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An Analysis of Two - Layered Flows In Pipelines

**A thesis presented in partial fulfillment of the
requirements for the degree of**

**Master of Science
in
Mathematics**

**at Massey University, Albany,
New Zealand.**

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2003

Abstract

In this thesis, we consider the two-layered fluid flow problem in a circular pipeline. The justification of the model we shall employ, namely the stratified flow model will be discussed.

Our mathematical model is built from first principles, the conservation relations of mass, momentum and energy will be analysed and applied to the various flow configurations. We start by considering the simplest case possible, i.e. the planar axial interface and then extend the model by considering first, the motion of a travelling jump in the pipe section and second, to allow a small interfacial gradient.

The stability of the steady flow configuration is investigated using a linear stability method. This method starts off by assuming a travelling wave solution for the perturbation and then observing whether there is growth (unstable) or decay (stable) in this perturbed state. A stability map is provided to indicate the various possible scenarios for the travelling perturbation wave.

Acknowledgements

I would like to thank my supervisor, Professor Robert McKibbin for his patience, consistent encouragement and never-failing advice as I worked through the thesis material. In addition, I wish to express my appreciation to the staff and Mathematics postgraduate students at the Institute of Information and Mathematical Sciences for their administrative and moral support that they have extended to me over the course of the preparation of this thesis.

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