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Modelling Avian Influenza In Bird-Human Systems

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Abstract

In 1997, the first human case of avian influenza infection was reported in Hong Kong. Since then, avian influenza has become more and more hazardous for both animal and human health. Scientists believed that it would not take long until the virus mutates to become contagious from human to human.

In this thesis, we construct avian influenza with possible mutation situations in bird-human systems. Also, possible control measures for humans are introduced in the systems. We compare the analytical and numerical results and try to find the most efficient control measures to prevent the disease.

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