

Copyright is owned by the Author of the thesis. Permission is given for a copy to be downloaded by an individual for the purpose of research and private study only. The thesis may not be reproduced elsewhere without the permission of the Author.

**Characterization of extremely low-frequency electromagnetic sources
in conducting media**

A thesis presented in partial
fulfilment of the requirements
for the degree
of Doctor of Philosophy
in Physics at
Massey University

Edward Ian Rumball
2001

Abstract

Physical fields whose sources exist within maritime vessels are of concern to ship-designers and military planners. Among the fields of most significance which have been studied intensively, are those of magnetic, acoustic and pressure sources. New technological developments now require analyses of the underwater electric and magnetic fields of onboard, extremely low-frequency electromagnetic sources.

This study investigated methods by which the electromagnetic sources of maritime vessels may be characterised during normal operations in typical coastal environments. It focused on situations where both the sources and field measurement points were located in a common seawater volume. At the electromagnetic frequencies of interest such a medium acts as a thin conducting layer with significant levels of wave reflection and refraction at the media boundaries.

To enhance propagation models of the electromagnetic fields over short ranges, the initial investigations aimed to characterise key parameters of the conducting media in shallow-water conditions. Conductivity values of seawater can be readily established by conventional methods. However, in the case of the seabed media, direct conductivity measurements are usually highly variable along horizontal and vertical sections due to aeons of land erosion, and the long-term effects of inshore waves and currents. Procedures are described which show how electromagnetic theory and indirect measurement techniques may be used to infer the characteristic values of key seabed parameters in shallow-water areas. This element of the study utilised both analytical and numerical electromagnetic models, and the efficacy of each in this context was examined.

Subsequent phases of the study analysed the nature of the electromagnetic sources. In some situations the sources were regarded as point dipoles, and in others they were assumed to have a finite length. Techniques were developed to characterise the dipole moment, length and the location of a typical ship-like source, when each is treated as an electric current dipole. This information was used in turn to demonstrate the likely accuracy in ranging operations on extremely low-frequency, electromagnetic sources over short ranges, and in shallow-water conditions.

Acknowledgements

The opportunity is taken to acknowledge with gratitude the encouragement of my study supervisors at the Institute of Fundamental Sciences and the Institute of Information and Mathematical Sciences.

The support of colleagues at the Defence Operational Technology Support Establishment (formerly the Defence Scientific Establishment) Auckland, and in the Royal New Zealand Navy is also acknowledged, and highly valued.

Table of contents

Acknowledgements	iii
Table of contents	iv
List of figures	vii
1. Introduction	i
1.1 Background	1
1.2 Research interests in extremely low-frequency electromagnetic fields	1
1.3 A new research requirement	4
1.4 Outline of the study	6
1.5 Underwater ELF electromagnetic sources and measurement facilities	8
1.6 Factors influencing the experimental measurement programme	13
1.7 Considerations in numerical modelling	15
2. Analytical models of electromagnetic fields	18
2.1 Characteristics of models for shallow water conditions	18
2.2 Desirable features of an analytical electromagnetic model	19
2.3 Basic equations of electromagnetic fields	20
2.4 Dipole in a layered medium	22
2.5 Vertical electric dipole	24
2.6 Horizontal electric dipole	32
2.7 Radial fields of electric dipoles	33
2.8 Observations concerning radial field predictions	34

3. Applications of analytical models	37
3.1 Characteristics of the primary test site	37
3.2 Computer simulations of seabed conductivity investigations	41
3.3 Deployment of the multi-influence sensor system	46
3.4 Measurements from serial 060	49
3.5 Aggregation of the survey results	51
3.6 Characteristics of the experimental results	53
3.7 Review of the analytical model	56
3.8 Potential for model enhancement	58
4. Numerical models of electromagnetic fields	60
4.1 General techniques for numerical modelling	60
4.2 Characteristics of numerical techniques	60
4.3 Finite element volumes of the hybrid model	63
4.4 Method of moments regions of the hybrid model	67
4.5 Design configuration of the discrete elements	70
4.6 Discrete elements of the FEM weak form	75
4.7 Discrete elements of the MOM surface integrals	76
4.8 Coupling of numerical techniques	77
4.9 Extended hybrid method	80
4.10 Comparison of results from analytical and numerical models	81
5. Applications of numerical models	88
5.1 Characteristics of simulated test sites	88
5.2 Computer simulation of a sloping seabed (<i>case I</i>) configuration	90
5.3 Computer simulation of a sloping sub-bottom (<i>case II</i>) configuration	95
5.4 ELFE source crossings of the sensor axis	99
5.5 Estimation of the dipole strength of onboard ELFE sources	105
5.5 Observations concerning the simulated experiments	106

6. Summary, conclusions and future issues	109
6.1 Summary	109
6.2 Conclusions	116
6.3 Future issues	118
 Appendix A: Estimating conductivity parameters for extremely low-frequency electromagnetic models	 121
 Appendix B: Rangings of extremely low-frequency electromagnetic sources over short ranges and in shallow-water conditions	 133
 Appendix C: Keywords for input data files of numerical models	 168
 Appendix D: Glossary of abbreviations	 174
 Bibliography	 177

List of figures

- Figure 1.A Schematic diagram of the equipment configuration used to measure multiple influence fields radiated from maritime vessels (page 9)
- Figure 1.B Diagram of the underwater frame showing the layout of the multi-influence sensor system (page 9)
- Figure 1.C Diagram of the buoyant surface platform to house the battery power system and the recording equipment (page 12)
- Figure 1.D Diagram of the towed catamaran hull used as a platform for the sources of selected influence fields (page 12)
- Figure 2.A Nomenclature for modelling fields of point-dipole sources in a one layer conducting medium (page 23)
- Figure 2.B Nomenclature for modelling fields of point-dipole sources in two layer conducting media (page 23)
- Figure 2.C Calculated values of radial electric fields for a VED point-source in a semi-infinite seawater medium (*) and in a medium of finite depth (o) (page 35)
- Figure 2.D Calculated values of radial electric fields for a HED point-source in a semi-infinite seawater medium (*) and in a medium of finite depth (o) (page 35)
- Figure 3.A Primary test site in the Pelorus Sound, showing ship tracks along which electromagnetic signal propagation experiments were conducted (page 38)

- Figure 3.B Vertical cross-section of the seawater and upper seabed layers, along a bearing of 48° through the position of the multi-influence sensor system (page 38)
- Figure 3.C Typical vertical cross-section from the chart record of the sub-bottom profiler where the vertical and horizontal scales are 1:200 and 1:5000, respectively (page 40)
- Figure 3.D Simulated analyses with a seawater depth of one skin-depth and with a seabed conductivity value set at 2 S.m^{-1} (page 45)
- Figure 3.E Simulated analyses with the seawater depth and horizontal ranges resembling site-like conditions and with the seabed conductivity value set at 2 S.m^{-1} (page 45)
- Figure 3.F Deployment of extremely low-frequency, electromagnetic dipole sources from HMNZS TUI in Tennyson Inlet (page 48)
- Figure 3.G Deployed extremely low-frequency, electromagnetic dipole sources being towed over the submerged multi-influence sensor system (page 48)
- Figure 3.H Recorded spectral levels of the 35 and 74 Hz electromagnetic signals at the multi-influence sensor system, during serial 060 (page 50)
- Figure 3.I Estimates of the characteristic seabed conductivity values (*) and the best-fit cubic polynomial approximation (o), for serial 060 (page 50)
- Figure 3.J Estimates of the characteristic seabed conductivity values from serial 060 (*) and complementary serials (o) at Tennyson Inlet (page 52)
- Figure 3.K Contour levels between 0.5 and 4 S.m^{-1} (in steps of 0.5 S.m^{-1}) for the characteristic seabed conductivity values at Tennyson Inlet (page 52)

- Figure 3.L Normal incidence view of the contour levels between 0.5 and 4 S.m⁻¹ (redrawn from figure 3.K) for the characteristic values of seabed conductivity, at Tennyson Inlet (page 54)
- Figure 4.A Original configuration (a) showing the geometry of an inhomogeneous dielectric body partially covered by conductors, with (b) its external equivalence and (c) its internal equivalence (page 65)
- Figure 4.B Node definition and edge numbering scheme of a tetrahedron (page 72)
- Figure 4.C Coordinates of a common edge associated with two triangles (a), and geometry showing normal component of basis function at the edge (b) (page 74)
- Figure 4.D Specification for the numerical model of a unit, 50 Hz 1 m, VED source in a one-layer conducting medium (page 83)
- Figure 4.E Calculated 50 Hz, vertical field values of a unit, VED point-source (*) and a unit, VED 1 m, source (o) in a one-layer conducting medium (page 83)
- Figure 4.F Specification for the numerical model of a unit, 50 Hz 5 m, VED source in a one-layer conducting medium (page 85)
- Figure 4.G Calculated 50 Hz, vertical field values from analytical (*) and numerical (o) models of unit, 5 m VED sources in a one-layer conducting medium (page 85)
- Figure 4.H Specification for the numerical model of a unit, 50 Hz 1 m, HED source in two-layer conducting media (page 87)

- Figure 4.I Calculated 50 Hz, horizontal field values of a unit, HED point-source (*) and a unit, HED 1 m source (o) in two-layer conducting media (page 87)
- Figure 5.A Nomenclature for numerically modelling fields of dipole sources in two-layer conducting media with a sloping interface (page 89)
- Figure 5.B Nomenclature for numerically modelling fields of dipole sources in three-layer conducting media with a sloping interface (page 89)
- Figure 5.C Specification for the numerical model of a unit, 50Hz 5 m, HED source in two-layer conducting media with a sloping interface (page 94)
- Figure 5.D Iterative simulated estimates of the seabed conductivity using a unit, 50 Hz 5 m, HED source in two-layer conducting media with a sloping interface (page 94)
- Figure 5.E Specification for the numerical model of a unit, 50Hz 5 m, HED source in three-layer conducting media with a sloping interface (page 98)
- Figure 5.F Iterative simulated estimates of the seabed conductivity using a unit, 50 Hz 5 m, HED source in three-layer conducting media with a sloping interface (page 98)
- Figure 5.G Graphical representation of experiments in which a towed HED source crosses the extended axis of the electric field sensors, at normal incidence (page 100)
- Figure 5.H Measurements of the electric field as a towed HED source crosses the extended axis of the electric field sensors, at normal incidence (page 100)

- Figure 5.I Specification for the numerical model of a unit, 50 Hz 1 m, HED source which crosses the extended axis of the electric field sensors, at normal incidence (page 102)
- Figure 5.J Predicted values of the horizontal field of a unit, 50 Hz 1 m, HED source which crosses the extended axis of the electric field sensors, at normal incidence (page 102)
- Figure 5.K Specification for the numerical model of a unit, 50 Hz 4 m, HED source which crosses the extended axis of the electric field sensors, at normal incidence (page 103)
- Figure 5.L Predicted values of the horizontal field of a unit, 50 Hz 4 m, HED source which crosses the extended axis of the electric field sensors, at normal incidence (page 103)
- Figure 5.M Specification for the numerical model of a unit, 50 Hz 16 m, HED source which crosses the extended axis of the electric field sensors, at normal incidence (page 104)
- Figure 5.N Predicted values of the horizontal field of a unit, 50 Hz 16 m, HED source which crosses the extended axis of the electric field sensors, at normal incidence (page 104)
- Figure 6.A Brief specification of project computer (page 115)

Figure 6.B Relationship between memory allocations required for numerical
models and the approximate program execution times (page 115)

1. Introduction

1.1 Background

The Study. Physical fields whose sources exist within maritime vessels are of concern to ship-designers and military planners. Among the fields of most significance that have been studied intensively are those of magnetic, acoustic and pressure sources. New technological developments now require analyses of the underwater electric (E-field) and magnetic (B-field) influences of onboard, extremely low-frequency, electromagnetic (ELFE) sources.

This study investigated methods by which the ELFE sources of maritime vessels or ship-like ELFE sources may be characterised during normal operations in typical coastal environments. It focused on situations where both the sources and field measurement points were located in a common seawater volume. At the electromagnetic frequencies of interest such a medium acts as a thin conducting layer with significant levels of wave reflection and refraction at the media boundaries.

In the context of this study a maritime vessel is a submarine or a surface ship with a direct-drive or motor-driven propulsion system.

1.2 Research interests in extremely low-frequency electromagnetic fields

Background. Except for specialised applications, electromagnetic frequencies in the ELF range of 5 Hz – 3 kHz are seldom used for wireless communications. Such a situation is partly due to the low data rates which are characteristic of communications at these frequencies. Another factor is the large antennae that would be needed for efficient radiation of the signal power. For instance, at an electromagnetic frequency of 50 Hz a one wavelength antenna in air would measure about 6000 km. In seawater with a conductivity of 4 S.m^{-1} the same type of antenna would have a length of 223.6 m.

Nevertheless, there are situations in which natural or man-made ELFE fields are used. Some of these applications have been subjects of considerable research and they are briefly described below:

Navigation. The propagation of ELFE waves in the seawater medium has been of practical interest for over eighty years. Shortly after the 1st World War, British Admiralty scientists¹⁻² investigated the experimental and theoretical aspects of the ELFE fields generated by alternating currents flowing through submarine cables. This work was motivated by the possibility of using the cable-generated magnetic fields as navigation aids in the approaches to key harbours. Further applications of this type were developed over the following decades³.

Communication. More recently, it was recognised that despite the absorption of electromagnetic energy in seawater, in some circumstances signals in the ELF band are able to propagate to moderately great distances across the ocean, and below its surface. Moreover, the signal-excess over the ambient ELF background noise⁴⁻⁵ in these conditions could provide an adequate margin for specialised communication purposes, despite the presence of Schumann⁶⁻⁷ resonance signals in the 5 – 50 Hz band - a consequence of world-wide discharges of lightning in the atmosphere.

Such phenomena utilise the leaky interfaces of the ocean with the atmosphere and seafloor, where the latter media are relatively efficient wave-guides. It is the leakage across these boundaries that enables ELFE signals to reach a range from the transmitter that is much greater than when the equivalent transmitted energy is propagated solely through the ocean medium. Characteristics of these up-over-down and down-over-up modes of ELFE propagation have been the topic of many investigations⁸⁻¹⁰.

About thirty years ago the phenomena were identified as possible means for long-range communication with submerged submarines¹¹, albeit at low data rates. This foresight led to subsequent theoretical and experimental studies, especially in the United States¹²⁻¹³ and Russia. In both countries ELFE signals are now routinely used for communicating with submerged submarines during deployments.

Geophysical research. Subsequent applications have exploited man-made ELF fields for geophysical prospecting. Electromagnetic signal propagation through the seafloor media is the basis of several exploration techniques. In the past decade, two practical methods that make use of seabed-to-seabed propagation have been developed for estimating the conductivity of the ocean floor^{14,15}. The first such technique is based on a vertical electric dipole (VED) source and horizontal magnetic-field detectors¹⁶. This method has been used for shallow sounding on the continental shelf and on the axis of a mid-ocean spreading centre.

The second method employs a horizontal electric dipole (HED) source and horizontal electric-field detectors¹⁷. Experimental work with this technique has yielded information about the electrical conductivity of the oceanic lithosphere to depths of 20 km. In particular, a relatively low conductivity (about 10^{-5} S.m^{-1}) region has been detected beneath the oceanic crust for moderate-age lithosphere. These results complement data concerning the shallower electrical structure of the oceanic crust obtained from DC experiments and logging measurements conducted in drill-holes.

Extremely low-frequency electromagnetic fields also occur within and around the earth as natural large-scale phenomena. These are identified as magnetotelluric fields. They are believed to result from the flow of charged particles in the ionosphere since fluctuations in the fields can be correlated with diurnal variations in the geomagnetic field caused by solar emissions. Magnetotelluric fields penetrate the ground and induce telluric currents to flow, especially in media of moderate conductivity. The fields are of various frequencies ranging from 10^5 Hz to the audio range, and have applications in geophysical prospecting¹⁸.

The relatively sparse direct-measurements of seafloor conductivity can be integrated with information on the electrical conductivity of the earth derived from land-based magnetotelluric experiments. Combined results suggest that marked vertical variations in the electrical conductivity of the seabed are commonplace¹⁸.

Health research. Finally, in the context of this review it is appropriate to consider studies of ELF waves in conducting media of an entirely different type – that of human tissue. The earliest studies of possible health effects from occupational

exposure to ELFE fields were published in the former Soviet Union in the 1960s. Subsequent public anxieties and the burgeoning of literature in this research topic led eventually to the promulgation of many guidelines for limiting human exposure to such fields¹⁹.

Although causative relationships have been established between human-cell changes and exposure to certain levels of ELFE fields much less progress has been made in modelling these processes²⁰. This situation arises because the long ELFE wavelengths involved mean the exposures must be considered as generated by orthogonal, near-field, electric and magnetic sources. Hence, the ELFE fields in living tissue behave independently and their net impacts are difficult to quantify. Despite this obstacle some aspects of health research have useful commonality with the numerical modelling of ELFE fields in the conducting media typically located in coastal environments.

1.3 A new research requirement

Ship fields. Detecting the presence of a mobile vehicle such as a surface ship or submarine²¹ by measurement of the induced electromagnetic fields was for many years regarded as being relatively ineffective. This view is changing because the acoustic silencing of modern submarines has greatly reduced the expected detection opportunities from both passive and active sonar detection systems. Similar observations could be made in respect of magnetic-silencing measures on new surface ships and submarines, and the consequential reduction in the likely detection ranges using conventional magnetometers. But now the sensitivity of electromagnetic field sensors²² has improved sufficiently to provide detection opportunities at useful ranges. Modern sensors of magnetic induction can provide sensitivities of 0.001 nT over the lower portion of the ELF band. In the same band, sensors of voltage gradient spectra can achieve sensitivities of $2 \cdot 10^{-11} \text{ V m}^{-1}/\text{Hz}^{0.5}$.

Opportunities to detect the E-field and B-field influences of a moving vessel may be divided into two categories; detection of a direct emission or radiation from an onboard source, and the detection of the vessel's wake. Radiated ELFE fields

associated with each vessel can often be traced back to rotating components within the onboard machinery. Sometimes these field variations can be detected several hundred metres from the vessel. On the other hand the wake is a very large feature extending up to 15 km in some cases and the motion of the conductive seawater in this volume induces a significant electromagnetic field.

Nevertheless, simulated wake studies²³ suggest that there is uncertainty concerning the practicability of detecting a submarine at assumed submerged depths of 15 - 30 m, and likely velocities of 10 m.s⁻¹ (or about 20 knots). Furthermore, as the depth of the submarine increases to the typical operating modes the effect of the surface wave-train diminishes very rapidly. But in a stratified medium as the depth of the vehicle increases the internal waves can be expected to induce a higher electromagnetic field, at least under water.

Clearly, some progress has been made in quantifying the likely wake-generated fields of a maritime vessel that is underway. But the ability to detect the corresponding ELFE field emissions from the wake is more uncertain. One factor that contributes to this uncertainty is the wide variation of the ambient ELF noise, where the primary sources include large near-shore currents and storms. Large variations in the operating modes of a vessel also contribute an element of uncertainty about detecting the wake effects.

In comparison the detection of onboard ELFE sources is primarily a near-field issue. And unlike the wake-generated fields referred to earlier these ELFE field emissions are only partially dependent on the velocity of the vessel. Detection opportunities will be largely determined by the characteristics of the media interposed between the sources and the distant field sensors, and their separation.

A key objective of this study was to find a means to detect and characterise such sources. It was recognised however, from the earlier observations that the detection opportunities were most likely to occur as a vessel moved through focal areas, choke points or enclosed shallow waters. Maritime environments such as these were central features in the study.

1.4 Outline of the study

Objectives. The initial aim of this study was to investigate propagation losses of ELFE signals over short ranges between immersed sources and field sensors in a common seawater medium. Both analytical and numerical modelling techniques were used in the investigations. Calculated propagation losses were then compared to actual ELFE signal losses at several coastal sites around New Zealand.

Information about the signal propagation losses was then used to estimate the characteristics of several ELFE sources. Comparisons were made with the measured characteristics of the same sources. Later, temporal variations in the received sensor signals were studied as a further means of characterising the dipole sources.

In some instances the mathematical models used point sources and sensors – in others these facilities had finite dimensions. The stratified media had a wide range of conduction properties. Analyses were extended to consider inter-media boundaries that are not horizontal surfaces; that is to say the effects of oblique interfaces between the conducting media were also investigated.

Study layout. In the **Introduction** a brief review highlights those topics where there have been major investigations of extremely, low-frequency electromagnetic fields in conducting media. Several applications are described.

A new research field has been opened by recent technological developments, and this development is described and reviewed. Study objectives within this new topic are then defined. To conclude the section there are abbreviated descriptions of both the experimental facilities used in the study and of the major factors such as the project costs which influenced the extent of the measurement programme.

Analytical models of electromagnetic fields are examined in the second section. Initial reviews consider the efficacy of a number of analytical formulae for the modelling tasks within this study. One such model was selected for development and testing. Computer simulations are described which provided a degree of

confidence in the technical goals of the project. These investigations were completed before the commencement of an expensive programme of equipment construction.

In the section titled the **Applications of analytical models**, the selected analytical model was used in computer simulations, and for examining a series of experimental measurements. The advantages, limitations and implications of this modelling approach are discussed at length.

Numerical models of electromagnetic fields were examined in the fourth section. Again, reviews were made of the possible approaches that are presently feasible for applying numerical models to the project tasks. One such model was selected for testing and development.

The study describes a series of computer simulations that show the comparative field predictions of the selected analytical and numerical models in similar configurations. Graphs are included which show the respective E-field predictions for sources far below the surface of a semi-infinite seawater volume. Subsequent comparisons describe the respective E-field predictions in layered conducting media, and where the ELFE sources may be either point-dipoles or dipoles of finite length.

The title of the fifth section is the **Application of numerical models**. Computer simulations were used to investigate the applicability of the selected numerical model. A feature of this work is that the modelling facilities were used to investigate more complex (and realistic) configurations of the environment such as a non-uniform seabed.

The final section contains the **Summary, conclusions and future issues**. New results and insights are described. Future areas of research are discussed. Technological factors are identified which are central to these types of investigations.

It was known from the outset that the project budget could not fund costly techniques such as offshore drilling operations to characterise seabed geophysical properties in areas of interest. However, prior to the construction of the E-field measurement facilities a series of computer simulations demonstrated a possible new technique for

estimating these properties. Details of the simulation methodology are described in **Appendix A: Estimating conductivity parameters for extremely low-frequency electromagnetic models**. It is a reprint of a paper²⁴ published in 1994 that provided the initial concepts for subsequent ELFE experiments in coastal environments.

In the course of the study some further results appeared in other publications²⁵⁻²⁶ including an invited paper which is reprinted as **Appendix B: Rangings of extremely low-frequency electromagnetic sources over short ranges and in shallow water conditions**. Comprehensive descriptions of the early experiments and the E-field measurement facilities are provided in the latter publication.

Because of the difficulties of describing and deciphering the three-dimensional numerical models using narrative or diagrammatic descriptions, in this study each configuration that is evaluated is defined exactly by the format of an input data file. A guide that explains the various terms used is provided in **Appendix C: Keywords for input data files of numerical models**.

In the penultimate section the common abbreviations used in the report are collated within **Appendix D: Glossary of abbreviations**.

The final section is the **Bibliography** that lists the various references used in the course of this report.

1.5 Underwater ELF electromagnetic sources and measurement facilities

Overview. The E-field measurements referred to throughout the study were obtained as a ship towing various field sources manoeuvred in the vicinity of electric field sensors that had been deployed on the seabed. In the main these manoeuvres consisted of the ship following assigned tracks at known distance offsets from the centre of the sensor configuration.

A diagrammatic representation of various equipment facilities involved in the measurement process is shown in figure 1.A (on page 9).

E-field measurements were made as the ship transited past the sensor array on straight tracks at 2 - 4 m.s⁻¹ (or 4 - 8 knots). A dan buoy was provided as a visual marker to assist ship navigation and to aid the recovery of the underwater frame. Preferred deployment sites had a surficial layer of sand, shingle or clay on the seabed.

Multi-influence sensor system. Over the last few years laboratory staff have developed the technology and equipment²⁷⁻²⁹ to measure the influence fields radiated from ships. Measurements of particular interest were those of the acoustic, pressure, seismic, three-axis magnetic and electric fields. The respective sensors were mounted on an underwater frame shown in schematic form in Figure 1.B (on page 9). An electric cable provided a link to the surface platform that contained the power and data recording facilities.

Like the sensors, the associated preamplifier components were also mounted on the underwater frame. Wherever possible non-magnetic and non-conducting materials were used to minimise any self-induced perturbations in the magnetic and electric fields. All pressure housings were manufactured from polyvinyl chloride (PVC) rod for a design depth of 100 m.

The equipment was modular, transportable and easily deployed in sheltered waters. Optimum sensor performance was obtained when the seafloor was relatively flat and at a water depth of 15 – 25 m. For a 35 Hz ELFE signal a water depth of 25 m corresponds to approximately 0.6 skin depths where such a depth is taken here as the effective depth of penetration. At such a depth a plane-wave signal that penetrated the surface at normal incidence would be attenuated by 37% (e^{-1}) of its amplitude at the surface. E-field measurements were made over horizontal ranges from the sensors that corresponded to 2 to 8 skin depths.

The multi-influence fields were measured using commercial devices linked to specialised equipment developed by staff from the laboratory. Sensors were chosen which could record the static and dynamic characteristics associated both with ship influences and with the ambient environmental fields. Standard design features

included the use of validation test signals and of dynamic, range-compression techniques³⁰⁻³¹.

Buoyant surface platform. The surface platform shown in figure 1.C (on page 12) housed battery power supplies and data recording facilities. A feature of this buoyant platform was that it had a catamaran hull-form to provide a stable base for the sealed equipment space in an above-water cabin.

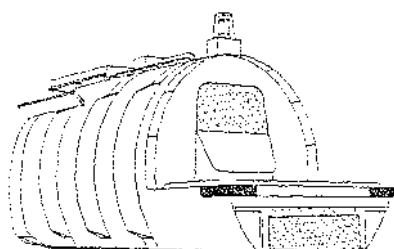
Electric field sources. Controlled ELFE sources were necessary to generate E-fields for the development and testing of the measurement equipment. Such sources not only assisted the processes of characterising the measurement sites they also provided data to verify models for each of the influence fields. No E-field signatures of actual shipboard equipment were available for this study.

The catamaran-hulled configuration shown in schematic form in Fig. 1.D (on page 12) was selected as the most suitable platform for the various sources. Like the sensor frame all construction materials were non-magnetic and non-conductive to simplify the operation and modelling of the magnetic and electric field sources.

A precise separation distance was maintained between the ship and towed source to provide temporal discrimination between the respective E-fields. Deployment depths of the sources were selectable in the range 0.5 - 2.5 m below the surface.

Electric field sensors. Measurement configurations and the relatively low-strength sources (approximately 4 A.m) required the use of low-noise electric sensors to measure the E-fields. Moreover, it was a planning requirement to measure not only the E-fields of ships and ship-like sources, but also the low-level ambient fields and phenomena such as the Schumann resonances.

A satisfactory performance was achieved by using dual Ag-AgCl³² cells linked to a low-noise differential amplifier. The Ag-AgCl cells acquired for this application had low internal resistance and very low self-noise particularly at low frequencies.

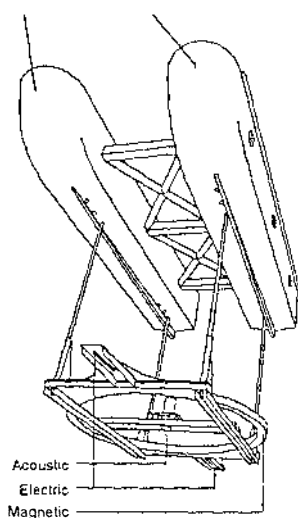
**Overall**

Length: 4.9 m. Beam: 2.5 m. Height: 2.4 m.
 Weight: 1.8 tonnes (fully equipped).
 Design: catamaran hulls, bridging cabin.
 Cabin: 2.2(l) x 1.2(b) by 1.5(h) m.
 Construction: marine plywood, fibreglass.
 Navigation: Fl(5) Y 10s 2m.

MIS Equipment Fit

StoreMouse plus interface.
 Battery supplies (18 x 6 V lead acid, 1000 A-Hr).

Figure 1.C. Diagram of the buoyant surface platform to house the battery power system and the recording equipment

**Catamaran Hull**

Length: 4.2 m. Beam: 2.2 m. Weight: 542 kg (fully equipped).
 Design: round nose, hard chine, 0.6 x 0.6 m cross-section, keeled.
 Construction: 13 mm 5-ply marine plywood, high density foam filler, pultruded fibreglass beams, fibreglass skin.

Underwater Frame

Depth: variable from 0.5 to 2.5 m.
 Construction: pultruded fibreglass beams, fibreglass sheet.

Acoustic Source

USRL J9 transducer.
 Maximum sound pressure level: 120 dB re 1 μ Pa at 1 m, > 40 Hz.

Magnetic Coil

Former: fibreglass, 25 by 90 mm slot, 1.93 m mean diameter.
 Wiring: automotive cable #150, ϕ 2.6 mm, 8A, 200 turns.
 Inductance: 0.18 H. Maximum level: 37 nT at 25 m, 4 A, 40 Hz.

Electric Electrodes

Material: Titanium rod. Length: 170 mm. Diameter: 15 mm.
 Spacing: 3.2 m. Maximum level: 4 V/m at 4 A.

Tow Cable

Strain member: 8 mm spectra, 30, 45, 60, 75, 90 m stops.
 Electrical: 3-core, type 24320, 2.5 mm², 20 A with heavy duty thermoplastic rubber insulation (ϕ 12.2 mm).

Figure 1.D. Diagram of the towed catamaran hull used as a platform for the sources of selected influence fields

Careful design of the cells was necessary to minimise electromagnetic noise interference. To this end screened rods were used in the electromechanical assembly. These rods formed a 2.75 m screened conductor from each cell to the frame-mounted amplifier.

A very low-noise matching preamplifier of high performance was developed specifically for this application. Final testing demonstrated a usable frequency bandwidth from 0.5 mHz - 100 Hz, whilst the noise-floor was maintained at -188 dBV.Hz^{-0.5} at 1 Hz, and -192 dBV.Hz^{-0.5} at 10 Hz.

Comprehensive details of the measurement facilities referred to in this section are provided in appendices A and B and the references therein.

1.6 Factors influencing the experimental measurement programme

Selection of measurement sites. From the outset the two trials were planned to acquire useful information from all measuring devices, not just the E-field sensors. Site choices were also greatly influenced by the ease of logistic support. Support for the RNZN diving team that was required for the deployment and recovery of the multi-influence sensor system was particularly important. Water depths of less than 30 m generally enable divers to operate without an on-site decompression chamber and adherence to this limit simplified the support planning.

To provide adequate margins for ship manoeuvring and short-term transit velocities in excess of 8 m.s⁻¹ (or 16 knots), the test sites required an absence of through-traffic and a clear area of about 1 km radius from the seabed sensors. In addition the measurement sites needed to be far from the electrical interference of built-up areas and from the national 50 Hz power-supply grid. A flat seabed was preferred to simplify the later modelling processes.

The first site referred to in this study is 100 km east of Auckland and at the northern end of Great Barrier Island in the well-protected harbour of Port Fitzroy. Ships of the Royal New Zealand Navy carried out the most recent hydrographic survey of the

area in 1964. Small pleasure craft comprised the bulk of the infrequent maritime traffic through the measurement site.

The second site selected was in the Pelorus Sound about 100 km west of Wellington. The specific location was at Tennyson Inlet. It is located about 35 km from the open sea, and again extremely isolated and an infrequent host to small pleasure craft. The Royal Navy vessel HMS PANDORA conducted the first and only hydrographic survey of this area in 1854.

Environmental measurements. An inescapable feature of such out-of-the-way sites was that there was minimal environmental information available prior to the measurement programme. For instance, the only offshore logs available were from seabed drill-sites many kilometres away.

Such shortcomings were significant as coastal areas usually have quite variable undersea topography and seabed properties. This characteristic is largely due to local tidal flows, the prevailing current-patterns and the runoff from the surrounding terrain. Aeons of erosion from the nearby landforms was also likely to have contributed to the variability in density and conductivity of sub-bottom layers³³ within both vertical and lateral sections.

The resistivity of a seabed sample can often be measured directly by diver-operated probes that utilise the four-electrode contact technique. The equipment consists of two pairs of electrodes; one pair applies the reference current and the other measures the resulting electrical potential. Figure 2 (on page 130) of appendix A shows a typical set of measurements.

However, with these hand-held systems it is difficult to obtain resistivity measurements at depths of more than several metres, even in surficial layers of mud. And many such measurements are needed to adequately map a small area.

To enable corrections to be made for tidal height variations it was appropriate to maintain continuous records of the shipboard echo-sounder measurements. Additional support for the environmental investigation at each site was provided

wherever possible by soundings from a Geopulse sub-bottom profiling system with peak energy output in the acoustic band of 0.5 – 5 kHz. This equipment provided qualitative sub-bottom data to a depth of about 50 m below the seafloor.

Seawater conductivity values typically lie within the range 3 - 5 S.m^{-1} , and are by contrast relatively simple to obtain. In New Zealand conditions the seawater conductivity is a slowly-varying function of temperature, salinity, frequency of the propagating electromagnetic wave and pressure, in that order of importance. Site measurements can be readily obtained from a range of onboard conductivity-temperature-density (CTD) instruments. Most of this environmental characterisation was carried out during the trial programme.

Position records. Accurate records of the ship's tracks were essential for most measurements, especially those involving magnetic field sensors where the B-field values change rapidly with varying ranges. These requirements were addressed in two ways. Firstly, the helmsman was provided with a head-up display showing the position of the ship relative to the required track. And secondly, the absolute position of the ship was accurately logged throughout using differential global positioning systems (DGPS).

Such systems can provide reasonable accuracy when post-processing techniques are used. In this instance, algorithms were developed for filtering the logged data to provide a positional accuracy of about 1 m. The data output was a set of computer files linking records of time with the range and bearing of the vessel from the E-field sensors during each phase of the measurement program.

1.7 Considerations in numerical modelling

Limits of computation tasks. Reference was made earlier to the variable nature of conductivity measurements in coastal areas, especially those of the seabed. Such measurements can vary widely across horizontal and vertical sections. Numerical modelling of ELFE wave propagation in these conditions is thus, unavoidably, a three-dimensional issue.

Models of these processes can entail lengthy computational tasks. To achieve the desired accuracy of the numerical solution it was therefore essential to pay close attention to the trade-offs between the assigned computer memory and the program execution time.

Throughout this study all of the calculations were carried out on a 1997 Compaq PRESARIO (166/200 MHz processor, 56 Mbyte RAM) computer. Accordingly, at certain points of the study (which are identified) the gross accuracy of some numerical solutions was relaxed in order to keep the program execution times within reasonable bounds. Generally, a period of 24 hours was used as a limit for individual calculations. A limit of 7 – 10 days was maintained for models that entailed lengthy iterative processes.

At first sight it may seem unworldly to have persisted with such lengthy calculations. However, an important consideration was a product announcement by IBM Corp. in mid-1999 describing the development of 3.3 – 4.5 GHz computer-chip processors. When these devices reach the component market the present lengthy calculations will become merely routine.

Specifications of the numerical models. Whereas the details of an analytical model can be simply stated in a diagram or a few lines of text, the same cannot be said of three-dimensional numerical models. For instance, the latter frequently require a non-uniform system of grids in the model space and different length scales along the three axes. In addition, although the parameters are uniform within each dielectric slab, the model space may involve asymmetric configurations of two, three or four dissimilar types of dielectric slabs. These considerations make the configuration of numerical models difficult to describe with simple diagrams and a few lines of text.

Throughout this study the situation was resolved by using a specialist vocabulary³⁴ of twelve strings of characters to define the configuration of each numerical model. To be precise, the primary purpose of the strings was to define the entire input data for the numerical models.

To make clear the precise nature of each numerical model every figure that shows a set of graphical results is always accompanied on the same page by a corresponding specification that describes the electromagnetic configuration of that model.

The specialist vocabulary of twelve strings of characters that was used to define the configurations of all numerical models is described in Appendix C.

2. Analytical models of electromagnetic fields

2.1 Characteristics of models for shallow water conditions

Introduction. For about eighty years research has been directed to calculating the electromagnetic fields of an electric dipole, harmonic in time, in the presence of a semi-infinite conducting medium. Much of the early work was focused on applications concerning radio propagation. Radiation patterns of dipoles placed above or on the conducting earth medium were accordingly of particular interest.

Recent developments in geophysics and other fields has led to an interest in the low-frequency and quasi-static fields of dipoles located not only above the earth's surface (or the ocean surface) but also within it. In this context the quasi-static field range is characterised by the condition that the distance from the dipole source to the field measurement point is very much smaller than the free-space wavelength.

Analytical models that describe the electromagnetic fields of an electric dipole embedded in a uniform conducting half-space have been developed thoroughly³⁵⁻³⁷. Subsequent extensions and applications arising from this research have led in turn to a plethora of reference texts and handbooks³⁸⁻⁴¹ on the topic.

Consequences of a two-layer conducting half-space. Early investigations indicated that if a dipole was located just below the surface of a semi-infinite conducting medium the resultant fields differed markedly from those when it was far below the same surface. If therefore the conducting medium is two-layered it follows that the presence of the lower layer may likewise affect the values of the electromagnetic fields. As a rule-of-thumb, at low electromagnetic frequencies the two sets of field values will differ if the upper layer is seawater with in a depth of less than one skin-depth.

In the context of this study the conductivity values of the seawater and the seabed media were clearly important parameters of the environment. It was thus fortuitous that measurements of electromagnetic wave propagation through the seabed had

recently been developed to become a practical tool for geophysical exploration⁴² and conductivity surveys. These studies used various configurations of electric dipole sources, and electric or magnetic field receivers⁴³, often deployed in water depths of several thousand metres. Most of the investigations appear to use seabed-to-seabed propagation modes. Survey results indicate that the uppermost sediment layer is rarely more than a few hundred metres thick. Within this layer the conductivity usually varies between 0.01 - 2.5 S.m⁻¹. Such variations are mainly due to differing degrees of sediment porosity to water.

In most cases the deep-water models differ significantly from the short range, shallow-water models needed for this project. One study⁴⁴ is an exception. It postulates that seabed conductivity may be determined over long ranges by measuring only the horizontal electric field components produced by a long horizontal magnetic dipole at the surface. However, for this model it is necessary that the horizontal range is much greater than the skin depth, and that a transmission line analogy can be applied.

2.2 Desirable features of an analytical electromagnetic model

Although the extension of the analytical models to include a two-layer conducting half-space is relatively straightforward it is algebraically much more complex. Perhaps this is the reason it has received relatively little attention. Moreover, researchers who have addressed the issue have often attempted to simplify the computations at the expense of adding constraints to the applicability of their algorithms. Constraints of this type were an unsought hindrance to the project.

For this study it was desirable to have available analytical models that could deal with both VED and HED sources in a seamless and consistent manner. Because of the nature of the trial areas an ability to include configurations with at least two conducting layers was essential.

If the same analytical models could also embody point source and finite-length dipoles, such a capability would be highly advantageous. Lastly, in this virtual wish

list, it was desirable that the analytical models be capable of including horizontal and oblique boundaries between the various layers. Such facilities would enable the model configurations to resemble more accurately the conditions encountered in real-world experiments.

There are at present no such analytical models capable of including all of the above characteristics. However, a series of basic equations⁴⁵ that were developed over thirty years ago have many of the necessary features. It is from this work that analytical models within the study have been largely developed.

2.3 Basic equations of electromagnetic fields

To derive the basic equations (in MKS units) it convenient to begin with the general field equations for an isotropic and homogenous conducting medium containing electric current sources having a harmonic time variation of radial frequency ω . All field vectors will therefore contain the factor $e^{i\omega t}$ which can always be cancelled in the equations that follow. It is then possible to regard the vectors as functions of position only and to replace all time derivatives by the factor $i\omega$.

Current sources will be denoted by the vector $\mathbf{j}$. In this section the ELF sources and environments considered will be such that displacement currents may be neglected (see page 92). A differential form of Maxwell's equations that relates the values of the electric field $\mathbf{E}$ and the magnetic field $\mathbf{H}$ may be expressed as follows:

$$\text{curl } \mathbf{E} = -i\omega\mu\mathbf{H}, \quad (2.1)$$

$$\text{curl } \mathbf{H} = \mathbf{j} + \sigma\mathbf{E}, \quad (2.2)$$

where the parameters σ and μ denote the electric conductivity and the permeability of the medium, respectively.

It is convenient to work with vector and scalar potentials $\mathbf{A}$ and ϕ defined by the equations:

$$\mathbf{H} = \text{curl } \mathbf{A} \quad (2.3)$$

$$\mathbf{E} = -i\omega\mu\mathbf{A} - \text{grad } \phi \quad (2.4)$$

with the connecting relation, where σ is a constant:

$$\text{div } \mathbf{A} = -\sigma\phi \quad (2.5)$$

From equations 2.2 and 2.3 it follows that $\mathbf{A}$ satisfies the equation:

$$\nabla^2 \mathbf{A} - i\mu\sigma\omega\mathbf{A} = -\mathbf{j} \quad (2.6)$$

For the ensuing analyses it is appropriate to focus solely on the electromagnetic fields produced by an electric dipole. This source can be described equivalently as a dipole current that for mathematical calculations is conveniently written in the idealised form:

$$\mathbf{j} = I\delta(\mathbf{r}_0) \quad (2.7)$$

where I (A.m) is the dipole moment or strength of the dipole, $\mathbf{r}_0$ is a position vector from the dipole, and δ represents the Dirac generalised function. By substituting equation (2.7) into equation (2.6), and using the condition that $\mathbf{A} \rightarrow 0$ as $r_0 \rightarrow \infty$ (where $r_0 = |\mathbf{r}_0|$), it is possible to solve the differential equation to obtain the well-known result for the field of a dipole in a conducting medium:

$$\mathbf{A} = \frac{I}{4\pi r_0} \exp(-\alpha r_0 \sqrt{i}) \quad (2.8)$$

where the parameter α is expressed as:

$$\alpha = \sqrt{(\mu\sigma\omega)}. \quad (2.9)$$

2.4 Dipole in a layered medium

Consider next a semi-infinite, conducting medium occupying the half-space $z > 0$ of a rectangular coordinate system (x, y, z) . The permeability of the conducting media is taken to have the free-space value μ_0 throughout, and its conductivity is assumed to be σ_1 for $(0 < z < d)$, and σ_2 for $(z \geq d)$, where σ_1 and σ_2 are constants.

Assume that the current dipole I is located at the point $(x = y = 0)$ and $(z = h)$ where $(0 < h < d)$. The region $z \leq 0$ is taken to be free-space. The subscripts 0, 1, and 2 respectively, are notations added to the environmental parameters to denote which of the three regions $(z \leq 0)$, $(0 < z < d)$ and $(z \geq d)$ they belong. The respective systems of coordinates and parameters for one-layer and two-layer conducting environments are shown in diagrammatic form in figures 2.A and 2.B (on page 23).

In the region $(z \leq 0)$ it follows that $\sigma_0 = 0$ and $j_0 = 0$, so that by equation (2.6), A_0 satisfies the equation:

$$\nabla^2 A_0 = 0 \quad (2.10)$$

subject to the condition:

$$\text{div } A_0 = 0 \quad (2.11)$$

by equation (2.5). Likewise, the equations satisfied by A_1 and A_2 are:

$$\nabla^2 A_1 - i\alpha_1^2 A_1 = -I \delta(\mathbf{r}_0) \quad (2.12)$$

and:

$$\nabla^2 A_2 - i\alpha_2^2 A_2 = 0 \quad (2.13)$$

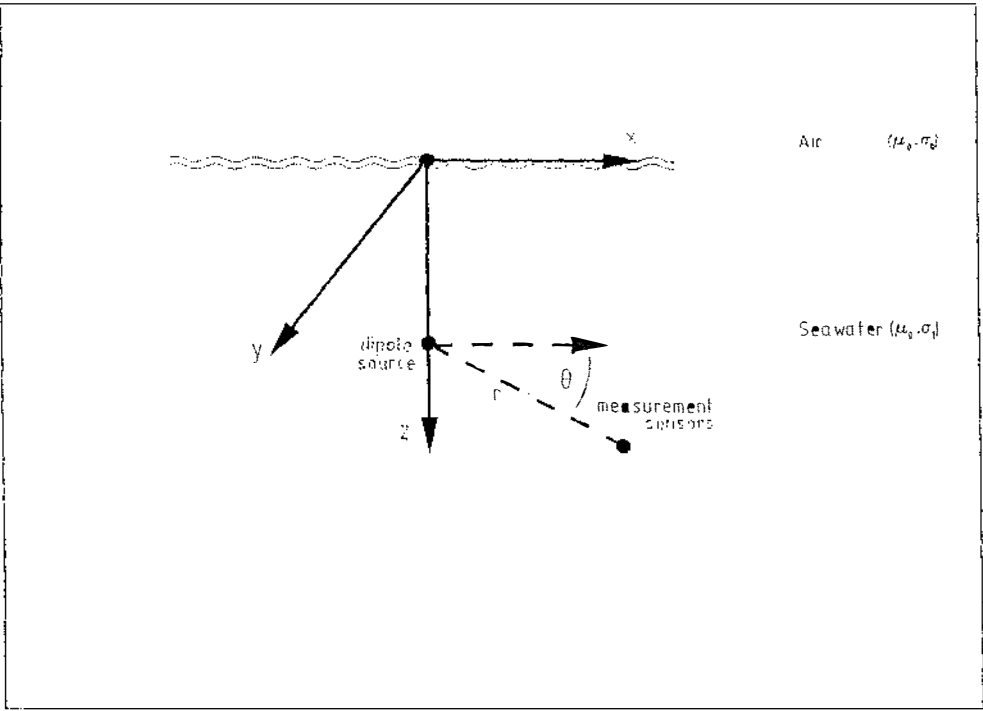


Figure 2.A Nomenclature for modelling fields of point-dipole sources in a one-layer conducting medium

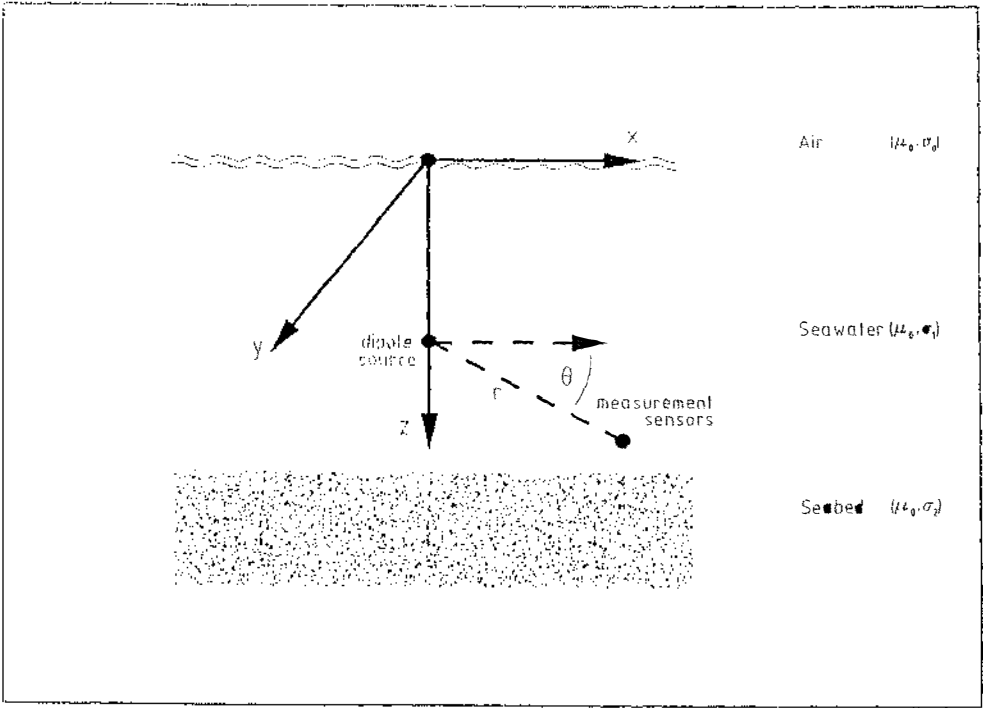


Figure 2.B Nomenclature for modelling fields of point-dipole sources in two-layer conducting media

respectively, where α_1 and α_2 are defined according to equation (2.9). The solution of equation (2.8) for the vector potential of the dipole source A^s represents a particular integral of equation (2.12). The vector potential A_1 can thus be written:

$$A_1 = A^s + A_1^0 \quad (2.14)$$

where A_1^0 satisfies the auxiliary equation:

$$\nabla^2 A_1^0 - i\alpha_1^2 A_1^0 = 0 \quad (2.15)$$

The complete solution for the field is obtained by solving for A^s , A_0 , A_1^0 and A_2 and applying the usual electromagnetic boundary conditions specifying the continuity of $k \cdot H$, $k \times E$ and $k \times H$ across the surfaces ($z = 0$) and ($z = d$), where k is a unit vector in the z direction. It is readily shown from equations (2.3), (2.4), and (2.5) that the corresponding conditions on the vector potential are:

$$[\text{curl}(A_1 - A_0)]_{z=0} = 0 \quad (2.16)$$

$$[\text{curl}(A_2 - A_1)]_{z=d} = 0 \quad (2.17)$$

$$[k \times \{\text{grad div}(A_2 / \sigma_2 - A_1 / \sigma_1) - i\mu_0 \omega(A_2 - A_1)\}]_{z=d} = 0 \quad (2.18)$$

Solutions for the two special cases of the dipole aligned with the z axis (i), and perpendicular to the z axis (ii), are obtained in the following analyses. These two configurations are referred to as the vertical electric dipole and the horizontal electric dipole, respectively. Field values for any other dipole orientation can be readily found by an appropriate superposition of the solutions for cases (i) and (ii).

2.5 Vertical electric dipole

For the case where the dipole is vertical, let $I = I k$. The model now possesses axial symmetry about the z axis, and it is therefore convenient to introduce cylindrical polar coordinates (r, θ, z) with $x = r \cos \theta$, $y = r \sin \theta$. In this system A is independent of the variable θ and it is possible to satisfy the boundary conditions by

taking $A_r = A_\theta = 0$. Hence, at all possible points $A = A(r, z)$. The source field A^s is given by equation (2.8), with:

$$r_0^2 = r^2 + (z - h)^2 \tag{2.19}$$

In this notation, the expression for A^s referred to in equation (2.14), can be given the Sommerfeld³⁸⁻⁴⁰ integral representation:

$$A^s = \frac{I}{4\pi r_0} e^{-\alpha_1 r_0} = \frac{I}{4\pi} \int_0^\infty \frac{\xi}{\nu_1} J_0(r\xi) e^{-\nu_1(z-h)} d\xi \tag{2.20}$$

where J_0 denotes the zero-order Bessel function of the first kind, and ν_1 is defined according to the general formula:

$$\nu_1(\xi) = \sqrt{(\xi^2 + i\alpha^2)},$$

the principal branch of the square root being understood, so that $R\{\nu > 0\}$. The form given in equation (2.20) is more convenient for the subsequent application of boundary conditions.

When using cylindrical coordinates, equations (2.10), (2.13), and (2.15) whose solutions are sought, are all of the type:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial A}{\partial r} \right) + \frac{\partial^2 A}{\partial z^2} - i\alpha^2 A = 0. \tag{2.21}$$

The solution of equation (2.21) can be found by applying the zero-order Hankel transform defined by the equation:

$$A(\xi, z) = \int_0^\infty r A(r, z) J_0(\xi r) dr.$$

Assuming that A and $\partial A / \partial r$ are $O(r^{-1/2})$ as $r \rightarrow \infty$, equation (2.21) reduces to:

$$\partial^2 A / \partial z^2 = \nu^2 A \quad (2.22)$$

Solving equation (2.22) and taking the inverse Hankel transform leads to the solution of equation (2.21) in the form:

$$A = P \int_0^\infty \xi \{ C(\xi) e^{z\nu_1} + D(\xi) e^{-z\nu_1} \} J_0(r\xi) d\xi \quad (2.23)$$

where $C(\xi)$ and $D(\xi)$ are arbitrary functions of ξ , and a constant factor $P = 1/4\pi$ has been taken out for algebraic convenience later.

General solutions for the vector potentials A_0 , A_1 and A_2 are given by equation (2.23), with appropriate subscripts inserted to distinguish between the air, seawater and seabed regions. It is noted, however, that since $\alpha_0 = 0$ (because $\sigma_0 = 0$) then $\nu_0 = \xi$. It is also clear that $(D_0(\xi) = 0)$ since $(A_0 \rightarrow 0)$ as $(z \rightarrow -\infty)$, and likewise that $(C_2(\xi) = 0)$. Hence, substituting for A^s in the integral form of equation (2.20), it is found that the required solutions for the vector potential are:

$$A_0 = P \int_0^\infty \xi C_0(\xi) e^{z\xi} J_0(r\xi) d\xi \quad (2.24)$$

$$A_1 = P \int_0^\infty \xi \left\{ \frac{e^{(-\nu_1 z + h)}}{\nu_1} + C_1(\xi) e^{z\nu_1} + D_1(\xi) e^{-z\nu_1} \right\} J_0(r\xi) d\xi \quad (2.25)$$

$$A_2 = P \int_0^\infty \xi D_2(\xi) e^{-z\nu_2} J_0(r\xi) d\xi \quad (2.26)$$

Equation (2.11) specifying the non-divergence of A_0 reduces in the present problem to the simple condition $(\partial A_0 / \partial z = 0)$, by which it follows from equation (2.24) that $(C_0 = 0)$, and hence that $(A_0 = 0)$. The remaining unknown quantities C_1 , D_1 and D_2 can be found by applying the boundary conditions at $(z = 0)$ and $(z = d)$. They also reduce to a particularly simple form for the problem under discussion. For example, from equation (2.16), with $(A_0 = 0)$ leads to $([\partial A_1 / \partial r]_{z=0} = 0)$, whence:

$$[A_1]_{z=0} = 0 \quad (2.27)$$

since $(A_1 \rightarrow 0)$ as $(r \rightarrow \infty)$. Similarly, it can be obtained from equation (2.17):

$$[A_1 - A_2]_{z=d} = 0 \quad (2.28)$$

and hence the boundary condition of equation (2.18) becomes:

$$[\sigma_2 \partial A_1 / \partial z - \sigma_1 \partial A_2 / \partial z]_{z=d} = 0 \quad (2.29)$$

Applying the condition of equation (2.27), leads to:

$$C_1 + D_1 = -e^{-h\nu_1} / \nu_1.$$

Remaining conditions in equations (2.28) and (2.29) yield the following equations:

$$C_1 e^{2d\nu_1} + D_1 - D_2 e^{d(\nu_1 - \nu_2)} = -e^{h\nu_1} / \nu_1,$$

$$C_1 e^{2d\nu_1} - D_1 + (\nu_2 / \varepsilon \nu_1) D_2 e^{d(\nu_1 - \nu_2)} = -e^{h\nu_1} / \nu_1,$$

where $(\varepsilon = \sigma_2 / \sigma_1)$. From these three equations in C_1 , D_1 and D_2 it is not difficult to establish that:

$$\begin{aligned} \nu_1 C_1 e^{z\nu_1} + \nu_1 D_1 e^{-z\nu_1} &= e^{-(z+h)\nu_1} + \frac{2e^{-2d\nu_1}}{1 - e^{-2d\nu_1}} \{ \cosh \nu_1 (z-h) - \cosh \nu_1 (z+h) \} \\ &+ \frac{8\varepsilon \nu_1 e^{-2d\nu_1} \sinh h\nu_1 \sinh z\nu_1}{(1 - e^{-2d\nu_1}) \{ \varepsilon \nu_1 + \nu_2 + (\varepsilon \nu_1 - \nu_2) e^{-2d\nu_1} \}}. \end{aligned}$$

It is now possible to substitute this expression in equation (2.25) and expand the factor $(1 - e^{-2d\nu_1})^{-1}$ in the second term, binomially. Then, after collecting all terms and recalling that in region 1 the inequality $(0 < |z-h| < z+h < 2d)$ holds, it is found that the solution can be expressed as follows:

$$\frac{A_1}{P} = \sum_{n=-\infty}^{\infty} \int_0^d \frac{\xi}{\nu_1} J_0(r\xi) \{e^{(-\nu_1)(2nd+z-h)} - e^{(-\nu_1)(2nd+z+h)}\} d\xi + Q \quad (2.30)$$

where:

$$Q = \int_0^d \frac{8\varepsilon\xi \sinh h\nu_1 \sinh z\nu_1 J_0(r\xi) e^{-2d\nu_1}}{(1 - e^{-2d\nu_1})\{\varepsilon\nu_1 + \nu_2 + (\varepsilon\nu_1 - \nu_2)e^{-2d\nu_1}\}} d\xi.$$

The integral in equation (2.30) is of the form described in equation (2.20), and it can therefore be evaluated to give the result:

$$\frac{A_1}{P} = \sum_{n=-\infty}^{\infty} \left(\frac{e^{(-\alpha_1 r_n \sqrt{i})}}{r_n} + \frac{e^{(-\alpha_1 R_n \sqrt{i})}}{R_n} \right) + Q \quad (2.31)$$

where:

$$r_n^2 = r^2 + (2nd + z - h)^2, \quad R_n^2 = r^2 + (2nd + z + h)^2. \quad (2.32)$$

The earlier definition of r_0 in equation (2.19) is automatically accommodated in this scheme. The physical interpretation of equation (2.31) is now clear. The infinite series refers to a simplified model in which ($\sigma_2 = 0$) and ($\varepsilon = 0$), so that the conducting medium is a slab of thickness d situated in vacuo. The terms in the series represent the effect at the point (r, z) for ($0 < z < d$), of the source field together with the sequence of fields reflected from the upper and lower surfaces of the slab. Hence, each term in the series gives the field of one of the image dipoles formed by considering the infinite sequence of geometrical images of the source in the two planes ($z = 0$) and ($z = d$). Those images whose distances r_n ($n \neq 0$) and R_n from the point (r, z) have nonnegative subscripts lie on the r axis above the plane ($z = 0$), while those for which r_n and R_n have negative subscripts are on the r axis beneath the plane ($z = d$). The other term Q in equation (2.31) obviously represents

the additional effect obtained by replacing the free space beneath the slab by a medium of conductivity σ_2 .

Since the concern is only with the electromagnetic field in the region $(0 \leq z \leq d)$, there will be no confusion if now the subscript 1 is dropped from the field quantities. The electric and magnetic field components in this region can all be determined from equation (2.31), for by equations (2.3), (2.4), and (2.5) it is possible to obtain the relations:

$$\begin{aligned} H_r + H_z &= 0, & H_\theta &= -\partial A / \partial r, \\ E_r &= \partial^2 A / \sigma_1 \partial r \partial z, & E_\theta &= 0, & E_z &= \partial^2 A / \sigma_1 \partial z^2 - i \mu_0 \omega A. \end{aligned}$$

It is convenient to express the solutions in dimensionless form by scaling all distances in lengths of $1/\alpha_1$. Hence, the following terms are now defined:

$$\begin{aligned} r' &= \alpha_1 r, & z' &= \alpha_1 z, & d' &= \alpha_1 d, \\ h' &= \alpha_1 h, & r_n' &= \alpha_1 r_n, & R_n' &= \alpha_1 R_n \end{aligned} \quad (2.33)$$

and substitute the new (dimensionless) variable $(u = \xi / \alpha_1)$. It is also convenient to introduce ψ_n and Ψ_n ($n = 0, \pm 1, \pm 2, \dots$) to denote the angles made with the positive z axis by the position vectors of lengths r_n and R_n respectively, from the relevant image dipoles. The new terms are defined as:

$$\psi_n = \arccot \frac{2nd + z - h}{r}, \quad \Psi_n = \arccot \frac{2nd + z + h}{r}.$$

Select now:

$$H' = 4 \pi H / I \mu_0 \sigma_1 \omega, \quad E' = 4 \pi E / I \mu_0^2 \sigma_1^2 \omega^2 \quad (2.34)$$

The resulting expressions for the non-vanishing electromagnetic field components can be written in the form:

$$H_{\theta}' = \sum_{n=-\infty}^{\infty} \{ F_1(r_n', \sin \psi_n) - F_1(R_n', \sin \Psi_n) \} - \frac{\partial Q_0}{\partial r'} \quad (2.35)$$

$$E_r' = \sum_{n=-\infty}^{\infty} \{ F_2(r_n', \sin 2\psi_n) - F_2(R_n', \sin 2\Psi_n) \} + \frac{\partial^2 Q_0}{\partial r' \partial z'} \quad (2.36)$$

$$E_z' = \sum_{n=-\infty}^{\infty} \{ F_3(r_n', \sin \psi_n) - F_3(R_n', \sin \Psi_n) \} + Q_2 \quad (2.37)$$

where, with $(\eta(u) = \sqrt{(u^2 + i)})$ and $(\eta_{\varepsilon}(u) = \sqrt{(u^2 + i\varepsilon)})$, it is possible to define:

$$Q_m = \int_0^{\infty} \frac{8 \varepsilon u^{m+1} \sinh h' \eta \sinh z' \eta J_0(r' u) e^{-2d' \eta} du}{(1 - e^{-2d' \eta}) \{ \varepsilon \eta + \eta_{\varepsilon} + (\varepsilon \eta - \eta_{\varepsilon}) e^{-2d' \eta} \}} \quad (2.38)$$

and where:

$$F_1(\rho, \chi) = \chi \rho^{-2} (1 + \rho \sqrt{i}) e^{-\rho \sqrt{i}} \quad (2.39)$$

$$F_2(\rho, \chi) = \frac{1}{2} \chi \rho^{-3} (3 + 3 \rho \sqrt{i} + i \rho^2) e^{-\rho \sqrt{i}} \quad (2.40)$$

$$F_3(\rho, \chi) = \rho^{-3} \{ (1 + \rho \sqrt{i}) (2 - 3 \chi^2) - i \rho^2 \chi^2 \} e^{-\rho \sqrt{i}} \quad (2.41)$$

Expressed in this form the right-hand sides of the solutions in equations (2.35), (2.36), and (2.37) are dimensionless and independent of frequency.

It is possible to calculate the field at the surface of the conductor by letting $(z' \rightarrow 0)$ in the above solutions. In the limit $(r_n' = R_{-n}')$ and $(\psi_n = \pi - \Psi_{-n})$, so that the terms of the infinite series in equations (2.35) and (2.37) exactly cancel each other. Since $(\partial Q_0 / \partial r')$ and Q_2 also vanish when $(z' = 0)$, it is found that $([H_{\theta}']_{z'=0} = 0)$ and $([E_z']_{z'=0} = 0)$. The expression for E_r can be simplified somewhat by noting that for this component the equal terms in the series add rather than subtract, to give the result:

$$\left[E_r' \right]_{z'=0} = 2 \sum_{n=0}^{\infty} F_2(r_n', \sin 2\psi_n) + \left[\frac{\partial^2 Q_0}{\partial r' \partial z'} \right]_{z'=0} \quad (2.42)$$

Likewise at the bottom surface ($z' = d'$) it is noted that $r_{n+1}' = R_{-n}'$ and $\psi_{n+1} = \pi - \Psi_{-n}$, so that again the infinite series in equations (2.35) and (2.37) vanish, leaving the solutions:

$$\left[H_\theta' \right]_{z'=d'} = - \left[\partial Q_0 / \partial r' \right]_{z'=d'}, \quad \left[E_z' \right]_{z'=d'} = \left[Q_2 \right]_{z'=d'} \quad (2.43)$$

The solution of equation (2.36) simplifies into the form:

$$\left[E_r' \right]_{z'=d'} = 2 \sum_{n=0}^{\infty} F_2(r_n', \sin 2\psi_n) + \left[\frac{\partial^2 Q_0}{\partial r' \partial z'} \right]_{z'=d'} \quad (2.44)$$

On the axis ($r' = 0$) only the vertical electric field E_z' is non-vanishing, a fact which is readily verified by substituting ($\sin \psi_n = \sin \Psi_n = 0$) and ($J_1(r'u) = 0$) in the solutions of equations (2.35) and (2.36).

The well-known expressions for the field of a current dipole situated in a half-space of uniform conductivity σ_1 can be recovered from the solutions by letting ($d \rightarrow \infty$). In this case the integral Q_m will vanish, as will all the terms in the infinite series except those for which ($n = 0$). This step leads to the following solutions:

$$H_\theta' = F_1(r_0', \sin \psi_0) - F_1(R_0', \sin \Psi_0) \quad (2.45)$$

$$E_r' = F_2(r_0', \sin 2\psi_0) - F_2(R_0', \sin 2\Psi_0) \quad (2.46)$$

$$E_z' = F_3(r_0', \sin \psi_0) - F_3(R_0', \sin \Psi_0) \quad (2.47)$$

which are in complete accord with previously obtained results. These results could have also been obtained (although not so readily) by making the conductivity values of the two layers equal, viz. by putting ($\varepsilon = 1$) in the general solutions. In this case it would be necessary to expand factor $(1 - e^{-2d\eta})^{-1}$ in integrand of Q_m and integrate by terms to produce the simplified results in equations (2.45), (2.46), and (2.47). The

infinite series so generated would then exactly cancel out the infinite series in the general solution except for the terms for which $(n = 0)$.

2.6 Horizontal electric dipole

The field of a horizontal dipole, aligned parallel to the x axis, say, can be found similarly by putting $\mathbf{I} = I \mathbf{i}$, where $\mathbf{i}$ is a unit vector in the x direction. Thus the source field can be written $\mathbf{A}^s = A^s \mathbf{i}$, where A^s can again be written in the form of equation (2.20). As the model no longer possesses the axial symmetry of the earlier vertical dipole it is necessary to retain the variable θ . In order to satisfy the boundary conditions it is also necessary to use a vector potential with two cartesian components.

Expressions for the non-dimensional electromagnetic fields associated with a horizontal electric dipole can then be derived by following the processes set out in the primary reference⁴⁵, and in the appendix of Appendix B. The corresponding formulae are as follows:

$$H_r'' = -F_1(r_0', \cos \psi_0) - F_1(R_0', \cos \Psi_0) + 2i F_4(R_0', \cos \Psi_0) + \frac{\partial^3 N}{\partial r'^2 \partial z'} + W_1 \quad (2.48)$$

$$H_\theta'' = -F_1(r_0', \cos \psi_0) - F_1(R_0', \cos \Psi_0) + \frac{2i}{r'} F_2(R_0', \sin 2\Psi_0) + \frac{\partial^2 N}{r' \partial r' \partial z'} + W_2 \quad (2.49)$$

$$H_z'' = F_1(r_0', \sin \psi_0) - F_1(R_0', \sin \Psi_0) + 2i F_4(R_0', \sin \Psi_0) + \frac{\partial^3 N}{\partial r' \partial z'^2} - i \frac{\partial N}{\partial r'} + W_3 \quad (2.50)$$

$$E_r'' = F_3(r_0', \cos \psi_0) + F_3(R_0', \cos \Psi_0) - \frac{2}{r'} F_1(R_0', \sin \Psi_0) - \frac{i}{r'} \frac{\partial N}{\partial r'} - W_4 \quad (2.51)$$

$$E_{\theta}'' = \frac{i e^{-r_0' \sqrt{i}}}{r_0'} - \frac{i e^{-R_0' \sqrt{i}}}{R_0'} + \frac{F_1(r_0', \sin \psi_0)}{r'} + \frac{F_1(R_0', \sin \Psi_0)}{r'} - 2 F_3(R_0', \cos \Psi_0) + i \frac{\partial^2 N}{\partial r'^2} + W_5 \quad (2.52)$$

$$E_z'' = F_2(r_0', \sin 2 \psi_0) + F_2(R_0', \sin 2 \Psi_0) - W_6 \quad (2.53)$$

Expressions (2.48 – 2.53) are of course identical to the formulae (A78 – A83) in Appendix B.

2.7 Radial fields of electric dipoles

Dimensional field expressions. It is useful to consider the graphical representations of the radial fields associated with electric dipoles in conducting media. But first the dimensional forms of these fields for a vertical electric dipole and a horizontal electric dipole are simply derived by substituting the second part of equation (2.34) into equations (2.46) and (2.51). Dimensional forms of the radial fields of the VED and HED sources are provided by $E_{r'}$ and $E_{r''}$, respectively:

$$E_{r'} = \left(\frac{I \mu_0^2 \sigma_1^2 \omega^2}{4 \pi} \right) * (F_2(r_0', \sin 2 \psi_0) - F_2(R_0', \sin 2 \Psi_0)) \quad (2.54)$$

$$E_{r''} = \left(\frac{I \mu_0^2 \sigma_1^2 \omega^2}{4 \pi} \right) * (F_3(r_0', \cos \psi_0) + F_3(R_0', \cos \Psi_0) - \frac{2}{r'} F_1(R_0', \sin \Psi_0) - \frac{i}{r'} \frac{\partial N}{\partial r'} - W_4) \quad (2.55)$$

Diagrammatic representation of radial electric fields. The next step considers predictions of radial electric fields in circumstances that are quite similar to those during the measurement phases of the study. For this purpose equations (2.54) and

(2.55) were used to provide estimates of the radial fields at various horizontal ranges from a unit, 50 Hz VED point-source and a similar HED point-source.

In the first set of calculations both sources were assumed to be 5 m below the surface of a semi-infinite volume of seawater with a uniform conductivity of 4 S.m^{-1} . The configuration is consistent with figure 2.A (on page 23) with the VED source aligned with, and lying in the z axis. For the purpose of the calculations the HED source was also taken to lie in the z axis, but in this instance its axis was aligned with the x axis of the coordinate system. Predictions were made of the electric fields of both the VED and HED sources at various horizontal ranges and at a depth of 20 m. The electric field values are those in the direction of the x axis (or along the radial axis with angle $\theta = 0$, using a cylindrical coordinate system).

For the second set of predictions there was assumed to be a uniform seawater medium with a finite depth of 20 m and conductivity 4 S.m^{-1} overlaying a semi-infinite homogeneous seabed with a conductivity of 2 S.m^{-1} . Other features of the earlier configuration were unchanged. This configuration resembles that shown diagrammatically in figure 2.B (on page 23), with field points on the seabed surface.

For a 50 Hz electromagnetic wave in a seawater medium of conductivity 4 S.m^{-1} , the skin-depth is 35.6 m. It therefore follows that the finite depth of 20 m selected for the second round of predictions corresponds to 0.56 skin-depths.

Calculated radial fields for the two configurations described above are shown in figures 2.C and 2.D (on page 35). To highlight certain features of the field values the two sets of results appear with identical vertical scales in the two figures. A similar option was exercised with the horizontal scales.

2.8 Observations concerning the radial field predictions

In an infinite conducting medium the electric field of a point dipole would decrease in a smooth exponential manner as the range from the source was increased. The calculated values of the electric field shown in figures 2.C and 2.D (on page 35).

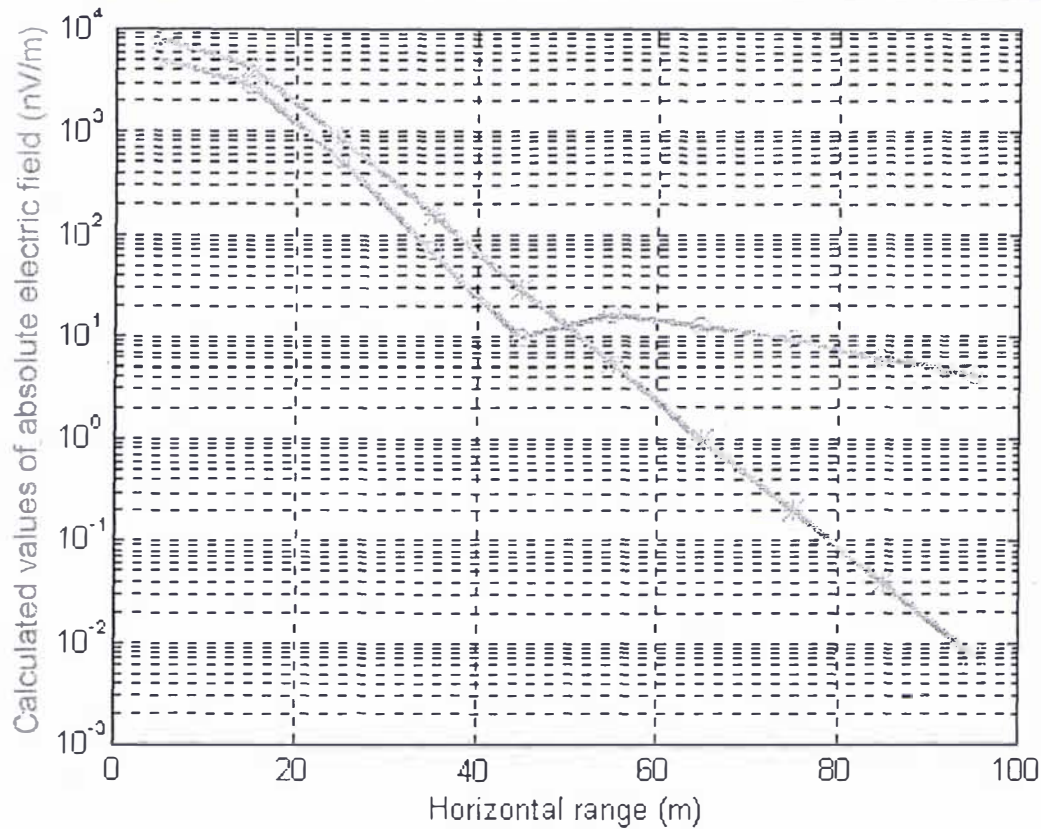


Figure 2.C Calculated values of radial electric fields for a VED point-source in a semi-infinite seawater medium (*) and in a seawater medium of finite depth (o)

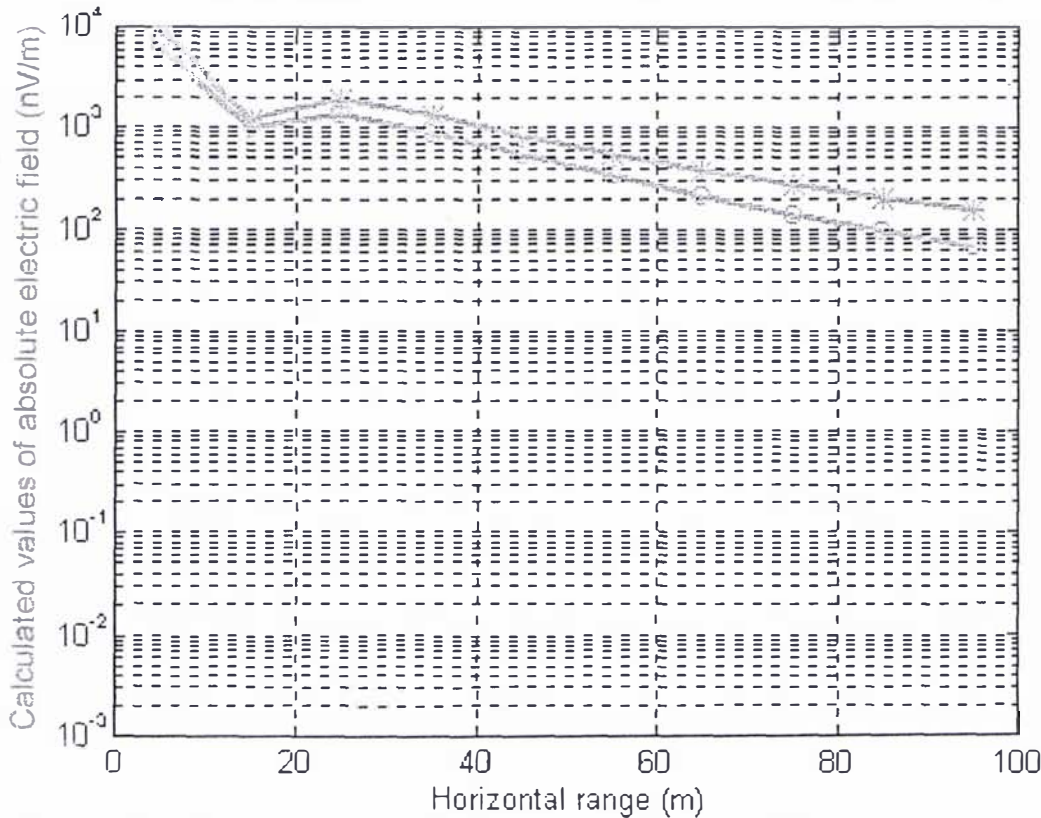


Figure 2.D Calculated values of radial electric fields for a HED point-source in a semi-infinite seawater medium (*) and in a seawater medium of finite depth (o)

are therefore clearly influenced by plane boundaries near the VED and HED point sources, and the position of the measurement points. In particular, the non-monotonic curves represent the aggregate effect of complex reflection and refraction processes near the air-seawater and seawater-seabed boundaries. A notable feature of this data is that in some instances the predicted field values at a measurement point will increase as the range from the point source is increased.

The data shown in figures 2.C and 2.D (on page 35) also demonstrate features that were significant during the construction phase for the equipment, and during the subsequent measurement programme. Key points were as follows:

- a. The configuration used in figure 2.D (on page 35) is fairly similar to the experimental environments foreseen in section 1, and in figures 1.A – 1.D (on pages 9 and 12). Signal detection equipment with extremely low-noise characteristics is necessary to measure such configurations:
- b. At identical horizontal ranges the predicted field of the HED source is typically greater than that of the VED source. Other factors being equal then measurements of the electric field from the HED source will be feasible at significantly greater ranges:
- c. Deployments of a VED source require that the equipment be maintained in a stable vertical configuration at each of the selected measurement points. Such operations usually require lengthy deployment periods. An HED source that is towed behind a ship can clearly occupy many more measurement points in a given period than could a VED source that must also be distant from the tow-ship hull, and water turbulence:
- d. Given a selection of measurement sites it is fortuitous that the source configuration (HED) that can be used at the greater ranges is also the one able to be deployed to the selected sites more rapidly. Such a coincidence minimises the necessary deployment time in the trial area, and hence the project costs.

3. Applications of analytical models

3.1 Characteristics of the primary test site

Background. Project experiments took place at two sites off the New Zealand coast. Measurements made at the southern site were supported by more extensive environmental information. It thus provided a preferred start-point from which the signal propagation phenomena in shallow water could be examined. Almost all of the measurements referred to in the study were made at this site.

The primary site was located within Tennyson Inlet of the Pelorus Sound, the former being about 35 km from the open sea and 100 km west of Wellington. Details of the site location and the ship tracks during the measurement program are shown in figure 3.A (on page 38). The actual site just north of the World's End passage was square-shaped, with sides of approximately 800 m.

Environmental factors. Pelorus Sound is on the western edge of the alpine-fault system, a transform fault on the boundary of the Pacific and Indian-Australian tectonic plates. It is dominated by much-broken mountainous country to the south. The principal rock structure in the area is greywacke sandstone with traces of schist. Tennyson Inlet itself extends in a southwest direction from the western end of Tawhitinui Reach. From this point to the open sea there is a maze of waterways with bays, coves and many islands of various sizes

Water depths at low-tide were quite uniform at 25 ± 0.5 m throughout the test site. To confirm this characteristic a series of line-surveys were performed using Ferranti-ORE sub-bottom profiling equipment deployed from the trial ship. For this purpose the equipment was operated at the low-power setting where the peak power output was in the frequency band 0.5 – 3 kHz.

From the subsequent records it was possible to assemble composite diagrams that described the gross features of the water depth and seabed sub-strata. An example of such a diagram is shown in figure 3.B (on page 38).

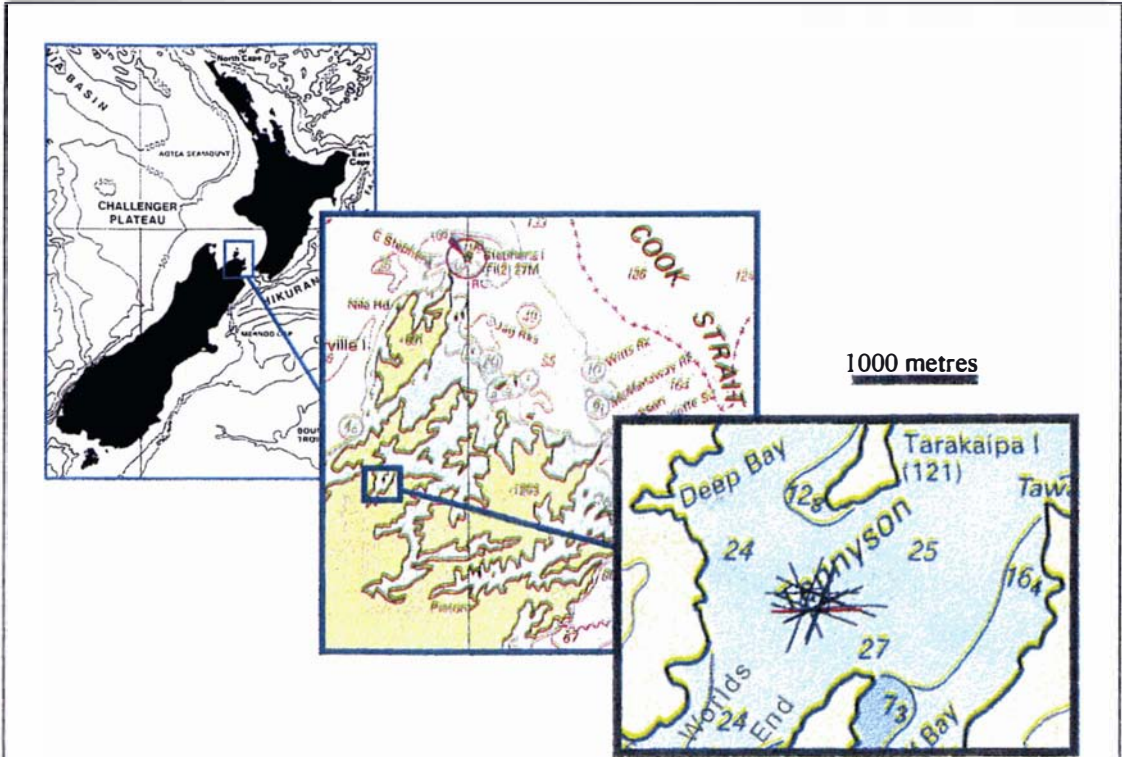


Figure 3.A Primary test site in the Pelorus Sound, showing ship tracks along which electromagnetic signal propagation experiments were conducted

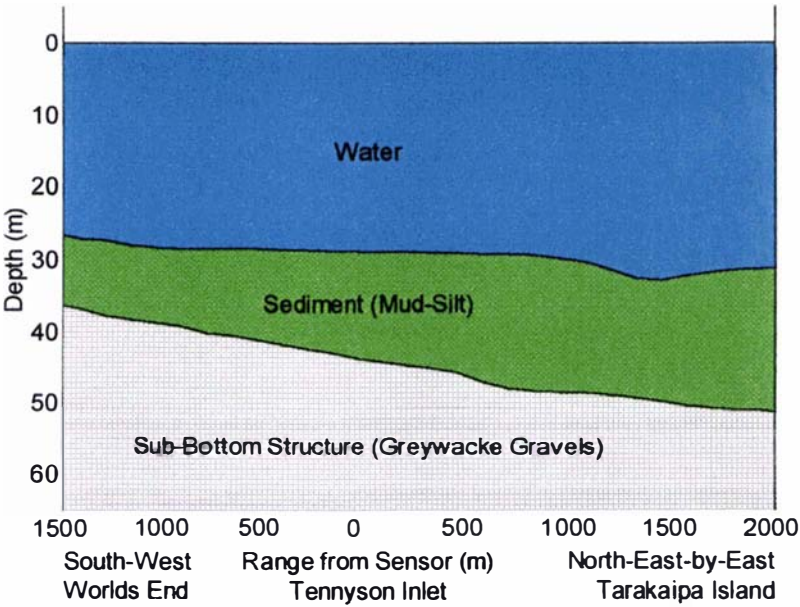


Figure 3.B Vertical cross-section of the seawater and upper seabed layers, along a bearing of 48 ° through the position of the multi-influence sensor system

A typical section of the chart record from the sub-bottom profiler is reproduced in figure 3.C (on page 40). In essence, the chart is comprised of time histories of the received signals for each vertical-incidence pulse of acoustic energy. The three horizontal features (from top to bottom) correspond to the 20 ms delay depth, the seabed level and the level of the dominant sub-bottom interface.

In situations where core data from the seabed had been obtained it is likely information concerning the sound velocities in the various seabed strata would also be available. Such information could then be used to provide precise estimates of the thickness of each layer. No such data were available for the test site. Thus the normal recourse of using the sound velocity in seawater (typically 1500 m.s^{-1}) was used to estimate the thickness of each layer.

For the chart section in figure 3.C (on page 40) it is then possible to establish that the vertical scale value is 1:200 (or $1 \text{ cm} \equiv 2 \text{ m}$) after photo-reduction. Also, after photo-reduction, the horizontal scale of 1:5000 (or $1 \text{ cm} \equiv 50 \text{ m}$) was established from the ship navigation records.

Chart records showed that the uppermost layer of the seabed comprised mud-silt sediments to a depth of about 15 m below the multi-influence sensor system. This low-density sediment overlaid a sloping sub-bottom structure understood to be greywacke gravel. Several areas of gas-bearing sediments, mostly outside the test site, were also observed 10 - 15 m below the seafloor.

Bearing-strength measurements were made with a locally-developed electronic sediment probe⁴⁶. During deployments the device sank approximately 0.6 m into the seabed, indicating that the mud-silt sediment was quite soft near the surface. Typically, the bearing strength increased linearly from 10 - 50 kPa, between depths of 0.1 - 0.5 m.

In the late autumn of 1995 when the trials were held, the weather was sunny and windless for nearly all the measurement programme. Sea conditions were flat and calm, except for small surface ripples late each day. Water temperature measurements made during the trial varied from $14.5 - 15.2^\circ\text{C}$ at mid-water depths.

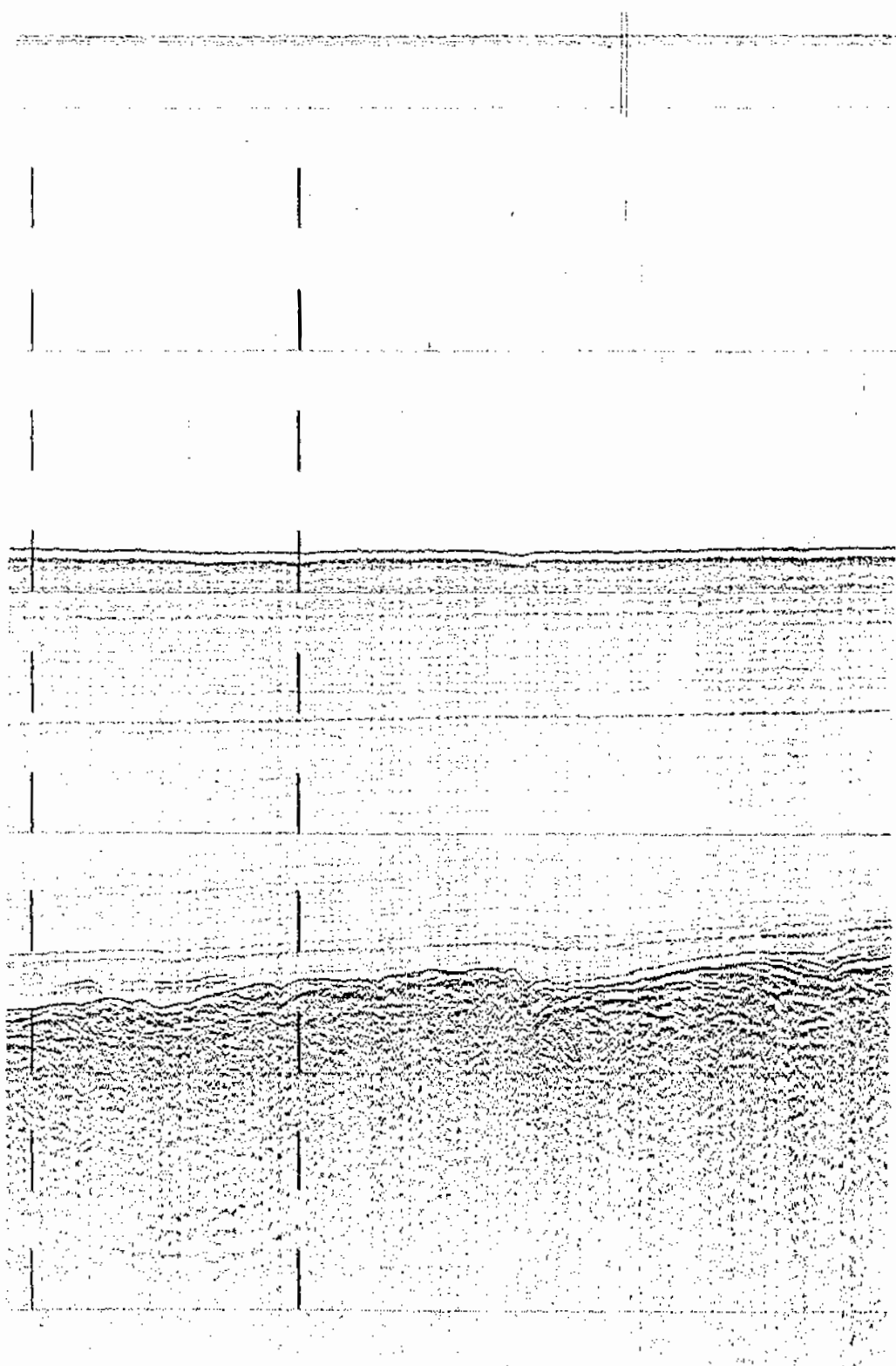


Figure 3.C Typical vertical cross-section from the chart record of the sub-bottom profiler where the vertical and horizontal scales are 1:200 and 1:5000, respectively

3.2 Computer simulations of seabed conductivity investigations

Introduction. An important prerequisite to characterize the ELFE sources was to enhance the wave propagation models of electromagnetic fields over short ranges. As a consequence the initial investigations were directed towards evaluating key parameters of the conducting media in shallow-water conditions.

An inspection of equations (2.45 – 2.53) (on pages 31 – 33) and their dimensional equivalents, would indicate that nearly all the parameters have known values (such as the permeability of free space μ_0) or they are terms that can be readily measured during site trials (such as the seawater conductivity σ_1 or depth z). The exceptions are the values for the seabed conductivity σ_2 , and the strength I of the point dipole.

The issues surrounding the seabed conductivity parameter σ_2 are discussed at length in section 1 (on page 14) and in appendix A (on page 123). There is a different set of issues associated with establishing the strength I for the real-world equivalent of a point dipole. For instance, the presence of shell-shaped electrodes or a dipole of short overall length, often make it difficult to estimate the parameter I .

It was therefore planned to design experiments (and computer simulations) that did not require prior knowledge of the dipole strength I . Moreover, no attempt would be made to make direct measurements of seabed conductivity σ_2 . Rather, the focus would be on inferring the values of seabed conductivity which were characteristic of ELFE wave propagation between a fixed sensor position and various source locations throughout the test site. The following paragraphs describe the underlying electromagnetic theory and the estimating techniques that were used.

Theoretical bases of the techniques. Given the coordinate system described in figure 2.B (on page 23) the electric and magnetic field values at a general point in the upper conducting layer due to a dipole source at any point in the same layer, may be calculated by using the analytical model⁴⁵ in section 2. The axis of the dipole source may be horizontal or vertical. Fields for any other dipole orientation can be found by an appropriate superposition of the solutions for these two cases.

Specifically, the radial and vertical electric fields due to a vertical electric dipole (VED) in equations (2.46) and (2.47) respectively (on page 31), may be expressed in the following non-dimensional and frequency-independent forms:

$$E_r' = \sum_n \{ F_2(r_n', \sin(2\psi_n)) - F_2(R_n', \sin(2\Psi_n)) \} + \frac{\partial^2 Q_o}{\partial r' \partial z'} \quad (3.1)$$

$$E_z' = \sum_{n=-\infty}^{\infty} \{ F_3(r_n', \sin(\psi_n)) - F_3(R_n', \sin(\Psi_n)) \} + Q_2 \quad (3.2)$$

Equations (3.1) and (3.2) are functions that also correspond to equations (A46) and (A47) in the appendix of appendix B.

In the first technique to be described the value which characterizes the seabed conductivity value σ_2 is found by solving the equation:

$$|E_r'| / |E_z'| - |E_{r,M}'| / |E_{z,M}'| = 0 \quad (3.3)$$

where the ratio $|E_r'| / |E_z'|$ contains expressions from equations (3.1) and (3.2), and $|E_{r,M}'| / |E_{z,M}'|$ is the ratio of the measured radial and vertical components of the electric field at the trial site. This notation system follows the earlier convention as far possible, with the additional subscript M denoting a measured value. Equation (3.3) therefore has a form that does not require prior knowledge of the source strength I of the vertical electric dipole, but it is dependent on the availability of both radial and vertical components of the electric field measurements.

An alternative approach that requires field measurements only along a single (radial) axis is described in the following paragraphs. For these experiments it was necessary that dual harmonic signals of equal known amplitudes be transmitted from the dipole source. It is not necessary to have prior knowledge of the strength I of the dipole - but it is necessary to know that the two current components, at f_1 and f_2 were equal. Again, such a feature can be useful where the dipole source is not calibrated or where

it is difficult to estimate the dipole moment because of shell-shaped electrodes, a short dipole length or similar impediments.

This latter approach uses equation (2.51) (on page 32) which is a non-linear and non-dimensional expression for the radial electric field $|E_r''|$ at a general point, due to a horizontal electric dipole (HED) located in the same seawater medium. It can be used in a reduction process similar to that outlined above provided the HED source includes dual-frequency ELFE transmissions of equal current amplitudes.

In this instance it necessary to combine equations (2.51) and (2.34) to provide a dimensional form of the radial electric field due to an HED source. The dimensional form of the radial electric field $|E_r''|$ can then be used in the following expression to estimate the unknown seabed conductivity value σ_2 :

$$\left(\frac{f_1}{f_2}\right)^{1.5} * |E_{r,f_1}''| / |E_{r,f_2}''| - |E_{M,f_1}''| / |E_{M,f_2}''| = 0 \quad (3.4)$$

here f_1 and f_2 designate the two transmitted ELFE frequencies. And the values $|E_{M,f_1}''|$ and $|E_{M,f_2}''|$ are the radial electric fields at each frequency, with the subscript M again denoting a measurement at the sensor location. To provide a degree of redundancy for lateral investigations it was project practice to include signals of at least four frequencies within the source dipole. A typical spectrum is shown in figure 11 (on page 145) of appendix B.

Although the expanded forms of expressions E_{r,f_1}'' and E_{r,f_2}'' (from page 166) have real and imaginary parts the real root quantifying the unknown seabed conductivity parameter σ_2 can be solved by any one of about a score of proven estimation methods. One technique that is robust and suited to this class of problem is Muller's iterative scheme of successive bisection and inverse parabolic interpolation⁴⁷

Using this technique a parabola polynomial is made to fit three points near a root. The proper zero of this polynomial, using the quadratic formula, is used as the improved estimate of the root. The process is then repeated using the set of three

points nearest the root being evaluated. This method was utilized extensively in the simulated experiments and throughout the measurement programme.

Configurations of the simulated investigations. Conductivity values in the upper layers of the seabed are highly variable, mainly because of the inclusion of different water contents⁴³. The examples that follow reflect this degree of variability by beginning the computer simulations at diverse start-points.

Both of the following simulations assumed a configuration similar to that shown in figure 2.B (on page 23). The seawater medium of conductivity 4 S.m^{-1} overlaid an isotropic seabed with a conductivity of 2 S.m^{-1} . Each conducting layer was assumed to have a permeability μ_0 equal to that of free space, namely $(4 \pi 10^{-7} \text{ H.m}^{-1})$.

Case I (VED source) configuration. In the initial simulation a VED source was assumed to be located at half the skin depth (56.26 m) for the source frequency of 5 Hz. Electric field sensors were sited on the seabed in a water depth of exactly one skin depth (112.54 m). The horizontal range between sensors and source was also one skin depth, measured along the radial axis. For these simulations the values of expressions $|E'_{r,M}|$ and $|E'_{z,M}|$ representing measured fields, were calculated using the two-layer analytical model with a correct value of the seabed conductivity σ_2 .

Results of the simulations using the VED technique are summarized in figure 3.D (on page 45). Initial guesses of the seabed conductivity (or seed values of the estimation algorithm) were 0.5, 1, 4 and 6 S.m^{-1} . In all instances the correct value of the conductivity parameter (2 S.m^{-1}) was identified in less than fourteen iterations of the previous-mentioned Muller⁴⁷ algorithm. A feature of the process was that it was robust enough to test negative values of the seabed conductivity ($\approx -4 \text{ S.m}^{-1}$).

Case II (HED source) configuration. The configuration of the second set of simulations resembled that for the primary test site. An HED source was assumed to be located 1.6 m below the surface in seawater of conductivity 4 S.m^{-1} and depth 27.5 m. The transmitted signal contained two equal 22 and 35 Hz harmonic components. Electric field sensors with a horizontal axis were assumed to lie 0.5 m

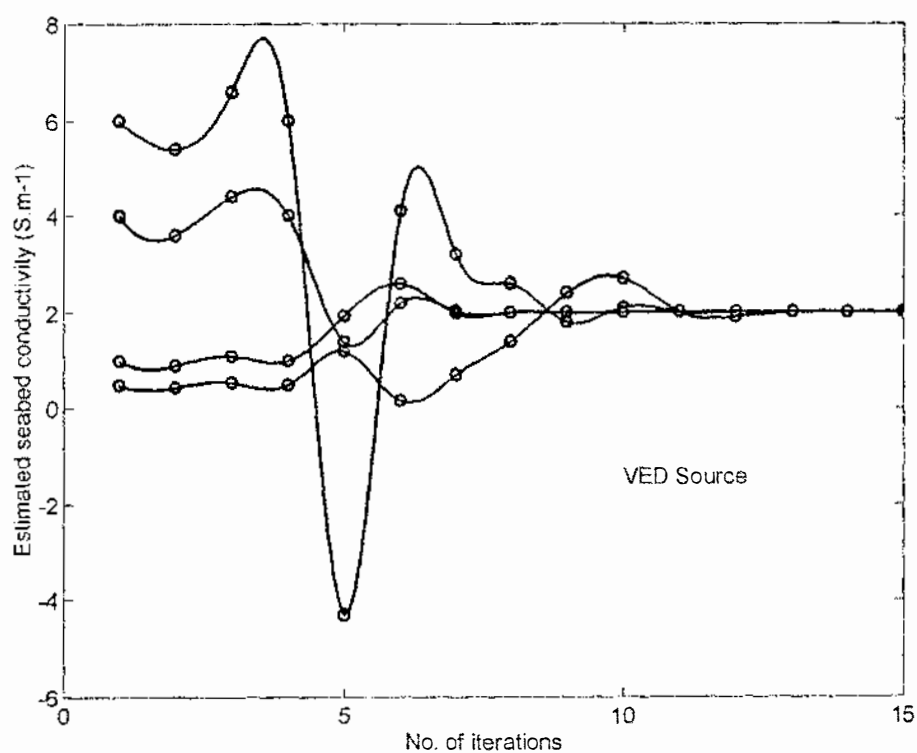


Figure 3.D Simulated analyses with a seawater depth of one skin-depth and with a seabed conductivity value set at 2 S.m^{-1}

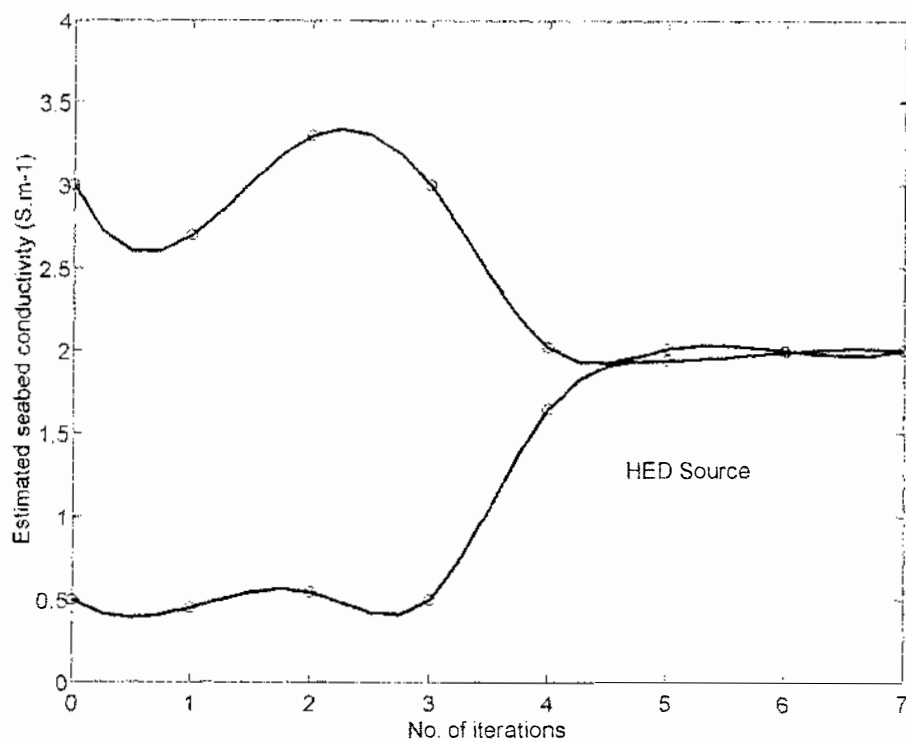


Figure 3.E. Simulated analyses with the seawater depth and horizontal ranges resembling site-like conditions and with the seabed conductivity value set at 2 S.m^{-1}

above a level isotropic seabed of conductivity 2 S.m^{-1} , and at a horizontal range of 100 m from the ELFE source.

For the simulations the values of expressions $|E_{M, freq_1}''|$ and $|E_{M, freq_2}''|$ representing measured fields, were again calculated using the two-layer analytical model with the correct value of the seabed conductivity σ_2 . In this example the seed values of the optimization algorithm were 0.5 and 3 S.m^{-1} . Results of the simulation are described in figure 3.E (on page 45). Once again, the correct value of the conductivity parameter (2 S.m^{-1}) was identified in less than fourteen cycles of the iterative search algorithm. As with the earlier simulations such searches typically required a program execution time of 20 s on the project computer.

It is important to appreciate that the expression for the radial electric field $|E_r''|$ due to an HED source, does not necessarily decrease exponentially with increases in the horizontal range. Figures 2.C and 2.D (on page 35) demonstrate this point. For this reason there will be instances where the iterative search processes fails, or indicates the possibility of more than one root for equation (3.4).

In conditions where more than one root is found the appropriate value at a specific measurement point can usually be found by simple inspections. This real-valued root will be referred to throughout the report as the characteristic value of the seabed conductivity between the electric field sensors and a specified source position within the test site.

Later sections show how these characteristic values of the seabed conductivity can be mapped over a large proportion of the test site.

3.3 Deployment of the multi-influence sensor system

Introduction. The sensor frame was laid so that the axis of the electric field sensor lay in the magnetic north-south direction. A need to expedite concurrent measurements of static magnetic fields set this requirement. The frame position was

fixed using DGPS data. In addition, the frame was clearly evident on the chart record of the ship's depth sounder, enabling a check of the estimated position.

All measurements were made with track offsets of between 50 - 100 m to avoid overloading the electric field sensors. On recovery five days later, the frame was found to have sunk into the mud up to the middle of longitudinal beams. The sensors however, were all in clear water.

Deployments of the ELFE source. Conductivity surveys of the test site using the HED technique were completed in the time needed for several straight-line traverses. In fact, during the measurement programme it was feasible to conduct simultaneous electromagnetic ranging and conductivity surveys by deploying the elementary HED sources about sixty metres behind the trial vessel. Photographs in figures 3.F and 3.G (on page 48) show views of the source deployments. Had the VED technique been used it would have been necessary to solve the more difficult tasks of deploying a vertical dipole that was kept clear of the ship hull, and the adjacent water turbulence.

Twelve radial runs were successfully made towing an HED source. Each run was made along a major magnetic axis, or along 45° inter-cardinal courses out to a range of 200 - 400 m from the sensors. The nominal depths of the electric field sensors and the HED source were 24.5 and 1.6 m, respectively. For these measurements the source was towed at a speed of about 4 knots. In most instances, four source signals were generated simultaneously at frequencies of 22, 35, 74 and 94 Hz.

Electric fields radiated by the trial vessel and the transmitted signals from the ELFE source were clearly observed during the twelve serials. Figure 11 (on page 145) in appendix B shows part of the trial record. An item of particular note is the 50 Hz interference component associated with the New Zealand national power grid, the nearest point of which was several kilometres away. The equivalent background noise floor was measured at $-170 \text{ dBV.m}^{-1}.\text{Hz}^{-0.5}$. Several spectral lines which denoted Schumann resonance frequencies were noted at times when the vessel was distant from the electric field sensors.

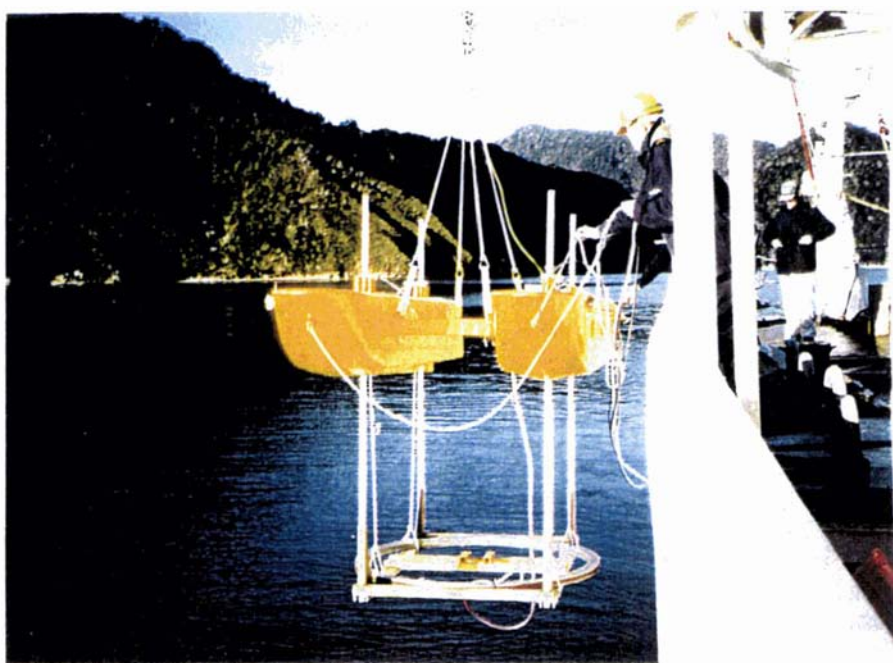


Figure 3.F Deployment of extremely low-frequency, electromagnetic dipole sources from HMNZS TUI in Tennyson Inlet

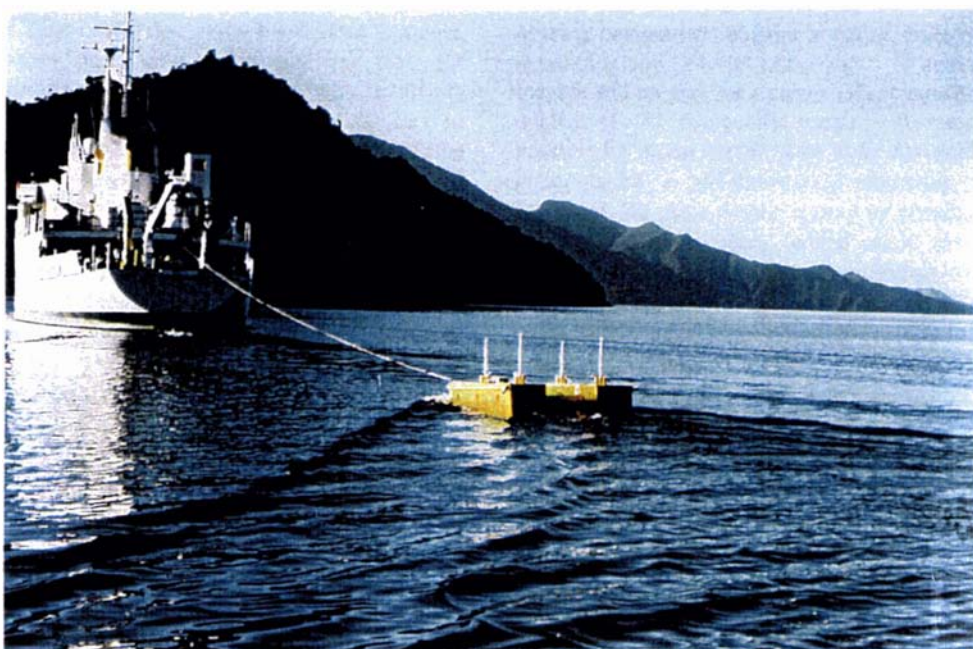


Figure 3.G Deployed extremely low-frequency, electromagnetic dipole sources being towed over the submerged multi-influence sensor system

3.4 Measurements from serial 060

General features. To demonstrate particular features of the measurements, the initial discussion of the surveys refers to a specific traverse (serial 060) of the Tennyson Inlet test site. Serial 060 appears as the red line in figure 3.A (on page 38).

During this serial ELFE transmissions of 35 and 74 Hz were used for the analyses. Figure 3.H (on page 50) shows the variation of the measured spectral levels at various horizontal ranges from the electric field sensors. In serial 060 as in most of the serials, the strong dipole sources within the tow ship tended to dominate the data records as the ship approached the sensor position. As the ship departed from this location the towed ELFE source became more distinct and amenable to resolution by the spectral line-tracking facility.

The asterisk symbols (*) in figure 3.I (on page 50) show the subsequent estimates of the characteristic conductivity of the seabed using the methodology described in equation (3.4). At the shortest horizontal ranges it is only to be expected that the differences in the ELFE propagation losses for similar frequencies would be quite small. In some instances these differences were less than 1 dB. The experimental error in measuring the ratio of $|E_{M,freq1}''|$ and $|E_{M,freq2}''|$ may therefore be relatively large at horizontal ranges of about 100 m.

To support these calculations three mid-water measurements of conductivity (σ_1) were made in the course of the trial. The mean value of 4.14 S.m^{-1} was within $\pm 1.5\%$ of all the measurements. It was this value which was used in all calculations associated with the conductivity surveys.

Approximation measures. From the earlier work it will also be apparent that at the shorter ranges the towed ELFE source no longer resembled a point dipole. This feature is a key assumption for the analytical model described in section 2. In these circumstances apparent values above 3 S.m^{-1} in figure 3.I (on page 50) for the seabed conductivity should be treated with caution.

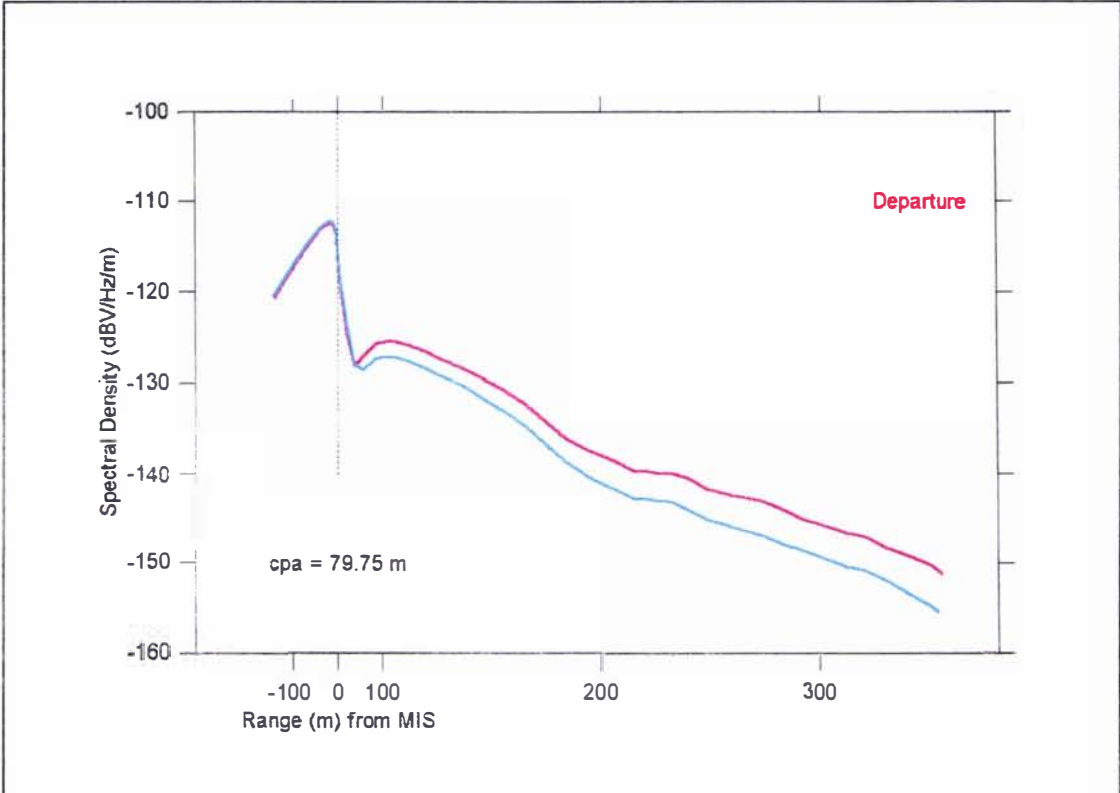


Figure 3.H Recorded spectral levels of the 35 and 74 Hz electromagnetic signals at the multi-influence sensor system, during serial 060

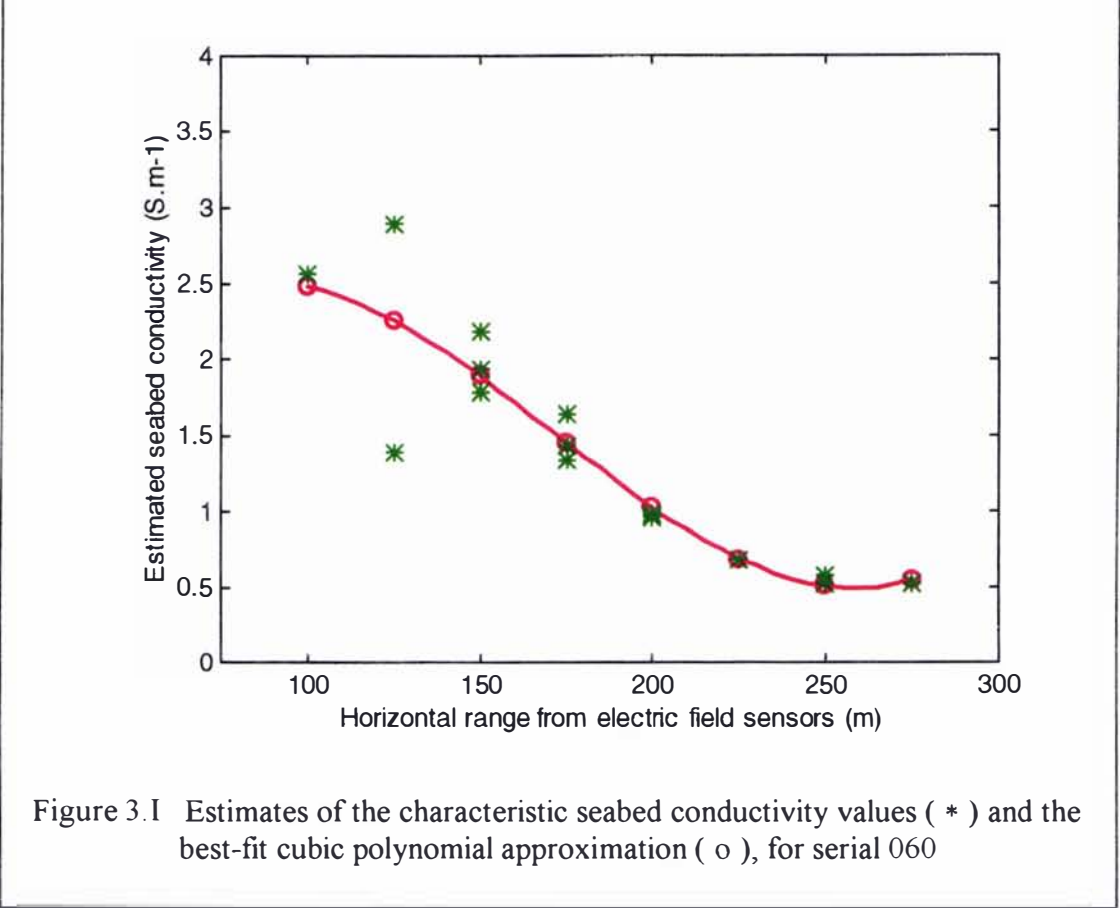


Figure 3.I Estimates of the characteristic seabed conductivity values (*) and the best-fit cubic polynomial approximation (o), for serial 060

Figure 3.I (on page 50) also clearly shows the increase in data scattering at the shorter ranges. It was a feature common to all serials. To provide a consistent approach to weighting the data, all measurements of each serial were smoothed. This recourse is a significant factor that is considered later in more detail.

Data smoothing steps were applied initially to the measurements in serial 060. The curve with the alternative symbols (o) in figure 3.I (on page 50) describes the least-squares fit by a cubic polynomial, to the sixteen measurements of the characteristic conductivity of the seabed, at eight values of the horizontal range.

3.5 Aggregation of survey results

Results from complementary surveys. Characteristic conductivity values of the seabed were similarly obtained from measurements made during complementary serials. These data together with that of serial 060, are shown in the form of a three-dimensional scatter diagram in figure 3.J (on page 52)

Measurements during the latter serials, like those of 060, were made after the ELFE sources had reached the closest point of approach (CPA) to the multi-influence sensor system.

Contours of characteristic conductivity. Given the spatial diversity of the estimates in figure 3.J (on page 52) it was a simple matter to develop contours of the characteristic values of the seabed conductivity throughout the test site. The first such representation is shown in figure 3.K (on page 52).

The different coloured contours represent increments of 0.5 S.m^{-1} over the range of values 0.5 to 4 S.m^{-1} . There is a small gap in the 0.5 S.m^{-1} contour where it lies beyond the axial bounds of the diagram.

In the uppermost contours the small perturbations are largely due to the inherent inaccuracy of the electric field measurements. Such a feature is accentuated by using

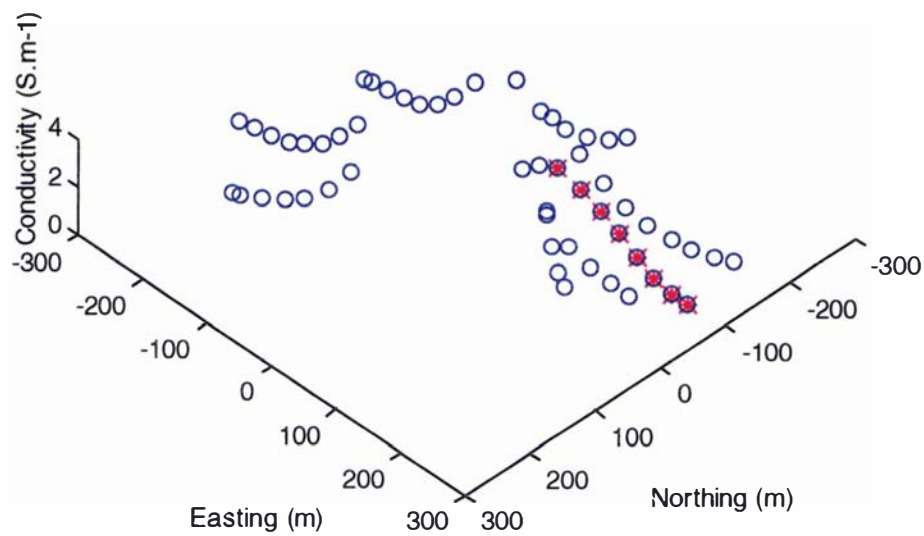


Figure 3.J Estimates of the characteristic seabed conductivity values from serial 060 (*) and complementary serials (o) at Tennyson Inlet

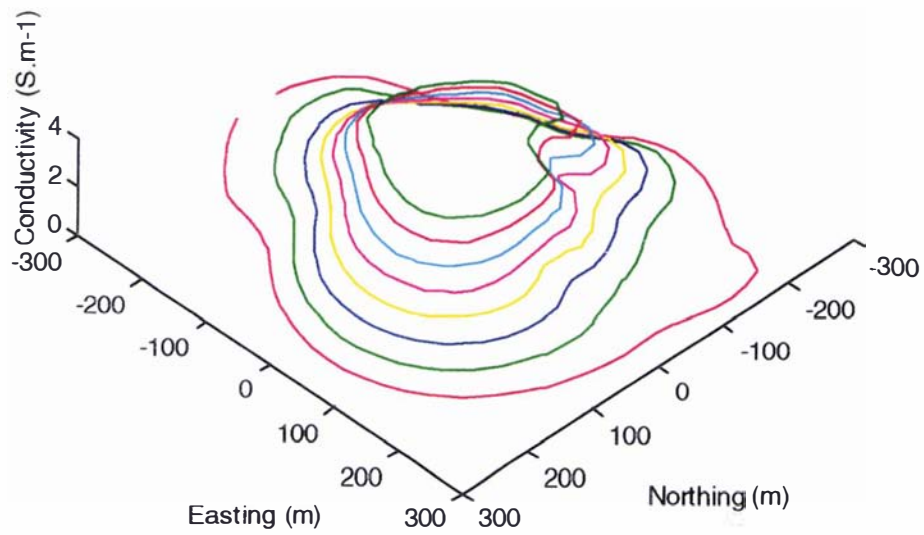


Figure 3.K Contour levels between 0.5 and 4 S.m⁻¹ (in steps of 0.5 S.m⁻¹) for the characteristic seabed conductivity values at Tennyson Inlet

the ratio of these fields in equation (3.4). Furthermore, inaccuracies were inescapable at the closest ranges when the ELFE dipoles could no longer be regarded as point sources – which is a basic assumption for the analytical model.

The final representation of this series of experimental results is shown in figure 3.L (on page 54). Here, the contours in the previous figure 3.K (on page 52) have been redrawn with the correct geographical alignments, and with the contours viewed at normal incidence. Again, the different colours represent contours at an increment of 0.5 S.m^{-1} over the range of values 0.5 to 4 S.m^{-1} .

A notable feature of the diagram is the variation in contour spacing along a diagonal bearing from the SW corner of the test site. It is discussed at length in the following paragraphs.

3.6 Characteristics of the experimental results

Sub-bottom characteristics. As indicated earlier, sub-bottom profiler records were compiled during the trial period to support acoustic experiments. These records were gathered through the centre and around the edges of the 800 m square site. Figure 3.B (on page 38) shows a typical vertical cross-section of the seawater, mud-silt and greywacke gravel sediments. This section is along a bearing of 48° on a straight track through the position of the electric field sensors.

A point of interest is the quite uniform slope of the sub-bottom structure through the test site. In particular, the mud-silt section is wedge-shaped, and varies uniformly from about 14 to 17.5 m through the area.

A comparison of figures 3.B (on page 38) and 3.L (on page 54) indicate a possible relationship between the thickness of the uppermost sediment (mud-silt) layer, and the spacing of the contours which describe the characteristic values of seabed conductivity at the same location. In particular, the close-spaced contours tend to lie above areas where the mud-silt layer is at its narrowest, and vice versa. Such a relationship suggests that ELFE signal propagation within the test site was influenced

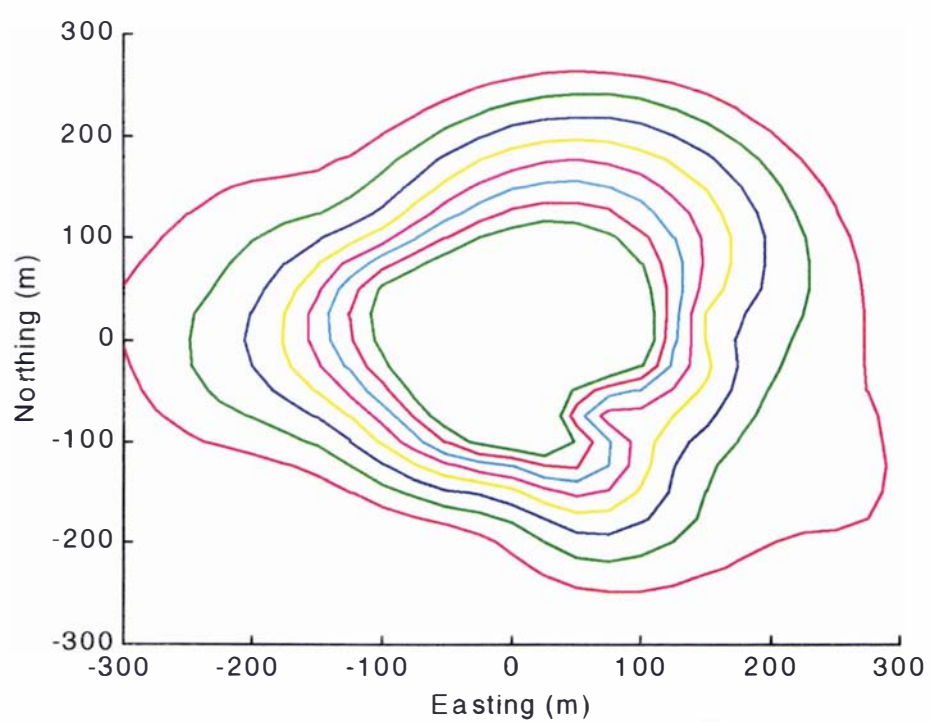


Figure 3.L Normal incidence view of the contour levels between 0.5 and 4 S.m^{-1} (redrawn from figure 3.K) for the characteristic values of seabed conductivity, at Tennyson Inlet

by more site features than just the two horizontal conducting layers which comprise the analytical⁴⁵ model. However, more comparative surveys of the sub-bottom structures are needed before such relationships can be verified.

Tests of accuracy. Earlier references to the 35, 74 and 94 Hz ELFE transmissions indicated that their current amplitudes were constant and of equal value, although the actual dipole moments were not known. It is a useful test of the conductivity survey to use this data to calculate the expected propagation loss between the electric field sensors and the ELFE sources. The propagation losses, when collated with the received signal levels, provide predictions of the dipole moments along a ship track. Results of such predictions are shown in figure 13 (on page 146) of appendix B.

Between the horizontal ranges of 100 - 275 m the mean-strength values of the 35, 74 and 94 Hz dipole sources were 13.3, 12.5 13.3 dB above a unit dipole (1 A.m). Also, over this horizontal range the standard deviation of the estimated dipole strength, was 2 dB. To place this performance in perspective it is relevant to note that the received signal levels of the 35, 74 and 94 Hz ELFE transmissions were reduced by 30.1, 37.5 and 38.6 dB respectively, between the horizontal ranges of 100 to 275 m.

It is useful to compare the above results (from serial 060) with those of a complementary serial at the World's End test site. Therefore, as a check for serial 060 corresponding analyses were carried out on serial 056 which is in the east-most position in figure 3.J (on page 52). Here, between horizontal ranges of 125 - 260 m, the mean-strength values of the 35, 74 and 94 Hz dipole sources were 11.7, 11.8 and 12.5 dB above a unit dipole. The standard deviation for the latter measurements was unchanged, at 2 dB.

At the shortest ranges the experimental errors could clearly be reduced by increasing the frequency spread of the ELFE transmissions. For instance, an alternative approach for serial 060 would have been to obtain conductivity estimates by analysing the 35/94 Hz transmissions at the shorter ranges (≈ 100 m). Analyses of the 35/74 Hz transmissions would preferably be used at the greater ranges (≈ 275 m).

For perfect survey results the three curves showing the estimated dipole strengths in figure 13 (on page 146) of appendix B, would of course be coincident and horizontal. The departure from such an ideal survey result is attributed partly to irregularities in the seabed surface and in its underlying structures. Indeed, the sub-bottom profiler records showed many small areas throughout the test site where the seabed density is non-uniform. This implies a likely non-uniformity in the conductivity characteristics of this medium. In such circumstances the test-site would not represent a true two-layer model. Other contributory factors include errors in the position estimates and in the measured levels of the transmitted and received ELFE signals.

Finite length and point sources. Clearly the experimental results raise concerns about the extent to which the analytical model can be used when dipole sources are of finite length. At this point it appears that dipoles several metres in length, can be regarded as point sources at all site locations outside the $2.5 \sim 3 \text{ S.m}^{-1}$ conductivity contours.

ELFE sources several tens of metres in length can be regarded as point sources for the purposes of the analytical model, only near the outer extremities of the test site.

3.7 Review of the analytical model

Applicability of the analytical model. Application of the two-layer (Weaver) model requires difficult evaluations of twelve analytical expressions. Each has real and imaginary components. Even after these computations it is now clear that for some seabed areas it is an over-simplification of the environment to use the two-layer concept as a global model with which to describe ELFE propagation effects. But extending the model to take account of three or more uniform conducting layers would markedly increase in its complexity.

The same observation would apply if it were planned to extend the model to take account of a sloping seabed. Such a change would provide an improved theoretical basis for considering long-range ELFE signal propagation. However, it would likewise greatly increase the complexity of the model.

Nevertheless, these experiments demonstrate that there are shallow-water areas in which ELFE propagation can be accurately described by a two-layer analytical model. It is preferable to apply such a model on a point-by-point basis to estimate the characteristic values of the seabed conductivity σ_2 at various ranges and bearings from the field sensors. Characteristic values of the seabed conductivity may then be mapped. If the sensor-source separation is sufficient for the latter to be considered point sources, the strength I of ELFE dipoles can then be accurately estimated.

Calibrated ELFE sources. Given additional resources, an alternative approach to the experiments would be to augment conductivity surveys at a test site using both a sensor spacing and dipole length of about 1 m. Ideally, such an ELFE dipole would be towed by a small workboat with a hull of non-conducting material.

Several advantages would accrue from such a procedure. First, assume that a short HED source can be successfully ranged using the method in equation (3.4), to provide an accurate calibration of its dipole moment I . This information can then be exploited to carry out conductivity surveys over the inner areas of the test site. Now however, the real root which characterizes the conductivity value (σ_2) of seabed can be found by using similar iterative search processes to evaluate the expression:

$$(|E_r''| I \mu_0^{1.5} \sigma_1^{0.5} \omega^{1.5}) / (4 \pi) - |E_{r,M}''| = 0 \quad (3.5)$$

in which $|E_r''|$ is as defined⁴⁵, $|E_{r,M}''|$ is the measured signal from an HED source at a point along the radial axis from the electric field sensors and ω is the angular frequency of the HED transmission. Equation (3.5) is a reordering of equation (A34) in appendix B but it is simplified with prior knowledge of parameter I .

Such an approach is expected to reduce the earlier loss of accuracy in measuring the seabed conductivity over the inner area of the test site. It would also extend the area coverage over which the HED source may be treated as a point source and within which the analytical model provided a valid representation of the ELFE signal propagation.

3.8 Potential for model enhancement

Representation of ELFE signal propagation. The conductivity measurements have shown that there are coastal sites where the two-layer analytical model would represent an undue simplification of the environment. However, the two-layer model can be effective if applied in a point-by-point manner with range-varying values of the seabed conductivity. This latter concept represents the environment with sufficient accuracy to satisfy some of the requirements for the ranging of ELFE sources over short ranges, and in shallow-water conditions.

At this stage it is appropriate to question whether the analytical model is sufficiently sensitive to detect the presence of unexpected propagation dependencies within the conducting media. In some circumstances it may be important to ascertain if there were frequency-related or polarization-related properties of the seabed which needed to be considered for modelling applications.

For instance, reference was made earlier to the single value of permeability that is used to characterize each of the two isotropic conducting layers of the analytical model. This simplification is appropriate in many areas to the east of New Zealand. The material that comprises the upper layers of the seabed has a relative permeability of approximately unity. Such a condition does not apply over large tracts of western coastal regions where the seabed composition often has a significant magnetite component. A brief review indicates that it would be difficult to extend the analytical model to accommodate such consequential variations of the environmental parameters.

Sloping interfaces within the conducting media. For this project the choice of test sites was based largely on the capabilities of the analytical model. A prime determinant was thus to find a coastal area in which the seabed was level, and of homogeneous composition.

Coastal areas with these characteristics, that were relatively free of maritime traffic, proved extraordinarily difficult to locate. However, it was also apparent that the

ELFE analytical model would require major enhancements before it was feasible to represent configurations with sloping media boundaries.

Finite length ELFE dipoles. Early measurements indicated that the ELFE sources on maritime vessels could rarely be treated as point sources at near-field ranges. Advanced analyses to identify shipboard ELFE sources and their locations would thus need to include cases where the sources were of finite length. The analytical model used in the study does not presently include such a facility. And it would be difficult to add such an enhancement.

A way ahead. All of the above points support the notion that advanced studies of ELFE wave propagation over short ranges, and in shallow-water conditions may be better carried out with numerical models. Such models could provide for the inclusion of more varied boundary conditions including multiple seabed layers of differing thickness and gradients. With these models it may also be practicable to include more accurate descriptions of conductivity, permittivity and permeability parameters.

4. Numerical models of electromagnetic fields

4.1 General techniques for numerical modelling

Introduction. Electromagnetic field configurations are sometimes described by partial differential equations, sometimes by integral equations and sometimes by the minimization of a function such as an energy integral. The unknown function is usually continuous and dependent on continuous independent variables.

Computers can clearly process only a finite set of numbers, whereas the analytical equations that describe electromagnetic fields involve a comparatively large set of numbers. By one means or another the accurate continuous equations must therefore be manipulated to produce a set of equations able to be processed by computers. The subsequent equations are usually algebraic, and frequently linear. Hence, there is a general requirement to solve sets of linear algebraic equations.

Research to provide numerical solutions of partial differential equations (PDE) and of integral equations has a long history. Many useful methods have been developed⁴⁸⁻⁴⁹. Since studies of electromagnetic fields often begin with such equations this research has often provided improved and more powerful techniques for obtaining numerical solutions. Some of the most frequently used methods are described in the following sections.

4.2 Characteristics of numerical techniques

General. Numerical techniques generally require more computational effort than analytical methods, but for extensive tasks they may be more flexible and powerful analysis tools. Moreover, such techniques analyze the entire configuration geometry provided as input data, without a need for a priori assumptions about which field interactions are the most significant. In addition they provide the numerical solutions to problems based on full-wave analyses.

The three basic techniques⁵⁰ presently used for numerical analyses of electromagnetic field configurations are usually referred to as the finite-difference time-domain (FDTD) method, the finite element method (FEM) and the method of moments (MOM). A large number of well-proven computer programs that utilize these techniques are available to expedite analyses of various electromagnetic configurations. Each may be conveniently characterised by the specific technique used to solve the complex equations associated with electromagnetic field analyses.

Method of moments technique. The method of moments technique (also known as the surface integral method, or the boundary element method) is used to solve Maxwell's equations in their integral form⁵¹. Such methods are efficient at solving open radiation problems involving long thin wires and/or conducting surfaces.

Other applications include the numerical modelling of resonant antennae, or large resonant structures such as ships or aircraft. However, if an electromagnetic configuration has a complex, arbitrary geometry (or inhomogeneous dielectrics) the corresponding method of moments model would entail intensive computations, and be relatively inefficient.

Finite element technique. An electromagnetic configuration involving a complex geometry with inhomogeneous dielectrics can often be efficiently modelled using techniques based on the numerical solution of Maxwell's equations in their differential form. One commonly used technique is referred to as the finite element method⁵²⁻⁵³. It requires the entire region under analysis be divided into elements or cells, each of which has specified electromagnetic properties.

Until recently, two inhibiting factors have tended to limit the usefulness of three-dimensional modelling using this technique. In the first place, practical three-dimensional vector problems require significantly more computation than two-dimensional problems, or scalar problems. Secondly, spurious solutions can result in unpredictable erroneous errors, and this is often referred to as the vector parasite problem. However, subsequent research has shown the latter errors can now be largely eliminated during the computations⁵⁴⁻⁵⁵. Moreover, faster computer-chip

processors and the reduced cost of memory facilities are continually extending the computation resources available.

A major weakness of the finite element technique is that it is relatively difficult to model open or unbounded electromagnetic configurations where the fields are not known at every point on a closed boundary. Various techniques such as absorbing boundaries⁵⁶⁻⁵⁷ have been used to reduce this deficiency. Although these methods are quite useful in two-dimensional problems they are of only limited effectiveness with three-dimensional configurations.

Finite-difference time-domain technique. The finite-difference time-domain (FDTD) method provides a direct solution of Maxwell's time-dependent curl equations in a linear medium⁵⁸. Hence, it also is an example of a technique involving partial differential equations. The FDTD method is a time-stepping procedure where the input data comprises time-sampled signals. Two interleaved grids of discrete points are required to represent the configuration region. One grid contains the points at which the magnetic field is evaluated. Points at which the electric field is evaluated are contained in the other grid.

This technique, like the finite element method, is limited by its inability to efficiently model unbounded configurations. For such configurations the entire volume under analysis must be meshed and it is likewise necessary to employ absorbing boundaries at each unbounded surface in the meshed region.

It was indicated in section 1, that in some conditions components of the ELFE signals will propagate through the air dielectric. To use the finite-difference time-domain technique in these circumstances would require inordinately many time-steps (about a billion) during the analyses to maintain the Courant stability condition. For this reason the FDTD method will be considered no further in this study.

Hybrid method of moments/finite element technique. Since method of moment techniques excel at modelling the types of problems the finite element techniques do not model well, and vice versa, a score or so of researchers⁵⁹⁻⁶² have combined the two methods in hybrid numerical models. These hybrid programs take advantage of

the strengths of each numerical technique in order to solve problems where each alone is unable to provide an efficient model.

A notable characteristic of the present approaches to hybrid numerical models is that each tends to be quite specific to its application, be it in physics, medicine or engineering. And a model that is used successfully in one field is not necessarily transportable to diverse applications.

The remainder of this section deals with the development and testing of a specific numerical model based on hybrid MOM/FEM techniques. Citations in the technical literature show this specific hybrid model has hitherto been used mainly for analyses of electromagnetic configurations with frequencies above 500 MHz.

Most of the development process follows the procedures described in earlier references^{61, 62} but with some adaptations for this project. The theoretical framework that follows similarly owes much to the group⁶³ that has been working together for the past ten years or so at the Electromagnetic Compatibility Laboratory of the University of Missouri, Rolla. Some of their work is reported directly at the web site of the laboratory (<http://www.emclab.umr.edu/>). The site also contains directories with links to many commercial and non-commercial versions of electromagnetic modelling codes available from diverse sources.

4.3 Finite element volumes of the hybrid model

Basic configuration. The first step in finite element analysis is to divide the model configuration into a small number of homogeneous pieces, or elements. In each finite element a simple (often linear) variation of the field quantity is assumed. Corners of the elements are referred to as nodes, and the goal of the analysis is to determine field values at the nodes, and if practicable the full volume fields.

Each element is then described in terms of its geometry (or part of the problem space it occupies), material constants, excitations and boundary constraints. A significant advantage of this technique stems from the fact that the electrical and geometric

properties of each element can be defined independently. Furthermore, the discrete elements can be small near a source where electric field values are high, or where fine construction features exist. Element sizes may be much larger elsewhere. This feature enables the problem to be set up with a large number of small elements in regions of complex geometry and fewer larger elements in relatively open regions. Hence, it is possible to model electromagnetic configurations that have complicated geometric features and many arbitrarily shaped dielectric regions, in a relatively efficient manner. Within this study the subdivision of the finite element region is by tetrahedral elements, and an edge-based basis function is utilized.

Formulation. Consider the inhomogeneous dielectric body that is partially covered by conductors shown in (a) of figure 4.A (on page 65). In this configuration the interior region of the dielectric structure is characterised by the environmental parameters $(\mu_0, \mu_r, \epsilon_0, \epsilon_r(\mathbf{r}))$ and the exterior region by (μ_0, ϵ_0) , where μ_0 and ϵ_0 are the free-space permeability and permittivity, respectively. The relative permeability μ_r is assumed to be constant in the dielectric structure. Since the dielectric region contains different materials, $\mathbf{r}$ is used to denote the spatial dependence of the relative permittivity ϵ_r .

This structure can either be illuminated by an incident field $(\mathbf{E}^i, \mathbf{H}^i)$ or be excited by a source internal to the dielectric region. The formulation derived in the following subsections is generalised for both types of excitation.

By introducing the equivalence principle, the entire region $V_0 (= V_1 \cup V_2)$ may be separated into the two sub-regions shown in (b) and (c) of figure 4.A (on page 65). The region external to the dielectric is denoted by V_1 and the region inside the dielectric by V_2 . The regions are coupled through proper boundary conditions.

For the interior region, the finite element method (FEM) is applied where the unknown boundary information is represented by equivalent electric currents $\mathbf{J}$ and the tangential electric fields $\mathbf{E}$. For the exterior region, the method of moments is applied and sets of surface integral equations are developed for the dielectric and conducting surfaces.

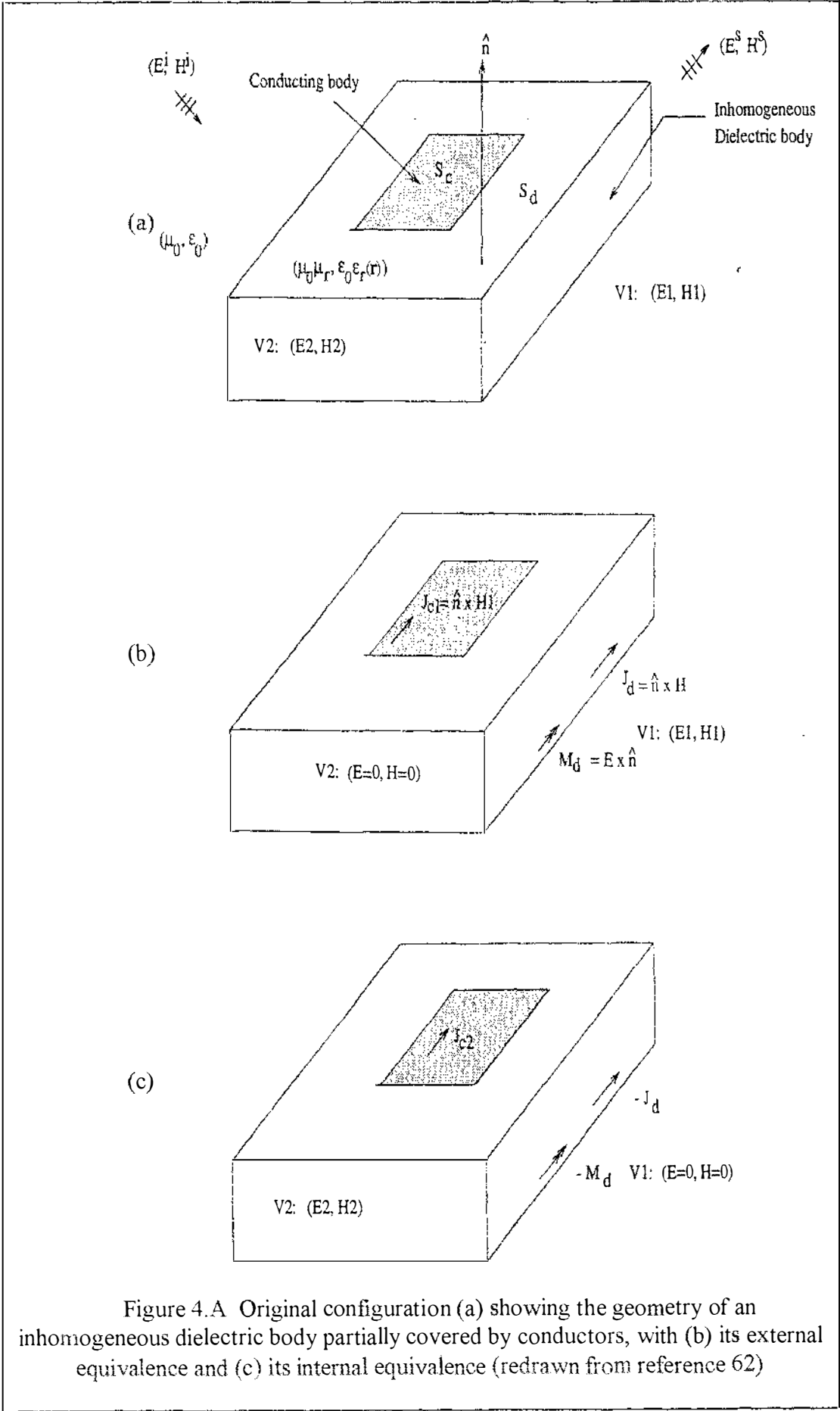


Figure 4.A Original configuration (a) showing the geometry of an inhomogeneous dielectric body partially covered by conductors, with (b) its external equivalence and (c) its internal equivalence (redrawn from reference 62)

To derive the finite element formulation for the interior region V_2 it is necessary to begin with the two basic Maxwell equations that govern the time-harmonic electromagnetic fields in this region:

$$\nabla \times \mathbf{E}(\mathbf{r}) = -\mathbf{M}^{\text{int}}(\mathbf{r}) - j\omega\mu_r\mu_0\mathbf{H}(\mathbf{r}) \quad (4.1)$$

$$\nabla \times \mathbf{H}(\mathbf{r}) = \mathbf{J}^{\text{int}}(\mathbf{r}) + j\omega\epsilon_r(\mathbf{r})\epsilon_0\mathbf{E}(\mathbf{r}) \quad (4.2)$$

where $\mathbf{M}^{\text{int}}$ and $\mathbf{J}^{\text{int}}$ are the internal magnetic and electric sources, respectively. From these equations it is possible to eliminate one field variable and obtain a *curl curl* equation in terms of the other field variable:

$$\begin{aligned} \nabla \times \left(\frac{1}{j\omega\mu_0\mu_r} \nabla \times \mathbf{E}(\mathbf{r}) \right) + j\omega\epsilon_0\epsilon_r(\mathbf{r})\mathbf{E}(\mathbf{r}) \\ = -\mathbf{J}^{\text{int}}(\mathbf{r}) - \frac{1}{j\omega\mu_0\mu_r} \nabla \times \mathbf{M}^{\text{int}}(\mathbf{r}) \end{aligned} \quad (4.3)$$

$$\begin{aligned} \nabla \times \left(\frac{1}{j\omega\epsilon_0\epsilon_r(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r}) \right) + j\omega\mu_0\mu_r\mathbf{H}(\mathbf{r}) \\ = -\mathbf{M}^{\text{int}}(\mathbf{r}) + \frac{1}{j\omega\epsilon_0\epsilon_r(\mathbf{r})} \nabla \times \mathbf{J}^{\text{int}}(\mathbf{r}) \end{aligned} \quad (4.4)$$

Both equations (4.3) and (4.4) involve second derivatives. They can be reduced to single derivative expressions by constructing their weak forms. Multiplying equation (4.3) by a set of real *vector* weighting functions $\mathbf{w}(\mathbf{r})$, and integrating over the finite element domain V_2 produces:

$$\begin{aligned} \int_{V_2} \left[\nabla \times \left(\frac{1}{j\omega\mu_0\mu_r} \nabla \times \mathbf{E}(\mathbf{r}) \right) \cdot \mathbf{w}(\mathbf{r}) + j\omega\epsilon_0\epsilon_r(\mathbf{r})\mathbf{E}(\mathbf{r}) \cdot \mathbf{w}(\mathbf{r}) \right] dV_2 \\ = - \int_{\Omega} \left[\mathbf{J}^{\text{int}}(\mathbf{r}) \cdot \mathbf{w}(\mathbf{r}) + \frac{1}{j\omega\mu_0\mu_r} \nabla \times \mathbf{M}^{\text{int}}(\mathbf{r}) \cdot \mathbf{w}(\mathbf{r}) \right] dV_2 \end{aligned} \quad (4.5)$$

Using the Gauss divergence theorem and a standard vector identity:

$$\int_V (\nabla \times \mathbf{a} \cdot \nabla \times \mathbf{b} - \mathbf{a} \cdot \nabla \times \nabla \times \mathbf{b}) dV = \int_S [\mathbf{a} \times (\nabla \times \mathbf{b})] \cdot \mathbf{n} dS \quad (4.6)$$

and letting $\mathbf{a} = \mathbf{w}$, and $\nabla \times \mathbf{b} = \left(\frac{1}{j\omega \mu_0 \mu_r} \right) \nabla \times \mathbf{E}$, yields a weak form⁶⁴ of equation (4.5):

$$\begin{aligned} & \int_{V_1} \left(\frac{1}{j\omega \mu_0 \mu_r} \nabla \times \mathbf{E}(\mathbf{r}) \cdot (\nabla \times \mathbf{w}(\mathbf{r})) + j\omega \varepsilon_0 \varepsilon_r(\mathbf{r}) \mathbf{E}(\mathbf{r}) \cdot \mathbf{w}(\mathbf{r}) \right) dV_1 \\ &= \int_S (\mathbf{n} \times \mathbf{H}(\mathbf{r})) \cdot \mathbf{w}(\mathbf{r}) dS - \int_{V_2} \left[\mathbf{J}^{\text{int}}(\mathbf{r}) + \frac{1}{j\omega \mu_0 \mu_r} \nabla \times \mathbf{M}^{\text{int}}(\mathbf{r}) \right] \cdot \mathbf{w}(\mathbf{r}) dV_2 \quad (4.7) \end{aligned}$$

This equation gives the relationship between the electric field inside the inhomogeneous medium and the tangential magnetic field at the boundaries.

4.4 Method of moments regions of the hybrid model

Basic configuration. The method of moments is a technique for solving integral equations by reducing them to a system of simple linear equations. The goal resembles that of the finite element method.

In contrast to the variational approach of the finite elements however, the moment methods employ a technique known as the method of weighted residuals. All weighted residual techniques begin by establishing a set of trial solutions functions with one or more variable parameters.

The residuals are a measure of the difference between the trial solution and the true solution. The variable parameters are determined in a manner that guarantees a best fit of the trial functions based on a minimization of the residuals.

Formulation. To derive the method of moments formulation for the hybrid problem, consider the external equivalence of figure 4.A (on page 65). The total tangential components of the electric field on surfaces S_d and S_c are obtained from the following surface E representation:

$$\left[E_d^l(J_d) + E_d^l(M_d) + E_d^l(J_{cl}) + E_d^{\text{inc}} \right]_{\text{tan}} = 0 \quad (4.8)$$

$$\left[E_c^l(J_d) + E_c^l(M_d) + E_c^l(J_{cl}) + E_c^{\text{inc}} \right]_{\text{tan}} = 0 \quad (4.9)$$

Since the electric field integral equation is used for the MOM formulation, the electric field E (after dropping the superscripts and subscripts representing the region) due to the electric current J and the magnetic current M is given by:

$$E(J, M) = -j\omega A(\mathbf{r}) - \nabla V(\mathbf{r}) - \left(\frac{1}{\varepsilon}\right) \nabla \times \mathbf{F}(\mathbf{r}) \quad (4.10)$$

where the vector potential functions A and F and the scalar potential function V are defined by:

$$A_d(\mathbf{r}) = \mu_d \int_{S_d} \mathbf{J}_d(\mathbf{r}') G_o(\mathbf{r}, \mathbf{r}') dS(\mathbf{r}') \quad (4.11)$$

$$V_d(\mathbf{r}) = \frac{1}{\varepsilon_d} \int_{S_d} \rho_d^e(\mathbf{r}') G_o(\mathbf{r}, \mathbf{r}') dS(\mathbf{r}') \quad (4.12)$$

$$F_d(\mathbf{r}) = \varepsilon_d \int_{S_d} \mathbf{M}_d(\mathbf{r}') G_o(\mathbf{r}, \mathbf{r}') dS(\mathbf{r}') \quad (4.13)$$

for $\mathbf{r}'$ on S_d and

$$A_c(\mathbf{r}) = \mu_o \int_{S_c} \mathbf{J}_c(\mathbf{r}') G_o(\mathbf{r}, \mathbf{r}') dS(\mathbf{r}') \quad (4.14)$$

$$V_c(\mathbf{r}) = \frac{1}{\varepsilon_o} \int_{S_c} \rho_c^e(\mathbf{r}') G_o(\mathbf{r}, \mathbf{r}') dS(\mathbf{r}') \quad (4.15)$$

for $\mathbf{r}'$ on S_c . The vectors $\mathbf{r}$ and $\mathbf{r}'$ are the source and the observation points, respectively, and:

$$G_o(\mathbf{r}, \mathbf{r}') = \frac{e^{-jk_o |\mathbf{r} - \mathbf{r}'|}}{4\pi |\mathbf{r} - \mathbf{r}'|} \quad (4.16)$$

is the free-space Green's function, and free-space propagation constant $k_o = \omega^2 \mu_o \epsilon_o$.

The equivalent electric and magnetic currents on surfaces S_d and S_c are related to the total electric and magnetic fields on the surface:

$$\mathbf{J}_d(\mathbf{r}') = \mathbf{n} \times \mathbf{H}(\mathbf{r}') \big|_{S_d} \quad (4.17)$$

$$\mathbf{M}_d(\mathbf{r}') = \mathbf{E}(\mathbf{r}') \times \hat{\mathbf{n}} \big|_{S_d} \quad (4.18)$$

$$\mathbf{J}_c(\mathbf{r}') = \hat{\mathbf{n}} \times \mathbf{H}(\mathbf{r}') \big|_{S_c} \quad (4.19)$$

where $\hat{\mathbf{n}}$ is an outward unit normal on the surface shown in figure 4.A (on page 65). The following continuity equations show the relationships between the equivalent electric charges and the currents:

$$\rho_d^{ve}(\mathbf{r}') = \frac{-1}{j\omega} \nabla_s' \cdot \mathbf{J}_d(\mathbf{r}'), \quad \rho_c^{ve}(\mathbf{r}') = \frac{-1}{j\omega} \nabla_s' \cdot \mathbf{J}_c(\mathbf{r}')$$

From equations (4.9) - (4.19) it is possible to obtain:

$$\begin{aligned} \mathbf{E}_d^{\text{inc}}(\mathbf{r}) &= \int_{S_d} \{ [\hat{\mathbf{n}}' \times \mathbf{E}(\mathbf{r}')] \times \nabla' G_o(\mathbf{r}, \mathbf{r}') + j k_o \eta_o (\hat{\mathbf{n}}' \times \mathbf{H}(\mathbf{r}')) G_o(\mathbf{r}, \mathbf{r}') \\ &\quad + \frac{j \eta_o}{k_o} \nabla' \cdot (\hat{\mathbf{n}}' \times \mathbf{H}(\mathbf{r}')) \nabla G_o(\mathbf{r}, \mathbf{r}') \} dS' \end{aligned} \quad (4.20)$$

$$\begin{aligned} \mathbf{E}_c^{\text{inc}}(\mathbf{r}) &= - \int_{S_c} \{ [\mathbf{E}(\mathbf{r}') \times \hat{\mathbf{n}}'] \times \nabla' G_o(\mathbf{r}, \mathbf{r}') + j k_o \eta_o (\hat{\mathbf{n}}' \times \mathbf{H}(\mathbf{r}')) G_o(\mathbf{r}, \mathbf{r}') \\ &\quad + \frac{j \eta_o}{k_o} \nabla' \cdot (\hat{\mathbf{n}}' \times \mathbf{H}(\mathbf{r}')) \nabla G_o(\mathbf{r}, \mathbf{r}') \} dS' \end{aligned} \quad (4.21)$$

These equations provide a relationship between the unknown tangential magnetic field quantities $\mathbf{n} \times \mathbf{H}$ and the unknown electric field quantities $\mathbf{E} \times \hat{\mathbf{n}}$ on the boundary surfaces S_d and S_c .

4.5 Design configuration of the discrete elements

Basic configuration. This section describes the processes by which discrete elements are configured in both the finite element weak form and the MOM surface integrals. To subdivide the finite element volume, tetrahedral elements were chosen as the basic building blocks and an edge-based basis function was used.

For the MOM surface integrals, the dielectric and/or conducting bodies are subdivided into triangular elements corresponding to the faces of the tetrahedral elements. A triangular patch function developed by Rao⁶⁵ is used to represent the equivalent currents at the boundaries.

However, prior to describing the discrete elements it is necessary to select suitable basis functions to expand the unknown quantities in the formulation.

Choice of basis functions. To avoid the possible occurrence of non-physical solutions, a class of tangentially-continuous, finite edge elements and consistent boundary surface elements were used. There are a number of different edge elements reported in the literature. For the interior domain of volume V_2 the vector basis functions chosen are those proposed in earlier work⁶⁶. These are defined within a tetrahedron and are associated with the six edges.

A unique feature of these basis functions is that they do not contain fictitious line or point electric charges. Such sources (especially at low frequencies) can be the dominant contributors to the electric fields and may introduce serious errors. The junction basis functions⁶⁷ are also used to couple the dielectric bodies to the conducting wires.

Assume the four nodes of the tetrahedron are numbered as shown in (a) and (b) of figure 4.B (on page 72). The vector basis function associated with the k th edge of that tetrahedron is then defined as:

$$\begin{aligned} \mathbf{w}_k(\mathbf{r}) &= \mathbf{f}_k + \mathbf{g}_k \times \mathbf{r} & \text{where } \mathbf{r} \text{ is in the tetrahedron,} \\ \mathbf{w}_k(\mathbf{r}) &= 0 & \text{otherwise,} \end{aligned} \quad (4.22)$$

with

$$\mathbf{f}_k = \frac{b_k}{6 V_e} (\mathbf{r}_{(7-k)_1} \times \mathbf{r}_{(7-k)_2}) \quad (4.23)$$

$$\mathbf{g}_k = \frac{b_k b_{(7-k)}}{6 V_e} \mathbf{e}_{(7-k)} \quad (4.24)$$

where $k = 1, 2, \dots, 6$, and

$$\begin{aligned} V_e &= \text{volume of the tetrahedron,} \\ \mathbf{e}_k &= \frac{(\mathbf{r}_{(7-k)_2} - \mathbf{r}_{(7-k)_1})}{b_k} = \text{unit vector of the } k^{\text{th}} \text{ edge,} \\ b_k &= |(\mathbf{r}_{(7-k)_2} - \mathbf{r}_{(7-k)_1})| = \text{length of the } k^{\text{th}} \text{ edge,} \end{aligned}$$

and $\mathbf{r}_{(7-k)_1}$ and $\mathbf{r}_{(7-k)_2}$ denote the location of the $(7-k)_1$ and the $(7-k)_2$ nodes, respectively. A more detailed explanation of the basis functions $\mathbf{w}_k$ can be found in earlier work at the University of Missouri⁶⁸.

Using these basis functions the electric field $\mathbf{E}$ in the interior region can be expanded as:

$$\mathbf{E}(\mathbf{r}) = \sum_{n=1}^{N_v} E_n \mathbf{w}_n(\mathbf{r}) \quad (4.25)$$

where $\{ E_n, n = 1, 2, \dots, N_v \}$ is a set of unknown complex scalars, and N_v the number of inner edges.

(a)

Edge#	k1	k2
1	1	2
2	1	3
3	1	4
4	2	3
5	4	2
6	3	4

(b)

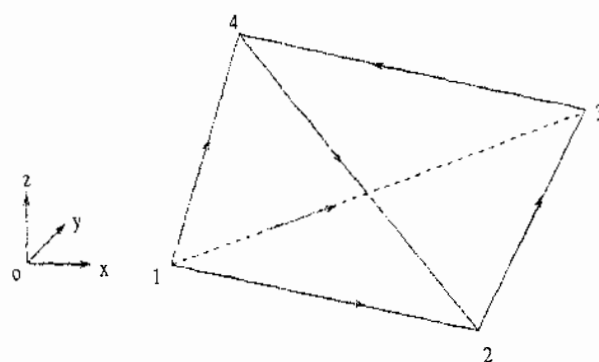


Figure 4.B Node definition and edge numbering scheme of a tetrahedron

The expansion functions for the unknowns on the surface S_d and S_c are chosen to be those proposed in earlier work⁶⁵ which are given as:

$$\begin{aligned} f_n(\mathbf{r}) &= \begin{cases} \frac{l_n}{2A_n^+} (\mathbf{r} - \mathbf{r}_n^+), & \text{for } \mathbf{r} \text{ in } T_n^+ \\ \frac{l_n}{2A_n^-} (\mathbf{r}_n^- - \mathbf{r}), & \text{for } \mathbf{r} \text{ in } T_n^- \\ 0, & \text{otherwise,} \end{cases} \end{aligned} \quad (4.26)$$

$$\begin{aligned} \nabla_S \cdot f_n(\mathbf{r}) &= \begin{cases} \frac{l_n}{A_n^+}, & \text{for } \mathbf{r} \text{ in } T_n^+ \\ \frac{-l_n}{A_n^-}, & \text{for } \mathbf{r} \text{ in } T_n^- \\ 0, & \text{otherwise,} \end{cases} \end{aligned} \quad (4.27)$$

where T_n^+ and T_n^- are two adjacent triangles with the n th edge common, l_n is the length of the n th common edge, $A_n^\pm$ is the area of the triangle $T_n^\pm$, and $\mathbf{r}^\pm$ are the position vectors of the nodes that are not related to the n th edge in triangle $T_n^\pm$.

Diagrams (a) and (b) of figure 4.C (on page 74) show the parameters associated with the basis function f_n . References^{65,69} provide a detailed discussion on the various properties of this basis function.

Using these basis functions, the unknown surface tangential field $\hat{\mathbf{n}} \times \mathbf{H}$ can be expanded as:

$$\hat{\mathbf{n}} \times \mathbf{H}(\mathbf{r}) = \sum_{n=1}^{N_s} \mathcal{J}_n f_n(\mathbf{r}). \quad (4.28)$$

It can be shown that on the boundary surface S , the basis function f_n and $\mathbf{w}_n$ are related by:

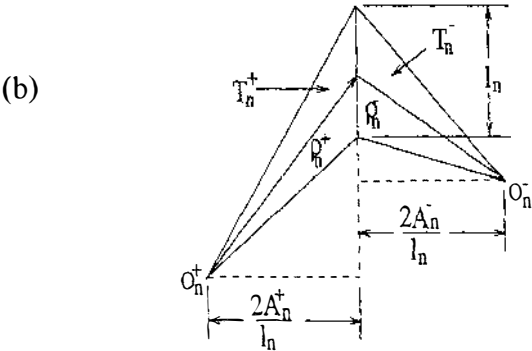
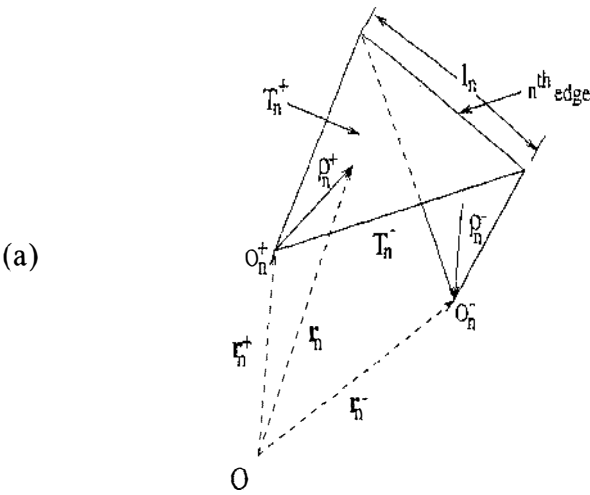


Figure 4.C Coordinates of a common edge associated with two triangles (a), and geometry showing normal component of basis function at the edge (b)

$$\mathbf{w}_n(\mathbf{r}) = \mathbf{n} \times \mathbf{f}_n(\mathbf{r})|_{\mathbf{r} \text{ on the surface } S} \quad (4.29)$$

A description of this relationship can be found in a well-known text⁷⁰.

4.6 Discrete elements of the FEM weak form

Formulation. To provide discrete elements for the finite element interior region V_2 , the volume is divided into several tetrahedral elements. The volume basis function $\mathbf{w}(\mathbf{r})$ is then used to expand the state variable $\mathbf{E}$ in this region, and the surface basis function $\mathbf{f}(\mathbf{r})$ to expand the intermediate quantity $\hat{\mathbf{n}} \times \mathbf{H}$ on the boundary surface:

$$\mathbf{E}(\mathbf{r}) = \sum_{n=1}^N E_n \mathbf{w}_n(\mathbf{r}) \quad (4.30)$$

$$\hat{\mathbf{n}} \times \mathbf{H}(\mathbf{r}) = \sum_{n=1}^{N_d} J_n \mathbf{f}_n(\mathbf{r}). \quad (4.31)$$

If the Galerkin procedure is then used to provide the discrete elements for the weak form of the $\mathbf{E}$ formulation, and assuming the weighting functions to be the same as the expansion functions, then the discrete version of equation (4.7) becomes:

$$[A] \{ \mathcal{E} \} = [B] \{ \mathcal{J} \} + \{ \mathcal{G}^{\text{int}} \}. \quad (4.32)$$

The elements of $[A]$, $[B]$ and $\{ \mathcal{G}^{\text{int}} \}$ are:

$$A_{mn} = \int_{V_2} \left[\frac{1}{j\omega \mu_0 \mu_r} (\nabla \times \mathbf{w}_n(\mathbf{r})) \cdot (\nabla \times \mathbf{w}_m(\mathbf{r})) + j\omega \varepsilon_0 \varepsilon_r \mathbf{w}_n(\mathbf{r}) \cdot \mathbf{w}_m(\mathbf{r}) \right] dV_2 \quad (4.33)$$

$$B_{mn} = \int_{S_d} \mathbf{f}_n(\mathbf{r}) \cdot \mathbf{w}_m(\mathbf{r}) dS \quad (4.34)$$

$$\mathbf{Q}_m^{\text{int}} \approx - \int_{V_2} \left[\mathbf{J}^{\text{int}} + \frac{1}{j\omega \mu_0 \mu_r} \nabla \times \mathbf{M}^{\text{int}} \right] \cdot \mathbf{w}_m(\mathbf{r}) dV_2 \quad (4.35)$$

where $[A]$ is a sparse, banded and symmetric $N \times N$ matrix and $\{\mathbf{E}\}$, $\{\mathbf{Q}^{\text{int}}\}$ and $\{\mathbf{J}\}$ are $N \times 1$ column vectors. Here, N is the total number of edges. The matrix $[B]$ is a sparse and symmetric $N \times N$ matrix, where only the bottom-right $N_d \times N_d$ sub-matrix contains nonzero elements. The term N_d is the total number of edges on the dielectric boundary. The unknown electric field vector $\{\mathbf{E}\}$ consists of all field expansion coefficients with respect to the element edges interior to the finite element region.

4.7 Discrete elements of MOM surface integrals

Formulation. To provide discrete elements for equations (4.20) and (4.21), equations (4.30) and (4.31) are used to expand the field quantities $\mathbf{E}(\mathbf{r})$ and $\hat{\mathbf{n}} \times \mathbf{H}(\mathbf{r})$ respectively. Then, testing these equations with the surface basis function $\mathbf{f}(\mathbf{r})$ and integrating over the boundary S , leads to:

$$[C_{dd}]\{\mathbf{J}_d\} + [C_{dc}]\{\mathbf{J}_c\} = [D'_{dd}]\{\mathbf{E}_d\} - \mathbf{E}_d^i \quad (4.36)$$

$$[C_{cd}]\{\mathbf{J}_d\} + [C_{cc}]\{\mathbf{J}_c\} = [D'_{cd}]\{\mathbf{E}_d\} - \mathbf{E}_c^i. \quad (4.37)$$

The elements of the C and D matrices and the $\mathbf{E}$ vector are:

$$c_{mn} = \left\langle \int_S \left[-j k_0 \eta_0 \mathbf{f}_n \cdot \mathbf{G} + \frac{j \eta_0}{k_0} (\nabla' \cdot \mathbf{f}_n) \nabla' \cdot \mathbf{G} \right] dS', \mathbf{f}_m \right\rangle \quad (4.38)$$

$$d'_{mn} = \left\langle \int_S \left[(\hat{\mathbf{n}}' \times \mathbf{w}_n) \times \nabla' \cdot \mathbf{G} \right] dS', \mathbf{f}_m \right\rangle \quad (4.39)$$

$$\mathbf{e}_m^i = \left\langle -\mathbf{E}^{\text{inc}}, \mathbf{f}_m \right\rangle \quad (4.40)$$

where $C_{dd}, C_{dc}, C_{cd}, C_{cc}, D_{dd}'$ and D_{cd}' are sub-matrices with dimensions $N_d \times N_d$, $N_d \times N_c$, $N_c \times N_d$, $N_c \times N_c$, $N_d \times N_d$ and $N_c \times N_d$, respectively. The terms N_d and N_c are the total number of edges on S_d and S_c respectively.

Equation (4.39) involves a singularity when the source and observation points are located on the same surface patch. For such cases, it is possible to evaluate the singularity contribution analytically⁷⁰ [see chapter 9], and having done this equation (4.39) can be written as:

$$\begin{aligned} d_{mn}' &= \frac{-1}{2} \int_S \mathbf{w}_m \cdot \mathbf{f}_n dS + \int_S \mathbf{f}_m \cdot \hat{\mathbf{n}}_S [(\hat{\mathbf{n}}' \times \mathbf{w}_n) \times \nabla' G] dS' dS \\ &= \frac{1}{2} \mathbf{b}_{mn} + d_{mn}'' \quad (\text{since } \langle \mathbf{w}_n, \mathbf{f}_m \rangle = -\langle \mathbf{f}_n, \mathbf{w}_m \rangle) \quad (4.41) \end{aligned}$$

where the bar across the integral indicates the singularity point has been removed. With the singularity removed, equations (4.36) and (4.37) become:

$$[C_{dd}]\{\mathcal{J}_d\} + [C_{dc}]\{\mathcal{J}_c\} = \frac{1}{2}[B_{dd}]\{\mathcal{E}_d\} + [D_{dd}'']\{\mathcal{E}_d\} - \mathcal{E}_d^i \quad (4.42)$$

$$[C_{cd}]\{\mathcal{J}_d\} + [C_{cc}]\{\mathcal{J}_c\} = \frac{1}{2}[B_{cd}]\{\mathcal{E}_d\} + [D_{cd}'']\{\mathcal{E}_d\} - \mathcal{E}_c^i \quad (4.43)$$

which can be written more concisely as:

$$[C_{dd}]\{\mathcal{J}_d\} + [C_{dc}]\{\mathcal{J}_c\} = [D_{dd}]\{\mathcal{E}_d\} - \mathcal{E}_d^i \quad (4.44)$$

$$[C_{cd}]\{\mathcal{J}_d\} + [C_{cc}]\{\mathcal{J}_c\} = [D_{cd}]\{\mathcal{E}_d\} - \mathcal{E}_c^i \quad (4.45)$$

where $D_{dd} = \frac{1}{2} B_{dd} + D_{dd}''$ and $D_{cd} = \frac{1}{2} B_{cd} + D_{cd}''$.

4.8 Coupling of numerical techniques

Formulation. After partitioning the elements of matrices $[A]$ and $[B]$ in equation (4.7) for inner and boundary edges, and setting the tangential $\mathbf{E}$ field on the

conducting walls to zero, the following form of the finite element equation is obtained:

$$\begin{bmatrix} A_{ii} & A_{id} \\ A_{di} & A_{dd} \end{bmatrix} \begin{Bmatrix} \mathcal{E}_i \\ \mathcal{E}_d \end{Bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & B_{dd} \end{bmatrix} \begin{Bmatrix} 0 \\ \mathcal{J}_d \end{Bmatrix} + \begin{Bmatrix} 0 \\ \mathcal{G}_i \end{Bmatrix} \quad (4.46)$$

where subscripts i and d refer to the interior and boundary edges in the finite element volume V_2 , respectively. Prior to coupling this equation with the MOM integrals, it is necessary to define the location of the sources with respect to the volume V_2 . The following two cases can be considered.

Case I (The source is outside V_2). For this case, the last term in equation (4.46) is eliminated and the new equation becomes:

$$\begin{bmatrix} A_{ii} & A_{id} \\ A_{di} & A_{dd} \end{bmatrix} \begin{Bmatrix} \mathcal{E}_i \\ \mathcal{E}_d \end{Bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & B_{dd} \end{bmatrix} \begin{Bmatrix} 0 \\ \mathcal{J}_d \end{Bmatrix} \quad (4.47)$$

From equation (4.45), it is possible to obtain:

$$\mathcal{J}_c = -C_{cc}^{-1} C_{cd} \mathcal{J}_d + C_{cc}^{-1} D_{cd} \mathcal{E}_d - C_{cc}^{-1} \mathcal{E}_c^i \quad (4.48)$$

Substituting the value of $\mathcal{J}_c$ from equation (4.48) into equation (4.44) produces:

$$[C_{dd} - C_{dc} C_{cc}^{-1} C_{cd}] \mathcal{J}_d = [D_{dd} - C_{dc} C_{cc}^{-1} D_{cd}] \mathcal{E}_d + C_{dc} C_{cc}^{-1} \mathcal{E}_c^i - \mathcal{E}_d^i \quad (4.49)$$

Solving for $\mathcal{J}_d$ yields:

$$\begin{aligned} \mathcal{J}_d &= [C_{dd} - C_{dc} C_{cc}^{-1} C_{cd}]^{-1} [D_{dd} - C_{dc} C_{cc}^{-1} D_{cd}] \mathcal{E}_d \\ &\quad + [C_{dd} - C_{dc} C_{cc}^{-1} C_{cd}]^{-1} (C_{dc} C_{cc}^{-1} \mathcal{E}_c^i - \mathcal{E}_d^i) \\ &= [C_{dd}']^{-1} [D_{dd}'] \mathcal{E}_d + [C_{dd}']^{-1} (C_{dc} C_{cc}^{-1} \mathcal{E}_c^i - \mathcal{E}_d^i) \end{aligned} \quad (4.50)$$

where $C'_{dd} = C_{dd} - C_{dc} C_{cc}^{-1} C_{cd}$ and $D'_{dd} = D_{dd} - C_{dc} C_{cc}^{-1} D_{cd}$. In matrix form:

$$\begin{Bmatrix} 0 \\ \mathcal{J}_d \end{Bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & [C'_{dd}]^{-1} [D'_{dd}] \end{bmatrix} \begin{Bmatrix} 0 \\ \mathcal{E}_d \end{Bmatrix} + \begin{Bmatrix} 0 \\ [C'_{dd}]^{-1} (C_{dc} C_{cc}^{-1} \mathcal{E}'_c - \mathcal{E}'_d) \end{Bmatrix} \quad (4.51)$$

This equation can easily be coupled to the finite element equation (4.47). The resulting hybrid matrix equation becomes:

$$\begin{bmatrix} A_{ii} & A_{id} \\ A_{di} & A'_{dd} \end{bmatrix} \begin{Bmatrix} \mathcal{E}_i \\ \mathcal{E}_d \end{Bmatrix} = \begin{Bmatrix} 0 \\ \mathcal{E}^{src} \end{Bmatrix} \quad (4.52)$$

where

$$A'_{dd} = A_{dd} - B_{dd} [C'_{dd}]^{-1} [D'_{dd}]$$

and

$$\mathcal{E}^{src} = B_{dd} [C'_{dd}]^{-1} (C_{dc} C_{cc}^{-1} \mathcal{E}^i_c - \mathcal{E}^i_d).$$

Once the solution of equation (4.52) is available, it is easy to obtain J_d and J_c from equations (4.50) and (4.48), respectively.

Case II (The source is inside V_2). In this case, equations (4.44) and (4.45) are simplified as:

$$[C_{dd}] \{\mathcal{J}_d\} + [C_{dc}] \{\mathcal{J}_c\} = [D_{dd}] \{\mathcal{E}_d\} \quad (4.53)$$

$$[C_{dd}] \{\mathcal{J}_d\} + [C_{cc}] \{\mathcal{J}_c\} = [D_{cd}] \{\mathcal{E}_d\} \quad (4.54)$$

From equation (4.54):

$$\mathcal{J}_c = -C_{cc}^{-1} C_{cd} \mathcal{J}_d + C_{cc}^{-1} D_{cd} \mathcal{E}_d \quad (4.55)$$

Substituting the value of $\mathcal{J}_c$ from equation (4.55) into equation (4.53) produces:

$$[C_{dd} - C_{dc} C_{cc}^{-1} C_{cd}] \mathcal{J}_d = [D_{dd} - C_{dc} C_{cc}^{-1} D_{cd}] \mathcal{E}_d \quad (4.56)$$

Solving for $\mathcal{J}_d$ yields:

$$\begin{aligned}\mathcal{J}_d &\approx [C_{dd} - C_{dc} C_{cc}^{-1} C_{cd}]^{-1} [D_{dd} - C_{dc} C_{cc}^{-1} D_{cd}] \mathcal{E}_d \\ &\approx [C_{dd}']^{-1} [D_{dd}'] \mathcal{E}_d\end{aligned}\quad (4.57)$$

where $C_{dd}' = C_{dd} - C_{dc} C_{cc}^{-1} C_{cd}$ and $D_{dd}' = D_{dd} - C_{dc} C_{cc}^{-1} D_{cd}$. In matrix form:

$$\begin{Bmatrix} 0 \\ \mathcal{J}_d \end{Bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & [C_{dd}']^{-1} [D_{dd}'] \end{bmatrix} \begin{Bmatrix} 0 \\ \mathcal{E}_d \end{Bmatrix}\quad (4.58)$$

This equation can easily be coupled to the finite element equation (4.46). The resulting hybrid equation becomes:

$$\begin{bmatrix} A_{ii} & A_{id} \\ A_{di} & A_{dd}' \end{bmatrix} \begin{Bmatrix} \mathcal{E}_i \\ \mathcal{E}_d \end{Bmatrix} = \begin{Bmatrix} 0 \\ \mathcal{G}^i \end{Bmatrix}\quad (4.59)$$

where $A_{dd}' = A_{dd} - B_{dd} [C_{dd}']^{-1} [D_{dd}']$. Once the solution of equation (4.59) is available it is easy to obtain J_d and J_c from equations (4.55) and (4.57), respectively.

4.9 Extended hybrid method

Basic configuration. It is now possible to extend the method developed in the previous sections to allow the conducting body to extend beyond the dielectric surface shown in figure 4.A (on page 65). This facility may be necessary to study a field configuration such as that of a conductor which extends beyond the edge of a printed circuit board. But as the extension is not central to this study the general approach is only outlined briefly.

Using the equivalence principles it is possible to obtain the interior and exterior equivalence similar to that shown in (b) and (c) of figure 4.A (on page 65). The

method of handling the interior region V_2 is identical to that described in the previous sections. However, the exterior equivalence is handled in a slightly different fashion. In this case, a 2-way junction basis function⁶⁷ is used to represent the equivalent surface currents at dielectric-conductor junctions. For this type of junction, two basis functions represent two types of current flowing across each junction edge.

The basis function f_a represents the component of current that flows from the external conductor surface to the exterior surface of the dielectric-conductor interface. The basis function f_b represents the current component that flows into the dielectric body from the external conductor surface. These two currents are related in such a way that they obey Kirchhoff's current law and maintain current continuity at the junction. Since the current J_b represents the dielectric equivalent current at the junction, this current is set equal to the interior equivalent current corresponding to that edge.

4.10 Comparison of results from analytical and numerical models

An essential part of the study was to compare the respective results when analytical and numerical models were applied to solve selected electromagnetic configurations. Three hypothetical scenarios were examined in this part of the study.

Case I (Point and finite length VED sources in a semi-infinite seawater volume). Both types of model were used with the *case I* configuration to provide estimates of the electric field values of VED sources, located far below the surface of a semi-infinite sea. Each source was a unit, 50 Hz dipole.

The analytical model described in section 2 was used to predict the vertical electric fields at designated points due to a unit, 50 Hz VED point source. Predictions of the vertical electric field at identical points due to a unit, 50 Hz 1 m ($\approx 0.0045\lambda$) VED source, were then made using the numerical model described earlier in this section. Estimates of the vertical field values provided by the numerical model are the mean values between two nearest nodes of the configuration grid. Figure 4.D (on page 83)

provides a precise description of the configuration for the hybrid numerical model using the specialist 12-string vocabulary³⁴ referred to earlier in section 1.7.

The overall configuration resembled that shown in figure 2.A (on page 23). The centres of the point source and the finite length source were assumed to successively occupy the same point in the seawater. Each model provided predictions of the vertical electric field at various horizontal ranges, measured from the centres of the dipole sources. A field value from the analytical model was therefore measured at a point midway between the two nodes from which the corresponding field estimates were obtained from the numerical model.

A uniform seawater conductivity of 4 S m^{-1} was assumed, from which it follows that the 50 Hz signal would have a skin-depth and wavelength of approximately 35.6 m and 235 m respectively, in seawater. Since the numerical model required terms that express the complex permittivity of each medium a relative permittivity value of 72 for seawater was assumed. However, it will be shown later that the real component of the complex permittivity parameter is of little significance in these applications.

Comparable estimates of the respective electric field values from the analytical and numerical models are shown in figure 4.E (on page 83). Symbol (*) denotes the estimated field values of the analytical model, with the point source. Field values from the numerical model with the 1 m dipole, are denoted by the symbol (o).

Notable features of the graphs include the generally close agreements between the two sets of estimated field values at close ranges, and the deterioration of the relationships at the longer distances. The latter characteristic may be a consequence of using a uniform 5 m grid along the z axis of the numerical model.

Case II (5 m finite length VED sources in a semi-infinite seawater volume). The electromagnetic configuration in this case resembled that described above, except that both dipole sources had a finite length of 5 m ($\approx 0.0224\lambda$). In this instance, the analytical model described in section 2 was inappropriate as it was suitable only for VED or HED point sources. For the *case II* configuration the analytical model was selected from a well-known handbook⁴¹ (equation 4.5).

unit	1.0	m							
boundary	0	0	0	10	11	70			
celldim	0	10	5	x					
celldim	0	5	5	y					
celldim	5	6	1	y					
celldim	6	11	5	y					
celldim	0	70	5	z					
dielectric	0	0	0	10	11	70	72.0	-1438006307.0	
isource	5	5	35	5	6	35	0.00005	y	1.000
output	5	5	25	5	6	70	y	dipole28A.out	

Figure 4.D Specification for the numerical model of a unit, 50 Hz 1 m, VED source in a one-layer conducting medium

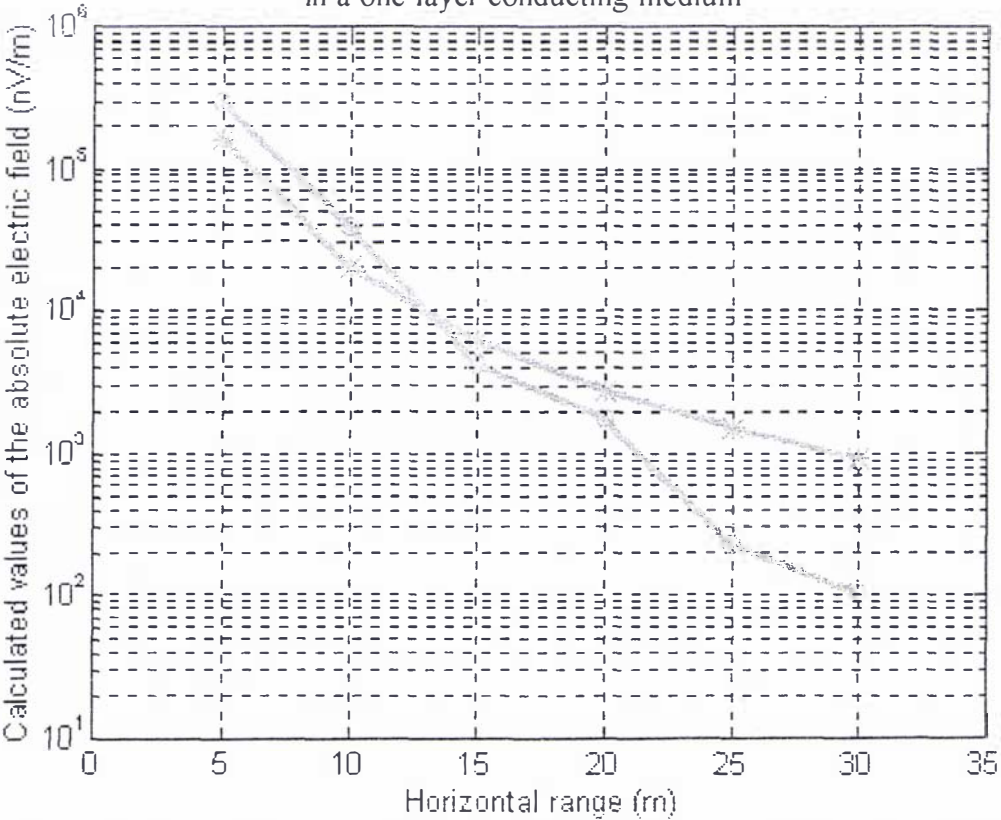


Figure 4.E Calculated 50 Hz, vertical field values of a unit, VED point-source (*) and a unit, VED 1 m source (o) in a one-layer conducting medium

Again, the general VED layout was similar to that shown in figure 2.A (on page 23). However, for the *case II* scenario a uniform 5 m grid was used for all three axes of the numerical model. Figure 4.F (on page 85) provides a precise description of the model configuration, which required a computer memory allocation of 38.5 Mbytes. This resource level was one of the highest such requirements throughout the study, and its implications are discussed later.

For the *case II* configuration the comparable estimates of the respective electric field values from the analytical and numerical models are shown in figure 4.G (on page 85). As in the *case I* graphs, symbol (*) denotes the estimated field values of the analytical model with the 5 m VED source. Comparable field values from the numerical model with the 5 m VED source, are denoted by the symbol (o).

Once again, the graphs indicate a close agreement between the two sets of predicted electric field values. However, this relationship clearly deteriorates at the longer ranges. The latter characteristic may be a consequence of using a uniform 5 m grid of node points, all of which were at least 5 m away from the 5 m dipole. Such comparisons can usually be improved by utilizing greater node densities near the source where field values change rapidly with changes in range.

Case III (Point and finite length sources in a three-layer configuration). The computer simulations for the *case III* configuration provided estimates of the radial field values of two HED sources in seawater, with a uniform depth 30 m. Each source was a unit, 50 Hz dipole, located at a depth of 15 m.

The analytical model used was that described in section 2, and in *case I*. Again, it entailed the use of a point source and provided estimates of the electric field value at designated points. The numerical (hybrid FEM/MOM) model used was that described earlier in this section. In this instance, the HED source had a finite length of 1 m (again, $\approx 0.0045\lambda$). Estimates of electric field values provided by this model were the mean values between two nodes of the configuration grid. Figure 4.H (on page 87) provides an exact description of this configuration.

unit	5.0	m							
boundary	0	0	0	4	5	18			
celldim	0	4	1	x					
celldim	0	5	1	y					
celldim	0	18	1	z					
dielectric	0	0	0	4	5	18	72.0	-1438006307.0	
isource	2	2	4	2	3	4	0.00005	y	0.200
output	0	3	5	4	3	18	x	dipole30.out	
output	2	0	5	2	5	18	y	dipole30.out	
output	2	0	5	2	5	18	z	dipole30.out	

Figure 4.F Specification for the numerical model of a unit, 50 Hz 5 m, VED source in a one-layer conducting medium

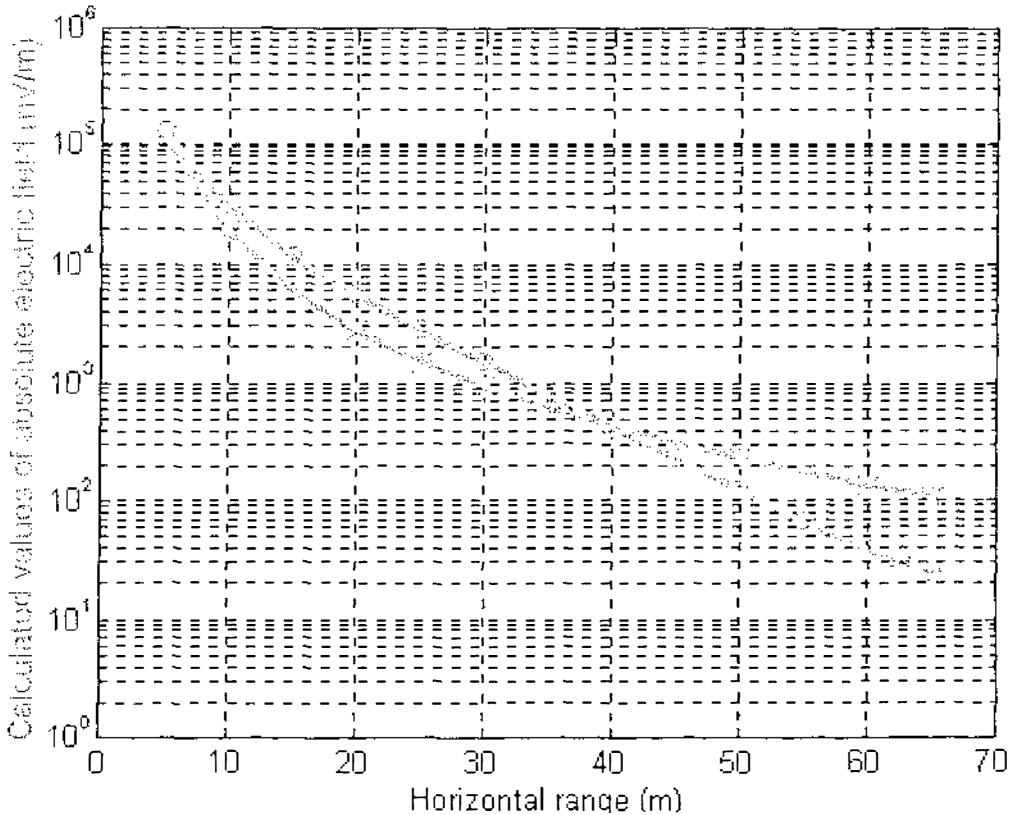


Figure 4.G Calculated 50 Hz, vertical field values from analytical (*) and numerical (o) models of unit, 5 m VED sources in a one-layer conducting medium

The overall configuration resembled that shown in figure 2.B (on page 23). The centres of the point source and the finite length source were assumed to successively occupy the same point in the seawater medium. Each model provided estimates of the radial electric field at various horizontal ranges, measured from the centres of the dipole sources. For the numerical model, the field values stated are those along a radial axis which is normal to the extended axis of the HED source. A comparable field value from the analytical model was therefore measured at a point midway between the two nodes from which the corresponding field estimates were obtained from the numerical model.

A uniform seawater medium of conductivity of 4 S m^{-1} , and relative permittivity of 72 were assumed. Corresponding parameters for the semi-infinite seabed medium were taken to be 2 S m^{-1} and 36, respectively.

Comparable estimates of the respective electric fields from the analytical and numerical models are shown in figure 4.I (on page 87). Once again symbol (+) denotes the estimated field values of the analytical model, with the ELFE point source. Radial electric fields from the numerical model with the 1 m ELFE dipole, are denoted by the symbol (o). Again, there is a reasonable degree of consistency between the two sets of estimated field values.

However, in all three cases there are notable differences in the field estimates close to the inner boundaries of the FEM volume, particularly with the sources of lowest λ -values. This is a concern. Reductions in the volume sub-divisions would enable this issue to be investigated further. But such changes would greatly increase the necessary memory allocations and would be better carried out on a faster machine.

Another useful avenue of investigation would be to run the numerical models on a modern workstation or mainframe computer with quad-precision facilities. Already it is clear that the inherent machine accuracy and the choice of operating system (Linux 5.0 or Windows 95) affects the number of reliable significant figures in estimates of the electric field values. Finally, it is observed that some of the basis functions referred to in section 4.5 (on page 70) are frequency-dependent, and they may need to be revised for use with numerical models of ELFE wave propagation.

unit	1.0	m							
boundary	0	0	0	11	50	170			
celldim	0	5	5	x					
celldim	5	6	1	x					
celldim	6	11	5	x					
celldim	0	20	10	y					
celldim	20	24	4	y					
celldim	24	26	1	y					
celldim	26	30	4	y					
celldim	30	50	10	y					
celldim	0	40	20	z					
celldim	40	50	5	z					
celldim	50	170	20	z					
dielectric	0	0	0	11	10	170	1.0		-0.0
dielectric	0	10	0	11	40	170	72.0	-1438006307.0	
dielectric	0	40	0	11	50	170	36.0	-719003154.0	
isource	5	25	45	6	25	45	0.00005	x	1.000
output	5	0	50	5	50	170	z		dipole28.out

Figure 4.H Specification for the numerical model of a unit, 50 Hz 1 m, HED source in two-layer conducting media

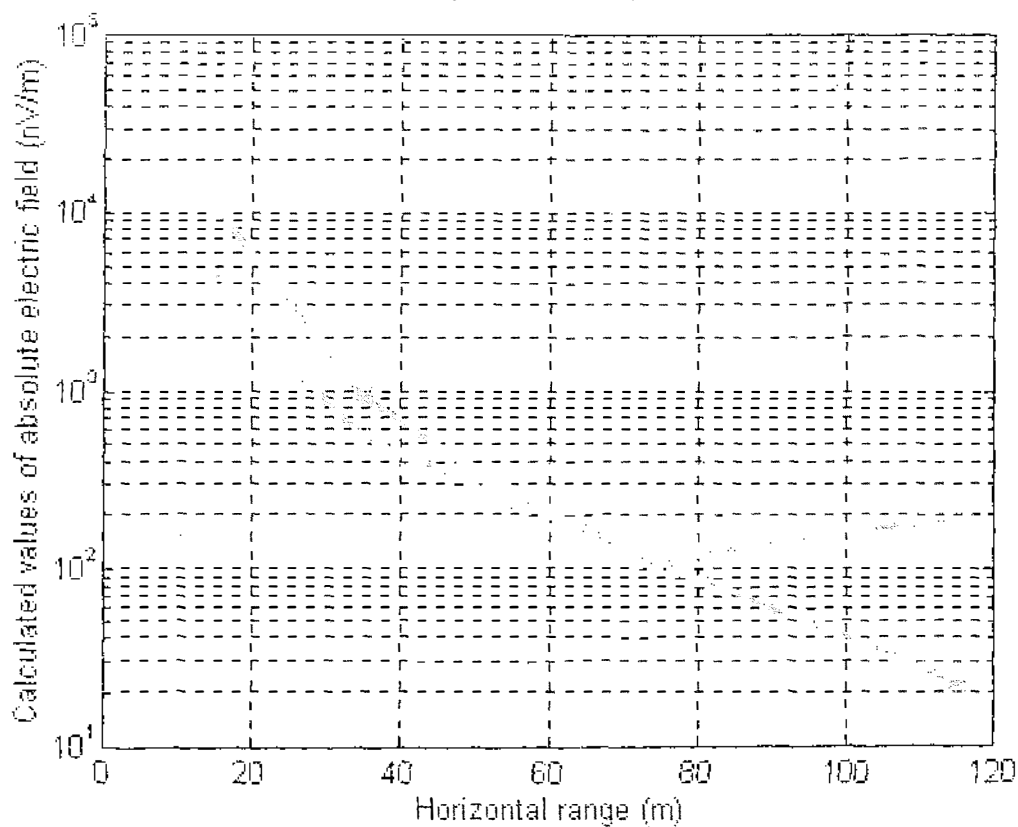


Figure 4.I Calculated 50 Hz, horizontal field values of a unit, HED point-source (*) and a unit, HED 1 m source (o) in two-layer conducting media

5. Applications of numerical models

5.1 Characteristics of simulated test sites

Introduction. Section 3 concluded by describing a series of factors that inhibited the use of analytical techniques for analysing propagation experiments in real environments. One such factor was the present inability of analytical models to accommodate configurations that include a sloping surface on the seabed. A second limitation of the model described in section 2 was its inability to include finite dimensions for the ELFE source and the electric field sensors. Thirdly, without major extensions the analytical technique in section 2 was unable to include more than two conducting layers in the model configurations.

This section considers the efficacy of numerical techniques to model ELFE propagation experiments. The initial objective was to define two simulated environments that resembled the conditions found within the test sites at Tennyson Inlet and Port Fitzroy. Hybrid numerical models were then used to simulate ELFE propagation experiments in these two environments.

Simulated coastal environments. The first of the simulated coastal environments was quite similar to that shown in figure 2.B (on page 23). However, for this computer simulation the previous level surface of the semi-infinite homogeneous seabed is now replaced by a uniform slope of 0.05 radians (or $\approx 2.8^\circ$).

To exploit the full capabilities of the numerical model it was also necessary to enlarge the set of environmental parameters. Additions included values of the characteristic permittivity for each medium. The extended parameter set and a diagram of the simulated environment is shown in figure 5.A (on page 89).

Figure 5.B (on page 89) describes the second of the simulated coastal environments. It has an obvious resemblance to the real-world conditions within Tennyson Inlet,

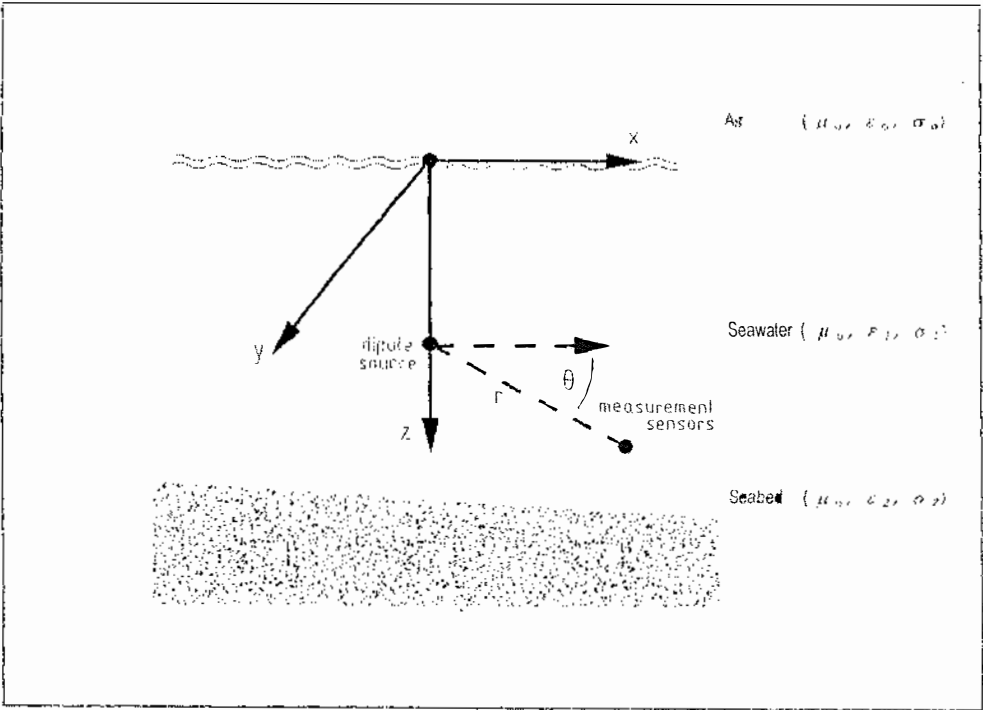


Figure 5.A Nomenclature for numerically modelling fields of dipole sources in two-layer conducting media with a sloping interface

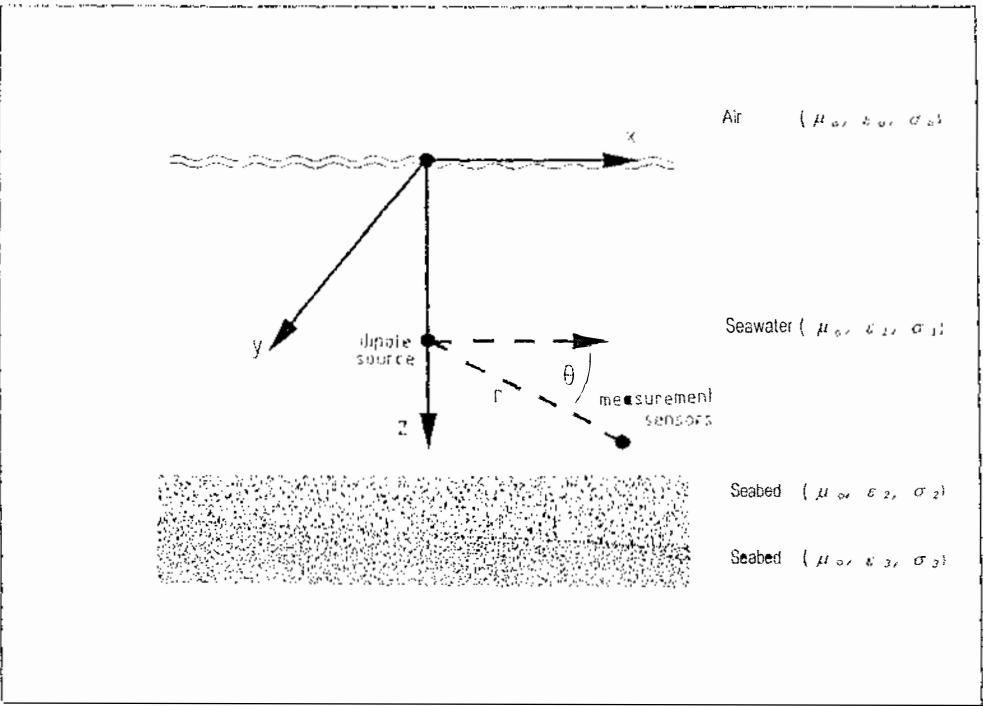


Figure 5.B Nomenclature for numerically modelling fields of dipole sources in three-layer conducting media with a sloping interface

which are shown in figure 3.B (on page 38). In this example the third or lower conducting medium was also assumed to have a uniform slope of 0.05 radians (or $\approx 2.8^\circ$). Again, it was necessary to use an extended set of environmental parameters to adequately describe the simulated environment. Details of the parameters are included in the figure.

Case I (sloping seabed surface) environment. Numerical values of the parameters used in the first simulated environment (with two conducting layers) were as follows:

Absolute permittivity $\epsilon = 8.8542 \cdot 10^{-12} \text{ F.m}^{-1}$	Permeability $\mu_o = 4\pi \cdot 10^{-7} \text{ H.m}^{-1}$
Relative permittivity $\epsilon_o = 1.0$	Conductivity $\sigma_o = 0.0 \text{ S.m}^{-1}$
Relative permittivity $\epsilon_1 = 72.0$	Conductivity $\sigma_1 = 4.0 \text{ S.m}^{-1}$
Relative permittivity $\epsilon_2 = 36.0$	Conductivity $\sigma_2 = 2.0 \text{ S.m}^{-1}$

Case II (sloping sub-bottom surface) environment. Numerical values of the parameters used in the second simulated environment (with three conducting layers) were as follows:

Absolute permittivity $\epsilon = 8.8542 \cdot 10^{-12} \text{ F.m}^{-1}$	Permeability $\mu_o = 4\pi \cdot 10^{-7} \text{ H.m}^{-1}$
Relative permittivity $\epsilon_o = 1.0$	Conductivity $\sigma_o = 0.0 \text{ S.m}^{-1}$
Relative permittivity $\epsilon_1 = 72.0$	Conductivity $\sigma_1 = 4.0 \text{ S.m}^{-1}$
Relative permittivity $\epsilon_2 = 18.0$	Conductivity $\sigma_2 = 1.0 \text{ S.m}^{-1}$
Relative permittivity $\epsilon_3 = 4.5$	Conductivity $\sigma_3 = 0.25 \text{ S.m}^{-1}$

5.2 Computer simulation of a sloping seabed (case I) configuration

Numerical model development. The computer simulations described in paragraph 3.2 (on pages 44 – 46) entailed analytical models and program executions times that were typically measurable in minutes. Similar programs for the numerical models described above were correctly forecast to run for several days on the project computer. For both computer simulations the actual execution times were to a large

extent determined by the number of iterations required during the processes of searching for appropriate root values of the seabed parameters.

Clearly, an important design aim for each numerical model was to minimize the program execution time, without compromising the accuracy of the solution. To this end non-uniform grids were used. In particular, higher densities of cells were used when field values varied at a very high rate with variations in ranges. Typically, these situations occurred near ELFE sources, and near the interfaces of dielectrics or conducting media of widely differing properties.

For the *case I* configuration, the homogeneous seabed with a slope of 0.05 radians (or $\approx 2.8^\circ$) was simulated by a terraced surface with steps 20 m wide, and 1 m deep. Also, for this model the horizontal electric field was generated by a unit, 50 Hz 5 m, HED source, situated 15 m below the surface in a water depth of 29 m, at that point.

Further, the horizontal field value was calculated between a pair of hypothetical sensors of 5 m spacing, with an axis parallel to the 5 m dipole. It was also assumed that these sensors were at a horizontal range of 65 m, and located 30 m below the surface in a water depth of 31 m, at that point. With this configuration there were two 1 m steps in the simulated seabed, each of breadth 20 m, between the source and sensors.

As with the earlier examples, the precise statement of the electromagnetic configuration has been described in terms of the specialist vocabulary of twelve character strings outlined in appendix C. Figure 5.C (on page 94) describes the subsequent specification for the numerical model.

Estimation of seabed parameters. Like the analytical models, almost all the terms of the numerical models were already known or could be readily measured during site trials. It was assumed that the strength I of the ELFE dipole could also be estimated or evaluated using techniques such as those described in section 4. In actual experiments the relative permittivity ϵ_3 and conductivity σ_3 , of the seabed would be unknown.

In the numerical model the seabed parameters ϵ_3 and σ_3 are subsumed within the following equation:

$$\epsilon^* = \epsilon_r - j(\sigma / \omega \epsilon_0) \quad (5.1)$$

where ϵ^* is the complex permittivity expression^{41,71} for the medium. Parameters ϵ_r and σ are the real part of the complex permittivity (also known as the relative permittivity or the dielectric constant) and the conductivity of the medium, respectively. Parameter ω is the radial frequency of the simulated ELFE signal.

Expressing the complex permittivity of the media for the *case I* configuration in terms of equation 5.1 leads to the following three complex expressions, which represent the simulated coastal environment encountered by a 50 Hz ELFE signal:

$$\begin{aligned} \epsilon_0^* &= 1.0 - j 0.0 && \text{(air medium)} \\ \epsilon_1^* &= 72.0 - j 1438006307.0 && \text{(seawater medium)} \\ \epsilon_2^* &= 36.0 - j 719003154.0 && \text{(seabed medium)} \end{aligned}$$

To prepare for the computer simulation, the electric field value at the sensor position was calculated using the hybrid numerical model and these environmental characteristics. The predicted value of the electric field at this location was (4.5207e-08, -5.3128e-09, 4.5518e-08) V.m⁻¹. For the computer simulation this voltage gradient may be considered as the equivalent of the measured field values referred to earlier in equations (3.3) and (3.4) (on pages 42 and 43). The number of reliable significant figures available from such calculations will invariably be influenced by the inherent machine accuracy.

An inspection of the complex permittivity expression ϵ_2^* above indicates that the imaginary component is more than seven orders of magnitude greater than the real component. Even with severe scaling of the real component it was obvious that the imaginary component of the complex permittivity expression ϵ_2^* would dominate an estimation process. And that even small variations in the imaginary component of ϵ_2^* during the process would mask large variations of the real term.

However, the justification for persisting with such an example was that off the west coast of New Zealand there are extensive seabed tracts containing magnetite and similar compounds of heavy metals. The complex permittivity of the seabed in these regions will differ markedly from the *case I* example, which can therefore be seen as a necessary precursor for investigating other electromagnetic configurations.

In the *case I* example, the object was to simulate a real experiment in which the only unknown term which required estimation to complete the model configuration, was the complex permittivity characteristic of the seabed ϵ_2^* . In essence the simulation process was required to estimate the roots of a univariate complex expression, namely ϵ_2^* .

Again, Muller's⁴⁷ iterative scheme of successive bisection and inverse parabolic interpolation described on page 43 – 44, was a technique which appeared to be robust and well suited to this class of problem. It was used in the following simulated experiment, which required a computer memory allocation of approximately 18 Mbytes.

For the computer simulation the initial (or seed) value of the complex permittivity expression ϵ_2^* was chosen to be identical to that of the seawater medium ϵ_1^* .

Numerical model results of sloping seabed configuration. Results of the computer simulation for the *case I* configuration are shown in figure 5.D (on page 94). A notable feature of the graph is that after seeding the optimization algorithm with a start-value of 4 S.m^{-1} , the actual value of the seabed conductivity (2 S.m^{-1}) was found within seven iterations of the search process.

As foreseen the search algorithm failed to identify a meaningful value for the real component of expression ϵ_2^* , namely the relative permittivity (or dielectric constant) of value 36.0. However, complementary experiments indicated that this inability was due entirely to the set of complex values of the seabed permittivity chosen for the *case I* example. It did not appear to be a systematic failure of the search technique selected for this computer simulation.

unit	1.0	m							
boundary	0	0	0	15	40	110			
celldim	0	15	5	x					
celldim	0	4	4	y	celldim	4	6	1	y
celldim	6	15	9	y	celldim	15	25	5	y
celldim	25	34	9	y	celldim	34	36	1	y
celldim	36	40	4	y					
celldim	0	20	20	z	celldim	20	30	5	z
celldim	30	110	20	z					
dielectric	0	0	0	15	5	110	1.0		-0.0
dielectric	0	5	0	15	34	110	72.0	-1438006307.0	
dielectric	0	34	0	15	35	50	36.0	-719003154.0	
dielectric	0	34	50	15	35	110	72.0	-1438006307.0	
dielectric	0	35	0	15	36	70	36.0	-719003154.0	
dielectric	0	35	70	15	36	110	72.0	-1438006307.0	
dielectric	0	36	0	15	40	110	36.0	-719003154.0	
isource	5	20	25	10	20	25	0.00005	x	0.200
output	0	35	30	15	35	110	x		dipole23.out

Figure 5.C Specification for the numerical model of a unit, 50 Hz 5 m, HED source in two-layer conducting media with a sloping interface

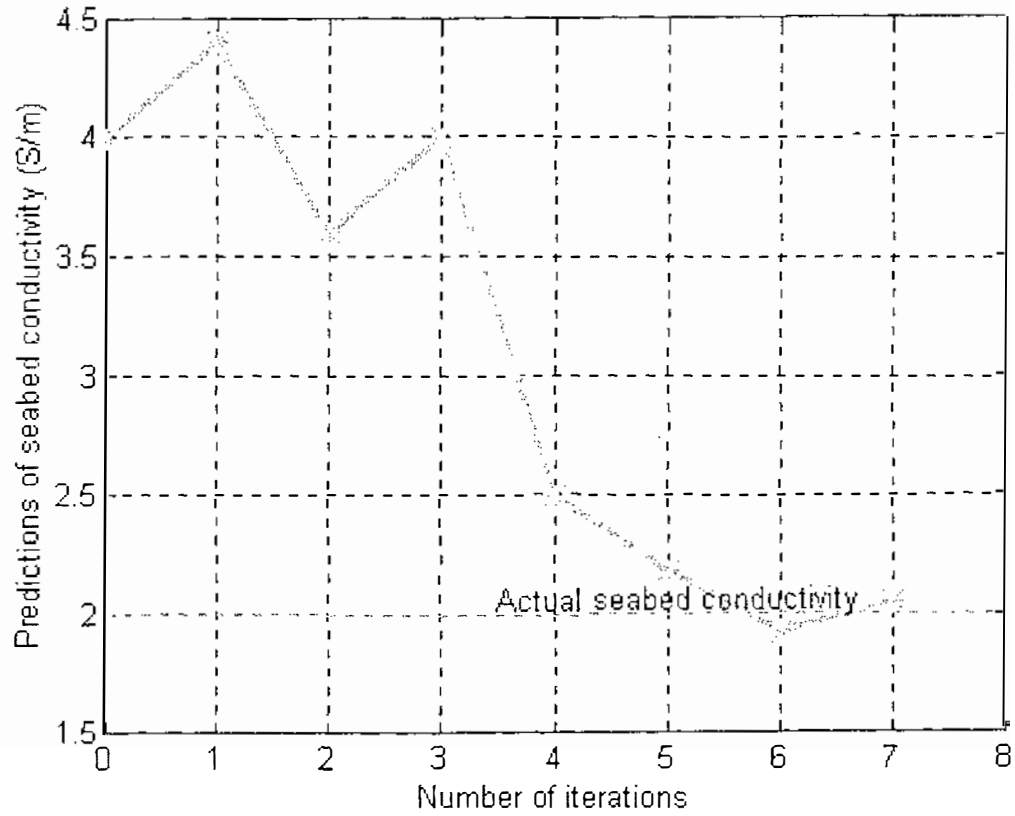


Figure 5.D Iterative simulated estimates of the seabed conductivity using a unit, 50 Hz 5 m, HED source in two-layer conducting media with a sloping interface

A less evident factor of the simulation is that the program execution times for typical *case I* configurations were approximately two days. Details of the necessary tradeoffs between the required accuracy for a numerical model and acceptable execution time of the computer simulation are discussed at length in section 6.

5.3 Computer simulation of a sloping sub-bottom (*case II*) configuration

Numerical model development. Like the earlier simulation that for the *case II* configuration aimed to minimize the program execution time without compromising the selected accuracy of the model solution. To this end higher densities of cells were used where field values varied at a very high rate with changes in ranges. Again, these situations occurred near ELFE sources, and near the boundaries of dielectrics or conducting media of widely differing properties.

For the *case II* configuration the upper layer of the seabed was assumed to have a level surface and a homogeneous composition defined by the parameters ϵ_2 and σ_2 (on page 90). Characteristics of the lower seabed layer were defined by the corresponding parameters ϵ_3 and σ_3 . Furthermore, for this configuration it was assumed that the boundary between the two homogeneous seabed layers formed a uniform slope of 0.05 radians (or $\approx 2.8^\circ$)

Also, for this configuration the electric field was generated by a unit, 50 Hz 5 m, HED source, situated 15 m below the surface of a seawater medium with a constant depth of 30 m. The horizontal field value for this computer simulation was calculated between a pair of hypothetical sensors of 5 m spacing, and an axis parallel to the 5 m dipole.

It was also assumed that these sensors were at a horizontal range of 45 m, and located 29 m below the seawater surface. The sloping interface between the layered seabed in this configuration was simulated by a terraced boundary with steps of breadth 20 m, and 1 m deep. As for the earlier examples of numerical modelling the precise statement of this configuration has been described in terms of the specialist

vocabulary of twelve character strings outlined in appendix C. Figure 5.E (on page 98) describes the specification for this numerical model.

Estimation of seabed parameters. Again, like the analytical models, almost all the terms of the numerical models were already known or could be readily measured during site trials. And again, it was assumed that the strength I of the ELFE dipole could also be estimated or evaluated using techniques such as those described in section 4. In actual experiments the relative permittivity parameters ϵ_2 and ϵ_3 and conductivity values σ_2 and σ_3 , of the seabed would be unknown.

Expressing the complex permittivity of the media for the *case II* configuration in terms of equation 5.1 leads to the following four complex expressions that represent the simulated coastal environment encountered by a 50 Hz ELFE signal:

$$\begin{aligned}\epsilon_0^* &= 1.0 - j 0.0 && \text{(air medium)} \\ \epsilon_1^* &= 72.0 - j 1438006307.0 && \text{(seawater medium)} \\ \epsilon_2^* &= 18.0 - j 359501577.0 && \text{(upper seabed medium)} \\ \epsilon_3^* &= 4.5 - j 89875394.0 && \text{(lower seabed medium)}\end{aligned}$$

To prepare for the computer simulation, the electric field at the sensor position was calculated using the hybrid numerical model and these environmental parameters. The predicted value of the electric field value at this location was $(-1.4268\text{e-}07, 1.4881\text{e-}08, 1.4346\text{e-}07) \text{ V.m}^{-1}$. Again, for the computer simulation this voltage gradient may be considered as the equivalent of the measured field values referred to earlier in equations (3.3) and (3.4) (on pages 42 and 43).

Results from the *case I* configuration showed that dielectric constants of the seabed layers were of little consequence in the numerical models of similar configurations. Moreover, there was no discernable difference in the predicted field values of the numerical model even after applying variations of $\pm 3000\%$ to the dielectric constant values (ϵ_2 and ϵ_3) of the two seabed layers. For this particular computer simulation it was thus appropriate to use only the two imaginary components of the

complex permittivity expressions (ε_2^* and ε_3^*) of the seabed layers, as the unknown environmental parameters.

In the *case II* example, the object therefore was to simulate a real experiment in which the only unknown terms which required estimation to complete the model configuration, were the imaginary terms of the seabed characteristics ε_2^* and ε_3^* . And the start-point of the estimation process was based on the premise that the upper and lower layers of the seabed had identical permittivity parameters. The seed value of seabed conductivity used to start the estimation process, was therefore the average value of the parameters σ_2 and σ_3 (on page 90), namely 0.625 S.m^{-1} .

Many processes were suitable to estimate the parameters σ_2 and σ_3 for the *case II* configuration. One such method was based on the minimization of a function of N variables by a direct search polytope algorithm^{72,73}. The (convex) polytope is defined to be the convex hull of a finite set of points, which is the generalization of a polygon in a plane, or a polyhedron in 3-D space. Polytopes of low dimensions were studied by the ancient Greeks, but the revived interest in the last fifty years is based on their importance in linear programming and game theory.

The polytope algorithm began with $n + 1$ points of the set $x_1, x_2, \dots, x_{n+1}$. At each iteration a new point was generated to replace the worse point x_j , which had the largest function value of the $n + 1$ points. New points were constructed according to specified criteria. This procedure was repeated until stopping criteria were satisfied.

The polytope algorithm used as the estimation process in evaluating the *case II* configuration required a computer memory allocation of approximately 20 Mbytes.

Numerical model results of sloping sub-bottom configuration. Results of the computer simulation for the *case II* configuration are shown in figure 5.F (on page 98). A feature of the solution is that after seeding the search algorithm with identical start-values (0.625 S.m^{-1}) for the parameters σ_2 and σ_3 , the actual conductivity values (1 S.m^{-1} and 0.25 S.m^{-1}) of the upper and lower seabed layers, were found within fourteen iterations.

unit	1.0	m							
boundary	0	0	0	15	40	90			
celldim	0	15	5	x					
celldim	0	4	4	y	celldim	4	6	1	y
celldim	6	15	9	y	celldim	15	25	5	y
celldim	25	34	9	y	celldim	34	38	1	y
celldim	38	40	2	y					
celldim	0	20	20	z	celldim	20	30	5	z
celldim	30	90	20	z					
dielectric	0	0	0	15	5	90	1.0		-0.0
dielectric	0	5	0	15	35	90	72.0	-1438006307.0	
dielectric	0	35	0	15	36	90	18.0	-359501576.8	
dielectric	0	36	0	15	37	30	4.5	-89875394.2	
dielectric	0	36	30	15	37	90	18.0	-359501576.8	
dielectric	0	37	0	15	38	50	4.5	-89875394.2	
dielectric	0	37	50	15	38	90	18.0	-359501576.8	
dielectric	0	38	0	15	40	90	4.5	-89875394.2	
isource	5	20	25	10	20	25	0.00005	x	0.200
output	0	34	30	15	34	90	x		dipole24.out

Figure 5.E Specification for the numerical model of a unit, 50 Hz 5 m, HED source in three-layer conducting media with a sloping interface

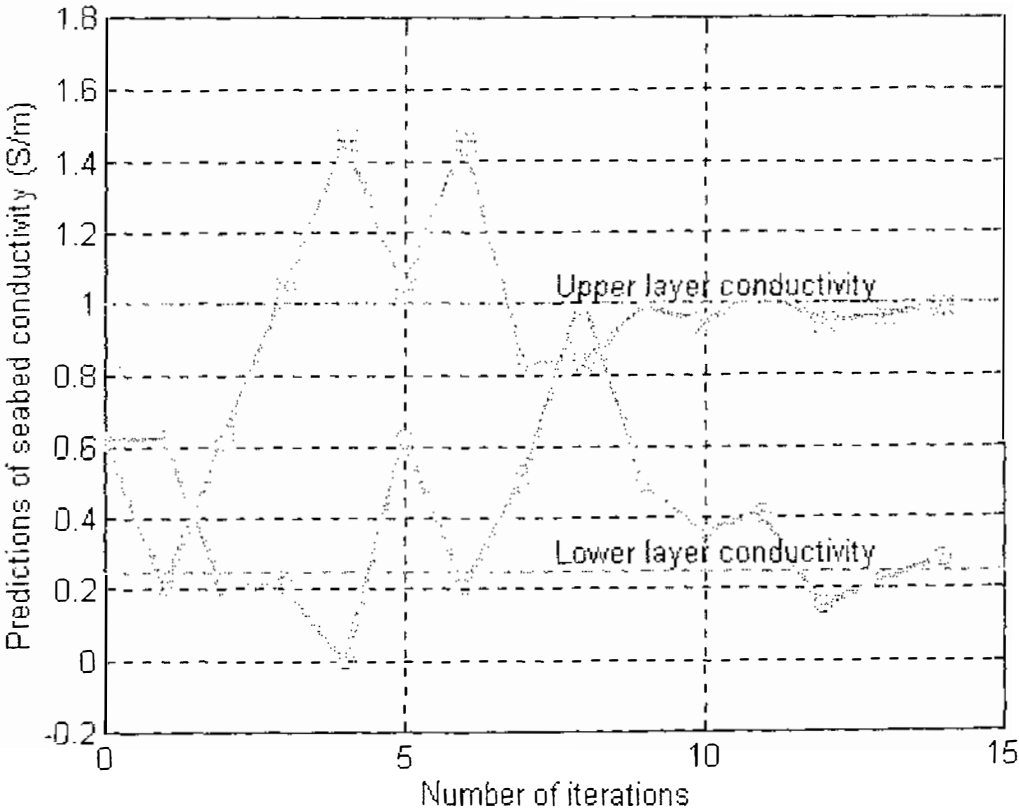


Figure 5.F Iterative simulated estimates of the seabed conductivities using a unit, 50 Hz 5 m, HED source in three-layer conducting media with a sloping interface

Since the search process used only function information at each step to determine a new approximate minimum, it would be inefficient for problems where derivative information can be included to assist the search processes. For this reason the algorithm is normally regarded as a robust rather than an efficient technique, and it is not necessarily the best choice for complementary computer simulations.

The fourteen iterations needed for the *case II* configurations typically required seven days of program execution time on the project computer. It was by far, the computer simulation of longest duration during the project. Section 6 contains a discussion of this issue.

5.4 ELFE source crossings of the sensor axis

Background. A stated objective in section 1 was to describe the shipboard ELFE sources in as much detail as possible. Ideally, the measurement data would include information about the location and the horizontal length of the onboard dipoles. Information of this type would be of value for identifying the possible origin of each source. Such insights would then enable means of source suppression to be found, or a countermeasure to be devised.

It was for these reasons that measurements of the electric field values were made when towed HED sources crossed the extended axis of the field sensors at normal incidence. The operation is shown in diagrammatic form in figure 5.G (on page 100). A typical smoothed record from Port Fitzroy, where the minimum horizontal range between the source and sensors was approximately 120 m, is also shown in figure 5.H (on page 100).

A noteworthy feature of the record was that the inverted cusp of the ELFE field occurred at the instant at which the centre of the towed source crossed the extended axis of the sensors. This relationship was established from the navigation records of the ship. It was verified by simulated experiments using ELFE point sources with the analytical model described in section 2. Similar inverted cusps in the records of ELFE signal occurred when the sensor axis was crossed on bearings other than that

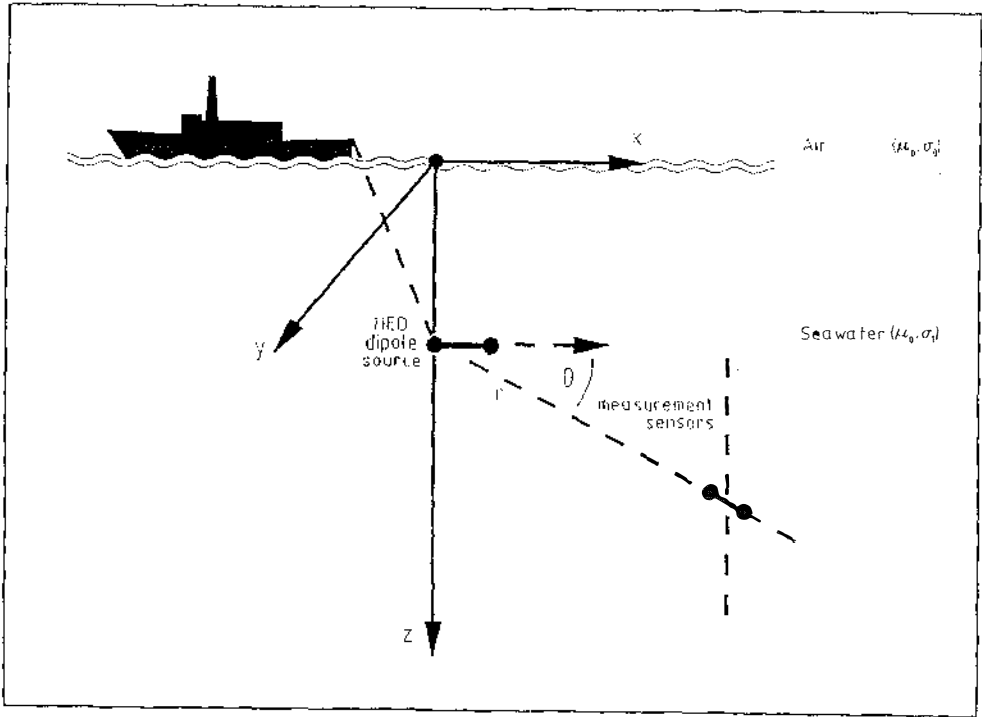


Figure 5.G Graphical representation of experiments in which a towed HED source crosses the extended axis of the electric field sensors, at normal incidence

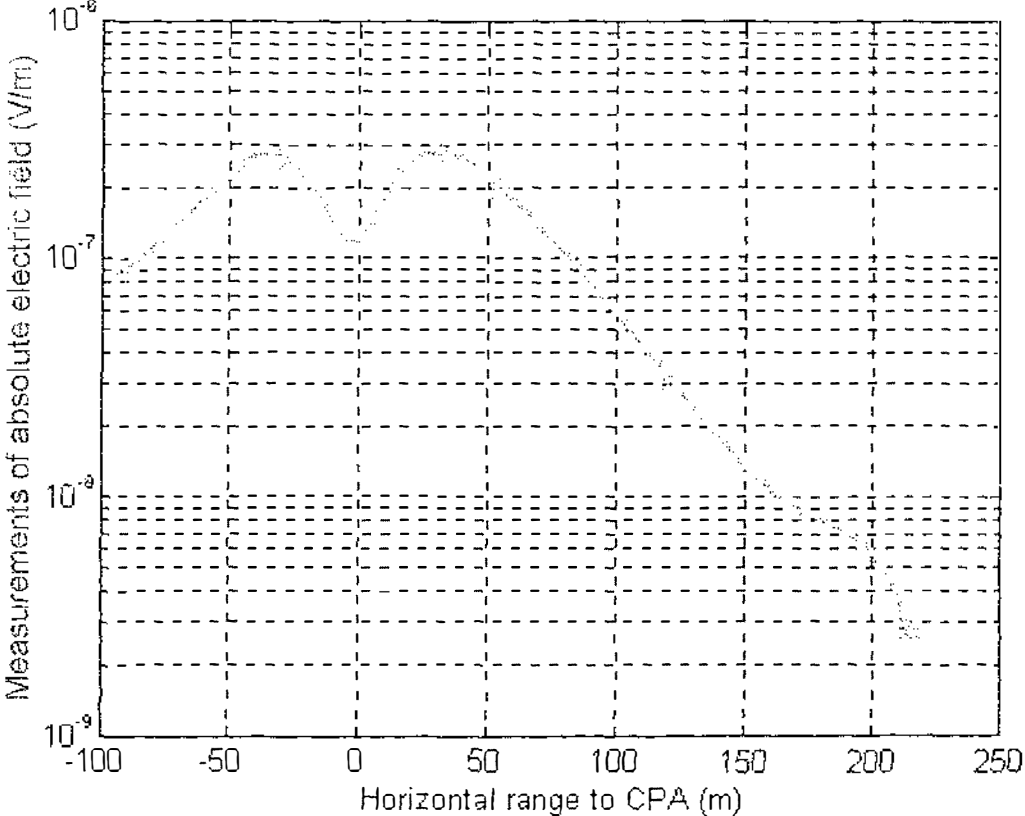


Figure 5.H Measurements of the electric field as a towed HED source crosses the extended axis of the field sensors, at normal incidence

of normal incidence. However, in these circumstances the momentary cusp of the ELFE signal did not necessarily coincide with the normal line from the centre of the ELFE source crossing the centroid of the pair of sensors. Such a relationship was sometimes precluded by the elongated radiation pattern from an ELFE dipole of finite length.

Results of three complementary computer simulations, using the numerical model with sources of finite length, are described in the following paragraphs. In each of the three simulated experiments a unit, 50 Hz HED source was assumed to cross the extended axis of the sensors at normal incidence, with a CPA of 25 m. Both source and sensors were assumed to be far below the surface of a semi-infinite seawater medium of uniform conductivity 4 S.m^{-1} . Predictions were made of the horizontal electric fields between the pair of sensors, as the sensor axis was crossed by ELFE dipoles with a finite length of 1, 4 and 16 m, respectively.

Specifications of the three numerical models with dipoles of length 1, 4 and 16 m, are shown in figure 5.I (on page 102), figure 5.K (on page 103) and figure 5.M (on page 104), respectively.

Numerical model results for dipole crossings of the extended sensor axis. The results from the three numerical models with ELFE dipoles of length 1, 4 and 16 m, are shown in figure 5.J (on page 102), figure 5.L (on page 103) and figure 5.N (on page 104), respectively.

A feature of the three figures is that on each side of the inverted cusps there are symmetrical peaks in the diagrams whose widths vary with the lengths of the hypothetical dipoles. And again, each inverted cusp denotes the instant at which the centre of the dipole crossed the extended axis of the sensors.

An implication of these results is that computer simulations of a measurement configuration provide the means to match the electric field of an onboard source, to that of a simulated source of the same finite length. In effect, a comparison technique can be used to estimate the characteristic length along the horizontal plane, of a shipboard ELFE source. Such information, together with measurements

unit	0.25 m						
boundary	0	0	0	40	160	216	
celldim	0	16	16	x			
celldim	16	24	4	x			
celldim	24	40	16	x			
celldim	0	78	26	y			
celldim	78	82	2	y			
celldim	82	160	26	y			
celldim	0	64	32	z			
celldim	64	72	4	z			
celldim	72	88	16	z			
celldim	88	216	32	z			
dielectric	0	0	0	40	160	216	72.0 -1438006307.0
isource	20	78	68	20	82	68	0.00005 y 1.000
output	0	80	72	40	80	216	x dipole25.out
output	20	0	72	20	160	216	y dipole25.out
output	20	0	72	20	160	216	z dipole25.out

Figure 5.I Specification for the numerical model of a unit, 50 Hz 1 m, HED source which crosses the extended axis of the electric field sensors, at normal incidence

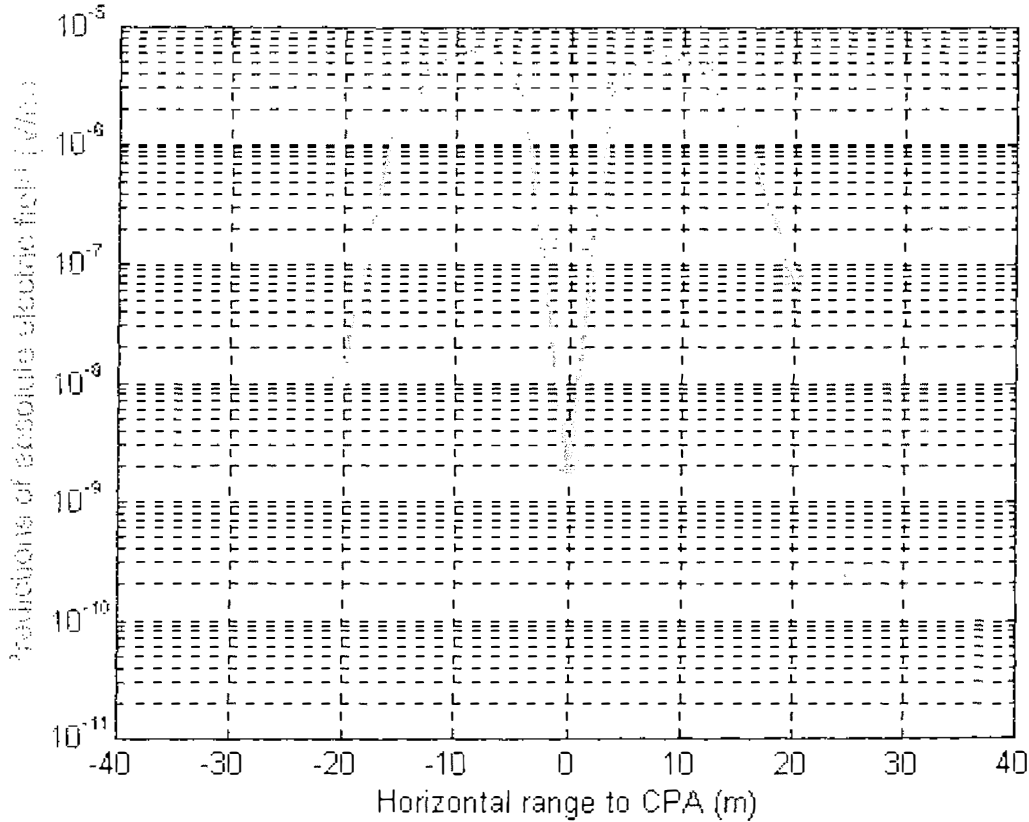


Figure 5.J Predicted values of the horizontal field of a unit, 50 Hz 1 m, HED source which crosses the extended axis of the electric field sensors, at normal incidence

unit	1.0	m							
boundary	0	0	0	10	40	54			
celldim	0	4	4	x					
celldim	4	6	1	x					
celldim	6	10	4	x					
celldim	0	16	8	y					
celldim	16	24	2	y					
celldim	24	40	8	y					
celldim	0	16	8	z					
celldim	16	18	1	z					
celldim	18	22	4	z					
celldim	22	54	8	z					
dielectric	0	0	0	10	40	54	72.0	-1438006307.0	
isource	5	18	17	5	22	17	0.00005	y	0.250
output	0	20	18	10	20	54	x	dipole16.out	
output	5	0	18	5	40	54	y	dipole16.out	
output	5	0	18	5	40	54	z	dipole16.out	

Figure 5.K Specification for the numerical model of a unit, 50 Hz 4 m, HED source which crosses the extended axis of the electric field sensors, at normal incidence

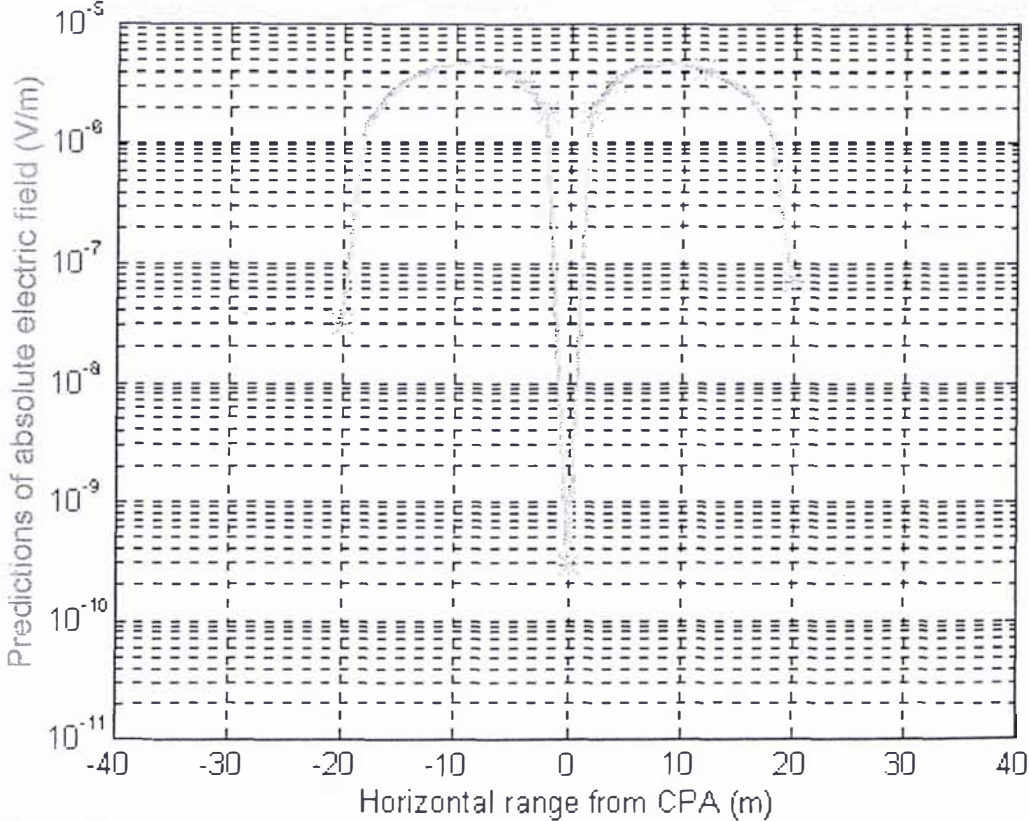


Figure 5.L Predicted values of the horizontal field of a unit, 50 Hz 4 m, HED source which crosses the extended axis of the electric field sensors, at normal incidence

unit	1.0	m							
boundary	0	0	0	10	64	54			
celldim	0	4	4	x					
celldim	4	6	1	x					
celldim	6	10	4	x					
celldim	0	64	8	y					
celldim	0	16	8	z					
celldim	16	18	1	z					
celldim	18	22	4	z					
celldim	22	54	8	z					
dielectric	0	0	0	10	64	54	72.0	-1438006307.0	
isource	5	24	17	5	40	17	0.00005	y	0.06667
output	0	32	18	10	32	54	x	dipole17.out	
output	5	0	18	5	64	54	y	dipole17.out	
output	5	0	18	5	64	54	z	dipole17.out	

Figure 5.M Specification for the numerical model of a unit, 50 Hz 16m, HED source which crosses the extended axis of the electric field sensors, at normal incidence

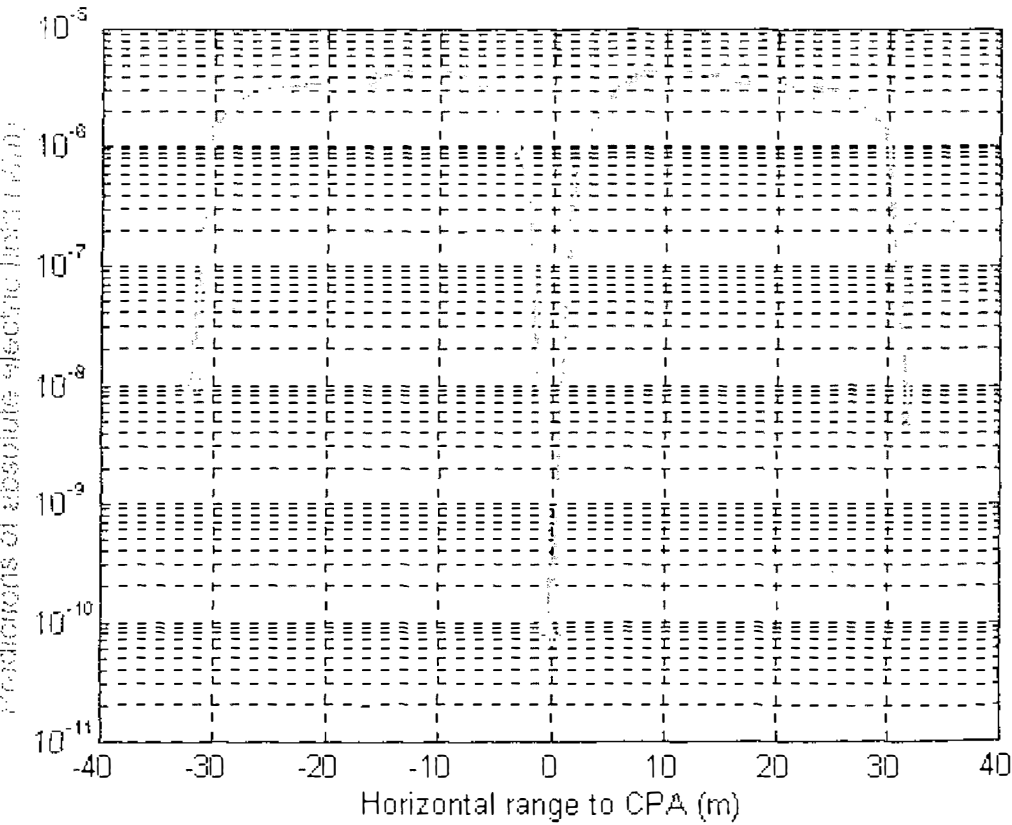


Figure 5.N Predicted values of the horizontal field of a unit, 50 Hz 16m, HED source which crosses the extended axis of the electric field sensors, at normal incidence

about the centre of the dipole, would be of benefit in locating and identifying onboard sources.

Nevertheless, practical issues suggest such comparison techniques need to be applied with caution. For instance, a sloping seabed may cause an asymmetric electric field about the axis of the sensors. A similar situation may arise if an onboard ELFE source is not symmetric about the longitudinal axis of the vessel. In both instances it would be necessary to duplicate the electric field measurements along a reciprocal bearing, and average the two estimates of source location to establish the correct shipboard position.

5.5 Estimation of the dipole strength of onboard ELFE sources

Introduction. In earlier sections numerical models and optimization algorithms were used to simulate the process for obtaining estimates of key environmental parameters. This information made it feasible to then estimate the various signal propagation losses between electric field sensors and ELFE sources at known locations throughout the test sites.

A subsequent series of computer simulations, again using a numerical model, demonstrated methods to establish the characteristic horizontal length and dipole centre, of an onboard ELFE source. These data can be collated to achieve the primary objective of the study, namely, that of characterizing the ELFE sources within maritime vessels during normal operations in typical coastal environments.

Onboard ELFE sources. The design of the numerical model required the dimensions of both an ELFE source and the system of electric field sensors. Similar dimensional information was required for the site configuration and the appropriate environment parameters, such as the seawater depth.

Given this information it is then possible to apply the numerical model to predict the electric field values at selected locations in the trial site. Alternatively, given the actual measurements of the electric field values it is possible to apply the reverse

process and estimate first the signal propagation loss, and then the strength of onboard ELFE sources that produced these field values.

In essence, the procedure for estimating the dipole strength of the onboard sources was identical to that used to calculate the dipole moment of the controlled ELFE sources. Estimates of the latter sources are described in figure 13 (on page 146) of appendix B.

5.6 Observations concerning the simulated experiments

Complex permittivity of the seabed. When an external time-varying electric field is applied to material bodies there may be small displacements of bound charges, which give rise to a volume density of polarization. This polarization vector will vary with the same frequency as that of the applied field.

As the frequency increases the inertia of the charged particles tends to prevent the particle displacements from remaining in phase with the field changes. This feature in turn leads to a frictional damping mechanism that causes a power loss because work must be done to overcome the damping forces. The phenomena of out-of-phase polarization can be characterised by a complex electric susceptibility, and hence a complex permittivity parameter^{41,71} which is defined in equation 5.1.

If, in addition the material body or medium has an appreciable amount of free charge carriers such as electrons in a conductor or the ions in an electrolyte, there will be ohmic losses. In analysing such media it is customary to include the effects of both damping and ohmic losses in the imaginary part of the complex permittivity.

Clearly, for the example in paragraph 5.2 (*case I*) the relative permittivity (or dielectric constant) of the conducting media was of little significance at the specified frequencies. But at higher frequencies, or in media with a higher density of magnetite or similar minerals, there remains a need to estimate both components of the complex permittivity parameter for the seabed material.

Therefore, in the longer term there will be a requirement to correct the earlier inability to provide an estimated value of the relative permittivity (or dielectric constant) ϵ_2 , in configurations similar to the *case I* example.

Limitations imposed by computer memory allocations. Sections 5.2 and 5.3 described configurations that included a sloping seabed (*case I*) and a sloping sub-bottom (*case II*). It is arguable that a more representative configuration of the coastal areas around New Zealand would have combined elements of the two examples. For instance, a single generalized configuration could have included a sloping seabed at a particular oblique angle, and a sub-bottom that sloped at some different oblique angle.

Similar comments could be made concerning the media interfaces in *case I* and *case II* where slopes of 0.05 radians (or $\approx 2.8^\circ$) were represented by terraced surfaces with steps of 20 m breadth, and depth 1 m. An improved configuration would have used for example, steps of 10 m breadth, with a depth of 0.5 m.

In each of the above variations the unknown environmental parameters (σ_2 and σ_3) would remain unchanged. But the more detailed grid systems would have required a considerable increase in the computer memory allocation required for the corresponding numerical model. As will be demonstrated later there is a close relationship between the computer memory allocation needed for a specific numerical model, and the required program execution time

Apart from one significant exception (for *case II* on page 84), the computer memory allocation for numerical models was limited throughout the project to about 35 Mbytes. A useful rule-of-thumb was that such a model would incur a program execution time of about twenty-four hours.

Limitations imposed by program execution times. The model enhancements described above entail no increases in the number of unknown variables to be estimated by the numerical models. Nevertheless, all the necessary increases in

computer memory allocations would in turn lead to substantial increases in the program execution times.

It is regarded as a sound design practice in numerical models to provide higher node densities in regions where electric field values change at a high rate with variation in ranges. Typically, these situations occur at field points close to either an ELFE source or at the boundaries of dissimilar media. In these circumstances it may be an option to utilize a non-uniform grid, at the expense of an increase in the memory allocation required for the model.

Within this study each of the numerical models that entailed optimization processes were designed so that the program execution times on the project computer were limited to 7 ~ 10 days. As a consequence the model enhancements described above remain issues for the future.

6. Summary, conclusions and future issues

6.1 Summary

Study objectives. The primary objective in this study was to investigate methods to characterise the ELFE sources within maritime vessels during normal operations in typical coastal environments. For this purpose it was appropriate to focus on situations where the sources and field measurement points were both located in the seawater volume. At the electromagnetic frequencies of interest such a medium acts as a thin conducting layer with significant levels of wave reflection and refraction at the media boundaries.

To characterize the sources an important prerequisite was to enhance the wave propagation models of ELFE fields over short ranges. As a consequence the initial investigations were directed towards evaluating key parameters of the conducting media in shallow-water conditions.

It was anticipated that the study would need to utilize both analytical and numerical models of the electromagnetic fields. An essential component of the study was to therefore examine the efficacy of each technique for modelling the propagation of ELFE signals in typical coastal environments.

Coastal environments and supporting measurements. An inescapable feature of the remote coastal areas that were used as trial sites was that there was minimal environmental information available prior to the measurement programme.

Such shortcomings were significant as coastal areas usually have highly variable undersea topographies and seabed properties. It is a characteristic due largely to local tidal flows, prevailing current-patterns and the runoff from the surrounding terrain. Aeons of erosion from coastal landforms was also likely to have produced large variations in the density and conductivity of sub-bottom layers, within both vertical and lateral sections.

Environmental factors of importance included the seawater depth, and the relative permittivity and conductivity of this medium. Similar information was required for the seabed, especially for the uppermost layers. Unfortunately, the measurement equipment that could be operated by divers was poorly suited to providing information about these features. Almost all the necessary environmental data was obtained from specialized onboard facilities during the measurement programme.

Multi-influence sensor system. The key component of the study was a sensor system, suitable for deployment on the seabed, and able to measure the influence fields radiated from maritime vessels. Although measurements of electric fields were of particular interest, information was also available from acoustic, pressure and three-axis magnetic sensors.

The equipment was modular, transportable and easily deployed in sheltered waters. Optimum sensor performance was obtained when the seafloor was relatively flat and at a water depth of 15 – 25 m. All sensors were mounted on an underwater frame. An electric cable provided the essential link to a surface platform containing power and data recording facilities.

Like the sensors, the associated preamplifier components were also mounted on the underwater frame. Wherever possible non-magnetic and non-conducting materials were used to minimize any self-induced perturbations in the magnetic and electric fields.

The multi-influence fields were measured using commercial devices linked to specialized electronics facilities. Sensors were chosen which could record the static and dynamic characteristics associated both with ship influences and with the ambient environmental fields. Standard design features included the use of validation test signals and of dynamic, range-compression techniques.

ELFE sources. Controlled ELFE sources were necessary to generate E-fields for testing the various sensor systems. These sources also supported the processes of characterizing the measurement sites and provided data to verify models for each of

the influence fields. Such sources were essential because E-field signatures of actual shipboard equipment were not available for the study.

A catamaran-hulled configuration was selected as the most suitable platform for the various sources. Again, to simplify the operation and modelling of the magnetic and electric field sources only non-magnetic and non-conductive materials were used in the construction. Deployment depths of the sources were selectable in the range 0.5 - 2.5 m below the surface.

Some of the measurements described in the study implied a prior knowledge of the dipole strength of the towed ELFE sources. Nevertheless, it was recognized that situations could arise where calibrated sources were not available. Furthermore, in some circumstances it would be difficult to estimate the dipole strength because of shell-shaped electrodes, a short overall length or similar impediments.

Techniques capable of providing information about signal propagation losses, and hence source calibration data, were described in the study. Where an analytical model was used during this process it was necessary that the source-sensor separation was sufficient for the current dipole to be regarded as a point source. Such a precondition was unnecessary for numerical models.

Analytical models of ELFE wave propagation. The choice of analytical model was influenced by the experiments planned in the coastal environments. Because of the nature of the trial areas an ability to include configurations with at least two conducting layers was clearly necessary. Additionally, for this study it was necessary to have available analytical models that could deal with both VED and HED sources in a seamless and consistent manner.

Although the extension of the analytical models to include a two-layer conducting half-space is relatively straightforward it is algebraically much more complex. Perhaps this is the reason it has received relatively little attention. Moreover, researchers who have addressed the issue have often attempted to simplify the computations at the expense of adding constraints to the applicability of their algorithms.

If the same analytical models could also embody point source and finite-length dipoles, such a capability would be highly advantageous. Lastly, in this virtual wish list, it was desirable that the analytical models be capable of including horizontal and oblique boundaries between the various layers. Such facilities would enable the model configurations to resemble more accurately the conditions encountered in real-world experiments.

There are at present no such analytical models capable of including all of the above characteristics. However, a series of basic equations that were developed in Canada over thirty years ago have many of the necessary features. It is from this work that analytical models within the study were largely developed.

Numerical models of ELFE wave propagation. Numerical techniques generally require more computation than analytical methods but they are more flexible and powerful analysis tools. Moreover, such techniques analyze the entire configuration geometry provided as input data, without making a priori assumptions about which field interactions are most significant. In addition they provide the numerical solutions to problems based on full-wave analyses.

There are many well-proven computer programs that utilise numerical models for the analysis of electromagnetic configurations. Each may be conveniently characterised by the specific technique used to solve the complex equations associated with electromagnetic field analyses.

Tests with several of the basic types of numerical models indicated that a recent hybrid model would suit the study requirements. Hybrid programs take advantage of the strengths of each numerical technique in order to solve problems where each alone is unable to provide an efficient model. For this study the most suitable option was a numerical model based on hybrid method of moments (MOM)/finite element (FEM) techniques.

The development process generally followed the procedures described in the earlier references within section 4, but with some adaptations for this project. The

theoretical framework owed much to the group that has been working together for the past ten years or so at the Electromagnetic Compatibility Laboratory of the University of Missouri, Rolla.

Estimation of the key environmental parameters. Several techniques to estimate the key environmental parameters were developed, simulated and utilized. The first method was suitable for analytical models. It required measurements of orthogonal electric fields associated with a single-frequency electromagnetic signal. However, the present configuration of the multi-influence sensor system, with a single pair of electric field sensors, precluded the use of this method during the site trials.

An alternative technique required measurements of the radial electric field associated with a dual-frequency electromagnetic signal, where the relative amplitudes of each spectral component were known. This latter technique was well suited to analytical models, and it was used extensively during the study.

Estimation of the dipole strength of unknown ELFE sources. Numerical and analytical models were combined with optimization algorithms in computer simulations to obtain estimates of key environmental parameters. Analytical models were subsequently used to establish the corresponding parameters at the test sites. This information made it feasible to then estimate the various signal propagation losses between electric field sensors and ELFE sources at known locations.

The design of a numerical model required the dimensions of both an ELFE source and the system of electric field sensors. A series of computer simulations demonstrated methods to establish the characteristic horizontal length and dipole centre, of an onboard ELFE source. In the normal course complementary information would be available concerning the site configuration and the appropriate environment parameters, such as the seawater depth.

Given this information it was then possible to apply the numerical model to predict the electric field values at selected locations in the trial site. Alternatively, given actual measurements of electric fields it is a simple operation to apply the reverse

process to estimate the dipole strength of onboard ELFE sources that produced these field values.

Performance limits of the project computer. The project computer was purchased at the beginning of the study in mid-1997. By the standards of the time it was good quality equipment of adequate performance for any of the candidate analytical models. In the course of the project all such models were evaluated in less than two minutes. A brief specification for the machine is set out in figure 6.A (on page 115).

Design processes for numerical models of electromagnetic wave propagation imposed additional constraints. Such a state occurred because the design of numerical models was invariably planned to minimise the program execution time without compromising the selected level of accuracy of the solution.

For instance, it was regarded as a sound design practice in numerical models to provide higher node densities in regions where electric field values change at a high rate with variations in ranges. Typically, these states occur at field points close to either an ELFE source or at the boundaries of dissimilar media. In these circumstances it is possible to utilize a non-uniform grid, at the expense of an increase in the memory allocation required for the model. An unavoidable consequence of this choice would be an increase in the program execution time.

An example of the essential tradeoff between model performance and execution time is shown in diagram form in figure 6.B (on page 115). Variations in execution time at a specific memory allocation arise because different model designs are not all equally efficient. Moreover, the numerical model was planned to degrade, gracefully. That is to say, if a planned level of accuracy was unattainable the model selected a best possible endpoint.

Within this study each of the numerical models which include iterative processes, were designed so that the program execution times on the project computer were limited to 7 – 10 days. As a consequence the model enhancements described above remain issues for the future.

Product	: Compaq PRESARIO, model 4170
Processor	: Pentium (R) / MMX at 166/200 MHz
Total physical memory	: 56 Mbytes, EDO RAM
Storage	: 2.1 + 1.6 + 1.6 Gbytes
Operating systems	: Microsoft DOS, version 7.0 Red Hat LINUX, version 5.0 Microsoft WINDOWS 95, version 4.0
Year of manufacture	: 1997

Figure 6.A Brief specification of project computer

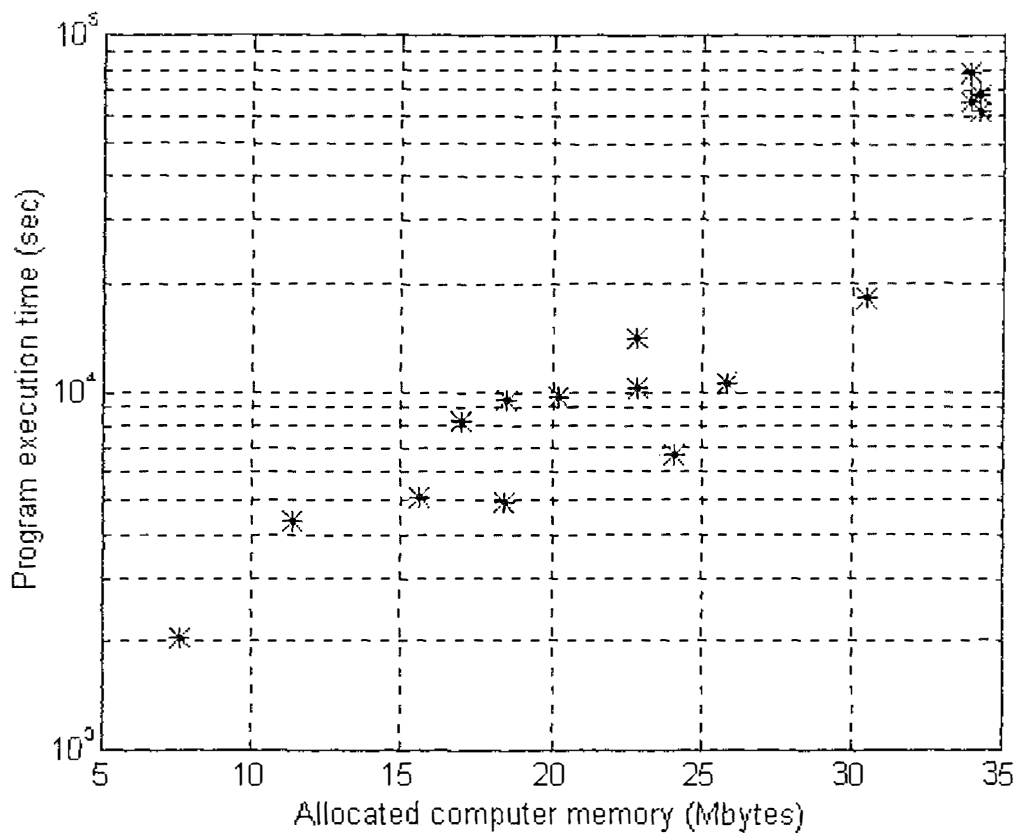


Figure 6.B Relationship between memory allocations required for numerical models and the approximate program execution times

6.2 Conclusions

Study milestones. An overview of the project can be obtained by comparing the earlier study objectives with the milestones listed below. Key findings from this project were as follows:

- a. Narrow-band spectral analyses of records from the multi-influence sensor system were used to successfully detect shipboard and controlled ELFE sources within several hundred metres of the sensors, against typical backgrounds of ambient noise;
- b. Close agreement was shown between the estimated values of electric field values from analytical and numerical models, when each was used to predict ELFE wave propagation within identical configurations;
- c. The analytical model was successfully used to characterize the seabed conductivity parameter within computer simulations, and in trial experiments where the configuration included two conducting layers with a horizontal boundary;
- d. Information describing the characteristic conductivity of the seabed at the Tennyson Inlet site was first assembled into contours, and then mapped. Tests of the mapped data indicated that a satisfactory level of accuracy had been achieved;
- e. The hybrid numerical model was successfully used to characterize the seabed conductivity parameters in computer simulations, where the configurations included two or three conducting layers with sloping interfaces;
- f. Techniques based on analytical models were used in several computer simulations to demonstrate means to calibrate controlled ELFE sources, provided the sensor-source separation was sufficient for the source to be regarded as a point dipole. One such technique, which required prior

knowledge of the relative amplitudes of a multi-frequency ELFE signal, was used successfully in experiments at the trial sites;

- g. Computer simulations using the hybrid numerical model demonstrated that the real component (dielectric constant) of the complex permittivity parameter at each test site was of little significance, at the extremely low frequencies of interest. Specifically, the imaginary component of the parameter, that included both damping and ohmic factors, exceeded the real component by more than seven orders of magnitude;
- h. Experiments and computer simulations using the analytical model demonstrated an accurate method for locating the centre of a point source that crossed the extended axis of the field sensors, at normal incidence. Similar results were obtained from computer simulations using the numerical model, where the dipole sources were of finite length. Additional simulations demonstrated a technique to estimate the characteristic horizontal length of an onboard ELFE source.

Broadband ELFE sources. Although the preceding analyses and computer simulations have focussed on narrow-band signals, these are not necessarily the dominant elements of the onboard ELFE sources. Indeed, in some circumstances it may be difficult to distinguish a narrow-band component.

Nearly all the analysis methods that have been developed in this study can, nevertheless, be used with broadband sources. For these sources it would first be necessary to divide the measured ELFE signal into say, 1/3-octave bands. All subsequent wave propagation losses could be estimated with reasonable accuracy, on the basis of the centre-frequency of each 1/3-octave band. Additionally, these propagation losses could be used with electric field measurements over the same bands, to obtain estimates of the strength of the onboard sources in each 1/3-octave band.

2-dimensional and 3-dimensional systems of electric field sensors. During the trials programme the configuration of the multi-influence sensor system was such that

electric field measurements were available only along a single horizontal axis. Measurements from this configuration were adequate to successfully characterize sources with a dominant horizontal component. In other circumstances the single-axis data would be inadequate.

Moreover, it is likely that some onboard ELFE sources may be subject to significant levels of amplitude modulation. Analyses of these sources and others with a strong vertical component would require data from field sensors along two or three axes. An enhanced sensor configuration would also require augmented facilities for data recording. However, the analytical and numerical models used in the study are suitable for extended analyses involving 2-axis or 3-axis field data.

Selection of measurement sites. Early in the study it was appropriate to set out the criteria that influenced the selection of the test sites at Port Fitzroy and Tennyson Inlet. Among the primary considerations was the requirement for seabed areas that were uniform and level.

Analyses using the numerical models, especially those involving a sloping boundary in the conducting media, indicate that in future such considerations can be set aside. Future selections of measurement sites are likely to be determined by the logistic advantages and the need for minimal maritime traffic in the candidate area. A precondition is however, that some technical issues which impact on the program execution time of the hybrid numerical models can be successfully resolved.

6.3 Future issues

Computer-chip processors. The chip processor embedded in the project computer has clearly had an impact on the program execution of the numerical models. The relationship is such that an increase in the processor clock speed would provide a consequential reduction in program execution time.

In mid-1999 a product announcement by the IBM Corp. forecast the development of computer-chip processors with clock speeds in the range 3.3 – 4.5 GHz. Similar

developments were projected a few months later by the INTEL Corp. in announcing the Pentium 4 chip, for which clock speeds of 10 GHz are planned. When these devices reach the component markets the program execution times of the numerical models will be greatly reduced.

Parallel-processing computer facilities. Research laboratories in universities and industry are presently investing considerable resources to enable the computing resources of many disparate computers to be linked, on demand, and operated simultaneously to accomplish large computational tasks.

A representative large-scale task is that of modelling the virtual Yucca Mountain⁷⁴. The actual mountain is the only candidate site for a national repository of high-level nuclear waste in the United States. Present plans are for the virtual mountain to be represented by a numerical model of 40 million cells each defined by 120 parameters.

It has been proposed that the numerical model would be run on 1400 parallel microprocessors controlled by the Blue Pacific supercomputer. Such developments, like those of the computer-chip processors, are expected to eventually have major impacts on the performance of the hybrid models described in the study.

Magnetic field components of ELFE sources. The extensive applications of the Weaver⁴⁵ analytical model and the finite-difference, time-domain (FDTM) model, are due partly to ability of each to provide estimates of both the electric and magnetic field components of an electromagnetic source. In the case of the FDTM model, two separate grid configurations are used to calculate the electric and magnetic fields.

In the longer term, it would be highly advantageous if similar facilities were to be added to the hybrid numerical model. Such an enhancement would extend the applications of the model. It would also, with shorter program execution times provided by the above developments, enable the remaining differences between predictions from analytical and numerical models to be more closely examined, and eventually resolved.

Volume sub-division errors of the numerical model. Although close agreement was obtained between analytical and numerical models with common configurations, there were some unresolved issues. In particular, it was not possible to ascertain if differences in predicted field values were due solely to errors arising from the volume sub-divisions within the numerical model. It is a topic that requires future investigation with enhanced computational resources.

Appendix A: Estimating conductivity parameters for extremely low-frequency electromagnetic models

(Appendix A contains a reprint of a paper²⁴ presented to the 20th meeting
of panel GTP-13 within The Technical Cooperation Program (TTCP),
in Victoria BC, Canada during October, 1994)

Estimating conductivity parameters for extremely low-frequency electromagnetic models

E I Rumball

Defence Scientific Establishment
Auckland, New Zealand

Abstract

Mine countermeasures tasks sometimes require knowledge of the conductivity parameters of the seawater medium and the seabed for use in extremely low-frequency electromagnetic (ELFE) models. Direct measurements of the seabed parameter are costly or difficult to obtain. Moreover, the subsequent measurements are often highly variable. This paper describes how ELFE theory may be implemented to provide the requisite data using a remote-sensing technique. Analysis results from several measurement scenarios are simulated and described.

Introduction

The extremely low-frequency electromagnetic (ELFE) models used in mine countermeasures (MCM) tasks simplify the geotechnical descriptions of the media. Usually these are represented as isotropic, two-layer, conducting half-spaces. The significant environmental parameters of such models are shown in figure 1. Here μ and σ denote the permeability and conductivity characteristics of each medium.

Reference 1 describes a typical mathematical model for calculating the electromagnetic fields in such an environment. This analytical model was subsequently converted to a FORTRAN source code by staff at the Defence Scientific Establishment (DSE).

Seawater conductivity values typically lie within the range 3 to 5 S.m^{-1} ($\text{S} \equiv \text{mho}$). Over practical ranges of seawater values it is a slowly varying function of temperature, salinity, the frequency of the propagating electromagnetic wave and pressure, in that order of importance. It can be readily obtained from a range of onboard conductivity-temperature-density (CTD) instruments.

Direct resistivity measurements (or its reciprocal, conductivity) of the seabed have long been used in geophysical surveying and offshore morphology studies. The instrumentation usually comprises electrical logging tools or diver-held electrode probe systems that are implanted in the seabed. Data from the logging tools may be obtained in the course of an offshore drilling programme. The information obtained from the probe technique is also costly to obtain, and records² often exhibit the high degrees of variability shown in figure 2.

In the last decade studies of electromagnetic propagation through the seabed have developed to become a practical tool for geophysical exploration³. These studies used various configurations of electric dipole sources and electric or magnetic field sensors⁴, deployed in water depths of several thousand metres. Most appear to use seabed to seabed propagation modes. The results indicate that the uppermost sediment layer is rarely more than a few hundred metres thick, and within this layer the conductivity varies between 0.01 to 2.5 S.m^{-1} . Such variations are mainly due to differing degrees of porosity.

The above deep-water models differ significantly from the short range, shallow-water models used for MCM investigations. Reference 5 is an exception. It postulates that seabed conductivity may be determined over long ranges by measuring only the horizontal electric field components produced by a long horizontal magnetic dipole at the surface. For this model it is necessary that the

horizontal range is much greater than the skin depth, and that “transmission line analogy” can be applied.

The DSE requirements to use such techniques are firstly, the technique must be inexpensive. Ideally, it should not require the use of RNZN divers or specialized electrode probe equipment. It must be applicable to shallow-water areas and the results must be useable in models that encompass isotropic, two-layer conducting half-spaces. Finally, it is preferable that the technique should be independent of the physical size and strength of the ELFE source, since DSE does not have either a small, or a calibrated, ELFE source.

Some researchers have noted that earlier magnetic field measurements made in shallow water have been contaminated by noise. This noise characteristic was attributed to mechanical motion or the movement of the seawater medium within the sensor structure. DSE has low-sensitivity sensors only with which to measure the magnetic components. For this project the preference was to use electric field sensors when low-noise measurements are required; and this is the case when the horizontal ranges exceed several skin depths.

Theoretical basis of the technique

Given the coordinate system described in figure 1, the electric and magnetic field values at a general point in the upper conducting layer due to an electric point-dipole at any point in the same layer, may be calculated by using the Weaver model¹. The axis of the electric dipole may be horizontal or vertical. Specifically, the non-dimensional and frequency-independent, radial and vertical electric fields due to a vertical electric dipole (VED) are:

$$E_r' = \sum_{n=-\infty}^{\infty} \{ F_2'(r_n', \sin(2\psi_n)) - F_2'(R_n', \sin(2\Psi_n)) \} + \frac{\partial^2 Q}{\partial r' \partial z'} \quad (1)$$

$$E_z' = \sum_{n=1}^{\infty} \{F_3'(r_n', \sin(\psi_n)) - F_3'(R_n', \sin(\Psi_n))\} + Q_z \quad (2)$$

Equations (1) and (2) above correspond to equations (36) and (37) in reference 1. These are the key equations in the remainder of this paper. The notations in (1) and (2) are described in equations (32), (33), (38), (40), and (41) in reference 1, and are therefore not repeated here.

Figures 3 and 4 show the non-dimensional values of $|E_r'|$ and $|E_z'|$ at short horizontal ranges, where the latter are expressed in skin depths. At a frequency of 5 Hz (for the selected sea water conductivity value of 4 S m^{-1}) the skin depth is 112.54 m. In each figure the water depth d is also one skin depth, and the vertical electric point-dipole is at the mid-water depth h . The nine curves in each figure show the estimated amplitudes of the non-dimensional electric fields when the electric field sensors are located at the surface ($z = 0$), at the mid-water depth ($z = h$) and at the seabed ($z = d$). In each receiver position the calculation is made for ratios of the {seabed conductivity/seawater conductivity} (eps), corresponding to 0.0, 2.0 and 4.0.

When the electric field sensors are on the seabed ($z = d$) it is clear from observations of figures 3 and 4, that the ratio $|E_r'| / |E_z'|$ will have a finite value at most horizontal ranges. This characteristic will apply when the {seabed conductivity/seawater conductivity ratios} (eps), are such that $0 < (\text{eps}) \leq 1$.

In this paper the estimated value of the sea bed conductivity is found by solving the equation:

$$|E_r'| / |E_z'| - |E_{r,M}'| / |E_{z,M}'| = 0 \quad (3)$$

where $|E_{r,M}'| / |E_{z,M}'|$ is the ratio of the measured electric field components at a trial site. Equation (3) is thus independent of the source strength of the vertical

electric dipole (VED). Since equation (3) utilises only those electric field components of a VED source, it is more easily calculated than similar methods that require the evaluation of horizontal dipole sources^{1,4}.

At a specific trial site the water depth, axial alignments, and the position coordinates of the receiver and source will be known. Also, subject to the qualifications described later, the permeability of the seabed would be known. The sole unknown parameter in equation (3) is the conductivity value for the seabed.

Although the expression $|E'_r| / |E'_z|$ has real and imaginary parts, the root of the conductivity parameter in equation (3) can be solved by the application of Muller's iteration scheme of successive bisection and inverse parabolic interpolation. Representative results are described below.

Results of the computer simulations

Conductivity values in the upper layers of the seabed are highly variable (see figure 2) mainly because of the inclusion of different degrees of water content⁴. Such a degree of variability is shown in the examples that follow. The two simulations apply to circumstances where the seabed conductivity is 2 S.m^{-1} and 0.5 S.m^{-1} , respectively. The permeability of all layers is assumed to be that of free space ($4 * \pi * 10^{-7} \text{ F.m}^{-1}$).

For simplicity in the simulations the VED is located at exactly half the skin depth (56.12 m) for the source frequency of 5 Hz and seawater of conductivity 4.0 S.m^{-1} . Electric field sensors are sited on the seabed in a water depth of exactly one skin depth (112.54 m). The horizontal range between sensors and source is also one skin depth, measured along the radial axis. Values of $|E'_{r,M}|$ and $|E'_{z,M}|$ were calculated accordingly.

Results of the first simulation are summarized in figure 5. As indicated the initial guesses of the seabed conductivity were 0.5, 1, 4 and 6 S.m^{-1} . In all instances the

correct value of the conductivity (2 S.m^{-1}) was identified in less than fourteen iterations of the algorithm.

Figure 6 summarizes the results of the second simulation. Initial guesses of the conductivity in this example were 0.1, 0.25 and 1 S.m^{-1} . Again, the correct value of the sea bed conductivity (0.5 S.m^{-1}) was identified in less than fourteen iterations.

Discussion

Reference was made earlier to the single value of permeability that is used to characterize each layer (of the Weaver model). This simplification is appropriate in many areas off the east coast of New Zealand. The material that comprises the upper layers of the seabed has a relative permeability (or dielectric constant) of approximately 1.

Such a condition does not apply over large tracts off the country's west coast. Corresponding material in the seabed often has a significant ferrous content. Other simulations have also revealed a clear deficiency in the ability of the Weaver model to predict subsurface ELFE propagation in such areas. Studies of ELFE propagation in these conditions would best be carried out with finite element models. These models provide for the inclusion of more detailed descriptions of the conductivity, permittivity and permeability parameters in each conducting layer.

Several options are available to test the above technique. The simplest test is to repeat the measurement and analysis process at a number of nearby locations. Each repetition should produce consistent estimates of the seabed conductivity value. A second option is to maintain the initial sensor and source locations, but to change the alignment from that of a vertical axis (VED) to a horizontal axis (HED) dipole. This change produces different ELFE propagation modes. The subsequent analysis task is longer and more complex, but it is still within the capabilities of the algorithm.

Conclusions

The simulations have demonstrated a relatively simple and inexpensive means to estimate conductivity values that characterize the seabed. Such knowledge is essential for ELFE models. Moreover, the method is independent of the source strength and it does not require measurements of the magnetic field components. This remote-sensing technique requires measurements of two electric field components ($|E'_{r,M}|$ and $|E'_{z,M}|$) of the ELFE transmissions from a VED source. The position coordinates of the source and sensors must be known.

Position coordinates of the sensors and ELFE source can be satisfactorily established by commercial microwave navigation systems or by precise differential calculations using the Global Positioning System (GPS). Both systems can provide a position accuracy of about 2 m.

The two sets of simulations were carried out on a 486DX, 66 MHz computer. Each computer simulation required a program execution time of less than 120 s.

References

1. J.T. Weaver, "The quasi-static field of an electric dipole embedded in a two-layer conducting half-space," *Can. J. of Phys.* **45**, 1981-2002 (1967).
2. R.A. Geyer, *CRC Handbook of geophysical exploration at sea* (CRC Press, Boca Raton, Florida, 1983), p. 353.
3. A.D. Chave, "Some comments on seabed propagation of ULF/ELF electromagnetic fields," *Rad. Scien.* **25**, 825-836 (1990).
4. C.S. Cox, S.C. Constable, A.D. Chave, and S.C. Webb, "Controlled-source electromagnetic sounding of the ocean lithosphere," *Nature* **320**, 52 (1986).

5. P.R. Bannister, "Determination of the electrical conductivity of the sea bed in shallow waters," *Geophys.* **33**, 995-1003 (1968).

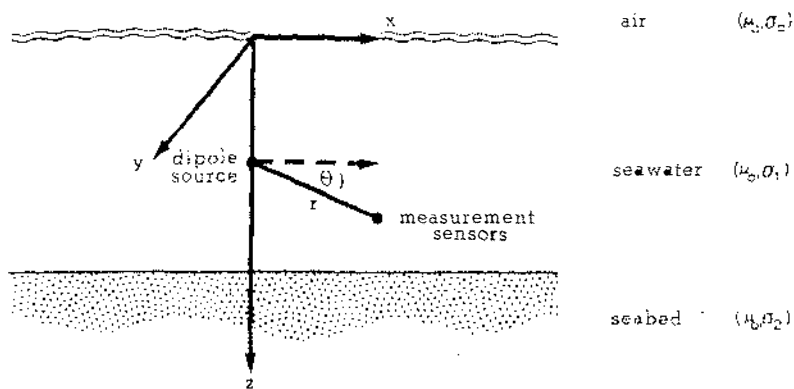


Figure 1. Coordinate system for the two-layer model

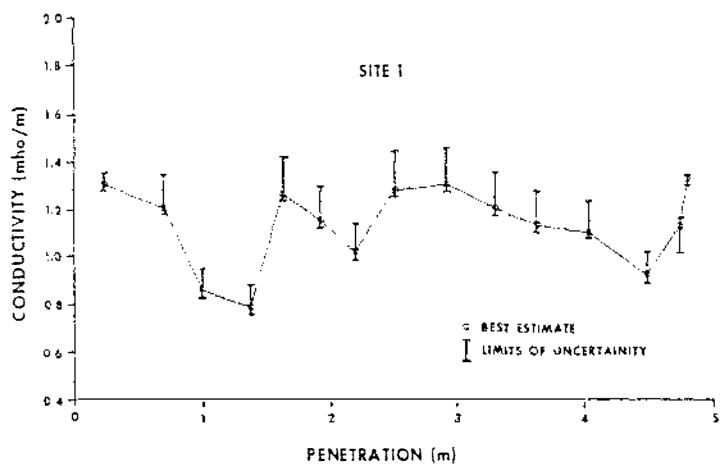


Figure 2. Conductivity value versus depth of seabed penetration (from reference [2], p353)

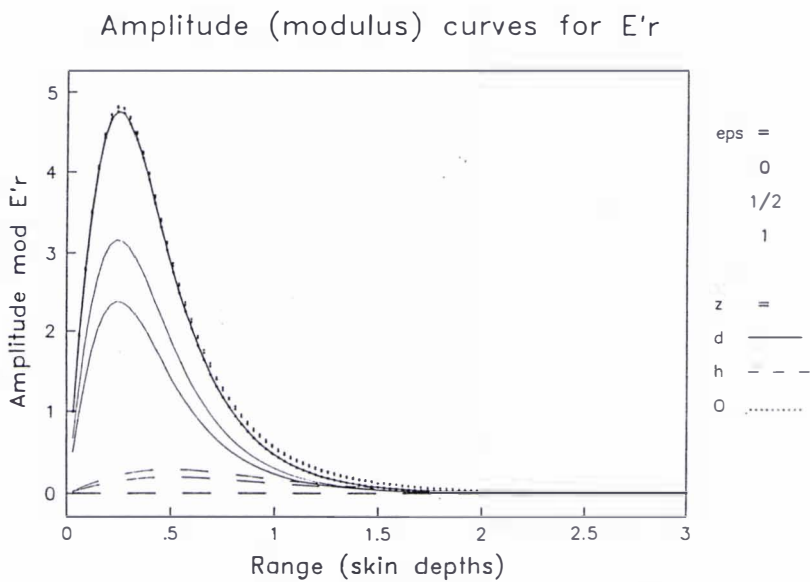


Figure 3. Moduli of non-dimensional values of radial electric fields
(see text for notations)

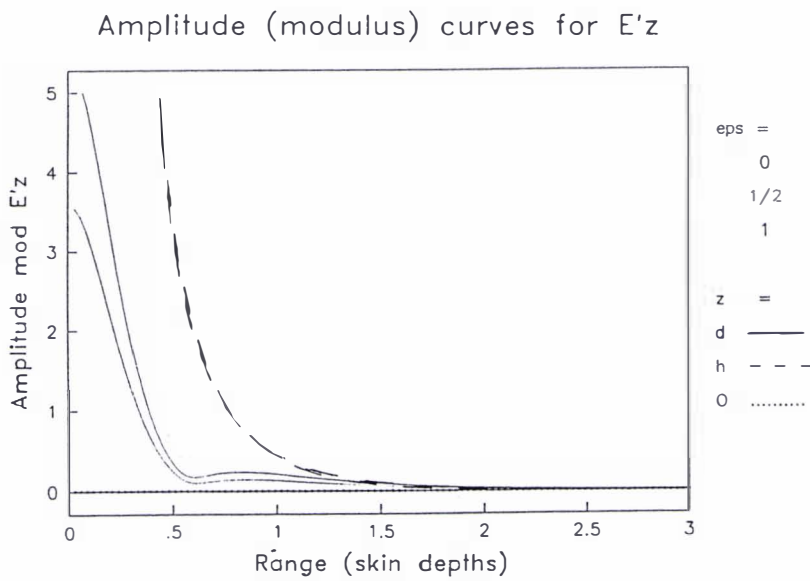


Figure 4. Moduli of non-dimensional values of vertical electric fields
(see text for notations)

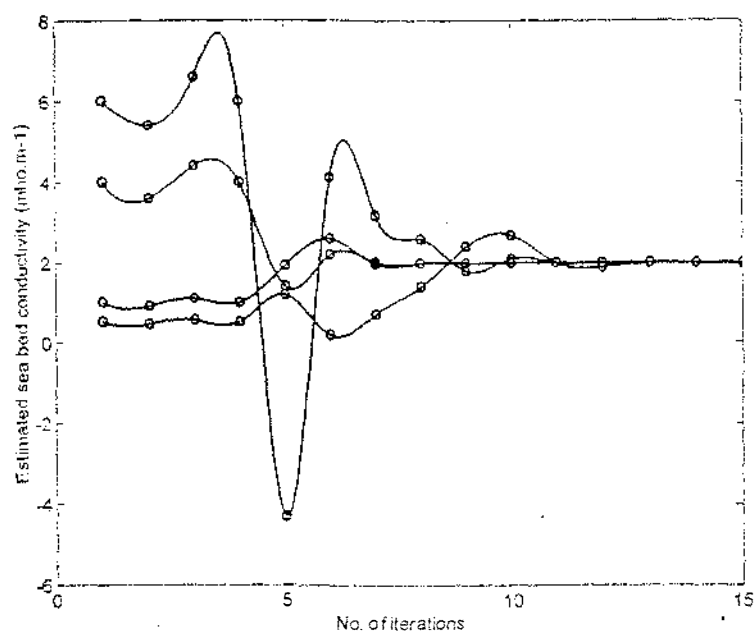


Figure 5. Simulated analyses with the actual seabed conductivity value set at 2.0 S m^{-1}

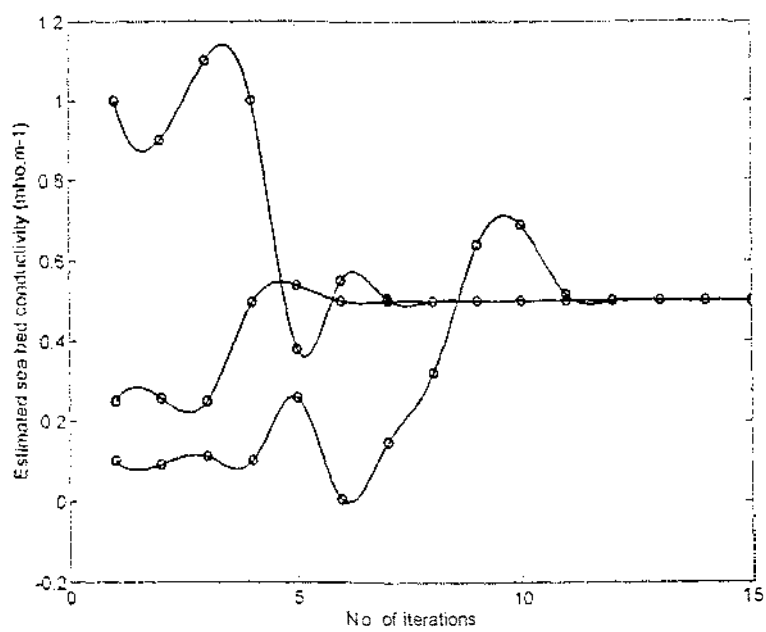


Figure 6. Simulated analyses with the actual seabed conductivity value set at 0.5 S m^{-1}

**Appendix B: Rangings of extremely low-frequency electromagnetic
sources over short ranges and in shallow water conditions**

(Appendix B contains a reprint of an invited paper²⁶ that describes some preliminary study results. The paper was published in a 1997 theme issue of the Journal of Underwater Acoustics, vol. 47, no. 2)

UNCLASSIFIED

U.S. Navy Journal of Underwater Acoustics

Volume 47, No. 2

April 1997

RANGINGS OF EXTREMELY LOW-FREQUENCY ELECTROMAGNETIC SOURCES OVER SHORT RANGES AND IN SHALLOW-WATER CONDITIONS

Edward I. Rumball and John S. Kay

Defence Scientific Establishment
Auckland Naval Base, Auckland
New Zealand

(Received July 30, 1996)

This paper describes procedures to characterize conductivity parameters of the seabed medium in shallow-water conditions, thus enhancing propagation models of extremely low-frequency electromagnetic (ELFE) effects over short ranges. Whereas values of the seawater conductivity can be readily measured, direct readings of the corresponding seabed parameter are costly and difficult to obtain. Moreover, the subsequent measurements are usually highly variable in both the horizontal and vertical planes. Procedures are described that show how electromagnetic theory and indirect measurement techniques may be used to infer the characteristic seabed conductivity values in a shallow-water area. In essence, iterative search processes are used to find a single unknown root (σ_2 seabed conductivity) in nonlinear equations given the levels of transmitted and received ELFE signals. This conductivity information is used in turn to demonstrate the inherent modeling accuracies for several rangings of elementary low-frequency electromagnetic sources over short ranges and in shallow-water conditions.

I. INTRODUCTION

In certain circumstances, a maritime vessel may become vulnerable to naval mines due to extremely low-frequency electromagnetic (ELFE) emissions that propagate outside its hull. Estimating the exact degree of vulnerability requires information on the strength of these ELFE sources

and, if possible, their spatial distributions. Processes for measuring and quantifying such source strengths are often referred to as rangings.

The ELFE models used in mine countermeasures (MCM) studies simplify the geotechnical descriptions of the media. Usually these are represented as isotropic, two-layer, conducting half-spaces.¹ For these investigations, it is usual for both the source and receiver to be located at general points in the upper conducting layer, which is the fluid medium. The significant environmental parameters of such models are shown in Fig. 1. Here μ and σ denote the permeability and conductivity characteristics of each medium. A computer source code for this (Weaver) model¹ was written by staff at the Defence Scientific Establishment (DSE) several years ago.

Seawater conductivity values typically lie within the range 3 to 5 S.m⁻¹ ($S \equiv \text{ohm}^{-1}$). Over practical ranges of seawater values, it is a slowly varying function of temperature, salinity, the frequency of the propagating electromagnetic wave, and pressure — in that order of importance. It can be readily obtained from a range of onboard conductivity-temperature-density (CTD) instruments.

Approved for public release; distribution unlimited.

UNCLASSIFIED

670

RUMBALL AND KAY

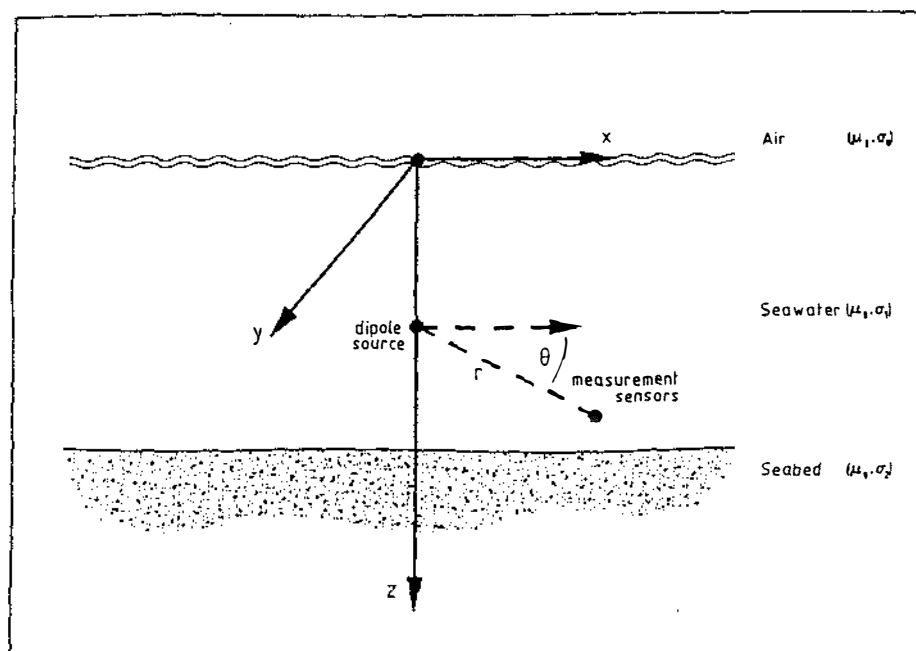


Fig. 1 — Environmental parameters and coordinate system for the two-layer model

Estimates of the seabed conductivity are more difficult to obtain, but these are needed before the ranging processes can begin. Conductivity measurements of the seabed have long been used in geophysical surveying and offshore morphology studies. In these circumstances, the measurements may be derived from diver-held probe systems that are implanted in the seabed. Additionally, core conductivity measurements may be obtained in the course of an offshore drilling program. This information, like that obtained from the probe technique, is costly to obtain.

Diver-operated conductivity probes often use the four-electrode contact technique to directly measure the resistivity. The equipment consists of two pairs of electrodes: one pair applies the reference current and the other measures the resulting electrical potential. Such measurements are characterized by the high degree of variability shown in Fig. 2.² With these hand-held systems, it is difficult to obtain measurements at seabed depths of more than several meters.

Electromagnetic propagation through the seabed has recently been developed to become a practical tool for geophysical exploration.³ These studies used various configurations of electric dipole sources and electric or

magnetic field receivers⁴ deployed in water depths of several thousand meters. Most appear to use seabed-to-seabed propagation modes. The results indicate that the uppermost sediment layer is rarely more than a few hundred meters thick. Within this layer, the conductivity usually varies between 0.01 and 2.5 S.m⁻¹. Such variations are mainly due to differing degrees of sediment porosity to water.

The deep-water models differ significantly from the short-range, shallow-water models needed for MCM investigations. Reference 5 is an exception. It postulates that seabed conductivity may be determined over long ranges by measuring only the horizontal electric field components produced by a long horizontal magnetic dipole at the surface. For this model, it is necessary that the horizontal range is much greater than the skin depth, and that a "transmission line analogy" can be applied.

Our requirements for such techniques are, first, that they should not require the use of divers or specialised electrode probe equipments. Second, the methods must be applicable to shallow-water areas, and the results must be usable in models that encompass isotropic, two-layer

UNCLASSIFIED

UNCLASSIFIED

RANGINGS OF ELFE SOURCES OVER SHORT RANGES

671

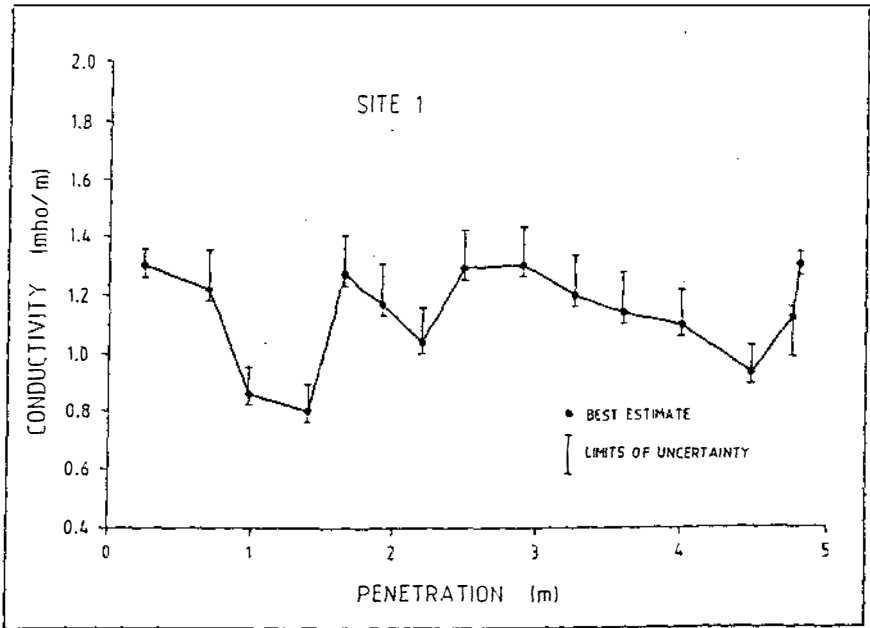


Fig. 2 — Seabed conductivity data vs depth (from Ref. 2, p. 353)

conducting half-spaces. Third, it is preferable that the technique should be independent of the physical size and strength of the ELFE source, since DSE does not have either a small or a calibrated ELFE source. To meet these objectives, DSE developed and applied the techniques described below to characterize the seabed medium and expedite the ranging of elementary dipoles.

II. THEORETICAL BASES OF THE TECHNIQUES

Given the coordinate system described in Fig. 1, the electric and magnetic field values at a general point in the upper conducting layer due to an electric point-dipole at any point in the same layer may be calculated by using the Weaver model.¹ The axis of the electric dipole may be horizontal or vertical. Specifically, the nondimensional and frequency-independent radial and vertical electric fields due to a vertical electric dipole (VED) are:

$$E_r' = \sum_{n=-\infty}^{\infty} \left\{ F_2(r_n', \sin(2\psi_n)) - F_2(R_n', \sin(2\Psi_n)) \right\} + \frac{\partial^2 Q_0}{\partial r' \partial z'}, \tag{1}$$

$$E_z' = \sum_{n=-\infty}^{\infty} \left\{ F_3(r_n', \sin(\psi_n)) - F_3(R_n', \sin(\Psi_n)) \right\} + Q_2. \tag{2}$$

Equations (1) and (2) correspond to Eqs. (36) and (37) in Ref. 1. The notations in Eqs. (1) and (2) are described in Eqs. (32), (33), (38), (40), and (41) in Ref. 1. Key equations of this approach are described in the Appendix. In the first technique to be described, the value that characterizes the seabed conductivity is found by solving the equation:

$$|E_r'|/|E_z'| - |E_{r,M}'|/|E_{z,M}'| = 0, \tag{3}$$

where $|E_{r,M}'|/|E_{z,M}'|$ is the ratio of the measured electric field components at a trial site. This notation system follows the earlier convention¹ as far as possible, with the additional subscript *M* denoting a "measured" value. Equation (3) is thus independent of the source strength of the VED. Nevertheless, for experiments described in later sections, it is necessary that a constant dipole moment is maintained for the VED source, albeit of unknown magnitude. In effect, this condition requires only that a constant dipole current is maintained. Since Eq. (3) uses only those electric field components of a VED source, it can be seen¹

UNCLASSIFIED

UNCLASSIFIED

672

RUMBALL AND KAY

to be more easily solved than the next method, which requires the evaluation of electric fields due to horizontal dipole sources.

Equation (74) of Ref. 1 can be used in a similar reduction process. It can provide the following nonlinear expression for the radial electric field $|E_r''|$ at a general point due to a horizontal electric dipole (HED) located in the same seawater medium. Again, the expression $|E_r''|$ refers to the nondimensional form of the signal. The HED source is assumed to maintain dual-frequency ELFE transmissions of equal and constant current magnitudes:

$$\left(\frac{freq_1}{freq_2} \right)^{1.5} * |E_{r,freq_1}''| / |E_{r,freq_1}''| - |E_{r,freq_1}''| / |E_{r,freq_1}''| = 0, \quad (4)$$

where $freq_1$ and $freq_2$ are the two transmitted ELFE frequencies. The values $|E_{r,freq_1}''|$ and $|E_{r,freq_2}''|$ are the radial electric fields at each frequency, with the subscript M again denoting a measurement at the sensor location. Equation (4) is likewise independent of the dipole moments of the HED sources. As the later results demonstrate, there can be clear, practical advantages from undertaking these more difficult calculations. Diagrammatic representations of the two methods for estimating the seabed conductivity are shown in Figs. 3 and 4. The sources and sensors are all located in the fluid medium.

At a specific trial site, the water depth, axial alignments, and the position coordinates of the receiver and source will be known. Also, subject to the qualifications described later, the permeability of the seabed would be known. The sole unknown parameter in Eqs. (3) and (4) is the conductivity value (σ_2) for the seabed.

Expressions containing $|E_r'|$, $|E_r''|$, $|E_z'|$ and $|E_z''|$ have real and imaginary parts. The real root describing the conductivity parameters (σ_2) in Eqs. (3) and (4) can, however, be solved by the application of Muller's⁶ iterative scheme of successive bisection and inverse parabolic interpolation. Representative results are described in the following sections.

III. COMPUTER SIMULATIONS OF THE MEASUREMENT PROCESSES

Conductivity values in the upper layers of the seabed are highly variable (see Fig. 2) mainly because of the

inclusion of different water contents.⁴ Such a variability will be reflected in the examples that follow by beginning the simulations at diverse start points. These computer simulations apply to circumstances where the actual conductivity of an isotropic seabed is 2 S.m^{-1} . The permeability of both conducting layers is assumed to be that of free space ($4 * \pi * 10^{-7} \text{ H.m}^{-1}$).

For simplicity in the simulations, a VED source is assumed to be located at exactly half the skin depth (56.26 m) for the source frequency of 5 Hz. The seawater conductivity is taken to be 4 S.m^{-1} . Electric field sensors are sited on the seabed in a water depth of exactly one skin depth (112.54 m). The horizontal range between sensors and source is also one skin depth, measured along the radial axis. The expressions $|E_{r,M}'|$ and $|E_{z,M}'|$, which in this instance simulate the measured values, were calculated accordingly using the two-layer (Weaver) model.

Results of the simulations using the VED technique are summarised in Fig. 5. As indicated, the initial guesses of the seabed conductivity were 0.5, 1.0, 4.0, and 6.0 S.m^{-1} . In all instances, the correct value of the conductivity parameter (2 S.m^{-1}) was identified in fewer than 14 cycles of the iterative search algorithm. Such searches typically required an execution time of 120 s on a 486DX-2, 66 MHz computer.

Results of other computer simulations are described in Ref. 7. An essential finding in the latter simulations is that the technique is also successful when dual-frequency ELFE transmissions from an HED source are received by single-axis sensors. It is important to appreciate that the expression for the radial electric field $|E_r''|$ due to a HED source does not necessarily decrease monotonically with increases in the horizontal range. Figure 2 of Ref. 1 demonstrates this point. For this reason, there will be instances where the iterative search processes indicate there are two real roots for Eq. (4)—a single root or, in some circumstances, the search may fail. In conditions where more than one root is found, the correct value at a specific measurement point can be found by simple inspections. This real-valued root will be referred to throughout the article as the characteristic value of the seabed conductivity between the electric field sensors and a specified point.

In later sections of the paper, it will be noted that DSE used three and sometimes four frequencies for the HED transmissions. This practice enables Eq. (4) to be evaluated for several pair-wise combinations of dual-ELFE transmissions at each measurement point. The results section that follows outlines further reasons to support the

UNCLASSIFIED

UNCLASSIFIED

RANGINGS OF ELFE SOURCES OVER SHORT RANGES

673

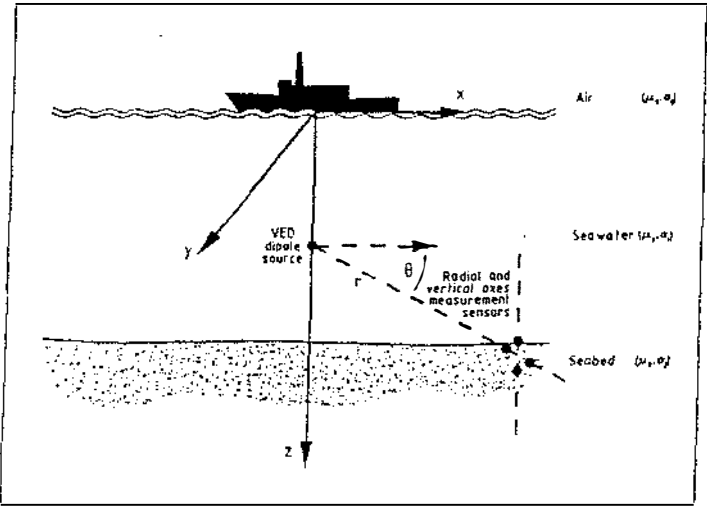


Fig. 3 — Conductivity estimation using a single-frequency VED and sensors on radial and vertical axes

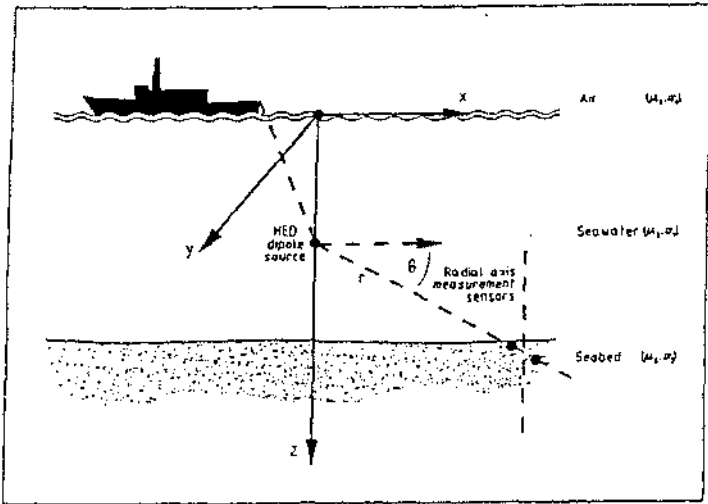


Fig. 4 — Conductivity estimation using a dual-frequency HED and sensors on a radial axes

UNCLASSIFIED

UNCLASSIFIED

674

RUMBALL AND KAY

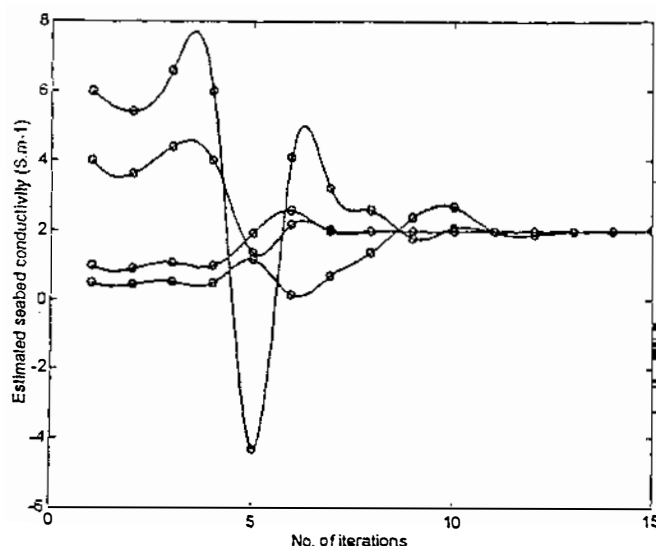


Fig. 5 — Simulated analysis using VED technique and with a seabed conductivity of 2 S.m⁻¹

practice of using multifrequency ELFE transmissions during the experiments.

IV. ENVIRONMENTAL FEATURES OF THE TEST SITE

The measurements described here were made in Tennyson Inlet, of the Pelorus Sound, during the late autumn of 1995. Tennyson Inlet is about 35 km from the open sea and 100 km west of Wellington. The test site, just north of the World's End passage, was square-shaped, with 600-m sides. Ship tracks during the electric field measurements and the site location are shown in Fig. 6.

Pelorus Sound is on the western edge of the alpine-fault system, a transform fault on the boundary of the Pacific and Indian-Australian tectonic plates. The sound was formed by the drowning of ranges and valleys of the much-broken mountainous country to the south. It is a maze of waterways with bays, coves, lesser sounds, and many islands of various sizes. The principal rock structure in the area is greywacke sandstone, with traces of schist. Tennyson Inlet itself extends in a south-west direction, at the western end of Tawhitinui Reach.

Water depths at low-tide are quite uniform at 25 ± 0.5 m throughout the test site. Subbottom profiling showed that the uppermost layer of the seabed comprised mud-silt sediments to a depth of about 15 m under the electric field

sensor. This sediment overlays a sloping subbottom structure understood to be greywacke gravels. Some gas-bearing sediments, mostly outside the test site, were observed 10 to 15 m below the seafloor.

Bearing-strength measurements were made with the DSE-developed electronic sediment probe (ESP).⁸ The probe sank approximately 0.6 m into the mud, indicating that the mud-silt sediment was quite soft near the surface. Typically, the bearing strength increased linearly from 10 to 50 kPa between depths of 0.1 to 0.5 m.

The weather was sunny and windless for most of the trial period. Sea conditions were flat-calm, except for small surface ripples late each day. Water temperature measurements made during the trial varied from 14.5° to 15.2° C at midwater depths.

V. MULTI-INFLUENCE SOURCE EQUIPMENT

Reference measurements, using known controlled sources, were crucial to the development and testing of the measurement equipment described in the next section and to the understanding of the ranging environment. These sources not only assisted with the processes for characterizing the test site, they also provided data to verify models for each of the measured influence fields. However, to minimize ship-induced distortions, the sources were separat-

UNCLASSIFIED

UNCLASSIFIED

RANGINGS OF ELFE SOURCES OVER SHORT RANGES

675

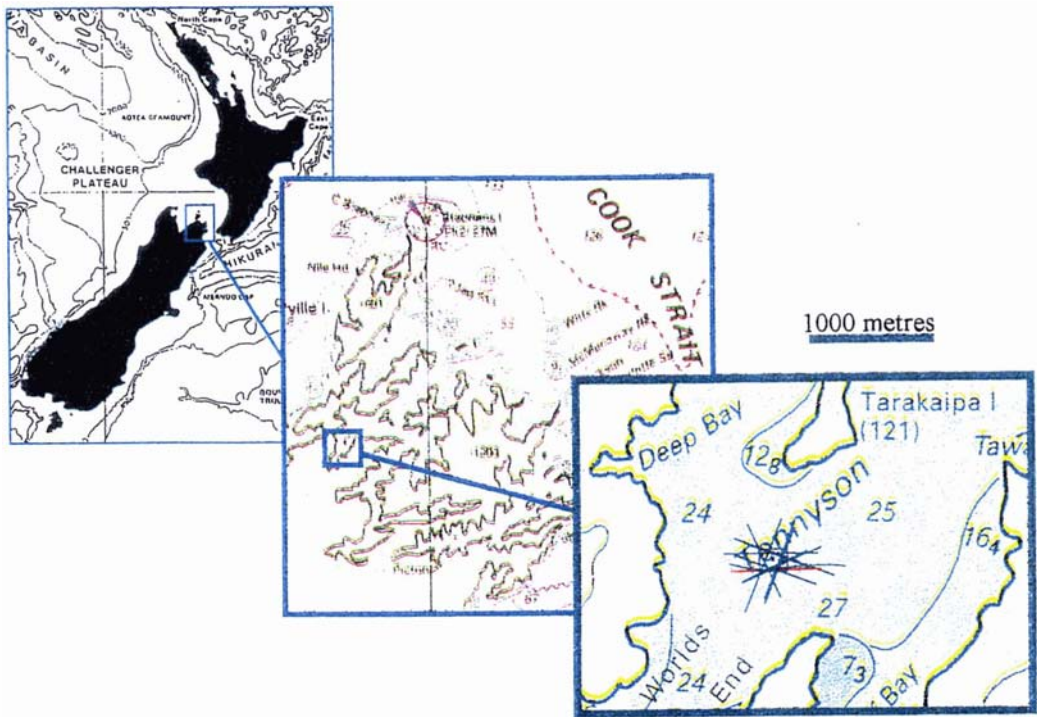


Fig. 6 — The electric field measurement tracks in Tennyson Inlet, Pelorus Sound at the northeastern corner of the South Island of New Zealand. Serial 060, shown in red, is the southernmost transverse track running from west to east.

ed from the towing ship by a distance of at least twice the water depth. In addition, the depth of the sources were selectable between 0.5 and 2.5 m below the surface.

The catamaran-hulled arrangement⁹ shown in Fig. 7 was selected as the most suitable platform. Like the sensor frame discussed below, all construction materials are nonmagnetic and nonconductive, to facilitate the modeling and operation of the magnetic and electric field sources.

The hulls are hard-chined, with small external keels to aid towing stability. The entrances are rounded to provide buoyancy up front to counteract nose diving caused by the drag of the underwater frame supporting the acoustic, magnetic, and electric source transducers. The depth of the frame below the surface is controlled by four fibreglass rods that pass through vertical bores in each hull.

The three frame-mounted sources are powered individually from a multifunction synthesiser connected to a power amplifier. This arrangement is capable of generating four

tones and of driving loads up to 230 V and 4 A. The acoustic source is an Underwater Sound Reference Laboratory (USRL) type J9 transducer.¹⁰ The magnetic source is a simple coil wound on a fiberglass former. The horizontal electric field source consists of two titanium rod electrodes separated by 3.2 m.

For a frequency of 35 Hz, the site water depth of 25 m corresponds to approximately 0.6 skin depths, where such a depth is taken here as the effective depth of penetration. Electromagnetic measurements were made over horizontal ranges from the sensors, which corresponded to 2 to 8 skin depths. This trial configuration and the relatively low-power sources (approximately 4 A.m) required the use of low-noise electric sensors to measure the ELFE fields.

VI. MULTI-INFLUENCE MEASUREMENT EQUIPMENT

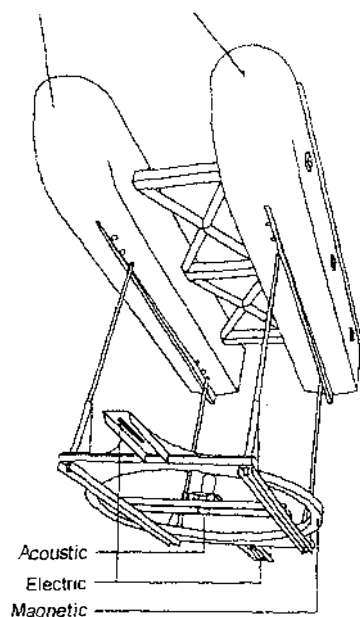
Over the last several years, the DSE staff has been developing the technology and the equipment^{9,11} to measure the influence fields radiated from ships. Measurements were

UNCLASSIFIED

UNCLASSIFIED

676

RUMBALL AND KAY

**Catamaran Hull**

Length: 4.2 m. Beam: 2.2 m. Weight: 542 kg (fully equipped).
Design: round nose, hard chine, 0.6 x 0.6 m cross-section, keeled.
Construction: 13 mm 5-ply marine plywood, high density foam filler, pultruded fibreglass beams, fibreglass skin.

Underwater Frame

Depth: variable from 0.5 to 2.5 m.
Construction: pultruded fibreglass beams, fibreglass sheet.

Acoustic Source

USRL J9 transducer.
Maximum sound pressure level: 120 dB re 1 μ Pa at 1 m, > 40 Hz.

Magnetic Coil

Former: fibreglass, 25 by 90 mm slot, 1.93 m mean diameter.
Wiring: automotive cable #150, ϕ 2.6 mm, 8A, 200 turns.
Inductance: 0.18 H. Maximum level: 37 nT at 25 m, 4 A, 40 Hz.

Electric Electrodes

Material: Titanium rod. Length: 170 mm. Diameter: 15 mm.
Spacing: 3.2 m. Maximum level: 4 V/m at 4 A.

Tow Cable

Strain member: 8 mm spectra, 30, 45, 60, 75, 90 m stops.
Electrical: 3-core, type 24320, 2.5 mm², 20 A with heavy duty thermoplastic rubber insulation (ϕ 12.2 mm).

Fig. 7 — The towed source platform. The acoustic transducer, the magnetic coil and electric electrodes are mounted on the frame suspended by four vertical rods below the catamaran hulls.

required of the acoustic, pressure, three-axis magnetic and electric fields. The equipment arrangement comprises a seafloor frame cabled to a surface platform, as shown in Fig. 8. Sensor performances are designed to register the static and dynamic characteristics associated both with ship influences and with the ambient environmental fields.

The equipment is modular, transportable, and easily deployed in sheltered waters. Optimum performance is obtained when the seafloor is relatively flat. Sand, shingle, or clay seafloor compositions are preferred.

The sensors and associated electronics are mounted on the underwater frame shown in Fig. 9. Again, nonmagnetic and nonconducting materials are used throughout to minimise any self-induced perturbations in the magnetic and electric fields. All pressure housings are manufactured from polyvinyl chloride (PVC) rod.

The frame itself is fabricated using beams and stiffening plates manufactured in pultruded fibreglass. These are resin glued and overlaid with fibreglass to form an integral bonded structure. The planar dimensions are 6.0 by 2.4 m. This provides an acceptable baseline for the electric field sensors and also adequate separation between sensors to

minimise cross-coupling effects. The frame is, however, sufficiently rigid to limit physical movement and vibration of the sensors. When deployed, the 250-kg weight of the fully equipped frame in seawater provides sufficient weight loading to settle each foot into a sand/mud seafloor and, thus, stabilize the platform.

All the instrumentation necessary for recording the sensor data and the battery power supply for the complete facility are housed in the surface platform described by Fig. 10. The recording medium is a battery-powered computer. On-line monitoring of the data is possible, and the display options include time-series, frequency spectra, and/or lofargrams.

The multi-influence fields are sensed using commercially available devices interfaced to specialised electronics developed at DSE. Features include the use of validation test signals and of dynamic range-compression techniques.¹² The acoustic sensor is an omnidirectional spherical hydrophone interfaced to low noise electronics. The pressure field is sensed with a silicon diaphragm transducer and the magnetic field with a triaxial fluxgate magnetometer. Both are interfaced to dynamic, range-compression facilities.

UNCLASSIFIED

UNCLASSIFIED

RANGINGS OF ELFE SOURCES OVER SHORT RANGES

677

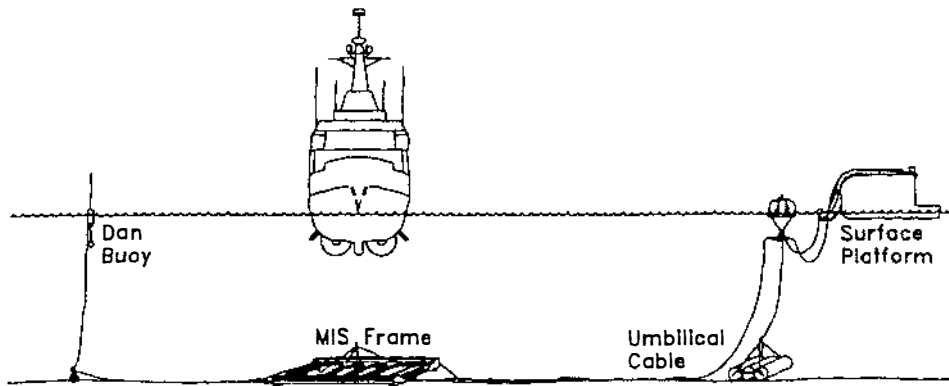


Fig. 8 — The equipment arrangement for measuring the multiple influence fields radiated from New Zealand naval vessels. The sensors are mounted on the underwater frame shown in Fig. 9. The umbilical cable carries the signals from and the electrical power to the sensor frame. The recording and some on-line analysis instrumentation, plus the battery power supplies for the complete rig, are housed in the surface platform shown in Fig. 10.

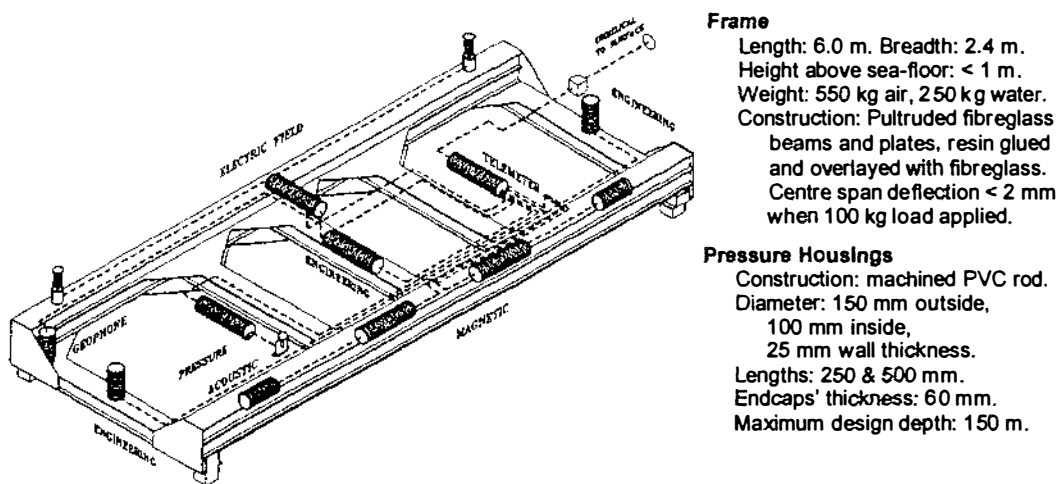
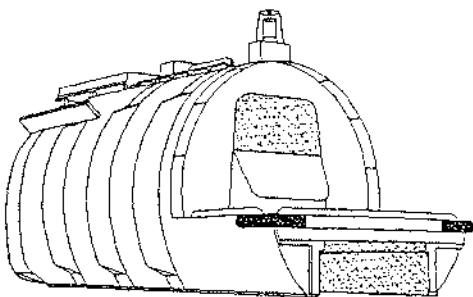


Fig. 9 — The underwater frame and sensor fit

**Overall**

Length: 4.9 m. Beam: 2.5 m. Height: 2.4 m.
 Weight: 1.8 tonnes (fully equipped).
 Design: catamaran hulls, bridging cabin.
 Cabin: 2.2(l) x 1.2(b) by 1.5(h) m.
 Construction: marine plywood, fibreglass.
 Navigation: FI(5) Y 10s 2m.

MIS Equipment Fit

StoreMouse plus interface.
 Battery supplies (18 x 6 V lead acid, 1000 A-Hr).

Fig. 10 — The surface platform

UNCLASSIFIED

UNCLASSIFIED

678

RUMBALL AND KAY

The electric field sensor is a dual Ag-AgCl cell-differential amplifier. The operational requirement is to measure not only the electric signatures of ships but also naturally occurring environmental phenomena, such as the Schumann resonances.^{13,14} The Ag-AgCl cells¹⁵ procured for this application have low internal resistance and very low self-noise, particularly at low frequencies. The source impedance of the cells was measured at DSE in a test tank filled with artificial seawater. It was found to decrease from 4.5 to 2 Ω between 0.1 and 80 Hz. A frequency exponent of -0.77 was calculated. This compared favorably with the standard value of -0.5 .¹⁴

Careful engineering of the cells was necessary to minimize electromagnetic noise interference. Rigid screened rods were used in the electromechanical assembly, which comprised the 2.75-m long conductor from each cell to the differential amplifier. A very low-noise matching pre-amplifier¹⁶ was developed specifically for this application. The frequency bandwidth is from 0.5 mHz to 100 Hz, and the noise floor is -188 dBV/√Hz at 1 Hz and -192 dBV/√Hz at 10 Hz.

The signals from and the electrical power to the underwater electronics are carried by the umbilical cable. It is assembled in sections. The 30-m section immediately adjacent to the frame is, like the frame, nonferrous. The remaining 150-m length of cable is further sectionalized. This practice not only facilitates deployment operations but also provides water-isolation barriers within the cable. Section replacement can be undertaken at sea if the cable becomes damaged. The cable is terminated in the surface platform shown in Fig. 10. This ocean data buoy houses all the instrumentation necessary for recording and for some on-line analysis of the sensor data,¹⁷ plus the battery power supply for the complete facility.

Accurate navigation is essential for ship-ranging measurements. The trial requirements were two-fold. First, the helmsman was provided with a head-up display, showing the position of the ship relative to the required track. Second, the position of the ship needed to be accurately logged. Different approaches using differential global positioning systems (DGPS) have been used. Such systems give extremely good accuracy when the outputs are filtered. Algorithms have been developed to reject and appropriately filter the logged data to provide a positional accuracy of about 1 m. The output is a set of files relating time, range, and bearing of the vessel from the frame, for each serial of the trial.

The sensor and positional data are fused in the analysis computer normally installed on the ship undergoing ranging. Both sets of data are routinely downloaded throughout the ranging exercise. The sensor data are first demultiplexed into separate influence/serial files, checked, and restored. Both time and frequency-domain processing follows. The results are converted, using precomputed calibration polynomials, to the equivalent influence field measurement. Interpolation routines are used in conjunction with the positional data to remove the time dependency. Thus the received signal level is found as a function of range and bearing from the sensor.

VII. ELECTROMAGNETIC MEASUREMENTS

The sensor frame was planned so that the axis of the electric field sensor lay in the magnetic north-south direction, and the position was fixed using DGPS data. In addition, the frame was clearly evident on the depth sounder of the tow ship, enabling a check of the estimated position. All measurements were made with track offsets of between 50 to 100 m, to avoid saturation of the electric field sensors. On recovery, 5 days later, the frame was found to have sunk into the mud up to the middle of longitudinal beams. The sensors however were all in clear water.

Although the computer computations for the HED technique are more difficult, they enable conductivity measurements to be made from an underway vessel. VED measurements from a moving vessel would require the dipole to be kept vertical and well clear of the hull and turbulence. Hence, a conductivity survey within a test area can be completed in the time needed for several straight-line traverses. Moreover, it is feasible to conduct simultaneous electromagnetic ranging and conductivity surveys by deploying the elementary HED sources about 50-m behind the trial vessel.

A set of 12 radial runs were successfully made towing an HED source. These runs were made along the major magnetic axes and along 45° intercardinal courses out to a range of between 200 and 400 m from the sensors. The depths of the sensors and HED dipoles were nominally 24.5 and 1.6 m, respectively. For these measurements, the source was towed at a speed of 4 kts. Four tonals were generated simultaneously at frequencies of 22, 35, 74, and 94 Hz.

During all serials, the electric field radiated by the passing of the trial vessel and the transmitted tonals from

UNCLASSIFIED

UNCLASSIFIED

RANGINGS OF ELFE SOURCES OVER SHORT RANGES

679

the source were clearly observed, as shown in Fig. 11. Of particular note is the 50 Hz interference tonal associated with the New Zealand national power grid, several kilometres distant. The equivalent background noise floor was measured at $-170 \text{ dBV}/\sqrt{\text{Hz/m}}$. Schumann resonances were noted at times when the vessel was distant from the sensors.

VIII. EXPERIMENTAL RESULTS

To demonstrate particular features of the results, our discussion of the surveys initially refers to a particular traverse (serial 060) of the World's End test site. During this serial, we used ELFE transmissions of 35, 74, and 94 Hz for analyses. Figure 12 shows the subsequent estimates of seabed conductivity using the methodology described in Eq. (4). To support these calculations, three midwater measurements of conductivity (σ_1) were made in the course of the trial. The mean value of 4.14 S.m^{-1} was within $\pm 1.5\%$ of all the measurements, and it was this value that was used in all calculations associated with the conductivity surveys.

At the shortest horizontal ranges, it is expected that the differences in the ELFE propagation losses for similar frequencies will be quite small. In some instances, these differences will be less than 1 dB. The experimental error in measuring the ratio of $|E''_{M, \text{freq}_1}|$ and $|E''_{M, \text{freq}_2}|$ may therefore be relatively large at horizontal ranges of about 100 m. In these circumstances, apparent values above 3.5 S.m^{-1} for the seabed conductivity should be treated with caution. Figure 12 also clearly shows this expected increase in data scattering at the shorter ranges. This is a significant factor, which is later considered in more detail.

Associated experimental errors can sometimes be reduced by increasing the frequency spread of the ELFE transmissions at the shortest ranges. A preferred choice for serial 060 would have been to obtain seabed conductivity estimates by analysing the 35/94 Hz transmissions at the shortest range (100 m). Analyses of the 74/94 Hz transmissions would preferably be used at the greatest range (275 m). In this rare instance, the iterative search algorithm failed to locate more than one valid solution at the 100-m range, using the processes described by Eq. (4). The solid curve in Fig. 12 describes the least-squares fit by a cubic polynomial to the 16 measurements of characteristic seabed conductivity at 8 values of the horizontal range.

Earlier references to the 35, 74, and 94 Hz ELFE transmissions indicated that their current amplitudes were constant and of equal value, although the actual dipole

moments were not known. It is a useful test of the conductivity survey to use this information to calculate the expected propagation loss between the electric field sensors and the ELFE sources. These propagation losses, when related to the received signal levels, provide predictions of the dipole moments along the surveyed track. Results of such predictions are shown in Fig. 13.

Between the horizontal ranges of 100 to 275 m, the mean-strength values of the 35, 74, and 94 Hz dipole sources were 13.3, 12.5, and 13.3 dB above a unit dipole (1 A.m). Also, over this horizontal range, the standard deviation of the estimated dipole strength was 2 dB. To place this performance in perspective, it is relevant to note that the received signal levels of the 35, 74, and 94 Hz ELFE transmissions were reduced by 30.1, 37.5, and 38.6 dB, respectively, between the horizontal ranges of 100 to 275 m.

For perfect survey results, the three curves showing the estimated dipole strengths in Fig. 13 would be coincident and horizontal. The departure from such an ideal survey result is attributed partly to irregularities in the seabed surface and in its underlying structures. Indeed, the sub-bottom profiler results described later show that seabed density at the test-site is nonuniform. This implies a likely nonuniformity in the conductivity characteristics of this medium. In such circumstances, the test site would not represent a true two-layer model. Other contributory factors include errors in the position measurements and in the levels of the transmitted and received ELFE signals.

It is useful to compare the results of the conductivity survey during serial 060 with those of other complementary serials over the World's End test site. The geographic distribution of these additional serials and their results are shown in Fig. 14. As a check for serial 060 above, corresponding analyses were carried out on serial 056, which is in the 9 o'clock position in Fig. 14. Here, between the horizontal ranges of 125 to 260 m, the mean-strength values of the 35, 74, and 94 Hz dipole sources were 11.7, 11.8, and 12.5 dB above a unit dipole. The standard deviation for the latter measurements was unchanged—at 2 dB.

The aggregate survey data are collated as smoothed grid-data in Fig. 15. The grid has been cropped in such a manner that dipole sources several meters in length can be regarded as point dipoles at all site locations represented by the cropped grid. Inside the cropped area, they should be regarded as having finite length. Dipole sources several tens of meters in length can be regarded as point dipoles only near the outer extremities of the gridded area. In other grid

UNCLASSIFIED

680

UNCLASSIFIED

RUMBALL AND KAY

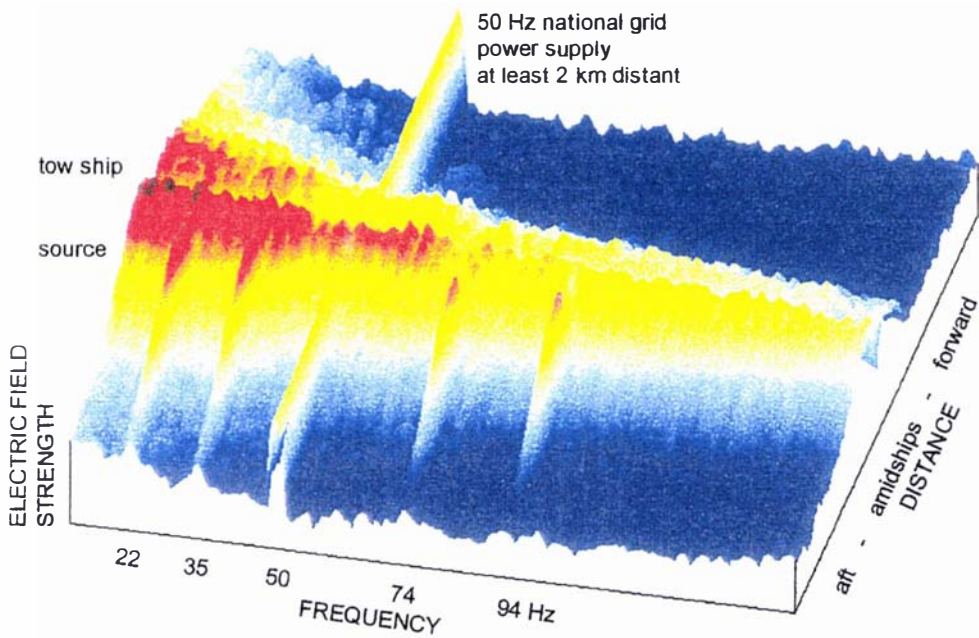


Fig. 11 — The electric field strength, as measured during serial 060. The four tonals at 22, 35, 74, and 94 Hz radiated by the electric field source can be clearly seen following the broadband noise associated with the tow ship. Also present is a 50 Hz interference tonal from the national grid power supply 2 km distant.

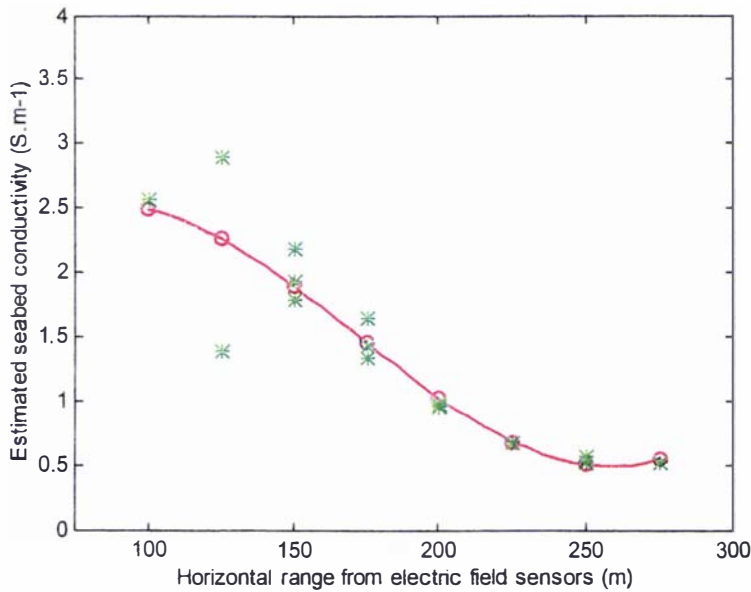


Fig. 12 — Estimated seabed conductivity values (.) and the least-squares fit by a cubic polynomial (o), for serial 060 at the World's End test site

UNCLASSIFIED

UNCLASSIFIED

RANGINGS OF ELFE SOURCES OVER SHORT RANGES

681

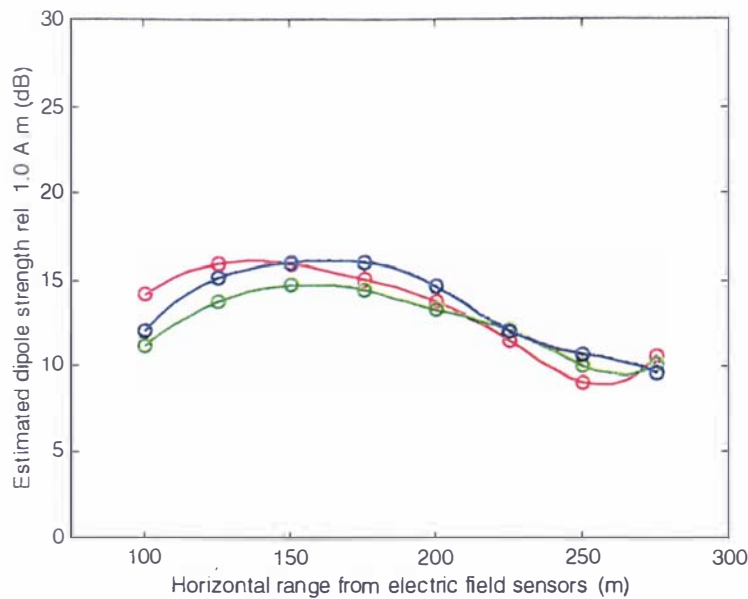


Fig. 13 — Variation with range of the estimated dipole strengths for 35 (red), 74 (green), and 94 (blue) Hz dipole sources in serial 060

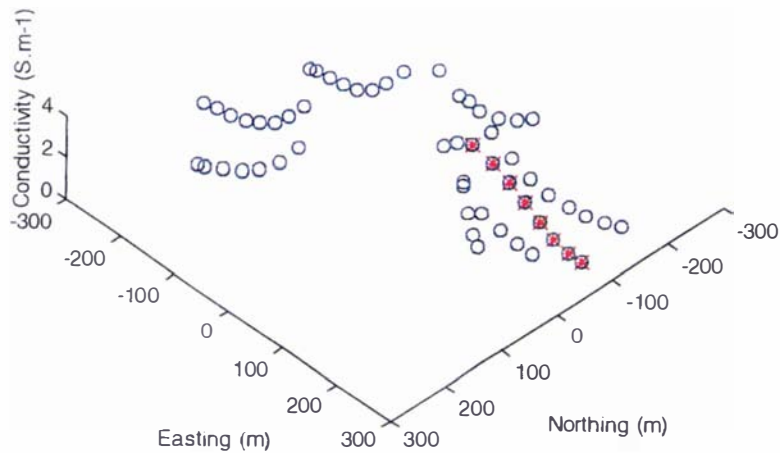


Fig. 14 — Estimates of the seabed conductivity from serial 060 (.) and complementary serials (o) at the World's End test site

UNCLASSIFIED

UNCLASSIFIED

682

RUMBALL AND KAY

areas, these latter sources should likewise be regarded as dipoles of finite length.

Contours of the characteristic values of the seabed conductivity throughout the area of the cropped grid are displayed in Fig. 16. The different colored contours represent increments of 0.5 S.m^{-1} over the range of values 0.5 to 4.0 S.m^{-1} . There is a small gap in the 0.5 S.m^{-1} contour, which lies beyond the axial bounds. In the uppermost contours, the small perturbation is due to an inability to obtain more than one conductivity estimate at the horizontal range of 100 m during serial 060. This factor was referred to earlier.

During the trial period, sub-bottom profiling records were compiled to support acoustic experiments. These records were gathered through the center and around the edges of the 600-m square site. Figure 17 shows a vertical cross-section of the seawater, mud-silt, and greywacke gravel sediments. This section is along a bearing of 48° on a straight-line track through the position of the electric field sensors. A point of interest is the quite uniform slope of the sub-bottom structure through the test site. In particular, the mud-silt section is wedge-shaped and varies uniformly from about 14 to 17.5 m through the test area.

To conclude this review of the experimental results, the data in Fig. 16 have been redrawn with the correct geographical alignments and with the contours viewed at normal incidence. Figure 18 shows the redrawn data. Once again, the different colored contours represent an increment of 0.5 S.m^{-1} over the range of values 0.5 to 4.0 S.m^{-1} .

Comparisons of Figs. 17 and 18 indicate a possible relationship between the thickness of the uppermost sediment (mud-silt) layer and the spacing of the contours that describe the characteristic values of seabed conductivity at the same location. Specifically, the closely spaced contours tend to lie above areas where the mud-silt layer is at its narrowest and vice versa. Such relationships suggests that ELFE propagation within the test site was influenced by more site features than just the two horizontal conducting layers that comprise the Weaver model. However, more comparative surveys of the sub-bottom structures are needed before such relationships can be verified.

IX. DISCUSSION

Application of the two-layer (Weaver) model requires difficult evaluations of 12 analytical expressions. Each has real and imaginary components. Even after these computa-

tions, it is clear that for some seabed areas, it is an oversimplification of the environment to use the two-layer concept as a "global" model with which to describe ELFE propagation effects. Extending the model to take account of three or more uniform conducting layers would markedly increase in its complexity.

The same observation would apply if it were planned to extend the model to take account of sloping seabeds. Such a change would provide an improved theoretical basis for considering long-range ELFE detections. However, it would likewise greatly increase the complexity of the model.

Nevertheless, these experiments have demonstrated that there are shallow-water areas in which ELFE propagation can be accurately described by the two-layer (Weaver) model, provided it is applied on a pointwise basis, with range-varying values of seabed conductivity. Many of the requirements for the electromagnetic ranging of vessels will be satisfied by using the Weaver model in this way.

A question may arise as to why the inner area of the grid in Fig. 15 is unsurveyed. Such a situation arose because the multifrequency ELFE dipoles were towed by a vessel that was also undergoing electromagnetic ranging. The electric field detectors were thus required to simultaneously provide undistorted records of both the vessel signatures and the ELFE transmissions. Such records could not be obtained at close ranges. Moreover, the presence near the sensors of a conductive body (the ship hull) several scores of meters in length and of several meters draft was believed to influence ELFE propagation in that vicinity. The two-layer model cannot be readily extended to include such boundary conditions.

With the benefit of experience and additional resources, another approach would be to augment conductivity surveys at a test site using sensor spacings and dipole lengths of about 1 m . Ideally, the ELFE sources would be towed by a small work boat with a hull of nonconducting material.

Several advantages would accrue from such practices. First, assume that a short HED source can be successfully ranged using the method in Eq. (4) to provide an accurate calibration of its dipole moment I . This factor can then be exploited to carry out conductivity surveys over the inner areas represented by the cropped grid. In this instance, the real root that characterises the conductivity value (σ_2) of seabed can be found by using similar iterative search processes to evaluate the expression:

UNCLASSIFIED

UNCLASSIFIED

RANGINGS OF ELFE SOURCES OVER SHORT RANGES

683

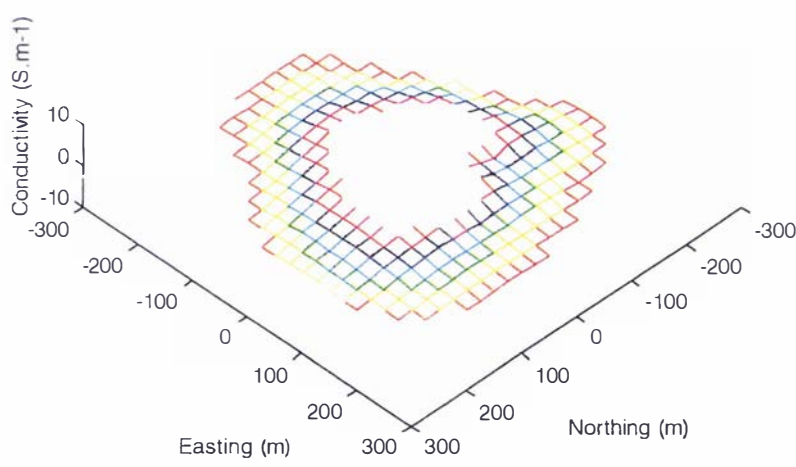


Fig. 15 — Cropped mesh grid indicating the coverage of characteristic seabed conductivity values between measurement points and the electric field sensors at the center of the grid

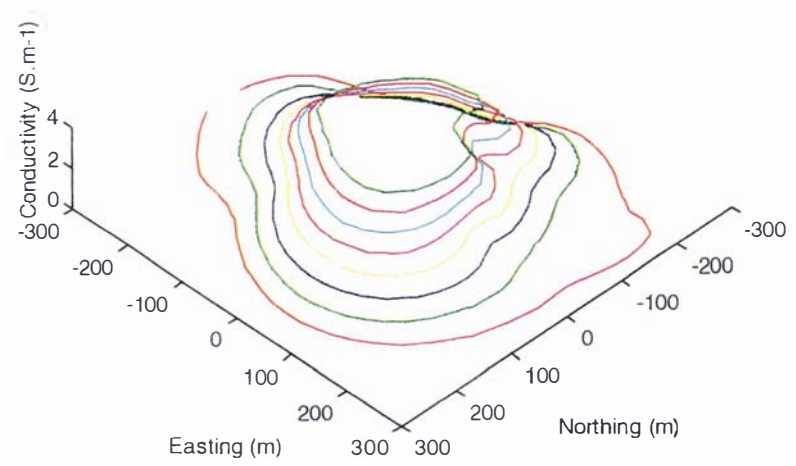


Fig. 16 — Contour levels between 0.5 and 4.0 S.m⁻¹ (in steps of 0.5 S.m⁻¹) for the characteristic seabed conductivity values at the World's End test site

UNCLASSIFIED

UNCLASSIFIED

684

RUMBALL AND KAY

$$\left(|E_r''| * I * \mu_0^{1.5} * \sigma_1^{0.5} * \omega^{1.5} \right) / (4 * \pi) - |E_{r,M}''| = 0, \quad (5)$$

in which $|E_r''|$ is as defined,¹ $|E_{r,M}''|$ is the signal from an HED source as measured along the radial axis of the electric field sensors, and ω is the angular frequency of the HED transmission. Equation (5) is a reordering of Eq. (34) in Ref. 1. Such an approach may reduce the earlier loss of accuracy in measuring the seabed conductivity over the inner area of the cropped grid.

In attempting to apply the global, two-layer model¹ at the World's End test site (and others), it has been found necessary to use range-varying values of seabed conductivity to describe ELFE propagation through media that are nonisotropic. The tests described in this article are not sufficiently sensitive to ascertain whether there are other propagation dependencies of the media that are not being detected. For instance, in some circumstances, it may be important to ascertain if there are frequency-related or polarisation-related properties of the seabed that need to be considered for modeling applications.

Reference was made earlier to the single value of permeability that is used to characterize each of the two isotropic conducting layers (of the Weaver model). This simplification is appropriate in many areas off the east coast of New Zealand. The material that comprises the upper layers of the seabed has a relative permeability of approximately 1.0.

Such a condition does not apply over large tracts off the country's west coast. Material in the seabed often has a significant magnetite content. Other simulations have revealed a clear deficiency in the ability of the Weaver model to predict subsurface ELFE propagation in such regions.

All of the above points support the notion that advanced studies of ELFE propagation over short ranges and in shallow-water conditions may be better carried out with finite element models. Such models provide for the inclusion of more detailed boundary conditions, which include more accurate descriptions of conductivity, permittivity, and permeability parameters. Multiple seabed layers of differing thicknesses and gradients can also be considered within the finite element models.

X. CONCLUSIONS

The simulations and experiments have demonstrated alternative means to estimate the conductivity values that characterise the seabed. Such knowledge is essential for the electromagnetic ranging of vessels at short ranges and in shallow-water conditions. Moreover, the methods are independent of the source strength of ELFE dipoles and do not require measurements of the magnetic field components. These estimation techniques require measurements of one or two electric field components of the ELFE transmissions from simple, uncalibrated sources.

To apply these techniques, the position coordinates of the source and sensors must be known. Coordinates of the sensors and ELFE sources can be satisfactorily established by commercial microwave navigation systems or by precise differential calculations using the Global Positioning System. Both systems can provide position accuracies of about 2 m.

If a three-axis configuration of electric field sensors is available, there will be no difference in the intrinsic accuracies of the VED and HED techniques. However, the disadvantage of maintaining stationary positions for the ELFE source (and its platform) using the VED method may be offset by the easier computation task. Since the HED technique can be used with the trial vessel under way, it provides the quickest means to characterise the seabed conductivity features in a trial area.

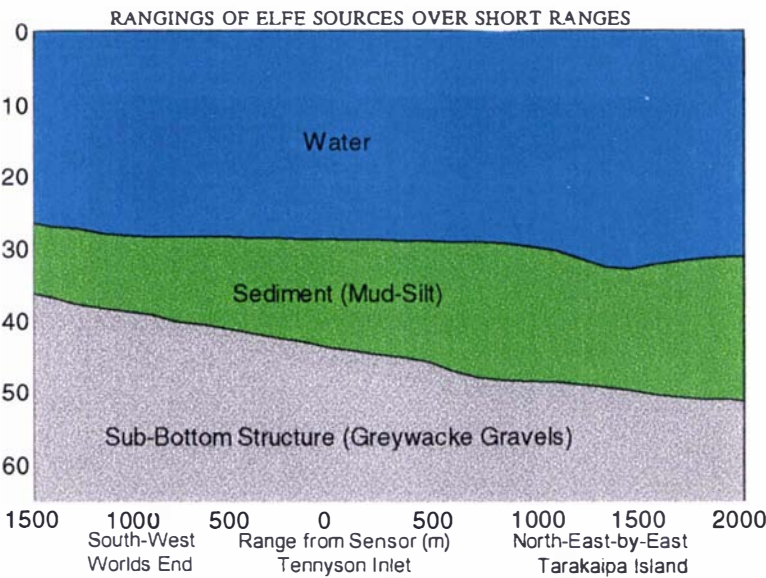
The conductivity measurements have been used in turn to demonstrate that there are seabed sites where to apply the global two-layer model would represent an over-simplification of the environment. Nevertheless, it is similarly shown that the two-layer model can be applied in a point-wise manner, with range-varying values of the seabed conductivity. This latter concept represents the environment with sufficient accuracy to satisfy most of the requirements for the ranging of ELFE sources over short ranges and in shallow-water conditions.

XI. REFERENCES

1. J.T. Weaver, "The Quasi-Static Field of an Electric Dipole Embedded in a Two-Layer Conducting Half-Space," *Can. J. Phys.* **45**, 1981-2002 (1967).
2. R.A. Geyer, *CRC Handbook of Geophysical Exploration at Sea* (CRC Press, Inc., Boca Raton, Florida, 1983), p. 353.

UNCLASSIFIED

UNCLASSIFIED



685

Fig. 17 — Vertical cross-section of the seawater, mud-silt, and greywacke gravel sediments along a bearing of 48°, on a straight-line track through the position of the electric field sensors

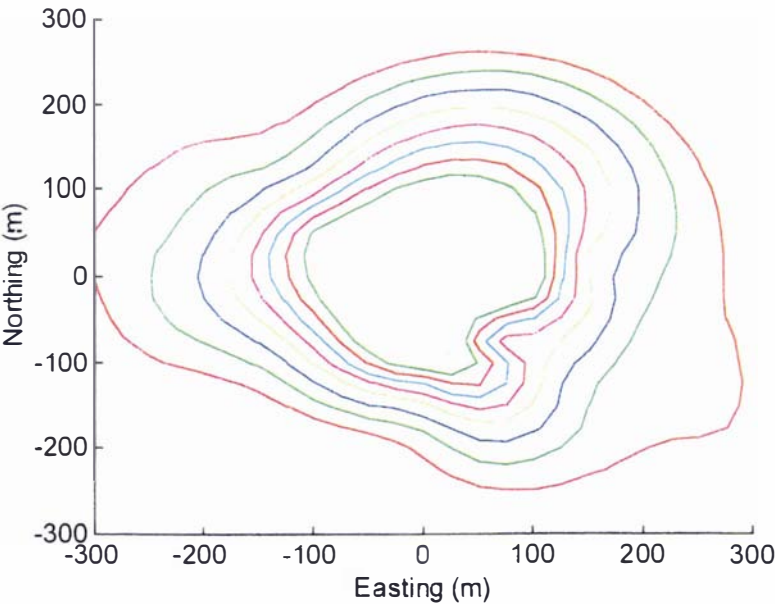


Fig. 18 — Normal incidence view of the contour levels between 0.5 and 4.0 S.m-1 (redrawn from Fig. 16) for the characteristic values of seabed conductivity at the World's End test site

UNCLASSIFIED

UNCLASSIFIED

686

RUMBALL AND KAY

3. A.D. Chave, "Some Comments on Seabed Propagation of ULF/ELF Electromagnetic Fields," *Rad. Scien.* **25**, 825-836 (1990).
4. C.S. Cox, S.C. Constable, A.D. Chave, and S.C. Webb, "Controlled-source Electromagnetic Sounding of the Ocean Lithosphere," *NATUA* **320**, 52 (1986).
5. P.R. Bannister, "Determination of the Electrical Conductivity of the Sea Bed in Shallow Waters," *Geophys.* **33**, 995-1003 (1968).
6. D.E. Muller, "A Method for Solving Algebraic Equations Using an Automatic Computer," *Mathematical Tables and Aids to Computation* **10**, 208-215 (1956).
7. E.I. Rumball and J.S. Kay, "Modelling of Extremely Low-Frequency Electromagnetic Propagation Over Short Ranges and Shallow-Water Conditions," presented at the 22nd meeting of TTCP Panel GTP-13, Naval Research Laboratory, Stennis Space Center, MS, October 16-20, 1995.
8. R.B. Hurst, J.F.F. Pope, R.H. Poeckert, and D.N. Lambert, "A Small Seabed Penetrometer for Measuring Sediment Bearing Strength," presented at the TTCP Subgroup G, 1996 Symposium, Defence Research Establishment Atlantic, Halifax, Nova Scotia (22-25 October 1996).
9. J.S. Kay, "The New Zealand Development of Equipment for the Measurement of Underwater Multi-influence Fields," in *Proceedings of the Institution Professional Engineers NZ (IPENNZ) Conference*, 7-9 August 1996, Dunedin, IPENNZ, Wellington, NZ, Vol. 1, 1996, pp 16-21.
10. C.C. Sims, "High-Fidelity Underwater Sound Transducers," *Proc. IRE* **47**, 866-871 (1959).
11. J.S. Kay and E. Marchant, "The New Zealand Multi-influence Sensor Experimental Measurement Facility," in *Proceedings of TTCP Subgroup G, 1994 Symposium*, July 11-15, 1994, Heriot-Watt University, Edinburgh, (Defence Research Agency (Bincleaves), Weymouth, United Kingdom), Vol. 1, 1994, pp. 133-140.
12. J.S. Kay, M.E. Bold, and A.R. Maggs, "Analog Signal Reduction," in *Proceedings 2nd Electronics NZ Conference*, 10-12 September 1995, Auckland, (Defence Scientific Establishment, Auckland, NZ), Vol. 1, 1995, pp. 77-82.
13. E.F. Soderberg, "ELF Noise in the Sea at Depths from 30 to 300 Meters," *J. Geophys. Res. Space Phys.* **74**, 2376-2387 (1969).
14. S.C. Webb, S.C. Constable, C.S. Cox, and T.K. Deaton, "A Seafloor Electric Field Instrument," *J. Geomag. Geoelectr.* **37**, 1115-1129 (1985).
15. AIS Electronics Inc., La Jolla, CA, EF sensor model J (private communication, September 1995)
16. J.M. Tricker, "A Sub-Nanovolt Chopper Amplifier," in *Proceedings 30th NZ National Conference*, 1-3 December 1993, Auckland, (Physics Department, Auckland University, Auckland, NZ), Vol. 1, 1993, pp. 125-132.
17. J.S. Kay, E. Marchant, and A.D. Dorrington, "Store-Mouse — A Versatile Data Collection, Storage, and On-line Analysis Facility," in *Proceedings 1st Electronics NZ Conference*, 23-25 November 1994, Hamilton, (Physics Department, Waikato University, Hamilton, NZ), Vol. 1, 1994, pp. 43-48.

UNCLASSIFIED

UNCLASSIFIED

APPENDIX

DEVELOPMENT OF THE TWO-LAYER (WEAVER) MODEL OF
EXTREMELY LOW-FREQUENCY ELECTROMAGNETIC PROPAGATION

This appendix is a shortened version of the mathematical content within a paper in the Canadian Journal of Physics **45** (1967).⁴¹ Here the aim is to describe key sections in the development of the two-layer (Weaver) model of electromagnetic propagation. The numbering system is identical to that of the reference.

ELECTROMAGNETIC FIELD EQUATIONS

We shall consider the general field equations in the meter-kilogram-second (MKS) system for an isotropic conducting medium containing electric current sources having an harmonic time variation of angular frequency ω . Thus all field vectors will contain the factor $e^{j\omega t}$, which can always be cancelled in the equations to follow so that we may regard the vectors as functions of position only and replace time derivatives by the factor $i\omega$.

The current sources will be denoted by the vector $\mathbf{j}$ and will be assumed to be oscillating with a sufficiently low frequency that displacement currents may be neglected. Then Maxwell's equations connecting the electric field $\mathbf{E}$ with the magnetic field $\mathbf{H}$ take the simple form:

$$\text{curl } \mathbf{E} = -i\omega\mu\mathbf{H}, \quad (\text{A1})$$

$$\text{curl } \mathbf{H} = \mathbf{j} + \sigma \mathbf{E}, \quad (\text{A2})$$

where σ denotes the electric conductivity and μ the permeability of the medium. It is convenient to work with vector and scalar potentials $\mathbf{A}$ and ϕ defined by

$$\mathbf{H} = \text{curl } \mathbf{A}, \quad (\text{A3})$$

$$\mathbf{E} = -i\omega\mu\mathbf{A} - \text{grad } \phi, \quad (\text{A4})$$

with the connecting relation:

$$\text{div } \mathbf{A} = -\sigma\phi. \quad (\text{A5})$$

It then follows from Eqs. (A2) and (A3) that $\mathbf{A}$ satisfies the equation:

$$\nabla^2 \mathbf{A} - i\mu\sigma\omega\mathbf{A} = -\mathbf{j}. \quad (\text{A6})$$

UNCLASSIFIED

688

RUMBALL AND KAY

In the ensuing analysis, we shall be concerned solely with the electromagnetic field produced by an electric dipole. This source can be described equivalently as a dipole current which, for mathematical calculations, is conveniently written in the idealized form:

$$\mathbf{j} = I \delta(\mathbf{r}_0), \quad (\text{A7})$$

where I is the strength of the dipole, $\mathbf{r}_0$ is a position vector from the dipole, and δ represents the Dirac generalized function. By substituting Eq. (A7) into Eq. (A6) and using the condition that $A \rightarrow 0$ as $r_0 \rightarrow \infty$ (where $r_0 = |\mathbf{r}_0|$), we can solve the differential equation to obtain the well-known result for the field of a dipole in a conducting medium:

$$A = \frac{I}{4\pi r_0} \exp(-\alpha r_0 \sqrt{i}), \quad (\text{A8})$$

where we have put:

$$\alpha = \sqrt{(\mu \sigma \omega)}. \quad (\text{A9})$$

DIPOLE IN A LAYERED MEDIUM

Let us now consider a semi-infinite conducting medium occupying the half-space $z > 0$ of a rectangular coordinate system (x, y, z) . The permeability of the conductor is taken to have the free-space value μ_0 throughout, but its conductivity is assumed to be σ_1 for $0 < z < d$, and σ_2 for $z > d$.

We shall suppose that the current dipole I is situated at the point $x = y = 0, z = h, (0 < h < d)$. The region $z < 0$ is taken to be free space. Subscripts 0, 1, and 2, respectively, will be used on quantities to denote to which of the three regions $z < 0, 0 < z < d$, and $z > d$ they belong.

In the region $z < 0$, we have $\sigma_0 = 0$ and $\mathbf{j}_0 = 0$, so that by Eq. (A6), A_0 satisfies the equation:

$$\nabla^2 A_0 = 0, \quad (\text{A10})$$

subject to the condition:

$$\text{div } A_0 = 0, \quad (\text{A11})$$

by Eq. (A5). Likewise, the equations satisfied by A_1 and A_2 are:

$$\nabla^2 A_1 - i \alpha_1^2 A_1 = -I \delta(\mathbf{r}_0), \quad (\text{A12})$$

and:

UNCLASSIFIED

UNCLASSIFIED

RANGINGS OF ELFE SOURCES OVER SHORT RANGES

689

$$\nabla^2 A_2 - i \alpha_2^2 A_2 = 0, \quad (\text{A13})$$

respectively, where α_1 and α_2 are defined according to Eq. (A9). The solution Eq. (A8) for the vector potential of the dipole source represents a particular integral of Eq. (A12). Thus, denoting it by A^s , we can write:

$$A_1 = A^s + A_1^0, \quad (\text{A14})$$

where A^0 satisfies the auxiliary equation:

$$\nabla^2 A_1^0 - i \alpha_1^2 A_1^0 = 0. \quad (\text{A15})$$

The complete solution for the field is obtained by solving for A_0 , A_1^0 , and A_2 and applying the usual electromagnetic boundary conditions specifying the continuity of $\mathbf{k} \cdot \mathbf{H}$, $\mathbf{k} \times \mathbf{E}$, and $\mathbf{k} \times \mathbf{H}$ across the surfaces $z = 0$ and $z = d$, where $\mathbf{k}$ is a unit vector in the z direction. It is readily shown from Eqs. (A3), (A4), and (A5) that the corresponding conditions on the vector potential are:

$$\left[\text{curl} (A_1 - A_0) \right]_{z=0} = 0, \quad (\text{A16})$$

$$\left[\text{curl} (A_2 - A_1) \right]_{z=d} = 0, \quad (\text{A17})$$

$$\left[\mathbf{k} \times \left\{ \text{grad div} (A_2 / \sigma_2 - A_1 / \sigma_1) - i \mu_0 \omega (A_2 - A_1) \right\} \right]_{z=d} = 0. \quad (\text{A18})$$

Only the solutions for the two special cases of the dipole aligned (i) along and (ii) perpendicular to the z axis will be obtained in the following analysis. We shall refer to these two configurations as the "vertical dipole" and the "horizontal dipole," respectively. The field for any other dipole orientation can be readily found by an appropriate superposition of the solutions for cases (i) and (ii).

Vertical Dipole

When the dipole is vertical, we put $\mathbf{I} = I \mathbf{k}$. The model now possesses axial symmetry about the z axis, and it is therefore convenient to introduce cylindrical polar coordinates (r, θ, z) with $x = r \cos \theta$, $y = r \sin \theta$. In this system, A is independent of the variable θ , and it is possible to satisfy the boundary conditions by taking $A_r = A_\theta = 0$. Thus we may write $\mathbf{A} = A(r, z) \mathbf{k}$ everywhere. The source field A^s is given by Eq. (A8), with:

$$r_0^2 = r^2 + (z - h)^2. \quad (\text{A19})$$

In this notation, the expression for A^s can be given the well-known Sommerfeld integral representation:

$$A^s = \frac{I}{4\pi r_0} \exp(-\alpha_1 r_0 \sqrt{i}) = \frac{I}{4\pi} \int_0^\infty \frac{\xi}{\nu_1} J_0(r\xi) \exp\{-\nu_1 |z - h|\} d\xi, \quad (\text{A20})$$

UNCLASSIFIED

UNCLASSIFIED

690

RUMBALL AND KAY

where J_0 denotes the zero-order Bessel function of the first kind, and ν_1 is defined according to the general formula:

$$\nu(\xi) = \sqrt{\xi^2 + i\alpha^2},$$

the principal branch of the square root being understood, so that $\text{re } \nu > 0$. The form given in Eq. (A20) is more convenient for the subsequent application of boundary conditions. In cylindrical coordinates, Eqs. (A10), (A13), and (A15), whose solutions are sought, are all of the type:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial A}{\partial r} \right) + \frac{\partial^2 A}{\partial z^2} - i\alpha^2 A = 0. \tag{A21}$$

The solution of Eq. (A21) can be found by applying the zero-order Hankel transform defined by the equation:

$$\hat{A}(\xi, z) = \int_0^\infty r A(r, z) J_0(\xi r) dr.$$

Assuming that A and $\partial A/\partial r$ are $O(r^{-1/2})$ as $r \rightarrow \infty$, we find that Eq. (A21) reduces to:

$$\partial^2 \hat{A} / \partial z^2 = \nu^2 \hat{A}. \tag{A22}$$

Solving Eq. (A22) and taking the inverse Hankel transform, we arrive at the solution of Eq. (A21) in the form:

$$A = P \int_0^\infty \xi \{ C(\xi) e^{\nu_1 z} + D(\xi) e^{-\nu_1 z} \} J_0(r\xi) d\xi, \tag{A23}$$

where $C(\xi)$ and $D(\xi)$ are arbitrary functions of ξ , and a constant factor $P = 1/4\pi$ has been taken out for algebraic convenience later.

The general solutions for A_0 , A_1^0 , and A_2 are given by Eq. (A23), with appropriate subscripts inserted to distinguish among the three regions. We note, however, that since $\alpha_0 = 0$ (because $\sigma_0 = 0$), we may write $\nu_0 = \xi$. It is also clear that $D_0(\xi) = 0$ since $A_0 \rightarrow 0$ as $z \rightarrow -\infty$, and likewise that $C_2(\xi) = 0$. Hence, substituting for A^s in its integral form Eq. (A20), we find that the required solutions for the vector potential are:

$$A_0 = P \int_0^\infty \xi C_0(\xi) e^{z\xi} J_0(r\xi) d\xi, \tag{A24}$$

UNCLASSIFIED

RANGINGS OF ELFE SOURCES OVER SHORT RANGES

691

$$A_1 = P \int_0^\infty \xi \left\{ \frac{\exp(-\nu_1 |z - h|)}{\nu_1} + C_1(\xi) e^{\nu_1 z} + D_1(\xi) e^{-\nu_1 z} \right\} J_0(r\xi) d\xi, \quad (A25)$$

$$A_2 = P \int_0^\infty \xi D_2(\xi) e^{-\nu_2 z} J_0(r\xi) d\xi. \quad (A26)$$

Equation (A11), specifying the *nondivergence* of A_0 , reduces in the present problem to the simple condition $\partial A_0 / \partial z = 0$, by which it follows from Eq. (A24) that $C_0 = 0$ and hence that $A_0 = 0$. The remaining unknown quantities C_1 , D_1 , and D_2 can be found by applying the boundary conditions at $z = 0$ and $z = d$. They also reduce to a particularly simple form for the problem under discussion. For example, from Eq. (A16) (with $A_0 = 0$), we have $[\partial A_1 / \partial r]_{z=0} = 0$, whence:

$$[A_1]_{z=0} = 0, \quad (A27)$$

since $A_1 \rightarrow 0$ as $r \rightarrow \infty$. Similarly, we obtain from Eq. (A17):

$$[A_1 - A_2]_{z=d} = 0, \quad (A28)$$

and hence the boundary condition Eq. (A18) becomes:

$$[\sigma_2 \partial A_1 / \partial z - \sigma_1 \partial A_2 / \partial z]_{z=d} = 0. \quad (A29)$$

Applying the condition Eq. (A27), we obtain

$$C_1 + D_1 = -e^{-h\nu_1} / \nu_1.$$

The remaining boundary conditions Eqs. (A28) and (A29) yield the following equations:

$$C_1 e^{2d\nu_1} + D_1 - D_2 e^{d(\nu_1 - \nu_2)} = -e^{h\nu_1} / \nu_1,$$

$$C_1 e^{2d\nu_1} - D_1 + (\nu_2 / \varepsilon \nu_1) D_2 e^{d(\nu_1 - \nu_2)} = -e^{h\nu_1} / \nu_1,$$

where $\varepsilon = \sigma_2 / \sigma_1$. From these three equations in C_1 , D_1 , and D_2 , it is not difficult to establish that:

UNCLASSIFIED

UNCLASSIFIED

692

RUMBALL AND KAY

$$\begin{aligned} \nu_1 C_\varepsilon e^{\nu_1 z} + \nu_1 D_1 e^{-\nu_1 z} &= e^{-(z+h)\nu_1} + \frac{2e^{-2d\nu_1}}{1 - e^{-2d\nu_1}} \{ \cosh \nu_1(z-h) - \cosh \nu_1(z+h) \} \\ &+ \frac{8\varepsilon\nu_1 e^{-2d\nu_1} \sinh h\nu_1 \sinh z\nu_1}{(1 - e^{-2d\nu_1}) \{ \varepsilon\nu_1 + \nu_2 + (\varepsilon\nu_1 - \nu_2)e^{-2d\nu_1} \}}. \end{aligned}$$

We substitute this expression in Eq. (A25) and expand the factor $(1 - e^{-2d\nu_1})^{-1}$ in the second term binomially. Then, after collecting all terms and recalling that in region 1 the inequality $0 < |z-h| < z+h < 2d$ holds, we find that we can express the solution as follows:

$$\begin{aligned} \frac{A_1}{P} &= \sum_{n=-\infty}^{\infty} \int_0^{\infty} \frac{\xi}{\nu_1} J_0(r\xi) \{ \exp(-\nu_1 |2nd + z - h|) \\ &- \exp(-\nu_1 |2nd + z + h|) \} d\xi + Q, \end{aligned} \quad (\text{A30})$$

where:

$$Q = \int_0^{\infty} \frac{8\varepsilon\xi \sinh h\nu_1 \sinh z\nu_1 J_0(r\xi) e^{-2d\nu_1}}{(1 - e^{-2d\nu_1}) \{ \varepsilon\nu_1 + \nu_2 + (\varepsilon\nu_1 - \nu_2)e^{-2d\nu_1} \}} d\xi.$$

The integral in Eq. (A30) is of the form Eq. (A20) and can therefore be evaluated to give the result:

$$\frac{A_1}{P} = \sum_{n=-\infty}^{\infty} \left[\frac{\exp(-\alpha_1 r_n \sqrt{i})}{r_n} - \frac{\exp(-\alpha_1 R_n \sqrt{i})}{R_n} \right] + Q, \quad (\text{A31})$$

where:

$$r_n^2 = r^2 + (2nd + z - h)^2, \quad R_n^2 = r^2 + (2nd + z + h)^2. \quad (\text{A32})$$

Our earlier definition Eq. (A19) of r_0 is automatically accommodated in this scheme. The physical interpretation of Eq. (A31) is now clear. The infinite series refers to a simplified model in which $\sigma_2 = 0$ (i.e., $\varepsilon = 0$) so that the conducting medium is a slab of thickness d situated in vacuo. The terms in the series represent the effect at the point (r, z) , $0 < z < d$ of the source field together with the sequence of fields reflected from the upper and lower surfaces of the slab. In fact, each term in the series gives the field of one of the "image dipoles" formed by considering the infinite sequence of geometrical images of the source in the two planes $z = 0$ and $z = d$. Those images whose distances r_n ($n \neq 0$) and R_n from the point (r, z) have nonnegative subscripts lie on the r axis above the plane $z = 0$, while those for which r_n and R_n have negative subscripts are on the r axis beneath the plane $z = d$. The other term Q in Eq. (A31) obviously represents the additional effect obtained by replacing the free space beneath the slab by a medium of conductivity σ_2 .

UNCLASSIFIED

UNCLASSIFIED

RANGINGS OF ELFE SOURCES OVER SHORT RANGES

693

Since we shall be concerned only with the electromagnetic field in the region $0 < z < d$, there will be no confusion if we now drop the subscript 1 from the field quantities. The electric and magnetic field components in this region can all be determined from Eq. (A31), for by Eqs. (A3), (A4), and (A5), we obtain the relations:

$$H_r = H_z = 0, \quad H_\theta = -\partial A / \partial r, \quad E_r = \partial^2 A / \sigma_1 \partial r \partial z, \quad E_\theta = 0, \\ E_z = \partial^2 A / \sigma_1 \partial z^2 - i\mu_0 \omega A.$$

It is convenient to express the solutions in dimensionless form by scaling all distances in lengths of $1/\alpha_1$. Thus we define

$$r' = \alpha_1 r, \quad z' = \alpha_1 z, \quad d' = \alpha_1 d, \quad h' = \alpha_1 h, \quad r'_n = \alpha_1 r_n, \quad R'_n = \alpha_1 R_n, \quad (A33)$$

and substitute the new (dimensionless) variable $u = \xi/\alpha_1$. It is also convenient to introduce ψ_n and Ψ_n ($n = 0, \pm 1, \pm 2, \dots$), denoting the angles made with the positive z axis by the position vectors of lengths r_n and R_n , respectively, from the relevant image dipoles; i.e., we define:

$$\psi_n = \arccot \frac{2nd + z - h}{r}, \quad \Psi_n = \arccot \frac{2nd + z + h}{r}.$$

If we also put:

$$H' = 4\pi H / I \mu_0 \sigma_1 \omega, \quad E' = 4\pi E / I \mu_0^{\frac{3}{2}} \sigma_1^{\frac{1}{2}} \omega^{\frac{3}{2}}, \quad (A34)$$

the resulting expressions for the nonvanishing electromagnetic field components can be written in the form:

$$H'_\theta = \sum_{n=-\infty}^{\infty} \left\{ F_1(r'_n, \sin \psi_n) - F_1(R'_n, \sin \Psi_n) \right\} + \frac{\partial Q_0}{\partial r'}, \quad (A35)$$

$$E'_r = \sum_{n=-\infty}^{\infty} \left\{ F_2(r'_n, \sin 2\psi_n) - F_2(R'_n, \sin 2\Psi_n) \right\} + \frac{\partial^2 Q_0}{\partial r' \partial z'}, \quad (A36)$$

$$E'_z = \sum_{n=-\infty}^{\infty} \left\{ F_3(r'_n, \sin \psi_n) - F_3(R'_n, \sin \Psi_n) \right\} + Q_2, \quad (A37)$$

where, with $\eta(u) = \sqrt{(u^2 + i)}$ and $\eta_\epsilon(u) = \sqrt{(u^2 + i\epsilon)}$, we define:

UNCLASSIFIED

UNCLASSIFIED

694

RUMBALL AND KAY

$$Q_m = \int_0^{\infty} \frac{8 \varepsilon u^{m+1} \sinh h' \eta \sinh z' \eta J_0(r' u) e^{-2d' \eta} du}{(1 - e^{-2d' \eta}) \{ \varepsilon \eta + \eta_e + (\varepsilon \eta - \eta_e) e^{-2d' \eta} \}}, \quad (\text{A38})$$

and where

$$F_1(\rho, \chi) = \chi \rho^{-2} (1 + \rho \sqrt{i}) e^{-\rho \sqrt{i}}, \quad (\text{A39})$$

$$F_2(\rho, \chi) = \frac{1}{2} \chi \rho^{-3} (3 + 3 \rho \sqrt{i} + i \rho^2) e^{-\rho \sqrt{i}}, \quad (\text{A40})$$

$$F_3(\rho, \chi) = \rho^{-3} \{ (1 + \rho \sqrt{i}) (2 - 3 \chi^2) - i \rho^2 \chi^2 \} e^{-\rho \sqrt{i}}. \quad (\text{A41})$$

Expressed in this form, the right-hand sides of the solutions Eqs. (A35), (A36), and (A37) are dimensionless and independent of frequency.

We can calculate the field at the surface of the conductor by letting $z' \rightarrow 0$ in the above solutions. In the limit, we have $r'_n = R'_n$, and $\psi_n = \pi - \Psi_n$, so that the terms of the infinite series in Eqs. (A35) and (A37) exactly cancel each other. Since $\partial Q_0 / \partial r'$ and Q_2 also vanish when $z' = 0$, we find that $[H'_\theta]_{z'=0} = 0$ and $[E'_z]_{z'=0} = 0$. The expression for E_r can be simplified somewhat by noting that, for this component, the equal terms in the series add rather than subtract to give the result:

$$[E'_r]_{z'=0} = 2 \sum_{n=-\infty}^{\infty} F_2(r'_n, \sin 2\psi_n) + \left[\frac{\partial^2 Q_0}{\partial r' \partial z'} \right]_{z'=0}. \quad (\text{A42})$$

Likewise, at the bottom surface $z' = d'$, we have $r'_{n-1} = R'_n$ and $\psi_{n-1} = \pi - \Psi_n$ so that again the infinite series in Eqs. (A35) and (A37) vanish, leaving the solutions:

$$[H'_\theta]_{z'=d'} = -[\partial Q_0 / \partial r']_{z'=d'}, \quad [E'_z]_{z'=d'} = [Q_2]_{z'=d'}. \quad (\text{A43})$$

The solution Eq. (A36) simplifies into the form:

$$[E'_r]_{z'=d'} = 2 \sum_{n=-\infty}^{\infty} F_2(r'_n, \sin 2\psi_n) + \left[\frac{\partial^2 Q_0}{\partial r' \partial z'} \right]_{z'=d'}. \quad (\text{A44})$$

On the axis $r' = 0$, only the vertical electric field E'_z is nonvanishing, a fact that is readily verified by substituting $\sin \psi_n = \sin \Psi_n = 0$ and $J_1(r'u) = 0$ in the solutions Eqs. (A35) and (A36).

UNCLASSIFIED

UNCLASSIFIED

RANGINGS OF ELFE SOURCES OVER SHORT RANGES

695

The well-known expressions for the field of a current dipole situated in a half-space of uniform conductivity σ_1 can be recovered from our solutions by letting $d \rightarrow \infty$. In this case, the integral Q_m will vanish, as will all the terms in the infinite series except those for which $n = 0$. Thus we obtain the solutions:

$$H'_\theta = F_1(r'_0, \sin \psi_0) - F_1(R'_0, \sin \Psi_0), \quad (\text{A45})$$

$$E'_r = F_2(r'_0, \sin 2\psi_0) - F_2(R'_0, \sin 2\Psi_0), \quad (\text{A46})$$

$$E'_z = F_3(r'_0, \sin \psi_0) - F_3(R'_0, \sin \Psi_0), \quad (\text{A47})$$

which are in complete accord with previously obtained results. We could also have obtained these results (although not so readily) by making the conductivities of the two layers equal, i.e., by putting $\epsilon = 1$ in our general solutions. In this case, we would have to expand the factor $(1 - e^{-2d\eta})^{-1}$ in the integrand of Q_m and integrate term by term to produce the simplified results Eqs. (A45), (A46), and (A47); the infinite series so generated would then exactly cancel out the infinite series in the general solution except for the terms for which $n = 0$.

HORIZONTAL DIPOLE

The field of a horizontal dipole, aligned parallel to the x axis, say, can be found similarly by putting $\mathbf{I} = I\mathbf{i}$, where $\mathbf{i}$ is a unit vector in the x direction. Thus the source field can be written $\mathbf{A}^s = A^s \mathbf{i}$, where A^s can again be written in the form Eq. (A20). The model no longer possesses axial symmetry, so that we must retain the variable θ , and in order to satisfy the boundary conditions, we must use a vector potential with two cartesian components. Hence we define:

$$\mathbf{A} = A_x(r, \theta, z)\mathbf{i} + A_z(r, \theta, z)\mathbf{k}.$$

We may separate the variable θ by putting:

$$A = F(\theta)G(r, z),$$

whence Eqs. (A10), (A13), and (A15), whose solutions are sought, can all be written in the form:

$$\frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{\partial G}{\partial r} \right] + \frac{\partial^2 G}{\partial z^2} + \frac{1}{F} \frac{d^2 F}{d\theta^2} \frac{G}{r^2} = i\alpha^2 G.$$

It follows that the single-valued solution, which has the required symmetry about the plane $\mathbf{y} = 0$, is:

$$A = \sum_{m=0}^{\infty} G(r, z) \cos m\theta, \quad (\text{A48})$$

where m is a separation constant, and G is the general solution of:

UNCLASSIFIED

UNCLASSIFIED

696

RUMBALL AND KAY

$$\frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{\partial G}{\partial r} \right] + \frac{\partial^2 G}{\partial z^2} = \left[i\alpha^2 + \frac{m^2}{r^2} \right] G. \quad (\text{A49})$$

Equation (A49) can be solved by using a Hankel transform of order m , yielding the result:

$$G = P \int_0^\infty \xi \{ C(\xi, m) e^{z_0} + D(\xi, m) e^{-z_0} \} J_m(r\xi) d\xi,$$

where $C = \mathfrak{A} \mathbf{i} + \mathfrak{B} \mathbf{k}$ and $D = \mathfrak{C} \mathbf{i} + \mathfrak{D} \mathbf{k}$ are arbitrary functions of ξ and m , and again, the constant factor $P = 1/4 \pi$ has been taken out for convenience. If we now label quantities with appropriate subscripts for the respective regions, as before, and substitute for A^i , we obtain the solutions:

$$A_0 = P \sum_{m=0}^\infty \cos m\theta \int_0^\infty \xi C_0(\xi, m) e^{z_0} J_m(r\xi) d\xi, \quad (\text{A50})$$

$$A_1 = P \int_0^\infty \xi \left[\frac{\exp(-\nu_1 |z - h|)}{\nu_1} J_0(r\xi) \mathbf{i} + \sum_{m=0}^\infty \cos m\theta J_m(r\xi) \{ C_1(\xi, m) e^{z_1} + D_1(\xi, m) e^{-z_1} \} \right] d\xi, \quad (\text{A51})$$

$$A_2 = P \sum_{m=0}^\infty \cos m\theta \int_0^\infty \xi D_2(\xi, m) e^{-z_2} J_m(r\xi) d\xi. \quad (\text{A52})$$

The boundary conditions Eqs. (A16), (A17), and (A18) again take a simple form in the present problem. By considering the x and z components in Eqs. (A16) and (A17) and using arguments similar to those employed in the vertical dipole case, we deduce that:

$$[A_0 - A_1]_{z=0} = 0, \quad [A_1 - A_2]_{z=d} = 0. \quad (\text{A53})$$

It then follows from the y components of Eqs. (A16) and (A17) that:

$$\left[\frac{\partial}{\partial z} \{ (A_0)_x - (A_1)_x \} \right]_{z=0} = 0, \quad \left[\frac{\partial}{\partial z} \{ (A_1)_x - (A_2)_x \} \right]_{z=d} = 0. \quad (\text{A54})$$

Applying these conditions to the x components of the solutions Eqs. (A50), (A51), and (A52) and equating coefficients of $\cos m\theta$, we obtain for $m \neq 0$ a system of four homogeneous linear equations that possesses the trivial solution:

UNCLASSIFIED

UNCLASSIFIED

RANGINGS OF ELFE SOURCES OVER SHORT RANGES

697

$$\mathfrak{A}_0(\xi, m) = \mathfrak{A}_1(\xi, m) = \mathfrak{C}_1(\xi, m) = \mathfrak{C}_2(\xi, m) = 0.$$

The only contribution to the vector potential comes from the case $m = 0$, when the system is nonhomogeneous. It follows that A_x is independent of θ everywhere, so that we may write:

$$\frac{\partial A_x}{\partial x} = \frac{\partial A_x}{\partial r} \cos \theta,$$

and hence state boundary condition Eq. (A18) in the component form:

$$\left[\sigma_1 \left\{ \frac{\partial(A_2)_x}{\partial r} \cos \theta + \frac{\partial(A_2)_z}{\partial z} \right\} - \sigma_2 \left\{ \frac{\partial(A_1)_x}{\partial r} \cos \theta + \frac{\partial(A_1)_z}{\partial z} \right\} \right]_{z=d} = 0. \quad (\text{A55})$$

Likewise the property Eq. (A11) expressing the nondivergence of A_0 becomes:

$$\frac{\partial(A_0)_x}{\partial r} \cos \theta + \frac{\partial(A_0)_z}{\partial z} = 0. \quad (\text{A56})$$

It is immediately apparent from Eqs. (A55) and (A56) that the z component of the solutions Eqs. (A50), (A51), and (A52) will be nontrivial only when $m = 1$. We can, therefore, simplify our notation at this stage by writing $\mathfrak{A}(\xi) = \mathfrak{A}(\xi, 0)$, $\mathfrak{B}(\xi) = \mathfrak{B}(\xi, 1)$, $\mathfrak{C}(\xi) = \mathfrak{C}(\xi, 0)$, and $\mathfrak{D}(\xi) = \mathfrak{D}(\xi, 1)$ everywhere.

It now follows from Eq. (56) that $\mathfrak{A}_0 = \mathfrak{B}_0$ and from the other conditions Eqs. (A53) to (A55) that:

$$\nu_1 \mathfrak{A}_0 - \nu_1 \mathfrak{A}_1 - \nu_1 \mathfrak{C}_1 = e^{-h\nu_1},$$

$$\mathfrak{A}_0 - \mathfrak{B}_1 - \mathfrak{D}_1 = 0,$$

$$\nu_1 \mathfrak{A}_1 e^{2d\nu_1} + \nu_1 \mathfrak{C}_1 - \nu_1 \mathfrak{C}_2 e^{d(\nu_1 - \nu_2)} = -e^{h\nu_1},$$

$$\mathfrak{B}_1 e^{2d\nu_1} + \mathfrak{D}_1 - \mathfrak{D}_2 e^{d(\nu_1 - \nu_2)} = 0,$$

$$\xi \mathfrak{A}_0 - \nu_1 \mathfrak{A}_1 + \nu_1 \mathfrak{C}_1 = e^{-h\nu_1},$$

$$\nu_1 \mathfrak{A}_1 e^{2d\nu_1} - \nu_1 \mathfrak{C}_1 + \nu_2 \mathfrak{C}_2 e^{d(\nu_1 - \nu_2)} = e^{h\nu_1},$$

UNCLASSIFIED

UNCLASSIFIED

698

RUMBALL AND KAY

$$\epsilon \nu_1 \mathfrak{B}_1 e^{2d\nu_1} + (1 - \epsilon) \xi \mathfrak{C}_2 e^{d(r_1 - r_2)} - \epsilon \nu_1 \mathfrak{D}_1 + \nu_2 \mathfrak{D}_2 e^{d(r_1 - r_2)} = 0.$$

These equations can be solved for $\mathfrak{A}_1$, $\mathfrak{B}_1$, $\mathfrak{C}_1$, and $\mathfrak{D}_1$ and substitution made in Eq. (A51) to give the vector potential in region 1. Since we are confining our attention solely to the field in this region, we may again drop the subscript 1 on all field components. Also, it is convenient to reintroduce our previous notation involving the dimensionless quantities Eq. (A33) and the dimensionless variable u , and to define:

$$A' = 4\pi A / I(\mu_0 \sigma_1 \omega)^{\frac{1}{2}}.$$

We may then write the solutions in the form:

$$A'_x = \frac{\exp \left(-r'_0 \sqrt{i} \right)}{r'_0} + \int_0^\infty \frac{u}{\eta} \left\{ f \exp \left(-\eta (h' + z') \right) + p_+(u, z') \right\} J_0(r'u) \, du, \tag{A57}$$

and $A'_z = A''_z \cos \theta$, where:

$$A''_z = \int_0^\infty \left\{ (1 - f) \exp \left(-\eta (h' + z') \right) + q_-(u, z) \right\} J_1(r'u) \, du. \tag{A58}$$

Here we have defined:

$$f = \frac{\eta - u}{\eta + u}, \quad g = \frac{\eta - \eta_\epsilon}{\eta + \eta_\epsilon}, \quad \gamma = \frac{\epsilon \eta - \eta_\epsilon}{\epsilon \eta + \eta_\epsilon}, \tag{A59}$$

$$p_\pm(u, z') = \frac{g(1 \pm f e^{-2\eta z'}) (1 + f e^{-2\eta h'})}{(1 - fg e^{-2\eta d'})} \exp \left(-\eta (2d' - z' - h') \right), \tag{A60}$$

and

$$\begin{aligned} q_\pm(u, z') = & \frac{(fg + \gamma) (1 \pm \exp [-2\eta (z' - h')]) \exp [-\eta (2d' - z' + h')]}{(1 - fg e^{-2\eta d'}) (1 + \gamma e^{-2\eta d'})} \\ & + \left\{ \frac{g(1 \pm f^2 \exp [-2\eta (z' + h')])}{(1 - fg e^{-2\eta d'})} + \frac{\gamma(1 \pm \exp [-2\eta (z' + h')])}{(1 + \gamma e^{-2\eta d'})} \right\} \\ & \times \exp [-\eta (2d' - z' - h')], \end{aligned} \tag{A61}$$

UNCLASSIFIED

UNCLASSIFIED

RANGINGS OF ELFE SOURCES OVER SHORT RANGES

699

By some rather tedious algebraic rearrangement, it is possible to display the solutions Eqs. (A57) and (A58) (as was done in the vertical dipole case) as an infinite series that is independent of ϵ plus a term $O(\epsilon)$, viz.

$$A'_x = \sum_{n=-\infty}^{\infty} \int_0^{\infty} \frac{u}{\eta} \{f^{|2n|} \exp(-\eta |2nd' + z' - h'|) + f^{|2n+1|} \exp(-\eta |2nd' + z' + h'|)\} J_0(r'u) du + L_x, \quad (\text{A62})$$

$$A'_z = \sum_{n=-\infty}^{\infty} \text{sgn}(n + \frac{1}{2}) \int_0^{\infty} \{(1 - f^{|2n|}) \exp(-\eta |2nd' + z' - h'|) + (1 - f^{|2n+1|}) \exp(-\eta |2nd' + z' + h'|)\} J_1(r'u) du + L_z, \quad (\text{A63})$$

where L_x and L_z are infinite integrals with rather complicated integrands $O(\epsilon)$, which we shall not write out in full here. The interpretation of the solution in terms of image dipoles may be exhibited by writing:

$$f = 1 - \frac{2u}{\eta + u}, \quad (\text{A64})$$

in Eq. (A62) and expanding $f^{|2n|}$ and $f^{|2n+1|}$ binomially, but this appears to offer no computational advantage.

A more useful form of the solutions is obtained by substituting f in the form Eq. (A64) directly into Eqs. (A57) and (A58) and employing the Sommerfeld integral Eq. (A20). Writing:

$$M = \int_0^{\infty} \frac{2J_0(r'u)}{\eta(\eta + u)} e^{-\eta(z' + h')} du, \quad (\text{A65})$$

and using the well-known recurrence relations for the derivatives of Bessel functions, we then have

$$A'_x = \frac{e^{-R'_0 \sqrt{i}}}{R'_0} + \frac{e^{-R'_0 \sqrt{i}}}{R'_0} + \frac{1}{r'} \frac{\partial}{\partial r'} \left(r' \frac{\partial M}{\partial r'} \right) + \int_0^{\infty} \frac{u}{\eta} p_+(u, z') J_0(r'u) du, \quad (\text{A66})$$

$$A'_z = \frac{\partial^2 M}{\partial r' \partial z'} + \int_0^{\infty} q_+(u, z') J_1(r'u) du, \quad (\text{A67})$$

Moreover, the integral M can be evaluated exactly by noting that $(\eta + u)^{-1} = i(u - \eta)$ and then applying the Sommerfeld integral again and the z' derivative of a standard result found in tables. Defining $2s = R'_0 - z' - h'$ and $2S = R'_0 + z' + h'$, we obtain:

UNCLASSIFIED

UNCLASSIFIED

700

RUMBALL AND KAY

$$M = \frac{2ie^{-R'_0\sqrt{i}}}{R'_0} + N, \quad (\text{A68})$$

where:

$$N = -\left(2i\right)^{\frac{3}{2}}/R'_0 \left\{ SI_0(s\sqrt{i})K_1(S\sqrt{i}) + sI_1(s\sqrt{i})K_0(S\sqrt{i}) \right\}, \quad (\text{A69})$$

and I_n and K_n are the modified Bessel functions of order n .

When $\epsilon = 1$, we have $g = \gamma = 0$ and, hence, by Eqs. (A60) and (A61), $p_+ = q_- = 0$. Thus the solutions for the special case of the horizontal dipole in a uniform half-space are given by Eqs. (A66) and (A67) with the last terms, involving the infinite integral, set equal to zero. Although expressed in a slightly different form, this result agrees with that given in previous work.

The electromagnetic field components can be found by taking the appropriate derivatives of the vector potential according to the defining relations Eqs. (A3), (A4), and (A5). The variable θ enters the resulting expressions only in a common sine or cosine factor that is conveniently removed by defining:

$$\begin{aligned} H'_r &= H''_r \sin \theta, & H'_\theta &= H''_\theta \cos \theta, & H'_z &= H''_z \sin \theta, \\ E'_r &= E''_r \cos \theta, & E'_\theta &= E''_\theta \sin \theta, & E'_z &= E''_z \cos \theta. \end{aligned} \quad (\text{A70})$$

We then have:

$$H''_r = \partial A'_x / \partial z' - A''_z / r', \quad (\text{A71})$$

$$H''_\theta = \partial A'_x / \partial z' - \partial A''_z / \partial r', \quad (\text{A72})$$

$$H''_z = -\partial A'_x / \partial r', \quad (\text{A73})$$

$$E''_r = \partial^2 A'_x / \partial r'^2 + \partial^2 A''_z / \partial r' \partial z' - iA'_x, \quad (\text{A74})$$

$$E''_\theta = -\partial A'_x / r' \partial r' - \partial A''_z / r' \partial z' + iA'_x, \quad (\text{A75})$$

$$E''_z = \partial^2 A'_x / \partial r' \partial z' + \partial^2 A''_z / \partial z'^2 - iA''_z. \quad (\text{A76})$$

The application of these differentiations to the expressions Eqs. (A66) and (A67) can be simplified somewhat by substituting for M in the form Eq. (A68) and noting that M satisfies the differential equation:

UNCLASSIFIED

UNCLASSIFIED

RANGINGS OF ELFE SOURCES OVER SHORT RANGES

701

$$\frac{1}{r'} \frac{\partial}{\partial r'} \left[r' \frac{\partial M}{\partial r'} \right] + \frac{\partial^2 M}{\partial z'^2} = iM,$$

as is easily verified by differentiating the integral Eq. (A65). The parts of the solutions not involving integrals can then be written in terms of derivatives of N and the functions F_1 , F_2 , and F_3 introduced in Eqs. (A39), (A40), and (A41), together with a new function F_4 , defined by

$$F_4(\rho, \chi) = \chi \rho^{-4} \left[(5\chi^2 - 4)(3 + 3\rho\sqrt{i}) + ip^2 \{1 - (1 - \chi^2) \times (6 + \rho\sqrt{i})\} \right] e^{-\rho\sqrt{i}}. \quad (A77)$$

The full solutions now become:

$$H_r'' = -F_1(r'_0, \cos \psi_0) - F_1(R'_0, \cos \Psi_0) + 2i F_4(R'_0, \sin \Psi_0) + \frac{\partial^3 N}{\partial r'^2 \partial z'} + W_1, \quad (A78)$$

$$H_\theta'' = -F_1(r'_0, \cos \psi_0) - F_1(R'_0, \cos \Psi_0) + \frac{2i}{r'} F_2(R'_0, \sin 2\Psi_0) + \frac{\partial^2 N}{r' \partial r' \partial z'} + W_2, \quad (A79)$$

$$H_z'' = F_1(r'_0, \sin \psi_0) - F_1(R'_0, \sin \Psi_0) + 2i F_4(R'_0, \sin \Psi_0) + \frac{\partial^3 N}{\partial r' \partial z'^2} - i \frac{\partial N}{\partial r'} + W_3, \quad (A80)$$

$$E_r'' = F_3(r'_0, \cos \psi_0) + F_3(R'_0, \cos \Psi_0) - \frac{2}{r'} F_1(R'_0, \sin \Psi_0) - \frac{i}{r'} \frac{\partial N}{\partial r'} - W_4, \quad (A81)$$

$$E_\theta'' = \frac{ie^{-r'_0\sqrt{i}}}{r'_0} - \frac{ie^{-R'_0\sqrt{i}}}{R'_0} + \frac{F_1(r'_0, \sin \psi_0)}{r'} + \frac{F_1(R'_0, \sin \Psi_0)}{r'} - 2F_3(R'_0, \cos \Psi_0) + i \frac{\partial^2 N}{\partial r'^2} + W_5, \quad (A82)$$

$$E_z'' = F_2(r'_0, \sin 2\psi_0) + F_2(R'_0, \sin 2\Psi_0) - W_6, \quad (A83)$$

where, after noting that from Eqs. (A60) and (A61):

$$\frac{d}{dz'} \{p_\pm\} = \eta \{p_\mp\}, \quad \frac{d}{dz'} \{q_\pm\} = \eta \{q_\mp\},$$

and again using the recurrence relations for Bessel functions as necessary, we may write:

UNCLASSIFIED

UNCLASSIFIED

702

RUMBALL AND KAY

$$W = \int_0^{\infty} \left\{ u p_-(u, z') J_0(r'u) - q_-(u, z') \frac{J_1(r'u)}{r'} \right\} du, \quad (A84)$$

$$W_2 = \int_0^{\infty} \left[u \{ p_-(u, z') - q_-(u, z') \} J_0(r'u) + q_-(u, z') \frac{J_1(r'u)}{r'} \right] du, \quad (A85)$$

$$W_3 = \int_0^{\infty} \frac{u^2}{\eta} p_+(u, z') J_1(r'u) du, \quad (A86)$$

$$W_4 = \int_0^{\infty} \left[u \eta \{ p_-(u, z') - q_-(u, z') \} J_0(r'u) - \left\{ \frac{u^2}{\eta} p_+(u, z') - \eta q_+(u, z') \right\} \frac{J_1(r'u)}{r'} \right] du, \quad (A87)$$

$$W_5 = \int_0^{\infty} \left[\frac{i u}{\eta} p_+(u, z') J_0(r'u) + \left\{ \frac{u^2}{\eta} p_-(u, z') - \eta q_-(u, z') \right\} \frac{J_1(r'u)}{r'} \right] du, \quad (A88)$$

$$W_6 = \int_0^{\infty} u^2 \{ p_-(u, z') - q_-(u, z') \} J_1(r'u) du. \quad (A89)$$

We note in particular that from Eqs. (A83) and (A89), it can be deduced that $[E_z'']_{z=0} = 0$, which is, of course, a result to be expected on physical grounds.

REFERENCE

- A1. J. T. Weaver, "The Quasi-Static Field of an Electric Dipole Embedded in a Two-Layer Conducting Half-Space," Can. J. Phys. **45**, 1981-2002 (1967).

Edward I. Rumball received a B.E. degree (University of Canterbury) in electrical engineering and a B.Sc. degree (University of Auckland) in mathematics. He is presently manager of the Countermeasures Section at the Defence Scientific Establishment, Auckland Naval Base, Auckland, NZ.

John S. Kay gained his Ph.D. degree in electrical engineering from the University of Auckland in early 1978. He then worked in the Engineer-in-Chief's Office and in the Regional Engineer's Office, Wellington, of the New Zealand Post Office until late 1979. Since then he has been with the Defence Scientific Establishment, New Zealand Defense Force. At the time of writing, he was the project manager for the development and at-sea application of the multi-influence sensor equipment described in this article. Dr Kay is a member of the Institution of Electrical Engineers, United Kingdom, and of the Institution of Professional Engineers, New Zealand. He is also a Chartered Engineer, United Kingdom, and a Registered Engineer, New Zealand.

UNCLASSIFIED

Appendix C: Keywords for input data files of numerical models

(Appendix C describes a set of keywords³⁴ for the input data files that are identical to those developed at the Electromagnetic Compatibility Laboratory, University of Missouri, Rolla, for the EMAP series of numerical models.)

Keywords for input data files of numerical models

Keyword formats. Figure 1 describes the set of keywords used to define configuration parameters that were essential to the operation of the hybrid numerical model used in this study. The data input file of each model used only a selection of keywords from this specialist vocabulary comprising twelve strings of characters.

Keyword	Position coordinates	Cell dimensions	Secondary attributes
#			
unit			Number in m, cm or mm
boundary	x1 y1 z1 x2 y2 z2		
celldim	p1 p2	Δp	Axis (x, y or z)
dielectric	x1 y1 z1 x2 y2 z2		ϵ' ϵ''
conductor	x1 y1 z1 x2 y2 z2	Δx Δy Δz	
resistor	x1 y1 z1 x2 y2 z2		Resistor value in ohms
eplane			Frequency q1 j1 q2 j2 magnitude
vsource	x1 y1 z1 x2 y2 z2		Frequency polarization (x, y or z) magnitude
isource	x1 y1 z1 x2 y2 z2		Frequency polarization (x, y or z) magnitude
output	x1 y1 z1 x2 y2 z2		Axis (x, y or z) filename
default out			Filename

Figure 1. Keywords for input data files

Keyword definitions. The following list provides a definition for each of the keywords.

The model will skip each line beginning with the symbol #. All such lines can be used to add comments to the input files.

unit Within the hybrid numerical model the metre unit is the measure of distance. However, users can select their own units from the list provided (m, cm or mm) as the input data files are converted to metric geometry as required. The keyword "unit" specifies the length of one standard mesh cell along all three axes. The following specifies a standard cell to be two centimeters:

 unit 2.0 cm

All geometry specified by other keywords is now relative to the unit specified above. If it is necessary to specify the geometry in terms of an inch, the following line should be include in the input data file:

 unit 2.54 cm

Now, all geometry defined by other keywords will have a unit of one inch.

boundary The keyword "boundary" specifies a rectangular box-shape which forms the FEM region. The FEM region should enclose all dielectric slabs. In addition, the boundary should be as close as possible to the dielectric surfaces to reduce the computation time and memory requirements. If only one dielectric object is present the boundary should normally coincide with the outer surface of the dielectric material.

celldim By default, the mesh size for the FEM volume is one unit along each axis. However, users can specify a mesh step in an interval along the x, y or z axes by using the keyword "celldim". For example, the following line defines the mesh step to be two units in the interval [0, 10] along the x axis:

```
celldim 0 10 2 x
```

If the unit is defined as one centimetre, five segments will be generated in the interval [0 10] along the x axis. Each segment will have a length of two centimetres.

dielectric The keyword "dielectric" defines a rectangular dielectric slab of uniform properties. The values (xi, yi, zi) where $i = 1, 2$ specify two nodes on opposing corners of the slab. Parameters ϵ' and ϵ'' refer to the real and imaginary parts of the complex permittivity, respectively. A configuration can include one or more dielectric slabs. However, the FEM region defined by "boundary" must include all dielectric slabs.

conductor The keyword "conductor" defines one of several classes of conductors. At present, the model can only generate mesh files for patch (viz. two-dimensional) conductors. Traces are typical patch conductors. Thin wires can be treated as patch conductors by using an equivalent width. The model will use (Δx , Δy and Δz) as mesh steps along the x, y and z axes to subdivide the patch conductor and to generate edge grids.

The conductors are classified within the model into three categories, namely, "internal" conductors, "boundary" conductors and "external" conductors according to the geometric relationship between the conductors and the FEM boundaries. For "internal" and "boundary" conductors the values of Δx , Δy and Δz specified

by the keyword "conductor" will be ignored. Instead, these conductors are meshed using the Δx , Δy and Δz specified by the keyword "celldim" in order to match the grid edges along the boundary.

resistor This keyword defines a lumped resistor. A resistor should be defined as a line along either the x, y or z axis. Each edge coinciding with the line should have a resistor equal to the value specified in the parameter list. If a 50 ohm resistor is modeled as two edges, each edge would have a resistor of 25 ohms. Thus, the value in the corresponding parameter list should be 25 rather than 50.

eplane The keyword "eplane" defines an incident plane-wave source. The unit of frequency is the value in MHz. Parameters q1 and j1 define the E-field unit vector while the values q2 and j2 define the direction of propagation. The following string defines a 300 MHz plane wave travelling along the positive z axis. The polarization of the E-field is along the x axis and the magnitude of the E-field is one volt.meter⁻¹:

```
eplane 300 90 0 0 0 1.0
```

vsource This keyword defines a delta-gap voltage source. The two nodes specified by values (x1, y1, z1) and (x2, y2, z2) in the parameter list specify a line which is the source location. The voltage source can be only defined on metal patches that are not within the FEM region. The source should be located on grid edges defined by keywords "conductor", "unit" or "celldim".

isource The keyword defines an impressed current source. The two nodes specified by values (x1, y1, z1) and (x2, y2, z2) in the parameter list specify a line which is the source location. The current source can be defined only within the FEM region. The source should be located on grid edges defined by keywords "unit" or "celldim".

output The keyword "output" defines the region in which the E-field values of interest will be printed by the model. The parameter strings (x1, y1 and z1) and (x2, y2 and z2) specify two nodes on opposite corners of a rectangle. All edges within this rectangle and parallel to the specified axis will be printed to the file specified in the parameter list. For example the following character string will generate a print-out of the E-field values in file "E1.out" along edges which are parallel to the x axis, and within the rectangle area specified by two diagonal nodes (0, 0, 0) and (10, 10, 0):

```
output 0 0 0 10 10 0 x E1.out
```

default out This keyword provides for all E-field values calculated within the hybrid numerical model to be printed to the file specified in the parameter list. For example, the following character string causes all field values to be printed to file "E1.out":

```
default_out E1.out
```

Appendix D: Glossary of abbreviations

Glossary of abbreviations

AC	alternating current (flow)
B-field	magnetic field (strength)
CPA	closest point of approach
CTD	conductivity-temperature-density (measuring instrument)
DC	direct current (flow)
DGPS	differential global positioning system
DSE	Defence Scientific Establishment (now renamed Defence Operational Technology Support Establishment)
E-field	electric field (strength)
ELF	extremely low-frequency (band)
ELFE	extremely low-frequency electromagnetic (field)
FEM	finite element method (volume)
FDTM	finite-difference, time-domain (model)
GPS	global positioning system
GRP	glass reinforced plastic (material)
HED	horizontal electric dipole

MCM	mine countermeasures
MISS	multi-influence sensor system
MKS	metre kilogram second (international system of units)
MOM	method of moments (region)
PDE	partial differential equation
PVC	polyvinyl chloride (material)
RAM	random access memory (of a computer)
RNZN	Royal New Zealand Navy
TTCP	The Technical Cooperation Programme (of Australia, Canada, New Zealand, the United Kingdom and the United States)
USRL	Underwater Sound Reference Laboratory
VED	vertical electric dipole

Bibliography

Bibliography

1. C.V. Drysdale, "The distribution of the magnetic field and return current round a submarine cable carrying alternating current, 1," *Philos. Trans. R. Soc. London* **224**, 95-140 (1924).
2. S. Butterworth, "The distribution of the magnetic field and return current round a submarine cable carrying alternating current, 2," *Philos. Trans. R. Soc. London* **224**, 141-184 (1924).
3. C. Wright, "Leader gear," in *Proceedings of the symposium on underwater electromagnetic phenomena*, edited by G.W. Wood (National Academy of Sciences National Research Council, Washington DC, 1953), pp. 177-184.
4. E.L. Maxwell, "Atmospheric noise from 20 Hz to 30 kHz," *Rad. Scien.* **6**, 637-644 (1967).
5. L.H. Ginsberg, "Extremely low-frequency atmospheric noise level statistics for Project Sanguine," *IEEE Trans. Commun.* **COM-22(4)**, 555-567 (1974).
6. E.F. Soderberg, "ELF noise in the sea at depths from 30 to 300 meters," *J. Geophys. Res. (Space Physics)* **74(9)**, 2376-2387 (1969).
7. S.C. Webb, S.C. Constable, C.S. Cox, and T.K. Deaton, "A seafloor electric field instrument," *J. Geomag. Geoelectr.* **37**, 1115-1129 (1985).
8. A.C. Fraser-Smith and D.M. Bubenik, "ULF/ELF electromagnetic fields generated at the sea surface by submerged magnetic dipoles," *Rad. Scien.* **11**, 901-913 (1976).
9. P.R. Bannister and R.L. Dube, "Numerical results for modified image theory quasi-static range, subsurface-to-subsurface and subsurface-to-air

- propagation equations,” Tech. Rept. 5775, Naval Underwater Systems Center, New London, Conn., 1977.
10. D.M. Budenik and A.C. Fraser-Smith, “ULF/ELF electromagnetic fields generated in a sea of finite depth by a submerged vertically-directed magnetic harmonic dipole,” *Rad. Scien.* **13**, 1011-1020 (1978).
 11. J.R. Wait, “Project SANGUINE,” *Science* **178**, 272-275 (1972).
 12. J.R. Wait, *Electromagnetic waves in stratified media* (Pergamon Press, Oxford, 1972), 2nd ed., Chap. 6.
 13. Multiple authors, “Special Issue on Extremely Low Frequency (ELF) Communications,” *IEEE Trans. Commun.* **COM-22(4)**, (1974).
 14. P.R. Bannister, “Determination of the electrical conductivity of the seabed in shallow waters,” *Geophys.* **33**, 995-1003 (1968).
 15. A.D. Chave, S.C. Constable, and R.N. Edwards, *Electrical exploration methods in applied geophysics*, edited by N.M. Nabighian (Society of Exploration Geophysicists, Tulsa, Okla., 1990), Vol. 2.
 16. R.N. Edwards, L.K. Law, and J.M. DeLauier, “On measuring the electrical conductivity of the oceanic crust by a modified magnetometric resistivity method,” *J. Geophys. Res.* **86**, 11609-11615 (1981).
 17. A.D. Chave and S.C. Cox, “Controlled electromagnetic sources for measuring electrical conductivity beneath the oceans, 1, forward problem and model study,” *J. Geophys. Res.* **87**, 5327-5338 (1982).
 18. V.R.S. Hutton, M.R. Ingham, and E.W. Mbipom, “An electric model of the crust and upper mantle in Scotland,” *Nature* **287**, 30-33 (1980).

19. Unspecified author(s), *Extremely low frequency (ELF) fields (Environmental Health Criteria 35)*, (World Health Organization, Geneva, 1984).
20. R.T. Hitchcock and R.M. Patterson, *Radio-frequency and ELF electromagnetic energies* (Van Nostrand Reinhold, New York, NY, 1995), Chap. 9.
21. J.B. Keller and W. Munk, "Internal wave wakes of a body in a stratified fluid," *Phys. Fluids* **13**, 1425-1430 (1970).
22. B.J. Marshall, "Undersea measurement ranges – the impact of technology," in *Proceedings of the Undersea Defence Technology Conference – UDT 1988*, October 1988, London (Undersea Defence Technology, London), pp. 681-686.
23. D. Madurasinghe and E.O. Tuck, "The induced electromagnetic field associated with submerged moving bodies in an unstratified conducting fluid," *IEEE J. Ocean. Eng.* **19(2)**, 193-199 (1994).
24. E.I. Rumball, "Estimating conductivity parameters for extremely low-frequency electromagnetic models," presented at the 20th meeting of TTCP panel GTP-13, Defence Research Establishment Pacific, Victoria, Can. (October, 1994).
25. E.I. Rumball and J.S. Kay, "Modelling of extremely low-frequency electromagnetic propagation over short ranges and shallow-water conditions," presented at the 22nd meeting of TTCP panel GTP-13, Naval Research Laboratory, Stennis Space Centre, MS (October 16-20, 1995).
26. E.I. Rumball and J.S. Kay, "Rangings of extremely low-frequency electromagnetic sources over short ranges and in shallow-water conditions," *J. Underwater Acoustics* **47(2)**, 669-702 (1997).

27. J.S. Kay, "The New Zealand development of equipment for the measurement of underwater multi-influence fields," in *Proceedings of the Institution of Professional Engineers of NZ Conference*, Dunedin, 1996, (Institution of Professional Engineers NZ, Wellington), pp. 16-21.
28. C.C. Sims, "High-fidelity underwater sound transducers," *Proc. IRE* **47**, 866-871 (1959).
29. J.M. Tricker, "A sub-nanovolt chopper amplifier," in *Proceedings of the 30th NZ National Electronics Conference*, 1993, Auckland (NZ National Electronics, Auckland), pp. 125-132.
30. J.S. Kay, E. Marchant, and A.D. Dorrington, "StoreMouse - a versatile data collection, storage and on-line analysis facility," in *Proceedings of 1st Electronics NZ Conference*, 1994, Hamilton (Electronics NZ, Auckland), pp. 43-48.
31. J.S. Kay, M.E. Bold, and A.R. Maggs, "Analog signal reduction," in *Proceedings of the 2nd Electronics NZ Conference*, Auckland, 1995, (Electronics NZ, Auckland), pp. 77-82.
32. AIS Electronics Inc., La Jolla, CA, EF sensor model J, (Private communication, September 1995).
33. R.A. Geyer, *CRC Handbook of geophysical exploration at sea* (CRC Press, Boca Raton, Florida, 1983), p. 353.
34. T.H. Hubing, C. Hong-Him Lim, and J. Drewniak, "A geometry description language for 3D electromagnetic analysis codes," in *Proceedings of the 10th annual review of Applied Computational Electromagnetics*, March, 1983, Monterey, CA, pp. 417-422.

35. W. von Aulock, "Low-frequency electromagnetic dipole fields in a semi-infinite conductor," Tech. Rept. 104, Navy Dept. Bur. Ships, Minesweeping Section, Washington, DC, 1952.
36. A. Banos and J.P. Wesley, "The horizontal electric dipole in a conducting half-space," Tech. Rept., Marine Phys. Lab., Univ. California, 1953.
37. J.R. Wait, "The electromagnetic fields of a horizontal dipole in the presence of a conducting half-space," *Can. J. Phys.* **39**, 1017-1028 (1961).
38. A.F. Peterson, S.L. Ray and R. Mittra, *Computational methods for electromagnetics*, (IEEE, New York, NY, 1998).
39. J.R. Wait, *Electromagnetic waves in stratified media* (Pergamon Press, New York, NY, 1962).
40. A. Sommerfeld, *Partial differential equations in physics* (Academic Press, New York, NY, 1949).
41. M.B. Kraichman, *Handbook of electromagnetic propagation in conducting media* (US Government Printing Office, Washington, DC, 1970).
42. A.D. Chave, "Some comments on seabed propagation of ULF/ELF electromagnetic fields," *Rad. Scien.* **25(5)**, 825-836 (1990).
43. C.S. Cox, S.C. Constable, A.D. Chave, and S.C. Webb, "Controlled-source electromagnetic sounding of the ocean lithosphere," *Nature* **320**, 52 (1986).
44. P.R. Bannister, "Determination of the electrical conductivity of the sea bed in shallow waters," *Geophys.* **33(6)**, 995-1003 (1968).
45. J.T. Weaver, "The quasi-static field of an electric dipole embedded in a two-layer conducting half-space," *Can. J. Phys.* **45**, 1981-2002 (1967).

46. R.B. Hurst, J.F.F. Pope, R.H. Poeckert, and D.N. Lambert, "A small seabed penetrometer for measuring sediment bearing strength," presented at the TTCP Subgroup G, 1996 Symposium, Defence Research Establishment Atlantic, Halifax, Nova Scotia (22-25 October 1996).
47. W. H. Press (et al), *Numerical recipes in FORTRAN ; the art of scientific computing* (Cambridge University Press, Cambridge UK, 1992), 2nd ed.
48. L. Lapidus and G.F. Pinder, *Numerical solutions of partial differential equations in science and engineering* (John Wiley and Sons, New York, NY, 1982).
49. Germund Dahlquist and Ake Bjorck, *Numerical methods* (Prentice Hall, Englewood Cliffs, NJ, 1974).
50. R.C. Booton, *Computational methods for electromagnetics and microwaves* (John Wiley and Sons, New York, NY, 1992).
51. R.F. Harrington, *Field computation by moment methods* (Macmillan Co., New York, NY, 1968).
52. P.P. Silvester and R.L. Ferrari, *Finite elements for electrical engineers* (Cambridge University Press, New York, NY, 1983).
53. C.W. Steele, *Numerical computation of electric and magnetic fields* (Van Nostrand Reinhold, New York, NY, 1987).
54. D.R. Lynch and K.D. Paulsen, "Origin of vector parasites in numerical Maxwell solutions," IEEE Trans. Microwave Theory and Tech. **MTT-39**, 383-394 (1991).

55. K.D. Paulsen and D.R. Lynch, "Elimination of vector parasites in finite element Maxwell solutions," *IEEE Trans. Microwave Theory and Tech.* **MTT-39**, 395-404 (1991).
56. J.G. Blaschak and G.A. Kriegsmann, "A comparative study of absorbing boundary conditions," *J. Comp. Phys.* **77**, 109-139 (1988).
57. K.D. Paulsen, D.R. Lynch, and J.W. Strohbehn, "Numerical treatment of boundary conditions at points connecting more than two electrically distinct regions," *Commun. Appl. Numer. Meth.* **3**, 53-62 (1987).
58. K.S. Yee, "Numerical solutions of initial boundary value problems involving Maxwell's equations in isotropic media," *IEEE Trans. Antennas Propagation* **AP-14**, 302-307 (1966).
59. A. Taflove and K. Umashankar, "A hybrid moment method/finite-difference time-domain approach to electromagnetic coupling and aperture penetration into complex geometries," *IEEE Trans. Antennas Propagation* **AP-30**, 617-627 (1982).
60. W.E. Boyse and A.A. Seidl, "A hybrid finite element and moment method for electromagnetic scattering from inhomogeneous objects," in *Proceedings of the 7th Annual Review of progress in Applied Computational Electromagnetics* (IEEE, New York, NY, 1991), pp. 160-169.
61. K.D. Paulsen, D.R. Lynch, and J.W. Strohbehn, "Three-dimensional finite, boundary, and hybrid element solutions of the Maxwell equations for lossy dielectric media," *IEEE Trans. Microwave Theory* **MTT-36**, 682-693 (1988).
62. M.W. Ali, T.H. Hubing, and J. Drewniak, "A hybrid FEM/MOM technique for electromagnetic scattering and radiation from dielectric objects with attached wires," *IEEE Trans. Electromag. Compat.* **39**, 304-314 (1997).

63. T. Hubing, M. Ali, and G. Bhat, "EMAP: Finite element modelling code for analyzing time-varying electromagnetic fields," J. Applied Computational Electromagnetic Society **8(1)**, 116-124 (1993).
64. J.S. Wang, "On edge based finite elements and method of moments solutions of electromagnetic scattering and coupling," Ph.D. dissertation, University of Akron, OH, 1992.
65. S.M. Rao, D.R. Wilton, and A.W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," IEEE Trans. Antennas Propagation **AP-30**, 409-418 (1982).
66. M.L. Barton and Z.J. Cendes, "New vector finite elements for three-dimensional magnetic field computations," J. Appl. Phys. **61**, 3919-3921 (1982).
67. *Patch code user's manual* (Houston Press, Univ. Texas, Houston, TX).
68. M.W. Ali, "Development of a hybrid 3D numerical modelling technique for analyzing printed circuit models with attached wires," Ph.D. dissertation, Univ. Missouri, Rolla, 1996.
69. K. Umashankar, A. Taflove, and S.M. Rao, "Electromagnetic scattering by arbitrary shaped three-dimensional homogeneous lossy dielectric objects," IEEE Trans. Antennas Propagation **AP-34**, (1986).
70. J. Jin, *The finite element method in electromagnetics* (McGraw-Hill, New York, NY, 1995).
71. D.K. Cheng, *Field and wave electromagnetics* (Addison-Wesley Publishing, Reading, MA, 1990), 2nd ed.

72. P.E. Gill, W. Murray, and M. Wright, *Practical optimization* (Academic Press, New York, NY, 1981).
73. J. Helder and R. Meads, "A simplex method for function minimization," *Computer Journal* **7**, 308–313 (1965).
74. Unspecified author(s), "A virtual Yucca mountain," *Nuclear Engineering International* **45**, 33 (2000).