

# An analytical framework for decision criteria validation in complex supply chains

Frank Michael Theunissen<sup>\*</sup> , Shafiq Alam, Aymen Sajjad

School of Management, Massey Business School, Massey University, Auckland, New Zealand

## ARTICLE INFO

### Keywords:

Topological Analysis  
Analytical Modeling  
Criteria Validation  
Supply Chain Strategy  
Geometric Distance Metric  
Decision Criteria

## ABSTRACT

Multi-Criteria Decision Making (MCDM) in supply chain management often applies rigorous methods for weighting and aggregation yet devotes little attention to the structural validity of the decision criteria that precede them. Even when organisations do not proceed to full MCDM model application, criteria are still elicited during problem structuring and used to justify initiative selection. This paper introduces a topological validation framework that addresses this asymmetry by representing criteria as a high-dimensional Decision Criteria Configuration (DCC). Using tools from Topological Data Analysis (TDA), we translate foundational MCDM axioms into measurable invariants: completeness through connectivity, non-redundancy through structural impact analysis, and logical consistency through cycle detection. Two industrial experiments demonstrate the framework's utility. In a supply chain strategy-setting workshop, TDA diagnosed the criteria set underpinning initiative selection as a "conceptual monolith," revealing significant redundancies and systemic feedback loops overlooked by conventional facilitation. In a subsequent inventory classification exercise, the audit resolved expert deadlock by reducing 32 proposed criteria to a minimal, non-redundant core of six operationally essential levers, providing an objective and defensible basis for moving forward. By transforming criteria sets into auditable decision architectures, this approach ensures that MCDM models and the initiatives they justify rest on a validated foundation before weighting or ranking alternatives. For managers, it functions as a pre-hoc "structural audit," reducing redundancy, exposing hidden interdependencies, and directing resources toward criteria that genuinely drive strategic and operational outcomes.

## 1. Introduction

Multi-Criteria Decision Making (MCDM) is widely used in supply chain management, engineering, and policymaking to evaluate alternatives against multiple objectives, offering a wide portfolio of methods for weighting criteria and ranking alternatives [1]. Yet a paradox persists: aggregation and weighting are rigorously formalized, but the criteria they rely on are often chosen informally, without structural validation [2,3]. In practice, initial criteria identification typically relies on expert judgment and pre-analytical facilitation [4]. This imbalance is especially evident in supply chain domains such as inventory classification and supplier selection.

The asymmetry constitutes a foundational weakness in MCDM. Computationally robust analyses frequently rest on criteria sets whose structural properties i.e., completeness, non-redundancy, and logical consistency lack formal validation. Some frameworks stress iterative refinement [5] or rely on implicit checks through axiomatic consistency

[6,7], yet the literature consistently notes the absence of explicit validation procedures independent of selection itself [2]. This gap is evident across recent supply chain studies, where methodologies often proceed directly from criteria identification to weighting, leaving inter-criterion relationships untested [8]. Causal mapping approaches can illustrate influence but are not designed to diagnose redundancy [9]; and many analytical techniques simply assume independence to fit their formal structures [10]. The recurring pattern is an immediate leap to calculation, where sophisticated models rest on the unverified assumption that criteria are structurally sound.

To address this gap, this research introduces a novel paradigm for criteria validation using Topological Data Analysis (TDA) [11]. The core contribution is the translation of foundational MCDM axioms into measurable topological invariants i.e., core properties of a configuration's "shape" that remain stable under transformation [12]. By modelling the criteria set as a high-dimensional *Decision Criteria Configuration (DCC)* derived from expert-led problem structuring, we

<sup>\*</sup> Corresponding author.

E-mail address: [frank.theunissen.1@uni.massey.ac.nz](mailto:frank.theunissen.1@uni.massey.ac.nz) (F.M. Theunissen).

enable a formal audit of its architecture. Completeness is tested through connectivity, identifying gaps; non-redundancy is evaluated through structural impact analysis, isolating essential criteria; and logical consistency is examined through cycle detection, distinguishing systemic feedback from circular redundancy. This approach extends beyond criteria selection to establish a validated and transparent decision-making architecture.

We test the framework in two contrasting industrial cases: a strategy-setting workshop, facilitated through cognitive mapping, where criteria selection informed the prioritisation of initiatives through urgency-impact assessments and whose outputs were retrospectively audited; and a subsequent inventory classification exercise where criteria selection followed a Delphi approach. Together, these cases demonstrate how pre-hoc validation can diagnose structural flaws, resolve expert deadlock, distil minimal non-redundant criteria sets, and assess alignment across decision models.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 presents the theoretical foundations of our topological approach, linking its invariants to MCDM axioms. Section 4 details the implementation methodology, and Section 5 describes its application to the two case studies. Results are presented in Section 6, followed by a discussion in Section 7, and a conclusion outlining limitations and future work in Section 8.

## 2. Literature review

Multi-Criteria Decision Making (MCDM) provides a formal structure for complex decision problems by evaluating alternatives against a set of criteria [13], typically represented as a decision matrix for subsequent aggregation [14]. A wide range of procedures exist: value-based methods such as AHP [15], calculate a total performance score for each alternative; distance-based methods such as TOPSIS [16] rank alternatives by their proximity to an ideal solution; and outranking methods such as ELECTRE [17] determine whether one alternative is credibly “at least as good as” another.

Yet a critical paradox persists: while the aggregation function has been the subject of extensive analytical rigor, the structural integrity of the criteria set the *Decision Criteria Configuration* (DCC) that forms the decision architecture is often assumed rather than formally validated [18,19]. This emphasis on aggregation is exemplified in the decision-aiding tradition pioneered by Roy [7] and significantly advanced by Bouyssou [6], which develops axiomatic foundations of preference modelling and aggregation. Their work ensures logical coherence in translating decision-maker preferences into recommendations. However, despite this rigor, scrutiny has historically focused on the aggregation stage, with the structural soundness of the criteria set largely taken for granted. A persistent gap therefore exists between the formal validation of aggregation and the informal, often subjective, “art” of selecting criteria.

Criteria identification, selection, and validation occur during the initial problem structuring phase of MCDM, which is widely acknowledged as subjective and reliant on qualitative deliberation. Belton and Stewart [20] describe this phase as a creative process where the main risk is an inappropriate problem representation. Marttunen et al. [2] similarly note the disconnect between qualitative structuring outputs and the quantitative MCDM models that follow. Although both emphasise the need for criteria sets to be complete and non-redundant, outcomes remain dependent on facilitator skill and decision-maker subjectivity [21], making structural validity difficult to guarantee [22, 23].

This gap is clearly evident in supply chain analytics. Many studies proceed directly from criteria selection to weighting, ignoring interdependencies [8]. Others use causal mapping tools such as Fuzzy DEMATEL, which capture directional influence but cannot diagnose redundancy [9]. Hybrid MCDM models may even amplify redundancy by rewarding duplicated criteria with higher weight [24]. Furthermore,

some methods assume independence to fit analytical requirements, as in TOPSIS, an unrealistic premise for complex strategic problems [10]. The common theme is a premature advance to calculation, with sophisticated models resting on unvalidated criteria sets.

The consequences are both practical and strategic [25]. Decisions based on unvalidated criteria risk systematic misallocation of resources [26], flawed strategic outcomes, erosion of managerial trust in quantitative tools, and loss of auditability [27]. Redundant criteria may reinforce biases or exaggerate a single dimension while presenting the illusion of balance [28,29]. Without a robust mechanism to validate the selected criteria set, even advanced MCDM models can offer only the appearance of analytical certainty while concealing a flawed decision architecture.

Existing methods for pre-hoc criteria validation are fragmented and incomplete. They can be grouped into two broad categories. The first are problem structuring methods, such as the Delphi method or Nominal Group Technique [30], which encourage brainstorming and consensus but provide no formal test of completeness, are weak at detecting redundancy, and often overlook logical consistency. Structural modelling approaches such as Interpretative Structural Modeling (ISM) fall into this category. Both map interdependencies and support checks for logical consistency [31] but cannot test completeness and only structure the criteria provided. The second are diagnostic and statistical tools, including Factor Analysis [32], which can detect underlying correlations, and Network Analysis [33], which can reveal clustering. However, Factor Analysis assumes linear relationships and requires large samples, while Network Analysis detects connectivity patterns but cannot assess completeness or logical consistency. In short, qualitative approaches remain subjective and sensitive to group composition [34], while statistical tools are often ill-suited to expert-elicited problems. This methodological gap highlights a central paradox within MCDM. The field’s axiomatic tradition, developed in the decision-aiding school pioneered by Roy [7] and advanced by Bouyssou [6], provides a powerful foundation for ensuring logical coherence. Rooted in classical multi-attribute utility [35] and social choice theories [36], this school establishes formal axioms such as independence, monotonicity, and transitivity to govern preference modelling and aggregation. Yet, this analytical rigor has been focused almost exclusively on these downstream procedures. The structural soundness of the criteria set itself has rarely been subjected to comparable scrutiny. This study resolves this paradox by extending axiomatic reasoning upstream. It treats completeness, non-redundancy, and logical consistency as intrinsic properties of the DCC, operationalizing them as computable topological propositions.

To address this methodological void, this study employs Topological Data Analysis (TDA). By focusing on the intrinsic “shape” and connectivity of data, TDA offers a unified mathematical lens to assess all three axioms simultaneously. Although TDA is emerging in supply chain research for analysing physical and transactional networks [37-41], our novel application is a formal, pre-hoc audit of the abstract, conceptual structure of the DCC itself. This topological approach provides a formal solution to the specific shortcomings of established methods. For instance, it formally tests for completeness by measuring connectivity, a task beyond the scope of problem structuring methods or Network Analysis. It diagnoses redundancy and logical consistency through structural impact analysis and the detection of topological cycles, overcoming the weaknesses of consensus-based methods like Delphi. Second, TDA avoids the restrictive assumptions that limit other tools. Unlike Factor Analysis, it requires no assumption of linearity and is robust with the small, expert-derived datasets typical of strategic problems. Furthermore, in contrast to the rigid hierarchies imposed by ISM, TDA is uniquely capable of revealing the authentic, and often non-hierarchical, structure of expert knowledge.

The primary contribution of this research is a framework that transforms the validation of MCDM criteria from a subjective, qualitative art into an objective, quantitative, and auditable science. This is achieved by establishing a direct theoretical link between foundational

MCDM axioms and TDA. Instead of treating criteria as a simple list to be debated, the framework models their interdependencies as a high-dimensional geometric object, a simplicial complex, allowing for a multi-scale analysis of its structural integrity. This topological approach yields three specific contributions. It provides the first formal, computable test for completeness by operationalizing it as topological connectivity  $\beta_0$ . It offers a rigorous method for ensuring non-redundancy through a structural impact analysis that measures the precise effect of removing any single criterion. Finally, it allows for a nuanced audit of logical consistency by detecting and classifying topological cycles  $\beta_1$  as either legitimate systemic feedback or problematic circularity. Collectively, these innovations provide the robust mechanism required to ensure that MCDM models rest on validated and coherent decision architectures, addressing a long-standing gap in the field.

### 3. Theoretical foundations

Building on the axiomatic tradition introduced in Section 2, this study extends axiomatic reasoning from aggregation rules to the structural validation of decision criteria. Completeness, non-redundancy, and logical consistency are reconceptualised as properties of the Decision Criteria Configuration (DCC) itself and are operationalised as computable propositions through Topological Data Analysis (TDA). To translate a set of decision criteria into a formal geometric “shape,” the framework represents it as a mathematical object known as a simplicial complex. This structure captures not only pairwise relations but also higher-order interactions among criteria, allowing the decision architecture to be interrogated for structural coherence. The following subsections provide the formal definitions required to construct this object and to analyse its topological properties.

#### 3.1. Formal definitions

To establish a rigorous foundation for our methodology, this section provides formal definitions for the key topological concepts and MCDM axioms central to our framework.

##### 3.1.1. Core topological concepts

- i. *Simplicial Complex*: A mathematical structure used to represent the shape of data. It is built from simple components called simplices: points (0 – simplices), lines (1 – simplices), triangles (2 – simplices), and their higher-dimensional counterparts. Unlike a graph, which only captures pairwise relationships (lines), a simplicial complex can explicitly model higher order, many-to-many relationships (triangles and above).
- ii. *Filtration*: A core process in TDA for analyzing data across multiple scales. It involves creating a nested sequence of simplicial complexes, typically by varying a threshold parameter such as the strength of a relationship. This allows us to observe how the structure of the data evolves, from a disconnected set of points to a highly connected object.
- iii. *Persistent Homology*: The primary analytical engine of TDA. It analyzes a filtration to identify topological features like connected components or cycles and track their “lifespan”, i.e., the range of scales over which they exist. This distinguishes robust, significant features demonstrating persistence from transient noise.
- iv. *Topological Invariants & Betti Numbers*: Properties of a shape that do not change under continuous deformation like stretching or twisting. We use the Betti numbers ( $\beta_i$ ) as our key invariants.  $\beta_0$  counts the number of connected components,  $\beta_1$  counts the number of one-dimensional cycles (loops), and  $\beta_2$  counts two-dimensional voids or holes.

#### 3.1.2. Application to MCDM axiom validation

- i. *Decision Criteria Configuration (DCC)*: We define a DCC not as a simple list, but as the complete architecture of the decision space, comprising the criteria and their interdependencies. Operationally, a DCC is represented as a simplicial complex,  $K$ , constructed from expert data.
- ii. *Completeness*: A foundational MCDM axiom requiring the criteria set to capture all relevant dimensions of a decision. We define a DCC as topologically complete if its corresponding simplicial complex consists of a single connected component ( $\beta_0(K) = 1$ ).
- iii. *Non-Redundancy*: An axiom requiring each criterion to contribute unique information. We define a criterion as topologically non-redundant if its removal causes a significant change to the key Betti numbers ( $\beta_i$ ) of the DCC.
- iv. *Logical Consistency*: An axiom requiring a non-contradictory framework. We define a DCC as topologically consistent when its cycles ( $\beta_1(K) > 0$ ) are identified and validated as either legitimate systemic feedback or problematic circularity.
- v. *Structural Persistence*: A measure of the robustness of the DCC’s architecture. The persistence of its topological features, e.g., its single component or key cycles across the filtration indicates the strength of expert consensus on those structural properties.

#### 3.2. From graph models to simplicial complexes

A rigorous analysis of a DCC requires moving beyond the pairwise focus of graph models used in MCDM. In a standard graph, each criterion is a vertex, and each edge records a binary relation, leaving higher-order dependencies invisible (Fig. 1a). For instance, in a study of sustainable food supply chains [24], the economic barriers of “high initial investment”, “uncertainty about return on investment”, and “lack of financial incentives” interact collectively. This joint effect, crucial to an investment decision, is not captured by any single pairwise link. A simplicial complex resolves this limitation by encoding every  $k$ -way interaction as a  $k$ -simplex: vertices ( $k = 0$ ), edges ( $k = 1$ ), triangles ( $k = 2$ ), tetrahedra ( $k = 3$ ), and so forth (Fig. 1(b)). The result is a mathematically explicit model that preserves the full combinatorial architecture of a DCC.

#### 3.3. Formal construction and filtration

Let  $C = \{c_1, c_2, \dots, c_n\}$ , be the set of candidate criteria. For a fixed similarity threshold  $\epsilon$  and a symmetric similarity function  $s(c_i, c_j)$ , we define the Vietoris–Rips complex  $K_\epsilon(C)$  as:

$$K_\epsilon(C) = \{\sigma \subseteq C \mid \forall c_i, c_j \in \sigma, s(c_i, c_j) \geq \epsilon\}$$

In this construction, each criterion is a 0–simplex (vertex). An edge  $(c_i, c_j)$  is added when  $s(c_i, c_j) \geq \epsilon$ . A higher-dimensional simplex  $\sigma$  joins the complex if all its faces are present, ensuring  $K_\epsilon(C)$  is a valid simplicial complex. Lowering the threshold through a descending sequence,  $\epsilon_1 > \epsilon_2 > \dots > \epsilon_m$ , generates a nested filtration:

$$K_{\epsilon_m}(C) \subseteq \dots \subseteq K_{\epsilon_2}(C) \subseteq K_{\epsilon_1}(C)$$

This filtration traces the topological evolution of a DCC from isolated vertices to an increasingly connected structure. Fig. 2 illustrates this process for a small subset of five criteria. At a high similarity threshold ( $\epsilon = 0.9$ ), the criteria are represented as disconnected vertices (a). As the threshold is lowered ( $\epsilon = 0.6$ ), edges begin to connect the vertices into a path (b). Finally, at a low threshold ( $\epsilon = 0.4$ ), the structure becomes more complex, forming a cycle and a filled simplex (a triangle) (c). This filtration is the input for persistent homology, which distinguishes robust structural features from transient artefacts.

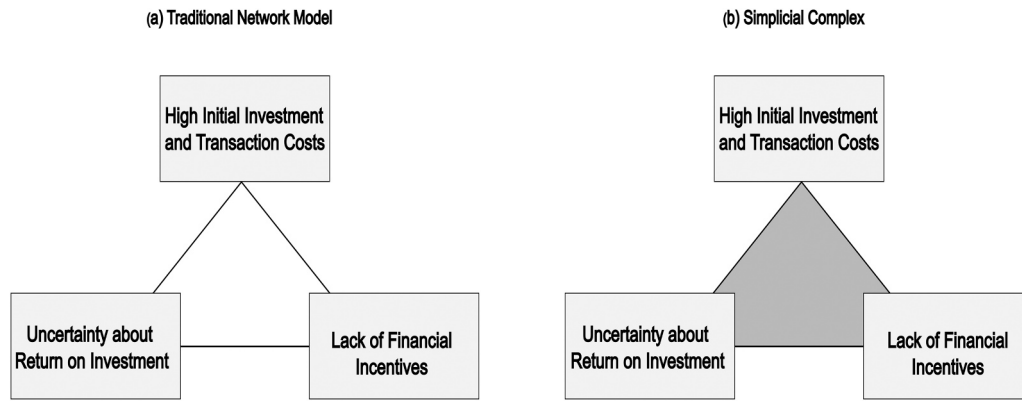


Fig. 1. Network versus simplicial representation of criteria interdependencies. Panel (a) shows a traditional network graph, where each criterion is a node and edges capture only pairwise associations (e.g., between high initial investment, uncertainty about returns, and lack of incentives). Panel (b) depicts the same configuration as a simplicial complex: the shaded triangle (2-simplex) encodes the higher-order interaction among all three criteria simultaneously. This extension moves beyond dyadic links to formally represent collective, multi-way dependencies within a Decision Criteria Configuration (DCC).

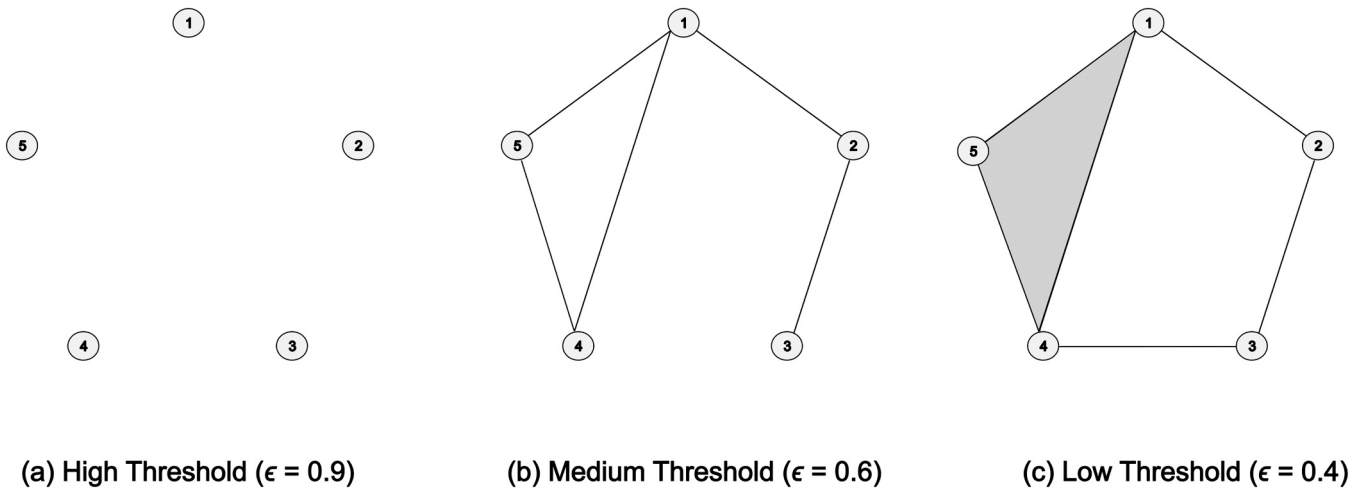


Fig. 2. Filtration of a Decision Criteria Configuration (DCC). The figure illustrates the Vietoris-Rips filtration of a criteria set across three decreasing similarity thresholds ( $\epsilon_1 > \epsilon_2 > \epsilon_3$ ). (a) At  $\epsilon_1$ , the complex consists of isolated vertices (criteria as 0 – simplices). (b) At  $\epsilon_2$ , edges appear, forming a one-dimensional hole (cycle). At  $\epsilon_3$ , a 2-simplex (triangle) closes the cycle, altering the topology of the complex. Tracking these changes across thresholds provides the input for persistent homology, distinguishing stable structural features from transient artefacts. (c) At  $\epsilon_3$ , a 2-simplex (triangle) closes the cycle, altering the topology of the complex. Tracking these changes across thresholds provides the input for persistent homology, distinguishing stable structural features from transient artefacts.

### 3.4. Topological invariants for structural analysis

The power of this representation lies in using topological invariants to measure the shape of the criteria architecture. The zeroth and first Betti numbers are two key invariants for validating a DCC. The zeroth Betti number  $\beta_0$  counts the number of connected components. A coherent framework should form a single component  $\beta_0 = 1$ . Multiple components  $\beta_0 > 1$  reveal a fragmented architecture, indicating missing criteria or an overly broad scope.

The first Betti number,  $\beta_1$ , counts the number of independent one-dimensional holes or cycles. In decision contexts, such cycles can represent legitimate feedback loops or problematic redundancy where multiple criteria measure the same underlying construct. The presence of cycles  $\beta_1 > 0$  therefore requires careful analysis to ensure the logical consistency of the DCC. Fig. 3 illustrates how the zeroth and first Betti numbers capture structural properties of a DCC, contrasting complete versus fragmented configurations ( $\beta_0$ ) and consistent versus redundant configurations ( $\beta_1$ ).

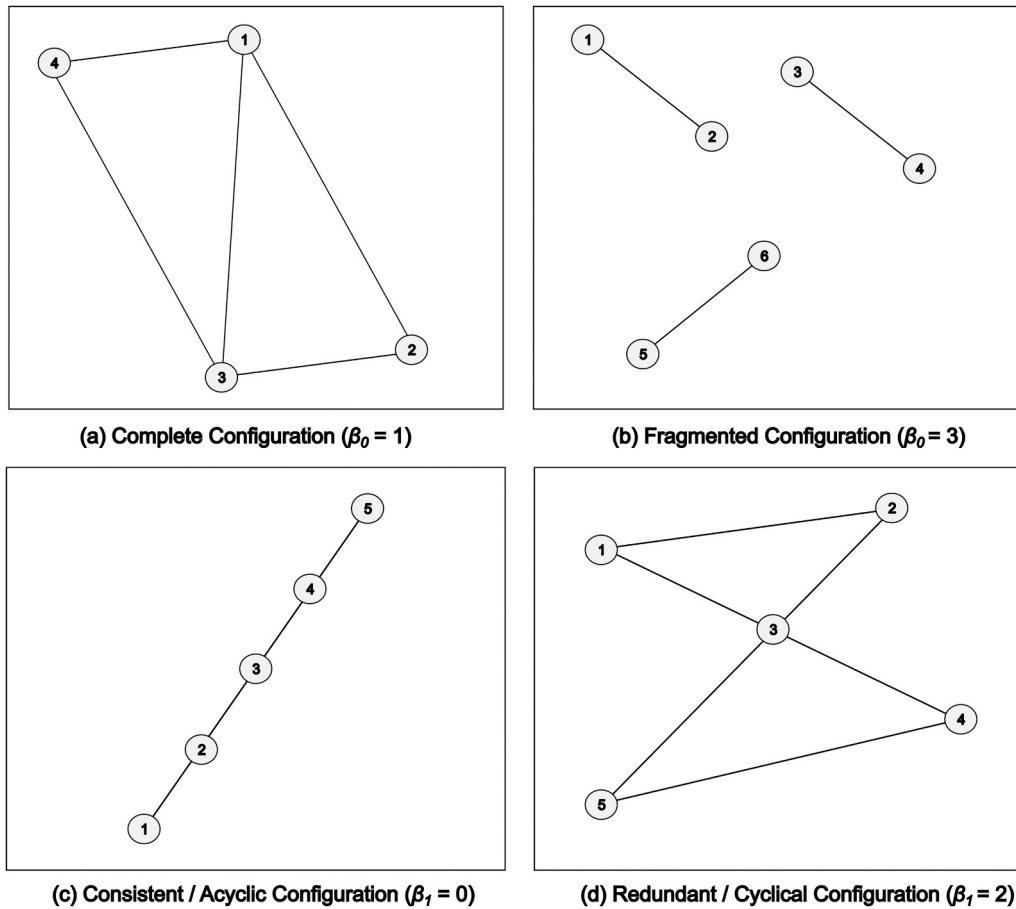
### 3.5. Ensuring MCDM requirements through topological analysis

Constructing a valid and coherent criteria set is a foundational aspect of MCDM, often seen as more art than science [42]. Our topological framework introduces formal validation into this process. By translating a DCC into a simplicial complex, we can diagnose flaws corresponding to the fundamental MCDM axioms of completeness, non-redundancy, and logical consistency. This section establishes a theoretical link between topological properties and these core axioms, showing how topological invariants can trigger the value-focused inquiry of the MAVT school [35] and the constructive dialogue of the European school [6,7].

#### 3.5.1. Completeness through connectivity analysis

**Proposition 1. (Topological Completeness):** A DCC is topologically complete if its corresponding simplicial complex  $K$  has a single connected component ( $\beta_0 = 1$ ) at a meaningful relationship threshold.

Completeness requires that criteria collectively capture all relevant dimensions of a decision. Topologically, this implies all meaningful criteria should be connected within a coherent framework. If a complex contains disconnected components ( $\beta_0 > 1$ ), the DCC is fragmented.



**Fig. 3.** Visualising Betti numbers as indicators of criteria architecture. Panels (a) and (b) illustrate completeness using the zeroth Betti number ( $\beta_0$ ). Panel (a) shows a fully connected configuration with  $\beta_0 = 1$ , representing a coherent set of criteria. Panel (b) shows a fragmented configuration with  $\beta_0 = 3$ , indicating three disconnected components and a structurally incomplete architecture. Panels (c) and (d) illustrate logical consistency using the first Betti number ( $\beta_1$ ). Panel (c) depicts an acyclic, tree-like configuration with  $\beta_1 = 0$ , consistent and non-redundant. Panel (d) depicts a configuration with  $\beta_1 = 2$ , containing two independent cycles that signal redundancy or circularity in the criteria set.

This structural flaw indicates a fractured value hierarchy from a value-focused perspective [42] or an incomplete shared understanding of the problem from a constructive viewpoint [7]. The finding prompts investigation into two potential issues:

- **Missing Bridging Criteria:** Key criteria linking disconnected components may have been omitted.
- **Overly Broad Scope:** The DCC attempts to integrate fundamentally unrelated dimensions that should be analyzed separately. Thus, the condition  $\beta_0 = 1$  serves as a formal, quantitative test for the architectural coherence of the decision model.

### 3.5.2. Non-redundancy through structural impact analysis

**Proposition 2. (Topological Non-Redundancy):** A criterion is non-redundant if its removal from the simplicial complex  $K$  meaningfully changes the complex's essential topological invariants ( $\beta_0$  or  $\beta_1$ ).

Non-redundancy requires each criterion to contribute unique information. Topologically, an essential criterion is one that affects the fundamental shape of the decision space. We assess this using Structural Impact Analysis, where for each criterion  $c_i \in C$ , we define a reduced complex  $K \setminus \{c_i\}$  and measure the impact of its removal. This analysis allows for a formal classification of criteria:

- **Essential Bridge Criteria:** Removal increases  $\beta_0$ .
- **Cycle-Critical Criteria:** Removal alters  $\beta_1$ .

- **Structurally Subsumed Criteria:** Removal produces no significant change in key invariants, flagging them as potentially redundant. This procedure provides an objective foundation for identifying a minimal set of criteria that preserves the essential structure of the decision space.

### 3.5.3. Logical consistency through cycle management

**Proposition 3. (Topological Consistency):** A DCC is logically consistent when its 1-cycles ( $\beta_1 > 0$ ) are identified, analyzed, and managed to distinguish legitimate systemic relationships from problematic circular dependencies.

Logical consistency requires a non-contradictory framework. While MAVT advocates for strict means-ends value hierarchies where cycles indicate flawed structure, the European school is more comfortable with non-hierarchical models representing complex interdependencies. Our framework accommodates this nuance by not demanding the elimination of all cycles. Instead, detecting them via  $\beta_1 > 0$  compels the analyst to investigate their nature:

- **Legitimate Systemic Cycles:** Represent genuine feedback mechanisms e.g., ‘Supplier Collaboration’ → ‘Innovation’ → ‘Profitability’ → ‘Supplier Collaboration’.
- **Problematic Circularity:** Indicate redundant measurement e.g., a cycle connecting ‘On-Time Delivery’, ‘Lead Time Reliability’, and ‘Schedule Adherence’. This topological transparency enforces logical

integrity by ensuring any cycles are defended as deliberate and meaningful modelling choices

#### 4. Implementation methodology

In this section, we operationalize the preceding theoretical foundations by presenting a systematic six-phase methodology for validating a DCC using TDA (Fig. 4). Our framework builds on established Python libraries, primarily *ripser* for computing persistent homology and *networkx* for graph-based visualization but the methodological contribution does not lie in the development of new TDA algorithms. Rather, its novelty is an interpretive and prescriptive framework that translates persistence outputs into actionable diagnostics aligned with MCDM axioms:  $\beta_0$ -curves for completeness (Proposition 1), structural impact analysis for non-redundancy (Proposition 2), and persistence diagrams of  $\beta_1$ -cycles for logical consistency (Proposition 3). In essence, while *ripser* computes the shape of the data, our framework renders these computations meaningful for MCDM practice by providing a coherent suite of diagnostics that enable a formal audit of the criteria set.

##### 4.1. Phase 1: Candidate criteria elicitation

The process begins with the elicitation of a comprehensive pool of candidate criteria  $C$ . This is typically achieved through structured methods such as Value Focused Thinking (VFT) workshops, systematic literature reviews, and expert panel techniques e.g., Nominal Group Process or Delphi method to ensure diverse stakeholder perspectives are captured. An additional and critical output is a glossary (Annexure A)

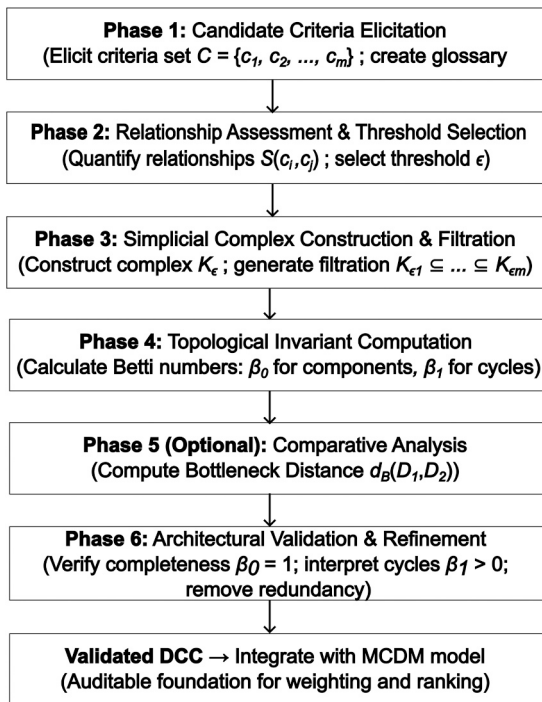


Fig. 4. Six-phase methodology for topological validation of a Decision Criteria Configuration (DCC). Phase 1 elicits candidate criteria and definitions. Phase 2 assesses inter-criteria relationships and selects the similarity threshold ( $\epsilon$ ). Phase 3 constructs the simplicial complex and generates a filtration. Phase 4 computes topological invariants ( $\beta_0$  for connectivity;  $\beta_1$  for cycles). Phase 5 (optional) compares persistence diagrams via bottleneck distance to test alignment across configurations. Phase 6 conducts architectural validation and refinement, verifying completeness ( $\beta_0 = 1$ ), managing cycles ( $\beta_1 > 0$ ), and removing redundancy. The outcome is a validated DCC that can be integrated with MCDM methods (e.g., AHP, TOPSIS, ELECTRE), providing a transparent, auditable foundation for weighting and ranking.

containing a precise operational definition for each criterion to ensure semantic clarity.

##### 4.2. Phase 2: Relationship assessment and threshold selection

In this phase, the interdependencies among criteria in  $C$  are quantified. The pairwise similarity score,  $s(c_i, c_j)$ , can be assessed through expert-elicited ratings, data-driven metrics, e.g., correlation, mutual information, or hybrid approaches. However, unlike traditional network analysis which requires the pre-selection of a single, arbitrary similarity threshold  $\epsilon$ , our TDA framework examines the DCC's structure across all possible thresholds simultaneously. This is achieved through a filtration, where the threshold is systematically varied, allowing us to observe the evolution of the structure from disconnected components to a fully connected complex via persistent homology. This multi-scale perspective distinguishes robust structural features from noise without being dependent on a single initial calibration choice. For specific diagnostic tests where a static graph is required (such as the cycle and centrality analyses in Section 6), a threshold can then be selected in a data-driven manner based on points of maximal topological interest, such as the peak of the  $\beta_0$  curve.

##### 4.3. Phase 3: Simplicial complex construction and filtration

This phase translates the relationship data into a formal topological model. For a given threshold  $\epsilon$ , a simplicial complex  $K_\epsilon$  is constructed by defining each criterion  $c_i \in C$  as a vertex ( $0$ -simplex). An edge ( $1$ -simplex) is formed between any two criteria if their relationship strength  $s(c_i, c_j) \geq \epsilon$ . Higher order simplices are then added where all constituent faces are present. To enable multi-scale analysis, a filtration is generated by systematically decreasing the threshold across a sequence  $\epsilon_1 > \dots > \epsilon_m$ , producing a sequence of nested complexes,  $K_{\epsilon_1} \subseteq K_{\epsilon_2} \subseteq \dots \subseteq K_{\epsilon_m}$ .

##### 4.4. Phase 4: topological invariant computation

The structural properties of the DCC are extracted by computing the topological invariants for each complex  $K_\epsilon$  in the filtration. This involves calculating the number of connected components ( $\beta_0$ ) and independent one-dimensional cycles ( $\beta_1$ ). This is accomplished using graph-theoretic algorithms for basic analysis or with specialized TDA Python library *ripser*.

##### 4.5. Phase 5 (Optional): comparative analysis via Bottleneck Distance

This optional phase provides a method for quantitatively comparing the structural similarity of two distinct DCCs, which is useful for measuring strategic alignment between different expert panels or tracking a model's evolution. The comparison is achieved by generating a persistence diagram for each DCC's filtration, which serves as a unique visual 'fingerprint' of the decision architecture's topological features. The bottleneck distance,  $d_B(D_1, D_2)$ , a single, quantitative score that measures the overall structural difference between the two decision architectures is then computed between the two persistence diagrams, a task for which we use the Python library *persim*.

- A small distance indicates high structural similarity and strong strategic alignment.
- A large distance reveals significant structural divergence and misalignment.

This phase is contingent upon having two or more DCCs to compare. In the present study, Phase 5 was not applied, as the case analyses focused on auditing each DCC independently. The comparative bottleneck distance is presented here as an optional extension for future

applications where alignment between multiple decision architectures is under investigation.

#### 4.6. Phase 6: Architectural validation and refinement

This pivotal phase applies the propositions from Section 3.5 to audit and refine the DCC's architecture, translating the computed topological invariants into an actionable diagnostic. The process is sequential, addressing each of the core MCDM axioms.

First, the *Completeness Audit* (Proposition 1) verifies that the DCC forms a coherent, single component ( $\beta_0 = 1$ ) at a meaningful similarity threshold. A fragmented structure ( $\beta_0 > 1$ ) provides a formal diagnostic that the criteria set is incomplete, prompting a targeted investigation into missing bridging criteria or an overly broad problem scope.

Second, the *Non-Redundancy Audit* (Proposition 2) is performed to identify and rank the criteria that are most essential to the DCC's architecture. We assess non-redundancy via a Structural Impact Analysis. For each criterion  $c_i$ , we build a reduced complex by removing  $c_i$ , recompute persistent homology, and measure the impact as the Wasserstein distance between the full and reduced persistence diagrams. Criteria with large impact scores are essential (bridges or cycle-critical); those with negligible scores are structurally subsumed. Computations use ripser (persistence) and persim.wasserstein (diagram distance). Centrality measures (e.g., betweenness) are optional corroborative diagnostics rather than the primary test.

The final step is the *Logical Consistency Audit* (Proposition 3), which involves a rigorous interpretation of the DCC's topological cycles ( $\beta_0 > 0$ ). To address the challenge of distinguishing legitimate feedback from problematic redundancy our framework moves beyond subjective assessment by implementing a structured, two-stage diagnostic process that leverages the data from the filtration.

##### 4.6.1. Stage 1: data-driven hypothesis generation

This stage uses the outputs of the persistent homology analysis (Phase 4) to classify each significant cycle.

1. *Filter for Significance*: The analyst first uses the persistence diagram to identify only the most structurally robust cycles, those plotted furthest from the diagonal, indicate a long lifespan across the filtration. This focuses the audit on features that are not mathematical noise.
2. *Classify by Formation Threshold*: For each significant cycle, we examine its birth time: the similarity threshold ( $\epsilon$ ) at which the cycle first appeared in the filtration. This data point provides a powerful heuristic for generating a testable hypothesis about the cycle's nature:

**Hypothesis. Problematic Redundancy.** A cycle that is highly persistent but forms at a very high similarity threshold, e.g.,  $\epsilon > 0.78$  connects criteria that are nearly synonymous. The structural loop emerges only because the criteria are so semantically close that they are effectively interchangeable, e.g., a cycle connecting 'On-Time Delivery', 'Lead Time Reliability', and 'Schedule Adherence'.

**Hypothesis. Legitimate Systemic Feedback.** A highly persistent cycle that forms at a more moderate similarity threshold connects criteria that are conceptually distinct but causally related. The relationship is not one of synonymy but of systemic influence, representing a valid feedback loop, e.g., 'Supplier Collaboration' → 'Innovation' → 'Profitability' → 'Supplier Collaboration'.

##### 4.6.2. Stage 2: expert-led confirmation

This data-driven hypothesis reframes the role of the domain expert from one of initial, unguided interpretation to one of targeted confirmation. The analyst is no longer asking "What does this cycle mean?" but

is presenting a specific, evidence-based hypothesis for validation. For example:

- To test for redundancy: "The analysis has identified a tightly-bound cycle between Criteria A, B, and C, suggesting they measure the same underlying concept. Do you agree that they are redundant?"
- To test for feedback: "The analysis suggests a persistent feedback loop exists between Criteria X, Y, and Z. Can you describe the operational or strategic process that this feedback represents?"

This structured process provides a robust and auditable method for managing the logical consistency of the DCC. It systematically leverages the quantitative outputs of the TDA to guide expert judgment, ensuring the final architecture is not only mathematically sound but also contextually valid.

#### 4.7. Final output and integration

The framework's final output is a validated Decision Criteria Configuration (DCC), refined into a minimal, non-redundant core set of criteria. The six-phase process serves as a comprehensive structural audit, diagnosing and enabling the correction of foundational flaws such as incompleteness, redundancy, and inconsistency. This ensures that the final criteria set used for strategic decision-making, whether for supply chain initiative prioritization as in Scenario 1, or for initiative selection such as multicriteria inventory classification as in Scenario 2 is not merely a subjective list, but a robust, defensible architecture built on a foundation of proven structural integrity.

## 5. Validation experiment

This section details the validation of the proposed topological framework through a retrospective analysis of two distinct industrial multicriteria decision scenarios from a single steel manufacturing company. The analysis is designed to demonstrate the framework's practical applicability and diagnostic utility in structuring and validating real-world DCCs. The first scenario examines a comprehensive set of strategic criteria derived from formal problem-structuring workshops aimed at defining a supply chain strategy. The second scenario, occurring 1.5 years later, analyzes a criteria set developed by internal experts for a multicriteria inventory classification project stemming from scenario 1. To clearly demonstrate the framework's capabilities, the Results section (Section 6) is structured around these three propositions. We will present the application of the tests for completeness (Proposition 1), non-redundancy (Proposition 2), and logical consistency (Proposition 3) in turn, drawing evidence from both industrial scenarios to illustrate each diagnostic audit.

### 5.1. Organizational setting and rationale

The context is a large steel manufacturing firm facing intense international competition and numerous intersecting pressures, including commodity price volatility, fluctuating demand, and evolving regulations. These external challenges are compounded by complex internal issues, particularly in inventory management, such as process inconsistencies, low forecast accuracy, and misalignment between sales and production schedules. This high-complexity environment provides an ideal testbed for the framework for several reasons:

- *High-Dimensionality*: The numerous intersecting financial, operational, and commercial challenges create a decision space where interdependencies are complex and non-obvious, making manual structural assessment exceptionally difficult.
- *High Risk of Flawed Architecture*: Traditional problem-structuring workshops are susceptible to producing incomplete or redundant DCCs in such a setting.

- *Potential for Expert Deadlock*: Competing stakeholder priorities often lead to subjective debate, creating a need for an objective, quantitative tool to audit and build consensus around a proposed criteria set.

## 5.2. Scenario 1: auditing a qualitatively derived strategic DCC

### 5.2.1. Criteria elicitation (Phase 1)

This scenario used outputs from a comprehensive review of the company's supply chain operations. A systematic analysis of workshop documentation, i.e., problem statements, strategic objectives, issue lists and cognitive maps, which are diagrams that illustrate an individual's beliefs about causal relationships was conducted. Following a rigorous standardization and de-duplication process, an initial DCC of 61 unique candidate criteria was extracted, forming the vertex set  $C$  for the analysis. Crucially, these criteria, framed as organisational "issues", formed the direct justification for a portfolio of six strategic initiatives subsequently selected by the working group: customer segmentation, capacity management, IBP design, a control tower, data integrity, and a core planning systems upgrade. The purpose of this retrospective audit is therefore to assess the structural validity of the foundational criteria upon which these high-stakes initiatives were built.

### 5.2.2. Relationship assessment (Phase 2)

Given the qualitative nature of the source documents, a similarity matrix,  $s(c_i, c_j)$ , was derived using a researcher-led, content-based proximity analysis. That is, the research team assigned similarity scores based on how closely linked the concepts were in the original workshop documents. A scoring heuristic was applied where, for example, issues explicitly linked in the source text were assigned a high similarity score, e.g., 0.9, while those merely appearing under the same broad heading received a weaker score, e.g., 0.4. This heuristic was designed to prioritize explicitly stated causal or thematic links over simple co-occurrence, ensuring that the resulting structure reflected the documented logic of the workshops. This process yielded a defensible matrix representing the latent structure within the strategic documents.

## 5.3. Scenario 2: refining an expert-driven inventory DCC

### 5.3.1. Criteria elicitation (Phase 1)

This scenario, taking place 1.5 years after the strategy review, utilized a DCC generated independently through an unstructured criteria elicitation with four company decision-makers (tenures of 20, 5, 2, and 1 years). This resulted in a list of 32 criteria, where one participant insisted all were essential, creating a classic case of expert deadlock.

### 5.3.2. Relationship data (Phase 2)

For this scenario, the relationship data was derived from expert assessments using a Fermatean fuzzy linguistic scale, where experts could use terms like 'Very High' or 'Low'. The four decision-makers were assigned weights based on tenure (with the highest weight of 0.4 assigned to the 20-year veteran and proportionally lower weights to more junior members), and their ratings for each criterion were aggregated using a Fermatean fuzzy weighted average. This process yielded a final importance score for each of the 32 criteria. To construct the input for the topological analysis, the pairwise similarity  $S_{ij}$  between any two criteria was then defined based on the proximity of their aggregated scores. This was defuzzified to create the required  $32 \times 32$  similarity matrix,  $S$ .

## 5.4. Analytical Plan

The analysis applies the topological framework (Phases 3–6) to these two scenarios. The subsequent Results section is structured to demonstrate the framework's application to each of the three core MCDM

axioms in turn, using a comparative analysis of the scenarios as the primary source of evidence.

- First, we present the results of the *Completeness audit* (Proposition 1), contrasting the structurally incoherent "conceptual monolith" discovered in the workshop-derived DCC (Scenario 1) with the well-structured, parsimonious architecture of the expert-derived DCC (Scenario 2).
- Second, we demonstrate the *Non-Redundancy audit* (Proposition 2), applying structural impact analysis to identify both keystone criteria and significant redundancies in Scenario 1, and to resolve the expert deadlock in Scenario 2 by isolating its minimal, non-redundant core set of criteria.
- Finally, we present the *Logical Consistency audit* (Proposition 3), again using a comparative approach to contrast the complex cyclical structure and systemic feedback loops found in Scenario 1 with the perfectly acyclic and hierarchical structure of the validated criteria set from Scenario 2.

## 6. Results

This section reports the application of the proposed six-phase methodology to the two industrial scenarios introduced in Section 5. The analysis is structured explicitly around the three propositions outlined in Section 3.5, which correspond to the MCDM axioms of completeness, non-redundancy, and logical consistency. In each case, topological invariants are operationalized through persistence-based diagnostics, with results interpreted in relation to the practical decision contexts of the steel manufacturer. Figs. 5–8 present the results of these three validation steps in sequence, showing how completeness, non-redundancy, and logical consistency are operationalised through topological diagnostics across the two industrial scenarios.

### 6.1. Completeness audit (Proposition 1)

The initial validation audit assesses topological completeness, which, according to Proposition 1, requires a Decision Criteria Configuration (DCC) to form a single connected component ( $\beta_0 = 1$ ) at a meaningful relationship threshold. The analysis, presented in Fig. 5, operationalises this proposition by plotting the evolution of the zeroth Betti number ( $\beta_0$ ) across a filtration of similarity thresholds ( $\varepsilon$ ). The critical threshold at which completeness is achieved ( $\varepsilon$ ) serves as the key diagnostic metric for evaluating the architectural integrity of each DCC.

The workshop-derived DCC (Scenario 1) exhibits rapid topological convergence, achieving a single connected component at a high similarity threshold of  $\varepsilon^* \approx 0.78$  ( $t^* \approx 0.22$ ). This quantitative result indicates a poorly differentiated structure, characteristic of a criteria set lacking discriminating validity. The high degree of interconnectivity suggests significant conceptual overlap and redundancy, where distinct criteria are not sufficiently independent. Consequently, while technically satisfying the condition of being a single component, this DCC fails a rigorous validation audit. Its structure is that of a conceptual monolith rather than a well-posed set of distinct decision criteria, signalling that the initial problem structuring phase produced a model of low analytical utility. Conversely, the expert-derived DCC (Scenario 2) displays a markedly different topological signature characterized by gradual convergence. Completeness is achieved only at a low similarity threshold of  $\varepsilon^* \approx 0.19$  ( $t^* \approx 0.81$ ).

The persistence of multiple components across a wide range of filtration values is not evidence of a flawed, fragmented model. Instead, it is a positive indication of a well-structured, parsimonious architecture. The step-wise reduction in  $\beta_0$ , visible as plateaus in Fig. 5, reflects a clear hierarchical structure whereby distinct conceptual clusters of criteria merges at meaningful, intermediate thresholds. This result provides strong evidence to validate the expert-derived DCC. The set is demonstrated to be both holistically coherent, as it eventually forms a single

Proposition 1: Completeness Audit

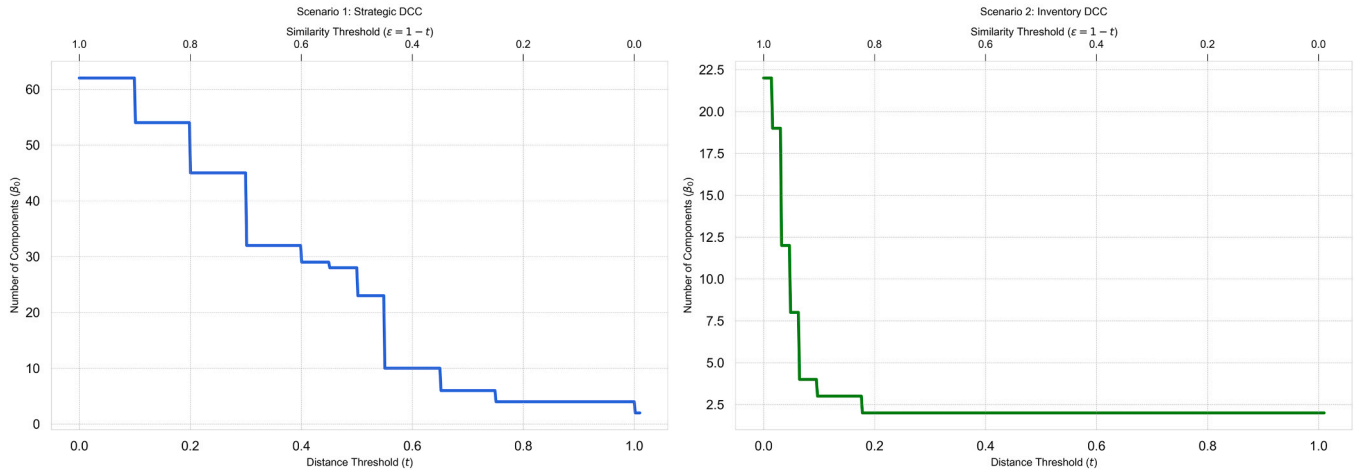


Fig. 5. Completeness audit of Decision Criteria Configurations (DCCs). The plots show the evolution of the zeroth Betti number ( $\beta_0$ ) across decreasing similarity thresholds for the two case scenarios. In Scenario 1 (left), the criteria rapidly merge into a single connected component only at a high threshold, indicating a monolithic but redundant structure. In Scenario 2 (right), completeness is achieved more gradually, suggesting a more differentiated yet coherent configuration. The critical threshold at which  $\beta_0 = 1 (e^*)$  provides the diagnostic measure of architectural completeness.

Proposition 2: Non-Redundancy Audit via Structural Impact Analysis

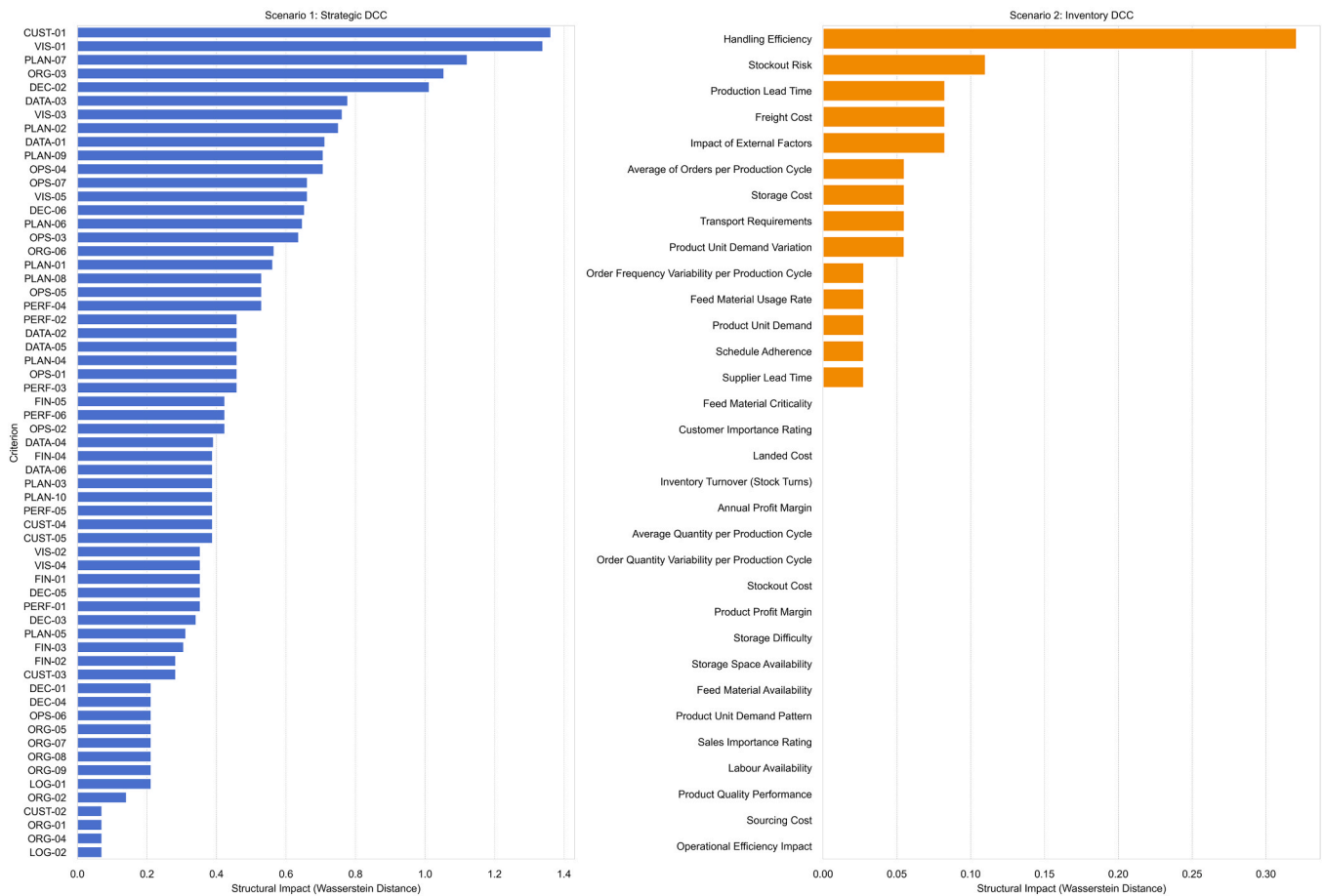
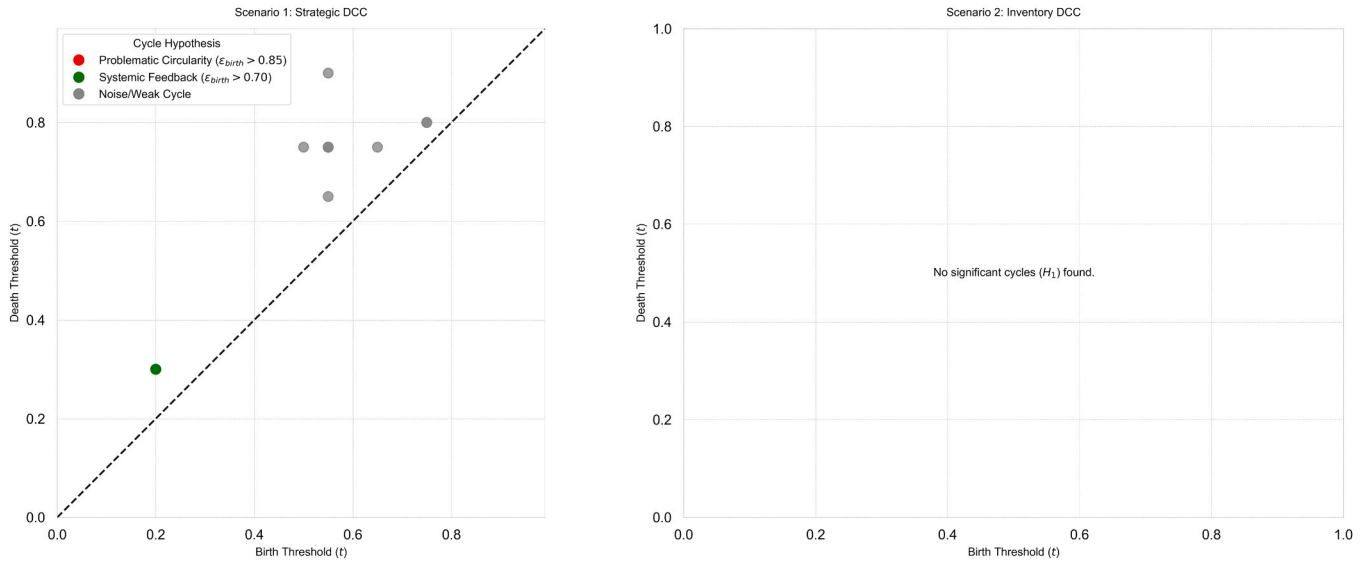


Fig. 6. Non-redundancy audit via Structural Impact Analysis. The plots rank criteria by their structural impact, measured as the topological perturbation (Wasserstein distance across  $H_0$  and  $H_1$ ) incurred by removing each criterion from the DCC. In Scenario 1 (left), the 61-criterion strategic DCC exhibits a gradual distribution of impact scores, with a long tail indicating that many criteria are structurally subsumed and redundant. In Scenario 2 (right), the 32-criterion inventory DCC yields a sharp drop after six dominant criteria, i.e., Handling Efficiency, Stockout Risk, Production Lead Time, Freight Cost, Impact of External Factors, and Average Orders per Production Cycle revealing a minimal, non-redundant core.

Proposition 3: Logical Consistency Audit ( $H_1$  Cycles)



**Fig. 7.** Logical consistency audit via persistence of  $H_1$  cycles. The persistence diagrams show the birth and death thresholds of one-dimensional cycles used to evaluate Proposition 3. In Scenario 1 (left), the strategic DCC exhibits multiple persistent cycles, confirming a systemic feedback structure. While no cycles emerged at very high similarity thresholds ( $\epsilon_{\text{birth}} > 0.85$ ), analysis at  $\epsilon \geq 0.8$  revealed 68 distinct loops, including a core cycle linking CUST-01, ORG-02, PLAN-06, DATA-04, and ORG-03. This pattern represents a legitimate feedback loop rooted in organizational dysfunction rather than problematic redundancy. In Scenario 2 (right), the inventory DCC displays no significant cycles, yielding an empty diagram and confirming the acyclic, logically consistent nature of its minimal six-criterion core.

component, and structurally sound, as it preserves the conceptual independence of its constituent criteria across a broad spectrum of relationship strengths.

In summary, this comparative analysis demonstrates the diagnostic power of Proposition 1. The completeness audit is shown to be more than a binary test; it is a quantitative method for assessing the architectural integrity of a proposed criteria set. The critical threshold,  $\epsilon^*$ , serves as a formal metric to distinguish between a validated, well-structured DCC (Scenario 2) and a poorly defined one requiring substantial revision (Scenario 1). This provides an evidence-based foundation for the initial and most critical phase of Multi-Criteria Decision Making.

### 6.2. Non-redundancy audit (Proposition 2)

To test for non-redundancy, a Structural Impact Analysis was performed, operationalizing Proposition 2. This audit quantifies the structural importance of each criterion by measuring the topological perturbation (Wasserstein distance across  $H_0$  and  $H_1$ ) incurred by its removal from the DCC. The results, shown in Fig. 6, rank each criterion by its contribution to the stability of the decision architecture, allowing for the identification of a minimal, non-redundant set.

In the case of the strategic DCC (Scenario 1), the analysis of the 61 criteria revealed a pronounced hierarchy of structural importance. As illustrated in Fig. 6 (Left Panel), the impact scores are characterized by a gradual distribution. While several criteria, such as CUST-01 and VIS-01, were identified as essential bridging elements, the distribution features a long tail. This indicates that a substantial subset of criteria exerted negligible topological influence upon their removal and can be classified as structurally subsumed, confirming significant redundancy within the workshop-derived set.

The audit of the inventory DCC (Scenario 2) yielded a more definitive result, providing a quantitative resolution to the expert deadlock over the initial 32 criteria. The analysis starkly delineated a minimal core set of six essential criteria, as shown in Fig. 6 (Right Panel). The structural impact is overwhelmingly concentrated in Handling Efficiency, Stockout Risk, Production Lead Time, Freight Cost, Impact of External Factors, and Average of Orders per Production Cycle. Following this core set,

there is a precipitous drop in impact scores, demonstrating that the remaining 26 criteria are topologically redundant. This provides an objective and defensible basis for model reduction, validating a minimal, non-redundant DCC that preserves the essential architecture of the expert-defined decision space.

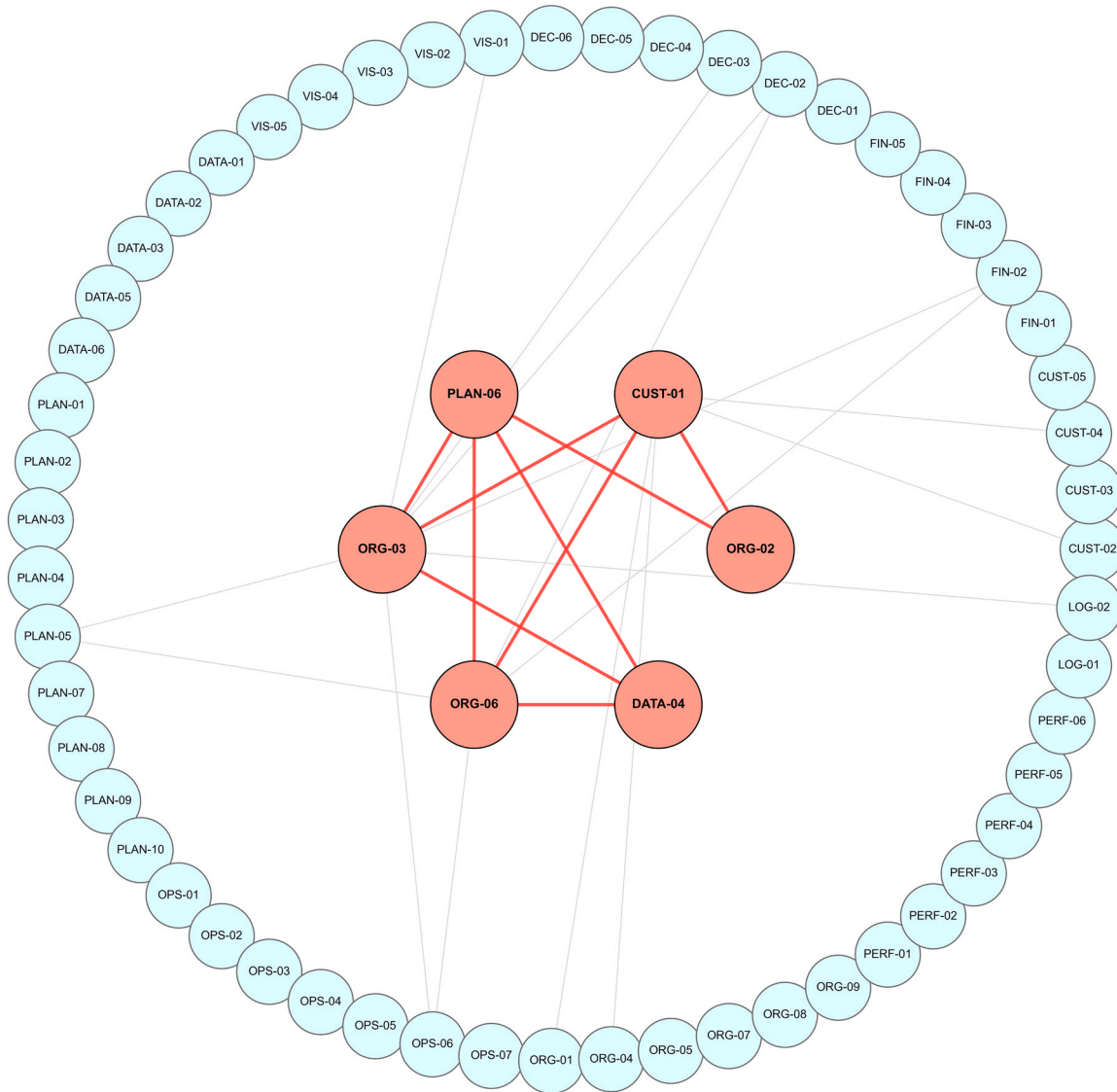
### 6.3. Logical consistency audit (Proposition 3)

The final validation step audits the logical consistency of the DCCs, in accordance with Proposition 3, by analyzing their 1-dimensional topological cycles ( $H_1$ ). This analysis is designed to distinguish between problematic circular dependencies and legitimate systemic relationships. The persistence diagram (Fig. 7) serves as the primary diagnostic tool, where a cycle's significance is determined by its persistence (distance from the diagonal) and its nature is hypothesized based on its birth similarity threshold ( $\epsilon_{\text{birth}}$ ).

The audit of the inventory DCC (Scenario 2) yielded a definitive result (Fig. 7, right panel), revealing a complete absence of one-dimensional cycles. The resulting persistence diagram is empty, indicating the minimal core set possesses a perfectly acyclic, non-redundant structure. This provides the strongest possible evidence for its logical consistency, confirming it as a well-posed, hierarchical framework for decision-making. In contrast, the analysis of the strategic DCC (Scenario 1) revealed a complex cyclical structure (Fig. 7, left panel) containing multiple persistent  $H_1$  features. An investigation into the cycle representatives was conducted to interpret these features. Notably, the audit did not detect any cycles born at very high similarity thresholds ( $\epsilon_{\text{birth}} > 0.85$ ), suggesting an absence of the most problematic form of circularity arising from synonymous criteria. Instead, the analysis of the graph at the  $\epsilon > 0.80$  threshold identified 68 distinct cycles, confirming the highly systemic nature of the DCC. A primary example is the cycle involving

CUST-01 (Inadequate Customer Segmentation & Understanding), ORG-02 (Misalignment between Sales & Planning), PLAN-06 (Deficient Capacity Planning & Allocation), DATA-04 (Constraining Legacy Systems), and ORG-03 (Conflicting KPIs). This structure is not indicative of problematic redundancy; rather, our analysis interprets it as a legitimate systemic feedback loop that describes a core organizational dysfunction

Systemic Hub in the Strategic DCC (at  $\epsilon \geq 0.8$ )



**Fig. 8.** Core–periphery topology of the strategic DCC. The network visualization at the critical similarity threshold reveals a distinct core–periphery structure, complementing the Proposition 3 audit of  $H_1$  cycles (Fig. 7). A tightly interconnected hub of criteria, i.e., CUST-01, ORG-02, ORG-03, ORG-06, PLAN-06, and DATA-04 forms the engine of systemic feedback, where organizational misalignment, planning deficiencies, and system constraints mutually reinforce one another. Surrounding this hub, peripheral criteria are weakly connected and dependent on the core, while several isolated nodes remain conceptually distant. This configuration both confirms the presence of a persistent feedback loop and pinpoints the central dysfunction that initiative selection must address.

where process gaps and system constraints foster conflicting metrics that prevent a clear understanding of the customer, which in turn reinforces the initial misalignment.

A network visualization of the DCC at this threshold further explicated this finding (Fig. 8), revealing a distinct core-periphery topology. The identified feedback loop forms the central, highly interconnected hub of the decision architecture, representing the engine of systemic feedback where key organizational, planning, and data criteria are shown to be mutually reinforcing. The peripheral nodes, representing more specific outcomes, are largely dependent on this central core, while several isolated nodes highlight criteria that are conceptually distant from the primary strategic conversation at this high level of abstraction. Given that the objective of the exercise was to select strategic initiatives, the topological audit successfully isolated the central, self-reinforcing problem that the new strategy must address.

#### 6.4. Synthesis

The three-part topological audit, applied to two distinct industrial scenarios, demonstrates the framework’s practical utility in operationalising foundational MCDM axioms. By translating the abstract concepts of completeness, non-redundancy, and logical consistency into measurable topological invariants, the analysis provided a comprehensive, evidence-based structural validation of each Decision Criteria Configuration (DCC).

First, the *Completeness Audit* (Proposition 1) quantified the architectural coherence of each DCC. It diagnosed the workshop-derived strategic DCC (Scenario 1) as a poorly differentiated "conceptual monolith" due to its rapid topological convergence. In stark contrast, it validated the expert-derived inventory DCC (Scenario 2) as a parsimonious and well-structured model with a clear hierarchical nature.

Second, the *Non-Redundancy Audit* (Proposition 2) provided an objective and defensible basis for model reduction. For the strategic

DCC, the Structural Impact Analysis identified significant distributed redundancy, classifying a substantial subset of criteria as "structurally subsumed." For the inventory DCC, the audit provided a definitive resolution to the expert deadlock by isolating a minimal, non-redundant core of six essential criteria from the initial set of 32.

Finally, the *Logical Consistency Audit* (Proposition 3) successfully investigated the cyclical structures within each DCC. For the strategic DCC, the audit identified a core, systemic feedback loop representing a key organisational dysfunction involving criteria such as CUST-01 (Inadequate Customer Segmentation & Understanding), ORG-02 (Misalignment between Sales & Planning), PLAN-06 (Deficient Capacity Planning & Allocation), DATA-04 (Constraining Legacy Systems), and ORG-03 (Conflicting KPIs). The audit confirmed the inventory DCC possessed a perfectly acyclic structure, providing the strongest evidence of its logical integrity.

Collectively, these results show that the topological framework moves beyond descriptive visualisations to deliver prescriptive, actionable structural audits. The analysis provides an auditable and reproducible methodology to ensure that the criteria sets underpinning multicriteria decision models are formally validated as complete, non-redundant, and logically consistent prior to subsequent weighting and ranking processes

## 7. Discussion

This section discusses the findings of the three validation audits, i. completeness (Fig. 5), ii. non-redundancy (Fig. 6), and iii. logical consistency (Figs. 7–8) and interprets them in relation to the two industrial scenarios. Together, these results show how topological diagnostics assess the structural validity of criteria and, in turn, the appropriateness of initiatives derived from them.

### 7.1. Principal findings: criteria, initiatives, and structural validity

The topological audit of the 61 criteria provides a rigorous, post-hoc evaluation of the structural soundness of the six strategic initiatives that were justified by them. As outlined in Section 5.2.1, the workshop process culminated in the selection of customer segmentation, capacity allocation, a core planning systems upgrade, a control tower, data integrity, and integrated business planning (IBP). The TDA results allow us to interrogate the foundational logic of these choices. Each initiative drew its justification from specific criteria: segmentation from CUST-01 (Inadequate Customer Segmentation); capacity initiatives from PLAN-06 (Deficient Capacity Planning & Allocation); visibility improvements from VIS-01 (End-to-End Supply Chain Visibility); system upgrades from DATA-04 (Legacy System Constraints) and DATA-01 (Data Integrity & Quality); and IBP design from ORG-02 (Sales–Planning Misalignment) and ORG-03 (Conflicting KPIs).

The *Completeness audit* (Proposition 1, Fig. 5) showed that these criteria collapsed into a "conceptual monolith" ( $\beta_0 = 1$  only at  $\epsilon^* \approx 0.78$ ), revealing that segmentation, capacity, visibility, data, and governance were not independent domains but overlapping expressions of a single dysfunction [2]. *Non-redundancy audit* (Proposition 2, Fig. 6) confirmed that while PLAN-06, DATA-04, and ORG-02 functioned as essential bridging criteria, many others especially visibility-related ones were structurally subsumed [20,21]. Elevating visibility to an initiative (control tower) therefore risked redundancy. *Logical consistency audit* (Proposition 3, Figs. 7–8) further revealed that segmentation, capacity, systems, and KPI alignment were bound together in a feedback loop, indicating that treating them as standalone initiatives ignored their interdependence.

When contrasted with Scenario 2, the six validated criteria, namely, Production Lead Time, Orders per Cycle, Stockout Risk, Freight Cost, Handling Efficiency, and External Factors distilled this systemic dysfunction into operational levers. Production Lead Time and Orders per Cycle directly mapped to the planning bottlenecks (PLAN-06, PLAN-

10); Stockout Risk and Freight Cost reflected consequences of segmentation and system unreliability; Handling Efficiency aligned with data and process quality. What was absent was redundancy: multiple segmentation and visibility criteria were collapsed into fewer, structurally essential drivers. As Figs. 5–8 collectively demonstrate, the workshop criteria were thematically appropriate but structurally incoherent, whereas the TDA-validated set was both parsimonious and strategically aligned.

### 7.2. Positioning relative to existing methods

As highlighted in the literature, established problem structuring approaches emphasise inclusivity and organisation but rarely provide a means to validate whether criteria sets are complete, non-redundant, and logically consistent. The facilitated workshop in this study relied on urgency–impact grids and cognitive maps, which broaden participation but cannot test structural distinctness. Consensus-building techniques such as Delphi or causal mapping approaches such as DEMATEL can capture perceived influence among criteria, yet they would not have revealed the collapse into a monolith [30]. Beyond these, diagnostic and statistical tools are sometimes used: factor analysis can extract latent dimensions from correlated inputs [32], and network analysis can detect clusters and connectivity patterns [33]. Yet both are limited, factor analysis assumes linear relationships and large samples, while network analysis provides no mechanism to assess completeness or logical consistency. The topological audit fills this methodological gap. By operationalising MCDM axioms as topological invariants, it identifies when criteria are genuinely independent, when they are redundant, and when they form feedback cycles that require joint treatment. TDA thus complements, rather than replaces, problem-structuring methods by auditing structural soundness before weighting, ranking, or initiative selection.

### 7.3. Theoretical and methodological contributions

These results contribute to the axiomatic foundations of decision analysis by extending them from preference aggregation to criteria validation. Whereas classical axioms ensure coherence in translating preferences into rankings or recommendations, the proposed topological propositions ensure that the criteria sets underlying those preferences are themselves structurally coherent. This reframing clarifies the methodological novelty of the framework: the contribution lies not in new TDA algorithms, but in the operationalisation of axioms at a neglected stage of decision modelling.

The study advances theory and method in three ways. First, it demonstrates how the axioms of completeness, non-redundancy, and logical consistency can be formalised as computable topological propositions, providing an objective standard for structural validity. Second, it bridges qualitative and quantitative traditions by showing how workshop-derived criteria can be codified and audited using persistent homology. Third, it introduces structural persistence, i.e., the endurance of cycles or stability of minimal cores as a measurable property that can be used to test alignment across decision models. These contributions extend MCDM by shifting validation from an implicit assumption to a formal, auditable process.

### 7.4. Managerial and practical implications

For managers, the implications are direct. Some initiatives, particularly IBP design and data integrity were validated as structurally essential (Fig. 6, Scenario 2). Others, such as a control tower and segmentation in isolation, were revealed as redundant or mis-sequenced (Figs. 6 and 8). The framework therefore functions as a pre-mortem governance tool: before committing resources, decision makers can test whether initiatives address structurally material levers or merely symptoms. Scenario 2 further showed that large, redundant sets often

collapse into a handful of validated criteria (Figs. 5–6). Concentrating resources on these levers increases coherence, reduces duplication, and enhances alignment between strategy and operations.

Importantly, these findings demonstrate that criteria validation is crucial even when organisations do not apply full MCDM models. Workshops and Delphi-style exercises still rely on elicited criteria to prioritise initiatives, yet without formal checks these foundations may be structurally unsound [30,31]. In hindsight, managers should have insisted on a structural audit immediately after the workshop process. Such an audit would have provided a structural basis to argue that visibility initiatives were derivative, that segmentation could not be treated in isolation, and that the real leverage lay in resolving the cycle between planning, KPIs, and systems (Figs. 7–8). A leaner, sequenced portfolio anchored on validated levers would have improved governance and reduced wasted effort.

The speculative lesson is clear: without validation, initiative selection risks building on conceptual sand [25–27]; with validation, managers can target structurally material levers with confidence, even at the earliest stage of problem structuring, before weighting or ranking models are applied. Moreover, redundancy can reinforce biases or exaggerate single dimensions (e.g., segmentation or visibility) while presenting the illusion of balance [28,29]. The contrast between the broad but redundant workshop set and the lean validated set (Figs. 5–8) underscores this risk and the value of embedding structural audits into governance practice.

## 8. Conclusion, limitations, and future research

### 8.1. Conclusion

This study set out to evaluate whether the criteria used to select initiatives in a complex supply chain decision context were structurally valid, and whether a topological audit could provide a more suitable basis. The findings show that the workshop criteria were thematically accurate but structurally incoherent, collapsing into a conceptual monolith that produced overlapping initiatives. By contrast, the TDA audit validated a minimal, non-redundant set of criteria that distilled the systemic dysfunction into operationally meaningful levers. Initiatives such as IBP design and data integrity were confirmed as appropriate, but others, particularly control tower and segmentation in isolation lacked structural validity. By embedding topological audits into governance processes, supply chain managers can ensure that decision criteria are formally validated for completeness, non-redundancy, and logical consistency before use. As Figs. 5–8 collectively showed, the workshop-derived initiatives were thematically appropriate but rested on structurally incoherent criteria, collapsing into a conceptual monolith with redundant visibility and segmentation drivers. In contrast, the validated criteria set distilled this complexity into a minimal set of operational levers that were both parsimonious and strategically aligned. The overarching lesson is that criteria validation must occur at the problem structuring stage: without it, initiative selection risks building on conceptual sand; with it, managers gain a defensible foundation for prioritisation, ensuring that initiatives address structurally material levers and that subsequent weighting or ranking models rest on sound decision architectures.

### 8.2. Limitations

This study is subject to several limitations, which fall into three main categories: study design, methodological considerations, and practical application.

First, concerning the study design, the validation was a retrospective simulation of pre-existing decision scenarios. This directly influenced the primary data limitation in Scenario 1, where similarity scores were derived from a single-researcher content analysis. This process, while guided by a scoring heuristic, lacks a formal calibration method, such as

using multiple raters to establish inter-rater reliability, e.g., Cohen's  $\kappa$  or Krippendorff's  $\alpha$ , and thus has a potential for subjective bias. This approach was deemed sufficient for the study's specific aim to use Scenario 1 as a diagnostic demonstration of the TDA framework's ability to extract latent structure from a complex qualitative archive but for prospective applications, a more rigorous, calibrated approach is essential.

Second, there are methodological considerations inherent to the TDA pipeline. A key choice is the selection of the similarity threshold ( $\epsilon$ ). While our filtration approach mitigates dependence on a single value, the development of robust, context-aware techniques for selecting optimal threshold ranges remains an open research area. Furthermore, computational complexity is a practical consideration as the number of criteria ( $n$ ) grows. For the medium-scale problems analyzed in this paper ( $n = 61$  and  $n = 32$ ), the entire computational process was completed in minutes on a standard laptop. However, the primary bottleneck is the construction of the Vietoris-Rips simplicial complex, which presents a scalability challenge for very large-scale problems. Established mitigation strategies, such as using computational approximations or subsampling techniques, would need to be incorporated into future work on large-scale, data-mined criteria sets to ensure tractability.

Finally, a limitation related to practical application is the challenge of visualizing high-dimensional conceptual structures. While the heatmaps and Betti curves used here are informative for analysts, developing more intuitive, interactive visualization tools is a critical next step for enabling broader exploratory analysis by non-expert managers. Closely related is the interpretation of the topological features themselves. Although our framework provides a structured, data-driven process for forming hypotheses about features like systemic cycles, the final validation of their meaning, distinguishing legitimate feedback from problematic redundancy still relies on confirmation from domain experts.

### 8.3. Future research

The findings of this study, while demonstrating the diagnostic power of a topological audit, also illuminate a clear and compelling path for future research. Having established the framework's potential, the research agenda must now turn towards validating its real-world impact, formalizing its interpretive power, refining its methodological core, and ensuring its practical accessibility for decision-makers.

The most critical next step is to move from retrospective audits to prospective, longitudinal validation. The ultimate test of this framework's value lies in its ability to improve decision outcomes. Future research should therefore conduct field studies that embed the topological audit into live strategy workshops, tracking not just the process but the long-term performance of the initiatives that result. This would provide direct, empirical evidence to test the paper's central hypothesis: that a structurally sound decision architecture leads to more efficient resource allocation, reduced project redundancy, and superior strategic performance.

Building upon the interpretive process in this study, a crucial research direction is to formalize the meaning of the topological features the framework reveals. Future work could build a diagnostic typology that maps specific signatures such as the "conceptual monolith" found in Scenario 1 or common systemic feedback cycles to known organizational dysfunctions. This would mature the analysis from a bespoke interpretation into a more standardized and scalable diagnostic process, directly addressing the challenge of cycle interpretability.

The framework also invites further methodological refinement at the front-end of data elicitation. The subjectivity inherent in our first scenario, a key limitation, points to a pressing need to develop structured protocols for eliciting similarity data directly from expert groups. Furthermore, building on the methods used in this study, future work should expand on the Fermatean fuzzy approach to better capture the uncertainty and hesitation inherent in expert judgments, thereby increasing the robustness of the input data.

Finally, for this framework to transition from a specialized analytical tool to a widely adopted managerial practice, the challenge of accessibility must be addressed. Future work should focus on developing intuitive, interactive decision-support dashboards. Such tools would empower managers to conduct "what-if" analyses in real time exploring the structural impact of adding or removing a criterion and could integrate the topological audit with outputs from methods like DEMATEL, offering a unified, multi-perspective interface for strategic dialogue. By pursuing these avenues, this research program can evolve the topological framework from a novel diagnostic method into an indispensable component of modern strategic governance.

**Appendix A. - Criteria Vertex Codes and Definitions**

Definitions of short codes used within figures and text in the main article. These codes were derived from documents provided by the case company which recorded outputs from a multi-stakeholder supply chain strategy workshop.

**CRedit authorship contribution statement**

**Frank Michael Theunissen:** Writing – original draft, Visualization, Software, Project administration, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Shafiq Alam:** Writing – review & editing, Supervision. **Aymen Sajjad:** Writing – review & editing, Supervision.

**Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Short Code	Short Label	Full Vertex Definition
CUST-01	Customer Segmentation	Inadequate Customer Segmentation & Understanding
CUST-02	Customer Experience	Suboptimal Customer Experience & Service Levels
CUST-03	Differentiated Service Offers	Lack of Differentiated Customer Value Proposition & Offers
CUST-04	Market Alignment	Misalignment with Market Needs & Influences
CUST-05	Customer Order Visibility	Low Customer Order Visibility & Tracking
FIN-01	Product/Customer Profitability	Unclear Product & Customer Profitability
FIN-02	Activity-Based / Cost-To-Serve Costing	Lack of Activity-Based Costing & Cost-to-Serve Data
FIN-03	Cost Control & Waste	Ineffective Cost Control & Waste Reduction
FIN-04	Earnings & ROIC Stability	Earnings Volatility & Unstable Profitability/ROIC
FIN-05	Revenue & Margin Growth	Suboptimal Revenue Growth & Margin Maximization
DEC-01	Decision-Making Speed	Slow & Ineffective Business Decision-Making
DEC-02	Decision Support & Scenario Tools	Lack of Robust Decision Support & Scenario Planning Capability
DEC-03	Risk Management & Resilience	Inadequate Business Risk Management & Resilience
DEC-04	Business Control & Predictability	Deficient Business Control & Outcome Predictability
DEC-05	Decision-Making Authority	Subjective & Over-Concentrated Operational Decision-Making
VIS-01	E2E Supply Chain Visibility	Lack of End-to-End Supply Chain Visibility
VIS-02	Inter-Plant & Sourcing Visibility	Poor Inter-Plant & Sourcing Process Visibility
VIS-03	WIP Visibility & Control	Insufficient Work-In-Process (WIP) Visibility & Control
VIS-04	Logistics Provider Visibility	Limited Visibility & Integration with Logistics Providers
VIS-05	Forward Schedule Visibility	Lack of Forward Schedule Visibility for Planning & Prioritization
DATA-01	Data Integrity & Quality	Poor Data Integrity, Consistency & Accuracy
DATA-02	Master Data Governance	Lack of Master Data Ownership & Governance
DATA-03	Single Source of Truth	Absence of a Single Source of Truth for Data
DATA-04	Legacy System Constraints	Constraining Legacy Planning & Production Systems
DATA-05	User System Proficiency	Inadequate User Understanding & Utilization of Systems
DATA-06	Inventory Management Systems	Ineffective Inventory Management Systems
PLAN-01	Demand Forecast Accuracy	Poor Demand Forecast Accuracy (Short & Medium Term)
PLAN-02	Demand Forecast Inputs	Inadequate Demand Forecasting Inputs & Insights
PLAN-03	Master Scheduling Capability	Lack of Master Scheduling Capability & Process
PLAN-04	Production Scheduling Tools	Ineffective Production Scheduling Tools & Practices
PLAN-05	Inventory Planning & Optimization	Suboptimal Inventory Planning & Optimization
PLAN-06	Capacity Planning & Allocation	Deficient Capacity Planning & Allocation (Operational & Tactical)
PLAN-07	Supplier Ordering Process	Lack of Granularity & Accuracy in Supplier Ordering Process
PLAN-08	Value-Based Forecasting	Forecasting by Volume Only (Lacking Value-Based Forecasting)
PLAN-09	Consumables Forecasting	Inadequate Forecasting & Planning for Consumables/Value-Add Products
PLAN-10	Grade-Level Forecasting	Lack of Grade-Level Forecasting in S&OP for Raw Materials
OPS-01	Reliance on Manual Intervention	High Reliance on Manual Intervention in Processes
OPS-02	Process Agility	Limited Operational Process Agility & Responsiveness
OPS-03	Invoicing & Dispatch Process	Inefficient Invoicing & Dispatch Processes
OPS-04	Production Disruptions	Production Disruptions from Small Orders or Uncoordinated Priorities
OPS-05	Production & Receiving Alignment	Misalignment between Production & Customer Receiving Capacity (Yard Congestion)
OPS-06	Network & Space Optimization	Suboptimal Network & Space Configuration & Management
OPS-07	Operational Bottlenecks	Operational Bottlenecks & Inefficiencies
ORG-01	Internal Collaboration	Poor Internal Alignment, Integration & Collaboration
ORG-02	Sales & Planning Alignment	Misalignment between Sales & Planning (S&OP Gaps)
ORG-03	KPI Alignment	Conflicting or Inadequate KPIs across Supply Chain
ORG-04	Role Clarity & Accountability	Lack of Clear Role Clarity, Accountability & Decision Ownership
ORG-05	Change Resistance & Discipline	Organizational Resistance to Change & Lack of Change Discipline
ORG-06	Project Prioritization	Ineffective Project Prioritization & Management Visibility
ORG-07	Decision Communication	Poor Communication of Decisions & Business Drivers
ORG-08	Workforce Engagement	Insufficient Workforce Engagement & Decisiveness

(continued on next page)

(continued)

ORG-09	Process Ownership	Unclear Ownership for Key Processes (e.g., Paint Ordering)
PERF-01	DIFOT Performance	Low DIFOT (Delivery In Full, On Time) Performance
PERF-02	Predictive Delivery Dating	Lack of Accurate Predictive Delivery Date Capability
PERF-03	STO/MTO Mix Strategy	Suboptimal STO/MTO (Stock-to-Order/Make-to-Order) Mix Strategy
PERF-04	Exception Management Planning	Inadequate Event & Exception Management Planning
PERF-05	Critical Supply Contingency	Uncertainty & Lack of Contingency in Critical Material Supply
PERF-06	Agile/Dynamic Capability	Lack of Agile & Dynamic Supply Chain Capabilities
LOG-01	Logistics Provider Influence	Limited Influence & Flexibility with Logistics Providers
LOG-02	Logistics Forecast Accuracy	Mismatch between Logistics Provider Forecasts & Actuals

Data availability

The data used in this study are derived from a private organization and are subject to confidentiality agreements; therefore, they cannot be publicly shared. The code and models were developed within this context and are not publicly available due to similar constraints. However, the analytical framework and methodological descriptions are sufficiently detailed to enable replication with appropriate data.

References

[1] A. Mardani, A. Jusoh, K. Nor, Z. Khalifah, N. Zakwan, A. Valipour, Multiple criteria decision-making techniques and their applications—a review of the literature from 2000 to 2014, *Econ. Res. Ekon. istraživanja* 28 (1) (2015) 516–571, <https://doi.org/10.1080/1331677X.2015.1075139>.

[2] M. Marttunen, J. Lienert, V. Belton, Structuring problems for Multi-Criteria decision analysis in practice: a literature review of method combinations, *Eur. J. Oper. Res.* 263 (1) (2017) 1–17, <https://doi.org/10.1016/j.ejor.2017.04.041>.

[3] E. Triantaphyllou, S.H. Mann, An examination of the effectiveness of multi-dimensional decision-making methods: a decision-making paradox, *Decis. Support Syst.* 5 (3) (1989) 303–312, [https://doi.org/10.1016/0167-9236\(89\)90037-7](https://doi.org/10.1016/0167-9236(89)90037-7).

[4] F.M. Theunissen, C.N. Bezuidenhout, S. Alam, Exploring the shortcomings in formal criteria selection for multicriteria decision making based inventory classification models: a systematic review and future directions, *Int. J. Prod. Res.* (2024) 1–21, <https://doi.org/10.1080/00207543.2024.2320680>.

[5] R.L. Keeney, *Value-focused thinking: a path to creative decisionmaking* / ralph L. Keeney, Harvard University Press, 1992.

[6] D. Bouyssou, *Editor building criteria: a prerequisite for MCDA*, Springer Berlin Heidelberg, Berlin, Heidelberg, 1990, [https://doi.org/10.1007/978-3-642-75935-2\\_4](https://doi.org/10.1007/978-3-642-75935-2_4).

[7] B. Roy, *Multicriteria methodology for decision aiding*, Kluwer Academic Publishers, 1996.

[8] A. Abdulla, G. Baryannis, A hybrid multi-criteria decision-making and machine learning approach for explainable supplier selection, *Supply Chain Anal.* 7 (2024) 100074, <https://doi.org/10.1016/j.sca.2024.100074>.

[9] R.K. Malviya, R. Kant, P. Kumar, S. Lahane, A.A. Pujara, A hybrid fuzzy decision-making trial and evaluation laboratory and multi-criteria decision-making approach for successful implementation of supply chain collaboration strategies, *Supply Chain Anal.* 5 (2024) 100053, <https://doi.org/10.1016/j.sca.2023.100053>.

[10] U. Tanveer, M.D. Kremantzis, N. Roussinos, S. Ishaq, L.S. Kyrgiakos, G. Vrontzos, A fuzzy TOPSIS model for selecting digital technologies in circular supply chains, *Supply Chain Anal.* 4 (2023) 100038, <https://doi.org/10.1016/j.sca.2023.100038>.

[11] E. Munch, A user’s guide to topological data analysis, 47–61, *J. Learn. Anal.* 4 (2) (2017) 47–61, <https://doi.org/10.18608/jla.2017.42.6>.

[12] G. Carlsson, M. Vejdemo-Johansson, *Topological data analysis with applications*, Cambridge University Press, Cambridge, 2021, <https://doi.org/10.7922/G20C4T4B>.

[13] M. Aruldoss, T.M. Lakshmi, V.P. Venkatesan, A survey on multi criteria decision making methods and its applications, *Am. J. Inf. Syst.* 1 (1) (2013) 31–43, <https://doi.org/10.12691/ajis-1-1-5>.

[14] T. Gal, T. Stewart, T. Hanne, *Multicriteria decision making: advances in MCDM models, algorithms, theory, and applications*, Springer Science & Business Media, 2013.

[15] N. Subramanian, R. Ramanathan, A review of applications of analytic hierarchy process in operations management, *Int. J. Prod. Econ.* 138 (2) (2012) 215–241, <https://doi.org/10.1016/j.ijpe.2012.03.036>.

[16] M. Behzadian, S.K. Otaghsara, M. Yazdani, J. Ignatius, A state-of-the-art survey of TOPSIS applications, *Expert Syst. Appl.* 39 (17) (2012) 13051–13069, <https://doi.org/10.1016/j.eswa.2012.05.056>.

[17] K. Govindan, M.B. Jepsen, ELECTRE: a comprehensive literature review on methodologies and applications, *Eur. J. Oper. Res.* 250 (1) (2016) 1–29, <https://doi.org/10.1016/j.ejor.2015.07.019>.

[18] Rd.F.S.M. Russo, R. Camanho, *Criteria in AHP: a systematic review of literature*, *Procedia Comput. Sci.* 55 (2015) 1123–1132, <https://doi.org/10.1016/j.procs.2015.07.081>.

[19] V. Pandey, Komal, H. Dincer, A review on TOPSIS method and its extensions for different applications with recent development, *Soft Comput.* 27 (23) (2023) 18011–18039, <https://doi.org/10.1007/s00500-023-09011-0>.

[20] V. Belton, T. Stewart, Problem structuring and multiple criteria decision analysis, *Trends Mult. Criteria Decis. Anal.* 209-39 (2010), <https://doi.org/10.1007/978-1-4419-5904-18>.

[21] L.A. Franco, G. Montibeller, Facilitated modelling in operational research, *Eur. J. Oper. Res.* 205 (3) (2010) 489–500, <https://doi.org/10.1016/j.ejor.2009.09.030>.

[22] D. Champion, J.M. Wilson, The impact of contingency factors on validation of problem structuring methods, *J. Oper. Res. Soc.* 61 (9) (2010) 1420–1431, <https://doi.org/10.1057/jors.2009.94>.

[23] C.M. Smith, D. Shaw, The characteristics of problem structuring methods: a literature review, *Eur. J. Oper. Res.* 274 (2) (2019) 403–416, <https://doi.org/10.1016/j.ejor.2018.05.003>.

[24] N. Gupta, P. Garg, N. Ahuja, An integrated pythagorean fuzzy delphi-AHP-CoCoSo approach for exploring barriers and mitigation strategies for sustainable supply chain in the food industry, *Supply Chain Anal.* 10 (2025) 100105, <https://doi.org/10.1016/j.sca.2025.100105>.

[25] A. Morton, B. Fasolo, Behavioural decision theory for multi-criteria decision analysis: a guided tour, *J. Oper. Res. Soc.* 60 (2) (2009) 268–275, <https://doi.org/10.1057/palgrave.jors.2602550>.

[26] A. Spyridakos, N. Tsotsolas, Y. Siskos, D. Yannakopoulos, I. Vryzidis, A visualization approach for robustness analysis in multicriteria disaggregation–aggregation approaches, *Oper. Res.* 20 (3) (2020) 1841–1861, <https://doi.org/10.1007/s12351-018-0394-1>.

[27] Momeni M.A., Kumar S., Jain V., Mostofi A., Chandra C. A Perspective on Improving the Multicriteria Decision-Making Environment. *IEEE Engineering Management Review, Engineering Management Review, IEEE, IEEE Eng Manag Rev. USA: IEEE; 2021. p. 196-203.*

[28] M.A. Matos, Decision under risk as a multicriteria problem, *Eur. J. Oper. Res.* 181 (3) (2007) 1516–1529, <https://doi.org/10.1016/j.ejor.2005.11.057>.

[29] G.D. Pelegrina, L.T. Duarte, M. Grabisch, J.M.T. Romano, Dealing with redundancies among criteria in multicriteria decision making through independent component analysis, *Comput. amp Ind. Eng.* (2022) 169, <https://doi.org/10.1016/j.cie.2022.108171>.

[30] S.S. McMillan, M. King, M.P. Tully, How to use the nominal group and delphi techniques, *Int. J. Clin. Pharm.* 38 (2016) 655–662, <https://doi.org/10.1007/s11096-016-0257-x>.

[31] S. Sorooshian, M. Tavana, S. Ribeiro-Navarrete, From classical interpretive structural modeling to total interpretive structural modeling and beyond: a half-century of business research, *J. Bus. Res.* 157 (2023) 113642, <https://doi.org/10.1016/j.jbusres.2022.113642>.

[32] B. Rolf, B. Alexander, J. Ilya, M. Marcel, R. Tobias, S. Heiner, et al., A review on unsupervised learning algorithms and applications in supply chain management, *Int. J. Prod. Res.* 63 (5) (2025) 1933–1983, <https://doi.org/10.1080/00207543.2024.2390968>.

[33] J. Li, S. Lai, Z. Shuai, Y. Tan, Y. Jia, M. Yu, et al., A comprehensive review of community detection in graphs, *Neurocomputing* (2024) 128169, <https://doi.org/10.1016/j.neucom.2024.128169>.

[34] L.A. Franco, E.A.J.A. Rouwette, Problem structuring methods: taking stock and looking ahead, in: S. Salhi, J. Boylan (Eds.), *The Palgrave Handbook of Operations Research*, Springer International Publishing, Cham, 2022, pp. 735–780, [https://doi.org/10.1007/978-3-030-96935-6\\_23](https://doi.org/10.1007/978-3-030-96935-6_23).

[35] R.L. Keeney, H. Raiffa, R.F. Meyer, *Decisions with multiple objectives: preferences and value Trade-Offs*. Cambridge, Cambridge University Press, 1993.

[36] K.J. Arrow, H. Raynaud, *Social choice and multicriterion decision-making*, 1, MIT Press Books, 1986.

[37] Carlsson J.G., Center M.T. Applying Topological Data Analysis to Logistics Systems Analysis. National Center for Sustainable Transportation (NCST)(UTC); 2024.

[38] W. Guo, A.G. Banerjee, Identification of key features using topological data analysis for accurate prediction of manufacturing system outputs, *J. Manuf. Syst.* 43 (2017) 225–234, <https://doi.org/10.1016/j.jmsy.2017.02.015>.

[39] H.M.A. Farid, M. Bouye, M. Riaz, N. Jamil, Fermatean fuzzy CODAS approach with topology and its application to sustainable supplier selection, *Symmetry* 15 (2) (2023) 433, <https://doi.org/10.3390/sym15020433>.

- [40] A. Razzaq, M. Riaz, M. Aslam, Efficient picture fuzzy soft CRITIC-CoCoSo framework for supplier selection under uncertainties in industry 4.0, *AIMS Math.* 9 (1) (2024) 665–701, <https://doi.org/10.3934/math.2024035>.
- [41] Mojdehi K.F., Amiri B., Haddadi A. A Novel Hybrid Model for Credit Risk Assessment of Supply Chain Finance Based on Topological Data Analysis and Graph Neural Network. *IEEE Access*, Access, IEEE: IEEE; 2025. p. 13101-13127.
- [42] R.L. Keeney, Value-focused thinking: identifying decision opportunities and creating alternatives, *Eur. J. Oper. Res.* 92 (3) (1996) 537–549, [https://doi.org/10.1016/0377-2217\(96\)00004-5](https://doi.org/10.1016/0377-2217(96)00004-5).