Supporting 5 – 6 Year Old Students To Know And Use Mathematical Practices

A thesis presented in partial fulfilment of the requirements for the degree of

Master of Education

in

Mathematics Education

at Massey University, Albany, New Zealand

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2024

Abstract

Supporting students to be mathematically proficient at school, begins from their first formal mathematics lesson. For students in Aotearoa, the refreshed curriculum outlines the expectation that students engage in mathematical explanations, justification, argumentation, representations and generalizing. Whilst there is sufficient literature on these mathematical practices with older students, there is limited research focusing on how to enact mathematical practices with young learners. This study examines the teacher actions used within students first formal mathematics lessons and the ways in which the teacher engaged the young learners to explain, justify, argue mathematically, represent their thinking and generalise mathematical ideas.

Drawing upon qualitative research methods within a single bounded case study this study was set within a semi-rural school in Aotearoa. One teacher was selected to participate with ten young students aged five years old and one student aged six years old. Data collection occurred during these students first seven formal mathematics lessons. A range of data were collected and analysed, including field notes, video recorded classroom observations, photographs of student work samples and a teacher questionnaire.

Findings revealed the complex nature of engaging young learners in mathematical practices. However, when students are expected to and provided with opportunities to reason mathematically, young learners can succeed. Initially the teacher actions included specific questioning and conversational moves to draw out the thinking from the students. Over the duration of the study these teacher actions shifted to include open prompts requesting the students engage in a mathematical practice.

This study provides insight to the progression of teacher actions used and offers a contribution to the literature regarding how young learners can mathematically reason. It is acknowledged that for such practices to occur teachers must value mathematical conversation and constantly provide opportunities for young learners to reason.

Acknowledgement

I would like to acknowledge and thank the number of people that have supported me throughout this journey.

Firstly, I would like to thank my two supervisors Professor Jodie Hunter and Doctor Generosa Leach. The advice, knowledge and support that you have both provided throughout this research project has been invaluable. Over the last five years, both yourselves and Professor Emerita Roberta (Bobbie) Hunter have encouraged and supported my passion to explore mathematics in early years education. I appreciate the mentoring, advice and opportunities you have all provided over the last five years. It has been an honor to learn from you all.

Secondly, I would like to thank the teacher participant for her work within this study. To allow me to sit alongside your students within their first mathematics lessons was an absolute privilege. Thank you to the principal and board of trustees for allowing me to peruse this research within your school. Starting school is a big moment, so I appreciate the willingness of the parents who gave consent for their young learners to participate in this study – thank you.

Finally, I would like to thank my whānau. Ryan, this journey could not have happened without your support and encouragement. Cooper and Frankie, I one day hope you are passionate and driven to achieve your goals, for now let us enjoy well earnt family time. To my wider whānau and friends, thank you for your support. To know that at any moment I could seek advice or motivation was immensely encouraging throughout this time.

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Chapter One: Introduction

1.1 Introduction

This chapter outlines the background to the study and provides the context of the study. Section 1.2 highlights the background to the study. Section 1.3 explains the rationale of the study. Following, the specific research question in Section 1.4 is presented. To conclude Section 1.5 provides an overview of the thesis.

1.2 Background to the study

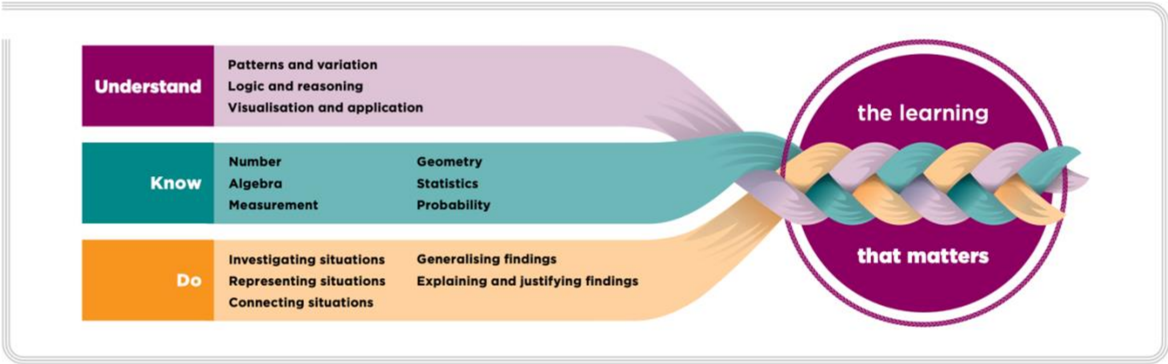
There have been ongoing concerns in New Zealand related to mathematics teaching and learning and ongoing disparities in achievement reflected in both national and international assessments (International Association for the Evaluation of Educational Achievement, 2020; Ministry of Education, 2022; OECD, 2024). In response, the New Zealand Ministry of Education asked the Royal Society Te Apārangi to convene an expert advisory mathematics panel to analyse the current mathematics curriculum and recommend changes. The expert advisory mathematics panel met from January to June in 2021 to support the ministry in this process (Royal Society Te Apārangi, 2021). Three key themes 'slippage, inequity and low teacher knowledge' emerged from this report outlining the critical state of mathematics in New Zealand classrooms. A key recommendation from the report was to provide students with access to:

> engage in mathematical and statistical practices that supports learners in developing both conceptual understanding of mathematics and statistics, and productive dispositions and identities as knowers and doers of mathematics. (Royal Society Te Apārangi, 2021, p. 39)

In March 2023, Te Mātaiaho The Refreshed New Zealand Curriculum, a draft curriculum was released with a focus on mathematical practices. This document divides each learning area into three elements: understand, know, do. More recently the August 2024 draft release provides specific detail, Figure 1 outlines these elements.

Figure 1

Understand, Know, Do Diagram



(Ministry of Education, 2024, p.8)

The figure demonstrates the interweaving of the three elements in teaching mathematics and importantly, the focus on the practices or process in the Do section. Currently, in the more recent refresh (Ministry of Education, 2024), the specific overview breaking down each element is under development. However, it is important to note that previous policy initiatives in New Zealand reflected the need for teachers to develop classroom cultures that included mathematical reasoning and presentation of argumentations (Ministry of Education, 2007). The new policy brings mathematical practices to the forefront of teaching and learning of mathematics. Included in the refreshed mathematics learning outcomes are indicators across all year levels to support teachers to facilitate students to use mathematical processes. For example, a student within their first year of school in algebra will "use the mathematical processes to … explain and justify how a pattern is repeating" (Ministry of Education, 2024, p. 30).

Both in New Zealand and internationally, curriculum documents have been including aspects of mathematical practices, however, the exact definition of mathematical practices varies across international curriculum documents. The Australian mathematics curriculum considers the following proficiency (process) strands: reasoning, understanding, fluency, and problem solving as important practices for children to learn (Australian Curriculum, Assessment and Reporting Authority, 2022). In the U.S.A., forty-five states base their mathematics programs on the Common Core State Standards for Mathematics (CCSS) which include the practices of problem solving, reasoning and proof, communication, and representations (Österholm, 2018). Similarly, the Singaporean Mathematics Framework incorporates five areas of mathematical proficiency; conceptual understanding, procedural fluency, strategic competence, adaptative reasoning, and productive disposition (Mehrjoo, Nourian, Norouzi, et al., 2022). Evident across all international curriculum documents is the need for students to mathematically reason and conceptually understand mathematics. Researchers (e.g., Grootenboer et al., 2023; Hunter & Hunter, 2018; Selling, 2016) contend that the use of mathematical practices provide the opportunities for students to understand mathematics, as well as do mathematics. While both researchers and curriculum documents emphasis mathematical practices, the challenge of enacting these practices in the classroom has been identified (Cobb et al., 2011; Schoenfeld, 2020). This highlights the need to examine ways in which both teacher and students can be supported to develop mathematical practices and the research around how these practices can be set up in classrooms.

1.3 Rationale

To be mathematically proficient, is to engage in mathematical practices that allow one to make sense of mathematics and to think mathematically (Grootenboer et al., 2023). Teacher enactment of mathematical practices in a classroom is complex due to the myriad of needs of the students, including mathematical language abilities and mathematical understanding. A key aspect of student engagement in mathematical practices relies on teacher actions. International and national studies (e.g. Cobb et al., 2011; Grootenboer et al., 2023; Hunter, 2008; Melhuish et al., 2015; Schoenfeld, 2020; Selling, 2016) demonstrate a variety of teacher actions to support mathematical practices within classrooms with older primary or high school students. However, studies of specific teacher actions to develop mathematical practices with students in the early years of primary schooling are limited.

Research studies (e.g., Bicknell et al., 2016; Cheeseman et al., 2014; Mulligan et al., 2020) have increasingly demonstrated the capabilities of young students thinking mathematically from an early age. There are studies with very young children aged four years old to six years old that discuss the capacity of such young learners to reason mathematically (e.g. Björklund et al., 2020; Nergård, 2023; Sumpter & Hedefalk, 2015). These studies conclude that, if students are given the opportunities to think and reason mathematically, young learners could succeed in using mathematical practices.

The current study aims to extend upon the research available and explore how one teacher can provide opportunities and engage in specific teacher actions to enact mathematical practices with young learners.

1.3.1 Young Learners in New Zealand

Internationally, there are a variety of terms used in research when undertaking research with young students. These include young learners, early learners, and the early years. Whilst the terms have similar meanings, the exact age of students involved in the variety of terms can cause confusion. For this study, the Organisation for Economic Co-operation and Development (OECD) (2015) definition will be used. They discuss a young learner as a child between birth to 8 years old. This definition encompasses both children in preschool and the first formal years of schooling. Internationally, in many countries children do not enter formal education until aged six or later, however in Aotearoa, New Zealand students can enter school anywhere between 5 to 6 years old and are classified as new entrant students. In this study, the term young learner will be used.

1.4 Objective

The purpose of the study is to identify the specific actions a teacher uses to position young learners to engage with or demonstrate the use of mathematical practices. In this study, there are ten student participants aged five years old (n = 9) and six years old (n = 1). The study occurs within the students' first seven formal mathematics lessons upon entering primary school.

In particular, the study addresses the following question:

- How does a teacher set up young learners to engage with mathematical practices during mathematics lessons?

1.5 Overview

Chapter Two summarises literature both national and international focused on mathematical practices. The literature review defines each mathematical practice, looking particularly at research with young learners engaging in the practices. Deliberate teacher actions are explored as well as teacher pedagogical moves to set up conversation classrooms and classroom environments to support students in engaging in deep mathematical reasoning.

Chapter Three outlines the methods used in the study and the justification for the method. The role of the researcher and data collection methods are explained. Following this, the research setting, and participants are introduced. The data analysis is outlined, and the ethical considerations highlighted.

Chapter Four interweaves both the findings and discussions of the findings to present the results of the study. The chapter explores the specific teacher actions used to set up the young learners to successfully engage in mathematical practices. The progression of the teacher actions are identified and research literature is used to discuss the findings.

To conclude the thesis, Chapter Five returns to the overall research question and outlines the key themes from the research study. Recommendations for educators are presented alongside the limitations of the current study. Finally, suggestions for future research are provided.

Chapter Two: Literature Review

2.1 Introduction

Chapter 1 introduced the background to the study and discussed the context of the research. This chapter summarises relevant literature firstly discussing mathematics in the early years in Section 2.2. Section 2.3 introduces each mathematical practice; explanations, justifications, argumentation, generalising and representation. Each practice is identified, with relevant studies discussed as well as any specific teacher actions required to engage students in each practice. Section 2.4 discusses the pivotal role of the classroom environment. Section 2.5 considers teacher actions looking particularly at the literature on responsive teaching, extending mathematical thinking and establishing mathematical practices. Communication of young learners and their language development is discussed in Section 2.6. Section 2.7 outlines the teacher tools to elicit conversation in a mathematics classroom. Finally, Section 2.8 summarises the literature review.

2.2 Mathematics in Early Years

> "Young children are naturally curious and observant, and they enter school ready to connect their rich informal understanding about the world around them to their experiences in classrooms" (Carpenter et al., 2017, p. 1)

Young students come to school with a wealth of intuitive understanding about mathematics. A growing body of research has highlighted that young children are capable of exploring complex mathematical concepts (Cheeseman, 2019; MacDonald, 2018). MacDonald (2018) argues that historically, "researchers used to think that very young children have very little knowledge of, or capacity to learn mathematics" (p. 4). However, in the last two decades, there is growing consensus that young children can do and think mathematically (Macdonald, 2018). Clemson and Clemson (2006) highlight how, upon school entry 5 year old students (in New Zealand) come equipped with at least five years' of life experience engaging in mathematics. These experiences may include sorting the shopping, setting the table, helping in the kitchen, being measured for shoes as well as many other experiences through play such as puzzles, threading beads or building construction toys and others (Clemson & Clemson, 2006). Similarly, Clements and Sarama (2018) emphasise that early mathematics is more than merely counting objects and that young learners can engage with meaningful mathematics opportunities.

In recent years, the New Zealand mathematics curriculum and related teaching programs for young children have focused predominantly on students constructing a body of mathematical knowledge related to number in order to lay the foundations for student mathematical learning (Ministry of Education, 2007). This focus began in 2000 with the launch by the Ministry of Education (MoE) of a nationwide pilot *Count Me In Too* which emphasised early stages of number development (Young-Loveridge & Peters, 2005). In following years, the MoE launched the Numeracy Development Project (NDP). The goal of the NDP was to ultimately improve the capability of teachers' mathematics knowledge and therefore see an improvement within student performance. However, the NDP focused heavily on number as a strand while emphasising differentiating between number knowledge and strategies and arguably this resulted in student learning across other mathematical strands being narrowed (Young-Loveridge & Peters, 2005). Additionally, the NDP did not build on the use of mathematical practices and communicative reasoning (Hunter, 2008c). The refreshed New Zealand Curriculum (NZC), set to be implemented by 2025, advocates the use of mathematical practices which are entitled "Dos" (Ministry of Education, 2023). By providing opportunities for young learners to learn and use mathematics practices as well as communicating their mathematical reasoning, young students can develop sound foundations in mathematical understanding.

The following sections will discuss mathematical practices and their role in the mathematics classroom.

2.3 Mathematical Practices

Mathematical practices extend beyond the knowledge and strategies children use to make sense of mathematics. The exact definition of mathematical practices varies across international curriculum documents. The Australian mathematics curriculum considers the following proficiency (process) strands: understanding, fluency, problem solving, and reasoning as important practices for children to learn (Australian Curriculum, Assessment and Reporting Authority, 2022). In the U.S., forty-five states base their mathematics programs on the Common Core State Standards for Mathematics (CCSS) which include the practices of problem solving, reasoning and proof, communication, and representations (Österholm, 2018). Similarly the Singaporean Mathematics Framework incorporates five areas of mathematical proficiency; conceptual understanding, procedural fluency, strategic competence, adaptative reasoning, and productive disposition (Mehrjoo, Nourian, Norouzi, et al., 2022). Several researchers (Grootenboer et al., 2023; Hunter & Hunter, 2018; Selling, 2016b) agree that the use of mathematical practices provide the opportunities for students to conceptually understand mathematics, rather than procedurally understanding mathematics. The teacher plays an important role in supporting students to know and use these practices effectively (Cobb et al., 2011; Schoenfeld, 2020).

The focus in this study is on the mathematical practices of explanation, justification, representation, generalisation, and argumentation. The following section will explore specific studies focused on particular mathematical practices. Links will then be made to specific teacher actions required to ensure students can engage with these practices.

2.3.1 Mathematical explanations

Explanations can be described as individuals or groups of students' ideas or statements highlighting their mathematics thinking. Making a mathematical explanation is a communicative act and is explicit and connects to the context of the task (Chapin & O'Connor, 2007; Franke et al., 2009; Hunter & Hunter, 2018). A reasoned explanation also requires the provision of evidence using materials, representations, or further elaborations to support reasoning. When students express their mathematical ideas as explanations they have the opportunity to transform their knowledge into rich and meaningful statements (Mueller et al., 2014; Selling, 2016b; Whitenack & Yackel, 2002; Yackel & Cobb, 1996). The process of constructing an explanation can be a cognitive challenge and students need support in developing this process. A study exploring student mathematical dispositions with students aged 7 – 8 years old by Yackel and Cobb (1996) demonstrate three steps in student explanations building towards what the teacher perceived as an acceptable explanation. The researchers describe the first step as students using mathematical explanations for mathematical understanding rather than responding to social situations. For example, when a teacher questions a students' thinking, the student may assume that they are incorrect. Yackel and Cobb (1996) discuss the second phase of establishing the use of mathematical reasoning as the base of an explanation. The third and final stage involves students making sense of the explanation and thinking critically about how their peers would make sense of it. This entails a shift from participating in explanation to making the *"explanation itself an object of reflection"* (Yackel & Cobb, 1996, p. 467). Key throughout Yackel and Cobb's study, was the role of the teachers in using clear and specific actions that positioned students to develop mathematical explanations. The idea of scaffolding to set up students for success relates back to the Vygotskian notion of scaffolding to help students to make connections - with the teacher providing steps and a clear outline for student success rather than reducing the cognitive demand of the task.

Whilst Yackel and Cobb's study describes how the teacher supports explanations with students aged 7 – 8 year olds, a study by Papandreou and Tsiouli (2022) in a kindergarten in Greece monitored the every-day mathematics that occurred as the 5 year olds played. Their study demonstrated the natural reasoning the students used throughout this play, such as "explaining why a figure is symmetrical, reasoning about quantities, age, volume and length…explaining how a pattern is made" (pg. 739). However, while these students were capable of explaining, there was a lack of sophistication to their mathematical ideas. Similarly a study by Sumpter and Hedefalk (2015) with preschool students aged between 1 – 5 years old saw play as the opportunities to construct and reconstruct mathematical knowledge. In two episodes the young learners were able to construct their own reasoning and explanations when comparing the heights of towers, however it was then through the guidance and scaffolds of the teacher that facilitated a deeper mathematical understanding. These scaffolds included confirming comments, that confirmed what the students were thinking and supporting comments, comments that encouraged the students to continue their discussions. It is important to note that discussed in this study and others such as Björklund et al. (2020) and Pramling et al. (2019) is the importance of teachers capabilities to respond and scaffold in the moment.

Recently, in early years studies with students aged 0 – 8 years old, mathematical explanations or explaining have been linked to research around mathematical thinking. However, definitions of mathematical thinking are varied. Woods et al., (2006) define mathematical thinking as the "abstraction and generalization" of mathematical ideas (p. 223). On the other hand, Monteleone and colleagues (2023) define critical mathematical thinking as the joining of critical thinking and mathematical thinking with critical thinking being defined into five themes; interpreting, analysing, evaluating, exampling, and creating with these themes applied to a mathematics setting. Within their Australian study, Monteleone et al., (2023) explored the critical mathematical thinking that young students (aged between 5 – 6 years and 8 months) exhibit before starting formal schooling. The study involved interviewing students one on one across a series of eight learning experiences. Explaining was identified as the most common critical mathematical thinking observed both verbally through sentences and written drawings. They emphasised the impact of using open ended tasks and questions that required the students to expand or discuss their thinking such as "What can we do to check?" as strategies to support students in explaining (Monteleone et al., 2023, p. 350).

In a New Zealand study by Anthony and Walshaw (2002) the challenges young students have in making sound explanations due to their use of language and levels of mathematical knowledge was highlighted. However, other studies from New Zealand and Australia (Herbert & Williams, 2023; Pearce & Hunter, 2022) have illustrated that specific teaching actions support student development of mathematical explanations. For example, one aspect of Herbet and Williams (2023) study with children aged 5-7 years old focused on how a teacher elicited deeper mathematical thinking. The following excerpt highlights how the teacher (Earl) used strategic questioning to support these young learners in their explanation.

Strategic questioning to support an explanation	
Student D	We think there is 10 people at the door.
Earl	What did that mean? {pose question to elicit mathematical meaning about group focus}
Student D	That means they get one cookie each. There's 2 people inside and 10 people outside [holding up 2 fingers] which makes 12 people {elicit connections between cookies and people}.
Earl	Is there a sum that you know that would show that? {elicit links between representations}
Student D	[Reads 10 + 2 = 12 from worksheet] Ten plus two equals 12
Earl	12 cookies altogether so what does that mean for the people? {elicit link between cookies and people in numeric representation}
Student D	They all get one each.
(Herbert & Williams, 2023, p. 87)	

Evident in the example above was the teacher action of using questioning to press for student reasoning in order to develop a clearer explanation for all learners. Similarly, a small-scale study with young children in a New Zealand classroom by Pearce and Hunter (2022) demonstrated that with teacher scaffolding and modelling of expectations of an accepted mathematical explanation, young students could be successful in constructing their own explanations. The process of developing a mathematical explanation provides students with the opportunities to take ownership of their thinking and support their peers' mathematical understanding as the explanation is shared (Esmonde, 2009). The mathematical practice of explanation is the stepping stone to students developing justification and mathematical argumentation (Krummheuer, 1995).

2.3.2 Mathematical justification

Justification requires a why for the claim, explanation, or decision and can be defined as providing proof and building on established facts to present an argument (Melhuish et al., 2015; Stylianides, 2007). Stylianides (2007) U.S study with students aged 8 – 9 years old provides a clear example of justification in the classroom. As can be seen in the excerpt below, when the students were asked to write number sentences for 10, the teacher pressed for justification:

Example of justification	
Teacher	How do we know that is it right?
Lucy	I have 10 divided--10 divided by one equals 10, and then I have it-and then I have like 50 divided by five equals 10, and Mei said that it was right, and so, like if you have a 100 divided by 10 it would be right because-because if you went from like five then it would be five more, and then-'cause 50 is five less than a 100 so-so it would be 10 'cause if it's five divided by-50 divided by five it would equal 10.
(Stylianides, 2007, p. 308)	

As was illustrated, the teacher pushed the student to prove the reasoning in a way that all students could understand. Lucy’s initial justification was not mathematical, the teacher continued to press for reasoning until the student had a mathematical reason for why 50 divided by 10 equals five. Stylianides (2007) outlines justification as consisting of a base argument → proof → mathematical argument → validated proof. Other researchers (Bieda, 2010; Civil & Hunter, 2015; Henningsen & Stein, 1997; Hunter, 2008a; Pearce & Hunter, 2022) have emphasised that key to developing justification is the opportunity for students to engage and grapple with sense-making to deepen student mathematical understanding. These opportunities were highlighted in a longitudinal study by Maher and Martino (1996) in the U.S., who followed students throughout five years of schooling (ages 6 – 11years old) examining their development of justifying or giving proof. These researchers concluded that the constant opportunities provided to the students to make sense of mathematical problems are important. As students progress through schooling and develop their mathematical understanding, teachers must provide ongoing opportunities to engage in justification to deepen mathematical knowledge. A constant expectation from teachers for justification from school entry allowed the development of more sophisticated reasoning over time. Similarly Krummheuer (2007) analysis of a interaction within a first grade classroom with students aged 6 years old demonstrated how students were positioned and expected to justify by being provided with space for reflection. As the teacher encouraged the students to examine and critique peers' ideas by creating intellectual space through pausing or questioning, explanations were pushed to justifications.

Young students need consistent support from teachers to develop sophisticated justification. In their Swedish study, Björklund and colleagues (2020) explored the dialogical interactions between students and teachers within three preschools (with students aged 4 – 6 years old). Whilst these young students showed early reasoning and justification skills and their answers made sense in the moment, they were not grounded in formal mathematical logic. For example, when justifying why a Beyblade changes colour as it spins, the student suggested "because I have magic fingers" (Björklund et al., 2020, p. 474). In that moment, the reasoning was plausible for the situation. Another student was able to use a notion to justify their thinking, (because it is this big with gestures), Björklund and colleagues (2020) defined the justification as imitative where the students use a notion that they were familiar with and where no further mathematics was explored in the situation. Similarly Pramling and colleagues (2019) working in play based settings discuss that young students required the need to differentiate, between what is relevant for their justification and how to express this in words. Developing this reasoning at a young age requires the specific support of teachers using actions such as highlighting, questioning, recapping ideas, and then if needed, instructing students on the words or concepts required to construct their justifications. Research literature (e.g., Björklund et al., 2020; Pramling et al., 2019) strongly suggests that there is a difference between simple pre-school capabilities of justification and formal schooling expectation of justification that is grounded in mathematical knowledge, what is missing from the literature is how a teacher sets up students to justify in their first term of schooling.

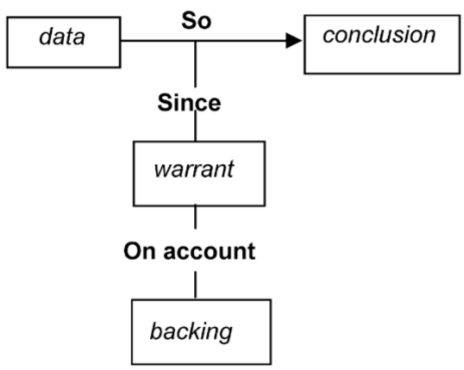
Another important mathematical practice that builds on both explanations and justification is argumentation. In the next section this mathematical practice will be described and discussed.

2.3.3 Mathematical argumentation

Argumentation is the act of students adjusting their ideas and interpretations by verbally presenting their logic (Krummheuer, 1995). Many researchers (Hunter & Hunter, 2018; Kosko et al., 2014; Krummheuer, 2009, 2013; Sumpter & Hedefalk, 2015; Whitenack & Yackel, 2002) draw on the framing of Toulmin's (1958) examination of 7 – 8 year old students' reasoning to construct an argument, the framing is explained in Figure 2.

Figure 2

Toulims Framing of Argumentation (as summarised by Krummheur 2007, p.65)



This metacommunicative activity (argumentation), occurs when an idea is doubted or challenged in the search for the truth. A study by Krummheuer (2009) across classrooms of 5 – 10 year olds, identified two common ways of arguing, namely, diagrammatic and narrative argumentation. Krummheuer (2009) defines diagrammatic argumentation as arguing through the use of materials or representations (discussed in Section 2.8), where ideas are made visible to manipulate and discuss, whereas narrative argumentation is defined as a sequence of sentences or logical implications with mathematical proof being seen as the narrative. In another study by Krummheuer (2013) with 5 – 7 year olds, the developmental use of diagrammatic and narrative argumentation was explored. When comparing arguments of kindergarten students (aged 4 – 5 years old) and primary students (aged 6 – 7 years old), it was noted that although kindergarten students are capable of mathematically arguing, their arguments contain diagrammatic aspects that are not yet consistently structured narratively. The primary aged students blended both diagrammatic and narrative arguments to resolve their mathematical discussion. Krummheuer (2013) highlighted the difference in speech and communication between the kindergarten and primary aged students., Krummheuer also discussed the development of language and complexity of a narrative that may not appear in children's language development until ages 5 – 6 years old. Similarly, Sumpter and Hedefalk's

(2015) study with students in a Swedish childcare (aged 1 – 5 years old) showed the natural way in which these young students used language relevant to their age to convincingly argue their reasoning. More recently, Breive (2017) explored argumentation with a small group of 5 year old students working on a task led by a primary school teacher. The findings showed that the students were able to use many of the elements in Toulmin's (1958) framing in their argument. Although the students' argument was structurally accurate, the words and gestures such as "there" "everywhere – waving hand" "there and there – pointing" highlight the appropriate language and mathematical reasoning for the age of the students (Breive, 2017, p. 5). The findings of the studies discussed in this section identify the need for establishing a classroom environment that supports mathematical argumentation. Authentic mathematical argumentation is possible when students are provided with many opportunities to develop explanations and critique the reasoning of others.

2.3.3.1 Setting up the classroom space for argumentation.

Developing mathematical argumentation can be challenging. The intellectual space in which students mathematically argue and debate must be done with high trust and mutual respect for all. Yackel and Cobb's (1996) study (see Section 2.4) specifically illustrated how teachers can establish this reflective space during discussions by directing students to listen to specific student explanations in order to respond to their reasoning. Similarly, in an Australian study with 5 – 7 year old students, Herbert and Williams (2023) highlighted how the teacher provided students with intellectual space to reason with a claim by specifically asking the students "I'm wondering if some people might like to think about that" (p., 88). The opportunity for space to think about the students reasoning, gave other students the right to ask questions or compare their reasoning. Additionally, there are cultural influences in relation to whether engaging in argumentation or disagreeing is viewed by students as acceptable. Research studies (Chazan, 2002; Hunter & Civil, 2021) with older students, highlight the importance of specific teacher acts of explicit teaching to ensure students are equipped with strategies to allow them to disagree, agree, or challenge in respectful ways. For example, Hunter and Hunter (2018) share a vignette of a teacher promoting argumentation.

Setting up respectful arguing
When you are doing your mathematical arguing it is because, for some reason, you don't agree with someone else's thinking, or perhaps the way someone else has done what they have done. Mathematicians engage in arguing, they do it all the time. It is good to have a healthy mathematical argument. (Hunter & Hunter, 2018, p. 14)

The use of social norms (see Section 2.4.1) and setting up mathematical conversation (see Section 2.7) provides students with opportunities to participate in rich mathematical argumentation. Similar to Hunter and Hunter (2018), Singletary and Conner (2015) focused on teachers supporting collective argumentation with students aged 14 – 15 years old. As the classroom had an existing norm of explaining or warranting mathematical thinking, the teacher was able to build on this and use questioning prompts to request evaluation such as "Laura says that is a right triangle. Do you agree?" (Singletary & Conner, 2015, p. 146). This provided intellectual space for students to collectively join the mathematical discussion and reason with mathematics. Despite this study being with older students, the prompt for evaluation could be adapted to use with younger students.

As students engage in mathematical explanation, justification, and argumentation, student reasoning develops and with teacher support can be extended into generalising where the mathematical properties or patterns are seen in a broader context or across several situations. Generalising is a mathematical practice that supports students to develop conceptual understanding of mathematics or extend mathematical reasoning. In the next section generalising will be discussed.

2.3.4 Generalising

Generalising in mathematics involves using intellectual tools and mental habits to recognise patterns, combine ideas, make connections, and confirm mathematical reasoning (Hunter & Hunter, 2018; Selling, 2016a). However, Carraher et al. (2008) emphasise that developing a valid generalisation also requires students to support their reasoning with proof and justification. An Australian longitudinal study by Mulligan et al. (2020) with 319 kindergartners (mean age 5 years old) found that the top third of the students demonstrated and showed an understanding of emergent generalisations within patterns. An Australian study by Papic and colleagues (2011) with students aged 3 years 9 months – 5 years old explored student representations to a variety of patterns. A key finding was that opportunities to discuss and explore a variety of patterns support the development of student generalisations such as "many patterns have a unit of repeat" (p.261). Similarly, Tzekaki and Papadopoulou (2019) analysed verbal responses to questions from 23 preschoolers aged 5 – 6 years old and found that students began to express generalised ideas when discussing the relationship between quantities and numbers. Whilst there is evidence of young students engaging with generalisations and in particular the success of generalising patterns (e.g. Mulligan et al., 2020; Papic et al., 2011; Rivera, 2012) or expressing generalising statements (Tzekaki & Papadopoulou, 2019), there is limited research studies that document specific teacher actions to support students in developing this mathematical practice.

Research studies (e.g., Blanton & Kaput, 2003; Carraher et al., 2008; Hunter, 2008b) which focus on teacher actions to promote generalisation with young students highlight the importance of careful teacher questioning. Blanton and Kaput (2003) suggest teachers use questions such as "Does this work with different numbers? Tell me what you were thinking? How do you know this is true" (p. 72). Furthermore, Hunter's (2008b) communication and participation framework, provides explicit models for teachers to draw on to develop generalisations. For example, "Look for patterns or connections" or "Make comparisons and explain" (Hunter, 2008b, p. 33). This explicit model builds on Blanton and Kaput (2003) questions to encourage teachers to make connections between mathematical ideas.

The next section will discuss the role of mathematical representations as a mathematical practice.

2.3.5 Mathematical representations

In mathematics representing an idea demonstrates mathematics visually. According to Diezmann and McCosker (2011) "representations play a key role in mathematical thinking: they offer a medium to express mathematical knowledge or organise mathematical information and to discern mathematical relationships" (p.162). At times, these representations can provide teachers the opportunity to understand student understanding. Opportunities to manipulate materials is critical to developing students' conceptual understanding of mathematics. As students develop mathematical understanding, their explanations and justifications deepen when asked to represent in multiple ways (Lesh et al., 2000). In their study, Murata and Stewart (2017) demonstrated how materials can support mathematical understanding. The following excerpt demonstrates how these students used materials to support them in solving an addition task:

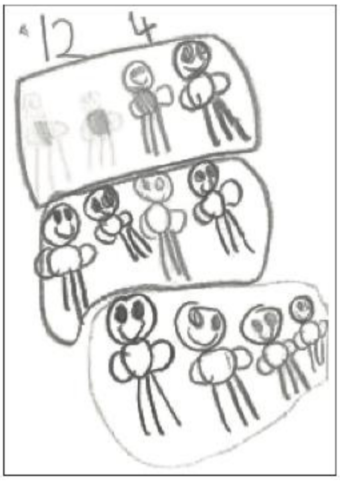
Multiple ways to solve an equation	
As the teacher placed a number line strip below the ten-frames	
Nina	I first did the jump of ten, then a jump of 8, then found the answer 18”.
Tia went to the board and also moved the three counters to the frame of seven to make ten…	
Tia	At first, I moved these to make ten; then I thought that there are eight in the ones place. . . .”
Teacher	Nina thought of it using a number line, and Tia thought of it using tens and ones.”
From Murata and Steward (2017)	

In this example, the teacher deliberately chose two students to expose all students to different material representations which could be used as a scaffold for representing in a written form. Similarly, other researchers (e.g. Blanton, 2008; Sullivan et al., 2015) demonstrated the importance of students being exposed to multiple ways to represent and solve a mathematical idea. By doing so, students have agency to choose meaningful representations.

For all students, but in particular, younger students, the process of drawing can provide the opportunity to represent mathematical reasoning (MacDonald, 2018). This was highlighted in an Australian study by Roche and colleagues (2020). In their investigation a group of 5 year old students were asked to group 12 little ducks equally. Figure 3 illustrates how one five year old student represented their reasoning:

Figure 3

12 Little Ducks Student Representation (as cited in Rohe et al., 2020 p.33)



This representation represents three groups of four little ducks. Whilst the mathematics is accurate in the example above, the mathematical thinking of the student (equal sharing or knowing 12 divided into three as a basic fact) is not present in the representation. Researchers (e.g., Bobis & Way, 2018; MacDonald & Murphy, 2019) contend that deeper insight into the meaning behind the representations can only occur as students are encouraged to talk about their thinking. Similarly, Selling (2016a) discuss representation as students attempt to communication their abstract ideas, using drawing or material representations is a reasoning tool. In an Australian study examining oral fluency with students aged 6 – 12 years olds by Cartwright (2023), talking as a means of communicating mathematical reasoning in conjunction with representation was examined. Cartwright concluded that verbal responses from students must be analysed in conjunction with the written responses of young students, as at times mathematical fluency may not be fully evident in their representation. While Cartwright (2023) study was with older students, a similar study in Australia by Bobis and Way (2018) with younger students (5 – 6 years old) yielded similar results. When analysing the representations of two of the participants, Bobis and Way concluded that while young learners have the capacity to generate their own representations it is important for teachers to notice the representations and support the progression or to highlight the connections between the student representations and key mathematical ideas.

On the other hand, Selling (2016a) outlined students' capabilities to go from learning to represent to representing to learn over a summer school period. In her study with students aged 12 – 13 years old, the two focus students demonstrated their abilities to construct sophisticated representations and reason with their mathematical thinking. This progression occurred as the students went from drawing their thinking to representing the mathematical thinking with numbers and equations. Another study by Jacobs and Empson (2016) with older students aged 9 – 10 years old highlighted the importance of visual representations as a means to connect students to the symbolic nature of their representations. In their study, the teacher prompted students to write an equation to match their explanation as the students verbalised their mathematical thinking. Similarly, in a study by Herbert and Williams (2023) with students aged 5 – 7 years old, the teacher explicitly prompted students to record their thinking using equations. Despite the difference in the age of the students, in both these studies, the teachers positioned students to use representations to reflect their thinking and, in this way, built on the student explanation and drawings to move them into a higher level of representation. For these practices to be enacted, there are specific processes that need to be established in classrooms.

The next section outlines classroom environments that provide multiple opportunities for students to develop mathematical practices and engage in mathematical discourse.

2.4 The Classroom Environment

The classroom environment plays a role in the success of engaging students in mathematical practices. Interactions between the teacher and students shape the classroom environment. The use of mathematical practices requires an environment where students are expected to share their ideas, reflect, negotiate, take risks, and feel safe to contribute at any time (Makar et al., 2015). A U.S based study by Franke and colleagues (2015) with students aged 4 – 12 years old (pre K classrooms up to grade 6) noted the teacher actions of inviting students into mathematical conversation, followed by scaffolding moves to develop a positive environment. These teachers were creating, alongside the students, a classroom environment to participate in mathematics and the safe space to support each other in their reasoning. These interactions set up by the teachers across all year levels, shaped the student to student interactions, and over time, the students supported each other's mathematical ideas without the prompts from the teacher in a classroom environment that naturally developed. Classroom norms and developing positive mathematical identity through the teacher assigning competency can be used as a way to develop a positive classroom environment. These are discussed below.

2.4.1 Classroom norms

Developing norms and expectations is imperative for students beginning schooling. For students to engage with their peers within mathematics by communicating and sharing their ideas or resources, it is particularly useful to provide opportunities for students to engage with and understand the social processes that must occur. Social norms can be defined as the characteristics or expectations of the classroom established by the teacher and students (Cobb et al., 2011). As Yackel and Cobb (1996) stated, students must learn and be supported to work in groups and within the wider classroom environment. Yackel and Cobb explored the idea of socio-mathematical norms in a study with 7 – 8 year old students. They defined the term socio-mathematical norms as the combination of social norms and expectations combined with the mathematical expectations when enacted on a mathematical activity. For example, the expectation to justify and share student thinking is a social norm however by adding the expectation of what is an acceptable mathematical justification it becomes a socio-mathematical norm. Yackel and Cobb (1996) emphasise the important role a teacher plays in setting up these socio-mathematical norms and establishing a level of expected mathematical quality within the classroom. Other researchers (e.g. Hunter & Hunter, 2018; Langer-Osuna, 2016) also emphasise the role of the teachers in explicitly outlining their expectations of group norms and consistently praising collaborative practices throughout the lesson.

2.4.2 Assigning competence

Assigning competence is a powerful act from a teacher with the goal to publicly and specifically praise students for their intellectual contribution. Teachers take a critical role in using actions to deliberately position students with the capability to develop a productive mathematical identity (Hunter, 2008c; Selling, 2016). A key element across both studies from Selling (2016b) and Hunter (2008c) is the earlier work by Cohen and Lotan (1995) in relation to assigning competence to students by explicitly drawing attention to their peers' reasoning and giving value to different students' intellectual contributions. Similarly, Leach and colleagues (2014) showed that both time and consideration was needed for teachers to set up the norms and expectations to provide status and equitable participation opportunities to students. Also, of importance, is the need to explicitly highlight student responsibility of their learning (Goos, 2004). The following section explores the types of teacher actions involved in facilitating students to engage with and successfully use mathematical practices.

2.5 Teacher Actions

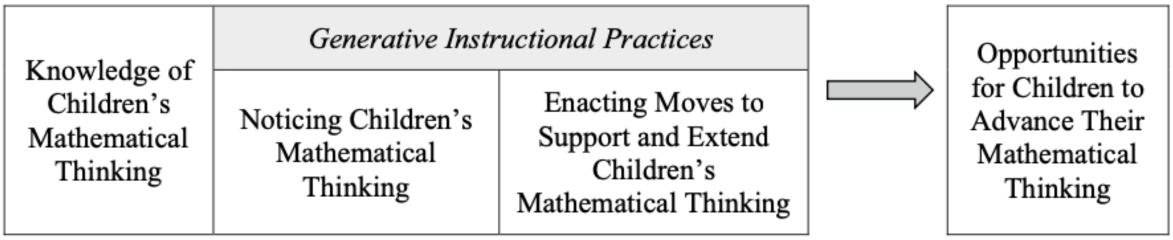
To engage with robust mathematical practices, young students need opportunities to build their understanding of mathematical practices (Civil & Hunter, 2015; Pearce & Hunter, 2022; RAND, 2003; Selling, 2016a). For teachers, this means noticing and responding to student participation in the lesson and providing opportunities to engage in mathematical practices. In this section of the chapter, the focus will be on research literature that outlines the responsive teacher actions that explicitly support the use of mathematical practices.

2.5.1 Responsive teaching

Responsive teaching in mathematics involves the teachers' instructional decisions about what to pursue in reaction to student thinking (Jacobs & Empson, 2016). Figure 4 illustrates a model of responsive teaching and emphasises the importance of teachers noticing in the moment.

Figure 4

A Model of Responsive Teaching (as cited in Jacobs & Empson, 2016, p. 186)



Teacher noticing of both mathematical thinking and engagement in mathematical practices is required to support students to further develop their mathematical thinking (Jacobs & Empson, 2016).

2.5.2 Extending mathematical thinking

An Australian study by Cengiz et al. (2011) with six teachers of students aged 8 – 12 years old focused on specific teacher moves to extend mathematical thinking. Three main categories of teacher instructional actions emerged: eliciting, supporting, and extending. Eliciting occurred as the teachers provided opportunities for the students to share their mathematical thinking. The instructional actions of extending encouraged further mathematical discussion from the students through prompts such as "What do you think? Do you agree? Do you think it's true?" (Cengiz et al., 2011, p. 363). Whereas, supporting instructional actions used by teachers in the study ensured the students made connections between the mathematical concepts being discussed for example the teachers "suggested an interpretation of a claim, recorded student thinking or repeated a claim" (Cengiz et al., 2011, p. 363). Further analysis discussed the complexity of extending mathematical thinking as more than one instructional action was needed for students to deepen their mathematical thinking through a whole class public discussion. For example researchers discuss an episode where the teacher used the following actions "inviting students to evaluate a solution, to provide reasoning for a claim, and to compare different solutions—the teacher also utilized supporting actions, such as suggesting an interpretation of an observation and repeating claims" (Cengiz et al., 2011, p. 371).

2.5.3 Establishing mathematical practices

The challenge of supporting students to develop mathematical practices was highlighted in a New Zealand study by Hunter (2008b). Her study involved working with teachers to design and use a Communication and Participation Framework (CPF) (see Appendix A1) to support and scaffold students use of mathematical practices. The flexible framework was designed to demonstrate a possible sequence of teacher actions to scaffold students to engage in mathematical practices (Hunter, 2008b). Importance was placed on focusing teachers' attention on specific communicative and performative actions required to support students in their engagement in mathematical practices. Selling's (2016) work identified the need for teachers to make mathematical practices explicit and developed a framework (see Figure 5) of eight types of teacher moves (reprising moves) that specifically encouraged students and provided opportunities to engage with mathematical practices.

Figure 5

Teacher Reprising Moves (as cited in Selling, 2016b, p. 524)

- Naming the mathematical practice(s)
- Highlight aspects of student engagement in mathematical practices
- Evaluating student engagement in mathematical practices
- Explaining the goal or rationale for engaging in a mathematical practice
- Connecting different students' engagement mathematical practices
- Framing student engagement in mathematical practices expansively

Notably, although Selling (2016) identified the need for explicit teacher moves, an important distinction is that this came after student engagement in the practice. It is important to note, that both Bieda (2010) and Henningsen and Stein (1997) documented the risk of teachers modelling or doing parts of the work for students through explicit instruction, which can limit the opportunities to engage within mathematical practices.

An initial step in developing mathematical practices is for teachers to notice and be aware of the mathematical practices students are using. A study by Melhuish et al., (2015) examined how teachers of 8 – 10 year olds noticed student discourse in a mathematics lesson by asking the teachers to identify the discourses used within the categories of justifying and generalising. The findings of their study identified a framework of noticing mathematical practices in relation to the mathematical content and strategies. The teachers in the study attended to one or the other, however, they rarely noticed when student discourse attended to both the mathematical practice and content. Consequently, Melhuish et al., (2015) contends along with other researchers (e.g., Hunter & Hunter, 2018; Selling, 2016b; Smith et al., 2009) that professional development is required to support teachers to develop knowledge of mathematical practices and recognise this in the moment. Utilising professional development to support teachers is a feature in a study by Davidson and colleagues (2019) who worked alongside four teachers working with 8 – 10 year old students. Their aim was to develop a set of prompts that would elicit mathematical reasoning during planned mathematics lessons. Figure 6 provides an example of these prompts that were designed to align with three key reasoning actions.

Figure 6

Teacher Prompts to Elicit Mathematical Reasoning (as cited in Davidson et al., 2019, p. 1157)

Analysing	Generalising	Justifying
What is the same and different about …?	How can you describe the pattern?	Is it [the conjecture] just sometimes true, or is it always true?
What stays the same and what changes?	If …then …	How can we be sure?
What do you notice?	Are there other examples that fit the rule?	Convince me.

The outcome of Davidson et al. (2019) study showed that with careful planning with the prompts above, the teacher had more opportunities to support reasoning within their mathematics classrooms. It also showed that there is a need for more support for teachers to notice reasoning actions and learn how to teach it or assess it. There were missed moments within the lessons where teachers failed to notice other reasoning actions such as justification or when the proficiency of the reasoning was varied, this highlighted that teachers required more support in eliciting further reasoning or assessing students in the moment. A key aspect of setting up successful opportunities for students to engage in mathematical practices is developing a discourse environment (Chapin & O’Connor, 2007; Otten et al., 2019). The following section will discuss communication in mathematics classrooms and the impact of classroom environment on student mathematical success.

2.6 Communication in Mathematics Classrooms

Learning mathematics and engaging in mathematical practices involves communication, a conversation between students and teachers. Dialogic pedagogies are complex and Kraatz et al. (2022) emphasis the idea that for classrooms to become conversation spaces, all questions must be considered as open no matter how the questions are phrased. Several research studies (Hunter & Civil, 2021; Hunter & Hunter, 2018) demonstrate that moving from a classroom in which the teachers role is to transmit information to the students to a classroom that requires student talk, argumentation, justification, and questioning can be challenging. Without engaging in mathematical discussion, students have limited opportunities to develop and extend mathematical practices (Goos, 2004; Herbert & Williams, 2023; Hufferd-Ackles et al., 2004a; Hunter & Hunter, 2018; Krummheuer, 1995; Melhuish et al., 2015; Murata & Karlin-Neumann, n.d.; Pearce & Hunter, 2022).

2.6.1 Mathematical communication of 5 year olds

Young students enter school with varying levels of oral language and communication skills (MacDonald, 2018). In recent years, researchers have focused on the correlation between language ability and mathematical development with some studies identifying a correlation between early language capabilities and mathematics (e.g. Hooper et al., 2010; LeFevre et al., 2010; Vukovic & Lesaux, 2013).

Clemson and Clemson (2006) suggest that communication in mathematics is challenging for young students, as mathematics has a powerful symbol system which students are required to learn. However, a U.S study by Purpura and Reid (2016) of 136 preschool students aged 3 – 5 years old demonstrated that even the youngest participants at the age of 3 showed an understanding of mathematical vocabulary, they argue that this shows the capabilities of young students when provided the opportunities to explore mathematical vocabulary.

2.6.2 Supporting the development of mathematical language

Teachers play a critical role in supporting students' development of mathematical language (Barwell, 2013; Chapin & O'Connor, 2007; Hufferd-Ackles et al., 2004b; Kazemi & Hintz, 2014). Teachers can support students to write using digits and symbols, however, these actions do not necessarily show evidence of understanding. Clemson and Clemson (2006) summarise this as "we cannot let children substitute the mechanics of mathematics symbolism for a real understanding of the ideas conveyed by those symbols" (pg. 85). The challenge with these symbols is the language that is used to explain them, for example the symbol '+' can be referred to as plus, add, addition, words such as difference or volume all which have a different meaning mathematically to everyday meanings (Barwell, 2013).

Cartwright (2023) examined students' oral responses and written/drawn representations and analysed these against each other to understand student knowledge. Findings from this study suggest that young students can successfully use everyday language and mathematical terms to express their thinking. An implication of this is that teachers should acknowledge the ideas and support students with technical mathematical terms rather than encouraging students to move away from using everyday language. Other researchers (e.g., Barwell, 2013; Cooke & Buchholz, 2005; Kazemi & Hintz, 2014) highlight that student progressing from using informal mathematical language to formal mathematical language only occurs with support and specific teacher actions including modelling, telling or scaffolding students to use formal language. Cooke and Buchholz (2005) examined the strategies used by a kindergarten teacher in the U.S. in promoting the use of mathematical language. Their investigation connected the development of successful communication patterns in the classroom to the opportunities provided by the teacher. These carefully planned opportunities such as encouraging students to use correct mathematical terms when playing with shapes, or accurately counting when making trains out of cubes allowed these preschool students to develop their mathematical communication. Cooke and Buchholz also highlighted the importance of giving students materials to generate and develop discussion. As discussed previously (see Section 2.8) use of materials in classrooms can also support mathematical representation. Hughes (1986) and Rubenstein and Thompson (2002) highlight how materials coupled with teacher modelling supported students to use mathematical representations as a means of building concepts before formally expressing their ideas. Similarly, Clements and Sarama (2020) echo the argument of Cooke and Buchholz (2005) stating "the more teachers talk about maths, the more their children develop math knowledge" (p. 363). As teachers are talking and facilitating conversations, the use of turn and talk, clearly repeating student ideas, and checking in for understanding supported the young students, the following section will discuss these ideas.

2.7 Teacher Tools to Elicit Conversation

Conversations amongst students in the classroom are common, however eliciting academic talk and discussions requires specific actions from teachers. There are multiple tools discussed in literature to facilitate meaningful classroom talk; teacher talk moves to engage students in academic productive talk (Chapin & O'Connor, 2007), five skills that focus and deepen academic conversations (Zwiers & Crawford, 2011), focusing on elicitations and extensions (Rubenstein & Thompson, 2002) among others. There is considerable overlap between many of these tools discussed below.

2.7.1 *Questioning and conversation moves*

Teacher questioning is a common strategy used to elicit conversation. However in a study by Di Teodoro et al. (2011) with four teachers of students aged 7 – 9 years old, the researchers noted that 76% of teacher questions elicited yes or no answers. These closed questions were considered surface level questions as they halted the conversation and added little value to the discussion. In contrast, conversation moves can be used to facilitate mathematical reasoning in which students engage with the ideas of their peers and reflect and critique these. Chapin and O'Connor (2007) suggest the use of the words "Do you agree or disagree and why?" (p. 122), with Zwiers and Crawford (2011) emphasising the idea of supporting students to use ideas together as building blocks, facilitating conversations using questions like "Do you agree? What might other points of view be?" (p. 33) these conversational moves open the discussion into a collective classroom discussion thus supporting meaningful mathematical talk.

Eliciting contributions from other students and inviting their ideas into the conversation can be beneficial to the development of mathematical communication. The simple statement of 'What can you add on?' (Chapin & O'Connor, 2007, p. 123) or using statements such as 'what was that', 'pardon' (Rubestein & Thompson, 2002) enables teachers to specifically choose or engage students in sharing their ideas in a public space. Similarly, Edward-Groves (2014) suggests asking students to add on after an answer or opinion as an opportunity to provide more depth to an idea using evidence, claims, or opinions. Franke and colleagues (2009) undertook a study with three teachers of 6 – 7 year olds in the U.S.A. and found that although teachers frequently questioned their students to build upon their reasoning, these questions did not consistently elicit deeper explanations. Franke et al. (2009) argue the need for teachers to use more than one question or conversation prompt to guide the students into deeper reasoning. A clear finding across the research studies is the importance of teachers using probing questions or statements to engage students in rich mathematical conversations.

2.7.2 *Revoicing and repeating*

Revoicing is a teacher talk move (Chapin & O'Connor, 2007) or paraphrasing (Zwiers & Crawford, 2011) with the goal to clarify thinking and improve understanding. This can be used to support students with the correct mathematical terminology and in making an idea more accessible to the others. Forman and colleagues (1998) highlighted how a middle-school teacher used revoicing to engage students in collective mathematical argumentation. Similarly, Pearce and Hunter (2022) focused on how teachers with young students used revoicing to introduce students to mathematical language in their explanations.

Repeating as a teacher talk move provides an opportunity to highlight key ideas to the class and in the same moment the conversation is opened between the teacher and student or student and student to the whole classroom (Chapin & O'Connor, 2007). Rubenstein and Thompson (2002) preschool study across 44 U.S. preschools focused on meaningful teacher-child conversations, found that extending (expanding) conversation by repeating student thinking provided the scaffold for the conversation to grow further deepening the students' vocabulary. Repeating could also be used as a tool for student ideas to be valued as resources for learning (Edwards-Groves, 2014).

2.7.3 Thinking time

Hunter (2008a) emphasised the importance of 'thinking time' (wait time) as a pause in the discussion to allow student ideas to be valued and then open opportunities to think critically and reason with peer ideas. Whilst this is not a 'talk move' as such, it is essentially a reminder to teachers to slow down the discussion and open up the space for students to engage and think carefully about their reasoning (Chapin & O'Connor, 2007; Kraatz et al., 2022). These specific 'talk moves' or teacher tools to elicit conversations provide some ideas for scaffolds for teachers when moving their classrooms into conversation spaces.

2.8 Summary

Mathematics as a subject involves both learning content and learning to engage with disciplinary practices. In New Zealand, the first year of schooling has often been content heavy with a strong focus on number knowledge skills. Evidence from the research studies demonstrate that young students have the capabilities to successfully engage in mathematical practices of explanations, justification, argumentation, generalisation, and representation.

This literature review highlights the important role of teachers in setting up the learning opportunities for all students to engage in a discourse community. Reprising moves (Selling, 2016b), teacher prompts (Davidson et al., 2019) and responsive teaching (Jacobs & Empson, 2016) are discussed as some of the many ways to elicit mathematical practices in studies with older students. The communication and vocabulary involved when engaging in these practices requires consideration from the teacher to build academic and meaningful conversations (Chapin & O'Connor, 2007; Kazemi & Hintz, 2014; Rubenstein & Thompson, 2002). Whilst there is considerable literature around mathematical practices and teacher actions, there is less research specifically focusing on how teachers of students beginning school facilitate these practices in their classrooms. Therefore, the objective of this study is to provide insight into the specific teacher actions that provide new entrant students the opportunities to explore and embed mathematical practices in their mathematics program.

Chapter Three: Methodology

3.1 Introduction

The previous chapter discussed the literature related to the current study. This chapter outlines the research design, and methods used in the study. Section 3.3 provides a justification for the use of case study research and qualitative methods used. Section 3.4 describes the role of the researcher and research setting, and participants are introduced in Section 3.5 In Section 3.6 data collection methods are explained. Section 3.7 describes the data analysis and outline the coding and themes. Section 3.8 and discusses the validity and reliability of the findings for the research. Finally, Section 3.9 outlines the ethical considerations for this study.

3.2 Research Aim

The aim of this investigation is to identify and illustrate the specific actions a teacher used to position young learners to engage with and demonstrate the use of mathematical practices. In this research there were ten student participants. Nine of these student participants were aged five years old (n=9) and one student participant was aged six years old (n=1). The study took place during the students' first seven formal mathematics lessons upon entering primary school. Specifically, this study aimed to address the following question: How does a teacher set up young learners to engage with mathematical practices during mathematics lessons?

3.3 Research Methodology

This study uses a qualitative methodological approach. A qualitative approach allows the researcher to focus on people, processes, and narrative information about their actions, beliefs, or behaviours in comparison to quantitative which solely focuses on the data of numbers (Punch & Oancea, 2014; Yin, 2015). This is appropriate given the socio-cultural perspective that will be taken and the focus on the exploration of teacher actions in a classroom of young learners to support mathematical practices (Punch & Oancea, 2014).

The choice of methodology aligns with the focus of this study. Given that a key focus is on the specific teacher actions used in the moment to support young students to engage with mathematical practice, a case study was selected as the chosen methodology. Firstly, qualitative case studies are common in the field of education as they allow the capture of in the moment observations and social data (Punch & Oancea, 2014). Secondly, use of case study methodology allows for the collection of detailed data through fieldwork in a bounded setting

(Barth & Thomas, 2012; Yin, 2011). A bounded case study means that there are boundaries and limits to the phenomenon being explored (Barth & Thomas, 2012; Gerring, 2006; Punch & Oancea, 2014). In this study, the classroom, teacher, and the students form this bounded system. The use of a bounded case study aims to provide a clear example in the authentic environment (social), supporting readers to surmise and understand the findings in more depth (Alpi & Evans, 2019; Cohen et al., 2017; Gerring, 2006; Merriam & Tisdell, 2015; Yin, 2015). Thirdly, Yin (2009) describes the three types of case studies as exploratory case studies collecting data and focusing on patterns; descriptive studies where theories are considered, and explanatory studies where the researcher focuses on explaining the how and why of the topic (Yin, 2009). With the focus of the current study being on teacher actions eliciting mathematical practices, there are a number of phenomena to be explored. Phenomena such as how to elicit discussion between young learners, how the teacher supports mathematical language development and the main focus of the study how the teacher develops mathematical practices with young learners. This study will illuminate specific teacher actions in context. Therefore it will be an exploratory case study. In this case study, as little is known, an understanding of the interpretation is required according to Denzin and Lincoln (2011). The exploration of this study can inform the audience about a case bound by a particular setting and time (Punch & Oancea, 2014).

Case studies allow the researcher to explore concepts more thoroughly rather than a reliance on hypothetical theories (Gerring, 2006). According to Swanborn (2023), this type of study can also be referred to as an intensive exploratory case study approach due to the researcher solely focusing on one specific aspect of the case; in this study the sole focus is on one teacher and the phenomenon occurring in one classroom. Whilst it is clear that there is variation between the definitions of specific case studies, researchers (Denzin & Lincoln, 2011; Punch & Oancea, 2014; Stake, 2005; Swanborn, 2023; Yin, 2009) all agree that a case study focuses on a phenomenon within the context of the study.

The generalisability of a case study is contentious (Gerring, 2006; Punch & Oancea, 2014; Yin, 2015), however, Punch and Oancea (2014) contend that a case study proposes concepts for further generalisation, or as Yin (2015) argues case studies generalise to develop or expand theory. Furthermore, Punch and Oancea (2014) argue that the researcher may put forward a proposition or hypotheses at the conclusion of their research. This hypotheses being the output of the research develops the link between how the research may be applied or is applicable to other situations. This contrasts with traditional research where the hypotheses or propositions are the input to the research (Punch & Oancea, 2014).

3.4 Role of the Researcher

In qualitative studies, the researcher's characterised as an independent data gatherer and analyst (Denzin & Lincoln, 2011; Merriam & Tisdell, 2015). However, in a case study approach, an active role may be taken by the researcher in the research process (Cohen et al., 2017). Opportunities such as direct observations, actions and events can be used to gain data (Barth & Thomas, 2012; Cohen et al., 2017). Accordingly, the researcher's role in this study was as an observer and collector of data. Specifically, the researcher sat during classroom mathematics lessons and closely observed both the teacher actions and student responses including both their observable mathematical understanding and discussions.

The relationship between the participant and researcher was professional and collaborative. The participant and researcher discussed the focus of the study. The participant was given opportunities to ask questions throughout as it allowed the participant a clear understanding into the phenomenon under exploration. The participant understood the researcher's role as an observer and collector of data. Data was openly collected.

The position of the researcher within this study was carefully considered. Due to the nature of the researcher's role as a mathematics mentor supporting the school to make pedagogical shifts, the students throughout the school and in this classroom were used to the researcher working in the classroom. This meant participants (teacher and students) felt comfortable when the observations took place. The researcher was previously an experienced primary school classroom teacher and this experience as a junior mathematics classroom teacher with young learners meant that she was familiar with potential practices and outcomes. The researcher's experience also gave her the ability to notice teacher actions specific to this year level. However, it also meant that the researcher entered the study with assumptions and biases which needed to be carefully monitored throughout the project. Concerns related to case study design are also noted by Yin (2015) related to the researcher letting preconceived notions impact the findings and not adhering to disciplined procedures (Yin, 2015). To address these concerns in this study, consistent and ongoing reflective discussions of the researchers' own assumptions and potential biases took place with the supervisory team.

3.5 Research Setting and Participants

To support the theoretical and practical constraints in a case study, sampling requires consideration (Groenewald, 2004; Punch & Oancea, 2014). The school was selected due to their current involvement with a three-year professional learning and development initiative called Developing Mathematical Inquiry Communities (DMIC), designed to support teachers to develop ambitious mathematics pedagogy (Hunter & Hunter, 2018). The teacher was an experienced new entrant teacher who had completed two years of DMIC professional learning and was purposefully selected. The teacher regarded their participation in this research as an opportunity to critically reflect upon and inform their practice.

The research was conducted with a group of ten students from one new entrant[1] class in an urban school in New Zealand. Nine of the students were aged five years old, and one student aged six years old. It is important to note that the six year old student had not attended formal schooling until starting at this school. Despite all students being in their first year of schooling, half of the students had only started school the week prior to the observations (n = 5), whilst the other students had started at various points throughout the previous term (n = 5) had only engaged in the warm up activities that the teacher did prior to each mathematics lesson. While the other students continued with the mathematics lesson these five students engaged in play activities.

3.6 Data Collection

Overall, data collection was conducted over two months (October – November 2023). Prior to data collection, several meetings between the researcher and the teacher occurred. The aim of these meetings was to outline and discuss the objective of the study and the research plan. The aim of the project was also shared with the students and their parents.

Both Punch and Oancea (2014) and Gerring (2006) suggest multiple data collection sources and methods to allow for effective case studies. Using multiple methods for data collection also allows for detailed information and enhances the credibility of the study (Punch & Oancea, 2014; Yin, 2015). In this study, a range of data collection tools were used including video

[1] In the New Zealand education system, the first classroom students enter into is called a New Entrant classroom. Children can enter into the class on their fifth birthdays or at any stage before they are six.

recorded classroom observations, a teacher written questionnaire, and photographs of student work or material representations. These will be discussed in the following sections.

3.6.1 Observations

In case study research, observations are an important tool for data collection (Punch & Oancea, 2014). The approach can provide opportunities to specifically observe actions and develop a deep understanding of the case (Renninger & Bachrach, 2015). Observations position the researcher to observe actions or behaviors that participants may overlook, which contributes to more valid data (Cohen et al., 2017; Punch & Oancea, 2014). In this current study, seven observations were made. Observations one to four occurred in the first two weeks of the students beginning school and with observation one capturing the young learners first formal mathematics lesson.

Date	Observation Number
16 October 2023	One
18 October 2023	Two
25 October 2023	Three
30 October 2023	Four
1 November 2023	Five
6 November 2023	Six
8 November 2023	Seven

The format of each of the lessons was similar aside from the variations that occur naturally within the classroom. The lessons were between 40 – 50 minutes in length and began with a warm-up activity followed by a task that students solved in small groups. The lesson ended with the teacher facilitating a conversation of how the students solved the problem and then supporting the students to make a connection to a mathematical idea.

Classroom observations were video recorded. Following the observation comprehensive field notes were written up to recount and provide a summary of what had been observed. These notes included a written commentary of the teacher actions, student response/action, materials used, and whiteboard recordings from the teacher that the videoing may have missed. Reflective comments and notes were added to the observational notes as the researcher noticed themes emerging or specific teacher actions related to the research question. This allowed the researcher to engage in preliminary data analysis as the data collection occurred and also provided opportunities to note particular aspects to observe in subsequent rounds of data collection.

3.6.2 Video recorded lesson observations

Video recorded observation allow the researcher to capture the social activity which cannot be done through observation alone (Cohen et al., 2017). Cohen et al., (2017) outline the following strengths of video recording as: video recording provides a sequential record (real time), recording of 'authentic' behaviours in the context of the study and a record of the observation that can be reviewed multiple times. In this study, to ensure a rich picture of the classroom learning was gathered, all lessons were video recorded. A video camera was set up to solely focus on the teacher throughout the mathematics lessons. The purpose was to investigate the specific teacher actions that led to the young students engaging in mathematical practices.

Video recorded lesson observations also allowed for moments to be captured that may otherwise have been missed, for example, non-verbal cues or reactions of the students. The recording also allowed multiple cycles of data analysis with the researcher able to reflect on what had been observed in conjunction with their written notes taken throughout the lesson. All video recorded lesson observations were recorded through the platform Iris Connect using the anonymise function to blur student faces to maintain confidentiality. The additional function of multiple microphones to capture student and teacher discussions. At the end of each lesson, these videos were uploaded and stored securely in the researchers' University Iris Connect account.

3.6.3 Written questionnaire

Written questionnaires are a way of gaining information related to the participants' experience and perspective (Punch & Oancea, 2014). The purpose of a written questionnaire is to find out information that cannot be gathered through a direct observation (Merriam & Tisdell, 2015). In this study, written questionnaires were used before and after the observations (see Appendix C). The questionnaire involved a series of open-ended questions exploring the participant's decision-making process as to the lesson plan and enactment. These questions provided an opportunity for the researcher to gain a lens into the decisions the participant made before and during the observations. The questionnaire also provided the participant an opportunity to share experiences relevant to the focus of the study. The questionnaire supported reciprocity as the participant could take the time to respond, thus highlighting respect for the participant's ideas.

3.6.4 Student work

Digital photographs of students' responses only (no children were photographed) were taken during the lesson to capture the representations they developed with scaffolding from the teacher. These photographs provided concrete evidence for the researcher to refer back to and analyse the students' use of mathematical practices in response to teacher actions.

3.7 Data Analysis

The goal of data analysis in a qualitative case study is to make sense of the phenomenon observed and to provide knowledge (Hsieh & Shannon, 2005). For this case study, analysis of the data meant making sense of the scaffolding or opportunities that the teacher provided for the young students to use mathematical practices. Specifically, this included noticing how and when the teacher set up the young students for success in these practices. To manage the study effectively, after each observed lesson, analysis occurred in order to generate themes and patterns within the lesson. This consisted of video footage analysis and comparing this with field notes alongside any artefacts (student work). Themes were identified and developed as the data was analysed. Once all observations and questionnaires were complete, the video footage was transcribed and alongside the field notes revisited many times in order to identify and confirm themes. Patterns were identified and categories generated to code the collected data, thus producing a comprehensive description of the case.

3.7.1 Coding and developing themes

Thematic analysis was used for data analysis for this study. The art of thematic analysis involves the researcher finding meaning from the data, in order to answer the research questions being addressed (Clarke & Braun, 2017). Thematic analysis also allows for the data to be organized systematically looking for patterns across the data (Braun & Clarke, 2006).

The systematic review of the data began with initial coding of the themes of mathematical practices. These codes were what Clarke and Braun (2017) term as identification codes or building blocks, to highlight when the students engaged with mathematical practices. In the next phase, the coding identified parent codes and child nodes. This involved taking a mathematical practice and identifying the teacher actions that occurred in that moment in time. The subsequent analysis involved the comparison of the developing themes to ensure they addressed the research question. Due to the nature of the data that emerged, individual codes were set for each mathematical practice, although there was considerable overlap between these codes, see Appendix B for the thematic coding.

3.8 Validity and Reliability

Research must be conducted with outcomes of validity, integrity, high quality and most importantly trustworthiness (Fereday & Muir-Cochrane, 2006; Gerring, 2006; Punch & Oancea, 2014). These are a researcher's ethical duties. In this study, care was taken to develop clear in-depth descriptions, detailing the setting, participants, and themes to ensure that transferability could be achieved. This technique enables readers to engage with the events being described and assess how the findings might apply to other contexts.

The use of one research method can lead to potential bias however the use of many data sources helps remove potential bias. Therefore, multiple sources of data collection were used in the current study. To allow for data triangulation wherever possible, the data collection method and analysis were systematic. For example, data from the questionnaire and observations were compared. Additionally, comparison between the field notes and video observation occurred to ensure all sets of data were interpreted accurately. Members checks, are an important procedure where the findings are shared with participants prior to completion to allow for feedback to improve the accuracy of the study (Gerring, 2006). In the current study, collaboration between the researcher and the teacher enabled the interpretations to be checked. This opportunity allowed for revision if needed after observations or the questionnaire. The researcher also consulted with supervisors to discuss emerging themes and findings, seeking their feedback on the plausibility of the results. As Punch and McGowan (2006) highlights, this adds another layer to the validity of the study maximising the quality of the data.

Of similar importance is the applicability and practical use of the data in relation to the participants in the study (Ryan et al., 2007). Outcomes of this study and the findings discussed highlighting the teacher actions supporting the use of mathematical practices with five and six year old students should be specifically helpful to teachers of similar age group. These findings should also be transferable to support mathematics classrooms of other aged students.

3.9 Ethical Considerations

In agreement with the Massey University Code of Ethical Conduct for Research, Teaching and Evaluations Involving Human Participants (Massey University, 2015) the research was designed and conducted in a responsible manner. Prior to the data collection, the project was considered and approved by the Massey University Human Ethics Committee prior to data collection. The researcher firstly ensured the fundamental ethical principle of doing no harm by reflecting on whether the research study design was best for the education and welfare of the students involved. Ethical considerations taken into account included informed consent (for the teacher participant), consent for participants (students) under seven, respect for privacy and confidentiality, as well as researcher relationship and time.

3.9.1 Written consent

All participants including the principal, Board of Trustees, the teacher, students and their guardians were provided a detailed information sheet and consent form to ensure participants had a clear understanding of why the research was occurring, what was involved in the research and what they were agreeing to (see Appendices D1 – D4). Written consent was obtained from all participants (see Appendices E1 – E4). As this research involved children under the age of 15 years old, informed consent from their parents or guardians was obtained. Participants were also made aware during the initial meeting and within their information sheets that at any stage of the research, participants had the right to withdraw.

3.9.2 Confidentiality & anonymity

It is important that the confidentiality and anonymity of the participants are guaranteed throughout a study (Cohen et al., 2017; Punch & Oancea, 2014). Care was taken to ensure that the confidentiality was respected due to the age of the participants in the study. To adhere to anonymity, the researcher took care whilst videoing to ensure that students not involved in the study were not accidentally videoed. Whilst videoing, the researcher used the anonymise function on Iris Connect to blur student faces to maintain confidentiality. All participants in the study were allocated pseudonyms. To maintain confidentiality throughout the written reports, no identifying information was included about the teacher, students, or school.

3.9.3 Researcher relationship

In accordance with the Treaty of Waitangi, respect for the participants was of upmost priority for the researcher throughout this study. To uphold a respectful relationship with all participants, trust was gained to ensure participants were not simply objects of research (Massey University, 2015; Punch & Oancea, 2014; Yin, 2015). Within a case study methodology, the researcher is obligated to ensure the relationship remains trusting and is initiated by the researcher (Punch & Oancea, 2014). So, the researcher spent time within the classroom space building this relationship prior to the research study commencing. There remained an ethical dilemma due to the change within the previous professional relationship between the researcher and participant. However, due to the focus of the study being on the specific teacher actions and student responses, the intended focus was not on the judgement of teacher practice but instead on highlighting the teacher actions that related to the research questions. To maintain the respect and trust in the participant, the participant (teacher) was involved in viewing parts of the analysis and the researcher ensured their opinions, suggestions or potential disagreements with interpretation were considered in the final report.

3.9.4 Time

Time is crucial to anyone therefore this was a consideration of the study. Video-recorded observations were scheduled within the normal routine classroom timetable to avoid disruptions to ensure harm to the students and teacher was reduced. Meetings for the written questionnaire were organised to best suit the teacher so as not to burden them with an increased workload.

3.10 Summary

A qualitative study with a single case study design was chosen as the most appropriate method for this study. Throughout the study multiple forms of data were collected by the researcher, including on site classroom observations that were filmed and transcribed, comprehensive observation notes, students' artefacts, and a teacher questionnaire. The researcher continuously maintained a high level of ethical consideration during the data collection and analysis to ensure the study-maintained validity and reliability. To ensure that no harm would come to any of the participants, ethical principles were maintained at all times.

Chapter Four: Findings and Discussion

4.1 Introduction

As highlighted in the literature review chapter, students of all ages can engage in mathematical practices, however, this requires specific teacher actions to set students up to engage with mathematical practices. In this chapter, a case study is presented of one teacher working with her class of ten young students. To develop a full description of the case, the chapter is organised into a timeline of three phases which showcase the notable shifts in the teacher actions to introduce mathematical practices. A further section then delves deeper into the use of mathematical discourse within the lessons. Finally, an overview of the specific teacher actions to support students in using mathematical practices is presented.

4.2 Phase One (Lesson One and Two)

In this phase the teacher noticeably used scaffolding and modelling to introduce mathematical practices. The teacher also placed a focus on the expectation that students shared their reasoning with their peers. These teacher actions are discussed below.

4.2.1 Developing explanations

The initial actions of the teacher focused on supporting students to develop mathematical explanations. The teacher actions included asking the students specific questions related to the mathematical concept being discussed to elicit understanding. In doing so, the teacher also provided time and intellectual space for students to answer questions specific to the mathematical task. An initial question prompted the students to think about their reasoning with further prompts carefully facilitating the students to construct their answers. When the students stated their response, the teacher utilised the talk moves of revoicing and repeating. The following vignette highlights the first whole class discussion. The are discussing patterns that had already been created during an independent play-based session prior to this lesson. The teacher actions are demonstrated below:

Developing an explanation	
Teacher:	Who can remember our snake pattern *[wait time]* and **what made it the pattern?** *[wait time]*
Luke:	We had one and then a different one
Liam:	Colours
Teacher:	That's right we had two different colours, today I have *[teacher pauses]* *[holds up a pink multilink cube]*
All students:	Pink
Teacher:	And *[wait time]*? *[holds up a white multilink cube]*
All students:	White
Teacher:	I am going to put them together just like Luke said because our snake last week had two different colours. So, if we had pink and white as our unit of repeat and we grew this pattern *[teacher pauses]*
Sophie:	Pink
Laura:	Start with pink
Teacher:	We need to add pink you are so right and then *[teacher pauses]*
Luke:	White
Teacher:	You are so good, the repeating pattern that is going pink, white, pink, white. Sophie, can you repeat that idea?
Sophie:	The repeating pattern is pink, white, pink

Firstly, the teacher used a specific question (shown in bold) to invite the students into the discussion and then asked further prompting questions to engage students in creating an explanation for a repeating pattern. Cengiz et al. (2011) refer to this teacher action as an eliciting action allowing the teacher to understand the mathematical thinking of the students before extending or supporting the students to further develop their explanation. In this instance, the teacher modelled an explanation to the students. Although Bieda (2010) states the risk of teacher modelling with older students reducing cognitive demand, in this case with younger students, modelling an explanation was important to support them to learn how to develop an appropriate explanation. Consistent with earlier findings of studies involving young learners (Herbert & Williams, 2023; Sumpter & Hedefalk, 2015), the use of strategic questioning supported the students to develop a collective explanation.

Evident in the vignette is the student engagement in collaborative discourse with multiple students participating at different times. This aligns with earlier research studies (e.g., Chapin & O'Connor, 2007; Franke et al., 2009; Kraatz et al., 2022) that demonstrated rich mathematical discussion can occur when the teacher allows the classroom to become a space for conversation. When constructing the explanation in the example above, the teacher allowed the students to discuss and join the public conversation at any stage rather than specifically asking individual students for their ideas. Similar to the findings by Hunter (2008a), and Kraatz et al.'s (2022), this allowed for student talk to develop. The teacher consistently used conversational moves such as described by Chapin and O'Connor (2007) to develop both mathematical discussion and student generated explanations. This included wait time after asking each question and the use of a repeat talk move to encourage the students to repeat a specific mathematical idea.

4.2.2 Developing justifications

In the first lesson, the teacher also used specific questioning relating to the mathematical content to support justification. The following vignette highlights the teacher actions used as the students worked in pairs and discussed what colour was going to be next in the pattern:

Specific question prompt for justification	
Teacher:	Can you talk with your buddies; **how do you know** it should be white at the end of the snake?
[All students start talking to their partners]	
Luke:	Because it is the pattern
[to his partner]	
Teacher:	Ooh I heard because over here, well done
[students continue to discuss – inaudible]	
Teacher:	Chloe, I heard some really great conversations in your group, why is it white on the end of the snake?
Chloe:	Cause pink, white, pink, white, and then pink next
Teacher:	That's right, because we are doing a repeating pattern. We have pink, white, pink, white, pink

Here, the initial prompt from the teacher "how do we know" supported the students to engage in justification. The teacher then publicly praised the student's use of the word 'because' when sharing their thinking. Additionally, the teacher's use of re-voicing to provide students with a model of mathematical language, "we are doing a repeating pattern". Instead of asking what the students did, the teacher instead framed the question as 'how do we know' to position the students to think about why white was the next colour in their pattern. This was a significant move as it placed a value on reasoning rather than the correct answer. In this way; the teacher was able to notice that most students had an understanding of a simple repeating pattern. Earlier studies with older students (e.g., Maher & Martino, 1996; Stylianides, 2007) have shown the importance of teachers placing value on justification by pressing and encouraging students to prove their thinking. While the longitudinal study by Maher and Martino (1996) focused on one student over time, a key similarity between their study and this one, is the consistent opportunities (multiple times in each lesson) to engage in justification. In the current study, the teacher stressed an expectation on justification from the first mathematics lesson which supported students to use this practice in the classroom. Similar to the teacher action discussed above, Stylianides (2007) found that with older students, the teacher was able to press for justification through questions such as 'how do we know that is true?' or 'how could you prove that to someone?'. The teacher used the talk move of revoicing to introduce the mathematical language to these young students and explicitly highlighted a key mathematical idea that patterns repeat. Earlier research with older students has shown how teachers used extending moves (Cengiz et al., 2011) or reprising moves (Selling, 2016b) to highlight and support the use of justification. A key finding of this study is that similar teacher moves can be used to support younger students. By modelling the practice, the teacher scaffolded the students to use justification by reasoning about mathematics. This finding is important as earlier studies (e.g., Björklund et al., 2010) identified that younger learners often do not sufficiently reason in justification with their justifications lacking mathematical logic.

Asking students to turn and talk with their buddy and provide a justification for their answer both invited the students into a mathematic conversation and provided the teacher with an opportunity to monitor student reasoning. A further teacher action to support justification was the public acknowledgement of student reasoning through the public statement "oh I hear because over here" and the comment "Chloe, I heard some really great conversations in your group". This action provided two important opportunities, firstly, it outlined the importance of students using the mathematical practice of justification, and secondly, praised the importance of sharing ideas between peers.

4.2.3 Developing collaborative reasoning and early argumentation

The teacher noticed and responded to collaborative mathematical reasoning as the young students worked in pairs. For example, as the students were beginning to select materials to solve the mathematical task the teacher noted a pair of students agreeing on a repeating pattern structure, as highlighted in the following vignette:

Teacher publicly naming agreeing during the student discussion	
Teacher:	Talk with your buddy and work out what colours you are going to use
[groups discuss]	
Laura	Let's do purple, blue, black
[to her partner Chloe]	
Chloe	Yeah *[nods her head]*
Teacher:	Laura and Chloe have discussed and **agreed on their idea**
[publicly to the whole class]	

An initial prompt from the teacher supported the students to share their ideas about the pattern and material with their buddy. As a result, a student stated "let's do purple, blue, black" explicitly describing the structure of the repeating pattern that she was proposing. The initial prompt of "what colours you are going to use" focused the students on the mathematical sequence and structure of a repeating pattern. There is some evidence here that both students understand and know the mathematical sequence of this pattern. The teacher then supported student reasoning by publicly naming the student response (underlined). This interaction simultaneously praised the sharing of ideas and explicitly named the action of agreement. Whilst this is a small moment within the lesson, it aligns with how Langer-Osuna (2016) defines this type of teacher action as the initial building blocks to students engaging in mathematical argumentation.

In the following lesson, the teacher provided an opportunity for the students to disagree with her reasoning by incorrectly drawing a pattern (Figure 7), in response the students all shouted "nooo" as she was drawing, encouraging the teacher to stop.

Figure 7

Incorrect Pattern

The teacher explicitly stated, "Oh I see that you disagree with my idea and that's okay, it is okay to disagree". Whilst she did not press the students to explain why they disagreed with her reasoning; the teacher used clear language to name the students' action of disagreement – "you disagree with my idea". While in Hunter's (2008c) study with older students, the teacher had an expectation for students to indicate agreement or disagreement, in this study the teacher scaffolded the young students into this practice by naming and modelling the language of acceptable argumentation. Earlier studies identify young students using agreement and disagreement as part of early argumentation; however, this study shows that teacher actions of naming and reinforcing argumentation can be used to set up the students to engage in this mathematical practice.

4.2.4 Developing mathematical representation

Throughout this study the teacher exposed the students to different mathematical representations as a way of connecting to mathematical concepts. This included both modelling and pressing students to use multiple representations. In the first lesson, the mathematical task was to repeat the pattern three times, the teacher used this as an opportunity to teach the students to represent three in different ways:

Teacher prompting multiple representations of a single number	
Teacher:	Can you say three?
Students:	Three
Teacher:	Can you show me three on your fingers? One, two, three. *[the teacher holds up three fingers and the students hold up three fingers]*
Teacher:	Tell your buddy, how many times are we continuing to repeat the pattern? *[writes the number three on the board]*
Students to their peers:	Three times

The example above illustrates the multiple opportunities set up by the teacher for students to learn and make connections to the written numeral three. The teacher encouraged the students to show the numeral on their fingers and wrote the number on the board. This finding aligns with Bobis and Way's (2018) recommendation of the importance of young students learning to represent. The teacher continued to scaffold student representations using a hundreds board[2] and by writing numbers on the white-board. For example, as students were counting aloud, the teacher would write the corresponding numerals on the board. Importantly, this provides an exemplar of how students can both be taught to identify and represent numerals while also learning to engage with mathematical practices.

After the students had repeated their pattern and a group shared their thinking to the class, the teacher modelled to the students how they could represent their pattern made with coloured unifix cubes[3]. Due to this being many of the students' first formal mathematics lesson, the

[2] Hundreds board is a chart divided into 100 squares, 10 by 10. Numbers 1 – 100 are written in sequential order.
[3] Unifix cubes are small coloured cubes that interlock together.

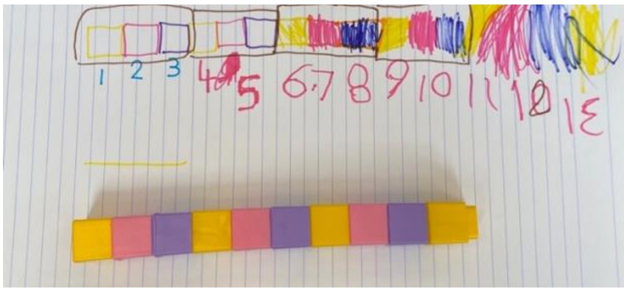
teacher carefully utilised scaffolding and modelling. The following vignette outlines this teaching episode:

Teacher modelling how to represent unifix patterns as a drawing	
Teacher:	Look what we could do, I could draw on my book what Liam and Sophie have done.
[*teacher starts to draw squares in a modelling book*]	
	Yellow *[draws yellow square],* Pink *[draws pink square],* Purple *[draws purple square]* What would come next Eleanor?
Eleanor:	Yellow
Teacher:	*[draws yellow square]* Raymond, what would come next?
Raymond:	Pink
Teacher:	And then what would come after that?
All students:	Purple.
Teacher:	How many cubes are in your <u>unit of repeat?</u> Tell your buddy?
[students talk with their buddies]	
Eleanor and Claire:	Three.
Teacher:	There are three, I am going to write that: one, two, three *[represents the numbers under the three cubes].*
Teacher:	Do you think you could now draw your snake, so you've made your patten lets copy it into our books.

Here, the teacher specifically modelled the pattern square by square to illustrate how a pattern could be translated from concrete materials to a drawing. The teacher then notated numbers under the cubes to support the mathematical understanding that each colour represents a position in the pattern. Following this, the teacher invited the students to represent their patterns in their books (Figure 8).

Figure 8

Students Multiple Representations of their Pattern



Evident here, is that the teacher modelling supported the students to successfully translate the pattern from the unifix cubes (materials) to a drawn representation. Throughout the lesson, the teacher continued to support students to model how materials or drawings could be used to represent ideas. As students shared their reasoning to the whole group, the teacher responded in the moment stating, “we could draw that” or “let us check on the hundreds board”. Cartwright (2023) highlighted students’ capacity to represent as well as provide verbal responses, this study illustrates the steps a teacher can take to begin scaffolding young students to represent through modelling. Reflecting on her actions after the lesson, the teacher emphasised the importance of all students having an opportunity to explore concrete materials, tens frames, and hundreds boards to both support representing but to also give the young learners something to manipulate when solving their tasks. In the first two lessons, the students were given multiple opportunities to represent their thinking with their peers. At times the opportunities were heavily scaffolded, with the teacher directing the students to use specific materials to create their patterns. If students finished making their patterns with materials before others, the teacher would ask, “is there a way you could now draw your pattern? You have made it – can you draw it?”.

4.2.5 Extending student thinking through representations

The teacher also used representations to extend student thinking. In the following vignette, the teacher demonstrates to the students how their counting could be represented as a mathematical equation. The students had repeated their pattern three more times using unifix cubes and Liam explained the group representation of the pattern:

Teacher modelling mathematical equation as a representation	
Liam:	…Our snake is now 12 cubes long, one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve *[counting and pointing to each cube]* [wait time]
Teacher:	*[sticks the unifix snake made by Liam on the whiteboard]* Hey, look we can also record this on the board *[pause]* Three *[teacher writes 3 + under the first unit of repeat]* another three blocks *[wrote 3 + under the second unit]* and three *[wrote 3 + under the third unit of repeat]* plus three more *[wrote 3 + under the final unit of repeat]* equals 12. Wow, three + three + three + three = 12. How many does it equal? How many blocks?
Claire:	12
Teacher:	I can also record that by saying *[pause]* hmm *[pause]* how many groups of three do I have? *[pointing to the threes]*
Sophie:	Four
Teacher:	So, we can say four groups of three equal 12 *[writing 4 x 3 = 12 on the board]*

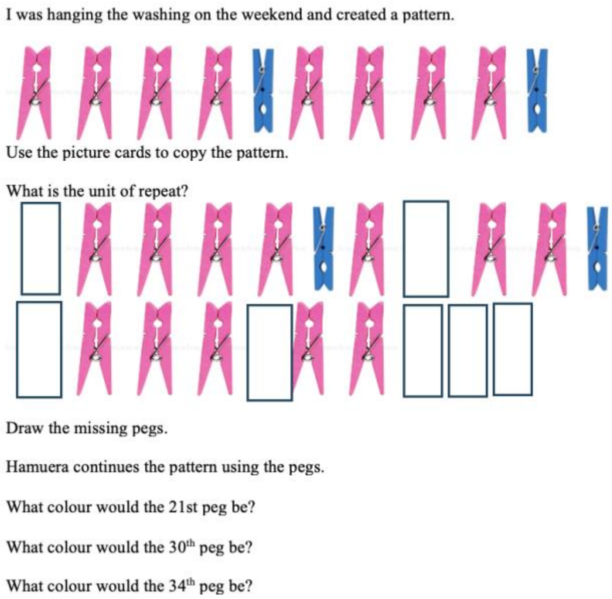
The teacher has built upon and extended the student generated explanation to incorporate and model representations using equations. Important to developing student understanding is providing opportunities for connections between visual representations and symbolic representations. Earlier studies with older students (e.g., Jacob & Empson, 2016) demonstrated students writing their own equations in response to their thinking, in contrast with these young students, the teacher specifically modelled the representations for students.

4.2.6 Providing opportunities to the students to reason with their peers.

In lesson two, the teacher had given students a task which progressed from a typical ABAB repeating pattern to AAAB which initially caused some confusion for the young students. Figure 9 shows the task.

Figure 9

Mathematical Task



The following vignette demonstrates the teacher revoicing Eleanor's understanding and asking the group of students whether they agreed or disagreed with her idea:

Teacher giving permission to disagree	
Teacher:	Eleanor thought that the pattern is four pink and then four blue, because that is what is happening in the pattern. What do we think? Do we agree or disagree with Eleanor? *[wait time]*
Teacher:	Remember it is okay to disagree with Eleanor, we all still like Eleanor as a friend. Sophie what do you think?

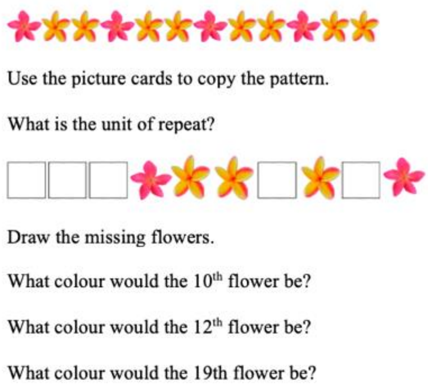
Two teacher prompts were used in this vignette. The first teacher prompt, "what do we think? Do we agree or disagree with Eleanor?", was used to position students to reason with their peer's idea. In this way, the teacher shifted the accountability to the students to justify their reasoning and the reasoning of others. Herbert and Williams (2023) refer to this action as opening the space for argumentation. The opportunity to reason was further extended when the teacher prompted a student "what do you think?". The second teacher prompt gave the students permission to disagree with their peers by reminding the students that their friendship with Eleanor would not change. Previous literature (Chazan, 2002; Civil & Hunter, 2015; Herbert & Williams, 2023; Yackel & Cobb, 1996) highlights the need for teachers to set up expectations around argumentation. In this study, an example has been presented of how a teacher can achieve this in the moment by being responsive to the social needs of these young learners. This teacher action also signaled to the students the importance of reasoning with their peers in a positive way. In these ways, the teacher has prepared the way for future mathematical argumentation.

4.3 Phase Two (Lesson Three)

In this phase the teacher noticeably used prompts to further develop mathematical practices. The teacher also placed a focus on deepening mathematical reasoning by pressing the students to extend explanations and provide justification. Figure 10 displays the mathematical task.

Figure 10

Mathematical Task Lesson Three



4.3.1 Teacher prompts to develop justification

Justification prompts were also used to intervene and support small groups as they were working together to solve a task. As Hunter (2008a) states collaborative interactions and discussion can be challenging for young learners without explicit adult support. In the current study, the teacher provided support specifically when she noticed that students had become unengaged, or the level of conversation had dropped within the collective group. The vignette below provides an example of a teacher intervention to promote justification that reengaged the group members and highlighted student mathematical thinking:

Justification during small group work	
Claire is recording an idea down in silence with her group members Raymond and Chloe sitting silently and unaware of the mathematical ideas that Claire is recording.	
Teacher:	Oh, it looks like you have a great idea there. Claire, can you tell Chloe and Raymond what you are doing?
Claire:	You have to put it this way *[pointing to the flowers in her pattern]*
Teacher:	Tell them **why** you have to put them that way
Claire:	Because it is the same
Teacher:	Are you saying it is like the pattern on the board, the same as the pattern?
Claire:	*[nods]*
Raymond:	[*picks up his flowers and adds them to Claires pattern]*
Teacher:	Looks like Raymond is moving his flowers now as well and is continuing the make the unit of repeat the same.

The teacher firstly shifted the explanation to a justification and then used the talk move of revoicing while modelling the mathematical language required to extend the justification. Cengiz et al. (2011) recommend instructional actions of extending when students are involved in deepening their own reasoning. However, in this instance the teacher engaged in this action to extend the justification as a model to the students. As the teacher highlighted Claire's thinking to the group, she also concurrently developed mathematical learning by supporting the students to recognise that the pattern can be re-created using the flowers. This explicit modelling is similar to the work by Pramling and colleagues (2019) who found when working with early childhood students, the teacher was required to instruct the students on the mathematical content required to form a justification.

As well as supporting students to share their reasoning, the teacher also reiterated the expectations that students share their ideas when working together. Langer-Osuna (2016) contends that the role of a teacher during small group inquiry can be complex and challenging. In the current study, the teacher played a pivotal role in the interaction by giving what Langer-Osuna (2016) describes as "intellectual authority" (p.121) in the moment. Similar to Langer-Osuna's study, assigning intellectual authority encouraged both Raymond and Chloe to engage with Claire's mathematical reasoning to solve the task. In contrast with studies with older students (e.g., DeJarnette et al., 2014; Langer-Osuna, 2016) where students were expected to assume responsibility for their own learning, in this classroom, the teacher worked to facilitate the discussion between the three learners and support collaborative work in a productive way.

4.3.2 Teacher prompts to develop explanations

Many of the opportunities to develop explanations occurred as the teacher directly questioned the students, however, she also used expanding prompts or open-ended questions to support the students to share their ideas, for example: "tell us what you are thinking or what did you notice?". Davidson et al. (2019) highlight how teacher prompts can be used to analyse the student reasoning, the teacher in this study used these prompts to support student reasoning. Kraatz et al.'s (2022) framework included similar prompts as conversation scaffolds however, these were only used with older students. In the current study, evidence shows that despite their age, young learners were supported to share their reasoning following the open-ended prompt. The following vignette demonstrates further questioning by the teacher to support a group of students sharing their explanation following an open-ended prompt:

Open ended prompt and further questioning.	
Teacher:	C**an you talk us through what you have made?** (open ended prompt)
Laura:	Flowers
Teacher:	What sort of flowers? (further questioning)
Leo:	A pattern
Teacher:	Tell us more?
Claire:	Red, red, red, yellow, red, red, red, yellow
Teacher:	So, our first unit of repeat was red, red, red, yellow and you repeated it?
Students:	[*nod]*

Evident is how the students build on their explanation of their pattern in response to the teacher pressing them beyond the simple answer of "flowers" and using a series of further questions to support an explanation. As suggested by Franke et al. (2009) and Sumpter and Hedefalk (2015), it is the teachers' role in the moment to support students to elaborate their explanations. As the teacher elicited student responses, the explanation developed allowing for the teacher to model an acceptable explanation, "so our first unit of repeat was red, red, yellow and you repeated it". While the students themselves were not yet making elaborate explanations, they were building the foundational skills to do so. The observed teacher actions were similar to those described in Hufferd-Ackles et al.'s (2004) framework designed to shift teachers from traditional teaching classrooms to students leading the conversations. Also illustrated is how the teacher concluded the discussion by revoicing the group explanation. Clarifying the explanation with the students as it was revoiced allowed the group to maintain ownership of the mathematical explanation. Revoicing is a tool to develop effective discourse (Chapin & O'Connor, 2007). In this case, the teacher's use of revoicing positioned the whole class to engage with the groups' reasoning.

Through framing the revoicing as a question, the students were encouraged to make sense of the explanation and respond to the teacher.

4.3.3 Teacher prompts to develop argumentation

The teacher specifically modeled how to agree with a mathematical idea and provide reasoning for agreement. In the vignette below, a pair of students have shared their explanation of the pattern and the total number of flowers in the first unit of repeat. The teacher supported the students to reason with the idea by asking them to indicate if they agree or disagree with the idea shared and then by providing a model for them of how to share reasoning to justify agreement or disagreement:

Teacher prompts to develop argumentation	
Teacher:	**Put your hands on your head if you agree with this group**, that four plus one equal five? *[all students put their hands on their heads]*
Teacher:	Yeah, I agree too, absolutely, there are five flowers in the first unit of repeat *[wait time]*
Teacher:	There are four red and one yellow, Laura can you repeat that idea?
Laura:	There is four red, and one yellow more makes… five
Teacher:	So, there are five in total, we all agree.

This example highlights the teacher prompts to develop argumentation. The teacher specifically asked the students whether they agree or disagree with the group that had just shared. All of the students indicated that they agreed, and the teacher provided the mathematical model for why this agreement was true. The teacher prompt in this example acts as a model for the students on how to justify why they agree with what another group has explained. This type of teacher modelling has been reported as beneficial in other research studies (e.g. Breive, 2017; Sumpter & Hedefalk, 2015) in demonstrating to young students how to use precise mathematical language. In this example, the teacher both modelled the type of mathematical language expected and prompted the students for agreement or disagreement.

Over the remaining lessons of the study, the teacher continued to encourage students to indicate their agreement or disagreement with a peers' idea. This was done in a positive way for these young learners with the use of prompts such as "hands on your heads if you agree" or "finger on your nose if you disagree". There are parallels in this teacher action to other research studies (e.g., Herbert & Williams, 2023; Hunter, 2008b; Yackel & Cobb, 1996) in which teachers established a reflective space for the students to reason, however, in this study the teacher also allows the students to gesture in a way that encouraged movement, an engaging, fun and enjoyable moment for these young learners.

4.3.4 Small group argumentation

Following the interaction in the previous vignette, the students began to reason with their peers. The whole class had discussed the repeating pattern having five flowers in the unit of repeat and were now creating their own patterns in small groups. The following vignette demonstrates Laura expecting her peers to reason with her idea:

Students reasoning in their small group discussion	
Laura:	Four plus one equals five, agree?
Claire:	Yeah, that's right
Laura:	Five because four and then one more is five?
Raymond:	Five five five *[yelling]*
Claire:	Yeah, that is what I was going to say

This example demonstrates how the teacher actions and modelling to facilitate argumentation had supported the students to begin to engage with each other's ideas. Initially, Laura invited her group to reason with her idea by asking "agree?", followed by her sharing her reasoning in an age-appropriate way. It is important to note, that in this example, whilst Raymond (English language learner) had not yet developed the mathematical language required to support his reasoning, he was participating in the wider discussion. Claire did not share her own reasoning; however, she supported Laura's mathematical claim by stating "yeah, that is what I was going to say". This contrasts with the findings of Brieve (2017) study who reported limits in young students' use of mathematical language. In this case, the young student used accurate mathematical language which reflected how the teacher had supported these students up to engage in mathematical argumentation.

4.4 Phase Three (Lesson Four, Five, Six and Seven)

In this phase the teacher noticeably named and praised the mathematical practices she noticed the students using. These actions highlighted the importance of the students knowing and using these practices to deepen their mathematical understanding.

4.4.1 *Naming and praising mathematical practices*

In the final phase of the study, the teacher transformed the language she used to prompt her students to engage in mathematical practices. First, she began explicitly naming and praising the students when they engaged in a practice. For example, as the students were sharing their ideas, a student stated: "eight because two, four, six, eight, so there are eight". The teacher then stated: "I love how you were able to justify your thinking as well Liam, as you used the word because". Selling's (2016b) study with older students developed a framework of reprising moves to support teachers to engage their students with mathematical practices. Similar moves to those documented in Selling's (2016) framework were used as the teacher named the practice "justify your thinking" and highlighted the practice "used the word because". Critical to the enactment of these reprisal moves is that the moves came after the student use of the mathematical practice allowing the students to engage with the mathematical practice rather than the teacher modelling the practice.

The teacher then combined naming and highlighting both explanation and justification with more open prompts such as "can you justify your thinking" "can you explain it for us?"; "can you explain your thinking to us"; "who can explain how many eyes there are?". At times, the teacher supported the students further by extending student reasoning through specific questions to elicit the accepted justification and explanation. Purposefully asking the students to explain or justify raised the expectations for students to provide mathematical explanations, rather than one-word answers. While previous studies with older students (e.g. Davidson et al., 2019; Hunter, 2008a) demonstrated the use of these prompts, the current study demonstrates that over time and after the use of specific questioning and explicitly naming the practice, young students can also engage in similar justification prompts. For example, the teacher in the current study asked the students "can you justify your thinking" and the students responded by saying "there are four tails because there are four bunnies, and each bunny has one tail".

A shift in language and the introduction of open-ended prompts occurred when the teacher requested the students to explain. The teacher used prompts which positioned the students to explain their thinking rather than simply provide an answer. When questioned about this shift towards naming the mathematical practice, the teacher stated that "naming the practice tells the students what they are doing, and I want them to explain. I don't know why I started in lesson four, just decided it was time".

4.4.2 Deepening student representations

To further develop student representations, the teacher consistently[4] exposed students to other ways of showing their patterns through colours, letters, and numbers. As the lessons progressed, the teacher encouraged students after using materials to translate their pattern: "have a think, what did we do last week to represent our pattern? *(*wait time) we represented our pattern in letters and numbers, talk with your buddy how could we *represent* that today?". At this point, students were not yet naturally translating their patterns with different representations, however, the prompt indicated an expectation that these young learners could learn to use multiple representations to show their mathematical ideas. A similar teacher prompt was noted by Hunter (2008c) and Herbert and Williams (2023) to explicitly prompt students to represent their thinking using equations. However, in this study, the teacher used an open prompt "how could we represent that today" providing the students with agency to decide on their choice of representation. For example, in Lesson Six, when the students were prompted to represent their pattern, one group used circles and colours to represent their pattern, another group used letters corresponding with the colours and one group used both colours, letters and numbers to represent (Figure 11).

Figure 11

Lesson Six Student Representation

Another teacher action was public praise to highlight a representation being used by a group. Statements such as "oh I love how Liam's group is representing; they are drawing letters to represent each flower*"* supported students to understand the expectation and benefits of using representations. In addition, the teacher was highlighting the mathematical understanding that patterns can be translated while drawing on the same pattern structure.

[4] After every share back or during whole class discussions representations were modelled.

4.4.3 Developing generalisation

Generalising is an important mathematical practice; however, it can be challenging for young students. In this study, the consistent focus on students explaining, justifying, and engaging with the reasoning of others were the initial steps in supporting future engagement in developing generalisations. There were two explicit teacher actions evident that were used to support the young students to engage in generalisation. These actions included teacher questioning for connections and teacher modelling how to make links across student representations. The teacher used questioning to support the young learners to make connections across the patterns they had been working on over the lesson sequence. The following vignette demonstrates the specific content related question the teacher used after a group shared their observation of the pattern growing:

Teacher question to support generalisation	
Liam:	The pattern is getting bigger and bigger
Teacher:	**Has this been the same with all of our patterns**, the more units of repeat, the bigger the pattern is? *[wait time]*
Eleanor:	Oh yeah *[pause]* that is the same
[Other students nod]	
Teacher:	That is right, the more units of repeat the bigger the pattern, even though the patterns are different, they are all getting bigger.

The teacher used the question "has this been the same with all of our patterns" to encourage the students to notice the connection between patterns. This supported the students to understand that patterns are predictable and grow in similar ways. This question is similar to those suggested in previous studies (e.g. Blanton & Kaput, 2003; Ellis et al., 2021) where the teachers used prompts such as "what is the same or different?" when comparing solution strategies. Ellis and colleagues (2021) also highlight the importance of balance between the teacher initially encouraging a generalisation through an open prompt followed by specific questioning seeking an answer. Although in this study, the teacher used a closed question, she still was supporting the students to recognise the similarities in their patterns. The teacher then explicitly stated the generalisation to the students "that is right, the more units of repeat, the bigger the pattern, even though the patterns are different, they are all getting bigger". The use of deliberate wait time was notable in this example, as it provided space in the conversation for the students to consider the discussion and reason with the teachers' ideas, as seen in Eleanor's comment “oh yeah, that is the same”. In highlighting this generalisation, the teacher made a connection to the mathematical idea that patterns are predictable.

4.4.4 Developing generalisation across representations

The teacher also highlighted connections between representations to provide a model to students of how to generalise. This occurred in the final lesson after two pairs had shared their patterns:

Developing generalisation across representations	
Teacher:	Raymond and Leo used numbers along their tower to show their thinking and Sophie and Liam used letters to represent their pattern. How cool. Wow, so are you telling me that even though we had the same pattern, different items to make it with, we could record the same pattern in a different way?
All students:	Yeah!
Claire:	And the other day too...
Teacher:	Yes, the other day you had flowers for the pattern and Eleanor you used numbers and letters to show that pattern, and today you all had blocks and recorded them using letters and numbers and drawings. So, it does not matter what are patterns are made of, we can represent them in the same way.

This vignette highlights how the teacher made clear connections between the context of the patterns and the similarities between the students’ representations. The teacher again used a closed question “are you telling me …we could record the same pattern in a different way?” to extend the explanation into a generalisation. Key to this teacher action was explicitly stating the generalisation, highlighting that patterns can be different (e.g. made of flowers or blocks) but represented in similar ways (e.g. using drawings, letters or numbers). Mulligan et al. (2020) highlighted the capability of kindergarteners demonstrating emergent generalisation through their replication of their pattern, however, in this study focused on teacher actions, e an example of how a teacher can highlight and model how to generalise to young students has been provided.

The following section will discuss how mathematical discourse is paramount to the enactment of mathematical practices.

4.5 Mathematical Discourse

Mathematics was a conversation with the teacher constantly providing opportunities for the students to share their thinking. While the teacher was the facilitator of the discussions, her questioning, prompts, and invitations to the students encouraged the students to generate the majority of talk within the lesson. Throughout the analysis of each practice, the use of talk moves; revoicing, repeating, and wait time have been discussed. Within these practices, the teacher also emphasised and supported the use of accurate mathematical language.

4.5.1 Modelling mathematical language

As expected with a group of learners starting school, there was variation in student capability with mathematical language. Throughout the seven observations, the teacher consistently modelled correct (formal) mathematical terminology to the students. For example, a student discussed the "pattern is happening again and again" and the teacher revoiced this to model mathematical language stating: "the pattern repeats, that is the unit of repeat in your pattern". Similarly, in another example, the teacher revoiced a student explanation using mathematical language:

Teacher revoicing students' mathematical language	
Liam:	Red, yellow, red, yellow
Teacher:	Are you saying Liam that your pattern is repeating over and over, red and yellow. So, the unit of repeat is red, yellow?
Liam:	Yeah *[nods head]*

By using the words, "are you saying", the teacher ensured that the student-maintained ownership of the mathematical idea while also adding another layer of meaning to their reasoning. This revoicing and use of mathematical language encouraged students to shift from using colloquial talk to engaging with and using specialised mathematical language, as appropriate to their age, in their explanations and justification. Evident in the data was how the teacher constantly revoiced or rephrased after a student shared their explanation or justification and extended the mathematical idea while modelling correct mathematical terminology. This is similar to the findings in the study by Cooke and Bulcholz (2005) where the teacher modelled and expected the use of accurate mathematical language.

Supporting students to become more proficient in using mathematical language allowed the students to engage in richer mathematical conversation. In the vignette below, Sophie and Raymond were using flowers to make a pattern, repeating four red and one yellow, however Sophie misplaced a flower in the second iteration of the pattern. Raymond identified this, and the following conversation occurred:

Maintaining student explanation	
Raymond:	Five, five, five *[pointing to the materials being used to make a pattern]*
Sophie:	*[pause]* Oh, we need five flowers in this unit of repeat?
Raymond:	Yes, yes

While Raymond was an English language learner, in this example, we can see how Sophie was able to model to Raymond the language to unpack his mathematical thinking. Evident is how Sophie could engage with formal mathematical language “unit of repeat” which the teacher had previously modelled. This links to a similar finding in Kazemi and Hintz’s (2014) study of teacher modelling to support reasoning. The expectation that these young students were to use precise language provided the space for students to support their peers in mathematical discourse.

4.5.2 Developing collaborative discourse

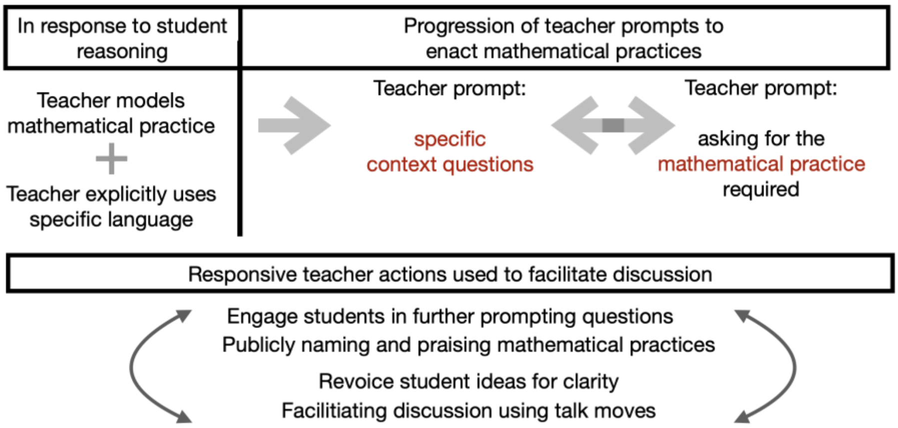
The teacher also established a consistent expectation that students would be asked to repeat key ideas. She placed a focus on this particular talk move to support the students to engage with and participate in small group discussions. The repeat talk move was used as both an individual and group expectation, for example, “Eleanor can you repeat that idea” or prompting the learners to “all together, repeat”. At times, individual students were not able to repeat, so the teacher directed other students to repeat but ensured that she returned to the initial student to ensure they had repeated the key mathematical language or idea.

4.6 Model of Teacher Actions

Throughout this chapter, the process for engaging the young learners in mathematical practices has been explained across three phases. Figure 12 is a model of the teacher actions to develop mathematical practices.

Figure 12

Model of Teacher Actions to Develop Mathematical Practices



The initial teacher prompt models the mathematical practice and expected mathematical language (see Section 4.2). By modelling both the practices and the expected language, the teacher supported the students to engage in the mathematical practices and use the correct terminology. Teacher prompts consisted of in-the-moment specific content questions which further supported the students (see Section 4.3). At times, (see Section 4.4) specific context questions were used interchangeably with open ended prompts to develop the use of a mathematical practice (as indicated by the double ended arrow in the diagram). Responsive teacher actions were used to facilitate and deepen mathematical discussions. These included the teacher asking further prompting questions, naming and praising the practice, revoicing student reasoning for clarity, and using talk moves to facilitate collaborative discourse. Notably, Jacobs and Empson (2016) emphasis responsive teaching as a model to notice student engagement in mathematical practices and then to extend student thinking. In the current study, the teacher responsively engaged in a variety of instructional actions that supported the engagement of the students in mathematical practices. The success of the mathematical practices would not have occurred without the use of the discussion moves.

4.7 Summary

This chapter has presented the findings in relation to how one teacher supported her students to engage with the mathematical practices of explanation, justification, argumentation, generalisation, and representation. The chapter mapped the teacher actions used over seven lessons and provided evidence that young students can be supported to enact mathematical practices in an age-appropriate way over a relatively short period. The chapter highlights the complex nature of the role of the teacher in being responsive in the moment and facilitating the classroom discussion by using talk moves such as revoicing, wait time, and repetition. It has described how the teacher plays a pivotal role in positioning young students to use mathematical language, and reason mathematically through modelling. Public praise was used by the teacher to promote group norms and begin to develop students' mathematical identity even during their first weeks of formal schooling. Finally, the chapter presented a model of the key teacher actions used throughout the seven observations. The next chapter will present a summary of the study, address the research questions and provide recommendations. Implications and limitations will also be discussed.

Chapter Five: Conclusion

5.1 Introduction

The previous chapter presented the findings and discussion in relation to how a teacher scaffolded her new young learners to engage in mathematical practices. This chapter concludes by reviewing the main findings in relation to the research question. A summary of the research questions is presented in Section 5.2 with key themes and recommendations presented in Section 5.3. The limitations of the study are addressed in Section 5.4 followed by suggestions for future research in Section 5.5

5.2 Summary of the Research Question

The focus of this study was to answer the following question: How does a teacher set up young learners to engage with mathematical practices during mathematics lessons?

An initial review of the research literature (e.g. Cobb et al., 2011; Hunter, 2008a; Melhuish et al., 2015; Whitenack & Yackel, 2002) provided an overview of studies that have previously shown how both young learners and older students engage in mathematical practices. This showed the differences between young learners' capability to engage in practices compared to older students. A further review of research literature (e.g. Henningsen & Stein, 1997; Hunter, 2008; Jacobs & Empson, 2016) on teacher actions to support students demonstrated the variety of prompts (such as, eliciting prompts, extending prompts) to enact mathematical practices in the classroom. Apparent in the literature is that there are limited studies that have investigated teacher actions in classrooms with students aged five years old.

This research study has demonstrated the complex nature of teaching. In particular, the teacher actions required to establish mathematical practices with young learners were not as straightforward and as simple as following a set of instructions, however, as demonstrated within the findings, it is achievable. Phase One demonstrated how the teacher extended students' responses to develop mathematical practices. Ongoing use of mathematical language and terms supported the students to enact important mathematical practices. Phase Two illustrated how the teacher provided the environment to facilitate students in mathematical argumentation and to reason with their peers' thinking. This was possible through the careful construction of social and socio-mathematical norms and maintaining high expectations that all students were capable of engaging in these practices. Phase Three continued to show the progression of the teacher actions to include specific prompts naming the mathematical practice expected. The transition in the student responses from reasoning with one or two-word responses to more sophisticated ways of constructing explanations or justifications emphasised how successful the teacher actions were. By the final phase of the study (Phase Three), students were representing their mathematical reasoning in multiple ways. This outcome highlights the impact of the initial modelling by the teacher.

Importantly, throughout all three phases of the study, the use of conversational moves were evident. Turn and talk teacher actions were used from the first mathematical lesson where the teacher constantly praised students for sharing their ideas. These deliberate teacher moves highlighted to the students the value of conversation and sharing of ideas in mathematics. Through the use of revoicing, the teacher was able to model accurate mathematical language. The use of the repeat talk move enabled students to engage with their peers' reasoning and engage with the mathematical content. Constant specific validation from the teacher was evident within all phases of the current study. This constant validation positioned the young learners to engage in mathematical opportunities. These teacher actions were summarised and presented in Figure 12.

5.3 Key Themes and Recommendations

This section presents a summary of the teacher actions supporting the students to develop mathematical practices and other key themes that were evident throughout the analysis. Recommendations developed from the study are also presented. The recommendations are pedagogical approaches that would support educators to provide opportunities for students to engage in mathematical practices, particularly within a classroom with young learners.

Model, facilitate, and enact mathematical practices from the beginning of mathematics teaching

Educators could provide constant opportunities to model and facilitate the use of practices to young learners including mathematical explanation, justification, and representation from the first lesson as the first step in the framework presented in Section 4.6. The Model of Teacher Actions (see Figure 11) outlines a potential cycle for educators to follow in enacting mathematical practices. It is important to note, that in order for students to respond to open prompts such as "Can you explain your thinking?", educators must name and praise mathematical practices used by the students prior to the open prompt.

Introduce mathematical language

Teachers could introduce young learners to formal mathematical language. This involves both noticing when to revoice a students' idea with the correct terminology and modelling appropriate mathematical language. The more students are exposed to mathematical language, the more they will engage with the language themselves. Consequently, educators should model and use correct mathematical terminology from the first lesson. As students begin to explain their reasoning and educators revoice using mathematical language, educators should then begin to expect their young learners to use mathematical language themselves.

Value mathematical discourse

A key part of learning mathematics is through mathematical discourse, and in order for discourse to be valued by young learners, it is important for educators to also value mathematical discourse. To demonstrate to students the value of discourse, educators should provide multiple opportunities for students to discuss their thinking. Once students have shared their ideas, educators should carefully notice and praise students for their mathematical reasoning. This supports classroom spaces to be set up as conversational spaces, where students are scaffolded to repeat ideas, add on, and reason with their peers thinking to create a shared communicative space.

5.4 Limitations of the Study

Despite the positive results shown in this study, this section acknowledges the limitations in the current study. The current study took place within one classroom with one teacher and one group of students. As a result, the generalisation of the findings for a different teacher or different cohort of students may vary. This means that the interpretation of the results can only provide a suggestion or an emerging understanding of the ways a teacher can set up young students to engage in mathematical practices.

5.5 Suggested Areas for Further Research

The current study investigated the teacher actions over seven lessons to set up mathematical practices and engage young students in these. A longitudinal study investigating further how these practices develop over time with young learners and how they are reflected in their mathematical learning is warranted. Additionally, it would be useful to have a study with a greater number of participants (teachers) to compare teacher actions across different settings. As new entrant students in Aotearoa enter school with varying mathematical experiences, monitoring different cohorts of students and the steps taken to set up mathematical practices is an important could be valid to support teachers.

Furthermore, a comparative study across classrooms with young learners of varying ages could support further interrogation of the teacher actions discussed in this study. Further research could also explore the impact of young learners engaging in mathematical practices and establish whether engaging in mathematical practices within earlier years of schooling impacts on mathematics results over time.

5.6 Final Thoughts

This study contributes to research in early years mathematics. Specifically, this study has focused on explicit teacher actions supporting young learners to engage in mathematical practices. These actions include teacher questioning and modelling, and the careful construction of a dialogic environment. While prior research in mathematical practices has focused on older students, this study highlights how young learners can develop mathematical practices upon entering school. Throughout this study, the students were responsive to the teacher's adaptive expertise resulting in the early development of mathematical explanation, representation, justification, argumentation, and generalisation. The teacher's use of dialogic pedagogies provided opportunities for students to engage in mathematical discussions. Of significance is evidence of the high expectations the teacher held for all students to learn mathematics. A further contribution of this study is the Framework developed (see Figure 11) capturing the ongoing and essential teacher actions required to support young learners to develop mathematical practices. Finally, to support students to develop mathematical practices, teachers must provide multiple opportunities for students to learn to know and use these.

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Appendices

Appendix A: Communication and Participation Framework

		Phase One	Phase Two	Phase Three
Communicative actions	Making conceptual explanations	Use problem context to make explanation experientially real.	Provide alternative ways to explain solution strategies.	Revise, extend, or elaborate on sections of explanations.
	Making explanatory justifications	Indicate agreement or disagreement with an explanation.	Provide mathematical reasons for agreeing or disagreeing with solution strategy. Justify using other explanations.	Validate reasoning using own means. Resolve disagreement by discussing viability of different solution strategies.
	Making generalisations	Look for patterns and connections. Compare and contrast own reasoning with that used by others.	Make comparisons and explain the differences and similarities between solution strategies. Explain number properties, relationships.	Analyse and make comparisons between explanations that are different, efficient, sophisticated. Provide further examples of number patterns, number relations, and number properties.
	Using representations and inscriptions	Discuss and use a range of representations or inscriptions to support an explanation	Describe inscriptions used to explain and justify conceptually as actions on quantities, not manipulation of symbols.	Interpret inscriptions used by others and contrast with own. Translate across representations to clarify and justify reasoning.
	Using mathematical language and definitions.	Use mathematical words to describe actions.	Use correct mathematical terms. Ask questions to clarify terms and actions.	Use mathematical words to describe actions (strategies). Reword or re-explain mathematical terms and solution strategies. Use other examples to illustrate.
Participatory actions		Active listening and questioning for more information. Collaborative support and responsibility for reasoning of all group members. Discuss, interpret, and reinterpret problems. Agree on the construction of one solution strategy that all members can explain. Indicate need to question during large-group sharing. Use questions that clarify specific sections of explanations or gain more information about an explanation.	Prepare a group explanation and justification collaboratively. Prepare ways to re-explain or justify the selected group explanation. Provide support for group members when explaining and justifying to the large group or when responding to questions and challenges. Use wait time as a think time before answering or asking questions. Indicate need to question and challenge. Use questions that challenge an explanation mathematically and draw justification. Ask clarifying questions if representation and inscriptions or mathematical terms are not clear.	Indicate need to question during and after explanations. Ask a range of questions including those that draw justification and generalised models of problem situations, number patterns, and properties. Work together collaboratively in small groups, examining and exploring all group members' reasoning. Compare and contrast and select most proficient (that all members can understand, explain, and justify).

(Hunter, 2008b, p. 84)

Appendix B1: Coding

<table>
<tr><th colspan="2">CODING</th></tr>
<tr><th colspan="2">PARENT CODE: EXPLANATION</th></tr>
<tr>
<td>CHILD NODES: whole class

Teacher invitation to make an explanation

Who can explain?
What do you notice?
Specific question in relation to context

Wait Time
Teacher Public Praise of..

Sharing reasoning with a buddy
Student use of mathematical ideas

Teacher invites student to share explanation
Student(s) respond by sharing mathematical thinking/process

First we did... (process)
One word answers

Teacher prompts with further questions

Teacher affirms knowledge and rephrases student explanation with mathematical language

Ensures student still holds ownership of mathematical ideas

Teacher invites students to repeat mathematical idea
Explicit Praise of

Mathematical language
Mathematical practice

Potential missed opportunity to name practice

</td>
<td>CHILD NODES: When students are solving problem in peers

Teacher publicly praises students :

for solving task naming the mathematical language (scaffolding future explanations).

Students respond by solving similarly using the mathematical language
Teacher invites student to explain their thinking with their group or teacher makes a statement
Student responds
Teacher supports by revoicing “are you saying that...)

</td>
</tr>
<tr><th colspan="2">PARENT CODE : JUSTIFICATION</th></tr>
<tr>
<td>CHILD NODES: whole class

Teacher invitation to make an justification

Who can justify?
How do you know?
What do you think
Because?
Why do you think....?
Tell your buddy why

Student responds with justification

To their peers
To the whole group

Teacher Public praise of...

Sharing reasoning to a buddy
Students using the word because
Student use of Mathematical ideas

Teacher Invite students to share justification back to the whole class

Through questioning.. Why...?

Student responds with their thinking
Repetition of student justification (either)

Teacher invites other students to repeat

</td>
<td>CHILD NODES: when students are solving problem in peers)

Teacher praises mathematical idea of a small group
Teacher invites student to share their thinking with their buddy

Tell them why...
How do you know

Student responds

Teacher supports by revoicing “are you saying that...
</td>
</tr>
</table>

<table>
<tr><td>o Teacher revoices with accurate mathematical language
• Teacher questions other students for mathematical understanding
• Teacher praises students use of justification [explicit]</td><td></td></tr>
<tr><td colspan="2">PARENT CODE : ARGUEMENTATION</td></tr>
<tr><td>CHILD NODES in response to students working with their buddies:
• Teacher invites students to share their idea with a buddy
• Student respond by sharing idea with a peer.
• Teacher explicitly and publicly labels discussion
o "Student has discussed and agreed on their idea"</td><td rowspan="2">CHILD NODES whole class
• Teacher invites students to reason with an idea
o What do we think do we agree or disagree with that idea
o Put your hands on your head if you agree
• Teacher reassures students that it is okay to disagree with the idea
• Teacher ask student what they think
• Student responds
o Agree
o Disagree
• Teacher invites student to justify why <u>or</u> teacher explains with correct mathematical language.
• Teacher affirms student knowledge
• Teacher invites other students to repeat mathematical idea</td></tr>
<tr><td>CHILD NODES when students are working with peers:
• Teacher invites student to reason with their buddies thinking
o Name do you agree with students thinking?
• Student responds
• Teacher prompts student to explain why to their buddy</td></tr>
<tr><td colspan="2">PARENT CODE : GENERALISATION</td></tr>
<tr><td colspan="2">CHILD NODES whole class
• Teacher verbally generalises
o representation [publicly to the whole class]
o mathematical idea [publicly to the whole class]</td></tr>
<tr><td colspan="2">PARENT CODE : REPRESENTATION</td></tr>
<tr><td rowspan="2">CHILD NODES whole class
• Teacher invites students to represent their thinking
• Individually
• In groups
• Verbally
• Students respond by representing their mathematical ideas.
o With materials
o Drawing in their books
o Using numbers
o Representing on their fingers
• Teacher publicly praises students use of representation
• Teacher invites student to share mathematical representation
• Students share representation
• Teacher models representation on the board
• Teacher adds onto students shared representation and models use of..
o mathematical equation
o numbers
• Teacher praises representation</td><td>• Teacher responds to student mathematical explanation/justification by representing idea (publicly).
o Using materials
o Drawing
o Representing numbers on the board
o Writing mathematical sentence on the board</td></tr>
<tr><td>• Teacher revoices mathematical idea
• Teacher asks student to repeat/read representation
• Teacher praises
o Mathematical thinking</td></tr>
</table>

Appendix B2: Example of Coding

Teacher [to the whole class]: oh I can see Eleanor and Raymond were talking about some great maths ideas, Eleanor what do you think?
 	
Eleanor: blue?
	• Teacher invitation to make an justification
	    ○ What do you think
	• Student verbal responds with justification
	    ○ To the whole class

Teacher: could you explain to us why you think its blue?
 	
Eleanor: because it goes pink pink pink pink, blue blue blue blue [misconception].
	• Teacher invitation to make an justification
	    ○ Why do you think
	• Student verbal responds with justification
	    ○ To the whole class

Teacher: oh wow I love you thinking Eleanor she did some great justifying with the word because. She thought four pink, and then four blue, because there are four pink and then one blue but you need four blue, interesting thinking, what do we think do we agree or disagree with that idea? Remember it is okay to disagree we still like Eleanor.
Sophie what do you think?
 	
Sophie: agree,
	• Teacher Public praise of...
	    ○ Students using the word because
	
	• Teacher invites students to reason with an idea
	• Teacher reassures students that it is okay to disagree with the idea
	• Teacher ask student what they think
	    ○ Student responds
	        ○ Agree

Teacher: Sophie why do you agree? Sophie: because I think It would be nice to have a balance of colours.	• Teacher invites student to justify why • Student responds with a justification ○ To the whole class

Appendix C: Teacher Questionnaire

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Supporting 5 – 6 Year Old Students To Know And Use Mathematical Practices

Teacher Questionnaire

- How do you group your students?
- How do you set up the group norms or expectations for your lessons as the teacher?
- How do you ensure all students have access to the maths? For example, brand new students that perhaps cannot identify all numbers?
- What do you consider when planning a lesson to allow students the opportunity to use mathematical practices?
- How do you support the development of representation?
- Do your teacher actions (the way you question or scaffold students) change over time as the students have been at school longer?
- When you get new students start, how do the older students support younger students use of mathematical practices?
- Your students are quite comfortable to talk with their peers, how do you set up the students to share their thinking?

Appendix D1: Principal information sheet

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Supporting 5 – 6 Year Old Students To Know And Use Mathematical Practices

INFORMATION SHEET

Dear Principal
My name is Emily Pearce and I am a Masters of Mathematics Education student at Massey University.

My masters study is titled *Supporting 5- 6 Year Old Students to know and use mathematical practices* and will be conducted within the context of a primary school mathematics classroom. The main purpose of this study is to explore specifically the teacher actions that are used the scaffold or support young students using mathematical practices. The practices of justification, explanations, generalisations, and representations are skills that deepen students mathematical understanding.

We would like to invite you to give permission for your school and new entrant teacher to be involved in this study. The duration of this study will be over one school term. During this project, seven mathematics lessons, during the normal classroom schedule, will be observed and filmed by me. The teacher, the students and their parents/caregivers will be given full information and consent will be requested. Specifically, permission to allow the students to be filmed will be sought from both the parents of the students. There will be one questionnaire with the teacher and the time involved for the teacher will be no more than 2 hours over the seven sessions. Work samples from each lesson may also be collected and photo-copied and I may take photographs of the students' representations that involve mathematics materials.
The time involved in the complete study for the teacher will be no more than 9 hours over the school term. There is no expectation that the usual classroom programme will be disrupted in any way.
All project data collected during individual interviews and filming will be stored in a secure location, with no public access and used only for this research and any publication arising from this research. After completion of five years, all data pertaining to this study will be destroyed in a secure manner. All efforts will be taken to maximize confidentiality and anonymity for participants. Names of all participants and the school will not be used once information has been gathered and only pseudonyms and non-identifying information will be used in reporting. The teacher participation in this study will not affect the teachers role or their appraisal in anyway.

Please note that the Board of Trustees is under no obligation to accept this invitation. If you decide to participate you have the right to:

- Decline to answer any particular question;
- Withdraw from the study at any time;
- Ask any questions about the study at any time during participation;
- Provide any information on the understanding that your name will not be used unless you give permission to the researcher
- Be given access to a summary of the project findings when it is concluded.

If you have any further questions about this project, you are welcome to discuss them with me personally:
Emily Pearce. Email. E.Pearce1@massey.ac.nz
Or contact my supervisors at Massey University

- Professor Jodie Hunter (09) 414 0800 ext. 43518. Email. J.Hunter1@massey.ac.nz Institute of Education, Private Bag 102 904, North Shore, Auckland 0745
- Dr Generosa Leach (09) 414 0800 ext. 49005. Email. G.Leach@massey.ac.nz Institute of Education, Private Bag 102 904, North Shore, Auckland 0745

This project has been reviewed and approved by the Massey University Human Ethics Ohu Matatika 3, Application OM3 23/24. If you have any concerns about the conduct of this research, please contact the Chairperson, Massey University Human Ethics Ohu Matatika 3, email humanethics3@massey.ac.nz.

Appendix D2: Board of Trustees information sheet

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Supporting 5- 6 Year Old Students to know and use mathematical practices

INFORMATION SHEET

Dear Board of Trustees

My name is Emily Pearce and I am a Masters of Mathematics Education student at Massey University.

My masters study is titled *Supporting 5- 6 Year Old Students to know and use mathematical practices* and will be conducted within the context of a primary school mathematics classroom. The main purpose of this study is to explore specifically the teacher actions that are used the scaffold or support young students using mathematical practices. The practices of justification, explanations, generalisations, and representations are skills that deepen students mathematical understanding.

We would like to invite you to give permission for your school and new entrant teacher to be involved in this study. The duration of this study will be over one school term. During this project, seven mathematics lessons, during the normal classroom schedule, will be observed and filmed by me. The teacher, the students and their parents/caregivers will be given full information and consent will be requested. Specifically, permission to allow the students to be filmed will be sought from both the parents of the students. There will be one questionnaire with the teacher and the time involved for the teacher will be no more than 2 hours over the seven sessions. Work samples from each lesson may also be collected and photo-copied and I may take photographs of the students' representations that involve mathematics materials.

The time involved in the complete study for the teacher will be no more than 9 hours over the school term. There is no expectation that the usual classroom programme will be disrupted in any way.

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Please note that the Board of Trustees is under no obligation to accept this invitation. If you decide to participate you have the right to:

- Decline to answer any particular question;

- Withdraw from the study at any time;
- Ask any questions about the study at any time during participation;
- Provide any information on the understanding that your name will not be used unless you give permission to the researcher
- Be given access to a summary of the project findings when it is concluded.

If you have any further questions about this project, you are welcome to discuss them with me personally:
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Appendix D3: Teacher Participant information sheet

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Supporting 5- 6 Year Old Students to know and use mathematical practices

INFORMATION SHEET

Dear

My name is Emily Pearce and I am a Masters of Mathematics Education student at Massey University.

My masters study is titled *Supporting 5- 6 Year Old Students to know and use mathematical practices* and will be conducted within the context of a primary school mathematics classroom. The main purpose of this study is to explore specifically the teacher actions that are used the scaffold or support young students using mathematical practices. The practices of justification, explanations, generalisations, and representations are skills that deepen students mathematical understanding.

I am formally inviting you to be a part of this research as I examine the specific teacher actions that provide students the opportunities to develop their mathematical practices. Your role in this project will be as the mathematics teacher of the student participants.

Permission to participate in the study will be sought from the parents/caregivers of the students in your class. The students and their parents/caregivers will be given full information and consent will be requested in due course.

I will also have a written questionnaire for you to answer. The time involved for the questionnaire will be no more than 2 hours in total.

The duration of this project will be over one school term. During this project, seven (none consecutive) mathematics lessons will be filmed. Work samples from each lesson may also be collected and photo-copied or student representations (using materials) may be photographed. The observations will take place in the classroom and be part of your normal mathematics programme to minimise disruption.

The time involved in the complete study for you will be no more than 9 hours over the whole term.

All project data collected during individual interviews and filming will be stored in a secure location, with no public access and used only for this research and any publication arising from this research. After completion of five years, all data pertaining to this study will be destroyed in a secure manner. All efforts will be taken to maximize confidentiality and anonymity for participants. Names of all participants and the school will not be used once information has been gathered and only pseudonyms and non-identifying information will be used in reporting. Participation in this study will not affect your employment and videos or evidence will not be shared with the Principal.

Please note that you are under no obligation to accept this invitation. If you decide to participate you have the right to:

- Decline to answer any particular question;
- Withdraw from the study at any time;
- Ask any questions about the study at any time during participation;
- Provide any information on the understanding that your name will not be used unless you give permission to the researcher
- Be given access to a summary of the project findings when it is concluded.

If you have any further questions about this project, you are welcome to discuss them with me personally:
Emily Pearce. Email. E.Pearce1@massey.ac.nz
Or contact my supervisors at Massey University

- Professor Jodie Hunter (09) 414 0800 ext. 43518. Email. J.Hunter1@massey.ac.nz
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Appendix D4: Parents of Participant information sheet

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MĀTAURANGA

Supporting 5- 6 Year Old Students to know and use mathematical practices
STUDENT/PARENT/CAREGIVER INFORMATION SHEET

Dear
My name is Emily Pearce and I am a Masters of Mathematics Education student at Massey University.
My masters study is titled *Supporting 5- 6 Year Old Students to know and use mathematical practices* and will be conducted within the context of a primary school mathematics classroom. The main purpose of this study is to explore specifically the teacher actions that are used the scaffold or support young students using mathematical practices. The practices of justification, explanations, generalisations, and representations are skills that deepen students' mathematical understanding.

We would like to invite you to provide permission for your child to participate in this project. For the research study, we plan to observe and video-record seven mathematics (none consecutive) lessons in your child's classroom over the duration of one school term. I may also collect work samples and take photographs of mathematical representations with materials. Your child's involvement for the mathematics lessons will be no more than that which occurs in normal daily mathematics lesson. If you do not wish for your child to be video-recorded, they will be seated in an area of the classroom that will not be visible on the video-recording and will be working independently in conjunction with the current classroom timetable. There will be no interruptions to the classroom timetable for this research to occur.

All data collected for the purpose of the study will be stored in a secure location, with no public access and used only for this research. After completion of five years, all data pertaining to this study will be destroyed in a secure manner. In order to maintain anonymity the school name will be assigned a pseudonym in any publications arising from this research. All efforts will be taken to maximize confidentiality and anonymity for participants. Names of all participants and the school will not be used once information has been gathered and only pseudonyms and non-identifying information will be used in reporting. Child's names and faces (from videos) will not be used in any form. Once the video is collected it will be run through an anonymise function on the recording software.

Please note that you and your child have the following rights in response to the request to participate in this study:
• decline to participate;
• withdraw from the study until the data analysis phase;

• ask any questions about the study at any time during participation;
• provide information on the understanding that your child's name will not be used;
• be given access to a summary of the project findings when it is concluded.

If you have any further questions about this project, you are welcome to discuss them with me personally:
Emily Pearce. Email. E.Pearce1@massey.ac.nz
Or contact my supervisors at Massey University

- Professor Jodie Hunter (09) 414 0800 ext. 43518. Email. J.Hunter1@massey.ac.nz
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This project has been reviewed and approved by the Massey University Human Ethics Ohu Matatika 3, Application OM3 23/24. If you have any concerns about the conduct of this research, please contact the Chairperson, Massey University Human Ethics Ohu Matatika 3, email humanethics3@massey.ac.nz.

Appendix E1: Principal consent form

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Supporting 5 – 6 Year Old Students To Know And Use Mathematical Practices

CONSENT FORM: PRINCIPAL

THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

I have read the Information Sheet and have had details of the study explained. Our questions have been answered to our satisfaction, and we understand that we may ask further questions at any time.

We agree to ___________________(school) and __________________________(teacher) participating in this study under the conditions set out in the Information Sheet.

Signature: ____ Date: ___________

Full Name – printed ____

Appendix E2: Board of Trustees consent form

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Supporting 5 – 6 Year Old Students To Know And Use Mathematical Practices

CONSENT FORM: BOARD OF TRUSTEES

THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

We have read the Information Sheet and have had details of the study explained. Our questions have been answered to our satisfaction, and we understand that we may ask further questions at any time.

We agree to ____________________(school) and ____________________________(teacher) participating in this study under the conditions set out in the Information Sheet.

Signature: ____ Date: ____________

Full Name – printed ____

Appendix E3: Teacher participant consent form

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Supporting 5 – 6 Year Old Students To Know And Use Mathematical Practices

CONSENT FORM: TEACHER PARTICIPANT

THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

I have read the Information Sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I agree to participate in this study under the conditions set out in the Information Sheet.

Signature: ____ **Date:** ___________

Full Name – printed ____

Appendix E4: Parents of student participant consent form

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MĀTAURANGA

Supporting 5 – 6 Year Old Students To Know And Use Mathematical Practices

PARTICIPANT CONSENT FORM – PARENT/CAREGIVER/CHILD
THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

I have read the Information Sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I agree/do not agree to my child being video-taped during mathematics lessons

I agree/do not agree to my child's assessment results being used in this study.

I agree/do not agree for my child/children to participate in this study under the conditions set out in the Information Sheet.

Child full name (printed) ____

Teacher name ______________________________ **Room**________________________

Parents/Caregivers full name (printed) ____

Parent/Caregivers signature _______________________________ **Date** ____________