NUMBER SENSE IN SECONDARY SCHOOL PUPILS

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Noel Francis Johnston

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ABSTRACT

Number sense refers to a quantitative appreciation of numbers and an ability to perform calculations in creative and original ways. A pupil with number sense will not be bound by learnt procedures when calculating, instead he/she will be able to create self-made solutions that reflect a good understanding of numbers and flexible ways of using operations on them. Number sense development is now a recognised as an integral component in the syllabi of many western countries and is promoted as an appropriate and productive way of teaching mathematics. Much of number sense revolves around computational mathematics, at which New Zealand pupils, on a comparative international basis, do not rate highly.

This study examines the number sense of a range of New Zealand secondary school-aged pupils by means of a questionnaire, and the processes that pupils use when performing calculations by a series of interviews. Affective factors that have influenced a pupil's mathematical development have also been studied. The qualitative and quantitative data generated enables judgements to be made about the development of number sense and what factors affect this development.

Results suggest that many pupils have had little experience with some of the computational aspects of number sense (estimation and mental arithmetic), only partly appreciate the potential of the distributive principle in numerical calculations and are too locked into algorithmic type solutions. By contrast, pupils with well developed number sense demonstrate good number operation knowledge which they employ in a variety of creative ways when solving problems. Indications are that the foundations for this expertise are put in place at an early age by examining the properties of numbers, performing number drill exercises and validating the processes so that a sense making aspect is built into mathematics education.

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INTRODUCTION

> *Number sense refers to an intuitive feeling for numbers and their various uses and interpretations; an appreciation for various levels of accuracy when figuring; the ability to detect errors, and a common-sense approach to using numbers....Above all, number sense is characterised by a desire to make sense of numerical situations. (Reys, 1992a, p.3)*

1.1 BACKGROUND

Results from the Second and Third International Mathematics Study conducted in 1981 and 1994 show that, compared with pupils from other developed western countries, New Zealand third formers do not perform well at arithematic computations and estimation exercises. Mediocre performances by our pupils suggest that the curriculum goals of developing problem solving skills and attaining confidence at mathematical calculations are barely being attained. Given that our intended curricula are similar to other countries, it must be assumed that the way the content is delivered results in this disappointing performance. Pupils who are algorithmic-bound, who are unable to call upon a flexible repertoire of arithematic operations or have no broad knowledge of number facts are unlikely to perform well at numeric calculations. A preliminary report of participating schools in the Third I.E.A International Mathematics and Science Study (1996) has number sense and estimation listed as examined aspects of mathematics knowledge for junior pupils. Preliminary findings show that, compared to other countries, our performance at number sense questions was average only.

The development of number sense now occupies a prominent position in the curricula of New Zealand and other western countries. However to date there has been very little investigation into the extent of this sense among New Zealand secondary school pupils or any identification of factors that promote or inhibit its development.

The number sense approach to mathematics education provides a path to mathematical competency by stressing that calculations should make sense. For too many pupils the algorithmic approach to performing mathematics calculations has left them ill-equipped for manipulating and applying mathematics to problems. This research exercise is in two parts; in the first the number sense of a range of secondary-aged pupils is examined using a questionnaire, and the second, based on interviews, is an in-depth examination of how pupils perform calculations and what attitudes and expectations they have developed. The study provides an opportunity to analyse mathematics education and to identify attributes that are sympathetic to the development of number sense.

> *When we know why we do something in the classroom and what effect it will have on our students, we shall be able to claim that we are contributing to the clarification of our understanding of our activity as if it were a science.* (Caleb Gattegno, cited in Jaworski, 1994. p. 2)

The advent of cheap, versatile and sophisticated calculators has meant that reliance on the paper and pencil approach to teaching mathematics is no longer appropriate. In the past many school curricula emphasised efficiency and speed in numerical computation with the result that pupils considered that the answer itself was more important that the process by which it was achieved, and guessing rather than proper calculations became common place. The focus on procedural knowledge is unproductive and does not in itself lead to good conceptual understanding (Narode, Board & Davenport, 1993). Pupils who do not have full understanding of numbers and what they mean must develop an extensive repertoire of rules to enable them to solve problems (Ekenstam, 1977). For example, a pupil who does not know that 0.45, 0.450 and $^{9}/_{20}$ are the same, or that $^{5}/_{8}$ is less than $^{4}/_{5}$, or that the relative difference between 1000 and 10,000 is much less than between 10,000 and 100,000, will find performing numerical calculations awkward and unproductive.

After twenty years of involvement with teaching secondary school I question whether sufficient numbers of pupils are leaving school with mathematics skills that allow them to function productively in the world. We need to develop instructional approaches which lead to greater competency at mathematical calculations and numeracy skills. Factors that influence pupils' achievement at mathematics are many and varied. This study examines number sense, what it is, how it can be developed, nurtured and refined. To achieve this a holistic consideration of the subject is presented, incorporating historical and philosophical development of mathematics, a framework for teaching mathematics that acknowledges constructivist learning and details of the attributes of number sense.

1.2 INTRODUCTION TO NUMBER SENSE

Contrary to the normal expectations of things mathematical, number sense does not lend itself to a precise all-embracing definition. Number sense emphasises number knowledge, operations on numbers and creative and flexible ways that the two can be incorporated for solving problems. Pupils with number sense possess in-depth number and number operation knowledge, can confidently perform calculations and are able to reflect on and evaluate answers. Number sense emphasises numbers as meaningful entities, and encourages a quantitative appreciation of numbers and an expectation that mathematical calculations give results which make sense and are reasonable. Thus pupils with number sense are able to call up checking procedures to judge the reasonableness of numerical calculations. The following two commentaries exemplify the spirit and essence of number sense. The first identifies the components of number sense.

> *Most characteristics of number sense focus on its intuitive nature, its gradual development, and the ways it is manifested. Manifestations include using numbers flexibly when mentally computing, estimating, judging number magnitude, and judging reasonableness of results; moving between number representations; and relating numbers, symbols, and operations, all stemming from a disposition to make sense of numerical situations.* (Markovits and Sowder, 1994, p.4)

The second focuses on the importance of understanding, how mathematical processes function and the need to be able to communicate mathematically.

> *Number sense refers to a person's understanding of number and the operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful strategies for handling numbers and operations. It reflects an inclination and an ability to use numbers and quantitative methods as a means of communicating, processing and interpreting information. It results in an expectation that numbers are useful and that mathematics has a certain regularity.* (McIntosh, A., Reys, B.J. and Reys, R.E, 1992, p. 3)

A pupil with developed number sense will not be restricted to a single approach with a computation but will employ a variety of processes that use the properties of numbers and operations in creative ways. The following non-algorithmic approaches to the problem of 38 x 24 demonstrate thinking that is indicative of someone with developed number sense.

- 38 x 24 is approximately 40 x 25 which is the same as 4 x 250 which is 1000
- 38 x 24 is 38(20 + 4) which is the same as (38 x 20) + (38 x 4) which is the same as (380 x 2) + (38 x 2) +(38 x 2) which gives 760 + 76 + 76 = 912.
- 38 x 24 will give an even answer as both the numbers being multiplied are even.
- 38 x 24 will have an answer that ends with a 2 because 4 x 8 gives 32.
- 38 x 24 is 2 x 19 x 2 x 2 x 2 x 3 which is the same as 2^4 x 3 x 19 using its prime factors.
- 38 x 24 is 2 x 24 x 19 which is 48 x 19 which is(48 x 20) - (48 x 1) which is (480 x 2) - 48 which gives 912.
- Would know that the answer should lie between 800 and 1000.

There may well be other ways of performing this computation. Taken together the above responses demonstrate an *at homeness with numbers* (Cockroft Report, 1982), as well as an approach to calculations which relies on the properties of the numbers, an understanding of the effects of operations and a realization of acceptable limits for the answer. A pupil with number sense could be expected to give any of the above as a response to the problem and would certainly comprehend and appreciate all the replies listed.

This contrasts with many students' reliance on using algorithms to solve problems. This procedural and memory-dependent approach discourages understanding and inhibits self-regulated approaches to solving problems. The result is that many pupils end up with compartmentalised knowledge and do not know how to put this knowledge together in successful ways for solving problems. The number sense approach to mathematics instruction encourages flexibility with calculations as opposed to traditional instruction that is algorithmic in design.

1.3 HISTORICAL DEVELOPMENT OF THE NUMBER SENSE APPROACH

The historical derivation of the term number sense is difficult to precisely determine. As early as the 1930s William Brownell advocated the need for meaningful learning to be an integral part of mathematics education. Brownell stressed that pupils should possess an:

> *intelligent grasp of number relations and the ability to deal with arithmetic situations with proper comprehension of their mathematical as well as their practical significance.*(p. 3)

The term *number sense* made its first appearance in print in the Cockroft Report (1982). Up to that time and subsequently, though to a lesser extent, the term *numeracy* was used in relation to working with numbers. *Numeracy* has been defined as those mathematical skills that enable an individual to cope with the practical demands of everyday life - comprehending and interpreting numbers, calculating and estimating.

The term *numeracy*, coined by Crowther in 1959, came to represent for mathematics education what the term *literacy* did for English teaching. Together, numeracy and literacy provided convenient references points for the popular *back to basics* calls of that time. *Numeracy* was to be developed, mainly by repeated practise of algorithms that would enable pupils to function in the everyday world. Little importance was attached to understanding or meaning nor to the benefits of considering alternate ways of using properties of numbers and number operations to solve numerical calculations. While the term *numeracy* is less frequently used in modern texts, a number of countries - America, Australia, England and New Zealand have incorporated the objectives of *number sense* into their resent curricula, justifying its inclusion on the basis that it promotes sense making in numerical calculations.

Number sense and problem solving are two of the modern thrusts in mathematics education. Both defy exact definition and are better considered as approaches to teaching rather than as actual topics that can be precisely structured and taught. Comprehensive number sense is beneficial for problem solving (Dougherty & Crites 1989), useful for establishing the magnitude and the expected number type for an answer and with helping to select the appropriate computation procedures for solving problems. While problem solving necessitates a high degree of number sense, the converse is not necessarily true. Problem solving helps to cement the conceptual and procedural aspects of number sense.

1.4 DECONSTRUCTION OF NUMBER SENSE

Despite previous suggestions that a precise definition of number sense is not possible, a number of authors have proposed that it is characterised by three principal attributes: number knowledge, knowledge of operations on numbers and an understanding of how to apply them in computational settings (Howden, 1989; Rowan & Thompson, 1989; Sowder, 1992a). McIntosh et al. (1992) has taken these three broad headings, teased out the fundamental components and elements that constitute each heading and presented them in a detailed, well organised framework that provides a very comprehensive overview of what constitutes number sense. For this study McIntosh's framework (Appendix A) has been modified to put more emphasis on the sense making attributes of number sense, focusing on pupils' grasp of the inherent logical implications of number and operation knowledge.

Figure 1 details the components of number sense that have been used in this study. The following text expands this framework in order to give a full description of the composition of number sense:

Number Knowledge
- **Sense of orderliness** - place value, ordering numbers
- **Number representation** decomposition/recomposition, equivalent numerical forms
- **Sense of magnitude** - comparative sizes, sense of size
- **Awareness of benchmarks** - both mathematical and physical

Operations Knowledge
- **Effect of operation** - appropriate use of operands, effect of operands on fractions and decimals
- **Properties of operations** - commutative, associative, distributive, use of inverses
- **Interrelationship of operations** - order of operations, use of repeated addition for multiplication and division for repeated subtraction

Numerical Computation
- **Computational estimation** - calculating with exact and approximate data
- **Mental computation** - use of multiple strategies, interpreting problems in context
- **Metacognition** - reflection on results and procedure
- **Use of logic** - awareness of the logical implication of number properties and operations on them

Fig 1. A number sense framework

The following is an in-depth consideration of the characteristics of the three components of number sense.

Number knowledge

Number knowledge involves the workings of the base 10 system, the structure of the number line, the numerical value of numbers, ways a numbers can be represented and an awareness of numerical benchmarks that will be useful for making everyday quantitative judgements. Specifically, important features include:

Sense of orderliness

- An understanding of the place value of numerals in a number including decimals and fractions. For example knowing what the 4 in 1.04 signifies.
- An awareness of the structure of the number system, so that 11 more than 689 is no more difficult to comprehend than 1 less than 20,000.
- An awareness of the patterns that numbers generate, particularly on the number line, for example that there are values between ⅓ and ⅔. Pupils should be able to generate patterns and understand the notion of density on the number line.
- An ability to order numbers, for example, ⅔, 0.61, ½, 0.49 and 0.492.

Number representation

- An ability to represent numerical values in a variety of ways, for example forty minutes can be thought of as ⅔ or 0.667 or 66.7% of an hour or that forty minutes implies that there is ⅓ of an hour left.
- An ability to express a number in other equivalent forms, for example 0.5 as ½ so as to simplify the calculation of a problems like 346 x 0.5.
- The ability to decompose and recompose a number so that it can be more easily manipulated. For example 452 x 71 could be re-expressed as (400 + 50 + 2) x 71 where the decomposition of 452 provides values that are easier to compute. Often exchanges of money involve manipulations that limit the number of coins

used, for example a fee of \$7.25 may be paid out with a \$10 note and 25c, so that the minimum number of coins is given.

Sense of magnitude

- Best exemplified with reference to a number line where relative numerical values can be easily demonstrated. For example that the relative sizes of 420 and −418 are almost the same or that 10,000 is ten lots of 1000.
- An awareness that the relative differences between 1000 and 100 are considerably less than that for 10,000 to 1000 or that by age fifteen years one has lived for just a little less than five and a half thousand days.
- Knowing that the difference between 53 and 78 is the same as that between 453 and 478.

Awareness of benchmarks

- Benchmarks provide a mental reference for making meaningful comparisons. For example, a door height is just under two metres, a normal walking pace is four kilometres per hour, a comfortable room temperature is seventeen degrees, and a popular sports stadium will accommodate fifty thousand people are all useful benchmarks. In a mathematical setting a realization that $\frac{5}{8}$ is close to $\frac{1}{2}$ or 2.98 is nearly 3 are examples of benchmarks that would be useful in estimations calculations.
- The use of knowledge of powers of ten for multiplication and division.

Operation on Numbers

By secondary school pupils are familiar with the algorithms for the four basic operations with integers, decimals and to some extent fractions. An understanding of the properties and interrelationships of the four basic operations enables a pupil to be flexible in the performance of mathematical computations. An appreciation of the effect of operations on numbers, for example, the fact that multiplication does not always give a greater result, as well as how they actually function, is vital for performing computations effectively. An understanding of how the properties of operations require them to be applied in a particular order and an awareness of the interrelationship between operations (multiplication can be considered as repeated addition or that division is equivalent to repeated subtraction) are examples of knowledge of operations on numbers.

Effect of operations

- Knowing that multiplying by 2 will increases the result faster than adding by 2, or that 5 + 5 + 5 + 5 + 5 is more conveniently calculated as 5 x 5 or that dividing by 10 is the same as moving the decimal point to the left.
- Multiplication by a number in the domain 0 to 1 will decrease the total while division with the same values increases the result.
- Effect of operating on numbers by the identities 1 and 0.
- Understanding the working of operations, for example to increase an amount by $12\frac{1}{2}$ % is the same as multiplying that value by 1.125.

Properties of operations

- Know the order of operations of the operands.
- The commutative, and associative principles and how they can be used in calculations.
- A sound knowledge of the distributive principle will provide a pupil with a powerful tool for carrying out estimation and mental calculations. The calculation 12 x 54 becomes 12 x (50 + 4) which moves to (12 x 50) + (12 x 4) which ends as (12 x 5 x 10) + (12 x 4) to give 648.

Interrelationship of operations

- That multiplication is repeated addition or that division can be considered as an application of subtraction.
- Knowledge of inverse relationship with operations - addition with subtraction and multiplication with division, for example computations like 84 - 50 is the same as asking *"50 plus what value"* gives 84, or 420 ÷ 6 can be evaluated as

"6 times what gives 420".

Numerical Computations

To engage in a mathematical activity with confidence it is important to have a well developed appreciation of numbers and the outcome of operating on them. The level of performance at estimation and mental computation problems is indicative of a pupil's depth of understanding of how numbers and operations on them can be applied and provides insights into their intuition for mathematical calculations.

Computational Estimation

- Approximating a number within the implied limits of a problem. For example, 0.94 is roughly 1, or $^{17}/_{32}$ is close to ½.
- Knowledge of strategies that can be used to perform a calculation such as reformation, translation or compensation. These strategies are listed in Appendix B and are derived from those proposed by Reys et al., (1982).
- An ability to make quantitative judgements about length, time, weight and volume.
- Be able to approximate shapes with standard ones. For example, that the South Island of New Zealand is approximately rectangular.

Mental Computation

- Be able to perform calculations without recourse to paper, pencils or calculator.
- Be able to translate a verbal problem into a mathematical one.
- Analyse the content and adopt strategies to perform the calculation mentally. These strategies entail being able to put together number and operation on numbers knowledge in creative ways. Mental computation strategies (Hope, 1987) are listed in Appendix B.

Metacognition

- Self-reflection on mathematical processes and correcting or adjusting strategies being used so as to improve the outcome of a calculation. Pupils can be taught to consider the appropriateness of what they are calculating and to reflect on their answer to see it *makes sense,* that it lies within bounds that are suggested by the problem.

Use of logic

- The properties of numbers and the way that operations on them function imposes logical consequences. A pupil with number sense will appreciate this logic and be able to use it to validate a calculation. He or she will be able to explain why subtraction cannot be commutative, why division by zero is not determinable or why an odd number times an even number is always even.

Number sense arises out of combining number knowledge, knowledge of operations on numbers and numerical computational knowledge and looking for the interrelationships between these components. While each of these considered in isolation will not foster the development of number sense, the three components taken together will.

Greeno (1991) has put forward a psychological perspective for the development of number sense. Greeno proposes a metaphor for number sense, in which a person knows the layout of a workshop (mathematics), where the materials (number facts and operation knowledge) are kept, and how to use the tools (combine conceptual and procedural knowledge) to make objects. The metaphor also extended to instruction so that the person would have enough confidence (accepted into a mathematics community) to consult a manual (invent new strategies) to produce new creations. Greeno's theoretical consideration of how cognitive activity can occur is a useful model for considering what number sense is. It contrasts well with the practical and tangible definitions proposed by other authors who identify attributes or dispositions of people with number sense.

1.5 NUMBER SENSE IN THE CURRICULUM

The genesis of the development of the ideas behind number sense can be traced to the 1982 Cockroft report *Mathematics Counts* which promoted a more expansive approach to how mathematics should be taught. The reference to an *at homeness* with numbers indicates a more humanist view of learning mathematics that encourages familiarity and flexibility with numbers and operations.

In America the 1989 National Council of Teachers of Mathematics report *Curriculum and Evaluation Standards for School Mathematics* includes number sense as a major theme throughout its recommendations. The tone and intent of the NCTM statement is sympathetic with the aims of the Cockroft report.

> *The greatest revisions to be made in the teaching of computation include the following: fostering a solid understanding of, and proficiency with, simple calculations; abandoning the teaching of tedious calculations using paper and pencil algorithms in favour of exploring more mathematics; fostering the use of a wide variety of computation and estimation techniques - ranging from quick mental calculations to those using computers - suited to different mathematical settings; developing the skills necessary to use appropriate technology and then translating computed results to the problem setting; and providing students with ways to check the reasonableness of computations promotes number and algorithmic sense, estimation skills* (p.95).

In 1991 the Australian Education Council published *A National Statement on Mathematics for Australian Schools* which adopts a constructivist philosophy to learning mathematics and encouraging teaching styles that recognize pupils' personal constructions of their own mathematics knowledge. Emphasis is given to encouraging pupils to reflect on their mathematical calculations and to the need to link new knowledge to existing knowledge, so that mathematical ideas can be refined and consolidated. In all strands and levels the importance of number sense is explicit.

All people need to develop a good sense of numbers, that is, ease and familiarity with and intuition about numbers. This requires a sound grasp of number concepts and notation, familiarity with number patterns and relationships, a working repertoire of number skills and, most importantly, confidence in one's capacity to deal with numerical situations (p. 107).

Teachers are encouraged to promote the historical and cultural aspects of the subject and provide opportunities for pupils to appreciate the structure and logical interconnection that occurs in mathematical calculations.

The *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) suggests a considerable change of emphasis in how mathematics is to be taught. Problem solving is extolled as the means by which the content should be learnt. Although there is no explicit reference to number sense in the introduction, aims or objectives, the need to foster numeracy skills, to reflect critically on methods used and to develop an understanding of number are mentioned. One of the five strands is devoted to number. In the first five of these levels number sense and its development is directly referred to under the heading *Suggested Learning Experiences.* The following quotes concerning number sense are from strand statements:

Exploring number

- *developing number sense by exploring number in the context of their own experiences and the world around them;* (Level 1, p.33)
- *developing a number sense by exploring number in the context of their everyday experience and the world around them;* (Level 2, p.37)
- *developing number sense by exploring number in the context of their everyday experiences and the world around them, and using number to explore events in their own lives;* (Level 3; p.41)

Exploring computation and estimation

- *developing number sense by exploring estimation and computation, in the context of their own experiences and the world around them, using concrete* *materials, mental strategies, and calculators;* (Level 1, p.33)

- *developing a number and computation sense by exploring estimation and computation in the context of their everyday lives;* (Level 3, p.41)

These objectives go some way to capturing the essence of number sense. Further reference to components of number sense that occur within the level descriptors include:

- *place value* (Level 2, p.37)
- *investigating odd and even numbers* (Level 2 p. 37)
- *extending their understanding of the number system* (Level 4 p.450)
- *developing instant recall of basic addition, subtraction, multiplication and division facts.* (Level 2 & 3, p.37 & 41)

1.6 THE RESEARCH OBJECTIVES

The principle aim of this study was to examine the level of number sense in secondary school pupils. In addition to measuring pupils' performance at the various components of number sense, the study examines affective factors which influence mathematical development.

The following objectives determined the enquiry of this research

1) To examine the number sense of a range of secondary school pupils and to identify any development.

A questionnaire was used to arrive at a quantitative basis upon which comparisons of number sense could be made. Identification of the development of number sense was examined by comparing year nine, year ten, year eleven and year twelve students knowledge of numbers, proficiency at operations and competency at computational arithmetic.

2) To investigate pupil's inclination to make sense of their mathematical thinking.

Attitudinal responses from a questionnaire and interviews were used to examine the extent of this inclination, whether pupils expected to make sense of calculations and how inclined they were to question the appropriateness of their answers.

3) To identify activities, attitudes and perceptions that contribute to or impair the development of a pupil's number sense.

Pupils with a range of mathematical abilities were interviewed to determine factors that contributed to or detracted from the development of their number sense. An examination of teaching approaches, classroom cultures and levels of base numerical knowledge was conducted to determine what factors contribute to number sense development.

1.7 JUSTIFICATION OF THE STUDY

Studies of the relationship between the acquisition of number concepts and pupils' proficiency in mathematics have been carried out in New Zealand by Graham (1991) and Young-Loveridge (1987, 1991), but these have been confined to primary and intermediate aged children. A number of studies into the mathematical needs of the *average* New Zealander point to the importance of computational skills and proficiencies at manipulating numbers. The Massey University report *The Mathematical Needs of New Zealand School Leavers* (Knight et al., 1992) noted that :

> *Most of the mathematics used in everyday lives can fairly be described as elementary. Arithmetic and basic geometry will suffice for most purposes. What is important is to be able to apply these elementary mathematical tools in a variety of situations in order to solve the many different kinds of problems that arise in everyday life* (p.28).

and that:

> *All school leavers should be familiar with the properties and manipulations of the number system*

Which leads on to the following recommendations;

> *Our education system should ensure that most of our students are competent in:*

(a) numeracy

(b) data measurement, storage, retrieval, manipulation and interpretation

(c) use of formula

(d)the basic arithmetical operations of +, −, x, ÷ and the use of % (p. 78)

The Massey findings are endorsed by a 1994 Lifelong Learning survey by Chartwell Consultants, involving eighty nine randomly selected adults from the Manawatu region. When questioned about the mathematics that they actually used in their every day life it was noted that:

> *the ability to make sensible quantitative judgements based on mental arithmetic and estimation was very important* (p. 30).

It seems that one of the most important functions of a mathematics education is to provide people with a wide knowledge of numbers, a sound appreciation of the ways that they can be manipulated so that everyday mathematics type problems can be calculated with confidence and ease. Numerical judgements of the appropriateness of a result (especially in computations involving quantities and money) are important and a useful skill for anyone wanting to function fluently in the everyday world.

The general lack of ability at making judgements about relative magnitudes has been well documented by Paulos (1988) in his text *Innumeracy*. Familiarity with commonly used benchmarks, like the average walking pace is 4 km/hr or a rugby ground is about half a hectare in area, enable pupils to apply mathematics to everyday problems in useful ways. Mental calculation has been identified by Biggs (1967) as one of the most commonly used mathematics techniques by the general populace and deserves to have a prominent place in pupils mathematics curricula. Estimation and mental calculations are integral components of number sense.

Number sense, flexibly taught and properly grasped will not only provide a sturdy platform for advanced mathematical studies but will also serve the general populace well with their day-to-day requirements for dealing with numbers.

SUMMARY

A consideration of the depth of development of number sense in secondary school pupils and the ways that it can be nurtured and developed is both timely and necessary. There can be little doubt that number sense plays an important role in post-secondary-school life and its development is crucial for people wanting to take an active part in everyday life.

Number sense is made up of number knowledge, number operations knowledge and numerical computation techniques. Each of these three can be divided up into a number of identifiable aspects which can be used as a basis for examining the extent of number sense in school pupils. There is sufficient literature to show that pupils with well developed number sense will be able to apply this knowledge to out-of-school problems. Chapter 2 will propose a historical, philosophical and theoretical basis upon which number sense can be considered, as well as ways to integrate its development into mathematics education which are sympathetic to its intent and goals.