Mathematical Modelling of Fluid Flow and Heat and Pollutant Transport in a Porous Medium with Embedded Objects

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Abstract

How does heat and/or pollutant transfer from objects embedded in the ground depend on their size, shape and burial depth, and how does the dispersion of heat and/or pollution in groundwater aquifers depend on the soil properties, the speed of the groundwater flow, etc.? In detail, the aims of present study are:

- To investigate how the size, shape and position of an object or set of solid or partially pervious objects, e.g., fluid tanks, pipes, etc., embedded in a porous medium affect the local speed and shape of the flow.

- If heat is ejected from the solid objects e.g., fuel storage cylindrical tanks, radioactive waste reservoirs in deep geological formations, etc., and/or a pollutant is released from, or removed by, the pervious object, e.g., septic tanks, disposal of drums of contaminants, etc., how does the subsequent dispersal through a groundwater aquifer depend on the various parameters involved (e.g., the object size, object's burial depth, perviousness of the object, the aquifer's depth, the fluid flow rates, etc.)?

- What is the effect of the non-homogeneity in matrix properties (e.g. permeability or hydraulic conductivity) on fluid flow, pollutant and heat transport rates?

This study pursues answers to these questions. The porous medium fluid flow equations, and the advection-dispersion equations that model the heat and/or species transport, have coefficients that depend mainly on depth. Generally, analytic solutions are not possible. In order to investigate the effects of various objects of different shapes embedded in a porous medium, I have developed numerical algorithms and used some special mathematical techniques for two-dimensional models, namely conformal mappings within the framework of complex analysis.

The velocity potential and (2-D) stream function satisfy Laplace's equation. Central and one-sided finite difference methods are applied to solve this equation subject to a chosen combination of constant-head or constant-flux boundary conditions. Results are discussed for various embedded shapes in homogeneous and layered groundwater aquifers. A Matlab command "contour" is used to depict the streamlines and equipotential lines, and the resulting temperature or pollutant concentrations.

Steady-state and time-dependent forced convection heat/pollutant transfer from some cylinders embedded in groundwater are explored numerically using finite difference methods. The results show that the size, shape, position, perviousness and burial depth of the cylinder affect the pressure drop, as well as the pollutant and/or heat transfer. Moreover advection and dispersion depend on the permeability structure and the fluid speed.

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Contents

Nomenclature

Symbol	Units	Description
Bi	[-]	Biot number
c	[J kg^{-1} K^{-1}]	specific heat
C	[kg m^{-3}]	concentration of pollutant in the fluid
D	[m^2 s^{-1}]	tensor coefficient of mechanical dispersion of a dissolved pollutant while it flows in the porous media
D	[m^2 s^{-1}]	scalar coefficient of mechanical dispersion of a dissolved pollutant while it flows in the porous media
D_{th}	[m^2 s^{-1}]	thermal diffusion/dispersion coefficient
g	[m s^{-2}]	gravitational acceleration
h	[m]	height of the vertical wall
k	[m^2]	isotropic permeability of the porous medium
K	[m s^{-1}]	hydraulic conductivity
n	[-]	porosity of the porous media
Nu_f	[-]	time-mean average fluid Nusselt number
Nu_s	[-]	time-mean average solid Nusselt number
P	[kg m^{-1} s^{-1}]	mass flux of a pollutant
p	[kg m^{-1} s^{-2}]	absolute pressure of the fluid
Q	[m^2 s^{-1}]	total flux through the whole aquifer per unit width of the aquifer, subscripts L, R and I stand for flux in, flux out, and net flow across the pervious rectangular/cuboidal cross section, respectively
Ra	[-]	Rayleigh number

Re	[-]	Reynolds number
t	[s]	time
T	[K]	temperature
$\mathbf{V}$	[m s^{-1}]	average three-dimensional Darcy velocity vector of fluid $\mathbf{V}=(u, v, w)$

Greek Symbols

α	[m]	dispersion length (dispersivity) of the porous medium
α_L	[m]	longitudinal dispersivity
α_T	[m]	transversal dispersivity
α_{th}	[m]	coefficient of thermal diffusion/dispersion
β_{ps}	[m]	coefficient of pressure difference (which is a measure of resistance of the object's surface to flow through it)
ϕ	[m]	two- and three-dimensional velocity potential function
Φ	[m^2 s^{-1}]	$\Phi = K\phi$
$\gamma = \alpha_L/\alpha_T$	[-]	ratio of longitudinal to transverse dispersivity
κ_{ms}	[W m^{-1} K^{-1}]	thermal conductivity
ω_{ps}	[s^{-1}]	constant of proportionality
μ	[kg m^{-1} s^{-1}]	dynamic viscosity of the fluid
ρ	[kg m^{-3}]	density of water (constant)
ψ	[m]	two-dimensional stream function
Ψ	[m^2 s^{-1}]	$\Psi = K\psi$
σ	[-]	coefficient of thermal advection
ξ, η	[m]	transformed coordinates

Subscripts

e	effective
f	fluid
$\hat{}$	transformed variable
I	internal
L	left

m	mixture (formation+fluid)
ms	mixture saturated
ps	porous surface
R	right