategy:

- As the designs make a complete rotation around the diagram, Design 5 will be upside down from Design 1, and Design 7 upside down from Design 3, and so on, with Design 2 and 6, and 8 and 4 also upside down from each other.
- Similarly, Design 3 is a quarter turn from Design 1 etc.
- There are eight diagrams. The degree of rotation is a one eighth turn.

This puzzle was too difficult for one SE 1 10 year old. The only other child in this study to attempt Figure 83 was the 10 year old AV girl who completed it accurately in 20 minutes, using a visual approach, not the above strategy. One 12 year old in a 1989 study has used this strategy.

Symmetry

Use of strategy, often involving rotation and symmetry, or awareness of rotation and symmetry in a puzzle such as Figure 83, indicates ability in a child. Figure 84, an advanced junior puzzle, (Plate), illustrates the issues of symmetry, strategy and rotation at a more elementary level:

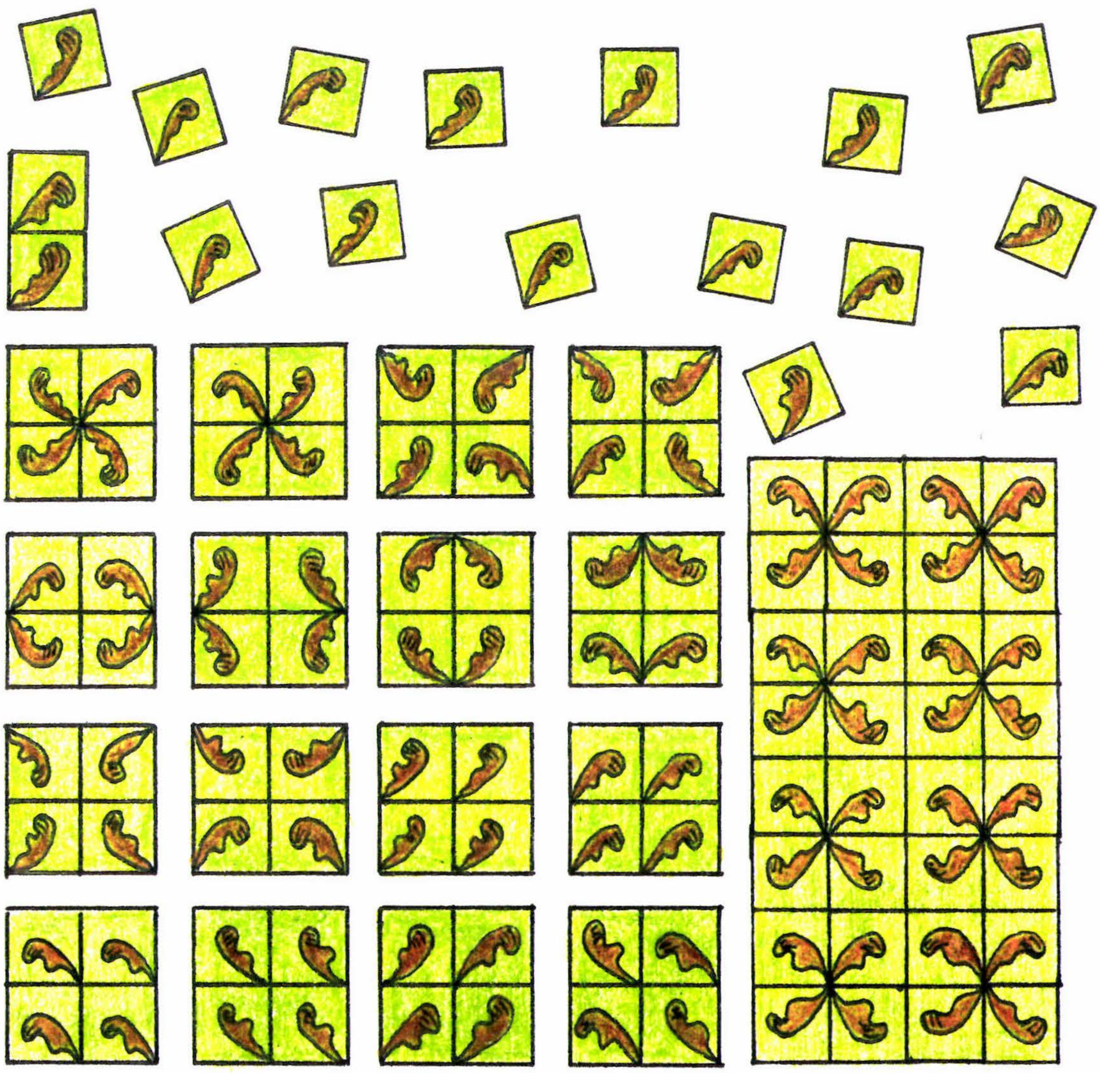


Figure 84: Sycamore Seeds:
Verbal Instructions:

(a) - Take four seeds at a time. Make each set of four seeds into a square.
Make as many different patterns in the squares as you can.

(b) - Make one big symmetrical design.

Table 38 shows levels of performance for Figure 84. Children are tabled in order of level of performance. About half of the children attempting Figure 84 created symmetrical patterns. There was no distinction between the mathematical group range of children who made symmetrical patterns and children who did not. It is interesting that all of the children who followed the different patterns of four with one large design of tessellated X shapes did not show any use of symmetry in their earlier different patterns of four. This could be an incubation effect, or it may be that the focus of attention on making all of the patterns of four different was a distraction element. No child used any kind of strategy to order and rotate their different patterns of four.

Table 38: Performance Levels for Figure 84: Sycamore Seeds: N = 18:

Age	Mathematical Group	Number of Different Patterns	Symmetry	Strategy	One Big Creative Design	Symmetry
10	SR	6	-	-	-	
9	SR	6	-	-	-	
6	JE	6	-	-	-	
6	JR	6	-	-	-	
10	SR	6	-	-	1	X shapes
10	SR	12	-	-	-	
8	SE 1	6	-	-	1	X shapes
7	JE	6	-	-	1	X shapes
8	SR	12	-	-	-	
9	SR	12	some	-	-	
7	JE	4	4	-	-	
6	JE	6	6	-	-	
7	JR	some	all	-	-	
6	JR	some	all	-	-	
6	JE	8	8	-	-	
10	AV	many	all	-	-	
8	AV	10	10	-	-	
9	SR	10	10	-	1	1

Summary

Transfer of learning in Chapter 6 is described as conscious or unconscious transfer of consciously or unconsciously previously learned concepts, or conscious transfer of a method used to solve one puzzle successfully, to another puzzle with similar elements. The Venn Diagram series of puzzles showed most children maintaining levels of performance across the first three puzzles, where small changes in difficulty level manifested as perceptual variables and numbers of items not being held constant across

Chapter 7

Diagnostic Progressions

Chapter 7 presents five puzzles demonstrating progressive conceptual levels of performance in children. Progressive conceptual levels of performance aid the diagnostic assessment of a child's level of conceptual understanding, and help specify perceptual and conceptual difficulties the child may be experiencing in early mathematical development. Each puzzle illustrates examples of some issues and concepts discussed in previous chapters. These include unconscious processing where a classification is made with the child unable to verbalise the classification; perceptual priority modes; perceptual difficulties such as problems with colours, lines, reversals, symmetry and rotations; problems with multiple concepts such as classifying and ordering, classifying with consecutive ordering, alternate ordering and rotational ordering; confabulation; transfer of learning; and conscious use of strategy. The first three puzzles do not involve enumeration and are presented in order of difficulty. The last two puzzles do involve enumeration and are also presented in order of difficulty.

Puzzles Without Enumeration

Figure 85, Fabric Butterflies, an advanced junior puzzle, is a pattern matched, colour grouped array involving bilateral symmetry. Each butterfly is divided into two separate wings to be matched together. Children give clear diagnostic information of perceptual difficulties and conceptual understanding by exhibiting different ability levels of matching within this puzzle. One 8 year old SR girl was unable to rotate the wings to make butterflies. She had difficulty even seeing butterfly wing shapes. Because of her absorption in wing rotation, she did not notice wing pattern or colour, and her butterflies were wing pattern and colour mismatched. Wing rotation was so difficult for her, she did not complete the set. This is similar to the level of performance demonstrated in Figure 28, Yachts, where some children could not rotate bilateral sails to make up the yachts. Some children were able to make butterflies without any difficulties with rotation or bilateral symmetry, but were unaware of pattern and colour matching possibilities. Other children who were able to pattern and colour match individual butterflies successfully, were unable to find pattern matched pairs of butterflies with different colours. Other children successfully matching butterfly pairs were unable to place colour groupings into a final colour / pattern matched pair array. Plates 11 and 12 show Figure 85 in incomplete form, and with some levels of matching.

Plate 11: Figure 85: Fabric Butterflies:
Verbal Instructions:

- Make butterflies.
- Find butterflies that belong together.

Plate 12: Figure 85: Fabric Butterflies.

Plate 13 shows a partially completed butterfly wing colour and pattern matched array.

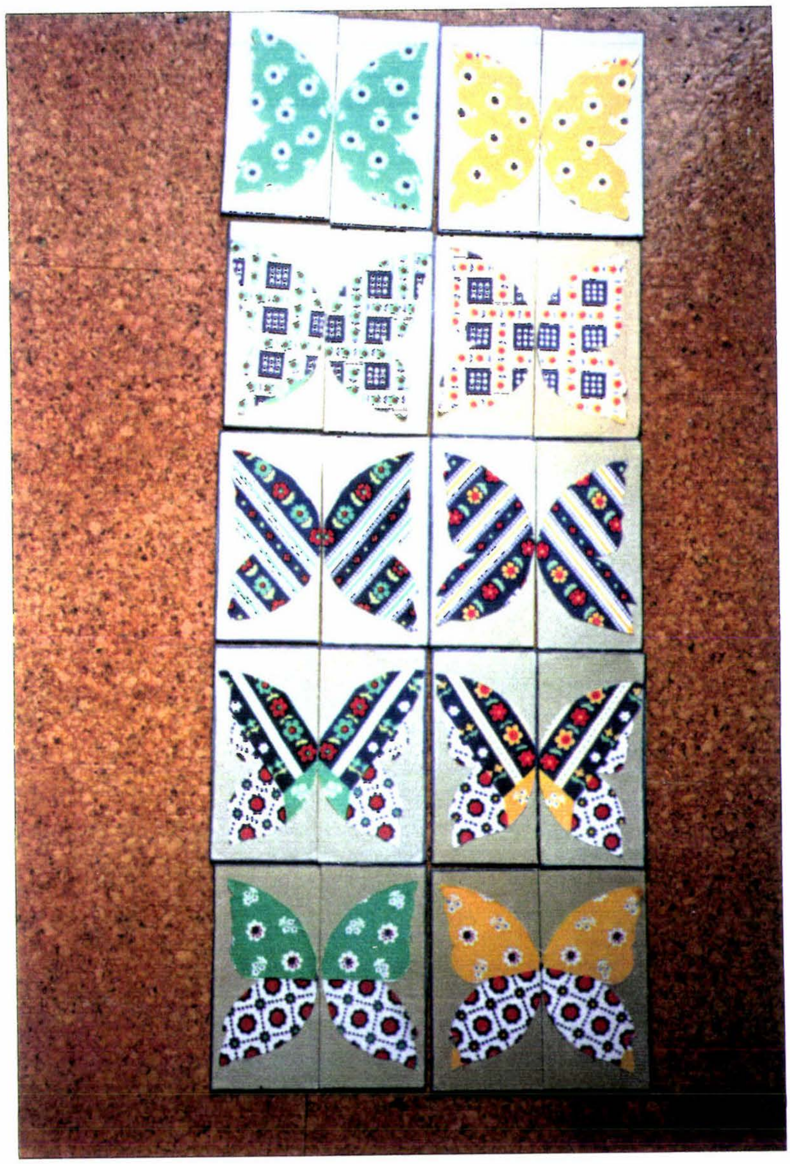


Plate 13: Figure 85a: Fabric Butterflies:
Verbal Instructions:

- If the child has made anything less than a complete array:
- Can you find another way to match your butterflies?
- If a child can not find another way, but has randomly matched correctly within the group of mismatches:
- Can you tell me why you matched these butterflies together?

Inability to match butterfly wings into a butterfly indicated difficulty with rotation of reversed shapes into bilateral symmetry. A butterfly given correct wing shape with each wing differently coloured and patterned showed lack of colour and pattern awareness. Pattern matching difficulties evidenced as correct colour and pattern matched wings formed into butterflies, with distraction from the two differing colour groups interfering with perceptual and conceptual processes preventing children finding purely pattern matched butterfly pairs. Inability to manipulate multiple concepts and tasks simulataneouly and consecutively contributed. Lack of awareness of two colour groupings eliminated the possibility of array formation. These perceptual and conceptual difficulties did not correlate well with mathematical groups. The highest performance was from junior children and an 8 year old AV girl; the lowest from an 8 year old SR girl. Subject numbers are too small to attribute reasons other than chance. Figure 85b shows performance levels across mathematical groups.

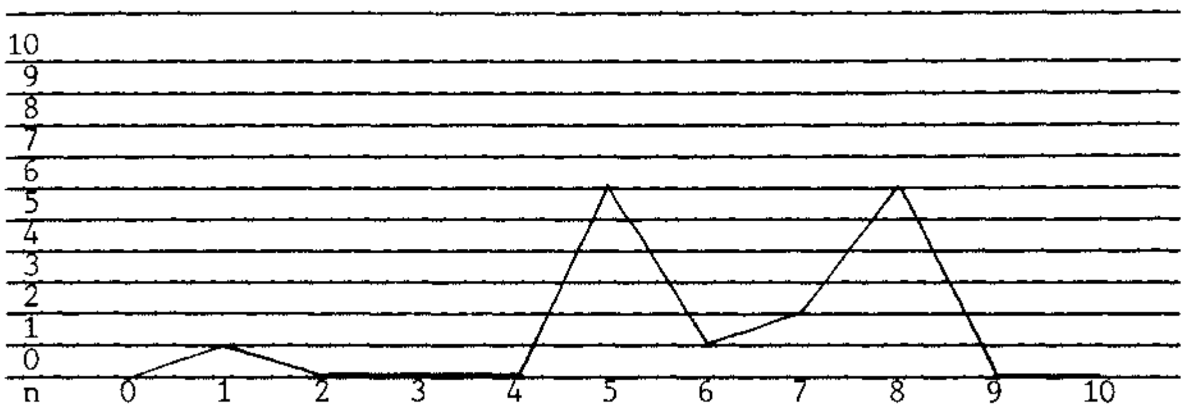
Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
10								
9								
8		3	2			1		
7		1	1					
6			1					
5		6						
4								
3								
2								
1					1			
0								

Levels of Performance

0 - Bilateral rotational wing shape matching too difficult
1 - Difficult and incomplete wing shape matching
2 - Difficult but complete wing shape matching
3 - Complete wing shape matching with no pattern matching
4 - Wing shape match: incomplete colour/pattern matching
5 - Wing shape matching: complete colour/pattern matching
6 - Pattern matched some butterfly pairs
7 - Pattern matched all butterfly pairs
8 - Colour array with questioning
9 - Colour array with no questioning
10 - Colour array with very fast completion

Figure 85b: Fabric Butterflies: Colour and Pattern Match Array: Levels of Performance Across Mathematical Groups. n = 16.

Figure 85c shows Figure 85 levels of performance:

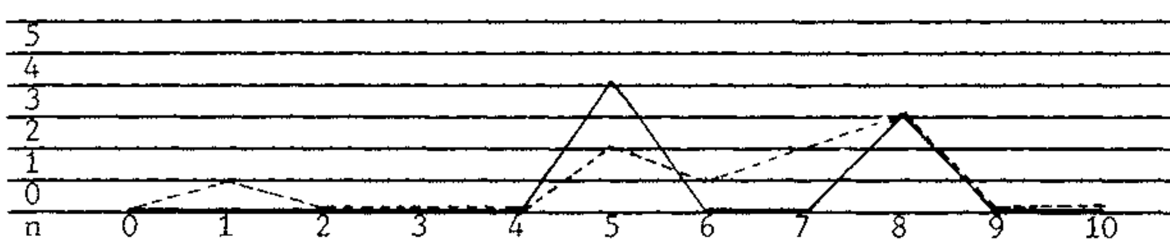


Levels of Performance

0 - Bilateral rotational wing shape matching too difficult
1 - Difficult and incomplete wing shape matching
2 - Difficult but complete wing shape matching
3 - Complete wing shape matching: no pattern matching
4 - Wing shape match: incomplete colour/pattern match
5 - Wing shape matching: complete colour/pattern matching
6 - Pattern matched some butterfly pairs
7 - Pattern matched all butterfly pairs
8 - Colour array with questioning
9 - Colour array with no questioning
10 - Colour array with very fast completion

Figure 85c: Fabric Butterflies: Colour and Pattern Match Array: Levels of Performance. n = 16.

Figure 85d examines gender performance across levels of performance: (Levels of performance as for Figure 85c).



n = 7 ________ Males
n = 9 ---------- Females

Figure 85d: Fabric Butterflies: Colour and Pattern Match Array: Gender Performance Across Levels of Performance. n = 16.

A Mann-Whitney U-Test shows gender differences in favour of males.

Figure 85e shows children at the highest level of performance for Figure 85 across mathematical groups and age. The most unexpected performance was from a 6 year old JR boy.

expected

Age	J SLD	JR	JE	S SLD	SR	AV	SE 2	SE 1
11								
10								
9								
8			1			1		
7		2	1					
6		1						

unexpected

(Age; Maths Groups)

Figure 85e: Fabric Butterflies: Colour and Pattern Match Array: Highest Level of Performance: Colour array with questioning: Mathematical Groups Across Ages. n = 6.

Children who show difficulties at any of these levels can be given other puzzles to help develop the concept there is difficulty with, irrespective of the child's mathematical group level or age.

A further puzzle showing a range of conceptual levels in children was the Adsum Shape Conservation Set and its extension with vinyl shapes. Four Adsum blocks fit together onto a series of different cardboard shapes to help develop ideas of conservation, as documented in Lee (1963). I have expanded the 5 shapes illustrated in Lee to a set of 25 shapes. Children place the four Adsum blocks onto each of the different cardboard shapes. Figure 86 shows the Adsum blocks to be fitted onto one cardboard shape.

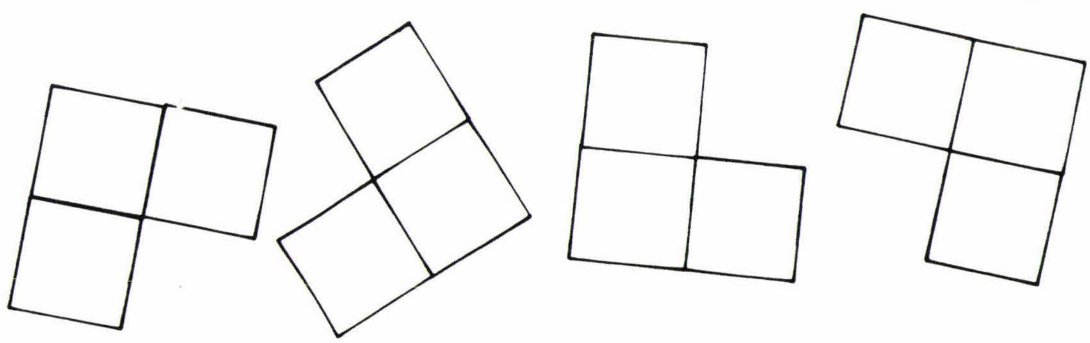


Figure 86: Adsum Shape Conservation Set.

Plate 14 shows the range of Figure 86 shapes in the Adsum Shape Conservation Set.

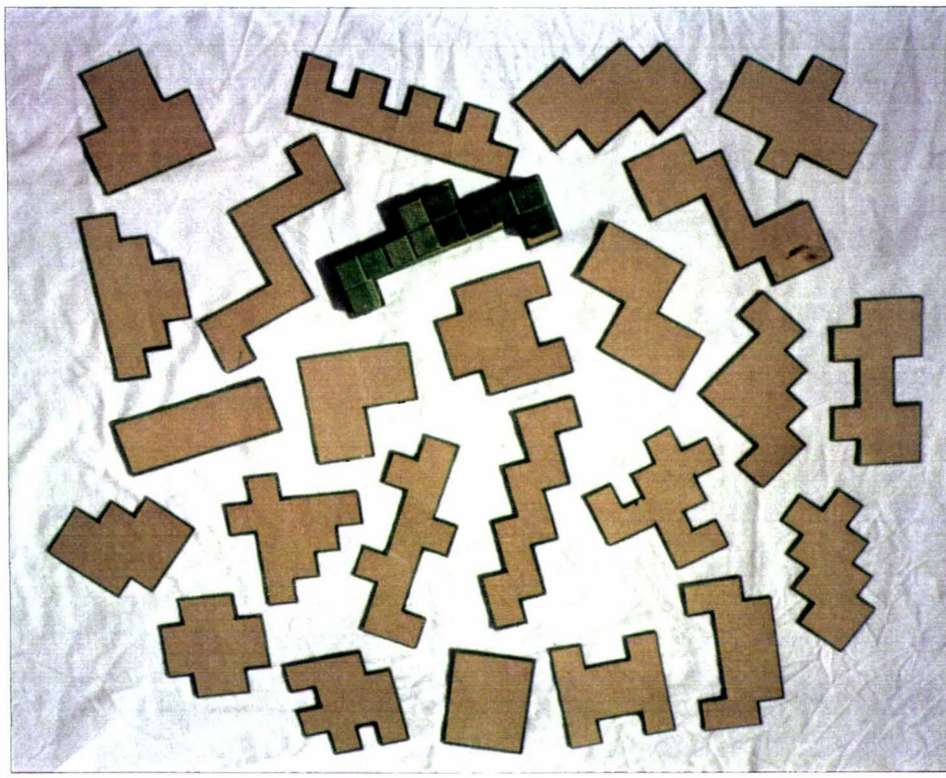


Plate 14: Figure 86a: Adsum Shape Conservation Set.

The transfer of learning required to place the blocks successfully and understand the idea of conservation of area was impossible for some children, and difficult for others who could not verbalise the meaning of all four blocks fitting exactly onto each of these very different shapes, despite having manipulated and rotated the blocks correctly onto the shapes. Understanding the concept of conservation was always more difficult for the child than the process of fitting the blocks onto the shapes. This reinforces Piaget's belief that children must manipulate and experience objects incorporating pre-mathematical concepts before formal mathematical understanding can take place.

Figure 86b shows Figure 86 levels of performance across mathematical groups. At the lowest level were children who could not rotate blocks to fit onto the shapes, followed by children who could rotate for some shapes but not for others. Some children were able to fit the shapes, but not complete the whole set through fatigue and loss of concentration or slowness. Lower levels of performance clustered around the junior groups, but children who completed these shapes correctly with no questioning ranged from the lowest to the highest mathematical groups. Junior children were also represented at the highest performance level.

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7			4		1	2		2
6	3	4	3	1	8			2
5								
4								
3			1					1
2		1			1			
1	1	2						
0	1	2						

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Correct but incomplete with some questioning
4 - Complete with some questioning
5 - Complete with minimal questioning
6 - Complete with no questioning
7 - Very fast completion with no questioning
8 - Very fast completion and use of strategy

Figure 86b: Adsum Shape Conservation Set:
Levels of Performance Across Mathematical Groups. n = 40.

Figure 86c shows children at the highest level of performance for Figure 86 across mathematical groups and age.

expected

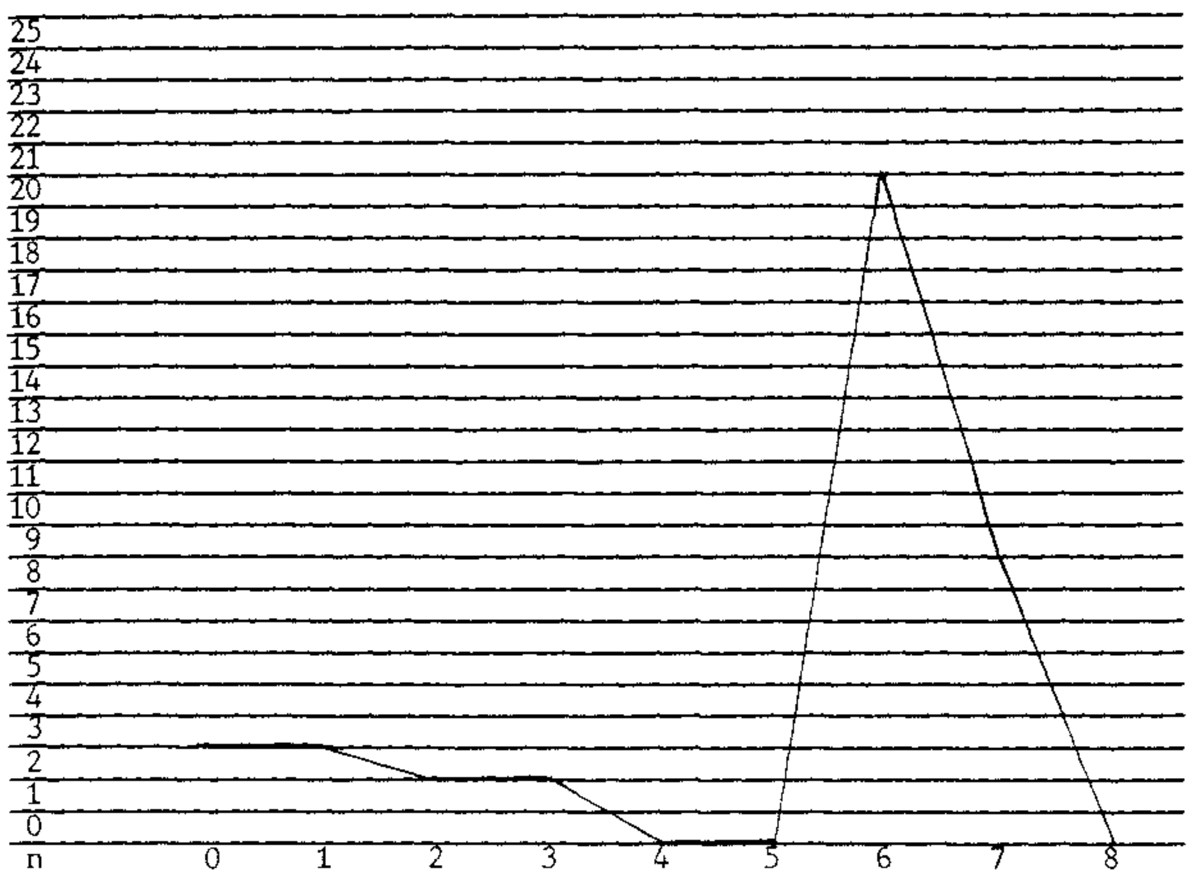
Age	Maths Groups J SLD	JR	JE	S SLD	SR	AV	SE 2	SE 1
11								
10						1		
9					1			2
8			2			1		
7			1					
6			1					

unexpected

Figure 86c: Adsum Shape Conservation Set:
Highest Level of Performance: Very fast completion with no questioning:
Mathematical Groups Across Ages. n = 9.

The most unexpected performance was from a 6 year old JE boy achieving the highest level of performance along with two 9 year old SE 1 children.

Figure 86d shows levels of performance across all children who attempted Figure 86. Most children peaked at level 6, complete with no questioning, with a smaller number at level 7, very fast completion with no questioning.

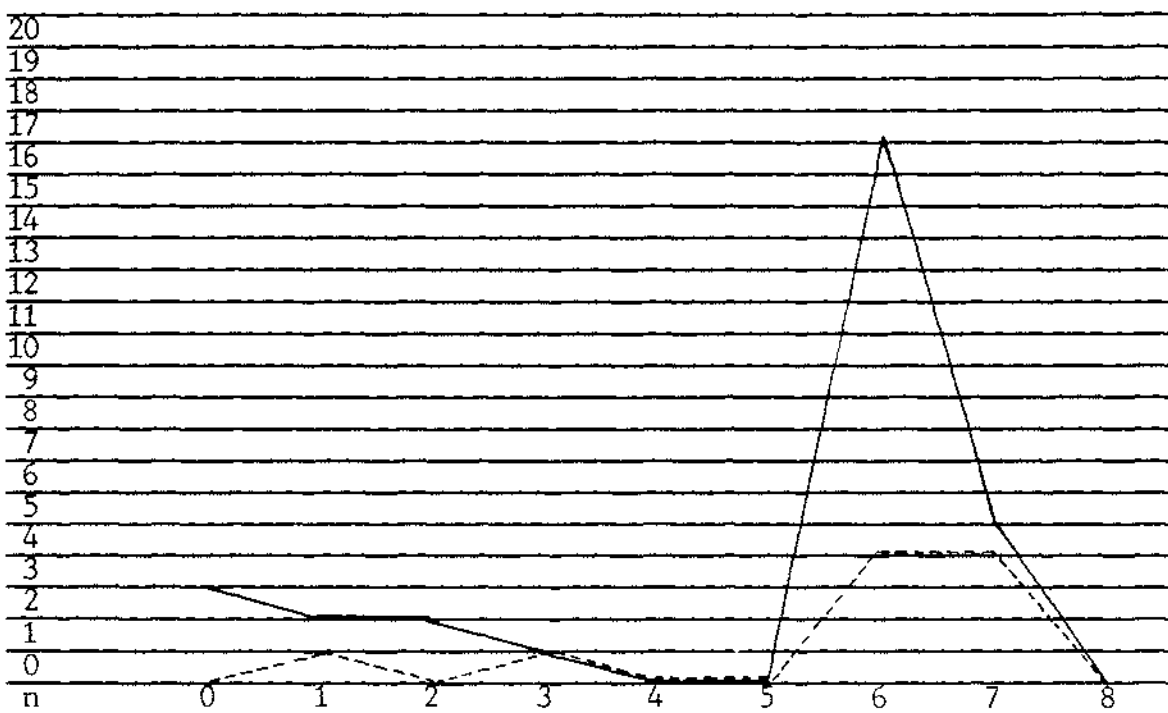


Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Correct but incomplete with some questioning
4 - Complete with some questioning
5 - Complete with minimal questioning
6 - Complete with no questioning
7 - Very fast completion with no questioning
8 - Very fast completion and use of strategy

Figure 86d: Adsum Shape Conservation Set:
Levels of Performance. n = 40.

Figure 86d shows how most children were able to complete this puzzle at level 6 with no questioning required, and how a smaller cluster of children were able to complete this puzzle at level 7, at speed. Figure 86e examines gender performance across levels of performance. (Levels of performance as for Figure 86d). Both boys and girls peaked at levels 6 and 7. A Mann-Whitney U-Test shows no gender differences.



Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Correct but incomplete with some questioning
4 - Complete with some questioning
5 - Complete with minimal questioning
6 - Complete with no questioning
7 - Very fast completion with no questioning
8 - Very fast completion and use of strategy

n = 30 ―――――― Males
n = 10 ---------- Females

Figure 86e: Adsum Shape Conservation Set:
Gender Performance Across Levels of Performance. n = 40.

Children who were able to complete the conservation shapes successfully were given a large set of loose enlarged vinyl shapes shaped like the Adsum blocks. The task was to take one conservation cardboard shape at a time, and recreate this shape with four of the enlarged vinyl Adsum look alike shapes. This was a very difficult task for many children. Ranges of levels of performance with the vinyl shapes as a continuum of the range of performance with the Adsum Shape Conservation Set gave a wide diagnostic view of a child's conceptual levels not only of ideas of conservation, but of shape awareness, rotational ability, awareness of symmetry, perceptual difficulties a size change created, and transfer of learning and reversal of task. Whereas most children who were able to transfer the learning from the conservation set shapes to the vinyl shapes needed to use the Adsum blocks fitted onto the card before they could see the rotational positions for the enlarged vinyl shapes, exceptional children were able to visualise these rotations mentally without use of the Adsum blocks. This was the upper level of performance. Plate 15 shows Figure 86 transfer from Adsum shapes to enlarged vinyl shapes.

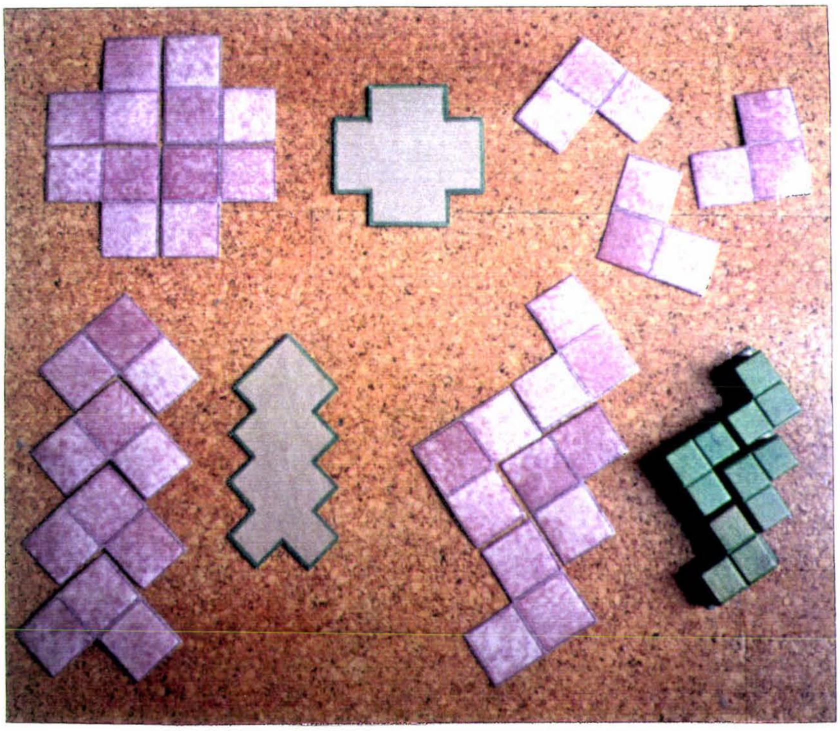


Plate 15: Figure 86: Adsum Shape Conservation Set Transfer to Vinyl Shapes.

Table 39 shows transfer of learning levels of performance between the Adsum Shape Conservation Set and the enlarged vinyl shapes. The table lists children from top to bottom in order of levels of performance for the enlarged vinyl shapes.

Table 39: Figure 86: Transfer of Learning Levels of Performance: Adsum Shape Conservation Set - Enlarged Vinyl Shapes. N = 22.

Adsum Shape Conservation Set - Enlarged Vinyl Shapes

Age	Mathematical Group	Adsum Level of Performance	Transfer Level of Performance	Creative Patterns
7	J-SLD	1		tessellation
6	JR	1		tessellation
6	JR	1		tessellation
6	JR	6		tessellation
10	SR	6	0	
6	JR	6	0	
6	JR	6	1	
9	SR	6	1	
10	SR	6	2	
8	SR	6	3	
10	SR	6	4	
7	JE	7	4	
6	JE	7	5	
10	S-SLD	6	6	
8	JE	7	6	
9	SE 1	7	6	
9	SR	7	7	
8	AV	7	7	
8	JE	6	8	
10	AV	7	8	
9	SR	6	9	
9	SE 1	7	10	

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete transfers with a lot of mismatches
2 - Difficult completion of transfers with many mismatches
3 - Difficult completion of transfers with some mismatches
4 - Few transfers attempted but all correct
5 - Some transfers attempted but all correct
6 - Most transfers attempted but all correct
7 - All transfers attempted and correct
8 - All transfers attempted and correct with some enlargements made observing cards alone without use of blocks
9 - All transfers attempted and correct with many enlargements made observing cards alone without use of blocks
10 - All transfers attempted and correct with most enlargements made observing cards alone without use of blocks

It is interesting that for children with levels of performance of 0 - 3 which include all performances with mismatches, all of the children were in a remedial mathematical group, whereas children performing correct vinyl enlargements using the Adsum blocks in levels 4 - 7, and children performing correct vinyl enlargements without using the Adsum blocks in levels 8 - 10, were represented by children across all mathematical levels except the two lowest groups of J-SLD and JR. This multiple concept task of transfer of learning involving rotation, ideas of conservation and reversal of task, combined with perceptual difficulties of shape discrimination, symmetry and size changes, made this puzzle difficult for many of the remedial children, irrespective of their age. Aspects of holding multiple ideas constant while manipulating them required much conscious focus of attention and concentration and ability to rotate and visualise both mentally and operationally. It is of note that some SR and JE children performed at the higher levels of performance for Figure 86, transferring learning to the enlarged vinyl shapes both mentally and operationally.

The third diagnostically progressive puzzle that does not involve emumeration is an above average senior puzzle, Figure 87, Transparent Triangles. The triangular design and use of colour is attributed to Childcraft (1980). I have given each size of triangle in this design its own transparent colour for individually cut coloured triangles, with five separate design boards for children to match the triangles onto and make creative designs with. This puzzle involves perceptions such as size discrimination, line and colour discrimination, rotation, symmetry and use of strategy. The differently sized triangles create a range of difficulty levels for children. Whereas the smallest yellow triangles were the easiest to place, followed by the two largest purple triangles, the next most difficult red triangles were easier to place if a child adopted a strategy of transfer of learning from the purple triangles. The inversion of two triangles superimposed on each other creates a star shape. Both purple and red triangles were placed in this kind of superimposition. In addition, a rotational strategy utilising symmetry needed to be used for successful placement of all internal red triangles, as well as the more difficult green and blue triangles where additional superimposed layers made it impossible to assess visually where individual triangles had been positionally and rotationally placed. This difficulty hierarchy for children unable to discriminate and place triangles on existent lines, through to the use of symmetry and strategy in rotational ordering, gave progressively diagnostic information on children's levels of conceptual understanding.

Plate 16 shows the triangular board design for Figure 87 with the transparent triangles to be fitted onto the five boards.

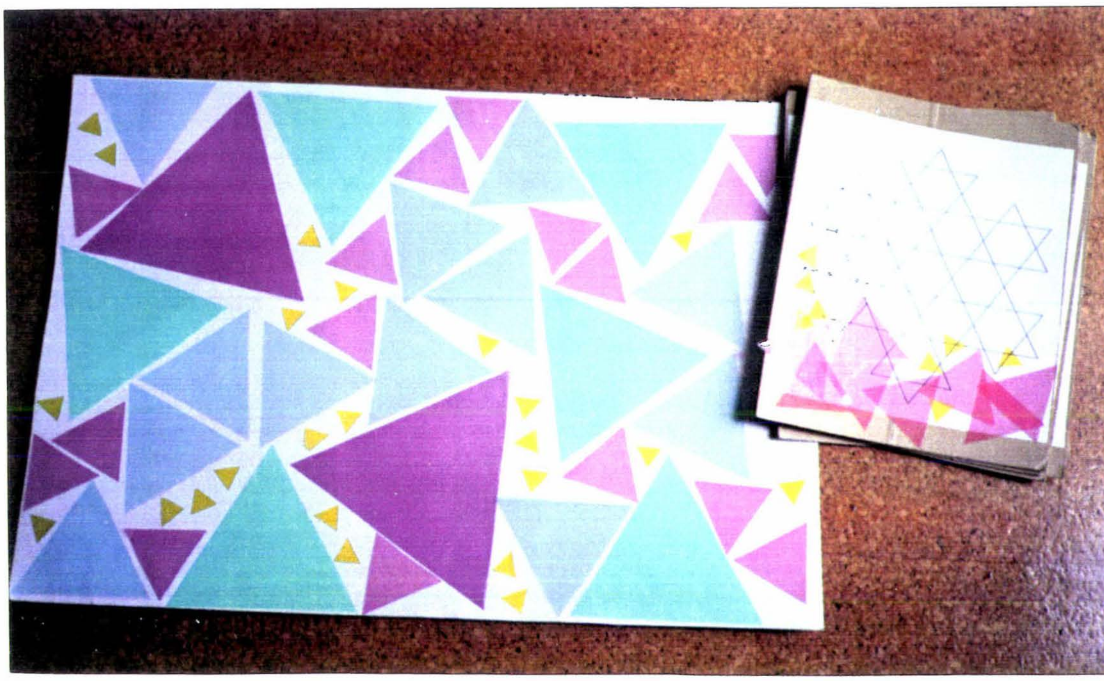


Plate 16: Figure 87: Transparent Coloured Triangles:
Verbal Instructions:

- Use one board for each colour.
- Find all of the triangles for each colour on the design boards.

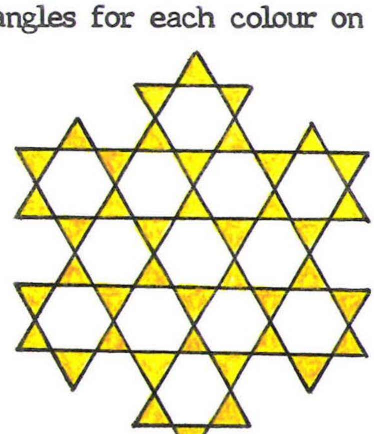


Figure 87a: Transparent Coloured Triangles: Yellow Board.

Plates 17 and 18 show Figure 87 colour boards complete or incomplete.

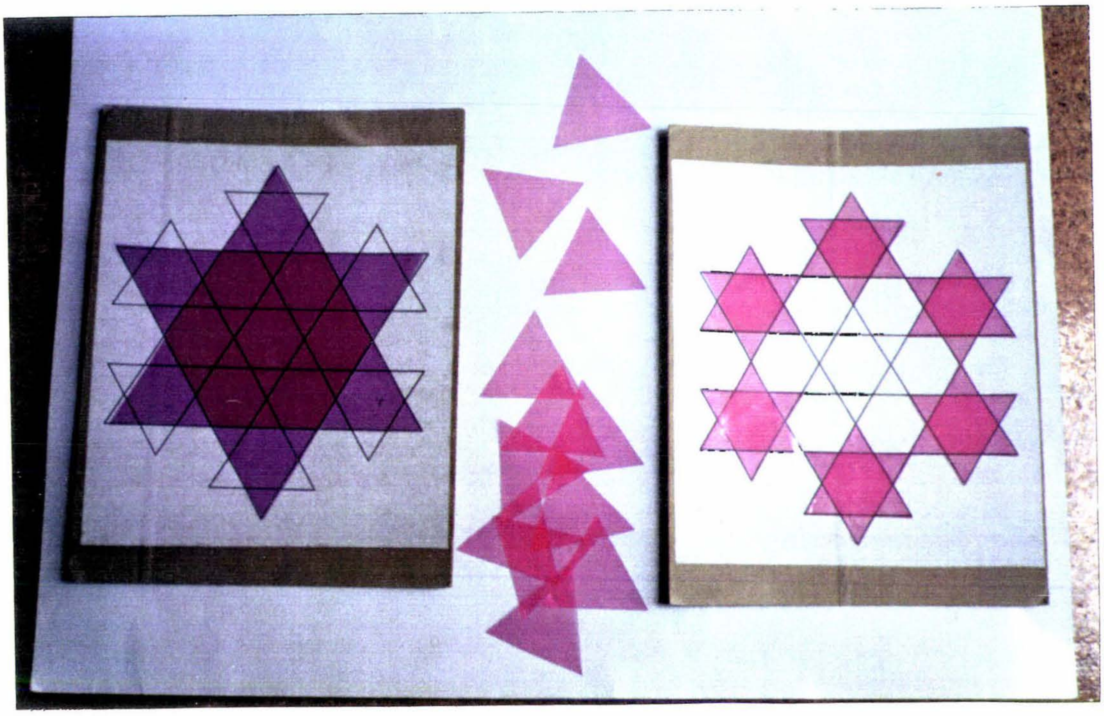


Plate 17: Figure 87b: Transparent Coloured Triangles: Purple; Red Boards.

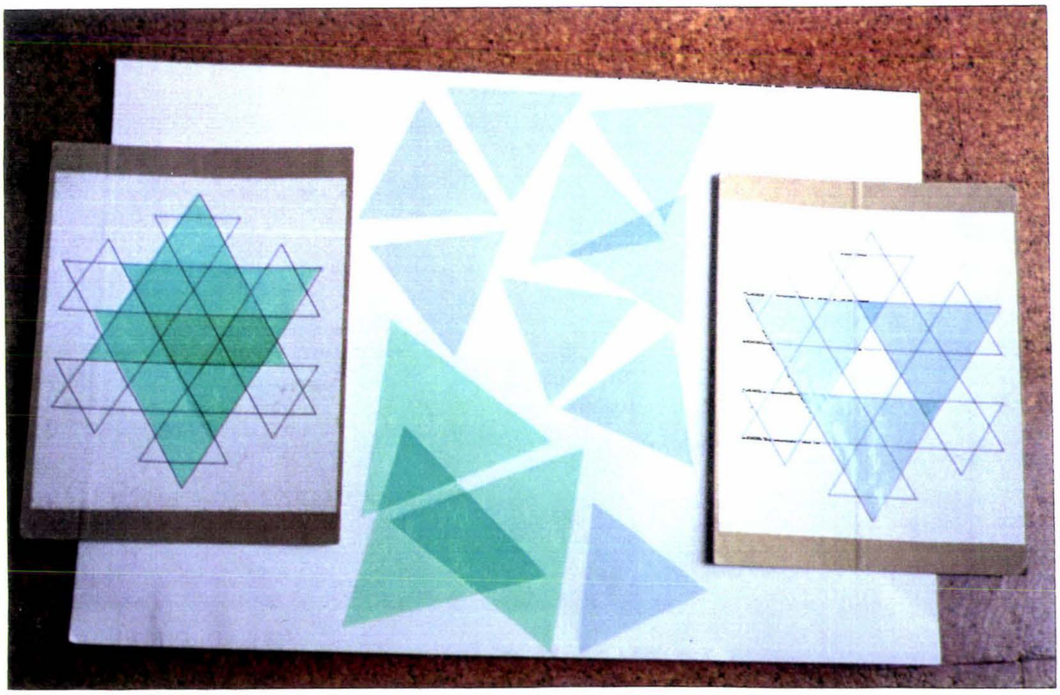


Plate 18: Figure 87c: Transparent Coloured Triangles: Green; Blue Boards.

Figure 87d shows levels of performance across mathematical groups:

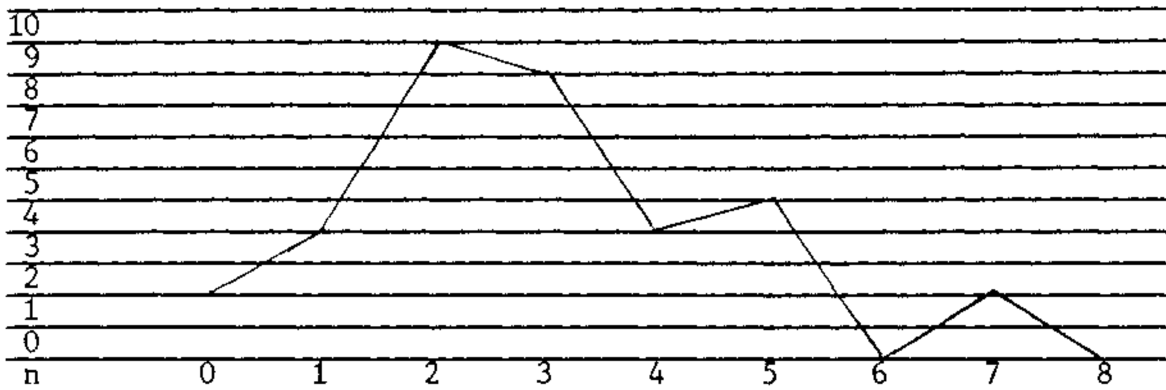
Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7			1	1				
6								
5		1			1	1		2
4					1	1		2
3	1		3		4			1
2			3		3		2	2
1	1	2	1					
0				1	1			

Levels of Performance

0 - Too difficult, with inability to stay on lines.
1 - Very difficult and incomplete with inability to stay on lines, and many inaccuracies.
2 - Difficult and incomplete with some inaccuracies.
3 - Difficult but complete with some inaccuracies.
4 - Difficult but accurate completion with questioning.
5 - Complete with no questioning.
6 - Complete with no questioning, and use of strategy and symmetry.
7 - Complete with no questioning, and use of strategy and symmetry. Creative patterns with symmetry.
8 - Exceptional or novel performance.

Figure 87d: Transparent Triangles:
Levels of Performance Across Mathematical Groups. n = 36.

The difficulty range was so wide there were only some children who attempted all of the colour boards. A number of children had difficulty perceiving lines, and it was surprising to see triangles placed over non existent lines. Questioning for the red board involved enabling the child to visually see and verbally describe the inverted triangular star pattern on the purple board that the same child had already just completed. Some children were unable to transfer this learning. Some children able to make symmetrical red star shapes around the periphery of the red board were unable to apply a symmetrical or rotational ordering strategy to complete the interior of the red board successfully, or to transfer this strategy to the green and blue boards. Figure 87e shows levels of performance across all children who attempted Figure 87.

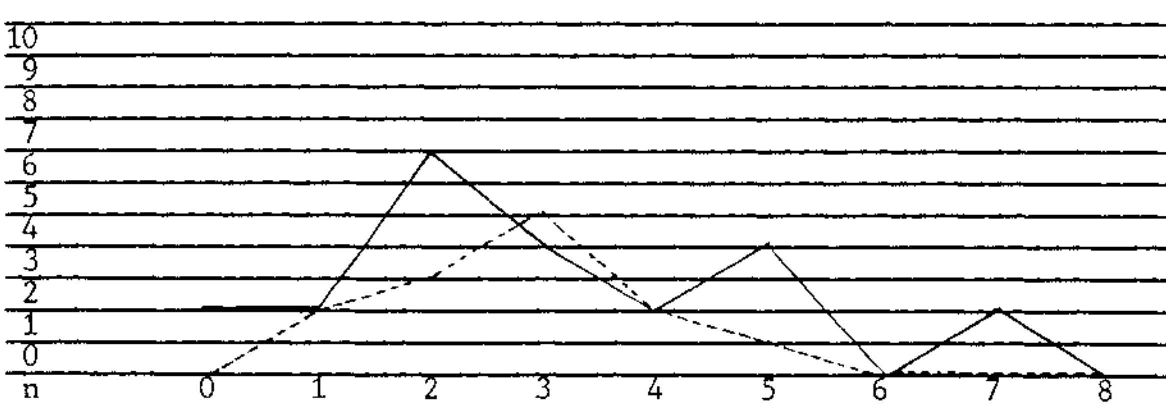


Levels of Performance

0 - Too difficult, with inability to stay on lines.
1 - Very difficult and incomplete with inability to stay on lines, and many inaccuracies.
2 - Difficult and incomplete with some inaccuracies.
3 - Difficult but complete with some inaccuracies.
4 - Difficult but accurate completion with questioning.
5 - Complete with no questioning.
6 - Complete with no questioning, and use of strategy and symmetry.
7 - Complete with no questioning, and use of strategy and symmetry. Creative patterns with symmetry.
8 - Exceptional or novel performance.

Figure 87e: Transparent Triangles:
Levels of Performance. n = 36.

Figure 87f examines gender performance across levels of performance: (Levels of performance as for Figure 87e).



n = 23 ________ Males
n = 13 ---------- Females

Figure 87f: Transparent Triangles:
Gender Performance Across Levels of Performance. n = 36.

Most children clustered in the difficult with some inaccuracies range, either after completion or incompletion of Figure 87. Levels of performance did not follow mathematical group categories. SE 1 children were represented in the middle range of levels of performance only, with the highest level of performance coming from a 7 year old JE boy. A Mann-Whitney U-Test shows no gender performance differences.

Table 40 lists children with specified colour difficulty areas and exceptional levels of performance. Difficulties included placing triangles on absentee lines, and incorrect placement or difficult but correct placement. Performance indicating ability includes correct placement without difficulty, transfer of learning of the star pattern in the purple triangles to the red triangles, and exceptional ability in the use of symmetry and rotation as a strategy for ordering red, green and blue triangles, and in symmetrical creative patterning. Colours are difficulty ordered from yellow to blue, from the easiest yellow triangles with no overlaying colours, to the medium difficulty reds incorporating symmetrical overlaying and transfer of learning from the purple triangles, to the most difficult green and blue triangles where application of a rotational and symmetrical strategy is necessary. Children are listed in order of level of performance.

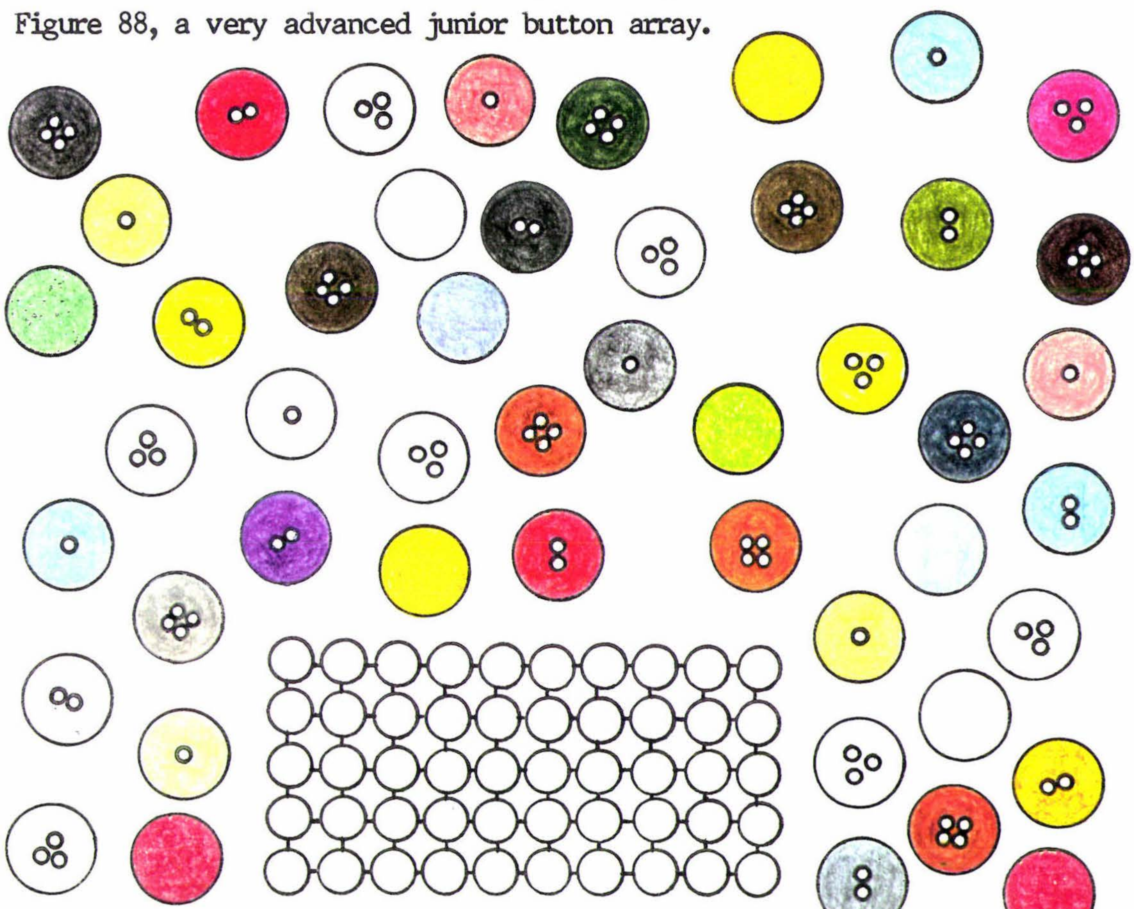
Table 40: Figure 87: Transparent Triangles: Line Performance, Colour Performance, Transfer of Learning, Use of Symmetry as Strategy for Ordering, and Creative Patterning: N = 16.

Age	Maths Group	Yellow	Purple	Red	Green	Blue	Creative Patterns
7	J-SLD	absentee	correct	absentee	absentee	absentee	
6	JE	correct	correct	transfer	absentee	absentee	
6	JR	correct	correct	transfer	absentee	absentee	
10	SR	correct			incorrect		
9	SE 1	correct	correct	difficult	difficult	difficult	
8	SE 2	correct	correct	incorrect	some	incorrect	
8	JE	correct	correct	some	some	some	
7	JE	correct		correct			
9	SR	correct	correct	correct	incorrect	incorrect	1
8	SR	correct	correct	incorrect	correct	incorrect	
7	JE	correct	correct	incorrect	incorrect	incorrect	
8	SE 1	correct	correct	correct	correct	incorrect	
11	SE 1	correct	correct	correct	difficult	difficult	
10	SE 1	correct	correct	correct	correct	difficult	
10	S-SLD	correct	correct	correct	strategy	correct	1 sym
7	JE	correct	correct	correct	strategy	correct	1 sym

Children performing correctly at an easier level sometimes have difficulty performing at a harder level, with correct or incorrect results. Transfer of learning may or may not be achieved, and a strategy may or may not be applied. At the lowest level, three children placed triangles on absentee lines. At the highest level, only two children applied a rotational or symmetrical strategy. These two children were also the only children to make symmetrical creative designs with the triangles. The use of strategy and symmetry is a useful measure of exceptional ability in a child.

Puzzles With Enumeration

The first of the two diagnostic progressive puzzles involving enumeration is Figure 88, a very advanced junior button array.



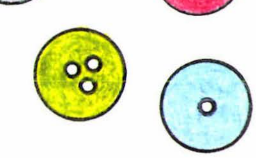


Figure 88: Button Array:
Verbal Instructions:

- Find which buttons belong together.
- Each button has its own circle on the diagram.
- Buttons belong together across rows on the diagram.
- Buttons belong together down columns on the diagram.
- Place buttons that belong together in rows and columns on the diagram.

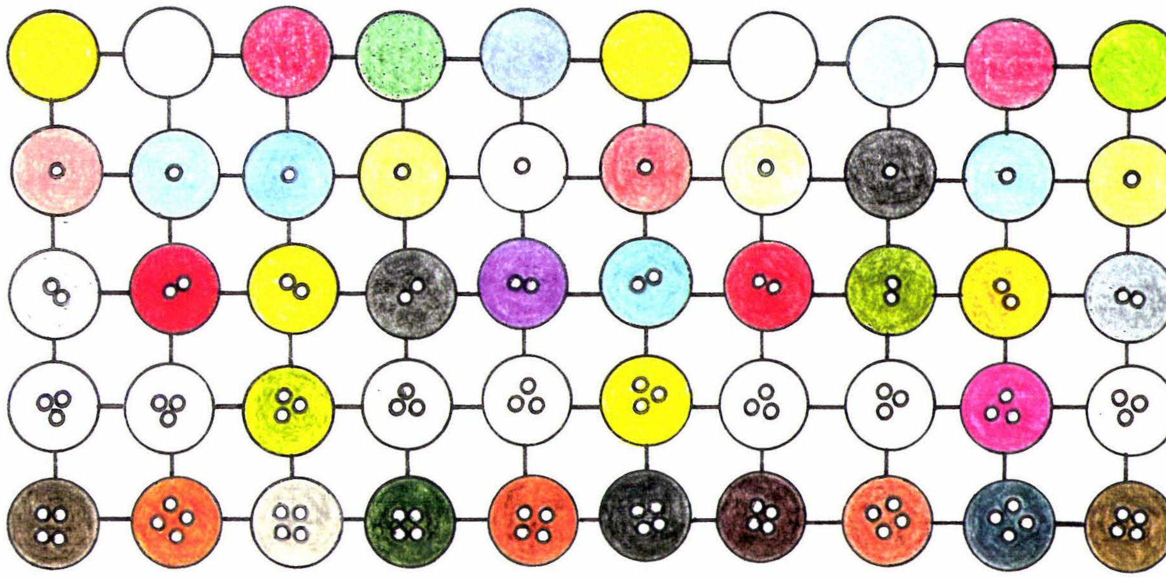


Figure 88a: Cardinal and Ordinal Button Holes Array:

This puzzle was very useful for checking difficulties in a child's movement from pre-mathematical conceptual levels of understanding to those of beginning formal mathematical thinking. Figure 88 gave examples of perceptual priorities and perceptual difficulties, mathematical conceptual levels of one to one correspondence and enumeration and the cardinal and ordinal meanings of numbers as a consecutive ordering task, and an example of a child unable to verbalise two classifications. Figure 88 was also a particularly useful puzzle for junior remedial children to process. Many of these children found the task difficult, and much questioning input was required. The following anecdotes illustrate levels of performance with the kinds of conceptual difficulties this puzzle is useful for detecting and remediating.

The lowest level of performance was from a 10 year old SR girl who placed the buttons on the diagram randomly at first. She then found all of the buttons with three holes, and took 30 more minutes to classify the remainder of the buttons according to the number of holes they had. Despite extensive questioning, she was unable to order her button rows in ordinal form.

A 7 year old JR girl separately classified buttons with three and four holes in them, but was unable to say what was the same about these buttons she had placed together. This is another example of a child making a classification at a possibly unconscious level with obvious conceptual understanding of the classification, but with an inability to verbalise the classification. After a lot of questioning she was able to name these sets. Only then could she continue classifying the remaining buttons she had not known how to classify. Following extensive questioning she was able to order her rows correctly. Namimg the sets brought unconscious perceptual and conceptual understanding to conscious attention, enabling conscious transfer of this learning for classification completion.

A 6 year old JR girl placed the buttons on the diagram randomly at first before noticing some buttons had no holes. I suggested she make a set of 'no holes' buttons. She spontaneously classified the remaining buttons into sets according to the number of holes they had, and arranged them into rows ordered 4's, 1's, 3's, 0's and 2's. I asked her to tell me about her rows. She named her ordered rows, '4's, 1's, 3's, nothings, 2's'. I asked her what another name for 'nothings' was. She did not know. Another child told her it was 'zero'. As she did not correct the order of her rows, I asked her how she would count her fingers. She began counting them, 'One, two, three, four, five, six.' I asked her what came before 'one.' She did not know. Another child told her it was 'zero'. I asked her to tell me the names of her rows again. She repeated her original ordering until she got to her 'zero' row, and said she had to put 'zero' at the top. She then ordered her rows, '0, 1, 2, 3, 4,' from top to bottom.

A 7 year old J-SLD boy attempted to colour match the buttons at first, showing a perceptual priority mode. I asked him to tell me about specific buttons I picked out before he noticed they had different numbers of holes in them. Without questioning, he arranged his rows ordered from buttons with 1 - 4 holes from the bottom of the board upwards, having no idea what to do with his holeless buttons, or what to do with his empty top row on the diagram. When asked to name the remaining holeless buttons, he called them 'zeros', and arranged them below the diagram beneath his row of buttons with one hole each. He could not perceive the possibility of moving his rows up so he could fit all of the sets onto the diagram.

A 7 year old JR boy placed all of the buttons with three holes in them along the top row of the diagram, with the remaining buttons placed at random. I asked him why he had put his top row of buttons together. He said it was because they all had three holes. I pointed to each other row and asked him why he had put these buttons together. He did not know. I asked him if he could see any buttons with a different number of holes in them than the three holes his top row buttons had. He said he could see buttons with four holes, two holes, one hole and no holes. I asked him if he could put all of the buttons with the same number of holes together. He put them into rows ordered 3's, 1's, 4's, 2's, 0's. I asked him to count from zero for me. He counted 0, 1, 2, 3, 4, then quickly ordered his rows correctly.

A 6 year old JR boy at first tried unsuccessfully to classify the buttons according to colour, and then size, showing perceptual priority modes. I asked him if he could find another way the buttons belonged together. He could not. He had randomly placed two buttons together with two holes in each of them. I asked him why he had put these two buttons together. He had no idea. After some moments he noticed there were different numbers of holes in different buttons, and he ordered them correctly into rows.

A 6 year old JR girl classified her button sets correctly and placed them in rows on the floor. She did not know how to place them on the diagram. I asked her to count how many buttons there were in each set, and how many sets she had. She said there were ten buttons in each set and five sets. I asked her to count how many circles there were in each row on the diagram, and how many rows there were. She said there were ten circles in each row and five rows. Without further questioning she was able to accurately order her sets onto the diagram.

An 8 year old AV girl placed the zero holed counters along a row. She then placed the one holed buttons around the outside of the four corners of the diagram in a decorative symmetrical pattern. I asked her to tell me about the row of buttons she had placed along a row. She said they all had no holes in them. I asked her to tell me about the buttons she had placed around the corners. She said they had one hole in them. I asked her if she could find another way to arrange them. She quickly placed the one holed buttons beneath the zero holed buttons, and completed the puzzle accurately.

Many of these anecdotes were placed in the level of performance of 2, of difficult completion with a lot of questioning, as were many of the children attempting Figure 88. Figure 88b shows Figure 88 levels of performance across mathematical groups:

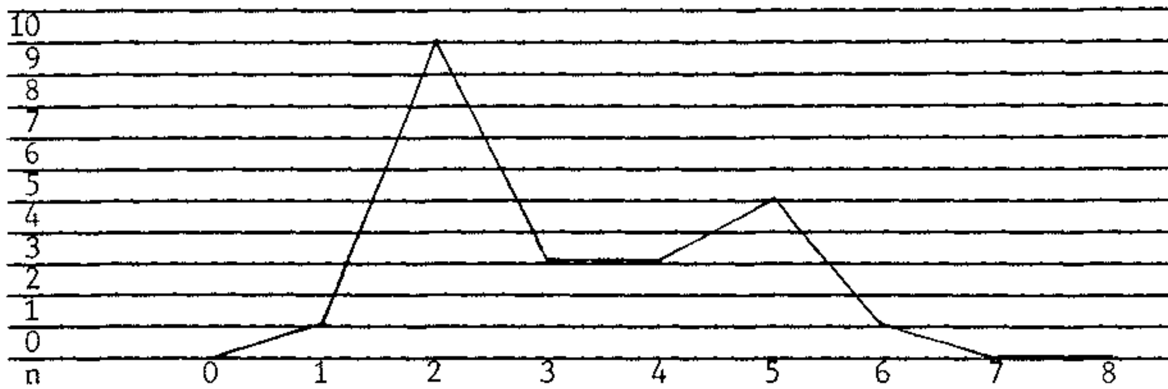
Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6						1		
5		1	1		2			1
4			1			1		1
3		2	1					
2	3	6	1					
1					1			
0								

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 88b: Button Holes: Cardinal and Ordinal Array: Levels of Performance Across Mathematical Groups. n = 23.

Figure 88c shows levels of performance across all children who attempted Figure 88:

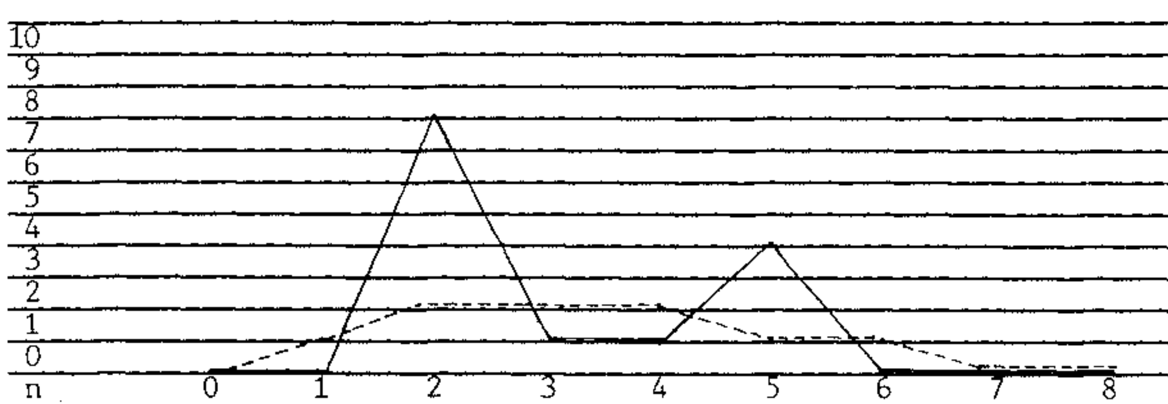


Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 88c: Button Holes: Cardinal and Ordinal Array: Levels of Performance. n = 23.

Figure 88d examines gender performance across levels of performance: (Levels of performance as for Figure 88c).



n = 14 __________ Males
n = 9 ---------- Females

Figure 88d: Button Holes: Cardinal and Ordinal Array: Gender Performance Across Levels of Performance. n = 23.

Two surprises in Figure 88 were a JR and JE child achieving the second highest level of performance with SR children and one SE 1 child, along with a 10 year old SR child achieving the lowest level of performance. A Mann-Whitney U-Test shows no gender performance differences for Figure 88.

Figure 89 is also a cardinal and ordinal array showing diagnostic progressions of conceptual understanding and levels of performance. In modified form it is suitable for children in any mathematical group. I reduced the number of cards in this puzzle from 121 to 55 on initial presentation to make it more manageable and less overwhelming for a child. This did not alter the conceptual processes involved. The initial modified form of 55 cards contained from 1 - 5 coloured candles on each card for the child to arrange onto a board containing a 55 card array diagram. If a child was able to process the concepts to produce a complete cardinal and ordinal colour array with the 55 cards, I extended this modified set to the full set of 121 cards, with from 1 - 11 candles on each card. Children then opted to extend from the modified diagram board or to use no board at all.

A very wide range of conceptual performance was shown by children attempting Figure 89. Levels of performance across mathematical groups, school classes and ages were significant, perhaps relating to cardinal and ordinal sequencing as in formal mathematics. Levels of performance, from ability or inability to make a one to one correspondence between cards and diagram, to various arrangements of cardinal, ordinal and colour sequencing, to awareness of diagonal relationships as well as vertical and horizontal sequencing, and to manipulating and rotating concepts mentally and verbally to produce correct and novel arrays, showed the great range of ability across children.

The full array could be arranged in several different ways including circular arrangement, although most children did not notice this possibility.

Plate 19 shows Figure 89 in the incomplete modified form of 55 cards as first presented to children, and Plate 20 (Figure 89a), in completed form.

Plate 19: Figure 89: Birthday Candle Cardinal and Ordinal Colour Array: Verbal Instructions:

- Find which cards belong together.
- Place the cards in order on the diagram.

Plate 20: Figure 89a: Birthday Candle Cardinal and Ordinal Colour Array.

Plate 21 shows Figure 89b with the full set of 121 cards.

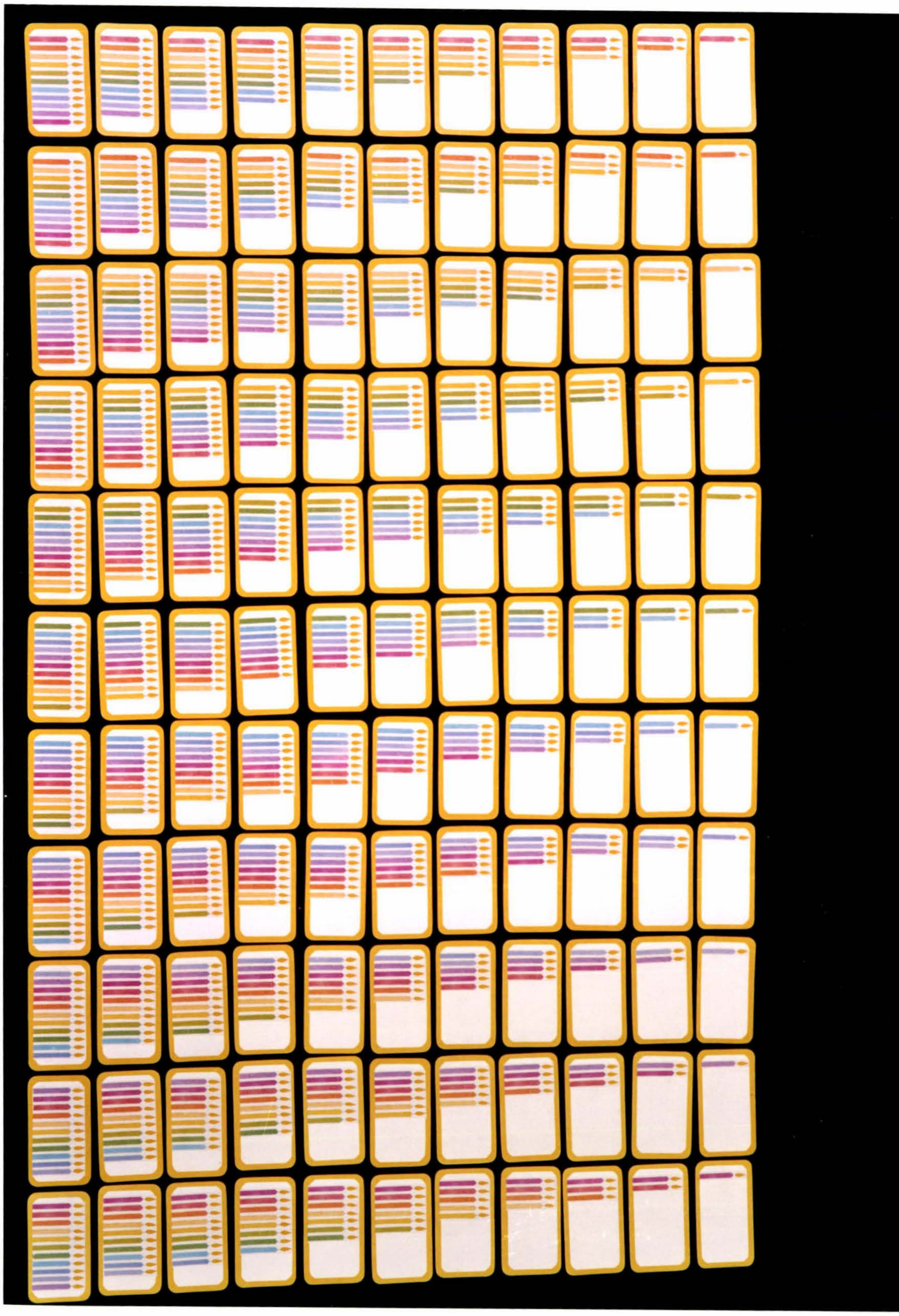


Plate 21: Figure 89b: Birthday Candle Cardinal and Ordinal Colour Array.

Figure 89c shows Figure 89 performance levels across mathematical groups. Levels range from no one to one to correspondence, to application of strategic and novel approaches. (Levels of performance as for Figure 89d).

Levels of Performance \ Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
21						1		
20								1
19					1			2
18			1			1		
17							2	
16					1			
15								1
14		1	1		1		1	2
13		1	2		1			
12					1			
11			1		3			
10								1
9		1						
8		1			1			
7			1		1			
6			1					1
5					1			
4	1		1					
3		1						
2	1							1
1		1						
0		1						

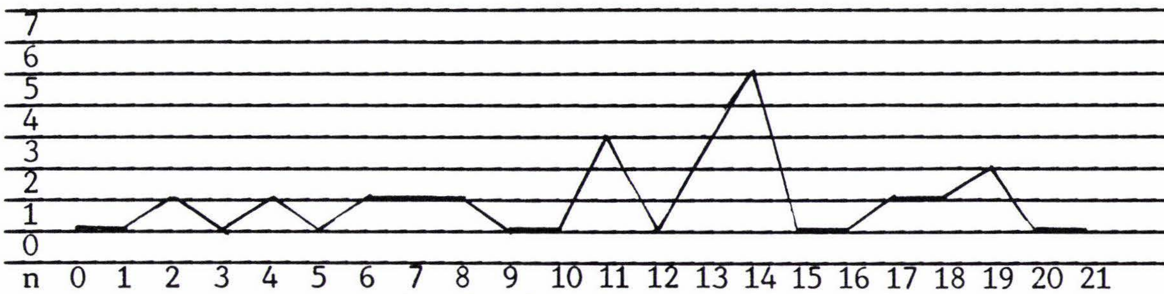
Figure 89c: Birthday Candle Cardinal and Ordinal Colour Array: Levels of Performance Across Mathematical Groups. n = 42.

Although performance levels are wide for different groups, they do increase in range with the upward movement of mathematical groups. Table 41 shows Figure 89 correlation coefficients between levels of performance and mathematical groups, school class levels, and ages.

Table 41: Spearman's Rho Correlation Coefficients:
Figure 89: Birthday Candle Cardinal and Ordinal Colour Array:
Levels of Performance: n = 42.

Across mathematical groups	$r_s = 0.520$	$p = <0.05$
Across school class levels	$r_s = 0.493$	$p = <0.05$
Across ages	$r_s = 0.503$	$p = <0.05$

Figure 89d shows Figure 89 levels of performance:



Levels of Performance

0 - No one to one correspondence. Random placement of cards into piles on the diagram board.
1 One to one correspondence. Random card placement.
2 - Numerical columns misplaced. No colour awareness.
3 - Cardinal and ordinal array. Some reversals.
4 - Correct cardinal and ordinal array, with difficulty.
5 - Correct cardinal and ordinal array. Cards upside down.
6 - Cardinal and ordinal array with no difficulty.
7 - Cardinal and ordinal array. Correct confabulated reasoning for a vertical colour placement. Unable to transfer concept to whole array.
8 - Cardinal and ordinal array. More than one correct confabulation. No transfer of concept to whole array.
9 - Cardinal and ordinal array. Correct confabulated reasoning for vertical colour placement. Transferred concept to some of the array. Many mistakes.
10 - Cardinal and ordinal array. Occasional vertical colour sequencing. Incomplete.
11 - Cardinal and ordinal array. Several columns in vertical colour sequencing.
12 - Partial cardinal and ordinal array. Incorrectly placed columns in two sections. Vertical colour sequencing
13 - Cardinal and ordinal array. Correct confabulated reasoning for one correct vertical colour sequence. Correctly transferred concept to whole array.
14 - Cardinal and ordinal array. Vertical colours correct.
15 - Cardinal and ordinal array. Horizontal colours correct.
16 - Cardinal and ordinal array. Vertical colours correct. Correct confabulated reasoning for occasional horizontal sequencing. Unable to transfer concept to whole array.
17 - Cardinal and ordinal array. Vertical colours correct. Correct confabulated reasoning for occasional horizontal sequencing. Transferred concept to whole array.
18 - Cardinal and ordinal array. Vertical colours correct. Correct confabulated reasoning for occasional horizontal sequencing. Corrected horizontal sequencing at speed.
19 - Cardinal / ordinal array. Vertical and horizontal colour.
20 - Use of strategy. Mentally worked out and verbalised complete array in advance.
21 - Novel array showing advanced conceptual thinking.

Figure 89d: Birthday Candle Cardinal and Ordinal Colour Array: Levels of Performance. n = 42.

Ranges of performance across mathematical groups shown in Figure 89c are widely spaced for several mathematical groups. JR children ranged from levels of performance of 0-14; JE children from levels of 4-18; SR children from levels of 5-19; and SE 1 children from levels of 2-20, the widest range of any group. Figure 89d shows the biggest number of children performing at level 14 with a correct cardinal and ordinal array and vertical colour, but incorrect horizontal colour sequencing. The next biggest number of children was in level 11 with incomplete vertical colour sequencing, and in level 13 where vertical colour sequencing was rearranged correctly, with both of these levels reached following confabulated reasoning for random or irregular sequencing. Several children also had both vertical and horizontal colour sequencing correct at level 19.

Figure 89e examines gender performance across levels of performance. (Levels of performance as for Figure 89d).

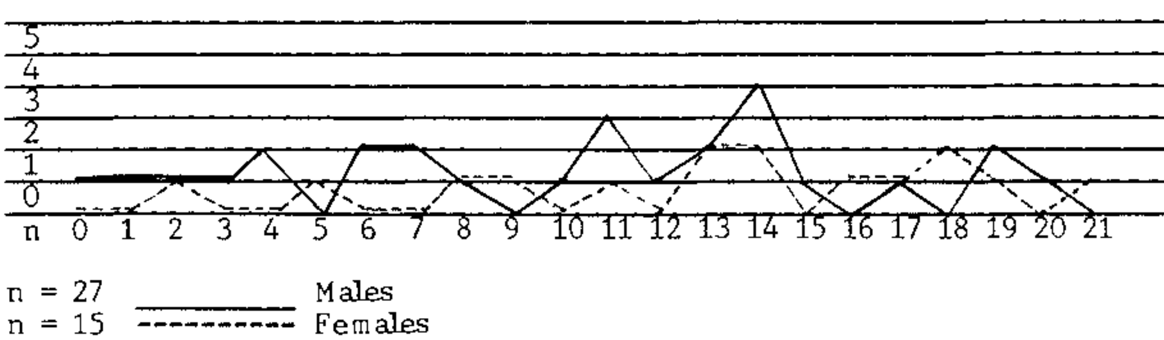


Figure 89e: Birthday Candle Cardinal and Ordinal Colour Array: Gender Performance Across Levels of Performance. n = 42.

A Mann-Whitney U-Test shows no gender performance differences.

The diagnostic value of observing the way a child processes this array can be understood more clearly from the following anecdotal range of levels of performance, beginning with the 6 year old JR boy who placed several piles of cards at random on the diagram board, showing no one to one correspondence or cardinal and ordinal or colour awareness. This puzzle was an exercise in one to one correspondence alone for the next 8 year old JR boy who placed one card to one box at random on the diagram. Similarly there was no cardinal or ordinal or colour awareness. Even at these two levels of performance of 0 and 1, it is evident the first child had not mastered the pre-mathematical skill of one to one correspondence, but the second child had, although without further movement into formal mathematical thinking that develops with the ability to enumerate.

Beginning enumeration featured in the next level of performance. Counting the number of candles on each card was one issue. A more difficult issue was the transference of the sets of cards onto the diagram. Synchronising the number of cards with the same number of candles on them and the number of different candle numbers, with the number of boxes on each row and down each column of the diagram was too difficult as a cardinal number task for some children. One 7 year old J-SLD boy with a level of performance of 2 made columns of similarly numbered cards in no particular order on the floor. He could not arrange these onto his diagram. I asked him how many cards had one candle, and how many boxes there were in a row on the diagram. He said there were 11 one's and 13 boxes. I asked him how many boxes there were in a column. He said there were 4. I asked him to count these again, which he answered correctly the second time. He was still unable to see how to arrange his cards onto the diagram. An 8 year old SE 1 girl with a level of performance of 2 placed her cards in a 1-5 order across her rows twice, as shown:

1	2	3	4	5	1	2	3	4	5
1	2	3	4	5	1	2	3	4	5
1	2	3	4	5	1	2	3	4	5
1	2	3	4	5	1	2	3	4	5
1	2	3	4	5	1	2	3	4	5
1	2	3	4	5					

As was apparent in Figure 88, the ordinal meaning of numbers is a later and more difficult concept for children to grasp. One 7 year old JE boy with a level of performance of 4, who was able to classify cardinal groups, needed extensive questioning to arrange these classifications ordinally into array form. This array was without colour awareness. One 6 year old JR boy with a level of performance of 3 made a cardinal and ordinal array with his fourth and fifth rows reversed. He also showed no colour awareness. A 10 year old SR girl with a level of performance of 5 first placed all cards upside down at random onto the diagram. She then made piles of cards holding the same number of candles (cardinal groups), placing them onto the diagram in correct cardinal and ordinal array form, with many cards still upside down, and no colour sequencing.

Children who were able to make correct cardinal and ordinal arrays with no colour awareness or beginning colour awareness often gave confabulated reasoning for randomly or inconsistently placed colour sequences. One 9 year old SR boy with a level of performance of 7 placed the cards at random onto the diagram before rearranging them into a cardinal and ordinal array. Two cards had been randomly placed in colour sequence. When asked why he had placed these two cards together, he said it was because they started with the same colour. He was unable to transfer this observation to place all of the cards into this kind of vertical sequence. Similarly, a 7 year old JE boy at the same level of performance who randomly placed three cards in vertical colour sequence gave the same confabulated reply. He said all of the cards could be arranged like this, but he too was unable to rearrange them.

Two further children showed partial colour sequencing awareness, but not in sufficiently conscious form for conscious transfer of learning to make the arrangement consistent, nor for the children to verbalise their reasons sufficiently. These are examples of concepts either not fully formed, or of inconsistent application. One 9 year old girl with a level of performance of 8 made a correct cardinal and ordinal array with occasional vertical and horizontal colour sequencing, but she was unable to transfer these kinds of sequences to all of the cards, and had some perceptual difficulties distinguishing between colour subtleties. A 6 year old JR boy with a level of performance of 8 made a correct cardinal and ordinal array with correct vertical colour sequencing in his first two columns only, with no horizontal colour sequencing. He was also unable to transfer this vertical colour arrangement to all of his columns.

Some children were able to transfer learning from these confabulations. A 6 year old JR girl with a level of performance of 9 made a cardinal and ordinal array with no colour awareness apart from one column with all candle sequences beginning with the same colour. This suggests partial and probably initially unconscious conceptual understanding, as she was able to confabulate correct reasoning for this and to rearrange all of her columns with candle sequences beginning with the same colour. Few of these were completely correct as she had further difficulty perceiving differences in colour subtleties. A 9 year old SE 1 boy with a level of performance of 10 also showed evidence of incomplete conceptual understanding that was insufficient for him to be able to show consistency. At first he could not see how to arrange his numerically vertically sequenced cards onto the diagram until he counted the number of cards for the number of boxes provided. His colour sequencing on the diagram was very inconsistent with occasional horizontal and vertical colour sequences made. He did understand why he had made these sequences, but was unable to extend them and became frustrated, only placing half of the 55 cards he had.

Fatigue can also be a factor, as evidenced by a 6 year old JE boy with a level of performance of 11. At first he made a cardinal and ordinal array with no colour sequencing apart from one column where he had randomly placed four cards beginning with an orange candle. He confabulated his reason for this, and began changing his columns to make all of the cards start with the same candle colour, but lost concentration after completing two columns. An 8 year old SR boy with a level of performance of 11 also lost concentration after making half cardinal and ordinal rows of 1-5 candles with vertical colours the same for each candle sequence. The remainder of the diagram was placed at random, both in cardinal and ordinal and colour sense. This reinforces the idea of concepts being inconsistent when they are not firmly established, creating the mental fatigue concentration produces. The partially established and partially conscious concepts also illustrate the gradual process of unconscious concepts slowly coming to conscious attention where they are then more likely to become consciously embedded and consciously able to be used and applied in a transfer of learning situation.

Sometimes a child may show evidence of one concept applied consistently, but another concept that is only partially complete, as in the performance of an 8 year old SR boy with a level of performance of 12 whose vertical colour columns were consistent, but whose cardinal and ordinal array was not. This boy placed all vertical columns with candles starting with the same colour, but his cardinal and ordinal array was the same as the SE 1 girl with a level of performance of 2 and the 8 year old SR boy mentioned above with a level of performance of 11, who each placed two sets of 1-5 cards across each row instead of placing these vertically.

Children with a level of performance of 13 were able to correct their vertical colour columns after giving a confabulated reason for their randomly placed colour sequences. None of these children were aware of horizontal colour sequencing possibilities. Two children had difficulties arranging their cardinal and ordinal array onto the diagram. One 7 year old JE girl mixed her four and five candle cards so that she had uneven numbers of cards to place along the rows. Once she had discovered and corrected this through questioning, she ordered her array correctly, and completed the vertical colours following further questioning. A 7 year old JR boy also had difficulties with the diagram at first with uneven numbers of cards in his cardinal columns. Once this was corrected he placed his array partly on and partly off the diagram, with ordinal sequences running across rows, and cardinal cards in vertical columns, a reverse arrangement from the way the diagram was drawn. This meant he had as many cards arranged beneath the diagram as there were empty boxes within. By asking him to count his sets and row and column boxes, he was able to see another way of arrangement and to correct this before moving on to his confabulated reasoning for his randomly placed colour sequences. One advantage of not using a diagram at all would be to allow for this alternate array arrangement. A further 7 year old JE boy within this level of performance placed every card upside down on the diagram. I referred to this by wondering aloud what would happen if my candles were on my cake that way. He said that they would all go out, and continued with his upside down placement.

Six children achieved a level of performance of 14, a correct cardinal and ordinal array with vertical colour sequencing completed without questioning, showing consciously applied conceptual awareness. None of these children were aware of horizontal colour sequencing possibilities. One 10 year old SE 1 boy with a level of performance of 15 reversed the above performance by placing all cards in correct horizontal colour sequence, a much more difficult task than the vertical colour grouping, but he did not notice the possibility of vertical colour placements.

Another series of children with a correct cardinal and ordinal array and complete vertical colour grouping gave confabulated reasons for an occasional and randomly placed horizontal colour sequence. One 10 year old SR girl with a level of performance of 16 confabulated the correct reason but was unable to make any conceptual transfer to rearrange the whole set. Two further 9 year old SE 2 children with a level of performance of 17 were able to make this transfer, and two more children were able to make this transfer at speed. These two children with a level of performance of 18 included an 8 year old AV girl and a 7 year old JE girl. These were the two highest levels of performance from younger children.

Three further children with a level of performance of 19 produced a cardinal and ordinal array with both vertical and horizontal colours correct as in Plate 21 (Figure 89b). These included 9 and 10 year old children at SE 1 and SR mathematical levels. One of these children placed his vertical colour cards with the same candle colour at the end of the sequence on each card instead of at the beginning as other children arranged them. Either is correct if consistent. All of these children were then able to see and enjoy diagonal as well as vertical and horizontal colour and cardinal and ordinal relationships and patterns.

The remaining two performances from children were truly exceptional. A 9 year old SE 1 boy with a level of performance of 20 applied a strategy to enable him to visualise and verbalise the whole array of 121 cards before he placed them on the diagram. This was the same boy who applied the visualising and verbalising strategy before manipulating and placing set pieces in Figure 57, the fimo Venn Diagram.

The final performance of 21 was from a 10 year old AV girl who gave a novel performance by making an intersecting brick array. She intersected the bottom number one cards with number two cards above, using both horizontal and vertical colour sequencing. She continued this pattern throughout the extended 121 set array, with the intersecting cards moving one along to the left with each increasing row. Each card above and overlapping two other cards contained at its beginning and end the colours presented in the two cards below it, giving an example of alternate ordering in conjunction with the consecutive ordering of the array. This gave a simple triangulation where the top point of a triangle contains the colours included in and coming between the two bottom points of the triangle. This triangular concept applies throughout the array, within every size of triangle present. The bottom points provide the start and finish of what is contained in the top point.

Within this intersecting array are also two cardinal and ordinal arrays with both vertical and horizontal sequencing. One, with a constant starting colour in each diagonal column flows in one direction, and the other with a constant finishing colour in each diagonal column flows in the opposite direction. This 10 year old AV girl verbalised all of these observations, including noticing that the end of the array linked with its beginning, meaning the array could be arranged in circular form. She did not attempt to transfer the array into circular form. Plate 22 (Figure 89f) shows this novel intersecting brick array.

There were several other ways of arranging these cards which were not discovered by the children in the 1985-1986 study. These are included here to give examples of continuing novel performance possibilities. One of these novel performance possibilities, also involving alternate ordering and consecutive ordering, was the result of a first attempt at Figure 89 by a 13 year old girl in 1987. She called this an 'odds and evens' array, and observed that this array could connect in circular form. She was given the full set of 121 cards and no diagram board. Plate 23 (Figure 89g) shows this novel odds and evens array. This arrangement is similar in concept to the hierarchy triangle button series where a set naming single element is added to as it progresses upwards through its hierarchy triangle. Symmetry is an important feature with the addition and subtraction of elements to and from the basic set colours. The vertical, horizontal and diagonal colour sequencing patterning in these two previous novel arrays is fascinating. An example is reversed colour sequencing along exterior edges of these triangles in Figure 89g.

There are further arrangement possibilities for Figure 89 not discovered or attempted by children. The simplest of these is a circular arrangement of Figure 89b, the full 121 card cardinal and ordinal vertical and horizontal colour array. Cards may radiate outwards with the smallest or largest numbers of candles at the centre, or the periphery of the wheel, as shown in Plate 24 (Figure 89h). The advantage of the circular array is continuity of pattern and colour. The disadvantage is the increasing separation of spokes towards the periphery of the wheel, making colour patterns and connectedness between spokes more difficult to observe. The positions and order of cards for a wheel array (Figure 89h), are easy to transfer from an already formed rectangular array (Figure 89b). If a wheel array is made from loose cards, those with the largest number of candles on them need to form the central ring to determine positional ordering of colour sequencing. Further cardinal concentric rings are then placed in outward ordinal direction from 11-1 candles on each card.

A further series of arrangements for the birthday candle cardinal and ordinal colour array cards is shown in Figure 89i, (Plate 25), as cardinal colour continuation circles. These involve making separate colour continuation circles for each cardinal set of cards. The order of colours on the candles can be seen in entirety on cards with 11 candles on them. The first colour on any card is to follow on from the last colour on the previous card for each cardinal colour continuation circle. The last card to be placed has colour continuation links with the first card.

If cardinal colour continuation circles are made directly from a cardinal and ordinal colour array in wheel form, a strategy can be applied for selecting cards which eliminates the slow visual checking approach for colour continuation. If a cardinal colour continuation circle using cards with nine candles on them is being processed, take one card with nine candles on it from the wheel array circle of nine candle cards, and begin counting around this circle from where the gap has been made. Each ninth card counted around the circle will be the correct colour continuation card for the cardinal colour continuation circle for nine candles on each card. If a circle of six candle cards is being processed, every sixth card in the six candle card circle on the wheel array will provide the correct colour continuation card, and every third card in the three candle card circle on the wheel array will provide the correct colour continuation. This strategy is consistent throughout the range of colour continuation circles, whichever cardinal number is being used.

Thought was given to reducing the number of coloured candles to the more conventional number of ten instead of eleven when considering this puzzle for possible manufacture. This was not a problem until it was discovered that with ten candle colours instead of eleven, this particular strategy only worked for cards with particular numbers of candles on them and not for others. The reason the strategy works consistently with an eleven candle set is because eleven is both an odd and a prime number, and is not divisible by other numbers as is the number ten.

A further arrangement moving on from cardinal colour continuation circles is to make one large colour continuation circle using most of the set of 121 cards. Beginning with the one candle cards, check the colour flow from an eleven candle card, and continue using every card in ordinal form from the set of one candle cards to the set of ten candle cards. As any eleven candle card contains the full colour range of the candles, only one eleven candle card needs to be used to link up the last placed number ten candle card with the first placed number one candle card.

There will be further possibilities for arrangement of these array cards that have not been discussed here or even thought of at this time. One such possibility might be to give a child the task of seeing if it is possible to create a two or three difference sequence as an extension exercise beyond the inherent one difference sequence between connecting cards arranged into this vertical and horizontal colour cardinal and ordinal array.

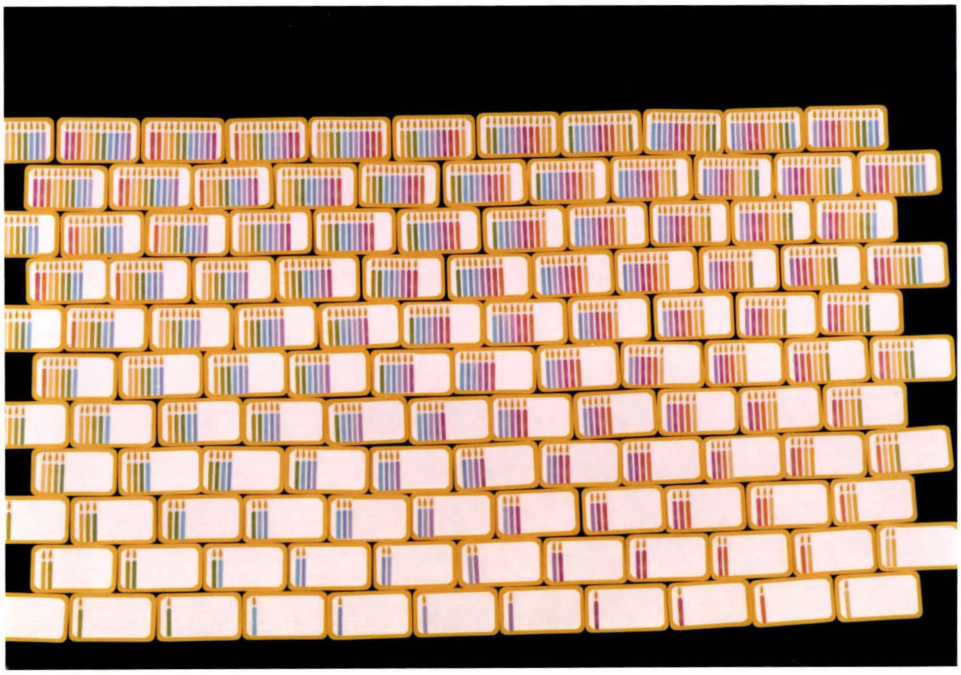


Plate 22: Figure 89f: Birthday Candle Cardinal and Ordinal Colour Array: Intersecting Brick Array.

Plate 23: Figure 89g: Birthday Candle Cardinal and Ordinal Colour Array: Odds and Evens Array.

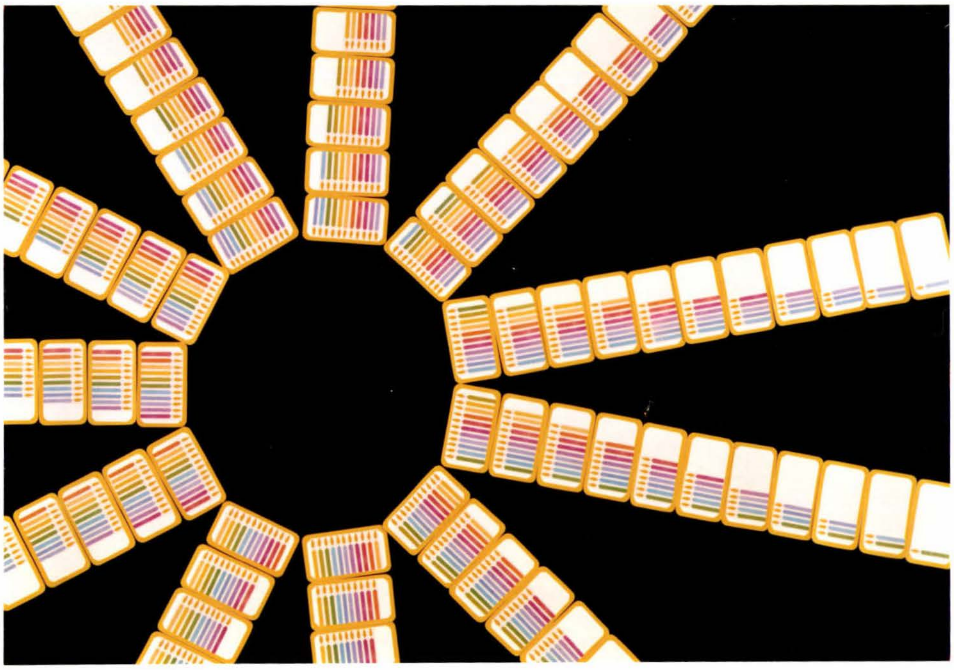


Plate 24: Figure 89h: Birthday Candle Cardinal and Ordinal Colour Array: Wheel Array.

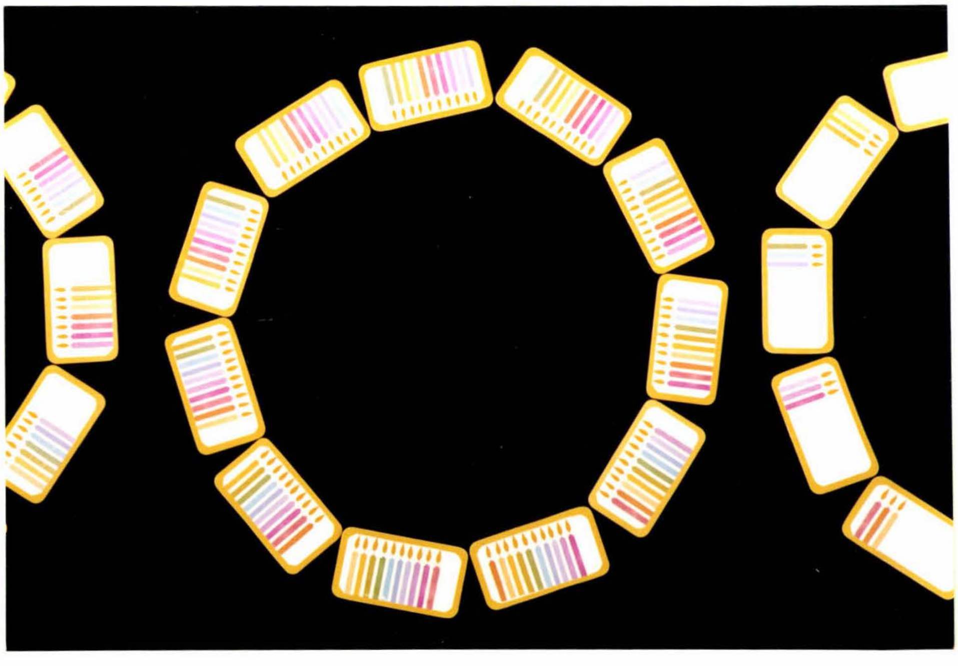


Plate 25: Figure 89i: Birthday Candle Cardinal and Ordinal Colour Array: Cardinal Colour Continuation Circles: 3's, 10's, 7's.

Gender differences were not significant for the diagnostically progressive puzzles in Chapter 7, with the exception of Figure 85, significant in favour of males. This is likely to be a chance result. Table 41A shows two tailed Mann-Whitney U-Tests testing gender differences in levels of performance across mathematical groups for Chapter 7, at 0.05, where U (M) = Males, U1 (F) = Females, and z shows use of normal approximation for larger samples.

Table 41A: Mann-Whitney U-Tests For Chapter 7: Gender Differences: Levels of Performance Across Mathematical Groups:

Figure	U (M)	U1 (F)	z	Males	Females
Puzzles demonstrating diagnostic progressions					
85	58.5	4.5		*	-
86	152.5	147.5	+ 0.07	-	-
87	154.5	144.5	+ 0.16	-	-
88	56	70		-	-
89	141	264	+ 1.61	-	-

* Significant at 0.05.

Puzzles discussed and presented in Chapter 7 show examples of most of the previously described conceptual levels and difficulties children exhibit in unconscious and conscious processing involved in the movement from pre-mathematical to formal mathematical conceptual understanding. These included unconscious processing where classifications were made with the child unable to verbalise the classifications; perceptual priority modes; perceptual difficulties, problems with lines, reversals, symmetry and rotations; problems with multiple concepts such as classifying and ordering, classifying with consecutive ordering, alternate ordering and rotational ordering; confabulation; transfer of learning; and conscious use of strategy.

Unconscious processing where classifications were made with a child unable to verbalise the classifications occurred in the button holes cardinal and ordinal set when a child unconsciously classified sets of buttons with three and four holes in them, but was unable to verbalise these classifications with no conscious awareness of them. Perceptual priority modes became apparent when a child first classified buttons according to colour and then size, before classifying the numbers of holes in the buttons. What was a familiar mode was processed first. A process of elimination followed from what was familiar to the child, to what was unexpected. Perceptual difficulties were evident when one child could not see butterfly shapes in the fabric butterfly wing array, and when another child could not make a one to one correspondence between cards and boxes on a diagram in the birthday candle cardinal and ordinal colour array. Difficulties between the subtleties of colours occurred for some children in the birthday candle array. Problems with lines occurred where several children attempted to place transparent coloured triangles on absentee lines.

Problems with reversals and symmetry occurred when one child could not visualise the bilateral symmetry in butterfly wing shapes, and in other children who found difficulty reversing triangles to make star shapes in the transparent coloured triangles set. Problems with multiple concepts such as rotational ordering occurred with the placement of transparent triangles. Examples of classifying and ordering with the butterfly wings, and classifying with consecutive ordering were evident in the button holes set and the birthday candle array. The birthday candle array also gave examples of alternate ordering and many examples of confabulated reasoning for a randomly placed correct answer observed after placement and during questioning.

Children also showed signs of difficulty with multiple concepts such as being able to hold one concept or another constant, but not both simultaneously. This was frequently observed throughout various grades of cardinal and ordinal sequencing, as well as stages of colour sequencing in the birthday candle cardinal and ordinal colour array. Sometimes a conceptual observation was made by a child who was unable to process the concept in a practical rearrangement of the birthday array. Further children became fatigued and lost both concentration and conceptual constancy. Transfer of learning difficulties and the conscious need for use of strategy were particularly noticeable with the transparent coloured triangles.

Summary

Examples given in Chapter 7 illustrate the inconsistently applied and improperly established form of some of these concepts in children, and show the immediate effects of 'on the spot' learning and transfer of learning. Perceptual and conceptual processes that are unconscious may or may not be drawn to conscious awareness allowing verbal introspection of such concepts and processes. Speed indicates firmly established conceptual understanding, whether consciously or unconsciously applied. The use of strategy to reduce the amount of visual checking required and increase the speed of the solution indicates a conscious application of conceptual understanding. This also indicates logical processing and sometimes a possible intuitive lateral thinking approach.

Chapter 8 looks at ranges in levels of performance of children within mathematics groups, and the unexpected lack of correlation between puzzle performance and P.A.T. results, as well as some unexpected performances from junior children.

Chapter 8

Range in Levels of Performance

Chapter 7 looked at the range of diagnostically progressive conceptual levels of performance within some puzzles. Chapter 8 looks at some puzzles showing an unexpected range of levels of performance from children within mathematical groups, and in three different areas. The first area looks at puzzles including children from a wide range of mathematical group categories, and the lack of correlation between these mathematical groups and levels of performance from the children. The second area looks at puzzles involving mostly children from one mathematical group range, and the wide range of levels of performance from these children. The third area examines levels of performance from specific children within a specific mathematical group.

The first puzzle showing unexpected levels of performance from children within a wide range of mathematical groups is a very advanced junior puzzle, Figure 90, Red and Black Geometric Shapes. This puzzle consists of a range of geometrically cut shapes which fit onto seven red squares.

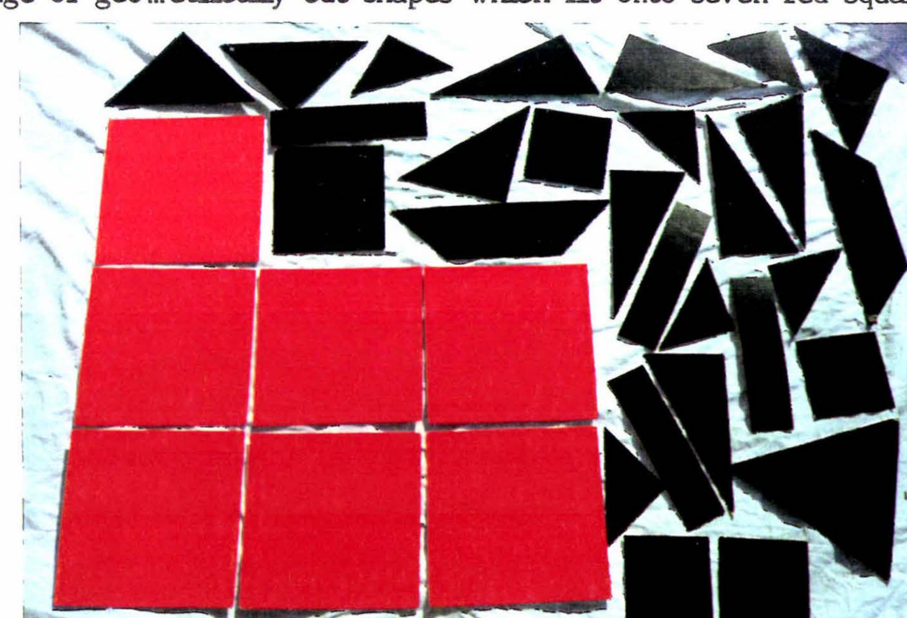


Plate 26: Figure 90: Red and Black Geometric Shapes:
Vrebal Instructions:

- Fit the black shapes onto the red squares.

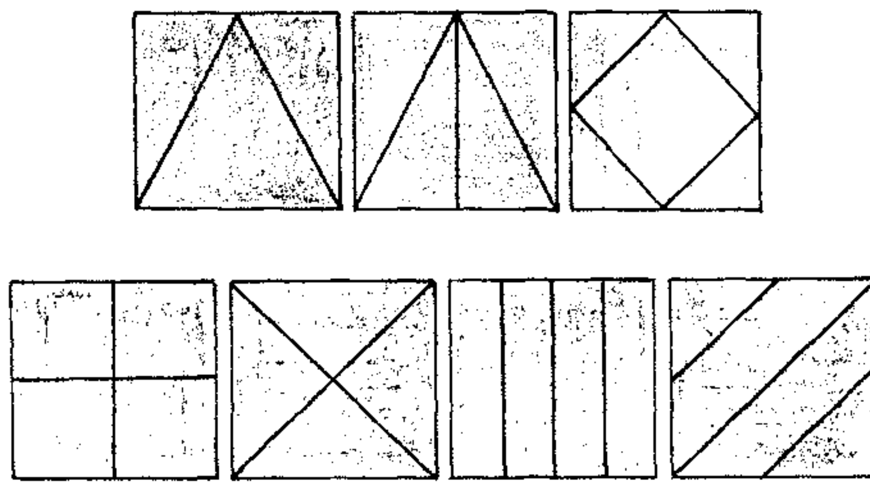


Figure 90a: Red and Black Geometric Shapes.

Figure 90b shows Figure 90 levels of performance across mathematical groups:

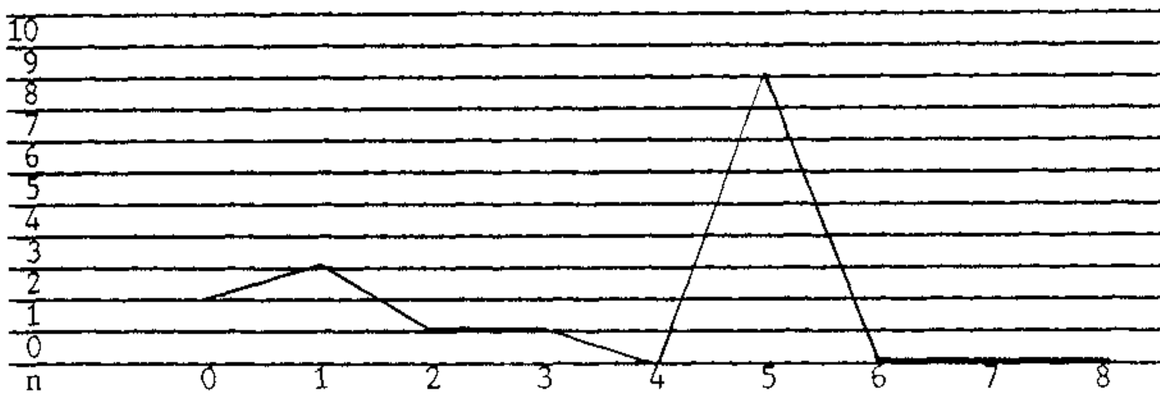
Levels of Performance \ Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6								
5		1	3			2		3
4								
3					1			
2					1			
1			1		2			
0			1					1

Levels of Performance

0 - Too difficult
1 - Difficult and inaccurate with a lot of questioning
2 - Difficult accurate completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 90b: Red and Black Geometric Shapes.
Levels of Performance Across Mathematical Groups. n = 16.

Figure 90c shows levels of performance across all children who attempted Figure 90:

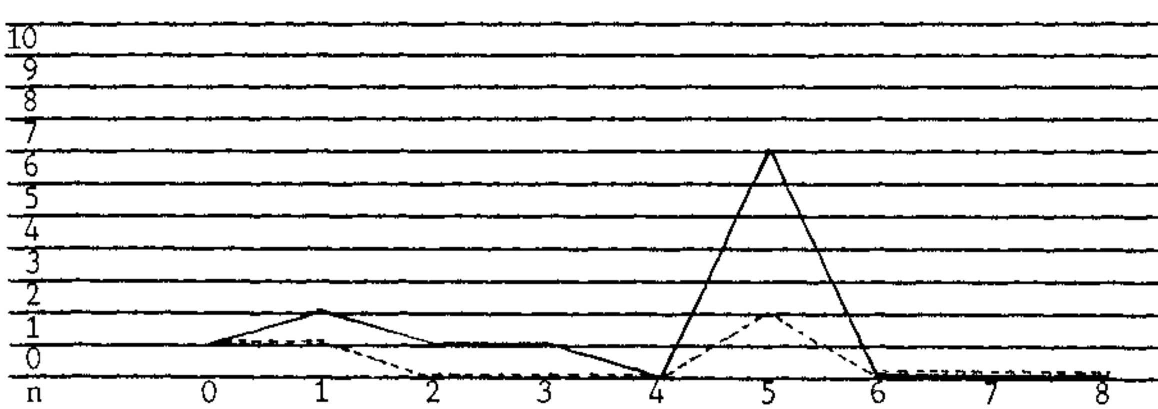


Levels of Performance

0 - Too difficult
1 - Difficult and inaccurate with a lot of questioning
2 - Difficult accurate completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 90c: Red and Black Geometric Shapes.
Levels of Performance. n = 16.

Figure 90d examines gender performance across levels of performance: (Levels of performance as for Figure 90c).



n = 12 ———— Males
n = 4 ---------- Females

Figure 90d: Red and Black Geometric Shapes.
Gender Performance Across Levels of Performance. n = 16.

Figure 90e shows children at the highest level of performance for Figure 90 across mathematical groups and age.

expected

Age	J SLD	JR	JE	S SLD	SR	AV	SE 2	SE 1
11								
10						1		
9								2
8			3			1		1
7		1						
6								

unexpected

Age / Maths Groups: J SLD, JR, JE, S SLD, SR, AV, SE 2, SE 1

Figure 90e: Red and Black Geometric Shapes.
Highest Level of Performance: Complete with no questioning:
Mathematical Groups Across Ages. n = 9.

Subject numbers were small for Figure 90, but the contrast between a 6 year old JR boy with a performance at the highest level and a 9 year old SE 1 boy with the lowest level of performance was striking, although may have been due to chance. This puzzle required application of spatial relationships. Items concerning spatial relationships are phased out at senior primary school level in P.A.T. formal mathematics in New Zealand schools. This may help explain the lack of correlation of P.A.T. mathematical groups and levels of performance. Figure 90 was not easy for children. No child performed at speed or used a strategy, but most children utilised symmetry in the way they placed the shapes onto the squares. A Mann-Whitney U-Test shows no gender performance differences.

A further puzzle showing a wide performance range across mathematical groups was Figure 91, a very advanced junior puzzle, Rainbow Colour Continuation Strips. The strips contained a set order of eleven rainbow colour bands. Each set of two strips started and finished with a particular colour that was different from the starting and finishing colours of each other set of two colour strips. Several possible arrangements included steps, a V shape and inverted V shape. Sets of two could be placed alongside or on top of each other. Although there was some correlation between mathematical groups and levels of performance, one 9 year old SE 1 boy performed below a 7 year old JE girl. Plates 26, 27, and 28 show Figure 91, 91a, and 91b arrangements; and Plate 29 shows Figure 91c depicting a 12 year old girl processing Figure 91 in a 1989 study.

Plate 27: Figure 91: Rainbow Colour Continuation Strips:
Verbal Instructions:

- Match the colour strips.

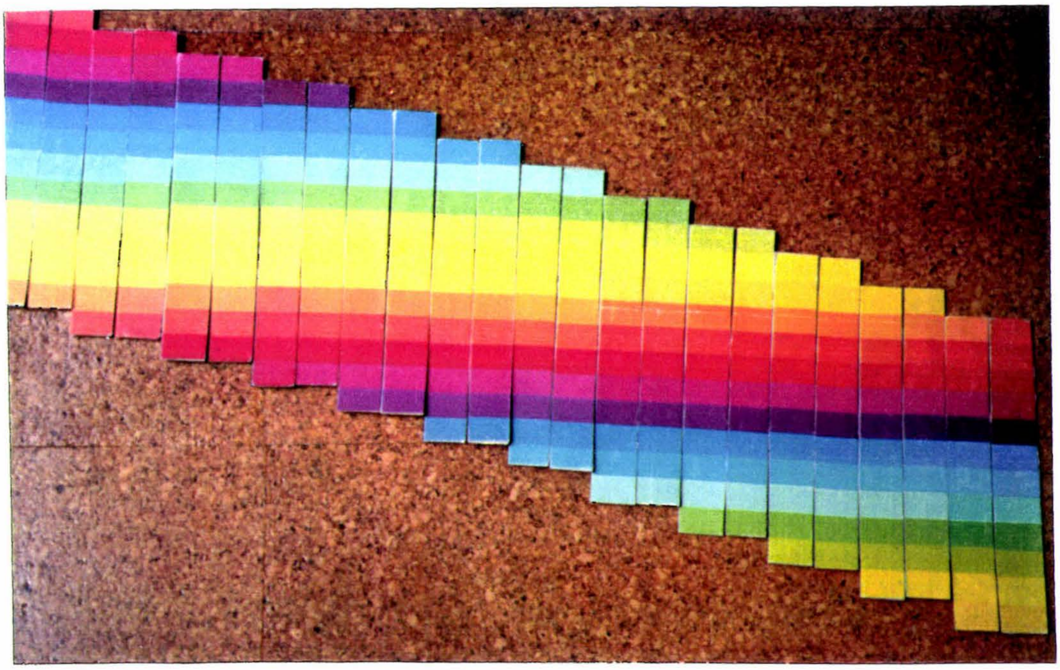


Plate 28: Figure 91a: Rainbow Colour Continuation Strips: Steps.

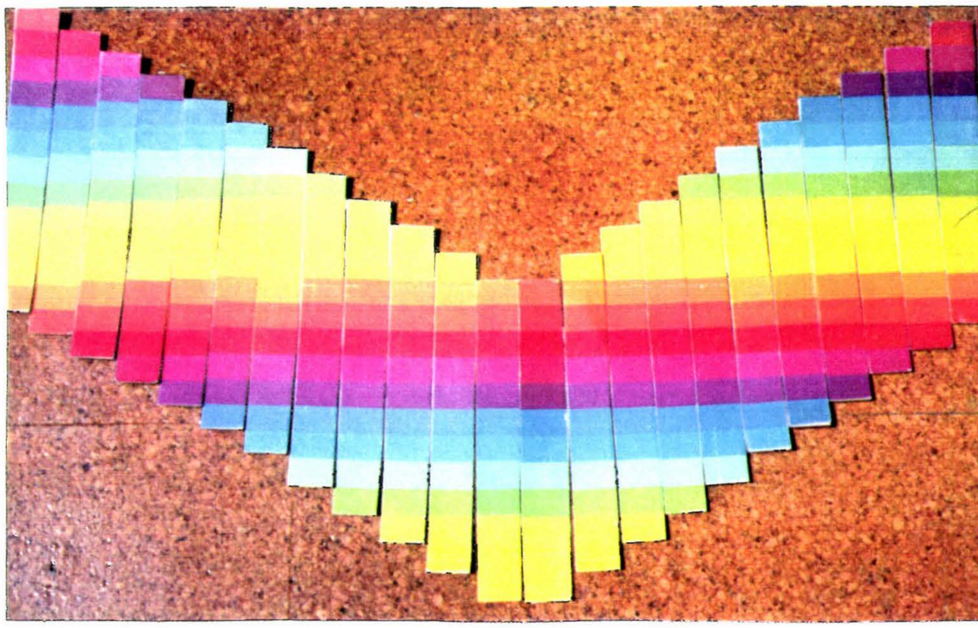


Plate 29: Figure 91b: Rainbow Colour Continuation Strips: V Shape.

Figure 91b shows Figure 91 levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6						1		
5			1		2	1		
4								
3								1
2					1			
1	1	1						
0	1	1	1					

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 91b: Rainbow Colour Continuation Strips:
Levels of Performance Across Mathematical Groups. n = 12.

Subject numbers may be too small for reasons other than chance for a wide performance range within and across mathematical groups. With a wide performance range in previously presented puzzles such as Figures 3, 4, 14, 20, 32, 34, 39, 42, 44, 57, 64, 85, 86, 87 and 89, further investigation into the reasons for this are implicated. Difficulties with colour perception may have created difficulty in Figure 91. The main difficulty seemed to be the dual multiple conceptual task of matching sets of two and an additional task of consecutive ordering. With multiple arrangements for possible ordering and no diagram provided, some children found the creation of an ordered symmetrical continuation shape a distraction from the colour continuation process.

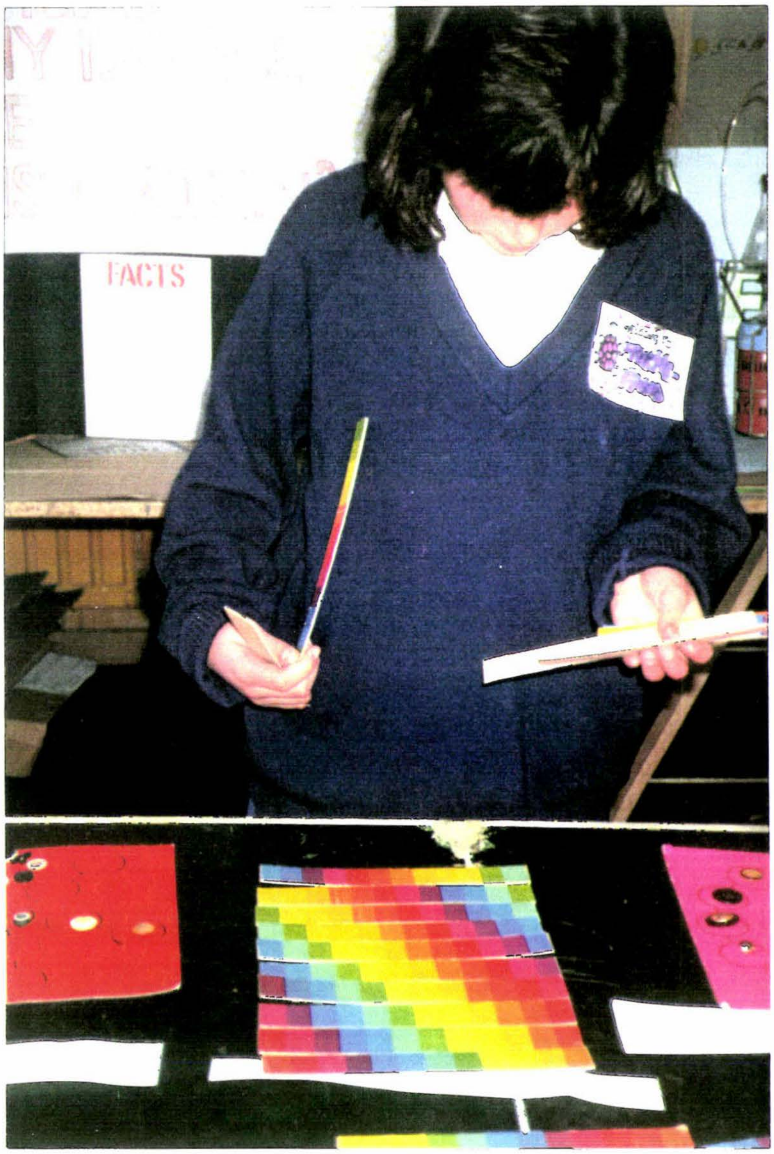


Plate 30: Figure 91c: Rainbow Colour Continuation Strips: Processing.

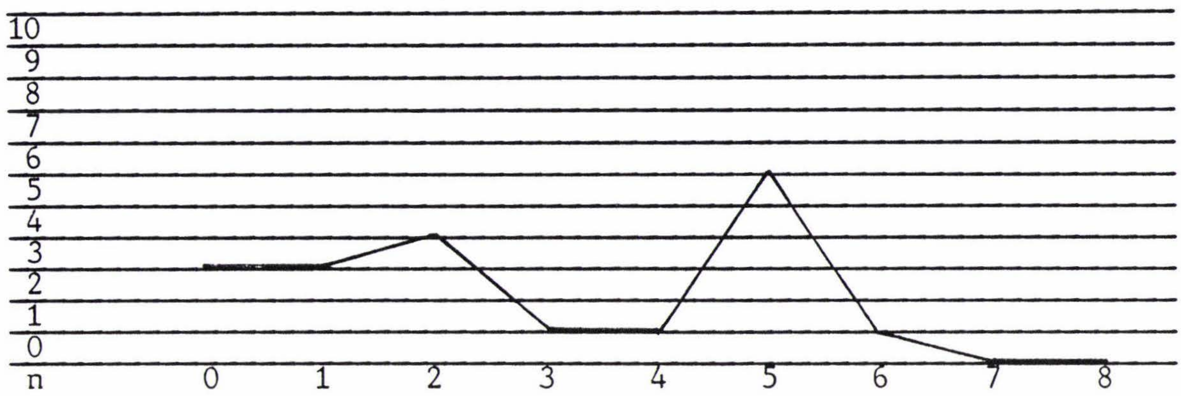
A further puzzle with mixed levels of performance across junior and senior children was a series of beginning senior puzzles of wallpaper flower pattern continuation strips requiring pure pattern matching skills. They are collectively presented in Plates 30 and 31 as Figure 92.

Plate 31: Figure 92: Wallpaper Flower Pattern Continuation Strips:

Plate 32: Figure 92: Wallpaper Flower Pattern Continuation Strips: Verbal Instructions:

- Join the pattern strips together to make one big picture.

Figure 92a shows Figure 92 levels of performance:

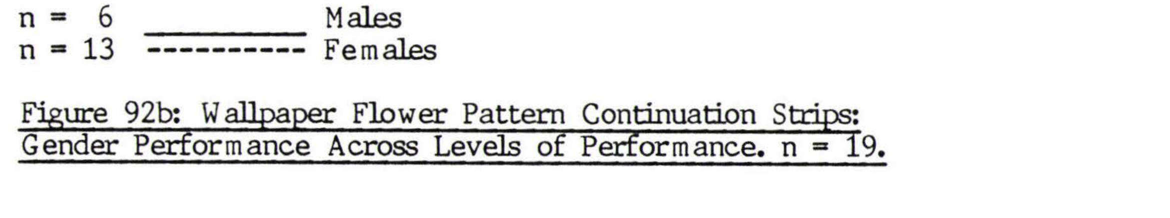


Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 92a: Wallpaper Flower Pattern Continuation Strips: Levels of Performance. n = 19.

Figure 92b examines gender performance across levels of performance: (Levels of performance as for Figure 92a).



n = 6 ________ Males
n = 13 ---------- Females

Figure 92b: Wallpaper Flower Pattern Continuation Strips: Gender Performance Across Levels of Performance. n = 19.

Figure 92c shows Figure 92 levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6						1		
5		2	1		2		1	
4					1			
3					1			
2			1	1	1	1		
1					3			
0		1	2					

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 92c: Wallpaper Flower Pattern Continuation Strips: Levels of Performance Across Mathematical Groups. n = 19.

The majority of the children attempting the series of pattern continuation strips found these puzzles difficult. Pure pattern matching seems to be one of the more difficult perceptual processes. What was surprising was not the range of children from JR to AV who experienced difficulty, but the equally high range of level of performance from JR, JE, SR and SE 2 groups. A Mann-Whitney U-Test shows no gender performance differences.

A further puzzle within the wallpaper pattern continuation strip series was Figure 93, an average senior puzzle. Figure 93 is the first of the puzzles showing a wide range of levels of performance within a mathematical group range. Puzzles of this type are more suitable for either junior or senior children. Previous examples include Figure 18, Reversed Sailing Ships, showing a wide range of performance from JR children, and Figure 40, One Colour Difference Button Figure Eight, where the wide performance range was across SE 1 and SE 2 children.

Plate 33 shows Figure 93:

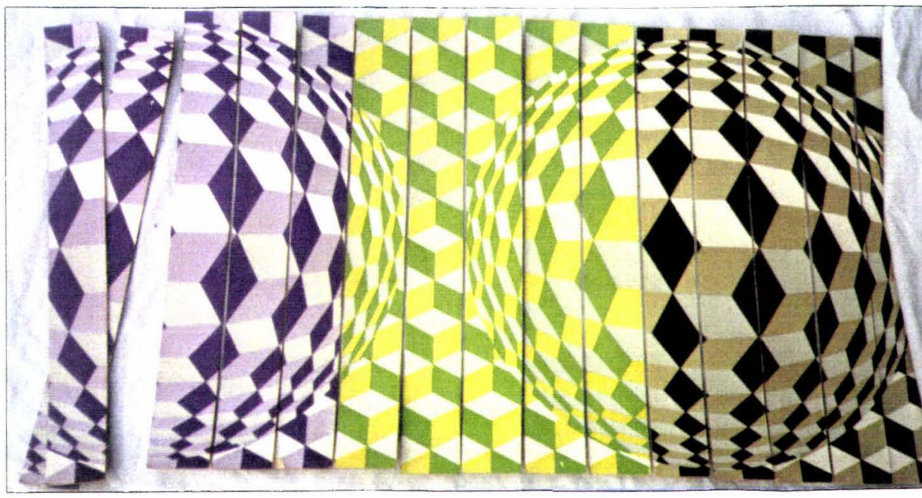


Plate 33: Figure 93: Wallpaper Sphere Pattern Continuation Strips.

Figure 93a shows Figure 93 levels of performance across mathematical groups:

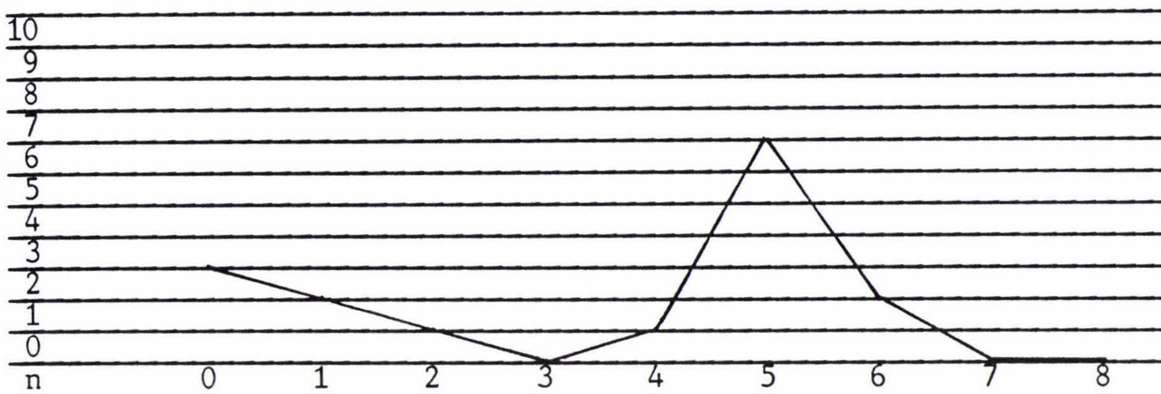
Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6								2
5					1	1	1	4
4								1
3								
2						1		
1					1		1	
0		1			2			

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of rearranging
2 - Difficult completion with a lot of rearranging
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 93a: Wallpaper Sphere Pattern Continuation Strips: Levels of Performance Across Mathematical Groups. n = 16.

Figure 93b shows Figure 93 levels of performance.

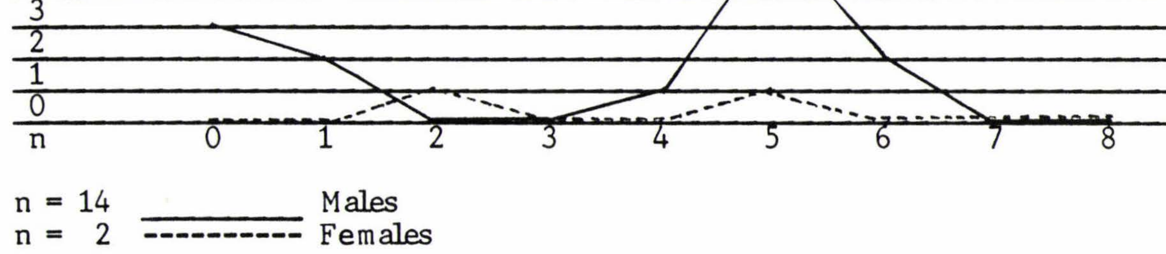


Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of rearranging
2 - Difficult completion with a lot of rearranging
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 93b: Wallpaper Sphere Pattern Continuation Strips: Levels of Performance. n = 16.

Figure 93c examines gender performance across levels of performance: (Levels of performance as for Figure 93b). A Mann-Whitney U-Test shows no gender differences in performance.



n = 14 ________ Males
n = 2 ---------- Females

Figure 93c: Wallpaper Sphere Pattern Continuation Strips: Gender Performance Across Levels of Performance. n = 16.

This senior puzzle showed similar levels of performance between SR and SE 2 children, as both groups exhibited low and high performance levels. This was an all boy performance range apart from the two AV girls. Spatial relationship skills were necessary for successful completion of this puzzle.

Figure 94, an above average senior puzzle was attempted only by SE 1 children apart from the two AV girls and one JE girl. As well as a full range of performance within the SE 1 group, the performance of the JE girl was at the highest level for this puzzle. Table 42 shows Figure 94 correlation coefficients between levels of performance and mathematical groups, school class levels, and ages.

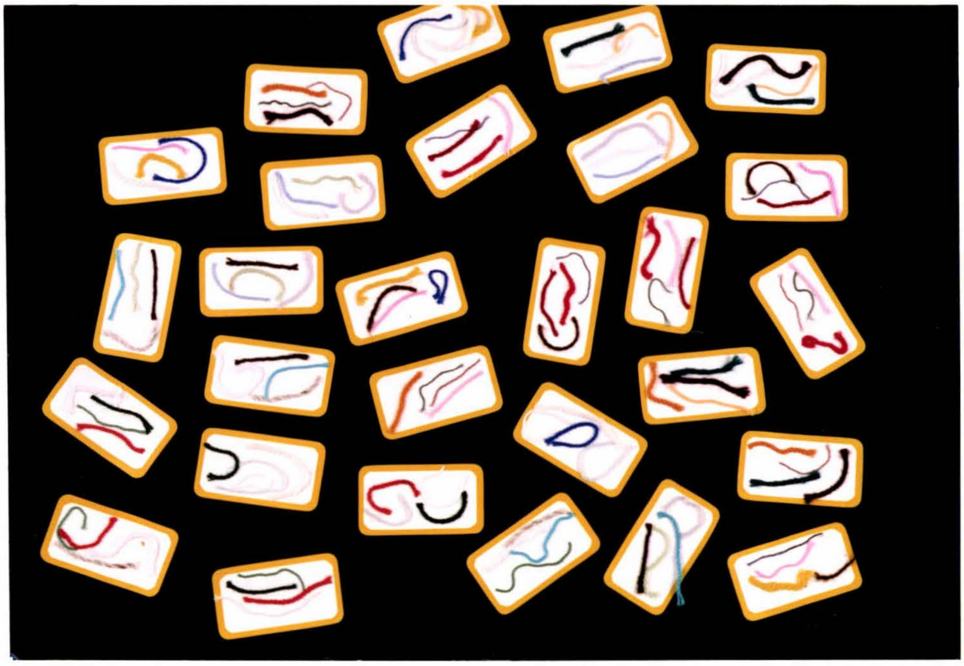
Table 42: Spearman's Rho Correlation Coefficients:
Figure 94: One Colour Difference Wool Circle:
Levels of Performance: n = 11.

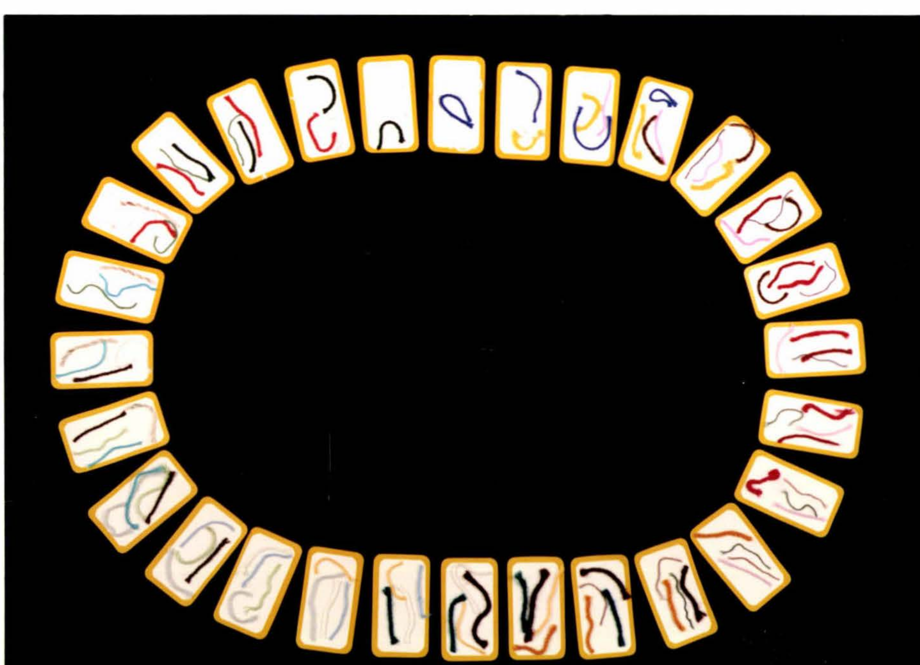
Across mathematical groups	$r_s = 0.111$	$p = >0.05$
Across school class levels	$r_s = 0.345$	$p = >0.05$
Across ages	$r_s = 0.265$	$p = >0.05$

These were not significant. This was not surprising as most of the children were from the SE 1 mathematical group, and ranged in levels of performance from too difficult to completing the puzzle with no questioning help. This highest level of performance was also achieved by the only junior child given Figure 94. Difficulties with colour and texture discrmination were one kind of difficulty, but the major difficulty lay in the inherent consecutive ordering pattern and the need to observe repeating patterns of colours and use this as a strategy to eliminate the amount of visual checking. Plates 34, 35 and 36 show Figure 94, One Colour Difference Wool Circle. Verbal instructions were as follows.

Verbal Instructions:

- Start with one card.
- Find a card with one colour difference to connect to the first card.
- Place cards in a one colour difference circle.
- The last card must link with the first card.





94:
Plates 34 - 35: Figure 94a: One Colour Difference Wool Circle:

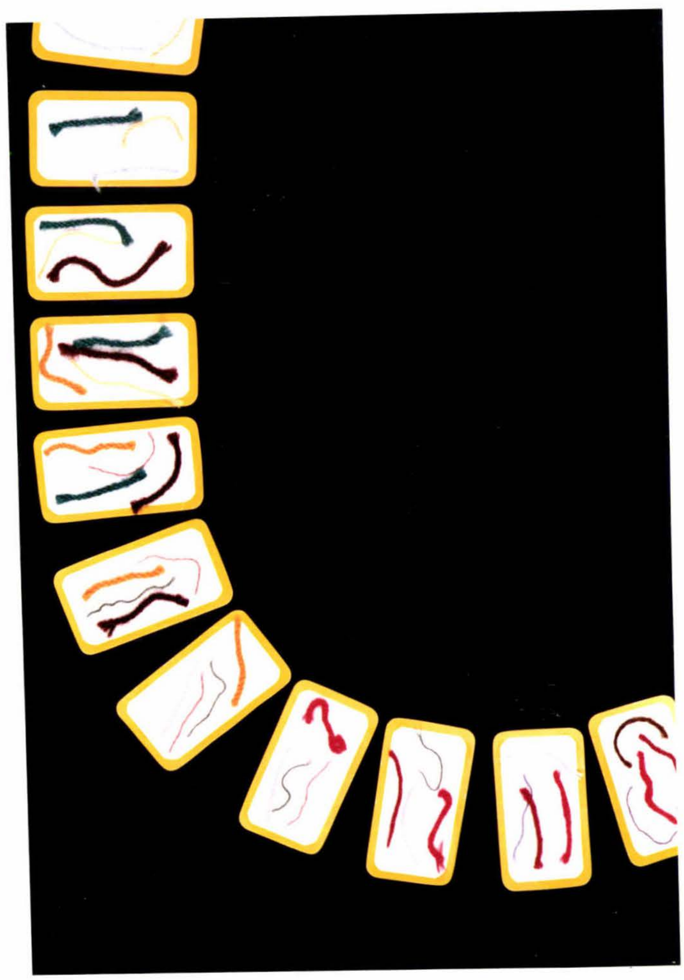


Plate 36: Figure 94a: One Colour Difference Wool Circle:

Figure 94b shows Figure 94 levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6								
5			1			1		4
4								
3								
2						1		2
1								
0								2

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete
2 - Difficult completion
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

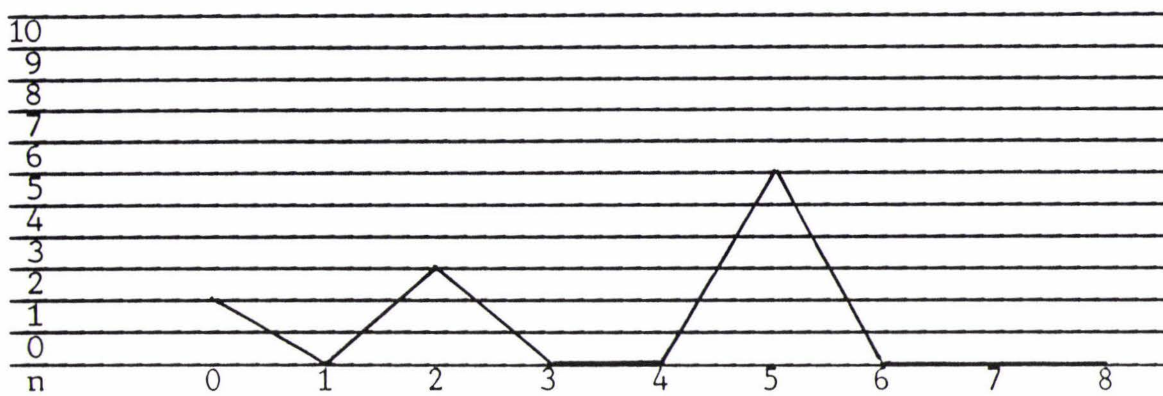
Figure 94b: One Colour Difference Wool Circle:
Levels of Performance Across Mathematical Groups. n = 11.

Figure 94c shows children at the highest level of performance for Figure 94 across mathematical groups and age.

Age	Maths Groups J SLD	JR	JE	S SLD	SR	AV	SE 2	SE 1
								expected
11								
10						1		2
9								2
8								
7			1					
6								
unexpected								

Figure 94c: One Colour Difference Wool Circle:
Highest Level of Performance: Complete with no questioning:
Mathematical Groups Across Ages. n = 6.

Figure 94d shows levels of performance across all children who attempted Figure 94:



Levels of Performance

0 - Too difficult
1 - Difficult and incomplete
2 - Difficult completion
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 94d: One Colour Difference Wool Circle:
Levels of Performance. n = 11.

Figure 94e examines gender performance across levels of performance: (Levels of performance as for Figure 94d).

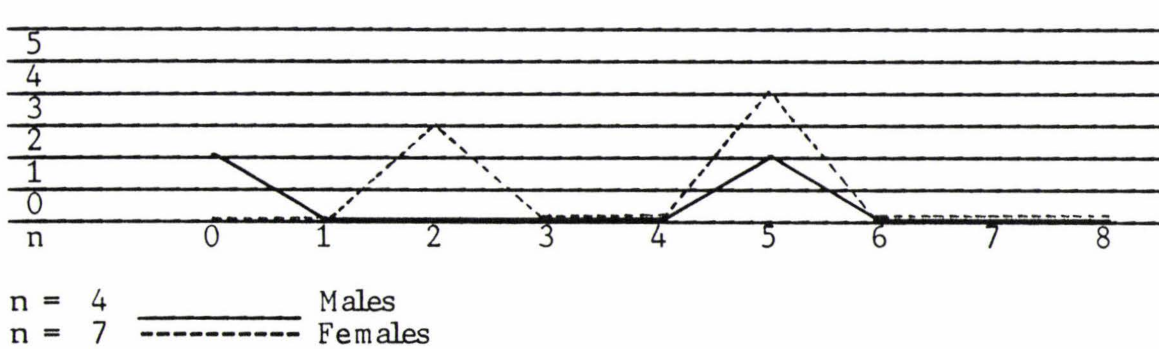


Figure 94e: One Colour Difference Wool Circle:
Gender Performance Across Levels of Performance. n = 11.

A Mann-Whitney U-Test shows no gender performance differences.

A further puzzle showing a wide range of levels of performance within a mathematical group range was Figure 95, an above average puzzle, Checked Fabric Pattern Repeat Array. All of the children attempting Figure 95 were SE 1 children with the exception of two SR children and the two AV girls. Plate 37 shows Figure 95 in incomplete form.

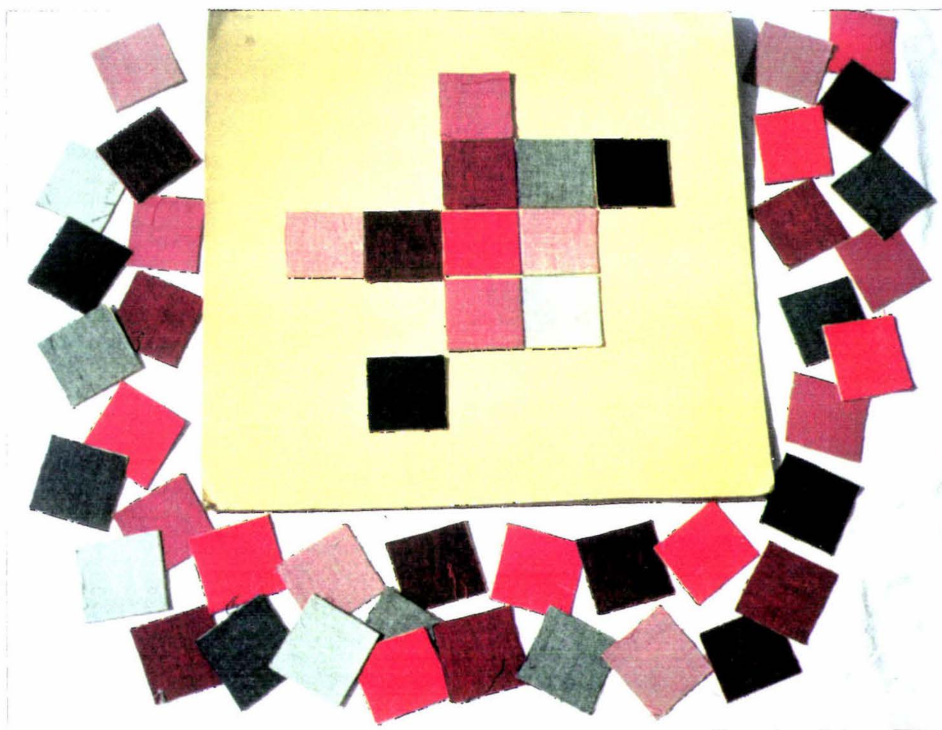


Plate 37: Figure 95: Checked Fabric Pattern Repeat Array:
Verbal Instructions:

- The glued down coloured squares form the beginning of a repeated colours pattern.
- Find patterns of repeated colours and complete them.

Some children needed initial or ongoing questioning to help them observe the repeated patterns of three colours within columns and rows, and to observe the repetition of rows and columns at every third row or column. This puzzle was also progressive as repeated colour patterns in some columns and rows could not be known until other columns and rows were completed. The final colour was placed by a process of elimination. Although the two SR children did not complete Figure 95 accurately, one SE 1 boy also found this too difficult. A wide range of performance came from SE 1 children. The most surprising performance came from a 10 year old AV girl who completed the puzzle very quickly with no questioning required. Plate 38 shows Figure 95a in complete form.

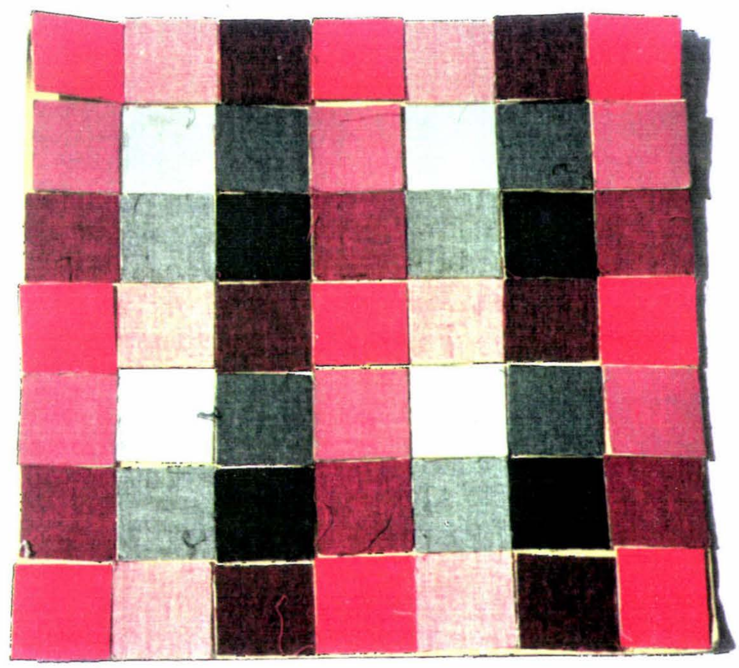


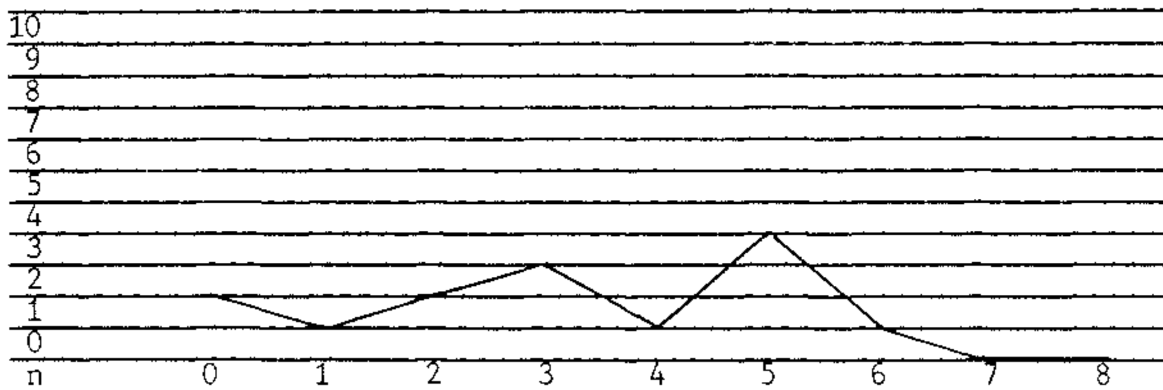
Plate 38: Figure 95a: Checked Fabric Pattern Repeat Array:

Figure 95b shows Figure 95 levels of performance across mathematical groups. (Levels of performance as for Figure 95c).

Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6						1		
5								4
4								1
3								3
2						1		1
1					1			
0					1			1

Figure 95b: Checked Fabric Pattern Repeat Array: Levels of Performance Across Mathematical Groups. n = 14.

Figure 95c shows levels of performance across all children who attempted Figure 95:



Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 95c: Checked Fabric Pattern Repeat Array: Levels of Performance. n = 14.

Figure 95d examines gender performance across levels of performance: (Levels of performance as for Figure 95c).

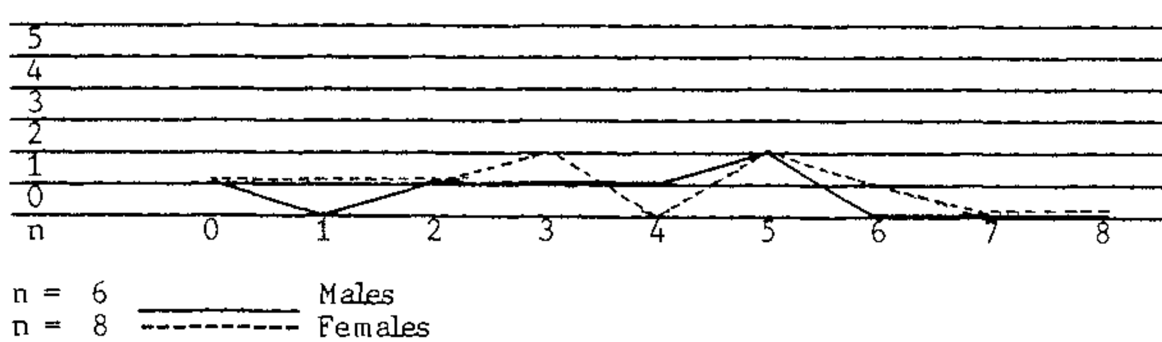


Figure 95d: Checked Fabric Pattern Repeat Array: Gender Performance Across Levels of Performance. n = 14.

A Mann-Whitney U-Test shows no gender performance differences.

The remaining puzzle showing a wide range of levels of performance for SE 1 children was an average senior puzzle, Figure 96, although subject numbers were small. Whereas one 8 year old SE 1 girl found Figure 96 too difficult, another 10 year old SE 1 girl completed it in 30 seconds, as did one 10 year old AV girl. It was too difficult for one 10 year old SR girl, and an 8 year old AV girl completed it in five minutes with no questioning help.

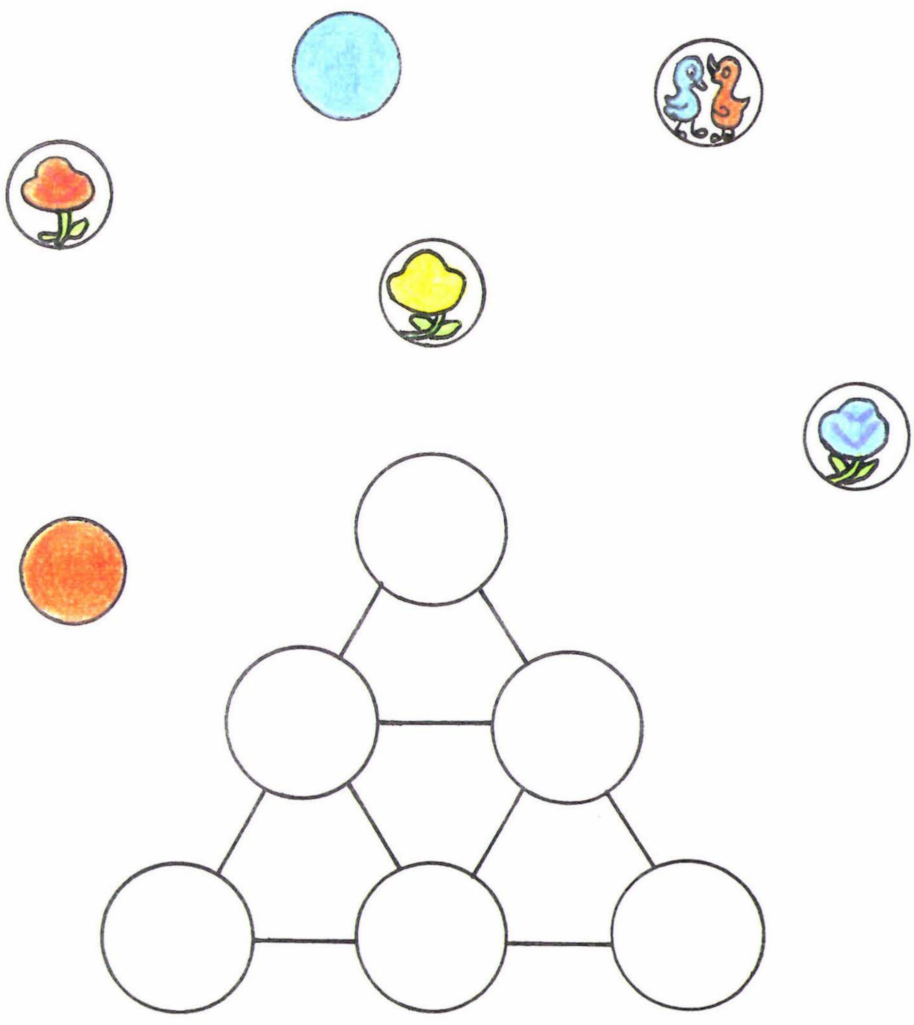


Figure 96: One Difference Button Triangle:
Verbal Instructions:

- Arrange the buttons with no more than a one difference relationship between any buttons you place in connecting circles.

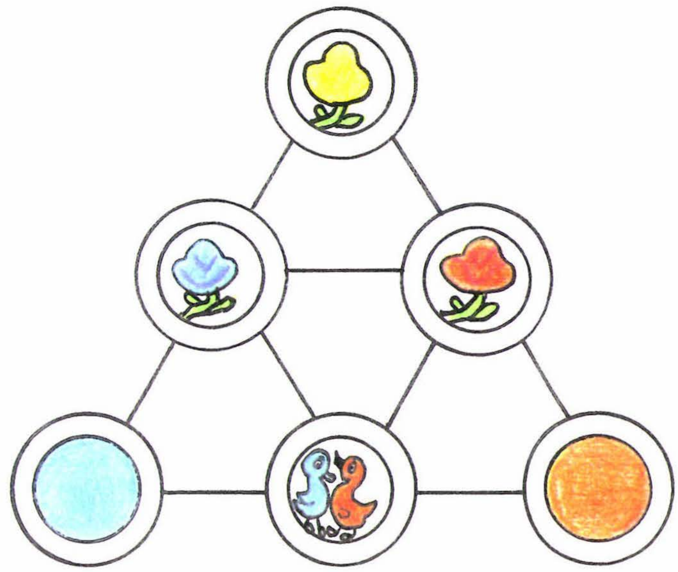


Figure 96a: One Difference Button Triangle:

Figure 96b shows Figure 96 levels of performance across mathematical groups:

Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8						1		1
7								
6						1		
5								
4								
3								
2								
1								
0					1			1

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 96b: One Difference Button Triangle:
Levels of Performance Across Mathematical Groups. n = 5.

The lack of correlation between mathematical groups and levels of performance was evident in children across mathematical groups achieving a range of levels of performance. It was also evident in some children from a particular mathematical group achieving a higher level of performance than expected. An example occurred with the two AV sisters, as evidenced in previously presented puzzles. The 8 year old AV girl achieved a high level of performance in previously mentioned Figures 1, 5, 17, 29, 32, 33, 39, 40, 52, 81 and 82. This girl also achieved the highest level of performance in Figure 97, a very advanced junior fabric array.

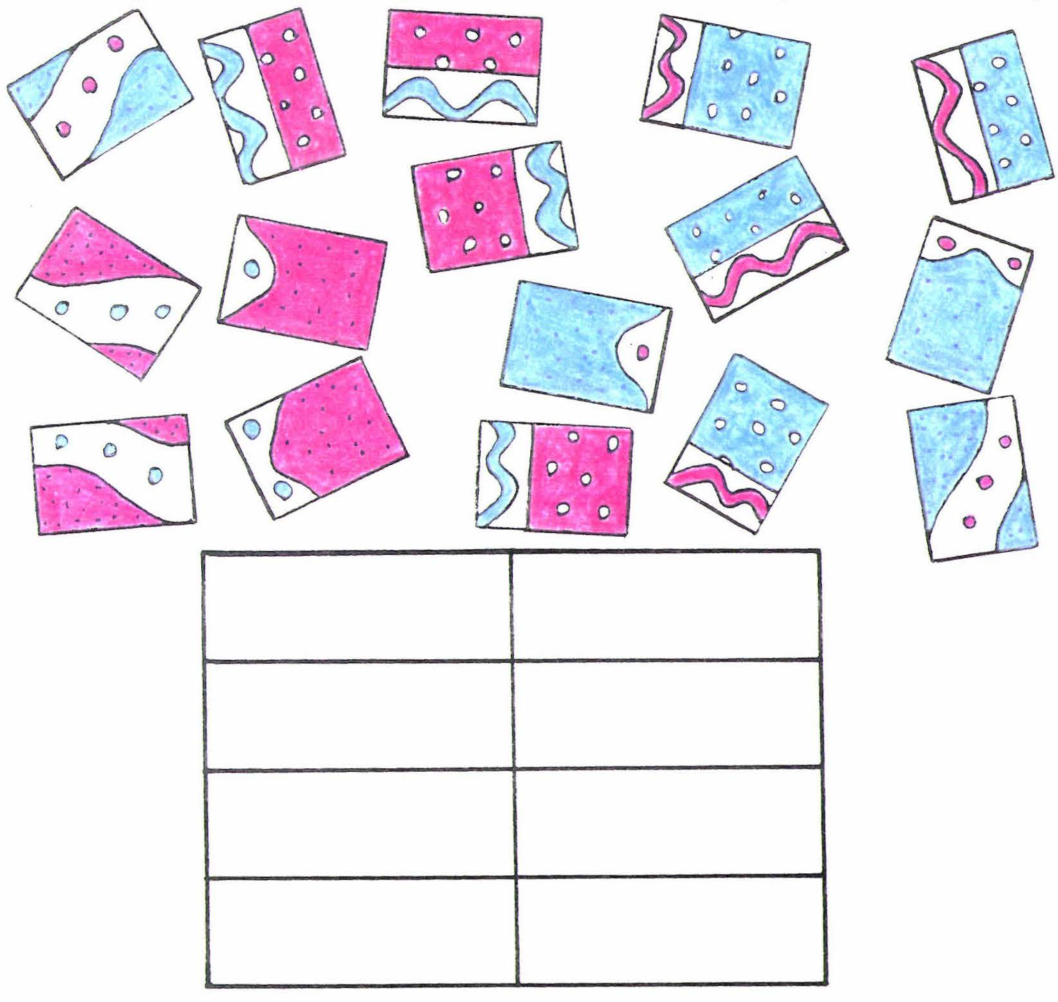


Figure 97: Pink /Turquoise Fabric Array
Verbal Instructions:

- Match the cards.
- Find sets.
- Make an array with the sets to make bigger sets.
- Name the sets.

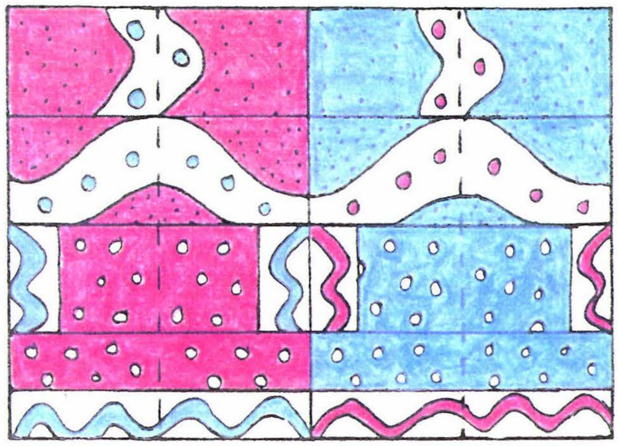


Figure 97a: Pink /Turquoise Reversed Colour Pattern Fabric Array.

Figure 97b shows Figure 97 levels of performance across mathematical groups:

Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
9						1		
8								
7								
6								
5		1				1		
4			1					1
3								1
2		1						
1								
0								

Levels of Performance

0 - Too difficult
1 - Matched some individual colour / pattern cards
2 - Matched all individual colour / pattern cards
3 - Matched some reversed colour pairs of individual cards
4 - Matched all reversed colour pairs of individual cards
5 - Made vertical colour array with horizontal reversed colour pairs with questioning
6 - Made vertical colour array with horizontal reversed colour pairs with no questioning
7 - Vertical colour array into crosscut pattern groups with questioning
8 - Vertical colour array into crosscut pattern groups with no questioning
9 - Very fast vertical colour array into crosscut pattern groups

Figure 97b: Pink /Turquoise Reversed Colour Pattern Fabric Array. Levels of Performance Across Mathematical Groups. n = 7.

There were three surprises in levels of performance for Figure 97. One was that SE 1 children performed at low levels. Another was that one JR child performed at a high level. The 8 year old AV girl not only performed this array at speed with no instructions or questioning, but was the only child to place crosscut pattern groups together, in addition to making a vertical colour array.

The 8 year old AV girl also gave the highest level of performance for Figure 98, a beginning senior puzzle.

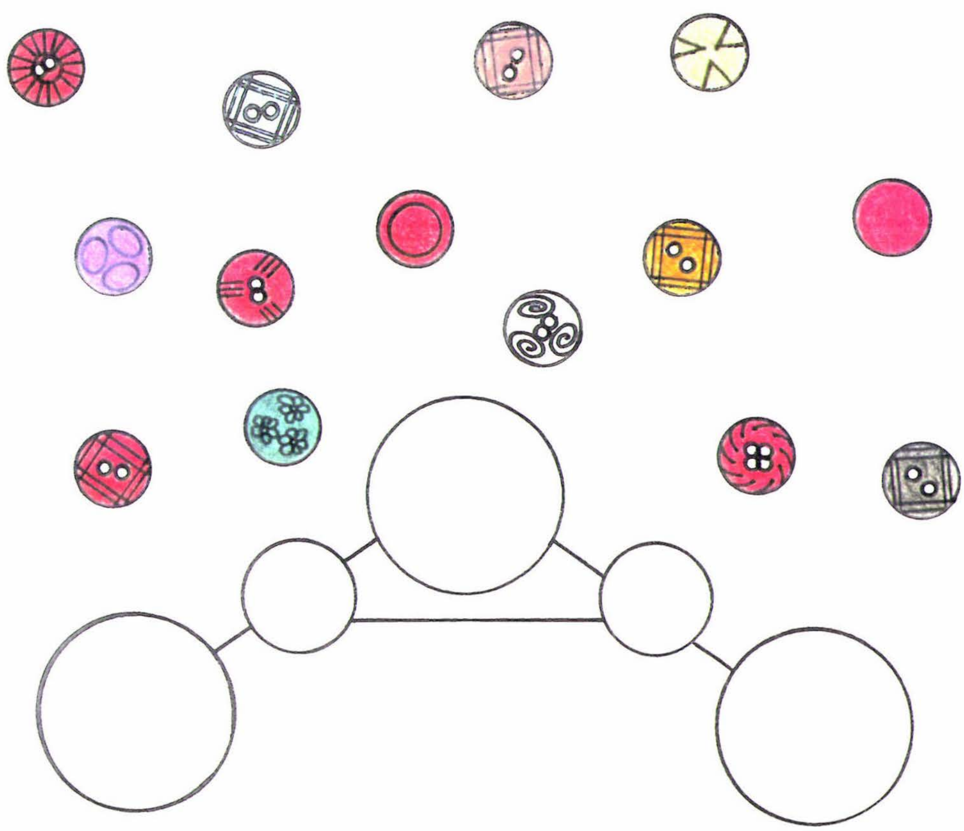


Figure 98: Three Linked Button Sets: A-Shape:
Verbal Instructions:

- Find which buttons belong together.
- Put each set of buttons that belong together in a big circle on the diagram.
- One button goes in each little circle. The button you put in each little circle must belong to (intersect) the buttons in each big circle it is connected to.

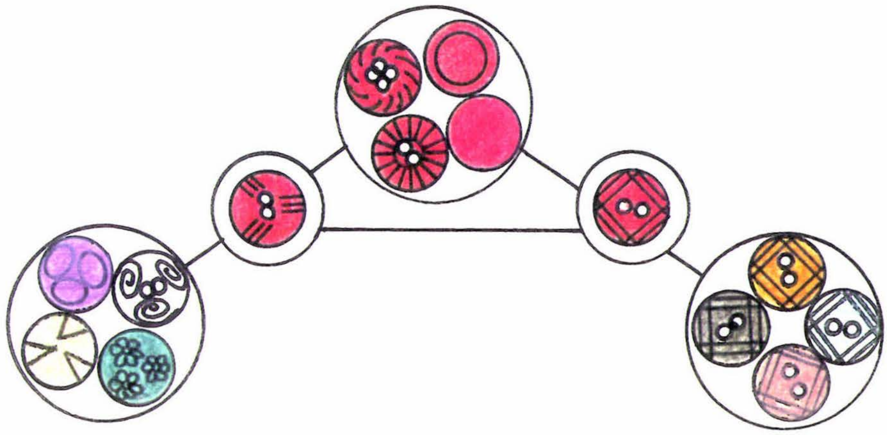


Figure 98a: Three Linked Button Sets: A-Shape:

Figure 98b shows Figure 98 levels of performance across mathematical groups:

Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6						1		
5								
4						1		
3								1
2							2	
1								
0								

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with minimal questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 98a: Three Linked Button Sets: A-Shape:
Levels of Performance Across Mathematical Groups. n = 5.

The 8 year old AV girl again achieved the highest level of performance in Figure 99, a beginning senior puzzle.

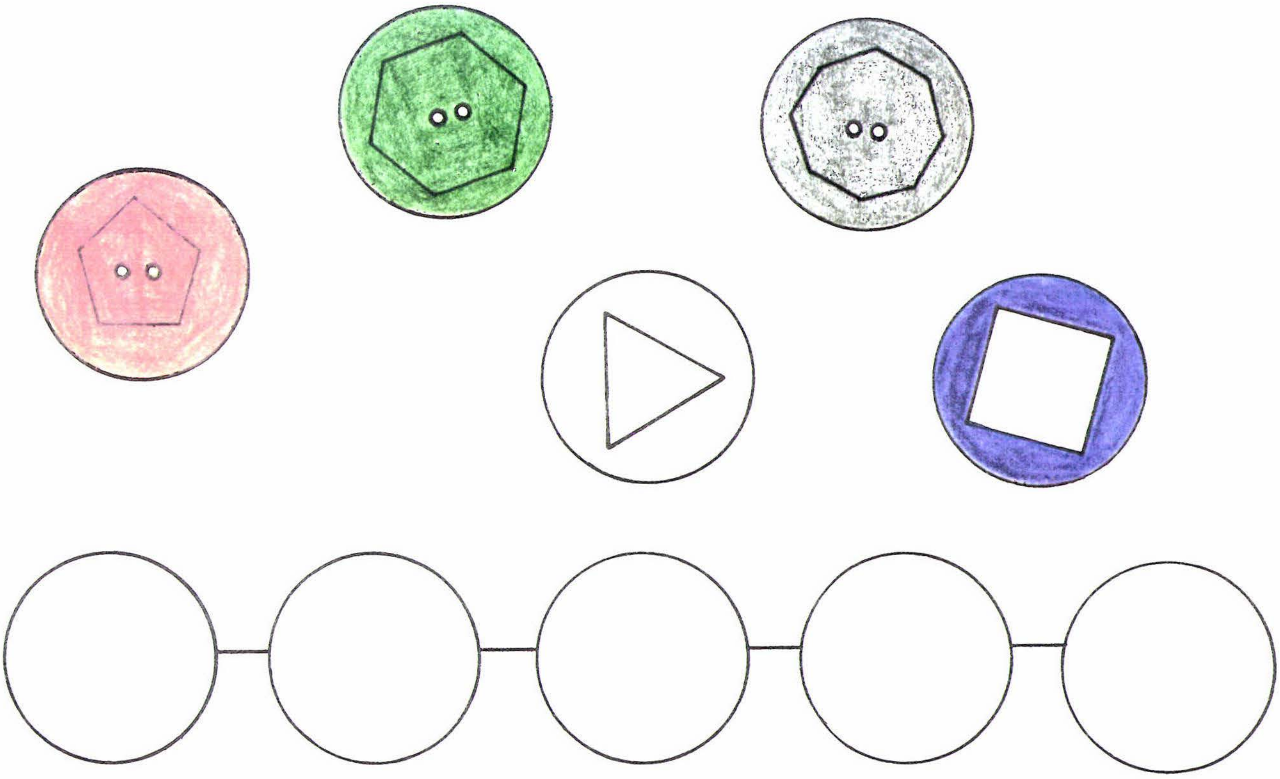


Figure 99: Circular Buttons with Ordinal Internal Shapes:
Verbal Instructions:

- Arrange the buttons in order on the diagram.

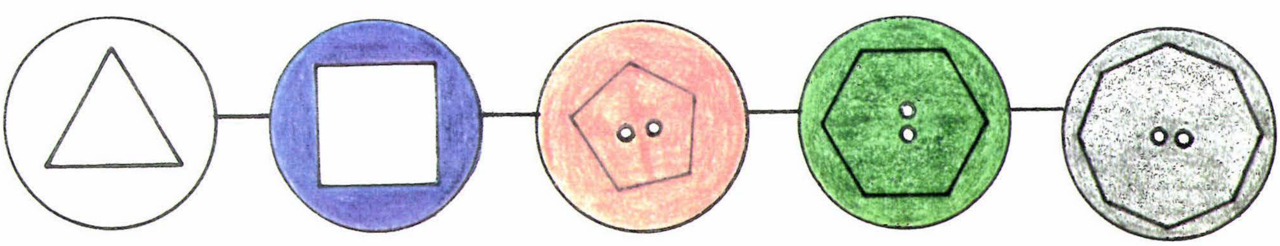


Figure 99a: Circular Buttons with Ordinal Internal Shapes.

Three other children completed this puzzle with some questioning required. These included one 7 year old JE girl, one 10 year old SE 1 girl, and one 10 year old AV girl. The 8 year old AV girl completed Figure 99 very quickly with no verbal instructions or questioning help.

Figure 99b shows Figure 99 levels of performance across mathematical groups:

Levels of Performance	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6						1		
5								
4								
3			1			1		1
2								
1								
0								

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 99b: Circular Buttons with Ordinal Internal Shapes. Levels of Performance Across Mathematical Groups. n = 4.

This 8 year old AV girl also achieved the highest level of performance for Figure 100, an above average senior puzzle of four intersecting button sets. Although all of the participating children required questioning to complete this puzzle, two SE 1 boys aged 9 and 10 years, as well as the 10 year old AV girl found this puzzle very difficult and time consuming with a lot of rearranging, whereas the 8 year old AV girl completed Figure 100 in five minutes, following minimal questioning.

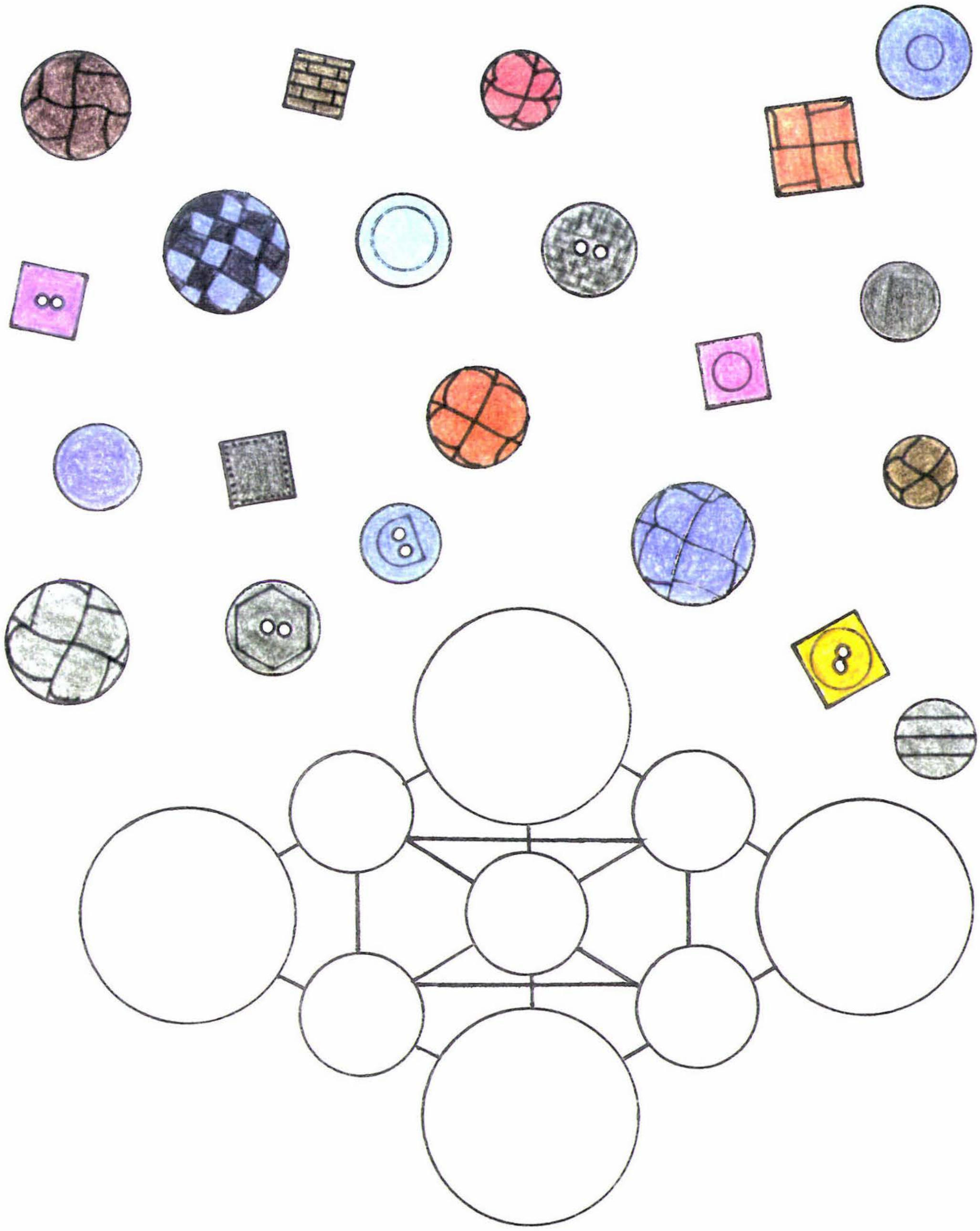


Figure 100: Four Intersecting Button Sets:
Verbal Instructions:

- Find which buttons belong together.
- Put each set of buttons that belong together in a big circle on the diagram.
- One button goes in each little circle. The button you put in each little circle must belong to (intersect) the buttons in each big circle it is connected to.
- Name the sets.

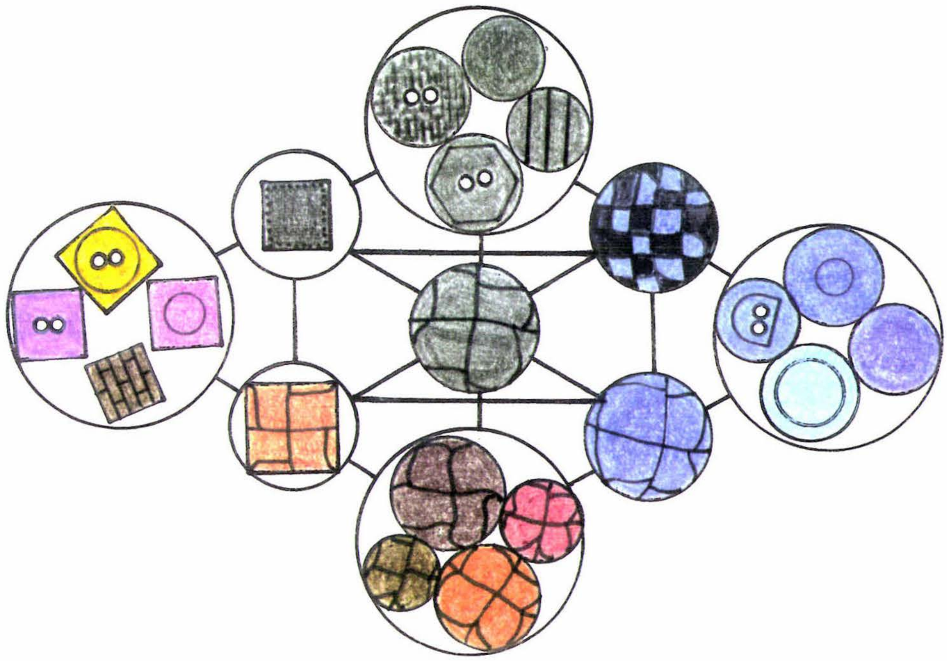


Figure 100a: Four Intersecting Button Sets: Black / Blue / Square / Leather. n = 5.

Figure 100b shows Figure 100 levels of performance across mathematical groups:

Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6						1		
5								
4								
3								1
2						1		2
1								
0								

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of rearranging
2 - Difficult completion with a lot of rearranging
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with minimal questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 100b: Four Intersecting Button Sets: Black / Blue / Leather / Square: Levels of Performance Across Mathematical Groups. n = 5.

A further puzzle where the 8 year old AV girl achieved a high level of performance was a three dimensional array I adapted from a two dimensional one difference probability array utilising four elements, designed by Z.P. Dienes. The original idea involved a two dimensional diagram on a piece of cardboard. I adapted the two dimensional diagram into a three dimensional diagram repeated on and connecting both sides of a tin. This allowed the element of circular polarity inherent in this array to become more easily visible. Buttons used in Figure 101 are magnetised.

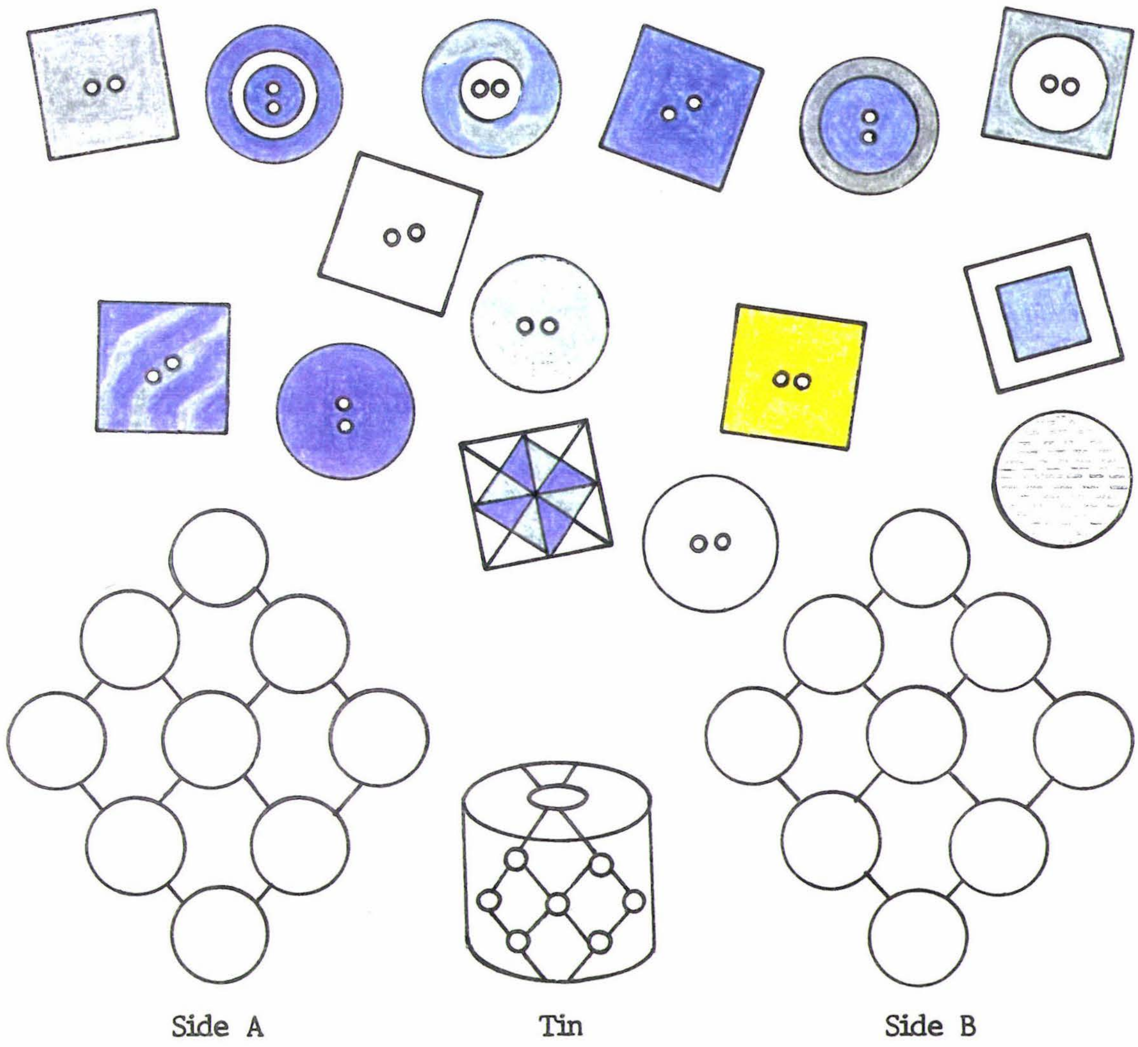


Figure 101: Three Dimensional One Difference Button Probability Tin Array:

Verbal Instructions:

- Find sets. Name the sets.
- Arrange the buttons with one element different between connecting buttons in circles on the diagram.

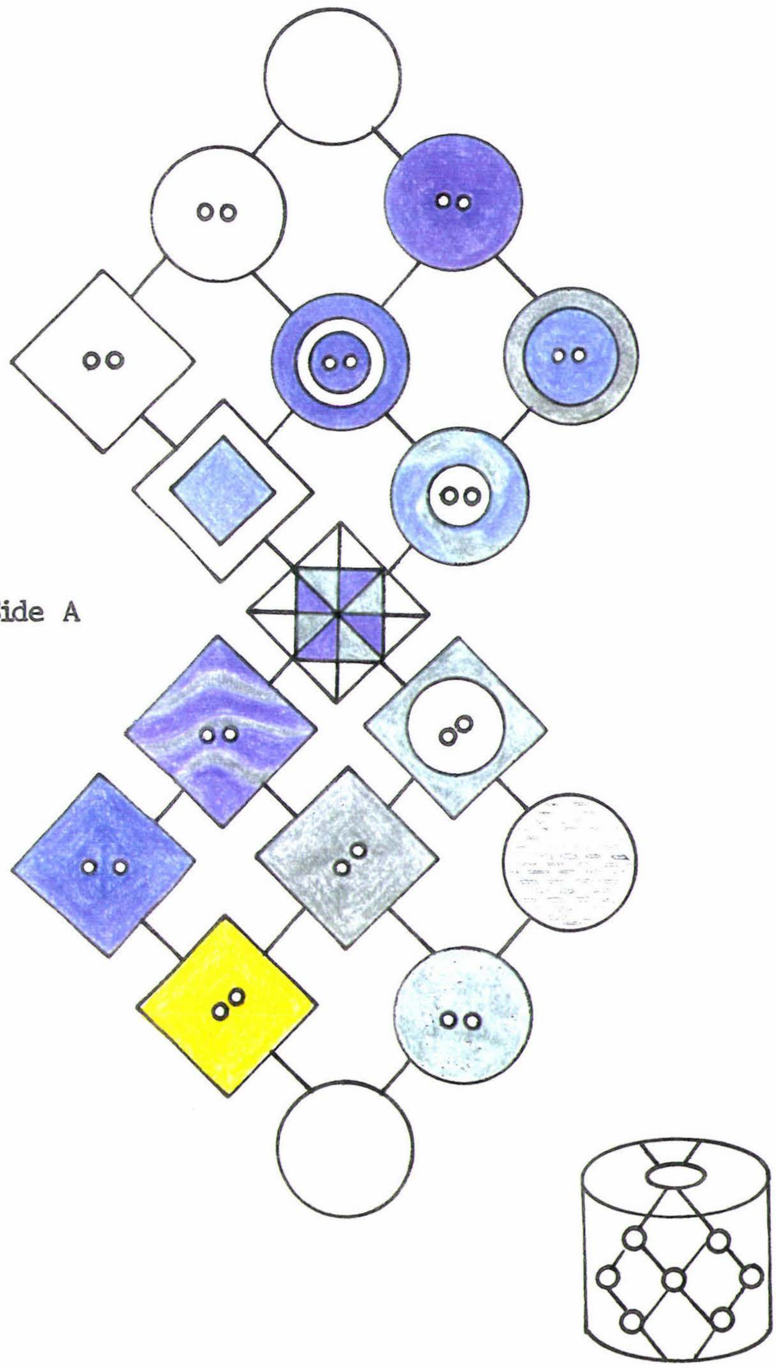


Figure 101a: Three Dimensional One Difference Button Probability Tin Array:

As this adult puzzle was so difficult, only the two AV girls attempted to solve it. Both girls solved it correctly. The 10 year old AV girl required quite a lot of questioning before she could name her four element sets. Once she named the four elements, she completed the arrangement without further questioning help. The 8 year old AV girl named the sets correctly, and arranged them without any questioning, apart from questioning required to draw her attention to two misplaced buttons which she corrected.

Two further related puzzles, Figures 102 and 103 also showed this 8 year old AV girl achieving a high level of performance. Figure 102 was too difficult for one 9 year old SE 1 boy. The three children who completed it, including one 10 year old SE 1 girl and the 8 and 10 year old AV girls, all required minimal questioning. The 8 year old AV girl was not only the youngest child to attempt this puzzle, but the only one to complete it at speed. Figure 102 shows levels of performance across mathematical groups:

Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6						1		
5								
4						1		1
3								
2								
1								
0								1

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with minimal questioning
7 - Very fast completion with no questioning
8 - Very fast completion and use of strategy
9 - Exceptional or novel performance

Figure 102: Domino Button Sets:
Levels of Performance Across Mathematical Groups. n = 4.

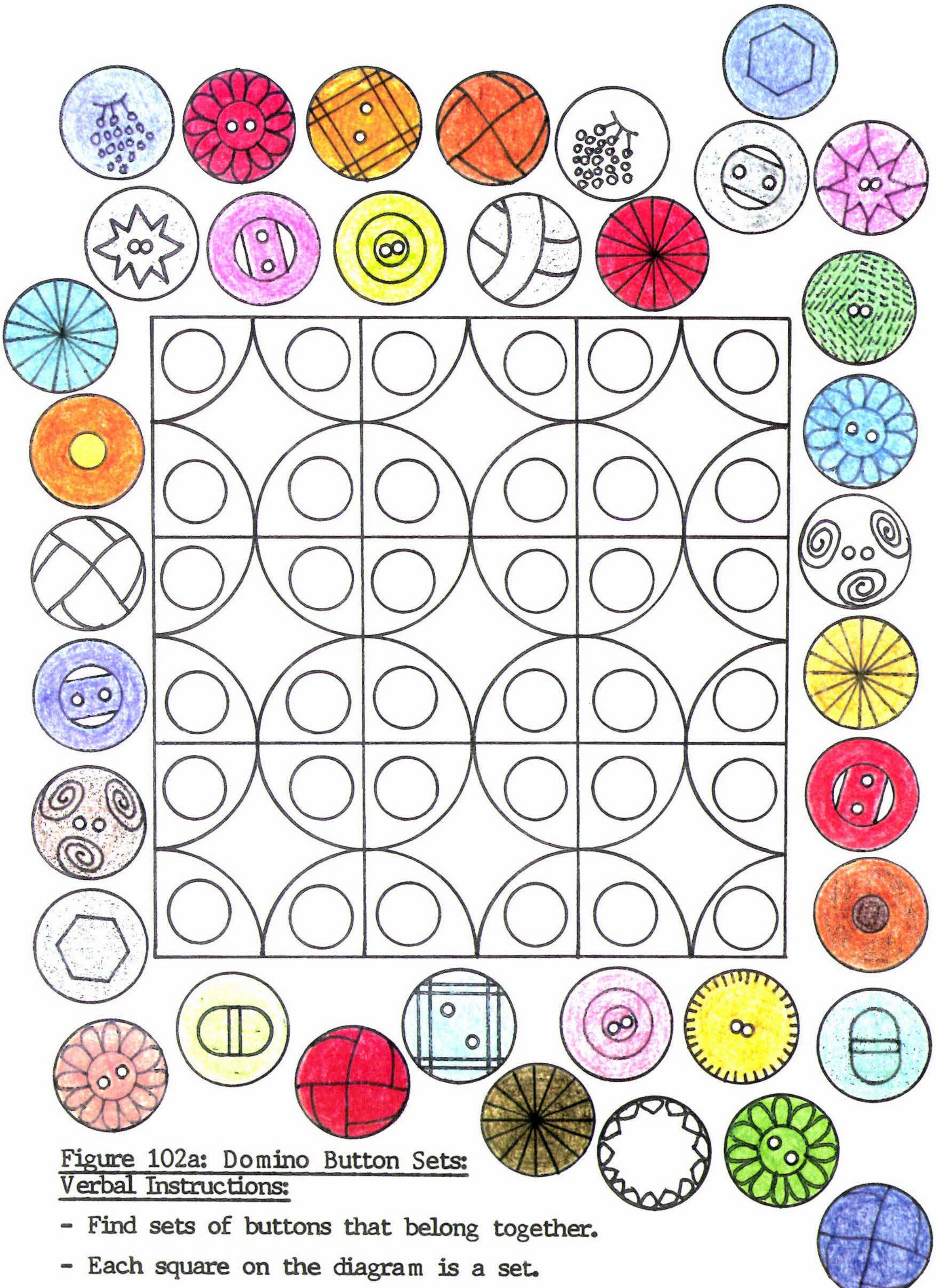


Figure 102a: Domino Button Sets:
Verbal Instructions:

- Find sets of buttons that belong together.
- Each square on the diagram is a set.
- Each circle on the diagram is a set.
- Each half circle (semicircle) and quarter circle on the diagram is part of a set.
- Put each set of buttons in its own circle or square.

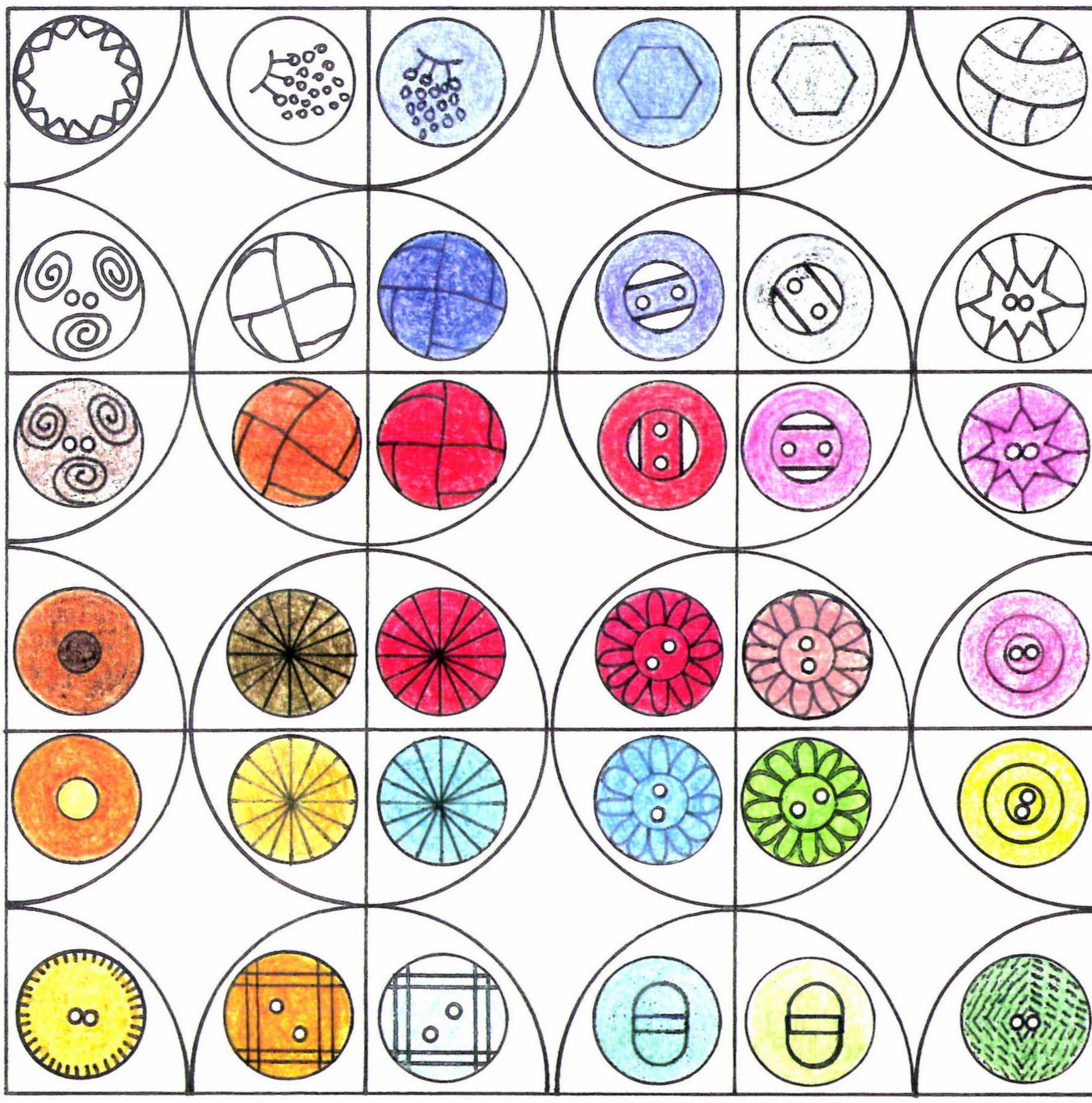


Figure 102b: Domino Colour / Pattern Button Sets.

Figure 103 was attempted only by the two AV girls. Neither girl was given verbal instructions or questioning, and the level of performance for both was exceptional. The 10 year old AV girl completed the puzzle in five minutes, and the 8 year old AV girl completed it in an amazing two minutes.

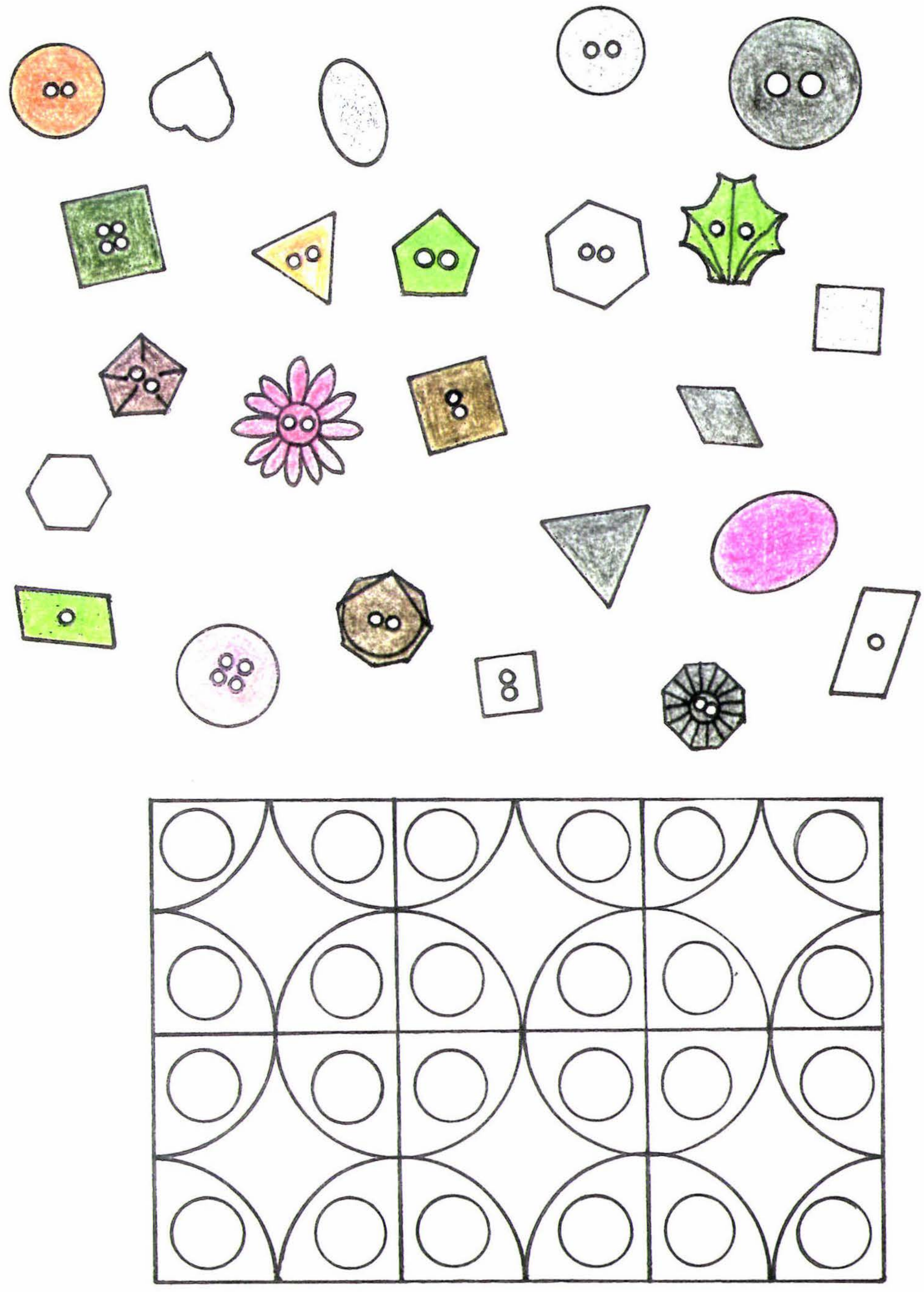
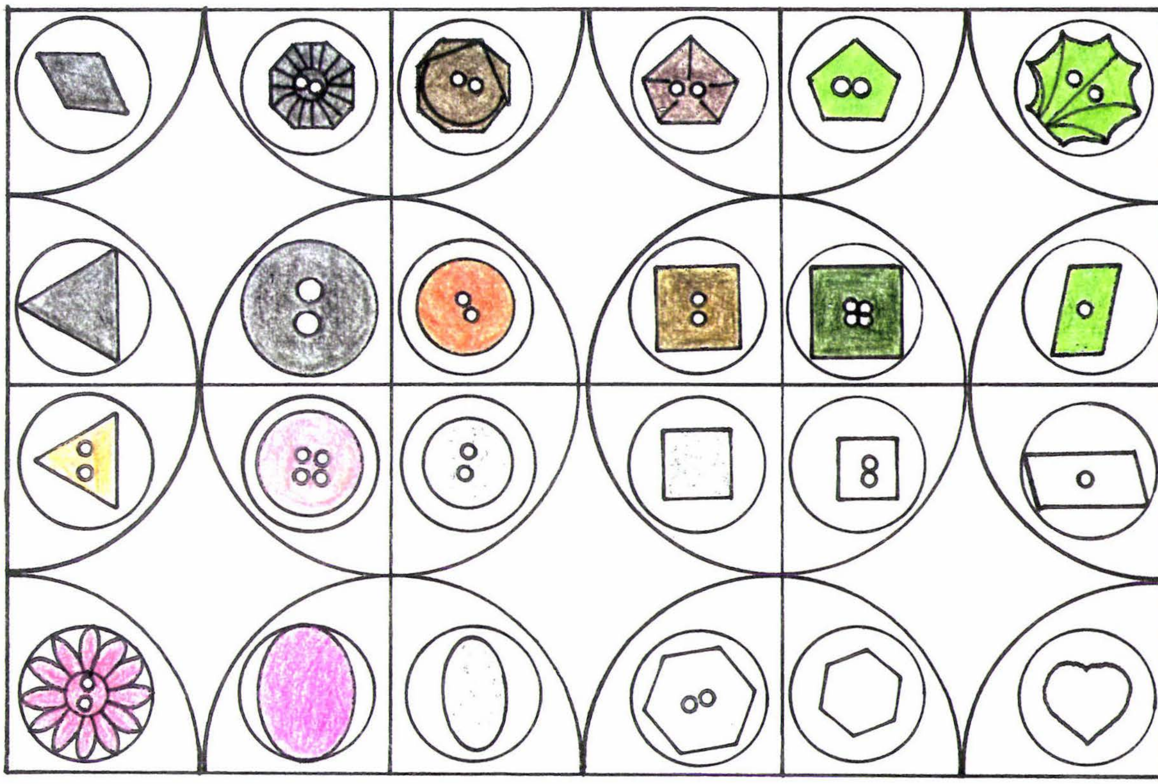


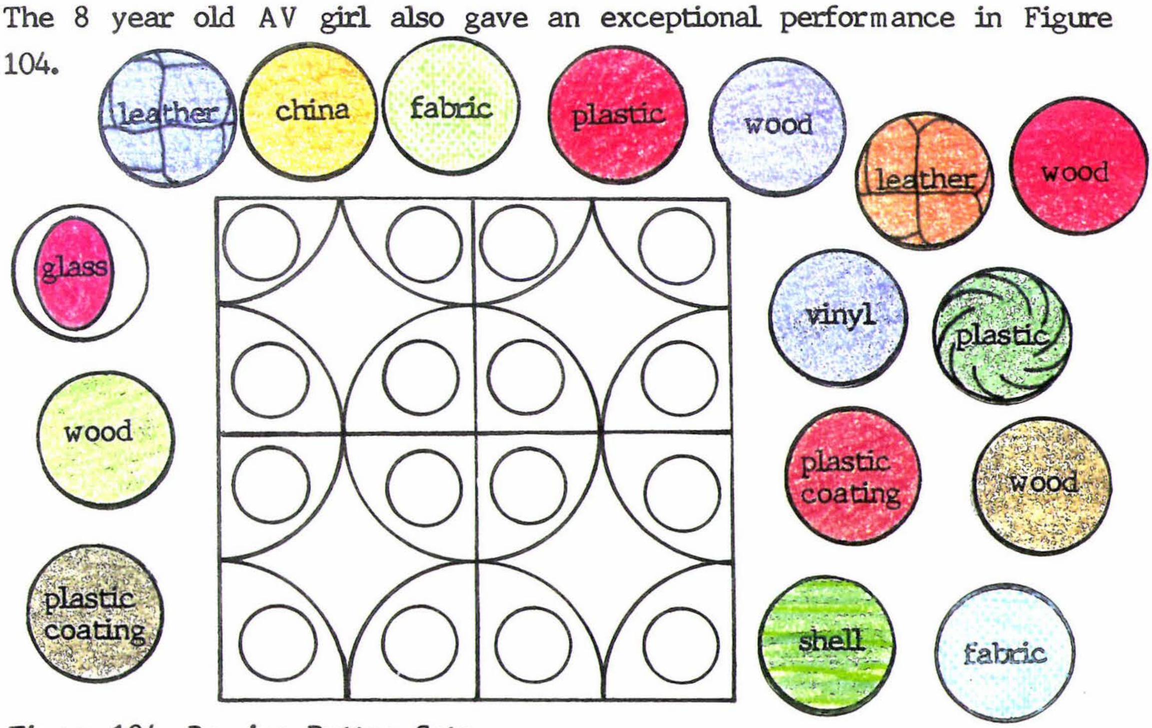
Figure 103: Domino Button Sets:
Verbal Instructions:

- Find sets of buttons that belong together.
- Each square on the diagram is a set.
- Each circle on the diagram is a set.
- Each half circle (semicircle) and quarter circle on the diagram is part of a set.
- Put each set of buttons in its own circle or square.



<u>Figure 103a: Domino Colour / Shape Button Sets.</u>

The 8 year old AV girl also gave an exceptional performance in Figure 104.



<u>Figure 104: Domino Button Sets.</u>
(Verbal instructions as for Figure 103).

Figure 104a: Domino Colour / Materials Button Sets.

Of the three children attempting Figure 104, one 10 year old SE 1 girl required minimal questioning, and both 8 and 10 year old AV girls completed Figure 104 at speed with no questioning or verbal instructions, with the 8 year old completing this puzzle accurately in five minutes.

Subject numbers have been small for many of the puzzles where the 8 year old AV girl performed at a high level. In Figure 105, an above average senior puzzle, this child's achievement was at the highest level of performance when taking into account her age, as both AV girls completed the puzzle accurately in one minute, with no verbal instructions or questioning help given. Figure 105 also gives a good example of a wide range of levels of performance within one mathematical group category. Of 20 children attempting this puzzle, all other children with the exception of one SR girl were SE 1 and SE 2 children. Six children found this puzzle too difficult, indicating the difficulty level, and also emphasising the exceptional performance from the 8 year old AV girl. The next highest level of performance came from an 8 year old SE 1 girl who completed Figure 105 accurately in five minutes. Only two other children completed Figure 105 fairly quickly.

Transfer of learning may have played a part in the exceptional performance of the two AV girls for this puzzle. Both girls utilised a strategy of counting the number of connections for each circle, and counting the number of times any particular colour occurred across the range of buttons in this set. They were then able to determine the central button and corner buttons.

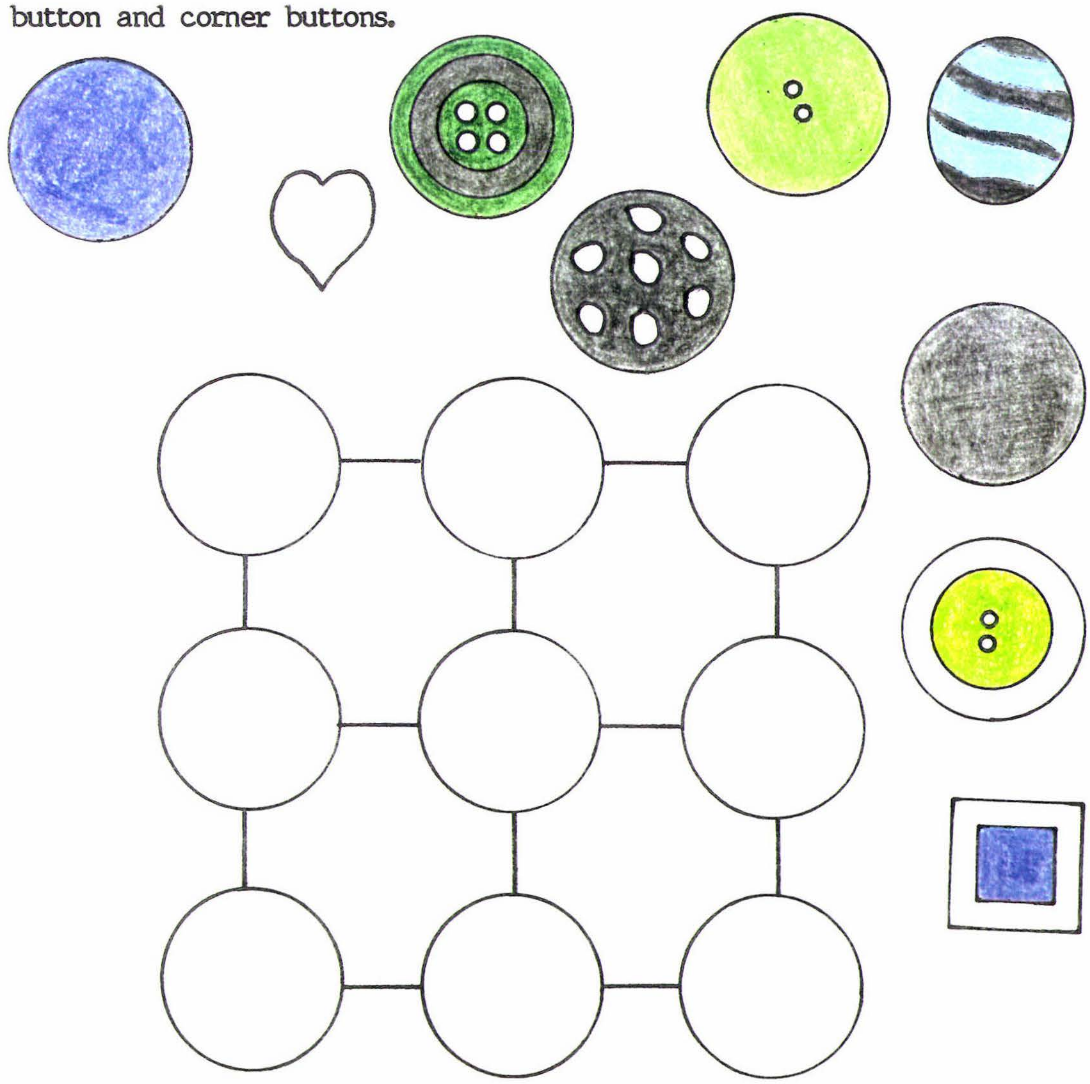


Figure 105: One Colour Difference Button Square:
Verbal Instructions:

- Arrange the buttons with one colour relationship and no more than one colour difference between any buttons you place in connecting circles.

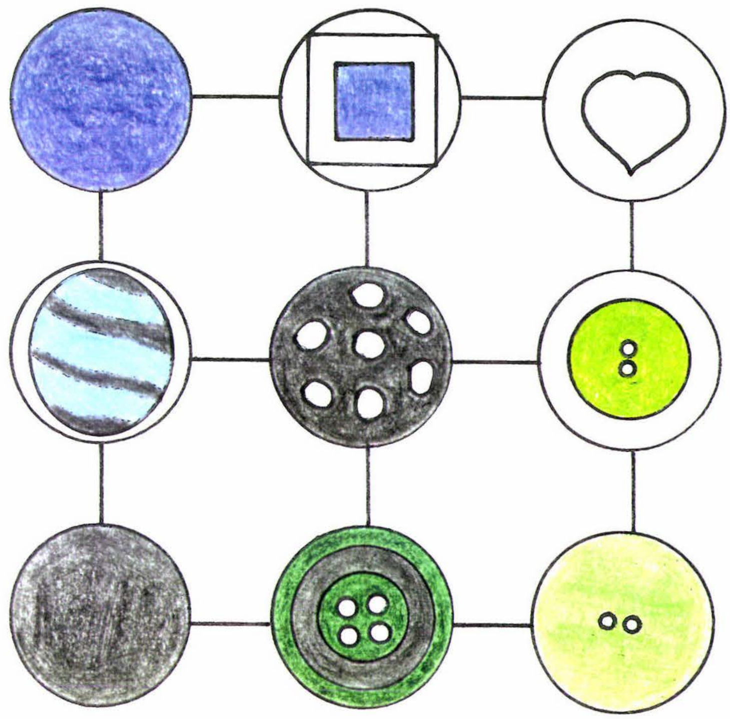


Figure 105a: One Colour Difference Button Square.

Figure 105b shows Figure 105 levels of performance across mathematical groups:

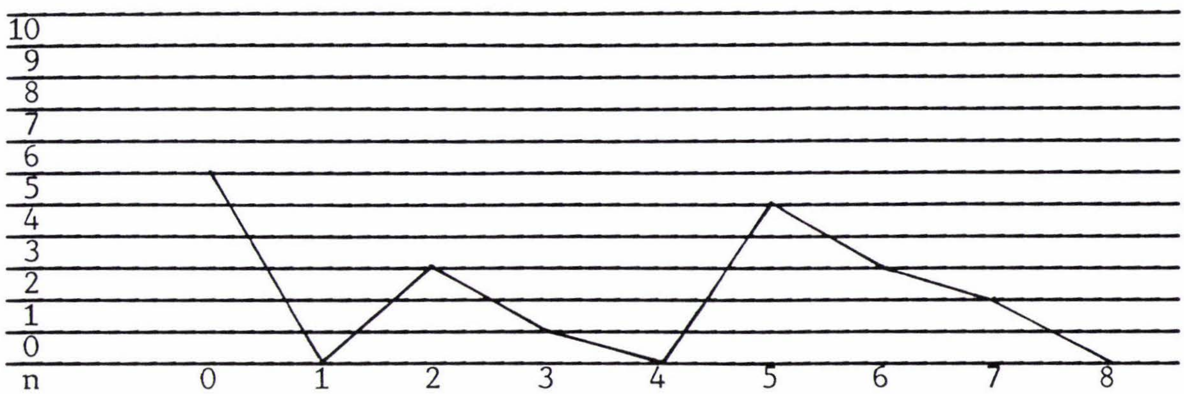
Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7						2		
6							1	2
5							2	3
4								
3							1	
2							1	2
1								
0					1		2	3

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 105b: One Colour Difference Button Square.
Levels of Performance Across Mathematical Groups. n = 20.

Figure 105c shows levels of performance for Figure 105:

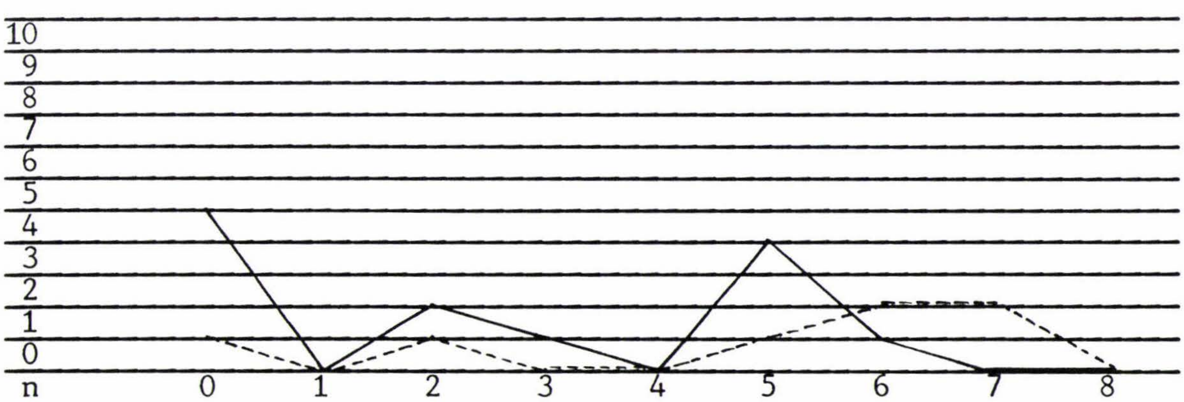


Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 105c: One Colour Difference Button Square: Levels of Performance. n = 20.

Figure 105d examines gender performance across levels of performance: (Levels of performance as for Figure 105c).



n = 7 ———— Males
n = 13 ---------- Females

Figure 105d: One Colour Difference Button Square: Gender Performance Across Levels of Performance. n = 20.

A Mann-Whitney U-Test shows no gender perfermance differences.

The 10 year old AV girl also achieved a high level of performance in previously mentioned Figures 2, 9, 16, 37, 40, 43, 45, 58, 61, 64, 67, 68, 73, 75, 80, 82, 83, 88, 89, 95, 96, 104, 105 and 108. As well she achieved high levels of performance in Figures 106, 107, 108, 109, 110, 111, 112 and 113. The first of these is Figure 106, an advanced senior puzzle. Whereas this puzzle was either too difficult or very difficult for other children, the 10 year old AV girl completed Figure 106 in two minutes with no questioning or verbal instructions.

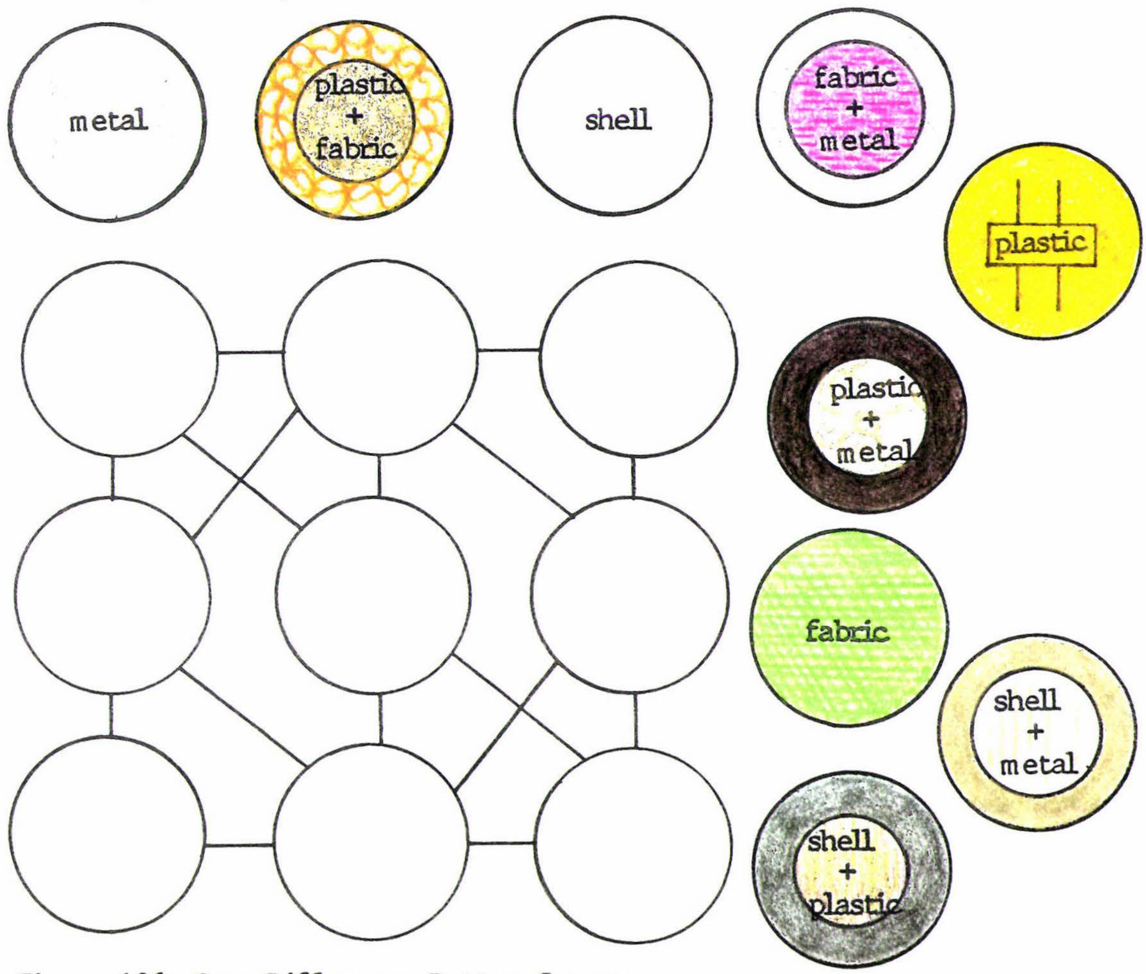


Figure 106: One Difference Button Square:
Verbal Instructions:

- Find sets of buttons. Name the sets. Use the set names for relationship connections between connecting buttons.
- Arrange the buttons with one relationship and no more than one thing different between any buttons you place in connecting circles.

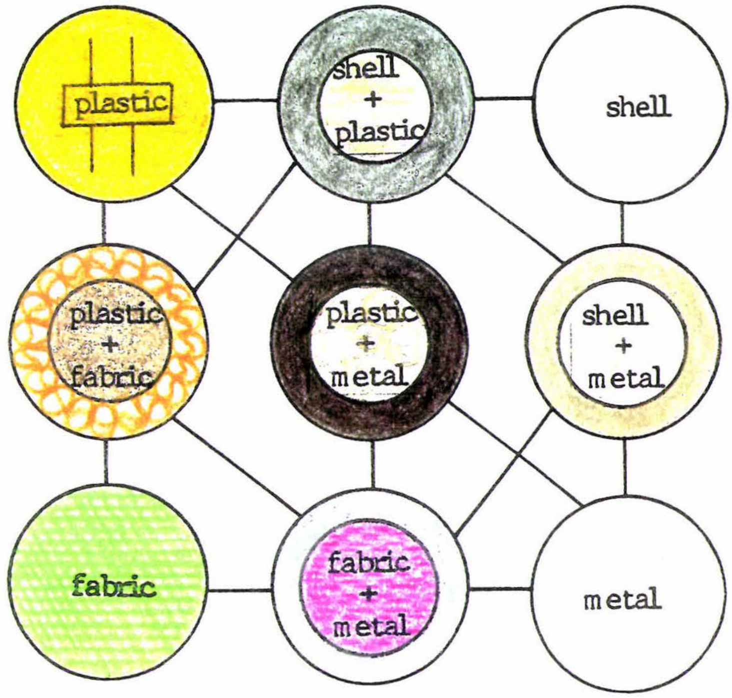


Figure 106a: One Material Difference Button Square.

Figure 106b shows Figure 106 levels of performance across mathematical groups:

Levels of Performance	Groups 1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6						1		
5								
4								
3								
2								2
1								
0						1	1	

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 106b: One Material Difference Button Square: Levels of Performance Across Mathematical Groups. n = 5.

Only the two AV girls attempted Figure 107, an average senior puzzle. The 8 year old AV girl required minimal questioning to begin this puzzle, and her 10 year old sister completed this quickly and accurately.

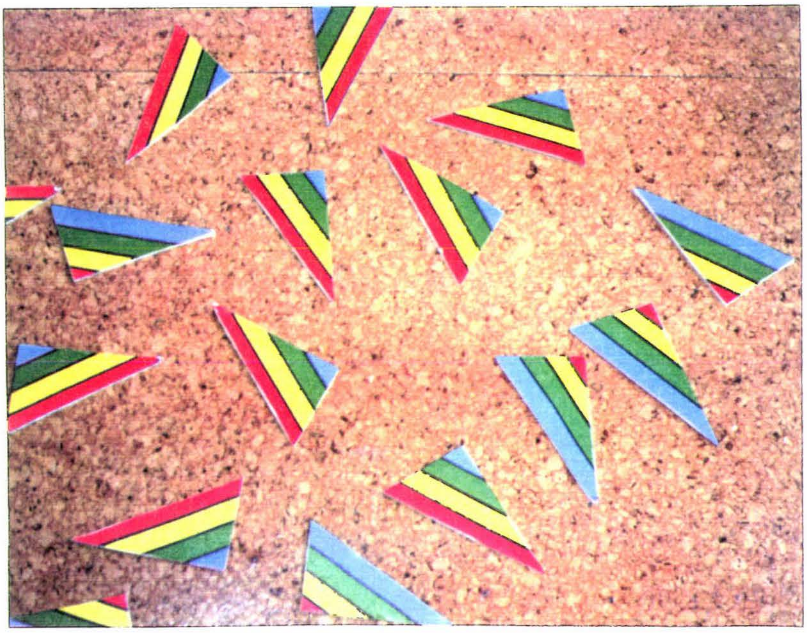


Plate 39: Figure 107: Triangles into Zigzags:
Verbal Instructions:

- Make the triangles into a zigzag pattern.

Plate 40: Figure 107a: Triangles into Zigzags.

Related puzzles Figures 108 A, an above average senior puzzle, and 108 B, an advanced senior puzzle, are shown collectively in Plate 40. The 10 year old AV girl achieved the highest level of performance for both puzzles. Figure 108 A shows levels of performance across mathematical groups:

Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6								
5						1		
4								
3								
2								
1								
0					1	1		

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 108 A: Red Flower Build Up Pattern Match Jigsaw: Levels of Performance Across Mathematical Groups. n = 3.

Figure 108 B shows levels of performance across mathematical groups:

Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6						1		
5								2
4								
3								
2								
1								
0							1	1

Figure 108 B: Yellow Flower Build Up Pattern Match Jigsaw: Levels of Performance Across Mathematical Groups. n = 5.

Plate 41: Figure 108 A: Red Flower Build Up Pattern Match Jigsaw.
Figure 108 B: Yellow Flower Build Up Pattern Match Jigsaw.

The 10 year old AV girl completed Figure 108 B in ten minutes, whereas the next level of performance was from a 10 year old SE 1 girl completing Figure 108 B in twenty minutes.

Figure 109, an advanced senior puzzle, was attempted by three children. The 10 year old AV girl again took ten minutes to complete this, and the above mentioned 10 year old SE 1 girl who took twenty minutes to complete Figure 108 B also took twenty minutes to complete Figure 109. Figure 109 shows levels of performance across mathematical groups:

Levels of Performance / Groups	1 J SLD	2 JR	3 JE	4 S SLD	5 SR	6 AV	7 SE 2	8 SE 1
8								
7								
6						1		
5								1
4								
3								
2								
1								
0						1		

Levels of Performance

0 - Too difficult
1 - Difficult and incomplete with a lot of questioning
2 - Difficult completion with a lot of questioning
3 - Complete with some questioning
4 - Complete with minimal questioning
5 - Complete with no questioning
6 - Very fast completion with no questioning
7 - Very fast completion and use of strategy
8 - Exceptional or novel performance

Figure 109: One Colour Relationship Buttons: Cork:
Levels of Performance Across Mathematical Groups. n = 3.

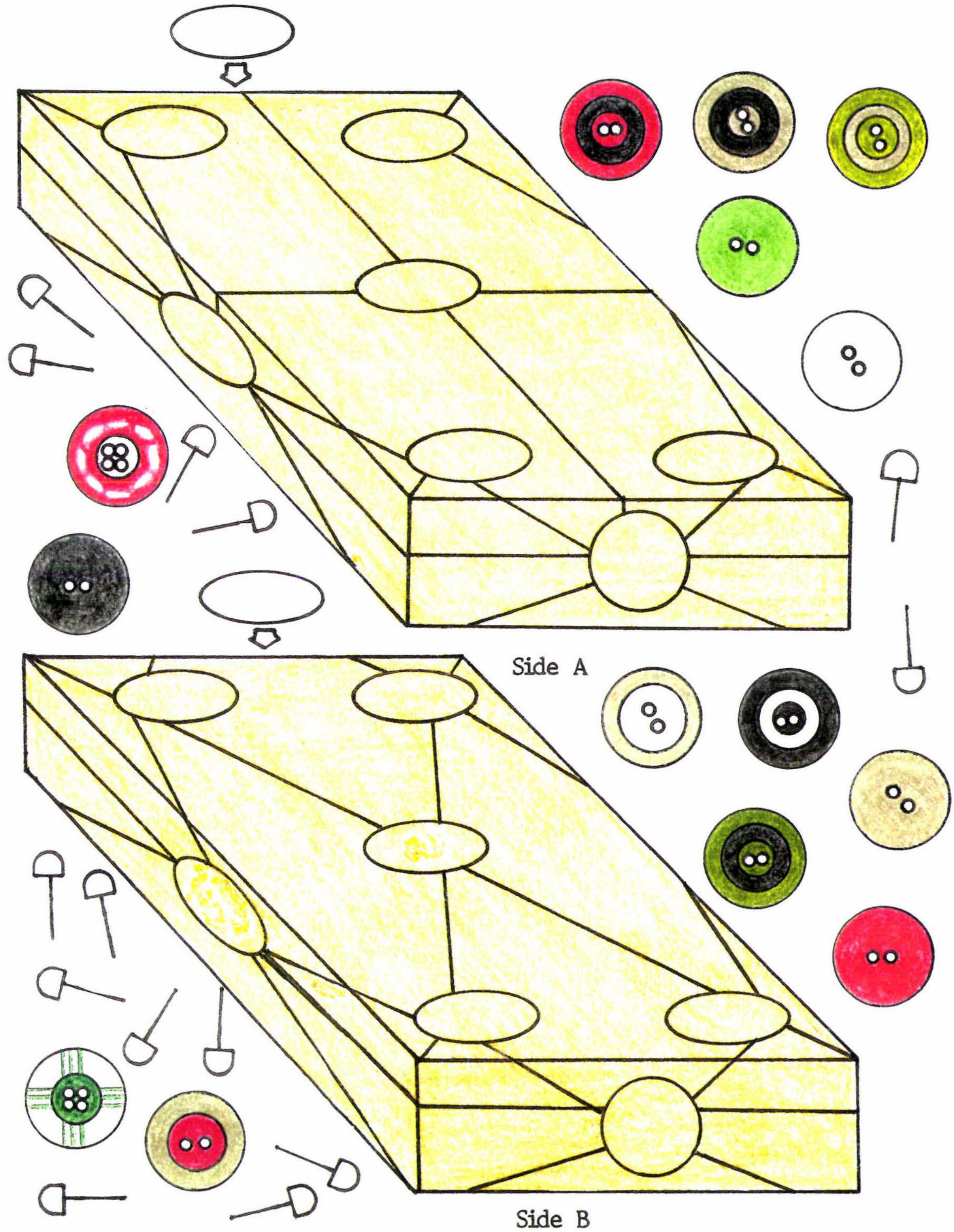
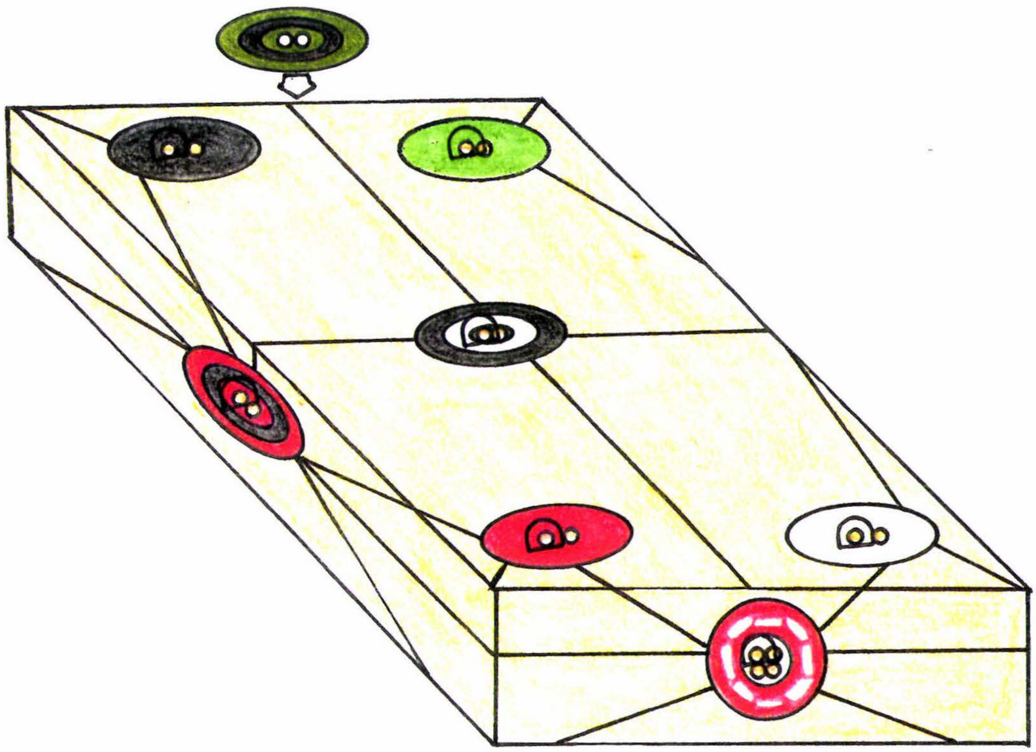
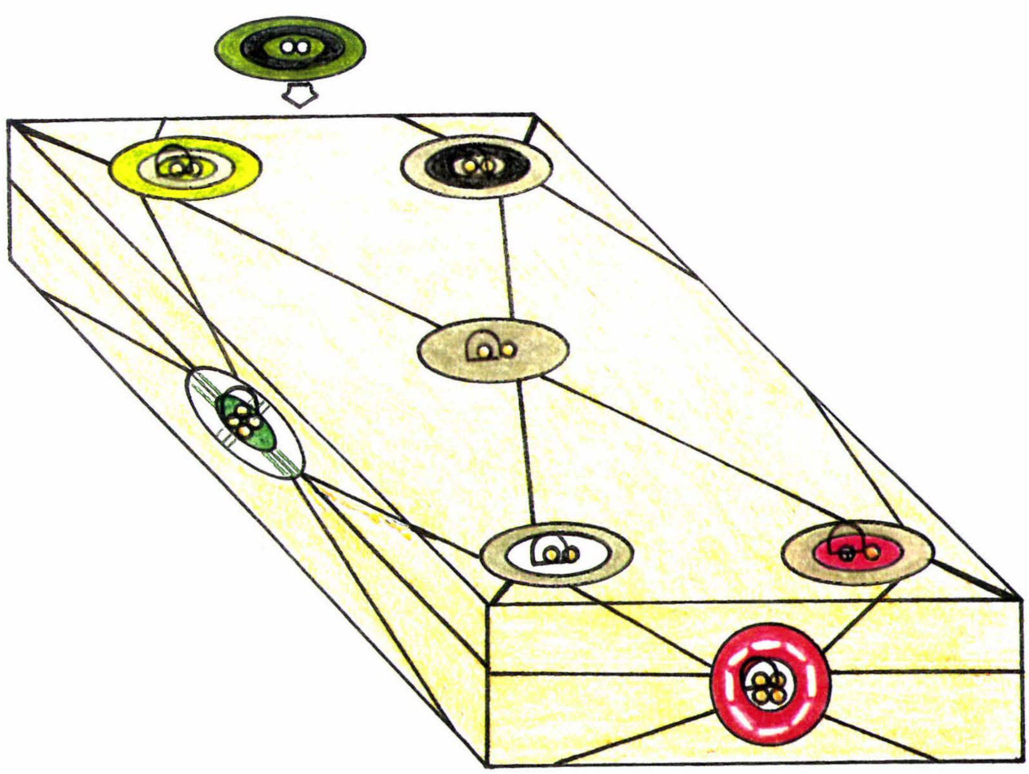


Figure 109a: One Colour Relationship Buttons: Cork:
Verbal Instructions:

- Arrange the buttons with one colour relationship between any buttons you place in connecting circles.



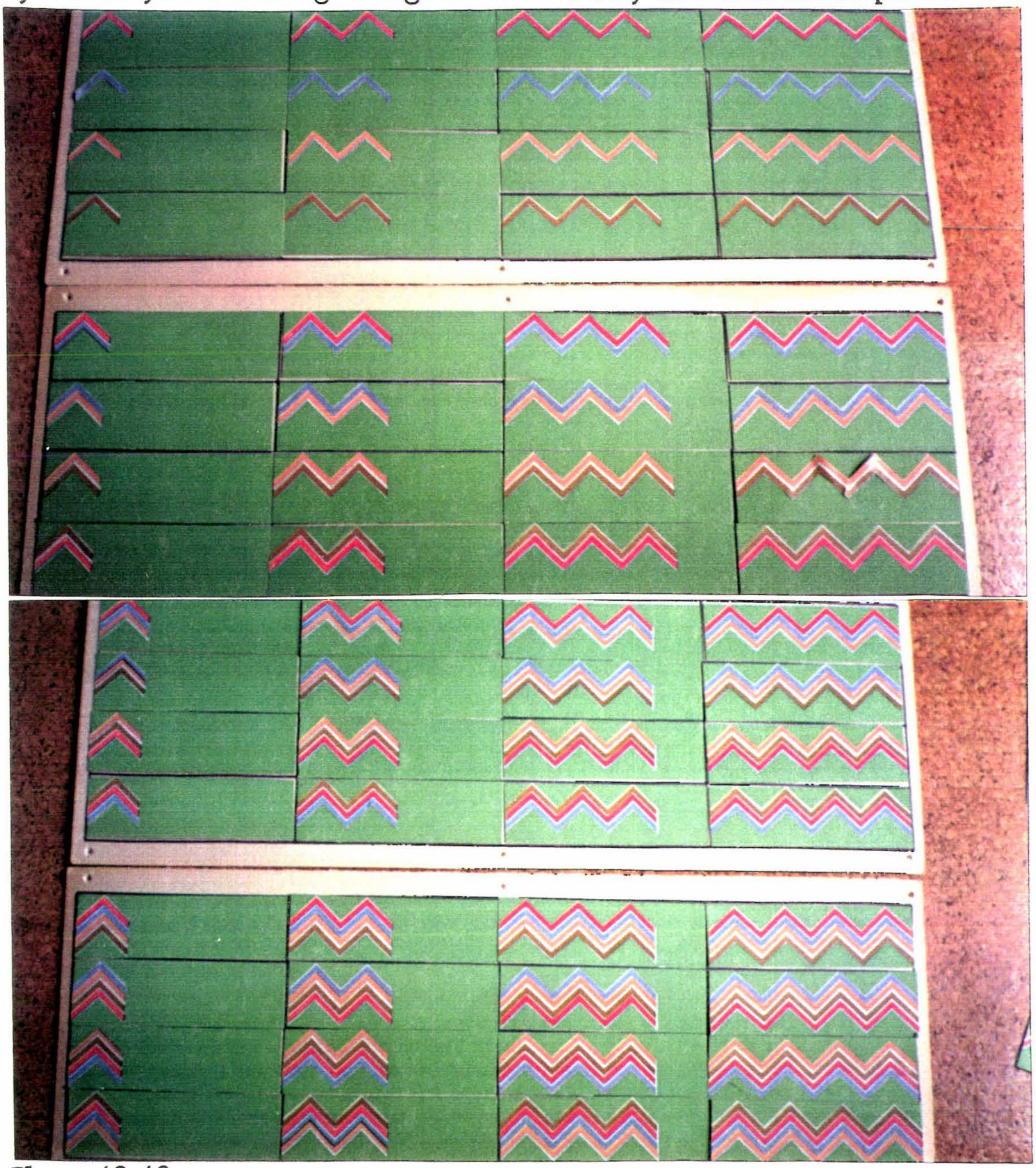
Side A



Side B

Figure 109b: One Colour Relationship Buttons: Cork:

The remaining difficult puzzles presented in Chapter 8 were completed only by the 10 year old AV girl. Figure 110 is a very advanced senior puzzle.



Plates 42-43:

Figure 110: Three Dimensional Cardinal and Ordinal Zigzag Colour Array: Verbal Instructions:

- Find sets of cards. Make a two dimensional (flat), or three dimensional (use golf tees to stack boards above each other) array, with no more than one difference between adjacent cards, horizontally, vertically and diagonally, within boards and between adjacent boards.

The 10 year old AV girl completed Figure 110 with no questioning help beyond initial verbal instructions. She formed the two dimensional array depicted in Plates 41-42. This is not the only arrangement for this array.

The final three puzzles, Figures 111, 112 and 113 are very difficult one difference probability triangles, all of which the 10 year old AV girl found difficult, but completed with questioning. The 8 year old AV girl also attempted Figure 111, an average senior puzzle, but found it too difficult.

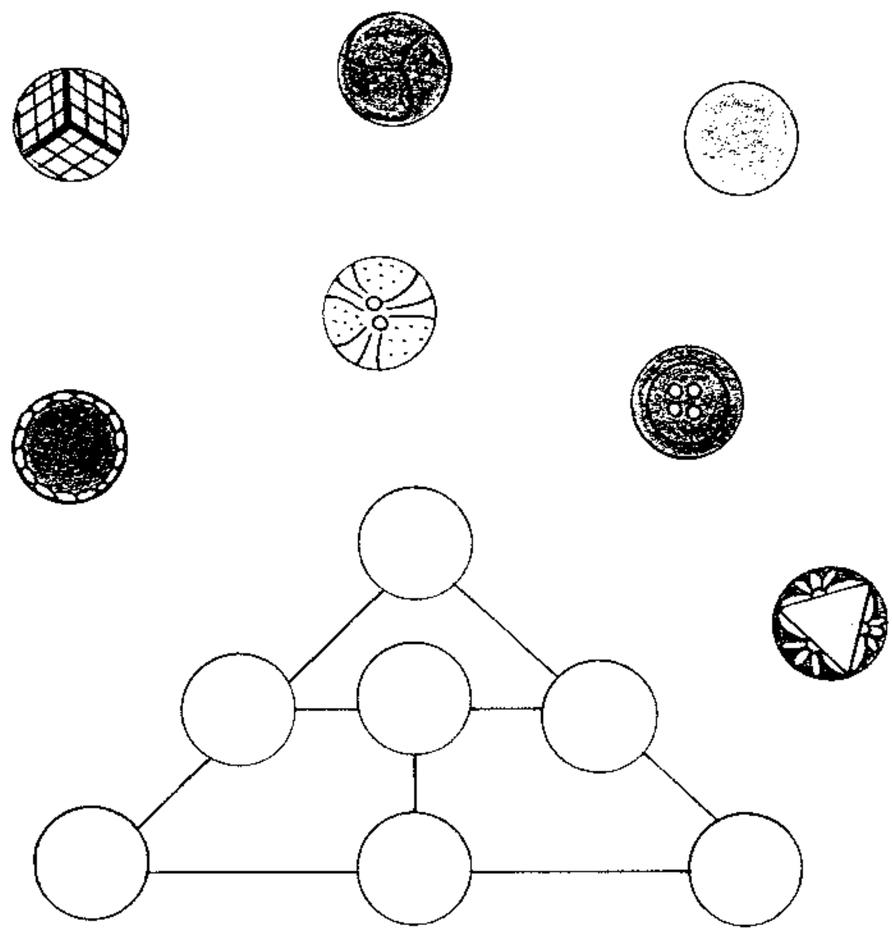


Figure 111: Three Set One Difference Probability Triangle:
Verbal Instructions:

- Find three sets. Name the sets.
- Place one button in each circle in the triangle. There must be a one difference relationship, with no more than one thing different between buttons in connecting circles.

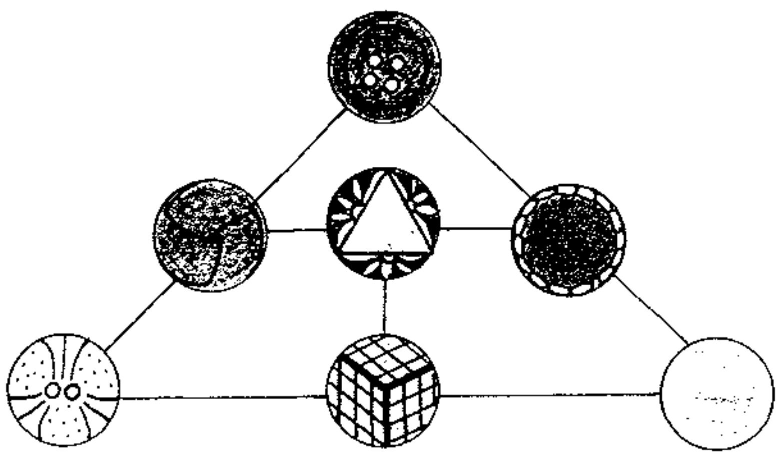


Figure 111a: Three Set One Difference Probability Triangle: Black / Silver / Patterns of Three.

This puzzle gives every possible combination of the three elements to be arranged. For this reason these three sets could also be arranged on a three circle Venn Diagram. The following probability triangle puzzles, Figures 112 and 113 utilise this single triangular formation used in Figure 111, intersecting single probability triangles with multiple probability triangular sets. Figure 112, an adult puzzle, has three intersecting probability triangles, and Figure 113, an adult puzzle, has four intersecting probability triangles.

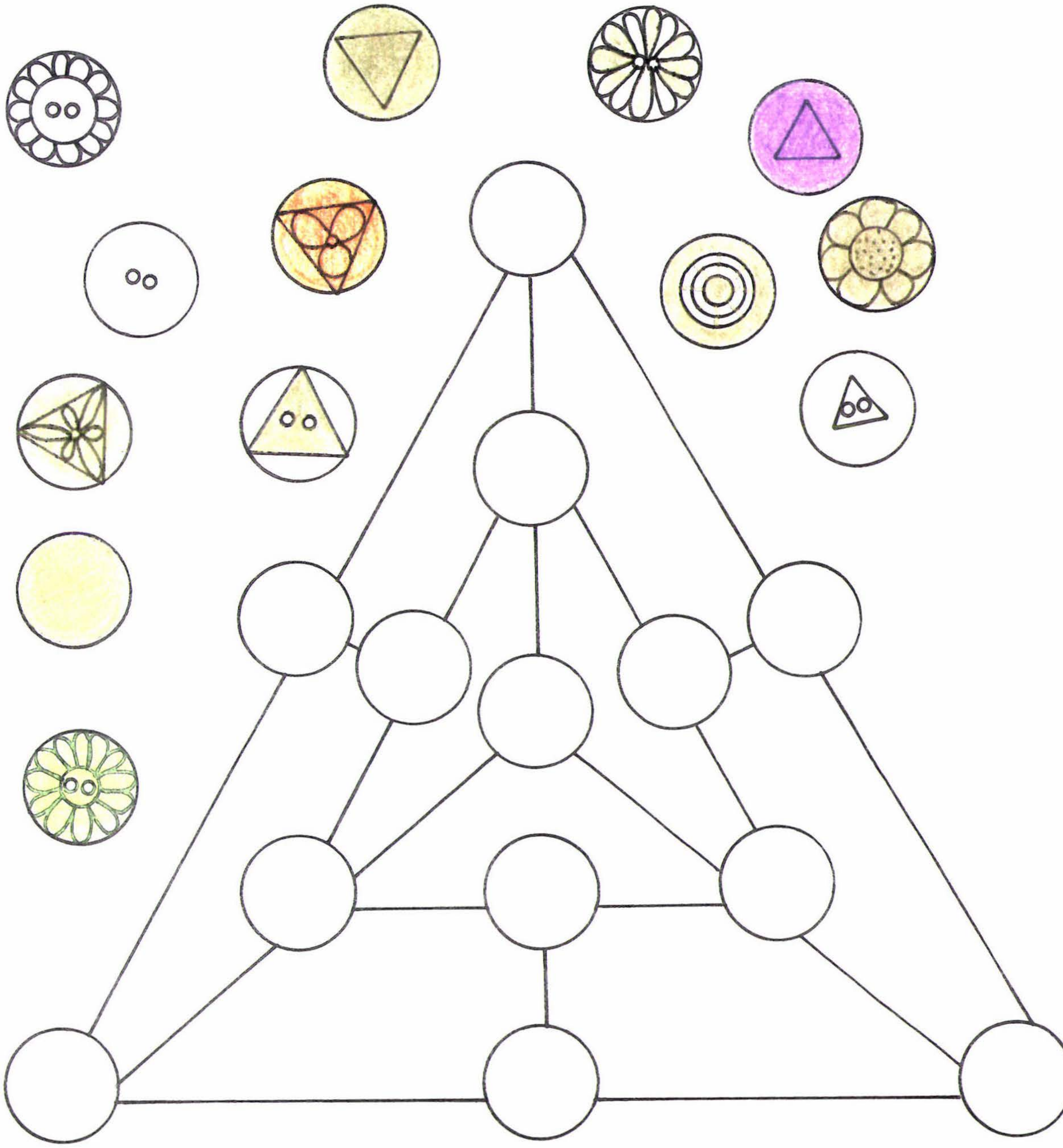


Figure 112: Three Intersecting Three Set One Difference Probability Triangles:

Verbal Instructions:

- Find sets. Name the sets.
- Place one button in each circle in each triangle. There must be a one difference relationship, with no more than one thing different between buttons in connecting circles.

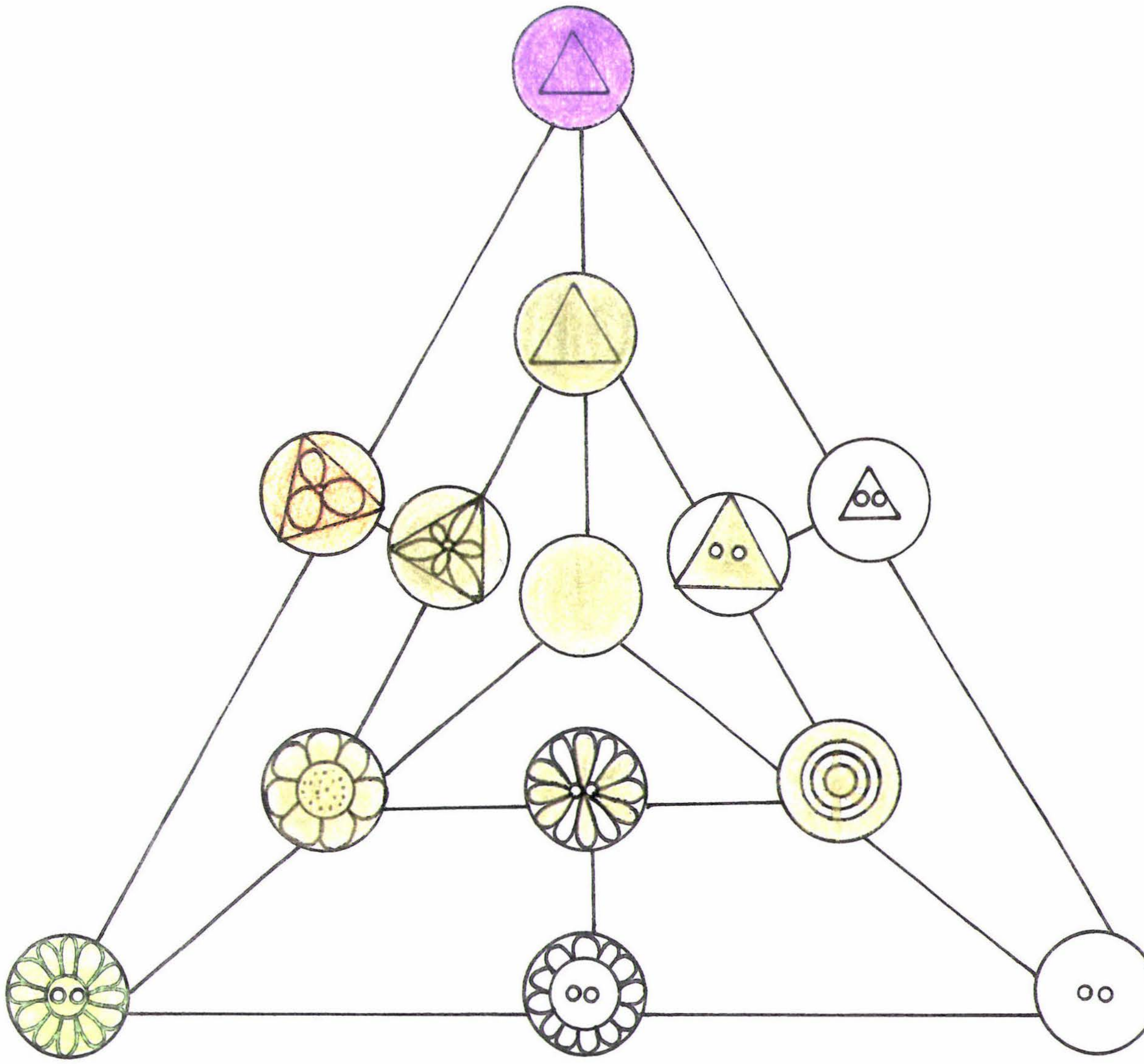


Figure 112a: Three Intersecting Three Set One Difference Probability Triangles:

1. Gold / Triangle in Circle / Flower.

2. Gold / Transparent / Triangle in Circle

3. Gold / Transparent / Flower.

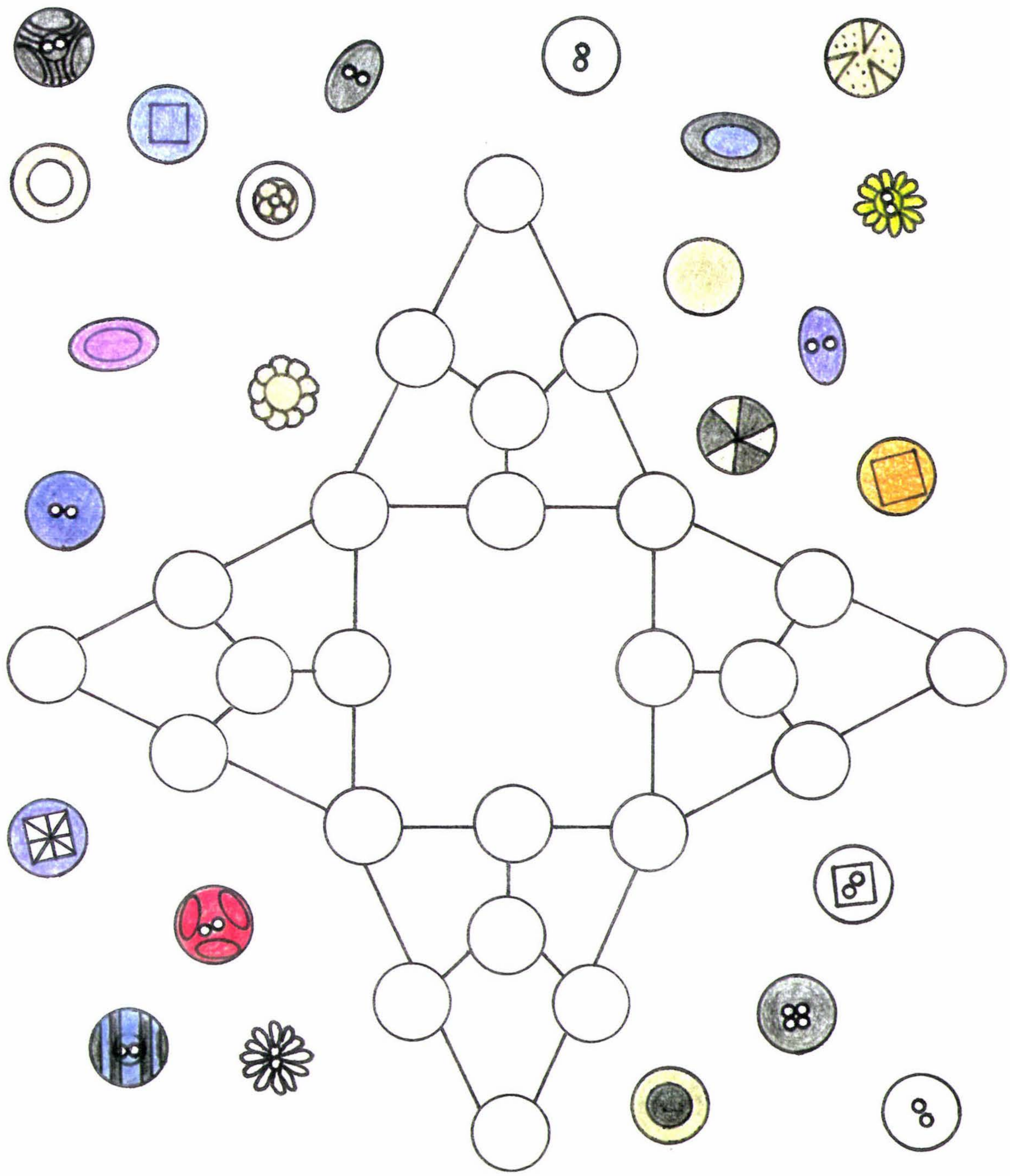


Figure 113: Four Intersecting Three Set One Difference Probability Triangles:

Verbal Instructions:

- Find sets. Name the sets.
- Place one button in each circle in each triangle. There must be a one difference relationship, with no more than one thing different between buttons in connecting circles.

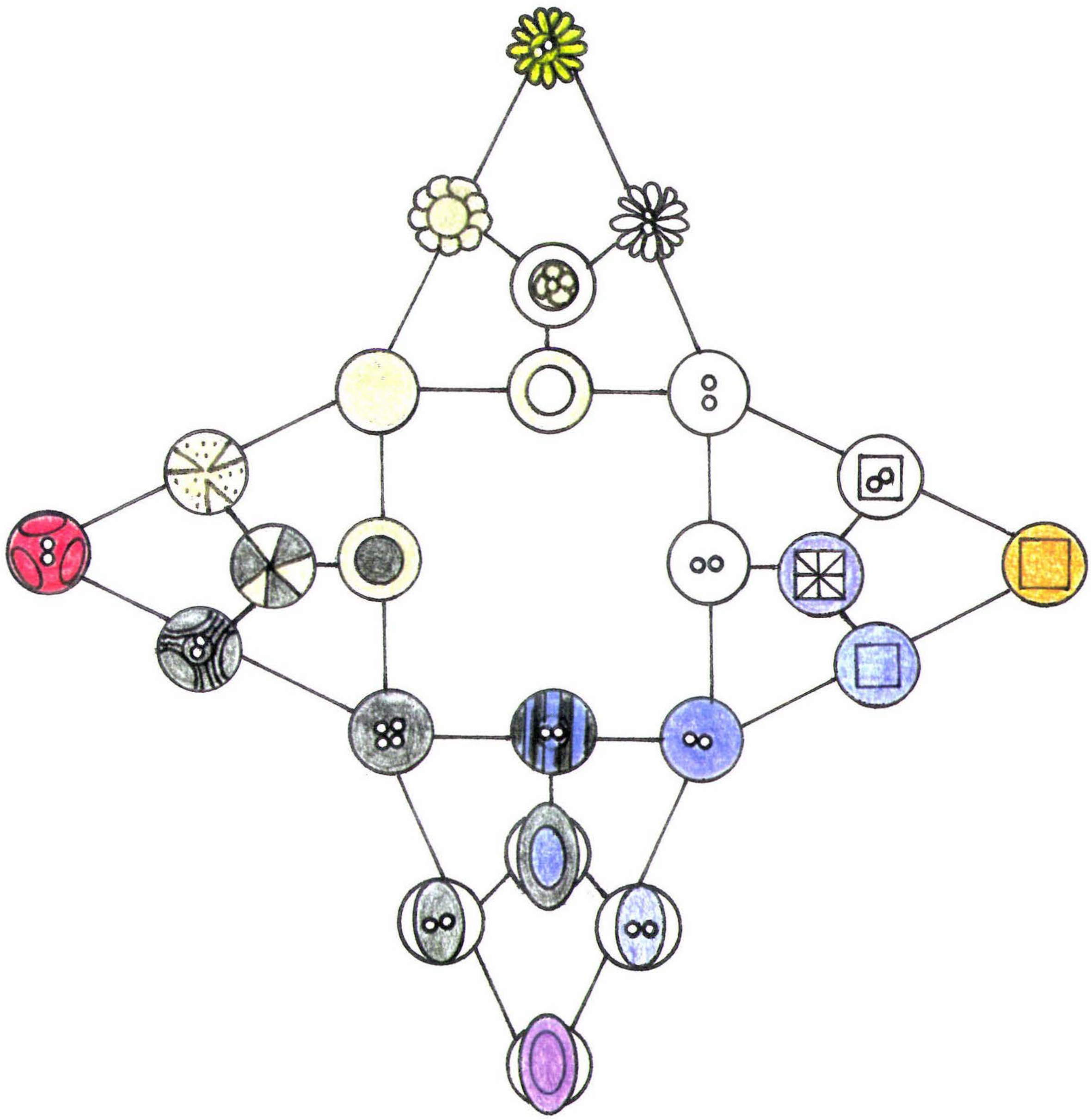


Figure 113a: Four Intersecting Three Set One Difference Probability Triangles:

1. Gold / Flower / Transparent

2. Transparent / Square in Circle / Blue

3. Blue / Oval / Black

4. Black / Patterns of Three / Gold.

Table 43 shows two tailed Mann-Whitney U-Tests testing gender differences in levels of performance across mathematical groups for Chapter 8, at 0.05, where U (M) = Males, U1 (F) = Females, and z shows use of normal approximation for larger samples. There were no gender differences in levels of perfromance for any of the puzzles tested.

Table 43 : Mann-Whitney U-Tests For Chapter 8: Gender Differences: Levels of Performance Across Mathematical Groups:

Figure	U (M)	U1 (F)	Males	Females
Puzzles demonstrating range of levels of performance				
90	28.5	19.5	-	-
92	46.5	31.5	-	-
93	14	14	-	-
94	10	18	-	-
95	24	24	-	-
105	21.5	69.5	-	-

* Significant at 0.05.

Summary

Chapter 8 has emphasised the wide range of levels of performance exhibited by children in this study, in puzzles attempted by children from a wide range of mathematical group categories, in puzzles attempted by children within a particular mathematical group range, and in the unexpected level of performance from two children within one mathematical group category. This general lack of correlation between P.A.T. Test mathematical group categories and ability to process the puzzles presented throughout this study will be discussed in Chapter 9.

Part 3

Implications

Correlation Results

Implications in the Teaching of Mathematics

Chapter 9

Correlation Results

Part 3 looks at correlation results and their implications for the teaching of mathematics. Chapter 9 presents correlation results, and Chapter 10 discusses the implications for the teaching of mathematics. Performance levels exhibited in puzzles presented in Chapter 8 highlight the general lack of correlation with children's ability to process puzzles and their formal mathematical test results in the Progressive Achievement Tests (P.A.T. mathematics) in New Zealand schools. This general lack of correlation appears through most of the puzzles presented in this thesis. The few puzzles showing significant correlation between mathematical groups, classes or ages with levels of performance were evenly distributed throughout puzzle difficulty levels. There were few gender differences in these levels of performance. Those few puzzles showing significant gender differences were evenly distributed in favour of males and females. With the large number of tests in this study, significance in correlation between mathematical groups and levels of performance, and in gender levels of performance has increasing potential for chance results.

Performance graphs and gender performance graphs showed a frequent tendency towards bimodal distribution. As children in this study represented high and low levels of mathematical ability, a bimodal distribution would be expected. As levels of performance for the puzzles did not generally correlate with mathematical groups, this bimodal distribution does not represent the ability of the children in the mathematical groups as they were originally selected, but reflects a distinction between children who had ability with the puzzles and children who did not. Bimodal distribution is common in the mathematical testing of children. An example is the New Zealand School Certificate Mathematics examination before marks are scaled, where frequent bimodal distribution reflects the difference between children who have real mathematical ability and children who do not.

Several analysis tables are presented in Chapter 9. These include Spearman's Rho correlations between P.A.T. mathematic groups, classes and ages respectively with levels of performance with the puzzles. Sometables detailing puzzle names according to correlation significance follow, with further tables placing puzzles in order of difficulty to see how correlations

Table 44 presents Spearman's Rho correlations between mathematical group categories, school classes, and ages across levels of performance. These are presented in order of appearance in the present study. There is generally no real difference in levels of performance in the puzzles across mathematical groups, classes, and ages respectively. Figure 15, Button Subsets and Buckle Classifications showed significance at 0.01 for mathematical groups across levels of performance. Reasons for the level of significance in this puzzle are unclear when the few other tests showing significance were at 0.05.

Tables 45, 46 and 47 show Table 44 named puzzles in order of r_s for mathematical groups; classes; and ages respectively across levels of performance.

Tables 48, 49 and 50 show approximate rank order of puzzle difficulty, along with r_s for mathematical groups, classes and ages across levels of performance. Puzzles are listed in approximate order from the least difficult to the most difficult. Puzzles that did show some correlation between mathematical groups, classes and ages appear to cluster nearer the least difficult puzzles rather than the most difficult. One reason for this may be that more older children in more advanced mathematical groups attempted the harder puzzles.

Table 51 shows two tailed Mann-Whitney U-Tests testing gender differences in levels of performance across mathematical groups at 0.05. These are presented in the order they appeared in throughout the puzzle chapters. Occasional gender significance in gender levels of performance are evenly distributed between the genders.

Table 44: Spearman's Rho Correlations (r_s) and Significance - (0.05) - Across Puzzles (In order of presentation in Thesis): Mathematical Groups; Classes; Ages; Across Levels of Performance.

Puzzle Figure Number	N	Mathematical Groups r_s (0.05)	Classes r_s (0.05)	Ages r_s (0.05)
1	37	0.263	0.238	0.198
2	21	0.363	0.459 *	0.411 *
3	19	0.345	0.424 *	0.492 *
4	22	0.169	0.094	-0.039
5	10	0.042	0.127	0.127
6	24	0.333	0.158	0.214
14	37	0.250	0.314 *	0.345 *
15	14	0.885 **	0.664 *	0.617 *
16	13	0.307	0.322	0.278
18	15	0.223	0.445 *	0.458 *
20	25	0.283	0.056	-0.014
27	12	0.141	0.005	0.003
28	19	0.483 *	0.374	0.512 *
29	15	0.247	0.243	0.325
31	8	0.440	0.434	0.434
32	12	-0.029	0.054	0.208
39	27	0.404 *	0.346 *	0.389 *
40	17	0.034	0.237	0.237
42	23	0.140	0.147	0.204
50	12	0.554 *	0.337	0.339
54	12	0.256	0.118	0.215
57	56	0.237 *	0.135	0.152
64	35	0.490 *	0.307 *	0.239
89	42	0.520 *	0.493 *	0.503 *
94	11	0.111	0.345	0.265

* Significant at 0.05. ** Significant at 0.01.

Table 45: Named Puzzles r_s (0.05) For Mathematical Groups Across Levels of Performance.

Mathematical Groups

Puzzle Figure Number	N	Maths Groups r_s	Name of Puzzle
15	14	0.885 **	Button Subsets and Buckle Classifications
50	12	0.554 *	Lego Ordinal Array
89	42	0.520 *	Birthday Candle Cardinal and Ordinal Colour Array
64	35	0.490 *	Venn Diagram: Three Intersecting Wallpaper Sets: Black & White / Small / Pattern
28	19	0.483 *	Yachts
31	8	0.440	Red / Silver / Blue Build Up Pattern Match Jigsaw
39	27	0.404 *	One Colour Difference Button Circle
2	21	0.363	Four Intersecting Button Sets: Leather / Square / Gold / Patterns of Three
3	19	0.345	Coloured Transparent Plastic Film - Addition Array
6	24	0.333	Four Intersecting Button Sets: Pattern / Red / White / Square
16	13	0.307	Button Subsets of Material Types
20	25	0.283	Reversed Tree: Reversed Shape / Pattern / Colour
1	37	0.263	Three Intersecting Button Sets: Blue / Transparent / Gold
54	12	0.256	Fabric Pattern Match: Red / Blue - Orange / Brown
14	37	0.250	Three Intersecting Button Sets: Pattern / Brown / Silver
29	15	0.247	Wallpaper: Three Intersecting Hexagons
57	56	0.237 *	Venn Diagram: Three Intersecting Fimo Colour Sets: Red / Yellow / Green
18	15	0.223	Reversed Sailing Ships
4	22	0.169	Four Intersecting Button Sets: Blue / Pink / Orange / Metal
27	12	0.141	Wallpaper: Ten Intersecting Hexagons
42	23	0.140	Venn Diagram: Four Intersecting Wallpaper Sets: Red / Small / Pattern / Triangle
94	11	0.111	One Difference Wool Circle
5	10	0.042	Four Intersecting Button Sets: Transparent / Plastic / Gold / Fabric
40	17	0.034	One Colour Difference Button Figure Eight
32	12	0.029	Chain Link: Intersecting Colour / Pattern Sets of Two Buttons

* Significant at 0.05.   ** Significant at 0.01.

Table 46: Named Puzzles r_s (0.05) For School Classes Across Levels of Performance.

Classes

Puzzle Figure Number	N	Classes r_s	Name of Puzzle
15	14	0.664 *	Button Subsets and Buckle Classifications
89	42	0.493 *	Birthday Candle Cardinal and Ordinal Colour Array
2	21	0.459 *	Four Intersecting Button Sets: Leather / Square / Gold / Patterns of Three
18	15	0.445 *	Reversed Sailing Ships
31	8	0.434	Red / Silver / Blue Build Up Pattern Match Jigsaw
3	19	0.424 *	Coloured Transparent Plastic Film - Addition Array
23	19	0.374	Yachts
39	27	0.346 *	One Colour Difference Button Circle
94	11	0.345	One Difference Wool Circle
50	12	0.337	Lego Ordinal Array
16	13	0.322	Button Subsets of Material Types
14	37	0.314 *	Three Intersecting Button Sets: Pattern / Brown / Silver
64	35	0.307 *	Venn Diagram: Three Intersecting Wallpaper Sets: Black & White / Small / Pattern
29	15	0.243	Wallpaper: Three Intersecting Hexagons
1	37	0.238	Three Intersecting Button Sets: Blue / Transparent / Gold
40	17	0.237	One Colour Difference Button Figure Eight
6	24	0.158	Four Intersecting Button Sets: Pattern / Red / White / Square
42	23	0.147	Venn Diagram: Four Intersecting Wallpaper Sets: Red / Small / Pattern / Triangle
57	56	0.135	Venn Diagram: Three Intersecting Fimo Colour Sets: Red / Yellow / Green
5	10	0.127	Four Intersecting Button Sets: Transparent / Plastic / Gold / Fabric
54	12	0.118	Fabric Pattern Match: Red / Blue - Orange / Brown
4	22	0.094	Four Intersecting Button Sets: Blue / Pink / Orange / Metal
20	25	0.056	Reversed Tree: Reversed Shape / Pattern / Colour
32	12	0.054	Chain Link: Intersecting Colour / Pattern Sets of Two Buttons
27	12	0.005	Wallpaper: Ten Intersecting Hexagons

* Significant at 0.05. ** Significant at 0.01.

Table 47: Named Puzzles r_s (0.05) For Ages Across Levels of Performance.

Ages

Puzzle Figure Number	N	Ages r_s	Name of Puzzle
15	14	0.617 *	Button Subsets and Buckle Classifications
28	19	0.512 *	Yachts
89	42	0.503 *	Birthday Candle Cardinal and Ordinal Colour Array
3	19	0.492 *	Coloured Transparent Plastic Film - Addition Array
18	15	0.458 *	Reversed Sailing Ships
31	8	0.434	Red / Silver / Blue Build Up Pattern Match Jigsaw
2	21	0.411 *	Four Intersecting Button Sets: Leather / Square / Gold Patterns of Three
39	27	0.389 *	One Colour Difference Button Circle
14	37	0.345 *	Three Intersecting Button Sets: Pattern / Brown / Silver
50	12	0.339	Lego Ordinal Array
29	15	0.325	Wallpaper: Three Intersecting Hexagons
16	13	0.278	Button Subsets of Material Types
94	11	0.265	One Difference Wool Circle
64	35	0.239	Venn Diagram: Three Intersecting Wallpaper Sets: Black & White / Small / Pattern
40	17	0.237	One Colour Difference Button Figure Eight
54	12	0.215	Fabric Pattern Match: Red / Blue - Orange / Brown
6	24	0.214	Four Intersecting Button Sets: Pattern / Red / White / Square
32	12	0.208	Chain Link: Intersecting Colour / Pattern Sets of Two Buttons
42	23	0.204	Venn Diagram: Four Intersecting Wallpaper Sets: Red / Small / Pattern / Triangle
1	37	0.198	Three Intersecting Button Sets: Blue / Transparent / Gold
57	56	0.152	Venn Diagram: Three Intersecting Fimo Colour Sets: Red / Yellow / Green
5	10	0.127	Four Intersecting Button Sets: Transparent / Plastic / Gold / Fabric
27	12	0.003	Wallpaper: Ten Intersecting Hexagons
20	25	-0.014	Reversed Tree: Reversed Shape / Pattern / Colour
4	22	-0.039	Four Intersecting Button Sets: Blue / Pink / Orange / Metal

* Significant at 0.05. ** Significant at 0.01.

Table 48: Approximate Difficulty Order of Puzzles:
Mathematical Groups r_s (0.05) Across Levels of Performance.

Mathematical Groups

Rank Order	Puzzle Figure Number	N	Maths Groups r_s	Name of Puzzle
least difficult				
1	18	15	0.223	Reversed Sailing Ships
2	28	19	0.483 *	Yachts
3	54	12	0.256	Fabric Pattern Match: Red / Blue - Orange / Brown
4	3	19	0.345	Coloured Transparent Plastic Film - Addition Array
5	50	12	0.554 *	Lego Ordinal Array
6	20	25	0.283	Reversed Tree: Reversed Shape / Pattern / Colour
7	57	56	0.237 *	Venn Diagram: Three Intersecting Fimo Colour Sets: Red / Yellow / Green
8	1	37	0.263	Three Intersecting Button Sets: Blue / Transparent / Gold
9	14	37	0.250	Three Intersecting Button Sets: Pattern / Brown / Silver
10	64	35	0.490 *	Venn Diagram: Three Intersecting Wallpaper Sets: Black & White / Small / Pattern
11	39	27	0.404 *	One Colour Difference Button Circle
12	4	22	0.169	Four Intersecting Button Sets: Blue / Pink / Orange / Metal
13	6	24	0.333	Four Intersecting Button Sets: Pattern / Red / White / Square
14	2	21	0.363	Four Intersecting Button Sets: Leather / Square / Gold / Patterns of Three
15	15	14	0.885 **	Button Subsets and Buckle Classifications
16	40	17	0.034	One Colour Difference Button Figure Eight
17	16	13	0.307	Button Subsets of Material Types
18	29	15	0.247	Wallpaper: Three Intersecting Hexagons
19	32	12	0.029	Chain Link: Intersecting Colour / Pattern Sets of Two Buttons
20	5	10	0.042	Four Intersecting Button Sets: Transparent / Plastic / Gold / Fabric
21	42	23	0.140	Venn Diagram: Four Intersecting Wallpaper Sets: Red / Small / Pattern / Triangle
22	89	42	0.520 *	Birthday Candle Cardinal and Ordinal Colour Array
23	94	11	0.111	One Difference Wool Circle
24	27	12	0.141	Wallpaper: Ten Intersecting Hexagons
25	31	8	0.440	Red / Silver / Blue Build Up Pattern Match Jigsaw
most difficult				

* Significant at 0.05. ** Significant at 0.01.

Table 49: Approximate Difficulty Order of Puzzles:
Classes r_s (0.05) Across Levels of Performance.

Classes

Rank Order	Puzzle Figure Number	N	Classes r_s	Name of Puzzle
least difficult				
1	18	15	0.445 *	Reversed Sailing Ships
2	28	19	0.374	Yachts
3	54	12	0.118	Fabric Pattern Match: Red / Blue - Orange / Brown
4	3	19	0.424 *	Coloured Transparent Plastic Film - Addition Array
5	50	12	0.337	Lego Ordinal Array
6	20	25	0.056	Reversed Tree: Reversed Shape / Pattern / Colour
7	57	56	0.135	Venn Diagram: Three Intersecting Fimo Colour Sets: Red / Yellow / Green
8	1	37	0.238	Three Intersecting Button Sets: Blue / Transparent / Gold
9	14	37	0.314 *	Three Intersecting Button Sets: Pattern / Brown / Silver
10	64	35	0.307 *	Venn Diagram: Three Intersecting Wallpaper Sets: Black & White / Small / Pattern
11	39	27	0.346 *	One Colour Difference Button Circle
12	4	22	0.094	Four Intersecting Button Sets: Blue / Pink / Orange / Metal
13	6	24	0.158	Four Intersecting Button Sets: Pattern / Red / White / Square
14	2	21	0.459 *	Four Intersecting Button Sets: Leather / Square / Gold / Patterns of Three
15	15	14	0.664 *	Button Subsets and Buckle Classifications
16	40	17	0.237	One Colour Difference Button Figure Eight
17	16	13	0.322	Button Subsets of Material Types
18	29	15	0.243	Wallpaper: Three Intersecting Hexagons
19	32	12	0.054	Chain Link: Intersecting Colour / Pattern Sets of Two Buttons
20	5	10	0.127	Four Intersecting Button Sets: Transparent / Plastic / Gold / Fabric
21	42	23	0.147	Venn Diagram: Four Intersecting Wallpaper Sets: Red / Small / Pattern / Triangle
22	89	42	0.493 *	Birthday Candle Cardinal and Ordinal Colour Array
23	94	11	0.345	One Difference Wool Circle
24	27	12	0.005	Wallpaper: Ten Intersecting Hexagons
25	31	8	0.434	Red / Silver / Blue Build Up Pattern Match Jigsaw
most difficult				

* Significant at 0.05. ** Significant at 0.01.

Table 50: Approximate Difficulty Order of Puzzles:

Ages r_s (0.05) Across Levels of Performance.

Ages

least difficult

Rank Order	Puzzle Figure Number	N	Ages r_s	Name of Puzzle
1	18	15	0.458 *	Reversed Sailing Ships
2	28	19	0.512 *	Yachts
3	54	12	0.215	Fabric Pattern Match: Red / Blue - Orange / Brown
4	3	19	0.492 *	Coloured Transparent Plastic Film - Addition Array
5	50	12	0.339	Lego Ordinal Array
6	20	25	-0.014	Reversed Tree: Reversed Shape / Pattern / Colour
7	57	56	0.152	Venn Diagram: Three Intersecting Fimo Colour Sets: Red / Yellow / Green
8	1	37	0.198	Three Intersecting Button Sets: Blue / Transparent / Gold
9	14	37	0.345 *	Three Intersecting Button Sets: Pattern / Brown / Silver
10	64	35	0.239	Venn Diagram: Three Intersecting Wallpaper Sets: Black & White / Small / Pattern
11	39	27	0.389 *	One Colour Difference Button Circle
12	4	22	-0.039	Four Intersecting Button Sets: Blue / Pink / Orange / Metal
13	6	24	0.214	Four Intersecting Button Sets: Pattern / Red / White / Square
14	2	21	0.411 *	Four Intersecting Button Sets: Leather / Square / Gold Patterns of Three
15	15	14	0.617 *	Button Subsets and Buckle Classifications
16	40	17	0.237	One Colour Difference Button Figure Eight
17	16	13	0.278	Button Subsets of Material Types
18	29	15	0.325	Wallpaper: Three Intersecting Hexagons
19	32	12	0.208	Chain Link: Intersecting Colour / Pattern Sets of Two Buttons
20	5	10	0.127	Four Intersecting Button Sets: Transparent / Plastic / Gold / Fabric
21	42	23	0.204	Venn Diagram: Four Intersecting Wallpaper Sets: Red / Small / Pattern / Triangle
22	89	42	0.503 *	Birthday Candle Cardinal and Ordinal Colour Array
23	94	11	0.265	One Difference Wool Circle
24	27	12	0.003	Wallpaper: Ten Intersecting Hexagons
25	31	8	0.434	Red / Silver / Blue Build Up Pattern Match Jigsaw

most difficult

* Significant at 0.05. ** Significant at 0.01.

Table 51 shows two tailed Mann-Whitney U-Tests testing gender differences in levels of performance across mathematical groups at 0.05, where U (M) = Males, U1 (F) = Females, and z shows use of normal approximation for larger samples.

Table 51: Mann-Whitney U-Tests: Two Tailed (0.05)
Gender Differences:
Levels of Performance Across Mathematical Groups:

Figure	U (M)	U1 (F)	z	Males	Females
Puzzles demonstrating unconscious processing					
1	149.5	186.5	+ 0.56	-	-
2	51.5	56.5		-	-
4	41.5	79.5		-	-
5	9.5	14.5		-	-
6	34	101		-	*
Puzzles demonstrating rote performance					
3	31	47		-	-
Puzzles demonstrating unexpected dimensions					
14	147.5	188.5	+ 0.62	-	-
15	20	28		-	-
16	5	35		-	*
Puzzles demonstrating reversals					
18	13.5	22.5		-	-
20	127.5	26.5		*	-
Puzzles demonstrating rotations					
27	10	25		-	-
28	18	66		-	*
Puzzles demonstrating distraction elements					
29	12.5	43.5		-	-
Puzzles demonstrating alternating concepts					
32	6	30		-	-
Puzzles demonstrating rotational ordering					
39	75.5	106.5	+ 0.75	-	-
40	31	41		-	-
42	53.5	72.5		-	-

continued

Table 51 continued: Mann-Whitney U-Tests: Gender Differences: Two Tailed (0.05): Levels of Performance Across Mathematical Groups:

Figure	U (M)	U1 (F)	z	Males	Females
Puzzles demonstrating consecutive ordering					
50	13.5	13.5		-	-
Puzzles demonstrating frustration with multiple concepts					
54	24.5	7.5		-	-
Puzzles demonstrating transfer of learning					
57	353.5	405.5	+ 0.43	-	-
64	108.5	191.5		-	-
Puzzles demonstrating diagnostic progressions					
85	58.5	4.5		*	-
86	152.5	147.5	+ 0.07	-	-
87	154.5	144.5	+ 0.16	-	-
88	56	70		-	-
89	141	264	+ 1.61	-	-
Puzzles demonstrating range of levels of performance					
90	28.5	19.5		-	-
92	46.5	31.5		-	-
93	14	14		-	-
94	10	18		-	-
95	24	24		-	-
105	21.5	69.5		-	-

* Significant at 0.05

Chapter 10

Implications in the Teaching of Mathematics

Chapter 10 looks at ways which support the hypothesis that children are able to understand mathematical concepts at much higher levels than is generally believed or taught within the educational system, and provides possible explanations for unconscious processing, with implications for the teaching of mathematics. Previously presented findings, previously unpresented findings, and findings relating specifically to unconscious processing are listed. A discussion examines some possible reasons for the general lack of correlation between P.A.T. mathematics scores and puzzle performance, with an exploration of P.A.T. mathematics and the mathematical puzzles and what they each measure, and the nature and degree of these different mathematical assessments. It includes a discussion on verbal and nonverbal processing and the validity of both P.A.T. mathematics and the mathematical puzzles in their ability to determine the acquisition of mathematical concepts in children. Left and right brain function, consciousness and the brain, and aspects of quantum consciousness lead into a discussion on the movement of percepts into concepts and the function of language in concept formation. Thoughts on Piaget's stages of cognitive development, and the place of intuitive, creative and divergent thinking are considered, followed by some theoretical findings regarding what mathematical ability is, issues of gender ability in mathematics, and descriptions of able children in general and how to aid the development of this ability. Acceleration and enrichment programmes are discussed, with implications for the teaching of mathematics. Limitations of the present study and suggestions for future studies with the puzzles are followed with a conclusion.

Previously Presented Findings

Several findings have been presented in this study:

1. Many children were able to process concepts conventionally considered above their age and understanding range.

2. Some children showed evidence of unconscious processing of concepts.
3. Puzzles had diagnostic and remedial value in identifying and remediating individual weaknesses and strengths. Weaknesses included difficulties with unfamiliar concepts and some types of perceptions as single concepts. The diagnostic aspect of the puzzles was to show when children had difficulty perceiving colours, shapes, patterns or sizes, or when they had difficulty with reversals, rotations or in perceiving lines, or had difficulty making a one to one correspondence or in seriating a group of objects. Weaknesses in the processing of multiple concepts included elements of distraction, difficulty observing patterning in alternating concepts, difficulties of rotation in rotational ordering, difficulties observing order in consecutive ordering, and in the general need for conscious focus of attention while processing multiple concepts. Inability to transfer learning or use a strategy indicated further difficulties. Strengths were indicated by perceptual acuity, speed, the use of symmetry, the ability to process multiple concepts, in transfer of learning, in the use of strategy, and in the mastery of difficulty levels above age groups.
4. Some senior remedial mathematics children overcame difficulties in their formal mathematics, even though the mathematical puzzles were not related to the kind of formal mathematics the children were having difficulty with.
5. Some junior remedial mathematics children began developing mathematical concepts in their school mathematics at a faster rate than previously, and were moved into average mathematical groups for normal classroom work.
6. P.A.T. mathematical groups, school classes, and ages of children did not generally correlate with mathematical puzzle performance. The large number of tests gives a greater likelihood of some chance significant correlations occurring.
7. Two children who were average in P.A.T. mathematics and other school subjects performed at a higher level than the top P.A.T. mathematics children.
8. There were no gender differences in levels of performance.

Unpresented Findings

Further unreported outcomes for children working with the mathematical puzzles were observed or reported by teachers:

1. Levels of performance amongst remedial children and children with special learning difficulties were variable, and related more specifically to individual problems within the children themselves than to any evenness or unevenness of mathematical groupings across schools.
2. Some children who previously disliked mathematics began to enjoy it.
3. Some children who previously disliked school began to enjoy school.
4. Many children at all levels gained general confidence in themselves and became more willing to attempt something new, and less afraid of difficulties or of making mistakes.
5. Several children who had previously been generally uncooperative at school became more cooperative and related better to their teachers.
6. Formal mathematics improved further for some top P.A.T. mathematics children.
7. Top P.A.T. mathematics children from the three schools performed at evenly high levels with the puzzles.
8. Children needing extension enjoyed the lateral problem solving enrichment challenges the puzzles provided.

Findings Relating to Unconscious Processing

- Performing a correct answer without conscious awareness of the process, and difficulty verbalising classifications. Nonverbal actions indicate possible unconscious processing of concepts. Children sometimes made classifications of less familiar concepts unconsciously. The construction of the puzzles concerned indicated a correct solution would be extremely unlikely by chance alone. Possibilities of this correct solution occurring through the use of a process of elimination, rote performance or an explanation with confabulated reasoning after a chance placement of sets, were eliminated from possibilities.

- The ability of children to perform above what is considered usual for their age, indicating children can act on higher level concepts if they are presented in manipulable and visual form. The children were not always able to verbalise what they had done, but clearly understood the structure of the problem at some nonverbal level, indicating unconscious processing of structure or conscious nonverbal processing. The ability of younger primary school children to process concepts such as set theory, probability, and tessellations as taught at 12 year old level in New Zealand schools showed children exhibiting mixed conceptual performance which Piaget's linear steps of conceptual development previously indicated did not occur. The uneven development of concepts is dependent on the child's natural processing modes and types of task experiences.

- High ability untapped and unknown in two children considered average. What are the P.A.T.'s and the curriculum missing? What is the unidentified ability? This specific ability may not be encompassed in the P.A.T.'s, as being generally unmeasurable and probably right brain oriented. The puzzles, with their visual-spatial, patterning and sequencing elements are more closely allied to items used in nonverbal tests which assess more right brain ability than the left brain verbal ability of formal mathematics.

- The incidence of upper and lower ranges of children's formal mathematics improving using the puzzles, particularly in remedial junior and senior children. This indicates the development of mathematical concepts through the broadening of experience from manipulating materials, and the development of conscious conceptualisation through language. Even though formal mathematics operating with symbols is very different from the items used in the puzzles, concepts integrated through processing the puzzles are the foundations for understanding formal mathematical concepts.

- The biggest question - how manipulating things is translated into, and the forerunner of formal mathematical work. Giving a multi-sensory experience base to draw from allows the concept to be integrated in a mulit-sensory manner, providing an increased chance the concept will be both accessed and applied in a wide variety of situations. Concepts are also more likely to be understood from the level of their basic structure, rather than from any memorised rote method of learning them. In this way they become more readily operational and transferable.

Discussion

To even begin the daunting task of attempting to define mechanisms operating in the lack of P.A.T. mathematics performance correlation with puzzle performance, and differences in modes of thinking associated with these two different types of mathematics, and why children can process some mathematical concepts beyond what they are exposed to in normal school work, it is necessary to evaluate what each of these two aspects of mathematics assesses. It must also be remembered that at senior levels in New Zealand schools, P.A.T. mathematics is a normal part of classroom practice, and the mathematical puzzles are not.

Test Validity and Validity of the Progressive Achievement Tests

Test reliability and validity are important in assessing the interaction between the psychological theory of P.A.T. mathematics and assessment practice, and in determining what P.A.T. mathematics versus the puzzles measure. Inferences from test scores need validating, bearing in mind the evolving property of validity, with validity evidence never complete (Messick 1990). Buros (1977) warns tests may be misused. Buros (1961) also finds the best tests still highly fallible instruments, and extremely difficult to interpret with assurance in individual cases. Mehrens & Lehmann (1978) find test problems stem more from over generalisation of use than from test invalidity, suggesting no test should be given if a better decision can be made without one. Reid (1980) emphasizes P.A.T. results should not be used as the sole criteria for grouping children. New Zealand schools have tended to express greater expectation from P.A.T. results than criticism of their limitations (Turnbull 1978).

P.A.T. mathematics used in New Zealand schools are standardised national achievement tests used for accurate, objective and comparative assessment (Reid & Hughes 1974). Although face validity shows P.A.T. mathematics measures what it intends to measure very well in recall of mathamatical ideas, computation, understanding and application, Messick (1990) claims that criteria used as measures which need to be evaluated in the same manner as the tests can not be used as standards for evaluating themselves. As such, the tests can measure what they set out to measure, but can not evaluate what should be in the tests to be evaluated. This process of construct validity requires several avenues of evidence which must not all be quantitative. All data emerging from theory, including criterion-related data are useful for construct validity, but construct validity can be threatened by under representation or over representation, and the P.A.T. tests must be erring at the former level of under representation. With only fifty questions covering all topics included, no area of knowledge is examined in depth. Cowie (1975) emphasises P.A.T. mathematics are standardized achievement tests of classroom instruction, broadly samplimg many areas of knowledge and skill. Although some diagnostic information can be gained from them, they differ from diagnostic tests by not isolating specific individual weaknesses in narrow fields of knowledge or skill. They also do not include all areas of mathemaucs, with no logic items in junior tests, and the phasing out of spatial dimensions in senior tests.

P.A.T. mathematics tests verbal capacities associated with what intelligence tests refer to as the 'g' factor (Croft 1982). Both school and P.A.T. mathematics are verbal in nature, and as reliant on verbal abilities as are verbal language tests with the assessments of mathematical recall, computation, understanding and application all being verbally dependent functions (Reid 1980). As P.A.T. mathematics involve reading skills, lower reading ability children are disadvantaged through being required to read the tests. If the test is administered orally (which it must not be because the norms were not obtained orally), up to six raw score points may be added (Reid & Hughes 1974). Messick (1990) suggests that to counter the possibility the tests are a reading test in disguise, correlations with reading scores should not be unduly high, loadings on a verbal comprehension factor should be negligible, and the reading level required by the item should not be taxing, otherwise discriminant evidence or test bias will occur, reducing test validity. Angoff (1987) considers this type of test bias to be a form of social bias. As the multiple-choice questions used in P.A.T. mathematics require selection rather than construction of an answer, P.A.T. mathematics are first reading tests and then mathematical tests of verbal mathematics (Lohman & Kyllonen 1983). Whereas Messick (1990) asks if multiple-choice questions reflect test-wiseness or commonsense, Lohman & Kyllonen (1983) consider that only by knowing how subjects solve items can the investigator know what it is that the task measures.

Buros (1977) considers it unfortunate so much attention is given to predicting success while less attention is given to determining what has been learned, although this does not seem to be a particular problem with P.A.T. mathematics. Whereas achievement tests measure present levels of knowledge or skill or performance, aptitude tests attempt to predict how well an individual may learn. As the best predictions for future performance are sought from past performance, aptitude and achievement tests can not differ from each other (Mehrens & Lehmann 1978). Popham (1980) reflects on the first measurers being interested in measures and teachers in teaching. Teachers later became evaluated by the results of the measures, then strove to teach to the dictates of the measurers. Whereas instructional objectives specify what students are expected to learn, tests were developed to evaluate intended learning outcomes (Nitko 1983).

Buros (1977) also finds it unfortunate tests are not trying to determine abilities in children previously undiscovered and uncatered for in the educational system. Traditional aptitude and intelligence test scores have limited diagnostic utility because they yield global norm-referenced scores. The problem of the limited diagnostic value of tests would be solved by moving from norm-referenced assessment to process oriented assessment identifying sources of developmental and individual differences on common verbal and spatial reasoning tests (Pellegrino & Goldman 1983). Ebel (1975) comments on a past tendency to deny the existence of what can not be measured. This is no longer valid, but if a process can not be clearly defined, it is still considered unable to be taught purposefully or measured validly. Le Rose, King & Greenwood (1979) cite Piaget's consideration that the identification of ability levels should not be quantitative as in intelligence tests but should be directed more to discovering new methods of thinking used by children of any age.

What the Mathematical Puzzles Measure

The mathematical puzzles are essentially diagnostic at pre-mathematical conceptual level in detecting perceptual difficulties such as identification of colour, shape, pattern, size, reversals, and rotations, or in detecting difficulties in making a one to one correspondence or in seriating, but provide remediation and enrichment and problem solving experiences for any level of mathematical development. They do not incorporate any form of written verbal numerical mathematics, as is the essential part of formal mathematics. The only verbal aspect of the puzzles is oral in the initial instructions, apart from occasional senior puzzles with written instructions, and in the questioning of children as to their processes and classifications. The present study has shown the mathematical puzzles to measure a child's ability to perceive primarily visually and in a tactile manner, and to manipulate objects and classify them to develop concepts of groups and order. The puzzles single out specific abilities of a child to match colour, size, shape and pattern, and to structure and order classifications, requiring increasing degrees of perceptual and conceptual acuity to process multiple concepts, transfer of learning and the use of strategy through the difficulty levels. As many concepts in the puzzles are unfamiliar, or structured in unfamiliar ways, the ability to perceive and creatively solve problems is challenged and enhanced. Whereas P.A.T. mathematics measures verbal and symbolic mathematics in a verbal and symbolic manner, the puzzles measure nonverbal and founding concept mathematics in a manipulative operational and mostly nonverbal manner, akin to items in nonverbal tests which are not used in New Zealand schools.

Nonverbal Tests

Nonverbal tests use items such as shapes, patterns, diagrams and sequences Croft (1982). These are the very mathematical materials used in the junior school to develop pre-mathematical concepts at symbolic and therefore verbal levels. These concepts involving shapes, patterns, diagrams and sequences are the very foundations upon which later verbal and formal mathematical concepts are built. The use of formal mathematical symbols is a verbal abstraction from these basic elements.

Nonverbal tests can not measure verbal capacities. Instead they measure a broader range of reasoning tasks and abstract concepts. These two divisions of intellect have only some general elements in common. Mathematics involving problem solving relates more highly to verbal than nonverbal tests, but mathematics stressing spatial relationships shows more positive relationships with nonverbal tests. However Lohman & Kyllonen (1983) consider it extremely difficult to devise spatial tasks that can not be partly solved by non spatial strategy. Nonverbal tests are not measures of the general ability required in a highly verbal classroom, nor can they predict verbal reading ability. Croft (1982) adds that where nonverbal test performance is better than verbal test performance, it is tempting to infer the nonverbal test is the more valid measure of underlying abilities and that verbal test results represent a form of underachievement. Whereas nonverbal tests are concerned with the broad domain of intelligence, schools are concerned with scholastic aptitude emphasizing verbal skills.

Formal Mathematics and Mathematical Puzzle Validity

The differences between the differing measures of verbal aspects of P.A.T. mathematics and nonverbal puzzle mathematics does not invalidate either P.A.T. mathematics findings or puzzle performance findings. Both measures validly measure what they intend to measure. Correct puzzle solution identifies conceptual understanding in the child of the mathematical ideas involved, and remediation effects indicate that the puzzles help achieve the acquisition of the mathematical ideas they incorporate. This validation of both measures does indicate that neither type of thinking alone encompasses the whole mathematical field, and that both types of mathematical experience are needed, not only for the development of one type of mathematics to aid the development of the other, but to expand hidden abilities in children that are not usually visible or enhanced in normal classroom practice. Evidence also suggests these two modes of mathematical learning may correspond closely to left and right hemisphere processing in the human brain. An exploration of left and right brain processing may help illuminate findings relating to unconscious processing in the present study. These include:

How children perform a correct answer without conscious awareness of the process and have difficulty verbalising classifications, indicating possible unconscious nonverbal processing of concepts.

The ability of children to perform above what is considered usual for their age, indicating children can act on higher level concepts if they are presented in manipulable and visual form.

The breaking of Piaget's linear steps of conceptual development with mixed conceptual performance in children, indicating uneven development of concepts is dependent on the child's natural processing modes and types of experiences.

High ability untapped and unknown in two children considered average, indicating P.A.T. mathematics and the curriculum are missing an unidentified ability not encompassed in the curriculum, as being generally unmeasurable and probably right brain oriented.

The incidence of upper and lower ranges of children's formal mathematics improving using the puzzles, particularly in remedial junior and senior children, indicating the development of mathematical concepts through the broadening of experience from manipulating materials, and the development of conscious conceptualisation through language.

How manipulating things is translated into, and the forerunner of formal mathematical work, giving an experience base to draw upon, indicating concepts are best developed and understood from the level of their basic structure, rather than in memorised rote form, becoming more readily operational and transferable when developed in this way.

Left Brain Right Brain

Blakeslee's (1980) explanation of mostly unconscious, nonverbal and intuitive processes as right brain activity, and mostly conscious, verbal and logical processes as left brain function have been cited in the present study, where the right brain thinks more in images and deals with spatial processes in a holistic view of the whole situation at once, and the left brain examines issues more sequentially and logically, with control normally passing to the hemisphere with the fastest answer. Kosslyn (1987) discusses right hemisphere processing as seeing in wholes, with the left hemisphere better at image generation and the right hemisphere better at image manipulation and transformation. The right hemisphere is critical for dealing with novel situations for which there is no existent code, and plays a part in assembling such codes. The code then transfers to the left. The left hemisphere controls all coded processes and deals with routine functions. A visual-spatial task, if embedded in a code, will come to show a left hemisphere superiority. The right hemisphere deals with the perceptual organisation of experimental situations (Goldberg 1989). Levy & Sperry (1972) found the right hemisphere specialised for gestalt perception, a synthesist in dealing with information input. Rausch (1985) finds the extent of mesial removal of the right temporal lobe correlates with deficits in learning an unfamiliar sequence. These descriptions help explain why and how children are more likely to process unfamiliar concepts nonverbally in the right hemisphere, with difficulty transferring the concepts into the left brain and into verbally expressible form. They also indicate the educational value of questioning children about what they have done, to help transfer and establish these concepts into a form useful for retrieval and application to further related concepts and problem solving experiences.

It is known from split half brain experiments that the two sides of the brain can be independently conscious (Sperry 1985). Sperry's split brain research showed the left brain did not know what the right brain had experienced. An epileptic child with the corpus callosum cut named all objects visually projected into the left brain, but nothing that was flashed to the right brain, as if the right hemisphere was blind. However the right hemisphere did see it, but was not conscious of it, and could not express the experience in words. The right brain was not able to speak what was there, but was able to write it or point to it, indicating the right brain can express itself if provided with a 'voice box' such as touching objects, pointing to a choice of objects, or spelling. When a direction is flashed to the right brain, that hemisphere performs the appropriate action. Since the left brain did not receive the command, it was unaware of what caused the person's behaviour. The left brain observed the change in behaviour and attempted to make sense out of it. When a command was flashed to the left hemisphere, the individual often spoke the command word. The right brain heard the direction and carried out the action. It is as if there is a double consciousness, with each side of the brain processing information independently, then selectively communicating with messages transmitted across the corpus callosum.

Asher (1993) discusses the meaning of language comprehended in the right hemisphere, with the right brain receiving and deciphering incoming messages and the left brain editing before speaking. The right hemisphere has better language comprehension than expression, and better auditory comprehension than reading ability (Zaidel 1985). The left brain processes verbally in serial step by step order, and the right brain processes nonverbally in patterns, and may process verbal information if it is presented in patterns such as a story, drama or experience. By simply asking a question, activity is automatically brainswitched from the right to the left hemisphere, especially if the expected response is either written or spoken (Asher 1993). Edwards (1979) also found people could draw using the right brain, or talk, but not undertake both tasks at the same time, indicating right versus left brain processing. This supports Brentano (1973) in considering the talking aloud method of introspection may alter the very phenomena under consideration.

The left brain focuses on memorisation and the right brain on internalisation (Asher 1993). Left and right brain thinking equate well with analytic and constructive thinking (Dienes (1959), reflective and intuitive thinking (Skemp (1971), paradigmatic left hemisphere and narrative right hemisphere modes of thought (Bruner 1986), explicit and implicit modes of thinking as described by Reber (1989), and the first and second modes of thinking as described by Perruchet, Gallego & Savy (1990).

Edwards (1979) makes a broad comparison of left and right hemisphere mode characteristics:

Left	Right
verbal (word descriptions)	nonverbal (nonverbal awareness)
analytic (step by step)	synthetic (creative)
symbolic (use of symbols)	concrete (relating in the present)
abstract (representational)	analogic (similarities, metaphor)
temporal (sequences)	nontemporal (timelessness)
rational (reasoning)	nonrational (suspending judgement)
digital (use of numbers)	spatial (relation of parts to the whole)
logical (algorithms)	intuitive (insight, hunches, images)
linear (convergence)	holistic (seeing the whole, divergence)

Sperry (1985) lists examples of some specific right brain functions discovered from split brain experiments, as the copying of designs, reading faces, fitting forms into molds, discrmination and recall of nondescript tactual and visual forms, spatial transformations and transpositions, judging whole circle size from a small arc, grouping a series of differently sized and shaped blocks into categories, perceiving whole plane forms from a collection of parts, and intuitive apprehension of geometric properties.

This description clearly defines the mathematical puzzles as tapping largely right brain function, as against formal mathematics, most of which is left brain processing. Cummings (1985) discusses left hemisphere specialisation for aspects of language comprehension, for language execution, verbal memory, and the numerical aspects of calculation. The right hemisphere specialises in visual-spatial and visual-perceptual function, nonverbal memory and comprehension, some aspects of language comprehension and the execution of speech prosody. Speech prosody includes functions such as rhythmic aspects of speech, tone or accent of a syllable, modulation of voice, and pronunciation. Yet it is noted both hemispheres have some part to play in both language and visual-spatial function. As largely right brain activity with the pre-mathematical puzzles clearly lays foundations for beginning formal left brain mathematical understanding, particularly when brought to conscious attention through the difficult verbal expression of the concepts involved, it makes good sense to consider expanding this mode of developing mathematical conceptual understanding further into the curriculum, into senior mathematics, where appropriate and possible.

The mechanisms for the enhanced development of pre-mathematical concepts from the structured use of environmental materials can be suggested. The manipulation of objects provides a kinaesthetic and multi-sensory experience to be associated with concepts inherent in the objects manipulated. Asher (1993) uses a right brain instructional strategy called total physical response (TPR) in the teaching of second languages, believing the assimilation of information and skills can be significantly accelerated through the use of the kinaesthetic sensory system. Using a kinaesthetic approach, Asher finds optimal conditions for learning are on the first exposure to the information to be internalised, and describes traditional learning procedures showing that the more exposures to a learning pattern before it is internalised, the more difficulty there is in later retention. Asher looks at the structure of the task rather than internal factors such as intelligence or aptitude.

For authentic learning to take place, the person must understand the inner structure and pattern of the solution. Asher describes this gestalt understanding labeled productive thinking by Wertheimer (1959), as in opposition to an associationist idea of each exposure to a task making for a stronger association. Where associationists used verbal tasks almost exclusively in their learning experiments, the Gestalts used nonverbal tasks such as patterns, pictures and experiences. These methods correspond closely to a left brain step by step multiple exposure which resists the novel, to a right brain one where a pattern is understood in a flash. Asher believes learning is the reverse of problem solving, where learning means to internalise an existing concept, and problem solving repairs an existing concept that has a tension producing flaw. The left brain resists the novel concept, but once the idea is incorporated, will resist any threat to remove or change the idea. Problem solving is therefore concept disruption. If the left brain is asked to retrieve information input into the right brain, results will be significantly less than if both input and output are from the right brain. This explains an unconscious solution. In school, input from the left and right brain is mixed, but output is usually measured from the left brain exclusively.

It is likely both gestalt and associationist approaches together give the best results in pre-mathematical classroom work. Multiple exposures to a concept, particularly in varied and multi-sensory form, enrich the meaning of the concept. Manipulating concepts in as many different forms as possible will aid both acquisition of the concept to be internalised, and its consolidation for retrieval and application to further tasks. This will input to both left and right brain function more readily, especially where expression of the concept into language is encouraged. Left and right brain input provide an explanation for the hypothesis that children are able to process mathematical concepts at much more difficult levels when the concepts are presented in manipulable and visual form. The child may not be able to fully describe the difficult concepts in verbal terms, but the hypothesis of the present study is supported by Bruner's (1960) notion that any subject can be taught effectively in some intellectually honest form to any child at any stage of development.

Consciousness and the Brain

With the emphasis on getting children to describe their understandings verbally, it is important all input is not in the same form as the required output. A verbal output may not most readily be accessed from a verbal input alone. An action output may well precede a verbal one, and an action input must precede a verbal one in the initial formation stages of a concept. It is too easy to believe that if a concept is not expressible in themselves would resist rearrangement. We can isolate features which in fact can not be isolated. We can juxtapose objects and events far separated in time and space' (pp. 43-44). This reflects the importance of encouraging language expression of experience, to symbolise it and expand it and entrench it in new forms that will connect with other experiences and build wider and increasingly connected concepts than before.

Vygotsky (1962) sees the development of language and mental powers as neither learned in a rote manner, nor emerging epigenetically, but as social and mediate in nature, a vehicle for transmitting information to others. It arises from the interaction of adult and child, and internalises the cultural instrument of language for the processes of thought. Sacks (1989) observes that the innate ability to acquire language is only activated by another who has linguistic competence, and this language is achieved only through transaction and negotiation with another. Vygotsky finds the leap from sensation to thought involves the right sort of questioning. It is vital both child and teacher ask these right sorts of questions. Irrespective of whether a solution is arrived at with conscious verbal awareness or conscious nonverbal awareness, any attempt to verbalise an existing concept will develop the concept further into conscious attention.

Sacks (1989) believes it is not just language but thought that must be introduced, believing neither language nor the higher forms of cerebral development occur spontaneously. They depend on exposure to language, communication, and proper language use. Language must be good enough to allow internal manipulation, then the normal shift to left hemisphere predominance can occur. Sacks (1989) finds in abstracting or generalising or theorising, the concrete is never lost. As it is seen from a broader and broader viewpoint, it has ever richer and unexpected connection, holding together and making sense as never before. Luria (1976) adds that science is the ascent to the concrete. It is the richness of the concrete that gives rise to the abstract. In abstracting or generalising the concrete is never lost. The concrete leads to the general and the general leads to the concrete, in intensified and transfigured form. Concrete experiences in some verbal form, it is absent or incomplete, or that every concept must be brought into conscious awareness to be functional. In the educational fervour for children to gain as much verbal and conscious translation of concepts as possible, more and more verbal inputs are presented to children, with less and less experiential ground work provided. Yet it is much more likely the child will remember an experience than a speech. Gazzaniga, LeDoux & Wilson (1977) found the verbal system attributes cause to behaviour produced. This process of attribution by the verbal system seems to be a major mechanism of consciousness. Behaviours are being continuously exhibited, the origins of which may come from coherent independent mental subsystems, and these actions are immediately interpreted by the verbal system. Consequently a large part of the sense of conscious reality comes from the verbal system attributing cause to exhibited behaviour. This explains confabulated reasoning for some actions.

With multi-modal thinking and multi-sensory information processing, the question of overall information integration and of what becomes conscious and why it becomes conscious remains unclear. It would not be correct to imply left brain processing being largely verbal is conscious and right brain processing being largely nonverbal is unconscious, as both hemispheres are known to have their respective shares of unconscious and conscious processing. It is also true the functions of vast tracts of the human brain are as yet unknown. Dennett (1991) finds it an open question whether any particular content eventually appears in conscious experience, and considers it a confusion to ask when it becomes conscious, considering there is no single point in the brain where all information funnels in. He believes discriminations only have to be made once. When a particular observation has been made in one specialised localised portion of the brain, the information does not have to be sent somewhere else to be rediscriminated. Discriminations form a stream or narrative sequence over time.

Dennett (1991) recommends the idea that consciousness is a mode of action of the brain rather than a subsystem of it, as all judgements are distributed around in time and space in the brain, and each act of discrimination or discernment or content fixation happens somewhere, but there is no one discerner doing all the work. The content of the judgement does not have to be expressed in propositional form. These are nobody's speech acts and they do not have to be in a language. They are like speech acts, with content, and have the effect of informing various processes with this content. Some of these content fixations have further effects, eventually leading to the utterance of sentences in natural language, either public or internal. Consciousness is not a matter of arrival at a point, but a representation exceeding some threshold of activation over the whole cortex. Dennett considers an element of content becomes conscious at some time by changing state right where it is through acquiring some property boosted above some threshold, not by entering some functionally defined and anatomically located system.

The idea that there is no central anatomical processor in the brain, and that each part carries out its own function takes the focus off conscious attention necessarily being defined as explicable in verbal terms. Each perception a child makes from any sensory system while processing the puzzles is a form of consciousness in its own right. Each action, even if originating from some unconscious process, is also a form of consciousness. When a child is unable to verbalise a classification processed unconsciously, the action of a correct solution is still an act of consciousness. In mathematics, the concept will become more conscious and more versatile for the purposes of transfer of learning to other situations if it is first translated into verbal terms where possible.

Quantum Consciousness

Physicists Goswami, Reed & Goswami (1993) believe people choose their conscious experiences, yet remain unconscious of the underlying processes. The quantum brain collapses certain events in order to focus attention on them and make them conscious, selecting what it is useful to be conscious about. In physics, the term quantum refers to a discrete, indivisible amount of some quantity, especially energy by which a given system may change in any process. With quantum objects as waves that spread in existence at more than one place, consciousness may be the agency that focuses the waves to be observed at one place, choosing the outcome of the collapse of any and all quantum systems. This outcome is a chosen conscious experience, the underlying processes of which remain unconscious. As there is a necessary ongoing sensory information input to be perceived and selected by the brain, some unconscious threshold mechanism selects which sensory information to make conscious in the form of a perception. These perceptual selections are unconsciously considered to be the necessary present focus of attention in the individual's experience of the moment.

The quantum brain operates on Heisenberg's uncertainty principle of an infinite number of possibilities coming into and out of existence at all times. The quantum brain collapses certain events in order to focus attention on them and make them conscious. This not only explains creativity, but the quantum brain is creativity, to create conscious observation of effects. This concept supports the idea that the normal classical ego-self arises from processes of secondary awareness of a conscious experience, with the time lag of secondary introspection allowing the ego experience of consciousness to feel continuous. The stream of consciousness is the result of mindless introspective clatter. There is little awareness of the immediacy of experience in the quantum mode more readily available through meditation. It is the quantum self which expresses the free will. The self 'I' is a relationship between conscious experience and the immediate physical environment.

This supports findings of conscious awareness of a stimulus following its sensory registration in the brain, also supporting the idea that unconscious processing is not only possible, but occurs before conscious awareness of the event. It may be that the manner in which children processed an unfamiliar concept unconsciously, fits along this hierarchy in a natural progression towards a more consciously aware consciousness of the result made available firstly through actions, and finally through a conscious verbal expression of the concept.

Connectionist models of epiphenomenalism, and bottom up theories of order within chaos are unable to prove a connection between neuronal states and states of mind experienced. Monistic idealists believe consciousness collapses the multifaceted structure, choosing one facet in the presence of mind-brain awareness. Awareness and choice are tangled hierarchies (which comes first?) It is the tangled hierarchical situation that gives rise to self reference and to the subject-object split of the world. Identification with an object and the 'I' of reflective awareness arise out of secondary awareness processes. Both primary and secondary awareness processes normally remain in the preconscious. The ego is the image constructed of the apparent experiencer of actions, thoughts and feelings, and is associated with secondary awareness experiences of self awareness, but not with the primary experience. With conditioning, certain responses gain in probability when a learned stimulus is presented to the brain-mind. Consciousness identifies with the apparent processor of the learned responses, the ego, but the identity is never complete. Consciousness always leaves room for unconditioned novelty, making free will possible.

This explains how unconscious choices of action may occur, and why learned patterns of recognition or behaviour are associated with greater conscious awareness. 'We see what we know.' Goswami et al believe the cognitive functionalist connectionist model has no explanation of self awareness. Attention is assumed to be a function of the ego. The quantum theory of self reference is that the self operates in both the secondary awareness ego mode, and the primary awareness quantum modality, and is associated with primary awareness experiences such as choice and direction of attention without self-awareness. It seems unconscious selections include not just sensory and perceptual and concept formation, but choice of action as well. Verbal consciousness comes from an observers role applied after nonverbal conscious awareness has occurred. This relates well to children performing correct solutions to unfamiliar concepts in a puzzle, then having great difficulty verbalising classifications and processes involved.

The Function of Language

Le Doux (1985) emphasises consciousness is not dependent on language. Nor are conscious processes synonymous with cognitive processes. Information encoded in the absence of language proved inaccessible to consciousness, though it was readily accessible through behaviour. Consciousness defined in language terms becomes only one avenue through which the brain accepts inputs and produces behaviour. Operating outside of conscious awareness are neural systems capable of comprehending external events and regulating organised interactions with the world. We are not consciously aware of all the information our mind processes or the cause of all our behaviour or the origin of all of our feelings. The conscious self uses these as data points to construct and maintain a coherent story and our subjective sense of self. Inputs registered by non conscious systems are not available to the conscious self. They are coded in such a way that can not be decoded by verbally dominant conscious mechanisms. The activity of non conscious mental systems is often not decodable internally in the brain by the conscious self, and can only be known and incorporated into the verbally constructed sense of self when exposed by behaviour. The finding that information encoded in the absence of language proved inaccessible to consciousness, though it was readily accessible through behaviour, supports the finding that children processed the concepts nonverbally when the concepts were unfamiliar. It explains why the six year old girl had to literally see through the transparent button again before she could verbalise that it was 'see through.'

Dennett (1991) finds the emergence of expression creates or fixes the content of the higher order thought expressed. Internal communications have the effect of organising the mind into an infinitely powerful reflective or self monitoring system. Such power of reflection is claimed to be at the heart of consciousness. The value of verbalising is a means to translate perceptions into conceptual language (Sacks 1989). Church (1961) finds language transforms experience and the individual to enable new things to happen, or old things to happen in a new way. Language permits things to be dealt with at a distance and acted upon without physically being handled. Sacks (1989) reports a description of what language can do, from Joseph Church, a deaf person who acquired sign language later than usual. 'Language transforms preverbal experiences... We can act on other people or on objects through people. We can manipulate symbols in ways impossible with the things they stand for, and arrive at novel and creative versions of reality. We can verbally rearrange situations which in manipulating the puzzles aid generalisation of the concept, adding enrichment and leading to new ideas, both concrete and abstract.

Thoughts on Piaget's Stages of Cognitive Development

In examining the pattern of human development, St. George & Shaw (1982) believe factors such as child rearing practices, forms of play and cognitive style are likely to be at least as important as language and social culture. Kogan (1971) finds modes or styles of thought are the products of socialisation patterns and experiential opportunities that cultures and ecologies provide. St. George & Shaw (1982) report that not all normal individuals in all cultures pass through all of Piaget's cognitive stages or achieve all of the cognitive tasks within a cognitive stage as a matter of course. Appropriate cultural support or environmental stimulation appears necessary for the achievement of certain levels of development. Price's (1978) review found the absence of formal operational thinking on some Piagetian tasks for some Papua New Guinea adults where a different thought mode had been evolved by the culture. Price-Williams, Gordon & Ramirez (1969) found the order of development of task mastery within a Piagetian cognitive stage is not invariant, such as conserving quantity before weight, and weight before volume. St. George & Shaw (1982) find the evidence of particular stages and concepts or processes within stages for Piaget's cognitive stages is much more task, experience and culture dependent than first thought. This moves away from a strict chronological and biologically based view of cognitive development, having important consequences in educational terms.

It also explains the finding Dienes reported in response to Becker & Rogers (1969), that children can and do make intuitive leaps, and that depending on the familiarity and usage and presentation of the particular concept involved, children may currently possess any mixture of intuitive, concrete operational and formal operational thinking, with these levels of conceptual understanding varying from one application of a concept to another. Dienes found developmental differences as well as differences in strategies used by children, and also found children transfer information more readily from a complex concept to a simple one than from a simple concept to a complex one, provided simpler concepts are embedded within the complex concepts presented to the children. These findings suggest Piaget's cognitive stages rigidly apply within certain cultural and task orientations only.

Clearly, the ascent through human developmental stages varies according to the input from the culture, environment and experiences presented to the individual along the way. Lack of exposure to a particular type of experience or way of thinking may well lead to the absence of aspects of any of the developmental stages described by Piaget. Furthermore, given enriched experiences at any stage of development, an individual also may develop concepts that are usually beyond the developmental level of that individual's age or stage. Where there is unevenness of experiences, unevenness of conceptual development may occur. Similarly, where there is enrichment at some level of experience, the individual may develop advanced thinking and functional ability within that particular context. This explains the findings of Dienes, and of the high performance of some younger children in the present study, where performance was mixed and uneven, depending on experiences, and more particularly, the form in which the experiences were presented to the child to enable these higher levels of function to establish.

Intuitive, Creative and Divergent Thinking

Bruner (1960) finds intuitive thinking does not advance in careful well defined steps, but involves manoeuvres based on implicit perception of the total problem, with little, if any awareness of the process by which the answer is arrived at. An adequate account of how an answer is obtained can rarely be provided and there may be lack of awareness as to which aspects of the problem situation were being responded to. Usually intuitive thinking rests on familiarity with the structure of the area of knowledge involved, making it possible for the thinker to 'leap about'.

Creative and divergent thinking must play a part in the process of cognitive problem solving. Ghisellin (1954) finds the process of creativity will not perform to command, as the central part of its activity is both unconscious and involuntary. There is a necessary period of intense concentration on the problem, then sudden insight, usually when the problem has been laid aside and the person is relaxed. Barron (1969) considers creativity a synectics process of joining together different and irrelevant elements to arrive at novel solutions. By aiming at the preconscious it is hoped insight will be gained into the underlying process as it occurs. The first stage depends on making the strange familiar or looking for similarities. The second stage looks at making the familiar strange and shedding preconceptions. Creativity has arrived.

Barron & Harrington (1981) ask what are the abilities creativity is tapping? How does creativity differ from divergent thinking? Divergent thinking is remoteness combined with cleverness and aptness going a distance from the obvious but not ruling out convergent thought. Divergent thinking goes with convergent thinking in every thought process that results in a new idea. Process is what is invisible in the divergent thinking test used in creativity research. The problem is set and the answer obtained. What happens in between is the guess of anybody except the respondent who has not been asked.

The creative and divergent processes appear to be an integral part of the puzzle solution process. In a creative sense, the child first looks for what is familiar and then for what is different. The intuitive and creative process enters into what is different, allowing the joining together of seemingly unrelated items into some coherent new idea. That this is likely to be a right brain nonverbal process is not surprising when the right hemisphere tends to see whole patterns at once. Furthermore, the right brain is more likely to decipher a new situation, where the left hemisphere will deal with what is already coded and familiar. The divergent aspect goes along with the convergent thinking processes, but adapts in a lateral way to encompass a new idea.

Approaches to Mathematics

Clearly, approaches to the pre-mathematical puzzles involve intuitive, creative and divergent thinking processes throughout Piaget's stages of cognitive development in the process of acquiring formal mathematical thinking. Bruner (1960) finds almost anything that moves away from the usual approach to natural numbers and their mechanical manipulation has the effect of freshening the students taste for discovering things for themselves. Discovery in mathematics is a byproduct of making things simpler. Where mathematics is concerned, the issue centres on how simplification occurs. It most often results from a succession of constructing representations of things. Something is done that is manipulative at the outset, literally providing a definition of something in terms of action. Having acted, the person is then able to turn around on their own action and represent it, observe the action, then represent it. The premature use of the language of mathematics and its end product of formalism makes mathematics seem like something new rather than something the child already knows. By interposing formalism the child is prevented from realising he has been thinking mathematics all along, and loses confidence in the ability to perform the process of mathematics. What is needed is ways to help children improve intuitive ways of thinking. Bruner (1960) quotes D. Page, 'Young children learn almost anything faster than adults do if it can be given to them in terms they understand.'

The Mathematically Able

The mathematically able think logically in quantitative and spatial relationships and with symbols; within curtailed structures; with flexibility; with solution clarity, simplicity, economy and variability; and with rapid free reconstruction, with reversibility of mental processes (Briars 1983). Skemp (1971) finds both visual and verbal symbols are used in mathematics together and apart. Despite visual imagery being commonly considered most favourable to the integration of ideas, verbal symbols are indispensible but visual symbols are not. The functions are different and perhaps complimentary. It is because of the indispensability of verbal symbols as against visual symbols that formal mathematical teaching can function with verbal and symbolic systems as abstractions of abstractions in the absence of simultaneous concrete experiential manipulations. If simultaneous concrete experiential manipulations could be utilised more frequently in the teaching of mathematics to children, mathematical ability and the enjoyment of mathematics by children may come to be viewed as less of an awesome acquisition for a select few, to a more natural transition towards the inherent mathematician already present in each child.

Gender Differences in Mathematics

The absence of gender differences in performance in the present study goes against a traditional belief that males exhibit superior mathematical ability, and females exhibit superior verbal skills, (Dweck & Licht 1980), but supports the absence of gender differences found in mathematical performance in primary school children (Tittle 1986). It may also support findings by Jacklin (1989) and Linn & Hyde (1989), who found previous beliefs of cognitive gender differences in verbal ability, spatial visualisation, and mathematical computation and concepts have declined, and no longer exist. In view of studies by Gold (1990) and Linn & Petersen (1985) showing secondary school males performing better than females in mathematics achievement tests, especially those involving problem solving, there may be clear differences between spatial and verbal tasks on the one hand, and general mathematical achievement and problem solving skills on the other. Another alternative may be that gender differences in these tasks emerge as a result of biological or other adolescent changes. Tittle (1986) and Jacklin (1989) find differences are more related to social expectations and educational experiences than true biological differences. Females have been shown to have lower perceptions of mathematical competence and lower performance expectations than males (Eccles, Adler & Meece 1984), creating loss of confidence in their mathematical ability. This attributional pattern discounts success and affirms failure, causing females to veer further away from mathematics (Weiner, Frieze, Kukla, Reed, Rest, & Rosenbaum 1971).

Able Children

Previously ability was thought of as an entity in itself. An individual either had ability or did not have it, and the aim was to prove or disprove it (Renzulli 1980). Hill (1977) finds a need for a concept of able behaviour rather than one of ability, considering a child only has ability when ability is displayed as such, and is only creative with creative production. Barbe's (1964) interpretation of the most universal characteristic of any able child is sensitivity, in kind and degree, directed into discovery, problem solving, particular areas of learning or to the feelings of others. Feldman (1985) cites Cox's opinion that interest is a good indicator of ability. Roeper (1977) finds real ability is the ability to think, generalise, and see connections and alternatives, abilities which may not necessarily translate into outstanding achievement. Renzulli (1978) believes above average ability, task committment and creativity are the only three things that are necessary to truly achieve in any area. McAlpine (1979) lists some of the characteristics of intellectually able children as keen observation and curiosity; periods of intense concentration; ability to understand complex concepts, perceive relationships and think abstractly; creative ability as ability to restructure and re-classify components in problem solving, divergent thinking and imagination; high energy; taking cognitive risks; and preference to work alone. Speed of thought and the ability to work quickly are additional characteristics of able children.

Identification of able children is more difficult where children may have ability in one or more areas but be above average or average in others. McAlpine (1986) finds that rarely, if ever is reliance of identification of ability placed on any single measure, and that different measures may show quite discrepant results. Robinson, Roedell & Jackson (1978) find children of various ages and intellectual levels may score the same on intelligence tests, yet have very different abilities and utilise quite different cognitive strategies. LaBenne & Greene (1969) consider observations over a period of time provide more data than test scores.

The child whose abilities fall in the narrow band required by school work is fortunate in gaining resultant recognition and emotional and social status as well as success. Achievement test performance declines in the child considered average whose real abilities are not recognised by the teacher (Sutherland & Goldschmid 1974). Lyon (1981) believes the concept of the average student is at best a myth and at worst a destroyer of human potential. Zaffran and Colangelo (1977) say many bright children never have the opportunity to realise their true capacity in doing truly outstanding work because their teachers never provide them with such challenges. Robinson et al (1978) find bright children may perform like average children if sufficiently challenging material is not included in the classroom. Williams (1979) makes the comment that it is alarming to see normally self confident able students freeze under the idea of accepting wrongness or working with the unknown. This reflects what happens to non academic children not coping with normal school work.

The Development of Ability in Children

Aspy & Bahler (1975) believe in the power of the teacher to affect achievement and growth in the child. The perception teachers have of their own ability and worth is most significantly related to the success of their students. Hollingworth (1942) finds not all intellectually advanced children develop equally well under all circumstances. Nor do grades give much indication of the future success of students (Clark 1983). Grading, along with teacher evaluation does not have a very good record for predicting the truly able and productive adult, nor does it contribute to the learning process. Walford (1979) cites Samuda as saying that conventional tests, among many inadequacies, measure a very narrow range of mental ability, and Yarborough & Johnson (1983) find the replacement of I.Q. tests by achievement tests merely narrows the field of learnt knowledge by eliminating the aptitude aspect. Robinson et al (1978) recommend that if all potentially able people are to be found, testing procedures must have a sufficient number of difficult items to allow those at the top of the scale to emerge. Bright children may perform like average children unless sufficiently challenging material is provided.

Acceleration Versus Enrichment Programmes

A further consideration is whether programmes should accelerate or enrich. Reid & McAlpine (1981) report the New Zealand Department of Education finding that few programmes enrich horizontally without leading vertically into higher levels. Reid (1976) cites Start who prefers more acceleration than enrichment, as enrichment can sometimes be more of the same work or different work at the same level. Feldman (1985) cites Cox's consideration that no one should need to choose between acceleration and enrichment. Both should be used, but pull out programmes are a part time solution to a full time problem. Raths (1979) believes it should not be necessary to either enrich or accelerate within a curriculum, providing the curriculum is based on the needs of all students. There is a difference between enriching or accelerating a deficient curriculum and designing an appropriate one in the first place. Callahan (1982) finds the issue the degree to which the needs of any child are being met. Reid (1978) believes teachers need to teach differently and facilitate learning, not direct it; to encourage process rather than product; to be learners more than leaders; and to be involved in the learning process. Birch (1984) sees that the more personalised education becomes, the less need there will be for formal identification procedures. All children will receive appropriate education. This may mean more small group work and some individualised work. Children in such a programme do not need assessment procedures. They identify themselves by the amount, the level and the quality of their work. Their identification is curriculum embedded. As the main function of tests is to predict potential, a test should not be needed to predict what is already being achieved.

Reid (1978) finds that although an unusually high performance will indicate possible high ability, a low performance does not have to indicate the reverse. It may mean that different strategies and approaches of assessment are needed. Renzulli (1980) emphasises that identification needs as much emphasis placed on the ways children interact with experiences as action and performance information, as on more formal methods. Schwartz (1981) considers the key to successful learning experiences lies in the provision of an optimal match between the child's skill level and the materials provided. Renzulli (1980) finds students can be more effectively served if the number and variety of opportunities is increased. Equally as important is that not every child should be made to follow through on every activity, but flexibility should be exercised in providing supplementary services at the time, and in the areas where the child is eager to follow through. Le Rose, King & Greenwood (1979) cite Gowan by stating that putting children in possession of their own learning gives them a consciousness of their own cognitive processing, termed meta thinking, and may be a crucial component in the development of ability.

Implications in the Teaching of Mathematics

The present study questions the validity of both curriculum embedded mathematics and New Zealand P.A.T. mathematics to truly teach and remediate, and identify and enrich all mathematically able children in the best manner possible. Whereas school mathematics and its associated P.A.T. tests rely heavily on verbal skills, nonverbal aspects of mathematics, despite aiding the foundation of mathematical concepts, are largely excluded from the curriculum, thus losing the means for remediating mathematical difficulties, and for identifying and enriching all mathematically able children.

The puzzles identify specific abilities and weaknesses. They provide acceleration and enrichment for the highly able, and remediation for those having difficulties. They provide a means for the development of enthusiasm for learning, and for potential embeddedness within the curriculum. As an extra-curricular activity they are provisions, but as an incorporated part of the curriculum their use would become a programme. Under the present mode of usage they are at best a pull out provision for which it is hard to deny exclusivity when some students are denied the opportunity to participate and other children have the privilege of attending. It is especially hard when these activities may appear more fun than regular classwork. As well is the let down when participation has ended (Callahan 1982).

A hypothesis emerging from the present study is that conscious processes are more difficult and unconscious learning is obtained more easily, perhaps because conscious processes largely use language expression, itself an abstraction, whereas unconscious processes make intuitive leaps and arrive at solutions using such things as visual imagery. The function of processing the puzzles is multi-faceted. It not only encourages logical processes, but with individual questioning during and after the procedure, aids the extremely difficult transfer of unconscious conceptual processes into conscious conceptual language. This crucial translation into language may be the key to conceptual development leading into the formation of formal mathematical concepts through facilitating the ability to handle multiple concepts, transfer learning and use strategy. For this reason it is essential children working with the puzzles are worked with individually or in small groups, where cognitions can be verbally 'caught' by teachers.

The whole of mathematics clearly involves both verbal and nonverbal skills. Therefore the school mathematics curriculum and with it P.A.T. mathematics should encompass this whole range, not only to identify all able children, but to broaden the dimensions of intelligence itself. It would have the effect of embedding an additional identification, remedial and enrichment system, and of increasing self esteem in children and the love of school work. The curriculum needs to cater for nonverbal and unconscious creative processes as well as verbal or conscious ones. A method to develop these would be to include within the mathematics curriculum more right brain experiences involving an extension of junior school methods into senior methods used on an individual teaching basis. If junior school methods are extended into senior levels, the need for tests, remedial and enrichment programmes would not be so necessary.

Limitations of the Present Study

The most glaring criticism that could be levelled at process and performance levels exhibited by children in the present study might be that children worked in small groups with the potential to absorb processing information from other children, perhaps influencing their own solutions and aiding them to achieve higher levels of performance than they might have done without this influence. Surprisingly, at consciously aware levels this phenomenon appeared to happen rarely. That this was a rare occurrence was determined by the very individual ways children approached the puzzles. Usually absorption in their own work meant they were not paying attention to what someone else was doing. One advantage of working with small groups was that children could be kept focused on their problem solving. However, there must be some subliminal perceptual effects. This may not be a good idea from a scientific point of view, but may enhance and move general conceptual thinking on to higher levels at faster rates. In general classroom use it would be unrealistic to work with children entirely on their own, except in exceptional circumstances. A further consideration is that children are individually questioned to elicit as much of their conceptual processing as they can manage. The purpose of the puzzles is not so much to solve them to their ultimate level possible, but to extend each child's thinking to the child's ultimate level possible in that moment. In this sense it is process which is valued much more than product. In order to evaluate puzzle performance in the present study more reliably, some children perhaps should have been worked with entirely alone to eliminate any of the above effects. This would still not prevent children from discussing the puzzles amongst themselves out of session. At all times children were encouraged not to discuss solutions with other children so that other children could come to each puzzle with their own approach. The children appeared to respect this request.

Further limitations from the point of view of a scientific evaluation of the puzzles was that children received varying numbers of lessons with the puzzles, and of differing time lengths. Furthermore, no child was given the same 'recipe' of puzzles, nor were the puzzles necessarily given to any child in any particular order. This is individualised and flexible teaching from an educational perspective, but from a scientific evaluative perspective of exact effects of the puzzles, lacked these controls.

Another limitation of the present study was the absence of groups of P.A.T. average children to help determine correlation levels between P.A.T. mathematics and the puzzles. An assumption was made that what was effective for children with high or low mathematical achievement, or special learning difficulties would be effective for children falling within these extremes.

A further consideration limiting the present study was that mathematical group levels were known by the researcher and children were worked in these ability groups. A blind study would eliminate any biases and expectations from the researcher, preventing subliminal or subtle influences from researcher expectations. It would quickly become apparent which children were more able with the puzzles, but these children might not necessarily exhibit high formal mathematical achievement in normal classroom work, as was the case with the two P.A.T. average children in the present study.

Another limitation of the present study was that senior children were selected for their mathematical groups from P.A.T. mathematics scores alone, without an additional input of teacher opinion regarding the mathematical ability of the children. It was not intended P.A.T. results alone should be used to place children into mathematical groups, but P.A.T. results provided a quantitative assessment that was much easier to determine than observing qualitative means of teacher assessment.

A reverse problem was that junior children were teacher assessed. There is no pre-mathematical concept test universally used in New Zealand schools, such as the comprehensive one by Churchill (1961) - (Appendix 2). Schools devise their own methods for junior assessment. This means junior children and senior children in the present study were assessed for participation on a different basis from each other, not only within schools, but between them.

A final limitation of the present study was that children were pulled out of their normal classroom programmme for the present study. Firstly this disrupted normal classroom work with the remainder of the class carrying on with normal classroom lessons, and secondly the work with the puzzles did not accurately simulate normal classroom conditions where children might not be given so much individual attention. This leaves open the question of whether this kind of puzzle work could practically be carried out within a normal classroom setting. However, small group work is a normal part of junior classroom teaching practice, and it is likely this method could be extended into senior classroom work.

Implications for Future Research

The best way of examining how the puzzles work may be to look at the results in children, and to focus on process rather than content, not only on what children can do with the puzzles, but on what else the puzzle experience may help them to do better. Even if each child was controlled for the number of puzzles attempted and the order of attempting them, and was worked in isolation from other children, such an attempt would need careful selection of participants to avoid the difficulties of placing a child into a situation that was inappropriate through being too difficult or too easy. Furthermore, large samples of participants would be needed to reduce chance results from specific individuals in the study. Each child is different, has different abilities, and may arrive at the same solution using a different approach. It is the approach that matters, the process of problem solving and its wider ramifications in the transfer of learning and application of strategy. For this reason future studies that focus on how children approach the puzzles would contribute more towards the research on how the mathematical puzzles work than studies attempting scientific controls .

A Study of 'Average' Children

A study of average P.A.T. mathematics children would find which chidren are more able with the puzzles than they appear to be in normal classroom work, separating children whose abilities show greater right brain than left brain strengths.

A Blind or Double Blind Study

The researcher is given random groups of children, having no prior knowledge of the formal mathematical ability status of the children. Perhaps one way of randomising groups would be to use a normal classroom of children and take them in groups of six alphabetically. This would allow for wild card variables, where some classroom ability groups might be more varied than others by chance alone. A tentative study has already been attempted in this manner, as is described below, with voluntary children. A double blind study where the researcher did not know the hypothesis or the mathematical ability of the children would show even less researcher bias.

A Study of Voluntary Children

An informal voluntary study has already shown P.A.T. mathematics performance did not always match puzzle performance. In 1989 a class of Form 2 children at Bohally Intermediate School in Blenheim set up a 'Science Scene' for one week for all of the visiting primary school children in Marlborough. Groups of children set up various science activities in the school hall such as a T.V. studio, for themselves and visiting children to explore. Thirty mostly senior puzzles from the present study collectively entitled 'Pattern and Rotation' provided one activity. Eleven hundred children visited. It was noted by the class teacher that children showing special interest in and ability with the puzzles were not always children with ability in normal classroom work. As the first child to successfully solve a particular puzzle contributed to a video produced by the children themselves, discussing the processes and solutions they found, it was also noted by the teacher that some of these children had previously experienced few opportunities to feel successful in normal classroom work, and develop confidence in their own ability.

A Study of Individual Children

Although unrealistic and not a normal use of classroom time and resource, a study with individual children would eliminate any possibility of a child absorbing process or solution information from other children in a group, and would also eliminate distractions from other children. Puzzle performance results would be more accurate using this method.

Reading Recovery

A preliminary attempt has been made to see if seven year old Standard one children undergoing a reading recovery programme might benefit from working with the pre-mathematical puzzles. The results were inconclusive because the study was not carried out for a long enough period to determine any reading recovery results. There is potential for controlled studies in this area, of children receiving puzzle work in addition to a reading recovery programme, and children receiving reading recovery only. If the puzzle group improves their reading more than the non puzzle group, further studies could be trialled in this area.

An Adolescent Study

An adolescent study would provide a more balanced view of gender differences in mathematical ability traditionally emerging at this time. Performance of the largely nonverbal puzzles utilising some visual-spatial aspects measured against formal largely verbal mathematics might provide some interesting data towards the validity or non validity of males performing visual-spatial tasks more successfully and females performing verbal tasks more successfully.

A Study of Deaf Children

Some preliminary work with several deaf children across the primary and secondary school age range has been undertaken in 1993-1994 with the aid of an interpreter using New Zealand sign language. As these children mostly had no English or spoken language, they showed great visual acuity and were quick to see relationships between elements in the puzzles. There is much potential for enrichment for these children using the puzzles, as so much is lost for them in a normal school classroom using spoken language.

A Study of Blind Children

It would not be difficult to adapt some of the puzzles for blind people, particularly with the use of such materials as button textures, types of material used for the buttons, sizes and weights of different buttons, and textured fabrics, graded sandpaper, stones and other tactile materials, or auditory puzzles involving sound matching, volume and pitch grading, rhythm matching and instrumental effects.

A Study of Dyslexic Children

The particular dyslexic children using the puzzles in the present study were quite able with the puzzles. As the number of children involved was so small, larger numbers of dyslexic children covering different types of dyslexia would be needed to determine how dyslexic children in general operated with the puzzles, and to determine any helpful effects that might emerge from their use.

A Study of Children with Attention Deficit Disorder

Preliminary puzzle work with children suffering from attention deficit disorder indicated more success when activities were action, movement and noise related. Use of the body and the voice in conjunction with one to one correspondence activites were effective for junior children.

A Study of Adults Following Brain Injury

The puzzles could be used along with neuropsychological testing procedures to determine the diagnostic and remedial aspects of the puzzles for different types of brain injury. Two people with a verified previous brain injury have worked with the puzzles. One person had a recovered left brain injury, and the other a right brain injury. Neither injury left functional effects that were too severe in the long term, and school work functioned at a reasonable level. Both participants were able to process many of the moderate difficulty level puzzles, given questioning help. No specific cognitive benefits over and above those acquired to solve the puzzles themselves were observed following this work.

A Study of the Elderly

Some work with the confused elderly could be attempted, using simple junior puzzles to determine any beneficial cognitive or occupational effects. A preliminary attempt has been made by someone using calendar pictures cut into jigsaw shapes on the basis of the junior puzzle concepts in the present study. The confused elderly were not only able to do these puzzles, but greatly enjoyed the occupational effect. The benefit to the person working with these elderly was that each day the participants could do the puzzles all over again with the same joy of discovery, having totally forgotten they had ever seen the puzzles before.

A Pre-School Study

Much preliminary work with the simplest junior puzzles has already been done with pre-school children. The more manipulative materials such as the sponges, magnetic shapes, clothes line and pegs, washer wheels and simple one to one correspondence and matching and seriated materials have been used (Appendix 1). A further study to find how far able children can be extended is of interest. Also of interest, is what very young children around two years of age can do.

Conclusion

Many children were able to process concepts conventionally considered above their age and understanding range, and some children showed evidence of unconscious processing of concepts. Puzzles had diagnostic value in identifying individual weaknesses and strengths. Difficulties occurred with unfamiliar concepts and some types of perceptions, with single concepts and multiple concepts, and in the transfer of learning and the use of strategy. Strengths were indicated by speed, the use of symmetry, the ability to process multiple concepts, in transfer of learning, in the use of strategy, and in the mastery of difficulty levels above age groups. Some senior remedial mathematics children overcame difficulties in their formal mathematics, even though the mathematical puzzles were not related to the kind of formal mathematics the children were having difficulty with. Some junior remedial mathematics children began developing mathematical concepts in their school mathematics at a faster rate than previously, and were moved into average mathematical groups for normal classroom work. P.A.T. mathematical groups, school classes and ages of children did not generally correlate with mathematical puzzle performance. The large number of tests gives a greater likelihood of some chance significant correlations occurring. Two children who were average in P.A.T. mathematics and other school subjects performed at a higher level than the top P.A.T. mathematics children. There were no gender differences in levels of performance. Levels of performance amongst remedial children and children with special learning difficulties were variable, and related more specifically to individual problems within the children themselves than to any evenness or unevenness of mathematical groupings across schools. Some children who previously disliked mathematics began to enjoy it. Some children who previously disliked school began to enjoy school. Many children at all levels gained general confidence in themselves and became more willing to attempt something new, and less afraid of difficulties or of making mistakes. Several children who had previously been generally uncooperative at school became more cooperative and related better to their teachers. Formal mathematics improved further for some top P.A.T. mathematics children. Top P.A.T. mathematics children from the three schools performed at evenly high levels with the puzzles. Children needing extension enjoyed the lateral problem solving enrichment challenges the puzzles provided.

In conclusion, children do show evidence of unconscious and conscious processing. Unconscious processing is more likely to relate to the movement of sensation into perception and perception into conceptual understanding, as is evident when unfamiliar tasks are presented. Unfamiliar concepts are more likely to be processed in the right hemisphere, transferring to the left hemisphere with the establishment of familiarity and conscious awareness through action and or language. Whereas pre-mathematical concept formation is a largely right brain function, utilising visual and spatial items, formal and symbolic mathematical function is a largely verbal left brain function using the symbols of mathematics. As right brain input both precedes and greatly aids left brain function, particularly when combined with verbal introspection aiding the acquisition of the language of mathematics, it makes good sense to extend right brain activity into senior classroom work to consolidate and speed the development of higher level concepts in children. This would effectively embed improved strategies for remediation and enrichment within the curriculum, reducing the need for further assessment tests and pull out programmes, giving children a multi-sensory experience base to draw from where concepts can be understood from the level of their basic structure rather than memorised in rote form. In this way they become operational and transferable.

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Appendices

Appendix 1 Junior Mathematical Puzzles

Appendix 2 A Number Readiness Test (Churchill 1961)

Appendix 3 Three Manufactured Adult Puzzles

Appendix 4 Puzzles in Approximate Order of Difficulty

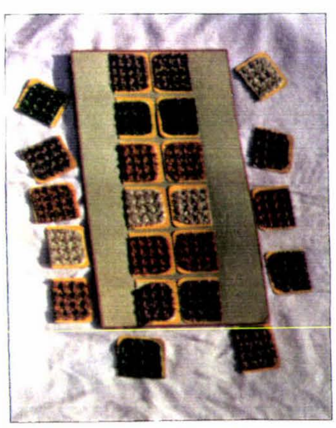
Appendix 1

Junior Mathematics Puzzles

Plate 44: Junior Mathematical Puzzle Display: Buttons.

Plate 45 : Junior Mathematical Puzzle Display: Shells; Miscellaneous.

Plate 46: Junior Mathematical Puzzle Display: Washers; Sponges.



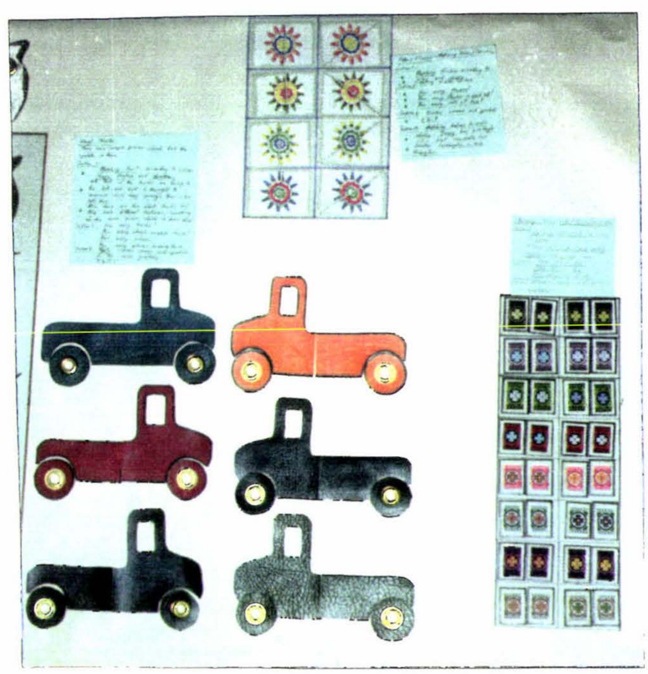


Plate 47: Junior Mathematical Puzzle Display: Fabric Sets.

Plate 48: Junior Mathematical Puzzle Display: Fabric Sets.

Plate 49: Junior Mathematical Puzzle Display: Fabric Sets.

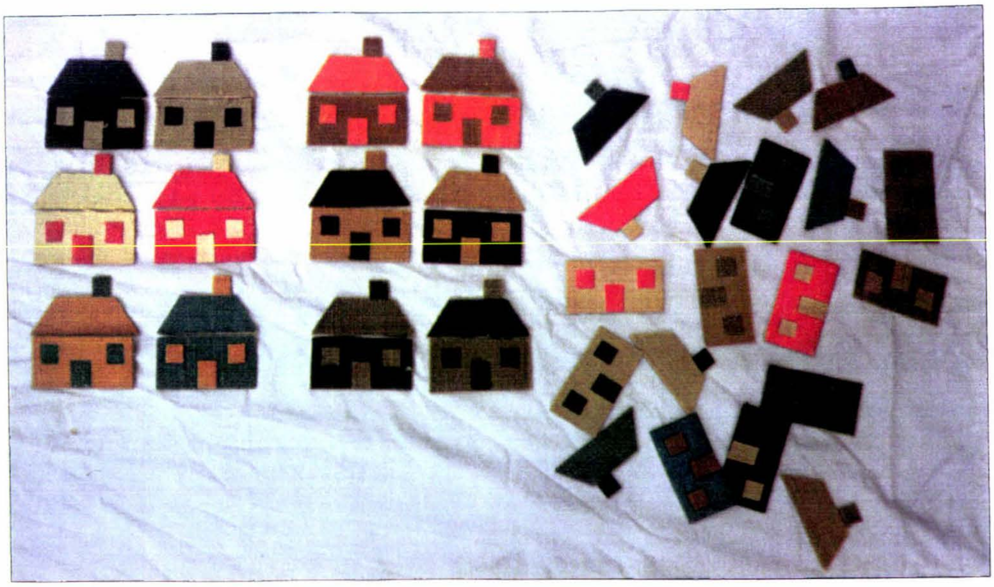


Plate 50: Junior Mathematical Puzzle Display: Fabric Sets.

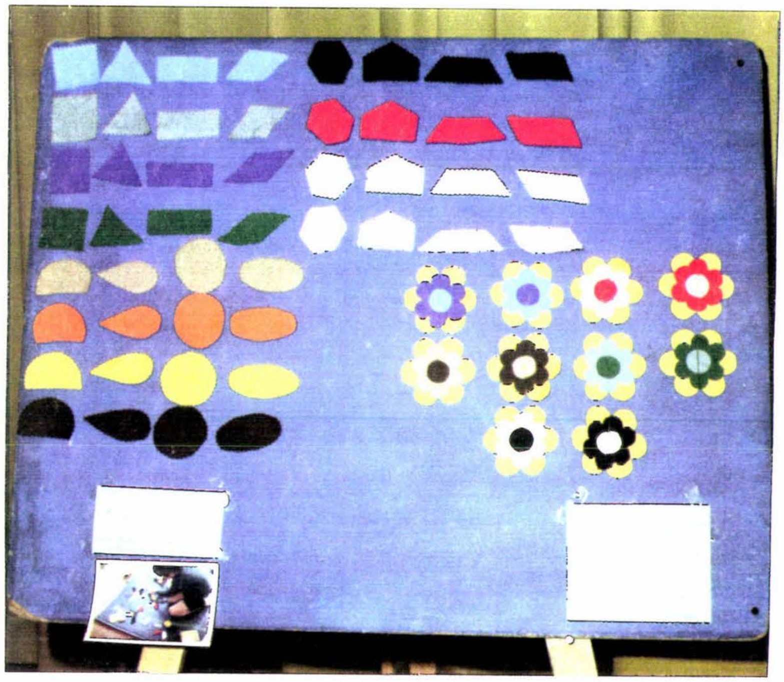


Plate 51: Junior Mathematical Puzzle Display: Fabric and Flannelboard Sets.

Plate 52: Junior Mathematical Puzzle Display: Flannelboard Sets.

Plate 53: Junior Mathematical Puzzle Display: Flannelboard Sets.

Plate 54: Junior Mathematical Puzzle Display: Vinyl; Miscellaneous.

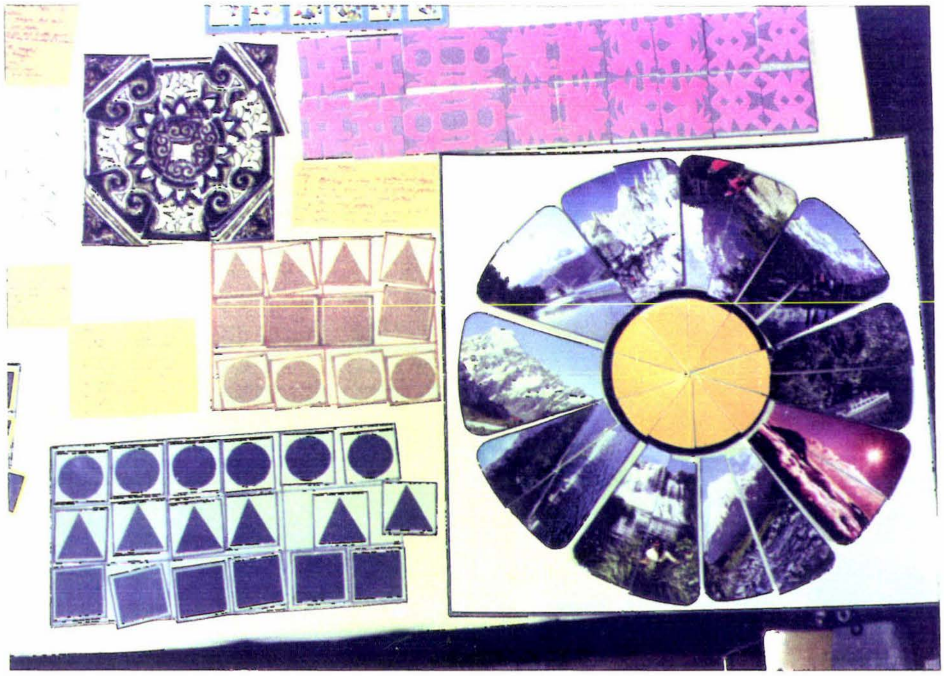


Plate 55: Junior Mathematical Puzzle Display: Miscellaneous; Pictures.

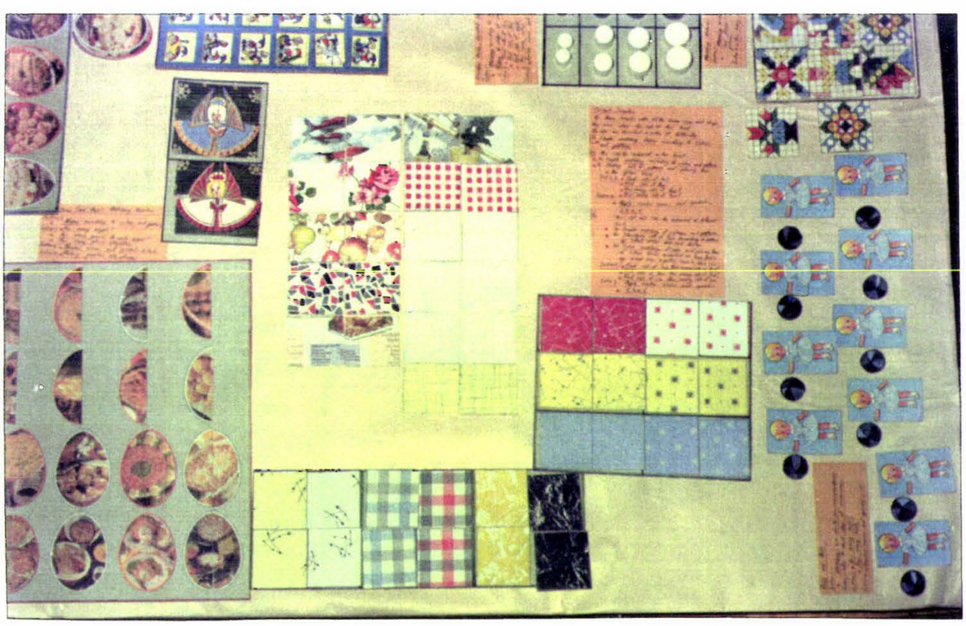


Plate 56: Junior Mathematical Puzzle Display: Miscellaneous; Pictures.

Plate 57: Junior Mathematical Puzzle Display: Miscellaneous; Pictures.

Plate 58: Junior Mathematical Puzzle Display: Miscellaneous; Pictures.

Plate 59 : Junior Mathematical Puzzle Display: Pictures.

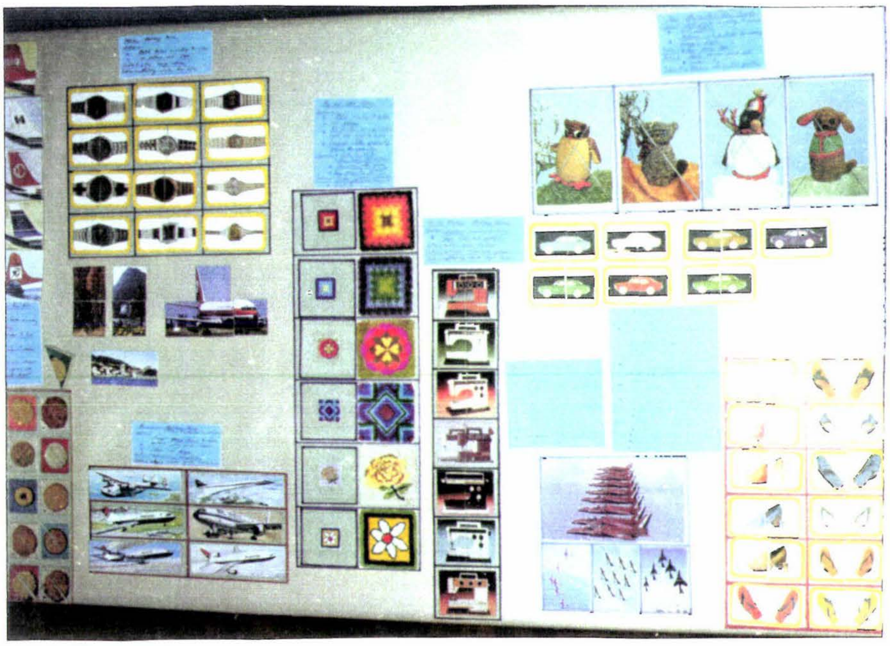


Plate 60: Junior Mathematical Puzzle Display: Pictures.

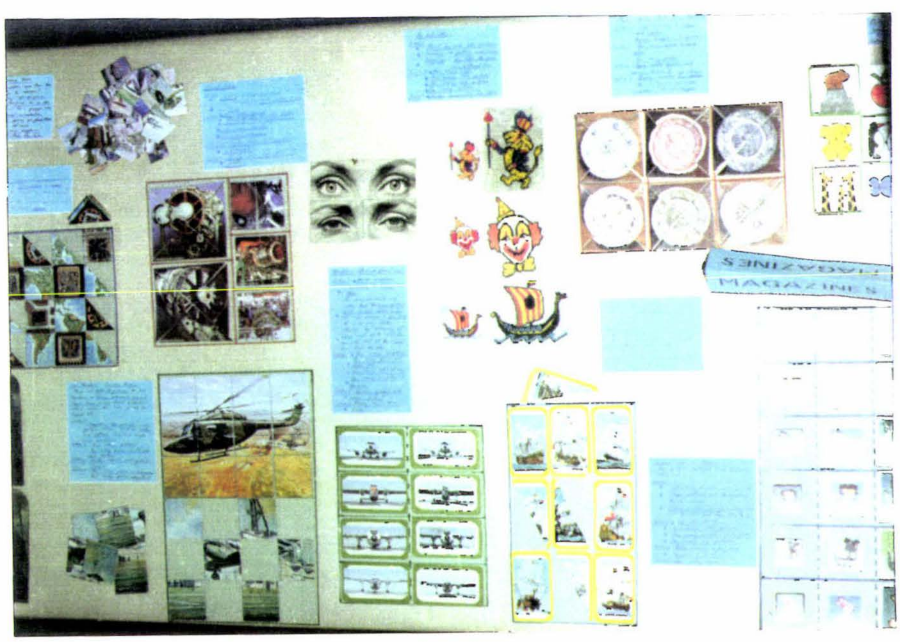


Plate 61: Junior Mathematical Puzzle Display: Pictures.

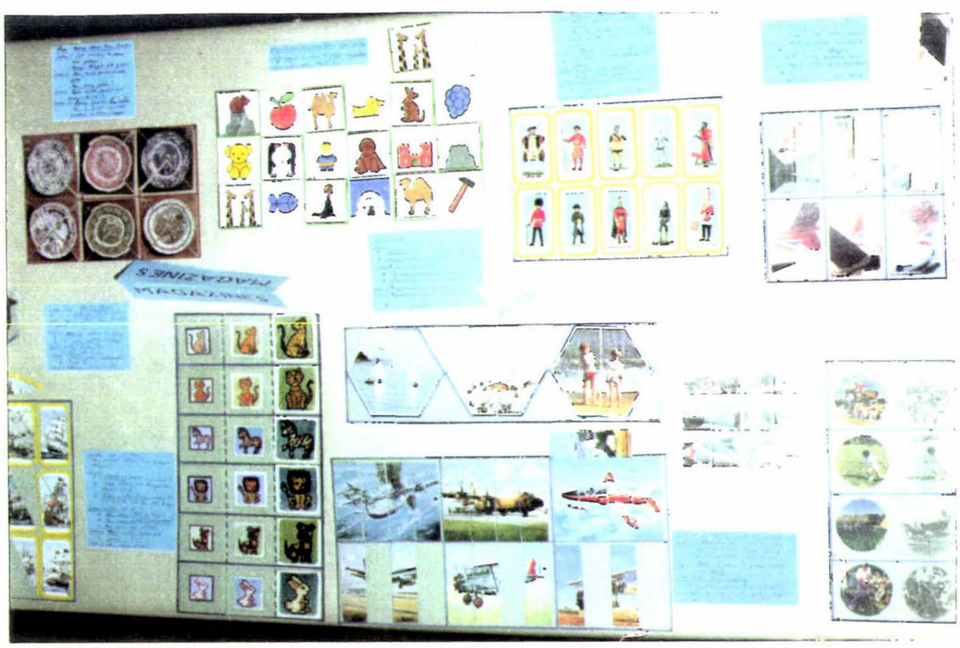
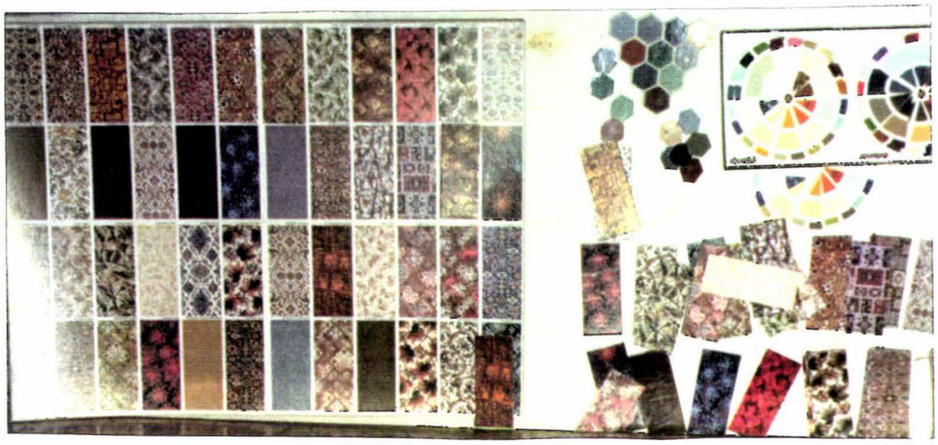
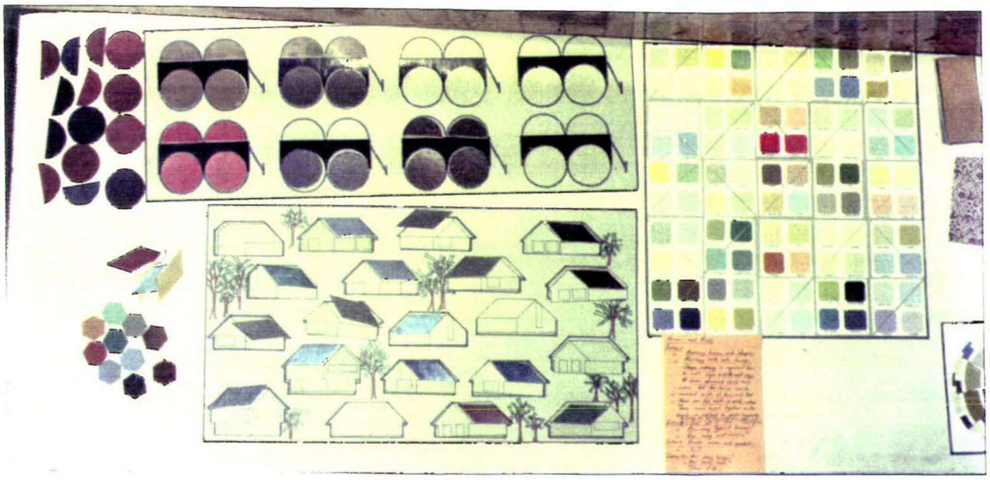


Plate 62: Junior Mathematical Puzzle Display: Pictures.





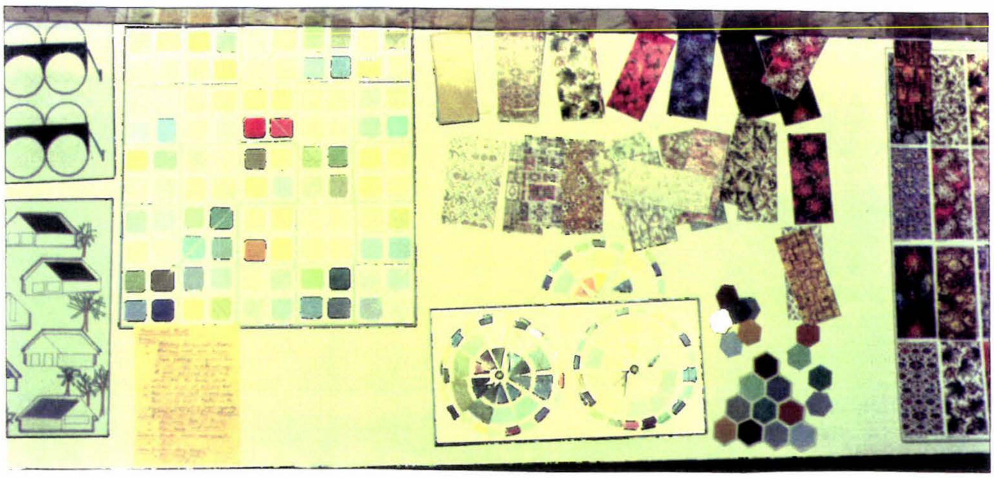


Plate 63: Junior Mathematical Puzzle Display: Pictures; Paint Charts.

Plate 64: Junior Mathematical Puzzle Display: Paint Charts; Miscellaneous.

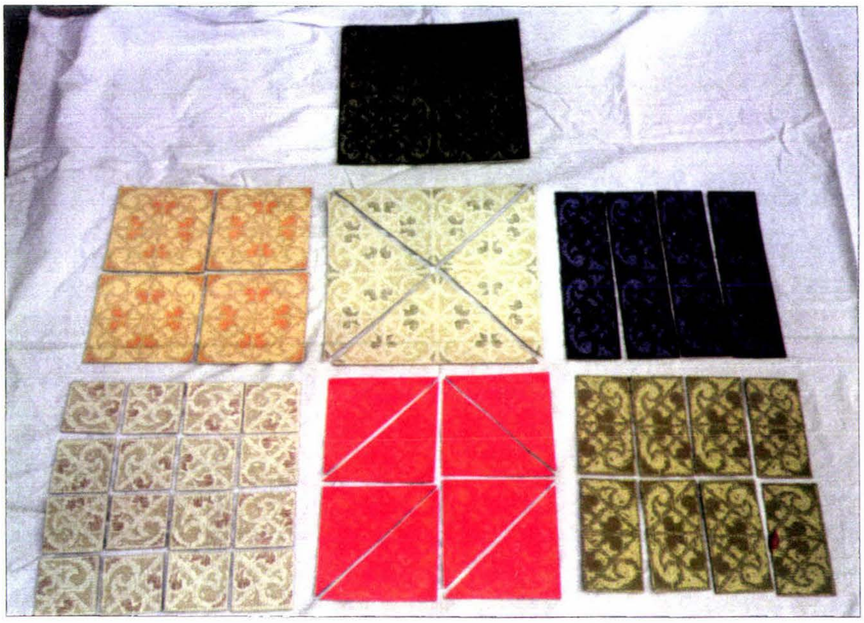


Plate 65: Junior Mathematical Puzzle Display: Wallpaper Sets.

Plate 66 : Junior Mathematical Puzzle Display: Wallpaper Sets.

Plate 67: Junior Mathematical Puzzle Display: Wallpaper Sets.

Plate 68 : Junior Mathematical Puzzle Display: Wallpaper Sets.

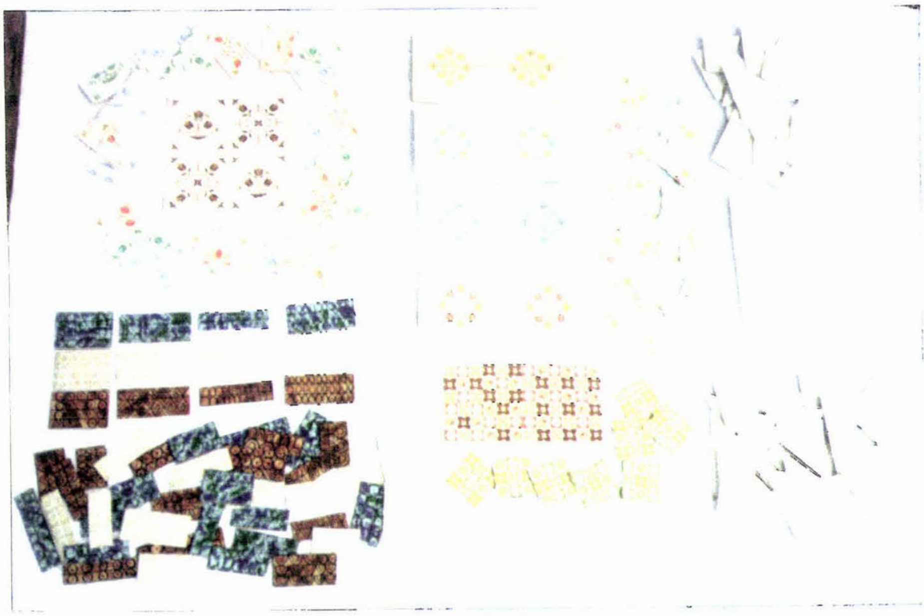


Plate 69: Junior Mathematical Puzzle Display: Wallpaper Sets.

Plate 70: Junior Mathematical Puzzle Display: Wallpaper; Miscellaneous.

Plate 71: Junior Mathematical Puzzle Display: Gift Wrapping Paper Sets.

Plate 72: Junior Mathematical Puzzle Display: Gift Wrapping Paper Sets.

Plate 73: Junior Mathematical Puzzle Display: Gift Wrapping Paper Sets.

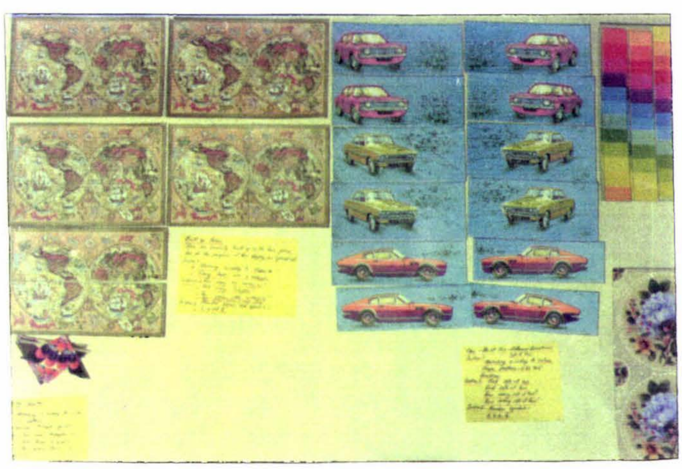


Plate 74: Junior Mathematical Puzzle Display: Gift Wrapping Paper Sets.

Plate 75: Junior Mathematical Puzzle Display: Gift Wrapping Paper Sets.

Plate 76 : Junior Mathematical Puzzle Display: Gift Wrapping Paper Sets.

Plate 77: Junior Mathematical Puzzle Display: Miscellaneous.

Appendix 2

A Number Readiness Test
(Churchill 1961).

FROM COUNTING TO CALCULATION

A NUMBER READINESS TEST

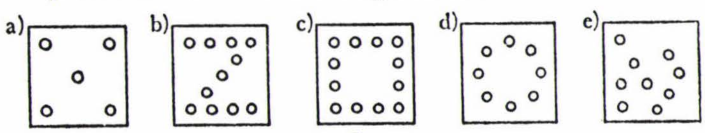
1. Put out two unequal-sized groups (e.g. 10 and 13) of sweets in front of the child and ask him which pile he would rather have. Then ask 'Why did you choose this pile?' Repeat with smaller groups (e.g. 6 and 4) if the child fails on first test.

2. Present child with two glass beakers of equal size and shape, one containing 20 small beads or balls. Ask him to put the same number of beads in the empty beaker and if his method is not clear ask him how he knew how many to put into his beaker.

3. Have ready a jug containing lemonade or some coloured liquid, and four glass beakers: two of the same size and shape, one taller and narrower, one shorter and wider. Place the two identical beakers in front of the child and after pouring some of the lemonade into one, ask him to pour the same amount in the other beaker. When he has finished ask 'Have the two beakers got the same amount of lemonade in them?' and then 'How do you know?' Then take the taller beaker and ask the child to empty the lemonade from his beaker into the new one. Place this when filled alongside the other beaker which still holds lemonade and ask, 'Which lot of lemonade would you like to drink?' By questioning discover the reasoning used to arrive at his answer.

4. Place a large pile of sweets (about 30) in front of the child and ask him to share them out with you so that both of you have the same number of sweets. Watch how he solves the problem (by rough handfuls, by building up in ones to form two equal groups, etc.). Then take the pile he has given you and spread them out on the table so that they occupy a much larger surface than the child's pile. Now ask him 'Which lot of sweets would you rather have?' and by questioning try to discover his mode of reasoning.

5. Have ready a box of counters and a series of cards with counters painted on them in the following manner:



Present each card in turn and say 'Put out as many counters as there are on the card.' Notice the way he achieves the correspondence (pattern-copying, counting, etc.).

6. Put out counters on the table in two dissimilar groups (e.g. 3 and 4.) Ask 'Where are there more counters?' Follow this, according to response to first question with either 'How do you know?' or 'How many more are there here?' pointing to the group the child has chosen. Repeat using groups of 8 and 7, 11 and 13.

7. (*a*) Give the child 7 pennies and ask 'How many pennies have you got?' Give him 5 more pennies and ask 'How many have you got now?' Notice the way he arrives at his answer (counting the whole group, counting on from 7, etc.).
(*b*) Give the child 8 small bricks and ask him to arrange them in a straight line. Then ask 'How many bricks are there?' Now add 9 more to the row yourself and then ask 'How many bricks are there altogether now?' (Leave a small space between your row and his so that counting-on is a possible method of obtaining the answer.)
(*c*) Ask the child to put 3 sweets into a paper bag and then to add 4 more. Ask him 'How many sweets are there in the bag now?' Repeat with 6 and 5 sweets.

8. Have ready the cards used in test No. 5 and a set of cards with the numerals printed on them, 1 to 12 are needed. Ask the child to find the numbers which go with each card.

9. Ask the child the following questions:

There are	4 cars,	how many	wheels?
,, ,,	6 children,	,,	,,	legs?
,, ,,	6 ,,	,,	,,	toes?
,, ,,	3 babies,	,,	,,	fingers?
,, ,,	5 chairs,	,,	,,	legs?

10. (*a*) Give the child a bag of 12 sweets and say 'I want you to share these out between 3 children so that each has the same number. Notice the method used (three rough groups, building up three equal piles in ones, etc.).
(*b*) Repeat using 30 sweets and 4 children. Notice method used and what child does with the remainder.

(*c*) Present child with a piece of ribbon or strip of paper 16 inches long and ask him to cut it into four equal pieces. Have a tape-measure on the table so that the child can use it if he wants to.

11. Present the child with a series of ten Tillich bricks or set of ten cardboard strips similarly graded in size and marked off in unit lengths, viz.:

Ask the child to arrange them in order and when he has finished point to the first in the series and ask 'Why did you put this one here?' Repeat question pointing to the last in the series, and then to one in the middle.

12. Arrange the series to form a 'stair' series if the child has not already done this, and ask the following questions:
(*a*) Pointing to the sixth stair 'Which stair am I on now?' Repeat with finger on first and eighth stairs.
(*b*) Put finger on bottom stair and ask 'How many stairs will my finger have to touch before it reaches the top of the stairs?'
(*c*) Put finger on bottom stair and ask 'How many stairs will my finger have to touch to reach the eighth stair?'

13. Repeat experiment in tests 11 and 12 with a series of unmarked strips of cardboard. Notice manner of response (whether attention is directed to size differences or each strip given a numerical value).

14. Have ready a series of picture friezes: a row of houses, a line of cars one behind the other, a line of cloakroom pegs; also the set of numerals used in test No. 8.
(*a*) Present frieze of houses and say 'I want you to give each house a number.' Then ask, pointing to the first house, 'Why have you given this house this number?' Repeat question pointing to fifth and eighth houses.
(*b*) Present frieze of cars and give child card with number 5 on it and say 'Which car should have this number?' Ask 'Which number will be needed for the last car?' and let him find and place it on the car. Repeat, using sometimes the first and sometimes the second type of question till all the cars are numbered.

(c) Present the frieze of clothes pegs and tell the child 'Your peg is No. 7. Find the number and put it on your peg.' Repeat using the names of other children he knows till several of the pegs are numbered. Then say 'Now tell me the names of the pegs which are not being used by any children.'

Comments on the Test Items

It is intended that these tests shall be administered to individual children in an informal setting, perhaps while the rest of the class are engaged in their freely chosen activities. Three or four might be administered on one occasion, and the others subsequently.

The purpose of giving the tests is to find out how the child approaches situations which for their successful solution require an understanding of the basic number concepts and skills. Piaget divides the responses he obtained in similar tests into three categories, according to the level of response, viz. (*a*) pre-operational, (*b*) intuitive, (*c*) operational, and this method of assessment may be found helpful. Any response which suggests that the child is not seeing the situation as a quantitative one, involving counting, will go into the pre-operational category, and any response in which the child uses counting or measuring operations appropriately, and for the successful solution of the problem, goes into the operational category. It will be found that these two levels of response are easy to distinguish but many children show an unevenness of response, some things understood and others not, which suggests an intermediate stage in development when the concepts are being acquired but are not yet stabilized. E.g. at one moment the child will establish the size of a group by counting and at another he will make global judgments of quantity based on spatial consideration. A series of tests of this kind is particularly helpful in ascertaining those children who, though on some occasions look as though they understand the quantitative aspects of a situation, when observed systematically reveal how uncertain they are as compared with children who have reached the operational level and are ready to proceed to the more abstract problem.

These tests have already been used by a small group of Infant Teachers in one area who have found them helpful in distinguishing the level at which children are thinking. However, we have found it necessary to question the children about why they have done things in certain ways in order to understand how they are looking at the situation. Sometimes the child's own spontaneous comment will give

the information needed as in the case of the little boy numbering houses who suddenly turned them into jockeys going for a race. More often this is not immediately clear as we found when asking children why they had chosen one pile of sweets rather than the other in Test No. 1. One child who had chosen the smaller pile explained that it would be greedy to choose the larger one.

It has been thought helpful to include some guidance as to what to look for in each of the tests, but these remarks should be regarded as suggestive only. It would not be possible to cover all the eventualities which can occur during testing and what follows is intended merely as a guide to the kind of concepts being examined in each test item.

The Group Concept.

No. 1. Children who have learned to think of groups of discrete items as comprising sets of different sizes whose numerical value can be established by counting are likely to use this method in solving these problems, particularly with the larger sized groups. Children who have not reached this stage are likely to base their choices on the size of space occupied by the various groups of sweets, responding to the question by some such reply as 'Because it's bigger' or 'Because it's more'.

No. 2 will give information of the same kind. The child who has attained the group concept will recognize that he must first count the number of beads in the filled beaker before he can know how many to put in the empty one. The child who has not reached this stage is likely to try and fill the empty beaker with beads until they reach the same level as those in the filled beaker, i.e. so that they look the same. Some children when asked how they knew how many to put in will now recognize the quantitative aspect of the situation and proceed to count the beads in the two beakers. This would be an example of the intermediate level response.

The Concept of Conservation of Quantity.

Nos. 3 and 4 will show those children who understand that a quantity (whether continuous as in No. 3, or discontinuous as in No. 4) always retains its quantitative value no matter what other changes may occur. The child who does not understand this is likely to choose the beaker which looks to him as though it contained the larger amount of lemonade and similarly the group of sweets which occupies the larger space. Usually these children will divide the group of sweets into two rough piles, perhaps making one or two adjustments till he is satisfied that they look the same. On the other hand the child who has attained the concept of conservation of quantity will recognize that the original quantity of lemonade or sweets is not altered because it now occupies a larger space. These children are likely to divide the sweets in test No. 4, by building up in ones to form two equal groups.

No. 5 will give further information about the attainment of the group concept. Some children feel no necessity to count the number of counters on any of the cards but concentrate on reproducing the pattern, often taking great care to reproduce the details of pattern and colour with great exactness. The more advanced children may also deal with this test in the same way, and it cannot therefore be regarded as useful diagnostically unless it is used in conjunction with other tests in the series. Some children will count to establish the size of the sets before putting out any counters, particularly in the later models in the series.

Enumeration

No. 6 is helpful in distinguishing those children who understand how to count. When the larger groups are involved correct answers can only be given after a counting operation and the difference between children who understand this and those who do not is very obvious.

Counting-on

No. 7. These situations can be dealt with either by counting the second group from one, or by counting on from the position in the number series already reached. The last item in this series is likely to be most informative since the group is hidden in the bag, unless the child takes them out to count before answering the question.

Knowledge of the Numerals

No. 8 is included to show whether the child has learned to associate the correct value with each numeral.

Ability to Count in Groups

No. 9. These items will readily be solved by children who have learned to count in groups. As has been noted already this is a useful skill and a helpful basis for later work in multiplication.

Ability to Share

No. 10. The first two items in this series can only be solved if the child

has attained the group concept and understands how to share a group of discrete items. Children who have not reached this stage are likely to be satisfied to divide the sweets into 3 and 4 rough groups respectively. As has been noted already in discussing division the last item is likely to be dealt with by folding the ribbon in half and then half again. Those experienced in measuring may use the tape-measure provided.

The Concept of Ordinal Number

So far in the test series all the counting has been used to establish the size of groups, i.e. cardinal number. The last three in the series test the child's appreciation of quantitative order.

No. 11. presents a situation in which the size relationship between the bricks can be used as a principle for ordering the series. Many children interpret the instruction 'arrange them in order' as a request to put the bricks tidy and arrange them in a long line or a tidy group, without respect to size difference. The more advanced children are likely to notice the size difference and use this as a basis for ordering, though some find difficulty with the larger bricks where the difference is not so easy to distinguish unless the child builds a stair series or counts the unit lengths on the bricks.

No. 12 will show whether the child understands the ordinal or positional function of number.

No. 13 provides further evidence about this. Children who have played a good deal with Tillich bricks and used them for ordering operations will often refer to them by their number names as represented by the number of units marked on them. In this test it has been found that some children will transfer this method of thinking to unmarked strips.

No. 14 presents situations of the kind which frequently occur in everyday life where numbers are used to indicate position in a series and where the cardinal aspect of number is not present as it is in the Tillich bricks. It will be found that these tests clearly differentiate between the more advanced and less advanced children.

Appendix 3

Three Manufactured Adult Puzzles

Plate 78 shows commercial versions of Figures 80, 75 and 79 respectively. (Moxham, 1993).

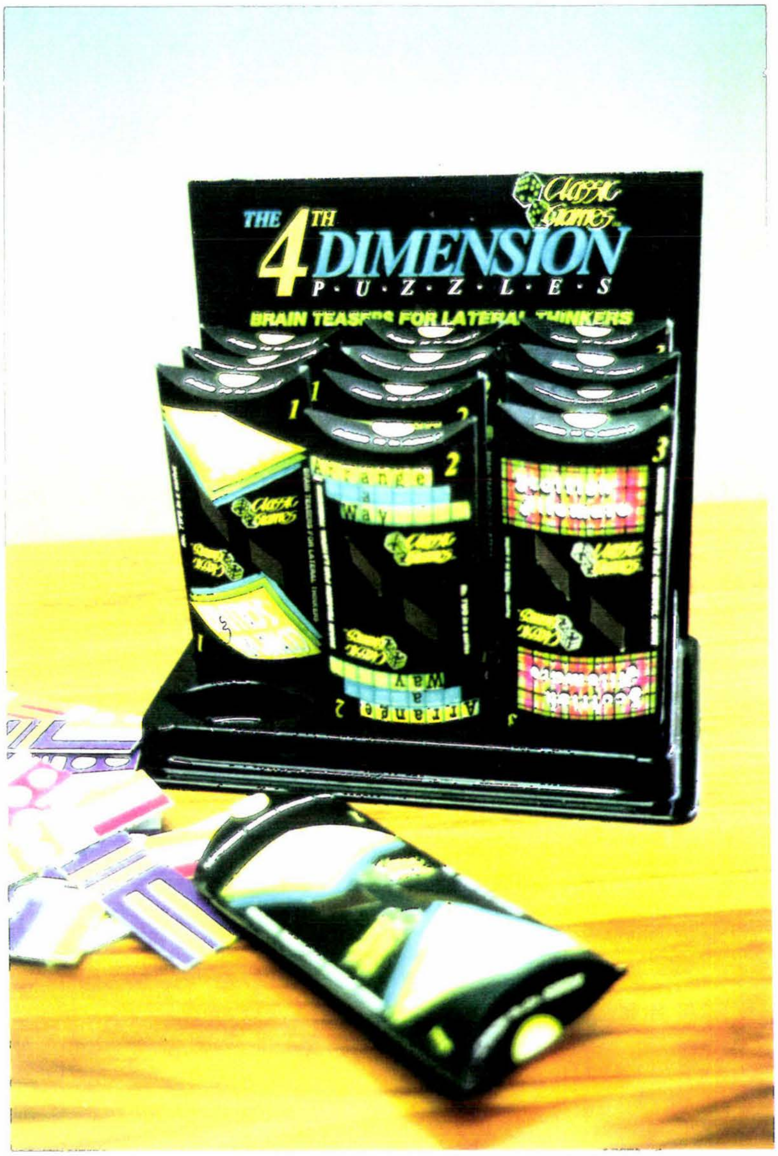


Plate 78 Figures 80, 75 and 79. (commercial versions)

1. Compare square (Figure 80)
2. Arrange a way (Figure 75)
3. Scottish nitemare (Figure 79)

Appendix 4

Table 52: Puzzles in Approximate Order of Difficulty.

Difficulty level columns graded left to right.
Difficulty levels within columns graded top to bottom.

Very Advanced Junior	Beginning Senior	Average Senior	Above Average Senior	Advanced Senior	Very Advanced Senior	Adult
Figure No.						
8	57	61	87	78	43	81
11	58	60	100	72	110	75
7	59	23	33	106	74	82
84	1	6	34	27	65	80
18	14	2	105	31	66	101
52	64	107	5	102	67	113
53	39	96	37	103	68	112
85	69	15	41	104	73	79
28	70	19	95	108b		83
44	63	40	42	48		
12	17	16	35	47		
38	62	29	94	109		
88	4	93	13			
54	71	49	89			
91	98	32	56			
25	10	26	55			
3	51	36	108a			
97	22	111	77			
50	9	46	76			
20	99	24				
90	21	30				
86	92					
	45					