THE DETERMINATION OF FUTURE OUTPUT FROM SHEEP AND CATTLE FARMS - AN INVESTMENT STUDY

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CHAPTER ONE

INTRODUCTION

Agricultural supply analysis is concerned with the practical and important problems of explaining historical and predicting future patterns of livestock and crop production. Production at the farm level is the foundation of supply at the regional or national level. Decisions which determine the production of the different agricultural products are made at the individual farm level. The collective results of these decisions are the aggregate supplies which are available for export, local consumption or further processing.

The objectives of supply analysis are to answer three questions: Why has production changed in the past? How may aggregate production be expected to change in the future? How may production be expected to respond to alternative controls contemplated by policy makers in Government? In developing countries policy makers need to know what is required to provide sufficient incentive for farmers to expand production. Highly developed countries such as the United States have sometimes suffered from an oversupply of particular products. Policy makers in these countries may need to know how production can be reduced, or diverted to more profitable products.

One approach to answer these questions could be to survey all farmers and/or experts in the field of agriculture to obtain their opinions on the answers to these questions. This would be a very expensive and time consuming exercise, particularly since in a dynamic world of continual change, repeated updating of information gathered would be required.

An alternative approach adopted by many supply analysts is that of mathematical modelling. Mathematical models define variables precisely and assumptions explicitly, so that complex relationships can be analysed and conclusions derived that cannot be derived by verbal or diagrammatic analysis. Abstract models can be developed to a very high degree of complexity limited only by knowledge of the system and by the ability of the researcher to translate his knowledge into functional forms. In general, increased complexity leads to added "realism", but in practice form and complexity need to be related to the objectives of the study, for interpretation of the results is highly dependent on both these attributes.

Techniques such as budgeting, mathematical programming, simulation and regression of time series data have been used in attempting to quantify supply responses to price changes and other variables of interest. In addition to the various techniques that can be used, there are various levels of aggregation from which the problem can be approached. These range from national aggregate responses to individual farmer responses.

1.1 Objectives of The Study

The objective of this study is to investigate the role of investment analysis as it relates to the processes which determine future livestock numbers and supplies of livestock products. The initial Chapters of this thesis are concerned with the principles of investment analysis and with their application to the individual farm situation. A model is then built that attempts to simulate the investment and output decisions made by a representative farmer. The representative farm modelled is the North Island Hill Country (Class 3N) farm derived from the New Zealand Meat and Wool Boards' Economic Service's Sheep Farm Surveys. If the methodology of this study is successful then similar models could be built for the other Farm Classes.

The behavioural hypothesis made is that the farmer will plan to use his productive assets to generate a stream of income of the greatest possible value to him for some period after he makes his investment/output decisions. The major productive asset of interest in this study is livestock which can be viewed as either a consumption good to be sold for slaughter or to another farmer for further fattening, or as an investment good in which case retained for further fattening and/or breeding.

The time period with which the farmer is concerned is the time period over which income (or business activity) is important to him i.e. his time horizon. Included in the behavioural hypothesis is the assumption that the farmer fully recognises the interdependence between changes in current output and potential future production.

1.2 The Production/Investment Model: Conceptual Framework

There are two aspects to the mathematical model of a farming system developed in this study. The production process describes the on-farm physical conditions that constrain the farmer's production possibilities or plans. The decision process involves the selection of the best plan according to the farmer's objectives and expectations.

The framework of the model is based on the notion of Hicks (1946) who developed a dynamic decision-making model of the firm under certainty. According to his view, just as in static theory, the firm is to choose from among alternative available courses of action, the one which is most conducive to the achievement of its goal. The decision problem faced by the firm at any given time is the selection of the best plan over the planning horizon. A fundamental way of measuring the preferred production plan involving costs and returns in future periods is that of capitalized value of the stream of surplus, which Hicks called the capitalized value of the production plan.

Following Modigliani and Cohen (1961) and Carvalho (1972), the notion developed by Hicks is modified for this study. Long-run plans are not necessarily made up in order to be implemented, but only to utilize all the available information to make the best plan for the current period. Since expectations in one period relative to economic and environmental conditions in future periods might be held with great uncertainty, the production and investment plans which are based on expectations must continuously be adjusted or revised with time. Emphasis is placed on the first move of the planning period which cannot be postponed and, hence, must be carried out.

1.3 Livestock Numbers or Product Supplies

A distinction should be made between models designed to project future livestock numbers and models designed to project future livestock product supplies. Approaches to both problems are very similar, the distinction usually being in the data used, with the latter requiring some sort of yield estimates. As an example, an increase in livestock numbers does not mean that at a particular time in the future, sales for slaughter will necessarily have increased as well. This will depend on market conditions at that particular time. However, an increase in livestock numbers over a period of time must affect product supplies eventually, provided that the increase does not significantly decrease yields. This study concentrates on attempting to explain increases in livestock numbers while recognising that the objectives and methodology of both types of problem may be similar.

1.4 Thesis Guide

Chapter Two is a review of the Theory of Investment as it relates to principles derived under conditions of certainty about the future.

Chapter Three applies some of the principles discussed in Chapter Two to the farm situation. The importance of viewing animals as consumption and/or investment goods is extensively discussed.

Chapter Four reviews previous work done on supply response analysis of New Zealand livestock and livestock products, and overseas studies that have the potential to improve the New Zealand studies. This is followed by a review of models of price expectation formation, and the expectation model used in this study.

Chapter Five discusses the problems involved in using representative farms in supply analysis and outlines the main features of the representative farm to be modelled.

Chapter Six describes in detail the model and its estimation.

Chapters Seven and Eight give the results and conclusions of the study.

CHAPTER TWO

THE THEORY OF OPTIMAL INVESTMENT DECISION

This chapter explains the fundamental notions of investment decision theory - time preference, production opportunity, and market opportunity; and discusses the various criteria, together with their associated rules, that can be used to guide an individual in selecting among a number of alternative investments.

A rational farmer, when faced with an investment decision will consider the alternatives open to him and then according to some criterion, will choose a course of action to be taken. Of interest, is the method he uses to analyse and choose between the alternatives open to him. It may be intuition, experience or some more objective method such as budgeting.

In attempting to model a farmer's investment behaviour it is necessary first to be able to describe the method and the criterion the farmer might use. The traditional investment criteria are, therefore, discussed both from a theoretical and a practical point of view.

This does not imply that the farmer will use a mathematical formula in his analysis of the alternatives open to him. However, it does imply that one or other of the mathematical formulas and associated criterion discussed in the following sections can be used in an attempt to model his investment behaviour.

2.1 Fundamental Notions

Traditionally, the theory of investment of the firm has always been associated with the theory of production, but it is also closely related to the theory of capital. The theories of investment, interest and capital represent an extension of economic analysis into the domain of time and of uncertainty. The questions of criteria for efficient investments and optimum financial budgeting are of such urgent practical interest that a large amount of work has been done in these fields, much of it only loosely connected with "mainstream" economic theory. Discussed in this study are the principles derived under conditions of certainty about the future.

The classical theory of interest was developed by Bohm-Bawerk, Wicksell and Fisher. Fisher's (1930) theory of interest is built upon three fundamental concepts mentioned earlier - time preference, production opportunity, and market opportunity.

Income provides some measurable concept of the pleasurable sensations (or consumption) available in a given period of time. The amount of income clearly depends upon the quantity of resources available and upon their utilisation. The best utilisation of resources is achieved where values of all marginal returns to these resources are equated.

As long as there are opportunities to defer consumption of current income to future time periods (i.e. there exists a range of productive opportunities) a person is concerned not only with equating the value of marginal returns from various possibilities at present but also with equating the value of marginal returns over time.

It is usual to describe the stock of resources existing at a point in time, and able to yield income in the future, as capital. The future stock of capital depends on the current allocation of present income between consumption and investment. In general, capital will be created until the desire for additional consumption at present is equated to a desire for additional consumption in the future (i.e. time preferences for consumption are satisfied).

Consider a situation in which the costs and returns of alternative individual investments are known with certainty. The investor wishes to select, according to some criterion, the scale and mix of investments to be undertaken. The two-period situation is analysed in the following two-dimensional diagram (Figure 2.1) and can be extended to the analysis of investments in multi-period situations.

In the case of a perfect capital market the individual (or firm) is faced with a situation in which the borrowing rate equals the lending rate, neither of which is affected by the amount of borrowing and lending.

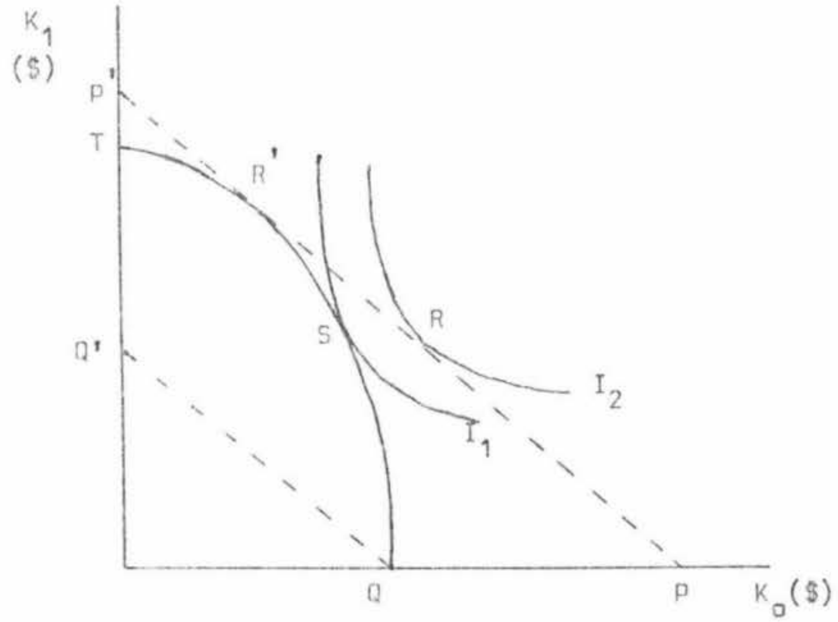

Figure 2.1 Fisher's Two Period Solution

The current year's income K_0, is measured along the horizontal axis and the following year's income K_1, along the vertical axis. Any point in the positive quadrant represents a given combination of the current year's and the following year's income. The individuals time-preference for income is represented by indifference curves I, the slopes of which are a measure of the marginal time preference of the individual at the corresponding combination of present and future incomes, i.e. they measure the rate at which the individual would forego future income in order to attain a higher current income and remain at the same level of satisfaction and vice versa.

Fisher (1930) separates the investment opportunities open to the individual into two components - production opportunities and market opportunities. The former are real productive transfers between income in the two time periods (e.g. planting a seed); the latter are transfers through borrowing or lending.

In Figure 2.1, an investor with a current income Q faces a market opportunity illustrated by the dashed line QQ'. Starting with all his income in time 0, he can lend at some given lending rate, sacrificing present for future income, any amount until his K_o is exhausted - receiving in exchange K_1 or income in period 1. Lines such as QQ' and PP' are in fact lines of constant present value with slope - (1 + i) where i is the market rate of interest (or discount rate).

The curve QST shows the range of productive opportunities available to an individual with current income Q. It is the locus of points attainable to an individual as he sacrifices more and more of K_o by productive investments yielding K_1 in return.

The equation of QST is:

$$-dK_o = \frac{dK_1}{1+p} \qquad (2.1)$$

or

$$p = \frac{dK_1}{(-dK_o)} - 1 \qquad (2.2)$$

Where:

dK_1, dK_o - changes in income for a movement along QST

p - productive rate of return

The productive rate of return is the slope of QST minus one. Since QST is concave to the origin a movement along it in the direction of T reduces p.

If each point on QST represents a different project, then p in effect ranks these projects, being the average productive rate of return for each project. It is also possible to consider the analysis as representing one project in which case p ranks infinitesimal increments to that project and is the marginal productive rate of return.

To maximize his utility an investor must reach as high an indifference curve as possible. The solution to this involves two steps. The "productive" solution - the point at which the individual should stop making additional investments - is at R'. R' represents the investment opportunity with the highest present value (when i is the discount rate). He may then move along his market line to a point better satisfying his time preferences, at R. He thus makes the best investment from the productive point of view and then "finances" it in the loan market.

This analysis provides a fundamental insight into what is involved in selecting the scale and mix of investments to be undertaken, under the assumptions made. The analysis can be generalised to the multi-period situation and some of the assumptions relaxed. In practice some sort of measure of the notions discussed is required to enable decisions to be made.

2.2 Criteria Commonly used in Investment Analysis

Three alternative measures of an investment's worth are usually advanced:

1. The Net Present Value
2. The Benefit Cost Ratio
3. The Internal Rate of Return.

2.2.1 Notation

Let $C_1, C_2, \ldots, C_n$ = the costs incurred during the years 1, 2, ..., n.

$b_1, b_2, \ldots, b_n$ = the benefits incurred during the years 1, 2, ..., n.

V = the present value of all benefits

C = the present value of all costs

i = the appropriate discount rate

r = the internal rate of return

N.P.V. = net present value

2.2.2 The Net Present Value

A chosen rate of discount is employed to value all benefits and costs as at the beginning of the investment. The "costs" are subtracted from the "benefits" to give a net present value.

$$N.P.V. = V - C$$

$$= \sum_{j=1}^{n} \frac{b_j}{(1+i)^j} - \sum_{j=1}^{n} \frac{c_j}{(1+i)^j}$$

The decision rules are:

iff $N.P.V. > 0$	investment worthwhile
iff $N.P.V. < 0$	investment unprofitable
iff $N.P.V. = 0$	indifferent as regards the investment
	for mutually exclusive projects
iff $N.P.V.(1) > N.P.V.(2)$	(1) is a better investment than (2)

2.2.3 The Benefit-Cost Ratio

The benefit-cost ratio is derived by dividing the present value of all benefits by the present value of all costs.

$$\frac{V}{C} = \frac{\sum_{j=1}^{n} \frac{b_j^{\bullet}}{(1+i)^j}}{\sum_{j=1}^{n} \frac{c_j}{(1+i)^j}}$$

The decisions rules are:

iff $V/C > 1$	investment worthwhile
iff $V/C < 1$	investment unprofitable
iff $V/C = 1$	indifferent as regards the investment
if $V/C(1) > V/C(2)$	and capital is not a limiting factor (1) is a better investment than (2).

A variant of this is:

$$\text{Net benefit/cost} = \frac{V - C}{C}$$

The decision rules are based around zero instead of one.

2.2.4 The Internal Rate of Return

A rate (or rates) of discount is calculated such that the present value of the benefits equals the present value of the costs.

When $V = C, \quad i = r$

or $$0 = \sum_{j=1}^{n} \frac{b_j}{(1+r)^j} - \sum_{j=1}^{n} \frac{C_j}{(1+r)^j}$$

The decision rules are:

if $r > i$	investment worthwhile
if $r < i$	investment unprofitable
if $r(1) > r(2)$	then (1) is a better investment than (2)

2.3 The Choice of a Criterion - Theoretical Aspects

Investment processes are characterized by the fact that time elapses between the application of inputs and the attainment of the resultant outputs. The choice between alternative investments is commonly known as the capital "widening" process. Capital "deepening" refers to choosing the optimum (according to some criteria) manner in which to conduct an individual investment. One aspect of a capital deepening problem is the optimum period over which an investment should take place, commonly labelled a duration problem. In such problems time is treated as a continuous variable. Duration analysis provides a framework for determining the optimum slaughter age for farm animals discussed in detail in the next chapter.

There are four types of these duration problems:

(a) Point input - point output
(b) Point input - continuous output
(c) Continuous input - point output
(d) Continuous input - continuous output.

The simplest form, point input - point output, involves only an initial current outlay and a terminating receipt. If the initial outlay is a given constant then the model is a variation of the two-period model discussed in Section 2.1.

A familiar example is the problem of when to cut a growing tree. (Lutz, 1945). Consider an entrepreneur who is investing in trees and has perfect foresight in that he knows the lumber value of each tree at all the future dates at which he may possibly cut the tree and sell the lumber. The value of the lumber increases but at a decreasing rate. The costs include the price paid for the sapling, labour and rent for the land and are all incurred instantaneously.

The entrepreneur might set up a series of equations equating the costs of the investment with the value of the tree at the various future dates discounted back to the present at the unknown rate of return. He could then solve all these equations for the rate of return, and find the one which gives him the highest rate of return, and choose the period of growth which characterizes this equation. This maximization of the internal rate of return is represented in Figure 2.2.

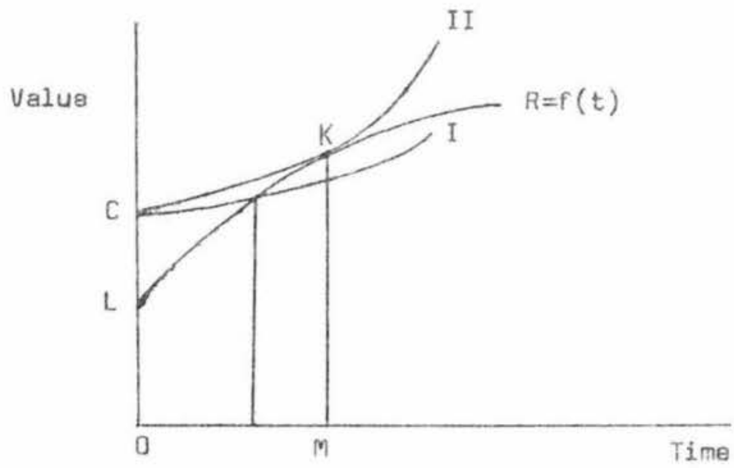

Figure 2.2 The Growing Tree Problem

$R = f(t)$	-	value of lumber as a function of time
OC	-	cost of the investment
I, II	-	discount curves

The steeper the discount curve, the higher is the discount rate on which it is based. Curve II is tangential to $R = f(t)$ at K, giving an optimum period of growth OM. The discount rate of Curve II is the maximized internal rate of return. At K then, the maximized internal rate of return is equal to the percentage rate of growth of the tree.

If the market rate of interest is lower than this maximized internal rate of return this implies that the percentage rate of growth of the tree at OM exceeds this market rate and the entrepreneur could increase the present value of his profits by extending the period of growth. This will be so until the point is reached where the percentage rate of growth equals the market rate of interest.

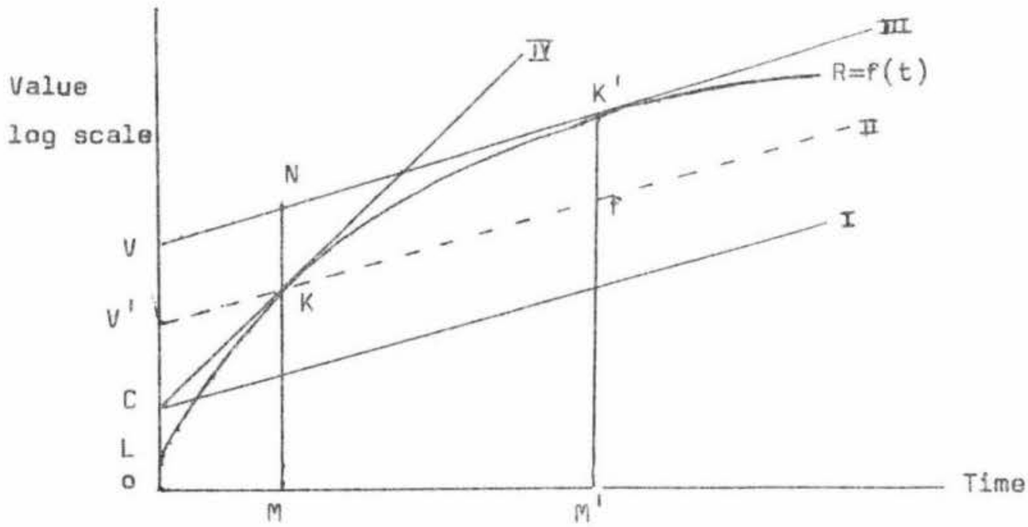

Figure 2.3 The Growing Tree Problem - Log Scale

Figure 2.3 is simply an extension of Figure 2.2 with the arithmetic vertical scale transformed into a logarithmic scale.

The general equation of the discount curves is:

$V_t = V_o e^{rt}$ where V_t - value at time t
V_o - an arbitrary present value

or $\ln V_t = \ln V_o + rt$ which is linear in t.

Lines I, II, III represent discount lines based on the same interest rate (the market rate of interest), while line IV represents a discount line based on the maximized internal rate of return.

It is obvious then, that unless the market rate of interest coincides with the maximized internal rate of return, the optimum period of growth will be different according to whether the entrepreneur maximizes the internal rate of return or the net present value.

Maximization of the internal rate of return dictates that OM is the optimal duration while maximization of the net present value dictates it as OM'.

Lutz points out that under competitive conditions the situation depicted in Figure 2.3 could not last, since the profits would attract other investors and speculators, thus altering the value of the costs and returns until the present value of the output becomes equal to the cost of investment. In this equilibrium situation the internal rate of return becomes equal to the market interest rate. Lutz (1945, p 70) concludes that:

"provided a market rate of interest exists, the entrepreneur investigating the profitability of a given investment opportunity should adopt total profits as his criterion of profitability, in preference to the internal rate of return".

If this is the case, then it can be shown that the optimum period of growth will be such that the marginal earnings from tree growth would exceed his earnings from investing the value of the lumber elsewhere if his investment period were slightly shorter than the optimum, and would be less than his earnings elsewhere if it were slightly longer than the optimum. In addition, an increase in the rate of interest will lead the entrepreneur to shorten his aging period, and a decrease will lead him to lengthen it (Henderson and Quandt, 1971, p 323).

2.3.1 Capital "Widening"

There are two main functions in which the choice between the different criteria must be considered:

(a) Accept - Reject (absolute function of a criterion)
(b) Ranking of alternative projects (relative function).

In either function, different criterion can select different projects. There is no generally applicable method of evaluation. The criterion to use in a particular situation depends on the conditions under which the decision is being made and the maximands, assumptions and constraints of the different criteria.

From a theoretical point of view Hirschleifer (1958) strongly criticises the internal rate of return as an investment criterion. He goes even further to say:

> "Since Fisher, economists working in the theory of investment decision have tended to adopt a mechanical approach - some plumping for the use of this formula, some for that. From a Fisherian point of view, we can see that none of the formulas so far propounded is universally valid."

For the two-period case (Figure 2.1) with a perfect capital market, the present value rule and the internal rate of return rule lead to identical answers (Hirschleifer, 1958). As previously noted the market opportunity lines are in fact lines of constant present value with slope - (1+i), where i is the interest rate. To maximize present value is to invest until the highest such line is attained i.e. R' in Figure 2.1. This rule says nothing, however, about the financing also necessary to attain the final optimum at R since knowledge of the decision-maker's utility isoquants would be required to determine R.

Even in the simplest possible situation, these rules only give the "productive solution" - only part of the way towards attainment of the utility optimum. In addition, this productive decision is optimal only when it can be assumed that the associated financing decision will in fact be made.

The analysis can be extended to the case where borrowing and lending rates are not equal, the borrowing rate being higher than the lending rate (Hirschleifer, 1958). This would be represented in Figure 2.1 by two market lines, the steeper one representing the borrowing rate and the flatter one the lending rate. Where the productive opportunity, time-preference, and market (or financing) opportunities stand in such relations to one another as to require borrowing (lending) to reach the optimum, the borrowing (lending) rate is the correct rate to use in the productive investment decision. However, when the borrowing and lending rates differ a third possibility exists. If a tangency of the productive opportunity locus and an indifference curve occurs when the marginal productive rate of return (Equation 2.2) is somewhere between the lending and the borrowing rate, neither borrowing nor lending is called for and some interest rate between the lending and borrowing rates would lead to the correct results. Hirschleifer (1958) calls this the marginal productive opportunity rate. In such a case neither rule (present value nor internal rate of return) is satisfactory in providing the productive solution without reference to the utility isoquants.

Investment opportunities that are non-independent result in complications in choosing from the alternative criteria. (Figure 2.4).

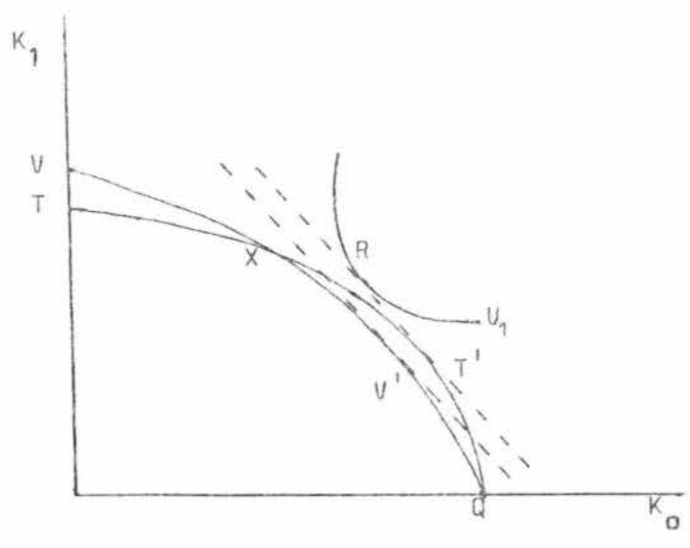

Figure 2.4 Non Independence

Consider two mutually exclusive investment opportunities given by the loci QV'V and QT'T. With the assumption of a perfect capital market the productive solutions are given by V' and T' respectively. However, the one attaining the highest present value line (QT'T at T') will permit the investor to reach the highest possible utility curve at R. The internal rate of return rule would locate the points T' and V' but would not discriminate between them.

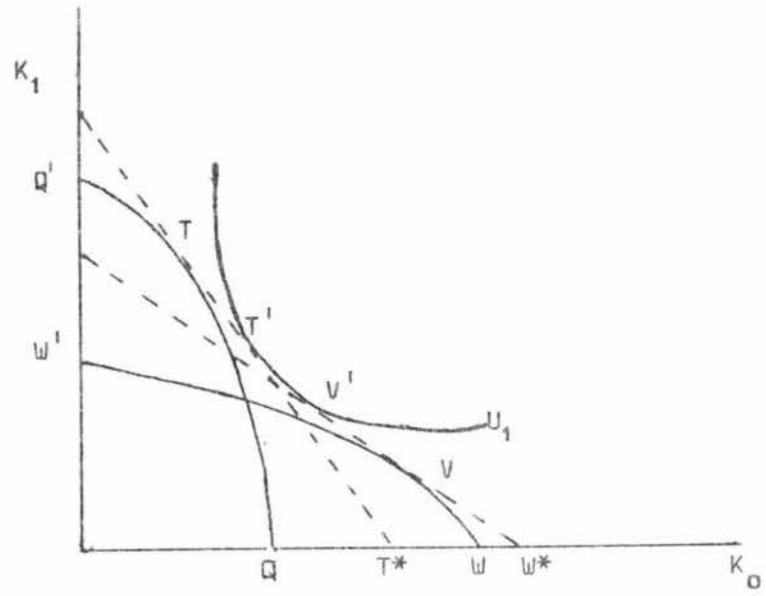

Figure 2.5 Non Independence, Different Borrowing and Lending Rates

Figure 2.5 can be used to show that under conditions of non-independence of investment opportunities, and different borrowing and lending rates the present value rule can also fail. If the production opportunity loci QQ' and WW' are interpreted as mutually exclusive alternatives, then it may be necessary to compare, say, a lending solution at V with a borrowing solution at T. The two solutions attain the same indifference curve U_1, but the present value of the solution $V(=W^*)$ at the relevant discount rate for it (the lending rate) far exceeds that of the solution T $(=T^*)$ at its discount rate (the borrowing rate), when the two are actually indifferent.

Hirschleifer's work provides a general theoretical solution against which to judge the methods. Despite his conclusion that none of the formulas is universally valid, in a practical context the decision-maker is forced to resort to one or other of the methods since he can rarely make a utility analysis.

2.4 Practical Aspects of the Criteria

Under certain rigid conditions (i.e. competitive general equilibrium) all three criteria will give the same solution to the same problem (Lutz, 1951). Since these conditions are generally not met in a practical environment the choice of an investment criterion will rest mainly with the value judgements of the investor, his maximands and the constraints in operation. Some of the main considerations are discussed briefly below.

2.4.1 Net Present Value versus Internal Rate of Return

The mathematical nature of both of these criteria can be a disadvantage, particularly when considering their adoption by an individual investor. The ingrained inclination of businessmen to assess projects in terms of a rate of return on capital favours the internal rate of return criterion.

In order to compare criteria it is important to determine the characteristics of an investment which are maximized. For example, maximization of the internal rate of return will achieve the fastest growth rate of the investment if the entrepreneur can forgo consumption of proceeds i.e. all income must be re-invested to achieve such a result - no consideration is given to consumption preferences. Other characteristics than maximization of overall profitability or growth rate, may be important, for example, the life of the project.

In addition to these broader aspects there are several more specific arguments for the use of a particular criterion:

(a) Ranking

A common criticism of the internal rate of return is that it fails to rank projects correctly. This criticism is based on the unstated assumption that a net benefit criterion must always be maximized which is perhaps not universally relevant. For example, if the aim is to accumulate assets as fast as possible then only the internal rate of return gives the true order.

(b) Re-Investment

Where discounting takes place over several time periods a common assumption is that during intervening periods any funds made available must be re-invested at the discount rate. While the same basic assumption is required for the present value approach the actual re-investment requirements are not so stringent. Thus, if the discount rate in the present value calculations is the market rate of interest, then re-investment at the same rate can be expected. However, if the internal rate of return is substantially above the market rate, it seems unrealistic to assume that re-investment can continually occur at this rate.

There is considerable controversy on this issue. From a practical point of view it is possible to argue that the re-investment assumption is unwarranted (Sinden, 1972). The benefits may not be available for re-investment, particularly in public investment projects. If the investor is interested in the intermediate cash flows and not just the final period assets, then the re-investment question would be relevant. The question is really only relevant when project selection is affected if net proceeds are re-invested or not.

Sinden (1972) argues that when two projects are being considered, if proceeds are re-invested and the internal rate of return is used, then the relative and absolute values are likely to be changed out of proportion because re-investment opportunities are likely to differ.

(c) Multiple Solutions

There are conditions under which the internal rate of return is not unique; the exact number depends on the number of changes of sign of the investment.

In many practical applications only one solution is possible, and the internal rate of return would be acceptable as a choice indicator.

2.4.2 Benefit/Cost Ratios versus Net Benefits

Lutz (1945) suggests that if outside capital is not available, the entrepreneur should maximize the benefit/cost ratio. The argument for the use of the ratio is based on two considerations, i.e. the limitation on capital and the scale problem. If capital is limited then net benefits will be maximized only by selection of the project which has the highest ratio of benefits to costs. It is then suggested that capital is usually, if not always, the limiting factor therefore the ratio is usually the correct measure.

The size of the benefit/cost ratio gives no indication of how worthwhile the investment is. For example, if there was a choice between mutually exclusive projects, A with V = \$120, C = \$100 and B with V = \$12, C = \$10, then in both cases the benefit/cost or V/C ratio would be 1.2, which by this criterion would indicate indifference. However, provided no capital restrictions applied, project A would be superior to project B as the present value of future net income from project A is \$20, whereas from project B it is only \$2.

2.5 Choice of Criterion - Conclusion

The various assumptions of the main criteria are rarely satisfied in practice. Hildreth (1964) summarises the valid basis for selection:

> "rational maximization criteria for individual investment opportunities are likely to depend on the technological possibilities and input limitations associated with each investment opportunity."

None of the three criteria are generally inappliçable. Selection of a method therefore depends upon specification of the objective and constraints of each project and selection of the method which best satisfies these conditions.

CHAPTER THREE

OPTIMAL INVESTMENT DECISIONS ON THE FARM

The previous Chapter outlined the criteria a farmer might use in evaluating alternative investments. The criterion to be chosen was shown to be dependent on the type of investment and the conditions under which it is to be undertaken. One of the important investment decisions a farmer is required to make is concerned with his livestock. He must decide whether to retain his current flock and herd sizes and structures, or alter them. His policy to some extent will be determined by physical factors such as climate and soil type, however economic conditions will be a major determinant of any changes made. These changes are brought about by sales and purchases. A purchase of an animal is an investment. Similarly, a decision to retain (rather than sell) an animal is a decision to further invest in that animal. Conversely, the sale of an animal is a disinvestment.

3.1 The Theory of the Firm

The farm manager in the theory of the firm is faced with the decision of the optimum allocation of a set of resources to the production of alternative products given factor and product prices. His animals can be regarded as end products (i.e. as slaughtered animals), or they can be regarded as resources. The livestock thus have a dual role, that of a finished product and that of an investment good. The farm manager also has a dual role; that of investor-producer. His set of resources (animals) is constantly changing and he is faced with the decision of holding on to them (further investing) or selling his stock at any point in time, either for slaughter (consumption) or to some other farmer-investor for further fattening.(1)

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(1) The ideas outlined here originate from Yver (1971) and Jarvis (1974).

At any time the typical farm[1] is holding a composite capital good represented by animals that differ in type, sex and age. The decisions faced by the farmer can be classified into three categories:

1) How much capital to own.

2) What the composition of the capital should be.

3) What the rate of utilization of capital should be in the alternative uses of producing further capital goods and consumption goods.

Under the classification of the previous Chapter, 1) and 2) are capital "widening" choices while 3) is a capital "deepening" (duration) choice. Farmers can therefore be viewed as portfolio managers seeking the optimal combination of different categories of animals to complement their non-animal assets, given existing conditions and future expectations.

3.2 The Capital Price of Male and Female Animals

The capital value of an animal indicates what the animal is worth to the farmer either as an investment good or as a consumption good to sell for slaughter, or to another farm. The capital value of an animal can be identified with the maximized net present value of that animal at birth or for its remaining lifetime. Conceptually this is found by jointly determining the optimum amount of feed and other inputs the animal should receive over its lifetime and the age at which the animal is to be sold.

Jarvis (1974) outlines how micro-models could be used to determine the optimum slaughter age and feed input for a steer, given growth functions for the animal and the following parameters faced by the producers: the price of beef, the interest rate, and the cost of inputs. Under the assumption that the animals are fed, the optimal

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(1) The farm is considered as a firm; the farmer is the manager.

ration, the criterion to find the optimum slaughter age becomes maximization of the present discounted "profit"[1] of the fattening process, which in perfect markets will be the value of the calf at birth. This is given in equation (3.1):

$$\pi(\theta) = p(i,\theta)w(i,\theta)e^{-r\theta} - ci \int_0^{\theta} e^{-rt}\, dt \qquad (3.1)$$

Where:

- θ - the age of the steer
- π - the present discounted profit of the fattening process
- i - a fixed bundle of daily inputs to the steer, independant of θ.
- c - the cost of the fixed bundle, i.
- p - price per kg of beef as a function of feed inputs and age.
- t - time
- r - interest rate.
- w - weight of steer as a function of feed inputs and age.

Both the weight and the price of the animal are assumed to be functions of i and θ, implying that the quality of the beef is reflected in the unit price received. The inputs required consist primarily of feed, but conceptually may include all inputs such as labour, shelter, fences, machinery and veterinary care. Jarvis (1974) notes that the assumption that the input bundle is fixed is unrealistic since for beef it varies over the animal's life, however to keep mathematical complication to a minimum these effects are not included in the analysis.

The first-order conditions for a maximization of π require that the producer select both the optimal slaughter age and the optimal input stream:

$$\frac{\partial \pi}{\partial \theta} = e^{-r\theta}\left(p\frac{\partial w}{\partial \theta} + w\frac{\partial p}{\partial \theta}\right) - re^{-r\theta}pw - cie^{-r\theta} = 0 \qquad (3.2a)$$

(1) i.e. Net Present Value. Under the classification of Section 2.3 this is a continuous input - point output duration problem.

$$\frac{\partial \pi}{\partial i} = e^{-r\theta} \left(p \frac{\partial w}{\partial i} + w \frac{\partial p}{\partial i}\right) - c \int_0^{\theta} e^{-rt}\, dt = 0 \qquad (3.2b)$$

Which yield:

$$p\frac{\partial w}{\partial \theta} + w\frac{\partial p}{\partial \theta} = rpw + c\hat{i} \qquad (3.2c)$$

$$p\frac{\partial w}{\partial i} + w\frac{\partial p}{\partial i} = ce^{r\hat{\theta}} \int_0^{\hat{\theta}} e^{-rt}\, dt = \frac{c}{r}\left(e^{r\hat{\theta}} - 1\right) \qquad (3.2d)$$

at $\hat{\theta}$, the optimum slaughter age, the change in value due to changing weight and quality (unit price) less the cost of feeding is equal to the current interest forgone. Similarly at i, the optimum input level, the present discounted value of the marginal net weight gain and price increase corresponding to a higher stream of inputs throughout the steer's life, less the present discounted cost of feeding the animal these inputs, must be zero.

A similar analysis can be carried out for cows. Cows can produce beef either directly, by being fattened for slaughter, or indirectly, by bearing calves which can themselves be fattened for slaughter. The analogous equation to (3.1) for cows is:

$$\pi(\theta) = \sum_{t=1}^{\theta} \frac{C(i,t)}{(1+r)^t} - ci \int_0^{\theta} e^{-rt}\, dt + p(i,\theta)w(i,\theta)e^{-r\theta} \qquad (3.3)$$

Where C(i,t) is the expected value of the calf born in year t, assuming the cow has been fed input stream i throughout its life. Maximization of (3.3) with respect to θ and i shows that the female may be withheld from slaughter, even though it has ceased to gain weight, on account of the (expected) value of its calf output.

The response of capital values at birth to a change in beef prices, input prices and the interest rate are important since these measure the effect exogenous changes will have on the future composition of a herd. This response is reflected in changes in the optimal feed ration and the optimal slaughter age.

It can be shown that the response of capital values to changes in beef prices is positive and that for input prices is negative. Both are greater in absolute terms for females (Jarvis, 1974). This means that female capital values at birth will increase (decrease) relatively more than male capital values when beef market prices increase (decrease) which implies that, *ceteris paribus*, an increase in beef market prices (expected to persist) would make it more profitable for a farm to hold relatively more female animals than male. Also of interest is the fact that there is a positive response of capital values of steers, to changes in beef prices. This indicates that a negative slaughter response for steers is expected in the "short run", *ceteris paribus*. Fewer steers are slaughtered temporarily because a higher price causes them to be withheld to be further fattened.

3.3 Determination of Market Prices

Equation (3.1) represents the present discounted profit of the fattening process for a steer. Assuming the steer receives the optimum amount of feed and other inputs (i.e. $i = \hat{i}$) equation (3.1) can be rewritten:

$$\pi^{*}(\theta) = p(\hat{i},\theta)w(\hat{i},\theta)e^{-r\theta} - c\hat{i}\int_{0}^{\theta} e^{-rt}\,dt \qquad (3.4)$$

Then, if θ is replaced by $\hat{\theta}$, the optimal slaughter age, the equation gives the maximum present discounted profit, or the capital value of the animal at birth.

$$\pi^{*}(\hat{\theta}) = p(\hat{i},\hat{\theta})w(\hat{i},\hat{\theta})e^{-r\hat{\theta}} - c\hat{i}\int_{0}^{\hat{\theta}} e^{-rt}\,dt \qquad (3.5)$$

$\pi^{*}(\hat{\theta})$ represents the calf's value at birth; that is, $\pi^{*}(\hat{\theta})$ is the amount which if invested at interest rate r would have the same money value at time $\hat{\theta}$ as the finished steer, less the total feed costs compounded from their time of input to $\hat{\theta}$, at rate r. Equation (3.4) is illustrated graphically in Figure 3.1 by the curve $\pi^*(\theta)$.

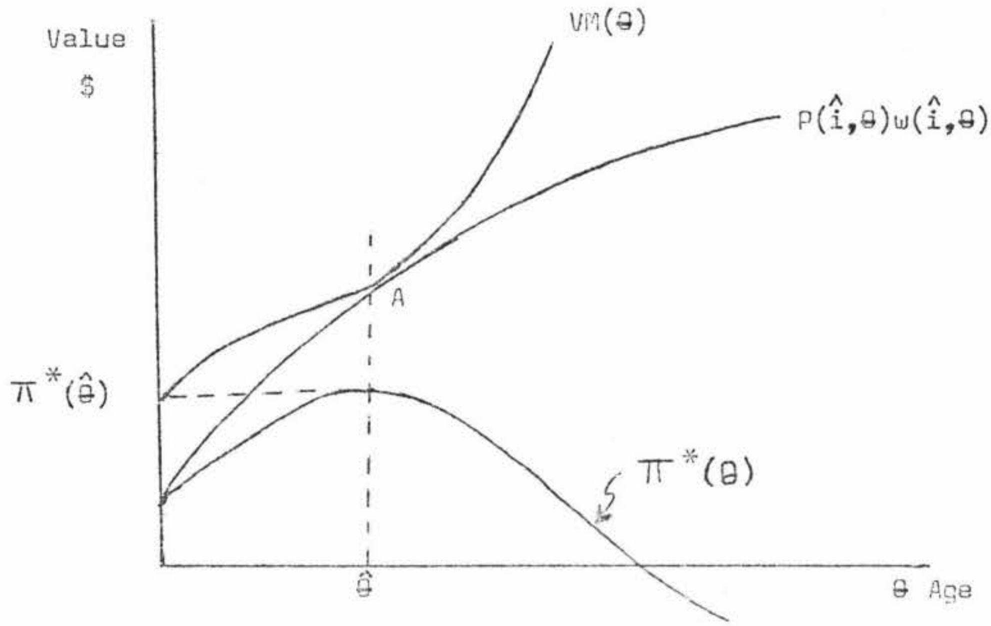

Figure 3.1 The Value of a Steer

Where:

$p(\hat{i},\theta)w(\hat{i},\theta)$	- market value of the animal over time.
$VM(\theta)$	- supply cost
$\pi^*(\theta)$	- present discounted profit
$\pi^*(\hat{\theta})$	- maximum present discounted profit (Equ. 3.5)
$\hat{\theta}$	- optimal slaughter age.

In deriving the optimal slaughter age and input stream, it was assumed that producers faced known functions for the rate of gain and the rate of change of price per unit for each animal. The product of these functions would, if graphed as a function of age, yield the locus shown as $p(\hat{i},\theta)w(\hat{i},\theta)$. This represents the market value of the animal over time. Given the previous simplifying assumptions, slaughter occurs only at one age $\hat{\theta}$, and because perfect competition is assumed, the market value of the animal at $\hat{\theta}$ must equal the cost of producing the animal.

An expression for the market value of the animal at the optimal age $\hat{\theta}$ can be obtained by rearranging (3.5).

$$p(\hat{i},\hat{\theta})w(\hat{i},\hat{\theta}) = \hat{\pi}^{*}e^{r\hat{\theta}} + \frac{c\hat{i}}{r}(e^{r\hat{\theta}}-1) \tag{3.6}$$

The cost of producing the animal (the supply cost VM (θ), which reflects the cost of feed inputs as well as the interest forgone on the value of the calf) is in fact the capital value of the animal at age θ. From birth on, the capital value VM (θ) will be equal to the sum of the initial capital value and the value of the feed intake, compoundedat rate r up to age θ.

$$VM(\theta) = \pi^{*}e^{r\theta} + \frac{c\hat{i}}{r}(e^{r\theta}-1) \tag{3.7}$$

Equating the market value and the supply cost at $\hat{\theta}$

$$VM(\hat{\theta}) = \hat{\pi}^{*}e^{r\hat{\theta}} + \frac{c\hat{i}}{r}(e^{r\hat{\theta}}-1) = p(\hat{i},\hat{\theta})w(\hat{i},\hat{\theta}) \tag{3.8}$$

which is indicated by point A in Figure 3.1. All transactions in animals will take place at the capital value WM(θ). Under the original assumption of perfect competition a calf's capital value dominates its slaughter value until age $\hat{\theta}$, and no calf will be slaughtered until this age.

Many animals are in fact sold for slaughter before their optimum slaughter age. If animals were sold at their capital values then a farmer would be indifferent between holding or selling any animal of age less than $\hat{\theta}$. This implies that for a transaction to occur at less than the optimum slaughter age, the market price must exceed the farmer's expectation of the future productive and slaughter value of that animal.

3.4 The Whole Farm Situation

The model discussed here has considered steers as consumption and investment goods and has indicated how cows could be analysed in a similar way. The feeding and selling decisions with regard to sheep can be subjected to a similar analysis. At any one point in time the farm holds a mixed portfolio of animals of different type, sex and age. A profit maximizing firm will try to equalize the rates of return to investment in its mixed portfolio. Under steady state conditions there will be a set of capital prices such that rates of return are indeed equalized.

A change in meat, wool or input prices alters not only the level of capital prices but also the relative capital prices of animals of different type, sex and age. The fact that animals are simultaneously a capital good and a consumption good imposes an additional complication to the way the adjustment to a disturbed equilibrium situation takes place. For example, a rise in beef prices would result in the capital price of a young female rising in relation to the capital price of an adult male. Since there can be only one price for a given animal, either as a capital good or consumption good, the adjustment process via changed capital prices would require that the consumption price of the young female rise relative to that of the adult male. This would require that consumers pay a higher price for female beef than for male beef and this would not be acceptable for beef of similar quality. As a result the move to a new equilibrium brought about by changed market conditions, would occur through altering the physical composition of the capital stock of animals. In the example above it would pay the farmer to purchase more young females since their capital value has increased relative to older male animals.

The model discussed here focuses on the partial equilibrium behaviour of producers facing exogenous changes in prices, although clearly such prices are endogenous to the economic system as a whole. The "instantaneous" increase in capital value which is reflected in this model is a partial equilibrium result. No account is taken of the fact that other adjustments will occur in response to a beef price increase. For example, as the capital value of an animal rises producers will respond by retaining more such animals to be used for future production, and will reduce the number currently slaughtered. The resulting reduction of current slaughter will increase even more the current price of beef but will also increase the future supply of beef, thereby lowering the expected future price of beef. As the capital value of an animal depends on expected as opposed to current prices, the movement of expected prices will dampen, at least at some point, the tendency for the relative prices of animals to change.

When considering retaining or buying-in a particular class of animal as an investment, the investment decision should be made by a comparison of its capital value (maximized present value) with the alternative slaughter value. If the capital value rises relative to the slaughter value following a price change, then this would indicate further investment in that type of animal.

In the production/investment model developed later in this thesis, it will be useful, following Yver (1971):

"..to regard the change in the composition of the sales as the manifestation of an adjustment process, triggered by changes in relative capital prices, leading to the equalization of rates of return to investment in different animals."

In the short-run, land area and technological constraints are placed on the possible investments the farm can make. Increasing meat and wool prices may call for an increase in all classes of stock. With fixed land areas and slow technological change this may not be possible. This desire to retain animals of all classes will cause a rise in the opportunity cost of feed, which in turn will prompt the slaughter of some. The animals most likely to be affected will be those which are nearing their time of slaughter, such as steers or wethers, for the capital values of animals with longer productive lives will be less sensitive to a short-run change in the cost of feed.

A rise in the prices of inputs which is expected to persist will have the opposite effect to that of a rise in meat and wool prices. This effect could be particularly important in the New Zealand livestock industry which has recently been suffering from the so-called "cost-price" squeeze.

3.5 Implications of this Analysis

The micro-models discussed in this Chapter indicate that a supply model of sheep and cattle products should be disaggregated to obtain a clear understanding of producer behaviour and improved estimates of future production. Animals of different age, sex and breeding ability have different economic functions within the herd or flock and their productive values will accordingly be differentially affected by exogenous shocks to the system.

The hypothesis is that the farmer responds to the changed structure of relative capital prices by altering the composition of his stock of animals in an attempt to equalize the rates of return to investment in his mixed animal portfolio. This emphasizes the need to treat the whole farm management system simultaneously. This is particularly important in the short-run when limitations to expansion exist, and a particular enterprise may only be able to expand at the expense of another.

The analysis indicates that the instantaneous response in the number of animals slaughtered to price increases should be negative for both cattle and sheep, since farmers may build up stock numbers when prices increase in the expectation of increased future income outweighing present income. The previous supply studies of Court (1967) and Bergstrom (1955) have derived negative short-run and long-run price elasticities of supply for mutton and beef in New Zealand. These studies have not been able to show that rising breeding herds and flocks, through higher calf and lamb crops would lead to rising slaughter over time. A reduction of slaughter one year (the transitory component) increases the size of the herd or flock in the next (and therefore the permanent component of slaughter), and it is the net effect of changes in both the permanent and transitory components which yields the true effect of price in lagged periods.

3.6 The Discrete Nature of the Data

The theoretical micro-models outlined have all been continuous. This implies that decisions are being made continuously by the farmer with regard to sales, purchases and input levels. In addition expenses are being paid and income received continuously. However, the model to which these investment principles are applied in Chapter Six is a discrete one. This arises because of the nature of the data available, which summarizes a whole year's activities by statements of the situation at the beginning of a year and the end of that year. In a sense, this implies that everything takes place on only one day of each year.

The continuous situation is the more realistic, however many activities do take place in a discrete way. The important thing is that the investment principles outlined above remain the same whether the model is continuous or discrete.

CHAPTER FOUR

AGRICULTURAL SUPPLY ANALYSIS

This Chapter reviews the Theory of Supply and follows this with a review of empirical work reported on supply analysis of New Zealand agricultural products. A number of overseas studies are discussed and the techniques used in these studies are contrasted with the technique developed in this thesis. The chapter concludes with a review of Price Expectation models, and a description of the Price Expectation model used in this thesis.

4.1 The Production Foundation

The Theory of Supply has its origins in profit maximizing principles and rational economic behaviour by individual entrepreneurs. Under conditions of perfect knowledge with respect to all variables, a firm's static supply function could be derived from the production function, given a goal of profit maximization for competitive firms.

Consider the case of a firm using two inputs (x_1 and x_2) to produce one output (Y). Its production function shows the maximum product output obtainable from various levels of factor inputs and can be written;

$$Y = f(x_1, x_2) \qquad (4.1)$$

Where
- f is a function
- Y, x_1, x_2 are divisible
- x_1, x_2 are continuous variables
- the production function f is continuous and twice differentiable.

The profit for this firm can be represented by the following function;

$$\pi = Py\ f(x_1, x_2) - P_1x_1 - P_2x_2 \tag{4.2}$$

Where: π - profit

Py - price of the product

$f(x_1, x_2)$ - the production function

P_1, P_2 - prices of the inputs x_1 and x_2 respectively.

The necessary condition for maximum profit is;

$$\frac{\partial \pi}{\partial x_1} = Py\ f_1 - P_1 = 0 \quad Py\ f_1 = P_1 \tag{4.3a}$$

$$\frac{\partial \pi}{\partial x_2} = Py\ f_2 - P_2 = 0 \quad Py\ f_2 = P_2 \tag{4.3b}$$

The product prices multiplied by the marginal product of an input is called the marginal value product of the input. Equations (3a) and (3b) state that for an optimum, the marginal value product must equal the input price. They imply:

$$\frac{P_1}{f_1} = \frac{P_2}{f_2} = Py \tag{4.4}$$

It can be shown that the input price divided by the marginal product is the marginal cost. Hence, for an optimum, marginal cost is equal in terms of each input, and the equalized marginal cost is equal to the product price. This result, based on the production function, is the same as that derived from the cost function.

In the short-run the firm's problem is to minimize total cost subject to the constraint of the production function.

The Lagrangean function L for this minimization problem is;

$$L = P_1x_1 + P_2x_2 + b + \lambda \left[Y - f\ (x_1, x_2) \right] \tag{4.5 }$$

$$= C + \lambda \left[Y - f\ (x_1, x_2) \right] \tag{4.5b}$$

Where: $C = P_1x_1 + P_2x_2 + b$ is the total cost

b = fixed costs

λ = Lagrangean multiplier

Solving the problem for x_1 and x_2 and the Lagrangean multiplier in terms of P_1 and P_2 and Y will give a cost function

$$C = g(Y) + b \tag{4.6}$$

Where: g = a function.

The marginal cost function is therefore:

$$\frac{dC}{dY} = g'(Y) \tag{4.7}$$

The firm as a market agent is producing an optimum amount of output when it maximizes its profits i.e. total revenue minus total cost:

$$\pi = P_yY - g(Y) - b \tag{4.8}$$

The assumption is made that the demand curve for the firm's production and its own demand curves for its factors of production are completely elastic. Under these assumptions profit in (4.8) is a function of output Y alone. For the firm to maximize its profits, the first-order condition is:

$$\frac{d\pi}{dy} = Py - g'(Y) = 0 \tag{4.9}$$

i.e. product price = marginal cost.

The second-order condition for maximum profits is:

$$\frac{d^2\pi}{dY^2} = -g''(Y) < 0 \tag{4.10}$$

i.e. marginal cost must be rising.

The marginal productivities f_1 and f_2 in (4.4) are functions of x_1 and x_2. Hence they make up a system of two equations in two unknowns x_1 and x_2. Solution of this system of simultaneous equations can yield the demand functions for the two factors:

$$x_1^* = g\ (Py,\ P_1,\ P_2) \quad x_2^* = h\ (Py,\ P_1,\ P_2) \qquad (4.11)$$

and the product-supply function:

$$y_j^* \equiv f_j\ (x_1^*,\ x_2^*) \equiv F_j\ (Py,\ P_1,\ P_2) \qquad (4.12)$$

where: g, h, f_j, F_j are functions

y_j^* – product supply from the $_j$th firm in the industry.

If, as assumed above, the demand curves for the inputs are infinitely elastic then the industry supply function is obtained by the horizontal summation of the individual firm's supply functions.

$$Y = \sum_{j=1}^{n} y_j^* = \sum_{j=1}^{n} F_j\ (Py,\ P_1,\ P_2) \qquad (4.13).$$

The static supply function derived from the relevant production function and set of commodity prices provides a conceptual starting point in any analysis of output responses. The analysis can be made more realistic by assuming that a firm is concerned with an optimal production plan over a number of years, rather than just a single period as has been considered up till now. In many instances the firm is free to borrow and lend money in addition to, or as an alternative to, purchasing inputs for a production process. Given these opportunities, the firm will generally desire to maximize the Present Value of its profit from production subject to the technical constraints imposed by its production function(s). As for the static situation discussed above, the individual firm's input demand and output supply functions can be derived. The individual firm's supply of a product through time can be expressed as a function of output prices, input prices and interest rates, with all prices to be received or paid in future periods being appropriately discounted (Henderson and Quandt, 1971 pp 311-312).

4.1.1 Empirical Supply Analysis

The equations (4.1) through to (4.13) provide the inventory of types of variables which are used in deriving actual output response functions. Techniques such as budgeting, mathematical programming, simulation and regression of time-series data have been used in attempts to quantify supply responses to price changes and other variables of interest. In addition to the various techniques that can be used, there are various levels of aggregation from which the problem can be approached. These range from national to individual farm response studies.

Nerlove and Bachman (1960) have reviewed the problems and approaches in supply analysis prior to the 1960's, and while a considerable amount of work has been done on supply analysis since then, the problems remain far from being fully solved. They list four important theoretical gaps in supply analysis:

1) An adequate theory of aggregation for firm supply functions.
2) An adequate theory of behaviour under uncertainty.
3) An adequate operational theory of investment of the firm, i.e. how so-called fixed factors are varied over time in response to economic and other forces.
4) A theory of, or at least techniques for measuring, the diffusion of technological changes.

The use of representative farms and the resulting aggregation problems are discussed in Chapter Five. A number of studies which have proceeded part of the way towards solving some of the other problems are discussed in Section 3 of this Chapter.

4.2 Positive and Normative Analyses

The major distinction between positive and normative analyses is adequately described by Heady (1961):

> "Positive analysis can be described as prediction of quantitative relationships among variables as they actually do exist at a point in time, or have existed over a period of time. In contrast, normative analysis refers to what ought to exist, under certain assumptions. It is an indication of what might be expected to happen if decision makers possess certain goals and knowledge and are free from certain resource and institutional restraints."

The most common type of positive supply analysis involves regression of time-series data. The major limitation of this technique is that if historical data are used then the model is necessarily tied to the past, and there are problems in incorporating major changes in technology, institutions and government policy in the model. In addition, because of statistical necessity, regression models are highly aggregate with respect to inputs and cannot reflect quantitative effects of many specific variables of interest.

Normative supply analysis techniques include budgeting, programming, judgement and related methods. Programming techniques usually take the individual farm as the unit to be analysed and thus aggregation problems arise if regional or national aggregates are required. A common criticism of normative supply analysis techniques is that they usually require a profit maximization assumption and this may not reflect the aspirations of a great many farmers.

The classifications discussed above are not mutually exclusive. Positive regression analyses are in fact based on the analysis in Section 4.1. In regressing the quantity supplied of a product against its price the implied assumption is one of profit maximization of the individual or firm involved. Conversely, a linear programming model of supply is considered normative in nature, however the input-output coefficients in the simplex tableau are usually obtained in a positive manner from observations drawn from "real world" situations.

4.3 Literature Survey

This section reviews previous attempts to model New Zealand Agricultural Supply and overseas studies that have potential to improve on these.

4.3.1 New Zealand Studies

Previous attempts to analyse the supply of New Zealand sheep and beef products have been reported by Bergstrom (1955), Court (1967), Rayner (1968) and Johnson (1970).

Bergstrom's (1955) supply model was an integral part of a system of simultaneous difference equations explaining supply of and demand for New Zealand's exports. Supply of agricultural products was estimated as a function of product prices and stock numbers. Past prices, rather than expected prices are among the explanatory variables.

Bergstrom derived negative short-term and long-term price elasticities of supply[1] for the main sheep and beef products. He explains this as being due to some behavioural aspects of farmers' investment processes i.e. preference for "normal" income rather than maximum profit.

Rayner (1968) used single equation multiple regression techniques to predict animal numbers in various sex/age categories. Explanatory variables used were product prices and time, to allow for technological change. The model was validated by comparing its

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(1) The price elasticity of supply is:

$$E_s = \frac{\Delta Q_{sj}}{Q_{sj}} \Big/ \frac{\Delta P_j}{P_j}$$

where: E_s - Elasticity

Q_{sj} - Quantity supplied of product j.

ΔQ_{sj} - Change in quantity supplied of product j.

ΔP_j - Change in price of product j.

P_j - Price of product j.

This is a <u>ceteris paribus</u> concept. i.e. it attempts to measure the response of supply of a product to a change in its own price, assuming all other things are unchanged. This is of doubtful theoretic value where such independencies do not exist. In a dynamic multi-product agricultural industry such independencies would be extremely rare.

Elasticities have remained of interest because many statistical single commodity models produce them as a single invariant parameter for response estimates. This thesis does not attempt to estimate any elasticities, however, it is useful to discuss previous studies which have estimated them.

predictions with actual 1967 data and the results found to be poor, especially in the case of ewes and ewe hoggets.

Court (1967) used regression techniques to estimate a system of simultaneous equations that attempts to explain both supply and current prices of lamb, mutton and beef. A theoretical model was developed first based on the hypothesis that a farmer will attempt to maximize his expected (discounted) total income up to some horizon, subject to restrictions on wool production and stock numbers and to his ability to vary stock numbers over time.

These restrictions are incorporated in the objective function through the use of Lagrangean multipliers. Differentiation of this objective function with respect to outputs, stock numbers and multipliers results in a system of simultaneous equations, from which expressions for meat and wool output and stock numbers can be obtained. The resulting equations explaining supply are functions of ratios of prices within the same season and between different seasons. A modified adaptive price expectation model was used to generate expected prices.

The final functions derived for meat supply, as an example, were:

$$x_{Lt} = S_L \left(\frac{P_{Lt}}{P_{mt}}, \frac{P_{wt}}{P_{Lt}}, \frac{P_{Lt}}{P_{Lt-1}}, E \right)$$

$$x_{mt} = S_m \left(\frac{P_{Lt}}{P_{mt}}, \frac{P_{wt}}{P_{Lt}}, \frac{P_{Lt}}{P_{Lt-1}}, E \right)$$

where: S_L, S_m = functions

x_{Lt} = lamb supplies in year t.

x_{mt} = mutton supplies in year t.

P_L = expected price of lamb.

P_w = expected price of wool

P_m = expected price of mutton.

E = time shift operator from the Price Expectations model.

In order to obtain suitable equations for estimation purposes the functions S_L and S_m must be approximated by convenient functions.

The theory is developed for an individual farmer, but the estimated model is an aggregate one. The theory suggests which variables should be included in the model to be estimated, however, it does not state explicitly how they should be included. Court (1967, p 295) states; "It is not assumed that there is any necessary or stable relationship between micro-parameters and macro-parameters, but only that the supply macro-parameters exist and are stable over time."[1]

Court also obtained negative short-term and long-term elasticities of supply for the main sheep and beef products. He suggested that if livestock numbers are used as explanatory variables rather than lagged supply of lamb and mutton, a better model could be developed. This would enable changes in livestock numbers over time to be better explained.

Under the definitions of Section 4.2 the three studies discussed so far represent positive approaches to supply analysis. All are statistical analyses based on aggregate time-series data.

In contrast Johnson's (1970) study is normative in nature. He used a linear programming (LP) model to make projections of future sheep and beef production in New Zealand. The model was based on the New Zealand Meat and Wool Boards' Economic Service's farm classification, the unit of analysis within each region being a representative farm.

The model assumed that farmers are profit maximizers. It also assumed farmers are aware of all the opportunities open to them and that they are sufficiently flexible to bring about changes that are required. The objective of the LP was to maximize total net revenues considering all relevant activities[2] over all the regions. Each region was represented by an average farm which incorporated the range of productive activities found in each region. Resource availabilities and activity requirements were estimated on a per farm basis. Once the activity levels for each average farm had been solved for, the regional totals were found by aggregation and finally the national totals by aggregation of the regional totals through the use of a raising formula which weighted each class according to some criteria.

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(1) Problems resulting from aggregation are discussed in Section 5.2.

(2) Example of an activity - sheep breeding.

Intra-regional constraints included the levels of resource supplies such as land and capital, and constraints on the present and potential limits of given crop and stock policies in each region. Inter-regional constraints were included to control stock movements between regions and the rate of growth of the national beef breeding herd. These satisfy the requirements that net sales of various classes of store stock from all regions are balanced by net purchases. Various combinations of high and low levels of expected prices for products were chosen and the resulting range of projections of stock numbers and wool production presented.

4.3.2 Overseas Studies

The studies of Evans (1971), Yver (1971) and Jarvis (1974) are similar to each other in approach. Each developed an extensive model of production response in the cattle industry, Evans for the U.K. and Yver and Jarvis for Argentina. In each case the national cattle industry is regarded simply as a scaled up version of an individual vertically-integrated beef enterprise, with the qualification that the 'aggregate' producer responsible for the enterprise also engages in external trade. The theoretical framework on which these studies are based has been discussed in Chapter Three of this thesis.

Evans' (1971) models are systems of regression equations linked together to represent the particular sequence of operations assumed to characterize the decision-making pattern of cattle production. Primary decision equations describe how producers respond to changes in values of economic variables by altering cattle inventory levels and flows. They are primary decisions because their outcome affects other decisions taken in later periods. Secondary decision equations also describe producer behaviour but the predicted values of the dependent variables are not fed back into the model as part of a causal chain. e.g. retaining a certain number of the calves born in any period for further rearing is a primary decision, while the secondary decision is to dispose of a certain number of calves by sending them to be slaughtered. The model also includes a number of price formation and technical equations.

Yver (1971) and Jarvis (1974) estimated systems of simultaneous equations using time series data. The theoretical models they developed provided the frameworks for econometric models of the Argentinian beef sector. Both studies obtained negative short and positive long-term price elasticities of slaughter, contrary to previous studies which had obtained negative long-term elasticities. Both argued that the previous studies using single equation estimation techniques were unable to show that a rising herd, through higher calf crops, would lead to rising slaughter over time.

These studies imply that an econometric model of livestock production should be disaggregated to obtain a clearer understanding of producer behaviour and better estimates of future output. Animals will have different economic functions (investment functions or output functions) and will be affected differently by exogenous variables according to their age, sex or breeding ability.

Carvalho (1972) reported a study of the United States cattle industry. He first derived a model based on a framework very similar to that of Yver (1971) and Jarvis (1974). The 'aggregate' cattleman is assumed to maximize profits for his entire productive life. i.e. maximize the expected net present value of his enterprise. The problem is to derive the producer's present value expression as a function of decision variables and maximize its expected value with respect to those decision variables. The approach used for representing the dynamic features of the problem is the concept of a desired stock (by class and age distribution). The cattleman compares a desired stock of cattle with his actual stock, assumed to be different from the desired, and his action is to adjust his stock towards the desired, in accordance with some allowance for adjustment costs.

This theoretical framework provides the inventory of variables used in a simultaneous equation regression model. Carvalho (1972) then suggested that construction of an explicit profit function and its maximization may be a better way to approach the problem. In this case, all the arbitrary assumptions are concentrated in the construction of the profit function, so the maximizing solutions can be estimated without additional simplifications. A simple profit function was constructed which was quadratic in its arguments, thus guaranteeing a unique maximum. The maximization was accomplished by dynamic programming techniques and was used to obtain the relevant behavioural functions, which were then estimated by Ordinary Least Squares regression. The major problem encountered by Carvalho was the severe computational burden in deriving the behavioural functions.

Shecter (1968) reported a complex study, the methodology of which could have application in supply analysis. It involves a model based on a behavioural theory of farm firms. Behavioural theories can account for the fact that while profit maximization may be hypothesized as the goal of a farmer, for various reasons this may not be achieved. It may stem from a cautious attitude on account of some unfortunate past experience when a similar situation existed; or, the farmer may simply wish to maintain the present size of an operation because it fits his objectives, which may or may not be strict profit maximization. Behavioural theories incorporate a theory of search in addition to a theory of choice. A decision maker faced with an uncertain situation would usually seek some additional amount of evidence and information to aid him in making his decisions. Behavioural models can allow for changing goals or aspirations of firms over time as a result of experience. The revision can be done through a feedback control mechanism. Shecter incorporated these basic propositions of a behavioural theory of the firm into simulation models, which attempt to represent the problem solving processes of the firm and the ensuing decisions.

Day (1963) used recursive programming (R.P) to estimate the production response of cotton and alternative field crops. R.P is the problem of optimizing an infinite set of resursively generated linear functionals subject to an infinite set of recursively generated linear constraints. This differs from multi-period linear programming which is the problem of finding a set of functions which optimize a single linear functional subject to a set of intertemporal linear constraints. Both approaches allow incorporation of important production dynamics. Basically the R.P procedure involves a series of linear programmes linked by "flexibility constraints". These constraints reflect such things as the needed delay for setting up new investments, the duration of production cycles, producer attitudes towards uncertainty and risk, and producers' passive resistance to change. The major criticism of this approach is that it stresses one-period decisions, rather than allowing for the possibility that a farmer will make his decisions on the basis of a multi-period plan.

However, Day (1963) discussed the possibility of using a multi-period linear programming model in a recursive way.

> "The actual process of planning over time can be simulated by embedding so called dynamic linear programming models into a recursive structure. This combines the anticipatory nature of planning with the necessity of continual re-evaluation and reformation."

This procedure was in fact adopted by Chien and Bradford (1974). The conceptual framework of their model is very similar to that used in the model developed in this thesis. The unit of analysis was the individual farm, and based on his expectations and goals the farmer was assumed to form an ex-ante long-run plan. Only the first step of this plan is definitely implemented. The farmer's expectations and thus his plans are changed as new information is obtained. Chien and Bradford's technique then takes into account the fact that in reality the farmer may not be able to, or may not wish to maximize net returns. This could arise because of uncertainty (in prices and yields), personal preference for keeping an established farming practice or a number of similar reasons. This means that the "optimal plan" which is based on the assumption of perfect rationality in the ex-ante planning model may be modified.

The technique used involved a multi-period linear programme/simulation combination. Only the first or current year solutions of the M.L.P. model are directly of concern, since they are the only activities of the current plan that are actually implemented.

A simulation model then adjusts the "optimal" levels of some of the first year activities found by the M.L.P. model to take account of factors such as those discussed above. These results are then passed to a further M.L.P./simulation model, and the process continued in a recursive manner. In this way the model can be used to make projections.

4.3 An Alternative Technique

The studies reviewed in the previous Section encompass a wide variety of alternative approaches. These range from the less complex statistical models to the very complex behavioural simulation approach. The approach used in this study incorporates features from several of these studies. The unit of analysis is a representative farm. Following Carvalho's suggestion the farmer's explicit multi-period objective function is written out and maximized with respect to decision variables, subject to constraints. The maximization is carried out by differentiation of the objective function which is quadratic with respect to the decision variables and thus a unique solution can be obtained. The constraints are production constraints and restrictions on the stock numbers attainable, in the form of stock reconciliations, which are incorporated by direct substitution into the objective function.

The approach is thus very similar in principle to a multi-period L.P. Model (or more strictly a multi-period quadratic programming model). However, an analytical solution is obtained, from which a numerical solution can be obtained by appropriate substitution. A programming model will only give a numerical solution.

The decision variables solved for at any point in time represent current decisions, in that they are the first year decisions of an envisaged five year plan. Only these decisions are actually implemented and since they are current they can be compared with actual values of the decision variables from available data. This means that the ability of the model to predict values for the decision variables can be improved in a pragmatic manner. Certain parameters within the model can be estimated so that the values for the decision variables estimated by the model are as close to the actual values as possible. It is then hypothesized that if the model can predict the decision variables accurately it should be able to predict the resulting stock numbers accurately.

If the model in this study was set up as a programming problem then the decision variables could be solved for in the usual way. The solution could then be compared with actual data and attempts made to improve the ability of the model to estimate the decision variables. These attempts would involve parametizing the model. This could only be done essentially in an ad hoc way, unlike the approach used if an analytical solution is available.

The framework outlined above enables the important investment principles discussed earlier to be incorporated in a model that can be used to make projections of livestock numbers. The estimation technique discussed above is described in greater detail in Chapter Six.

4.4 Models of Price Expectation Formation

To make dynamic economic models complete, various expectations formulae have been used. Early this century Economists recognised that for some commodities the current price was determined by the size of the current crop; while the current crop was influenced by the previous year's price. This indicated that the farmers must have some expectation of the future price, when they plant their crop. The recognition of this lag effect led to the formulation of the Cobweb model. Since then various alternatives have been suggested to explain price expectations, however it is unlikely that there is a unique explanatory mechanism.

4.4.1 Cobweb Models

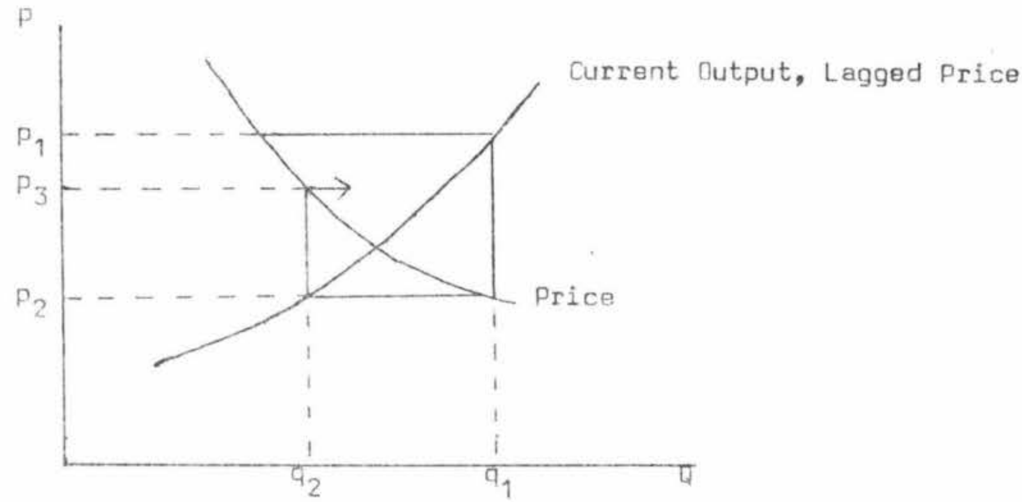

Figure 4.1 A Simple Cobweb Model

In Figure 4.1 the price curve shows how current prices are related to current production. The output curve shows how current output is related to past prices. These curves are not the Cournot-Marshall demand and supply curves, nor are they simultaneous. Ezekiel (1938) was careful to point out that the price curve showed how current production affected current price - not the amounts some group of consumers would buy if the price were set at various levels. He emphasized especially that the lagged-output curve differed greatly from a supply curve indicating how much of a commodity sellers would be willing to offer currently at various current prices.

Provided both curves are linear a model will converge to an equilibrium if the lagged-output curve is steeper than the price curve; will oscillate continuously if the slopes of the two curves are equal; or will diverge if the price curve is steeper than the lagged-output curve. As Ezekiel points out, the Cobweb theory can only apply to commodities which fulfil three conditions:

1) Production is completely determined by the producer's response to price, under conditions of pure competition.

2) Where the time needed for production requires at least one full period.

3) Where the price is set by the supply available.

These restrictive conditions, the requirement for linear or well behaved curves, and the poor empirical support of the theory have been frequently presented as arguments against it. Since farm production decisions must be made in advance (the period of production is at least one year) these decisions must be based on expected prices. The Cobweb theorem postulates that these expected prices, P_t^*, are the current prices at the time of the production decisions, P_{t-1}.

4.4.2 Extrapolative Expectations

The naive price expectations assumption of the Cobweb model led Metzler (1941) to propose an alternative, the extrapolative expectation model, the purpose of which was to make the Cobweb theory take into account the most recent trend in prices.

$$P_t^* = P_{t-1} + \eta(P_{t-1} - P_{t-2}) \qquad (4.14)$$

Where:

P_t^* - expected price for period t at period t-1.
P_{t-1} - observed price in period t-1.
P_{t-2} - observed price in period t-2.
η - coefficient of expectation.

The extrapolative property of this expectation formation model holds only if $1 > \eta > 0$. If $\eta = 0$ the model becomes the traditional Cobweb model and expectations are said to be static. If $\eta < 0$ the expected price will be a weighted average of the past two prices with weights $1 - |\eta|$ and $|\eta|$ for P_{t-1} and P_{t-2}, respectively.

4.4.3 Adaptive Expectations

The concept of adaptive expectations was used extensively by Nerlove (1958). Under the adaptive expectation hypothesis, the individuals are assumed to revise their expectations according to their most recent experience.

$$P_t^* - P_{t-1}^* = \gamma(P_{t-1} - P_{t-1}^*) \quad \text{for} \quad |\gamma| < 1 \tag{4.15}$$

Where:

P_t^* - expected price for period t at period t-1.

P_{t-1}^* - expected price for period t-1 at period t-2.

P_{t-1} - observed price in period t-1.

γ - coefficient of expectation.

Expansion of (4.15) yields a useful expression for P_t^*.

$$P_t^* - (1-\gamma)\, P_{t-1}^* = \gamma\, P_{t-1} \tag{4.15a}$$

Replacing $(1-\gamma)$ by β and introducing the lag operator B, such that $B^i x_t = x_{t-i}$

$$(1-\beta B)\, P_t^* = \gamma\, P_{t-1} \qquad \text{where } \gamma = 1-\beta \tag{4.15b}$$

so that:

$$P_t^* = \left(\frac{1-\beta}{1-\beta B}\right) P_{t-1}$$

Now provided $0 < \beta < 1$

$$\frac{1}{1-\beta B} = 1 + \beta B + \beta^2 B^2 + \beta^3 B^3 + \ldots\ldots,$$

$$\therefore \quad P_t^* = (1-\beta) \sum_{i=0}^{\infty} \beta^i\, P_{t-1-i} \tag{4.15c}$$

Under the adaptive expectation hypothesis, the expected price may be expressed by an infinite weighted average of past realized prices with weights which decline geometrically with the lag. Much of the criticism of the adaptive expectations theory has to do with its implication of geometrically decaying lag structure. There is no economic explanation for this lag structure other than the form of the expectation model. Despite this, adaptive expectations have been popular because of their simplicity, because maximum likelihood estimates for β can be obtained(1) and because they appear to work well in a number of empirical studies.

(1) Although not from expression (4.15c).

4.4.4 Rational Expectations

Muth (1961) advanced the hypothesis that expectations that are formed are essentially the same as the predictions of the relevant economic theory. If a producer operating under free competition has some idea of market conditions, he will use the information available to him about demand and supply conditions in generating his expectations about the relevant variables for decision purposes. Muth (1961) uses a simple example to explain the hypothesis:- Short-period price variations in an isolated market with a fixed production lag of a commodity which cannot be stored.

The market equations take the form:

$$\begin{aligned}
C_t &= -\beta p_t && \text{(demand)} \\
Q_t &= \gamma p_t^{*} + u_t && \text{(supply)} \\
Q_t &= C_t && \text{(market equilibrium)} \\
p_t^{*} &= E_{t-1}(p_t) && \text{(expectation formation)}
\end{aligned} \qquad (4.16a)$$

Where:

C_t = quantity demanded in period t

Q_t = quantity supplied in period t

p_t = market price in period t.

p_t^{*} = expected price for period t in period t-1.

E_{t-1} = an expectation function.

The expectation function results from Muth's suggestion that the expected price for period t+1 at period t must be equal to the expected value of the market equilibrium price for period t+1 as expected in period t.

$$p_{t+1}^{*} = E_t\,(p_{t+1})$$

i.e. Expectations are unbiased and the expected price is treated as endogenous to the system.

In the model (4.16a) above there are four endogenous variables: C_t, Q_t, p_t, and p_t^{*}. Solving the system for p_t^{*}. (Muth 1961, pp 318-320):

$$P_t^* = \frac{\beta}{\gamma} \sum_{j=1}^{\infty} \left(\frac{1}{1+\frac{\beta}{\gamma}}\right)^j P_{t-j} \tag{4.16b}$$

This equation is very similar to the adaptive expectation formulation (4.3c). The difference is that here the analysis states that the "coefficient of adjustment" in the expectations formula should depend on the demand and supply coefficients, whereas the adaptive expectation model arbitrarily defines an expectation coefficient.

4.5 Price Expectation Formation - New Zealand Farmers.

To test the hypotheses discussed, in the New Zealand situation, data from a random survey of farmers were analysed. The survey was designed to obtain information as to how farmers assess the future profitability of their enterprises from the information available to them.[(1)(2)]

4.5.1 The Survey

The survey was conducted over 130 randomly selected farmers from four different farm classes during April of 1975. The respondents were asked for their price expectations (budget prices) for commodities they would buy and sell during the 1975/76 year. They were also asked the prices they had previously expected to receive during the 1974/75 year.

Responses were generally good in that the farmers were consistent in their answers, willing to provide the information and held similar ideas on 1975/76 prices. Initially a sample of 180 farmers was drawn randomly from a list of all sheep farmers in New Zealand. The sample

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(1) I am indebted to Mike Davey who conducted the survey.

(2) For a comprehensive survey conducted in the U.S. see Heady and Kaldor (1954). According to them (p35) "no single procedure was employed by all farmers. Moreover, the same farmer often used more than one procedure, depending upon the amount of information possessed and upon the degree of confidence attached to it."

was stratified in that there were 45 farmers belonging to each of four farm classes:

1) Hard hill country, North Island.
2) Intensive fattening farm, North Island.
3) Fattening-breeding farms, South Island.
4) Intensive fattening farms, South Island.

The response rate was 72%, reasons for non-response being mainly retirement from farming or the farmer having changed his type of farming enterprise.

4.5.2 Survey Results

The survey results were analysed to provide an answer to the question:

How do farmers assess the future profitability of their enterprises from the information available to them?

Two commonly accepted explanations of how farmers assess future profitability were tested. The first explanation is that farmers assess future profitability by extrapolating recent trends in prices. This explanation was rejected as it did not give consistent results when tested. The second explanation is that farmers adapt their assessment of profitability in the next period according to the errors that they made in their previous assessment for the current period.

The second explanation was found to be consistent and was a good description of how farmers formulate their future price expectations. This explanation is described mathematically by the adaptive expectations model discussed in Section 4.4.3. No attempt was made to test Muth's rational expectations hypothesis since an expectation model of this sort would not fit into the framework of the production/investment model developed in Chapter Six of this thesis.

The results obtained are of a very tentative nature and a continuing survey over a number of years would be required to obtain the true nature of price expectation formation. However, the results are adequate enough to provide estimates of the parameters of an adaptive expectations model to be used in this study.

4.6 The Price Expectation Model

The Adaptive Expectations Model used is:

$$EP_t - EP_{t-1} = \gamma (P_{t-1} - EP_{t-1})$$

where

EP_t = expected price for period t at period t-1
EP_{t-1} = expected price for period t-1 at period t-2
P_{t-1} = observed price in t-1
γ = coefficient of expectation

From the survey data, γ was estimated by Ordinary Least Squares regression for the following products:

Product	$\hat{\gamma}$
Lamb	0.8551
Cull ewes	0.9145
Wool	0.9199
Beef	0.8563
Two-tooth ewes	0.6802
Weaner steers	0.8504
Yearling steers	0.6914

CHAPTER FIVE

THE REPRESENTATIVE FARM TO BE MODELLED

This Chapter is introduced by a discussion of the different farming systems in New Zealand. The use of representative farms to describe these different systems, and the New Zealand Meat and Wool Board's Economic Service's farm classification, are outlined. Details of one such class, the North Island Hill Country farm, are discussed as an introduction to the production/investment model developed in Chapter Six.

5.1 Farming in New Zealand

Exports of pastoral products have historically made up over 80% of New Zealand's total exports. Agricultural production has grown in a way which has resulted in increased quantities of dairy products, meat and wool for export. A general improvement in management practices and a heavy rate of investment in land improvement have been the principle factors responsible for the expansion of output. The pattern is one of more intensive farming, the area used for farming having not expanded much since early this century.

The types of farming carried out in any particular area are mainly influenced by climate and topography, although soil fertility, location of markets and servicing industries and historical usage also are major determinants. In the high country of the South Island breeding flocks of fine-woolled sheep dominate. There is some scope for varying the proportion of dry sheep, and there are possibilities for breeding cattle. In the foothill country of the South Island, breeding medium-fine wool sheep also predominates, but there is considerable potential for both breeding and finishing cattle. Most lambs are now fattened. The mixed hill and light land country of the South Island is similar to the foothill country, but crossbred sheep now become important, a higher proportion of lambs are fattened, and a small area is available for cash or forage crops.

In Southland, the choice is whether to breed crossbred wool sheep or to buy in mixed age crossbred replacements, to produce fat lambs along with both cattle alternatives, and a small crop area. Carrying capacities are much higher in this region and most surplus lambs fattened. In Canterbury cropping is dominant and other alternatives include buying-in fine woolled replacements, as well as a choice of cattle breeding and fattening.

In the North Island, cattle are frequently more important, and most sheep are crossbred, although some distinctive breeds (e.g. Perendale, Coopworth) are becoming important. On hard hill country, the farmer is restricted to a choice of expanding his breeding flock or breeding cattle. In either case a considerable proportion of income comes from store stock and wool with very little finishing of stock being possible.

On North Island hill country, farmers have a choice of breeding or buying their own replacements of both sheep and beef cattle. Stock can be turned off in fat or forward condition. On the best North Island pastoral country, fattening stock and dairy farming are the main alternatives, plus cash cropping for specialised crops like maize, potatoes, peas and some grain.

In general, there is a wide diversity of enterprises undertaken by New Zealand farmers. Despite this, there are reasonably well defined systems based on combinations of these enterprises. These systems can be classified and a representative farm chosen or hypothetically constructed to describe both the productive processes and the decision processes of each system.

5.2 Representative Farms and the Aggregation Problem

A considerable amount of work has been done on the problems of using representative farms, and the associated aggregation problems when regional or national supply response estimates are made.

Following Barker and Stanton (1965), the procedure most commonly used is:

1. Stratify the region into areas based upon type of farming or other relevant characteristics such that the farms within the area might be expected to have similar enterprise alternatives, and to be faced with similar yield potentials, prices, and costs.

2. Sample each area to provide a basis for sorting farms into homogeneous groups based upon size, soil type, or other relevant factors.

3. Define a representative farm for each stratum (actual or hypothetical).

4. Derive supply functions for each such farm.

5. Aggregate the supply functions.

Aggregate supply response is obtained by the summation of weighted representative farm supply responses. Farm input/output relationships are assumed to be the same at all levels of aggregate production.

5.2.1 Selection of Representative Farms

In any farming region there is usually a diverse variety of farmer managerial abilities, financial and economic circumstances, soil and physical characteristics and farm resources. Almost every individual farm departs widely in one or more important characteristics from the so-called norm. Thus the objective in defining a representative farm is to isolate the primary characteristics of the farms and farmers that tend to dominate or strongly influence the particular decision under study. Some of the apparently more important characteristics such as managerial ability or risk aversion, which affect supply responses are the most difficult to quantify.

Frequently it is assumed that each stratum (Barker and Stanton's Step 2) being considered is homogeneous or normally distributed with respect to a relevant attribute. The representative farm (actual or hypothetical) is defined to be that which is characterized by the mean[(1)] level of the attribute being considered. A decision must be made as to what is an acceptable degree of variability (measured by variance) around the mean of the attribute. If the variability is unacceptable then a further stratum may have to be defined with another representative farm. As the number of strata for a given population of farms is increased the variance of the estimate of the mean attribute level for each stratum can be reduced. However, increasing the number of representative farms undermines the purpose of using them in the first place, and increases costs and time required for analysis.

The representative farm becomes much more difficult to define as economic time (and calendar time) becomes longer. A typical farm however selected, remains typical only as long as the technology, institutions, and other attributing factors remain static.

Two main problems face the economist in defining representative farms. The first is the criteria to be used in sorting the farms. The second is the question of the number of representative farms that are necessary to reduce the aggregation error to a tolerable degree.

5.2.2 Aggregation Error

Aggregation error is the difference between an area supply function as developed from the summation of supply functions for each individual farm in the area, and summations for a smaller number of representative farms.

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(1) In some instances the mode may be a better measure of central tendency to use e.g. Adeemy and MacArthur (1968).

An example should help to clarify this:

Consider an area with two farms A and B.

Table 5.1

Resources	Farms		Average
	A	B	
Labour	50 units	150	100
Capital	100 units	100	100

Each farm has two resources, capital and labour and produces one product, say Y. The production function is such that it requires 10 units of labour and 10 units of capital to produce 1 unit of Y. Farm A can produce 5 units of Y before its labour is exhausted, Farm B 10 units before its capital is exhausted.

Total production = 15 units of Y

If the resources are averaged the representative farm would have 100 units of capital and 100 units of labour.

Production = 10 units of Y

Aggregation involves multiplication of this by two.

Total production = 20 units of Y

∴ Aggregation error = 5 units of Y

The error in this example resulted because the farms in the sample differed according to their most limiting resource - labour for farm A and capital for farm B. The method of selecting representative farms according to their most limiting resource is a common one (Frick and Andrews, 1965). In general, aggregation error is a result of how well the representative farm describes the population from which it is derived.

5.2.3 Other Sources of Error

Stovall (1966) considers two other types of error in addition to aggregation error;

a) Specification error arises because the model does not accurately reflect the conditions actually facing the farm firm. It may include errors in the technical coefficients, the resource restraints, or the prices (product or input).

b) Sampling error arises because the parameters which characterize the population of farms are estimated from a sample.

The three types of error are not independent. Specification error, for example, may result in part from sampling error. Either sampling or specification errors may lead to aggregation error, though there may be still other sources of aggregation error which are independent of both.

5.2.4 Aggregation Bias

The previous sections have discussed the aggregation problem when the aggregate supply response is obtained by the summation of weighted representative farm supply functions. However, a number of supply studies (e.g. Court (1967), Yver (1971)) develop theoretical supply models based on individual farmer maximization behaviour and then estimate the model using aggregate data. This is called the analogy approach (Theil, 1954). The aggregates are averages, index numbers, or national totals and macrotheories are derived from microtheories(1) by means of analogy considerations. On this method of aggregation, Theil (1954) states; "It is clear that the principal merits of this approach are to be found in its simplicity, not - at least not necessarily - in its intrinsic qualities. Nevertheless it is highly important, first because of its almost universal application, secondly because lack of data often prevents other solutions." The problem then becomes one of discovering under what conditions consistent aggregation is possible, where aggregation is said to be consistent, "when the use of information more detailed than that contained in the aggregates would make no difference to the results of the analysis of the problem at hand" (Green, 1964 p 3).

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(1) Relations postulated from theories of individual economic behaviour.

Unfortunately, many studies ignore the implications of inconsistent aggregation. As a result a number of problems arise in this aggregate type of analysis. The number of separate variables that may be considered has to be determined in terms of data availability, and also with regard to the resources available for estimation. As the number of variables increases, the work involved in computation grows more than proportionately. In addition, it is necessary to use a functional representation that is statistically manageable, both in terms of estimation and of testing. The necessity of confining attention to a few relevant variables in time-series analysis invariably results in several distinctive variables being aggregated into a single variable category. This can lead to serious problems of aggregation bias in the estimation of parameters.

Consider the following function (5.1) and assume it represents the true aggregate supply function for a particular product.

$$Y = \beta o + \sum_{i} \beta_{1i} X_{1i} + \sum_{j} \beta_{2j} X_{2j} \quad i+j = K \qquad (5.1)$$

Where:

Y - aggregate supply

X's - independent variables e.g. prices, weather.

β's - parameters.

However, for estimation purposes insufficient data are available and the model is estimated as:

$$Y = \overline{\beta_0} + \overline{\beta_1}\ \overline{X_1} + \overline{\beta_2}\ \overline{X_2} \qquad (5.2)$$

Where $\overline{X_1} = \sum_{i} X_{1i}$ and $\overline{X_2} = \sum_{j} X_{2j}$

In matrix notation the $\overline{\beta}$ estimates are, (Johnson, 1972)

$$\hat{\overline{\beta}} = \left(\overline{X}'\overline{X}\right)^{-1}\overline{X}'Y \qquad (5.3)$$

Rewriting (5.1) in matrix notation the true regression model is:

$$Y = X\beta + \epsilon \tag{5.4}$$

Where:

Y is a vector of dependent variable observations

X is a matrix of independent variable observations

ϵ is a vector of disturbance terms.

Substituting (5.4) into (5.3) and taking expected values

$$\hat{\overline{\beta}} = \left(\overline{X}'\overline{X}\right)^{-1}\overline{X}'X\beta + \left(\overline{X}'\overline{X}\right)^{-1}\overline{X}'\epsilon \tag{5.5}$$

$$\therefore \quad E\left(\hat{\overline{\beta}}\right) = \left(\overline{X}'\overline{X}\right)^{-1}\overline{X}'X\beta \quad \text{since it is assumed that } E(\epsilon) = 0$$

$$= \left(P\right)\beta \tag{5.5a}$$

Where $\left(P\right) = \left(\overline{X}'\overline{X}\right)^{-1}\overline{X}'X$

is a $(3 \times K+1)$ matrix

The matrix $\left(P\right)$ is in effect the matrix of regression coefficients of the columns of X on all the variables included in $\overline{X}$.

Rewriting (5.5a):

$$E\begin{pmatrix} \hat{\overline{\beta}}_0 \\ \hat{\overline{\beta}}_1 \\ \hat{\overline{\beta}}_2 \end{pmatrix} = \left(P\right)\begin{pmatrix} \beta_0 \\ \beta_{11} \\ \beta_{12} \\ \beta_{21} \\ \beta_{22} \end{pmatrix} \tag{5.6}$$

In equation (5.6) the expected value of the estimated parameters actually derived are expressed in terms of the true parameters and the available data $(\overline{X})$. It can be shown that each $\overline{\beta}$ is influenced by all the β's. In fact the expected value of each aggregate coefficient ($\overline{\beta_1}$ and $\overline{\beta_2}$) is equal to the arithmetic mean of its corresponding coefficients (the βi's or βj's), plus the weighted arithmetic mean of its corresponding coefficients, plus a sum of weighted arithmetic means of the "non-corresponding" coefficients (Green, 1964). All terms other than the arithmetic means are defined as forms of aggregation bias. If the weights and the true supply function parameters are uncorrelated there will be no bias due to aggregation, however, such a situation is not to be expected. It can be shown (Theil, 1954 p 119) that this bias is in general different for different statistical methods of estimation.

From the preceding discussion it is clear that any attempt to obtain estimates of national supply responses by modelling techniques will face aggregation problems whether the approach is to use representative farms and aggregate the results from these, or whether it is to use aggregate data in the first place. The approach chosen will depend on a number of factors such as the objectives of the analyst and the availability of data and computing facilities.

5.3 The Meat and Wool Boards' Economic Service's Survey[1]

The Economic Service carries out a sample survey of New Zealand sheep farms based on a random sample stratified by geographical regions and by sheep numbers. The sampling unit is the farm and the prime source of data the farmer. The survey results are presented in eight farming sub-groups and include:

(a) Physical features of the farms.
(b) The livestock and general management policy.
(c) Quantities of meat and wool produced.
(d) Financial results, including capital invested and details of annual revenue and expenditure.

Stratification divides the "population" of eligible[2] farms into groups of units, in such a manner that the units in each group are as homogeneous as possible. Variable sampling fractions[3] are

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(1) Sheep Farm Survey - an annual publication.

(2) Flocks of over 500 sheep.

(3) Geographical and Flock size stratifications are used initially. Each of the groups or strata is sampled at random.
The need to have at least 25 to 30 farms in a stratum before it is of any analytical use has resulted in the variation of the sampling fraction as between strata i.e. different numbers of farms in each strata.

used for different strata and this can lead to considerable gains in accuracy in cases where the material is shown to have a wide deviation from the mean. Since the survey results are presented in eight farming sub-groups a hypothetical farm can be constructed to represent each such group.

5.4 Class 3N - Hill Country, North Island

This class represents the easier hill country of the North Island of New Zealand. The holdings average about 800-1000 acres and carry 2 to 3 sheep to the acre with a high proportion of breeding ewes. Cattle are an important adjunct, with a general average of 1 beast to 10 sheep. Sales of wool have been slightly more important in recent years than sales of sheep and cattle (Figure 5.1). As a result of aerial topdressing much of the surplus stock (other than breeding ewes and heifers) is now turned off in fat or forward condition.

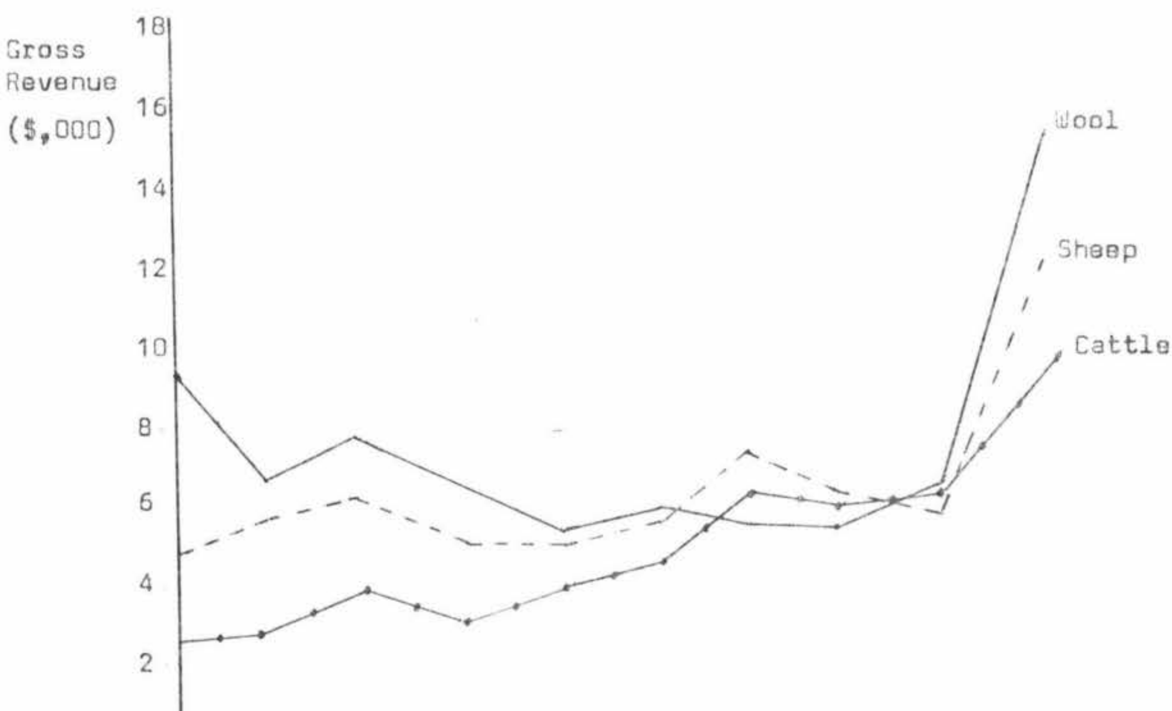

Figure 5.1 Gross Revenue - Class 3N[1] Farm

(1) Source: N.Z. M. & W. Boards' Economic Service

5.4.1 Productivity

Production levels have fluctuated only slightly over the period considered, and there has been no upward trend. This could indicate that technology increases over this period have had very little apparent effect, for this particular representative farm. If the figures in Table 5.2 are compared with those in Table 5.3, it is clear that while the stocking rate has markedly increased (3.55 EE/Acre - 4.17 EE/Acre) from 1963/64 to 1972/73, real expenditure per ewe equivalent has not increased and even decreased in some years. This effect coupled with a probable small technological advance would explain the apparent constant levels of production per animal.

Table 5.2 Productivity on Class 3N Farms

	Stocking Rate (EE/Acre)	Lambing %	Calving %	Wool Production (lbs) (per sheep)	(per acre)
1963/64	3.55	94.2	86.0	11.4	30.9
64/65	3.57	91.5	81.0	12.2	33.8
65/66	3.64	93.6	77.0	13.4	37.6
66/67	3.91	95.1	85.0	12.4	37.8
67/68	4.12	96.7	84.0	12.2	40.0
68/69	4.28	90.8	75.9	11.1	36.6
69/70	4.23	96.8	81.8	10.1	39.7
70/71	4.18	90.8	80.9	11.7	38.5
71/72	4.09	92.7	82.0	11.9	37.3
72/73	4.17	95.0	85.1	11.8	37.0

Source: N.Z. M. & W. Boards' Economic Service

5.4.2 Expenditure

Real expenditure per stock unit increased until 1965/66 at which point sheep farmers' terms of exchange declined dramatically for the next three years followed by a rapid fall in expenditure per stock unit. It was not until the 1972/73 season that real expenditure per stock unit approached the level reached in 1965/66. This is shown in Table 5.3

The Economic Service uses three groupings to classify expenditure:

1) Working Expenses
   - Wages and rations
   - Farm requisites
   - Shearing expenses
   - Fertilizer, lime and seeds
   - Vehicles, fuel and power
   - Feed and grazing
   - Contract
   - Repairs and maintenance
   - Railage and cartage
   - General expenses

2) Standing Charges
   - Insurance
   - Rates and land tax
   - Managerial salaries
   - Rent

3) Depreciation

Working expenses plus standing charges represent total cash expenditure.

Since few farm accounts show development expenditure as a separate item the Economic Service points out the difficulty in separating maintenance from development expenditure.(1) As a result the normal set of farm accounts obscures the true rate of re-investment in farms. The expenditure figures presented here are assumed to include both maintenance and development expenditure.

Real expenditure is obtained by deflating actual expenditure. Deflation is the process of removing the effects of price changes from the dollar values measuring the level of expenditure.

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(1) N.Z. M. & W. Boards' Economic Service - Sheep Farm Survey 1972-73 p 25.

$$\text{Deflated dollar value} = \frac{\text{Original dollar value}}{\text{An appropriate price index}}$$

In this case the appropriate price index is the Index of Prices Paid by Sheep Farmers (Base 1960-61 = 1000) which is compiled by the Economic Service.

Table 5.3 Expenditure on Class 3N Farms

	Working Expenditure	Total Cash Expenditure	Deflated Working Expenditure	Deflated Working[1] Expenditure E/E
1963/64	8220	10038	8019	2.81
64/65	8062	9970	7656	2.79
65/66	9164	11252	10389	2.92
66/67	8862	11122	7926	2.54
67/68	8041	10319	6950	2.14
68/69	9714	12130	8142	2.31
69/70	10600	13027	8618	2.47
70/71	10174	13028	7880	2.28
71/72	11031	13938	8034	2.34
72/73	14712	17935	10188	2.83

Source: N.Z. M. & W. Boards' Economic Service.

5.5 General Management Policy on Class 3N Farms

5.5.1 Sheep

Over the ten years considered, the number of mixed-age ewes carried on the average farm has risen from approximately 1000, to 1400 in 1972/73. The number of two-tooth ewes has risen from 350 to 500 and the number of hoggets from 500 to 620. The two-tooth and mixed-age ewes account for approximately 70% of the total sheep carried. The

(1) Deflated by an Index of Prices Paid by Sheep Farmers (1960/61 prices).

majority of replacements are bred on the farm with a small number of replacement ewes being purchased each year.

The lambing percentage has ranged from 90-96%. Most of the wether lambs are sold fat. In recent years an increasing proportion of the ewe lambs have been retained with a few tail-enders being sold as stores. A small number of wether hoggets are also carried.

5.5.2 Cattle

The number of breeding cows carried has increased from 72 in 1963/64 to 114 in 1972/73. Dry stock are comparatively more important in the cattle system than they are in the sheep system, with approximately equal numbers of weaner, yearling and two-year old steers being carried. Replacement cows are bred on the farm. A small number of weaner and yearling steers are usually purchased and sold mainly as two-year-olds, probably to take advantage of surplus feed in good seasons. The calving percentage is usually between 80-85% with most of the weaner calves born being retained.

5.6 Summary

The previous Sections have sketched the major characteristics of the farm to be modelled. Since the farm is only a hypothetical one, more detailed information is not available. The general management policy represents a summary of stock reconciliations(1) for ten years of data.

The information obtained from the reconciliations, and the expenditure and productivity data, can be used to build a model that attempts to simulate the production processes on the representative farm, and the decision processes of the representative farmer. This is outlined in Chapter Six. There are many decisions the farmer makes, however the decisions of interest are the ones which will directly affect the numbers of the different

(1) N.Z. M. & W. Boards' Economic Service - Sheep Farm Reconciliations by Type of Farm. - Class 3N, North Island Hill country Farm.

classes of stock carried, both at present and in the future. The basis of the behavioural hypothesis is that the farmer fully recognises the interdependence between changes in current output and potential future production. Thus, the decisions he makes regarding livestock sales and purchases can be regarded as either investment decisions, in which case the farmer will be prepared to forgo some present income for the sake of future income, or, as consumption decisions, in which case present income needs are being satisfied.

The decisions the model attempts to simulate are:

Ewe lamb sales	(ELS)
Cull ewe sales	(CE)
Weaner cow sales	(WCS)
Cull cow sales	(SALE)
Working Expenditure	(EXP)

The values for these decision variables over a ten year period are given in Table 5.4.

Table 5.4 The Decision Variables

	ELS	CE	WCS	SALE	EXP $
1963/64	120	297	8	16	8019
64/65	100	275	5	16	7656
65/66	156	240	7	26	10389
66/67	113	290	5	19	7926
67/68	150	327	7	22	6950
68/69	163	404	6	32	8142
69/70	208	403	6	33	8618
70/71	27	401	6	28	7880
71/72	45	406	7	25	8034
72/73	27	453	10	31	10188

Source: N.Z.M.& W. Boards' Economic Service - Sheep Farm Reconciliations by Type of Farm and Sheep Farm Surveys.

CHAPTER SIX

THE SHEEP AND BEEF FARM INVESTMENT MODEL

In this chapter the concepts discussed in the previous four chapters; investment theory, expectations formations, and the management systems observed on a North Island hill country representative farm are incorporated in a model that simulates the production and decision making processes of the farmers concerned. Since these decisions determine the likely output response to various exogenous changes, the model can be used to make projections of livestock numbers for that particular class of farm.

Section 6.1 is a general outline of the model, including explanation of how it can be used to make projections. The remaining sections discuss the model in greater detail.

6.1 Model Outline

It is useful to divide the model into two sub-models -

(1) Production model
(2) Decision model

The distinction between the two is to some degree artificial since they are not physically separate models. The production model can, however, be treated as a complete entity in itself.

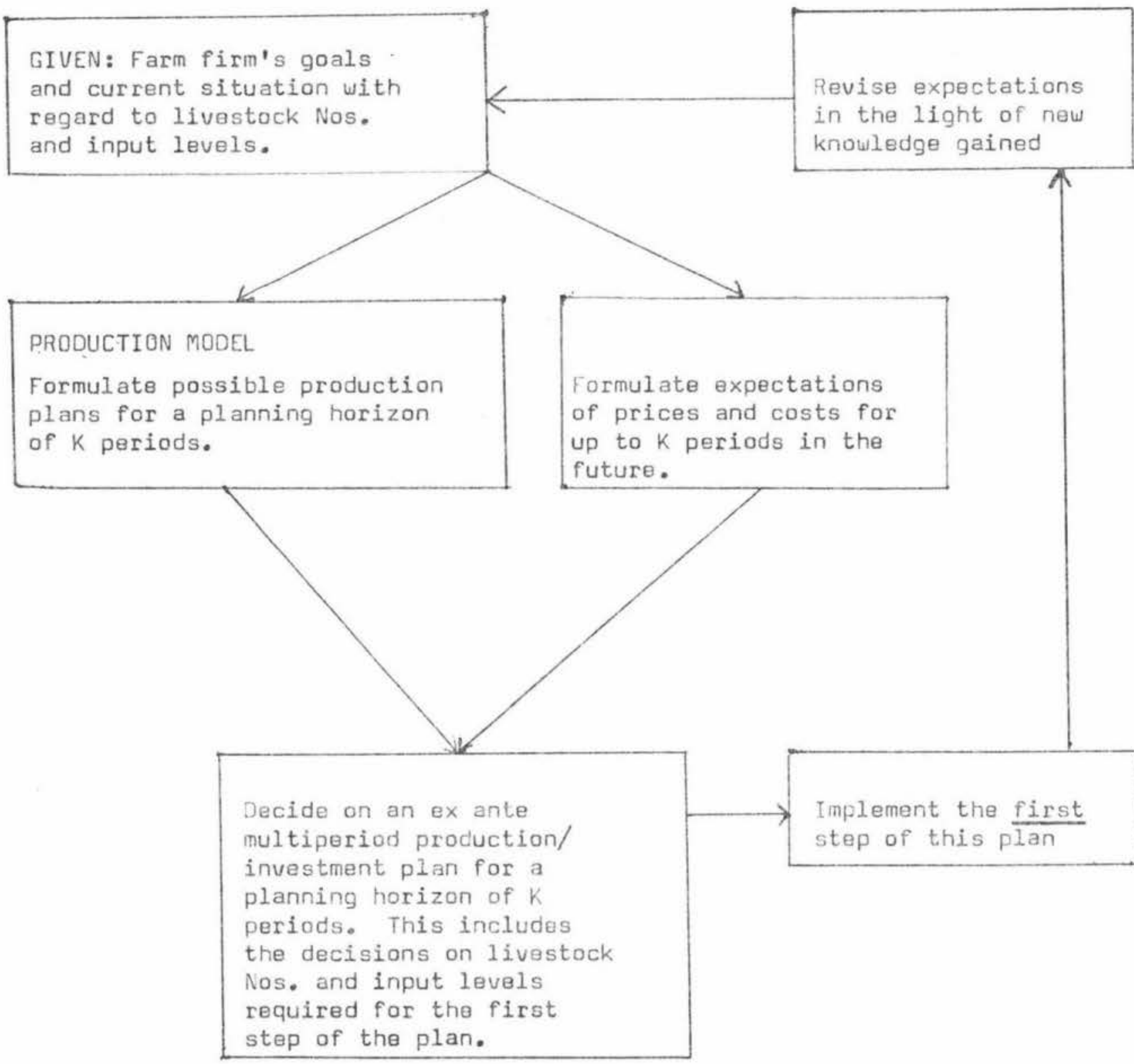

Figure 6.1 Conceptual Model of Production/Investment Process

The production model by itself indicates what the farmer envisages the outputs will be if certain decisions are made. The complete model indicates what the likely decisions will be, in addition to the resulting outputs. The livestock numbers that result from the first step of the plan are the projections made by the model. This is shown more clearly in Figure 6.2.

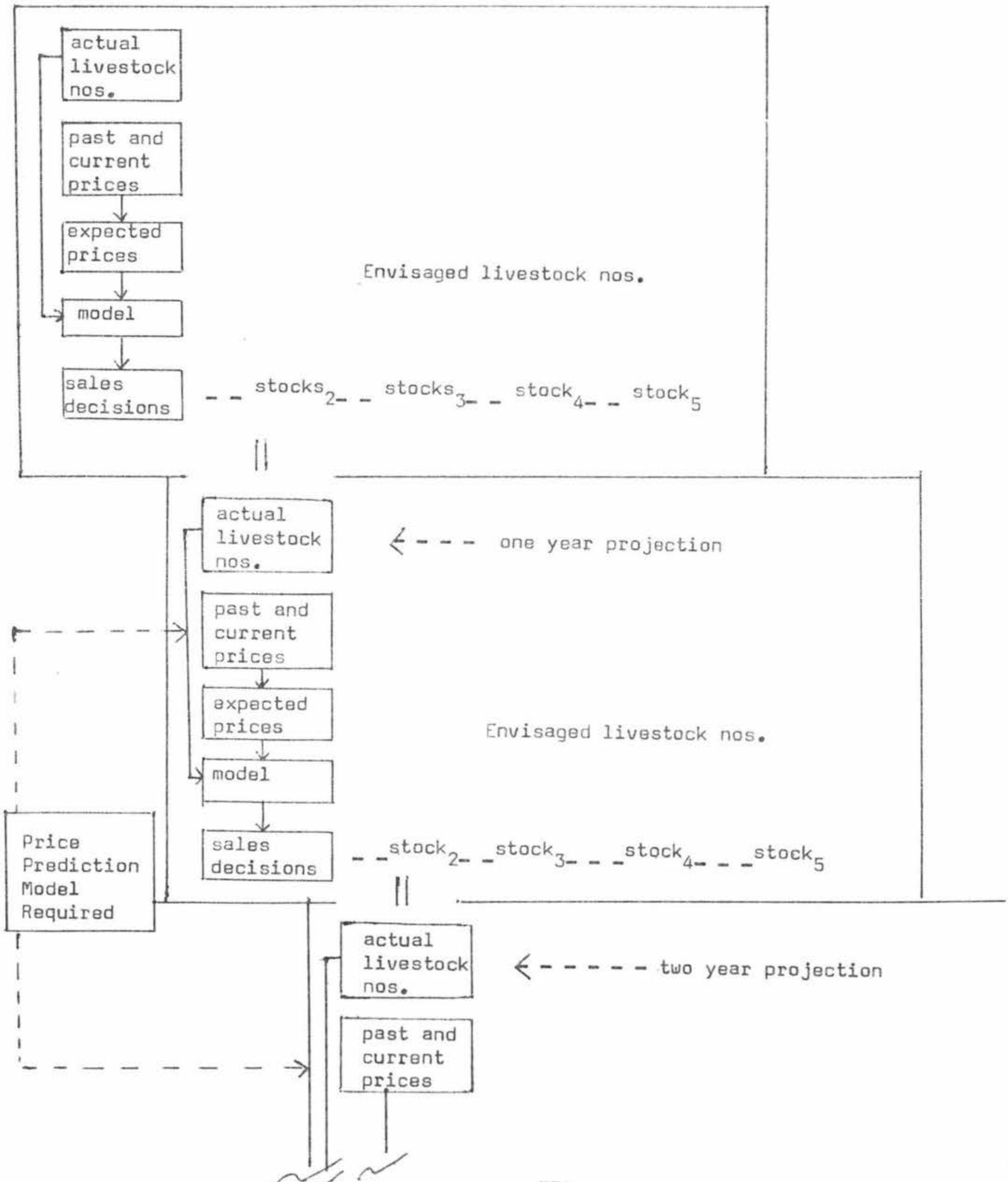

Figure 6.2 Projections

In any given year the farmer knows what his current stocks are and current and past prices for these stocks. He has an expectation of future prices based on these current and past prices, and envisages a plan for the following four years based on his current knowledge and expectations.

In the following year, he will have a new set of stock determined by the decisions made in the preceding year; this constitutes the first step of the envisaged plan. However, he will now have a new set of prices and therefore price expectations, and will form a revised five year plan based on this latest information. The plan in years two through to five initially envisaged by the farmer will only materialise if he has perfect foresight with regard to prices, therefore these plans represent conditional predictions.

The model is recursive[1] since current year decisions determine the following year's stocks. However, it is not fully recursive as following year's prices (and hence the farmer's further expectations) are not known unless a price prediction model is available.

In summary, the model suggests that the farmer envisages a five year plan based on his expectations of future prices and costs, and according to his objectives and time preference for income he makes his decisions concerning stock levels and input expenditure for the following year. The hypothesis is made in this study that the farmer's objective is to maximize the net present value of his income stream.

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(1) This framework is very similar to Chien and Bradford's (1974), the difference being that the model in this study is not solved by programming methods (and does not involve a simulation model).

The model can be conveniently summarised algebraically:

Objective:

$$\text{Maximize} \quad Z = Y_1 + d_2Y_2 + d_3Y_3 + d_4Y_4 + d_5Y_5 \tag{6.1}$$

Where:

$$Y_t = f_t\ (P_t,\ C_t,\ SS_t) \quad t = 1,\ldots\ldots,5 \tag{6.2}$$

and

$$St_t = f_s\ (St_{t-i},\ SS_{t-i},\ \gamma_j) \quad \begin{array}{l} t = 2,\ldots\ldots,5 \\ i = 1,\ldots\ldots,4 \\ j = 1,\ldots\ldots,n \\ i < t \end{array} \tag{6.3a}$$

$$\therefore\ St_t = f_1\ (St_1,\ SS_1,\ \gamma_j\ ) \quad t = 2,\ldots\ldots,5 \tag{6.3b}$$

and

$$SS_t = f_2\ (St_1,\ SS_1,\ \gamma_j\ ) \quad t = 2,\ldots\ldots,5 \tag{6.4}$$

$$\therefore\ Y_t = g\ (P_t,\ C_t,\ St_1,\ SS_1,\ \gamma_j) \quad t = 2,\ldots\ldots,5 \tag{6.5}$$

Where:

- Z – Future income stream.
- Y_t – Net income in year t.
- St_t – Stock numbers
- SS_t – Stock sales
- P_t – Product prices
- C_t – Costs
- d_t – Discount rates
- γ_j – Parameters (from production functions, cost functions, and discount rates).
- f,g – Functions

Substituting (6.5) into (6.1)

$$Z = Y_1 + g\ (P_t,\ C_t,\ St_1,\ SS_1,\ \gamma_j) \tag{6.6}$$

or

$$Z = Y_1 + g\ (P_t,\ C_t,\ St_1,\ SS_1,\ DV_k,\ \gamma_j) \tag{6.7}$$

Where: DV_k – Decision variables $k = 1,\ldots\ldots,5$

From Equation (6.7), the first-order conditions for a maximum are:

$$\begin{aligned} \frac{\partial Z}{\partial DV_1} &= h_1' \; (St_1, SS_1, DV_k, P_t, C_t, \gamma_j) = 0 \\ &\vdots \\ \frac{\partial Z}{\partial DV_5} &= h_5' \; (St_1, SS_1, DV_k, P_t, C_t, \gamma_j) = 0 \end{aligned} \tag{6.8}$$

This results in 5 simultaneous equations which can be solved for the decision variables.

$$\hat{DV}_k = h_k \; (St_1, SS_1, P_t, C_t, \hat{\gamma}_j) \qquad j = 1,\ldots,n \quad k = 1,\ldots,5 \tag{6.9}$$

The decisions solved for are current decisions in terms of current stocks and sales, prices, costs, and the parameters. Since they are current decisions they can be compared with actual decisions from the Economic Service data.

It is important to note that Equations (6.9) are explicit analytical solutions. This enables some of the parameters γ_j to be estimated so that the estimated $\hat{DV}_k$ are as close as possible to the actual DV_k. This is done by an iterative technique which is described in Section 6.5.

Once the "best" parameters in the above sense have been found, they are retained and the model is set to make projections in the way described in Figure 6.2. The appropriate price information is fed into the model and the decision variables estimated. These decisions determine the following years stocks through the stock reconciliations. This results in a one year projection and further projections can be made in the recursive way outlined. The whole process is fully automated - the computer programme for making projections is described in Appendix I. The computer programme for estimating the parameters $\hat{\gamma}_j$ is also described in Appendix I.

6.2 The Production Model

The production model is a simple model of a sheep and beef farm production process over time. It represents the process as it is envisaged by the farmer. The relationships between animals of different age classes are defined, and represented by a series of stock reconciliations. The envisaged relationships between expenditure on inputs and output of each product are described by production functions. These functions are not the true ones, rather they represent the functions as the farmer sees them.

6.2.1 Income

Income is derived from sales of:

Wool
Lambs
Cull ewes
Weaner, yearling and cull cows
Weaner, yearling and two-year-old steers.

6.2.2 Stock Classes Defined

6.2.2.1 Sheep

Lambs- ewe	(EL)
- wether	(WL)
Ewe hoggets	(HG)
Two-tooth ewes	(2TH)
Four-tooth ewes	(4TH)
Six-tooth ewes	(6TH)
Mixed-age ewes	(MAE)
Old ewes	(OE)

The assumption is made that all wether lambs are sold and no wethers are purchased.

6.2.2.2 Cattle

Weaner cows	(WC)
Yearling cows	(YC)
Breeding cows	(BC)
Weaner steers	(WS)
Yearling steers	(YS)
Two-year-old steers	(2S)

6.2.3 Reconciliations[1]

6.2.3.1 Sheep

The planning horizon assumed is five years. The farmer has some desired flock structure he would like to attain. Whatever his envisaged total number of sheep, the different age classes will be in some predetermined ratio. This, he envisages, will ensure that a balanced flock results. The process can be represented diagrammatically as in Figure 6.3.

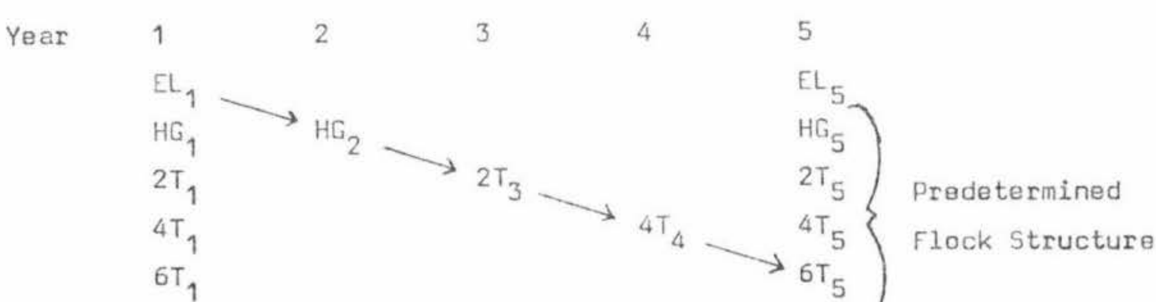

Figure 6.3 Sheep Reconciliations

A ewe lamb born in year t=1 (the current year) will become a six-tooth in year t=5. Therefore, in the absence of a buying or selling policy for younger ewes, the number of six-tooth ewes in year t=5 is dependent on the number of ewe lambs retained in year t=1.

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(1) The reconciliations are listed in APPENDIX III.

All other age classes in year t=5 (except for old ewes) will have numbers in some specified proportion (as desired by the farmer) of six-tooth ewes in year t=5.

There are two decisions the farmer can make regarding the envisaged future size and structure of his flock.

(1) The number of ewe lambs to sell(ELS)

(2) The number of mixed-age ewes to cull (CE)

The exclusion of hogget and two-tooth ewe sales as decisions is justified on the grounds that the basic decisions regarding future flock size and structure are made at the most fundamental levels. For example, if the farmer envisages building up his flock, his first actions are to retain more ewe lambs and/or cull fewer mixed age ewes.

From Figure 6.3 the following relationships can be derived:

$$HG_2 = \gamma_1 (EL_1 - ELS_1) \tag{6.10}$$

where γ_1 is the survival rate.

This says that the number of hoggets envisaged in year t=2 is the number of ewe lambs retained in year t=1 that survive. It is assumed that the survival rates are the same for all classes.

Similarly,

$$2T_3 = \gamma_1 HG_2 = \gamma_1^2 (EL_1 - ELS_1) \tag{6.11}$$

$$4T_4 = \gamma_1 2T_3 = \gamma_1^3 (EL_1 - ELS_1) \tag{6.12}$$

$$6T_5 = \gamma_1 4T_4 = \gamma_1^4 (EL_1 - ELS_1) \tag{6.13}$$

The other classes in year t=5 are likewise given by:

$$HG_5 = \alpha_1\ 6T_5 = \alpha_1\ \gamma_1^{\ 4}\ (EL_1 - ELS_1) \tag{6.14}$$

$$2T_5 = \alpha_2\ 6T_5 \tag{6.15}$$

$$4T_5 = \alpha_3\ 6T_5 \tag{6.16}$$

$$MAE_5 = \alpha_4\ 6T_5 \tag{6.17}$$

where:

$\alpha_1, \ldots\ldots, \alpha_4$ - proportion constants reflecting the desired flock structure.

The assumption is made that all old ewes (OE) are culled each year together with a proportion of the mixed age ewes. The proportion of mixed age ewes culled in year t=1 is a decision made by the farmer. The number of mixed age ewes envisaged as culled in other years is given by the relationship

$$CE_t = \frac{CE_1}{MAE_1}\ MAE_t \qquad t = 2,\ldots\ldots,5 \tag{6.18}$$

Total culls in any year is:

$$TCE_t = CE_t + OE_t \qquad t = 1,\ldots\ldots,5 \tag{6.19}$$

It follows then, that the number of old ewes in any year is:

$$OE_t = \gamma_1(MAE_{t-1} - CE_t) \qquad t = 2,\ldots\ldots,5 \tag{6.20}$$

Referring back to Figure 6.3 it should be obvious that these reconciliations enable envisaged stocks to be expressed in terms of current stocks and sales, for which data are available. The way in which these expressions are used is indicated by equations (6.1) to (6.6).

Because a predetermined desired flock structure is envisaged in year t=5, the envisaged intermediate year stocks and sales are determined and expressions can be obtained for these as follows:

$$\begin{aligned} \text{Stocks}_t &= f(\text{Stocks}_5) \qquad t = 2,3,4 \\ &= g(\text{Stocks}_1,\ \text{Sales}_1,\ \gamma_1) \end{aligned} \tag{6.21}$$

An example:

From Figure 6.3

$$4T_5 = \gamma_1\ 2T_4$$

$$\therefore \quad 2T_4 = \frac{1}{\gamma_1} 4T_5$$

from (6.16) and (6.13) $\qquad = \alpha_3\ \gamma_1^{\ 3}\ (EL_1 - ELS_1) \qquad (6.22)$

Envisaged income is obtained from envisaged sales of stocks, therefore expressions must be obtained for these in each year.

The number of culls sold in any year is given by equation (6.19).

Other intermediate year sales are determined in the same way as intermediate year stocks. In general:

$$\begin{aligned} \text{Sales}_t &= f_1(\text{Stocks}_5) \qquad t = 2,3,4 \\ &= g_1(\text{Stocks}_1,\ \text{Sales}_1,\ \gamma_1) \end{aligned} \qquad (6.23)$$

An example:

Ewe lamb sales envisaged in year $t = 2$

given $HG_3 = \gamma_1(EL_2 - ELS_2)$ $\qquad$ (6.24a)

i.e. lambs retained in year $t=2$ that survive will become hoggets in year $t=3$.

Re-arranging (6.24a)

$$ELS_2 = EL_2 - \frac{1}{\gamma_1}\ HG_3 \qquad (6.24b)$$

From Figure 6.3

$$2T_4 = \gamma_1\ HG_3$$

$$\therefore \quad HG_3 = \frac{1}{\gamma_1} 2T_4$$

from (6.22) $\qquad = \alpha_3\ \gamma_1^{\ 2}\ (EL_1 - ELS_1) \qquad (6.24c)$

Substitute in (6.24b)

$$\therefore \quad ELS_2 = EL_2 - \alpha_3\ \gamma_1\ (EL_1 - ELS_1) \qquad (6.24d)$$

EL_2 is the number of ewe lambs envisaged born in year t=2 and is obtained from the lambing production function which is discussed in detail in Section 6.2.4.2.

Envisaged wool sales are obtained from expressions involving wool production functions which are also discussed in Section 6.2.4.1.

Finally, expressions for year t=5 stock sales must be derived. Since it is assumed that a stable flock structure is envisaged in year t=5, the structure will be identical in year t=6.

This means that year t=5 stock sales are determined and expressions for them can be easily found.

6.2.3.2 Cattle

The basic ideas involved are the same for cattle as for sheep. The farmer has in mind some desired herd structure he would like to attain. The planning horizon assumed is three years. The steer enterprise is considered a subsidiary to that of the breeding cows. Any decisions made regarding cattle concern only females directly. However, a decision to retain more weaner cows to build up the breeding cow herd will eventually result in more steers being born and raised, so decisions regarding females indirectly affect males.

(a) Females

$$\begin{tikzcd}
\text{Year} & 1 & 2 & 3 \\
& WC_1 \arrow[dr] & WC_2 & WC_3 \\
& YC_1 & YC_2 \arrow[dr] & YC_3 \\
& COW_1 & COW_2 & COW_3
\end{tikzcd}$$

Figure 6.4 Cow Reconciliations

Replacements are bred on the farm, with two-year-old cows entering the breeding herd, or being sold as culls along with the other breeding cows culled. Sales of weaner cows (WCS) and yearling cows (YCS) are also made. (To avoid confusion with sheep culls, cattle culls are denoted as $SALE_t$).

From Figure 6.4 the expression for breeding cow numbers in year t=3 is:

$$BC_3 = \gamma_5(YC_2 - YCS_2) + \gamma_5(BC_2 - SALE_2) \tag{6.25}$$

γ_5 - cattle survival rate

The breeding herd in year t=3 is thus the herd from year t=2, less deaths and culls, plus the yearlings retained which enter the herd as two year olds.

The number of yearling cows in year t=3, (YC_3) is some desired proportion of breeding cows.

$$YC_3 = \alpha_6 BC_3 \tag{6.26}$$

The breeding herd in year t=2 is given by an expression analogous to (6.25).

$$BC_2 = \gamma_5 YCR_1 + \gamma_5(BC_1 - SALE_1) \tag{6.27}$$

where:

YCR - yearling cows retained

Yearling cows in year t=2

$$YC_2 = \gamma_5(WC_1 - WCS_1) \tag{6.28}$$

The number of weaner cows (WC_t) is obtained from the calving production function discussed in Section 6.2.4.3. Again it is necessary to find expressions for sales.

Weaner cow sales in year t=2 (WCS_2) is easily obtained from the requirement that yearling cows in year t=3 be in some desired proportion to breeding cows in year t=3.

$$YC_3 = \gamma_5(WC_2 - WCS_2) \tag{6.29}$$

Re-arranging:

$$WCS_2 = WC_2 - \frac{1}{\gamma_5} \; YC_3$$

$$= WC_2 - \frac{\alpha_6}{\gamma_5} \; BC_3 \qquad (6.30)$$

where: α_6 is defined in (6.26) and further substitution in (6.30) results in an expression in terms of year t=1 stocks and sales.

Yearling cow sales in year t=2, (YCS_2) is assumed to be some proportion of the number of yearling cows in year t=2.

$$YCS_2 = \alpha \, YC_2 \qquad (6.31)$$

Cull cow sales in year t=2, ($SALE_2$) is assumed to be equal to sales in the previous year.

In an analogous way to that for sheep, all sales in year t=3 are calculated according to the requirement that stocks in year t=4 equal stocks in year t=3.

(b) Males

$$\begin{tikzcd} \text{Year} & 1 & 2 & 3 \\ & WS_1 \arrow[dr] & WS_2 & WS_3 \\ & YS_1 & YS_2 \arrow[dr] & YS_3 \\ & 2S_1 & 2S_2 & 2S_3 \end{tikzcd}$$

Figure 6.5 Steer Reconciliations

Sales of weaner steers (WSS), yearling steers (YSS) and two year old steers (2SS) are made.

The assumption is made that half the weaners retained in year t=1 are sold as yearlings in year t=2 and half as two-year-old steers in year t=3. Thus no steers older than two years are carried. Yearling steers in year t=3 are carried in proportion to two-year-old steers.

$$YS_3 = \alpha_7 \, 2S_3 \tag{6.32}$$

Sales in year t=3 are calculated as for female sales.

6.2.4 The Production Functions

The requirement for an analytical solution to the maximization problem (see Section 6.1) means that the production functions cannot be too complex. Similar types of study to this one have used extremely simple functions. For example, Carvalho (1972 p.67) uses a calving function of the form:

$$Bn = \lambda Kn$$

where:

- Bn – calves born
- Kn – the reproductive herd
- λ – calving percentage

The use of fairly simple functions is further justified since the functions for years t=2,...,5 are in fact those envisaged by the farmer, which could not be considered to be the true functions. Rather, they are some fairly simple sort of function which reflects the farmer's weighting given to different variables affecting envisaged production.

Therefore, in estimating these relationships no attempt is being made to estimate the true production functions.

With these points in mind the following functions are defined.

6.2.4.1 Wool

$$Y = AY + \beta_4 \frac{(EXP)}{SPW} - \beta_5 \frac{(EXP)^2}{SPW} \tag{6.33}$$

where:

- Y – wool yield (kg/sheep)
- AY – average wool yield (kg/sheep)
- EXP – total working expenditure (deflated by a Prices Paid by Farmers index)
- SPW – number of sheep producing wool
- β_4, β_5 – parameters

6.2.4.2 Lambs

$$LP = ALP + \beta_6 \left(\frac{EXP}{BE}\right) - \beta_7 \left(\frac{EXP}{BE}\right)^2 \qquad (6.34)$$

where:

- LP - lambing percentage
- ALP - average lambing percentage
- EXP - total working expenditure (deflated by a Prices Paid by Farmers index)
- BE - number of breeding ewes
- β_6, β_7 - parameters

6.2.4.3 Calves

$$CP = ACP + \beta_8 \left(\frac{EXP}{BC}\right) - \beta_9 \left(\frac{EXP}{BC}\right)^2 \qquad (6.35)$$

where:

- CP - calving percentage
- ACP - average calving percentage
- EXP - total working expenditure (deflated by a Prices Paid by Farmers index)
- BC - number of breeding cows
- β_8, β_9 - parameters

6.2.4.4 The Functions for Year t = 2,....,5

The level of expenditure on inputs is a decision made by the farmer.

Envisaged expenditure in years t=2,....,5 is related to the current expenditure decision in the following way:

$$EXP_t = EXP_1 + \delta \, TSN_t^{(1)} \qquad t = 2,....,5 \qquad (6.36)$$

where:

- EXP_t - expenditure in year t
- TSN_t - total stock numbers in year t
- δ - a parameter relating the level of envisaged expenditure to stock numbers

(1) A better relationship would have been:

$$EXP_t = EXP_1 + \delta(SN_t - SN_1) \qquad t=2,...,5.$$

The inclusion of this would have increased the size of the model to such an extent that it was not considered.

The general form of equations (6.33) - (6.35) is:

$$P_t = AP + \beta \left(\frac{EXP_t}{SN_t}\right) - \beta_0 \left(\frac{EXP_t}{SN_t}\right)^2 \qquad (6.37)$$

where:

P_t - production in year t
AP - average production
EXP_t - deflated expenditure in year t
SN_t - the appropriate stock numbers in year t (i.e. BE, BC or SPW)

Substituting (6.36) into (6.37) the general form of the envisaged production functions is obtained:

$$P_t = AP + \beta \frac{(EXP_1 + \delta\, TSN_t)}{SN_t} - \beta_0 \frac{(EXP_1 + \delta\, TSN_t)^2}{SN_t} \qquad (6.38)$$

where the variables and constants are as previously defined.

The assumption is made that the farmer envisages technology to remain unchanged over the five year planning horizon, therefore the production function parameters are the same over the five years.

6.2.4.5 Estimation

Equations (6.33) - (6.35) were estimated by Ordinary Least Squares Regression using time-series data from the Economic Service Survey data.

The equations were estimated as:

$$P_t - AP = \beta_1 \frac{(EXP_t)}{SN_t} - \beta_2 \frac{(EXP_t)^2}{SN_t} \qquad (6.39)$$

In forcing the equations to the origin calculation of the usual test statistics was not undertaken. The estimated functions are presented in Appendix IV.

6.2.4.6 Interpretation

The unusual nature of these functions has already been outlined. A further problem in the interpretation of the estimated functions requires consideration (Lewis, 1971).

Production functions are generally represented as in Figure 6.6.

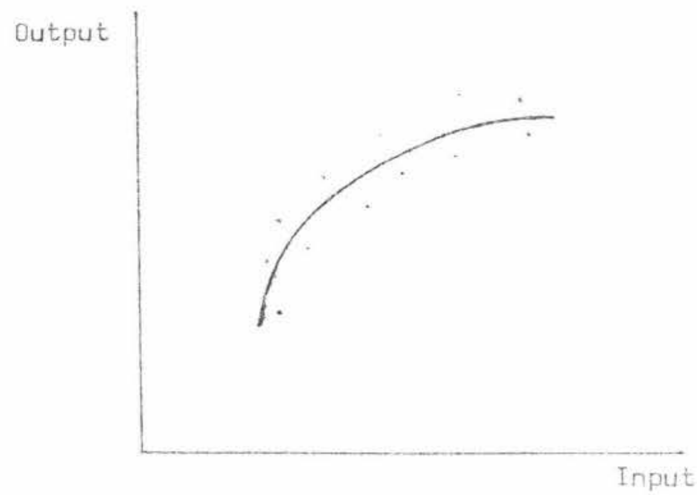

Figure 6.6 A Production Function

The scatter of points which enables such a line to be drawn can be thought of as being brought about by different levels of response from the same amounts of inputs (i.e. differences of a technical nature) or by farmers, with identical farms and responses, for no apparent reason using different levels of the input (i.e. decision differences). Of concern is the reason for the variation in input use and the variation in response to identical levels of input use.

If cross-section data are used, then drawing or estimating a line as above is suggesting that all farmers have the same production function and know it, and therefore if they all seek to maximize profit, they would all use the same amount of input and achieve the same level of production. This would result in a single point on the graph and it follows that the above line could not be estimated.

What in fact the above line is attempting to represent is the average production function. This situation is depicted in Figure 6.7.

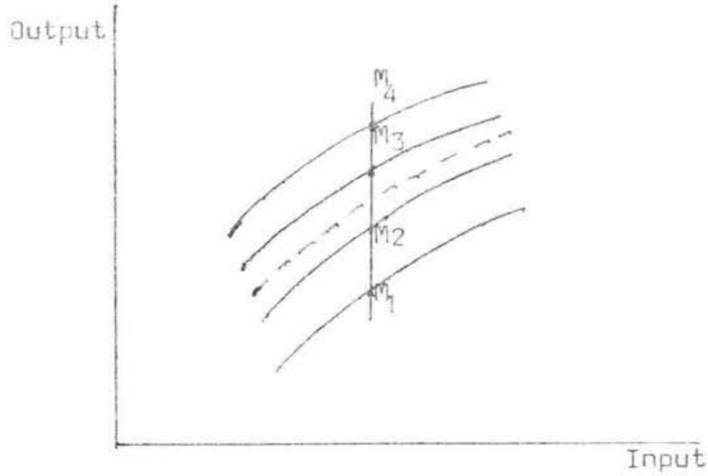

Figure 6.7 The Average Production Function

The dashed line is the average production function. The line through the points M_1, M_2, M_3, and M_4, which would be the observations secured, in no way represents the average production function.

If in fact farmers do not know what their production functions look like and in seeking to maximize profit make mistakes in choosing the level of inputs, then a conglomeration of points will result which would only by accident give the average production function. The direction the line will take will depend on which has the greater variation - the production function or the decision function.

If by some means, all but the decision variation could be removed then a line very close to the average production function could be obtained. However, this will still be biased because of the different managerial abilities of farmers, and this cannot easily be measured.

When time-series data are used the reason for different levels of input use in different years might be changing prices of inputs and product. Because of this, the identification problem is not so acute, although problems of bias still arise if technological change has occurred over the period the data cover.

Keeping in mind that the parameters estimated represent only weights applied by the farmer to the effect that envisaged expenditure will have on envisaged production, and that time series data are used then the estimation of the parameters in the way described is satisfactory. An important property is that the estimated functions exhibit diminishing marginal returns as in Figure 6.8.

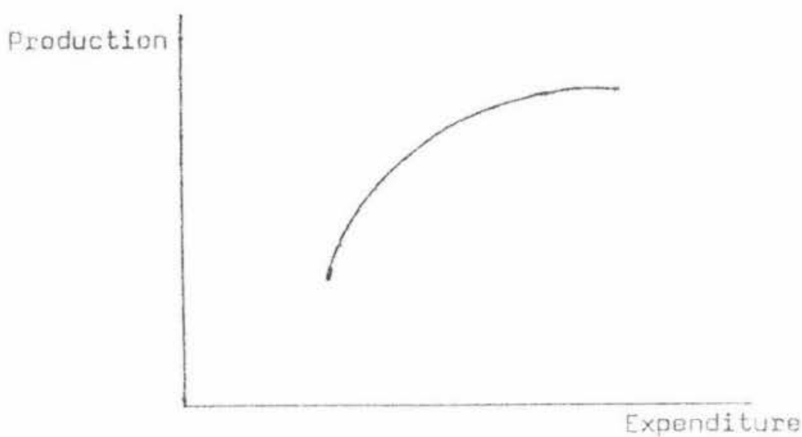

Figure 6.8 General Form of the Functions

6.3 The Decision Model

6.3.1 The Objective Function[1]

Through the production model, expressions have been obtained for all envisaged stocks and sales over the five year planning horizon. Information can be obtained about the farmer's expectations of future prices from the adaptive expectations model discussed in Chapter 4.5.

With the incorporation of a cost function it is possible to write out an explicit analytical net income function representing the farmer's envisaged net income stream.

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(1) The complete objective function is listed in APPENDIX III.

Depending on his "time preference" and "investment opportunity" the farmer can alter his income stream to achieve desired goals. If, as is hypothesized, his objective is to maximize his returns over his entire lifetime, then this is best achieved by maximizing the net present value of his income stream.

Under normal circumstances, future income is not as desirable as current income, therefore future income is discounted. The discount factors are estimated by the iterative technique discussed in Section 6.5. The resulting estimates are conditional on the values of all other parameters in the model. If the parameters all have their true values, and the model is an exact representation of the production and decision making processes on the farm then the discount factors so estimated would represent the farmer's time preference for income.

In summary, the problem is to derive the farmer's present value expression as a function of decision variables and maximize it with respect to these decision variables.

6.3.2 The Cost Functions

Costs are incorporated in the objective function as follows:

$$C_t = FC_t + \beta_1 SN_t + \beta_2 SN_t^2 \qquad t = 1, \ldots, 5 \qquad (6.40)$$

where:

C_t - total costs in year t
FC_t - fixed costs (standing charges)
SN_t - stock numbers in year t

6.3.3 The Decision Variables

These are all current (year t = 1) decisions.

ELS_1 - ewe lamb sales
CE_1 - cull ewe sales
WCS_1 - weaner cow sales
$SALE_1$ - cull cow sales
EXP_1 - expenditure on inputs

6.4 The Procedure

6.4.1 Differentiation

The full objective function is differentiated with respect to each of the five decision variables.

The assumption that the farmer wishes to maximize Z requires the first-order conditions:

$$\frac{\partial Z}{\partial ELS_1} = \frac{\partial Z}{\partial CE_1} = \ldots = , \frac{\partial Z}{\partial EXP_1} = 0 \qquad (6.41)$$

These conditions yield five equations describing simultaneously the five decision variables in terms of each other, initial stocks in year t=1, current and expected prices, discount factors and the constants previously defined.

6.4.2 Solution

For any time period, given values for initial stocks, the parameters and prices, the values for each of the decision variables can be determined. Data are available for initial stocks and current prices for the periods 1963/64 to 1972/73. The same data can be used to estimate the constants previously discussed.

The model should therefore predict values for the decision variables in period t=1 that bear a close relationship to the observed values for the data period. This relationship can be improved by an estimation procedure to be discussed in Section 6.5.

The five simultaneous equations resulting from (6.41) above can be represented as:

$$AB = C$$

i.e. $$\underset{(1\times5)}{(ELS_1, \ldots, EXP)} \underset{(5\times5)}{\begin{pmatrix} B_{1,1} \ldots B_{1,5} \\ \\ B_{5,1} \ldots B_{5,5} \end{pmatrix}} = \underset{(1\times5)}{(C_1 \ldots C_5)}$$

A - a vector of decision variables

B - a matrix of coefficients

C - a vector of constants

This system of equations is solved by inverting the matrix B and postmultiplying the vector of constants by the resultant inverse. The computer programme is written in FORTRAN.

6.4.3 The Coefficient Matrix B

This matrix is symmetric thus providing a check as to whether the differentiation and coefficient extraction have been performed accurately. A failure to find an inverse of B would also indicate possible linear dependence between columns of the matrix, indicating that two or more of the decision variables are not independent.

6.5 The Estimation Procedure

Earlier it was stated that the model should predict values for the decision variables in year t=1 that bear a close relationship to the observed values for that year.

Once initial predictions have been made an iterative procedure is then used to improve these predictions. Essentially, this involves finding values for certain parameters which it is desirable to estimate within the model (e.g. discount rates), which minimize the differences between the predicted and the observed values of the decision variables.

More explicitly, it involves minimizing the determinant (F) of the matrix (SS) of sums of squares and cross products of the differences between the observed and the predicted values of the decision variables. This criterion for estimating parameters in a multi-response model was suggested by Box and Draper (1965).

$$SS = \begin{bmatrix} \sum_{I}^{10} (ME(I,1) - MA(I,1))^2 \ldots\ldots\ldots\ldots \\ \sum_{I}^{10} (ME(I,2) - MA(I,2))(ME(I,1)-MA(I,1)) \ldots\ldots \\ \vdots \qquad \ddots \\ \sum_{I}^{10} (ME(I,5)-MA(I,5))(ME(I,1)-MA(I,1)) \quad \ldots\ldots \end{bmatrix}$$

where:

F = |SS| i.e. the determinant of the matrix SS.

ME = matrix of estimated values of the decision variables.

MA = matrix of actual values of the decision variables.

I = 1,...,10 number of years of data.

Since there is more than one decision variable, a change in a parameter value may reduce the difference between the actual and estimate for one decision variable but increase the difference for another. The Box and Draper (1965) criterion attempts to overcome this problem by ensuring that each decision variable receives an appropriate weighting such that a new parameter value improves the overall fit of the model. The procedure is an unconstrained minimization and is carried out by a computer routine called MODFIT. This minimizes a function of several variables by changing one parameter at a time and is based on the idea of conjugate directions. Powell (1964) explains the algorithm used. The procedure does not require derivatives to be calculated. The properties of the method and the errors associated with each estimated parameter are unknown.

6.6 Simple Analytics of the Problem

It is not possible to represent such a complex procedure on a two dimensional diagram. However, it can be simply explained using a partial approach.

For any one particular year and one particular decision variable, say ewe lamb sales, the situation is depicted in Figure 6.9.

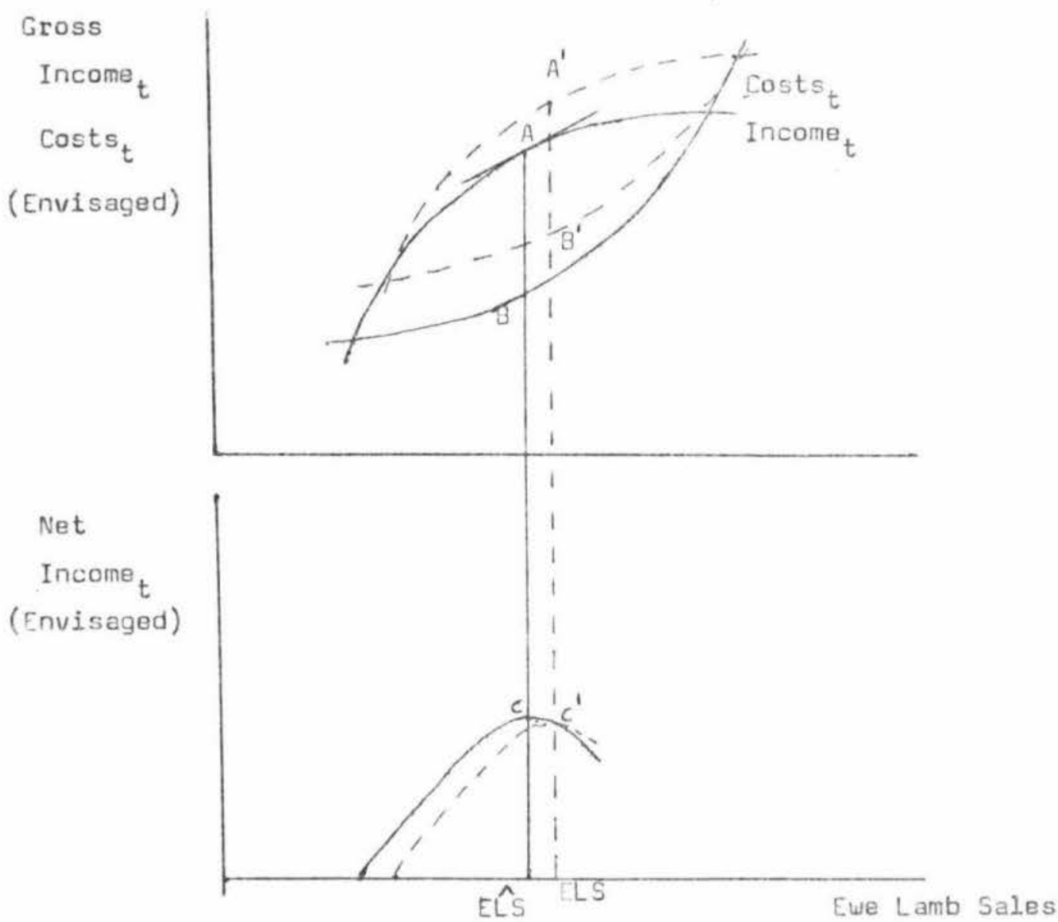

Figure 6.9 Simple Analytics of the Estimation Procedure

The dashed lines represent the actual functions and ELS the actual sales. The solid lines represent the functions as derived from the model. The true shape of the functions is not known but they can be represented by the above functions since they are assumed to be quadratic with respect to the decision variables.

The iterative estimation technique works by moving the solid lines towards the dashed line or more correctly $A \longrightarrow A^1$, $B \longrightarrow B^1$, and $C \longrightarrow C^1$.

Points A and B represent points where the slope of the income function equals the slope of the cost function.

i.e. Marginal Revenue = Marginal Cost for lamb sales

The net income function is also quadratic and negative definite with respect to the decision variables which guarantees a unique solution to the maximization problem.

The complete problem is far more complex. As well as equating marginal revenue to marginal cost for each productive opportunity, it involves equating marginal revenues between different productive opportunities both at present and over time.

CHAPTER SEVEN

RESULTS

7.1 The Production Model

The production model has been extensively discussed in Chapter Six. It represents the production process over time as it is envisaged by the farmer. The relationships between animals of different age classes are specified, and represented by a series of stock reconciliations. The envisaged relationships between expenditure on inputs and wool, lamb and calf production are described by production functions.

The production model can be treated as a complete entity in itself. The realism[(1)] of the model can be tested by substitution of different values of the decision variables; in effect "making the farmer's decisions for him". The results obtained with a particular set of values of the decision variables represent an envisaged five year plan.

This type of analysis is a simple simulation and thus shares the verification and validation problems encountered by most simulation studies. Anderson (1974,p 16) describes verification as "checking the correctness of the model as conceived in earlier stages", and validation as "deciding the adequacy of the model to mime the behaviour of the system being modelled". Since the model produces a plan that is envisaged by the farmer, the only way to truly validate the model would be to ask the farmer what he envisages his plan would be if he made the particular decisions substituted into the model. Since this is clearly impossible with a hypothetical farmer, resort is made to Anderson (1974,p 16):

> "A thorough review of a model to determine if its behaviour is as anticipated during construction can be regarded as an essay in applied commonsense."

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(1). Realism - The ability of the model to accurately describe the true process.

In addition the model should satisfy certain conditions which are discussed in Section 7.1.2.

7.1.1 A Simulated Comparison of Two Envisaged Five Year Plans.

The difficult nature of validating a simulation has been discussed in Section 7.1. In general, farmers do not make radical changes in their operations over a short period of time. For this reason a comparison of the envisaged plans resulting from the decision model with historical data can be of some use. This type of comparison can indicate the likely magnitude of changes the farmer could envisage making over a period of time, however its usefulness is limited in that it is purely a subjective evaluation.

In Chapter Three the concept of animals as investment and/or consumption goods was discussed. There, it was argued that a farmer will invest in potential breeding stock by retaining greater numbers of young female animals to gain future increases in total livestock numbers. The importance of this investment function of young animals in the determination of future livestock numbers requires that it be explicitly accounted for in a model which makes projections of livestock numbers.

Using actual livestock and expenditure data from the 1963/64 and 1972/73 reconciliations and Sheep Surveys, a comparison of two different sets of decisions and the resulting simulated plans can be made. These are detailed in Tables 7.1 (a) and 7.1 (b). This enables subjective evaluation of the plans by comparison with historical data, and investigation as to whether the investment principles outlined above can be accounted for.

Table 7.1 (a) Envisaged Plans - Stock Numbers

Decisions (Year t=1)		PLAN 1 1963/64 data		PLAN 2 1972/73 data	
			(2)		(2)
Ewe lamb sales		120	17%	27	3%
Cull ewe sales (1)		14	5%	98	27%
Weaner cow sales		8	25%	10	21%
Cull cow sales		16	21%	31	37%
Expenditure $		8019		10188	
Resulting Plan (Years t=2,...,5)					
Sheep Stock	Year		(3)		(3)
Numbers	1	1705		2209	
(E.E)	2	1756		2418	
	3	1946		2796	
	4	2239		3325	
	5	2627	35% ↑	3947	44% ↑
Cattle Stock					
Numbers	1	758		1096	
(E.E)	2	751		1008	
	3	533		669	
	4	533		669	
	5	533	29% ↓	669	39% ↓
Total Stock					
Numbers	1	2463		3305	
(E.E)	2	2507		3426	
	3	2479		3464	
	4	2772		3994	
	5	3159	28% ↑	4616	39% ↑

(1) As defined earlier this does not include old ewes (OE) which are automatically culled.

(2) Sales as a percentage of initial stocks

(3) Percentage increase or decrease in stocks from year t=1.

Table 7.1 (b) Envisaged Plans - Stock Sales

Decisions as in Table 7.1(a)

Resulting Plan (Years t=2,..,5)	Year	PLAN 1 1963/64 data	PLAN 2 1972/73 data
Lamb sales	2	784	1041
	3	1052	1532
	4	1045	1532
	5	1078	1676
Cull ewe sales	2	272	341
	3	260	327
	4	318	467
	5	482	744
Weaner cow sales (1)	2	21	24
	3	2	5
Yearling cow sales	2	8	13
	3	11	13
Cull cow sales	2	16	31
	3	8	10
Weaner steer sales	2	20	26
	3	13	14
Yearling steer sales	2	11	18
	3	0	-1 (2)
2 year old steer sales	2	26	28
	3	11	18

(1) Sales in years t=4,5 are the same as in year t=3 for all cattle.

(2) Negative indicates a purchase.

7.1.2 Discussion

In Plan 1,17% of ewe lambs born are sold and 5% of the mixed age ewes[1] are sold as culls. In Plan 2, 3% of ewe lambs born are sold and 27% of the mixed age ewes sold as culls.

If the investment principles outlined above are accounted for, then Plan 2, which involves retaining comparatively more ewe lambs than Plan 1, will result in comparatively higher future sheep numbers. This is in fact what happens as Table 7.1 (a) shows. Under Plan 2, a 44% increase in sheep stock numbers from year t=1 to year t=5 is envisaged while under Plan 1 the envisaged increase is only 35%.

Both plans envisage decreasing cattle numbers. Under Plan 1, 25% of weaner cows born are sold and 21% of the breeding cows are culled, while under Plan 2,21% of weaners are sold and 37% of the breeding cows are culled. The difference between the two plans in the percentage of weaner cows sold in this case is negligible, however the higher cow culling rate under Plan 2 means that comparatively lower future cattle numbers are envisaged than for Plan 1. Table 7.1 (a) indicates that under Plan 2 there is a 39% decrease, from year t=1 to year t=3, envisaged in cattle stock numbers while under Plan 1 the decrease is only 29%.

Finally, both plans envisage increasing total stock numbers over the following five years, Plan 1 by 28% and Plan 2 by 39%.

---

(1) The stock classes are defined in Section 6.2.2 e.g. mixed age ewes here is a narrower definition than normally used. See Appendix II.

7.2 The Decision Model

The method of solving the decision model is discussed in Section 6.4.2. The model aims to predict values for the five decision variables in year t=1 that bear a close relationship to the observed values for the data period. This relationship can be improved by an estimation procedure (MODFIT) discussed in Section 6.5. Initial results are presented in Table 7.2. The predicted values for the five decision variables for ten separate years of data are presented with the actual values of the decision variables from the Economic Service Reconciliations alongside each for comparison.

Table 7.2 Initial Results from the Decision Model

	ELS (1)	$\hat{ELS}$ (2)	CULL	$\hat{CULL}$	WCS	$\hat{WCS}$	SALE	$\hat{SALE}$	EXP	$\hat{EXP}$
1963/64	120	425	14	2663	8	20	16	-46	8019	4529
64/65	100	209	3	2590	5	43	16	-8	7656	4511
65/66	156	174	-41	2647	7	49	26	5	8461	4512
66/67	113	260	-15	2859	5	58	19	-18	7926	4510
67/68	150	208	27	3012	7	70	22	-5	6950	4489
68/69	163	-110 (3)	69	3316	6	45	32	87	8142	4490
69/70	208	276	58	3283	6	71	33	-17	8618	4492
70/71	27	424	52	3302	6	24	28	-38	7880	4501
71/72	-45	372	57	3326	7	43	25	-33	8034	4519
72/73	27	288	54	3485	10	46	31	-2	10188	4519

(1) Source: N.Z. M.& W.B's Economic Service - Reconciliations by Type of Farm.

(2) Estimated by the model.

(3) A negative indicates a purchase.

7.2.1 Estimation by MODFIT

The MODFIT routine estimates parameters within the model which minimize the determinant F of the matrix of the sums of squares and cross products of the differences between the estimated and the actual values of the decision variables. (See Section 6.5).

For the initial results given in Table 7.2, $F = 4.1 \times 10^{25}$.

The values of the parameters estimated by using MODFIT are conditional on the values of the other parameters. A number of parameters (usually between one and five) are chosen to be estimated for a particular run and MODFIT alters these one at a time until a local minimum is reached. These parameter estimates are then retained and a different set of parameters are estimated in a new run. Two problems arise in using MODFIT for such a large scale problem:

1) It is not possible to know whether a movement to a local minimum is in fact a movement towards the global minimum.

2) No constraints can be placed on the values of the parameters being estimated, thus while F may be at a minimum (local or global), the parameter estimates resulting may not be sensible. Using this method a model that is in fact a poor representation of the true process can be made to perform well (predict the decision variables accurately) by using perverse values for the estimated parameters. Ideally a model should be able to predict the decision variables accurately and have sensible values for the parameters.

A large number of computer runs using MODFIT were undertaken in an attempt to improve the models performance, however the improvements were negligible. For illustrative purposes one run is reported.

7.2.2 Estimation of the Discount Factors Using MODFIT

Table 7.2 gives the results from the decision model with:

$$d_t = \frac{1}{(1 + i)^t} \qquad t = 2,\ldots,4$$

Where: d_t – discount rate in year t.
i – the interest rate. (Set at 6%)

and

$$d_5 = \frac{1}{(1+i)^5} \bigg/ i$$

The fifth year's envisaged income is discounted and capitalised. Although the farmer's immediate planning horizon may be only five years, he is also concerned with his longer term future. Capitalisation allows for this.

These discount rates d_t (not the interest rate i, although this could be done if desired) are re-estimated to try and improve the model's predictions of the decision variables. Table 7.3 (a) shows the effect of estimating the discount factors using MODFIT.

Table 7.3 (a) Results from the Decision Model - Estimating the Discount Rates

	ELS	$\hat{ELS}$	CULL	$\hat{CULL}$	WCS	$\hat{WCS}$	SALE	$\hat{SALE}$	EXP	$\hat{EXP}$
1963/64	120	333	14	2417	8	14	16	8	8019	4223
64/65	100	241	3	2049	5	44	16	26	7656	4259
65/66	156	256	-41	1936	7	59	26	32	8461	4255
66/67	113	311	-15	2144	5	60	19	25	7926	4246
67/68	150	326	27	2082	7	80	22	30	6950	4260
68/69	163	251	69	1919	6	115	32	46	8142	4263
69/70	208	354	58	2413	6	73	33	32	8618	4268
70/71	27	334	52	3028	6	13	28	23	7880	4282
71/72	45	327	57	2888	7	30	25	28	8034	4275
72/73	27	313	54	2676	10	15	31	29	10188	4230

Table 7.3 (b) gives the values of the discount rates estimated.

Table 7.3 (b) The Estimated Discount Rates

$$d_2 = 0.86$$
$$d_3 = 13.15$$
$$d_4 = 3.30$$
$$d_5 = 43.30$$

For the new results given in Table 7.3 (a), $F = 5.1 \times 10^{23}$.

Estimation of the discount factors has reduced F by a factor of approximately 10^2. However, this improvement is only very small and the resulting values for the discount factors are extremely suspect. A value greater than 1.0 would be expected for d_5 because of capitalisation. Discount factors greater than 1.0 for years t=3,4 are unlikely. These results are not surprising, since the initial results before estimation of the discount factors by MODFIT were very poor. In order for MODFIT to be successful the initial results need to be quite close to the actual values for the decision variables. MODFIT then can be used as a "fine tuner".

These results are reported for illustrative purposes only. Attempts were also made to re-estimate the production function and cost function parameters using MODFIT, however none of the results were any better than those reported here.

7.3 Model Performance Under Different Price Combinations

The decision model's performance in predicting values for the decision variables is disappointing. Despite this, it is interesting to study the performance of the model under different price situations. The direction and magnitude of responses of the model to price changes are reflected in the values of the decision variables.

Changes in values of the observed prices affect the farmer's expectations of prices. The hypothesised relationship between observed and expected prices is given by the adaptive expectations model:

$$EP_t = EP_{t-1} + \gamma\,(P_{t-1} - EP_{t-1}) \qquad (7.1)$$

Where:

EP_t - Expected price for period t at period t-1.
EP_{t-1} - Expected price for period t-1 at period t-2.
P_{t-1} - Observed price in t-1.
γ - Coefficient of expectation.

The variables of interest are the observed or actual current prices. The responses to changes in the observed prices of the different products are given in Table 7.4 Using 1972/73 data the model is run under seven different price combinations:

Medium Sheep[1]	–	Medium Cattle[2]
Low Sheep	–	Low Cattle
High Sheep	–	High Cattle
Low Sheep	–	High Cattle
Medium Sheep	–	High Cattle
High Sheep	–	Low Cattle
High Sheep	–	Medium Cattle

The Medium Sheep – Medium Cattle combination represents the "normal" situation for comparative purposes. For each product the medium price is set such that:

$$P_{t-1} = EP_{t-1}$$

$\therefore$ from (7.1)

$$EP_t = EP_{t-1}$$

i.e. expectations do not change from one year to the next.

The high prices are set at actual 1972/73 prices (which were very high compared to previous years' prices). The low prices are set at values equal to half that of the medium prices.

---

(1) Sheep prices – Wool, lamb, cull ewe.

(2) Cattle prices – Weaner, yearling, and cull cow.
– Weaner, yearling, and 2yr old steers.

e.g. Medium Sheep means all three sheep prices are set at medium prices.

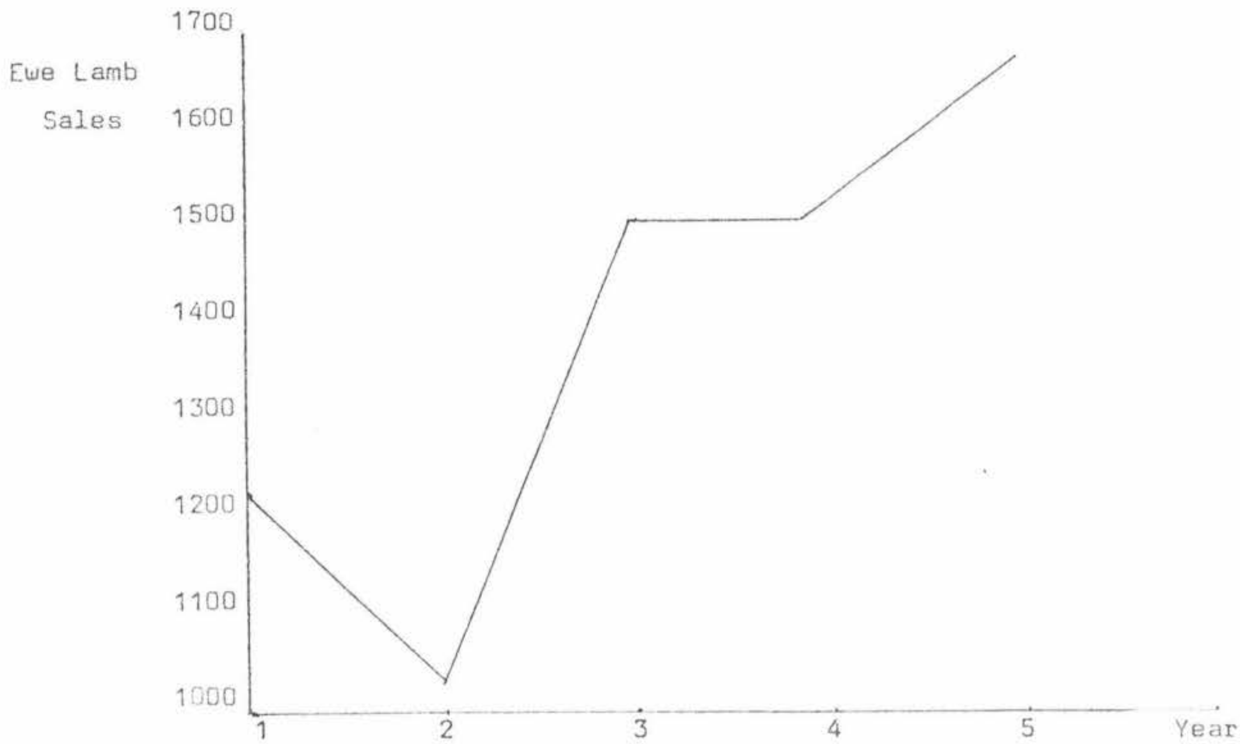
Table 7.4 Decision Variable Predictions - Different Price Combinations

		Cattle Prices											
		LO				MED				HI			
		ELS	CULL	WCS	SALE	ELS	CULL	WCS	SALE	ELS	CULL	WCS	SALE
Sheep Prices	LO	309	3530	36	-5	.	.	.	.	348	3579	24	-14
	MED	.	.	.	.	315	3528	47	-15	331	3540	36	-13
	HI	249	3437	57	6	271	3473	57	-4	288	3485	46	-2

A study of Table 7.4 indicates that the short-term responses of the model to different price changes are largely as would be expected from the principles outlined in Chapter Three. The changes in observed prices are reflected in changes in the farmer's expectations of future prices. As an example, a rise in Sheep Prices (which are expected to persist) with low or medium expected Cattle Prices results in fewer ewe lambs and cull ewes being sold. Thus, the response to a high expected price for sheep products is to build up the flock size to take advantage of these high prices in future years. However, if the expected prices for cattle products are also high then sheep numbers would not increase to the same extent. This emphasizes the misleading nature of elasticities estimated under ceteris paribus conditions.

7.4 Response Time Paths

The envisaged response over time, of sales of animals, to an increase in product prices can be obtained from the production model as outlined in Section 1.2 of this Chapter. For example, the 1972/73 prices for wool, lambs and ewes represented a substantial increase over the previous year's prices. The envisaged effect (Plan 2) of these increases is shown in Figure 7.4.



Source: Tables 7.1 (a) and (b).

Figure 7.4 Envisaged Response to Sheep Price Increases

Initially the price increases result in envisaged lamb sales decreasing as a large proportion of the ewe lambs are retained. (The transitory component). As the resulting breeding flock builds up the envisaged lamb sales increase. (The permanent component).

The actual[(1)] response over time, of sales of animals, to an increase in product prices, can only be obtained from the Projection Model. For example, the response to an increase in lamb prices is obtained in the following way:

---

(1) Actual refers to the model's predictions - not the response as it occurs in the real world; hopefully both would be the same.

The new high price is substituted into the Projection Model[1] and the decision variables solved for. These recursively determine the next year's stocks. The same high price is retained and the model solved again, this time with the new stocks. This process is continued for the desired number of steps. At each step the resulting lamb sales can be easily calculated.

Again it is emphasised that different response time paths will result depending on what assumptions are made about the other product prices. If all Cattle Prices were assumed to remain unchanged then a similar time path to that in Figure 7.4 would be obtained for lamb sales. However, if cattle prices increased at the same time as the lamb prices, then a very different time path to that in Figure 7.4 would be obtained.

Figure 7.5 is the result of making four year projections with the Projections Model, under different price combinations. IS/M is the result of increasing Sheep Prices (i.e. lamb, cull ewe and wool) over time according to :

$$P_t = P_o + 0.25T$$

Where:

P_t - Price of product in year t.
P_o - Price in current year
T - time in years.

The other projections are thus the result of once and for all price changes that are expected to persist through to year 5.

---

(1) The Projection Model is outlined in Appendix I.

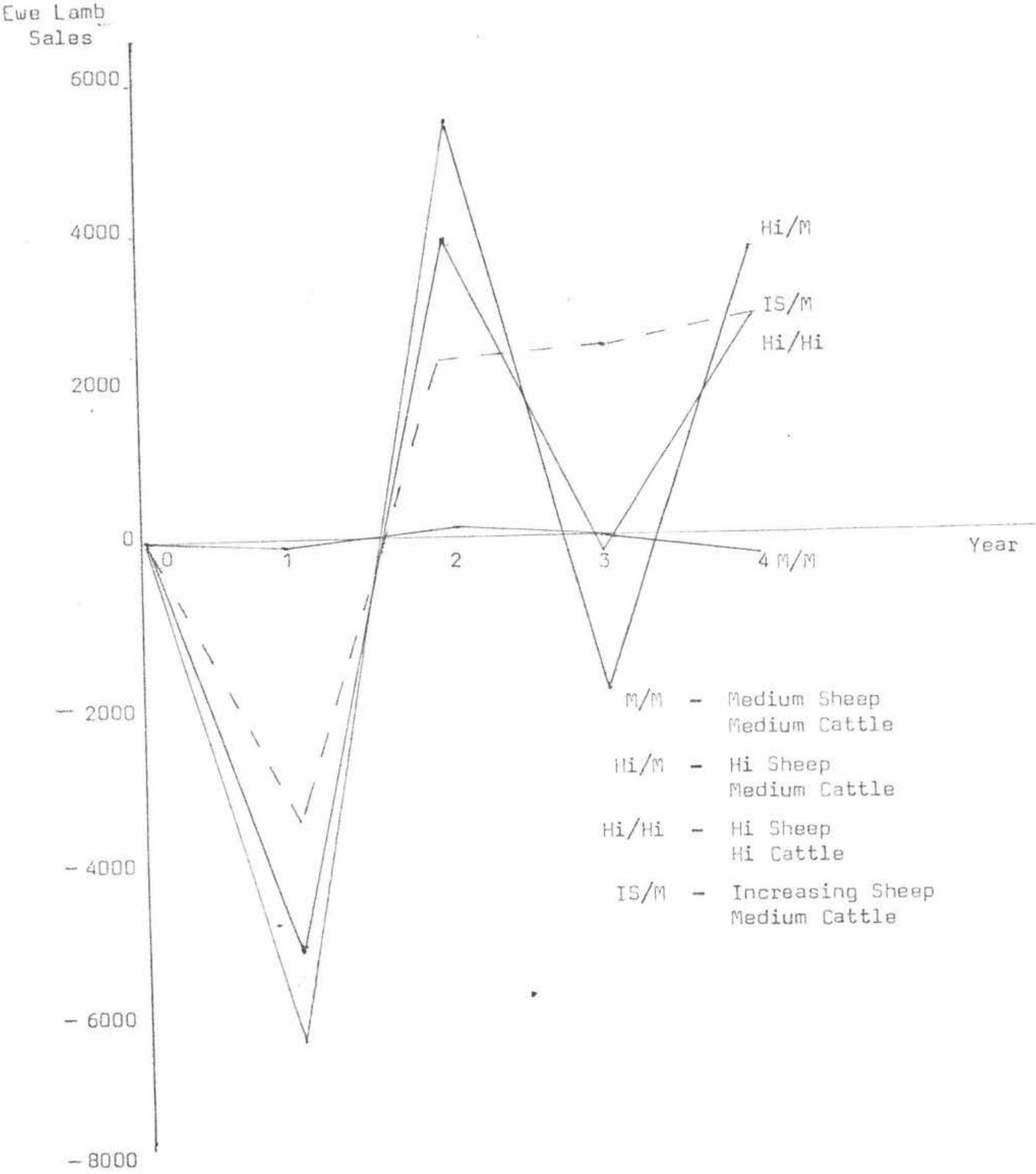


Figure 7.5 Time Path of Ewe Lamb Sales Response to Different Price Changes

7.5 Projections of Livestock Numbers

The use of the model for making projections of future livestock numbers was extensively outlined in Chapter Six. The ability of the model to make these projections is dependent on its ability to predict the decision variables. In view of the model's disappointing performance in predicting these, as outlined in Section 2 of this Chapter, no attempt is made to make any more projections.

Even in its simplest form the model turned out to be very complicated, conceptually and computationally. Unfortunately any simplification returns the model to models of the conventional type which have proved unsuccessful in the past.

CHAPTER EIGHT

CONCLUSIONS

The objective of this study was to investigate the role of investment analysis as it relates to farmers' decision making processes which determine future livestock numbers and supplies of livestock products. Some aspects of the Theory of Investment were reviewed and the application of investment principles to the farm situation outlined. Emphasis was placed on the importance of recognising that livestock has a distinctive dual role; as a consumption good or as an investment good.

Animals of different type, age and sex have distinctive economic functions within the livestock system of a particular farm, and their productive values will be affected differently by changes in prices and costs. At any time, a typical farm is holding a composite capital good represented by animals that differ in type, sex and age. Farmers can therefore be viewed as portfolio managers seeking the optimal combination of different categories of animals to complement their non-animal assets, given existing conditions and future expectations.

The supply models developed in a number of previous studies have been insufficiently disaggregated and have not been able to account for the different economic functions of animals of various types and ages. These models have been unable to show that an expanding flock or herd, through higher lamb and calf crops, would lead to an increase in the number of animals slaughtered over time.

In this study, a model was built that attempted to simulate the investment and consumption decisions made by a representative individual farmer. Satisfactory performance of the model would have enabled projections of livestock numbers to be made. The approach used required a large amount of information, much of it not available, concerning the decision processes of an individual representative farmer and the production relationships of his farm. A survey was conducted to obtain information on the way in which farmers form their price expectations, however further surveys over a number of years would be required before the results from the initial survey could be accepted with any real degree of confidence.

The ultimate aim of many farmers would be to maximize profit from their farming operations. However, a number of farmers would have other objectives, for example, minimization of the variation in income between years subject to some minimum income level being achieved. There are a number of possible objectives, and many farmers may even have multiple objectives. Lack of reliable information on these and the difficulties inherent in modelling when alternative objectives are specified, resulted in profit maximization being chosen initially as the farmer's objective for this study.

It was the intention at the outset to make a rough predictive model then to refine it till further refinements gave little improvement to its predictive power. Thus the effects of taxation, of various government policy measures and of farmer objectives other than profit maximization were ignored in this initial stage. However, the mathematics and computer programming for the simplified model described turned out to be so complicated and so wide of the mark that further refinement was not attempted. This is not to say that the final model presented here has not undergone extensive revision. It has, and the revisions from version to version have been substantial.

The performance of the model was very disappointing. Despite this, some success has been achieved in that the important investment principles outlined in the earlier Chapters of this study were explicitly accounted for in the results generated by it. The model was able to show that the short-term response of a North Island hill country farmer to a price increase for sheep or sheep products that was expected to persist, <u>ceteris paribus</u>, would be to retain more young breeding stock (those with the highest capital value). In addition it showed that the resulting expansion in flock size, through higher lamb crops, would lead to an increase in the number of sheep and lambs slaughtered over time.

The methodology involved in building and using the model, while conceptually similar to a number of previous studies, differed markedly in a number of important respects. The advantages of obtaining an explicit analytical solution to the problem of solving for the decision variables has been described previously. If these decisions, which are the investment and/or consumption decisions made by the farmer, can be predicted with accuracy, then the resulting projections of livestock numbers are likely to be more reliable. Thus, there are real advantages in a methodology that enables the ability of the model to predict values for the decision variables to be improved in a pragmatic manner.

In practice, however, a number of difficulties arose with the use of this methodology. Much of this difficulty arose in the derivation of the explicit analytical solution which required that a large number of algebraic manipulations be performed by the model builder. As the sophistication of the model increases so do the number of manipulations. The result is an extremely difficult and tedious checking problem. The objective function(1) is differentiated with respect to each decision variable and the resulting system of simultaneous equations solved for the decision variables. The algebraic statements that result must then be programmed for the computer. The model is very much more difficult to construct and check than a similar type of model based on a numerical procedure such as linear programming would be.

It is the complexity of the production model and the number of decision variables to be simultaneously solved for that determines how large the decision model will be. Since a sheep and beef farming system, such as the one described in this study, is an extremely

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(1) This function is a very long and awkward algebraic statement. See Appendix III.

complex system a number of simplifying assumptions are required in order that the model can be kept to a manageable size. These assumptions are reflected in the results from the production model which have been discussed in the previous Chapter. While these results appear to be reasonable there is no way in which they can be properly validated. The poor predictive performance of the decision model could well be traced back to simplifying assumptions made in the production model.

Estimation of parameters using the MODFIT routine was outlined in Chapter Six. It was pointed out that by using this routine, a model that is in fact a poor representation of the true process can be made to perform well (predict the decision variables accurately) by using perverse values for the estimated parameters. Ideally, a model should be able to predict the decision variables accurately and have sensible values for the parameters. A routine such as MODFIT, which enabled _a priori_ bounds on the values of the parameters being estimated would be a significant improvement.

The ultimate aim of building models such as the one described in this study is to provide projections of national livestock numbers and/or agricultural product supplies, and to evaluate the probable effects of various Government policies. To enable this, eight models such as the one described in this study[(1)] would have to be formulated and estimated and the results from these reconciled with each other. Alternatively, one large model with sufficient flexibility to incorporate all eight farm classes could be built. The amount of work involved in either undertaking would be extensive.

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(1) There are eight farm classes in the N.Z. M.& W. B s'. Economic Service Classification.

APPENDIX I

THE COMPUTER PROGRAMMES

Figure A1.1 outlines the programme used to estimate the parameters of the model.

Differentiation of the objective function results in five simultaneous equations. (Chapter 6.3.3.1).

$$AB=C \qquad \begin{array}{l} A(1\times 5) \\ B(5\times 5) \\ C(1\times 5) \end{array}$$

Subroutine Calc calculates the elements of the matrix B and the vector C. This is done ten times for ten years' data. Subroutine Model then inverts the matrix B and solves for A, the vector of predicted values for the decision variables. As each year's solution vector is found it is placed in a matrix ME(10x5).

$$ME= \begin{bmatrix} ELS_1 & CULLS_1 \;\cdots\cdots & EXP_1 \\ \vdots & & \\ ELS_{10} & \cdots\cdots\cdots\cdots & EXP_{10} \end{bmatrix}$$

A matrix MA(10x5) of actual values of the decision variables is also set up.

From these two matrices, a 5x5 matrix SS is derived according to the procedure outlined in Section 6.5.

SS is the matrix of the sums of squares and cross-products of the differences between the estimated and the actual value of the decision variables.

The determinant of SS, F is calculated in Subroutine Calcfx, and Subroutine Modfit then proceeds to find the X(I), the values of the parameters to be estimated, such that F is minimized. It does this by iteration, altering the X(I) one at a time.

Figure A1.2 outlines the programme for making projections. The programme is written such that projections of up to and including ten years can be made. An N year projection, however, requires N-1 predicted prices for each product, therefore the length of projection is determined by the availability of these price predictions.

Figure A1.1 The Computer Programme - For Estimating the Parameters

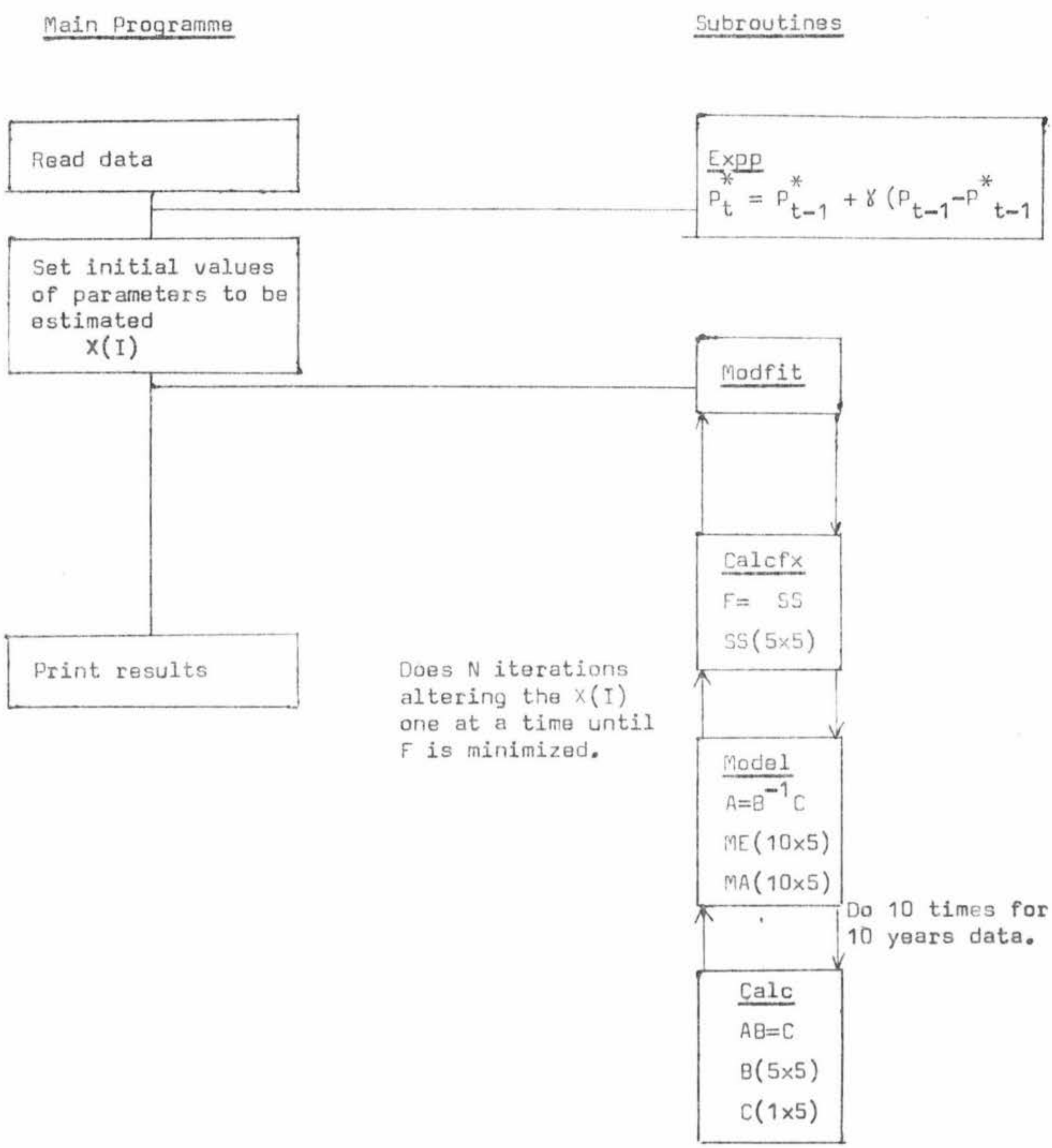
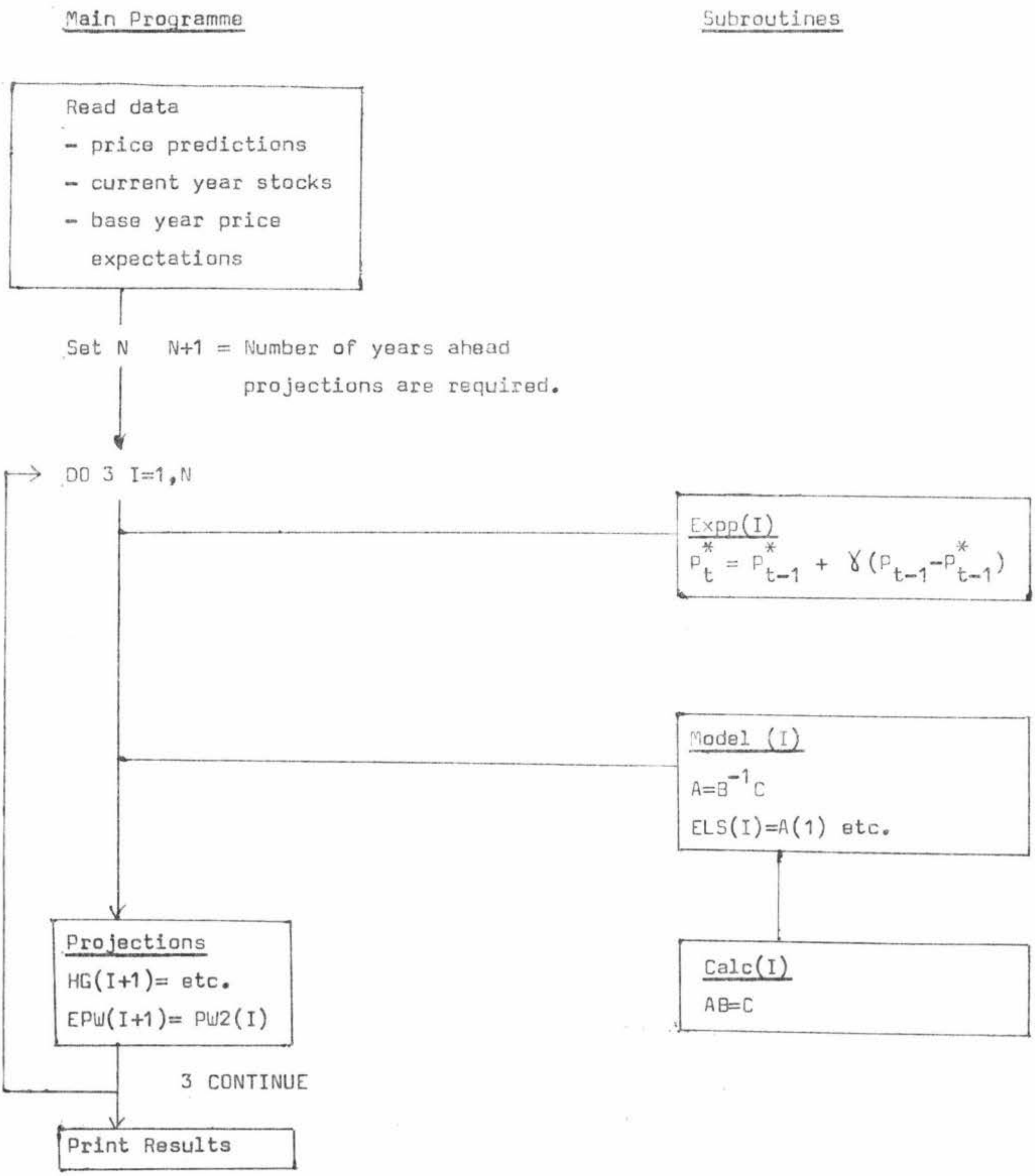


Figure A1.2 Projection Programme



APPENDIX II

THE DATA

The source of all non-price data is the New Zealand Meat and Wool Boards' Economic Service. The Sheep Farm Surveys from 1963/64 to 1972/73 provide data on production, expenditure, income and capital structure of representative farms for eight different farm classes. The Flock and Herd Reconciliations By Type of Farm from 1963/64 to 1972/73 provide data on stocks and sales of animals.

Calculation of Constants

In Chapter Six a number of constants are defined. The values assigned to these are derived from the Economic Service data.

γ_1 Sheep survival rate = 0.96

γ_5 Cattle survival rate = 0.96[1]

δ Expenditure parameter = 0.1[1]

Proportion constants reflecting the desired flock structure.

$\alpha_1 = 1.78$

$\alpha_2 = 1.4$

$\alpha_3 = 1.0$

$\alpha_4 = 1.0$

Proportion constants reflecting the desired herd structure.

$\alpha_6 = 0.35$

$\alpha = 0.5$

$\alpha_7 = 1.0$

Mixed-age Ewes

The Economic Service Reconciliations do not disaggregate mixed-age ewes further into age classes. The separate age classes are obtained in the following way:

Mixed-age ewes = 4TH + 6TH + MAE + OE

(1) Varied in the MODFIT routine

Prices

Actual Prices $

	PW (1)	PL (2)	PE (2)	PWC (1)	PYC (3)	PCC (3)	PWS (2)	PYS (3)	P2S (3)
1963/64	0.91	4.75	3.08	34	44	56	42	56	68
64/65	0.72	6.00	3.90	44	60	72	56	72	86
65/66	0.69	5.44	3.50	44	64	64	60	64	80
66/67	0.57	4.56	2.96	32	50	54	36	54	70
67/68	0.44	5.46	3.54	32	50	55	45	55	70
68/69	0.51	5.45	2.30	40	50	60	50	60	80
69/70	0.47	5.76	4.68	45	70	75	55	75	100
70/71	0.45	5.70	3.90	55	90	80	70	80	100
71/72	0.59	4.92	3.33	60	90	95	80	95	100
72/73	1.25	9.34	12.00	90	120	115	100	115	160

Sources: (1) Wool Marketing Corporation
(2) Schedules
(3) Representative prices - auctions at Te Kuiti and in the Hawkes Bay.

Expected Prices generated by the model $

	PW	PL	PE	PWC	PYC	PCC	PWS	PYS	P2S
1963/64 (1)	0.70	4.00	3.00	24	36	48	28	48	56
64/65	0.89	4.65	3.07	33	42	54	50	54	64
65/66	0.73	5.81	3.80	42	54	66	55	66	79
66/67	0.70	5.49	3.56	44	61	65	59	65	80
67/68	0.58	4.69	3.01	34	53	57	39	57	73
68/69	0.45	5.35	3.49	32	51	56	44	56	71
69/70	0.51	5.44	2.41	39	50	59	49	59	77
70/71	0.47	5.71	4.50	44	64	70	54	70	98
71/72	0.45	5.70	3.96	53	82	77	68	77	99
72/73	0.58	5.03	3.40	59	88	89	78	89	100
73/74	1.24	8.75	11.23	85	110	112	96	107	141

(1) Arbitrarily set.

APPENDIX III

THE OBJECTIVE FUNCTION (1)

$$
\begin{aligned}
Z = {} & (0.6HG_1 + 2TH_1 + 4TH_1 + 6TH_1 + MAE_1 + OE_1).AWY.PW_1 + B_4.EXP_1.PW_1 \\
& - B_5.EXP_1^2.PW_1 + (ELS_1 + WLS_1).PL_1 + CULL_1.PE_1 + OE_1.PE_1 \\
& + WCS_1.PWC_1 + YCS_1.PYC_1 + SALE_1.PCC_1 + WSS_1.PWS_1 + YSS_1.PYS_1 \\
& + 2SS_1.P2S_1 - FC - B_1.SN_1 - B_2.SN_1^2 \\
& \\
& + (1.54.EL_1 - 1.54.ELS_1 + 0.96.2TH_1 + 0.96.4TH_1 + 0.96.6TH_1 \\
& + 0.96.MAE_1 - 0.96.CULL_1).AWY.PW_2.D_2 + B_4.PW_2.D_2.EXP_1 \\
& + B_4.PW_2.D_2.D_8.SN_2 - B_5.PW_2.D_2.EXP_1^2 - 2.B_5.PW_2.D_2.D_8.SN_2.EXP_1 \\
& - B_5.D_8^2.PW_2.D_2.SN_2^2 + (0.05.ELS_1 - 0.05.EL_1 - 0.91.CULL_1 \\
& + 0.91.(2TH_1 + 4TH_1 + 6TH_1 + MAE_1).PL_2.D_2 + B_6.PL_2.D_2.EXP_1 \\
& + B_6.PL_2.D_2.EXP_1 + B_6.D_8.PL_2.D_2.SN_2 - B_7.PL_2.D_2.EXP_1^2 \\
& - 2.B_7.PL_2.D_2.D_8.EXP_1.SN_2 - B_7.D_8^2.D_2.PL_2.D_2.SN_2^2 + 0.96.6TH_1.PE_2.D_2.CULL_1/ \\
& MAE_1 + 0.96.MAE_1.PE_2.D_2 - 0.96.PE_2.D_2.CULL_1 + (0.04.YCR_1 \\
& - 0.17.WC_1 + 0.5.WCS_1 + 0.4.BC_1 - 0.67.SALE_1).PWC_2.D_2 \\
& + 0.5.B_8.D_2.EXP_1.PWC_2 + 0.5.B_8.PWC_2.D_2.D_8.SN_2 - 0.5.B_9.D_2.EXP_1^2.PWC_2 \\
& - B_9.PWC_2.D_2.D_8.EXP_1.SN_2 - 0.5.B_9.D_8^2.D_2.SN_2^2.PWC_2 + (0.48.WC_1 \\
& - 0.48.WCS_1).PYC_2.D_2 + SALE_1.PCC_2.D_2 + (0.38.YCR_1 + 0.38.BC_1 \\
& - 0.38.SALE_1 - 0.48.WSR_1).PWS_2.D_2 + 0.5.B_8.D_2.EXP_1.PWS_2 \\
& + 0.5.B_8.D_2.D_8.PWS_2.SN_2 - 0.5.B_9.D_2.EXP_1^2.PWS_2 - B_9.PWS_2.D_2.D_8.EXP_1.SN_2 \\
& - 0.5.B_9.D_8^2.D_2.PWS_2.SN_2^2 + 0.46.WSR_1.D_2.PYS_2 + 0.96.YSR_1.D_2.P2S_2 \\
& - FC_2.D_2 - B_1.D_2.SN_2 - B_2.D_2.SN_2^2 \\
& \\
& + (2.39.EL_1 - 2.39.ELS_1 + 0.92.2TH_1 + 0.92.4TH_1 + 0.92.6TH_1 \\
& - 0.92.6TH_1.CULL_1/MAE_1).AWY.PW_3.D_3 + B_4.D_3.PW_3.EXP_1 \\
& + B_4.PW_3.D_3.D_8.SN_3 - B_5.D_3.PW_3.EXP_1^2 - 2.B_5.D_3.D_8.PW_3.EXP_1.SN_3 \\
& - B_5.D_8^2.D_3.PW_3.SN_3^2 + (0.46.EL_1 - 0.46.ELS_1 + 0.87.(2TH_1 + 4TH_1 \\
& + 6TH_1)).PL_3.D_3 + B_6.PL_3.D_3.EXP_1 + B_6.D_8.PL_3.D_3.SN_3
\end{aligned}
$$

CONTINUED---

(1) The survival rates and proportion constants have been substituted.

OBJECTIVE FUNCTION CONTINUED....

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$- \mathrm{B_7.D_3.EXP_1^2.PL_3 - B_7.D_8^2.PL_3.D_3.SN_3^2 + 0.92.4TH.D_3.CULL_1.PE_3/MAE_1}$

$+ \mathrm{0.92.6TH_1.D_3.PE_3 - 0.92.6TH_1.D_3.CULL_1.PE_3/MAE_1 + (0.02.WC_1}$

$- \mathrm{0.06.WCS_1. + 0.04.YCR_1 + 0.04.BC_1 - 0.08.SALE_1).D_3.PWC_3}$

$+ \mathrm{0.5.B_8.D_3.PWC_3.EXP_1 + 0.5.B_8.PWC_3.D_3.D_8.SN_3 - 0.5.B_9.PWC_3.D_3.EXP_1^2}$

$- \mathrm{B_9.PWC_3.D_3.D_8.SN_3.EXP_1 - 0.5.B_9.D_8^2.PWC_3.D_3.SN_3^2 + (0.08.WC_1}$

$- \mathrm{0.24.WCS_1 + 0.16.YCR_1 + 0.16.BC_1 - 0.32.SALE_1).PYC_3.D_3}$

$+ \mathrm{(0.06.WC_1 - 0.18.WCS_1 + 0.12.YCR_1 + 0.12.BC_1 - 0.24.SALE_1).D_3.PCC_3}$

$+ \mathrm{(0.18.WC_1 - 0.55.WCS_1 + 0.37.YCR_1 + 0.37.BC_1 - 0.74.SALE_1}$

$- \mathrm{0.48.WSR_1).PWS_3.D_3 + 0.5.B_8.D_3.EXP_1.PWS_3 + 0.5.B_8.PWS_3.D_3.D_8.SN_3}$

$- \mathrm{0.5.B_9.PWS_3.D_3.EXP_1^2 - B_9.PWS_3.D_3.D_8.EXP_1.SN_3 - 0.5.B_9.D_8^2.PWS_3.D_3.SN_3^2}$

$- \mathrm{0.02.WSR_1.PYS_3.D_3 + 0.46.WSR_1.P2S_3.D_3 - FC_3.D_3 - B_1.SN_3.D_3}$

$- \mathrm{B_2.SN_3^2.D_3}$

$+ \mathrm{(3.38.EL_1 - 3.38.ELS_1 + 0.88.2TH_1 + 0.88.4TH_1 - 0.88.4TH_1.CULL_1/MAE_1).}$

$\mathrm{AWY.PW_4.D_4 + B_4.PW_4.D_4.EXP_1 + B_4.PW_4.D_4.D_8.SN_4 - B_5.PW_4.D_4.EXP_1^2}$

$- \mathrm{2.B_5.PW_4.D_4.D_8.EXP_1.SN_4 - B_5.D_8^2.PW_4.D_4.SN_4^2 + (0.93.EL_1 - 0.93.ELS_1}$

$+ \mathrm{0.84.(2TH_1 + 4TH_1 + 6TH_1) - 0.84.4TH_1.CULL_1/MAE_1).PI_4.D_4}$

$+ \mathrm{B_6.PL_4.D_4.EXP_1 + B_6.D_8.PI_4.D_4.SN_4 - B_7.PL_4.D_4.EXP_1^2}$

$- \mathrm{2.B_7.PL_4.D_4.D_8.SN_4.EXP_1 - B_7.D_8^2.PL_4.D_4.SN_4^2 + 0.88.2TH_1.PE_4.D_4.CULL_1/ MAE_1}$

$+ \mathrm{0.88.4TH_1.PE_4.D_4 - 0.88.4TH_1.PE_4.D_4.CULL_1/MAE_1 + (0.02.WC_1 - 0.06.WCS_1}$

$+ \mathrm{0.04.YCR_1 + 0.04.BC_1 - 0.08.SALE_1).PWC_4.D_4 + 0.5.B_8.D_4.EXP_1.PWC_4}$

$+ \mathrm{0.5.B_8.PWC_4.D_4.D_8.SN_4 - 0.5.B_9.PWC_4.D_4.EXP_1^2 - B_9.PWC_4.D_4.D_8.EXP_1.SN_4}$

$- \mathrm{0.5.B_9.D_8^2.PWC_4.D_4.SN_4^2 + (0.08.WC_1 - 0.24.WCS_1 + 0.16.YCR_1}$

$+ \mathrm{0.16.BC_1 - 0.32.SALE_1).PYC_4.D_4 + (0.06.WC_1 - 0.18.WCS_1 + 0.12.YCR_1}$

$+ \mathrm{0.12.BC_1 - 0.24.SALE_1).PCC_4.D_4 + (0.18.WC_1 - 0.55.WCS_1 + 0.37.YCR_1}$

$+ \mathrm{0.37.BC_1 - 0.74.SALE_1 - 0.48.WSR_1).PWS_4.D_4}$

CONTINUED.....

OBJECTIVE FUNCTION CONTINUED....

$$+ 0.5.B_8.D_4.PWS_4.EXP_1 + 0.5.B_8.PWS_4.D_4.D_8.SN_4 - 0.5.B_9.PWS_4.D_4.EXP_1^2$$
$$- B_9.PWS_4.D_4.EXP_1^2 - B_9.PWS_4.D_4.D_8.SN_4.EXP_1 - 0.5.B_9.D_8^2.PWS_4.D_4.SN_4^2$$
$$- 0.02.WSR_1.PYS_4.D_4 + 0.46.WSR_1.P2S_4.D_4 - FC_4.D_4 - B_1.SN_4.D_4$$
$$-B_2.D_4.SN_4^2$$

$$+ (4.61.EL_1 - 4.61.ELS_1 + 0.84.2TH_1 - 0.84.2TH_1.CULL_1/MAE_1).AWY.PW_5.D_5$$
$$+ B_4.PW_5.D_5.EXP_1 + B_4.PW_5.D_5.EXP_1 + B_4.PW_5.D_5.D_8.SN_5 - B_5.PW_5.D_5.EXP_1^2$$
$$- 2.B_5.PW_5.D_5.D_8.EXP_1.SN_5 - B_5.D_8^2.PW_5.D_5.SN_5^2 + (1.93.EL_1 - 1.93.ELS_1$$
$$+ 0.81.2TH_1 - 0.81.2TH_1.CULL_1/MAE_1).PL_5.D_5 + B_6.PL_5.D_5.EXP_1$$
$$+ B_6.D_8.PL_5.D_5.SN_5 - B_7.PL_5.D_5.EXP_1^2 -2.B_7.PL_5.D_5.D_8.SN_5.EXP_1$$
$$- B_7.D_8^2.PL_5.D_5.SN_5^2 + (0.84.EL_1 - 0.84.ELS_1 - 0.88.2TH_1$$
$$+0.88.2TH_1.CULL_1/MAE_1).PE_5.D_5 + (0.84.2TH_1 - 0.84.2TH_1.CULL_1/MAE_1).$$
$$PE_5.D_5 + (0.02.WC_1 - 0.06.WCS_1 + 0.04.YCR_1 + 0.04.BC_1 - 0.08.SALE_1).$$
$$PWC_5.D_5 + 0.5.B_8.D_5.PWC_5.EXP_1 + 0.5.B_8.PWC_5.D_5.D_8.SN_5$$
$$- 0.5.B_9.PWC_5.D_5.EXP_1^2 - B_9.PWC_5.D_5.D_8.SN_5.EXP_1 - 0.5.B_9.D_8^2.PWC_5.D_5.SN_5^2$$
$$+ (0.08.WC_1 - 0.24.WCS_1 + 0.16.YCR_1 + 0.16.BC_1 - 0.32.SALE_1).PYC_5.D_5$$
$$+ (0.06.WC_1 - 0.18.WCS_1 + 0.12.YCR_1 + 0.12.BC_1 - 0.24.SALE_1).PCC_5.D_5$$
$$+ (0.18.WC_1 - 0.55.WCS_1 + 0.37.YCR_1 + 0.37.BC_1 - 0.74.SALE_1$$
$$- 0.48.WSR_1).PWS_5.D_5 + 0.5.B_8.D_5.PWS_5.D_5.EXP_1 +0.5.B_8.PWS_5.D_5.D_8.SN_5$$
$$- 0.5.B_9.PWS_5.D_5.EXP_1^2 - B_9.PWS_5.D_5.D_8.SN_5.EXP_1$$
$$- 0.5.B_9.D_8^2.PWS_5.D_5.SN_5^2 - 0.02.WSR_1.PYS_5.D_5 + 0.46.WSR_1.P2S_5.D_5$$
$$- FC_5.D_5 - B_1.SN_5.D_5 - B_2.SN_5^2.D_5$$

STOCK NUMBERS IN YEARS t=1,...,5

$$SN_1 = 0.6.HG_1 + 2TH_1 + 4TH_1 + 6TH_1 + MAE_1 + OE_1 + 4.YC_1 + 6.BC_1 + 4.YS_1 + 4.2S_1$$

$$SN_2 = 1.54.EL_1 - 1.54.ELS_1 + 0.96.(2TH_1 + 4TH_1 + 6TH_1 + MAE_1 - CULL_1) + 3.84.WC_1 - 3.84.WCS_1 + 5.76.(YCR_1 + BC_1 - SALE_1 + WSR_1 + YSR_1)$$

$$SN_3 = 2.39.EL_1 - 2.39.ELS_1 + 0.92.(2TH_1 + 4TH_1 + 6TH_1 - 6TH_1.CULL_1/MAE_1) + 3.40.WC_1 - 10.21.WCS_1 + 6.81.YCR_1 + 6.81.BC_1 - 13.6.SALE_1 + 2.48.WSR_1$$

$$SN_4 = 3.38.EL_1 - 3.38.ELS_1 + 0.88.(2TH_1 + 4TH_1 - 4TH_1.CULL_1/MAE_1) + 3.4.WC_1 - 10.21.WCS_1 + 6.81.YCR_1 + 6.81.BC_1 - 13.62.SALE_1 + 2.48.WSR_1$$

$$SN_5 = 4.61.EL_1 - 4.61.ELS_1 + 0.84.(2TH_1 - 2TH_1.CULL_1/MAE_1) + 3.4.WC_1 - 10.21.WCS_1 + 6.81.YCR_1 + 6.81.BC_1 - 13.62.SALE_1 + 2.48.WSR_1$$

AIII.2 THE RECONCILIATIONS

AIII.2.1 Sheep

$HG_2 = \gamma_1 (EL_1 - ELS_1)$

$HG_3 = \alpha_3 \gamma_1^2 (EL_1 - ELS_1)$

$HG_4 = \alpha_2 \gamma_1^3 (EL_1 - ELS_1)$

$HG_5 = \alpha_1 \gamma_1^4 (EL_1 - ELS_1)$

$2TH_2 = \alpha_4 \gamma_1 (EL_1 - ELS_1)$

$2TH_3 = \gamma_1^2 (EL_1 - ELS_1)$

$2TH_4 = \alpha_3 \gamma_1^3 (EL_1 - ELS_1)$

$2TH_5 = \alpha_2 \gamma_1^4 (EL_1 - ELS_1)$

$4TH_2 = \gamma_1 \; 2TH_1$

$4TH_3 = \alpha_4 \gamma_1^2 (EL_1 - ELS_1)$

$4TH_4 = \gamma_1^3 (EL_1 - ELS_1)$

$4TH_5 = \alpha_3 \gamma_1^4 (EL_1 - ELS_1)$

$6TH_2 = \gamma_1 \; 4TH_1$

$6TH_3 = \gamma_1^2 \; 2TH_1$

$6TH_4 = \alpha_4 \gamma_1^3 (EL_1 - ELS_1)$

$6TH_5 = \gamma_1^4 (EL_1 - ELS_1)$

$MAE_2 = \gamma_1 \; 6TH_1$

$MAE_3 = \gamma_1^2 \; 4TH_1$

$MAE_4 = \gamma_1^3 \; 2TH_1$

$MAE_5 = \alpha_4 \gamma_1^4 (EL_1 - ELS_1)$

$OE_2 = \gamma_1 (MAE_1 - CULL_1)$

$OE_3 = 6TH_1 \; \gamma_1^2 (1 - CULL_1/MAE_1)$

$OE_4 = 4TH_1 \; \gamma_1^3 (1 - CULL_1/MAE_1)$

$OE_5 = 2TH_1 \; \gamma_1^4 (1 - CULL_1/MAE_1)$

Sheep Sales

$$TLS_2 = EL_2 + WL_2 - \alpha_3 \gamma_1 (EL_1 - ELS_1)$$
$$TLS_3 = EL_3 + WL_3 - \alpha_2 \gamma_1^2 (EL_1 - ELS_1)$$
$$TLS_4 = EL_4 + WL_4 - \alpha_1 \gamma_1^3 (EL_1 - ELS_1)$$
$$TLS_5 = EL_5 + WL_5 - \alpha_1 \gamma_1^3 (EL_1 - ELS_1)$$

$$CULL_2 = \gamma_1 \; 6TH_1 . CULL_1 / MAE_1$$
$$CULL_3 = \gamma_1^2 \; 4TH_1 . CULL_1 / MAE_1$$
$$CULL_4 = \gamma_1^3 \; 2TH_1 . CULL_1 / MAE_1$$
$$CULL_5 = \alpha_4 \gamma_1^4 . (EL_1 - ELS_1) - \gamma_1^3 . (2TH_1 - 2TH_1 . CULL_1 / MAE_1)$$

AIII.2.2 Cattle Stocks

$$YC_2 = \gamma_5 (WC_1 - WCS_1)$$
$$YC_3 = \alpha_6 (\gamma_5^2 - \gamma_5^2 \alpha) WC_1 - \alpha_6 \gamma_5^2 (1-\alpha) WCS_1 + \alpha_6 \gamma_5^2 YCR_1 + \alpha_6 \gamma_5^2 BC_1$$
$$-2\alpha_6 \gamma_5^2 \; SALE_1$$
$$BC_2 = \gamma_5 YCR_1 + \gamma_5 (BC_1 - SALE_1)$$
$$BC_3 = \gamma_5^2 (1-\alpha) WC_1 - \gamma_5^2 (1+\alpha) WCS_1 + \gamma_5^2 \; YCR_1 + \gamma_5^2 \; BC_1$$
$$-2 \gamma_5^2 \; SALE_1$$

$$YS_2 = \gamma_5 \; WSR_1$$
$$YS_3 = \tfrac{1}{2} \alpha_7 \gamma_5^2 \; WSR_1$$
$$2S_2 = \gamma_5 \; YSR_1$$
$$2S_3 = \tfrac{1}{2} \gamma_5^2 \; WSR_1$$

Cattle Sales

$$WCS_2 = \gamma_5 (\tfrac{1}{2} \delta_2 - \alpha_6) \; YCR_1 - \alpha_6 \gamma_5 (1-\alpha) \; WC_1 + \alpha_6 \gamma_5 (1+\alpha) WCS_1 + \gamma_5 (\tfrac{1}{2} \delta_2 - \alpha_6) BC_1$$
$$-2\alpha_6 \gamma_5 \; SALE_1 + 0.5 . B_8 . EXP_1 - 0.5 . B_9 . EXP_1^2$$
$$WCS_3 = \gamma_5^2 (\tfrac{1}{2} \delta_2 - \frac{\alpha 6}{\gamma_5}) YCR_1 + (\gamma_5^2 - \gamma_5^2 \alpha)(\tfrac{1}{2} \delta_2 - \frac{\alpha_6}{\gamma_5}) WC_1 - (\gamma_5^2 + \gamma_5^2 \alpha)(\tfrac{1}{2} \delta_2 - \frac{\alpha_6}{\gamma_5}) WCS_1 + \gamma_5^2 (\tfrac{1}{2} \delta_2 - \frac{\alpha_6}{\gamma_5}) BC_1$$
$$-2 \gamma_5^2 (\tfrac{1}{2} \delta_2 - \frac{\alpha 6}{\gamma_5}) SALE_1 + 0.5 . B_8 . EXP_1 - 0.5 . B_9 . EXP_1^2$$

$$YCS_2 = \gamma_5 \alpha (WC_1 - WCS_1)$$

$$YCS_3 = \alpha_6 \gamma_5^2 (1-\alpha) \; WC_1 - \alpha\alpha_6\gamma_5^2 (1+\alpha) \; WCS_1 + \alpha\alpha_6\gamma_5^2 \; YCR_1 + \alpha\alpha_6 \gamma_5^2 \; BC_1 - 2\alpha\alpha_6 \gamma_5^2 \; SALE_1$$

$$SALE_2 = SALE_1$$

$$SALE_3 = \gamma_5^2 (1-\alpha)(\alpha_6 - \alpha_6\alpha + 1 - \frac{1}{\gamma_5}) \; WC_1 - \gamma_5^2 (1+\alpha)(\alpha_6 - \alpha_6\alpha + 1 - \frac{1}{\gamma_5}) WCS_1 + \gamma_5^2 (\alpha_6(1-\alpha) + (1 - \frac{1}{\gamma_5})) YCR_1 + \gamma_5^2 (\alpha_6(1-\alpha) + 1 - \frac{1}{\gamma_5}) \; BC_1 - 2\gamma_5^2 (\alpha_6(1-\alpha) + 1 - \frac{1}{\gamma_5}) \; SALE_1$$

$$WSS_2 = 0.5 \; ACP. \; \gamma_5 YCR_1 + 0.5.ACP.\gamma_5 \; (BC_1 - SALE_1) - 0.5.\gamma_5\alpha_7 WSR_2 + 0.5.B_8.EXP_1 - 0.5.B_9.EXP_1^2$$

$$WSS_3 = 0.5.ACP.(\gamma_5^2 (1-\alpha) \; WC_1 - \gamma_5^2 (1+\alpha) \; WCS_1 + \gamma_5^2 \; YCR_1 + \gamma_5^2 BC_1 - 2\gamma_5^2 \; SALE_1) - 0.5.\alpha_7 \gamma_5 WSR_1 + 0.5.B_8.EXP_1 - 0.5.B_9.EXP_1^2$$

$$YSS_2 = 0.5 \gamma_5 WSR_1$$

$$YSS_3 = 0.5 \gamma_5 (\alpha_7 \gamma_5 - 1) WSR_1$$

AIII.3 DEFINITIONS OF VARIABLES

STOCKS

EL	Ewe lambs
WL	Wether lambs
HG	Ewe hoggets
2TH	Two-tooth ewes
4TH	Four-tooth ewes
6TH	Six-tooth ewes
MAE	Mixed-age ewes
OE	Old ewes
WC	Weaner cows
YC	Yearling cows
BC	Breeding cows
WS	Weaner steers
YS	Yearling steers
2S	Two-year-old steers
SN	Stock numbers (E.E)
BE	Total breeding ewes
SPW	Sheep producing wool

FLOWS

ELS	Ewe lamb sales
TLS	Total lamb sales
WLS	Wether lamb sales
CULL	Cull ewe sales
WCS	Weaner cow sales
YCS	Yearling cow sales
YCR	Yearling cows retained
SALE	Cull cow sales
WSR	Weaner steers retained
YSR	Yearling steers retained
2SS	Two-year-old steer sales
YSS	Yearling steer sales

MISCELLANEOUS

EXP	Expenditure
FC	Fixed costs
C	Total costs
AWY	Average wool yield
ACP	Average calving %
ALP	Average lambing %

PRICES

PW	Wool
PL	Lamb
PE	Ewe
PWC	Weaner cow
PYC	Yearling cow
PCC	Cull cow
PWS	Weaner steer
PYS	Yearling steer
P2S	Two-year-old steer

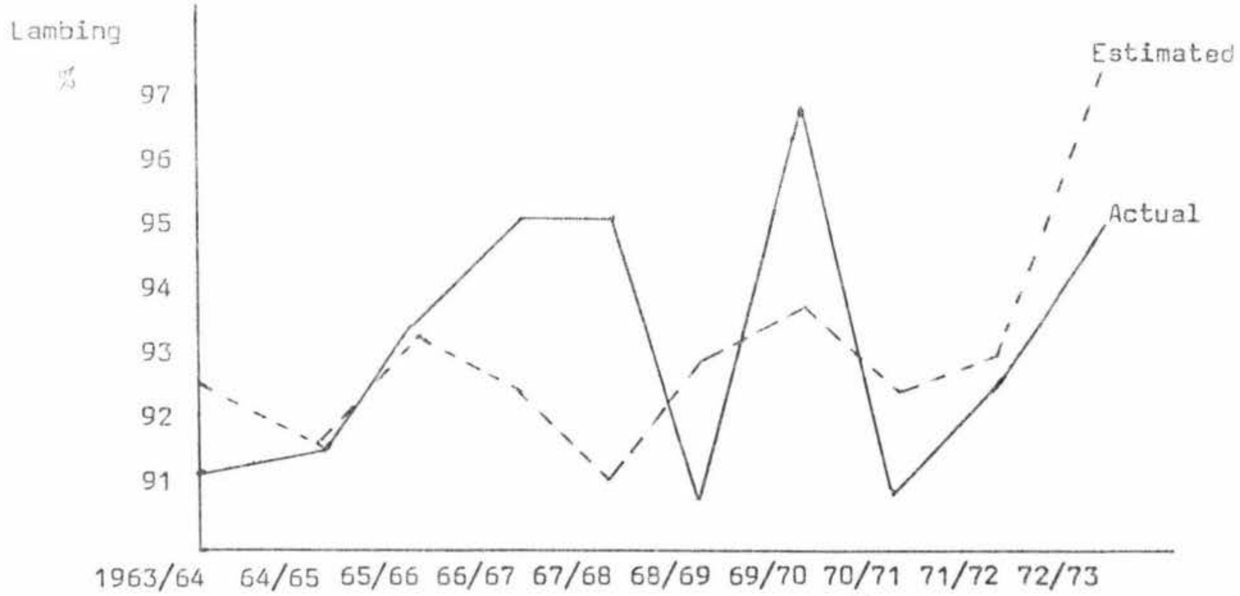
PARAMETERS

B_1, B_2	Cost	
B_4, B_5	Wool	
B_6, B_7	Lamb	
B_8, B_9	Calf	
γ_1	Sheep survival rate	
γ_5	Cattle survival rate	
α_1	$HG_5/6TH_5$	Proportion constants reflecting the desired flock structure.
α_2	$2TH_5/6TH_5$	
α_3	$4TH_5/6TH_5$	
α_4	$MAE_5/6TH_5$	
α	YCS_2/YC_2	Proportion constants for cattle.
α_6	YC_3/BC_3	
α_7	$YS_3/2S_3$	
D_2, D_3, D_4, D_5		Discount rates
δ		Constant relating envisaged expenditure to envisaged stock numbers.

APPENDIX IV

THE PRODUCTION FUNCTIONS

Section 6.2.4 outlined the production functions used in the model. All lamb and calf functions represent functions as envisaged by the farmer. There are two types of wool production function - current[1] and envisaged. Data are available on current wool production, and the production estimated by the model can be compared with actual data. This is shown in Figure A.4.3. Since the lamb and calf functions represent only envisaged functions their predictions can not strictly be compared with actual data. However, it is assumed that the farmer does not envisage the parameters of these functions changing over the five year planning horizon, hence they can be estimated from actual data. The results from these functions are shown in Figures A.4.1 and A.4.2.



Source: Economic Service.

Figure A.4.1 Lamb Production

(1) Current refers to the first year of the five year envisaged plan.

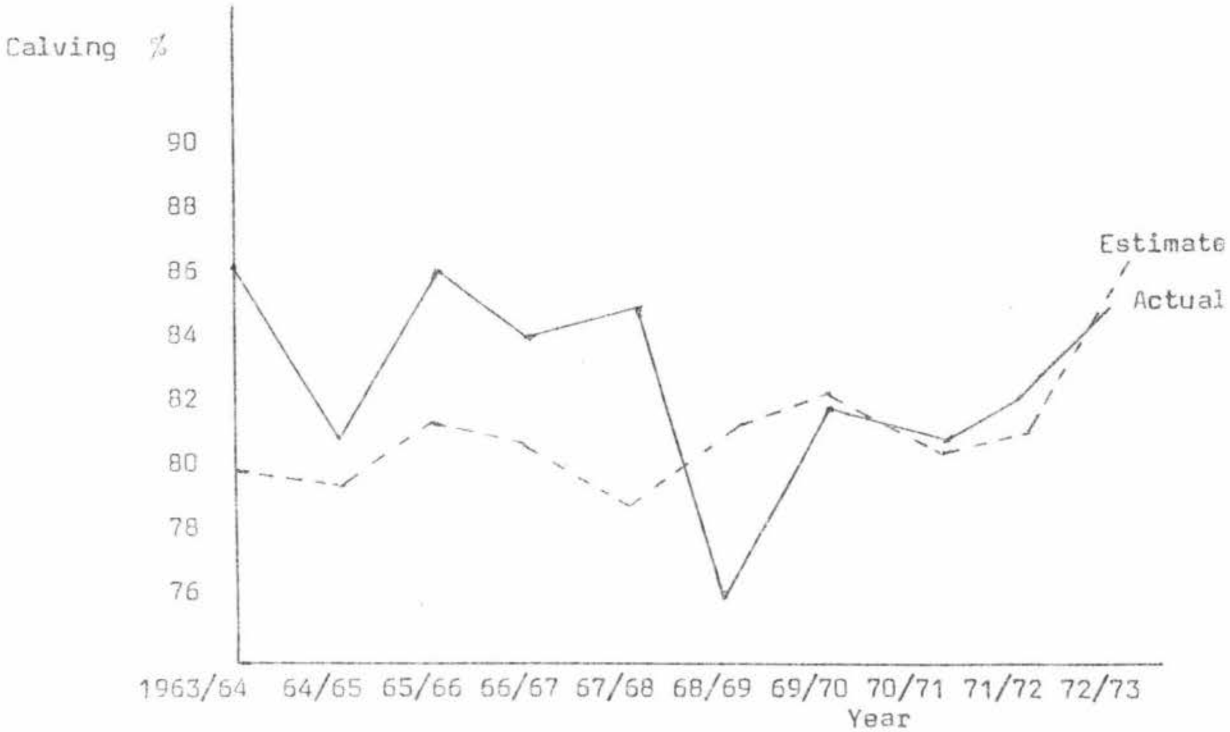
The estimated function is:

$$LP/100 = 0.938 + 0.036\left(\frac{EXP}{BE}\right) - 0.0000042\left(\frac{EXP}{BE}\right)^2$$

Where: LP - Lambing %

EXP - Expenditure as earlier defined.

BE - Number of breeding ewes.



Source: Economic Service.

Figure A.4.2 Calf Production

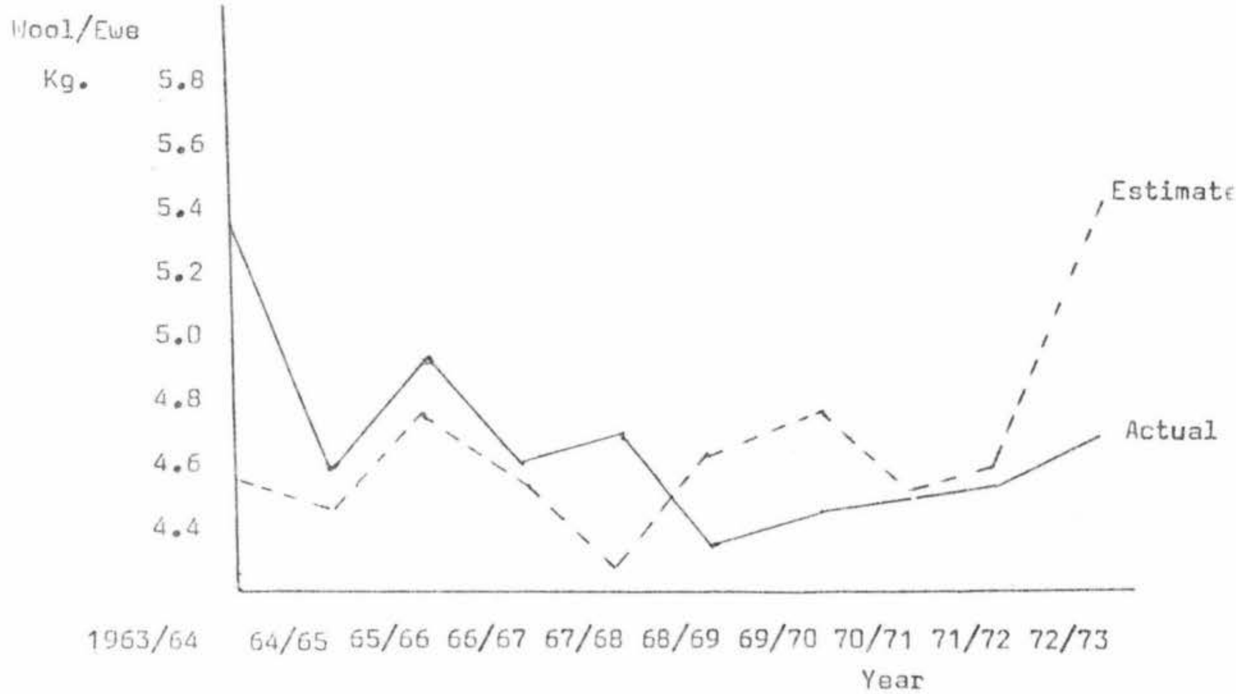
The estimated function is:

$$CP/100 = 0.834 + 0.0023\left(\frac{EXP}{BC}\right) - 0.00000026\left(\frac{EXP}{BC}\right)^2$$

Where: CP - Calving %

EXP - Expenditure as previously defined.

BC - Number of breeding cows.



Source: Economic Service

Figure A.4.3 Wool Production

The estimated function is:

$$W = 4.62 + 0.686\left(\frac{EXP}{SPW}\right) - 0.000084\left(\frac{EXP}{SPW}\right)^2$$

Where: W - Wool per ewe, kg.

EXP - As previously defined.

SPW - Sheep producing wool (weighted according to age).

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