TRANSFER OF THREE DIMENSIONAL SPATIAL VISUALISATION TO TWO DIMENSIONS

AN INVESTIGATION INTO THE EFFECT OF USING MANIPULATIVES ON THE TRANSFER OF THREE DIMENSIONAL SPATIAL VISUALISATION

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ABSTRACT

This investigation attempted to determine the effect of using manipulatives on the transfer of spatial visualisation of three dimensions to its two dimensional representation.

The research was carried out in three sections investigating the three topics of the School Certificate prescription that require the visualisation of three dimensions.

These topics were 1. Volume of prisms

2. Isometric drawings

3. Three dimensional trigonometry

The method used was to qualitatively assess students' three dimensional spatial visualisation before and after using appropriate manipulatives.

The theoretical basis of the study was linked to the van Hiele levels as it was believed that these are a valuable tool for teachers to evaluate students mathematical thinking and to develop appropriate teaching programmes. The lack of appropriate descriptors and assessments for determining levels of spatial visualisation made it difficult to carry out quantitative testing. As a consequence of the study suitable descriptors were proposed which should enable teachers to determine their students thinking processes.

The study was carried out in a secondary school with approximately 75 fifth formers and the worksheets formed part of the normal teaching programme for the three topics being investigated. This caused the least disruption for the students but led to some difficulties with administration and collection of their responses.

The results are inconclusive except for the visualisation of angles between lines and lines and between lines and planes in the third topic, this showed an improvement after constructing three dimensional models. This may have been due to the fact that identifying angles in three dimensions is a new concept to students and therefore affected to a greater extent by the use of manipulatives. There was not the same improvement shown for angles between planes, teachers comments supported the conclusion that the manipulatives used were not as appropriate for reinforcing this concept.

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CHAPTER 1 INTRODUCTION

As a teacher of 5th form mathematics classes I became concerned with the difficulty that my students had in interpreting diagrams used in three dimensional trigonometry problems. They had mastered the mathematics of using Pythagoras' theorem to solve for the length of sides and trigonometry to solve for angles and sides but experienced difficulty in determining the relationship between the relevant sides and angles.

I attempted to remedy this by using various objects within the classroom and then tried getting the students to build their own models of cuboids and pyramids using flexi straws. This activity was useful for identifying the quantity to be calculated, reinforcing the concept of height, but the students preferred to work with their model in front of them.

Similarly when working with interlocking blocks and drawing them isometrically students often asked if they were able to take the blocks into the examination with them. When I had to reply in the negative I began to consider whether the time spent working with blocks had been of benefit when the students had to interpret two dimensional diagrams in the examination.

The students I taught were girls and during readings for earlier studies I found that there was a lot of evidence that females have poorer spatial ability than males. Other research linked spatial ability with mathematical achievement and this gender issue is also of prime concern. It seemed important to determine whether improving spatial ability could improve mathematics achievement.

1.Geometry in the New Zealand Mathematics Curriculum

A new mathematics curriculum was introduced in New Zealand in 1984, incorporating 5 strands, Algebra, Geometry, Measurement, Number and Statistics and 8 levels of achievement. The curriculum document states (MINZC, p 13)

> "The importance of the use of apparatus to help students form mathematical concepts is well established. Using apparatus provides a foundation of practical experience on which students can build abstract ideas. It encourages them to be inventive, helps to develop their confidence, and encourages independence. "

For the geometry strand it states (MINZC, p 91)

> " The mathematics curriculum intended by this statement will provide opportunities for students to:
> ...gain a knowledge of geometrical relations in two and three dimensions, and recognise and appreciate their environment;
> ...develop spatial awareness and the ability to recognise and make use of the geometrical properties and symmetries of everyday objects
> ... develop the ability to use geometrical models as aids to solving practical problems in time and space"

The second of these objectives is of particular interest in that it expects students to develop "spatial awareness" and later in the document various learning experiences are suggested to help develop this ability. One of the these suggested learning experiences for Level 4 is the use of interlocking cubes to aid in the drawing of different views of a three dimensional object. Another suggested learning experience is the drawing of nets and using them to construct a three dimensional figure. These techniques require students to use spatial visualisation and it was this particular aspect of spatial ability that I decided to study.

The emphasis that the suggested learning experiences of the new curriculum place on the use of manipulatives such as cubes, nets etc may assist in spatial visualisation but is this ability transferred to visualising objects when they are drawn in 2 dimensions? This is of particular interest at 5th form level when the formal external examination is of pen and paper type and therefore tests the interpretation of 2 dimensional representations of three dimensional objects.

The three topics in the school certificate prescription that involved three dimensions are

- volume
- isometric drawing
- three dimensional trigonometry

This study incorporates the use of manipulatives in each of these topics and qualitatively assesses their effectiveness when students are required to interpret two dimensional representations of three dimensional objects.

The curriculum document also outlines the lower participation rate of girls and Maori in higher level mathematics, and suggests that their confidence in their abilities must be enhanced by providing suitable learning experiences.

It is not only in the new N.Z. mathematics curriculum that the importance of geometry is realised. Reviewers in other countries have also commented on its relevance, Leiva (1992) states that in the American mathematics curriculum there is an emphasis on the study of two and three dimensional geometry. The curriculum also outlines that teachers should encourage and use materials such as building blocks.

This international awareness of the importance of geometry also extends to an increased emphasis on spatial skills, and the use of manipulatives within these curricula. However the link, between the use of these tools and the changes in spatial ability required to interpret two-dimensional representations, needs greater investigation.

Despite this changing emphasis the assessment of spatial awareness is still normally conducted in a formal situation by using two-dimensional diagrams in pen and paper mode. Clements and Ellerton (1995) showed in their research that correct answers given by students in this type of assessment does not always indicate an understanding of the required mathematical concepts or relationships that the tests were designed for.

Clements and Battista (1983, p 457) provide an excellent review of spatial reasoning and in their conclusion state "Research is needed to identify the specific cognitive constructions that children make at all levels, especially in the context of supportive environments (for example, those including manipulatives, computer tools, and engaging tasks.)"

Another aspect of this instruction is how specific the training program needs to be in order to be effective. Does the training in a specific spatial skill affect other spatial abilities? Bishop (1980) suggests that research into the transfer effect of training in spatial tasks is necessary.

Bishop (1980) also proposes that there are two different directions of research into teaching spatial abilities. Firstly those studies which focus on individualised teaching programs so that students can benefit from the abilities that they do possess. An alternative approach is to identify and then teach specific spatial abilities. This seems to Bishop to be the more rewarding approach and a considerable amount of research has been successful with this. The latter is the format followed by this investigation, whereby the specific skills required for each concept were identified and taught.

It may also be beneficial to consider investigating different forms of testing that have greater validity than the formal pen and paper assessments used currently. Usiskin and Senk (1990) suggest that there is a need for a geometry test that is consistent with a theoretical model of learning, has both descriptive and predictive power, satisfies psychometric concerns and is easy to administer. This investigation did not develop such a test but suggests some sample problems that may be developed into such a resource.

2. The van Hiele levels

The van Hiele levels of thinking have received considerable recognition as a tool for identifying students' thinking and hence improving their learning. The sequential and hierarchical aspects of the theory have been rigourously studied but most of the research that has been conducted on them has been with polygons.

Despite the dominance of polygons in previous research it was decided to use the van Hiele levels as a theoretical basis for this research. In order to improve students' spatial visualisation their teachers needed a clearer understanding of the processes involved in the development of the skill. To enable this to occur there is a need for descriptors for the skill at each level and also sample problems. These were not available in the literature but were developed as a consequence of the investigation.

Other reviewers have recognised the need for descriptors and assessments for evaluating and improving students' learning in other mathematical concepts. Senk (1989) suggests that content specific tests should be used in research that is attempting to differentiate between the thinking processes that are characteristic of the van Hiele levels.

The emphasis of this investigation was to study techniques that will improve the spatial ability of students and it was felt that this would be assisted by a greater awareness and understanding of their thinking using the van Hiele levels .

The specific question that this study seeks to answer is

Does the use of manipulatives assist in the spatial visualisation of three dimensional objects in their two dimensional representation?