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# Ramanujan-type series for $\frac{1}{\pi}$ with quadratic irrationals

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## Abstract

In 1914, Ramanujan discovered 17 series for  $1/\pi$ , 16 are rational and one is irrational. They are classified into four groups depending on a variable  $\ell$  called the level, where  $\ell = 1, 2, 3$  and 4. Since then, a total of 36 rational series have been found for these levels. In addition, 57 series have been found for other levels. Moreover, 14 irrational series for  $1/\pi$  were found. This thesis will classify the series that involve quadratic irrationals for the levels  $\ell \in \{1, 2, 3, 4\}$ . A total of 90 series are given, 76 of which are believed to be new. These series were discovered by numerical experimentations using the mathematical software tool “Maple” and they will be listed in tables at the end of this thesis.

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