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Mathematical Modelling of Induced Resistance to Plant Disease

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Abstract

The underlying theory of induced resistance (IR) is concerned with the situation when there is an increase in plant resistance to herbivore or pathogen attack that results from a plant's response triggered by an agent such as elicitors (also known as "plant activators"). This mechanism has been well studied in plant pathology literature. In this thesis, a mathematical model of induced resistance mechanism using elicitors is proposed and analysed. An adaptation of traditional Susceptible-Infected-Removed (SIR) model, this proposed model is characterised by three main compartments, namely: susceptible, resistant and diseased. Under appropriate environmental conditions, susceptible plants (S) may become diseased (D) when it is exposed to a compatible pathogen or able to resist the infection (R) via basal host defence mechanisms. The application of an elicitor enables the signal activation of plant defence genes to enhance the basal defence responses and thereby affecting the relative proportion of plants in each of the S , R and D compartments. In literature, induced resistance is described as a transient response and this scenario is modelled using reversible processes to describe the temporal evolution of the compartments. The terms in the equations introduce parameters which are determined by fitting the model to matching experimental data sets using MATLAB "fminsearch". This then gives the model's outcome to predict the relative proportion of plants in each compartment and quantitatively estimates the elicitor effectiveness. Extensions of the model are developed, which includes some factors that affect the performance of IR such as elicitor concentration and multiple elicitor applications. This IR model is also extended to include a scenario of post-pathogen inoculation for elicitor treatment. Finally, an application of optimal control theory is derived to determine the best strategy for a continuous elicitor application. Numerical evaluations of this IR model provide a potential support tool for the development of more potent elicitors and its application strategies. The model is generic and will be applicable to a range of plant-pathogen-elicitor scenarios.

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Abbreviations

IR	Induced R esistance
SIR	Susceptible-Infected- R emoved
HR	Hypersensitive R esponse
BCA	Biological C ontrol A gent
SAR	Systemic A cquired R esistance
ISR	Induced S ystemic R esistance
PAL	Phenylalanine A mmonia L yase
MeJA	Methyl J asmonate
SSE	Sum of S quares of E rror

Notations

R	Proportion of plant population able to express resistance to infection.	dimensionless
D	Proportion of plant population being infected and becoming diseased.	dimensionless
S	Proportion of plant population which is susceptible.	dimensionless
t	Time	[days]
α	The specific rate at which untreated plants lose their resistance due to the pathogen attack.	[days ⁻¹]
β	The specific rate at which the disease spreads.	[days ⁻¹]
γ	The specific rate the resistant plant becomes susceptible.	[days ⁻¹]
$e(t)$	The effectiveness of the elicitor at a single application.	[days ⁻¹]
k	Determines the effectiveness of the elicitor.	dimensionless
L	The time where the elicitor effectiveness is at the peak.	[days]
t_p	The induction time of the pathogen i.e. the time interval between the elicitor application and the pathogen challenge.	[days]
R_i	The proportion of the plant population that exhibits natural resistance at the initial time $t = 0$.	dimensionless
D_i	The proportion of the plant population which becomes infected immediately after the pathogen challenge.	dimensionless
a	The scaled dimensionless elicitor concentration.	dimensionless
r	The parameter determines the sub-linear effect of elicitor concentration.	dimensionless

$E(t)$	The cumulative effectiveness of the elicitor at daily application.	[days]
$c(t)$	The continuous elicitor application.	[mass days ⁻¹]