An Analysis of the Interval of Observation and the Risk in Stocks

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An Analysis of the Interval of Observation and the Risk in Stocks

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Abstract

This research examines how the interval of observation affects the assessment of risk in stocks. I do this by analysing the economic and statistical significance of the worst returns on stocks, and by analysing the relationship between the interval of observation and factors which are thought to affect the return on stocks. This research shows the interval of observation used to assess the risk in stocks is important and the conclusions change considerably depending on how the data is drawn. In addition, the results indicate an investor's time horizon is important in deciding their asset allocation and the style of investment should be suitable for the time horizon selected.

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1 Introduction

How bad can it get when investing in stocks? This is a question investors ask themselves and they know for short time horizons it can get very bad. For the S&P 500 Index in the US, during the Great Depression the annualised real return was -43.7 percent from August 1929 to May 1932; during the 1973 oil crisis the annualised real return was -32.9 percent from December 1972 to September 1974; and during the financial crisis of 2008 the annual real return was -38.9 percent to the end of October.

For long time horizons, it is not as bad. Siegel (1992; 2002) shows the probability the real return on stocks is negative is zero for time horizons of 17 years or more. For time horizons of 30 years or more, the probability the real return on stocks is less than the real return on bond and bills is almost zero too. Many studies support the view that investing for long time horizons is beneficial for investors (see Connelly, 1996; Malkiel, 2003; Reichenstein & Dorsett, 1995; and Thorley, 1995 for details).

Nevertheless, investors seem reluctant to invest for long time horizons. Quill (2001) examines how long investor time horizons are and finds they range from 2.9 to 5.5 years. This indicates investor time horizons are not as long as those used in the studies stated. It is undoubtedly difficult to be a ‘buy-and-hold’ investor when the immediate future looks imminently bleak. In addition, many studies do not support the view that investing for long time horizons is beneficial for investors (see Bodie, 1995; Jorion, 2003; Samuelson, 1969, 1994; and Taleb, 2005, 2007 for details). These studies suggest the risk in stocks and the impact of the highly improbable are not well understood. Samuelson (1994) and Taleb (2005; 2007) assert that assessing the history of stocks leads investors to misunderstand risk as “we have only one history of capitalism” (Samuelson, 1994, p. 17).

In this research, I examine how time horizon affects the assessment of risk in stocks. Time horizon relates to the interval of observation and the importance of this is recognised by Jacobsen, Marshall & Visaltanachoti (2008). They show there are theoretical reasons to carefully consider the interval of observation in research. They show using different intervals of observation influences the level of noise in the data and impacts on the implications which are drawn. I show the distribution of annualised stock returns decreases as the interval of observation increases, and converges towards the median. In addition, the annualised standard deviation of stocks returns decreases with increases in the interval of observation. This is the first contribution to the literature in this research.

The second contribution is to expand these studies by using tests of statistical significance on the relationship between the interval of observation and the worst returns on stocks. I show the worst returns are negative and both statistically and economically significant when the interval of observation is less than 15 years. However, when the interval of observation is 22 years or more, the worst returns are positive and both statistically and economically significant as they converge towards the median.

The third contribution is to expand these studies by analysing the worst returns on stocks and the factors which are thought to affect them. I examine the worst returns on stocks and what relationships there are with economic and commodity factors as the interval of observation changes. The relationships are determined by defining the worst returns on stocks as bleak investment periods and comparing them with non-bleak investment periods. I show the relationship between economic factors and bleak investment periods in stocks is as expected when compared to non-bleak investment periods in stocks; both are bleak. As the interval of observation increases this relationship remains. The exception is unemployment which changes in bleak investment periods from being worse when the interval of observation is shorter to being better when the interval of observation is longer.

The relationship between metal and agricultural factors, and bleak investment periods in stocks is not as expected. At shorter intervals of observation, bleak investment periods are associated with metal and agricultural factors being worse compared to non-bleak investment periods; however, at longer intervals of observation, the relationship changes. Most agricultural factors are better in bleak investment periods whereas some metal factors are worse and some are better.

In addition, the difference between all factors in bleak and non-bleak investment periods changes as the interval of observation changes; and at longer intervals of observation the difference is less. This indicates as the interval of observation increases, factors do not affect the returns on stocks as much, as bleak and non-bleak investment periods are not as different.

The results of this research indicate the interval of observation is important in the assessment of risk in stocks. The longer the interval of observation is, the less risk in stocks there is, and even the worst returns should be satisfactory. The results also indicate some factors which affect the return on stocks at short intervals of observation, do not affect the return on stocks in the same manner at long intervals of observation. These results suggest an investor's time horizon is important when deciding their asset allocation and the style of investment should be suitable for the time horizon selected.

In this chapter, I introduced the research motivation, findings and implications. In chapter two, I review literature on the interval of observation, stocks' risk and return as time horizon changes and factors which affect the return on stocks. I describe the data and methodology used in this research in chapter three. Then I present and discuss the results in chapter four. Finally, in chapter five, I present the conclusions of this research.

2 Literature Review

2.1 Introduction

An introduction to this research is given in chapter 1. In this chapter I review the literature for this research. Section 2.2 discusses literature on the interval of observation. Section 2.3 discusses time diversification literature. Section 2.4 discusses literature on factors which affect the return on stocks. Section 2.5 summarises the chapter.

2.2 The Interval of Observation

Jacobsen et al. (2008) show the interval of observation affects the analysis of stocks. They examine the predictability of stock returns and find they are more predictable when improved intervals of observation are used. Monthly observations are traditionally used to predict monthly stocks returns; however, they state that "a full month of data may increase noise and understate or completely obscure the shorter term predictability that is present" (Jacobsen et al., 2008, p. 36). Their results show monthly stock prices are predictable when price changes in industrial metal commodities are taken over intervals of less than one month whereas price changes in energy commodities take more than one month to be reflected in stock prices. Their results for stock return predictability support the conclusion that "the interval of observation may be very important" (Kendall, 1953, p.18 cited in Jacobsen et al., 2008, p. 5).

Siegel (1992; 2002) examines the return on stocks, bonds and bills from 1802 to 2001. He shows as the interval of observation increases, the probability stocks outperform bonds and bills increases. For 1 year time horizons, the return on stocks is greater than the return on bonds and bills over 60 percent of the time. For 30 year time horizons, it is almost 100 percent of the time. With the risk in stocks measured as the probability of stocks underperforming less risky assets, these results indicate the risk in stocks decreases as the interval of observation increases. The implications of these results relate to the debate on time diversification which is discussed next.

2.3 The Time Diversification Debate

2.3.1 Time Diversification

Time diversification is summarised as "the dual proposition that, over long periods of time stock market returns have dominated returns of less risky assets, and that negative returns tend to be offset by positive" [1] (Connelly, 1996, p. 20). Figure 2.1 shows a simulated distribution of this adapted from Dimson, Marsh & Staunton (2004). Based on this, investors are recommended to increase their portfolio's stock allocation as their time horizon increases (Malkiel, 2003; Siegel, 2002). Figure 2.2 shows a guide of this adapted from Malkiel (2003). These studies support the view that investing for long time horizons is beneficial for investors.

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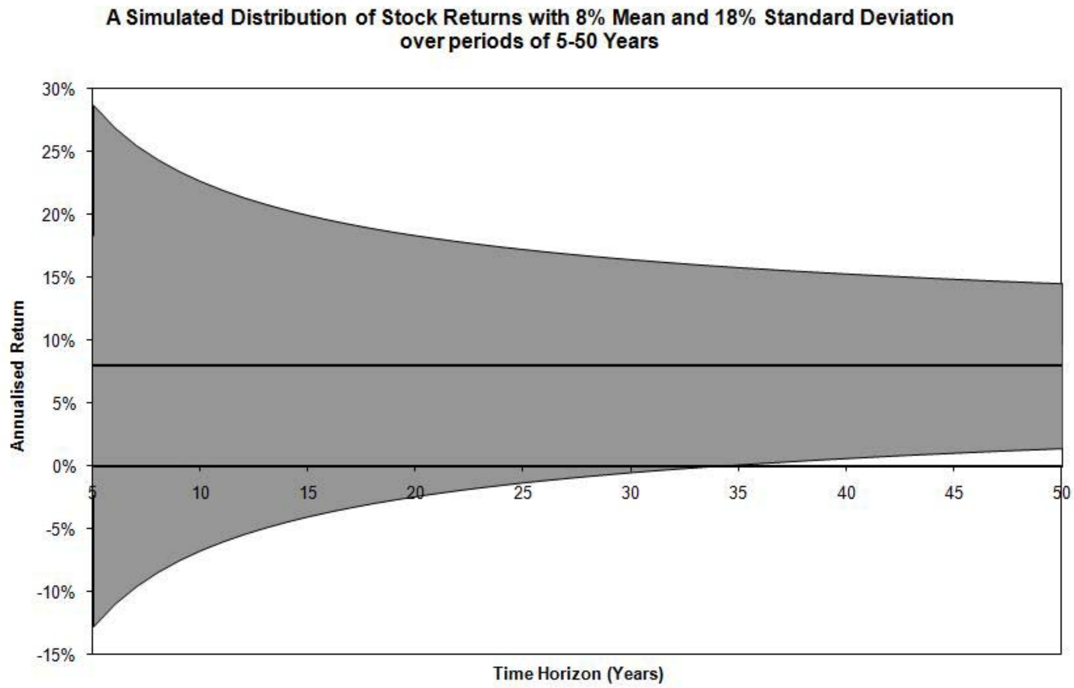
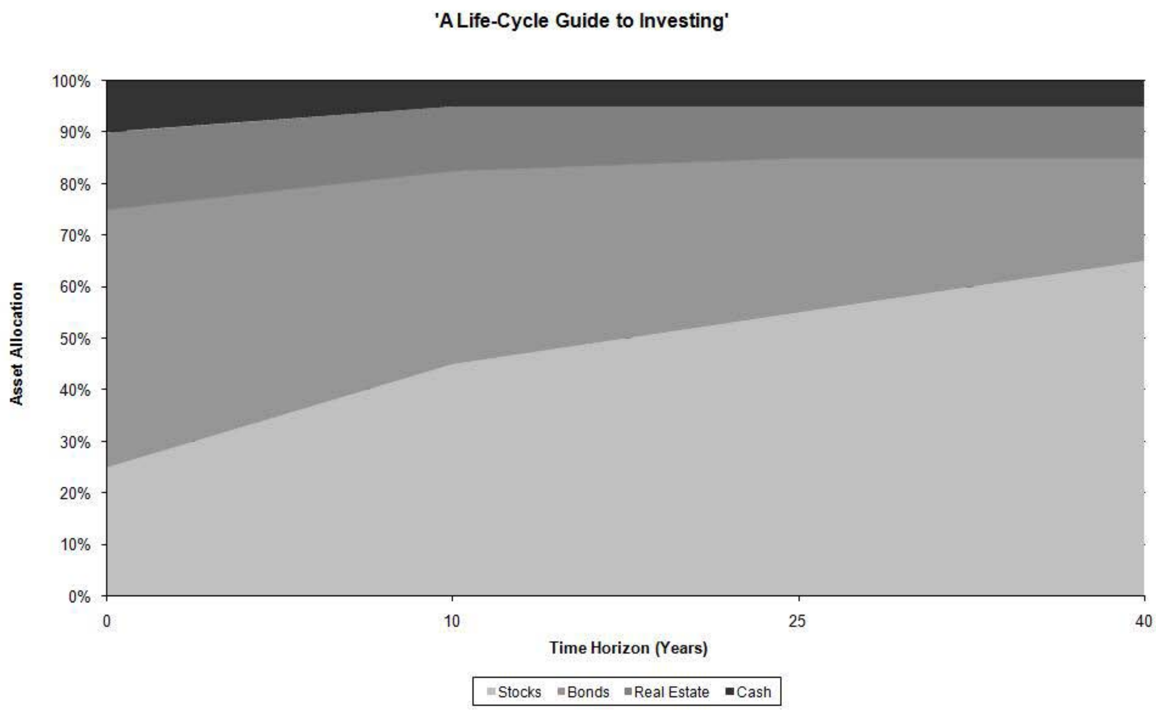
[1] Note: This relates to mean reversion which is when returns move back to a common trend and this indicates risk decreases as the interval of observation increases (Jorion, 2003).

Figure 2.1 A Simulated Distribution of Stock Returns[a]



[a] adapted from Dimson et al. 2004.

Figure 2.2 Recommended Asset Allocations[b]



[b] adapted from Malkiel, 2003.

Quill (2001) studies whether investors do this. Using redemption rates of mutual funds to calculate implied mutual fund holding periods from 1996 to 2000, he finds the implied holding period ranges from 2.9 to 5.5 years.

Whether investing for long time horizons is beneficial for investors is debated too. This debate is based on utility theory, options pricing theory, simulations and research comparing countries. Utility theory and time diversification are discussed first.

2.3.2 Time Diversification and Utility Theory

Samuelson (1994) cites a model (see Samuelson, 1969 for details) which shows time horizon cannot have any effect on an investor's portfolio proportions between riskless and risky assets. If an investor wants to maximise the expected value of their logarithm of wealth outcomes then they will act to maximise the geometric mean of their terminal wealth in all its likely possible outcomes which leads to age and time horizon being ignored. The model assumes investors have a utility function which is risk-averse and displays constant relative risk aversion[2], and act to maximise the utility function's expected value. In addition, the model assumes investors face a probability process that is stationary and each period's probabilities are independent.

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[2] Note: Constant relative risk aversion: "Percentage invested in risky assets is unchanged as wealth increases" (Elton, Gruber, Brown & Goetzmann, 2003, p. 219).

Samuelson (1994) states some exceptions to this:

- First, workers can invest more of their portfolio in stocks when they are young and they can hope to work harder later to compensate for losses in stocks.
- Second, if stock returns display mean reversion, then risk-averse investors should invest more in stocks when they are young.
- Third, if investors display decreasing relative risk aversion[3], then the stock proportion of a portfolio should increase as time horizon increases.

In addition, Kritzman (1994) states if an investor prefers a riskless asset to a risky asset over a short time horizon, then the investor should also prefer a riskless asset to a risky asset over a long time horizon. This is demonstrated with an example in table 2.1 which presents the distribution of a risky asset's outcomes over three periods, with the expected wealth and utility[4]. The expected utility of the risky asset always equals 4.61; therefore, diversifying across time does not create additional satisfaction.

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[3] Decreasing relative risk aversion: "Percentage invested in risky assets increases as wealth increases". (Elton et al., 2003, p. 219)

[4] Note: The formulae for these calculations are in appendix A. This example assumes an investor has 100 units of wealth with a utility function equal to the logarithm of wealth. This equals 4.61 units of utility (ln(100)=4.61)). In addition, it is assumed there is an investment that has a 0.5 probability of a ⅓ gain and a 0.5 probability of a ¼ loss.

Table 2.1 An Example of a Utility Function and Time Horizon[c]

Utility = ln(wealth)		**Distribution of Wealth After**	
Initial Wealth	**One Period**	**Two Periods**	**Three Periods**
		¼ × 177.78	⅛ × 237.04
	½ × 133.33		⅛ × 133.33
		¼ × 100.00	⅛ × 133.33
100			⅛ × 75.00
		¼ × 100.00	⅛ × 133.33
	½ × 75.00		⅛ × 75.00
		¼ × 56.25	⅛ × 75.00
			⅛ × 42.19
Expected Wealth 100	104.17	108.51	113.03
Expected Utility 4.61	4.61	4.61	4.61

[c] taken from Kritzman, 1994.

Kritzman (1994) states some exception to this:

- First, if an investor believes a risky asset displays mean reversion and an investor is more risk averse than a log wealth investor, then they will rationally increase the proportion of risky assets in their portfolio as time horizon increases
- Second, if an investor believes the returns of an investment are random, they might accept more risk over longer time horizons than shorter time horizons as they can change their level of consumption and work[5].

[5] Note: Challenges to time diversification assume an investor's terminal wealth depends on investment performance only.

- Third, if an investor has a discontinuous utility function and requires a minimum level of wealth to maintain a certain standard of living, then given a risky asset, an investor might be more likely to break their required minimum level of wealth over shorter time horizons than longer time horizons.
- Fourth, if an investor believes an event which causes an extremely bad outcome in a risky asset might also cause an extremely bad outcome in a riskless asset, then challenges to time diversification based on the size of potential loss of risky assets should be applied to riskless assets as well.

These studies are criticised though. Van Eaton & Conover (1998) address whether economic theory is in disagreement with time diversification and they state the assumptions made by Samuelson (1969; 1994) which lead to the optimal asset allocation being the same for a range of time horizons addresses the question of how an increase in the number of fixed holding periods affects the optimal asset allocation at each fixed holding period. However, the question of how a longer time horizon affects the asset allocation decision remains unaddressed. The example of a utility function by Kritzman (1994) showing an investor's preference for a risky asset is invariant to time horizon is also criticised. By changing the utility function and the return processes, it is shown the applicability of these examples is dependent on the inputs; and utility is shown to increase, decrease and remain constant as time horizon increases.

Thorley (1995) suggests models used to criticise time diversification have been inappropriately applied. Models of investor preference which are adequate for short time horizons have been applied to asset allocation preferences over long time horizons inappropriately. It is stated “if rational and informed investors make choices that contradict a given model of investor preference, then the application of the model is wrong, not the investors” (Thorley, 1995, p. 68). The optimal risky asset allocation at different time horizons, given lognormal returns, is explored under two expected utility functions; constant relative risk aversion and decreasing relative risk aversion. The optimal risky asset allocation at different time horizons under constant relative risk aversion is shown to be constant. However, under decreasing relative risk aversion it is shown to increase with time horizon. For decreasing relative risk aversion, an investor will allocate proportionally more of their portfolio to risky assets as time horizon increases and therefore, time diversification is not a fallacy.

On the other hand, Thorley (1995) uses mean-variance utility analysis to show as time horizon increases, the optimal risky asset allocation decreases. It is stated mean-variance utility analysis is useful in understanding investor choices in portfolio theory over fixed holding periods, but does not lead to reasonable results over changing holding periods.

Van Eaton & Conover (1998) use mean-variance utility analysis to show it is a special case when time horizon increases, the optimal risky asset allocation decreases. When the parameters are changed, mean-variance utility analysis is used to show that the optimal risky asset allocation increases, decreases, and increases then decreases when time horizon increases. Van Eaton & Conover (1998) conclude the examples used show the effect of time horizons on optimal risky asset allocation is ambiguous and depends on individual preferences and the opportunities present.

The literature on time diversification and utility theory shows there is much debate. Time diversification and options pricing theory are discussed next.

2.3.3 Time Diversification and Options Pricing Theory

Bodie (1995) uses the cost[6] of insuring a portfolio against earning less than the riskless rate to show that the risk in stocks increases with increases in time horizon. It is argued "if stocks were truly less risky in the long run than in the short run, then the cost of insuring against earning less than the risk-free rate of interest should decline as the investment horizon lengthens" (Bodie, 1995, p. 19). This is demonstrated with an example in table 2.2 and figure 2.3 which shows the cost of shortfall[7] insurance as a function of time horizon increases as time horizon increases and therefore, the risk in stocks increases with time horizon[8].

[6] Note: The cost of insuring a portfolio against earning less than the riskless rate is calculated using the Black-Scholes option pricing model, in which the cost of insurance is a put option.

[7] Note: A shortfall is defined as when the terminal value of a stock portfolio is less than some value determined by a specified target rate of return such as the terminal value of default-free zero-coupon bond.

[8] Note: The formulae for these calculations are in appendix B. This example assumes the annualised standard deviation of stock returns is 20 percent. The cost of insurance is assumed to be independent of the riskless rate.

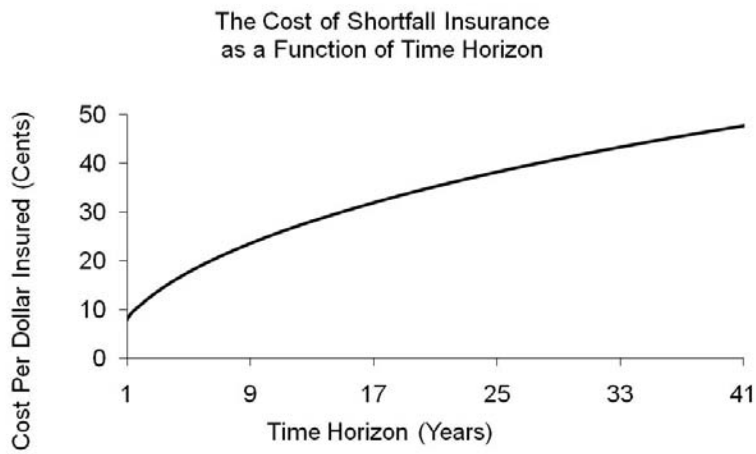
Table 2.2

An Example of the Cost of Shortfall Insurance as a Function of Time Horizon[d]

Time Horizon	Cost per Dollar Insured in Cents
1 Year	7.97
5 Years	17.69
10 Years	24.82
20 Years	34.53
30 Years	41.61
50 Years	52.05
75 Years	61.35
100 Years	68.27

[d] adapted from Bodie, 1995.

Figure 2.3 An Example of the Cost of Shortfall Insurance as a Function of Time Horizon[d]



As time horizon increases, the cost of shortfall insurance increases; however, this is criticised. Taylor & Brown (1996) state assuming a constant standard deviation for stocks over all time horizons may not be appropriate. It is argued the most important variable in the Black-Scholes options pricing model is the standard deviation of annualised stock returns and empirical evidence has been presented showing the annualised standard deviation of a well-diversified stock portfolio decreases as time horizon increases. They show as time horizon increases the cost of shortfall insurance decreases. This is demonstrated with an example in table 2.3 and figure 2.4[9].

[9] Note: The formulae for these calculations are in appendix B. This example assumes the annualised standard deviation of stock returns is 20 percent and it decreases as time horizon increases. *i* is a positive number equal to 0.05 and T=1. The cost of insurance is derived from a simplified Black-Scholes options pricing formula with the cost of insurance being independent of the riskless rate.

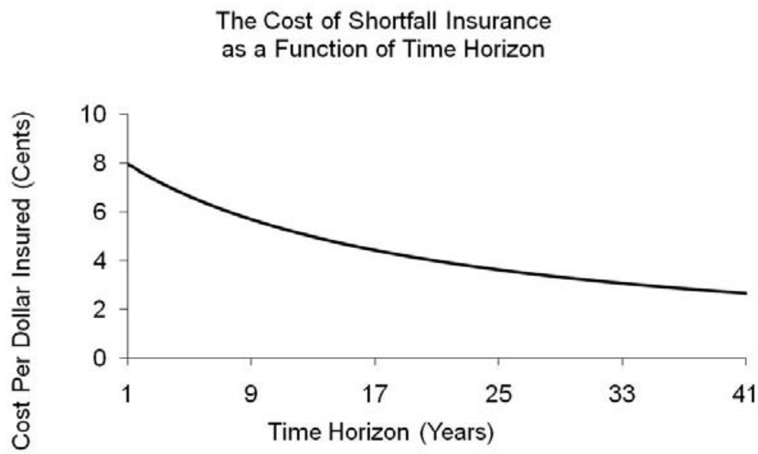
Table 2.3

An Example of the Cost of Shortfall Insurance as a Function of Time Horizon[e]

Time Horizon	Cost per Dollar Insured in Cents
1 Year	7.97
5 Years	6.64
10 Years	5.50
20 Years	4.09
30 Years	3.26
50 Years	2.31
75 Years	1.70
100 Years	1.34

[e] adapted from Taylor & Brown, 1996.

Figure 2.4 An Example of the Cost of Shortfall Insurance as a Function of Time Horizon[e]



Options pricing theory is used in other studies as "option pricing theory provides an objective and empirically verifiable cost associated with risk elimination, where risk is defined as the possibility that the investment's final value will fall outside a specified range" (Merrill & Thorley, 1996, p. 13). Merrill & Thorley (1996) use protected equity notes (PEN), in which a PEN is the name of a product offered by investment banks which guarantees a minimum rate of return and a proportion of any stock increase above the guaranteed minimum rate of return. A PEN can be constructed from a riskless asset and a call option in which the riskless asset guarantees a minimum annual rate of return (as long as the guaranteed rate of return is less than or equal to the riskless rate) and the call option allows the investor to participate in a proportion of any stock return above the guaranteed rate of return. It is shown for any guaranteed rate of return less than the riskless rate, the participation rate increases with time horizon, indicating the cost of insurance is lower on a per annum and value basis for long time horizons, and investing in stocks is less risky for long time horizons than for short.

Merrill & Thorley (1996) use a self-funding collar-cap as an alternative to a PEN in which a collar-cap is an options strategy which guarantees a minimum rate of return by holding a put option, limits a maximum rate of return by writing a call option and is self-funding. It differs from a PEN as a PEN provides a proportion of any stock increase above the guaranteed minimum rate of return whereas a collar provides all of any stock increase above the guaranteed minimum rate of return to a limit. As time horizon increases the maximum possible annual return increases and at the same time the minimum rate of return is guaranteed; therefore, it is less costly to insure against underperformance over a longer time horizon than a shorter one. “As with a PEN, the lower cost of risk reduction suggests that risk itself is lower, consistent with the principles of time diversification” (Merrill & Thorley, 1996, p. 17).

The use of options pricing theory in the time diversification debate is criticised though. Van Eaton & Conover (1997) look at put prices and PEN participation rates at long time horizons and argue there is not an unambiguous answer which provides convincing evidence about how the risk in stocks changes at long time horizons. The result from Bodie (1995), uses the riskless rate as the growth rate of the exercise price, but for exercise price growth rates less than the riskless rate, the cost of shortfall insurance reaches a maximum and then decreases at longer time horizons. This “calls into question the generality of the Bodie (1995) put-pricing result” (Van Eaton & Conover, 1997, p. 69). For the Merrill &

Thorley (1996) derivative strategies, it is argued there is an error "in interpreting certain features of the derivative strategies they examined as proxies for equity risk" (Van Eaton & Conover, 1997, p. 67).

The literature on time diversification and options pricing theory shows there is much debate about how the risk in stocks changes with time horizon. Time diversification and simulations are discussed next.

2.3.4 Time Diversification and Simulations

Kritzman (1994) states that for long time horizons, the decreasing probability of a loss is offset by the increasing size of potential loss. The distribution of annualised stock returns converges towards the expected return as time horizon increases. However, the distribution of terminal wealth diverges from the expected terminal wealth as time horizon increases; therefore, investors are less likely to lose money over long time horizons than over short. However, the size of potential loss increases as time horizon increases. Using hypothetical investments and simulations, this is demonstrated with an example in table 2.4[10].

[10] Note: The formulae for these calculations are in appendix C. This example assumes an investor has $1000 and there are two investment choices – a riskless asset with a continuous annual riskless return of 3 percent and a lognormally distributed risky asset with a continuous annual expected return of 10 percent and standard deviation of 15 percent.

Table 2.4 The Distribution of Riskless and Risky Asset Terminal Wealth Outcomes[f]

Terminal Wealth of Riskless and Risky Assets

Time Horizon	Terminal Wealth of Riskless Asset	Risky Asset with 95% Confidence Interval and Expected Terminal Wealth		
		Lower Boundary	**Expected**	**Upper Boundary**
1 Year	$1,030	$820	$1,100	$1,476
5 Years	$1,159	$835	$1,611	$3,108
10 Years	$1,344	$1,024	$2,594	$6,572
15 Years	$1,558	$1,338	$4,177	$13,044
20 Years	$1,806	$1,807	$6,728	$25,053

[f] taken from Kritzman, 1994.

Kritzman (1994) shows after 20 years, even though the terminal wealth of the lower boundary of a 95 percent confidence interval of the risky asset is greater than the terminal wealth of the riskless asset, the lower boundary of a 99 percent confidence interval is less than the riskless asset ($1,195).

Thorley (1995) uses ‘Practitioner Risk Measures’ to demonstrate a contrasting point in table 2.5[11]. ‘Practitioner Risk Measures’ include: the 10th and 90th percentiles of a risky asset; the probability a risky asset underperforms a riskless asset; and the expected value of a risky asset given it underperforms the riskless asset. It is concluded an investor can be almost certain that a risky asset will make them wealthier than a riskless asset at a 40 year time horizon. The risk associated with the value of a risky asset after 40 years is uncertainty as to how much more the risky asset will be worth compared to the riskless asset as there is

[11] Note: The formulae for these calculations are in appendix C. This example assumes an investor has $1000 and there are two investment choices – a riskless asset with a continuous annual riskless return of 4 percent and a lognormally distributed risky asset with a continuous annual expected return of 12 percent and standard deviation of 16 percent.

a 99.9 percent probability that the risky asset will outperform the riskless asset. These results are based on two important assumptions: first, the expected return of the risky asset is more than the return on the riskless asset and second, the risky asset's expected returns are temporally uncorrelated.

Table 2.5 'Practitioner Risk Measures' of a Risky Asset[g]

		Risky Asset			Underperformance	
Time Horizon	**Riskless Value**	**Mean**	**10th Percentile**	**90th Percentile**	**Probability**	**Risky Mean**
1 Year	$1,041	$1,142	$918	$1,384	30.9%	$942
5 Years	$1,221	$1,1943	$1,152	$2,882	13.2%	$1,032
10 Years	$1,492	$3,773	$1,736	$6,350	5.7%	$1,222
20 Years	$2,226	$14,239	$4,406	$27,578	1.3%	$1,776
40 Years	$4,953	$202,755	$33,220	$444,451	0.1%	$3,875

[g] taken from Thorley, 1995.

"Unless some logical flaw exists…, the example demonstrates that the same investor with the same two options will be more inclined to choose the riskier alternative the longer the investment horizon. If true, then any model of investor preferences that does not have this property (as time horizon increases, risk decreases) is not useful in analysing variable horizon choices" (Thorley, 1995, p. 70).

These simulations assume the risky asset is a normally distributed variable and the probabilities are calculated based on the standard normal cumulative density function. This leads to extreme events causing extremely negative returns being mostly ignored as the probability of them occurring is very low. In addition, looking back through the history of US stocks shows there have not been any extreme events which resulted in extremely negative returns for long time horizons. However, “we have only one history of capitalism” (Samuelson, 1994, p. 17). There are academics who suggest the law of large number and the impact of the highly improbable are not well understood and therefore, neither is the risk in stocks (see Kritzman & Rich, 1998; Samuelson, 1963, 1994; and Taleb, 2005, 2007 for details).

The literature on time diversification and simulations shows there is much debate on how the risk in stocks is assessed. Time diversification and research comparing countries are discussed next.

2.3.5 Time Diversification and Research Comparing Countries

Time diversification is examined in other countries as previous empirical studies focus mainly on the US. “There are reasons to suspect that these estimates are subject to survivorship bias, as the US is arguably the most successful capitalist system in the world” (Jorion & Goetzmann, 1999, p. 953). In addition, as time horizon increases the number of independent observations decreases. Investigating time diversification in more countries increases the number of independent observations. Jorion & Goetzmann (1999) use a sample of 39 countries with monthly capital appreciation returns measured in nominal terms in the local currency, in real terms deflated by the Wholesale Price Index (WPI) and in US dollar returns, over most of the 20th century. Stock markets which survived, experienced temporary interruptions and experienced permanent interruptions are included. It is found from 1921 to 1996, US stocks have an annual real return of 4.3 percent compared to a median of 0.8 percent for other countries, which is the highest. It is concluded if only ‘winner’ stock markets are included and ‘loser’ stock markets are not, then a biased view of history is provided “which ignores important information about actual investment risks” (Jorion & Goetzmann, 1999, p. 979).

Jorion (2003) uses a sample of 30 countries with long return series. The data uses returns measured in real terms, deflated by the WPI, to adjust for the changing rates of inflation experienced by many countries over the long-term. Variance ratios are calculated to determine evidence of mean reversion. Poterba and Summers (1988) show that variance ratios of long-term returns to short-term returns are lower than expected under the assumption of a random walk which is used as evidence of mean reversion; therefore, long-term returns should not be as risky as short-term returns and increased proportions of stocks in a portfolio are justified for long time horizons. Variance ratios lower than one are evidence of mean reversion; however, Jorion (2003) shows based on simulations, variance ratios are expected to decrease with time. Some variance ratios from the countries sampled are lower than the median variance ratios from the simulations, but they are not statistically significant; therefore, “there is no evidence of mean reversion” (Jorion, 2003, p. 12). In addition, stock markets which experience an interruption display mean aversion. The risk of investment in non-US stocks measured by the probability of loss and 95 percent Value at Risk (VaR) is greater than the risk of investment in US stocks. It is concluded that time diversification is less beneficial, the probability of losses on stocks decreases slowly with time horizon and “across-country diversification is more effective than across-time diversification” (Jorion, 2003, p. 25).

Dimson et al. (2004) study real stock returns in 16 countries from 1900 to 2002. All returns are deflated by the Consumer Price Index (CPI), include reinvested dividends and are gross of transaction costs, fees and taxes. It is shown in the US, real stock returns are higher and volatility is lower than in many other countries and therefore, this is a favourable history. It is stated that investors should be cautious about extrapolating these patterns into the future without consideration, because of survivorship bias and sampling error. It is argued the success of stocks in the US is an example of survivorship bias, and this is not typical of other countries or of the future. This survivorship bias is called a 'success bias'. It is also argued there is sampling error as the number of historical outcomes is limited whereas the number of future outcomes is possibly infinite.

A number of interesting results are presented by Dimson et al. (2004). These include:

- Autocorrelation of annual real stock returns in the US over 103 years was 0.01 and therefore, they are actually independent, contrary to beliefs of mean reversion or mean aversion.
- The US, Canada, Australia and Denmark have never experienced a negative 20 year real stock return whereas Germany, Italy, Belgium, France and Spain have experienced a negative 20 year real stock return with a frequency of 1 in 4 or greater.
- Global diversification via a world index will lower US investor risk, but overseas stocks underperform US stocks; therefore, global diversification will also lower their return.
- The future distribution of real returns on stocks is explored under various assumptions as real stock returns are projected to be lower and the probability of a

negative real return in stocks over long time horizons is 'substantial' regardless of the time horizon.

- Finally, it is stated "investors or corporations who assume… that stocks are safe, so long as they are held for 20 years are optimists. Their optimism is irrational" (Dimson et al., 2004, p. 24).

The literature on time diversification and research comparing countries shows there is a 'success bias' in US stocks when compared to non-US stocks. Literature on factors which affect the return on stocks are discussed next.

2.4 Factor which Affect the Return on Stocks

Chen, Roll & Ross (1986) test whether economic factors have a systematic influence on stocks and find several to be significant in explaining expected returns. Macroeconomic factors are usually considered to cause a response in stock prices; however, the relationship is not exclusively in one direction (Chen et al., 1986). All data are monthly and from 1953 to 1983 macroeconomic factors including industrial production, changes in the risk premium, changes in the yield curve, changes in expected inflation and unanticipated inflation are determined to be significant in explaining expected stock returns, whereas real per capita consumption and oil price changes are not.

Fama & French (1992; 1993) use a three factor model to explain the returns on stocks. The three factors used are the excess return on the market portfolio of stocks, firm size and book-to-market equity. They show these three factors proxy for risk and explain average monthly returns on stocks.

Santa-Clara & Valkanov (2003) investigate stock returns under Republican and Democratic presidents from 1927 to 1998. They show “the average excess return of the value-weighted CRSP index over the three-month Treasury bill rate has been about 2 percent under Republican and 11 percent under Democratic presidents” (Santa-Clara & Valkanov, 2003, p. 1841). The results are both statistically and economically significant. In addition, “the difference is due to real market returns being higher under Democrats by more than 5 percent, as well as to real interest rates being almost 4 percent lower under Democrats” (Santa-Clara & Valkanov, 2003, p. 1841). The difference in returns across Republican and Democratic presidents is largely a difference of unexpected returns rather than expected returns.

Driesprong, Jacobsen & Maat (2008) investigate oil prices and stocks as “the impact of oil price changes on the world economy is large” (Driesprong et al., 2008, p. 2) and find changes in oil prices predict stock returns worldwide. Monthly stock data from 48 countries, a world market index and price series of several types of oil over almost 30 years are used to investigate whether changes in oil prices predict stock returns. The results show after oil price increases, stock returns are likely to be lower, and after oil price decreases, stock returns are likely to be higher. In many countries and the world market index, this predictability is both statistically and economically significant. There is evidence “consistent with an under-reaction hypothesis, as it appears to take time before information about oil price changes becomes fully reflected in stock market prices” (Driesprong et al., 2008, p. 2). In addition, as stated in section 2.2, Jacobsen et al. (2008) examine the predictability of stock returns and find changes in price of commodity factors predict returns on stocks.

The literature shows factors related to the economy, firm-specific risk, the party of the President, oil and commodities, affect the return on stocks. In these studies, intervals of observation of 1 month are used to examine factors which affect the return on stocks.

2.5 Summary

This chapter reviewed literature on the interval of observation, time diversification and factors which affect the return on stocks. I describe the data and methodology next.

3 Data and Methodology

3.1 Introduction

A review of the literature is given in chapter 2. In this chapter I describe the data and methodology used in this research. Section 3.2 discusses the data and methodology used in examining the interval of observation and the return on stocks. Section 3.3 discusses the tests of statistical significance used. Section 3.4 discusses the data and methodology used to examine the relationship between the interval of observation and factors which are thought to affect the return on stocks. Section 3.5 summarises the chapter.

3.2 The Interval of Observation and the Return on Stocks

The objective of this research is to examine how the interval of observation affects the assessment of risk in stocks. In order to do this, I calculate the return on stocks as the interval of observation changes. The intervals of observation range from 1 to 30 year time horizons, and are calculated on a rolling monthly basis. This allows for all time horizons to be calculated with little bias to the initial month. All data used in this research are from Global Financial Data (GFD) and the frequency is monthly. For the stocks sample, the data are the 'S&P 500 Total Return Index with GFD extension' and begins in November 1832 and ends in March 2008 (see table 3.1). The data are deflated by the Consumer Price Index (CPI), are gross of transaction costs and include reinvested dividends. I do not make any deductions for fees and taxes. This approach to the data is consistent with the study by

Dimson et al. (2004) and allows for concerns about the effects of inflation on distorting returns. To calculate the return on stocks for a range of intervals of observation, I use the following formulae:

$$R^{a}_{t-k,t} = \ln\left(P_t \big/ P_{t-k} \right) \big/ n \qquad (1)$$

and

$$R_{t-k,t} = \left(\frac{P_t}{P_{t-k}} \right) - 1 \qquad (2)$$

where

$R^{a}_{t-k,t} =$ the annualised return at time t over $t-k$

$P_t =$ the value of a variable at month t

$P_{t-k} =$ the value of a variable at month $t-k$

$k =$ the interval of observation in months

$n =$ the number of years in the interval of observation, i.e. $k/12$

$R_{t-k,t} =$ the total return at time t over $t-k$

Continuously compounded returns are used for annualised returns, because of their additive properties (equation 1). There is a 'smoothing effect' when total returns are continuously compounded; therefore, arithmetic returns are used as it shows the 'full effect' (equation 2).

Table 3.1 Summary of Stock Data

Variable	**GFD Source Name**	**From**	**To**
Stocks	S&P 500 Total Return Index (w/GFD extension)	11/1832	03/2008

I assess the risk in stocks by examining the worst returns on stocks and the annualised standard deviation of stocks returns. I calculate the 0^{th}, the $2½^{th}$ and the 5^{th} percentile of annualised stock returns as the interval of observation changes. In addition, I calculate the annualised standard deviation of stocks returns as the interval of observation changes. These calculations are used as assessments of the risk in stocks. The statistical significance of these are calculated next.

3.3 Tests of Statistical Significance

The *t*-test is used to examine whether the annualised stock returns at the 0^{th}, the $2½^{th}$ and the 5^{th} percentile are statistically significantly different from zero as the interval of observation changes. In addition, the Wilcoxon signed rank sum test is used to examine whether the annualised stock return at the 2½th percentile is statistically significantly different from zero as the interval of observation changes. The *t*-test is a parametric test whereas the Wilcoxon signed rank sum test is a non-parametric test; therefore, by using both tests, assumptions about the sample being normally distributed are less of a concern. However, both tests assume the sample is independent. The accuracy of this assumption in this sample is limited as the return on stocks is calculated from overlapping periods; nevertheless, the tests are used as an indication of how statistically significantly different the worst returns on stocks are from zero as the interval of observation changes.

To calculate the *t*-test, I use the following modified formula:

$$t = \frac{Percentile_n}{\left(s_n / \sqrt{N_n}\right)} \qquad (3)$$

where

$t =$	the *t*-test
$n =$	the interval of observation in years
$Percentile_n =$	the annualised stock return at the 0th, the 2½th and the 5th percentile for an *n* year interval of observation
$s_n =$	the annualised standard deviation of stock returns for an *n* year interval of observation
$N_n =$	number of observations at an *n* year interval of observation

In addition, I use a modified *t*-test to examine whether the annualised standard deviation of stock returns at a certain interval of observation is statistically significantly different from the standard deviation of stock returns at a 1 year interval of observation. This test of statistical significance is novel and the accuracy of it is limited; nevertheless, it is used as an indication of how statistically significantly different annualised standard deviation is from 1 year standard deviation as the interval observation changes.

I use the following formula:

$$t = \frac{s_1 - s_n}{\sqrt{\frac{s_1^2}{N_1} + \frac{s_n^2}{N_n}}} \qquad (4)$$

where

$t =$	the *t*-test
$n =$	the interval of observation in years
$s_1 =$	the standard deviation of stock returns at a 1 year interval of observation
$s_n =$	the annualised standard deviation of stock returns at an *n* year interval of observation
$N_1 =$	number of observations at a 1 year interval of observation
$N_n =$	number of observations at an *n* year interval of observation

In the results, all tests in bold indicate statistical significance at the 5 percent level.

3.4 Factors which are thought to Affect the Return on Stocks

I look at how the interval of observation affects the assessment of risk in stocks by examining what relationships there are between factors relating to the economy and commodities, and the worst returns on stocks. In addition, I examine whether there is any effect from the interval observation on the relationships. The worst returns on stocks are defined as bleak investment periods and are classified as returns on stocks which are less than or equal to the 5th percentile[12]. I use the following algorithm:

$$\text{If} \quad R^a_{t-k,t} \leq 5^{th}_n \quad \text{then} \quad R^a_{t-k,t} \equiv Bleak_n \qquad (5)$$

where

$R^a_{t-k,t} =$	the annualised return at time t over $t-k$
$k =$	the interval of observation in months
$n =$	the number of years in the interval of observation, i.e. $k/12$
$5^{th}_n =$	the annualised 5^{th} percentile return of an n year interval of observation
$Bleak_n =$	a dummy variable indicating if it is an n year bleak investment period

[12] Note: The 5th percentile is the median risk measure from the literature reviewed in this research: Kritzman, (1994) uses a 95 percent confidence interval; Thorley (1995) uses the 10th percentile; Jorion (2003) uses 95 percent VaR; and Dimson et al. (2004) use the minimum and the 10th percentile. However, limiting the measure of risk to the 5th percentile is not appropriate as it ignores returns which are less than this; nevertheless, of concern; therefore, the risk measure is for returns which are less than or equal to the 5th percentile.

Factors which are thought to affect the return on stocks are used to characterise the bleak investment periods and to examine what relationships there are. I apply an algorithm which calculates the annualised change in the factors at time t over $t-k$ (using equation 1) when the return on stocks is classified as being in a bleak investment period at time t ($R^{a}_{t-k,t} \equiv Bleak_n$). As the interval of observation changes, I examine whether the relationships changes too.

A control is necessary to determine what relationships there are and what the effect from the interval of observation is. I define the control as non-bleak investment periods and they are classified as returns on stocks which are greater than the 5th percentile. I use the following algorithm:

$$\text{If} \quad R^{a}_{t-k,t} > 5^{th}_{n} \quad \text{then} \quad R^{a}_{t-k,t} \equiv Non-Bleak_n \qquad (6)$$

where

$R^{a}_{t-k,t} =$	the annualised return at time t over $t-k$
$k =$	the interval of observation in months
$n =$	the number of years in the interval of observation, i.e. $k/12$
$5^{th}_{n} =$	the annualised 5^{th} percentile return of an n year interval of observation
$Non-Bleak_n =$	a dummy variable indicating if it is an n year non-bleak investment period

Then I apply an algorithm which calculates the annualised change in the factors at time t over $t-k$ (using equation 1) when the return on stocks is classified as being in a non-bleak investment period at time t ($R^{a}_{t-k,t} \equiv Non-Bleak_{n}$).

The means of the annualised change in the factors at *k* length intervals of observation in bleak and non-bleak investment periods are compared to determine what relationships there are. A *t*-test of the difference between the means is calculated to assess the statistical significance of what difference there is[13].

Factors which are thought to affect the return on stocks relate to the economy and commodities (metals and agriculture). All data are from GFD with a monthly frequency[14] and are deflated by the CPI to allow for the distorting effects of inflation[15]. After the data are deflated by the CPI, equation 1 is used to calculate the return on the factors as the interval of observation changes as stated[16]. The length of the data available differs on a factor by factor basis which means, in some cases, direct comparisons at the same point in

[13] Note: A linear regression model using factors which are thought to affect the return on stocks as independent variables and the return on stocks as dependent variables was applied. The results were inconsistent and revealed little insight into the interval of observation.

[14] Note: Interpolation is necessary for some data: GDP data are annual from 1832 to 1920 and quarterly thereafter; CPI data are annual from 1832 to 1874; unemployment data are annual from 1890 to 1929; and butter data are annual from 1832 to 1890. The ‘smoothing effect’ of interpolation is less of a concern when the interval of observation is long.

[15] Note: The Unemployment data are not deflated by the CPI. Of course, neither is the CPI data.

[16] Note: The Unemployment data are a rate; therefore, a moving average is calculated. The Bonds and T-Bill data are total return indices which are gross of transaction costs, and I do not made any deduction for fees and taxes.

time are not possible. However, the data are selected so the minimum sample length is at least 118 years which is over two thirds the stocks' sample length.

The economic factors include: Gross Domestic Product (GDP), Unemployment, the CPI, Bonds, T-Bills and Oil. These factors are commonly cited in discussions on the economy. A summary of the data length and the source name used by GFD is in table 3.2.

Table 3.2 Summary of Economic Data

Variable	**GFD Source Name**	**From**	**To**
GDP	United States Gross Domestic Product	11/1832	03/2008
Unemployment	United States Unemployment Rate	12/1890	03/2008
CPI	United States of America BLS Consumer Price Index	11/1832	03/2008
Bonds	USA 10-Year Government Bond Total Return Index	11/1832	03/2008
T-Bills	USA Total Return Commercial/T-Bill Index	12/1835	03/2008
Oil	West Texas Intermediate Oil Price (US$/Barrel)	01/1863	03/2008

The metal factors include: Copper, Gold, Lead, Silver, Steel, Tin and Zinc. A summary of the data length and the source name used by GFD is in table 3.3.

Table 3.3 Summary of Metals Data

Variable	**GFD Source Name**	**From**	**To**
Copper	Copper Electrolytic Wirebar Price (US Cents/Pound)	11/1832	04/2007
Gold	Gold Bullion Price-New York (US$/Ounce)	11/1832	03/2008
Lead	Lead Bar Spot Price	11/1832	03/2008
Silver	Silver Cash Price (US$/Ounce)	11/1832	03/2008
Steel	Heavy Melting Steel Scrap in Chicago Price (USD/Metric Ton)	09/1894	04/2007
Tin	Tin (Straits, Pigs) Prices (US Cents/Pound)	11/1832	03/2008
Zinc	Zinc Spot Price (USD/Ton)	01/1840	03/2008

The agricultural factors include: Barley, Butter, Cattle, Cocoa, Coffee, Corn, Cotton, Eggs, Hides, Hogs, Lard, Milk, Oat, Rubber, Rye, Sheep, Sugar, Wheat and Wool. A summary of the data length and the source name used by GFD is in table 3.4.

Table 3.4 Summary of Agricultural Data

Variable	**GFD Source Name**	**From**	**To**
Barley	Barley No.2 (Cents/Pound)	01/1886	01/2008
Butter	Butter, Average Price (Cents/Pound)	11/1832	02/2008
Cattle	Live Cattle Spot Price (US Cents/Pound)	01/1858	03/2008
Cocoa	Cocoa Spot Price (USD/Metric Ton)	11/1832	03/2008
Coffee	Brazil Santos Arabicas Spot Price (Cents/Pound)	11/1832	03/2008
Corn	Corn Spot Price (US$/Bushel)	01/1860	03/2008
Cotton	Cotton Spot Price (Cents/Pound)	11/1832	03/2008
Eggs	Eggs, Lage (Cents/Dozen)	01/1890	03/2008
Hides	Hides, Heavy Native Steers (Cents/Pound)	01/1890	05/2007
Hogs	Live Hog Prices (US Cents/Pound)	01/1886	03/2008
Lard	Lard, Average Wholesale Price (Cents/Pound)	01/1877	03/2008
Milk	Milk, Average Price to Farmers (USD/CWT)	01/1890	03/2008
Oat	Oat Spot Price (US$/Bushel)	11/1832	03/2008
Rubber	Rubber Spot Price (US Cents/Pound)	01/1890	03/2008
Rye	Rye, No.2, Minneapolis (Cents/Bushel)	01/1886	12/2005
Sheep	Sheep, Average Price (USD/CWT)	01/1874	03/2008
Sugar	Sugar #11 Spot Price (US Cents/Pound)	11/1832	03/2008
Wheat	Wheat #2 Cash Price (US Dollars/Bushel)	07/1841	03/2008
Wool	Wool, 64s, Staple 2¾ and Up US (Cents/Pound)	01/1890	03/2008

Three intervals of observation are examined in detail: 1 year time horizons, 10 year time horizons and 20 year time horizons. When studying the return on stocks and long intervals of observation, Jorion (2003) uses a 10 year time horizon and Dimson et al. (2004) use a 20 year time horizon. In addition, investors' implied mutual fund holding period is at the most 5.5 years (Quill, 2001). This suggests time horizons which are 'too long' are of little relevance to investors. This allows me to examine how the interval of observation affects the risk in stocks, and to use time horizons which are relevant to investors.

It is expected bleak investment periods are associated with economic factors which indicate the economy is 'bleak', and metal and agricultural factors which indicate commodities are not 'bleak'.

In addition, it is expected as the interval of observation increases, the factors in bleak and non-bleak investment periods become more alike, because the return on stocks becomes more alike as the interval of observation increases.

In this section, I described the data and methodology used to examine the relationship between the interval of observation and factors which are thought to affect the return on stocks in bleak and non-bleak investment periods. The next section summaries the chapter.

3.5 Summary

In this chapter I described the data and methodology used in this research. The next chapter discusses the results.

4 Results

4.1 Introduction

A discussion of the data and methodology is given in chapter 3. In this chapter I discuss the results from the data and methodology used. Section 4.2 presents the results from the analysis of how the interval of observation affects the assessments of risk in stocks. Section 4.3 presents the results from the tests of the statistical significance. Section 4.4 presents the results from the analysis of factors which are thought to affect the return on stocks. Section 4.5 summarises this chapter.

4.2 The Interval of Observation and the Assessment of Risk in Stocks

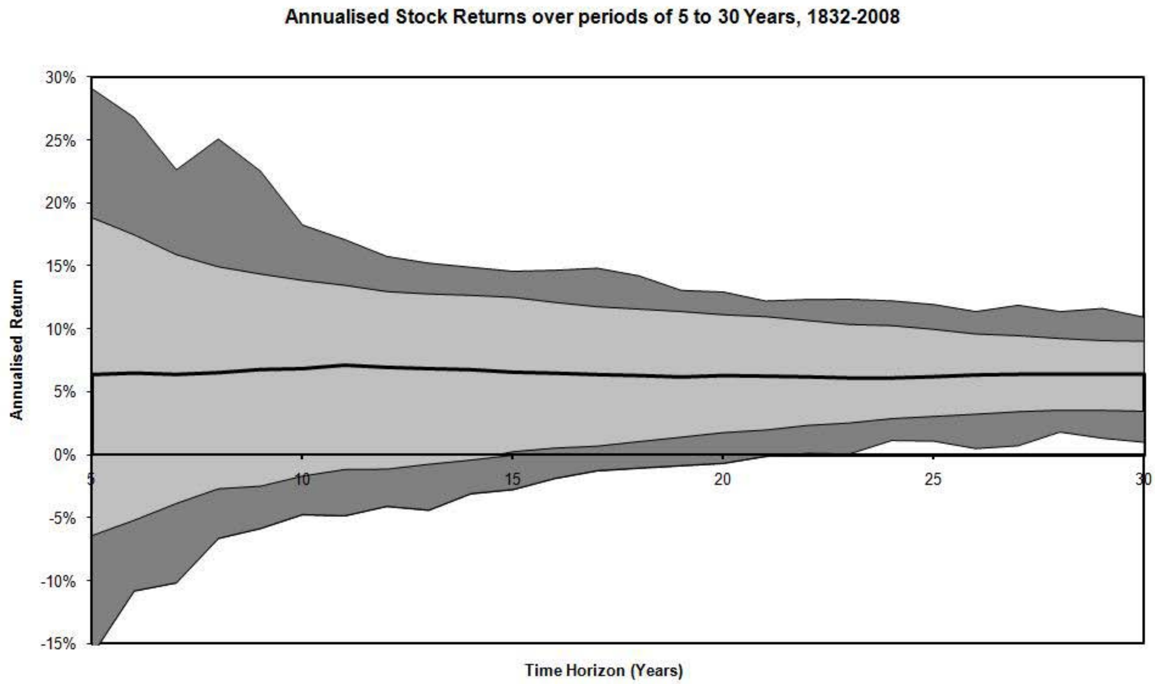
Table 4.1 presents the summary statistics of annualised stock returns as the interval of observation changes. As the interval of observation increases, the mean and median remain constant as expected. Standard deviation decreases in a manner of exponential decay which is expected as it is annualised standard deviation. The range (the maximum return minus the minimum return) responds in a manner consistent with standard deviation. Skewness is positive and negative 50 percent of the time for time horizons of 1 to 30 years. Kurtosis is positive for time horizons of 1 to 4 years indicating a relatively peaked distribution with 'fat tails'. For time horizons of 5 to 30 years, it is negative indicating a relatively flat distribution with 'thin tails' which implies the probability of an extreme return is less than for a normally distributed variable.

Table 4.1 The Summary Statistics of Annualised Stock Returns

Time Horizon	Mean	Median	Standard Deviation	Range	Skewness	Kurtosis	N
1	0.0630	0.0713	0.1871	2.0565	-0.38	1.74	2094
2	0.0630	0.0662	0.1304	1.1356	-0.45	1.40	2082
3	0.0625	0.0621	0.1013	0.8174	-0.34	1.10	2070
4	0.0625	0.0615	0.0865	0.6103	-0.04	0.12	2058
5	0.0629	0.0638	0.0758	0.4495	-0.01	-0.19	2046
10	0.0650	0.0685	0.0464	0.2300	-0.14	-0.54	1986
15	0.0645	0.0656	0.0360	0.1733	-0.07	-0.49	1926
20	0.0638	0.0630	0.0275	0.1360	0.02	-0.49	1866
30	0.0634	0.0642	0.0163	0.0990	-0.07	-0.35	1746

Figure 4.1 presents the distribution of annualised stock returns over intervals of observation of 5 to 30 years. The lowest dark area represents returns that occur 5 percent of the time (returns less than or equal to the 5^{th} percentile); the highest dark area represents returns that occur 5 percent of the time (returns greater than or equal to the 95^{th} percentile); the light area represents returns that occur 90 percent of the time; and the black line in the middle represents the median return. As the interval of observation increases, the distribution of annualised returns decreases and converges towards the median. At the 5^{th} percentile, returns are greater than zero at intervals of observation of 15 years or more, and at the minimum, returns are greater than zero at intervals of observation of 22 years or more.

Figure 4.1 The Distribution of Annualised Stock Returns



This indicates the risk in stocks decreases as the interval of observation increases. However, when the interval of observation is less than 15 years, the worst returns on stocks are negative.

Table 4.2 presents the summary statistics of the worst returns on stocks annualised as the interval of observation changes. The worst returns are measured by the 5^{th}, the $2½^{th}$ and the 0^{th} percentiles. At the 5^{th} percentile, returns are less than zero at intervals of observation of up 15 years, at the $2½^{th}$ percentile at intervals of observation of up to 16 years, and at the 0^{th} percentile at intervals of observation of up to 22 years.

Table 4.2 The Worst Returns on Stocks Annualised

Time Horizon	5th Percentile	2½th Percentile	0th Percentile
1	-0.2588	-0.3239	-1.0302
2	-0.1580	-0.2098	-0.6858
3	-0.1049	-0.1355	-0.4798
4	-0.0763	-0.1026	-0.2699
5	-0.0637	-0.0874	-0.1582
10	-0.0166	-0.0272	-0.0470
15	0.0030	-0.0049	-0.0271
20	0.0183	0.0119	-0.0063
30	0.0353	0.0311	0.0106

Figure 4.2 presents the worst returns on stocks annualised at intervals of observation of 0 to 30 years. The worst returns can get very bad[17]. The difference between the 0th and the 2½th percentile is greater than the difference between the 2½th and the 5th percentile at intervals of observation of less than 5 years. This indicates it is important to examine the 0th percentile when assessing the risk in stocks as the economic implications of this are significant. As the interval of observation increases, the worst returns increase, and the 0th percentile becomes more like the 2½th which becomes more like the 5th percentile which becomes more like the median. This indicates the worst returns on stocks become less different from all returns on stocks as the interval of observation increases; and therefore, the risk in stocks decreases.

[17] Note: Annualised returns are continuously compounded which causes negative returns to be overstated. In this case, the annualised return of the 0th percentile at an interval of observation of 1 year is -103.0 percent which is not possible. This is -64.3 percent when returns are not continuously compounded. Using continuously compounded returns overstates the worst return on stocks in this case; however, the implications of the results remain.

Figure 4.2 The Worst Returns on Stocks Annualised

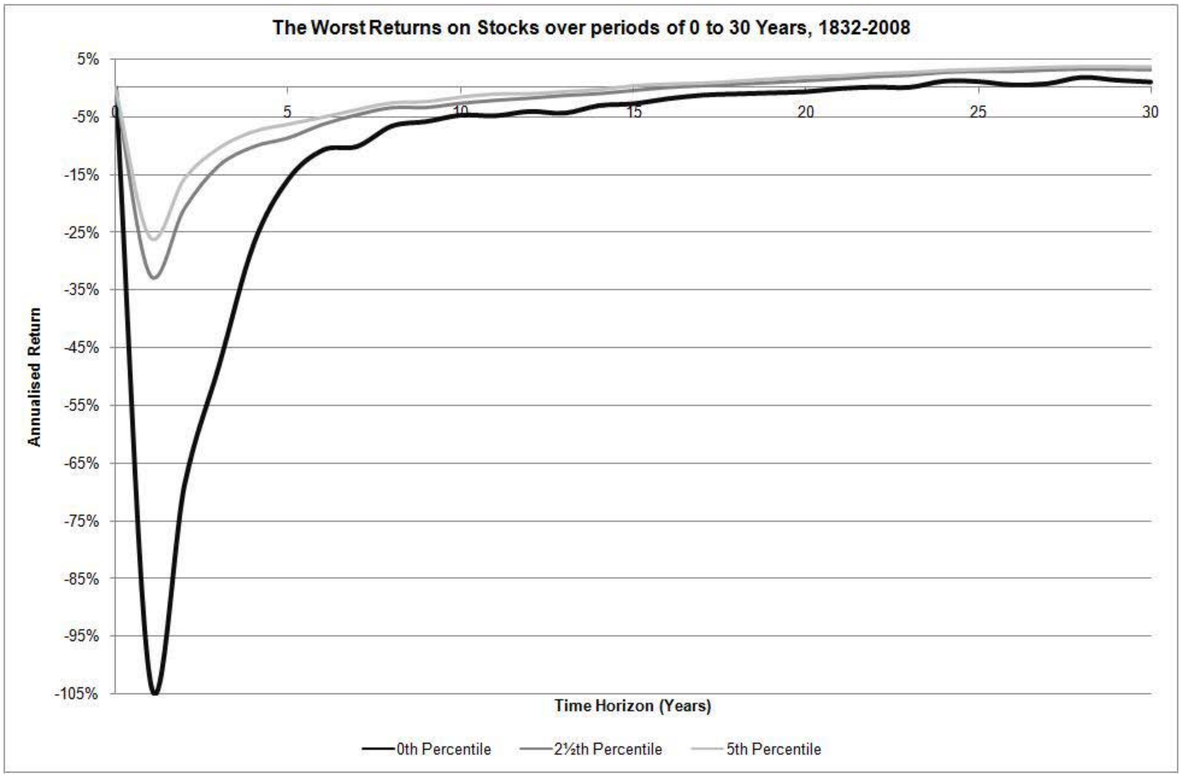


Figure 4.3 presents the standard deviation of annualised stock returns as the interval of observation changes. As the interval of observation increases, standard deviation decreases in a manner of exponential decay. This indicates the risk in stocks decreases as the interval of observation increases, and at intervals of observation of 25 years or more, the standard deviation changes little. Total stock returns are discussed next.

Figure 4.3 The Annualised Standard Deviation of Stock Returns

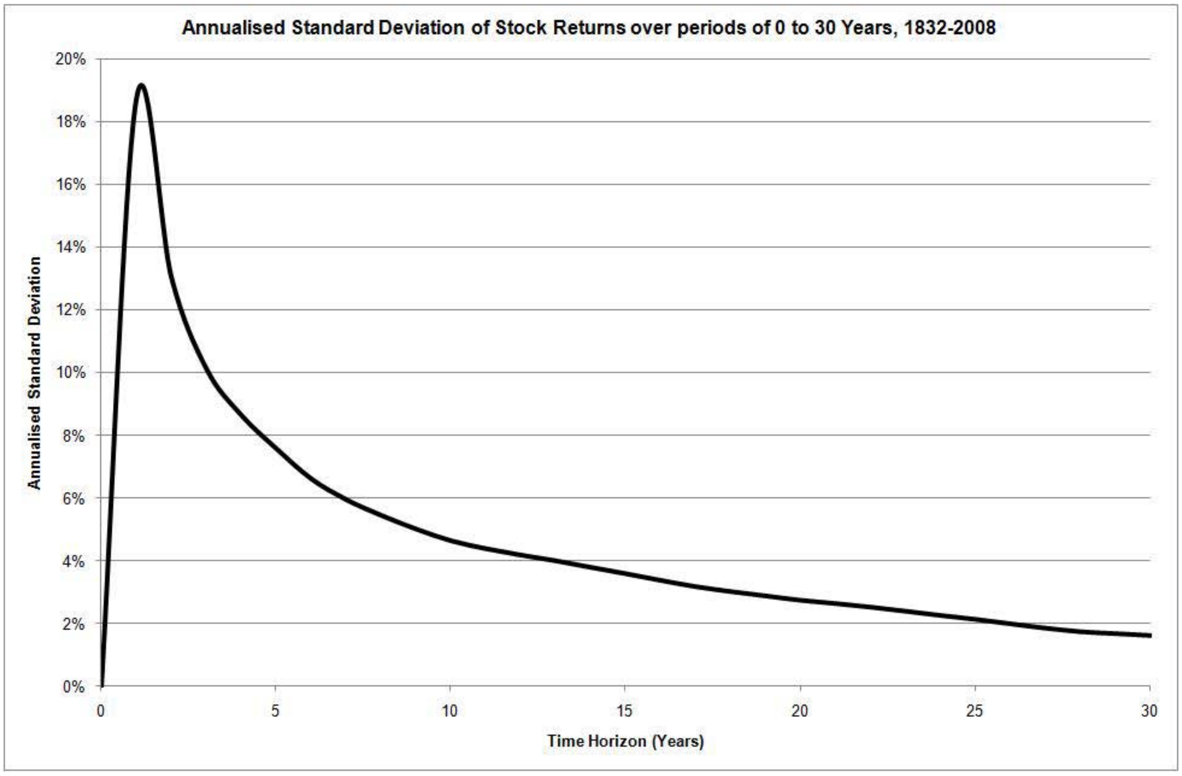


Table 4.3 presents the summary statistics of total stock returns as the interval of observation changes. As the interval of observation increases, the mean, median, standard deviation and range of total returns increase in an exponential manner. This is in contrast to annualised returns. In addition, as the interval of observation increases, the distribution of annualised returns converges towards the median, but the distribution of total returns diverges. This indicates the risk in stocks increases as the interval of observation increases. The skewness of total returns is positive 100 percent of the time, for time horizons of 1 to 30 years, which indicates a distribution with an asymmetric tail towards total returns which are greater than the mean. The kurtosis of total returns is positive 100 percent of the time too indicating a relatively peaked distribution with ‘fat tails’. This implies the probability of an extreme

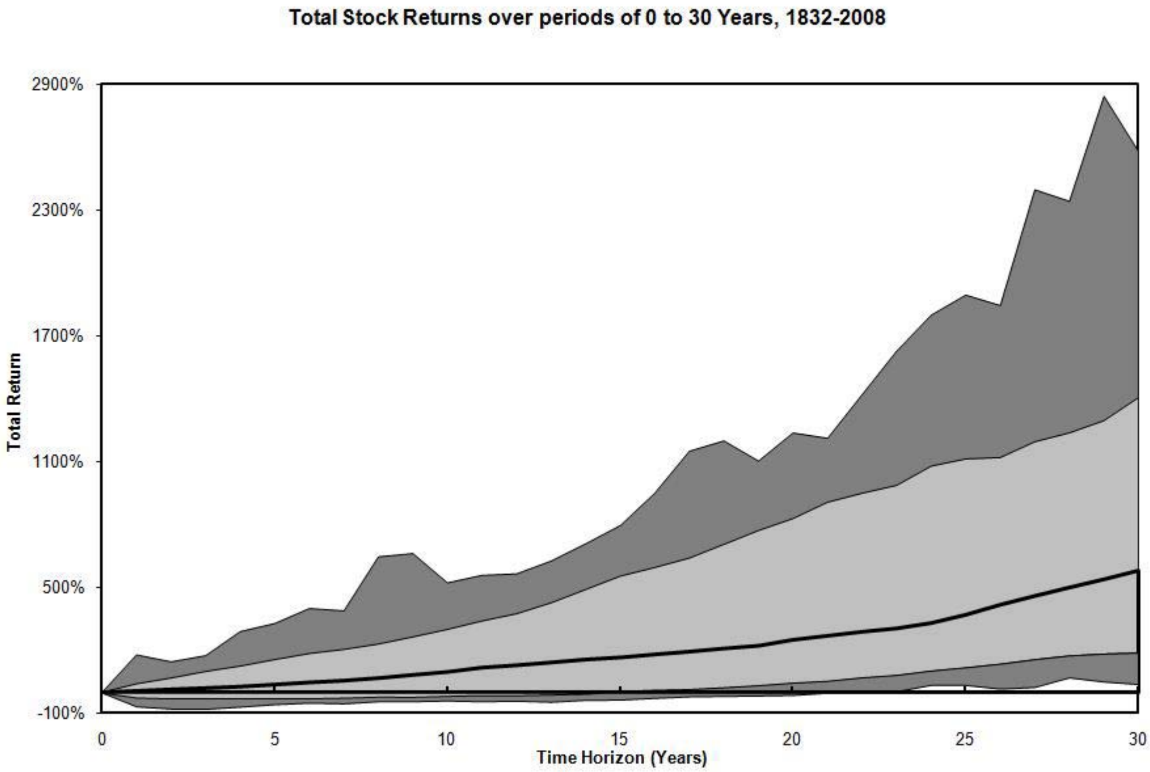
return is greater than for a normally distributed variable and the extreme return is more likely to be greater than the mean.

Table 4.3 The Summary Statistics of Total Stock Returns

Time Horizon	Mean	Median	Standard Deviation	Range	Skewness	Kurtosis	N
1	0.0835	0.0740	0.2005	2.434	0.67	3.85	2094
2	0.1722	0.1416	0.2982	2.205	0.57	0.83	2082
3	0.2616	0.2047	0.3787	2.516	0.73	0.69	2070
4	0.3629	0.2790	0.4824	3.564	1.02	1.53	2058
5	0.4716	0.3756	0.5731	3.838	1.08	1.68	2046
10	1.1269	0.9834	0.9768	5.608	0.92	0.84	1986
15	2.0344	1.6771	1.6495	8.303	1.08	0.73	1926
20	3.1630	2.5251	2.3603	12.517	1.19	0.97	1866
30	6.5238	5.8541	3.7321	25.477	1.14	1.50	1746

Figure 4.4 presents the distribution of total stock returns over intervals of observation of 0 to 30 years. The lowest dark area represents returns that occur 5 percent of the time (returns less than or equal to the 5th percentile); the highest dark area represents returns that occur 5 percent of the time (returns greater than or equal to the 95th percentile); the light area represents returns that occur 90 percent of the time; and the black line in the middle represents the median return. As the interval of observation increases, the distribution of total returns increases and diverges from the median. However, when the interval of observation is 15 years or more, the risk in stocks is related to how much greater than zero the total stock return becomes.

Figure 4.4 The Distribution of Total Stock Returns



The results from the analysis of total stock returns as the interval of observation changes indicate the risk in stocks decreases as the interval of observation increases to 15 years or more. However, when the interval of observation is less than 15 years, the worst returns are negative. The worst total return is -76.3 percent over a 3 year interval of observation.

An analysis of how the interval of observation affects the assessment of the risk in stocks indicates risk decreases as the interval of observation increases. When the interval of observation is less than 15 years, the worst returns are negative and the economic implications are significant; however, when the interval of observation is 22 years or more the worst returns are always positive and the economic implications of this are significant too. Tests of statistical significance are discussed next.

4.3 Tests of Statistical Significance

Table 4.4 presents the *t*-test of the statistical significance of the difference between the worst returns on stocks and zero as the interval of observation changes. The 5^{th}, the $2½^{th}$ and the 0^{th} percentiles of annualised returns are used. The 5^{th} percentile is statistically significantly less than zero at intervals of observation of 14 years or less; the $2½^{th}$ percentile is statistically significantly less than zero at intervals of observation of 15 years or less; the 0^{th} percentile is statistically significantly less than zero at intervals of observation of 20 years or less. These results indicates the risk in stocks decreases as the interval of observation increases and the decrease in risk is statistically significant as well as being economically significant[18].

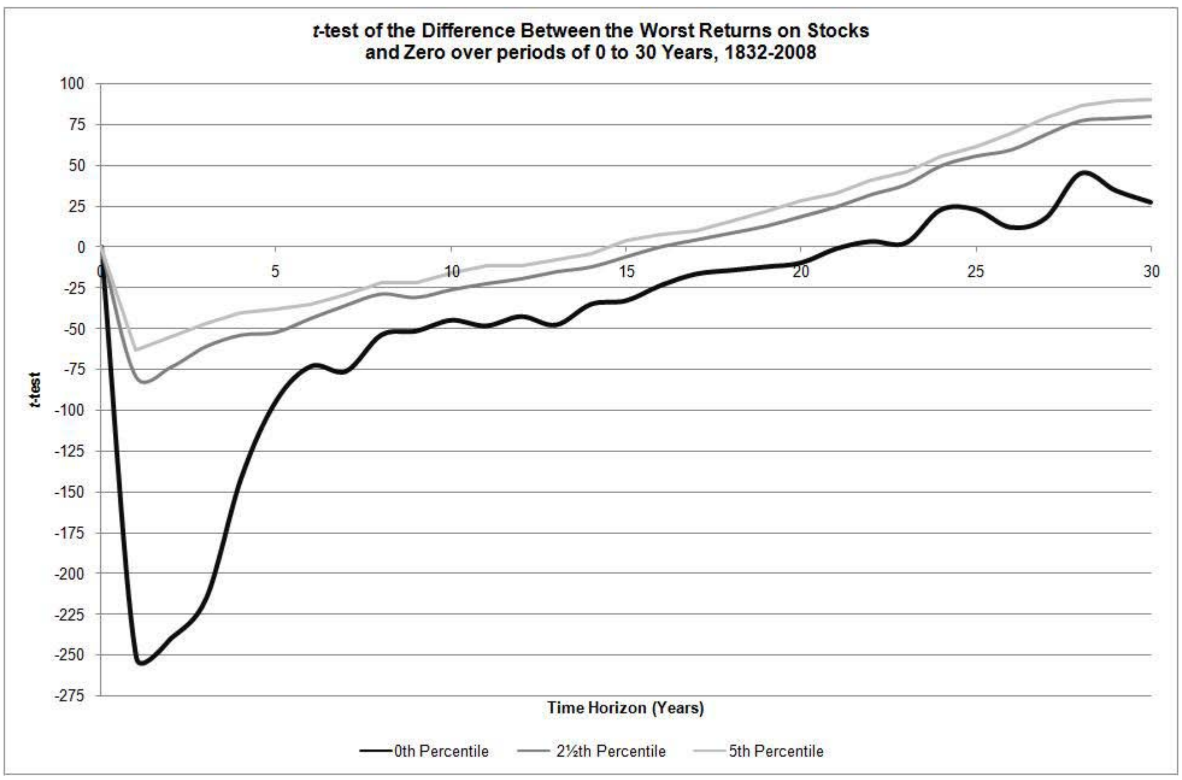
Table 4.4 The *t*-test of the Difference between the Worst Returns on Stocks and Zero

Time Horizon	**5^{th} Percentile *t*-test**	**$2½^{th}$ Percentile *t*-test**	**0^{th} Percentile *t*-test**
1	**-63.29**	**-79.23**	**-251.99**
2	**-55.29**	**-73.42**	**-239.95**
3	**-47.11**	**-60.87**	**-215.54**
4	**-40.06**	**-53.86**	**-141.62**
5	**-37.99**	**-52.12**	**-94.39**
10	**-15.92**	**-26.10**	**-45.08**
15	**3.71**	**-5.97**	**-33.07**
20	**28.69**	**18.66**	**-9.88**
30	**90.57**	**79.92**	**27.32**

[18] Note: The accuracy of this test of statistical significance is limited as the test assumes the sample is independent. The samples are not as the returns are calculated on a rolling monthly basis. Nevertheless, the tests are an indication of how statistically significant the results would be if the samples were independent.

Figure 4.5 presents the *t*-test of the statistical significance of the difference between the worst returns on stocks and zero at intervals of observation of 0 to 30 years. To start with, the worst returns on stocks are statistically significantly less than zero; however, as the interval of observation increases, the worst returns on stocks become less statistically significantly different from zero. Between intervals of observation of 15 and 22 years some of the worst returns on stocks are not statistically significantly different from zero and after this the worst returns are statistically significantly greater than zero.

Figure 4.5 The *t*-test of the Difference between the Worst Returns on Stocks and Zero



The *t*-test is a parametric test and assumes the samples are normally distributed. The accuracy of this assumption for the sample used is limited. Therefore, a non-parametric test is necessary. A Wilcoxon signed rank sum test is used to examine the statistical significance of the difference between the annualised stock return of the 2½th percentile and zero as the interval of observation changes[19].

Table 4.5 presents the Wilcoxon signed rank sum test used to examine the statistical significance of the difference between the 2½th percentile and zero as the interval of observation changes. To start with, the 2½th percentile is statistically significantly less than zero. As the interval of observation increases, the Wilcoxon signed rank sum test increases and at intervals of observation of 17 years or more the 2½th percentile is statistically significantly greater than zero. Figure 4.6 presents this result. The results from this test of statistical significance support the results from the *t*-test of statistical significance which shows some of the worst returns on stocks are statistically significantly less than zero when the interval of observation is less than 15 to 22 years. At intervals of observation greater than this, the worst returns on stocks are statistically significantly greater than zero.

Table 4.5 The Wilcoxon Signed Rank Sum Test of the Difference between the 2½th Percentile and Zero

Time Horizon	Wilcoxon Signed Rank Sum Test
1	**-2782.5**
2	**-2782.5**
3	**-2730**
4	**-2678**
5	**-2678**
10	**-2525**
15	**-1961.5**
20	**2170.5**
30	**1958**

[19] Note: Only the 2½th percentile is used as the methodology of the Wilcoxon signed rank sum test uses a distribution to examine the statistical significance. This distribution uses the 2½th percentile as the median and the range is from the 0th to the 5th percentile.

Figure 4.6 The Wilcoxon Signed Rank Sum Test of the Difference between the 2½th Percentile and Zero

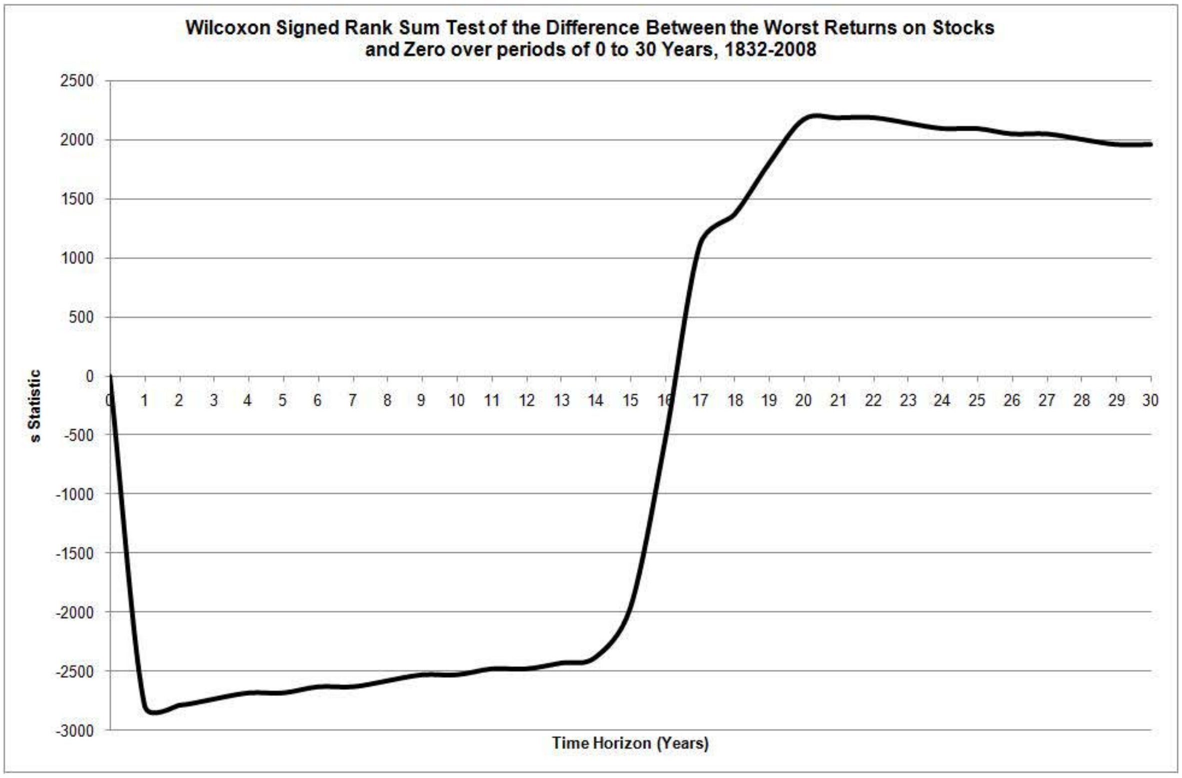


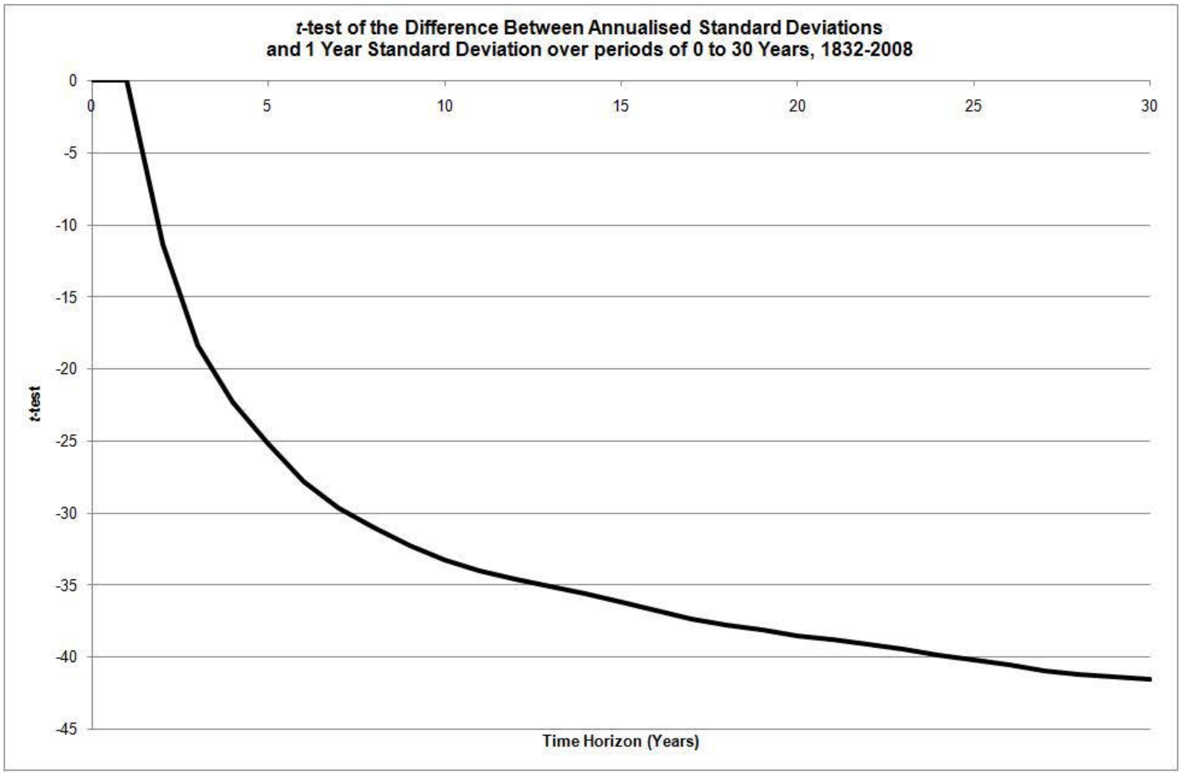
Table 4.6 presents the *t*-test of the statistical significance of the difference between the annualised standard deviation at an interval of observation of 1 year and as the interval of observation changes. At an interval of observation of 1 year, the standard deviation is not statistically significantly different from the standard deviation at an interval of observation of 1 year, which is expected. However, as the interval of observation increases, the annualised standard deviation becomes statistically significantly less than the annual standard deviation.

Table 4.6 The *t*-test of the Difference between the Annualised Standard Deviation of Stock Returns

Time Horizon	***t* Statistic**
1	0.00
2	**-11.36**
3	**-18.43**
4	**-22.31**
5	**-25.18**
10	**-33.34**
15	**-36.24**
20	**-38.57**
30	**-41.59**

Figure 4.7 presents the *t*-test of the statistical significance of the difference between the annualised standard deviation and the standard deviation at an interval of observation of 1 year. As the interval of observation increases, the statistical significance increases in absolute terms which indicates the annualised standard deviation becomes less like the 1 year standard deviation.

Figure 4.7 The *t*-test of the Difference between the Annualised Standard Deviation of Stock Returns



4.4 Factors which are thought to Affect the Return on Stocks

Tables 4.7, 4.8 and 4.9 present the summary statistics from the analysis of the interval of observation at 1, 10 and 20 years, and the mean annualised rates of change in economic factors which are thought to affect the return on stocks. At an interval of observation of 1 year, the bleak investment periods in stocks are characterised with a lower rate of GDP growth, a higher rate of unemployment and inflation, lower returns on bonds and bills, and a higher rate of oil price growth compared with non-bleak investment periods in stocks. However, the differences between unemployment, bills and oil in bleak and non-bleak investment periods is not statistically significant.

As the interval of observation increase to 10 years, the relationship between economic factors and bleak and non-bleak investment periods remains, with the rate of unemployment being the exception (it is lower in bleak investment periods compared to non-bleak investment periods; although, the difference in not statistically significant). The differences between the economic factors in bleak and non-bleak investment periods increase as the interval of observation increases to 10 years, with the rate of GDP growth and the unemployment rate being the exceptions. At an interval of observation of 20 years, the relationship remains as it is at 10 years; however, the differences decrease.

Table 4.7 The Summary Statistics of Economic Factors in Bleak and Non-Bleak Investment Periods in Stocks at Intervals of Observation of 1 Year

Investment		**Mean**	***t***	**Standard Deviation**	**Skewness**	**Kurtosis**	**N**
GDP	Bleak	0.0250		0.1353	-0.70	-0.22	105
	Non-Bleak	0.0552	**-2.03**	0.0719	-0.85	5.23	1989
Unemployment	Bleak	0.0756		0.0489	1.01	-0.28	75
	Non-Bleak	0.0663	1.37	0.0426	1.92	3.73	1322
CPI	Bleak	0.0462		0.0915	-0.18	-1.00	105
	Non-Bleak	0.0195	**2.66**	0.0500	0.16	1.87	1989
Bonds	Bleak	-0.0133		0.1004	-0.13	-1.15	105
	Non-Bleak	0.0290	**-3.68**	0.0767	0.34	2.25	1989
Bills	Bleak	0.0118		0.0905	-0.51	-0.85	101
	Non-Bleak	0.0297	-1.74	0.0562	0.39	2.17	1955
Oil	Bleak	0.0407		0.3516	0.38	2.05	86
	Non-Bleak	-0.0029	1.13	0.2926	0.21	2.79	1645

Table 4.8 The Summary Statistics of Economic Factors in Bleak and Non-Bleak Investment Periods in Stocks at Intervals of Observation of 10 Years

Investment		**Mean**	***t***	**Standard Deviation**	**Skewness**	**Kurtosis**	**N**
GDP	Bleak	0.0222		0.0112	-0.75	0.08	100
	Non-Bleak	0.0332	**-7.38**	0.0157	0.83	2.27	1886
Unemployment	Bleak	0.0599		0.0220	4.02	16.40	96
	Non-Bleak	0.0653	-1.69	0.0301	2.06	3.74	1073
CPI	Bleak	0.0612		0.0215	-2.38	6.79	100
	Non-Bleak	0.0186	**15.53**	0.0258	0.05	-0.56	1886
Bonds	Bleak	-0.0229		0.0261	1.43	2.79	100
	Non-Bleak	0.0295	**-15.55**	0.0329	-0.10	-0.69	1886
Bills	Bleak	-0.0051		0.0208	2.20	7.00	100
	Non-Bleak	0.0288	**-11.76**	0.0345	0.06	0.00	1848
Oil	Bleak	0.0797		0.0464	-0.62	0.25	96
	Non-Bleak	-0.0121	**14.63**	0.0603	-0.15	1.53	1527

Table 4.9 The Summary Statistics of Economic Factors in Bleak and Non-Bleak Investment Periods in Stocks at Intervals of Observation of 20 Years

Investment		**Mean**	***t***	**Standard Deviation**	**Skewness**	**Kurtosis**	**N**
GDP	Bleak	0.0265		0.0066	-1.32	1.50	94
	Non-Bleak	0.0331	**-7.30**	0.0092	-0.16	0.66	1772
Unemployment	Bleak	0.0621		0.0192	1.76	1.55	94
	Non-Bleak	0.0663	-1.64	0.0192	1.00	-0.08	1075
CPI	Bleak	0.0444		0.0155	-0.78	-0.64	94
	Non-Bleak	0.0204	**11.55**	0.0199	0.19	-0.68	1772
Bonds	Bleak	-0.0087		0.0142	0.71	-0.56	94
	Non-Bleak	0.0268	**-17.16**	0.0251	-0.10	-1.01	1772
Bills	Bleak	0.0034		0.0077	-0.25	0.13	94
	Non-Bleak	0.0265	**-15.63**	0.0284	0.17	-0.44	1734
Oil	Bleak	0.0251		0.0348	-0.43	-1.05	94
	Non-Bleak	-0.0061	**6.89**	0.0350	-0.30	0.20	1409

Figures 4.8, 4.9 and 4.10 present the mean annualised rates of change in economic factors in bleak and non-bleak investment periods in stocks at intervals of observation of 1, 10 and 20 years. The results indicate there are bleak investment periods in stocks when the economic factors are ‘bleak’ and as the interval of observation increases from 1 to 10 and 20 years this relationship remains, with the rate of unemployment being the exception. The difference between economic factors in bleak and non-bleak investment periods is small at long intervals of observation.

Figure 4.8 The Mean Annual Rate of Change in Economic Factors in Bleak and Non-Bleak Investment Periods in Stocks at Intervals of Observation of 1 Year

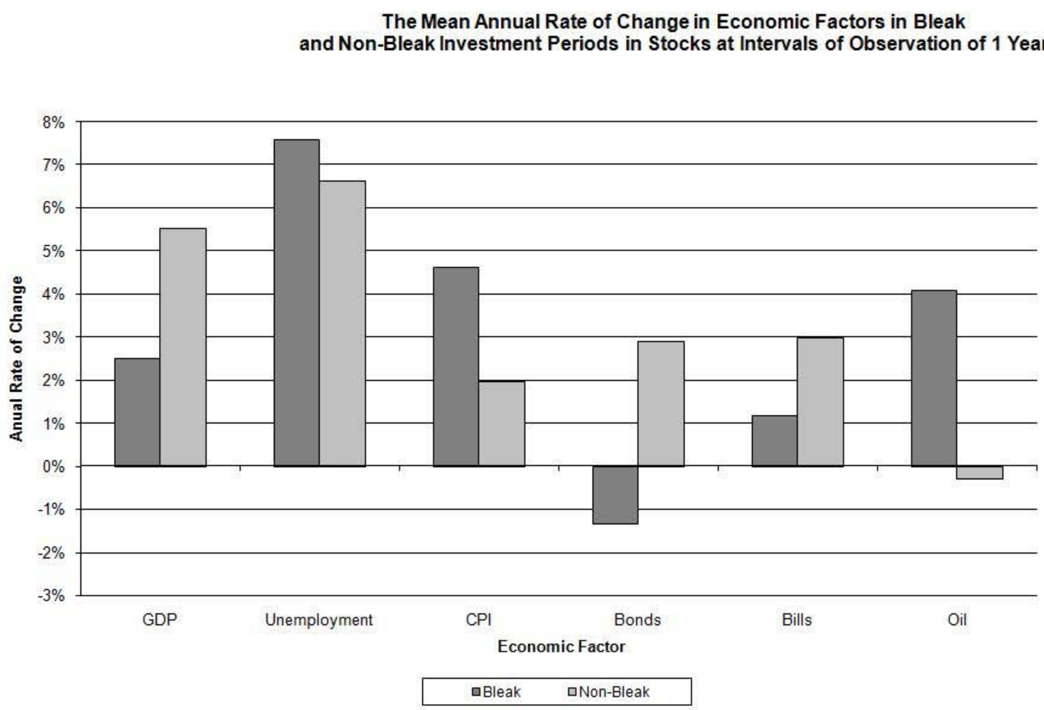


Figure 4.9 The Mean Annualised Rate of Change in Economic Factors in Bleak and Non-Bleak Investment Periods in Stocks at Intervals of Observation of 10 Years

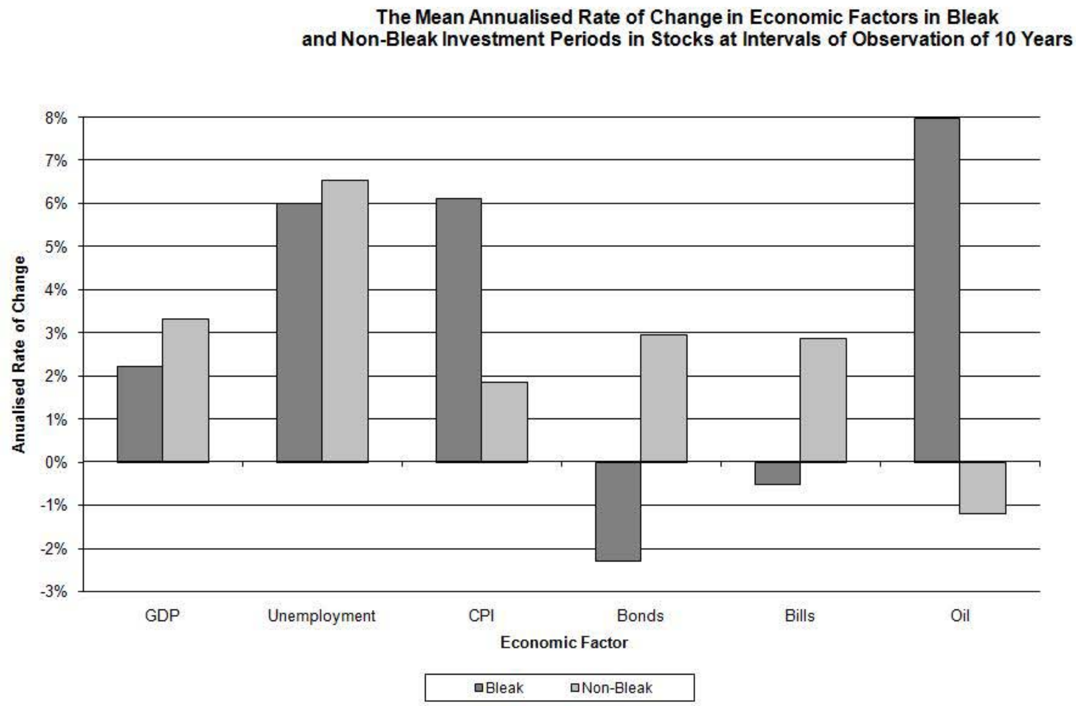
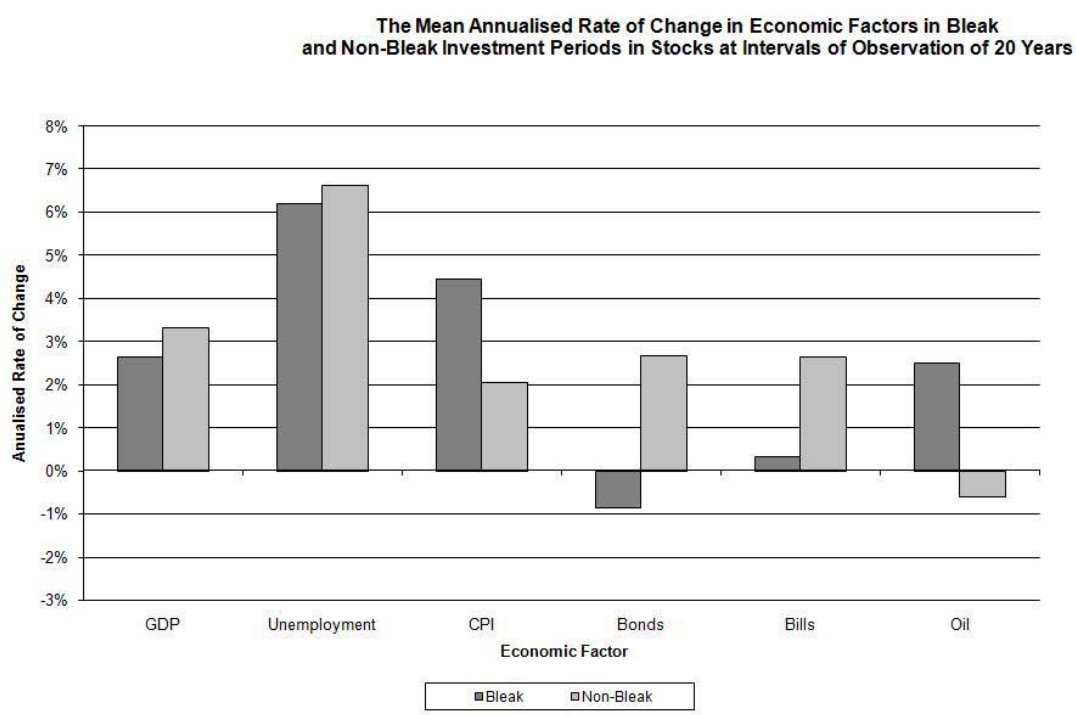


Figure 4.10 The Mean Annualised Rate of Change in Economic Factors in Bleak and Non-Bleak Investment Periods in Stocks at Intervals of Observation of 20 Years



Tables 4.10, 4.11 and 4.12 present the summary statistics from the analysis of the interval of observation at 1, 10 and 20 years, and the mean annualised rate of change in metal factors which are thought to affect the return on stocks. At an interval of observation of 1 year, the bleak investment periods in stocks are characterised with lower rates of price growth in metals compared with non-bleak investment periods in stocks. For copper and zinc the differences are large: 13.5 and 19.8 percent.

As the interval of observation increases to 10 years, the relationship changes and bleak investment periods are characterised with higher rates of price growth in metals. Copper and zinc are the exceptions. For gold and silver the difference are large: 4.7 and 4.2 percent. As the interval of observation increases to 20 years, bleak investment periods are characterised with higher rates of price growth in gold, silver and tin; and lower rates of price growth in copper, lead, steel and zinc, compared with non-bleak investment periods.

Table 4.10 The Summary Statistics of Metal Factors in Bleak and Non-Bleak Investment Periods in Stocks at Intervals of Observation of 1 Year

Investment		**Mean**	***t***	**Standard Deviation**	**Skewness**	**Kurtosis**	**N**
Copper	Bleak	-0.1335		0.2571	0.10	-0.91	105
	Non-Bleak	0.0012	**-4.59**	0.1911	0.05	1.37	1978
Gold	Bleak	-0.0330		0.1876	-0.68	3.16	105
	Non-Bleak	0.0019	-1.65	0.1293	1.19	8.43	1989
Lead	Bleak	-0.0599		0.2513	0.49	1.55	105
	Non-Bleak	0.0013	**-2.11**	0.1997	0.23	2.34	1989
Silver	Bleak	-0.0598		0.1994	-0.20	1.99	105
	Non-Bleak	-0.0041	**-2.35**	0.1891	0.64	9.34	1989
Steel	Bleak	-0.0352		0.3283	0.70	-0.17	74
	Non-Bleak	0.0027	-0.82	0.2893	0.03	0.38	1266
Tin	Bleak	-0.0518		0.2799	-0.26	0.48	105
	Non-Bleak	0.0042	-1.75	0.2082	0.07	2.21	1989
Zinc	Bleak	-0.1909		0.2384	0.08	1.07	94
	Non-Bleak	0.0076	**-6.53**	0.2538	1.01	4.67	1913

Table 4.11 The Summary Statistics of Metal Factors in Bleak and Non-Bleak Investment Periods in Stocks at Intervals of Observation of 10 Years

Investment		**Mean**	***t***	**Standard Deviation**	**Skewness**	**Kurtosis**	**N**
Copper	Bleak	-0.0274		0.0315	0.25	-0.48	100
	Non-Bleak	-0.0112	**-5.15**	0.0359	-0.36	0.95	1875
Gold	Bleak	0.0413		0.0866	-0.34	-1.61	100
	Non-Bleak	-0.0054	**5.39**	0.0418	1.69	5.71	1886
Lead	Bleak	-0.0072		0.0345	0.15	-0.76	100
	Non-Bleak	-0.0090	0.55	0.0351	0.50	1.31	1886
Silver	Bleak	0.0288		0.0497	0.02	-0.61	100
	Non-Bleak	-0.0130	**8.41**	0.0430	-0.04	4.45	1886
Steel	Bleak	-0.0002		0.0451	0.14	-1.19	96
	Non-Bleak	-0.0075	1.58	0.0445	-0.11	-0.29	1136
Tin	Bleak	0.0121		0.0576	-0.76	-0.46	100
	Non-Bleak	-0.0044	**2.85**	0.0458	-0.62	0.34	1886
Zinc	Bleak	-0.0089		0.0415	-0.50	-0.72	100
	Non-Bleak	-0.0068	-0.51	0.0407	-0.08	1.19	1799

Table 4.12 The Summary Statistics of Metal Factors in Bleak and Non-Bleak Investment Periods in Stocks at Intervals of Observation of 20 Years

Investment		**Mean**	***t***	**Standard Deviation**	**Skewness**	**Kurtosis**	**N**
Copper	Bleak	-0.0300		0.0219	-0.28	-0.72	94
	Non-Bleak	-0.0107	**-8.54**	0.0247	-0.07	-0.34	1761
Gold	Bleak	0.0120		0.0488	-0.06	-1.71	94
	Non-Bleak	-0.0036	**3.09**	0.0286	0.77	0.94	1772
Lead	Bleak	-0.0133		0.0266	0.54	-0.71	94
	Non-Bleak	-0.0087	-1.69	0.0214	0.42	0.76	1772
Silver	Bleak	0.0032		0.0394	0.52	-0.28	94
	Non-Bleak	-0.0130	**4.00**	0.0289	0.54	2.18	1772
Steel	Bleak	-0.0199		0.0231	0.42	-0.13	94
	Non-Bleak	-0.0055	**-6.05**	0.0262	0.38	-0.36	1018
Tin	Bleak	-0.0036		0.0296	0.02	-1.11	94
	Non-Bleak	-0.0038	0.07	0.0320	-0.63	0.27	1772
Zinc	Bleak	-0.0106		0.0240	-0.09	-0.62	94
	Non-Bleak	-0.0060	-1.85	0.0243	0.20	0.97	1685

Figures 4.11, 4.12 and 4.13 present the mean annualised rates of change in metal factors in bleak and non-bleak investment periods in stocks at intervals of observation of 1, 10 and 20 years. The relationship between metal factors in bleak and non-bleak investment periods changes as the interval of observation changes with only copper and zinc having lower rates of price growth at all three intervals of observation in bleak investment periods (only the differences in copper are statistically significant though). In addition, the differences between metal factors in bleak and non-bleak investment periods decrease, from an average difference of 8.3 percent at 1 year intervals of observation to 1.9 and 1.1 percent at 10 and 20 year intervals of observation.

Figure 4.11 The Mean Annual Rate of Change in Metal Factors in Bleak and Non-Bleak Investment Periods in Stocks at Intervals of Observation of 1 Year

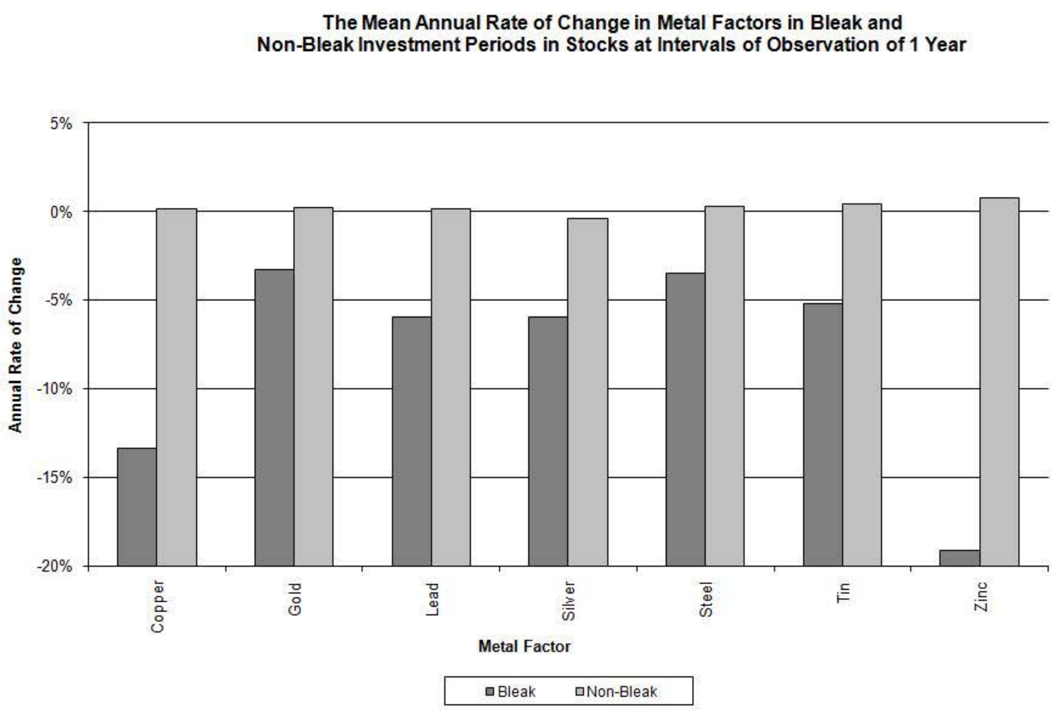


Figure 4.12 The Mean Annualised Rate of Change in Metal Factors in Bleak and Non-Bleak Investment Periods in Stocks at Intervals of Observation of 10 Years

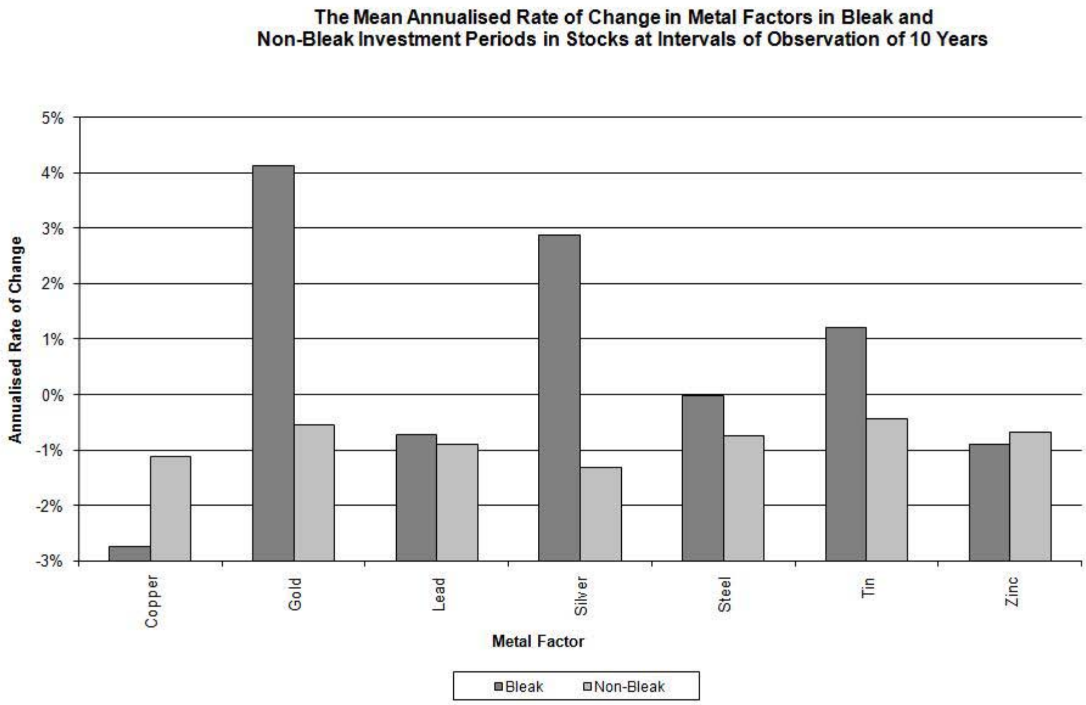
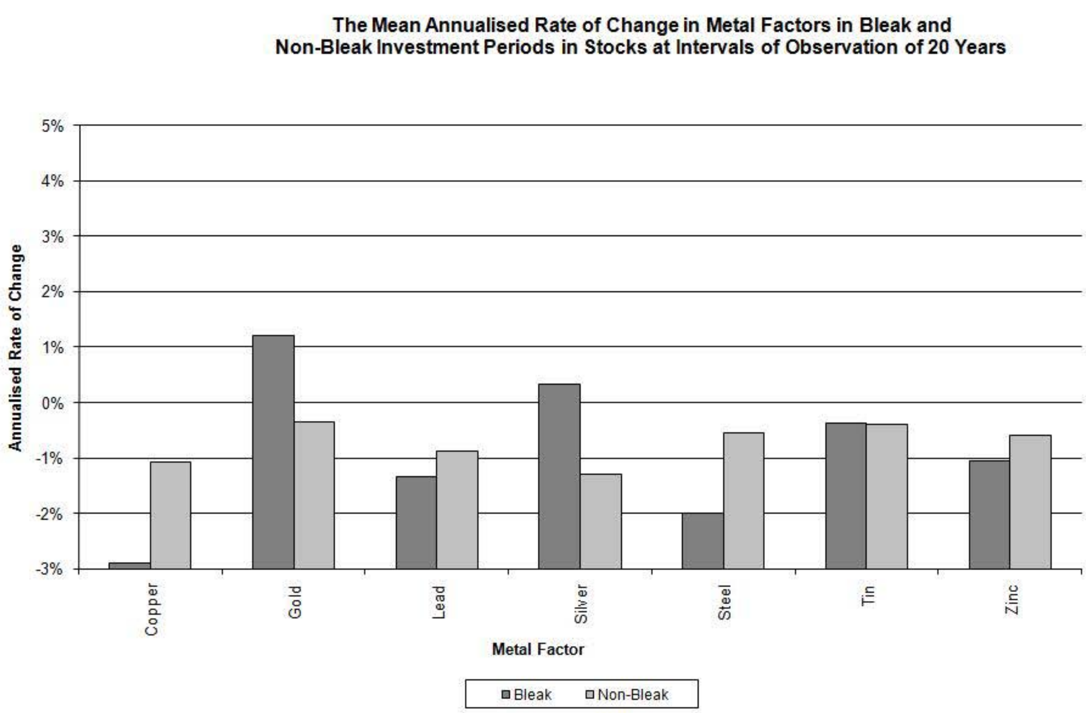


Figure 4.13 The Mean Annualised Rate of Change in Metal Factors in Bleak and Non-Bleak Investment Periods in Stocks at Intervals of Observation of 20 Years



Tables 4.13, 4.14 and 4.15 present the summary statistics from the analysis of the interval of observation at 1, 10 and 20 years, and the mean annualised rates of change in agricultural factors which are thought to affect the return on stocks. At an interval of observation of 1 year, the bleak investment periods in stocks are characterised with lower rates of price growth in agriculture for 16 out of 19 factors compared with non-bleak investment periods in stocks, with barley, rye and sugar being the exceptions. For hides and rubber the differences are large: 22.7 and 32.2 percent.

As the interval of observation increases to 10 years, the relationship changes and the bleak investment periods are characterised with higher rates of price growth in agriculture for 16 out of 19 factors compared with non-bleak investment periods. The exceptions are butter, cattle and rubber. For rubber and sugar the differences are large: 4.9 and 6.7 percent.

As the interval of observation increases to 20 years, the bleak investment periods are characterised with higher rates of price growth in agriculture for 15 out of 19 factors compared with non-bleak investment periods. The exceptions are cattle, cocoa, cotton and rubber. For rubber and sugar the differences are large: 2.0 and 3.0 percent.

Table 4.13 The Summary Statistics of Agricultural Factors in Bleak and Non-Bleak Investment Periods in Stocks at Intervals of Observation of 1 Year

Investment		**Mean**	***t***	**Standard Deviation**	**Skewness**	**Kurtosis**	**N**
Barley	Bleak	0.0840		0.2567	-0.27	0.25	75
	Non-Bleak	-0.0157	**2.71**	0.2643	0.17	1.62	1378
Butter	Bleak	-0.0413		0.1674	0.60	3.53	105
	Non-Bleak	-0.0066	-1.76	0.1479	0.37	5.17	1988
Cattle	Bleak	-0.0806		0.1664	0.20	-0.47	87
	Non-Bleak	0.0060	**-4.04**	0.1487	-0.11	0.83	1704
Cocoa	Bleak	-0.0476		0.3689	0.76	1.17	105
	Non-Bleak	-0.0075	-0.96	0.2527	0.59	2.18	1989
Coffee	Bleak	-0.1158		0.2741	0.30	-0.16	105
	Non-Bleak	-0.0022	**-3.46**	0.2722	0.08	1.85	1989
Corn	Bleak	-0.1305		0.3970	-0.28	-0.95	87
	Non-Bleak	-0.0025	**-2.59**	0.2844	0.08	0.74	1680
Cotton	Bleak	-0.1830		0.3800	-0.51	1.59	105
	Non-Bleak	-0.0019	**-4.21**	0.2632	0.07	2.85	1989
Eggs	Bleak	-0.0832		0.1505	0.10	-0.55	75
	Non-Bleak	-0.0101	**-3.16**	0.2091	0.00	0.70	1332
Hides	Bleak	-0.2237		0.2902	-0.22	0.00	75
	Non-Bleak	0.0034	**-5.54**	0.2728	-0.22	2.85	1322
Hogs	Bleak	-0.1618		0.2616	0.28	-0.44	75
	Non-Bleak	0.0029	**-4.45**	0.2532	0.14	0.30	1380
Lard	Bleak	-0.0511		0.3815	0.21	-0.69	75
	Non-Bleak	-0.0134	-0.74	0.2617	0.17	1.00	1488
Milk	Bleak	-0.0503		0.1823	0.09	-0.67	75
	Non-Bleak	-0.0019	**-2.00**	0.1130	0.42	3.05	1332
Oat	Bleak	-0.0541		0.2753	-0.01	-0.55	105
	Non-Bleak	-0.0068	-1.44	0.2672	0.08	0.57	1989
Rubber	Bleak	-0.3307		0.2927	0.52	-0.11	75
	Non-Bleak	-0.0092	**-7.51**	0.3291	0.71	2.85	1332
Rye	Bleak	-0.0075		0.3088	-0.78	-0.07	75
	Non-Bleak	-0.0125	0.12	0.2713	0.35	1.11	1353
Sheep	Bleak	-0.1428		0.2221	-0.42	-0.45	80
	Non-Bleak	-0.0031	**-4.66**	0.2017	-0.24	1.56	1519
Sugar	Bleak	0.0595		0.3823	1.95	4.31	105
	Non-Bleak	-0.0241	1.90	0.2947	-0.50	4.28	1989
Wheat	Bleak	-0.0951		0.2752	-0.13	-0.85	94
	Non-Bleak	-0.0045	**-2.66**	0.2475	0.31	1.00	1895
Wool	Bleak	-0.0675		0.3346	0.38	0.03	75
	Non-Bleak	-0.0137	-1.17	0.2605	0.35	2.52	1332

Table 4.14 The Summary Statistics of Agricultural Factors in Bleak and Non-Bleak Investment Periods in Stocks at Intervals of Observation of 10 Years

Investment		**Mean**	***t***	**Standard Deviation**	**Skewness**	**Kurtosis**	**N**
Barley	Bleak	-0.0061		0.0556	-0.79	0.52	96
	Non-Bleak	-0.0165	1.84	0.0437	-0.28	-0.02	1249
Butter	Bleak	-0.0140		0.0148	-0.65	0.69	100
	Non-Bleak	-0.0085	**-3.72**	0.0298	-0.80	2.28	1885
Cattle	Bleak	-0.0095		0.0204	0.02	-0.56	96
	Non-Bleak	0.0005	**-4.82**	0.0304	0.30	-0.53	1587
Cocoa	Bleak	0.0160		0.0823	-0.06	-1.32	100
	Non-Bleak	-0.0122	**3.43**	0.0563	0.41	0.68	1886
Coffee	Bleak	0.0029		0.0742	-0.48	-0.60	100
	Non-Bleak	-0.0096	1.68	0.0601	0.09	-0.28	1886
Corn	Bleak	-0.0025		0.0362	-0.78	0.64	96
	Non-Bleak	-0.0131	**2.89**	0.0419	0.12	-0.23	1563
Cotton	Bleak	0.0008		0.0351	-0.85	0.23	100
	Non-Bleak	-0.0122	**3.70**	0.0562	0.61	2.69	1886
Eggs	Bleak	-0.0064		0.0186	-0.57	0.57	96
	Non-Bleak	-0.0195	**6.92**	0.0311	0.28	-0.27	1203
Hides	Bleak	0.0095		0.0561	0.11	-0.92	96
	Non-Bleak	-0.0112	**3.61**	0.0463	-0.36	-0.23	1193
Hogs	Bleak	0.0040		0.0408	-0.08	-1.46	96
	Non-Bleak	-0.0076	**2.80**	0.0441	0.52	0.78	1251
Lard	Bleak	0.0100		0.0445	0.04	-1.45	96
	Non-Bleak	-0.0215	**6.95**	0.0444	0.07	-0.26	1359
Milk	Bleak	0.0052		0.0181	-2.24	8.05	96
	Non-Bleak	-0.0077	**6.97**	0.0253	-0.09	-0.62	1203
Oat	Bleak	-0.0048		0.0317	-0.59	0.06	100
	Non-Bleak	-0.0120	**2.29**	0.0402	-0.08	-0.09	1886
Rubber	Bleak	-0.0788		0.1180	-0.62	-1.37	96
	Non-Bleak	-0.0300	**-4.05**	0.0637	0.03	1.49	1203
Rye	Bleak	0.0093		0.0308	-0.99	0.81	96
	Non-Bleak	-0.0152	**7.76**	0.0492	-0.01	-0.43	1224
Sheep	Bleak	0.0077		0.0340	0.05	-0.88	96
	Non-Bleak	-0.0103	**5.17**	0.0397	-0.08	0.59	1395
Sugar	Bleak	0.0417		0.0897	0.20	-0.83	100
	Non-Bleak	-0.0249	**7.42**	0.0515	-0.56	5.17	1886
Wheat	Bleak	0.0073		0.0304	-0.04	-0.52	100
	Non-Bleak	-0.0129	**6.66**	0.0406	0.12	-0.37	1781
Wool	Bleak	-0.0093		0.0293	0.84	0.36	96
	Non-Bleak	-0.0206	**3.77**	0.0442	-0.50	0.97	1203

Table 4.15 The Summary Statistics of Agricultural Factors in Bleak and Non-Bleak Investment Periods in Stocks at Intervals of Observation of 20 Years

Investment		**Mean**	***t***	**Standard Deviation**	**Skewness**	**Kurtosis**	**N**
Barley	Bleak	-0.0064		0.0212	-0.01	-0.93	96
	Non-Bleak	-0.0152	**4.04**	0.0258	0.10	-0.53	1129
Butter	Bleak	-0.0095		0.0103	0.05	0.79	100
	Non-Bleak	-0.0099	0.43	0.0179	-0.53	0.13	1765
Cattle	Bleak	-0.0090		0.0127	0.72	0.23	96
	Non-Bleak	-0.0001	**-6.89**	0.0201	0.30	-0.48	1467
Cocoa	Bleak	-0.0123		0.0433	-0.27	-1.18	100
	Non-Bleak	-0.0109	-0.32	0.0365	0.08	0.31	1766
Coffee	Bleak	-0.0040		0.0268	-0.31	-1.13	100
	Non-Bleak	-0.0073	1.25	0.0345	-0.17	-0.31	1766
Corn	Bleak	-0.0067		0.0229	0.12	-0.30	96
	Non-Bleak	-0.0117	**2.13**	0.0267	0.15	0.04	1443
Cotton	Bleak	-0.0104		0.0226	0.92	0.10	100
	Non-Bleak	-0.0101	-0.11	0.0331	0.49	2.20	1766
Eggs	Bleak	-0.0122		0.0130	0.00	-0.64	96
	Non-Bleak	-0.0202	**6.02**	0.0187	0.35	-0.45	1083
Hides	Bleak	-0.0001		0.0243	-0.46	-0.46	96
	Non-Bleak	-0.0113	**4.50**	0.0303	-0.09	-0.80	1073
Hogs	Bleak	0.0065		0.0215	-0.58	0.21	96
	Non-Bleak	-0.0081	**6.68**	0.0252	-0.07	-0.26	1131
Lard	Bleak	-0.0045		0.0246	-0.34	0.60	96
	Non-Bleak	-0.0211	**6.60**	0.0269	0.03	-0.13	1239
Milk	Bleak	0.0039		0.0077	-1.68	6.84	96
	Non-Bleak	-0.0077	**14.86**	0.0147	-0.25	-0.56	1083
Oat	Bleak	-0.0060		0.0163	-0.30	0.13	100
	Non-Bleak	-0.0126	**4.04**	0.0239	-0.07	-0.25	1766
Rubber	Bleak	-0.0546		0.0411	-0.67	-0.98	96
	Non-Bleak	-0.0346	**-4.76**	0.0493	-0.69	1.60	1083
Rye	Bleak	-0.0023		0.0157	-1.23	2.26	96
	Non-Bleak	-0.0150	**7.91**	0.0291	0.35	-0.30	1104
Sheep	Bleak	-0.0023		0.0183	-0.42	0.01	96
	Non-Bleak	-0.0100	**4.11**	0.0234	0.05	0.95	1275
Sugar	Bleak	0.0072		0.0380	0.56	0.03	100
	Non-Bleak	-0.0227	**7.87**	0.0268	0.01	0.94	1766
Wheat	Bleak	-0.0114		0.0198	-0.07	-0.83	96
	Non-Bleak	-0.0119	0.27	0.0247	-0.13	-0.10	1665
Wool	Bleak	-0.0179		0.0139	0.44	-0.01	96
	Non-Bleak	-0.0192	0.90	0.0258	-0.59	1.22	1083

Figures 4.14, 4.15 and 4.16 present the mean annualised rates of change in agricultural factors in bleak and non-bleak investment periods in stocks at intervals of observation of 1, 10 and 20 years. The relationship between agricultural factors in bleak and non-bleak investment periods changes as the interval of observation changes with only cattle and rubber having lower rates of price growth at all three intervals of observation in bleak investment periods (the differences are statistically significant too). As the interval of observation increases, the differences between agricultural factors in bleak and non-bleak investment periods decrease, from an average difference of 10.4 percent at 1 year intervals of observation to 2.0 and 0.9 percent at 10 and 20 year intervals of observation. Bleak investment periods at short intervals of observation are characterised with agricultural factors which are mostly ‘bleak’ whereas at long intervals of observation, they are characterised with agricultural factors which are mostly not ‘bleak’.

Figure 4.14 The Mean Annual Rate of Change in Agricultural Factors in Bleak and Non-Bleak Investment Periods in Stocks at Intervals of Observation of 1 Year

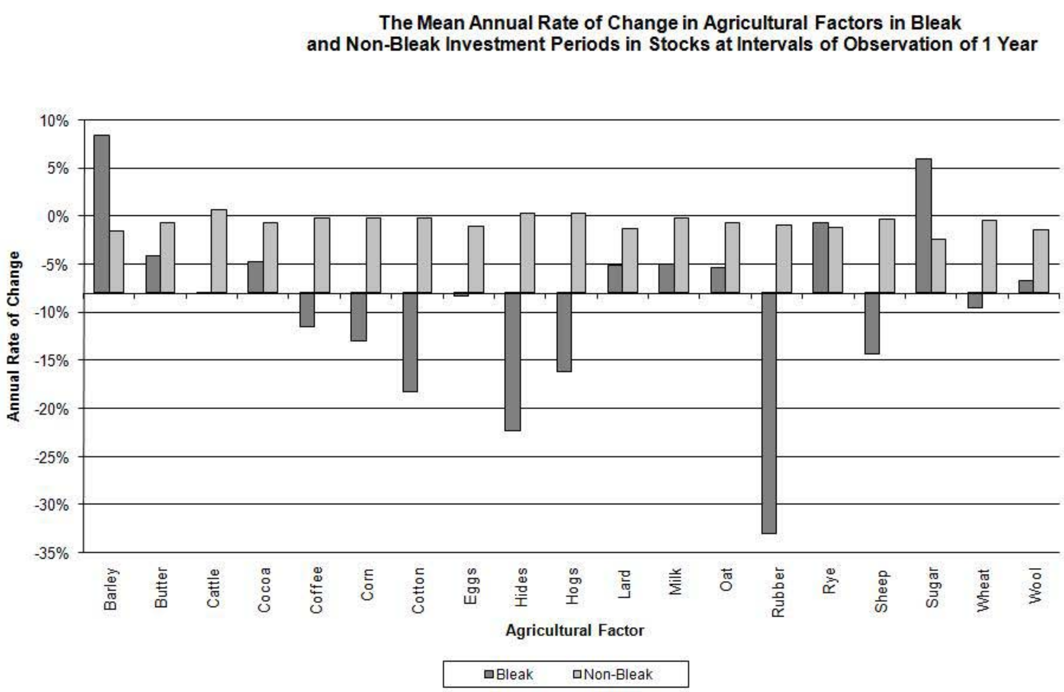


Figure 4.15 The Mean Annualised Rate of Change in Agricultural Factors in Bleak and Non-Bleak Investment Periods in Stocks at Intervals of Observation of 10 Years

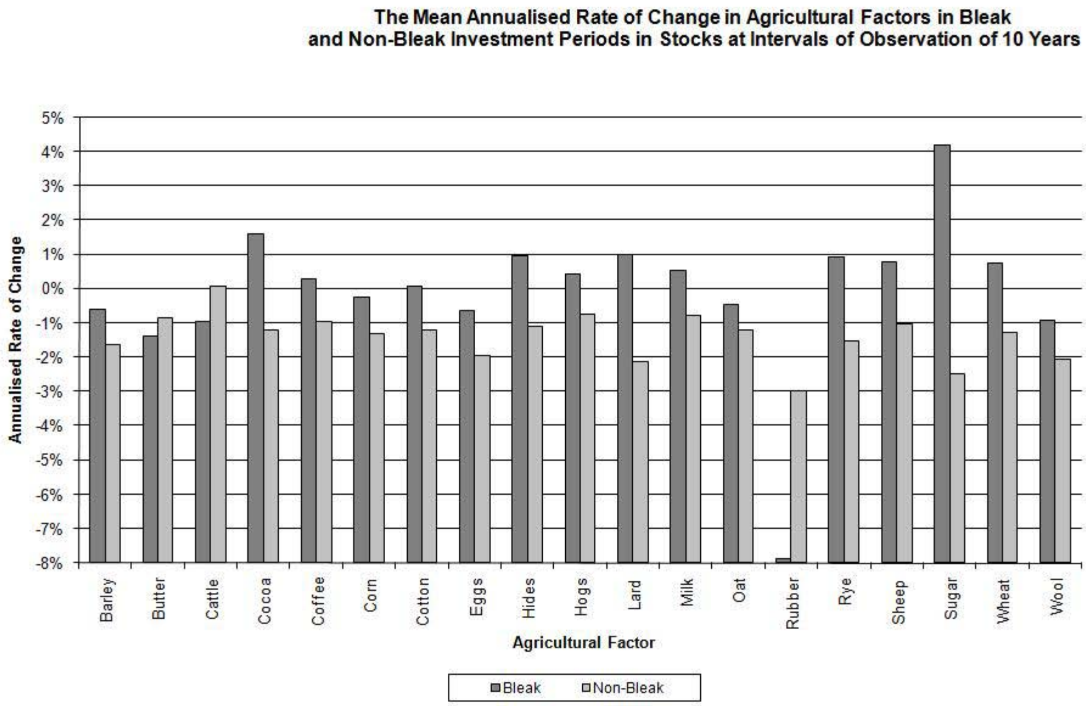
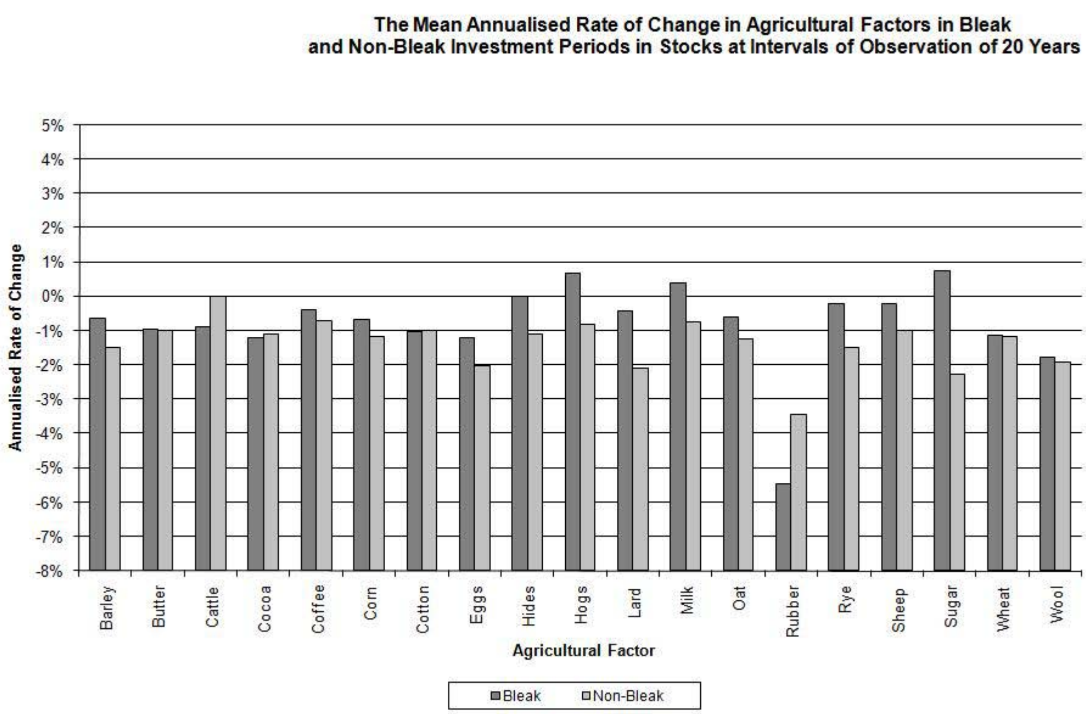


Figure 4.16 The Mean Annualised Rate of Change in Agricultural Factors in Bleak and Non-Bleak Investment Periods in Stocks at Intervals of Observation of 20 Years



4.5 Summary

In this chapter I presented and discussed results for the analysis of the interval of observation and the assessment of risk in stocks. The next chapter discusses the conclusions.

5 Conclusions

5.1 Research Summary

In this research I analyse how the interval of observation affects the assessment of risk in stocks. The risk in stocks is assessed by examining the worst returns on stocks and what changes there are as the interval of observation changes. Tests of statistical significance are used to examine, not only the economic implications, but the statistical implications of these results. In addition, factors which are thought to affect the return on stocks are analysed to determine what relationships there are and what changes there are as the interval of observation changes.

Using real returns on US stocks from 1832 to 2008, I show as the interval of observation increases, the distribution, standard deviation and range of annualised stock returns decrease. The worst returns on stocks can get very bad, with the worst annual return being -64.3 percent and the worst total return being -76.3 percent over a 3 year interval of observation. However, as the interval of observation increases, the worst returns become better and between intervals of observation of 15 to 22 years, the worst returns change from negative to positive. The economic implications of a long interval of observation are significant as are the statistical implications.

In addition, I show there are relationships between factors which are thought to affect the return on stocks and bleak and non-bleak investment periods in stocks. When there is a bleak investment period, economic factors are ‘bleak’ too; and as the interval of observation increases, this relationship remains. When there is a bleak investment period, metal and agricultural factors are ‘bleak’ at short intervals of observation; however, this changes and at long intervals of observation, some metal factors are ‘bleak’ and some are not whereas most agricultural factors are not. At long intervals of observation, the worst returns on stocks occur when the economy is ‘weaker’ and commodity inflation is ‘stronger’.

As the interval of observation increases, the differences between all factors in bleak and non-bleak investment periods decrease. This is expected, because as the interval of observation increases, the returns on stocks in bleak and non-bleak investment periods become more alike (the distribution of annualised stock returns converges towards the median); therefore, the factors are expected to become more alike. Depending on how risk is assessed, this indicates the risk in stocks decreases as the interval of observation increases and the factors reflect this.

The results of this research show the economic and statistical implications of the interval of observation on the risk in stocks are significant, and indicate, as the interval of observation increases, the risk in stocks decreases. Of course, it depends on investors’ perceptions of risk and how long a long interval of observation is. As the interval of observation increases to 15 years or more, there is less and less risk in stocks when risk is assessed by: the distribution, standard deviation, and range of annualised stock returns; how much greater than zero total stock returns become; and to an extent, factors which are thought to affect the return on stocks. This does not guarantee the future will resemble the past, and a description of the past risk in stocks is not prescription of the future risk in stocks. A 15 year time horizon is undoubtedly too difficult for some investors.

5.2 Research Limitations

There are limitations in this research. First, these relate to the number of independent observations and the tests of statistical significance used. The returns calculated from the sample are not independent as overlapping periods are used; therefore, the appropriateness of these tests is limited. In addition, the *t*-tests of statistical significance are novel and the robustness of these has not been proved with previous research. Nevertheless, I use these tests to give a general guide as to how much statistical significant there is. Second, the samples differ in length; therefore, direct comparisons are not possible at all times meaning some information is lost. Third, the analysis of factors which are thought to affect the return on stocks only examines relationships based on means and does not explain to what extent the return on stocks is caused by the factors analysed. Last, this research is a description of what has happened in the past. Prescriptions based on this for what will happen in the future are limited.

5.3 Further Research Questions

In this research, I examine how the interval of observation affects the assessment of risk in stocks in the US. Further research on non-US stocks is possible, especially examining factors which are thought to affect the return on stocks. Research comparing stocks with stocks' opportunity costs in countries with less successful stock histories should answer some questions about the risk in stocks. In addition, there are incentives for investors to miss the worst returns on stocks at short and long intervals of observation; however, questions about the possibility and the practicality of this are unanswered. A test of predictability and setting up a trading rule based on the relationships examined between factors which are thought to affect the return on stocks and the interval of observation should answer some of these questions.

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Appendices

Appendix A

Formulae used for Utility Functions

The calculations are based on the following formulas (Olsen & Khaki, 1998).

$$E(w) = \sum_{i=1}^{N} (p_i)(w_i)$$

and

$$E(u) = \sum_{i=1}^{N} (p_i)[u(w_i)]$$

where

$p_i =$	the probability of the i^{th} outcome
$w_i =$	the wealth associated with the i^{th} outcome
$u(w_i) =$	the utility of wealth associated with the i^{th} outcome
$E(w) =$	expected wealth
$E(u) =$	expected utility

Appendix B

Formulae used for Options Pricing

The cost of shortfall insurance as a function of time horizon is calculated as the price of a put option using a simplified form of the Black-Scholes options pricing model (Bodie, 1995). It is as follows:

$$\frac{P}{S} = N(d_1) - N(d_2)$$

$$d_1 = \frac{\sigma\sqrt{T}}{2}$$

and

$$d_2 = \frac{-\sigma\sqrt{T}}{2}$$

where

$P =$	price of put
$S =$	price of stock
$T =$	time to maturity of the option in years
$\sigma =$	standard deviation of the annualised continuously compounded rate of return
$N(d) =$	the probability that a random draw from a standard normal distribution will be less than d

For decreasing standard deviation over time the following equation is substituted into the above calculations (Taylor & Brown, 1996).

$$\sigma_{\mathrm{T}} = \frac{\sigma}{\left\{\sqrt{\mathrm{T}}\left[1 + i(\mathrm{T} - 1)\right]\right\}}$$ where i is a positive number

Appendix C

Formulae used for Simulations and 'Practitioner Risk Measures'

The following formulae are from Thorley (1995).

"Given a continuous annual risk-free rate, *f*, and initial wealth, W_0, the value of the risk-free fund after *t* years, F_t, is

$$F_t = W_0 e^{ft}$$

Given a normally distributed, continuous, annual risky return with mean, μ, and standard deviation, σ, the value of the risky fund after *t* years, G_t, is

$$G_t = W_0 e^{\left(\mu t + \sigma \sqrt{t} z\right)}$$

where z is the standard normal random variable. Under these two fund options the following formulas are derived. $\Phi(\ )$ denotes the standard normal cumulative density function, and $\Phi^{-1}(\ )$ is the inverse standard normal cumulative density function.

The expected risky fund value a time *t*, $\mathrm{E}(G_t)$, is the solution to

$$\ln\left[\frac{E(G_t)}{W_0}\right] = \left(\mu + \frac{1}{2}\sigma^2\right)t$$

The time *t* risky fund percentile *p* value, $G_t(p)$, is the solution to

$$\ln\left[\frac{G_t(p)}{W_0}\right] = \mu t + \sigma\sqrt{t}\,\Phi^{-1}(p)$$

The probability that the risky fund will be less than the risk-free fund at time *t* is

$$\mathrm{Prob}(\mathrm{G_t} < F_t) = 1 - \Phi\left(\frac{\mu - f}{\sigma}\sqrt{t}\right)$$

If the expected risky return is greater than the risk-free rate, $\mu > f$, then the probability goes from $1-\Phi(0)=\frac{1}{2}$ to $1-\Phi(\infty)=0$ as the time horizon, *t*, gets large. Thus, the probability that the risky fund will be greater than the risk-free fund approaches certainty at very long investment horizons. The time *t* expected value of the risky fund, given that it is less than the risk-free fund, $\mathrm{E}(G_t|G_t < F_t)$, is the solution to

$$\ln\left[\frac{E(G_t|G_t < F_t)}{W_0}\right]=\left(\mu+\frac{1}{2}\sigma^2\right)t+\ln\Phi\left[-\left(\frac{\mu-f}{\sigma}+\sigma\right)\sqrt{t}\right]-\ln\Phi\left[-\left(\frac{\mu-f}{\sigma}\right)\sqrt{t}\right]"$$

(Thorley, 1995, p. 74).