COMPUTER CONTROL OF THE MANUFACTURE OF SPRAY DRIED MILK POWDER

A thesis presented in fulfilment
of the requirements for the degree
of Doctor of Philosophy in
Industrial Management and
Engineering at Massey University

Owen Wong BE (HONS) (Electrical)
ME (Eng. Science)

July, 1979

To my parents, brothers and sister

in appreciation of their love,
understanding and encouragement

ABSTRACT

This thesis describes the development and application of modern control techniques to an industrial processing plant. The plant is made up of a pilot scale triple effect Wiegand falling film evaporator and a De Laval tall form spray drier situated at the laboratories of the New Zealand Dairy Research Institute.

The continuous version of the standard state space and measurement equations for linear, time-invariant, multivariable systems provides the basis for the development. This standard representation is not well suited for direct application to the plant because of the difficulty in defining a suitable state vector and the need for a precise knowledge of the process dynamics. The standard representation is therefore enhanced to provide a modified structure that exhibits several significant features:-

i) only measurable outputs, control inputs and measurable disturbances are involved in the process descriptions;

ii) the equation structure is similar to that of the standard discrete state space equations, so that state space time domain control system design methods are applicable;

iii) the structure is suitable for the direct application of statistical modelling;

iv) maximum flexibility is offered for selection of discrete intervals for measurement samples and controls;

v) precise knowledge of process time delays is not required.

Application of the control strategies is performed using an IBM System/7 minicomputer. A suite of computer programs, developed in FORTRAN, is used. These programs are suitable for transferring to any other minicomputer supporting a multi-tasking mode of operation with the major system dependent area being the actual sampling of process data. Whilst a control engineer would be required for the installation of the software and initial control system design, further tuning of the controllers could be performed by a suitably trained operator.

The procedures described allow an operator to apply sophisticated control techniques by following a sequence of relatively simple steps:-

a) apply perturbations to normal plant operating conditions;
b) use the interactive graphics facility to identify process models using the generalised least squares technique;
c) choose appropriate design weightings and use a dynamic programming algorithm to obtain controller gains;

d) simulate operation of the controller using the process model;

e) apply the controller to the actual plant.

Steps b), and c) and d may need to be repeated until a suitable controller is obtained.

The results of the implementation of controllers on the pilot scale plant are presented. Consistent control of the product density and flowrate from the evaporator is obtained. As an hierarchical control structure has been used on the evaporator, the development of lower level controllers on variables such as steam flowrate, preheat temperature, inlet milk flowrate and condenser water temperature is also described. On the spray drier environmental control on concentrate flowrate, gas flow-rate and drier chamber pressure is obtained.

ACKNOWLEDGEMENTS

The work described in this thesis was carried out as part of a joint project between the following organisations:-

Industrial Management and Engineering Department, Massey University;
New Zealand Dairy Research Institute;
Department of Scientific and Industrial Research;
New Zealand Dairy Board;
IBM (N.Z.) Limited.

I gratefully acknowledge these organisations for their provision of equipment and resources.

Within these organisations are individuals with whom I worked on a close, personal basis and to them I wish a very special thank-you.

Dr David Sandoz, Professor Kelvin Scott and Tim Hesketh of the IME Department for their continual support, enthusiasm and guidance of the work.

Chris Bloore and Tony Baucke of the NZDRI for the time and effort they spent with me in many invaluable technical discussions.

I wish to acknowledge also the Postgraduate Scholarship awarded by the University Grants Committee and Massey University.

Lastly, thanks to Coral Reid for the preparation of the final manuscript.

TABLE OF CONTENTS

Page

CHAPTER ONE

INTRODUCTION

Over the past twenty years there have been few technological developments which cannot be attributed either directly or indirectly to a corresponding development in the area of digital computers. The variety of applications in which computers are used is diverse and continually increasing as the cost and size of computers decreases. The computer has had an impact on development in established technologies as well. The increasing processing power of the computer has enabled the solution of problems in areas of work which had been slowed or halted because of the sheer numerical complexity of analysis. In other technologies, the impact of the computer has given rise to reappraisal and innovation in the methods of problem solution. This is well illustrated by developments in the field of control theory.

Much effort from sources throughout the world has been put towards the advancement and application of control theory. Limited resources in terms of energy supplies and raw materials coupled with inflationary costs means that many of today's complex manufacturing processes could benefit from the application of sophisticated modern control techniques. For various reasons however, there are relatively few instances where theory has been applied to a real system.

The work in this thesis attempts to show the application of modern techniques in an industry in which control has traditionally been based on experience. An outline of the development of control theory is presented, followed by an overview of the New Zealand dairy industry, with particular relation to the manufacture of milk powder. The second chapter details the identification of a least squares model for a generalised system. An output feedback algorithm utilising dynamic programming techniques is developed from the model structure. Chapter three concerns the development of two software systems for the implementation of computer control. Simulation methods are used to validate the operation of the identification and control algorithms. The application and implementation of the software systems and control techniques to a pilot scale milk powder manufacturing plant situated in the laboratories of the New Zealand Dairy Research Institute is described in the fourth chapter.

Control theory became established some thirty or forty years ago with the advent of classical control theory. This involved Bode, Nyquist and root locus techniques which were directed to the assessment of the stability and behaviour of linear scalar systems.[1,2,3,4] A system is defined by a differential equation expressed in terms of the Laplace operator s. Control is obtained by introducing some form of feedback compensation and the configuration is analysed and corrected until a satisfactory design is achieved. This procedure works well for simple system configurations but as the system order and complexity increases, the number and degree of equations to be solved quickly becomes excessive. This is particularly the case when multiple inputs and outputs are involved.

State space analysis was introduced into control theory relatively recently. Analysis by this means systems are represented by equations expressed in the time domain.[1,5,6,7] Linear systems of any order may then be expressed as a first order matrix vector differential equation of the form

$$\dot{\underline{x}} = f(\underline{x}, \underline{u}, t) \qquad (1)$$

where $\underline{x}$ is the state vector of the system, $\underline{u}$ is the input vector and t is the scalar representation of time. Any quantity which uniquely reflects some dynamic property of the process may be defined as a state variable. The order of the process is in turn defined by the number of state variables required to completly determine the dynamic behaviour of the system. Equation (1) can be solved for both continuous and sampled data systems.[8] The presence of digital computing techniques has lead to the discrete form of system representation taking precedence in control theory. The discrete solution to (1) is commonly known as the state predictor equation and related the current values of $\underline{x}$ and $\underline{u}$ to the system states at the beginning of the next discrete interval. This equation has the form

$$\underline{x}_{(k+1)T} = \underline{\alpha} \begin{bmatrix} \underline{x}_{kT} \\ \underline{u}_{kT} \end{bmatrix} \qquad (2)$$

where $\underline{\alpha}$ represents the system dynamics of the discrete interval T. The suffixes to the other vectors relate them to the time at which their values were sampled.

Efficient use of the state space analysis techniques coincided with the introduction of the early digital computers. Initial usage was as a tool for the completion of the laborious numerical calculations associated with systems and control design. It was not long however before the potential of the computer, to act as both the numerical tool and perform as an online controller, was exploited. Computers and software systems specifically designed for process control applications had a significant and far reaching impact.[9,10] Control system design became associated with the programming of a computer thus enabling the application of complex control techniques which would be difficult to implement using conventional hardware.

These techniques included Lyapinov stability analysis, calculus of variations, probability and statistical theory. Pontryagin's Maximum Principle provided an analytical solution to the optimal control problem. In practice, however, this method had limitations and it was not until the introduction of Bellman's dynamic programming optimisation algorithm that application of optimal control theory became a reality.[1,5] Though the analysis involved is complex, the final algorithm is straightforward and easy to implement using a computer. A requirement for use of this algorithm is a performance criterion. A quadratic performance criterion was shown to be extremely useful as it lead to an optimal policy that is a linear function of the current system states for all points in time. Quadratic performance criteria have therefore been widely used in control theory with suitable parameter selection[11] enabling various aspects of the overall system performance to be optimised.

Before an optimal control system can be designed, an accurate representation of the system dynamics is required. This may be obtained from the equations governing the physics of the process, but in many cases, particularly for real systems, it is difficult to achieve, and it is far easier to obtain the dynamics directly from operational plant data - a procedure called identification.[12] Many mathematical methods have been devloped to perform identification[13,14,15] including correlation analysis, spectral analysis, hill climbing, maximum likelihood and least squares. Identification (modelling) in this thesis is performed using the latter technique which has been extensively investigated by others[16,17,18,19,20] and has been found to offer some advantages over the other methods. In practice the system states may not all be accessible. If the system parameters are known an estimation procedure may be used to determine the inaccessible states. Alternatively the system may be described in terms of its input and output states only. The equations obtained are similar in form to the state space equations and the parameters may be obtained directly using standard identification procedures. Many of the established optimisation procedures can also be applied directly, thus eliminating any need to define and measure states. This form of representation is used throughout the thesis.

The control techniques described in this thesis have been applied to a pilot scale milk powder manufacturing plant situated in the laboratories of the New Zealand Dairy Research Institute.[37,38] Detailed process descriptions will be given later but at this point an overview of the New Zealand dairy industry would be useful. The dairy industry accounts for approximately 25% of New Zealand's total export earnings. Factory production has diversified from a single basic product and almost half the milk processed is received by companies equipped to manufacture butter and fat-related products, cheeses, spray dried products and protein-products, such as casein and its derivatives, all within the one unit. In achieving this diversification the number of spray-drying plants has increased to about three times as many as existed in the early 1960's.[21] One of the main reasons for the growth of the spray-drying section of the industry has been the demand for specification milk powders from Japan, South East Asia and Latin America. The requirements of these customers are rigid and the New Zealand Dairy Board ensures quality control by providing financial incentives for dairy companies to produce milk powder conforming to the desired specification.

Some 40 different specifications have currently been set. In many cases, however, there are only small qualitative differences between a premium grade powder and lower grade product but large financial drawbacks. This places pressure on managerial and technical staff to maintain product quality in the face of seasonal variations in milk properties and a general lack of management information. Studies have shown the benefit arising from improved control in manufacture.[22]

CHAPTER TWO

GENERALISED EQUATION STRUCTURE FOR THE REPRESENTATION OF INDUSTRIAL PROCESS DYNAMICS

2.1 Introduction

Many industrial processes, or subsystems of such processes, may be represented schematically by figure 2-1. In general such systems incorporate a number of measurable outputs, controllable inputs and disturbances which cause undesirable variations. Control inputs and disturbances often have an associated time delay, particularly when the process is one involving material feed mechanisms. This makes the derivation of analytical descriptions difficult because of the effort required to develop a comprehensive description of the internal mechanisms. The alternative is to apply statistical analysis to data recorded directly from the plant. These methods normally evaluate linearised approximations of the process dynamics unless prior knowledge of plants non-linear characteristics is available. Statistically derived models are often adequate for the purpose of control system design although they do not necessarily provide the accuracy or insight that may be possible with analytical derivations.

Statistical identification of process dynamics has been long established and widely applied. Most effort has concentrated on the modelling of systems with a single control input, a single output and, occasionally, a single disturbance signal. Representation frequently takes the form of a transfer function and extension to the multivariable situation is often considered by modelling each input to output path in isolation. These models are then combined in a matrix transfer function expression to form a complete system description. The methods can

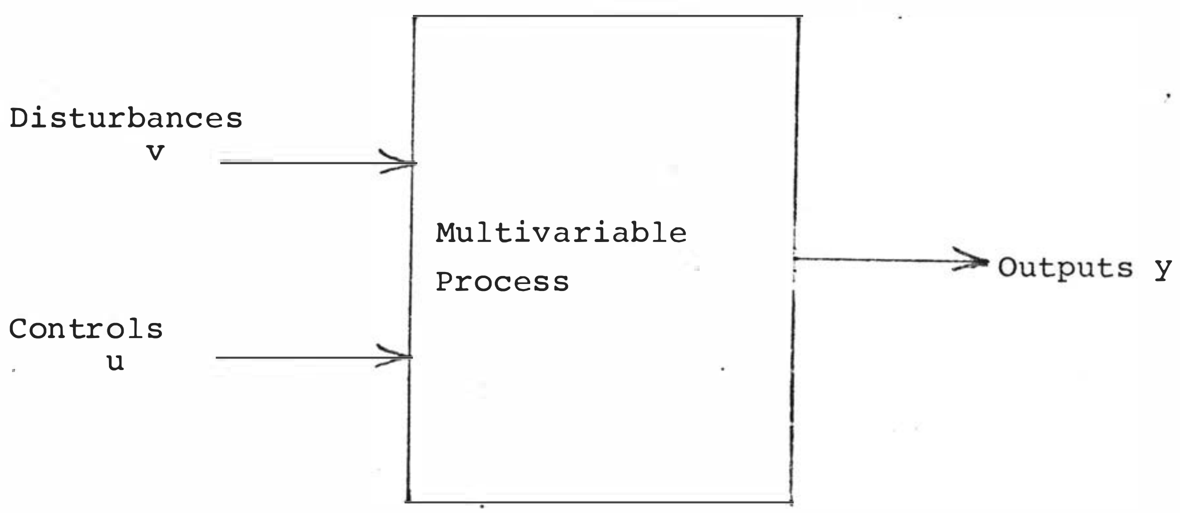


Figure 2-1 System schematic

become complicated in such circumstances and control system design is difficult. The matrix structure of the state space representation is more convenient for use with multivariable systems. The input to output inter-connections are not treated individually but are defined within the structure of the state, input and measurement matrices.

State space methods have not been widely used in industrial applications for control system design despite the fact that such design, in the time domain, has many advantages. The reason for lack of application is most often the difficulty in selecting a suitable state vector. With complex processes, particularly those involving transport delays, a state vector is not easily defined. Also, in a practical environment, the states, though known, are often difficult or expensive to measure directly. Thus statistical modelling of state space descriptions of industrial process dynamics has only received narrow application.

The equation structure presented resolves some of these difficulties. The continuous version of the standard state space and measurement equations for linear and time invariant multivariable systems, forms the basis for the analysis. The representation is enhanced to incorporate continuously varying disturbances and pure time delays. The objective of the derivation is to produce a discrete time domain equation with the following properties:

i) only measurable outputs, control inputs and measurable disturbances are involved in the process description;

ii) the equation structure is similar to that of the standard discrete state space equation so that state space time domain control system design methods are applicable;

iii) the structure is suitable for the direct application of statistical modelling;

iv) maximum flexibility is offered for selection of discrete intervals for measurement sampling and control; and

v) precise knowledge of process time delays is not required.

Satisfaction of these properties leads to a structure that is well suited for the approximation of industrial plant dynamics and subsequent control system development using computer aided design techniques.

2.2 Basis for Analysis

The basis for analysis is the continuous form of the linear, time invariant, multivariable state space and measurement equations.

$$\dot{\underline{x}} = \underline{A}\ \underline{x} + \underline{B}\ \underline{u} + \underline{D}\ \underline{v} \tag{1}$$

and

$$\underline{y} = \underline{c}\ \underline{x} \tag{2}$$

It is necessary that approximations in terms of these equations are valid. The following definitions apply: $\underline{x}$, nxl state vector; $\underline{v}$, pxl output vector; $\underline{A}$, nxn state matrix; $\underline{B}$, nxm input matrix; $\underline{C}$, pxn measurement matrix; and $\underline{D}$, nxq disturbance matrix. The controls $\underline{u}$, mxl, are freely adjustable. The disturbances, $\underline{v}$, qxl, are measurable but very continuously in an uncontrolled fashion.

To create a model suitable for the design of computer control systems, a discrete version of equation (1) is obtained. For this purpose, a discrete interval of t seconds is defined. The controls are constrained to be constant across this interval. The disturbances are approximated as ramp functions, with constant gradients across the discrete interval. Fig.2-2 illustrates the control and disturbance signal characteristics. The ramp approximations are acceptable if the disturbances vary slowly relative to the interval t. Piecewise ramp approximations of the continuously varying signals permit a more accurate process representation than piecewise steps.

The discrete solution to equation (1) may be derived as[7],

$$\underline{x}_{k+1} = \underline{\phi}.\underline{x}_k + \underline{\Delta}.\underline{u}_k + \underline{\Delta 1}.\underline{v}_k + \underline{\Delta 2}.V_{k+1} \qquad (3)$$

with $\underline{\phi}$, nxn = $e\underline{A}^t$, and $\underline{x}_k$, $\underline{u}_k$ and $\underline{v}_k$ the values of $\underline{x}$, $\underline{u}$ and $\underline{v}$ respectively, at instant kt (see fig. 2). Also,

$$\underline{V}_{k+1} = \underline{v}_{k+1} - \underline{v}_k \;,\quad \underline{\Delta}\;,\quad nxm = \underline{A}^{-1}.(\underline{\phi}-\underline{I}).\underline{B},\quad \underline{\Delta 1}\;,$$

$$nxq = \underline{A}^{-1}.(\underline{\phi}-\underline{I}).\underline{D} \quad \text{and} \quad \underline{\Delta 2}\;,\quad nxq = \underline{A}^{-1}.(\underline{\Delta 1}-\underline{D}.t).$$

To increase the generality of the representation, it is specified that all controls and disturbances are subject to a time delay of τ_d secs. In reality, individual delays will differ. Generalisation to this situation is considered at a later stage. Equation (3) is therefore rewritten;

$$\underline{x}_{k+1} = \underline{\phi}.\underline{x}_k + \underline{\Delta}.\underline{u}_{k-d} + \underline{\Delta 1}.\underline{v}_{k-d} + \underline{\Delta 2}.\underline{V}_{k+1-d} \qquad (4)$$

with $d = \dfrac{\tau_d}{t}$. Interval t is selected so that d is integer. It is advantageous, from control system aspects, if the process model does not incorporate absolute values of controls and disturbances.[8] Equation (4) is therefore restructured to give;

$$\underline{X}_{k+1} = \underline{\Phi}.\underline{X}_k + \underline{\psi}.\underline{U}_{k-d} + \underline{\Omega}.\underline{V}_{k-d} + \underline{n}.\underline{V}_{k+1-d} \qquad (5)$$

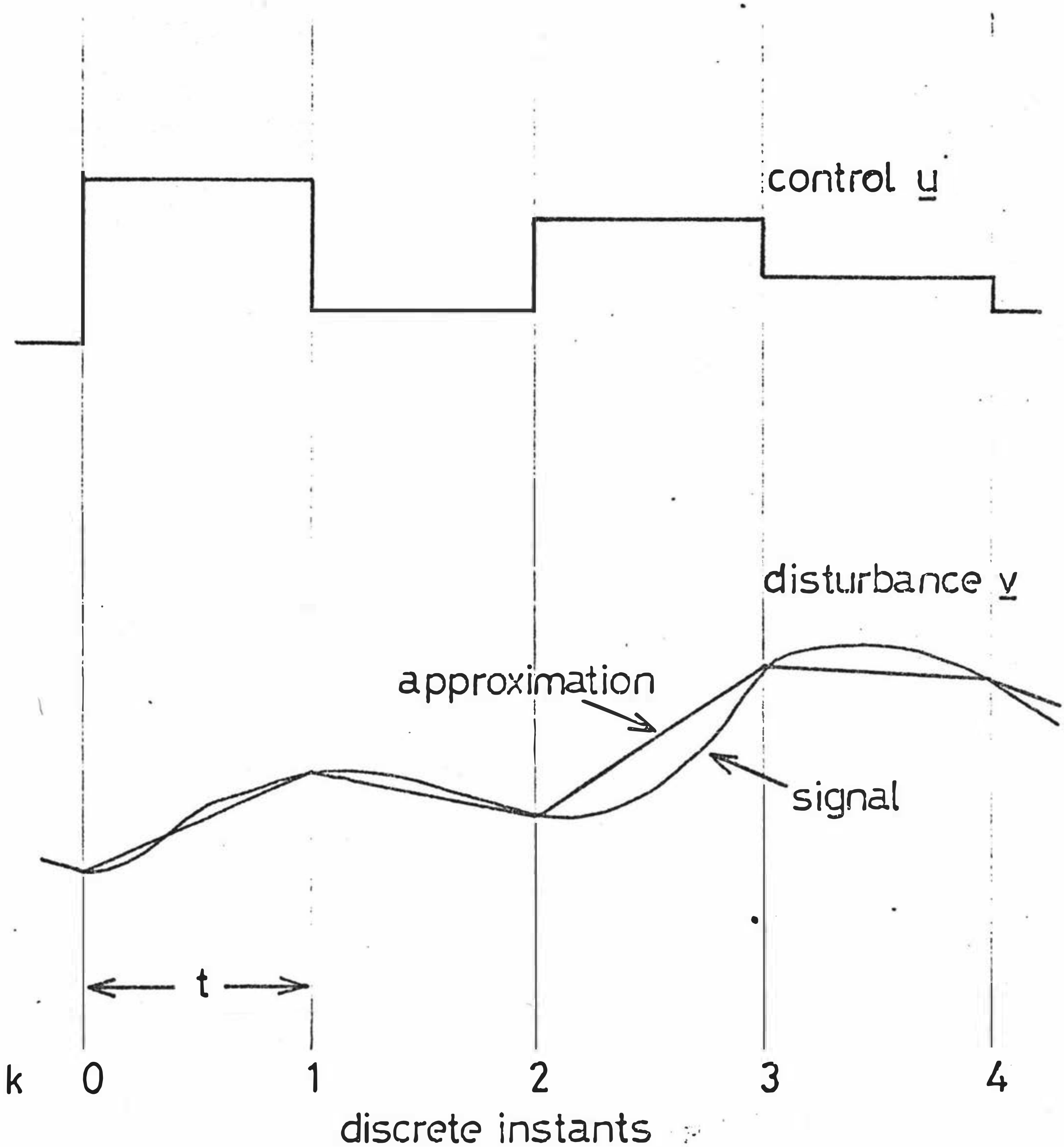


Figure 2-2 Disturbance and input characteristics

with
$$\underline{\Phi}\ ,\ sxs = \begin{bmatrix} \underline{\phi} & \underline{\Delta 3} \\ \underline{O} & \underline{I} \end{bmatrix} ; \quad \underline{\eta}\ ,\ sxq = \begin{bmatrix} \underline{\Delta 2} \\ \underline{O} \end{bmatrix} ;$$

$$\underline{\Omega}\ ,\ sxq = \begin{bmatrix} \underline{\Delta 1} \\ \underline{O} \\ \underline{I} \end{bmatrix} ; \quad \underline{\psi}\ ,\ sxm = \begin{bmatrix} \underline{\Delta} \\ \underline{I} \\ \underline{O} \end{bmatrix} ;$$

$$\underline{X}_k\ ,\ sx1 = \begin{bmatrix} \underline{x}_k \\ \underline{w}_{k-d-1} \end{bmatrix} ; \quad \underline{w}_k,\ \overline{m+q}+1 = \begin{bmatrix} \underline{u}_k \\ \underline{v}_k \end{bmatrix} ;$$

$$\underline{\Delta 3}\ ,\ nx\overline{m+q} = \begin{bmatrix} \underline{\Delta}\ ,\ \underline{\Delta 1} \end{bmatrix} ; \quad s = n+m+q ;$$

and
$$\underline{U}_k\ ,\ mx1 = \underline{u}_k - \underline{u}_{k-1} .$$

Equation (2) is rewritten to give

$$\underline{Y}_k = \underline{G}.\underline{X}_k\ ,\ \text{with } \underline{G},\ pxs = \begin{bmatrix} \underline{C}\ ,\ \underline{O} \end{bmatrix} \qquad (6)$$

Equations (5) and (6) provide the basic system specification from which analysis proceeds.

A discrete control system operates by computing control corrections at specific instants in time. Measurement samples for control calculations are normally taken only at the instants of control correction. This can place an unnecessary restriction on model structure. A more generalised scheme of measurement is here defined to give maximum flexibility in choice of a sampling pattern. In an application described in Reference 38, in some circumstances the full flexibility has had to be utilised to establish effective process models. Figure 2-3 illustrates this generalised control and sampling scheme. The control interval, the period between control instants, is τ_c secs. Each control interval is divided into a number of sampling intervals. The sampling intervals, referenced backwards from the control instant, define the instants at which measurement samples are collected. The

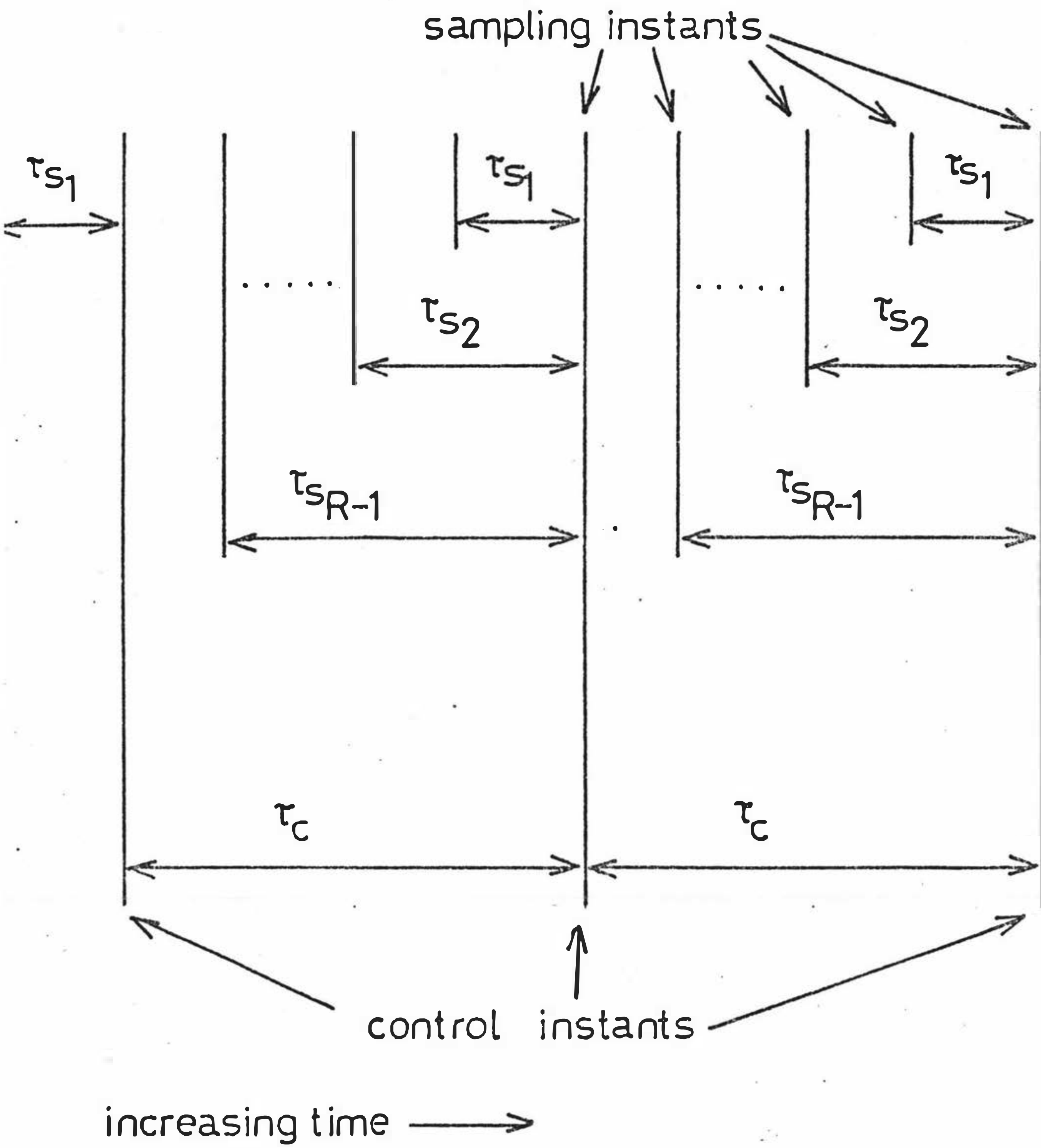


Figure 2-3 Control and sampling scheme

sampling pattern is identical across each control interval. R samples are defined per control interval, one being taken at the control instant and R-1 taken at intervals τs_i, i = 1,2 ...R-1, prior to the control instant (see fig. 2-4).

The objective of the analysis that follows is to establish a process model that is compatible with the control and sampling structure of fig. 2-3 The model is to predict all samples across the future control interval, utilising measurements from the current and preceding control intervals.

Interval t ($k \rightarrow k+1$) is selected to be small enough so that the ratios

$$\frac{\tau s_i}{t} = c \quad \text{and} \quad \frac{\tau s_i}{t}\ , \ i = 1,2, \ldots R-1\ , = s_i \qquad (7)$$

are all integer, with d also remaining integer. t may be selected quite arbitrarily so this condition does not imply any constraint. The requirement c>d is now introduced. The situation d>c is treated later. The controls are constant across a control interval so that, if k is a control instant,

$$\underline{U}_{k-i} = \underline{O} \quad \text{and} \quad \underline{U}_{k+1} = \underline{O}\ , \ i = 1,2,\ldots , c-1 \qquad (8)$$

Finally, the disturbance gradients are constrained to be constant across the control intervals so that

$$\underline{V}_i = \underline{V}_{i+1}\ , \ 1 = k-c+1 \rightarrow k-1 \qquad (9)$$

Since the latter is an approximation, disturbance behaviour can be an important factor in the selection of control interval.

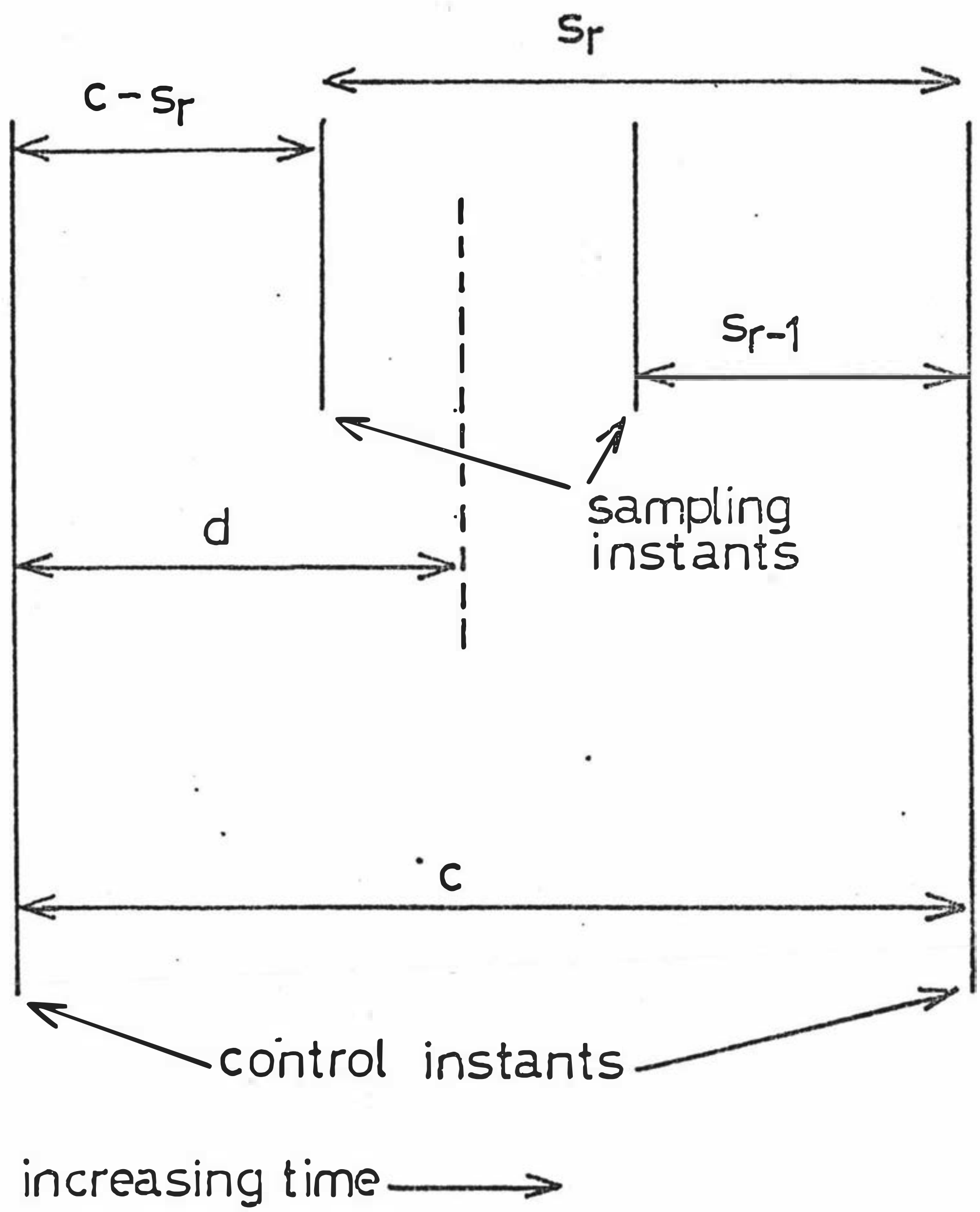


Figure 2-4 Control and sampling intervals

2.3 Analysis

This section develops analysis to transform the state space representation of equation (5) to a structure that satisfies the criteria described in sections 1 and 2.

2.3.1 Development of basic equations

With reference to fig. 2-4, specify integer r so that

$$c - s_{r-1} > d > c - s_r \ , \ R \geqslant r > 1 \ . \qquad (10)$$

The delay interval thereby terminates in the interval $k\text{-}s_r \rightarrow k\text{-}s_{r-1}$ as shown.

Consider the interval $k\text{-}s_i \rightarrow k\text{-}s_{i-1}$, $R \geqslant i > 1$, noting that $s_R = c$ and $s_o = 0$. Equation (11) may be derived by successively using equation (5) to eliminate $\underline{X}_{k-s_i+1} \rightarrow \underline{X}_{k-s_{i-1}-1}$.

$$\underline{X}_{k-s_{i-1}} = \underline{\Phi}^{s_i-s_{i-1}} \cdot \underline{X}_{k-s_i} + \sum_{j=1}^{s_i-s_{i-1}} \underline{\Phi}^{j-1} \cdot \Big\{ \underline{\Psi} \cdot \underline{U}_{k-s_{i-1}-d-j} + \underline{\Omega} \cdot \underline{V}_{k-s_{i-1}-d-j} + \underline{n} \cdot \underline{V}_{k-s_{i-1}-d-j+1} \Big\} \qquad (11)$$

Equation (11) may be considerably reduced because of the property of equations (8) and (5). There are three cases to consider:

Case 1, $i < r$

Since $c - s_i > d$ (see fig. 4), it follows that $c > s_i + s$, so that

$$\underline{U}_{k-s_{i-1}-d-1} = \underline{U}_{k-s_{i-1}-d-2} = \cdots = \underline{U}_{k-s_i-d} \qquad (12)$$

and

$$\underline{V}_{k-s_{i-1}-d} = \underline{V}_{k-s_{i-1}-d-1} = \cdots = \underline{V}_{k-s_i-d} = \underline{V}_k \tag{13}$$

from inspection of equations (8) and (9).

Therfore, equation (11) reduces to

$$\underline{X}_{k-s_{i-1}} = \underline{\Phi}^{s_i-s_{i-1}} \cdot \underline{X}_{k-s_i} + \sum_{j1=1}^{s_i-s_{i-1}} \underline{\Phi}^{j1-1} \cdot \underline{\Omega 1} \cdot \underline{V}_k \tag{14}$$

$$i < r \quad \text{with} \quad \underline{\Omega 1} = \underline{\Omega} + \underline{\eta} \, .$$

Case 2, $i = r$

The control and disturbance adjustments of instant k-c begin to influence the outputs at instant k-c+d. From equation (11), the equation

$$k - s_{r-1} - d - j = k - c \tag{15}$$

applies at the instant k-c+d. Therefore, at this instant

$$j = c - s_{r-1} - d \tag{16}$$

Thus, by application of equations (8), (9) and (16), equation (11) becomes

$$\begin{aligned}\underline{X}_{k-s_{r-1}} = {} & \underline{\Phi}^{s_r-s_{r-1}} \underline{X}_{k-s_r} + \underline{\Phi}^{c-s_{r-1}-d-1} \underline{\psi} \, \underline{U}_{k-c} \\ & + \left(\sum_{j1=1}^{c-s_{r-1}-d-1} \underline{\Phi}^{j1-1} \underline{\Omega 1} + \underline{\Phi}^{c-s_{r-1}-d-1} \underline{\eta} \right) \underline{V}_k \\ & + \underline{\Phi}^{c-s_{r-1}-d-1} \left(\underline{\Omega} + \sum_{j1=1}^{s_r-c+d} \underline{\Phi}^{j1} \underline{\Omega 1} \right) \underline{V}_{k-c} ,\end{aligned} \tag{17}$$

$$i = r$$

Since t may be selected arbitrarily small, the situations with $s_r - s_{r-1} = 1$ and $c - s_r = d$, are discounted.

Case 3, $i > r$

Equation (18) is obtained in analogous fashion to equation (14).

$$\underline{X}_{k-s_{i-1}} = \underline{\Phi}^{s_i - s_{i-1}} \underline{X}_{k-s_i} + \sum_{j1=1}^{s_i - s_{i-1}} \underline{\Phi}^{j1-1} \underline{\Omega 1}\, \underline{V}_{k-c} \,, \quad i > r . \tag{18}$$

The identity of equation (19) permits the series expressions of equations (14), (17) and (18) to be replaced.

$$\sum_{j=1}^{N} \underline{\Phi}^{-j} = (\underline{I} - \underline{\Phi}^{-N})(\underline{\Phi} - \underline{I})^{-1} \tag{19}$$

Equations (14), (17) and (18) may be inverted and combined together to form the set

$$\left.\begin{aligned}
\underline{X}_{k-s_1} &= \underline{\Phi}^{-s_1} \underline{X}_k - (\underline{I} - \underline{\Phi}^{-s_1}) \underline{\Omega 2}\, \underline{V}_k \\
&\quad \vdots \qquad\qquad \vdots \\
\underline{X}_{k-s_{r-1}} &= \underline{\Phi}^{-(s_{r-1}-s_{r-2})} \underline{X}_{k-s_{r-2}} \\
&\quad - \left(\underline{I} - \underline{\Phi}^{-(s_{r-1}-s_{r-2})}\right) \underline{\Omega 2}\, \underline{V}_k \\
X_{k-s_r} &= \underline{\Phi}^{-(s_r - s_{r-1})} \underline{X}_{k-s_{r-1}} - \underline{\Phi}^{c-s_r-d-1} \underline{W}_{k-c} \\
\underline{X}_{k-s_{r+1}} &= \underline{\Phi}^{-(s_{r+1}-s_r)} \underline{X}_{k-s_r} - \left(\underline{I} - \underline{\Phi}^{-(s_{r+1}-s_r)}\right) \underline{\Omega 2}\, \underline{V}_{k-c} \\
&\quad \vdots \qquad\qquad \vdots
\end{aligned}\right\} \tag{20}$$

$$\begin{aligned} &\vdots \qquad\qquad \vdots \\ \underline{X}_{k-c} &= \underline{\Phi}^{-(c-s_{R-1})}\underline{X}_{k-s_{R-1}} - \left(\underline{I}-\underline{\Phi}^{-(c-s_{R-1})}\right)\underline{\Omega 2}\ \underline{V}_{k-c} \end{aligned}$$

$$\text{with} \qquad \underline{\Omega 2} = (\underline{\Phi} - \underline{I})^{-1}\underline{\Omega 1} \tag{21}$$

$$\begin{aligned} \text{and} \quad \underline{W}_{k-c} &= \underline{\psi}\ \underline{U}_{k-c} + \left((\underline{I}-\underline{\Phi}^{-c+s_{r-1}+d+1})\underline{\Omega 2} + \underline{\eta}\right)\underline{V}_k \\ &+ \left(\underline{\Omega} + (\underline{\Phi}^{-c+s_r+d} - \underline{I})\ \underline{\Phi}\ \underline{\Omega 2}\right)\underline{V}_{k-c} \end{aligned} \tag{22}$$

The expression for $\underline{W}_{k-c}$ arises by considering the terms in $\underline{V}_k$ and $\underline{V}_{k-c}$ in equation (17). Upon inversion, these become

$$\begin{aligned} &-\left((\underline{\Phi}^{-(s_r-s_{r-1})} + \ldots + \underline{\Phi}^{c-s_r-d-2})\ \underline{\Omega 1} + \underline{\Phi}^{c-s_r-d-1}\ \underline{\eta}\right)\underline{V}_k \\ &-\underline{\Phi}^{c-s_r-d-1}\ \underline{\Omega}\ \underline{V}_{k-c} - \left(\underline{\Phi}^{c-s_r-d} + \ldots + \underline{\Phi}^{-1}\right)\underline{\Omega 1}\ \underline{V}_{k-c} \end{aligned} \tag{23}$$

The terms of expression (23) combine to give those of equation (22), following application of the identity of equation (19) and extraction of $\underline{\Phi}^{c-s_r-d-1}$.

The r.h.s. of equation (20) is now related to $\underline{X}_{k-s_i}$. There are two cases to consider:

<u>Case 1</u>, $i < r$

$$\left.\begin{aligned} \underline{X}_{k-s_{i+1}} &= \underline{\Phi}^{-(s_{i+1}-s_i)}\underline{X}_{k-s_i} - \left(\underline{I}-\underline{\Phi}^{-(s_{i+1}-s_i)}\right)\underline{\Omega 2}\ \underline{V}_k, \quad i < r \\ &\vdots \qquad\qquad \vdots \\ \underline{X}_{k-s_{r-1}} &= \underline{\Phi}^{-(s_{r-1}-s_i)}\underline{X}_{k-s_i} - \left(\underline{I}-\underline{\Phi}^{-(s_{r-1}-s_i)}\right)\underline{\Omega 2}\ \underline{V}_k \end{aligned}\right\} \tag{24}$$

$$\underline{X}_{k-s_r} = \underline{\Phi}^{-(s_r-s_i)} \underline{X}_{k-s_i} - \underline{\Phi}^{c-s_r-d-1} \underline{W1}_{k-c,s_i}$$

$$\underline{X}_{k-s_{r+1}} = \underline{\Phi}^{-(s_{r+1}-s_i)} \underline{X}_{k-s_i} - \underline{\Phi}^{c-s_{r+1}-d-1} \underline{W1}_{k-c,s_i} - \left(\underline{I}-\underline{\Phi}^{-(s_{r+1}-s_r)}\right) \underline{\Omega 2}\ \underline{V}_{k-c}$$

with

$$\underline{W1}_{k-jc,s_i} = \underline{\Psi}\ \underline{U}_{k-jc} + \left\{\left(\underline{I}-\underline{\Phi}^{-c+s_i+d+1}\right) \underline{\Omega 2} + \underline{\eta}\right\} \underline{V}_{k-(j-1)c} + \left\{\underline{\Omega} + \left(\underline{\Phi}^{-c+s_r+d} - \underline{I}\right) \underline{\Phi}\ \underline{\Omega 2}\right\} \underline{V}_{k-jc}\ ,$$

$$j = 1,2... \qquad (25)$$

The expression for $\underline{W1}_{k-c,s_i}$ results from consideration of row r in equations (20). On transforming to reference $\underline{X}_{k-s_i}$, this becomes

$$\underline{X}_{k-s_r} = \underline{\Phi}^{-(s_r-s_i)} \underline{X}_{k-s_i} - \underline{\Phi}^{c-s_r-d-1} \underline{W}_{k-c} - \underline{\Phi}^{-(s_r-s_{r-1})} \left(\underline{I}-\underline{\Phi}^{-(s_{r-1}-s_i)}\right) \underline{\Omega 2}\ \underline{V}_k \qquad (26)$$

Substitution for $\underline{W}_{k-c}$ with combination of the term in $\underline{V}_k$, results in the expression for $\underline{X}_{k-s_r}$ in equation (24). The procedure is now extended to obtain equations that relate to the control interval k-2c → k-c. These are:

$$\underline{X}_{k-c} = \underline{\Phi}^{-(c-s_i)} \underline{X}_{k-s_i} - \left(\underline{I}-\underline{\Phi}^{-(c-s_r)}\right) \underline{\Omega 2}\ \underline{V}_{k-c} - \underline{\Phi}^{-d-1} \underline{W1}_{k-c,s_i}$$

$$\left.\begin{aligned}
\underline{X}_{k-c-s_1} &= \underline{\Phi}^{-(c+s_1-s_i)}\,\underline{X}_{k-s_1} - \left(\underline{I}-\underline{\Phi}^{-(c+s_1-s_r)}\right)\underline{\Omega 2}\;\underline{V}_{k-c} \\
&\quad - \underline{\Phi}^{-d-s_1-1}\;\underline{W1}_{k-c,s_i} \\
&\;\;\vdots \qquad\qquad\qquad\qquad \vdots \\
\underline{X}_{k-s_r-c} &= \underline{\Phi}^{-(c+s_r-s_i)}\,\underline{X}_{k-s_i} - \underline{\Phi}^{-d-s_r-1}\;\underline{W1}_{k-c,s_i} \\
&\quad - \underline{\Phi}^{c-d-s_r-1}\;\underline{W2}_{k-2c} \\
&\;\;\vdots \qquad\qquad\qquad\qquad \vdots \\
\underline{X}_{k-s_{R-1}-c} &= \underline{\Phi}^{-(c+s_{R-1}-s_i)}\,\underline{X}_{k-s_i} - \underline{\Phi}^{-d-s_{R-1}-1}\;\underline{W1}_{k-c,s_i} \\
&\quad - \underline{\Phi}^{c-d-s_{R-1}-1}\;\underline{W2}_{k-2c} \\
&\quad - \left(\underline{I}-\underline{\Phi}^{-(s_{R-1}-s_r)}\right)\underline{\Omega 2}\;\underline{V}_{k-2c}\,, \quad i < r\,.
\end{aligned}\right\} \tag{27}$$

with

$$\underline{W2}_{k-jc} = \underline{\psi}\;\underline{U}_{k-jc} + \left\{\left(\underline{I}-\underline{\Phi}^{-2c+s_r+d+1}\right)\underline{\Omega 2} + \underline{\eta}\right\}\underline{V}_{k-(j-1)c}$$
$$+ \left\{\underline{\Omega} + \left(\underline{\Phi}^{-c+s_r+d} - \underline{I}\right)\underline{\Phi}\;\underline{\Omega 2}\right\}\underline{V}_{k-jc}\,,$$
$$j = 2,3\ldots \tag{28}$$

The equation for $\underline{W2}_{k-jc}$ is obtained by replacing s_i in equation (25) by $-c+s_r$. This is appreciated by writing out the expression for $\underline{X}_{k-s_r-c}$ as

$$\underline{X}_{k-s_r-c} = \underline{\Phi}^{-(c+s_r-s_i)} \underline{X}_{k-s_i} - \underline{\Phi}^{-d-s_r-1} \underline{W1}_{k-c,s_i} - \underline{\Phi}^{-(s_r-s_{r-1})} \cdot \left(\underline{I}-\underline{\Phi}^{-(c+s_{r-1}-s_r)}\right) \underline{\Omega 2}\ \underline{V}_{k-c} - \underline{\Phi}^{c-s_r-d-1} \underline{W}_{k-2c} \qquad (29)$$

and making comparison with equation (26).

Case 2, $i \geqslant r$

In this case, the impact of the inputs and disturbances from instant k-c has not occurred. It therefore follows that in equation (27),

$$\underline{W1}_{k-c,s_i} = \underline{O} \text{ and } \underline{W2}_{k-2c,s_i-c} \qquad (30)$$

if $i > r$.

Equations (27) may be generalised to express the samples collected over any preceding control interval in terms of $\underline{X}_{k-s_i}$. Equations (31) may be written to define the control interval f stages back from instant K.

$$\left.\begin{aligned}
\underline{X}_{k-fc} &= \underline{\Phi}^{-(fc-s_i)} \underline{X}_{k-s_i} - \left(\underline{I}-\underline{\Phi}^{-(c-s_r)}\right) \underline{\Omega 2}\ \underline{V}_{k-fc} \\
&\quad - \sum_{j=o}^{f-2} \underline{\Phi}^{-d-1-jc} \underline{W2}_{k-(f-j)c} \\
&\quad - \underline{\Phi}^{-(f-1)c-d-1} \underline{W1}_{k-c,s_i} \\
\underline{X}_{k-fc-s_1} &= \underline{\Phi}^{-(fc+s_1-s_i)} \underline{X}_{k-s_i} - \left(\underline{I}-\underline{\Phi}^{-(c+s_1-s_r)}\right) \underline{\Omega 2}\ \underline{V}_{k-fc} \\
&\quad - \sum_{j=o}^{f-2} \underline{\Phi}^{-d-1-s_1-jc} \underline{W2}_{k-(f-j)c} \\
&\quad - \underline{\Phi}^{-(f-1)c-d-1-s_1} \underline{W1}_{k-c,s_i} \\
&\quad\ \vdots \qquad\qquad\qquad \vdots
\end{aligned}\right\} (31)$$

$$
\left.
\begin{aligned}
&\vdots && \vdots \\
\underline{X}_{k-fc-s_r} &= \underline{\Phi}^{-(fc+s_r-s_i)} \, \underline{X}_{k-s_i} \\
&\quad - \sum_{j=-1}^{f-2} \underline{\Phi}^{-d-1-s_r-jc} \, \underline{W2}_{k-(f-j)c} \\
&\quad - \underline{\Phi}^{-(f-1)c-d-1-s_r} \, \underline{W1}_{k-c,s_i} \\
&\vdots && \vdots \\
\underline{X}_{k-fc-s_{R-1}} &= \underline{\Phi}^{-(fc+s_{R-1}-s_i)} \, \underline{X}_{k-s_i} \\
&\quad - \left(\underline{I}-\underline{\Phi}^{-(s_{R-1}-s_r)}\right) \underline{\Omega 2} \, \underline{V}_{k-(f+1)c} \\
&\quad - \sum_{j=-1}^{f-2} \underline{\Phi}^{-d-1-s_{R-1}-jc} \, \underline{W2}_{k-(f-j)c} \\
&\quad - \underline{\Phi}^{-(f-1)c-d-1-s_{R-1}} \, \underline{W1}_{k-c,s_i} \, , \\
&\qquad f = 2,3....
\end{aligned}
\right\} \quad (31)
$$

Equation (27) and (31) form the basis from which output representations of process dynamics are established.

2.3.2 Formation of output equations

Analysis is now applied to eliminate state variables in favour of outputs. Application of equation (6) to the second row of equation (27) gives

$$\underline{y}_{k-c-s_1} = \underline{G}\,\underline{\Phi}^{-(c+s_1-s_i}\underline{x}_{k-s_i} - \underline{G}\,\underline{\Phi}^{-d-1-s_1}\,\underline{W1}_{k-c\ ,s_i}$$

$$- \underline{G}\left(\underline{I} - \underline{\Phi}^{-(c+s_1-s_r)}\right)\underline{\Omega 2}\ \underline{V}_{k-c} \qquad (32)$$

The identity

$$\underline{\Phi}^{-N} = \begin{bmatrix} \underline{\Phi}^{-N} - (\underline{I} - \underline{\Phi}^{-N})\ \underline{\Delta 4} \\ \underline{O} \qquad \underline{I} \end{bmatrix} \quad , \quad N > 1 \qquad (33)$$

with $\underline{\Delta 4} = (\underline{\Phi} - \underline{I})^{-1}\,\underline{\Delta 3}$, (34)

leads to

$$\underline{G}\ \underline{\Phi}^{-N} = \left|\underline{C}.\underline{\Phi}^{-N}\ ,\ -\underline{C}(\underline{I} - \underline{\Phi}^{-N})\underline{\Delta 4}\right| \qquad (35)$$

Equation (32) may therefore be written, on eliminating $\underline{X}_{k-s_i}$,

$$\underline{y}_{k-c-s_1} = \underline{C}.\underline{\Phi}^{-(c+s_1-s_i)}\ \underline{x}_{k-s_i}$$

$$- \underline{C}.\ \left(\underline{I} - \underline{\Phi}^{-)c+s_1-s_i)}\right)\underline{\Delta 4}\ \underline{w}_{k-s_i-1-d} - ..(36)$$

The term in $\underline{w}$ may be eliminated from equation (36) thereby removing reference to the absolute controls and disturbances. Two cases apply.

<u>Case 1</u>, $i < r$

For this case

$$\underline{w}_{k-s_i-1-d} = \underline{w}_{k-c} \qquad (37)$$

It is possible to develop equation (38) by relating equation (4) to the interval $k\text{-}c \rightarrow k\text{-}s_i$.

$$\underline{x}_{k-s_i} = \underline{\phi}^{c-s_i} \cdot \underline{x}_{k-c} + \sum_{j=1}^{c-d-c} \underline{\phi}^{j-1} \cdot (\underline{\Delta 3} \cdot \underline{w}_{k-c} + \underline{\Delta 2} \cdot \underline{V}_k)$$

$$+ \underline{\phi}^{c-s_i} \cdot \sum_{j=1}^{d} \underline{\phi}^{j-1} \cdot (\underline{\Delta 3} \cdot \underline{w}_{k-2c} + \underline{\Delta 2} \cdot \underline{V}_{k-c}) \quad (38)$$

$\underline{w}_{k-2c}$ can be eliminated from equation (38) and $\underline{U}_{k-2c}$ introduced giving

$$\underline{x}_{k-s_i} = \underline{\phi}^{c-s_i} \cdot \Big\{ \underline{x}_{k-c} + \left(\underline{I} - \underline{\phi}^{-(c-s_i)}\right) \underline{\Delta 4} \cdot \underline{w}_{k-c}$$

$$- (\underline{I} - \underline{\phi}^{-d}) \cdot (\underline{\phi} - \underline{I})^{-1} \cdot \left[\underline{\Delta} \cdot \underline{U}_{k-c} + (\underline{\Delta 1} - \underline{\Delta 2}) \cdot \underline{V}_{k-c}\right]$$

$$+ \underline{\phi}^{-d} \cdot \left(\underline{I} - \underline{\phi}^{-(c-d-s_i)}\right) \cdot (\underline{\phi} - \underline{I})^{-1} \cdot \underline{\Delta 2} \cdot \underline{V}_k \Big\} \quad (39)$$

A relationship for $\underline{\Delta 4} \cdot \underline{w}_{k-c}$ may be extracted from equation (39). This substituted into equation (36), with incorporation of equation (37), leads to

$$\underline{y}_{k-c-s_i} = \Big\{ \underline{C} \cdot \underline{\phi}^{-(c+s_1-s_i)} - \underline{C} \cdot \left(\underline{I} - \underline{\phi}^{-(c+s_1-s_i)}\right) \cdot$$

$$\left(\underline{I} - \underline{\phi}^{-(c-s_i)}\right)^{-1} \cdot \underline{\phi}^{-(c-s_i)} \Big\} \cdot \underline{x}_{k-s_i}$$

$$+ \underline{C} \cdot \left(\underline{I} - \underline{\phi}^{-(c+s_1-s_i)}\right) \cdot \left(\underline{I} - \underline{\phi}^{-(c-s_i)}\right)^{-1} \cdot \underline{x}_{k-c}$$

$$+ \underline{W4}_{k-c,i,1} + \ldots \quad (40)$$

with

$$\underline{W4}_{k-c,i,h} = \underline{C} \cdot \left(\underline{I} - \underline{\phi}^{-(c+s_h-s_i)}\right) \left(\underline{I} - \underline{\phi}^{-c-s_h}\right)^{-1}$$

$$\Big\{ \underline{\phi}^{-d} \left(\underline{I} - \underline{\phi}^{-(c-d-s_i)}\right) \cdot (\underline{\phi} - \underline{I})^{-1} \underline{\Delta 2} \, \underline{V}_k$$

$$- (\underline{I} - \underline{\phi}^{-d}) (\underline{\phi} - \underline{I})^{-1} (\underline{\Delta} \, \underline{U}_{k-c} + (\underline{\Delta 1} - \underline{\Delta 2}) \underline{V}_{k-c}) \Big\} \quad (41)$$

$$h = 1,2,3\ldots\ ,\ i < r\ .$$

Equation (40) can be expressed in a more concise form. The term in $\underline{x}_{k-s_i}$ in equation (40) may be written

$$\underline{C}\left|\underline{\phi}^{-(c+s_1-s_i)}\left\{\underline{I} + \left(\underline{\phi}^{c-s_i}-\underline{I}\right)^{-1}\right.\right.$$

$$\left.\left. - \left(\underline{\phi}^{c-s_i}-\underline{I}\right)^{-1}\right\} + \underline{I}-\underline{I}\ \right|\ \underline{x}_{k-s_i} \qquad (42)$$

and since

$$\underline{I} + \left(\underline{\phi}^{c-s_i}-\underline{I}\right)^{-1} = \left(\underline{I}-\underline{\phi}^{-(c-s_i)}\right)^{-1} \qquad (43)$$

it follows that equation (40) may be written;

$$\underline{y}_{k-c-s_1} - \underline{y}_{k-c} = \underline{C}\left|\underline{I} - \left(\underline{I}-\underline{\phi}^{-(c+s_1-s_i)}\right)\left(\underline{I}-\underline{\phi}^{-(c-s_i)}\right)^{-1}\right|$$

$$(\underline{x}_{k-s_i} - \underline{x}_{k-c}) + \underline{W4}_{k-c,i,1} - \underline{G}\ \underline{\Phi}^{-d-s_1-1}\ \underline{W1}_{k-c,s_i}$$

$$- \underline{G}\left(\underline{I}-\underline{\phi}^{-(c+s_1-s_r)}\right)\underline{\Omega 2}\ \underline{V}_{k-c} \qquad (44)$$

<u>Case 2</u>, $i \geqslant r$

For this case

$$\underline{w}_{k-s_i-1-d} = \underline{w}_{k-2c} \qquad (45)$$

and

$$\underline{x}_{k-s_i} = \underline{\phi}^{c-s_i}\underline{x}_{k-c} + \sum_{j=1}^{c-s}\underline{\phi}^{j-1}\left(\underline{\Delta 3}\ \underline{w}_{k-2c} + \underline{\Delta 2}\ \underline{V}_{k-c}\right) \qquad (46)$$

It is straightforward to show, by following the procedure leading to equation (40), that

$$\underline{W4}_{k-c,i,h} = \underline{C}.\ \underline{I}-\underline{\phi}^{-(c+s_h-s_i)}\ .\ \underline{I}-\underline{\phi}^{-(c-s_h)\ -1}\ .$$
$$.(\underline{I}-\underline{\phi}^{-d}).(\underline{\phi}-\underline{I})^{-1}.\underline{\Delta 2}.\underline{V}_{k-c} \qquad (47)$$
$$h = 1,2,\ \ldots \qquad i \geqslant r$$

Equation (40), and hence equation (44), still applies but subject to the conditions of equation (30).

The generalised matrix equation

$$\underline{Y1}_{k-c} = \underline{F}_i.(\underline{x}_{k-s_i}-\underline{x}_{k-c}) + \underline{W5}_{k-c,i} \qquad (48)$$

may be established from equations (27), (31) and (44). In equation (48)

$$\underline{Y1}_{k-c} = \begin{bmatrix} \underline{y}_{k-c-s_1} - \underline{y}_{k-c} \\ \cdot \\ \cdot \\ \cdot \\ \underline{y}_{k-(f-1)c-s_r} - \underline{y}_{k-c} \\ \cdot \\ \cdot \\ \cdot \\ \underline{y}_{k-fc} - \underline{y}_{k-c} \\ \cdot \\ \cdot \end{bmatrix},\ \underline{F}_i = \begin{bmatrix} \underline{C}.(\underline{\phi}^{-s_1}-\underline{I}) \\ \cdot \\ \cdot \\ \underline{C}.(\underline{\phi}^{-(f-2)c-s_r}-\underline{I}) \\ \cdot \\ \cdot \\ \cdot \\ \underline{C}.(\underline{\phi}^{-(f-1)c}-\underline{I}) \\ \cdot \\ \cdot \end{bmatrix}.(\underline{\phi}^{c-s_i}-\underline{I}$$

and

$$\underline{W5}_{k-c,i} = \begin{bmatrix} \underline{W4}_{k-c,i,0} \\ \cdot \\ \cdot \\ \cdot \\ \underline{W4}_{k-c,i,(f-2)c-s_r} \\ \cdot \\ \cdot \\ \cdot \\ \underline{W4}_{k-c,i,(f-1)c} \\ \cdot \\ \cdot \end{bmatrix} + \begin{bmatrix} \underline{G}.\underline{\Phi}^{-d-s_1-1} \\ \cdot \\ \cdot \\ \underline{G}.\underline{\Phi}^{-d-(f-2)c-s_r-1} \\ \cdot \\ \cdot \\ \cdot \\ \underline{G}.\underline{\Phi}^{-d-(f-1)c-1} \\ \cdot \\ \cdot \end{bmatrix}.\underline{W1}_{k-c,s}$$

$$
+ \begin{bmatrix} \underline{G1}\cdot\underline{V}_{k-c} \\ \underline{G2}\cdot\underline{V}_{k-2c} \\ \vdots \\ \underline{G2}\cdot\underline{V}_{k-fc} \\ \vdots \end{bmatrix}
$$

$$
+ \left\{ \begin{bmatrix} \underline{O} \\ \vdots \\ \underline{G}\cdot\underline{\Phi}^{c-d-s_r-1} \\ \vdots \\ \underline{G}\cdot\underline{\Phi}^{-d-1} \\ \vdots \\ \underline{G}\cdot\underline{\Phi}^{-d-(f-2)c-s_r-1} \\ \vdots \\ \underline{G}\cdot\underline{\Phi}^{-d-(f-2)c-1} \\ \vdots \end{bmatrix} \cdot \underline{W2}_{k-2c} + \ldots + \begin{bmatrix} \underline{O} \\ \vdots \\ \underline{O} \\ \vdots \\ \underline{O} \\ \vdots \\ \underline{G}\cdot\underline{\Phi}^{c-d-s_r-1} \\ \vdots \\ \underline{G}\cdot\underline{\Phi}^{-d-1} \\ \vdots \end{bmatrix} \cdot \underline{W2}_{k-(\underline{f}-1)} + \ldots \right\}
$$

$$
f = 2,3, \ldots \infty \qquad (49)
$$

In equation (49),

$$
\underline{G1} = \begin{bmatrix} \underline{G}\cdot\left(\underline{I}-\underline{\Phi}^{-(c+s_1-s_r)}\right) \\ \vdots \end{bmatrix} \cdot \underline{\Omega 2} \quad \text{and} \quad \underline{G2} = \begin{bmatrix} \underline{O} \\ \underline{G}\cdot\ \underline{I}-\underline{\Phi}^{-(s_{r+1}-s_r)} \\ \vdots \end{bmatrix} \cdot \underline{\Omega 2}
$$

$$\left\lfloor \underline{G}.\left(\underline{I}-\underline{\Phi}^{-(c+s_{r-1}-s_r)}\right)\right\rfloor \qquad\qquad \left\lfloor \underline{G}.\left(\underline{I}-\underline{\Phi}^{-(c+s_{r-1}-s_r)}\right)\right\rfloor$$

Equations (48) are now truncated at row nl (nl $\geqslant$ n). If Rl is the total number of control stages incorporated, then

$$p.R.(Rl-1) + n2 - p = nl \tag{50}$$

where n2 is the total number of scalar samples collected in the final control stage. Equation (51) is extracted from equation (48).

$$\underline{Yl}_{k-c,nl} = \underline{F}_{i,nl}.(\underline{x}_{k-s_i} - \underline{x}_{k-c}) + \underline{W5}_{k-c,i,nl} \tag{51}$$

with $\underline{Yl}_{k-c,nl}$, $\underline{F}_{i,nl}$ and $\underline{W5}_{k-c,i,nl}$ representing the first nl rows of $\underline{Yl}_{k-c}$, $\underline{F}_i$ and $\underline{W5}_{k-c,i}$ respectively. Inverting equation (51) and applying equation (2) leads to:

$$\underline{y}_{k-s_i} - \underline{y}_{k-c} - \underline{Fl}_i.\underline{Yl}_{k-c,nl} - \underline{Fl}_i.\underline{W5}_{k-c,i,nl} \tag{52}$$

with $$\underline{Fl}_i = \underline{C}.\underline{F}^{-1}_{i,nl} \text{ if } nl=n, \tag{53}$$

and $$\underline{Fl}_i = \underline{C}.\ \underline{F}^T_{i,nl}.\underline{F}_{i,nl}{}^{-1}\ \underline{F}^T_{i,nl} \text{ if } nl>n \tag{54}$$

The structure of equation (54) indicates that equation (52) generates a least squares estimate to $\underline{y}_{k-s_i} - \underline{y}_{k-c}$. It is necessary that $\underline{F}_{i,nl}$ has rank n.

Equation (52) is now applied to obtain equation (55) which relates to the complete control interval k-c→k, with i covering the range 0→R-1.

$$\begin{bmatrix} \underline{y}_k - \underline{y}_{k-c} \\ \\ \underline{Yl}_{k,n3} \end{bmatrix} = \underline{\alpha l}.\underline{Yl}_{k-c,nl} + \underline{\beta}_{k-c} \tag{55}$$

with

$$\underline{\alpha 1},\ n4 \times n1 = \begin{bmatrix} \underline{F1}_o \\ \cdot \\ \cdot \\ \cdot \\ \underline{F1}_{R-2} \\ \underline{F2}_{R-1} \end{bmatrix} \text{ and } \underline{\beta}_{k-c},\ n4 \times 1 = \underline{\alpha 1}. \begin{bmatrix} \underline{W5}_{k-c,o,n1} \\ \cdot \\ \cdot \\ \cdot \\ \underline{W5}_{k-c,R-1,n1} \end{bmatrix}$$

and $n4 = n3 + p.$

$n3 = p.(R-1)$ if $R>1$ and $n2-p$ if $R = 1.$

$\underline{F2}_{R-1}$ represents the first p1 rows of $\underline{F1}_{R-1}$ with

$$p1 = n3 - p.(R-2)$$

The $\underline{y}_{k-c}$ terms may all be combined on the r.h.s. of equation (55) to give:

$$\underline{Y}_k = \underline{\alpha}. \begin{bmatrix} \underline{y}_{k-c} \\ \cdot \\ \cdot \\ \cdot \\ \underline{y}_{k-(R1-1)c} \\ \underline{y}_{k-R1c} \end{bmatrix} + \underline{\beta}_{k-c} \tag{56}$$

with

$$\underline{Y}_k,\ n4 \times 1 = \begin{bmatrix} \underline{y}_k \\ \cdot \\ \cdot \\ \cdot \\ \underline{y}_{k-s_{R-1}} \end{bmatrix} \text{ and } \underline{Y2}_k$$

representative of the first n2 rows of $\underline{Y}_k$.

$$\underline{\alpha},\ n4 \times n1 = \left[\underline{\alpha 3},\ \underline{\alpha 1}\right] \text{ with } \underline{\alpha 3}.\underline{y}_{k-c} = \{[\underline{I},\underline{O}] - [\underline{O},\underline{\alpha 1}]\} \begin{bmatrix} \underline{y}_{k-c} \\ \cdot \\ \cdot \\ \cdot \\ \underline{y}_{k-c} \\ \underline{y1}_{k-c} \end{bmatrix}$$

and $\underline{y1}$ equal to the first p2 rows of $\underline{y}$, where

$$p2 = n2 - (R2-1).p$$

and R2 is n2/p rounded up to the nearest integer.

Combining like terms, the expression for $\underline{\beta}_{k-c}$ can be condensed into an equation with the structure

$$\underline{\beta}_{k-c} = \sum_{j=1}^{R1+1} \underline{\beta 1}_j . \underline{U}_{k-jc} + \sum_{j=0}^{R1+1} \underline{\gamma 1}_j . \underline{V}_{k-jc} \tag{57}$$

with the matrices $\underline{\beta 1}_j$ and $\underline{\gamma 1}_j$ resulting from combination of terms associated with equations (55), (49), (41), (28) and (25). Finally, the disturbances must be written in terms of values at the control instants. Let

$$\underline{V1}_{k-c} = \underline{v}_{k-c} - \underline{v}_{k-2c} = c.\underline{V}_{k-c} \text{ (see equation (9)).} \tag{58}$$

Equation (57) becomes,

$$\underline{\beta}_{k-c} = \sum_{j=1}^{R1+1} \underline{\beta 1}_j . \underline{U}_{k-jc} + \sum_{j=1}^{R1+1} \underline{\gamma 2}_j . \underline{V1}_{k-jc} \tag{59}$$

with

$$\underline{\gamma 2}_j = \underline{\gamma 1}_j \div c$$

The following features relate to equation (56):

(i) If R1 = 1, so that sampling covers only one back control stage, then equation (56) reduces to

$$\begin{aligned} \underline{y}_k = \underline{\alpha}\ \underline{Y2}_{k-c} + \underline{\beta 1}_1\ \underline{U}_{k-c} + \underline{\beta 1}_2\ \underline{U}_{k-2c} \\ + \underline{\gamma 2}_o\ \underline{V1}_k + \underline{\gamma 2}_1\ \underline{V1}_{k-c} + \underline{\gamma 2}_2\ \underline{V1}_{k-2c} \end{aligned} \tag{60}$$

(ii) If $c-s_{R-1} > d \geqslant o$, thesituation with i<r applies throughout. This implies that all measurements taken in a control interval are collected after the

preceding control correction is seen to be influencing the process. From equation (49), it can be established that there is, in this case, no term in $\underline{W2}_{k-(R1+1)c}$ or $\underline{V}_{k-(R1+1)c}$. It follows that $\underline{\beta 1}_{R1+1}$ and $\underline{\gamma 1}_{R1+1}$, in equation (59) are zero.

(iii) For the case with $i \geqslant r$, it follows from equation (30) that there can be no associated terms in $\underline{V}_k$ or $\underline{U}_{k-c}$. Therefore, if $r<R$, the bottom $n4-r.p$ rows of $\underline{\beta 1}_o$ and $\underline{\gamma 1}_o$ are zero.

(iv) If samples are collected only at the control instants, i.e., $k-jc$, $j=1, \ldots, r1$, then

$$s_{R-1} = s_o = 0 \tag{61}$$

Since $c>d$ by definition, both $\underline{\beta 1}_{R1+1}$ and $\underline{\gamma 2}_{R1+1}$ are zero.

(v) If samples are collected at the control instants only, and the delay interval d is zero, then since

$$\underline{G}\ \underline{\Phi}^{-1}\ \underline{\psi} = \underline{O} \quad \text{and} \quad \underline{G}\ \underline{\Phi}^{-1}\ \underline{\Omega} = \underline{O} \tag{62}$$

(see equations (5) and (34)), from equations (49) and (28) it is apparent that $\underline{G}\ \underline{\Phi}^{-1}\ \underline{W2}_{k-R1c}$ only incorporates a term in $\underline{V}_{k-(R1-1)c}$. There is no term in $\underline{V}_{k-R1c}$ or $\underline{U}_{k-R1c}$. Therefore, for this situation, $\underline{\beta 1}_{R1}$ and $\underline{\gamma 2}_{R1}$ are both zero. (NB: R1 must be >1 for this to apply).

2.3.3 Generalisation of consideration of time delays

Equation (56) has been derived by considering a single value of time delay, with $c>d$, that is associated with all the controls and disturbances. Consideration is now given to the situation without these constraints.

Firstly, suppose that each control and disturbance element has a different associated delay but that each delay is subject to the condition $c>d\geq 0$. Inspection of equation (56) shows that the output, control and disturbance vectors are independent of the values of any pure time delays. The elements of the matrices of equation (56) are partly defined by the values of the delays, however, the structure is independent of these values. The listed features of equation (56) must now be treated as applying to the individual control and disturbance signals, taken one at a time. It has been demonstrated that certain matrices may be equated to zero if particular conditions relating to the sampling and delay intervals apply. It now follows that columns of these same matrices, that relate to a particular control or disturbance signal, may be equated to zero if the delay associated with the signal satisfies the same conditions in relation to the sampling intervals.

If a pure time delay associated with a particular signal is greater than the control interval, then the delay must be divided into two postions as follows:

$$d = d1.c + d2 \qquad (63)$$

with $c>d2\geq 0$ and $d1$ integer ≥ 0. From equation (56) it is seen that the controls and disturbances are referenced backwards from instants $k-c$ and k respectively. If the condition of equation (63) applies to a particular signal, then this signal must be referenced backwards from instant $k-c\,.\,(d1+1)$ if it is a control, and instant $k-d1.c$ if it is a disturbance. The delays thereby become features of the control and disturbance vectors, but only as direct multiples of the control interval c.

2.4 Appreciation of Model Structure

The difference form of equation (55) lends itself well to incorporation wth statistical modelling techniques. Features of the equation structure are:

1. The equation is totally general for a representation of a multivariable linear process with associated disturbances and timedelays, these delays being associated solely with the control input and disturbance signal. Only measurable plant outputs are included.

2. Control inputs are assumed to be constant across each prediction stage. Disturbance signals are approximated as ramps across the prediction stage (if appropriate). Because of this approximation, disturbance signal characteristics can be an important consideration in the selection of a prediction interval.

3. The structure offers a great deal of flexibility for selection of prediction interval, total range of sampling and distribution of samples. Sampling instants may be freely selected and are not constrained by the discrete interval of the equation.

4. If the process order is n, then $n_1 \geq n$. If $n_1 > n$ then more samples than the minimum necessary are incorporated giving a least squares filtering effect on the measurements and helping to reduce the impact of noise.

5. The sampling pattern chosen can affect the number of terms required in the vector $\underline{\beta}_{k-c}$.

6. The structure incorporates a disturbance sample at the prediction instant k. This takes into account

the fact that disturbances impact upon the process during the prediction interval c and inclusion of this aspect generates improved models.

7. The equation is expressed in difference form and is therefore independent of absolute levels of process operation. This is an important feature if the plant is subject to drift and is an aid to the tolerance to non-linarity.

8. The equation can be easily transformed to that of equation (56) in which measurements are in absolute form but retaining control and disturbance input differences. This is advantageous for control system design and implementation.

The problem of identification is to establish suitable values of the elements of the matrice $\underline{\alpha}_1$, $\underline{\beta}_1$ and $\underline{\gamma}_1$ so that equation (55) provides a valid representation of the process behaviour. An interactive computer facility using least squares techniques has been developed. If the variable quantities associated with equation (55)

$$R,\ Rl,\ c,\ s_i (i=1...R-1),\ n_1,\ n_4 \text{ and } d$$

are correctly chosen the identification problem is straightforward. The interactive facility is described in greater detail in Chapter 3.

2.5 Derivation of Control Algorithm

Equation (56) is restructured into a form suitable for the application of discrete dynamics programming.[8] Rewrite equation (56) as $y_k = \underline{\alpha}\ z_k + \beta_{k-c}$. The system is first augmented so that $\underline{Z}$ appears on both sides of the equation.

$$\underline{Z}_k = \underline{\alpha 2}.\underline{Z}_{k-c} + \underline{\beta 2}. \begin{bmatrix} \underline{U}_{k-c} \\ \cdot \\ \cdot \\ \cdot \\ \underline{U}_{k-(R1+1)c} \end{bmatrix} + \underline{\gamma 2}. \begin{bmatrix} \underline{V}_{k} \\ \cdot \\ \cdot \\ \cdot \\ \underline{V}_{k-(R1+1)c} \end{bmatrix} \qquad (65)$$

with

$$\underline{\alpha 2} = \left|\begin{array}{cccc} \underline{\alpha} & & & \\ \hdashline \underline{I} & \underline{O} & \cdots & \underline{O} \\ \underline{O} & \underline{I} & \cdots & \underline{O} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \underline{O} & \underline{O} & \underline{I} & \underline{O} \end{array}\right| , \qquad \underline{\beta 2} = \left|\begin{array}{ccc} \underline{\beta 1}_1 & \cdots & \underline{\beta 1}_{R1+1} \\ \underline{O} & \cdots & \underline{O} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \underline{O} & & \underline{O} \end{array}\right| \quad \text{and}$$

$$\underline{\gamma 2} = \left|\begin{array}{ccc} \underline{\gamma 1}_o & \cdots & \underline{\gamma 1}_{R1+1} \\ \underline{O} & \cdots & \underline{O} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \underline{O} & \cdots & \underline{O} \end{array}\right|$$

Time delays are considered later. Equation (65) is iterated R1 times to give

$$\underline{Z}_{k+(R1-1)c} = \underline{\alpha 3}.\underline{Z}_{k-c} + \underline{\beta 31}.\underline{W}_{k+(R1-2)c} + \underline{\beta 32}.\underline{W}_{k-2c}$$

$$+ \underline{\gamma 3}. \left|\begin{array}{c} \underline{V}_{k+(R1=1)c} \\ \cdot \\ \cdot \\ \cdot \\ \underline{V}_{k-(R1+1)c} \end{array}\right| \qquad (66)$$

with

$$\underline{W}_{k-2c} = \left|\begin{array}{c} \underline{U}_{k-2c} \\ \cdot \\ \cdot \\ \cdot \\ \underline{U}_{k-(R1+1)c} \end{array}\right| \quad \text{and } \underline{\alpha 3} = \underline{\alpha 2}^{R1} .$$

$\underline{\beta 31}$, $\underline{\beta 32}$ and $\underline{\alpha 3}$ are derived as a result of successive manipulation of $\underline{\alpha 2}$, $\underline{\beta 2}$ and $\underline{\gamma 2}$. The elements of $\underline{Z}_{k+(R1-1)c}$ are now distinct from those of $\underline{Z}_{k-c}$. Equation (66) gives a new system definition with an update interval of Rl.c stages. Neglecting the disturbance terms in equation (66), the condensed version

$$\underline{Z}_{j+1} = \underline{\alpha 3}.\underline{Z}_j + \underline{\beta 31}.\underline{W}_j + \underline{\beta 32}.\underline{W}_{j-1} \tag{67}$$

may be written. The interval $j \rightarrow j+1$ is Rl.c times the interval $k \rightarrow k+1$.

Control system design is achieved by minimising the cost function

$$I = \sum_{i=1}^{M} \left[\underline{Z}_i^T.\underline{Q}.\underline{Z}_i + \underline{W}_i^T.\underline{P}.\underline{W}_i \right] \tag{68}$$

$\underline{Q}$ and $\underline{P}$ are positive definite or semi-definite weighting matrices. The design utilises dynamic programming and the derivation is very similar to that described elsewhere.[4] The algorithm resulting is

$$\underline{W}_j = \underline{K}_M. \{\underline{\alpha 3}.\underline{Z}_j + \underline{\beta 32}.\underline{W}_{j-1}\} \tag{69}$$

with $\underline{K}_M$ a matrix of control coefficients. If M is chosen large enough, $\underline{K}_M$ converges to the constant matrix $\underline{K}_\infty$.

The bottom m rows of equation (69) may be extracted to give the control equation that relates to the interval $k-c \rightarrow k$. Thus

$$\underline{U}_{k-c} = \underline{F}_\infty.\{\underline{\alpha 3}.\underline{Z}_{k-c} + \underline{\beta 32}.\underline{W}_{k-2c}\} \tag{70}$$

with $\underline{F}_\infty$ representing the bottom m rows of $\underline{K}_\infty$.

The disturbance terms of equation (66) may now be incorporated into the control However, disturbances are not known ahead of stage j in equation (69). It results that it is only possible to consider disturbances in the stage corresponding to i=1 in equation (68). Disturbances for the stages 2→M must be assumed zero. The full control equation thereby results as

$$\underline{U}_{k+(R1+2)c} = \underline{K}_{\infty}.\ \{\underline{\alpha 3}.\underline{Z}_{k-c} + \underline{\beta 32}.\underline{W}_{k-2c} + \underline{\gamma 3} \begin{bmatrix} \underline{V}_{k+(R1-1)c} \\ \vdots \\ \underline{V}_{k-(R1+1)c} \end{bmatrix} \} \qquad (71)$$

Set point control, in this case to zero the outputs, is based upon an M stage minimisation and disturbance rejection (regulator control), is based upon a single stage minimisation.

There is a realisibility problem associated with equation (71) because of the presence of disturbance elements which require to be referenced at stages ahead of the control instants. Since $\underline{K}_{\infty}$ is a converged matrix ($\underline{K}_M = \underline{K}_{M+1}$) it follows that control should be identical whether the algorithm is chosen from the top or bottom m rows of equation (71). The top m rows are preferable since the number of disturbance terms ahead of the control instant is minimised. The best algorithm is, therefore,

$$\underline{U}_{k+(R1-2)c} = \underline{G}_{\infty}.\ \{\underline{\alpha 3}.\underline{Z}_{k-c} + \underline{\beta 32}.\underline{W}_{k-2c} + \underline{\gamma 3}. \begin{bmatrix} \underline{0} \\ \underline{V}_{k+(R1-2)c} \\ \vdots \\ \underline{V}_{k=(R1+1)c} \end{bmatrix} \} \qquad (72)$$

$\underline{G}_{\infty}$ represents the top m rows of $\underline{K}_{\infty}$. $\underline{V}_{k+(R1-1)c}$ is not known at instant k+(R1-2)c and must therefore be taken as zero. This factor reflects the uncertainty of the controller because of lack of knowledge of what the disturbance is doing across the dead interval associated with controller update. There is still a difficulty with equation (71) because the measurement instant k-c does not correcpond to the control instant k+(R1-2)c. Equation (71) may be adjusted to align these instants. From equations (65) and (66) it can be established that

$$\underline{U}_{k+(R1-2)c} = \underline{G}_{\infty}.\Big\{\underline{\alpha 2}.\underline{Z}_{k+(R1-2)c} + \underline{\beta 21}.\underline{W}_{k+(R1-3)c} + \underline{\gamma 2}\begin{bmatrix} \underline{O} \\ \underline{V}_{k+(R1-3)} \\ \cdot \\ \cdot \\ \cdot \\ \underline{V}_{k-2c} \end{bmatrix}\Big\}. \tag{73}$$

Or, writing in the same notation as equation (70)

$$\underline{U}_{k-c} = \underline{G}_{\infty}.\ \{\underline{\alpha 2}.\underline{Z}_{k-c} + \underline{\beta 21}.\underline{W}_{k-2c} + \underline{\gamma 2}\begin{bmatrix} \underline{O} \\ \underline{V}_{k-c} \\ \cdot \\ \cdot \\ \cdot \\ \underline{V}_{k-(R1+1)c} \end{bmatrix}\} \tag{74}$$

$\underline{\beta 21}$ is equal to $\underline{\beta 2} - \begin{bmatrix} \underline{\beta 32} \\ \underline{O} \end{bmatrix}$ with the top m rows, which in fact result as zero, deleted.

2.6 Extension to cater for time Delays

The control algorithm of equation (74) needs to be extended to cater for the presence of time delays associated with the inputs $\underline{U}$ and disturbances $\underline{V}$. The structure of equation (64) has the advantage that delay values may be rounded down to the nearest multiple of c, the update interval.

Therefore, a delay less than c may be ignored for the purposes of control system design and implementation. This factor can be important in the initial selection of c.

First consider the situation with the delays associated with the controls $\underline{U}$ all the same. This is not as strict a requirement as it at first appears since the actual requirement is that all associated delays must fall within the update interval $d.c \rightarrow (d+1)c$, with d the delay expressed as a multiple of c. The more general situation is discussed later. Ignoring disturbance delays, the control algorithm of equation (74) becomes

$$\underline{U}_{k-c} = \underline{G}_{\infty}.\{\ \underline{\alpha 2}.\underline{Z}_{k-(1-d)c} + \underline{\beta 21}.\underline{W}_{k-2c} + \underline{\gamma 2}.\begin{bmatrix} \underline{V}_{k+d-c} \\ \cdot \\ \cdot \\ \cdot \\ \underline{V}_{k-(R1\_1-d)c} \end{bmatrix} \} \quad (75)$$

Disturbance term $\underline{V}_k$ is reintroduced from equation (74).

Equation (75) requires transformation, similar to that applied to equation (73), in order to synchronise the measurement and control instants for realisibility. Equation (65) is therefore iterated d-1 times and substituted into equation (75) to give

$$\underline{U}_{k-c} = \underline{G}_{\infty}.\{\ \underline{\alpha 4}_{k-c}.+ \underline{\beta 4}.\begin{bmatrix} \underline{U}_{k-2c} \\ \cdot \\ \cdot \\ \cdot \\ \underline{U}_{k-(R1+d+1)c} \end{bmatrix} + \underline{\gamma 4}.\begin{bmatrix} \underline{V}_{k+d} \\ \cdot \\ \cdot \\ \cdot \\ \underline{V}_{k-(R1+1)c} \end{bmatrix} \} \quad (76)$$

$\underline{\alpha 4} = \underline{\alpha 2}^d$ and $\underline{\beta 4}$ and $\underline{\beta 4}$ are derived by successive manipulation of $\underline{\alpha 2}$, $\underline{\beta 2}$, $\underline{\beta 21}$ and $\underline{\gamma 2}$.

If there is a delay associated with the disturbance elements, say d1, then equation (76) becomes, presuming all disturbances are subject to the same delay

$$\underline{U}_{k-c} = \underline{G}_{\infty} . \{ \ldots\ldots\ldots + \underline{\gamma 4} . \begin{bmatrix} \underline{V}_{k-(d1-d)} \\ \cdot \\ \cdot \\ \cdot \\ \underline{V}_{k-(R1+1+d1)c} \end{bmatrix} \} \tag{77}$$

For equation (77) to be realisable, all disturbances ahead of instant k-c must be set zero. Therefore, if d1>d then equation (77) holds. Otherwise, equation (77) must become

$$\underline{U}_{k-c} = \underline{G}_{\infty} . \{ \ldots\ldots\ldots + \underline{\gamma 41} \begin{bmatrix} \underline{V}_{k-c} \\ \cdot \\ \cdot \\ \cdot \\ \underline{V}_{k-(R1+1+d1)c} \end{bmatrix} \} \tag{78}$$

by setting $\underline{V}_k \rightarrow \underline{V}_{k-(d1-d)c}$ zero. $\underline{\gamma 41}$ is the truncated form of $\underline{\gamma 4}$.

If the delays associated with the disturbance elements in $\underline{V}$ differ, then the arguments used previously to relate to the zeroing of forward values of $\underline{V}$ now relate to the zeroing of particular elements within $\underline{V}$. If delays associated with the control elements in $\underline{U}$ differ, then it is necessary to select the largest of the associated delays as that defining the number of iterations to make the control equation realiable. Control actions for certain elements with shorter delays are then computed ahead of actuation time and must be saved until their actuation instants arise. For example, if $\underline{U}_{k-c}$ is constituted of two elements U1 and U2 such that

$$\underline{U}_{k-c} = \begin{bmatrix} U1_{k-c} \\ U2_{k-2c} \end{bmatrix} ,$$

then to be realisable the controller must compute $\begin{bmatrix} U1_k \\ U2_{k-c} \end{bmatrix}$.

The calculated U2 is implemented immediately at instant k-c. The calculated Ul is saved to be implemented at instant k.

2.7 Appreciation of Control Algorithm

Equation (77) represents the generalised control law to minimise the cost of equation (68) with $M \to \infty$. The equation relates to a linear approximation of process dynamics. It caters for multivariable interactions, time delays and feedforward compensation of disturbances. Equation (77) does not involve process set points. It is straightforward to show that the vector of set points $\underline{y}_{ss}$ (pxl) may be introduced into equation (56) to give[23]

$$\underline{YE}_k = \underline{\alpha}.\underline{ZE}_{k-c} + \underline{\beta}_{k-c} \tag{79}$$

with

$$\underline{YE}_k = \begin{bmatrix} \underline{y}_k - \underline{y}_{ss} \\ \cdot \\ \cdot \\ \cdot \\ \underline{y}_{k-s_{R-1}} - \underline{y}_{ss} \end{bmatrix} \quad \text{and} \quad \underline{ZE}_{k-c} = \begin{bmatrix} \underline{YE}_{k-c} \\ \underline{YE}_{k-2c} \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

Therefore, $\underline{Z}$ in equation (77) may be replaced by $\underline{ZE}$, which is a compound error vector. The controller is therefore now acting to zero $\underline{ZE}$ and thereby drive the outputs to set points. Considering equation (77) and replacing the difference terms $\underline{U}$ and $\underline{V}$ the final controller results as

$$\underline{u}_k - \underline{u}_{k-c} = \underline{G}_\infty \{\underline{\alpha 4}.\underline{ZE}_{k-c} + \underline{\beta 4}. \begin{bmatrix} \underline{u}_{k-2c} - \underline{u}_{k-3c} \\ \cdot \\ \cdot \\ \cdot \\ \underline{u}_{k-(R1+d+1)c} - \underline{u}_{k-(R1+d+2)c} \end{bmatrix}$$

$$+ \underline{\gamma 4} . \left[\begin{array}{c} \underline{v}_{k-(d1-d)c} - \underline{v}_{k-(d1-d+1)c} \\ \vdots \\ \underline{v}_{k-(R1\_1\_d1)c} - \underline{v}_{k-(R1+2+d1)c} \end{array} \right] \} \qquad (80)$$

Difficulties arise if the number of outputs is greater than the number of inputs. In this case, the set points vector becomes constrained. It is only possible to freely set as many set points as there are control inputs. Most control problems consider systems with the same number of inputs as outputs - the outputs being the parameters to be controlled. There is, therefore, usually no difficulty. Should it not be desirable to treat the problem in this way, then it is necessary to either have knowledge of plant steady state characteristics or to establish some other scheme for incorporation of the set points.

Equation (80) has properties that make it attractive as a control law.

(a) The controller computes a correction in input, not an absolute value.

(b) The r.h.s. is also based solely upon difference terms, not absolutes.

(c) Output quantities are only differenced with respect to set points, not with respect to each other.

(d) The structure of equation (80) can cater for the wide variety of situations, from the very simple first order, single input and output situation to the complex one with multiple inputs and outputs.

Factors a, b and c give the controller properties similar to the conventional industrial regulator, namely, integral action with error feedback. The control structure permits tolerance to non-linearity and time variance in the process, and performance should not be subject to offset in the steady state. Factor c is useful if noise is present on measurements. Noise impact is not aggravated by successive differencing as, for example, is necessary with conventional differential control action. In its most simple form, the application to a first order system with no delays or disturbances, the control law re reduces to

$$u_k - u_{k-c} = \underline{\alpha}_1(Y_k - Y_{ss}) + \underline{\alpha}_2(Y_{k-s_1} - Y_{ss}) \tag{81}$$

Only two computations of error are required and there are no input difference terms on the right hand side of the equation. Thus, although equation (79) appears complex, in many practical situations it reduces to a very simple structure indeed. An interactive computer facility incorporating a graphics display facility and simulation software has been developed for the design and implementation of control systems using this algorithm. The facility is described in greater detail in the following chapter.

CHAPTER THREE

COMPUTER SYSTEMS FOR MILK POWDER MANUFACTURE

3.1 Introduction

An IBM System 7 computer has been interfaced with a milk powder manufacturing plant situationed at the New Zealand Dairy Research Institute, Palmerston North. Software has been written to allow computer aided control system design methods to be applied and then allow operation of the plant using the controllers so designed. A description of the processing plant, instrumentation, computer and software follows.

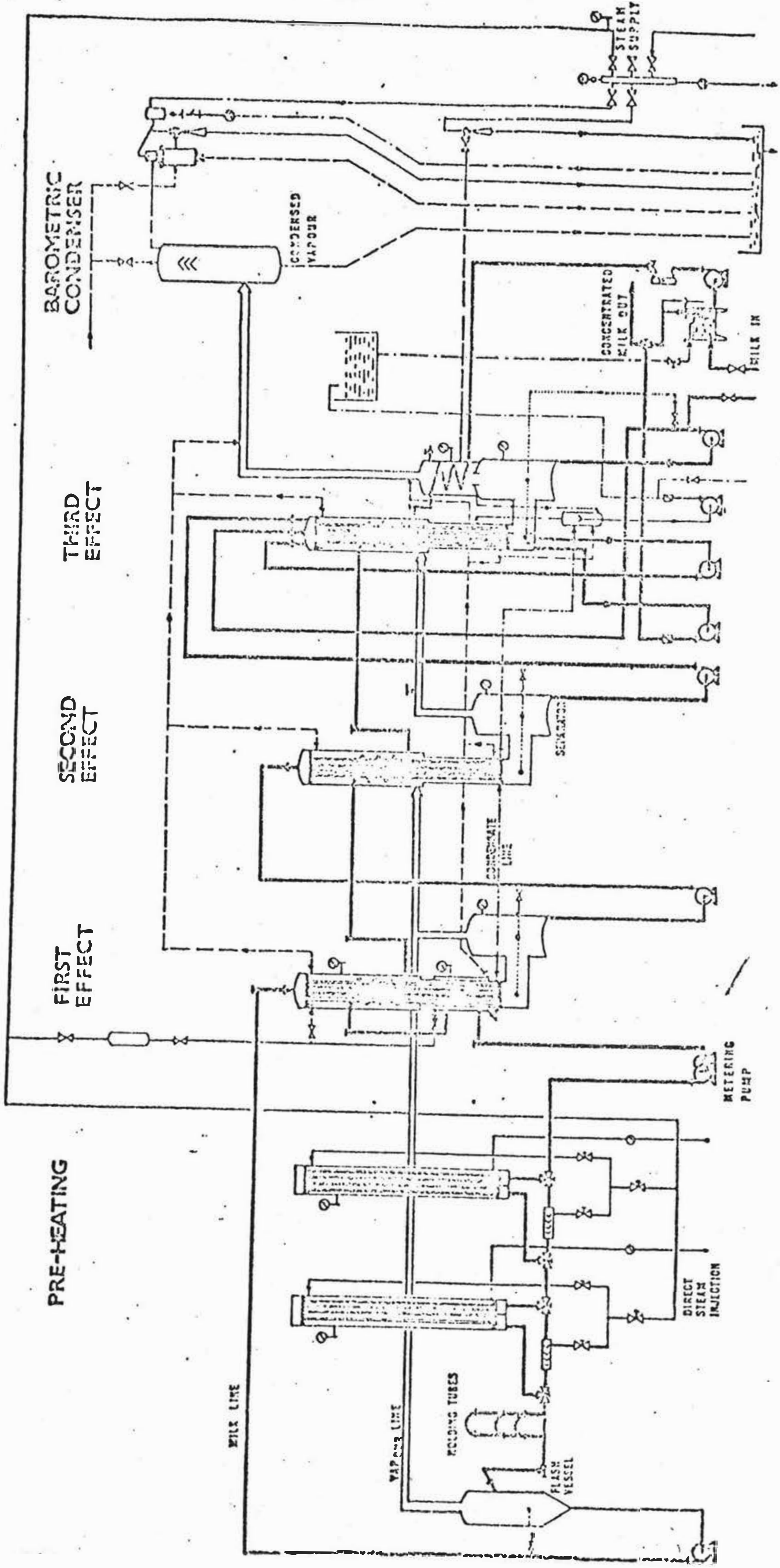
3.2 Processing Plant

The processing plant consists of a pilot scale Wiegand three effect falling film evaporator coupled to a De Laval tall form spray drier. A diagram of the evaporator is shown in figure 3-1.

The pilot plant is approximately 1/10th the capacity of a commercial system but is designed so that it behaves in the same way. Skim milk initially at 9% total solids concentration is processed to about 48% total solids at a rate of 1650 kg/hr.

Wiegand[24] gives a general description of the operation of falling film evaporators. A simplified block diagram of the NZDRI plant is given in figure 3-2.

The incoming milk is passed through the steam jacket in each of the three evaporating effects. This raises



WIEGAND EVAPORATOR

Figure 3-1

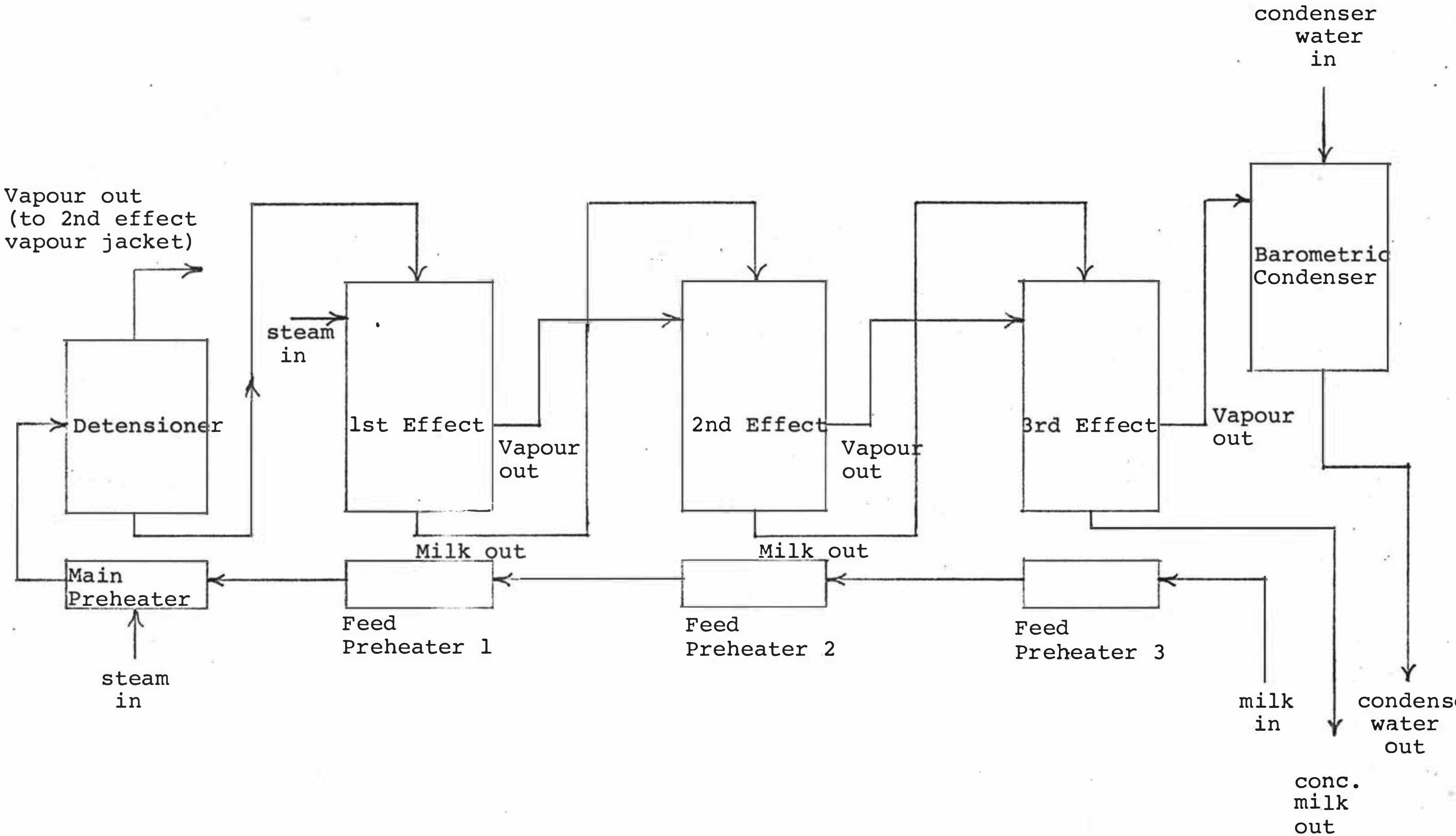


Figure 3-2 Block Diagram
Falling Film Evaporator

the milk temperature from a temperature of less than 10 $^{\circ}C$ up to a temperature of approximately 70 $^{\circ}C$. The milk is then passed through a direct steam injection unit operating at a temperature of 105 $^{\circ}C$. The milk is held at this temperature by means of a variable number of holding tubes for a period of time. This preheating of the milk serves the double purpose of destroying micro-organisms in the milk and determining some of the properties of the final product.

Because of the large temperature difference that exists between the preheated milk and the first effect a detensioner is used to isolate the holding tubes from the first effect and thus reduce the amount of flashing (instantaneous boiling) which occurs within the first effect. The detensioner operates at a pressure close to that of the first effect, thus the milk leaves at a temperature close to that of the first effect. The vapour flashed off in the detensioner is combined with vapour leaving the first first effect and used to heat the second effect. The milk leaving the detensioner is pumped to the top of the first effect where it is distributed across the first effect callandria tubes.

Flashing occurs when the milk enters the first effect and evaporation continues as the liquid flows down the tubes under the action of gravity and the vapours given off. The first effect callandria tubes are situated within a vapour jacket fed directly from a low pressure steam supply. At the base of the first effect tubes the liquid-vapour mix is passed to a separator. Here the vapour is passed to the vapour jacket surrounding the second effect callandria tubes and the liquid is pumped to the top of the tubes.

The second and third effects function in a similar manner to the first but there is a variation in the liquid flow path through the third effect. With increasing end-concentration the product becomes more viscous, thus causing a greater heat transfer resistance. This is accounted for by passing the product through three sets of tubes in succession. A single partitioned liquid-vapour separator is used. The product leaves the evaporator at approximately 48% total solids concentration. The vapours from the third effect are removed by condensation using a barometric mixing condenser. The vapours are mixed directly with a stream of cooling water and the condensate discharged under gravity. The non-condensible vapours are collected, compressed in a two-stage process using a high pressure steam inter-stage ejector and then discharged to atmosphere.

The spray drier is a De Laval tall form drier. It is matched to the capacity of the evaporator. A diagram of the spraydrier is shown in figure 3-3. Concentrate is taken from the drier to a high pressure feed pump. Holding tanks situated in the line between the evaporator and spray drier may be plumbed in as required. The concentrate is pumped through a spray nozzle at a pressure of 200-340 Bar. The nozzle characteristics are changeable and ensure that the concentrate is atomised and mixed with a flow of incoming heated air.

The airflow into the drier is heated by a direct fired natural gas burner to a temperature of about 200 ^{o}C. Air flowrate is controlled by damper vanes in the inlet and outlet ducts of the spray drier. The atomiser concentrate droplets are dried within the heated airstream to 3-5% moisture content dependent on the type of powder being manufactured. The powder is collected from the bottom of the drier and passed to a cyclone for packing.

3.3 Hardware System

The process control computer used is an IBM System 7 model. This unit has a 16-bit processor and 32K words of memory. Internal parity checking of stored data is

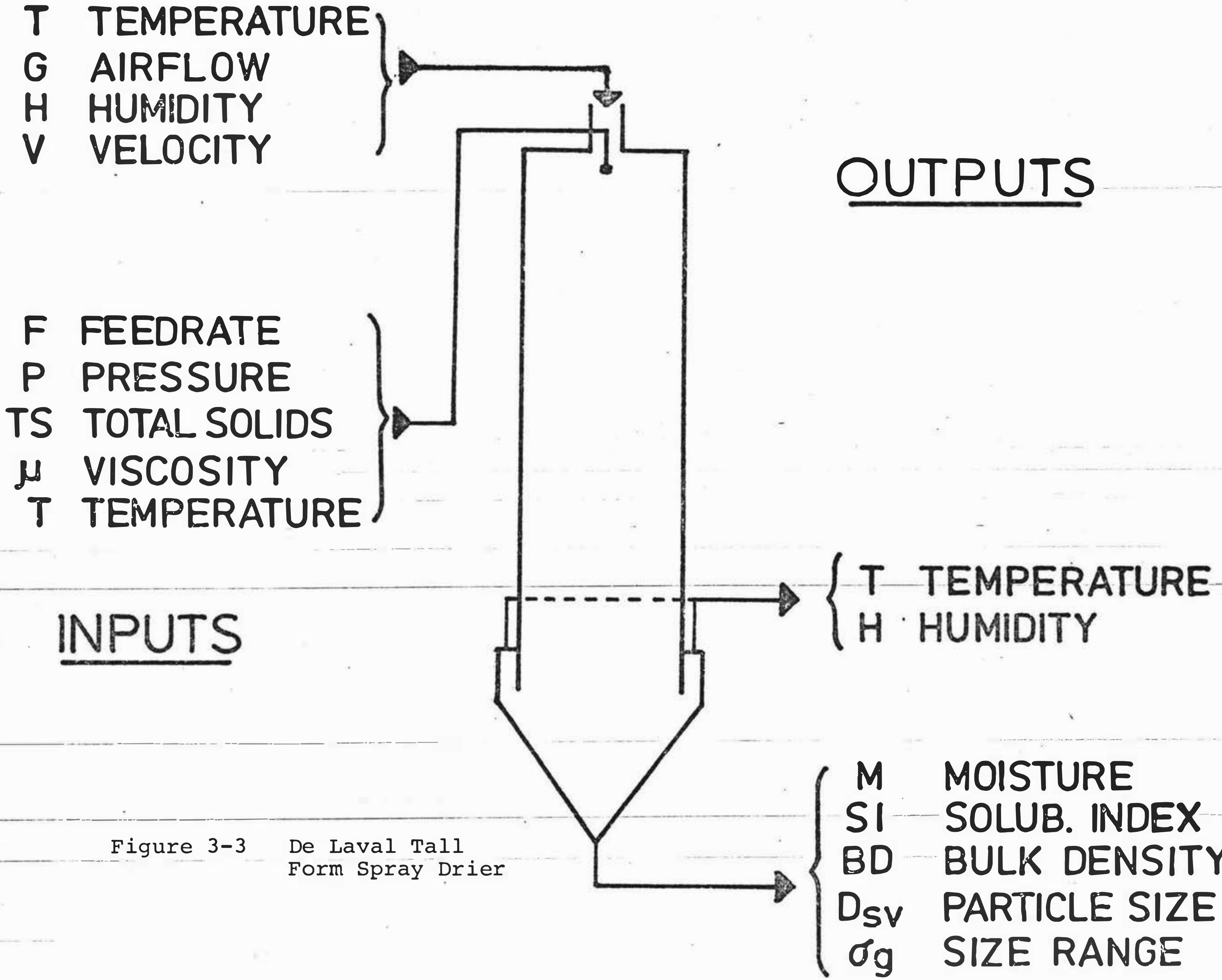


Figure 3-3 De Laval Tall Form Spray Drier

carried out. Background storage is available on magnetic disk and a 2 megaword capacity is provided. The disk unit comprises two disk cartridges, one fixed and one removable using a single drive spindle. A 5012 multifunction module provides the data interface between the computer and the instrumentation on the processing plant. The multifunction module incorporates 32 analogue input channels each with 12-bit resolution, 2-process interrupt groups, 6 digital input and 4 digital output groups (each with 16-bits/group) and 2 analogue output points (0± 10.24 volts d.c.). The module operation is software controlled.

Operator input is through a Tektronix 4010 visual display terminal. The terminal is driven using a process interrupt group, a digital input group and a digital output group on the System 7 computer. Hardcopy of printed information is obtained on a 5028 Teletype. The Tektronix terminal also has a graphics facility which is used extensively by the software operating systems. Hard copy of graphic displays is obtained by outputting information onto papertape and reprocessing on a Hewlett Packard 9830 minicomputer coupled to an X-Y plotter.

3.4 Instrumentation

The evaporator and spray drier at the NZDRI are both fully instrumented[25]. The locations of the various measuring devices are as shown in figures 3-4 and 3-5.

Temperature measurement using thermocouples is done at 22 locations on the evaporator and 18 locations on the spray drier. Each thermocouple is connected to one input channel of a Hewlett Packard 3485A digital voltmeter/ scanning unit. Under control of the computer a particular input channel is accessed and the voltage read. An appropriate binary coded decimal (BCD) value is then output to the computer. Conversion to a temperature value

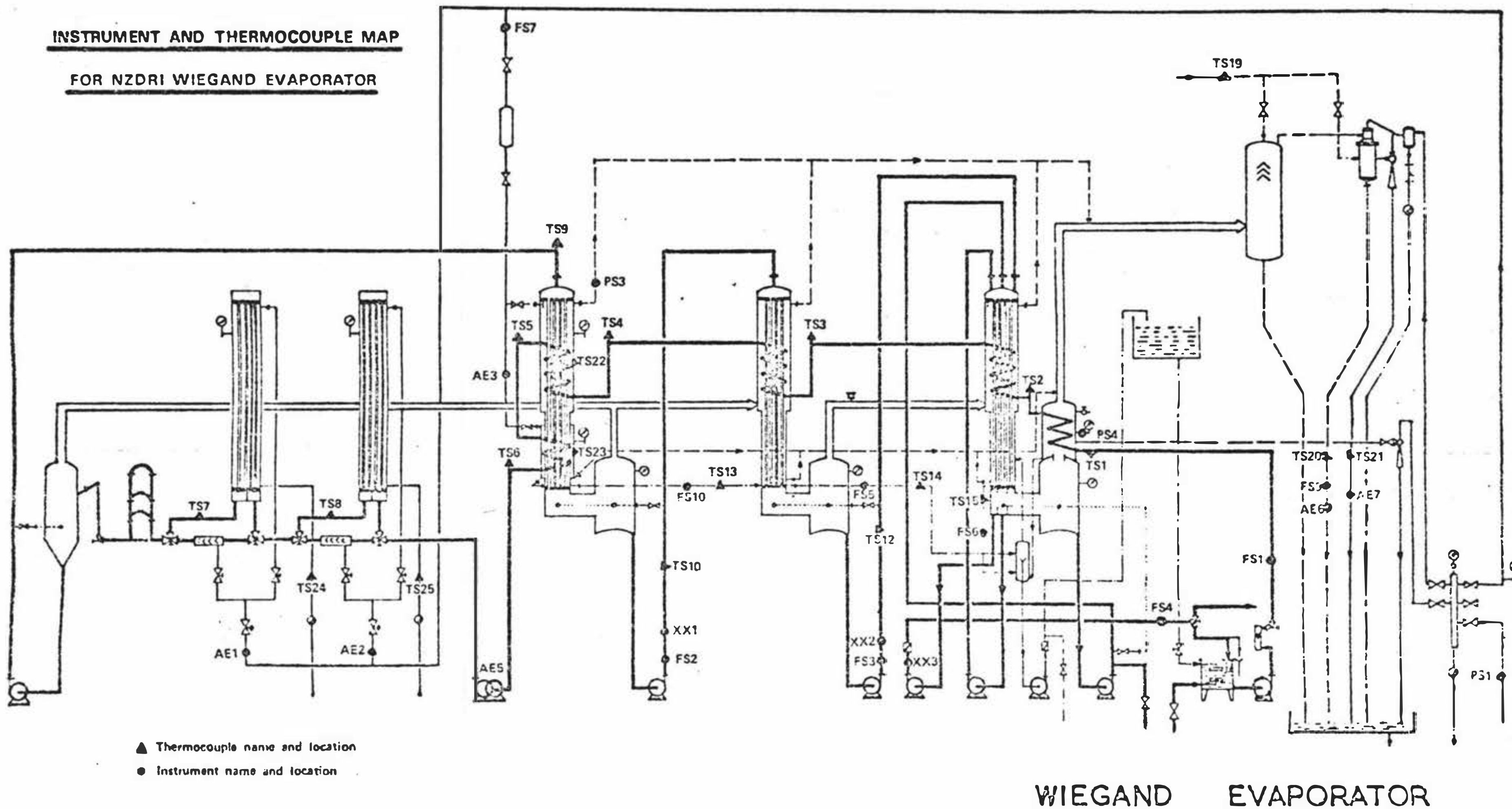

INSTRUMENT AND THERMOCOUPLE MAP
FOR NZDRI WIEGAND EVAPORATOR
FS7
TS19
TS9
PS3
TS5
TS4
TS3
TS22
AE3
TS2
PS4
TS1
TS6
TS23
TS13
FS10
FS5
TS14
TS15
FS6
TS12
TS20
TS21
AE7
AE6
TS7
TS8
TS24
TS25
TS10
FS1
FS4
XX1
FS2
XX2
FS3
XX3
AE1
AE2
AE5
PS1
▲ Thermocouple name and location
● Instrument name and location
WIEGAND EVAPORATOR

Figure 3-4a

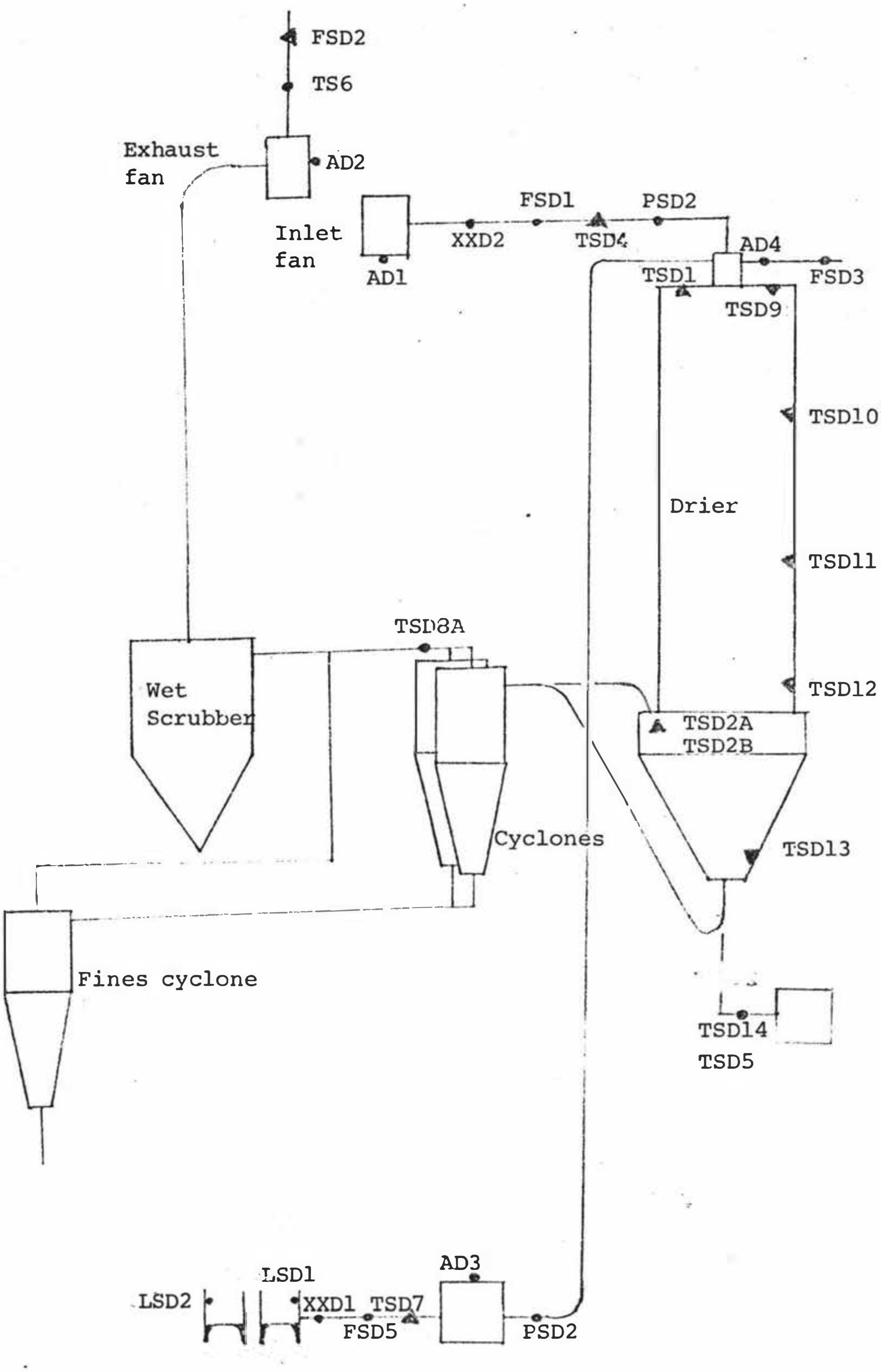


Figure 3-4b Instrument and Thermocouple Map for N.Z.D.R.I. Spray Drier

INSTRUMENT CODES AND DESCRIPTIONS

PS1	Steam supply pressure
PS3	1st effect chest pressure
PS4	3rd effect separator pressure
FS1	Flow to 1st effect
FS2	Flow from 1st effect
FS3	Flow from 2nd effect
FS4	Flow from 3rd effect
FS5	2nd effect condensate flow
FS6	3rd effect condensate flow
PSD1	Drier chamber pressure
FSD3	Burner gas flow
FS7	1st effect steam flow
FS9	Condenser water flow
FS10	1st effect condensate flow
XX1	Concentrate density from 1st effect
XX2	Concentrate density from 2nd effect
XX3	Concentrate density from 3rd effect
AE1	Preheater 1 steam valve controller
AE2	Preheater 2 steam valve controller
AE3	1st effect main steam valve controller
FSD1	Inlet air flow
FSD5	Drier feed flow
AE5	Evaporator feed pump controller
AE6	Main condenser water flow controller
AE7	Interstage condenser water flow controller
AD1	Drier Inlet air damper controller
AD2	Drier Outlet air damper controller
AD3	High pressure pump controller
AD4	Gas to Burner Controller
LSD1 & 2	Concentrate tank levels
PSD2 & 3	Drier inlet and outlet pressures
FSD2	Exhaust air flow
XXD1 & 2	Milk viscosity and inlet humidity

THERMOCOUPLE CODES AND DESCRIPTIONS

TS1	Milk feed to Spiral heater
TS2	Milk from Spiral heater
TS3	Feed to 3rd effect heater
TS4	Feed to 2nd effect heater
TS5	Feed to 1st effect heater
TS6	Milk from 1st effect jacket
TS7	Milk from 1st preheater
TS8	Milk from 2nd preheater
TS9	Milk to 1st effect
TS10	Milk to 2nd effect
TS11	Milk to 3rd effect
TS12	Concentrate from 3rd effect
TS13	1st effect condensate temperature
TS14	2nd effect condensate temperature
TS15	3rd effect condensate temperature
TS19	Main condenser feed
TS20	Main condenser tail
TS21	Interstage condenser tail
TS22	Steam in 1st effect chest
TS23	Steam in 1st effect jacket
TSD14	Chilled conveying air temperature
TS24	1st preheater condensate temperature
TS25	2nd preheater condensate temperature
TSD1	Atomiser feed temperature
TSD2	(A & B) exhaust bustle air temperature
TSD3	Throat temperature
TSD4	Ambient air temperature
TSD5	Heated conveying air temperature
TSD6	Exhaust venturi air temperature
TSD7	Low pressure concentrate feed temperature
TSD8	(A & B) cyclone exit temperatures
TSD9–13	Chamber wall temperature

Figure 3-5 Instrumentation description

is then performed within a computer program.

The remaining plant variables are measured using analogue signals. The instruments are calibrated to the standard range of 4-20 mA. The signal lines connect to 32 ohm resistors at the System 7 computer termination cards, thus producing a 128-640 mV reading across the analogue input channels of the 5012 multifunction module.

Flowmeasurement in the plants is performed using either differential pressure meters or electromagnetic flowmeters. The latter method is used for the measurement of milk flows, where sanitation is a prime requirement. Pressure measurement is performed using differential pressure cells except when sanitary construction is required. In these cases an isolating diaphragm type instrument is used.

Product density at various points through the plant is measured using densitometers. These instruments rely on variations in natural resonance frequency to give an indication of density. A high density fluid produces a lower resonance frequency than one of low density. A 4-20 mA signal proportional to the density is sent to the process control computer.

Process activation functions are implemented using standard Honeywell computer output stations. Each station is individually addressable and can sample a voltage output by the computer and convert it into a 4-20 mA current signal. This signal then drives an electro-pneumatic positioner thus obtaining mechanical movement proportional to the computer voltage signal. The output stations have the ability to "hold or maintain" their signal until an updated value is provided by the computer. Manual override is available if required.

Initially all sensor lines are directed to a control panel located in the process control room. This panel houses the Hewlett Packard digital voltmeter/scanning unit, frequency to current conversion units for the densitometers and power supplies for the analogue instrumentation as well as providing a convenient collection point for interfacing with the computer.

3.5 Software Systems

Two software systems (a real time system and an off-line system) have been developed for the IBM System 7 computer.[26,27] During the initial development stages it was found that the memory storage available (16 K words) would not allow a single system to be implemented, hence the division into two separate but inter-related parts. Both systems are written using the assembler and FORTRAN IV languages. The real time system is used to link the computer to the milk pwder plant and control its operation, whilst the off-line system is used for setting parameters, modelling the plant and designing and simulating the control systems for the plant.

3.5.1 Real time System

The real time system is made up of a number of different programs or tasks each of which is responsible for a particular operation, e.g., sampling of an input signal or allowing the plant operator to access parameters. The tasks utilise the System 7 interrupt driven, 4-level, priority structure.[28,29] Under this structure each task is assigned to a priority level according to the relative importance of the function it is to perform. 0 is the highest priority, 3 is the lowest. Table 3-1 shows the priority levels and the functions of the tasks associated with the particular level.

The real time system supports 15 thermocouple readings, 20 analogue input signals and 10 output stations. Up to

Table 3-1
Priority Structure

Level	Program Functions
0	Disk driver Real time clock Hewlett Packard DVM acknowledge
1	Analogue output to output stations Read digital input signals, convert to engineering units Read analogue input signals, convert to engineering units
2	Set level of automatic control on plant Perform control calculations Emergency plant shutdown
3	Operator input/output, graphics Plant status messages

twenty control loops are available, nominally 12 for controlling the evaporator and 8 for control of the spray drier. System configuration is performed using the off-line system. Parameters are then passed to the real time system using an external data file.

The various programs making up the real time system are interrupt driven. These interrupts can occur from the real time clock (i.e., tasks are scheduled to run at particular points in time), as a consequence of a request from the operator, or as a result of a request from another task. The programs allocated to level 1 are executed on a time scheduled basis. The analogue input channels are sampled every 0.25 seconds. A channel address is specified and the appropriate voltage signal recorded. The signal is processed to give a reading in engineering units. Filtering of the reading using the exponential smoothing algorithm is carried out if required. The procedure is repeated for each of the analogue input signals.

Sampling of temperature signals is also performed every 0.25 seconds. This time, however, the multiplexing of input channels is done by the Hewlett Packard Scanning Unit. The required channel address is sent to the scanning unit which then reads the corresponding signal. Once this voltage signal has been converted to its equivalent BCD form an acknowledge ready interrupt is sent to the computer. The digital input group is read and the engineering units conversion is performed as is filtering if required.

Filtering is performed using the well known formula

$$\hat{y}_k = (1-a) \,.\, \hat{y}_{k-1} + a\, y_k$$

The interval $k \rightarrow k+1$ is 0.25 seconds, Y_k is the measured value and $\hat{y}_k$ is the filtered estimate. The filter constant a is selected on the basis of visual inspection of the characteristic of variable $\hat{y}$, to give a reasonable smoothed response.

Analogue output signals (1-5V) are sent to the output stations every second. The output stations are connected to a common bus line and the receiving station is specified via a separate digital output group. On receipt of the required signal the input terminal of the selected output station is connected to the common bus line. The signal is sampled and an equivalent current signal passed on by the output station.

The program for calculation of control actions runs every second. Each control loop has its own internal clock counter and control calculations are synchronised to this counter. Data for use in these calculations is, however, being collected at the requisite points in time. An option is available to store plant and control data onto disk storage. The frequency of storage is specified by the operator.

The remaining programs are activated on request by the operator. On receiving a request interrupt the computer issues a prompt to which the operator must reply with a 3-letter code corresponding to the program function he requires. Functions which may be requested by the operator include printouts of plant variable status, control loop parameters including setpoints, control limits, mode and level of operation, graphics and level of plant operation. The computer can be directed to perform safety checks in which case warning messages and/or complete plant shutdown occurs if an abnormal situation is encountered.

The graphics display facility is an important feature of the real time system. It allows the operator to observe the history of the plant operation and compare the present state of operation with previous states. The time scale of the graph, variable displayed (maximum of six per graph), vertical scale and resolution can all be directly adjusted by the operator. Hard copies of particular graphs are obtained by outputting data onto papertape and reprocessing the papertape using a Hewlett Packard 9830 minicomputer coupled to an X-Y plotter. A typical graphics display is shown in figure 3-6. The graph scales represent the value in engineering units * 10.0.

The control portion of the real time system has been developed so as to be effectively plant independent and capable of supporting any desired structure of control hierarchy.

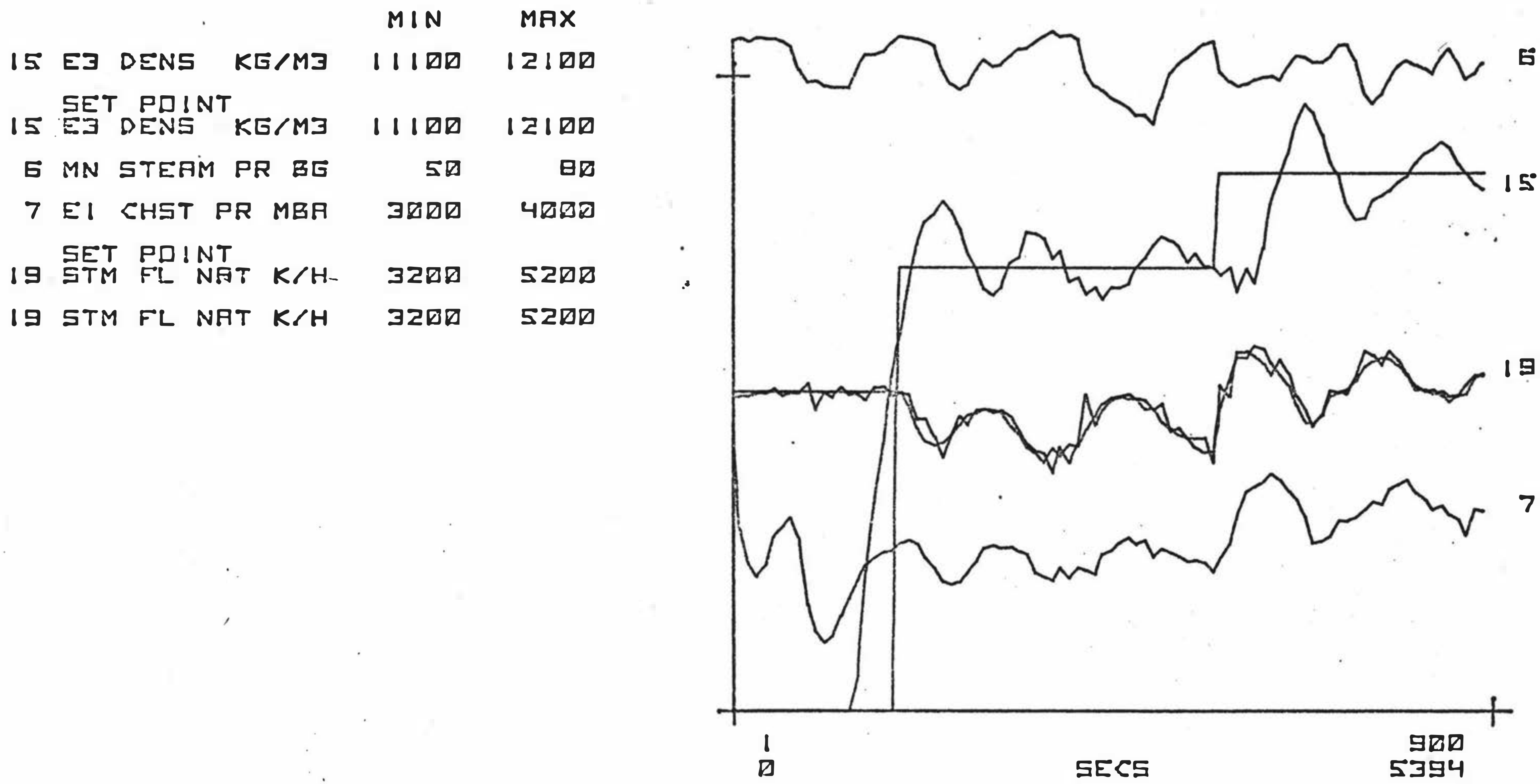


Figure 3-6 Typical graphics display

3.5.2 Offline System

The offline software system is designed to be used in conjunction with the real time system and serves two main purposes.

(i) The system allows the operator to specify the plant configuration to be used by the real time system,

(ii) The system allows the operator to access data and design appropriate controllers using least squares modelling, dynamic programming and simulation techniques. The data used by these techniques may be either actual plant data obtained whilst monitoring/ controlling the plant using the real time system or data generated from within the offline system.

A common structure is used between the two systems with respect to data areas and various subroutines. Data is passed from one system to the other by way of two disk files - one for the system configuration parameters and the other for plant data.

The system parameter file (SSDATA) can only be changed using the offline system. Some of the parameters read from this file by the real time system may be changed online. These parameters generally relate to the input/output functions, such as setpoints, graphics and data storage intervals, which a plant operator must have access to online. This ensures the integrity of operation of the real time system as parameters which could cause a system failure are inaccessible.

The ability to reconfigure the plant from the keyboard is an important feature of the software systems. Not all the plant variables measured need to be monitored during a particular run thus various system parameter files can be set up depending on the tupe of product to be manufactured

A great deal of flexibility is therefore available as far as structuring of the control loops. Twenty loops are available and these can be configured to be primary (i.e., adjusting a value position), secondary (i.e., adjusting the setpoint of some other control loop), single loop or multi-variable as required.

Control system design is performed using an interactive design procedure utilising the graphics display features of the 4010 Tektronix terminal. A recursive least squares procedure is used to estimate coefficients given a combination of parameters that define the model structure. Plant input and output data recorded on a disk file as a result of previous experimental runs is used. The designer selects the inputs and outputs associated with the model and proposes initial combinations of structure parameters, perhaps on the basis of desired response times of controller or because of disturbance characteristics. Initial delay values can be estimated by inspection of the graphical data. The least squares procedure is then called in to attempt an estimation of the coefficients. Model effectiveness is assessed by using the model and regenerating the input and output data. If the model is ineffective, it is up to the designer to postulate reasons (e.g., incorrect process order or exclusion of a disturbance) and to adjust parameters so as to attain an improved model.

The modelling facility relies almost entirely upon the pattern recognition abilities of the designer. No attempt has been made to provide additional facilities to help the designer in choice of parameters, apart from simple covariance/correlation quantities, generated by the least squares procedure. There are techniques available that can assist, for example, in selection of process order and time delays.[30,31]

Having obtained a suitable model, control system design is performed using a discrete dynamic programming procedure. The designer selects the diagonal weighting matrices $\underline{P}$ and $\underline{Q}$ as well as the number of stages the algorithm will be applied over. The computer then calculates the controller gains corresponding to these parameters. At this point the designer is then able to specify setpoints and disturbance characteristics before simulating the time response of his model when implementing this particular controller. The graphics facility enables these time responses to be studied in detail and assessment of control effectiveness to be established. If the simulated response is not satisfactory the control design can be repeated. Once a suitable controller has been designed and simulated the parameters are written to a disk file to be accessed by the real time system and the controller performance evaluated online.

3.6 Offline Simulation

The use of simulation was an important part in the development of the oeprating systems. By this means the various aspects of modelling, control system design and implementation could be extensively tested before actually using the real time system on the plant. The response of the test system is known and if the simulated response deviated from this "time" response the error in implementation could be traced through the various design stages.

The base system used is representative of a fourth order mass, spring and dashpot system as shown in figure 3-7.

Two external forces, U_1 and U_2 are required to maintain the masses M_1, M_2 at their respective equilibrium positions x_1 and x_2. If the masses are assumed to be 1 unit, spring constants k_1 and k_2 equal to unity and damping coefficient equal to 0.5 then the system can be

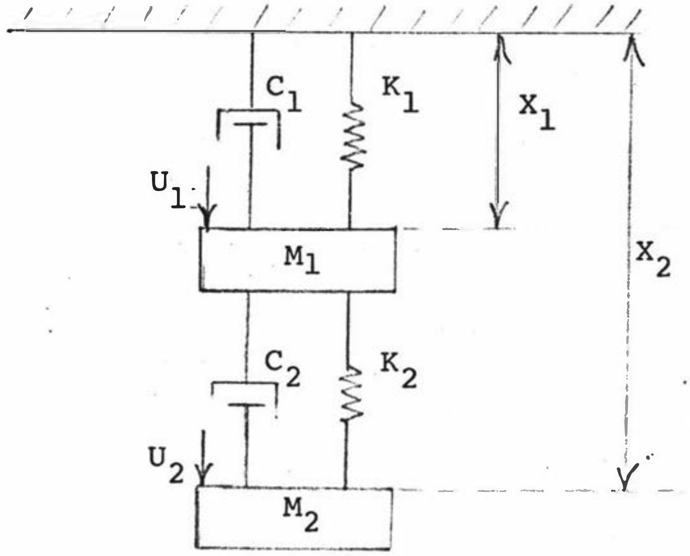


Figure 3-7 Mass Spring System (4th order)

represented by the state equation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 1 & -1 & 0.5 \\ 1 & -1 & 0.5 & -0.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$$\underline{\dot{x}} = \underline{A}\,\underline{x} + \underline{B}\,\underline{u}$$

Using the relations $\underline{\phi} = e^{\underline{A}t}$, $\underline{\Delta} = A^{-1}(\underline{\phi}-\underline{I})B$ for a sample period of 1 sec and assuming that the inputs are realisable in a time much shorter than the time constants of the system, so that any change in value is seen by the system as a step change gives

$$\begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}_{k+1} = \begin{bmatrix} .428 & .246 & .489 & .225 \\ .246 & .674 & .225 & .714 \\ -.753 & .264 & .052 & .378 \\ .264 & -.489 & .378 & .429 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}_k$$

$$+ \begin{bmatrix} .326 & .08 \\ .08 & .406 \\ .489 & .225 \\ .225 & .714 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_k$$

This equation has the form $\underline{x}_{k+1} = \underline{\phi}\ \underline{x}_k + \underline{\Delta}\ u_k$ and can be used to generate discrete data corresponding to the previously defined fourth order system.

Figure 3-8 shows a sequence of data generated for this system. With inputs u_1 and u_2 set to 300 the equilibrium values for x_1 and x_2 are 600 and 900 respectively. The inputs u_1 and u_2 have been randomly perturbed at 12 second intervals. The amplitude range of the perturbations is defined from the keyboard and in this example is 300±50. A 10 second delay period has been specified between the application of an input and the resultant response.

Superimposed on the responses of x_1 and x_2 is the piecewise characteristic response of the least squares model, derived from the perturbation data, when subjected to the same sequence of inputs. The model is fourth order and has an update interval of 6 seconds and a sample interval of 3 seconds. To cater for the delay in the system the model incorporates a delay of 1 update interval (6 second) plus an additional back input sample.

The least squares model has the form

		MIN	MAX
23	VARIABLE X1	0	10000
23	MODEL VARIABLE X1	0	10000
1	VARIABLE X2	0	10000
1	MODEL VARIABLE X2	0	10000
37	INPUT U1	1000	7000
38	INPUT U2	2200	8200

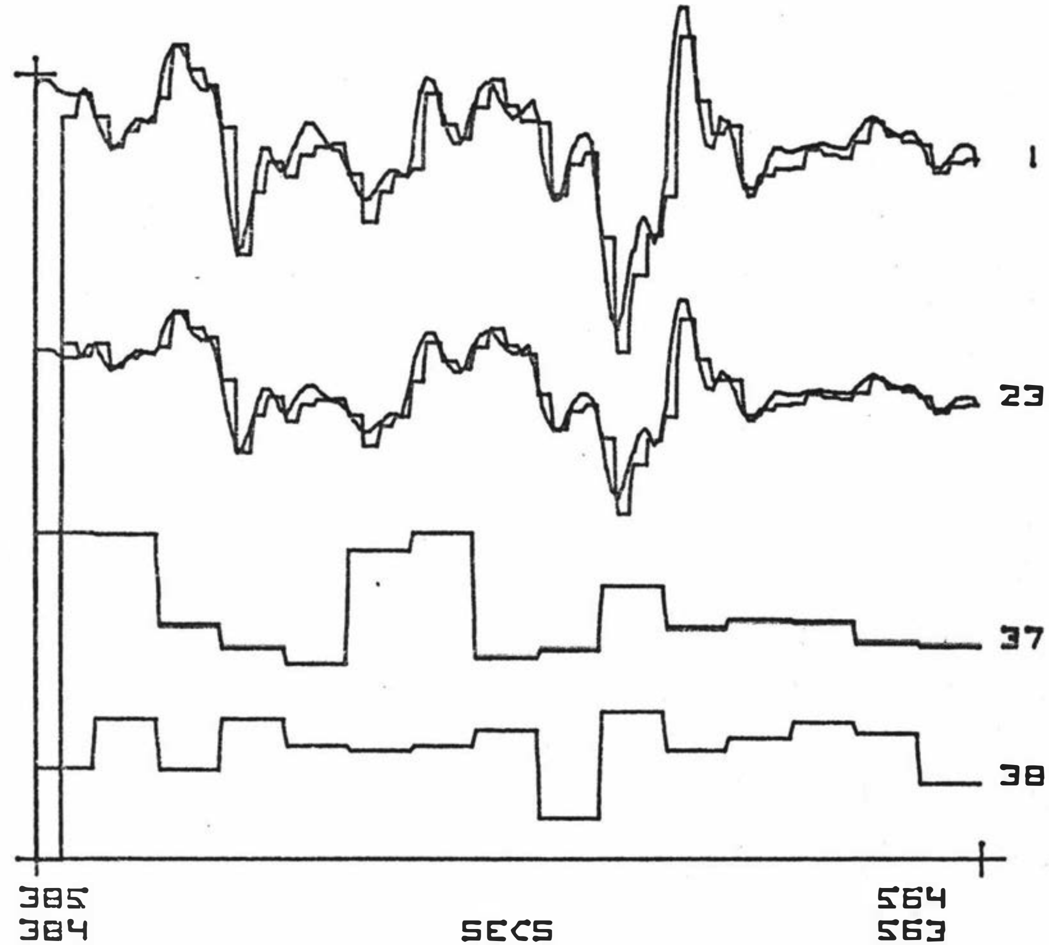


Figure 3-8

MODEL OF SIMULATED 4TH ORDER SYSTEM WITH 10 SECS DELAY
CONTROL =6, SAMPLE = 3, 1 DELAY STAGE, 1 BACK INPUT SAMPLE

$$\underline{X}_{(k+1)T} = \underline{\alpha}\,\underline{X}_{kT} + \underline{\beta}\,\underline{U}_k$$

where

$$\underline{X}_{kT} = \begin{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{kT} \\ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{kT-\tau} \end{bmatrix}$$

$$\underline{U}_k = \begin{bmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}_{kT} - \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}_{(k-1)T} \\ \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}_{(k-1)T} - \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}_{(k-2)T} \end{bmatrix}$$

$$\underline{\alpha} = \begin{bmatrix} .77 & -.03 & .23 & .03 \\ -.35 & .94 & .35 & .06 \\ 1.03 & .27 & -.03 & -.27 \\ .06 & 1.42 & -.06 & -.42 \end{bmatrix}$$

$$\underline{\beta} = \begin{bmatrix} .78 & .51 & .46 & .73 \\ .50 & 1.29 & .90 & 1.10 \\ .02 & .03 & .60 & 1.00 \\ .03 & .05 & 1.13 & 1.53 \end{bmatrix}$$

$T = 6$ seconds, $\tau = 3$ seconds

If the order of the model is made less than 4 or the delay altered from 1 update interval plus the back input sample a poorer model is obtained.

The next step in the procedure is to design a controller. The interactive facility allows the designer to specify the diagonal elements of the weighting matrices $\underline{P}$ and $\underline{Q}$. A dynamic programming procedure is then performed to minimise the quadratic performance criteria.

$$J = \sum_k \left[\underline{e}_k^T \, \underline{P} \, \underline{e}_k + \underline{U}_k^T \, \underline{Q} \, \underline{U}_k \right]$$

leading to a controller with the form

$$\underline{U}_{kT} = \underline{G}\ \underline{e}_{kT} + \underline{H}\ \underline{w}_{kT}$$

$\underline{e}_{kT}$ = error vector
$\underline{w}_{kT}$ = past input vector

For weights $\underline{P}$ = diag. $[1, 1, 1, 1]$ and $\underline{Q}$ = diag. $[0, 0]$ 10 iterations of the algorithm gives,

$$\underline{U}_{kT} = \left[\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}_{kT} - \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}_{(k-1)T} \right]$$

$$\underline{e}_k = \begin{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{kT} - \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_d \\ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{kT-\tau} \quad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_d \end{bmatrix}$$

$$\underline{w}_{kT} = \begin{bmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}_{(k-1)T} - \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}_{(k-2)T} \\ \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}_{(k-2)T} - \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}_{(k-3)T} \end{bmatrix}$$

$$\underline{G} = \begin{bmatrix} 1.72 & .91 & -.03 & .04 \\ .96 & -.90 & -.04 & .04 \end{bmatrix}$$

$$\underline{H} = \begin{bmatrix} -.75 & .21 & .02 & -.28 \\ .07 & -.77 & -.37 & -.38 \end{bmatrix}$$

The response of the least squares model with this controller acting is shown in figures 3-9 and 3-10. The setpoints in each case are 500 and 900 for x_1 and x_2 respectively. Figure 3-10 shows the control response when the measured value is subject to a 5% noise level. The values in the weighting matrices may be adjusted until

		MIN	MAX
23	VARIABLE X1	0	10000
23	SET POINT VARIABLE X1	0	10000
1	VARIABLE X2	0	10000
1	SET POINT VARIABLE X2	0	10000
37	INPUT U1	1000	7000
38	INPUT U2	2200	8200

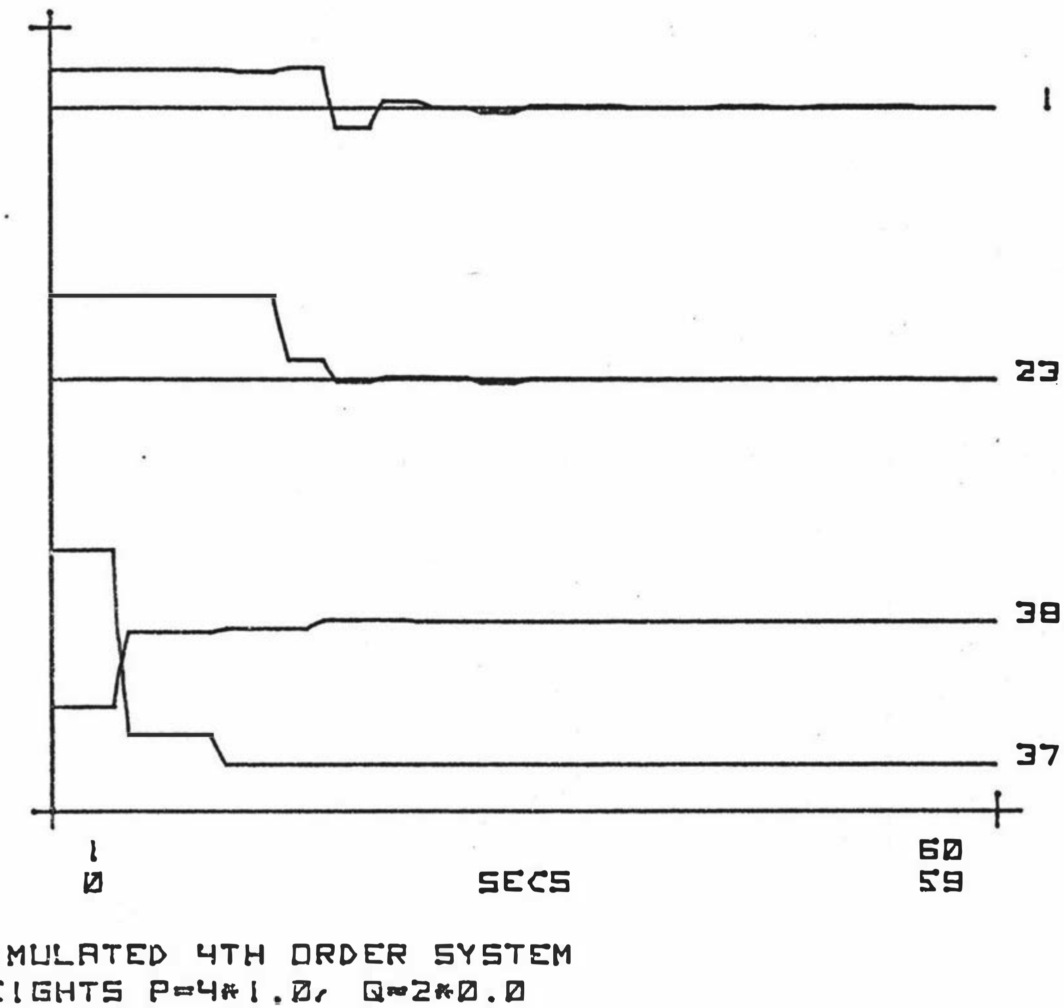
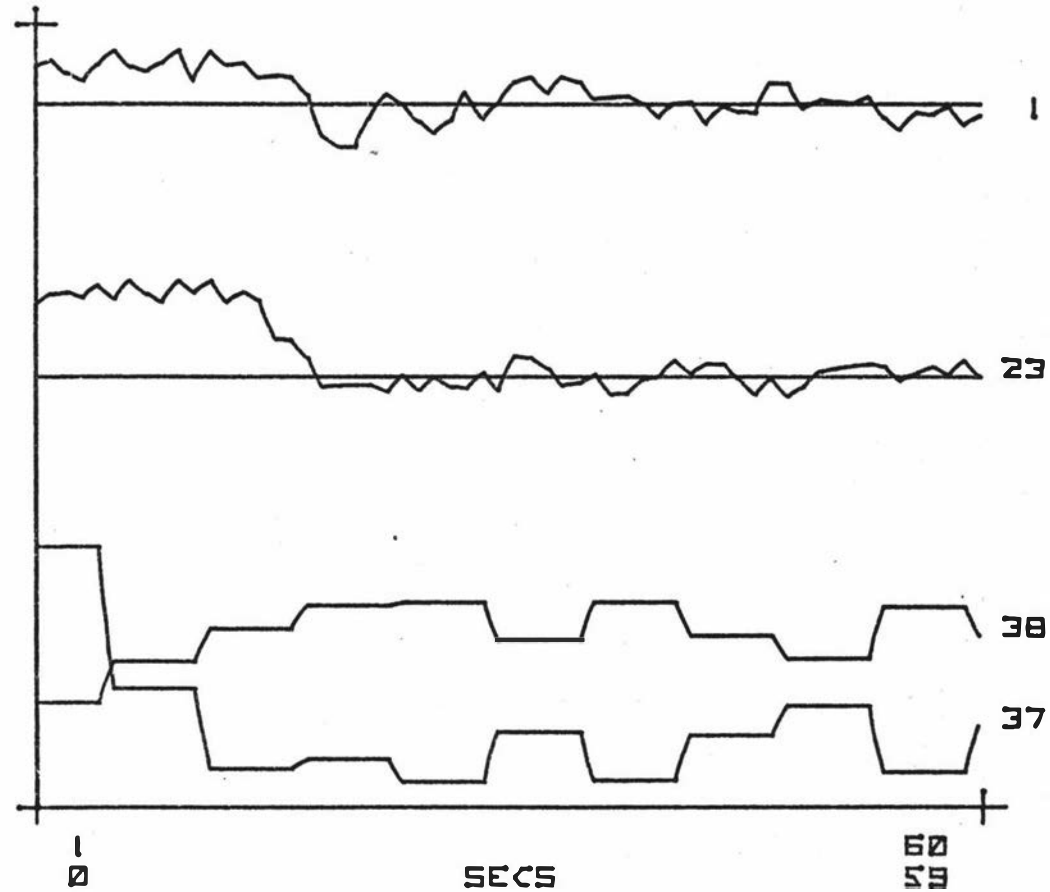


Figure 3-9

	MIN	MAX
23 VARIABLE X1	0	10000
23 SET POINT VARIABLE X1	0	10000
1 VARIABLE X2	0	10000
1 SET POINT VARIABLE X2	0	10000
37 INPUT U1	1000	7000
38 INPUT U2	2200	8200

Figure 3-10



SIMULATED 4TH ORDER SYSTEM
WEIGHTS P=4*0.0, Q=2*0.0, NOISE = 5%

the desired control response is obtained. Figure 3-11 shows the control response for weights $\underline{P}$ = diag. |1,1,1,1|, $\underline{Q}$ = diag. [1,1]. As there is a weighting on the control effort the control response is slower than previously. The control matrices for these weights are -

$$\underline{G} = \begin{bmatrix} -.81 & .33 & -.03 & .03 \\ .40 & -.53 & -.04 & .04 \end{bmatrix}$$

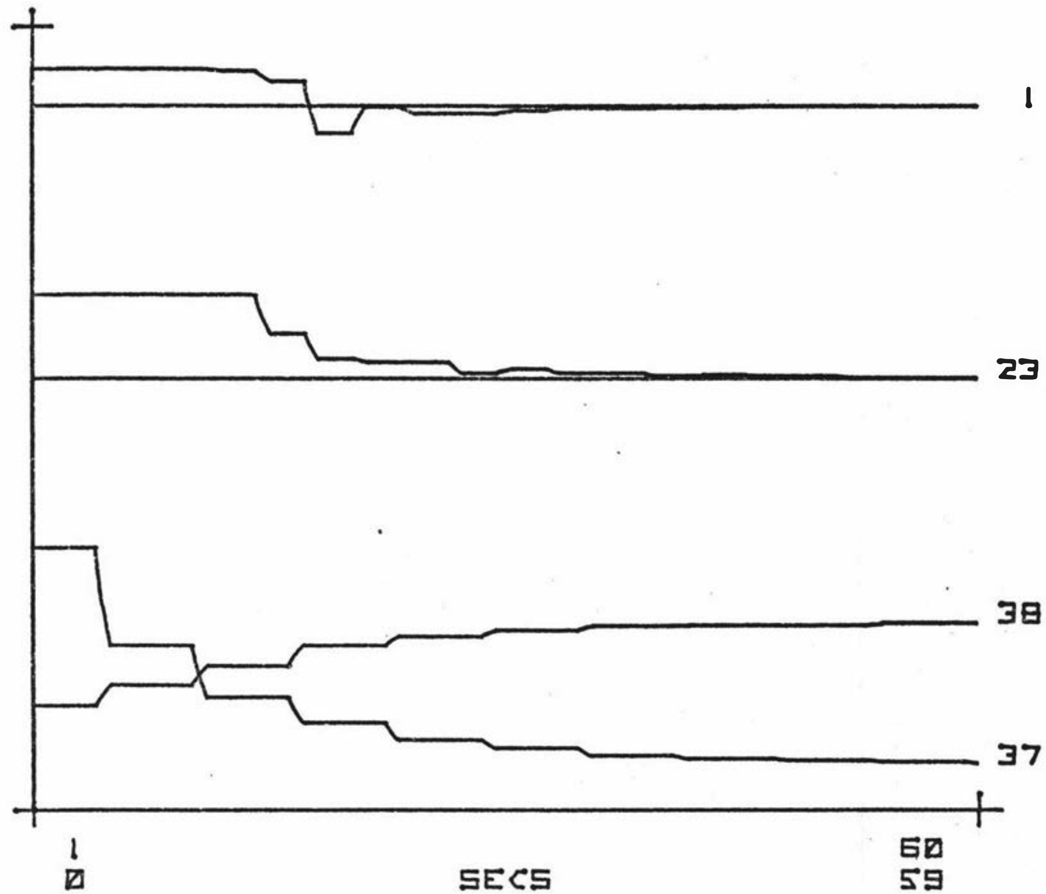
$$\underline{H} = \begin{bmatrix} -.45 & -.08 & -.08 & -.25 \\ -.10 & -.58 & -.30 & -.33 \end{bmatrix}$$

This example has demonstrated the design of a multivariable control system. The flexibility of the interactive facility allows other control configurations to be designed and tested without additional restraints.

Controllers incorporating disturbance terms may be simulated by generating data with up to 4 inputs. As an example, consider the data as shown in figure 3-8. If variable x_1 is regarded as the only output variable of concern, a control system with the single input u_1 and u_2 as the disturbance could be designed.

		MIN	MAX
23	VARIABLE X1	0	10000
23	SET POINT VARIABLE X1	0	10000
1	VARIABLE X2	0	10000
1	SET POINT VARIABLE X2	0	10000
37	INPUT U1	1000	7000
38	INPUT U2	2200	8200

Figure 3-11



SIMULATED 4TH ORDER SYSTEM

CHAPTER FOUR

REAL TIME APPLICATION OF CONTROL SYSTEMS

4.1 Introduction

The software systems previously described are used to develop and implement computer control systems on a milk powder manufacturing plant comprising a triple effect evaporator and a tall form spray drier. Plant models from measured input and output records are obtained using least squares techniques. These models incorporate time delays and disturbance effects if required. Controllers are designed using the derived plant models and tested in simulation. Control design is based on the minimisation of a quadratic performance index using dynamic programming. The modelling and control design utilise a graphics display unit allowing fast interactive development.

Controllers so designed are implemented using the real time software. The control systems may be developed in an hierarchical fashion following procedures similar to those conventionally used for the development of direct digital control (DDC) cascade heriarchies. A simulation model of the evaporator is used to aid in the choice of a hierarchical structure. The full facility, incorporating modelling, control system design and heirarchical implementation, is demonstrated to provide a very effective basis for the development of control system strategies for application to an industrial plant.

4.2 Hierarchical Control Schemes

DDC systems for large processes are usually configured in an hierarchical fashion[32,33]. Each controller is assigned an associated level of operation. In general, the higher the level of operation, the more difficult is the control task and the closer is the controlled variable to the product. The first level of control normally relates to the material and energy feeds to/from the plant (e.g., milk inlet flowrate, steam flowrate). These systems usually have relatively fast dynamics. Controllers to regulate plant or intermediate parameters within the process, usually with slower dynamics, are then designed to adjust the setpoints of the first level systems. There may be a number of levels in the final control scheme.

It is very often the case that, given the first level of control, there are a number of configurations of hierarchy that might appear realistic for overall plant control. The best such configuration is usually established by experimentation on the actual plant. The final form of the control system is therefore not always known initially.

These factors apply equally to the control approach described in this chapter. The method provides a basis for the control of generalised input and output systems. It does not restrict the inputs and/or outputs that may be associated with any particular control system. It does not provide a basis to establish a single plant control system by considering the whole plant as a multivariable black box (as is so often done with state space control designs). Instead, the method follows the well established control system design approach of regarding the plant as being comprised of a number of subsystems. Each subsystem is controlled independently and the overall plant operation is co-ordinated using a structured control system hierarchy.

The advantages of the method in comparison with the conventional DDC approach include

(a) the catering of aspects which degrade the performance of conventional DDC systems, e.g., interaction and time delays;
(b) the design approach for each control system, irrespective of its complexity and/or position in the hierarchy is uniform;
(c) it is possible to standardise the software, both for design and implementation to cater for a generalised hierarchical structure with generalised control systems within that structure; and
(d) online and interactive graphics allows the control engineer to quickly assess performance. This feature is a powerful aid in steering decisions to the most appropriate subsystem and hierarchy structure.

The control software has been developed so as to be effectively plant independent and capable of supporting any desired hierarchical structure. The concept of a control loop has been retained and in the multivariable context, a group of control loops is synchronised to utilise the same measurement set. Each control loop incorporates most of the parameters normally associated with control - measurement, set point, actuation signal, limits, etc. The actuation signal may be associated with an actual plant adjustment (e.g., valve position) or the setpoint of a controller active at some lower level in the hierarchy, according to the specification of the control engineer. Multivariable control is achieved by chaining loops together. Each loop has two associated link numbers - a base loop reference and a reference to the next loop in the multivariable chain.

The base loop is the first loop in any chain. If the link to the next loop points to itself, the end of the chain is signified. If both links are the same, a single input, single output control system is specified. This chaining mechanism permits a single control algorithm to be written that caters for both simple and complex modes of operation. The single input/single output controller is a special case of the n input/m output control scheme.

Control system operations may therefore be completely defined from a data base reference. Control system complexity and hierarchical structure are therefore very flexible. There is a much smaller risk, than with conventional d.d.c. systems, that software innovation might be required should a more complex control situation arise than had been anticipated initially.

4.3 Development of Total Plant Control Systems

Initially the computer has no information relating to control systems for the plant. The control engineer first selects subsystems that are to operate at the lowest hierarchy level. The input, output and disturbance structures are specified. Each subsystem is taken in turn and an experiment is carried out to collect plant data for modelling purposes. The order in which the subsystems are treated may depend on factors particular to a process. As each controller satisfies the design criteria it is incorporated into the hierarchical structure. Thus, the first controller at the lowest level is obtained with the plant operating under manual control, whilst the last control system at the first level is determined with all the other level 1 controllers operating. The first level controllers may then be redesigned if required.

Given a complete set of first level controls, experimentation can then commence to develop the second level of control. The procedure outlined is repeated for each of the subsequent higher levels. Control systems at a level must all be fully operational before commencing experimentation at the next higher level. It is important that the lower level controls remain fixed as the process models at a level incorporate controllers operational at a lower level. Adjustment would necessitate re-definition of all controllers on higher levels.

Many plants may not lend themselves to control development using this procedure. The nature of the plant may rule out the possibility of perturbation experiments or it may be necessary to have a satisfactory first level of control before any experimentation can be carried out. For example, a preliminary level of d.d.c. may be required to bring the plant to an operational status. The d.d.c. systems could then be replaced by more effective control systems. Control of the plant at the first level is usually straightforward so this aspect is not a major difficulty. Restricted experimentation may cause a problem however. It is equally possible to create acceptable data for analysis by injecting only small perturbations into the plant and thereby restricting output deviations to within normal operating values. If the practical constraints (e.g., safety, cost, etc), are such that perturbation experiments are out of the question, then the design of controllers must rely on analytically derived models of plant dynamics, thereby abandoning the statistical approach. Alternatively, it may be possible to establish models from the normal day to day operating records of the plant rather than as a result of a special experiment.

On the milk powder manufacturing plant this procedure has passed through four stages - preliminary work using an analytically derived simulation model, first or primary level control of the evaporator, second and higher level controls for the evaporator, primary level controllers for the spray drier.

The following sections detail the development of the control structure at each of the stages.

4.4 Simulation Model of Evaporator

Prior to the installation of instrumentation on the plant, and consequent linking to the process control computer, a simulation model of the evaporator was used to develop and test possible control strategies for used on the actual plant. The non-linear lumped parameter model[34] has been developed by writing mass and energy balances for the various component sections of the evaporator, i.e., preheaters, detensioner, effects 1, 2 & 3, barometric condenser.

Process state variables included in the model are:

(i) concentration of milk leaving the 1st effect, C;
(ii) temperature inside the 1st effect, T_1;
(iii) solution hold up inside the 1st effect, W;
(iv) concentration of milk leaving the 2nd effect, C_2;
(v) temperature inside the 2nd effect, T_2;
(vi) solution hold up inside the 2nd effect, W_2;
(vii) concentration of milk leaving the 3rd effect, C_3;
(viii) temperature inside the 3rd effect, T_3;
(ix) solution hold up inside the 3rd effect, W_3;

Control variables are defined as:

(i) temperature inside the 1st effect steam jacket, T_s;
(ii) flowrate of milk into the evaporator, F_D;

(iii) flowrate of condenser water, F_W;
(iv) preheat temperature of milk, T_{FD}.

Of the four control variables defined for the model, three were available to act as control inputs for the evaporator. The preheat temperature, T_{FD}, affects the final properties of the spray dried milk powder and thus is normally constant over a particular production run.

Previous simulation studies with the model have shown that there is interaction between all plant inputs and process variables. In particular, the 1st effect steam jacket temperature, T_s, affects the 1st effect milk temperature, T_1, heavily and F_W affects the 3rd effect milk temperature, T_3. These responses occur relatively quickly with time constants of the order of 5-10 seconds. In contrast, responses to changes in the inlet milk flowrate, F_M, are very much slower because of transport delays between effects and the subsequent propogation time through the evaporator.

In keeping with the established method of considering subsystems within a large system for control development, the temperatures with their fast responses are considered first. The simulation is used to generate data by applying random perturbations to both T_s and F_W, whilst keeping F_M and T_{FD} constant. A model is obtained from the data using least squares techniques and a multivariable feedback controller with minimum time characteristics can then be derived. Simulation of this controller shows that good control over T_1 and T_3 may be obtained.

From the application viewpoint, however, a controller of this form is seen to be unsuitable. The responses of T_1 and T_3 are both quick but there is a delay in the propogation of changes in the 1st effect through to the 3rd making choices of suitable setpoint values difficult. Also, the product state may be defined by any two of the properties temperature, flowrate and concentration. Thus a control system based on temperatures only, bears little relation to the product status.

A possibility is to have a control system defining the product status at some point in the evaporator and then use steady state relationships between that point and the final product out of the evaporator. Since the energy input occurs at the 1st effect a system is developed to give tight control over this section of the plant, T_1 and F_1, the milk flowrate out of the 1st effect are the variables to be controlled. Concentration is not regarded as control variable at this stage as it was thought that measurement of concentration would be more difficult and less accurate than measurement of the other two variables, flow and temperature. Subsequent operation on plant proves this to be incorrect with product flow measurement being much noisier than concentration measurement. The control inputs chosen are T_s and F_W, F_M being disregarded because of the effect of changes propogating through the plant.

Data is generated as before and a multivariable model found.

$$\begin{bmatrix} \underline{x}_{(k+1)T} \\ \underline{x}_{(k+1)T-t} \end{bmatrix} = \underline{\alpha} \begin{bmatrix} \underline{x}_{kT} \\ \underline{x}_{kT-t} \end{bmatrix} + \underline{\beta}\, \underline{W}_{kT} \qquad (1)$$

where

$$\underline{x}_{kT} = \begin{pmatrix} T_1 \\ F_1 \end{pmatrix}_{kT}$$

$$\underline{W}_{kT} = \begin{pmatrix} T_{s_{kT}} - T_{s_{(k-1)T}} \\ F_{W_{kT}} - F_{W_{(k-1)T}} \end{pmatrix}$$

T = 8 seconds   t = 4 seconds

$$\underline{\alpha} = \begin{bmatrix} -1.95 & 1.33 & -.95 & -1.33 \\ -.03 & 10.48 & .03 & -.48 \\ 1.50 & 1.00 & -.50 & -1.00 \\ -.03 & 10.36 & .28 & -.36 \end{bmatrix}$$

$$\underline{\beta} = \begin{bmatrix} .45 & -.18 \\ -.02 & -.03 \\ .36 & -.05 \\ -.01 & -.01 \end{bmatrix}$$

Figures 1 and 2 show the original data with the response of the least squares model superimposed on top. The top portion of each figure indicates the error between the true value and the modelled value. The temperature data has been modelled better than the flow. This is in accordance with the earlier simulation work which has shown that T_s and F_W affect the T_1, T_s pair more than they do T_1, F_1.

With weightings $\underline{P}$ = diag $[1,1,1,1]$ and $\underline{Q}$ = diag. $[0,0]$ and 2 iterations of the dynamic programming design algorithm, the following minimum time controller is obtained.

$$\underline{W}_{kT} = \underline{\gamma}\ \underline{YE}_{kT} \tag{2}$$

where $\underline{W}_{kT}$ is as before,

$$\underline{YE}_{kT} = \begin{bmatrix} \begin{pmatrix} T_1 - T_{sp} \\ F_1 - F_{sp} \end{pmatrix}_{kT} \\ \begin{pmatrix} T_1 - T_{sp} \\ F_1 - F_{sp} \end{pmatrix}_{kT-t} \end{bmatrix} \quad \text{error vector}$$

$$\underline{\gamma} = \begin{bmatrix} -4.05 & 24.24 & 1.23 & 1.50 \\ .69 & 7.78 & -2.02 & -4.53 \end{bmatrix}$$

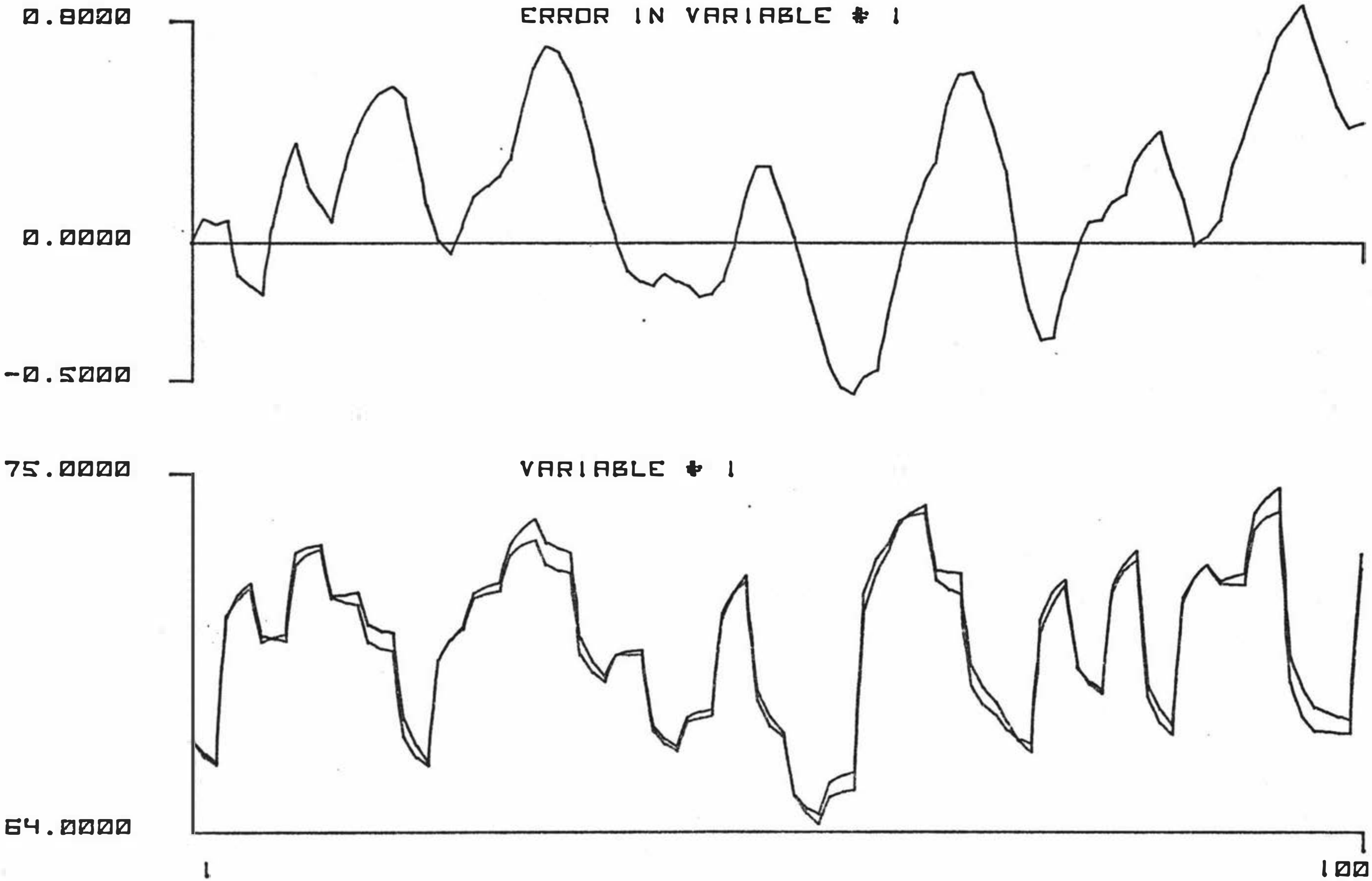


Figure 1 Simulated perturbation data

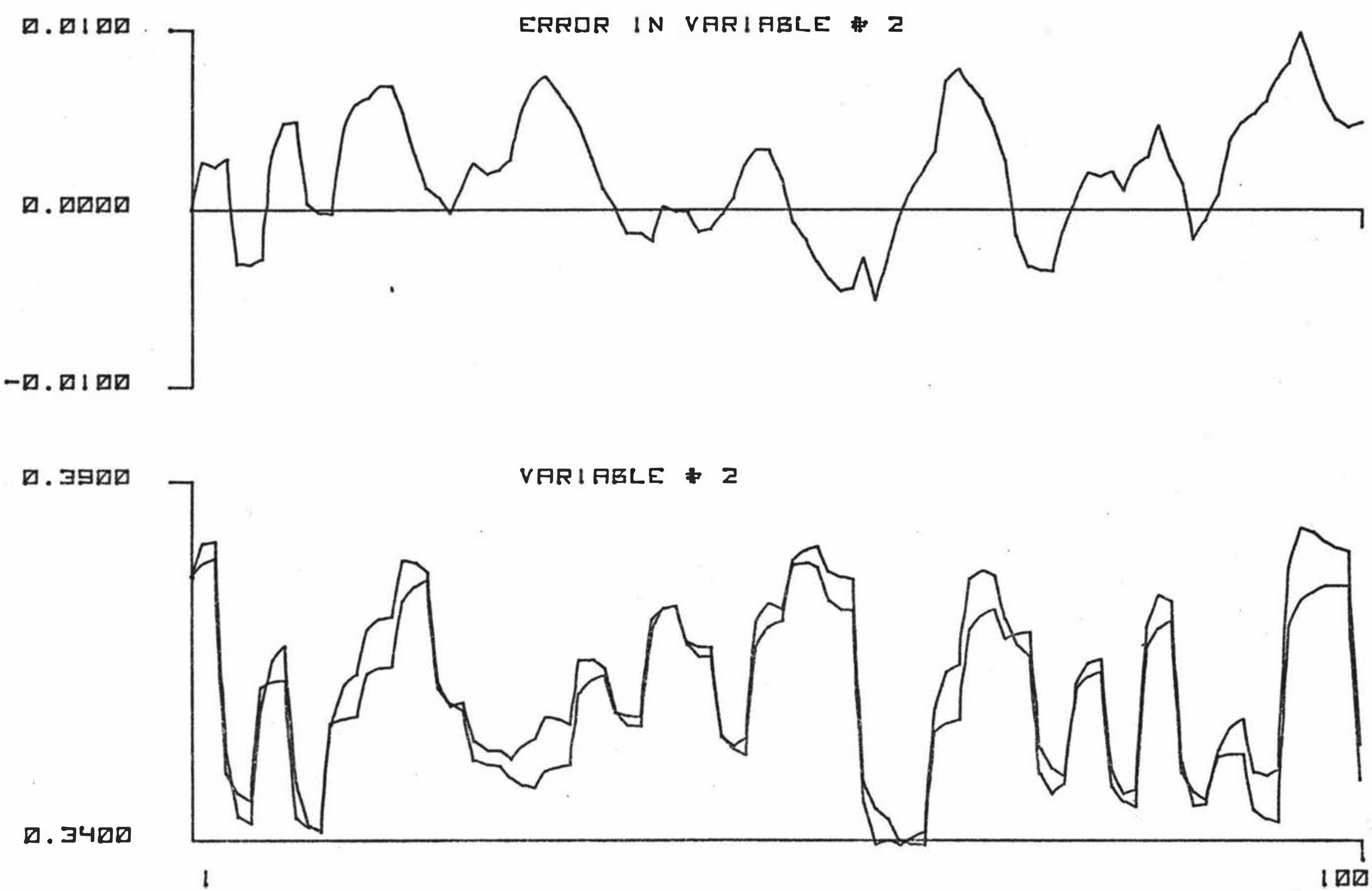


Figure 2 Simulated perturbation data

A series of simulation runs using the controller developed/ is performed to determine its effectiveness. The setpoints and corresponding response value after 10 controller cycles are summarised in Table 1. Limits are placed on the range of allowable control input values thus ensuring a degree of realism. These limits are set at

steam temperature, T_S	60 $^{\circ}C$ to 95 $^{\circ}C$
water flow, F_W	0 Kg/sec to 7.0 Kg/sec

The initial input values used are 83.0 $^{\circ}C$ and 3.5 Kg/sec. These give initial values of 69.43 $^{\circ}C$ and .362 l/sec. The inlet flowrate, F_M, is set to 0.5 l/sec and the preheat temperature. T_{FD}, to 105 $^{\circ}C$. The latter two inputs are constant over each of the simulation runs.

Table 1

Simulation Responses

	Setpoints		Final Value After 10 Control Cycles	
Figure No.	T_1	F_1	T_1	F_1
3	73.0	.362	72.79	.356
4	73.0	.34	72.99	.342
5	73.0	.33	73.01	.331
6	73.0	.32	73.09	.329
7	69.5	.38	69.57	.376
8	69.5	.36	69.50	.360
9	69.5	.34	69.51	.341
10	66.0	.40	66.39	.403
11	66.0	.38	66.17	.379
12	66.0	.362	66.01	.357

Figures 3 to 12 show the results of the simulation runs. In each case variable 1 is the temperature T_1, variable 2 is the 1st effect flow, F_1. In all cases the temperature setpoint is reached within 2-3 control cycles. The temperature responses in figures 3 to 6 and 10 to 12 exhibit overshoot. This has arisen because one input of the multivariable controller, the condenser water flowrate F_W has saturated.

The flowrate response characteristic is seen to be a function of the temperature setpoint. Figures 3 to 6 show flow response for a temperature higher than the initial value, improves as the setpoint is lowered. Figures 7 to 9 show a much improved flow response when the temperature, T_1, remains close to its initial value. Figures 10 to 12 indicate flow responses of a quality between those of the others. The saturation of the condenser water input masks some of the interaction between the flow and temperature setpoints. The diagrams show that effective control of T_1 and F_1 is available subject to reasonable choices in setpoints. Remembering that the input flow has been kept constant the setpoints should be compatible in terms of a mass balance, i.e., an increase in T_1 (implying a greater evaporation rate) should be accompanied by a drop in F_1 setpoint or vice versa.

A method for setting these setpoints is now developed. The objective of control on the evaporator is to define the characteristics of the final product, i.e., the concentration and flowrate. Consider the evaporator is operating at some steady state condition. Energy for evaporation in the third effect is obtained from the vapours leaving the second effect. This in turn depends on a vapour flow from the 1st effect and thus the temperature in the 1st effect. Similarly the flow out of an effect is a function of the flow leaving the previous effect. Therefore, provided tight control is maintained

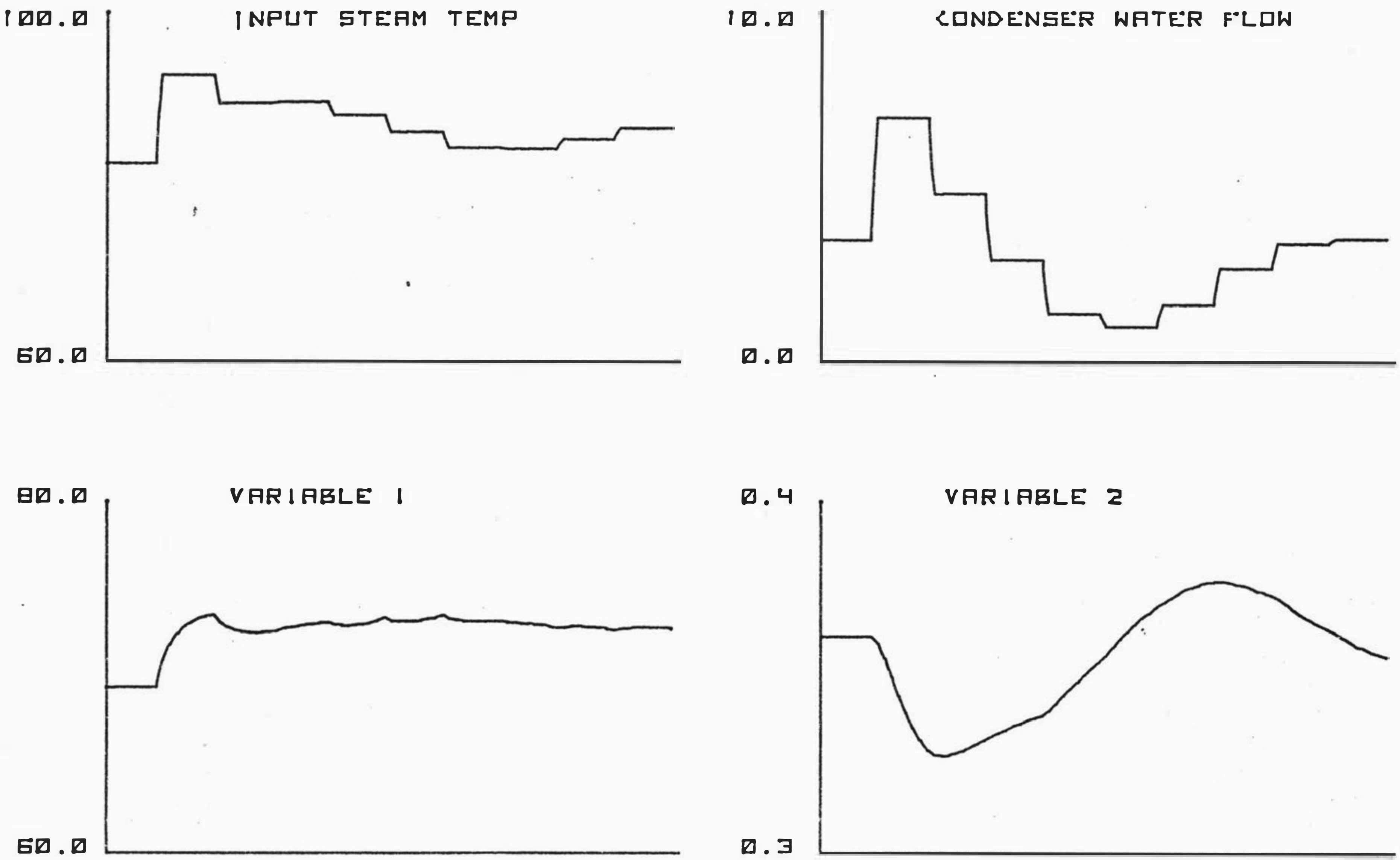


Figure 3 Simulated control responses

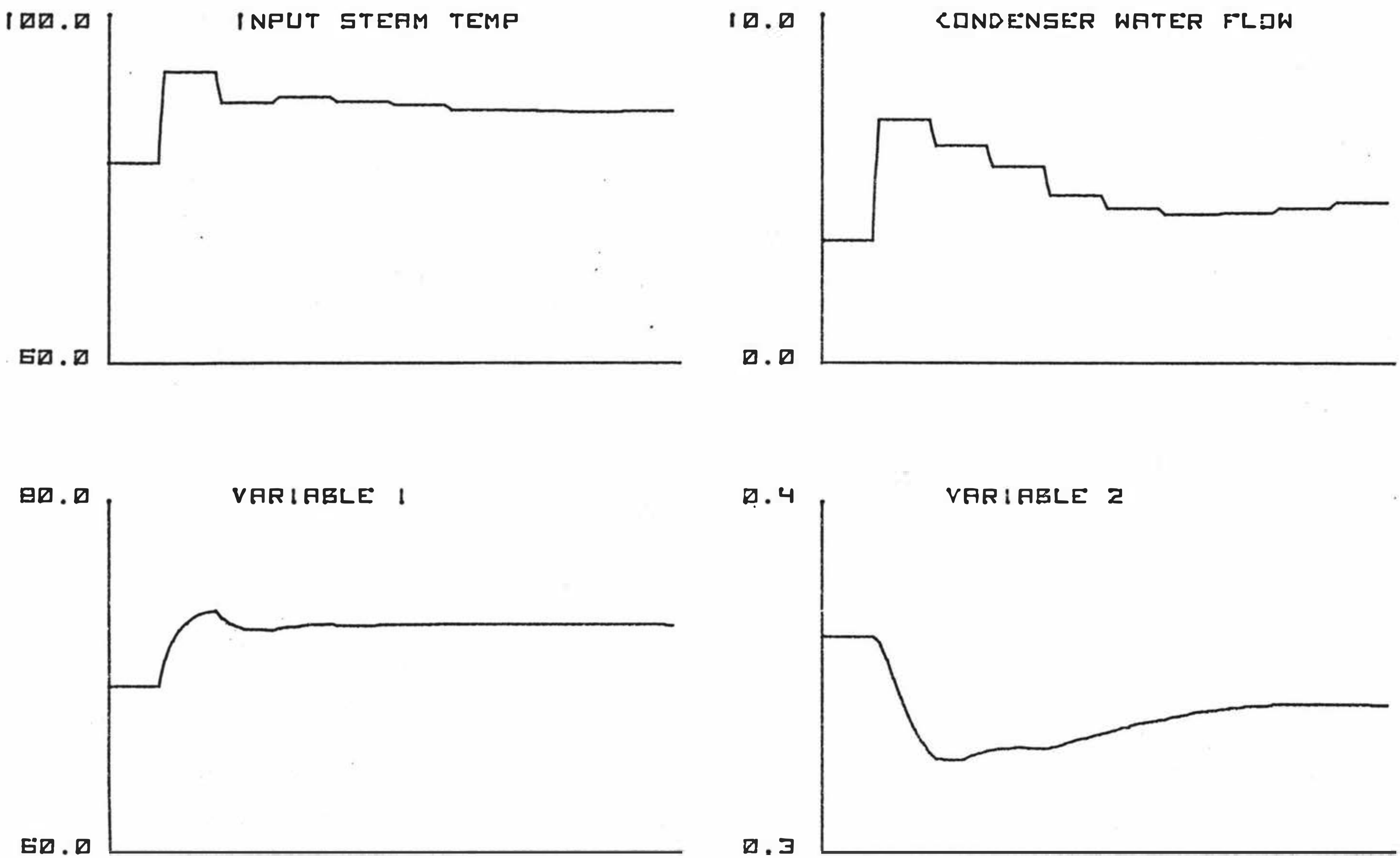


Figure 4 Simulated control responses

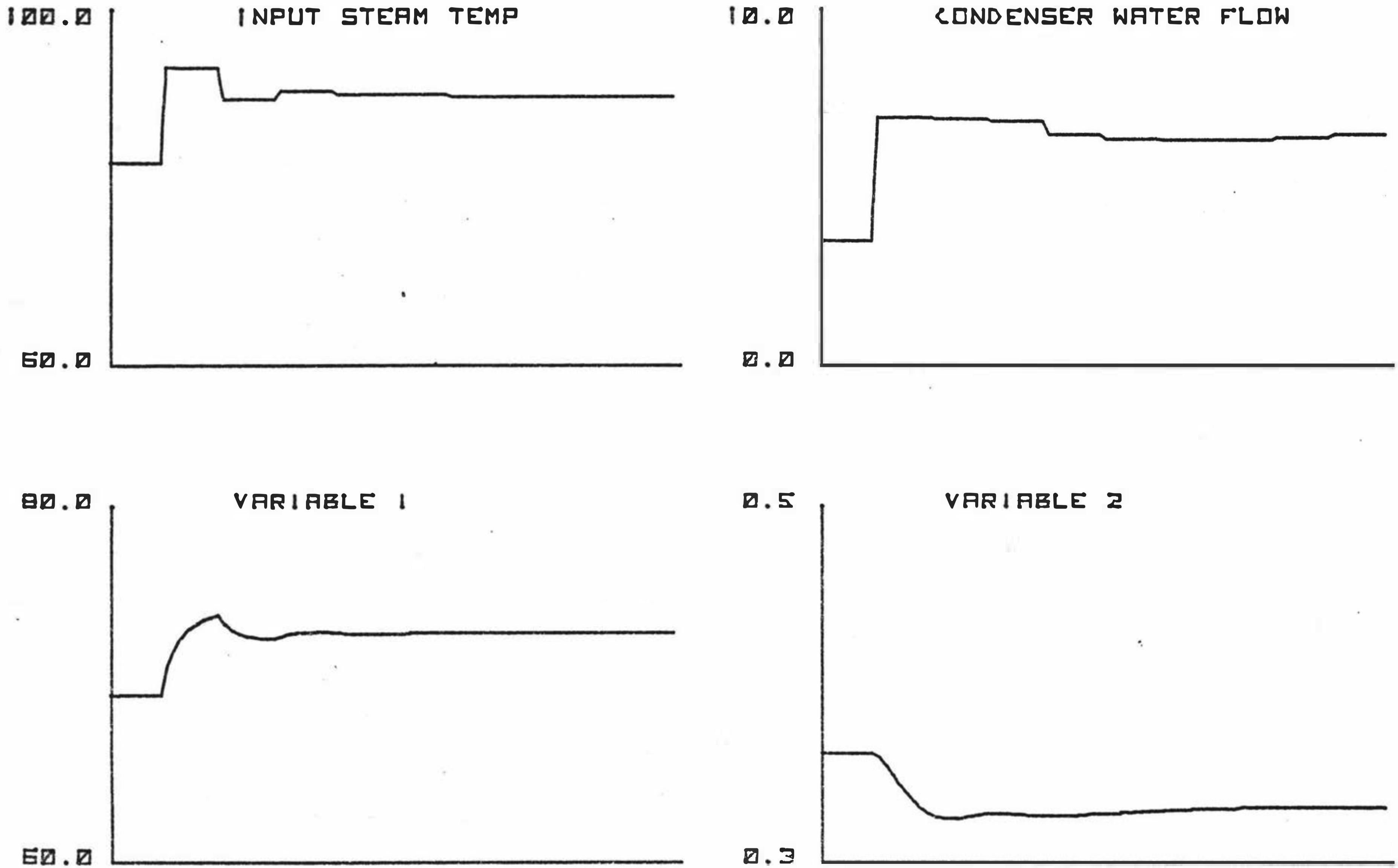


Figure 5 Simulated control responses

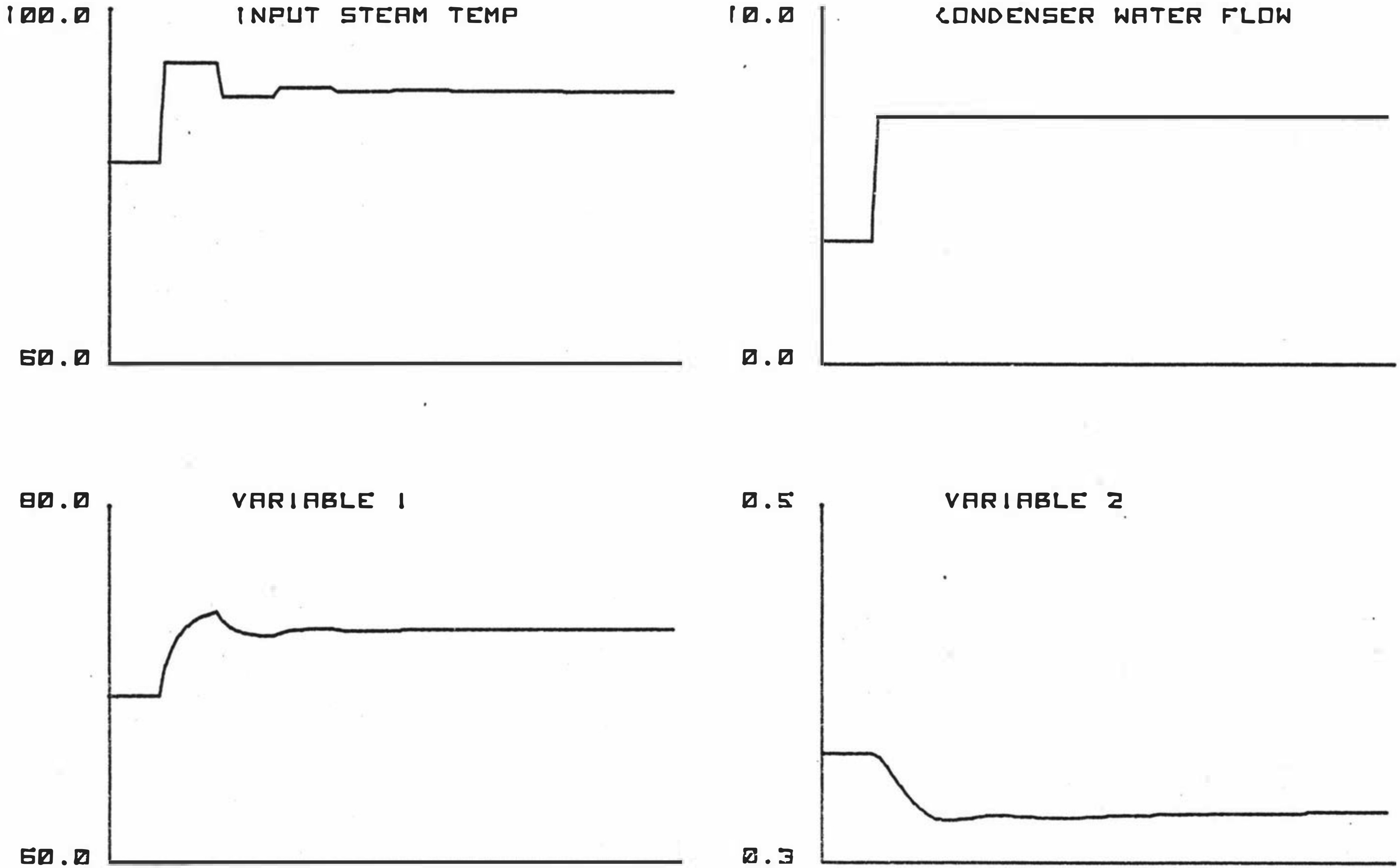


Figure 6 Simulated control responses

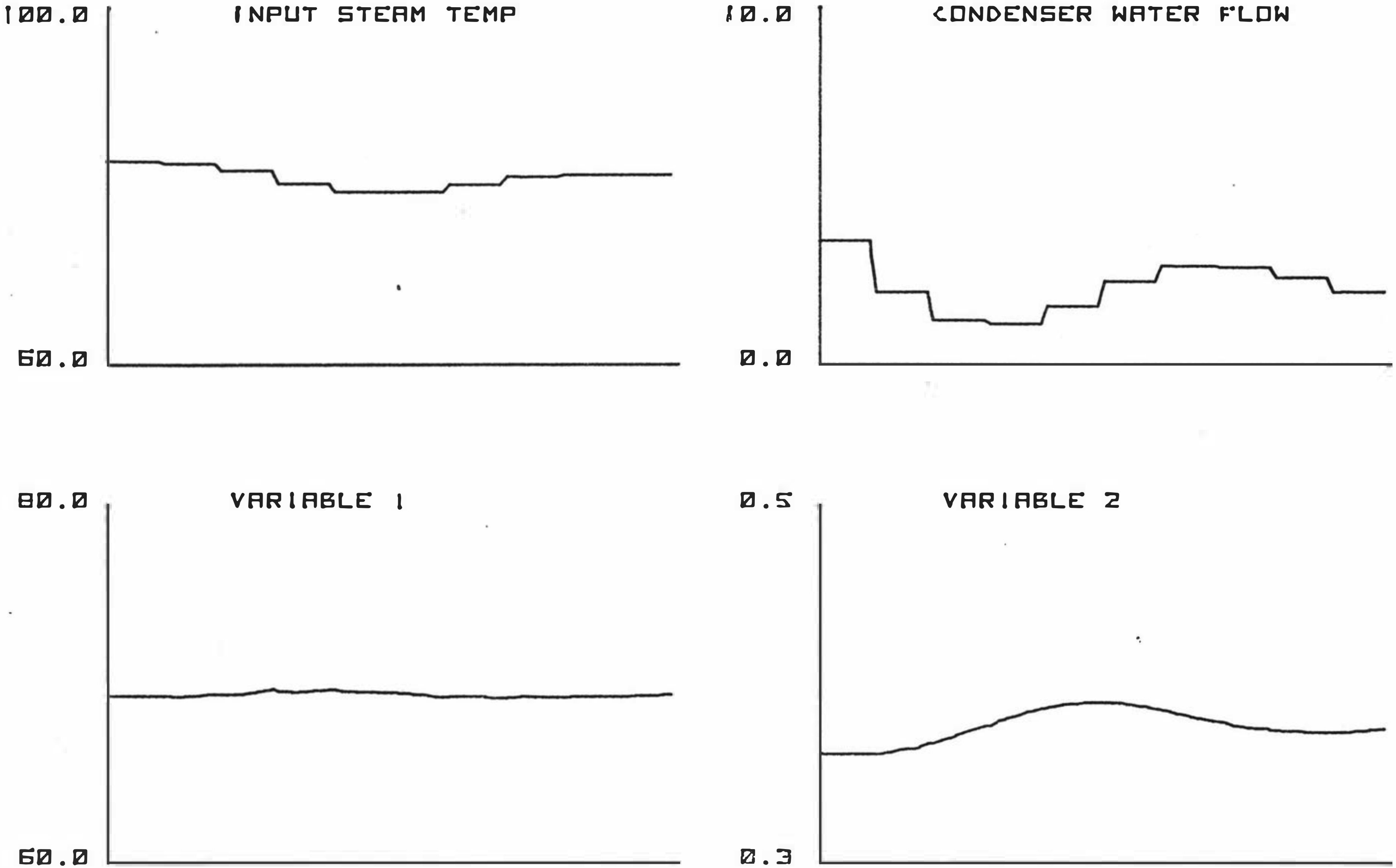


Figure 7 Simulated control responses

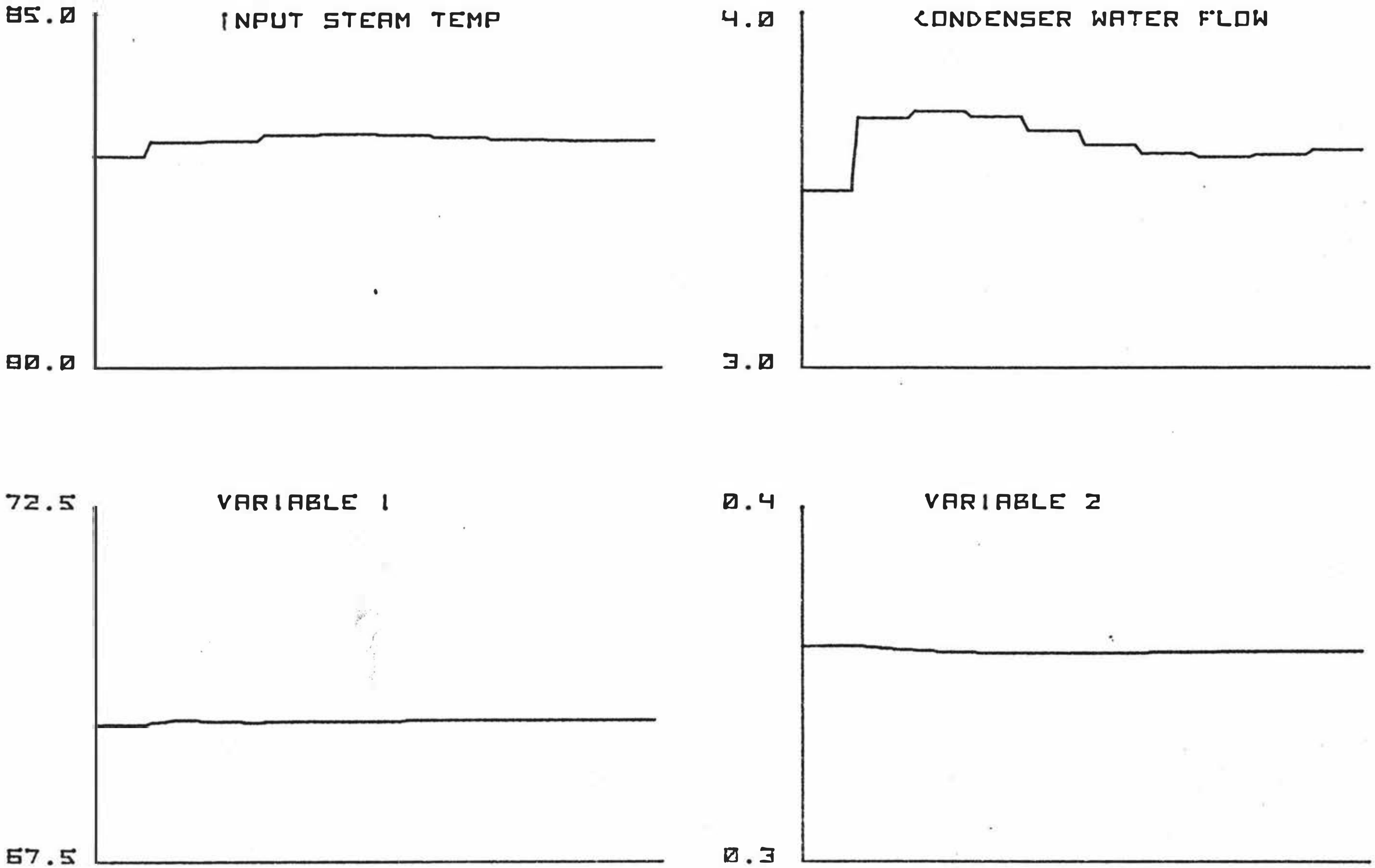


Figure 8 Simulated control responses

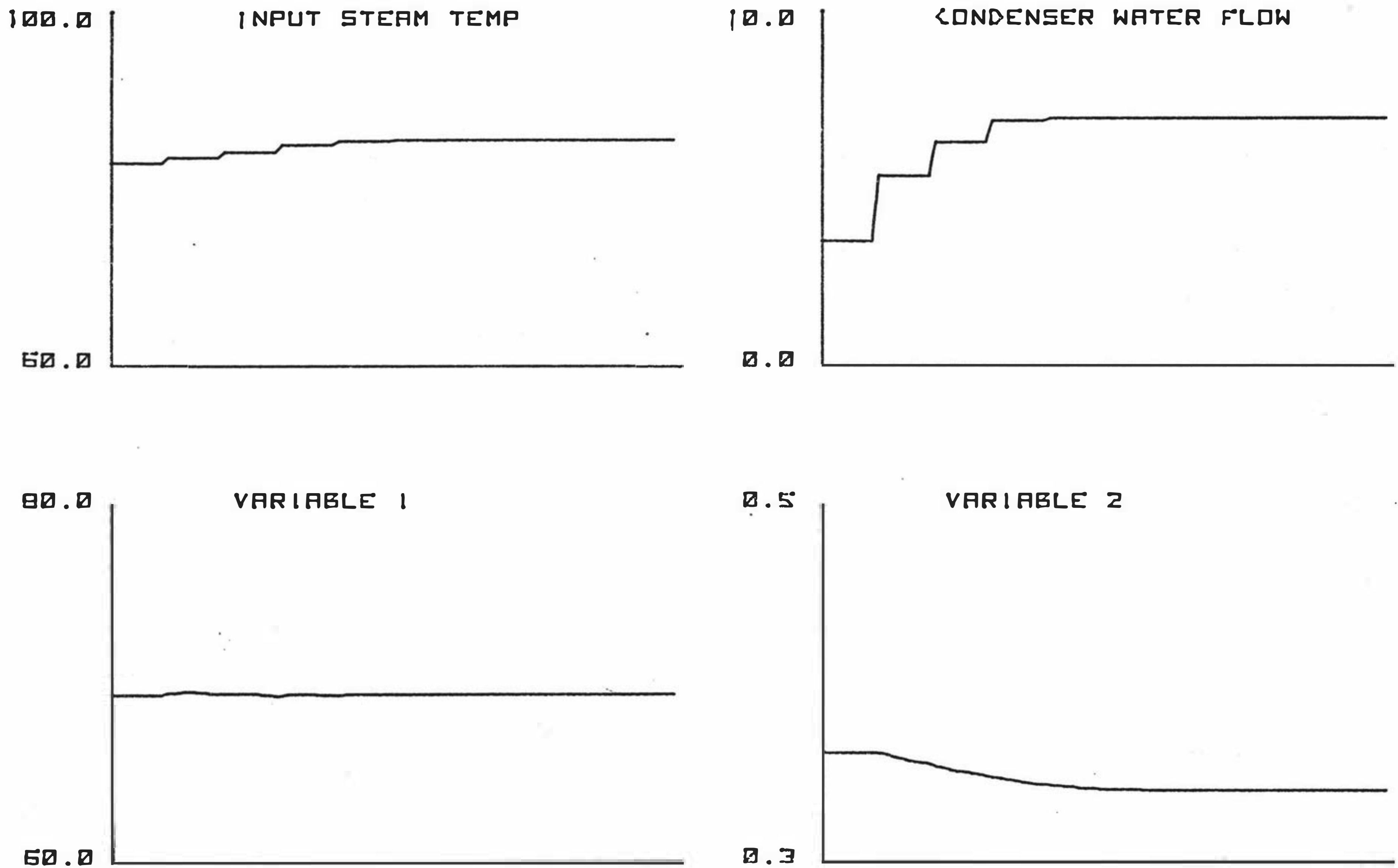


Figure 9 Simulated control responses

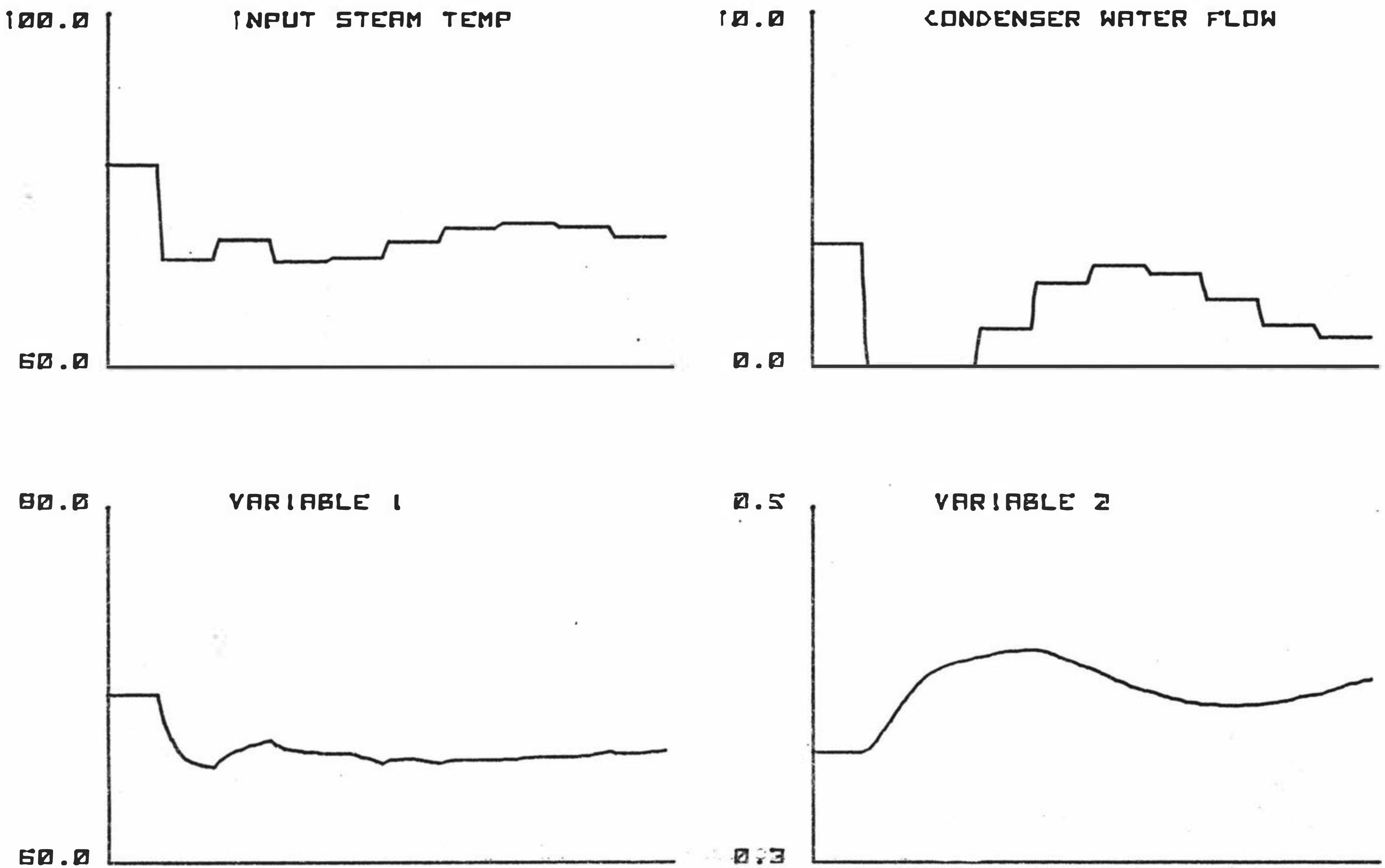


Figure 10   Simulated control responses

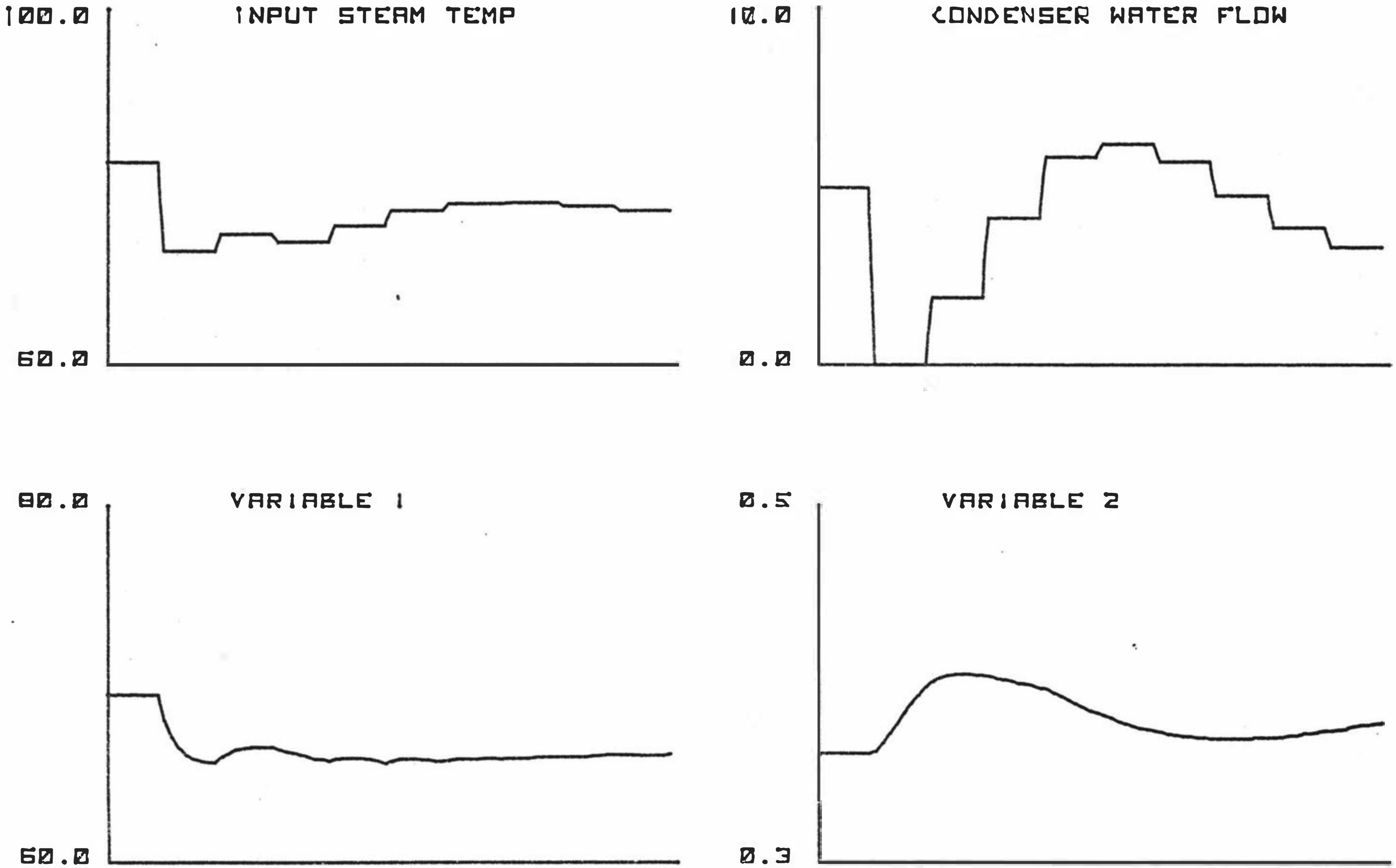


Figure 11 Simulated control responses

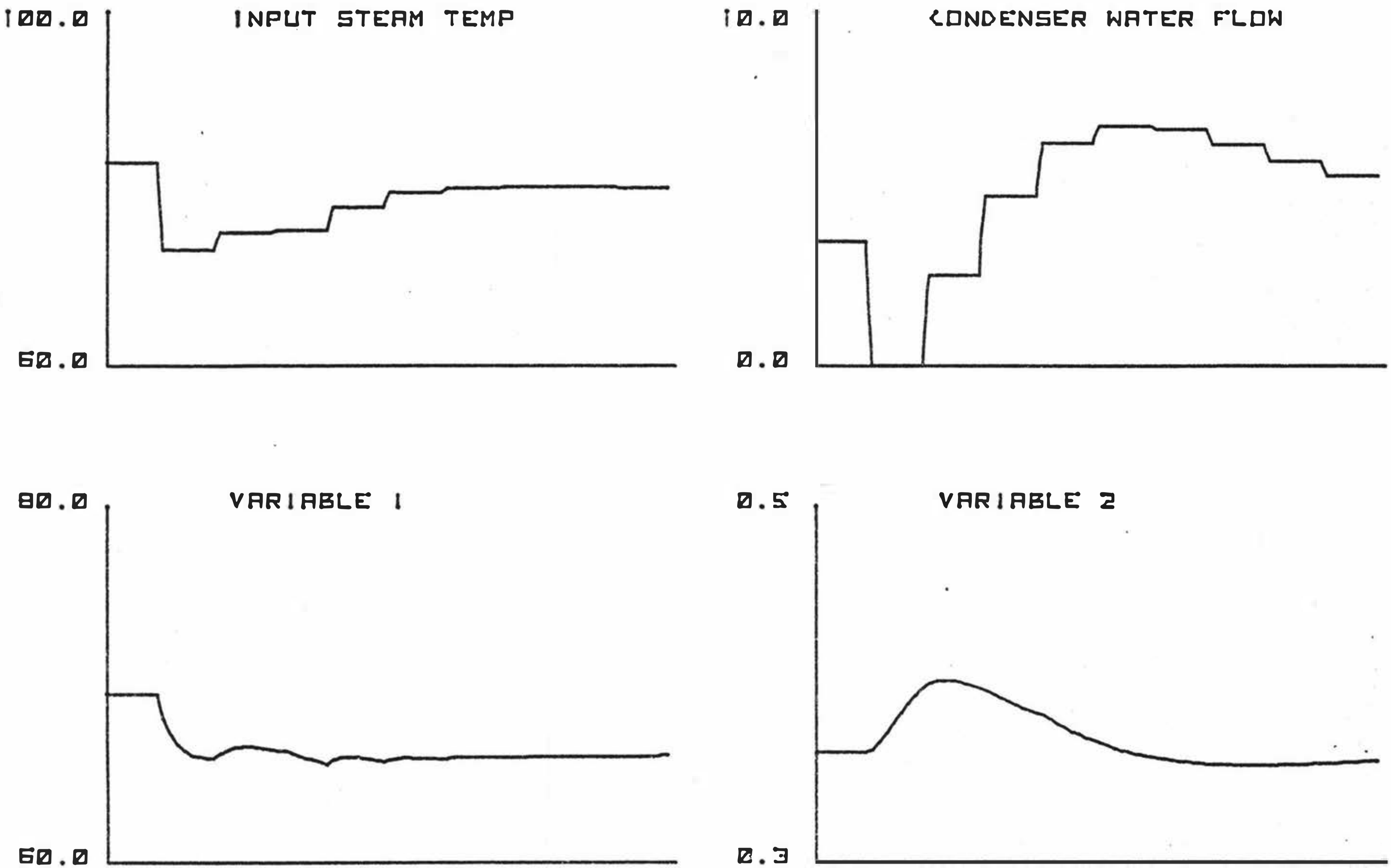


Figure 12 Simulated control responses

over the 1st effect, steady state relations can be used to determine suitable setpoints for T_1 and F_1 based on desired outflow.

The steady state equations obtained are:

$$F_1 = 0.0347\, F_M + 0.3777\, F_3 + 0.033 \tag{3}$$

$$T_1 = 121.4197\, F_M - 179.9037\, F_3 + 28.5128 \tag{4}$$

Figures 13 and 14 indicate the accuracy of the two equations obtained. The top portion of each figure gives the error between the true value and that estimated using equations (1) and (2).

4.5 Primary Controls for a Triple Effect Evaporator

The overall objective of control on the evaporator is to produce a consistent product, at some desired specification, with a steady production rate. Achievement of this objective simplifies some of the control problems that might arise on the spray drier, leading in turn to a higher quality milk powder. This objective results in a requirement for control of the density (and hence concentration) and flowrate of the concentrate leaving the third effect of the evaporator.

Table 2 outlines the hierarchy of control systems relating to a triple effect evaporator. This section deals with the first level of the control hierarchy.

The controllers on the first level are concerned with the operating environment of the evaporator. This includes the supply of energy and raw material to the plant. Four controllers have been developed to operate at this

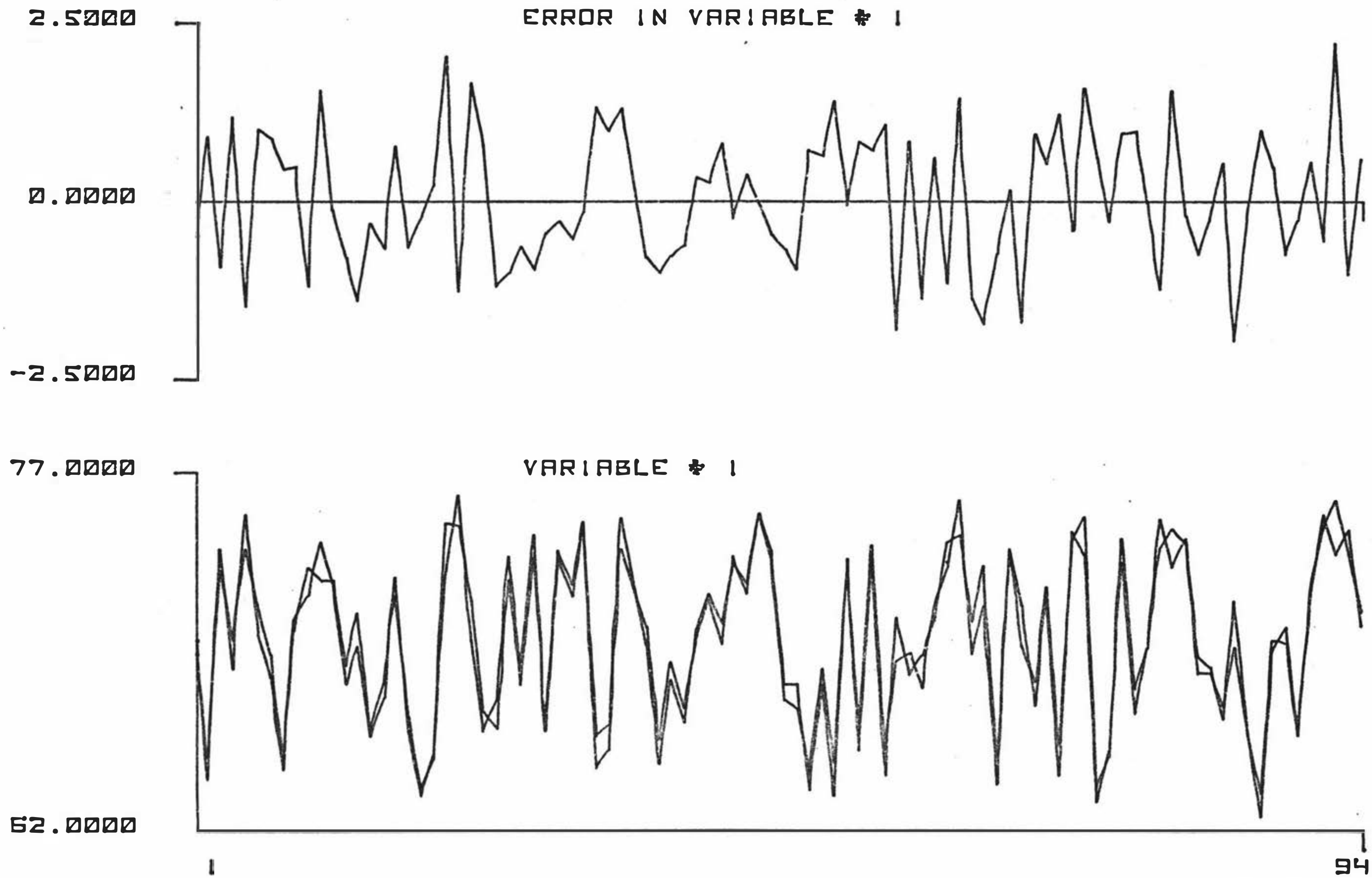


Figure 13 Steady state relation for T_1

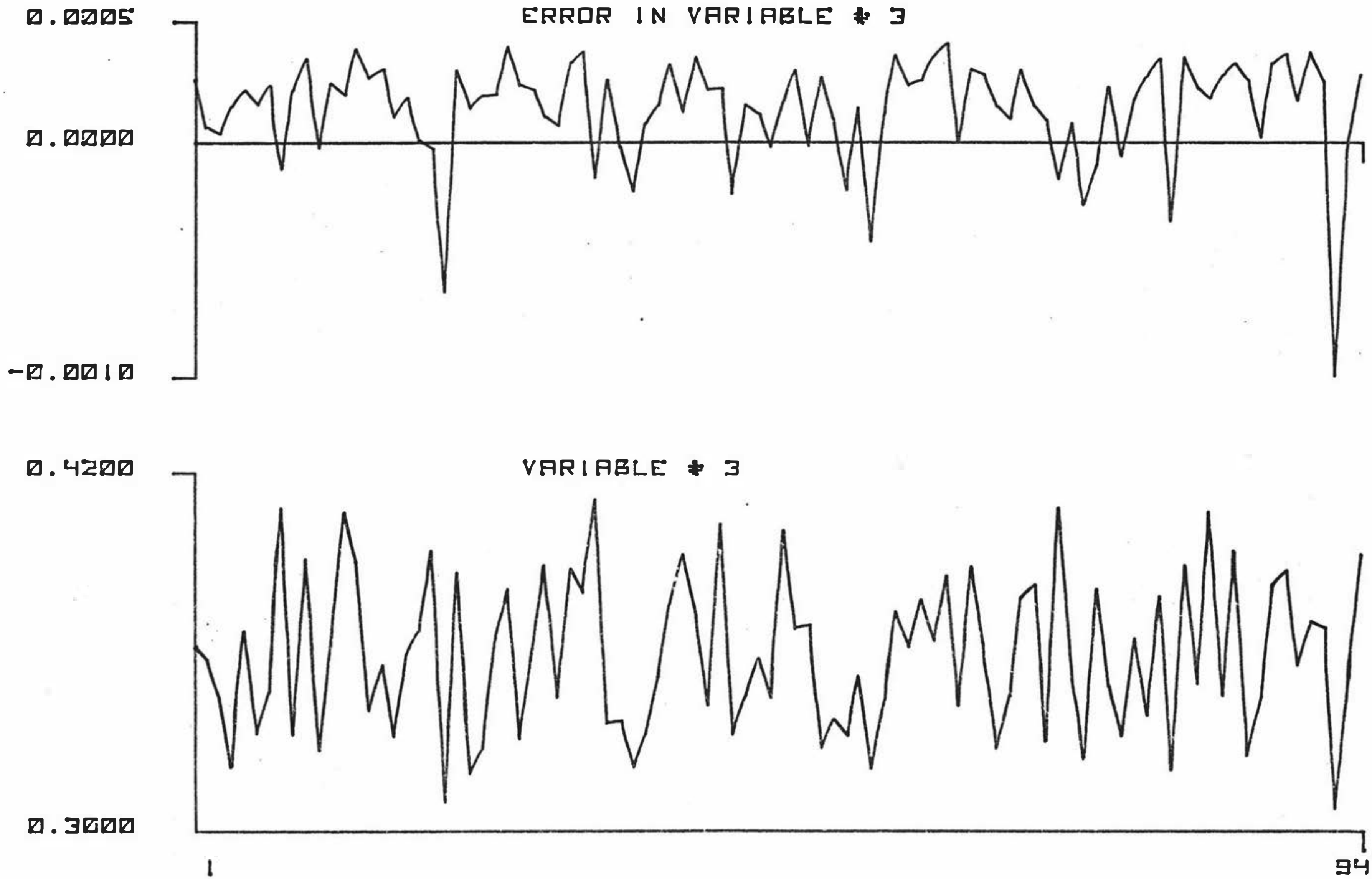


Figure 14 Steady state relation for F_1

Table 2

Control Hierarchy for a Triple Effect Evaporator

Secondary	control of product density and flowrate	level 3
	↓	
	control of evaporator pressure	level 2
	↓	
Primary	control of evaporator environment	level 1
	↓	
	↓	
	Plant actuators	process

level - control of the steam flowrate to the 1st effect, control of the water flowrate to the barometric condenser, control of the temperature of the water leaving the inter-stage ejector condenser and multivariable control of the milk inlet feedrate and preheat temperature.

Each loop is considered in isolation. The nature of the plant permits the plant to be operated on water while developing the first level of control. Manual operation is used to bring the plant to its normal operating level. As each loop is developed in turn, it is incorporated into the control scheme.

4.5.1 Steam Flowrate

Figure 15 is a schematic diagram of the steam supply system. The steam pressure is measured on the upstream side of the valve and the flowrate on the downstream side.

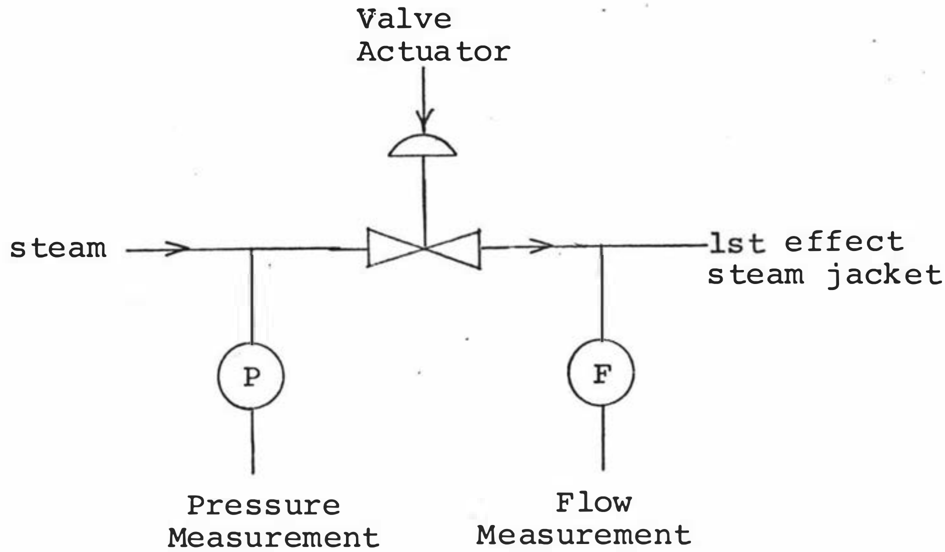


Figure 15 Steam flow to 1st effect

The objective is to control the flowrate of steam to the first effect. Manual operation of the valve shows there is no delay between actuation of the valve and the corresponding steam flowrate response.

The valve is set at a position corresponding to a 2400 mV signal from the computer (allowable values are 1000-5000 mV dependent on the particular valve). The position of the valve is changed at 12 second intervals over the range 240 $\pm$ 40 units (1 unit - 10 mV). A 1st order model (2 samples per control interval) with an update interval of 4 seconds and a sample interval of 2 seconds is obtained by applying the least squares procedure to the data generated. The response of the model obtained, to the original input sequence, is shown in figure 16.

Rapid control of variations in steam flowrate is desirable to minimise the impact of disturbances at later stages of the process. Thus a minimum time controller[35] is designed by setting weights of $\underline{Q}$ = diag.|1,1|, $\underline{P}$ = 0 and performing 2 iterations of the dynamic programming

		MIN	MAX
5	STM FL COM K/H	0	5000
5	MODEL STM FL COM K/H	0	5000
39	STEAM VALVE	1000	5000

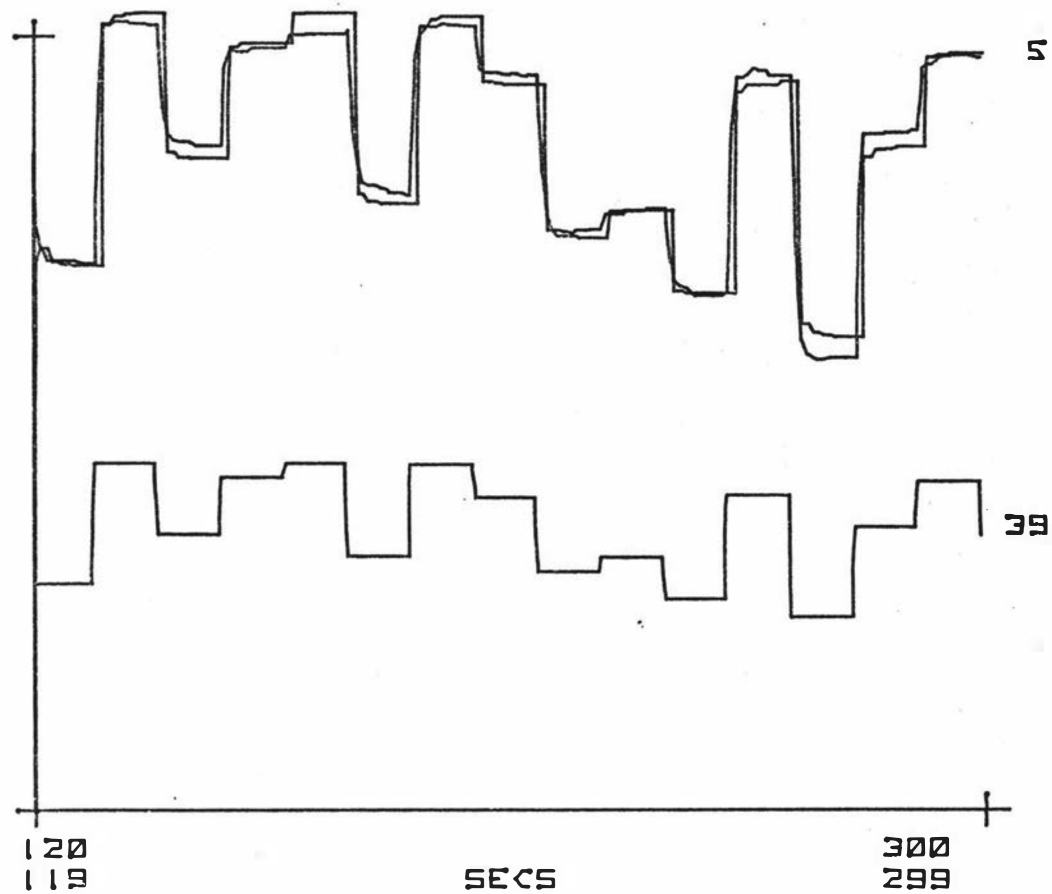


Figure 16 Perturbation data for steam flow

design algorithm. The operation of the controller is seen in the accompanying diagrams. Figures 17a and 17b are simulated control responses. The offline system is used for these simulations. Setpoint is reached within 2 control intervals. Figures 18a and 18b reflect the operation of the steam controller on the evaporator. Figure 18a shows a setpoint change (continuous straight lines) from 400 Kg/hr to 430 Kg/hr. The target is reached within 2 control cycles followed by minor changes in the valve position to keep the flow at setpoint. Figure 18b shows the effect of variations in the steam supply pressure on the flow controller. Variable 18 is the pressure reading indicated in the schematic of the steam system whilst variable 5 is the pressure at the header pipe junction to the boiler. Variations in the steam flowrate are kept small. Table 3 summarises the steam loop parameters.

Table 3

Steam Loop Parameters

Model Matrices

$$\underline{\alpha} = \begin{bmatrix} 1.37 & -.37 \\ 1.28 & -.28 \end{bmatrix}$$

$$\underline{\beta} = \begin{bmatrix} 2.63 \\ 2.52 \end{bmatrix}$$

control interval 4 secs
sample interval 2 secs

Control Design Parameters

$\underline{Q} = \text{diag.}\ (1,1) \quad \underline{P} = 0$ 2 iterations

Control Matrix

$$\underline{K} = \begin{bmatrix} -.50 & .089 \end{bmatrix}$$

		MIN	MAX
5	STEAM FLW KG/H	0	5000
39	STEAM VALVE	1000	5000
5	SET POINT STEAM FLW KG/H	0	5000

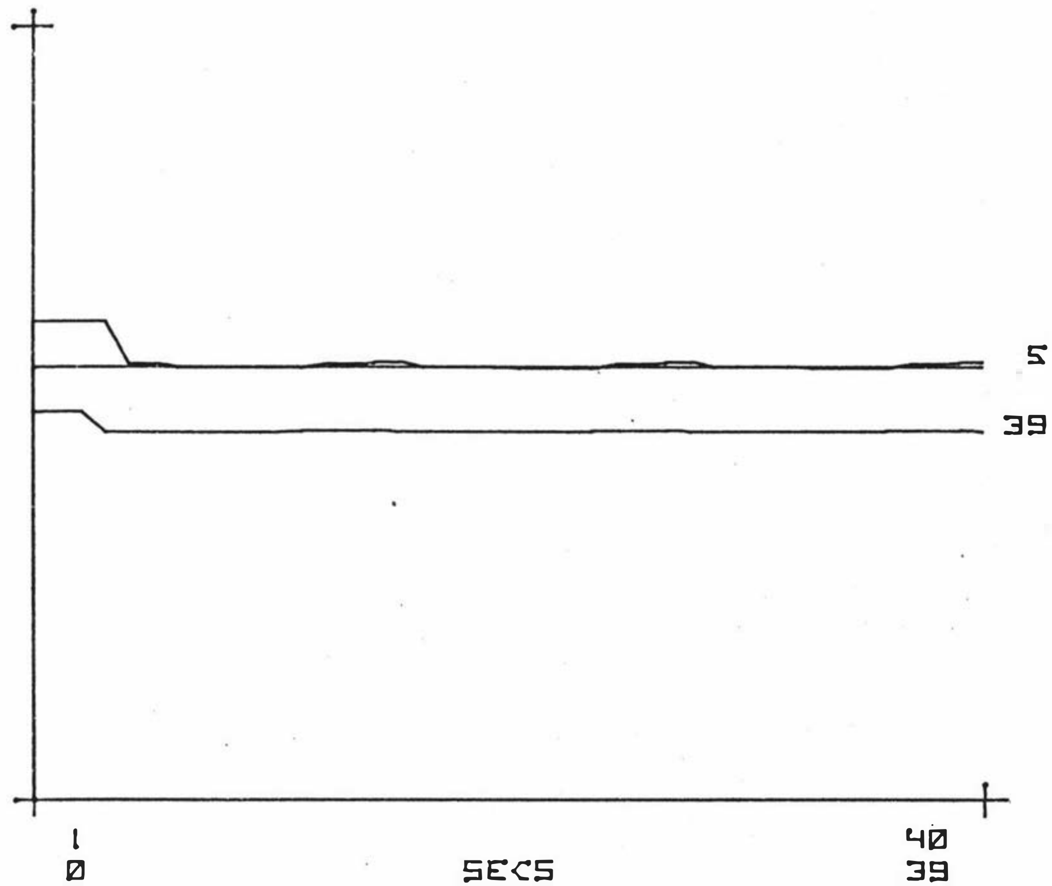


Figure 17a Simulated steam flow control response

		MIN	MAX
5	STEAM FLW KG/H	0	5000
39	STEAM VALVE	1000	5000
5	SET POINT STEAM FLW KG/H	0	5000

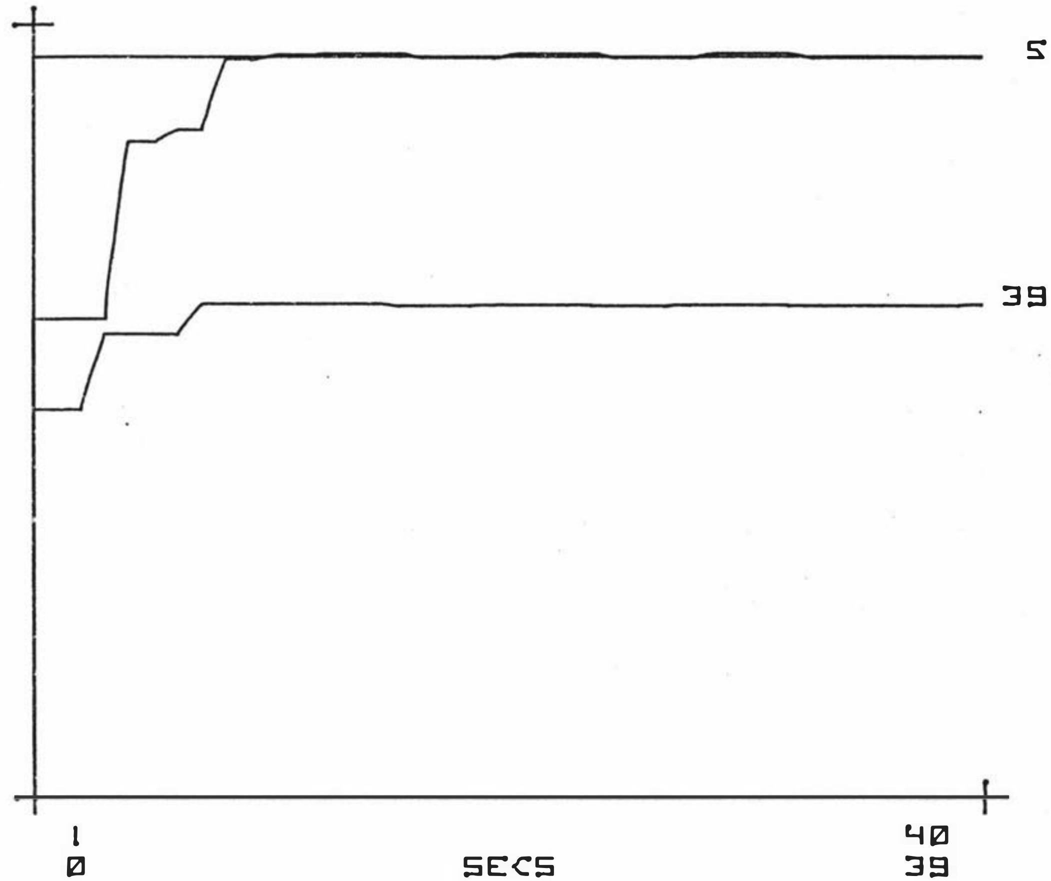


Figure 17b   Simulated steam flow control response

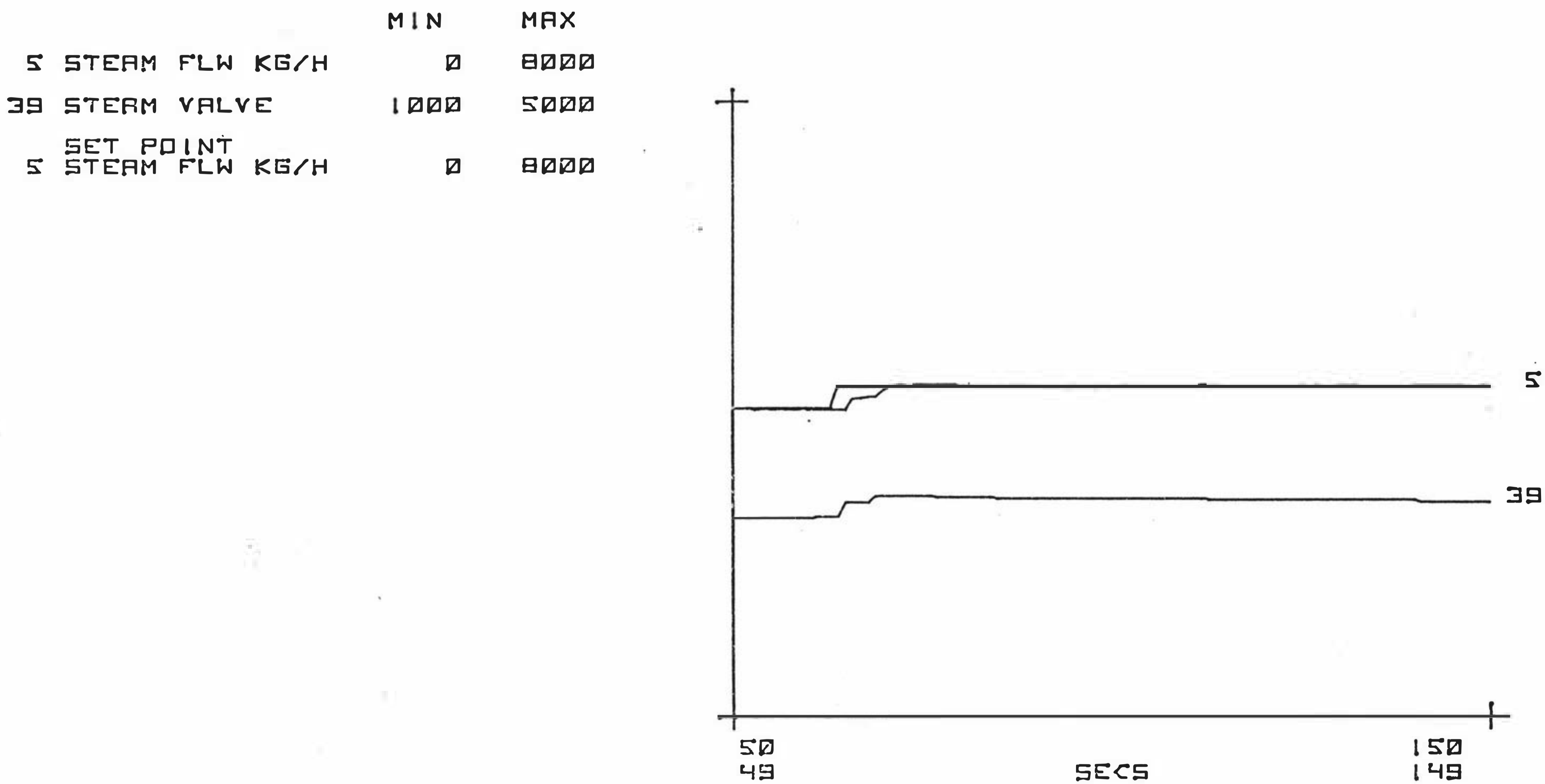


Figure 18a Online steam flow control response

	MIN	MAX
5 STEAM FLW KG/H	0	5000
39 STEAM VALVE	1000	5000
SET POINT 5 STEAM FLW KG/H	0	5000
6 MN STEAM PR BG	50	100
18 ST FLW COMP BG	50	100

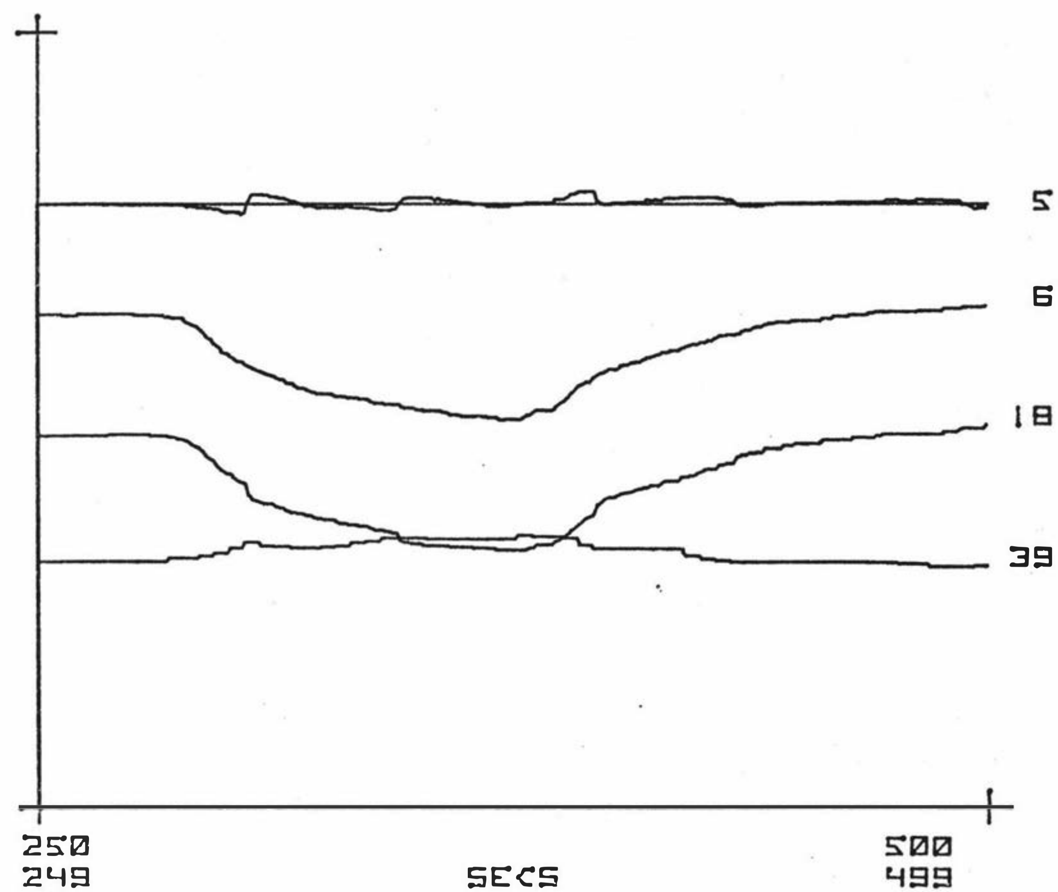


Figure 18b Online steam flow control response

4.5.2 Water Flowrate to Barometric Condenser

Figure 19 is a schematic diagram of the barometric condenser system. Control is required on the flow of cooling water to the condenser. Negligible delay exists between a change in valve position and a change in flow-rate.

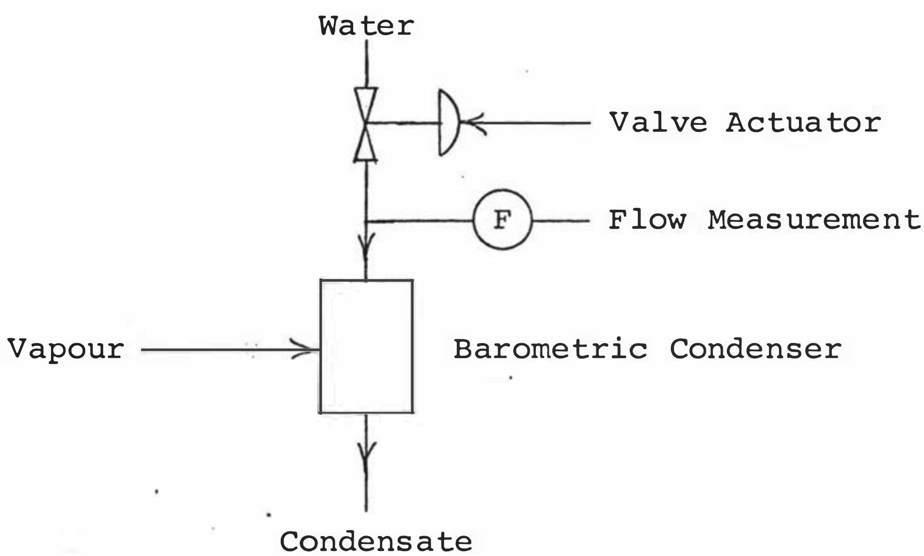


Figure 19 Water flow to condenser

The valve position is set to a mean value of 325. The valve position is then perturbed in the range $\pm$ 175 about the mean value at 12 second intervals. Figure 20 shows the perturbation data with data regenerated from a derived model superimposed on the flow. This model is 1st order with an update interval of 6 seconds and a sample interval of 3 seconds. Fast control action is again desirable for this control loop so a minimum time controller is designed by setting weights as for the steam loop. Simulated control responses are shown in figures 21a and 21b. Note in figure 21b the effect of

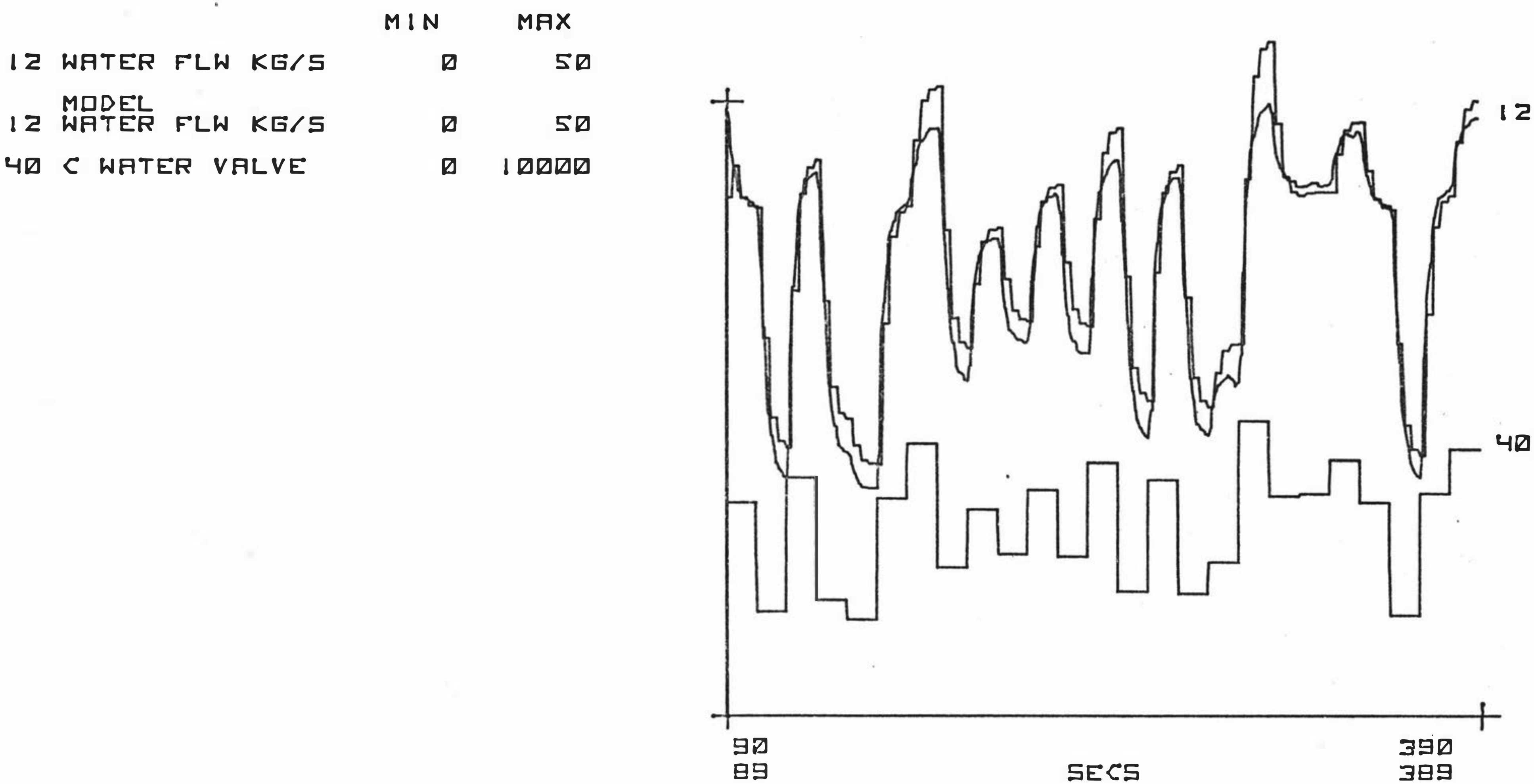


Figure 20 Perturbation data for condenser water flow

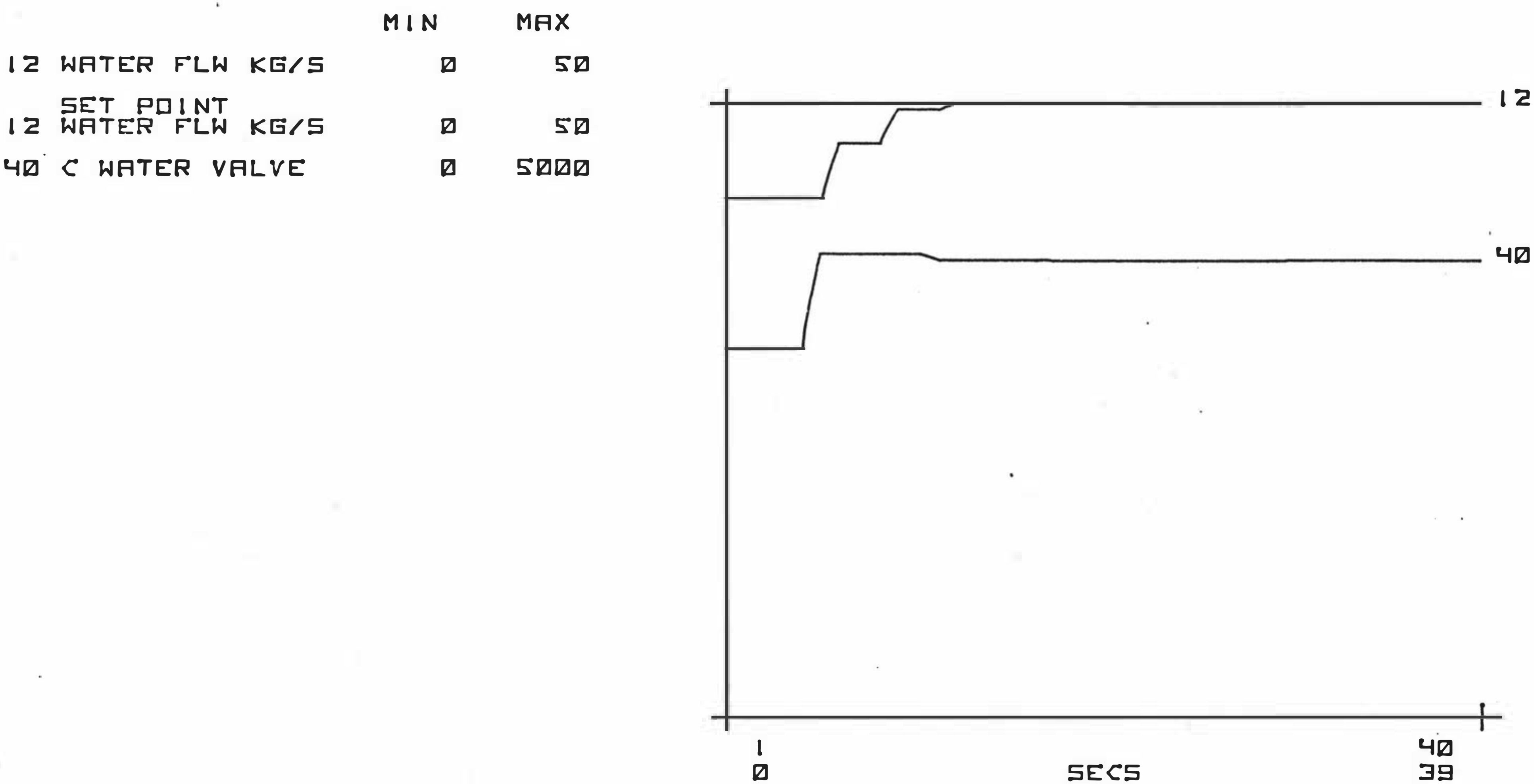


Figure 21a Simulated condenser water flow control response

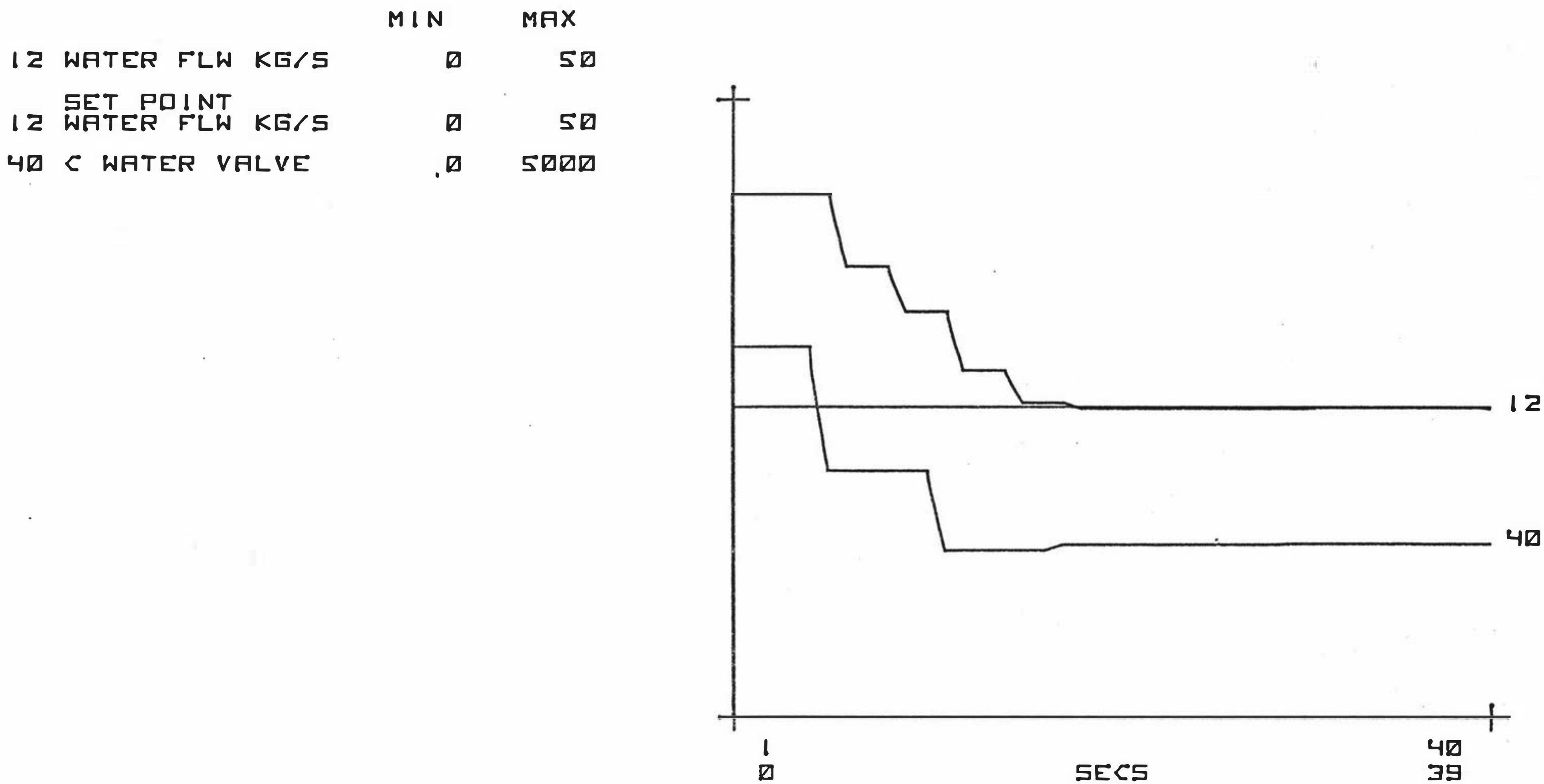


Figure 21b Simulated condenser water flow control response

a perturbation limit on the control action. The limit is used to restrict the maximum change in valve position that may be achieved in one control interval, thus prolonging the life of the valves. Figure 22 illustrates the operation of the controller online to the plant. The response of the controller to a sequence of setpoint changes is excellent. Setpoint is reached within two control cycles except for the second setpoint change. Here a perturbation limit is reached, thus reducing the speed of response slightly.

The parameters associated with the water flowrate control loop are summarised in Table 4.

<u>Table 4</u>

Water Flowrate Loop Parameters

<u>Model Matrices</u>

$$\underline{\alpha} = \begin{bmatrix} 1.36 & -.36 \\ 1.29 & -.29 \end{bmatrix}$$

$$\underline{\beta} = \begin{bmatrix} .009 \\ .006 \end{bmatrix}$$

control interval 6s
sample interval 2s

<u>Control Design Parameters</u>

$\underline{Q}$ = diag. (1,1), $\underline{P}$ = 0   2 iterations

<u>Control Matrix</u>

$$\underline{K} = \begin{bmatrix} -127.3 & 37.4 \end{bmatrix}$$

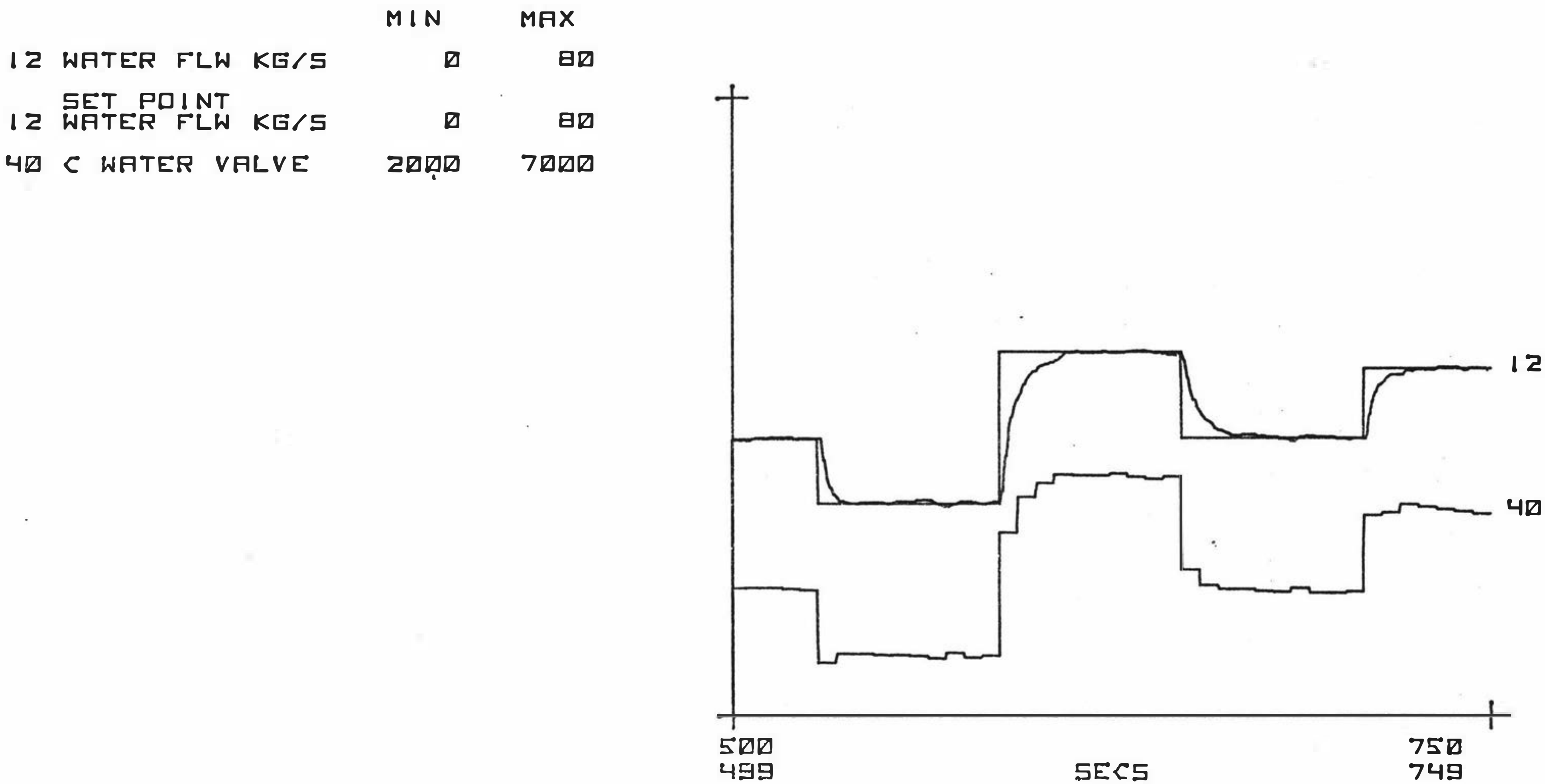


Figure 22 Online condenser water flow control response

4.5.3 Interstage Ejector Condenser

Figure 23 is a schematic diagram of the operation of the interstage ejector condenser. The flow of steam to the condenser is varied, thus affecting the rate at which the non-condensible vapours (i.e, nitrogen, oxygen) in the 3rd effect separator are drawn off. The temperature of the condensate is a measure of the rate of withdrawal of the non-condensibles.

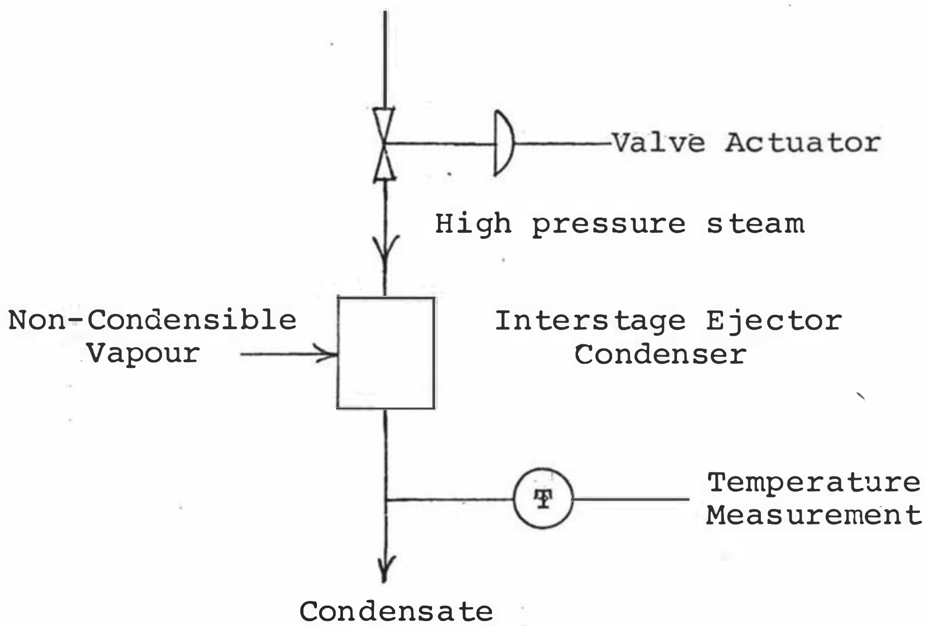


Figure 23 Interstage ejector condenser

The design of the interstage ejector system on this evaporator is such that the valve is always close to its fully open position. This restricts the allowable range of valve positions and coupled with the freeplay in the actuating mechanism, has caused some difficulty in implementation of a controller. Figure 24 shows the data from an experiment performed on this system. The valve position is varied in the range 115 $\pm$ 10 every 60 seconds. A first order model with an update interval of 30 seconds

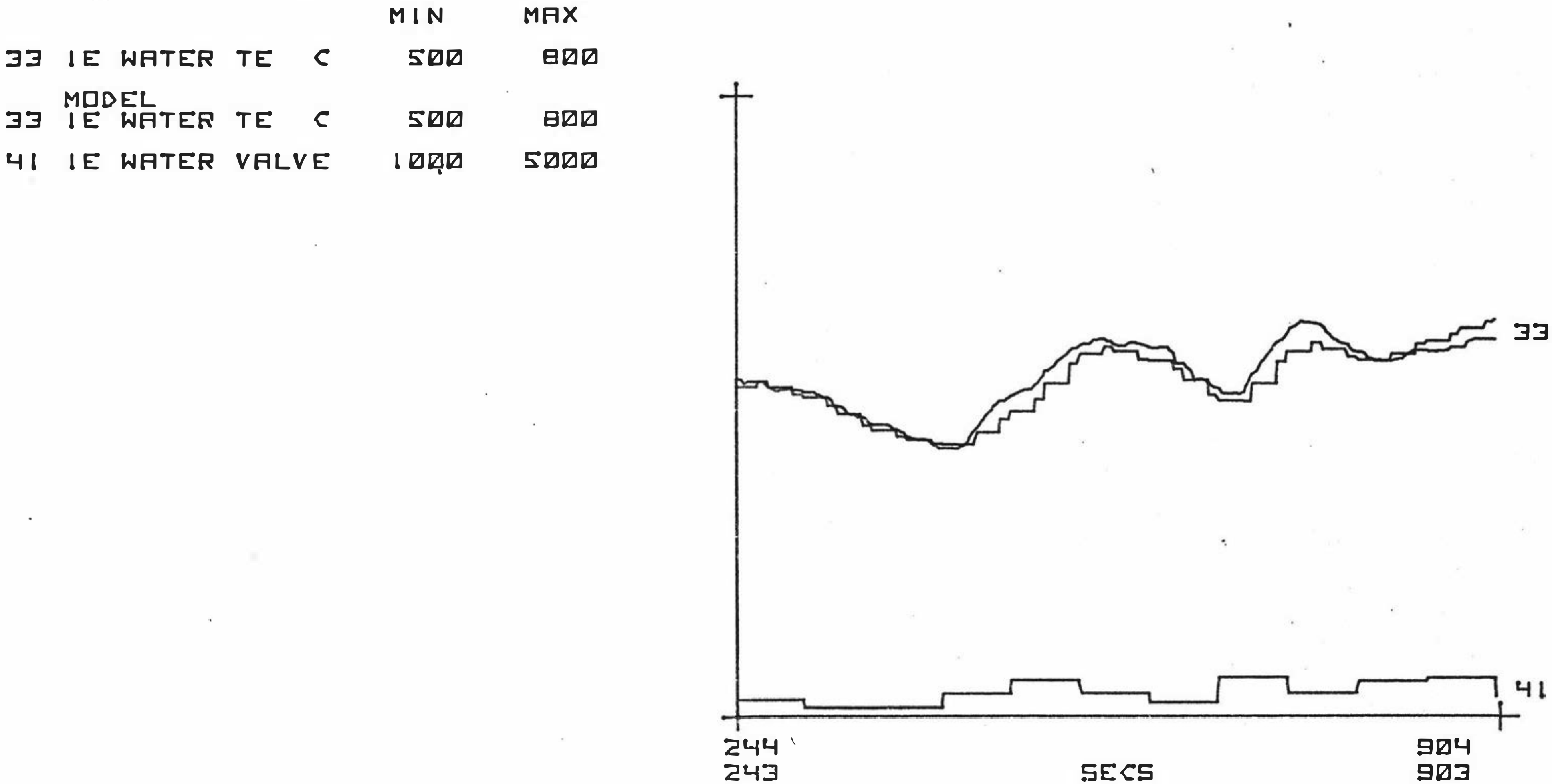


Figure 24 Perturbation data for interstage condenser temperature

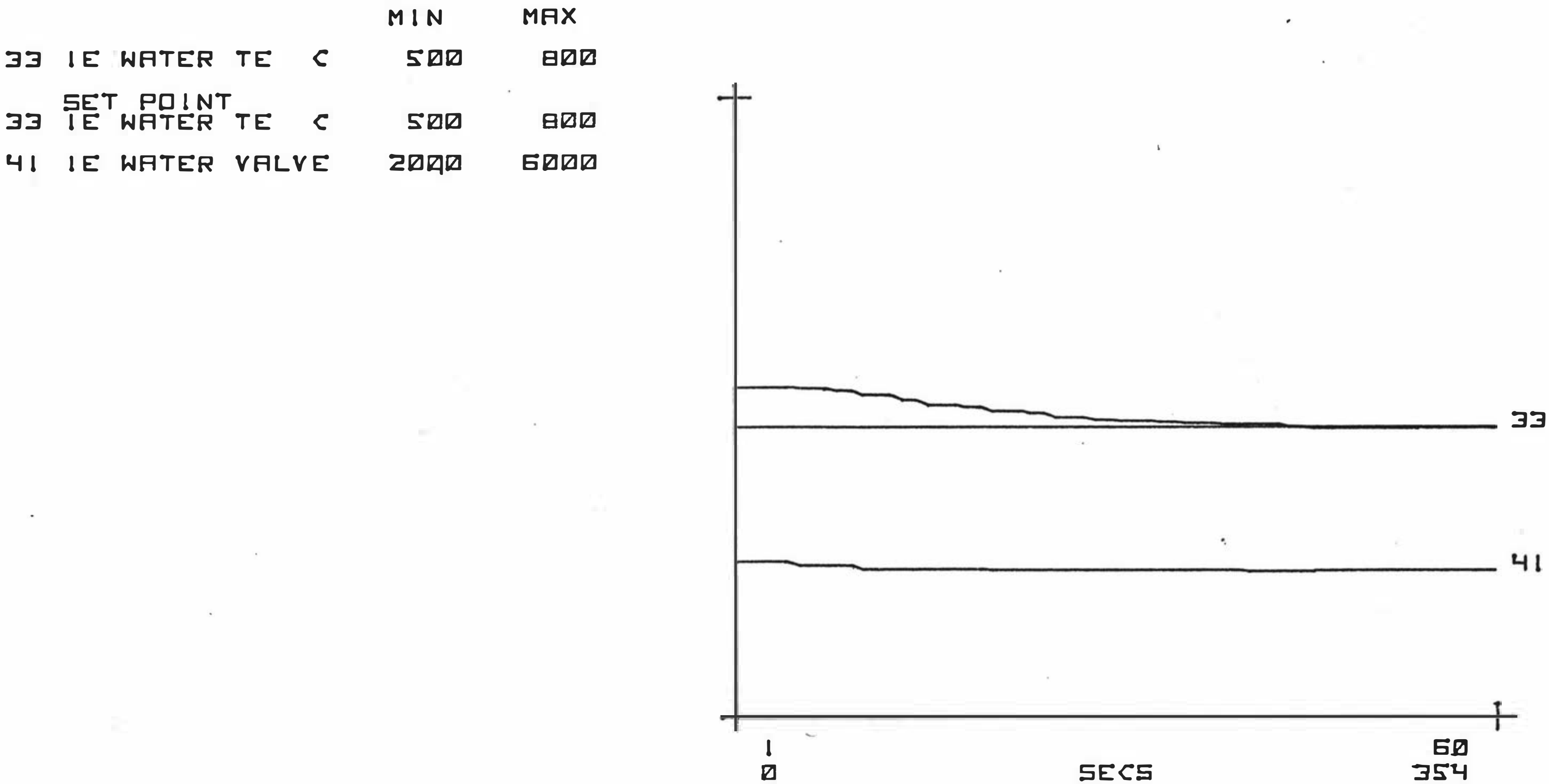


Figure 25 Simulated interstage ejector temperature control response

and a sample interval of 10 seconds is found. Increasing the sample interval to 15 or 20 seconds results in a degradation of the model accuracy. This confirms the presence of the delay which is evident from inspection of figure 24. A control system with weights $\underset{\sim}{Q}$ = diag. (1,1) $\underset{\sim}{P}$ = 1 is designed. The simulated response of this control system may be seen in figure 25. Slight problems in the implementation of this plant have been encountered. When the temperature error is small ($< 1\ ^{o}$C) the change in valve position calculated is less than 1 unit. However, integer arithmetic is used by the computer in outputting the signals, for valve positioning, to the output stations. With downward truncation this resulted in zero change in valve position. Rounding of the digital value to the nearest integer value lead to the problem of valve sensitivity. For reasonable temperature errors, changes in position of 1-5 units may be requested. This corresponds to valve movements of fractions of a millimeter. Friction and linkage freeplay made changes of this order of magnitude difficult. Fortunately tight control on the loop was not too critical and deviations of $\pm .75\ ^{o}$C from setpoint are acceptable during operation.

The ejector temperature loop parameters are summarised in Table 5.

4.5.4 Preheat temperature and inlet flow controls

Figure 26 is a schematic illustration of the preheat temperature and inlet flow system. A high pressure pump feeds fluid into the evaporator. The fluid is heated by direct steam injection downstream from the pump. Computer output stations position the steam valve and adjust the pump speed through a mechanical gear linkage. Control of the flowrate and temperature of the fluid downstream from the driect steam injection unit. However,

Table 5

Interstage Ejector Temperature
Loop Parameters

Model Matrices

$$\underset{\sim}{\alpha} = \begin{bmatrix} 2.81 & -1.81 \\ 2.23 & -1.23 \end{bmatrix}$$

$$\underset{\sim}{\beta} = \begin{bmatrix} .089 \\ .021 \end{bmatrix}$$

control interval 30 secs
sample interval 10 secs

Control Design Parameters

$\underset{\sim}{Q}$ = diag. (1,1), $\underset{\sim}{P}$=1 10 iterations

Feedback Gain Matrix

$$\underset{\sim}{K} = \begin{vmatrix} -5.21 & 4.05 \end{vmatrix}$$

the steam injection gives rise to a back pressure which tends to retard the flow. Also, a change in flow affects the temperature because the mass of steam injected into each unit volume alters. The system therefore has two inputs, two outputs and involves interactions.

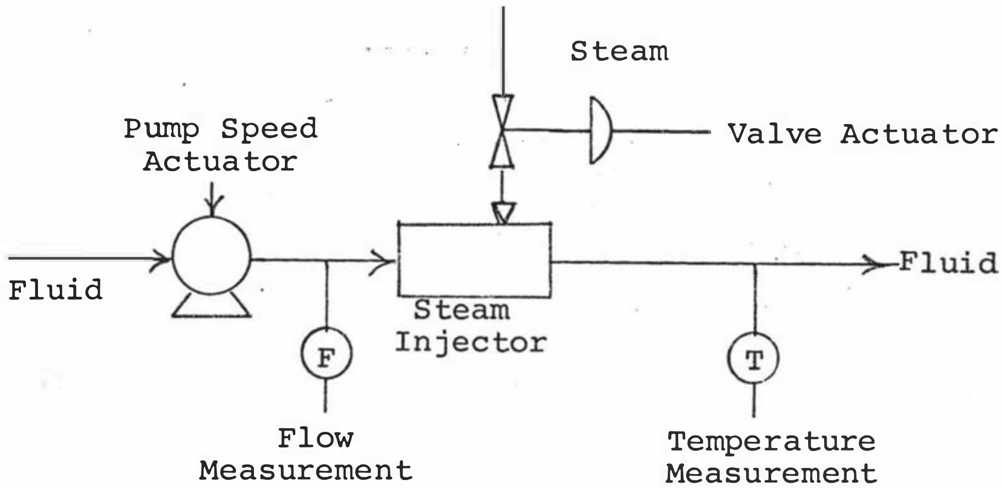


Figure 26 Preheat temperature and inlet flow system

The outputs are the fluid flowrate and its temperature whilst the inputs consist of the actuation signals to the pump and steam valve.

The perturbation data generated is shown in figure 27. The system is set manually to an operating level of approximately 1650 l/hr inlet flow and 105 ^{o}C preheat temperature. This corresponds to pump and steam valve positions of 307 and 259 units respectively. The inputs are varied randomly in the range $\pm$25 units of their mean values at 32 second intervals. A 2nd order multivariable model is fitted to this data. An update interval of 8 seconds and a sample interval of 4 seconds gives best results. No significant improvement in model accuracy is obtained with higher order models. The regenerated data gives a good fit to the temperature data but does not coincide as well as for the flow. This can be attributed partially to the non-linearities present in the pump speed adjustment linkage. The approximation to the flow characteristic is sufficient however to provide a basis for control design.

A control system with design weights Q = diag.(1,1,0,0) P = diag.(.1,0) is designed. This puts weighting on the latest error samples and constrains the action of the preheat steam valve slightly. The simulated control response is shown in figure 28. Flow response is seen to be quick while the temperature response is slower because of the weighting placed on the steam valve action. This is desirable to reduce the level of interaction and maintain tight control. Figures 29a and 29b show the control response when implemented on the plant. Good temperature control is maintained in the face of flow setpoint changes. There is some disturbance in flowrate after a temperature setpoint has been specified. Flow measurement is noisy but additional filtering of the signal is not desirable as it would increase the time lag associated with the

	MIN	MAX
1 MILK INFLW L/H	7000	17000
23 PHEAT 2 TEMP C	0	2000
MODEL 23 PHEAT 2 TEMP C	0	2000
MODEL 1 MILK INFLW L/H	7000	17000
37 PHEAT 2 VALVE	1000	7000
38 MILK PUMP STN	2500	8500

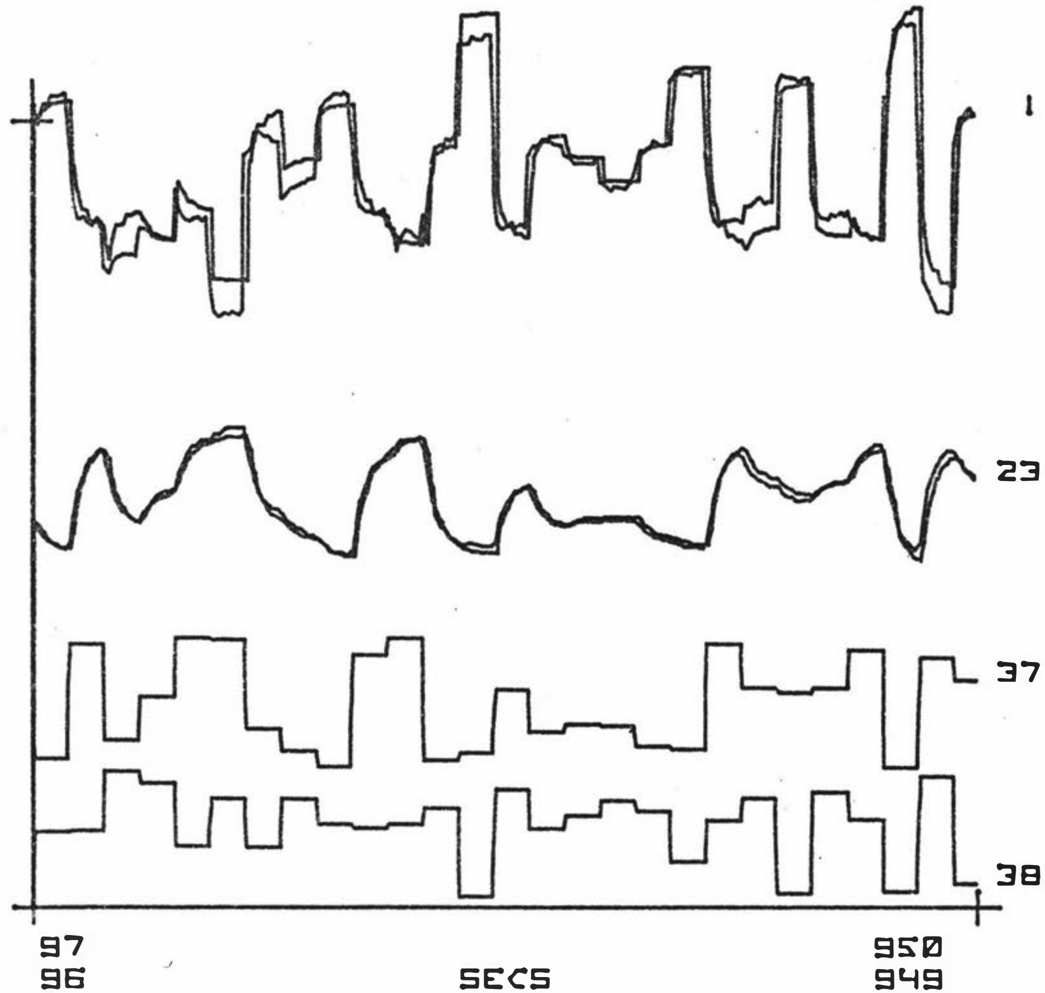


Figure 27 Perturbation data for multivariable preheat and flow control

		MIN	MAX
1	MILK INFLW L/H	7000	17000
23	PHEAT 2 TEMP C	0	2000
	SET POINT		
23	PHEAT 2 TEMP C	0	2000
	SET POINT		
1	MILK INFLW L/H	7000	17000
;7	PHEAT 2 VALVE	1000	7000
38	MILK PUMP STN	2500	8500

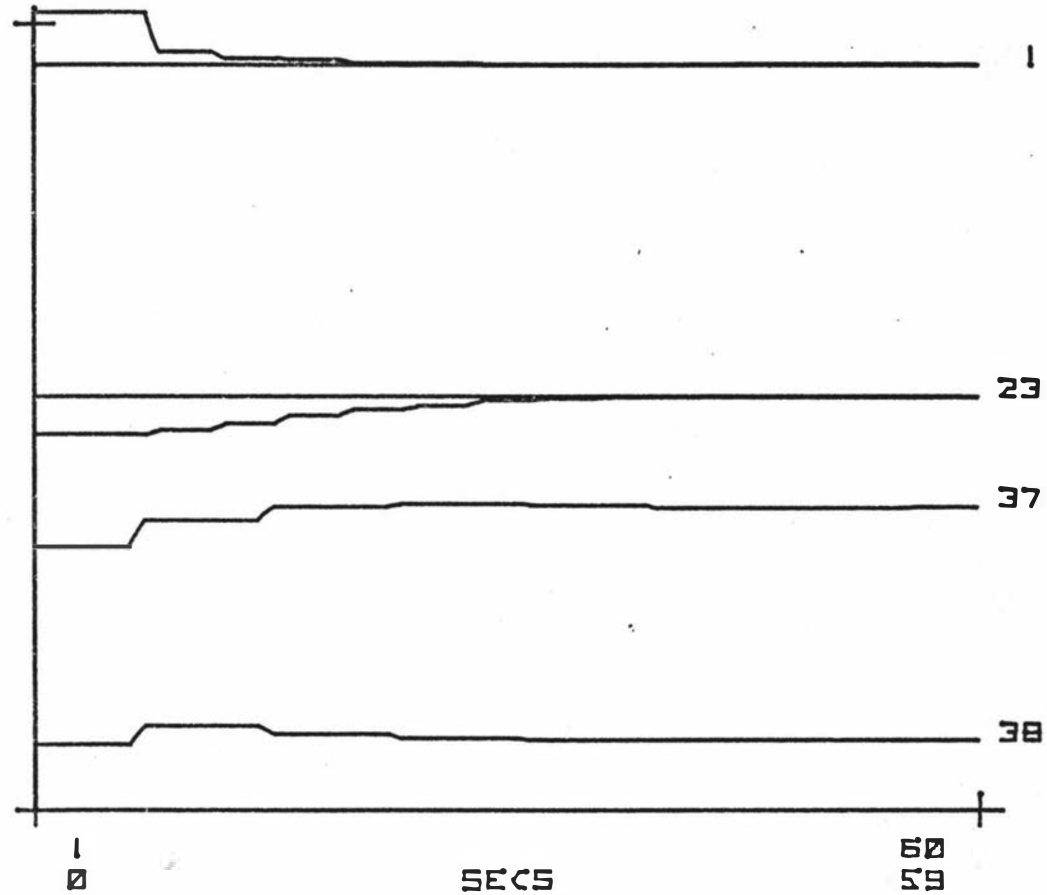


Figure 28 Simulated multivariable control responses

		MIN	MAX
1	MILK INFLW L/H	12000	18000
23	PHEAT 2 TEMP C	0	2000
	SET POINT		
23	PHEAT 2 TEMP C	0	2000
	SET POINT		
1	MILK INFLW L/H	12000	18000
37	PHEAT 2 VALVE	1000	6500
38	MILK PUMP STN	2500	8000

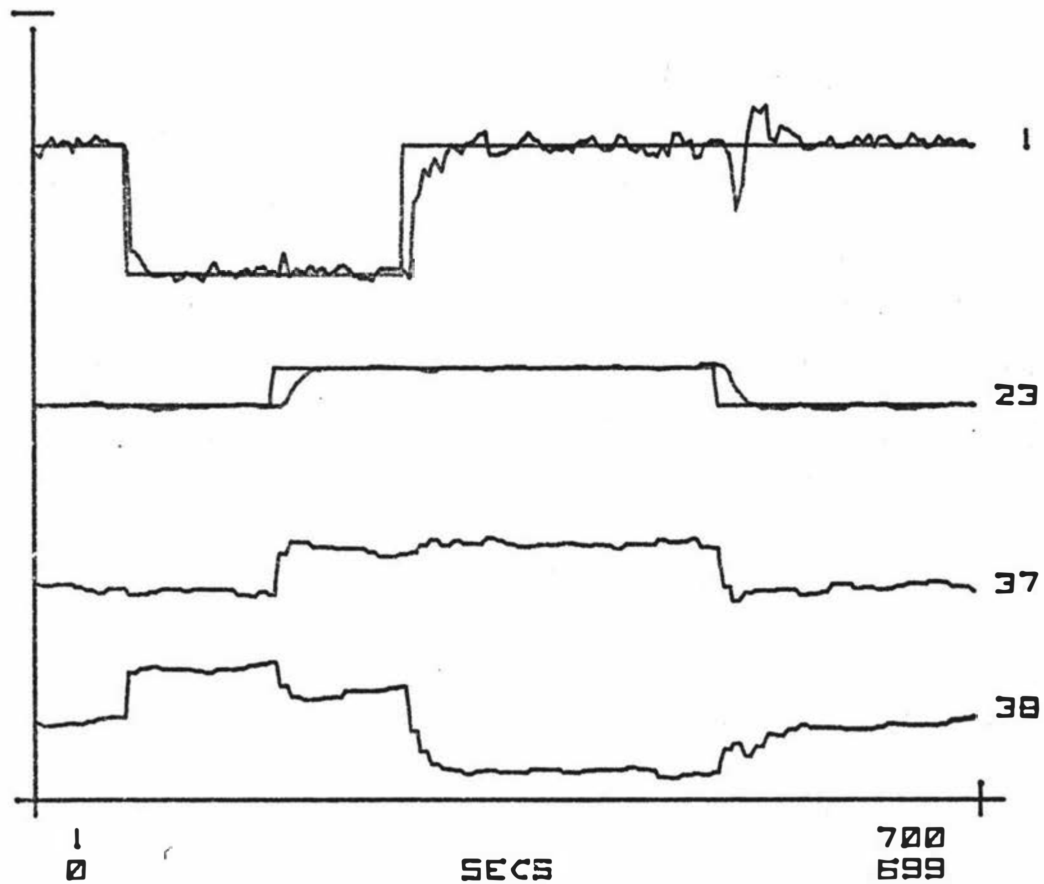


Figure 29a Online multivariable control responses
Rate factor = 0.7

		MIN	MAX
1	MILK INFLW L/H	12000	18000
23	PHEAT 2 TEMP C	0	2000
23	SET POINT PHEAT 2 TEMP C	0	2000
1	SET POINT MILK INFLW L/H	12000	18000
37	PHEAT 2 VALVE	1000	6500
38	MILK PUMP STN	2500	8000

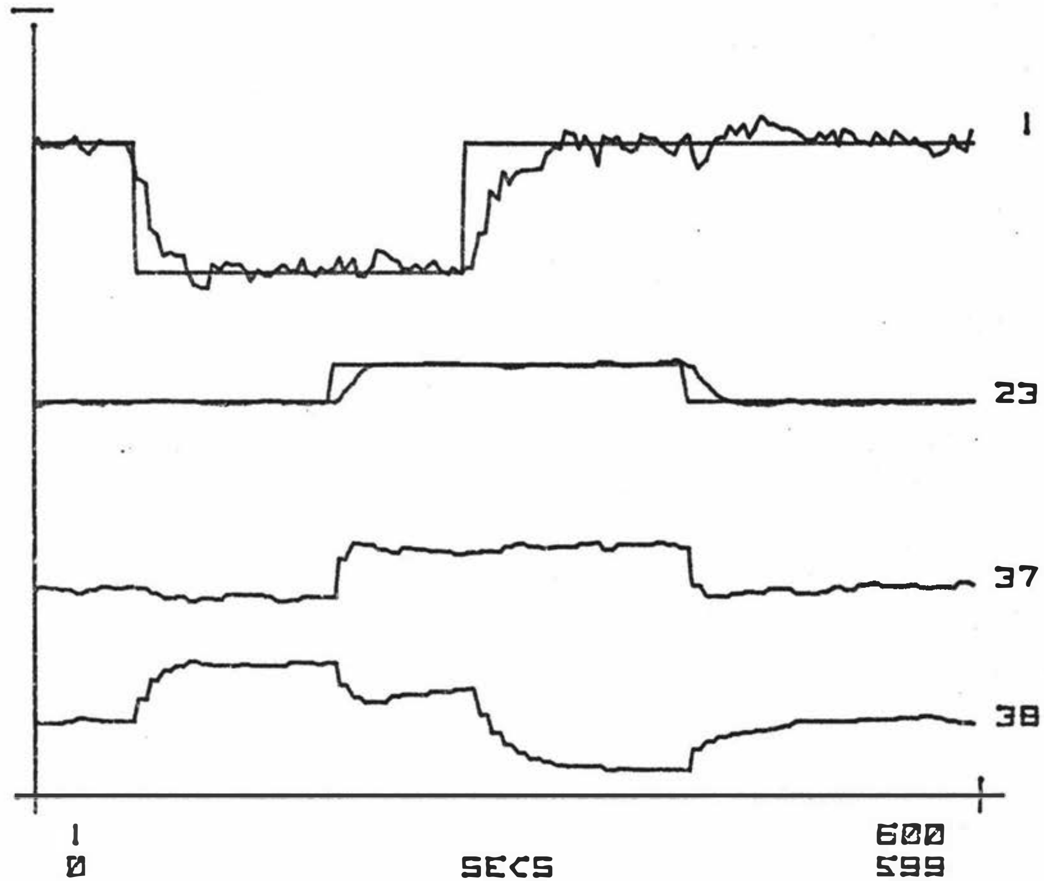


Figure 29b Online multivariable control responses
Rate factor = 0.4

measurement. A rate factor of 0.4 in figure 29b versus 0.7 in 29a on the flow error term reduces fluctuations only slightly but degrades the flow response to setpoint changes.

The parameter associated with this multivariable system are summarised in Table 6.

Table 6

Preheat Temperature and Inlet flow Parameters

Model Matrices

$$\underline{\alpha} = \begin{bmatrix} 2.08 & -.03 & -1.08 & .03 \\ -.04 & .96 & .04 & .04 \\ 1.65 & -.02 & -.065 & .02 \\ -3.97 & .81 & 3.97 & .19 \end{bmatrix}$$

$$\underline{\beta} = \begin{bmatrix} .13 & .02 \\ -1.27 & -2.38 \\ .05 & .01 \\ -.92 & -2.33 \end{bmatrix}$$

control interval 8 secs
sample interval 4 secs

Output matrix $\underline{y} = \begin{bmatrix} y_1 & \text{preheat temperature} \\ y_2 & \text{inlet flowrate} \end{bmatrix}$

Input matrix $\underline{u} = \begin{bmatrix} u_1 & \text{preheat temperature} \\ u_2 & \text{milk feed pump} \end{bmatrix}$

Control Design Parameters

$\underline{Q}$ = diag (1,1,0,0) P = diag (.1,0)

20 Iterations

Feedback Gain Matrix

$$\underline{K} = \begin{bmatrix} -7.00 & .09 & 4.06 & -.12 \\ 2.16 & .35 & -.59 & .08 \end{bmatrix}$$

All of the first level systems of the evaporator have proved to be straightforward to design and result in very simple feedback algorithms, the multivariable controller having the same basic structure as the single loop controllers.

4.6 Secondary Controls for a Triple effect Evaporator

Implementation of the full set of first level control systems enables the evaporator to operate at an equilibrium state that is a function of the setpoints to those controllers. Higher level systems, termed secondary controllers, are developed to adjust the set-points to the primary controls according to the desired properties of the concentrate leaving the evaporator.

Inspection of plant data obtained whilst developing and implementing the primary controls precluded the use of the control scheme developed using the previously described simulation model of the plant. The measurements of the concentrate flow, from each effect, and particularly those from the 1st and 2nd effects, were found to be noisy even with heavy filtering applied. This arises because the lines between effects are kept filled with fluid. Over-capacity scavenging pumps at the bottom of each effect ensure this resulting in noisy flow readings. Temperature changes measured within each effect were found to be slower than expected as a result of the thermal capacity of the plant. Changes in the 1st effect environment as a result of condenser water variations were also found to be slower than anticipated.

The alternative structure as shown in figure 30 has therefore been implemented. Instead of controlling the product properties at the 1st effect and then applying steady state relations over the remainder of the plant controllers relating directly to the concentrate leaving the evaporator are used. Time delays as a result of the propogation time through the evaporator are now important and are catered for in the control algorithm used. The 3rd effect separator pressure is used as an output rather than the temperature for the same reasons as before.

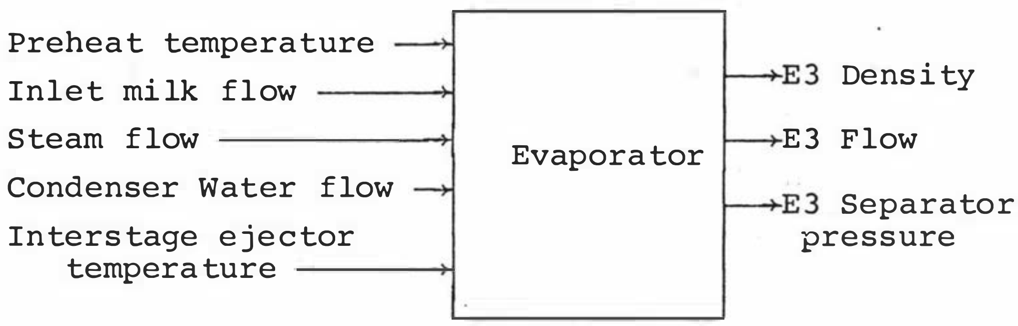


Figure 30 Secondary control structure for a triple effect evaporator

During normal plant operation the preheat and interstage ejector temperatures are maintained at constant levels, thus the secondary structure becomes one with 3 inputs and 3 outputs. Experiments on plant show this is best handled by further simplifying to a system for the 3rd effect pressure and a system for the density flow configuration. The development of these systems is described.

4.6.1 3rd Effect Separator Pressure Control

Control of the third effect separator pressure is required for a number of reasons. The pressure, which is easily read and relatively noisefree, affects the concentrate temperature. It is desirable that the temperature is as low as possible to maximise the efficiency of a multi-effect evaporator. The increased viscosity of the concentrate, however, limits the minimum temperature possible.[39] A constant value of pressure is also desirable to minimise flashing of the milk in the 3rd effect. An empirical relation is used to determine the percent total solids of the concentrate from the density and temperature measurements. Excessive flashing causes frothing within the lines leading to incorrect density readings and temperature compensation.

Figure 31 shows the perturbation data for the controller design. The condenser water flowrate is set to 3.5 Kg/sec using the 1st level flow controller. The setpoint to this loop is varied in the range $\pm$ 1 Kg/sec at 60 second intervals. A 2nd order model with an update interval of 30 seconds and sample interval 10 seconds gives the best fit. The simulated minimum time control response is shown in figure 32. Implementation of this controller is very effective. Figure 33 shows the response to setpoint changes. The slight delay arises because the point in time at which the change in setpoint is entered does not correspond with the time for a control action.

Separator pressure control parameters are shown in Table 7.

4.6.2 3rd Effect Density and Flow Controls

Control of the 3rd effect density and flow is important both for reasons of cost and the properties of the powder produced from the drier. In terms of water removal per unit of heat used, evaporators are far more efficient than spray driers, (of the order 10:1), thus it is important that maximum concentration is achieved within

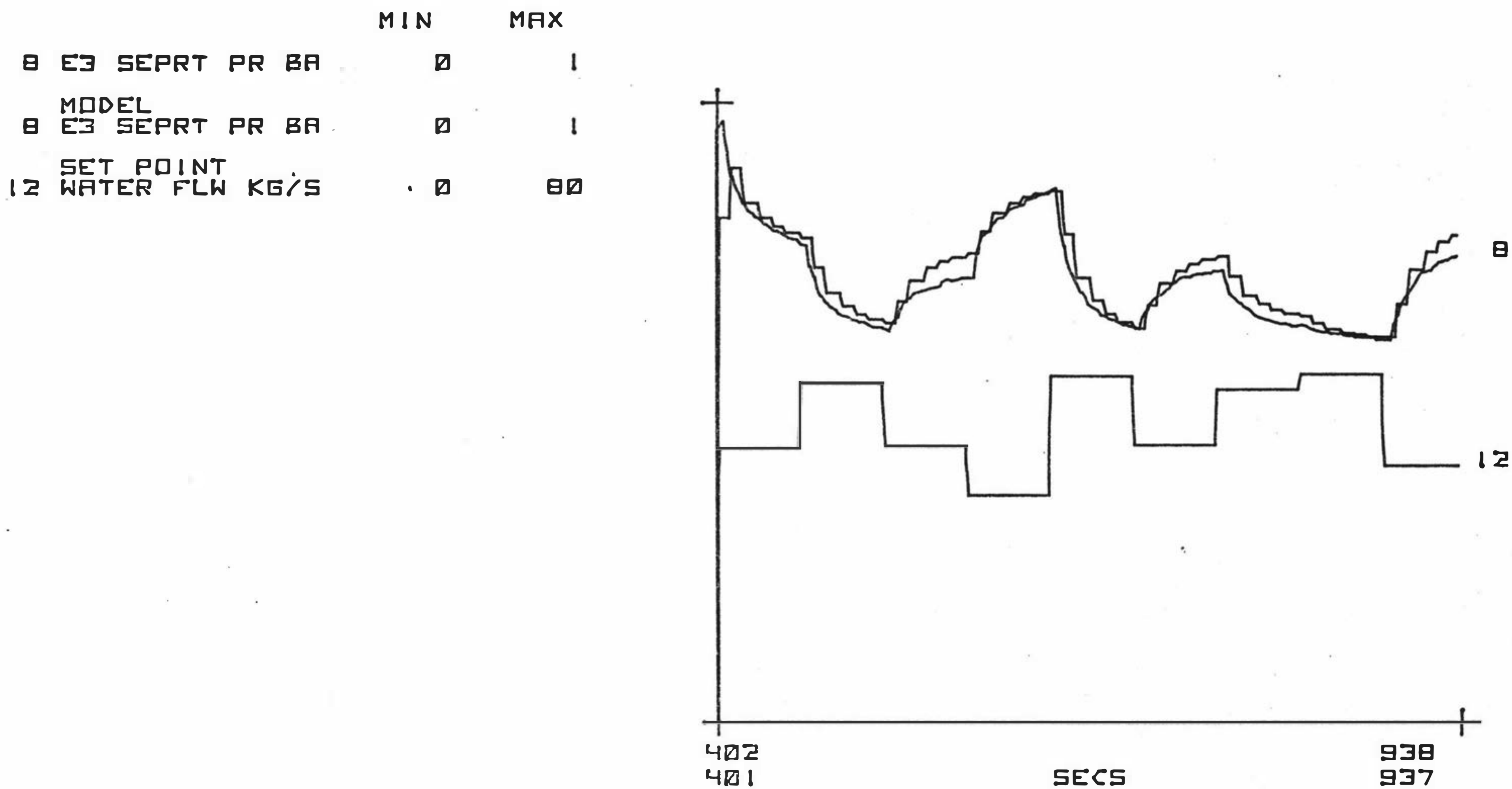


Figure 31 Perturbation data for E3 separator pressure control

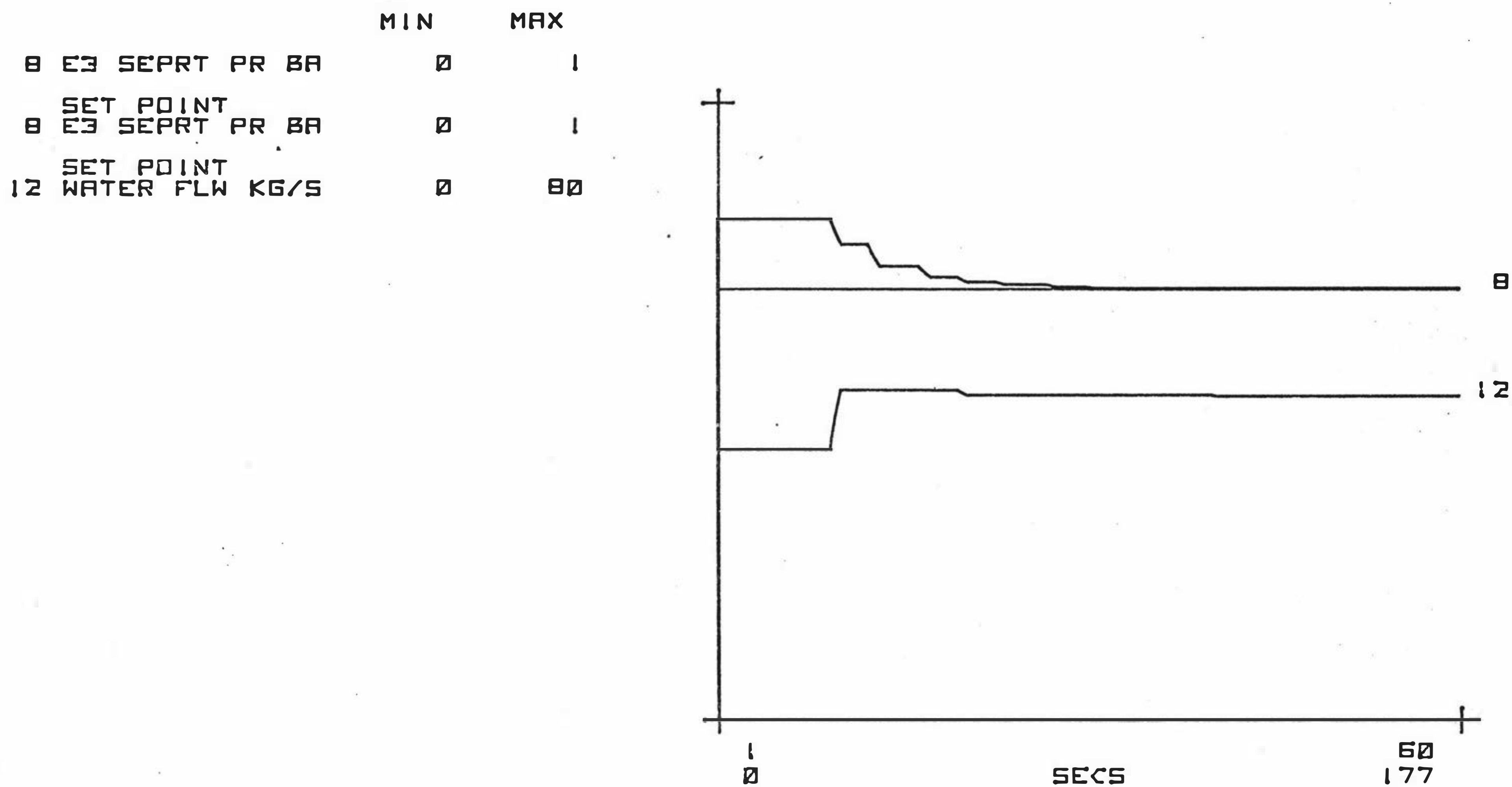


Figure 32 Simulated control response for E3 separator pressure

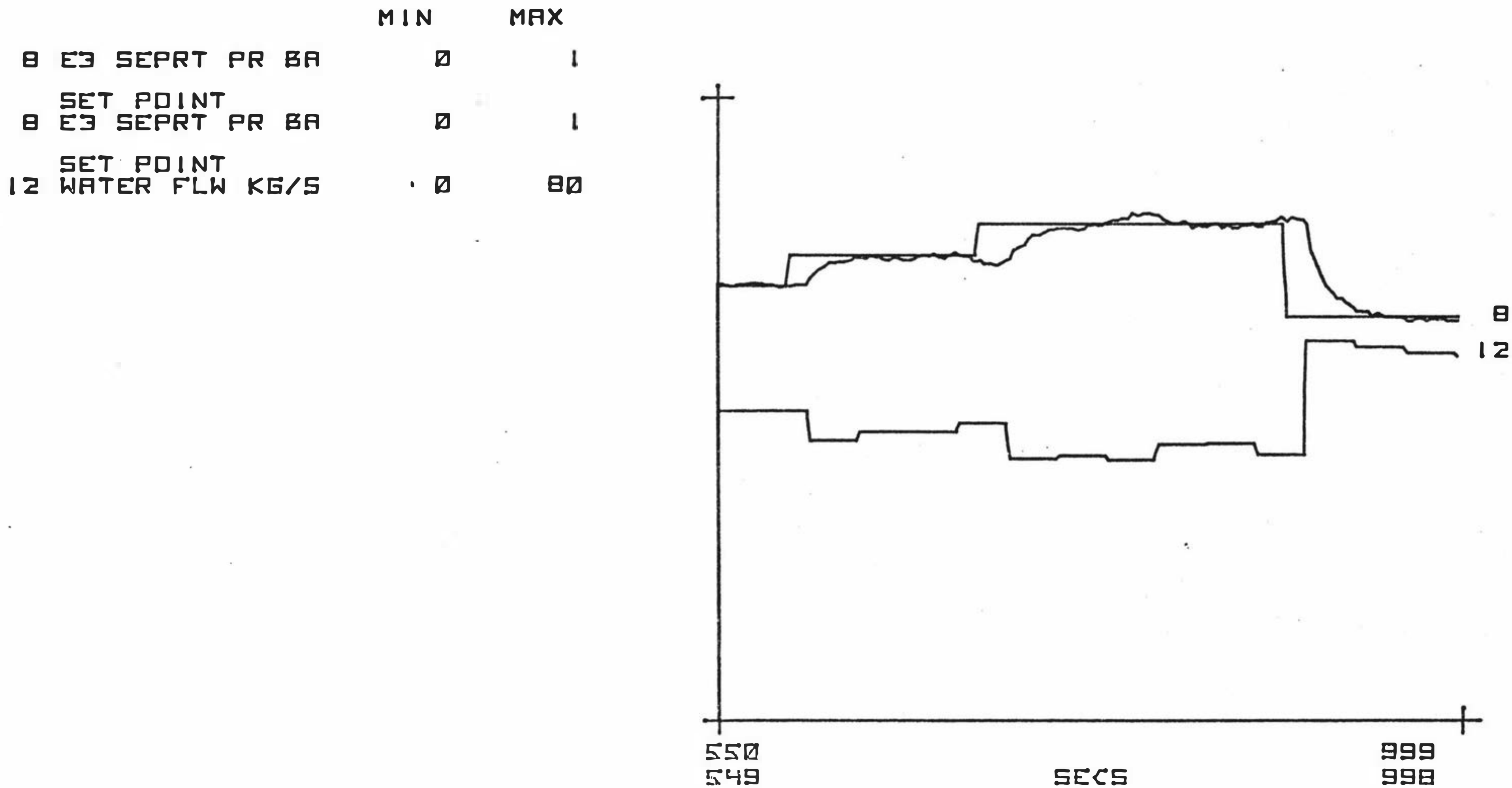


Figure 33 Online E3 separator pressure control response

Table 7

3rd Effect Separator Pressure Control Parameters

Model Matrices

$$\underline{\alpha} = \begin{bmatrix} 2.12 & -1.06 & -.06 \\ 1.99 & -1.0 & .01 \\ 1.68 & -\ .71 & .04 \end{bmatrix}$$

$$\underline{\beta} = \begin{bmatrix} -.012 \\ -.009 \\ -.005 \end{bmatrix}$$

control interval 30 secs
sample interval 10 secs

Control Design Parameters

$Q = \text{diag.}(1,1,1) \quad P = 0$ 2 iterations

Control Gain Matrix

$$\underline{K} = \begin{bmatrix} 177.9 & -86.85 & -7.61 \end{bmatrix}$$

the evaporator. Fouling of the callandria tubes as a consequence of scorching the concentrate puts an upper-limit on the concentration that can be achieved. Control allows a safe compromise to these requirements to be maintained over an extended period of time. A steady concentration in the feed to the drier also reduces variation in the powder bulk density.

Control in the flow of product leaving the evaporator allows close coupling of the spray drier evaporator plant. The capacity of the evaporator is usually greater than that of the drier. This means that high solids concentrate must be stored for a period of time before processing in the drier. Under these conditions the concentrate viscosity increases with time, again resulting in variation in the final powder properties.[36]

A single input, single output controller is considered first. The plant is brought to its normal operating level using the primary level controls and allowed to run for about twenty minutes to ensure steady operation. The setpoint to the steam loop is then perturbed randomly about its mean value at a specified frequency. Models incorporating differing control and delay intervals are fitted to the data generated. Figure 34 shows a model with an update interval of 60 seconds and a sample interval of 30 seconds. Additional samples are taken over the previous two control intervals. Delay is specified as being between 150 and 180 seconds. Figure 35 shows the regenerated data from a model with an update interval of 180 seconds and a sample interval of 30 seconds. Six samples are taken and the delay is specified as less than 180 seconds. The effect of these parameters is to give a fit that is almost identical to the model of the previous figure. This is not surprising as the effective sampling patterns for each model cover the same data spread. Figure 36 shows a model with an update interval of 180 seconds and a sample interval of 90 seconds, delay as before. The model is significantly worse, thus showing the beneficial effect of having more samples. Models with other parameter configurations have been determined but found to give poor representations of the plant dynamics.

The residence time of the milk in the evaporator is approximately five minutes with about 2½ minutes delay to a steam setpoint change before a density response is seen. Consideration of these factors leads to the choice of the first model as the basis for control design. Figure 37 shows the simulated control response obtained. Design weights are $\underline{Q}$ = diag (1,1,1,1,1,1), $\underline{P}$ = diag.(1,1,1). Implementation of the controller is shown in figures 38 and 39. The spikes in figure 39 on variable 15 are caused by the densitometer temporarily loosing synchronism

				MIN	MAX
15	E3 DENS	KG/M3		11250	12250
15	MODEL E3 DENS	KG/M3		11250	12250
5	SET POINT STEAM FLW	KG/H		1500	4500

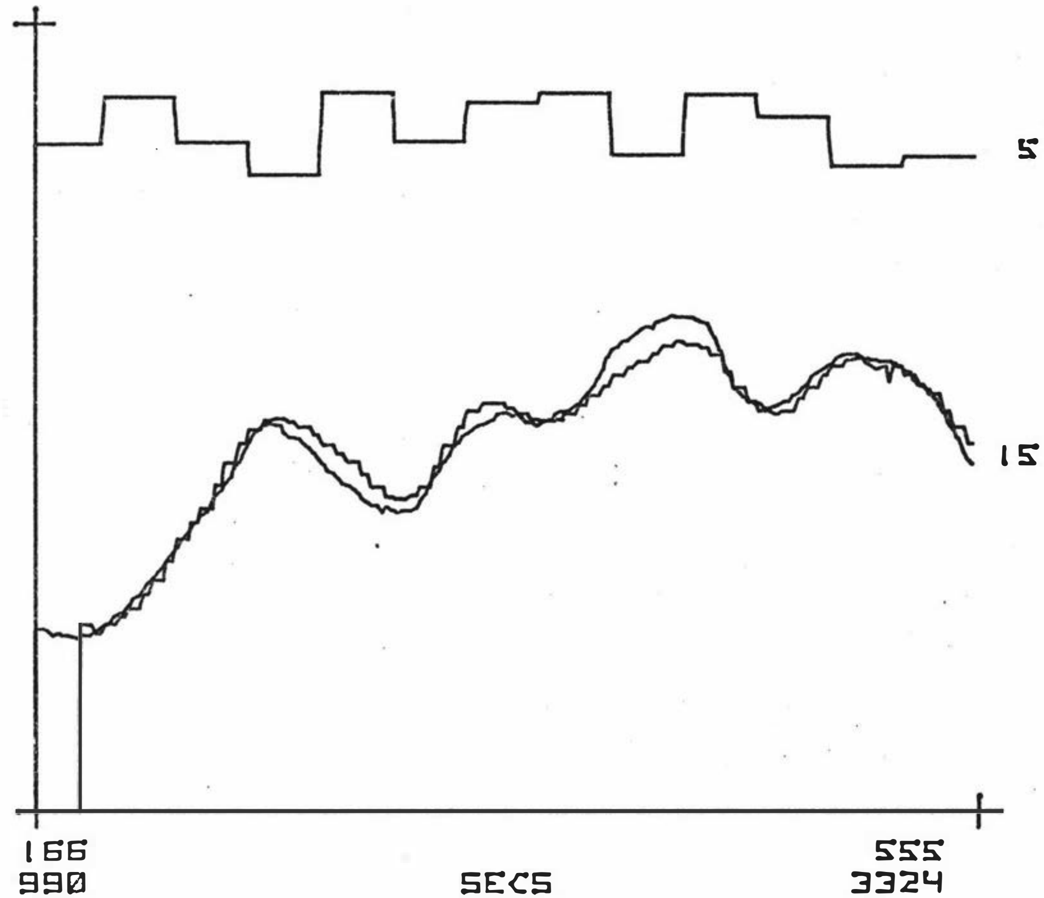


Figure 34 Perturbation data for 3rd effect density control

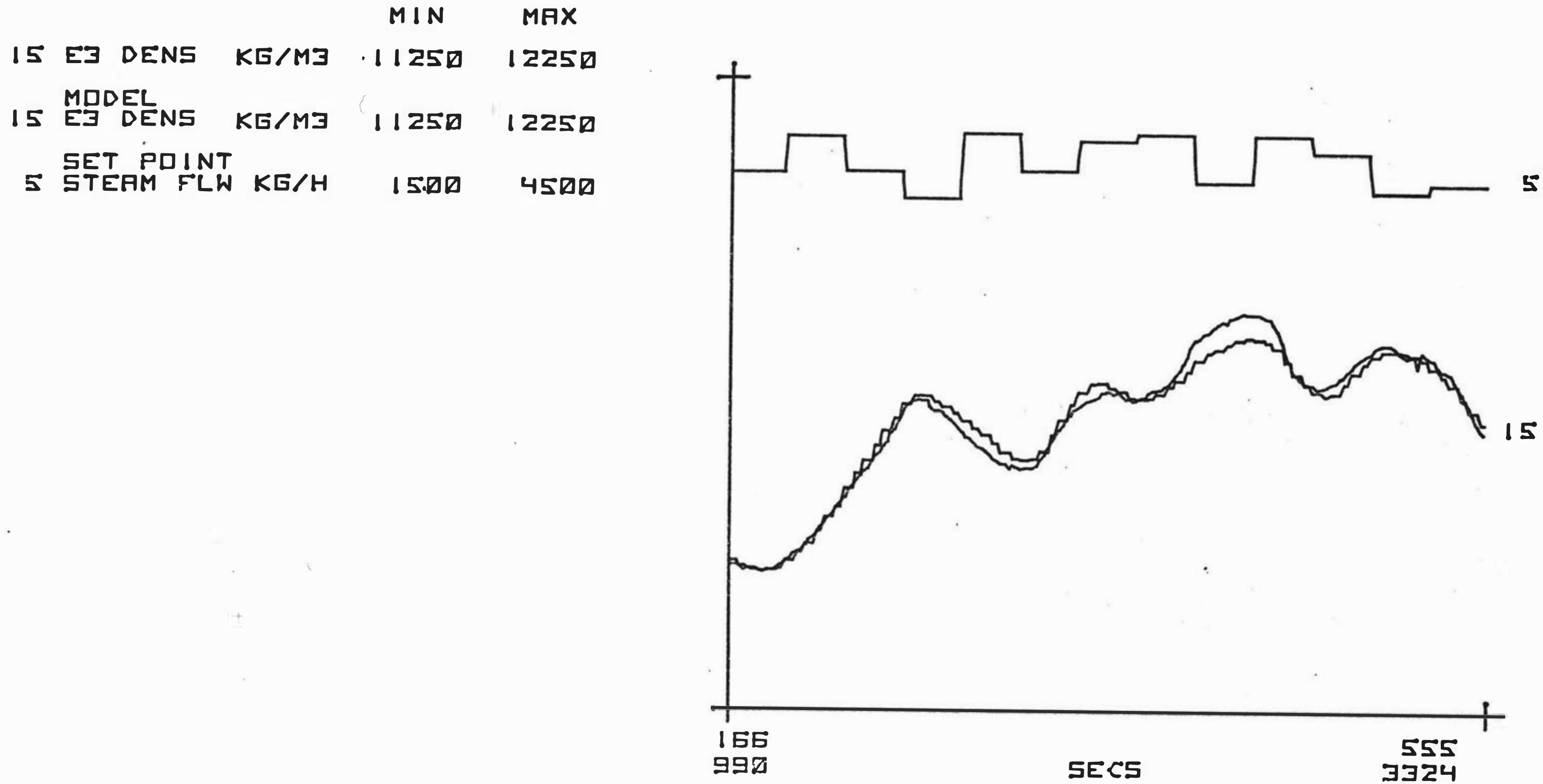


Figure 35 Perturbation data for 3rd effect density control

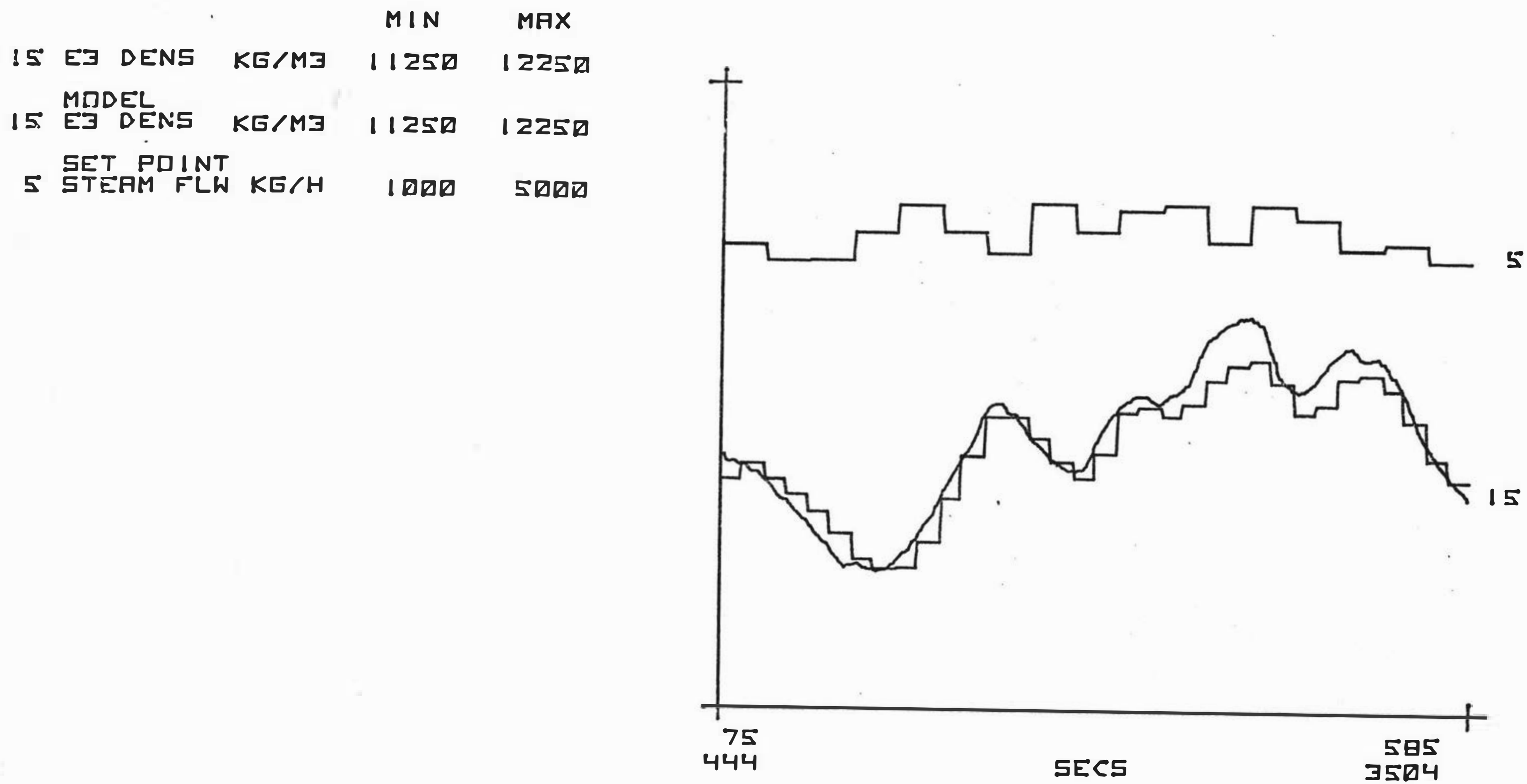


Figure 36 Perturbation data for 3rd effect density control

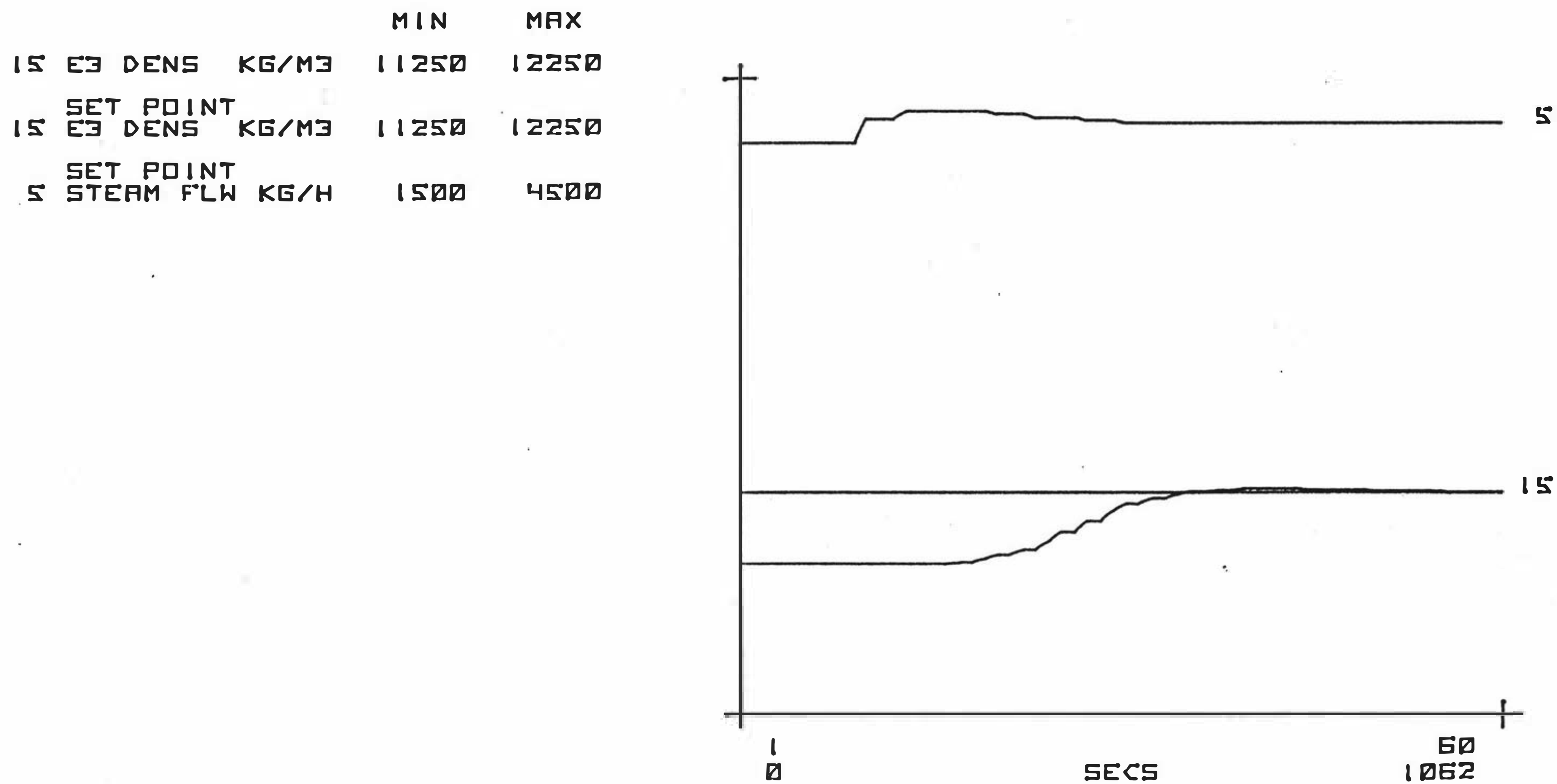


Figure 37 Simulated 3rd effect density control response

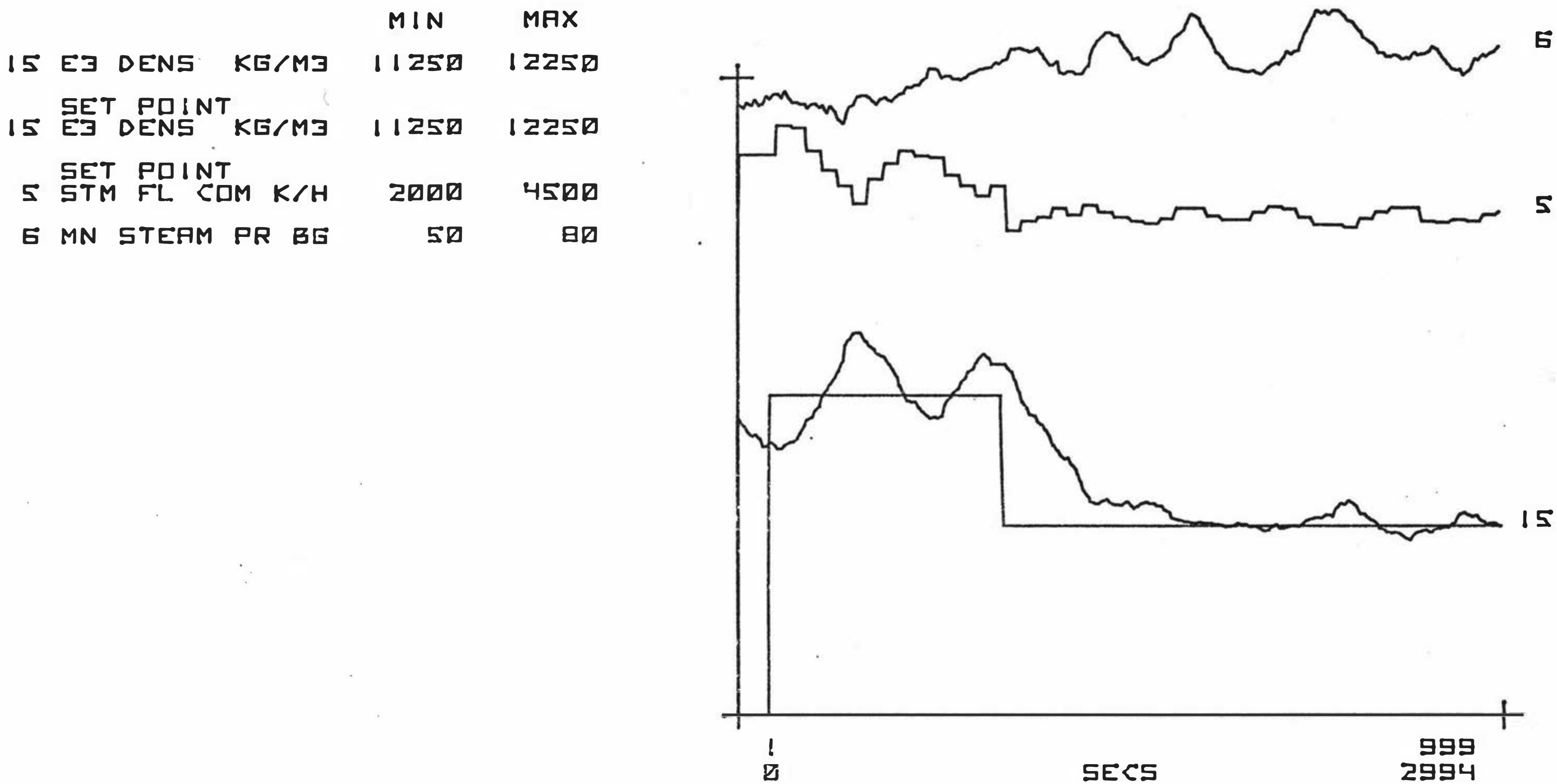


Figure 38 Online control response for 3rd effect density

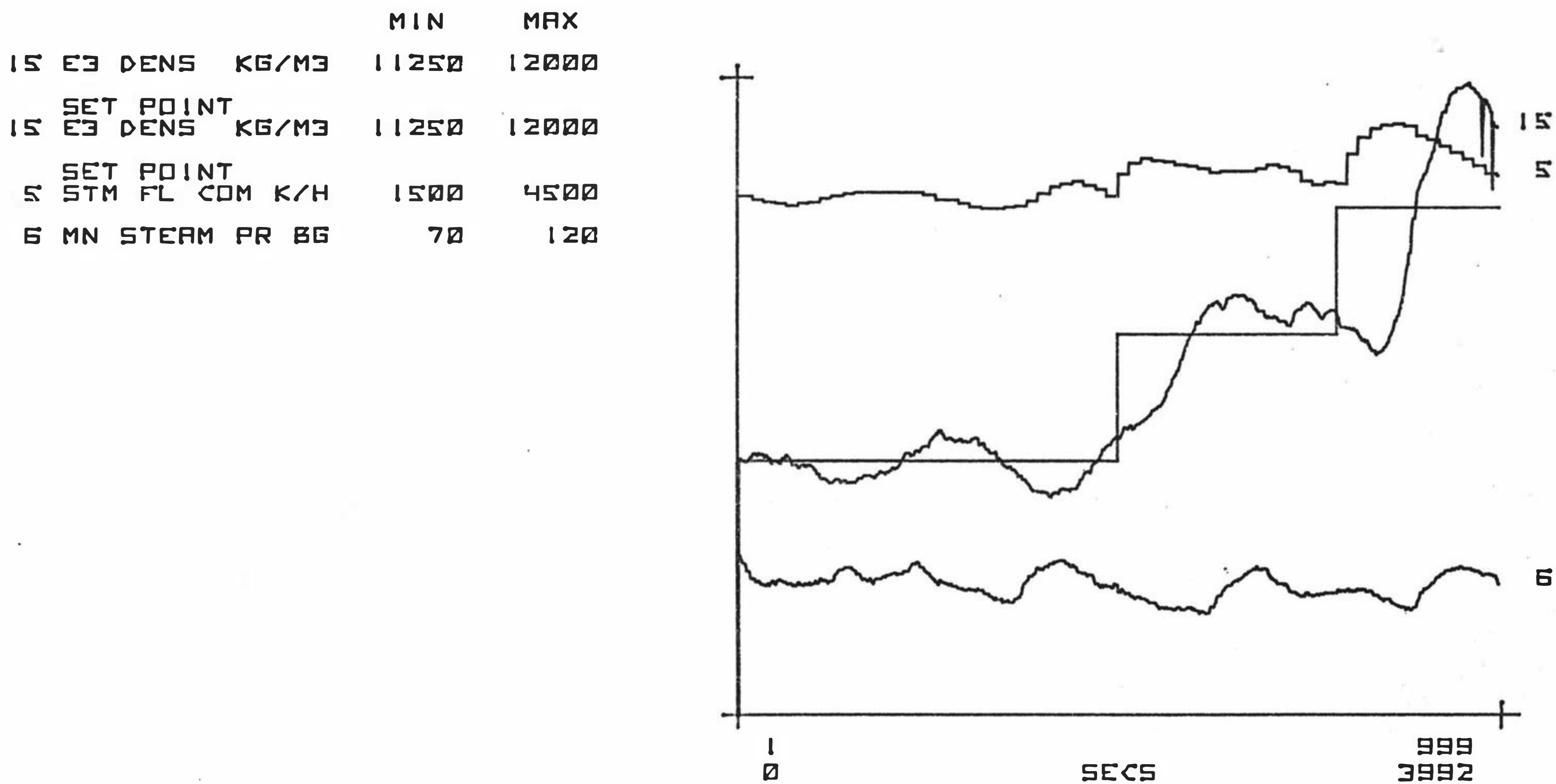


Figure 39 Online control response for 3rd effect density

because of frothing in the fluid line.

It is apparent from the control responses that some other disturbance effect is still impinging on the density. Analysis of plant data and further investigation of the plant suggested that variations in the pressure of the main steam supply should be monitored. A model with the parameters of figure 34 but including the main steam supply pressure as a disturbance factor is obtained. The regenerated response shown in figure 40 gives good agreement with the original data. Implementation of a controller based on this model does not, however, provide adequate protection from disturbance (see figure 41).

An additional loop operating on the 1st effect chest pressure was therefore set up. It was thought that if a constant pressure balance were maintained across the three evaporator effects, the impact of the steam supply pressure would be much reduced. A density control loop could then be set up to provide setpoints for the 1st effect pressure loop. Operation of this control scheme proved however to be ineffective. Though fluctuations were reduced they were still too large to allow close coupling of the evaporator and spray drier. Changes in the steam supply to the evaporator were then made to alleviate the problem. The steam supply line was rerouted and a knock-out vessel installed to remove entrained water droplets. As well, a pressure regulator was installed in the supply line. Test runs with the evaporator using primary level controls only suggested that the modifications had resolved the problems. New controllers were developed and these proved more effective.

The controllers are single input, single output, incorporating two disturbance factors. Figure 42 shows the perturbation data used for design of the controller for 3rd effect density. The steam setpoint is varied in

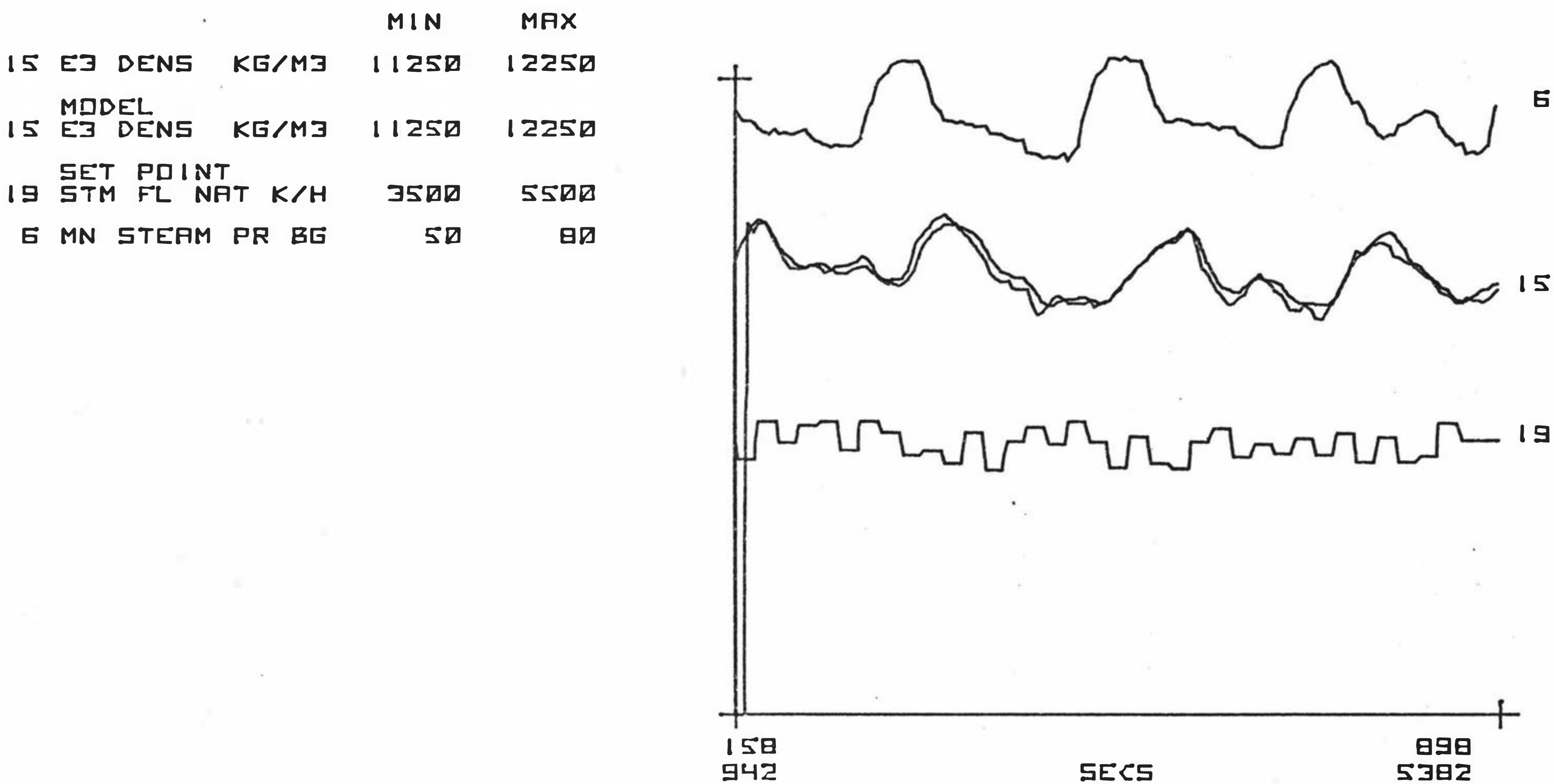


Figure 40 Perturbation sequence for 3rd effect density control

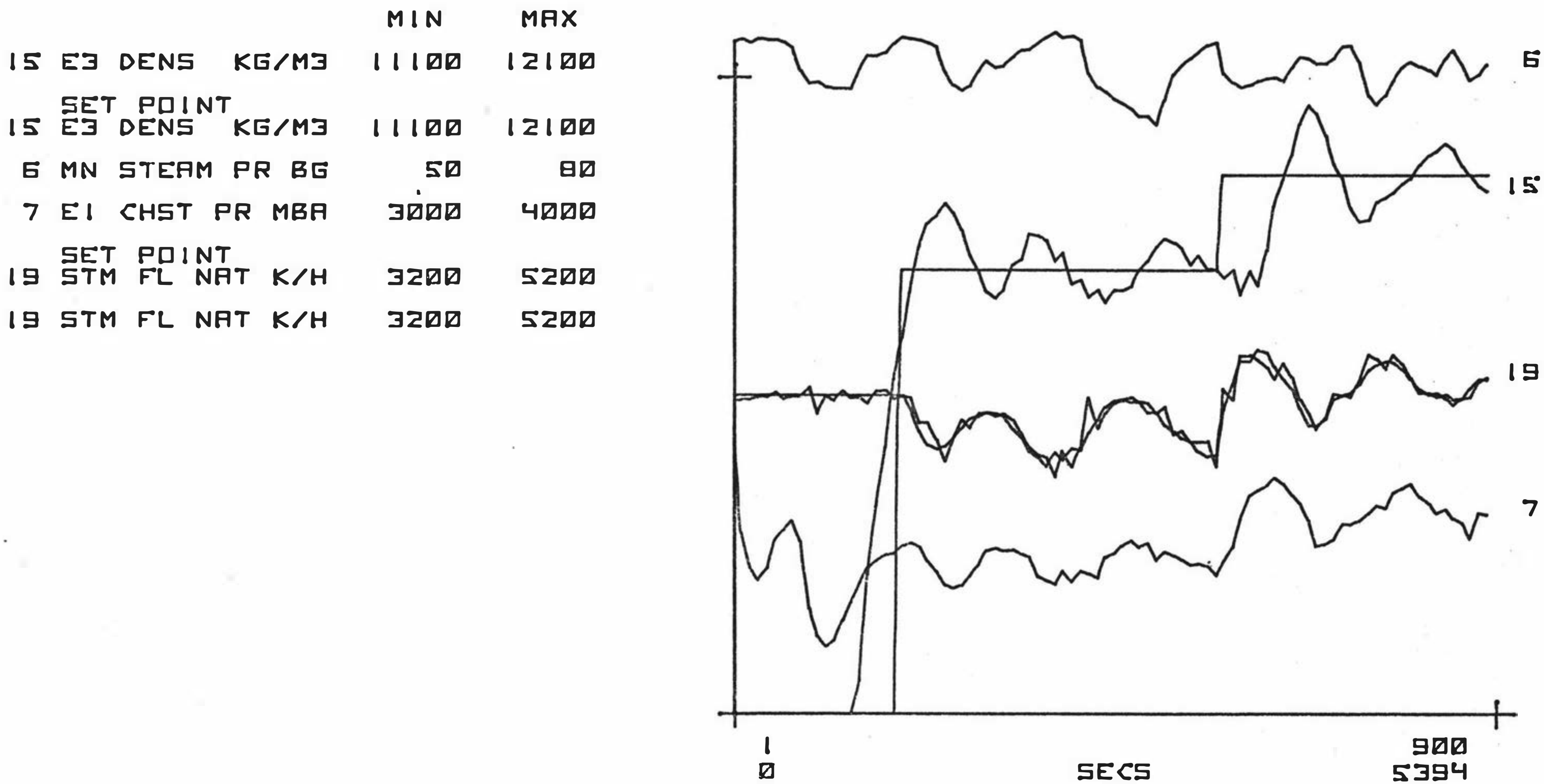


Figure 41 Online 3rd effect density control response

	MIN	MAX
15 E3 DENS KG/M3	11400	12000
MODEL 15 E3 DENS KG/M3	11400	12000
SET POINT 5 STM FL COM K/H	4000	5200
SET POINT 1 MILK INFLW L/H	14500	17500
14 MILK DEN KG/M3	10300	10700

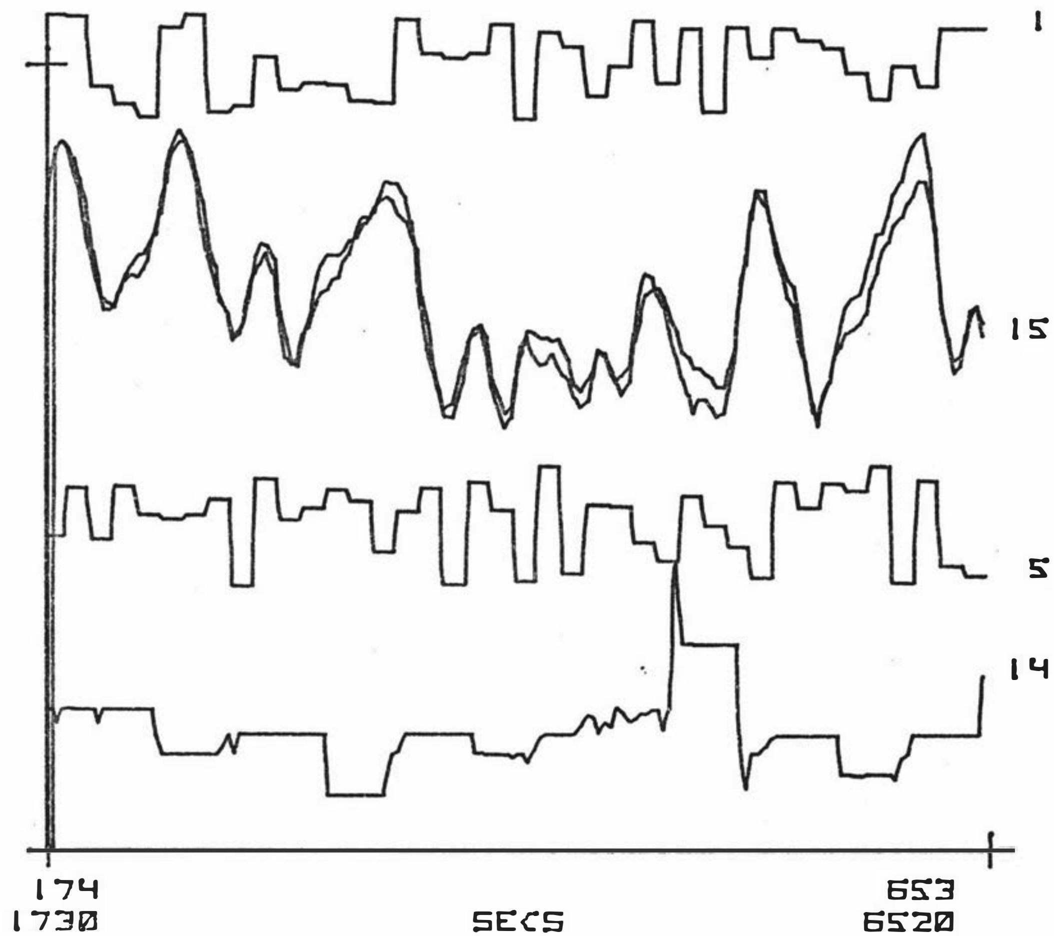


Figure 42 Perturbation data for 3rd effect density control

in the range 440 ± 10 Kg/hr while the milk inflow is varied ± 20 l/hr about 1750. Variable 14 is the density of the milk supplied to the evaporator and lies in the range 9 - 11 % T.S. Density fluctuations are introduced by the addition of water or high solids milk concentrate. The model fitted has an update interval of 60 seconds and a sample interval of 20 seconds. Three samples are taken per control interval with a total of 4 control intervals considered at each update. The inlet milk flow setpoint and inlet milk density are the two disturbance factors. Delay is specified as 2 control intervals, i.e., 180 seconds.

Figures 43a, b, and c show the simulated control responses with respectively no disturbances acting, inlet density fluctuations and both density and inlet flow variations. The responses indicate the best compromise between setpoint response and disturbance rejection. Table 8 details the control gains for this loop. It is clear that the structure of the controller is more complex than those of the level 1 and 2 controllers. Operation of the controller is illustrated in figures 44a and b. Disturbance effects are much reduced and the maximum variation of density from setpoint is approximately 5 Kg/m^3 (1% TS). Over the major portion of the run deviations are markedly less than this.

Figure 45 illustrates the perturbation sequences and regenerated model response for the product flow controller. The sample and update intervals are as for the density system. Disturbances this time however are inlet milk density and the setpoint to the 1st level steam loop. The model is as good as could be hoped for in view of the noisiness of the flow signal. The signal is already heavily filtered. Control design weights of Q = diag (12*1.0) $\underline{P}$ = diag (50,50,50,50) give a controller with the gains as shown in Table 9.

		MIN	MAX
15	E3 DENS KG/M3	11400	12000
15	SET POINT E3 DENS KG/M3	11400	12000
1	SET POINT MILK INFLW L/H	14500	17500
5	SET POINT STM FL COM K/H	4000	5000
14	MILK DEN KG/M3	2950	3350

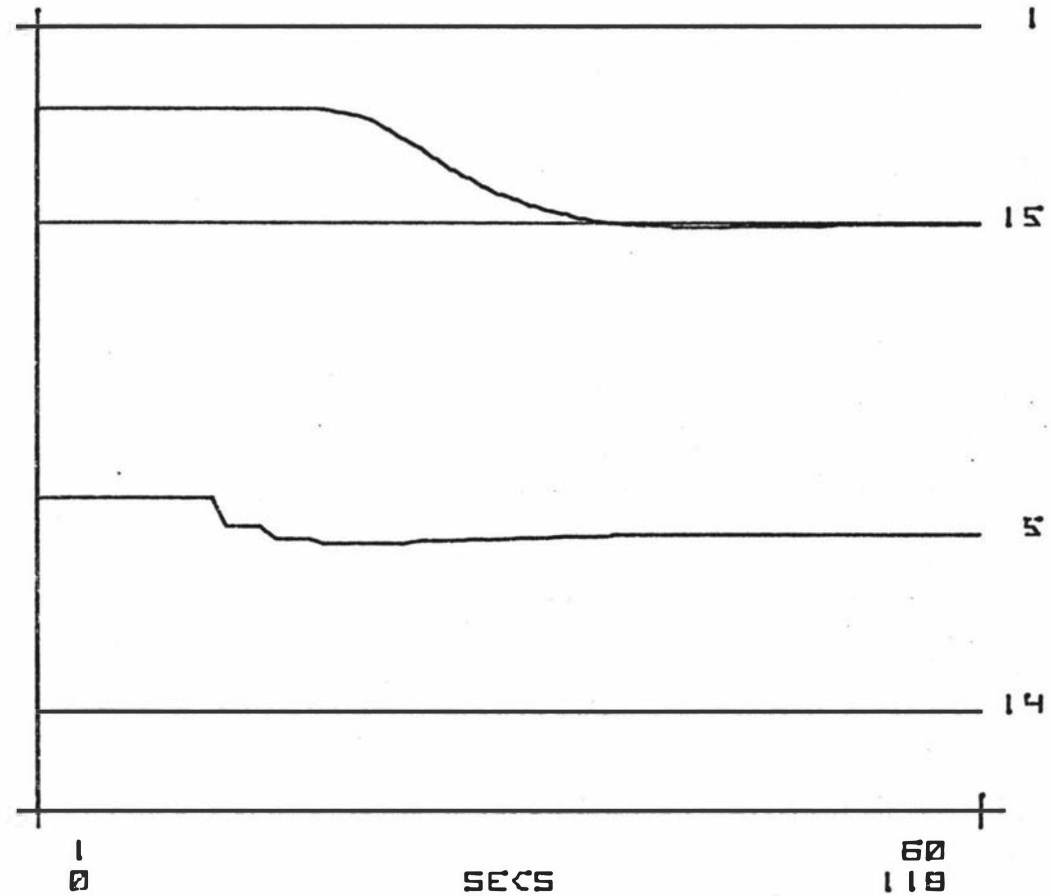


Figure 43a Simulated 3rd effect density control response

	MIN	MAX
15 E3 DENS KG/M3	11400	12000
SET POINT 15 E3 DENS KG/M3	11400	12000
SET POINT 1 MILK INFLW L/H	14500	17500
SET POINT 5 STM FL COM K/H	4000	5000
14 MILK DEN KG/M3	2950	3350

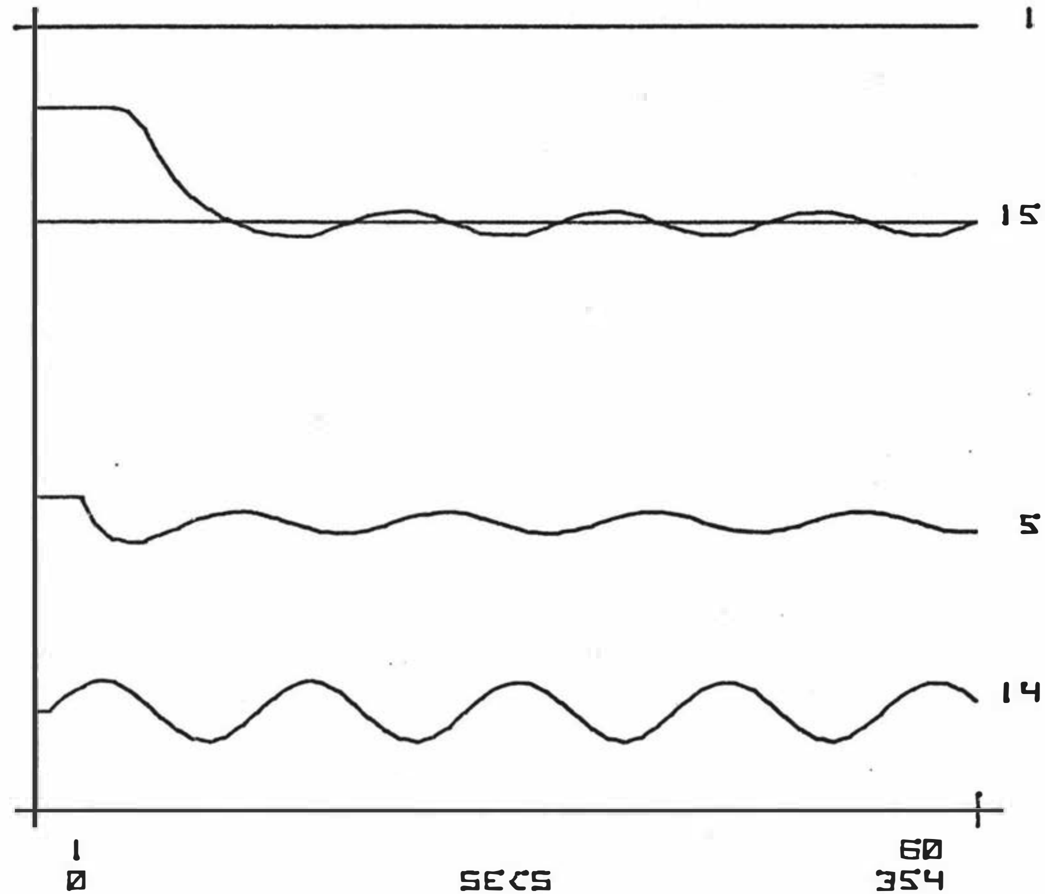


Figure 43b Simulated 3rd effect density control response

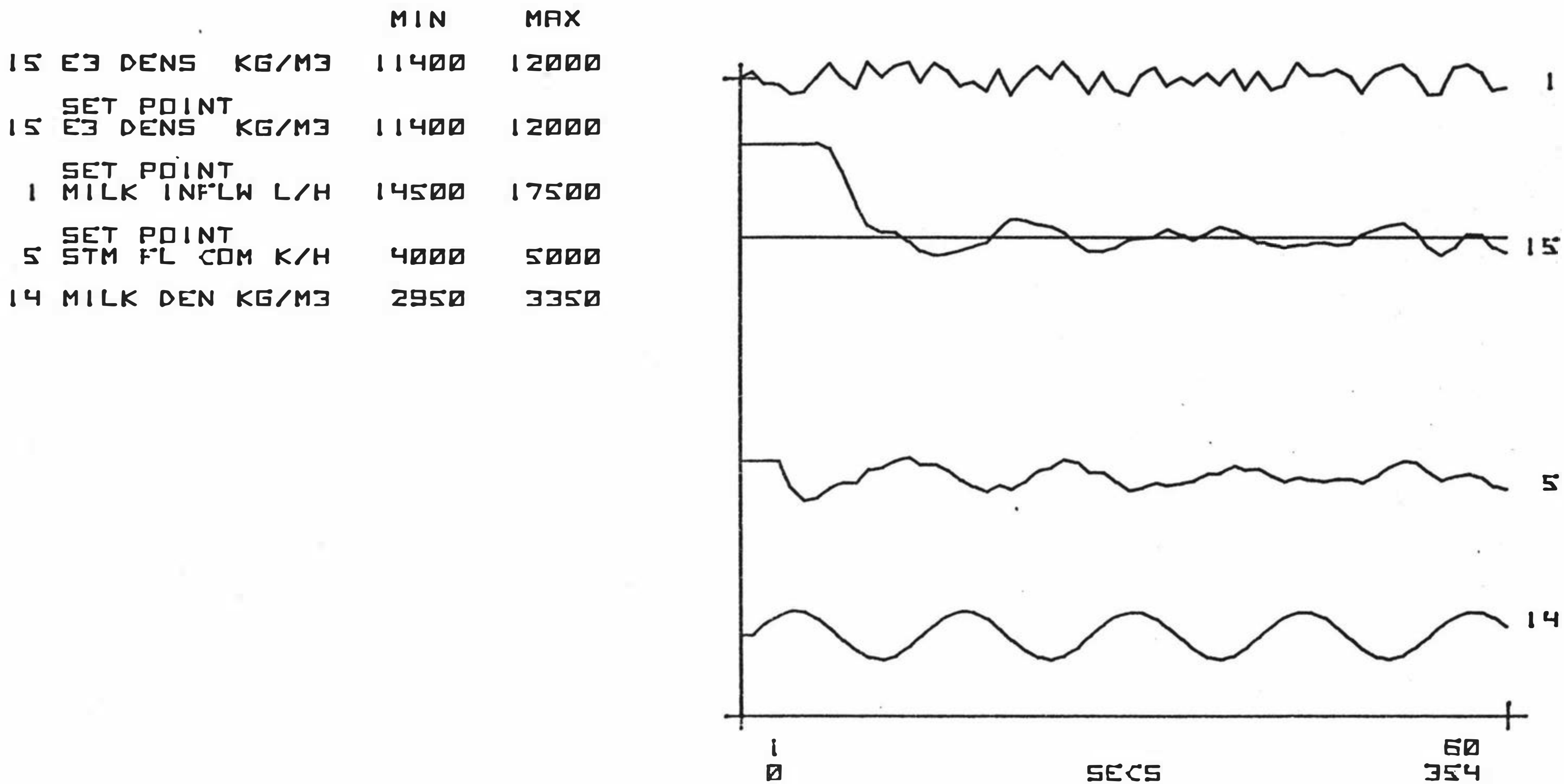


Figure 43c Simulated 3rd effect density control response

TABLE 8

E3 Density Control Gains

c = 60 seconds s_1 = 20 seconds

s_2 = 40 seconds delay = 160-180 seconds

Controller Gains

$$
\begin{aligned}
U_k = \; & -1.25E_k + .77E_{k-s_1} - .11E_{k-s_2} - .08E_{k-c} - .004E_{k-c-s} \\
& - .13E_{k-c-s_2} + 38E_{k-2c} + .20E_{k-2c-s_1} \\
& - .37E_{k-2c-s_2} + .12E_{k-3c} + .09E_{k-3c-s_1} \\
& - .02E_{k-3c-s_2} + .14V_k + .15V_{k-c} + .15V_{k-2c} \\
& + .09V_{k-3c} + .01V_{k-4c} - .02V_{k-5c} - .25W_k \\
& - .42W_{k-c} - .71W_{k-2c} - .90W_{k-3c} - .78W_{k-4c} \\
& - .49W_{k-5c} - .53U_{k-c} - .60U_{k-2c} - .53U_{k-3c} \\
& - .26U_{k-4c} - .04U_{k-5c}
\end{aligned}
$$

where, U_k = change in steam setpoint at time k

V_k = change in inlet flow at time k

W_k = change in inlet density at time k

			MIN	MAX
15	E3 DENS	KG/M3	11400	12000
15	SET POINT E3 DENS	KG/M3	11400	12000
5	SET POINT STM FL COM	K/H	4000	5000
1	SET POINT MILK INFLW	L/H	14000	17500
14	MILK DEN	KG/M3	10350	10750

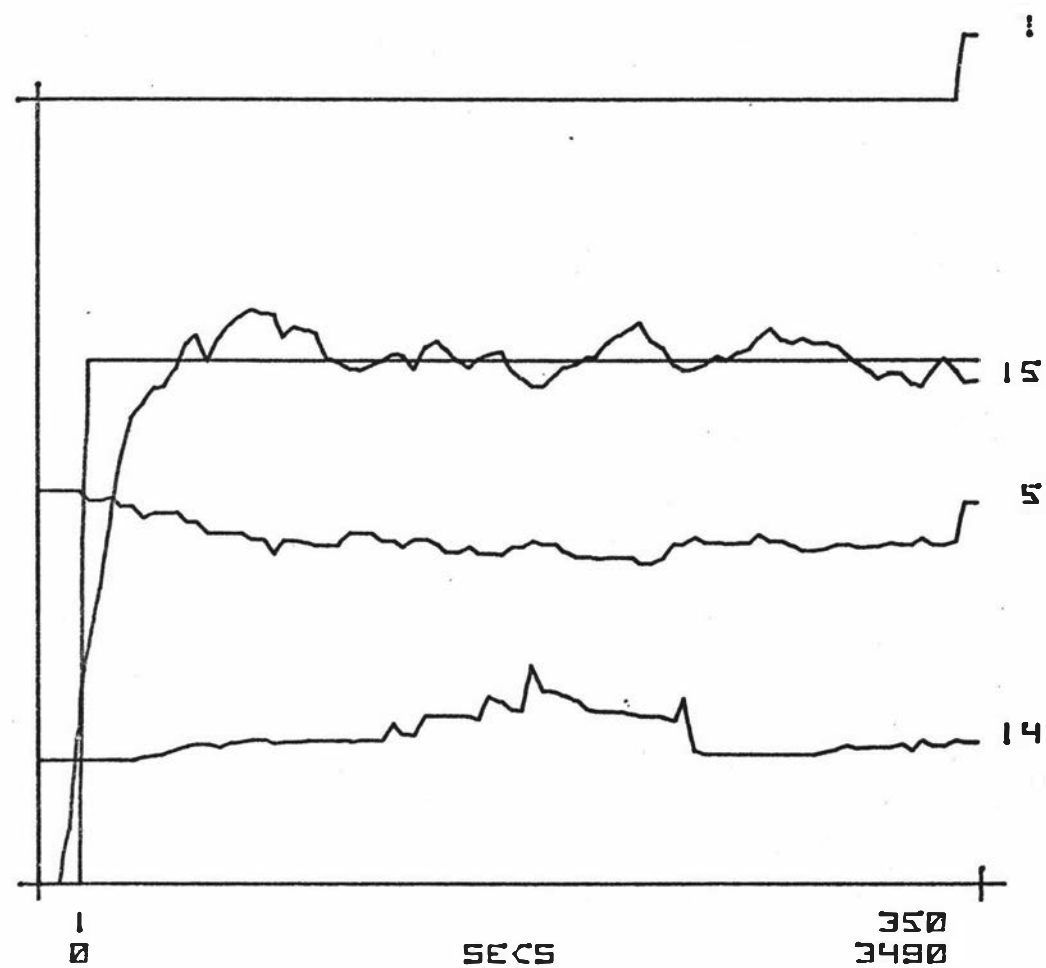


Figure 44a Online 3rd effect density control response

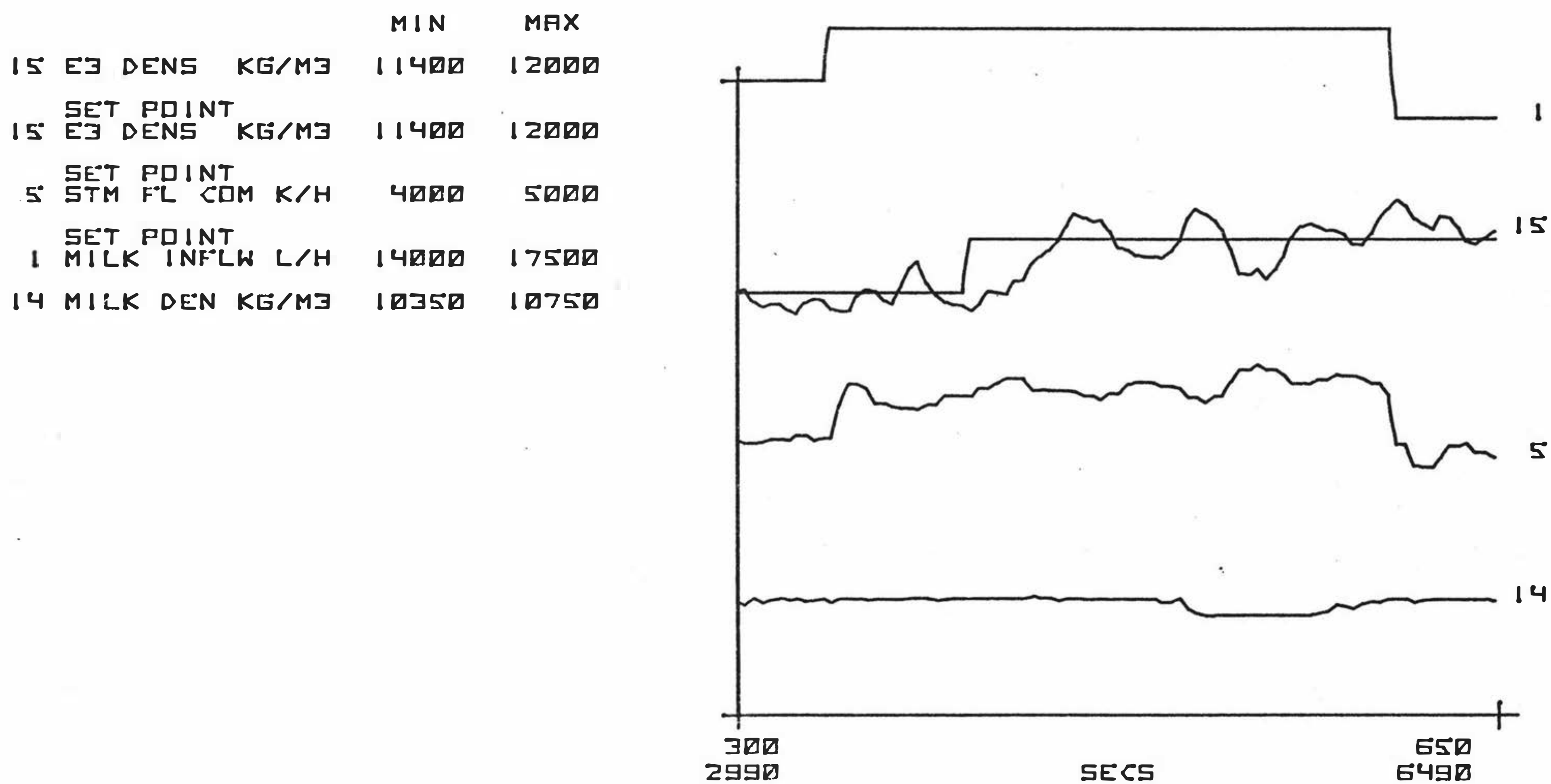


Figure 44b Online control response for 3rd effect density

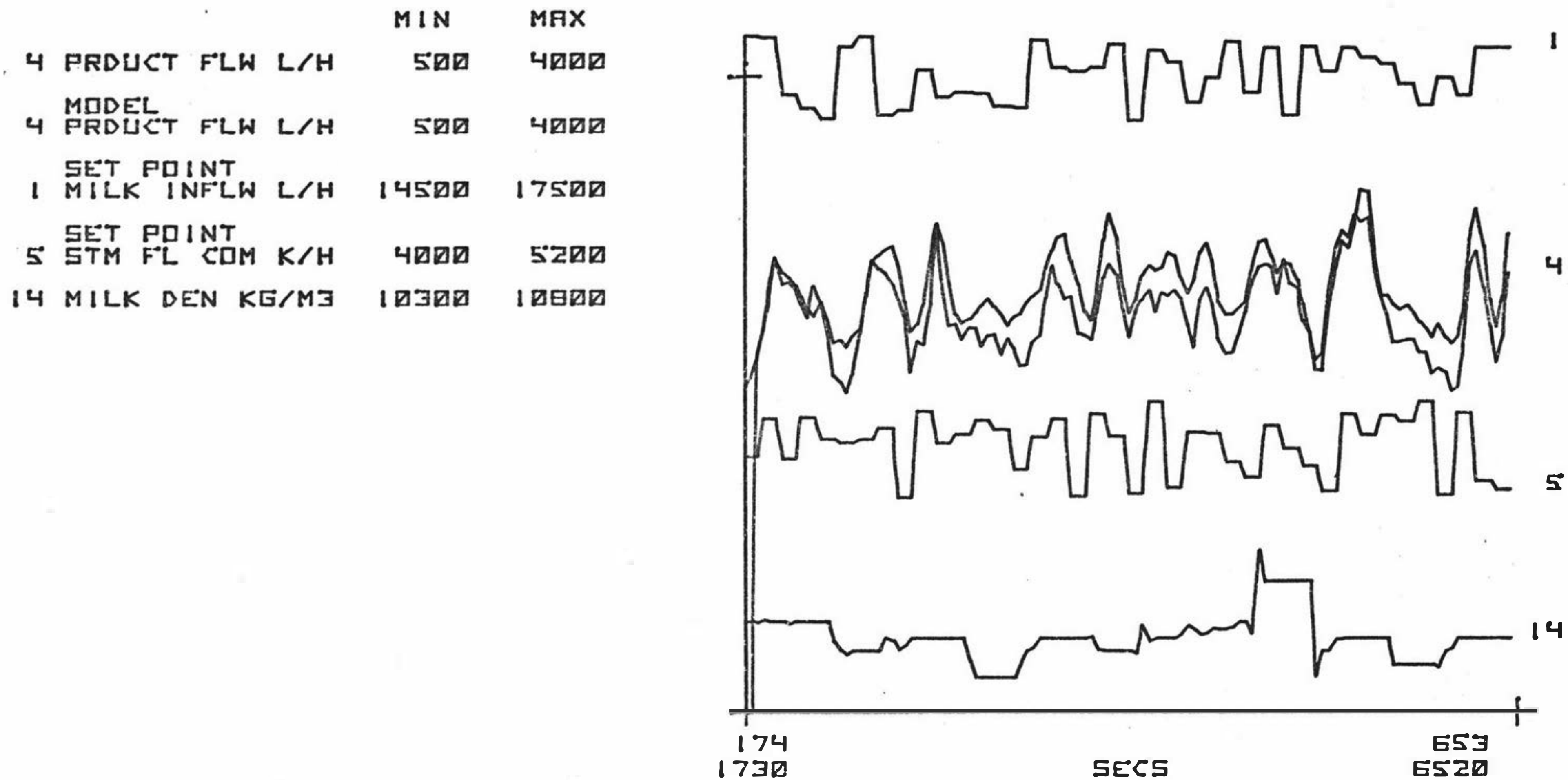


Figure 45 Perturbation data for product flow control

TABLE 9

E3 Product Flow Control Gains

c = 60 seconds $\quad$ s_1 = 20 seconds

s_2 = 40 seconds $\quad$ delay 160-180 seconds

Controller Gains

$$
\begin{aligned}
U_k = {} & -.15E_k - .02E_{k-s_1} + .06E_{k-s_2} - .13E_{k-c} \\
& + .02E_{k-c-s_1} + .04E_{k-c-s_2} - .01E_{k-2c} \\
& - .08E_{k-2c-s_1} + .06E_{k-2c-s_2} + .03E_{k-3c} \\
& - .11E_{k-3c-s_1} + .07E_{k-3c-s_2} + .97V_k \\
& + 1.07V_{k-2c} + .74V_{k-3c} + .31V_{k-4c} + .17V_{k-5c} \\
& + .65W_k + .81W_{k-c} + .91V_{k-2c} + .89V_{k-3c} \\
& + .29V_{k-4c} + .17V_{k-5c} - .16U_{k-c} - .15U_{k-2c} \\
& - .04U_{k-3c} + .01U_{k-4c} + .02U_{k-5c}
\end{aligned}
$$

where U_k = change in inlet flow setpoint at time k

V_k = change in steam flow setpoint at time k

W_k = change in inlet density at time k

Simulated control responses are shown in figures 46 and 47. Figure 46 illustrates the effect on control of a noisy measurement signal, obtained using a random number sequence. Figure 47 shows the effect of steam flow changes on the flow controller. This is important as the density and flow controls will both be working. Operation of the controllers is illustrated in figures 48 and 49. A dead zone of 5 l/hr is used on the flow controller.

4.7 Primary Controls for a Tall Form Spray Drier

Computer control systems for the tall form spray drier have been developed at the primary level only. They relate to the feedrate of concentrate to the atomizing nozzle, the gas flowrate to the burner, the inlet air flowrate and the chamber pressure. The latter two variables are controlled using a multivariable controller.

4.7.1 Drier Feedrate

The structure for this control loop is single input, single output. Flowrate is altered by changing the stroke of the high pressure pump. The rate of flow affects the degree of atomization at the spray nozzle. The pump is initially set to operate with a stroke corresponding to 400 units at the computer output station. Perturbations in the range ± 100 about the initial position are applied at 30 second intervals. Figure 50 shows a schematic diagram of the Drier feedrate system. The flow characteristics obtained from the above perturbations are illustrated in figure 51. The fit of the 1st order model with an update interval of 9 seconds, sample interval 4 seconds is also shown. Evidence of non-linearity in the flowrate may be seen.

		MIN	MAX
4	PRODUCT FLW L/H	500	4000
4	SET POINT PRODUCT FLW L/H	500	4000
1	SET POINT MILK INFLW L/H	15000	18000
5	SET POINT STM FL COM K/H	4000	5200

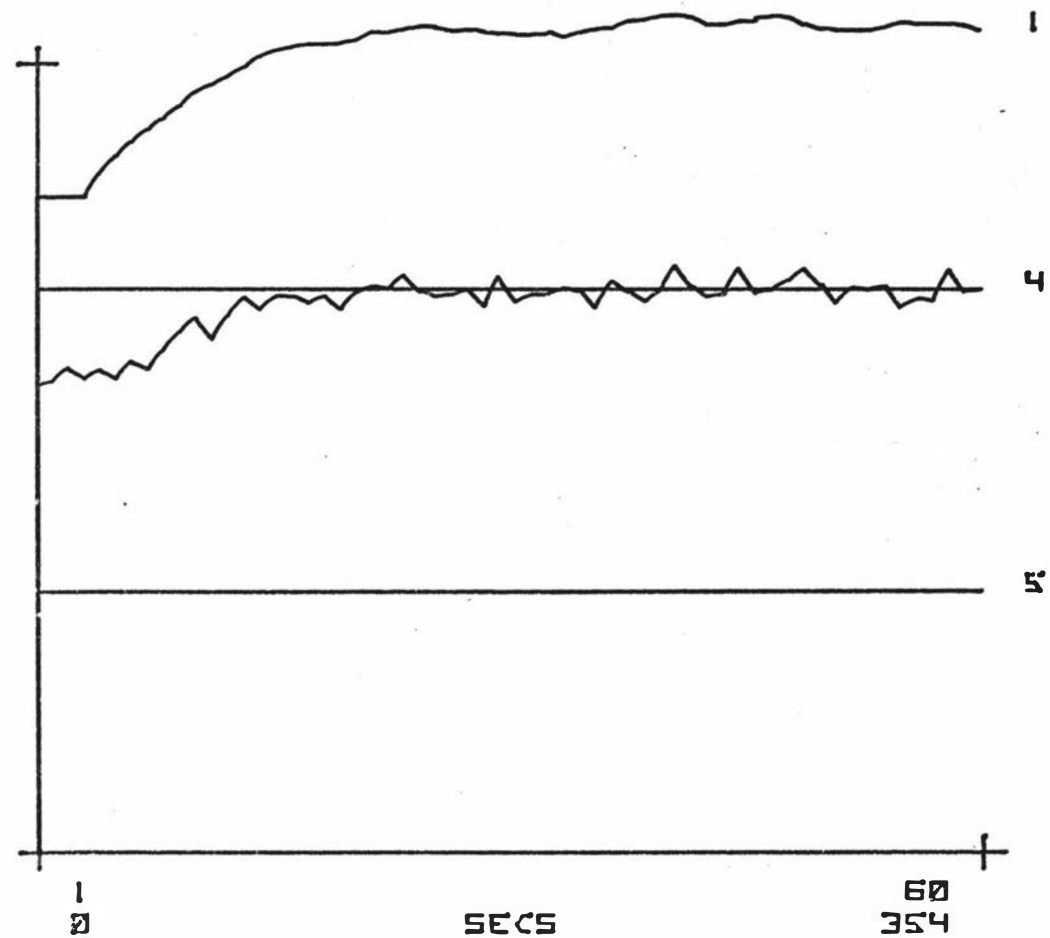


Figure 46   Simulated product flow control response

		MIN	MAX
4	PRODUCT FLW L/H	500	4000
4	SET POINT PRODUCT FLW L/H	500	4000
1	SET POINT MILK INFLW L/H	15000	18000
5	SET POINT STM FL COM K/H	4000	5200

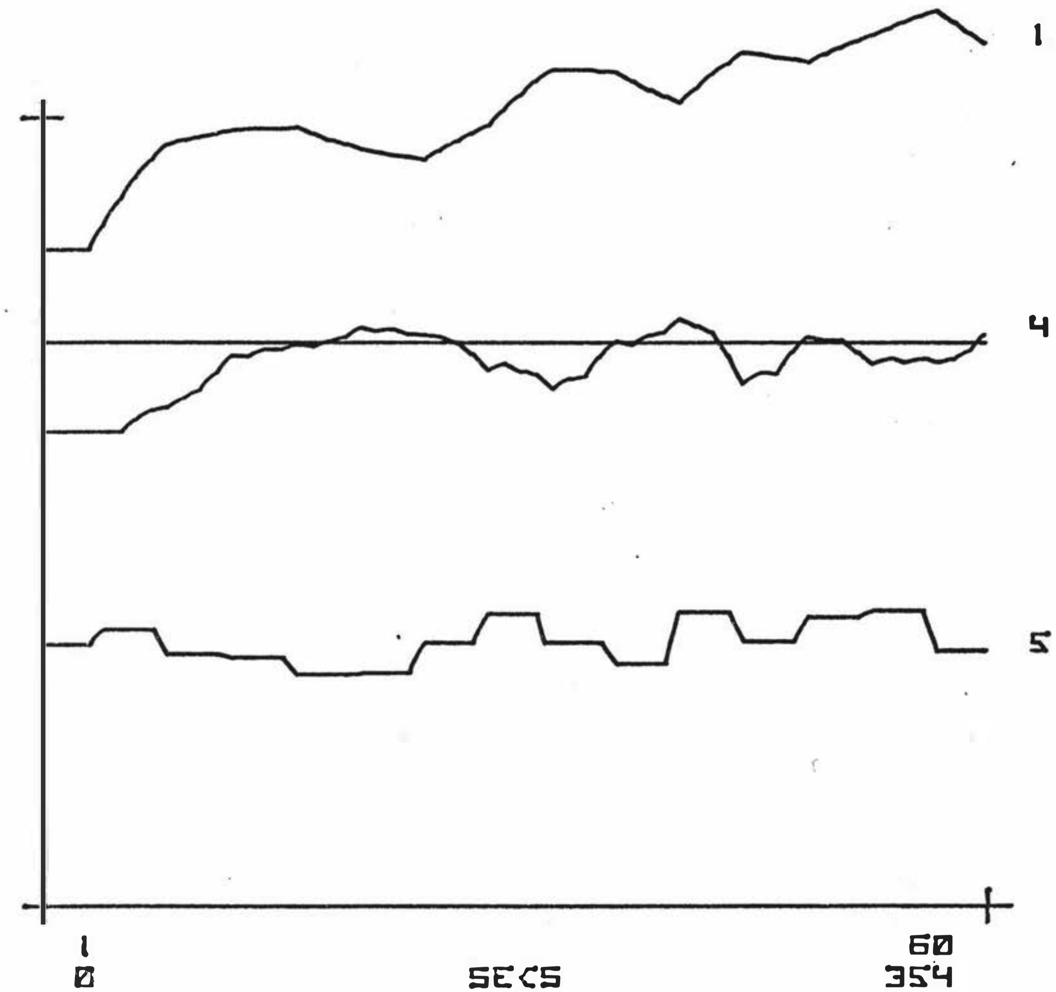


Figure 47 Simulated product flow control response

		MIN	MAX
4	PRDUCT FLW L/H	1850	4850
4	SET POINT PRDUCT FLW L/H	1850	4850
1	SET POINT MILK INFLW L/H	13750	17250
15	E3 DENS KG/M3	11400	12000
15	SET POINT E3 DENS KG/M3	11400	12000
5	SET POINT STM FL COM K/H	4000	5000

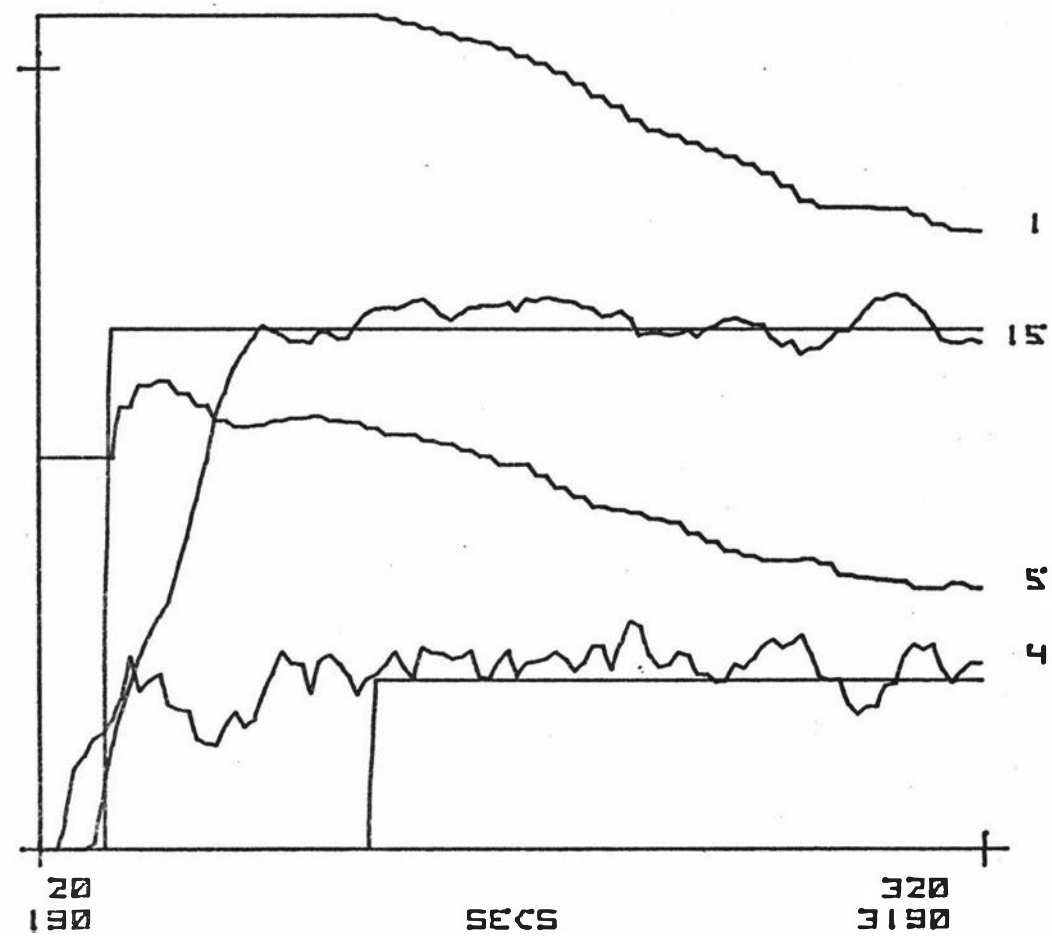


Figure 48 Online control response for product flow

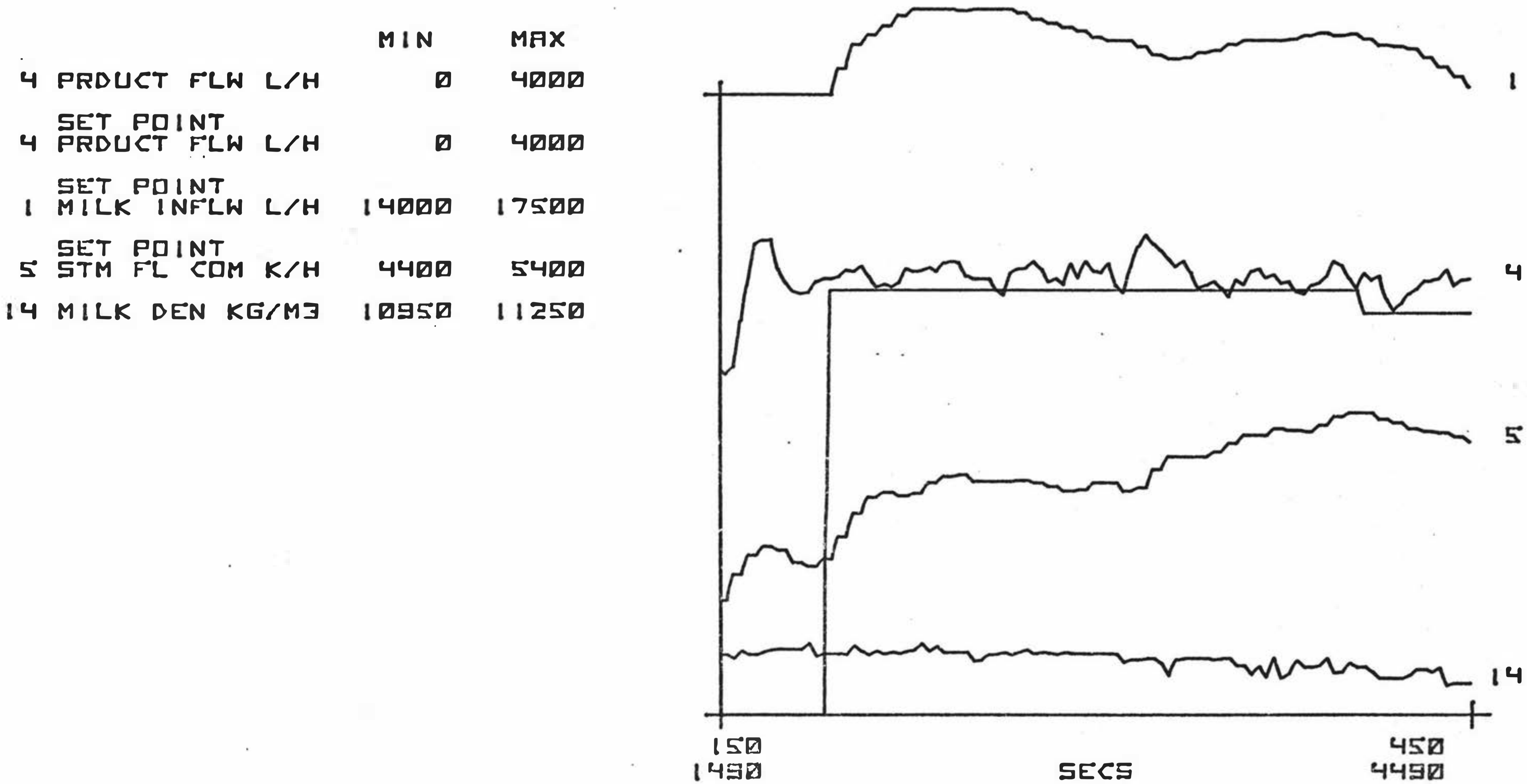


Figure 49 Online control response for product flow

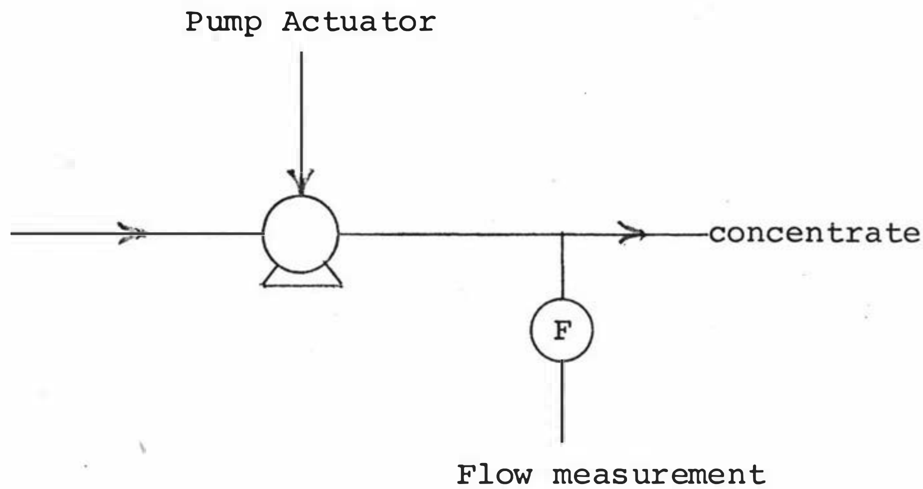


Figure 50 Schematic diagram of drier feedrate system

A minimum time control system is designed. Simulated and online responses are shown in figures 52 and 53 respectively. The slowness in response to the 1st setpoint change is caused by freeplay and friction in the linkage to the stroke adjusting mechanism in the pump. When the larger setpoint change is specified better adjustment of the stroke is obtained giving improved flow control. The parameters associated with the drier feedrate control are summarised in Table 10.

4.7.2 Gas Flow Control

The drying air to the drier is heated directly using a gas fired burner. A schematic diagram is shown in figure 54. A simple controller is operated on the gas flow rate. The valve in the gas line is perturbed in the range 345 $\pm$ 80 at 12 second intervals. Gas flow responses to valve perturbations are quick as may be seen in figure 55. The model fitted is 2nd order with sample interval 1 second and update interval 3 seconds. A reasonable representation of the flow dynamics is obtained.

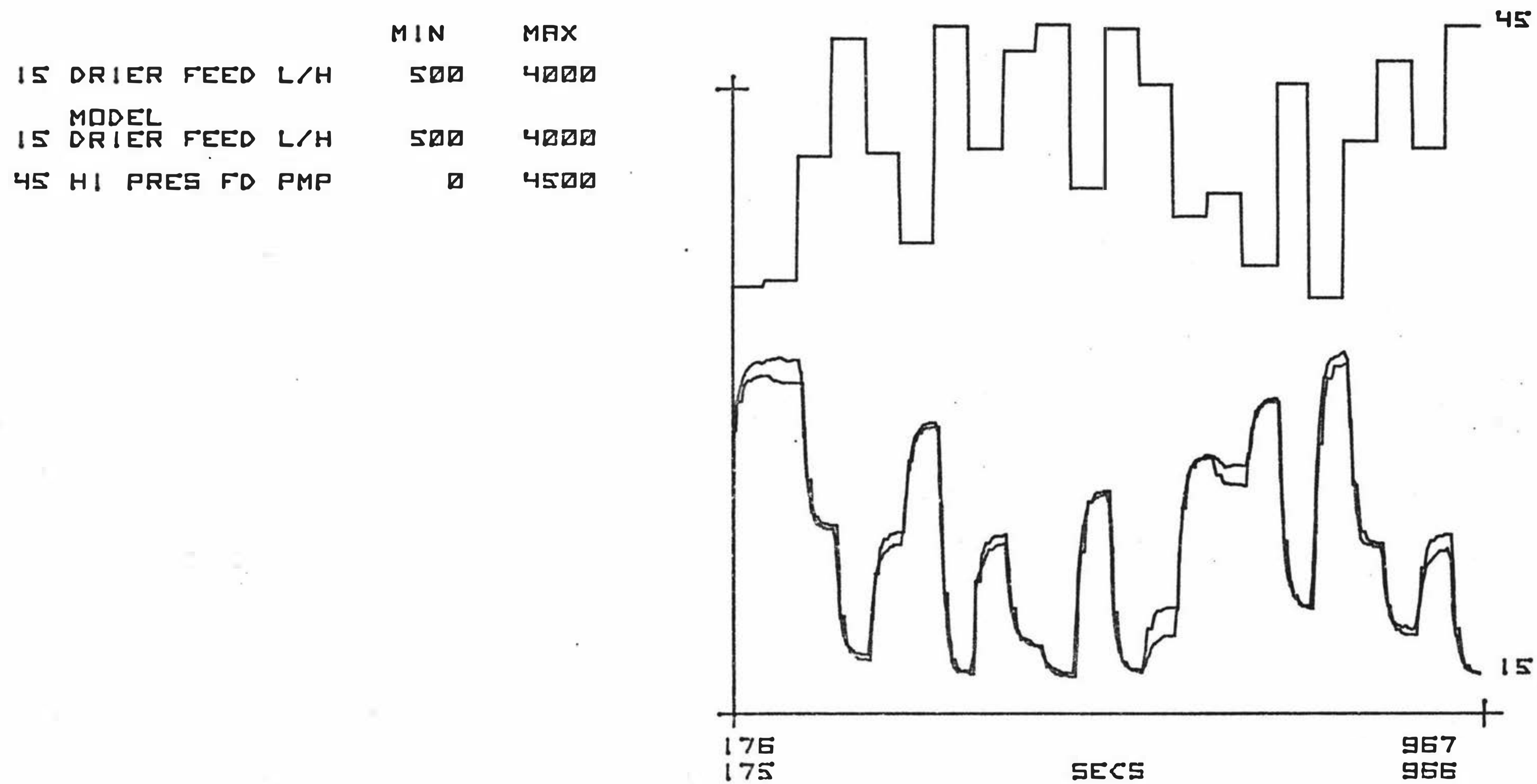


Figure 51 Perturbation sequence for drier feedrate control

	MIN	MAX
15 DRIER FEED L/H	500	4000
SET POINT 15 DRIER FEED L/H	500	4000
45 HI PRES FD PMP	0	4500

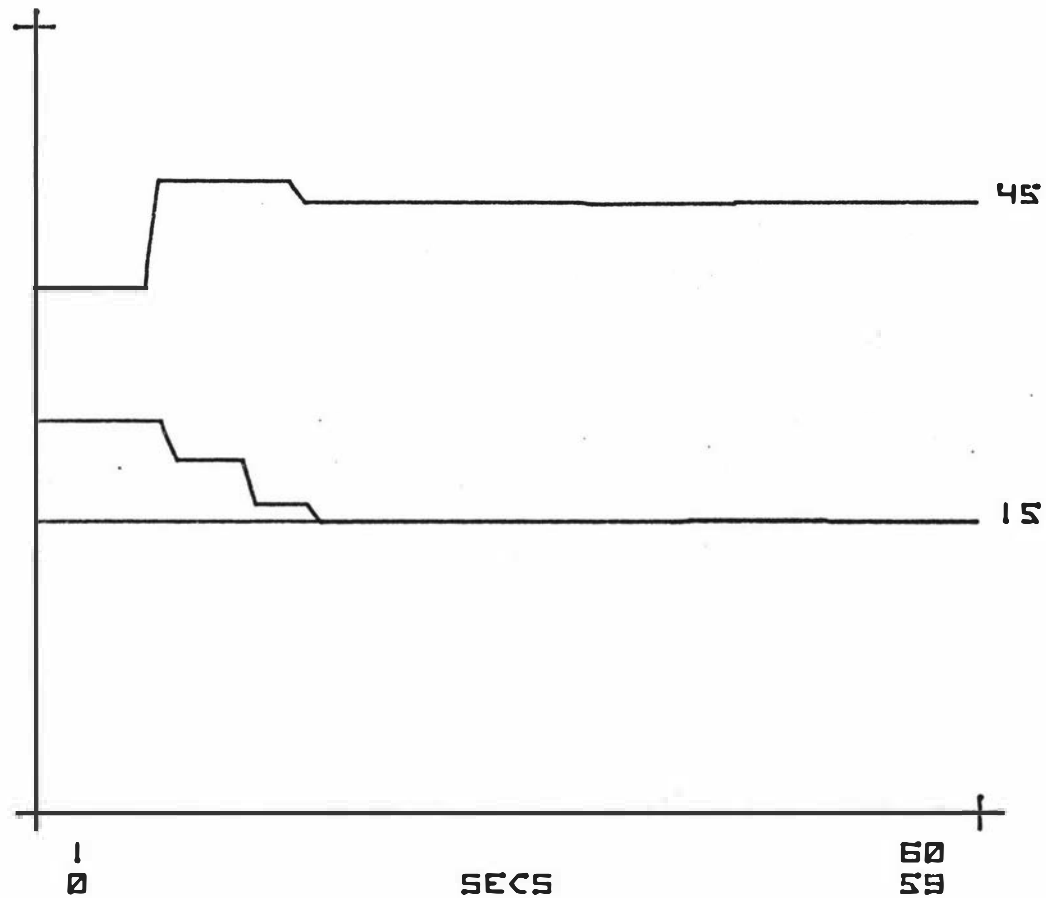


Figure 52   Simulated drier feedrate control response

	MIN	MAX
15 DRIER FEED L/H	1500	3000
SET POINT 15 DRIER FEED L/H	1500	3000
45 HI PRES FD PMP	0	4500

Figure 53 Online drier feedrate control response

Table 10

Drier feedrate control parameters

Model Matrices

$$\underline{\alpha} = \begin{bmatrix} 1.75 & -.75 \\ 1.56 & -.56 \end{bmatrix}$$

$$\underline{\beta} = \begin{bmatrix} -.59 \\ -.28 \end{bmatrix}$$ control interval 9 secs
sample interval 4 secs

Control Design Weights

$$\underline{Q} = \text{diag }(1,1) \quad \underline{P} = 0$$

Control Gains

$$\underline{K} = \begin{bmatrix} 2.54 & -1.15 \end{bmatrix}$$

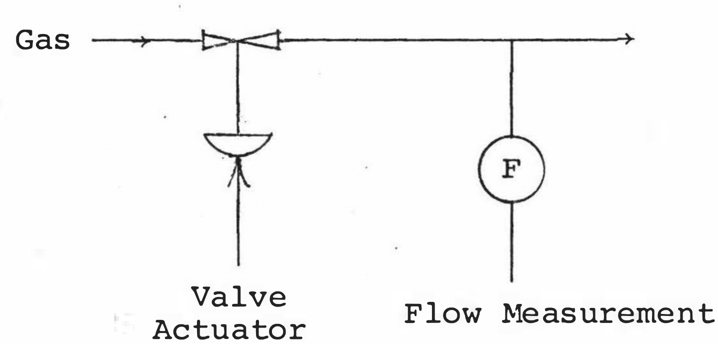


Figure 54 Schematic diagram of Gas Flow System

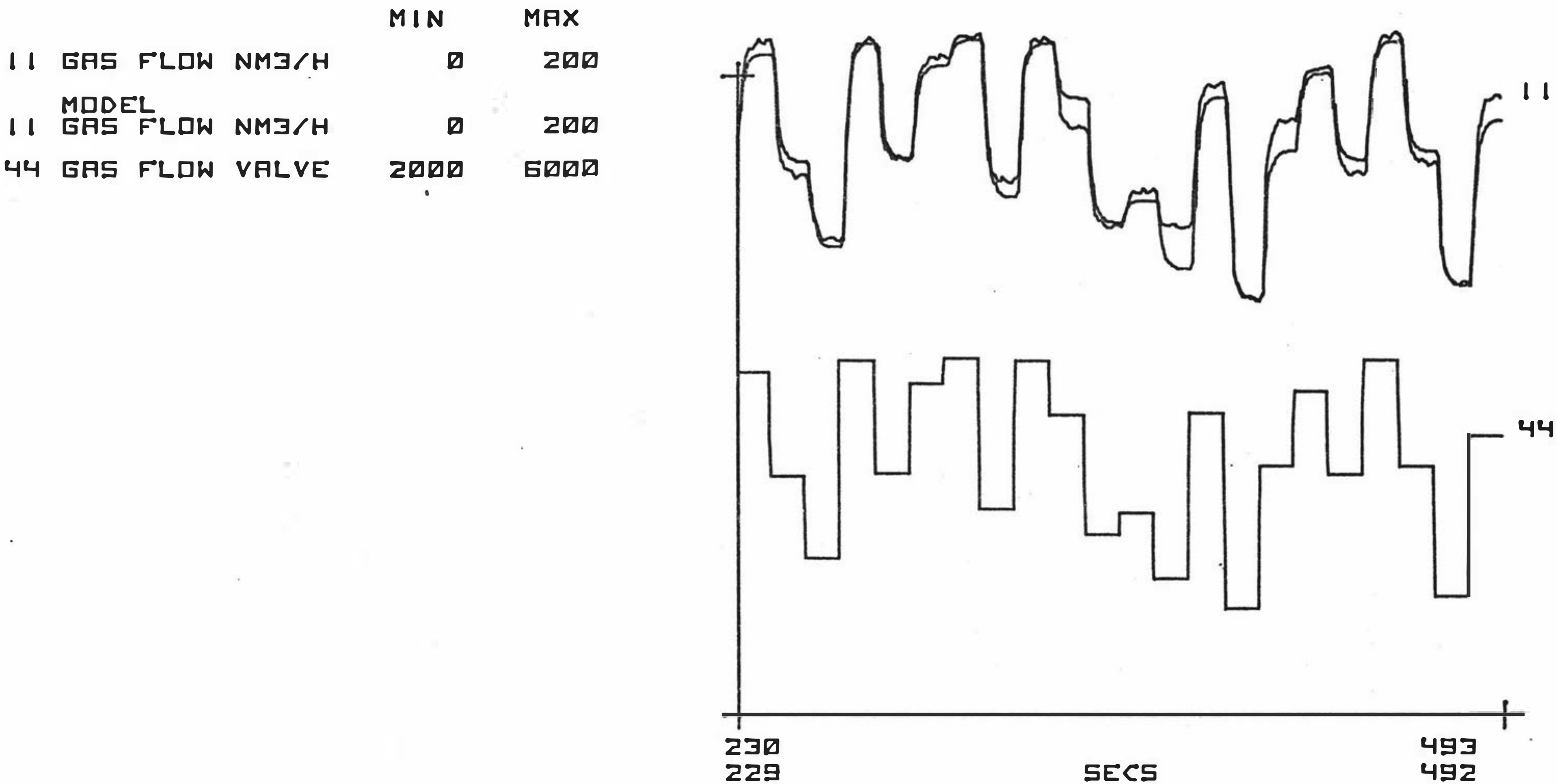


Figure 55 Perturbation data for gas flowrate control

A minimum time controller is designed. The simulated and online responses are illustrated in figures 56 and 57. In spite of the inaccuracies of the model over some parts of the original data set the two characteristics are virtually identical. The flow is quickly brought to setpoint and held there with slight fluctuations in the valve position. Restrictions are placed on the minimum and maximum values of the valve travel to ensure that no-burn or over-burn conditions will not occur. The parameters associated with the gas flow control loop are detailed in Table 11.

Table 11

Gas flow control Loop parameters

Model Matrices

$$\underline{\alpha} = \begin{bmatrix} 1.88 & -.96 & .08 \\ 1.77 & -.87 & .10 \\ 1.61 & -.71 & .11 \end{bmatrix}$$

$$\underline{\beta} = \begin{bmatrix} .036 \\ .019 \\ .002 \end{bmatrix}$$

control interval = 3 secs
sample interval 1 sec

Control design weights

$\underline{Q}$ = diag. $|1,1,1|$ $\underline{P} = 0$ 3 iterations

Control Gain Matrix

$$\underline{K} = [-42.1 \quad 21.9 \quad -1.5]$$

4.7.3 Chamber Pressure and Inlet Airflow Control

Figure 58 is a schematic diagram of the inlet airflow, chamber pressure system. Two fans, one in the inlet duct and one in the exhaust duct, force air through the spray drier. Damper vanes in the ducts are used to control

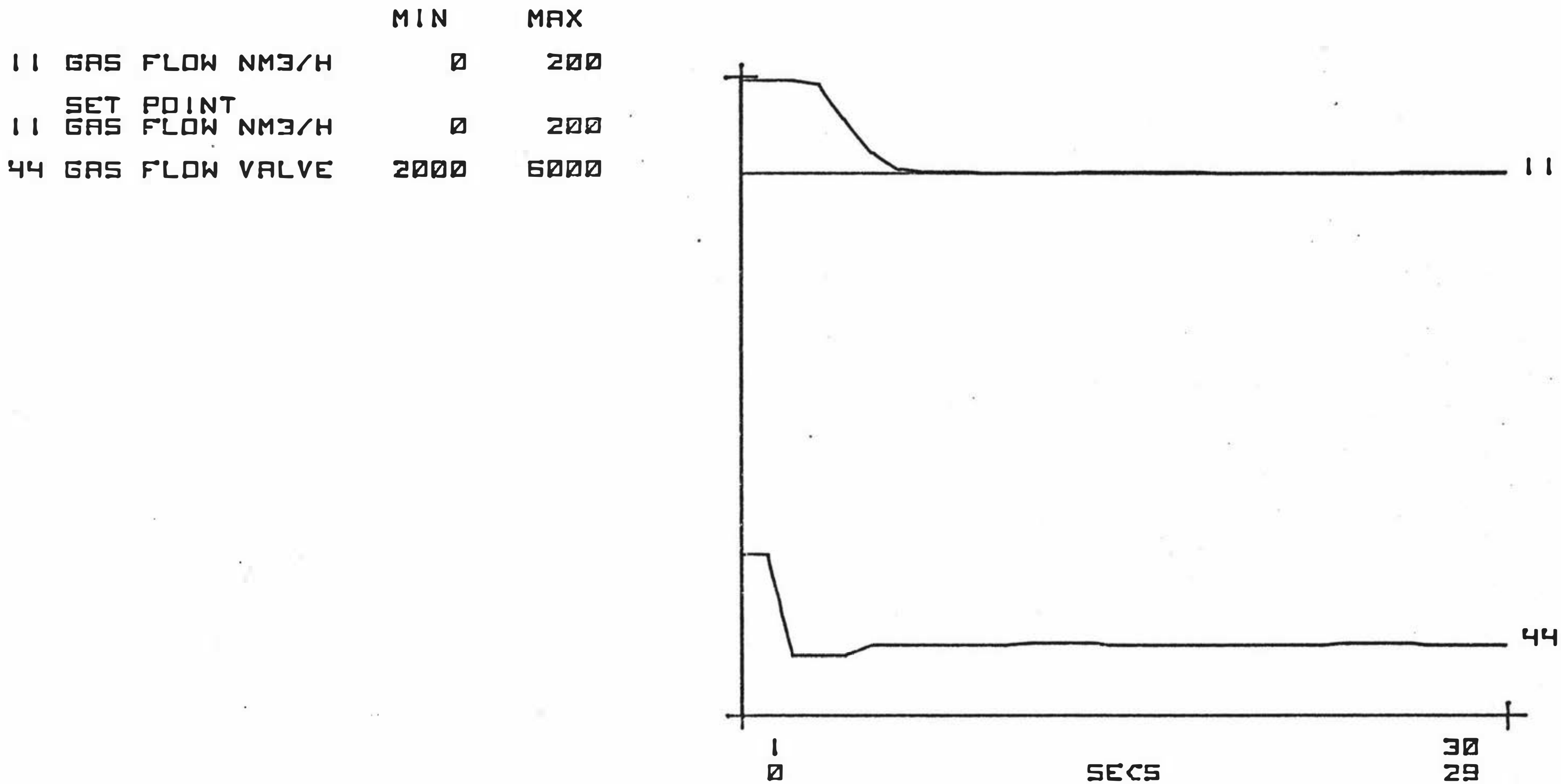


Figure 56 Simulated gas flowrate control response

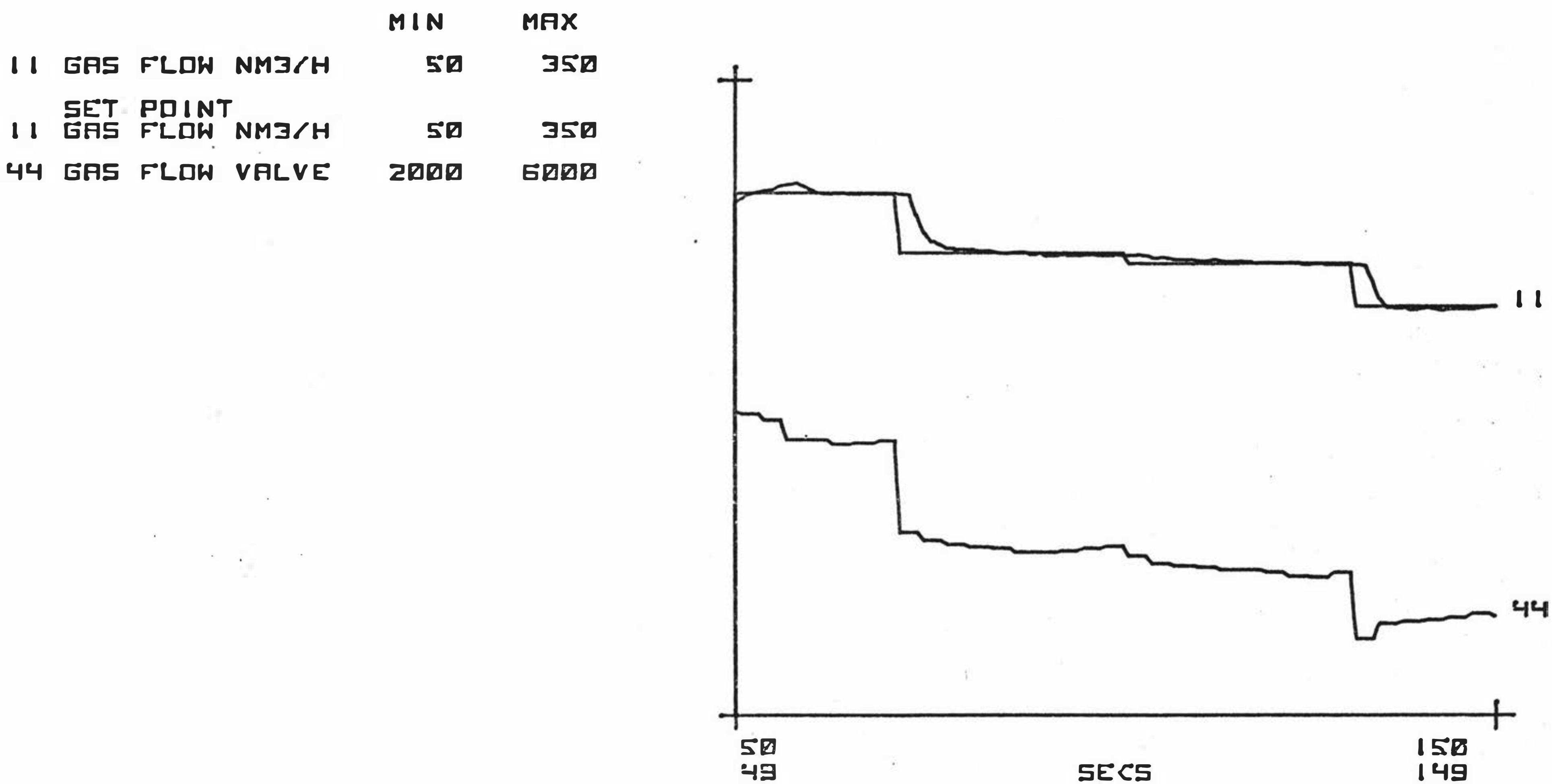


Figure 57 Online gas flowrate control response

the airflow and pressure in the drier. The pressure transducer gives a reading of 'bar gauge'. Operation of the direr may require a negative gauge pressure setpoint. As the real time system cannot operate with negative set-points (they reference a different control function) the transducer reading is modified by the computer software so that positive readings only are obtained. The unit used is defined as 'pseudo-millibar-absolute' (PMBA) with a reading 1000 PMBA equivalent to 0 bar gauge.

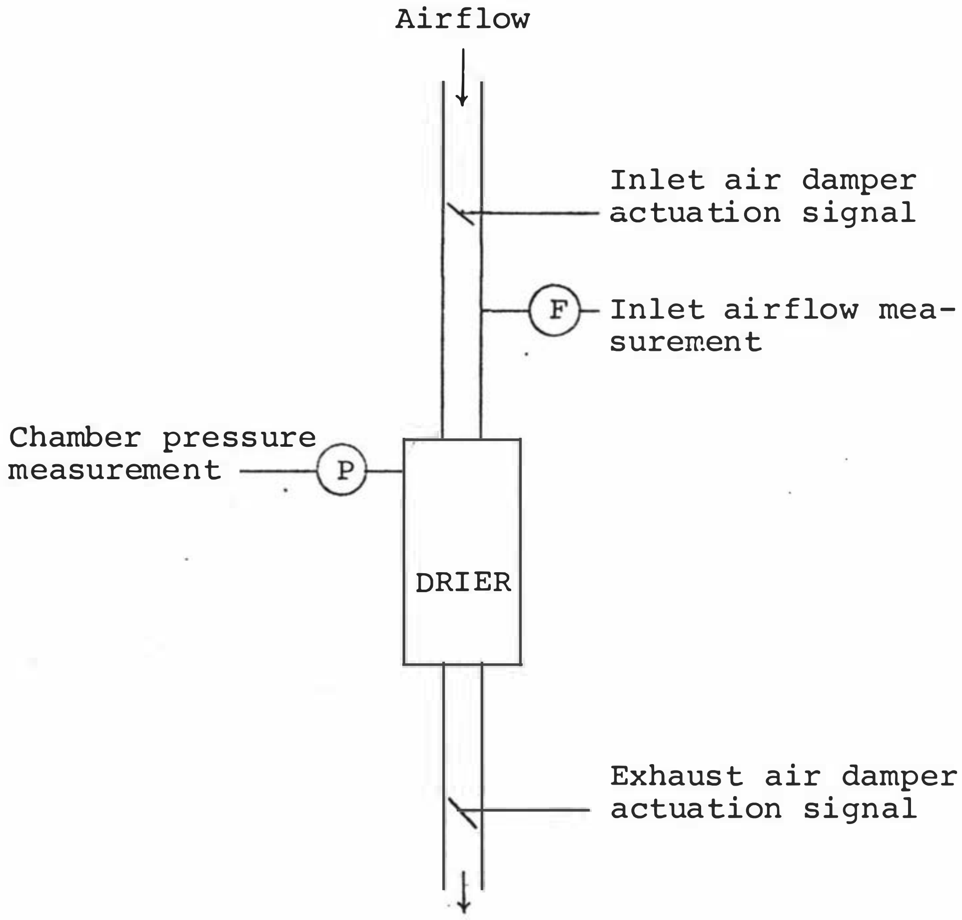


Figure 58 Schematic diagram of chamber pressure, Inlet airflow system

Figure 59 illustrates the perturbation data used to derive the multivariable model. The inlet damper position is varied in the range 160 $\pm$ 60 while the exhaust damper is varied in the range 420 $\pm$ 50. The perturbation intervals was 48 seconds. The model obtained is 2nd order, 2 input, 2 output. Update interval is 8 seconds with a 4 second sample interval. The model matrices are detailed in Table 12.

The simulated control response is shown in figure 60. Control design weights are $\underline{Q}$ = diag. (1,1,1,1) $\underline{P}$ = diag. (.005, .005). The weighting in the P matrix gives a slight reduction in the speed of the control response. The online control response is illustrated in figure 61. Variations appear large but note should be taken of the expanded graph scales. Airflow variations are of the order of .5 Kg/min and pressure variations .5 mbar. Interaction is observed when a setpoint change is specified but the disturbance is quickly damped out.

		MIN	MAX
13	INL AIR KG/MIN	800	1100
13	MODEL INL AIR KG/MIN	800	1100
18	CH PRESS PMBA	9800	10100
18	MODEL CH PRESS PMBA	9800	10100
42	INLET AIR DMPR	1000	5000
43	EXHST AIR DMPR	2500	6500

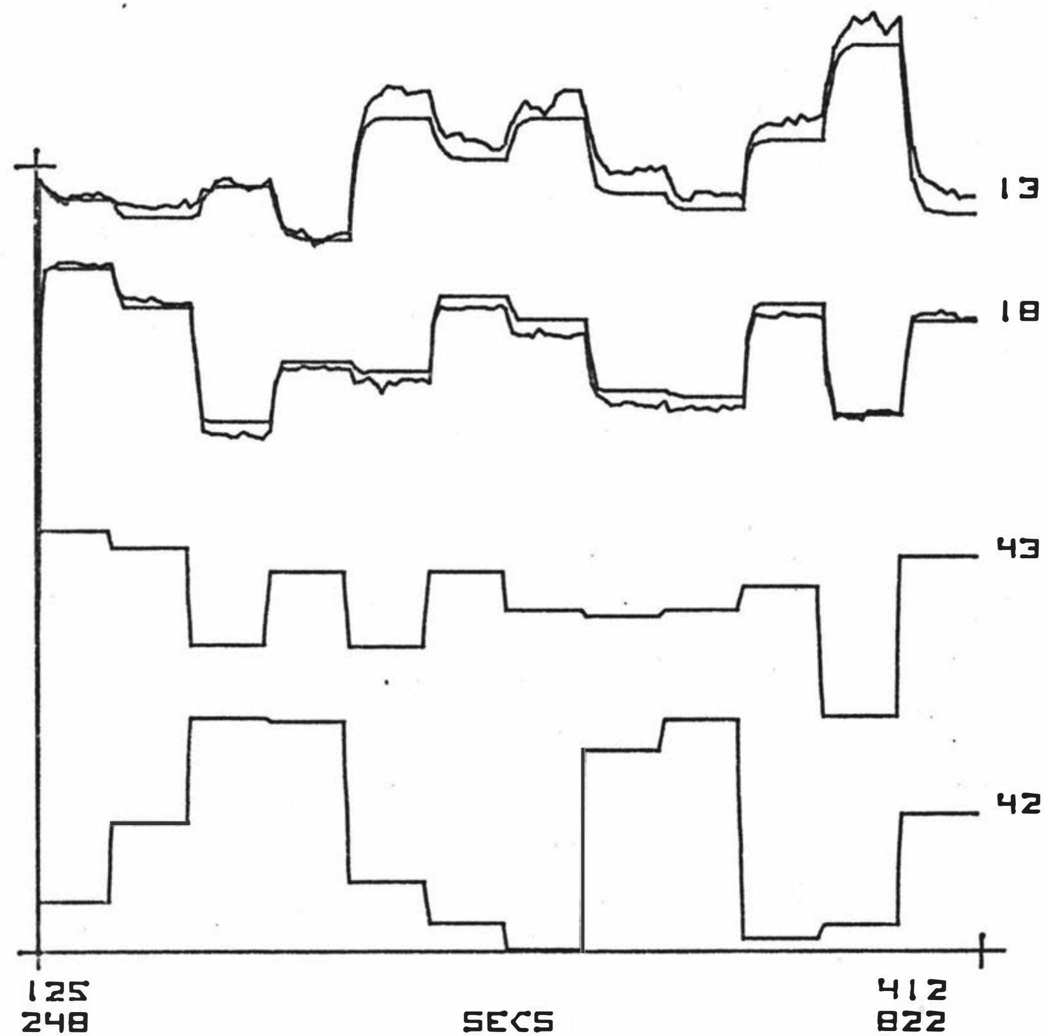


Figure 59 Perturbation data for inlet airflow, chamber pressure control system

TABLE 12

Inlet airflow, chamber pressure loop parameters

Model Matrices

$$\underline{\alpha} = \begin{bmatrix} 1.42 & -.12 & -.42 & .12 \\ .02 & 1.04 & -.02 & -.04 \\ 1.29 & -.07 & .29 & .07 \\ .03 & 1.04 & -.03 & -.04 \end{bmatrix}$$

$$\underline{\beta} = \begin{bmatrix} -.027 & -.045 \\ -.024 & .061 \\ -.018 & -.022 \\ -.015 & .038 \end{bmatrix}$$

control interval 8 secs
sample interval 4 secs

$$\text{output vector, } Y_k = \begin{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_k \end{bmatrix}$$

$$\text{input vector} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

where x_1 = inlet air flow
x_2 = chamber pressure
u_1 = inlet damper position
u_2 = exhaust damper position

Control Design Weights

$\underline{Q}$ = diag. (1,1,1,1) P = diag. (.005, .005)

Feedback Gain Matrix

$$\underline{K} = \begin{bmatrix} 15.69 & 8.17 & -4.91 & .93 \\ 9.07 & -9.00 & -.253 & 1.11 \end{bmatrix}$$

		MIN	MAX
13	INL AIR KG/MIN	1000	1300
13	SET POINT INL AIR KG/MIN	1000	1300
18	CH PRESS PMBA	9900	10200
18	SET POINT CH PRESS PMBA	9900	10200
42	INLET AIR DMPR	0	4000
43	EXHST AIR DMPR	2500	6500

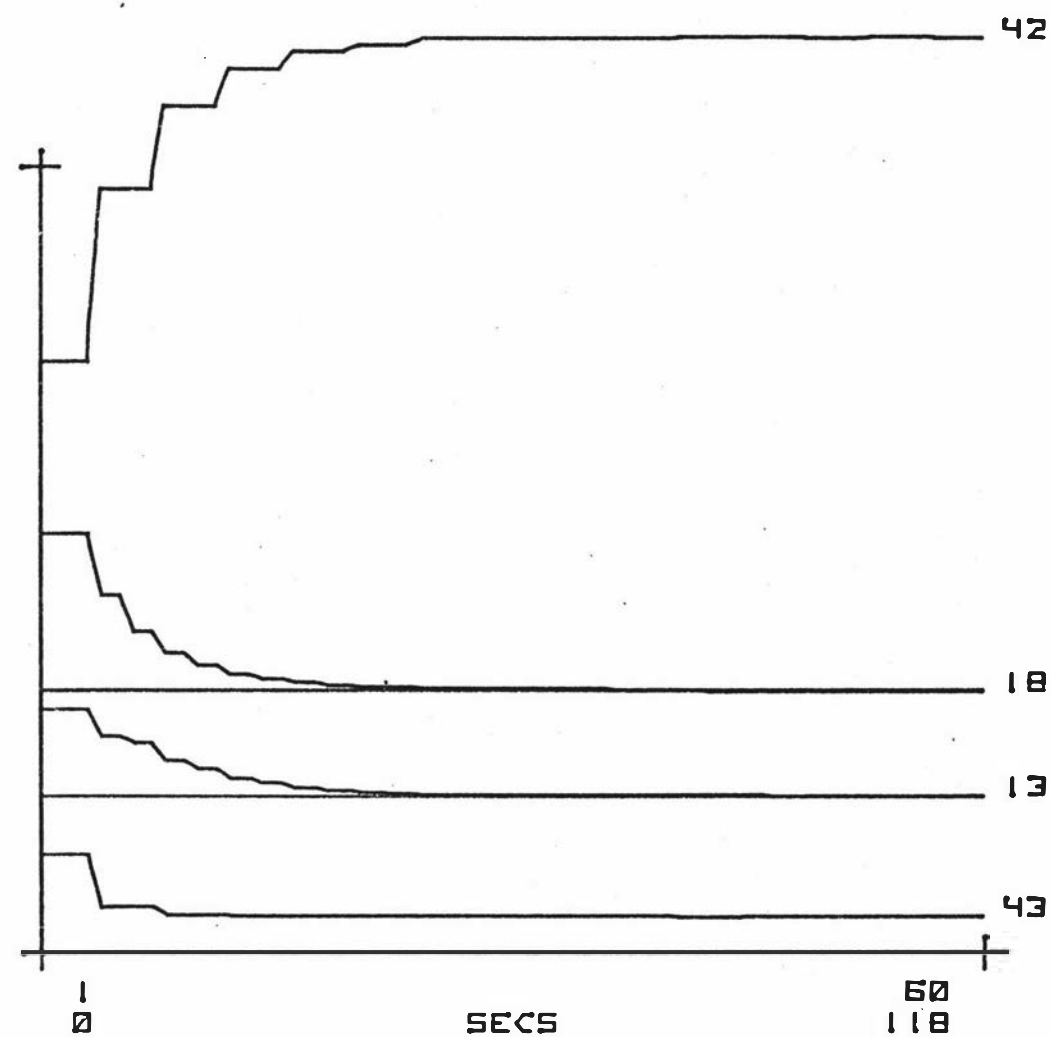


Figure 60 Simulated inlet airflow, chamber pressure control responses

		MIN	MAX
13	INL AIR KG/MIN	870	970
18	CH PRESS PMBA	9870	10070
13	SET POINT INL AIR KG/MIN	870	970
18	SET POINT CH PRESS PMBA	9870	10070
42	INLET AIR DMPR	0	4000
43	EXHST AIR DMPR	2600	6600

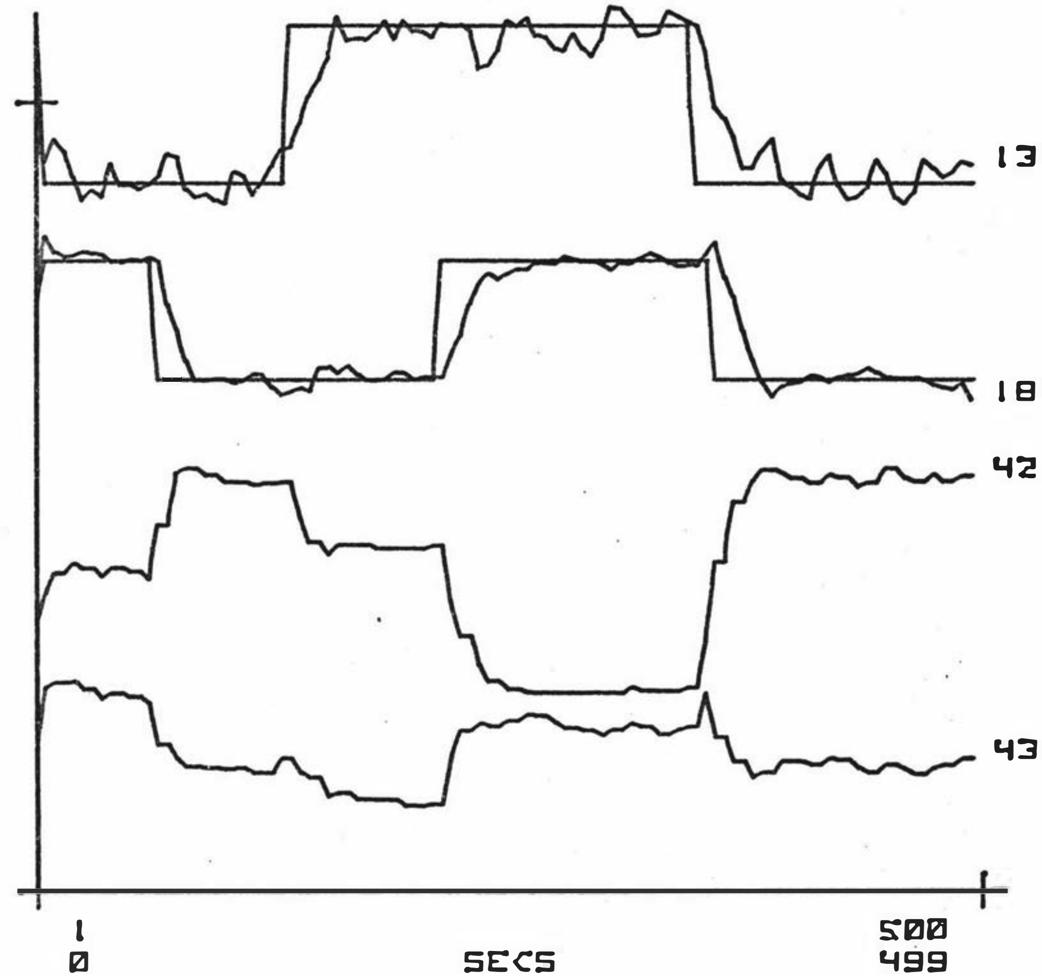


Figure 61 Online inlet airflow and chamber pressure control responses

CHAPTER FIVE

CONCLUSIONS

This thesis has described the application of modern state space control techniques to an industrial plant. A generalised equation structure has been developed to represent the process, dynamics of such a plant. The structure is based on the standard state space representation but offers significant advantages. In particular, only measurable inputs and outputs are required. Further, the discrete measurements obtained are used in difference rather than absolute terms, thus providing a degree of resilience to drift in the measured values. Precise knowledge of the process dynamics is not required and the computer programs written enable the control and sample intervals to be quickly adjusted to their most appropriate values.

The techniques for the design of a control system is the same regardless of the order and/or complexity of the system to be controlled. A perturbation run is performed first. The input variable is randomly purturbed within its normal operating range. The information collected is analysed offline and models obtained using least squares identification. Once a satisfactory model has been obtained, a controller may be designed and tested, first in simulation and then on the actual plant. Parameters may be quickly modified to give the designed control response characteristics.

Application of these techniques to the manufacture of spray dried milk powder has proved successful. At the lower level of the hierarchical structure, implementation of the controllers has allowed good environmental control of both the Wiegand evaporator and the De Laval spray drier. At the upper end of the structure consistent control of the flowrate and density of the concentrate leaving the evaporator has been achieved. This in turn has enabled the operation of the spray drier to be more closely matched to that of the evaporator. Consistent production of powder to a desired specification, therefore, results. Probably the best measure of the success of the implementation is the manner in which the staff of the Dairy Research Institute are utilising the control systems in their day to day activities.

The results achieved do not however preclude further research on the implementation of control to the process. As with many other processes involving chemical changes, the plant characteristics alter depending upon the properties of the raw material and the desired product. For example, fouling of the evaporator tubes occurs as milk burns on them during the process of normal operation. As well, the controllers described have been used for the production of skim milk powder. When used on whole milk powder adjustment of the parameters is required to give satisfactory results. This leads to the possible requirement of an adaptive control mechanism using online process estimation. Coupled in with such a system would be the effect non-linearity. The systems described have been designed relative to an environment which was regarded as 'typical' for this type of processing in the New Zealand Dairy industry.

Further work is also possible in the area of the operator - computer interface. The design procedure could be enhanced to aid the control engineer to achieve the best from the system. While work of this nature is essential to bring the interactive control system design methods to the level of integrity demanded by an industrial user, it is felt that the work described demonstrates that there is a role for new technology to play in the control of induatrial equipment and processing and that alternative control design techniques should be considered.

APPENDIX 1

Generalised least squares procedure

For an equation of the form

$$\underline{Y} = \underline{\Omega}\underline{X}$$

the least squares procedure is

$$\underline{\Omega}_{k+1} = \underline{\Omega}_k + \left[\underline{Y}_{k+1} - \underline{\Omega}\underline{X}_k\right] \frac{\underline{X}_k \cdot \underline{P}_{k-1}}{d_k}$$

$$d_k = 1 + \underline{X}_k^T \cdot \underline{P}_{k-1} \cdot \underline{X}_k$$

$$\underline{P}_k = \underline{P}_{k-1} - \frac{\underline{P}_{k-1} \cdot \underline{X}_k \, \underline{X}_k^T \, \underline{P}_{k-1}}{d_k}$$

Data collected is processed sequentially and Ω converges as k increases.

Initially for $k = 0$

$$\underline{\Omega}_o = 0$$

$$\underline{P}_{-1} = h.\underline{I} \qquad h \text{ is scalar} = .001$$

APPENDIX II

Bibliography

1. K. OGATA, 'Modern Control Engineering', Prentice Hall, 1970.

2. F. J. HALE, 'Introduction to Control System Analysis and Design', Prneitce Hall, 1973.

3. M. HEALEY, 'Principles of Automatic Control', English Universities Press, 1970.

4. DISTEFANO III, STUBBERUD AND WILLIAMS, 'Feedback and Control Systems', Schaum, McGraw-Hill, 1967.

5. J. T. TOU, 'Modern Control Theory', McGraw-Hill, 1964.

6. O.I.ELGERD, 'Control Systems Theory', McGraw-Hill, 1967.

7. K. OGATA, 'State Space Analysis of Control Systems', Prentice Hall, 1967.

8. B.C. KUO, 'Discrete Data Control Systems', Prentice Hall, 1970.

9. H. A. SPANG III, 'The Structure and Comparison of Three Real-Time Operating Systems for Process Contro', Automatica, Vol.8, 1978, pp. 49-64.

10. J. GERTLER and J. SEDLAK, 'Software for Process Control - A survey', Automatica, Vol.11,

1975, pp.613-625.

11. YEN-PING SHEN and CHIH-JIAN CHEN, 'On the weighting factors of the quadratic criterion in optimal control', International Jrnl of Control, 1974, Vol.19, pp.947-955.

12. M.S.BECK, 'Control; its present status and future prospects', Control & Instrumentation, May 1969, Vol.1, No.1, pp.34-37.

13. I. GUSTAVSSON, 'Comparison of Different Methods for Identification of Industrial Processes', Automatica, Vol.8, 1972, pp.127-142.

14. I. GUSTAVSSON, 'Survey of Applications of Identification in Chemical and Physical Processes', Automatica, Vol.11, 1975, pp.3-24.

15. K.J.ASTROM and P. EYKHOFF, 'System Identification - A Survey', Automatica, Vol.7, 1971, pp. 123-162.

16. J.R.HASTINGS and M.W. SAGE, 'Recursive generalised least squares procedure for online identification of process parameters,' Proceedings I.E.E., 1969, Vol.116, No.12, pp. 2057-2062.

17. T.C. HSIA, 'On least squares algorithms for system parameters identification', IEEE Transactions on Automatic Control, Vol.21, Feb. 1976, pp.104-108.

18. D.J.SANDOZ and B.H. SWANICK, 'A recursive least squares approach to the online adaptive control problem', Intern.Jrnl of Control, Vol.16, No.2, 1972, pp.243-260.

19. P. STOICA and T. SODERSTROM, 'A method for the identification of linear systems using the generalised least squares principle', IEEE Transactions on Automatic Control, Vol.AC-22, No.4, 1977, pp.631-634.

20. P.A.WALKER and M.D. DUNNE, 'Direct formation of a time optimal controller by least squares identification', IEEE Transactions on Automatic Control, Vol.AC-22, No.4, 1977, pp. 669-670.

21. W.A. McGILLIVRAY, 'The New Zealand Darying Scene 1970-74'. N.Z.Jrnl of Dairy Science and Technology, Vol.9, 1974, pp.58-105.

22. A.J.BALDWIN, 'The Assessment of the Benefits of Process Control', NZDRI Internal Publication.

23. D.J. SANDOZ, 'An Application of Computer-aided Control System Design', Proceedings of the IEE, Vol.123, No.11, 1976, pp.1263-1268.

24. J. WIEGAND, 'Falling Film Evaporators and Their Applications in the Food Industry', Jrnl of Applied Chemistry and Biotechnology, Vol.21, Dec. 1971, pp.351-358.

25. B.K. MARLOW, 'Instrumentation of the Milk Powder Plant at the New Zealand Dairy Research Institute', Physics and Engineering Lab., D.S.I.R., Manual 151, Feb. 1978.

26. D.J.SANDOZ, 'Real-time Operating System for Computer Control of a Milk Powder Plant', Internal report, Industrial Management and Engineering Dept., Massey University, November 1977.

27. O. WONG, 'Offline Operating System for Computer Control of a Milk Powder Plant', Internal report, Industrial Management and Engineering Dept., Massey University, Feb, 1978.

28. 'IBM System and Functional Characteristics Manual', GA34-0003-6.

29. 'IBM System and System Summary Manual', GA34-0002-4.

30. R. AL-THIGA and N.E.GOUCH, 'Artificial Intelligence Applied to the Discrimination of the Order of Multivariable Linear Systems', Int.Jrnl of Control, Vol.20, No.6, 1974, pp. 961-969.

31. D.J. BATEY, et al., 'The Design and Implementation of an Interactive Data Analysis Package for a Process Computer', Computer Aided Design, Vol.7, No.4, 1975, pp.265-269.

32. W.D.T. DAVIES, 'Control Algorithms for DDC', Instrument Practice, Vol.21, No.1, 1967, pp 70-77.

33. I.W.LEONHARD, 'Cascade Control', Control and Instrumentation, Vol.2, No.7, July 1970, pp. 28-31.

34. R.T. PULLMAN, 'A Computer Model of a 3 Effect Falling Film Milk Evaporator, DSIR Report 466,Feb.1975.

35. D.J.SANDOZ, 'An Application of Computer Aided Control System Design', Proceedings of the IEE, Vol.123, No.11, Nov. 1976, pp.1263-1268.

36. D.J. WOODHAMS and M.J. MURRAY, 'Properties of Spray Dried Milk Powders', N.Z.Dairy Research

Institute, Publication No. 650.

37. D.J.SANDOZ, O. WONG and C.G.BLOORE, 'Computer Control of the Manufacture of Spray Dried Milk Powder', Proceedings of the 4th International Congress on Chemical Engineering, Copenhagen, 1977, pp.79-92.

38. D.J. SANDOZ and O. WONG, 'Design of Hierarchical Computer Control Systems for Industrial Plant', Parts 1 & 2, Proceedings of the IEE, Vol.125, No.11, Nov. 1978, pp. 1290-1298, pp. 1299-1306.

39. D.W. KING, W.B. SANDERSON, D.J. WOODHAMS, 'Spray Drying of Dairy Products', Dairy Industries, Vol 19 No.1/2, Jan/Feb 1974, p.13-16.