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**A Test of Historical and Shrinkage Estimates of Expected
Returns in International Portfolio Selection.**

**A THESIS PRESENTED IN PARTIAL FULFILMENT OF THE
REQUIREMENTS FOR THE DEGREE OF MASTER OF BUSINESS STUDIES
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Abstract

A number of researchers have chosen internationally diversified portfolios using the Mean-Variance approach to portfolio selection. Typically, the estimates of expected returns, variances and covariances are taken from historical data. Recently this approach has come under criticism due to the poor performance of these portfolios out of the sample period. A suggested improvement is to use "shrinkage" estimators to improve the estimates, particularly for expected returns. This statistical adjustment leads to less emphasis being placed on increasing expected return and more on risk reduction.

The researchers to test shrinkage estimates internationally have had conflicting results, possibly due to the methodology used. Jorion (1985) found support for shrinkage estimators outperforming historical estimates, with short sales unconstrained. A single period model with a five year sample was used. Grauer and Hakansson use a multi-period model, with short sales restricted. The sample period is eight years in this instance, and the opposite result is obtained.

This study tests both types of mean estimate in a single period model, with short sales restricted. The difference in out of sample performance is insignificant with both four and eight year samples. Additionally, a naive strategy of weighting the portfolio equally between countries, thereby ignoring the historical data, outperforms the other methods. Thus, the use of four year sample periods appears to be of no use.

With the eight year sample the performance of all methods is remarkably similar, with a portfolio chosen to minimise variance having the best performance, although only slightly.

The use of historical data, whether or not shrinkage estimates are used, has proved to be of very little benefit in this study.

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CHAPTER 1

Introduction

It has been common to use the Mean-Variance approach for portfolio selection, as developed by Markowitz (1952)¹, to select internationally diversified portfolios. Wide diversification among securities of low correlation, as pointed to by the Mean-Variance approach, provides a logical impetus to consider international markets. Lower correlations typically occur between markets, when compared to correlations of the securities within the markets. International portfolios provide the widest possibilities for diversification, and this combined with lower correlations should lead to superior portfolios.

Early work in this area concentrated on using risk and return parameters estimated from historical data². These estimates were used to derive optimal portfolios, with little consideration of the effect of errors in the estimates. This approach has come under much criticism in recent times, for several reasons. The most significant criticism is the poor *ex-post* performance of these portfolios. Jorion (1986) finds that naive strategies, such as an equally weighted portfolio, often outperform the portfolio selected. It follows that errors in the estimates based on historical data have resulted in inferior performance.

Of greatest importance in improving the performance of portfolios selected by the Mean-Variance approach is the accurate estimation of expected returns. This is due to the large effect on portfolio composition of small variations in expected return, in comparison to the more moderate effects of variance and covariance³. Unfortunately, expected returns have been the more difficult parameter to estimate as well.

¹ Markowitz, Harry M., "Portfolio Selection. " Journal of Finance, (March, 1952).

² Grubel, H. G., "Internationally Diversified Portfolios: Welfare Gains and Capital Flows.", American Economic Review , (December, 1968).

Levy, H. and Sarnat, M., "International Diversification of Investment Portfolios.", American Economic Review (September , 1970).

³ Chopra, Vijay K., and Ziemba, William T., "The Effect of Errors in Means, Variances and Covariances on Optimal Portfolio Choice", The Journal of Portfolio Management, (Winter, 1993).

The need to improve expected return estimates has inspired the use of shrinkage estimates⁴. Shrinkage estimates adjust the sample means, in this case expected returns, towards a common value. The effect of this, in a portfolio context, is to reduce the likelihood that an optimal portfolio will largely consist of securities with high sample expected returns estimated with considerable error. Additionally, a security that has performed poorly in the past is less likely to be rejected as a contender for the portfolio. Portfolios selected by this method are thus more diversified, with more emphasis put on reduction of portfolio risk than on increased portfolio return.

The purpose of this thesis is to compare historical and shrinkage estimates of expected returns when choosing optimal portfolios. The performance of these portfolios is compared with two alternative strategies, these being minimum variance portfolios, based on historical estimates of variance and covariance, and equally weighted portfolios.

This topic has been considered by other researchers with mixed success when international markets are considered. Reasons for the differing results may include the methodology and data used. This report considers an intermediate methodology to those giving the conflicting results, and a different database. This will provide further insight into the usefulness of the Mean-Variance approach and historical or shrinkage estimates of expected returns.

⁴ Jorion, P., "International Portfolio Diversification With Estimation Risk.", *Journal of Business*, (No. 3, 1985).

CHAPTER 2

Literature Review

2.1 INTRODUCTION.

The methods used in this research draw from various areas of financial economics and statistics. The following articles have considered the Mean-Variance criterion, selection of efficient and optimal portfolios and the extension of the theory to international portfolio selection. Criticism of previous applications of the model and the reasons for the poor performance of the Mean-Variance approach follows. The application of shrinkage estimates, a statistical technique, to portfolio selection provides the final area of research to be considered. These three areas are looked at in turn.

2.2 THE MEAN-VARIANCE APPROACH TO PORTFOLIO SELECTION.

*Portfolio Selection (Markowitz, 1952)*¹.

The Mean-Variance approach to portfolio selection was introduced by Markowitz in 1952. Markowitz defines risk as the variance of expected return, thus enabling risk to be quantified. Furthermore, efficient portfolios are defined as those portfolios of investment assets that for any given level of expected return have minimum variance, or alternatively, for any given level of variance have maximum return. By the Mean-Variance rule, investors who consider return favourably and risk unfavourably, would select a portfolio from within the set of efficient portfolios according to their own particular degree of risk aversion.

Maximising expected return is discounted as an investment rule as it never implies diversification. Using this rule an investor would merely invest all funds in a single investment, irrespective of risk. However, diversification is a commonly observed investment behaviour, in an attempt to reduce risk.

The Mean-Variance approach has appeal for explaining investment behaviour, and also as a normative guide to investment. It will typically imply diversification as a risk reduction technique for investors who are risk averse. Thus, it provides an explanation for the observed behaviour of investors. Additionally it implies diversification among securities with low correlations to each other. Diversification among assets that are only weakly correlated provides a higher degree of risk reduction than would be possible if

¹ *op. cit.*, Markowitz (1952).

correlations were high. However, as security returns tend to be positively correlated, risk cannot be eliminated completely by diversification and the law of large numbers does not apply.

Stochastic Linear Programming: A Study in Resource Allocation Under Risk. (van Moeseke, 1965)²

Van Moeseke's paper builds on the Mean-Variance approach. Efficient portfolios can be constructed with negative asset holdings (short-sales) allowed or disallowed. Markowitz provided quadratic programming as a method of finding efficient short sale restricted portfolios³. Van Moeseke introduced an alternative approach to finding Mean-Variance efficient portfolios when short sales are restricted. The truncated minimax criterion is proposed as follows:

$$\max: \phi f(x) = Ef(x) - m\sigma f(x) \quad (1)$$

where: x = an output or, in the portfolio selection instance, a security or portfolio of securities.

$Ef(x)$ = expected return of $f(x)$

$\sigma f(x)$ = standard deviation of return $f(x)$

m = a risk preference term

The risk preference term, m , may be interpreted in two ways. First, it can be considered as giving a relative weighting to risk and return of m and 1 respectively⁴. Alternatively, given the assumption of a normal distribution of returns, a confidence interval interpretation can be made. Van Moeseke gives an example that with $m = 1.65$, a .05 lower confidence limit is found⁵.

Homogeneous programming makes use of the truncated minimax criterion to allocate resources under risk. In the particular case of portfolio investment, it allows the selection of efficient portfolios with short sales restricted. The entire range of efficient

² Moeseke, Paul v. "Stochastic Linear Programming: A Study in Resource Allocation Under Risk.", *Yale Economic Essays*, 5, (1965)

³ Markowitz, Harry M., *Portfolio Selection*, (1959), Wiley, New York.

Due to the constraint on short sales, the efficient portfolios chosen with short sale restrictions will typically have inferior Mean-Variance performance to those chosen without this restriction.

⁴ The substitution of standard deviation for variance does not effect the composition of portfolios on the efficient frontier. It does however enable the confidence interval interpretation to be made.

⁵ *op. cit.* Moeseke (1965), p. 209.

portfolios can be found by varying m ⁶. An algorithm for solving the homogeneous programme is given by van Moeseke, and will be used in this study.

The duality theorem of homogeneous programming provides for an optimal solution to the portfolio selection problem. By the duality theorem, a dual solution to the programme is found. Furthermore, an optimal solution is found where the primal, $\phi(x)$, equals the dual solution, λ . It follows from the primal solution that at this point the investor is indifferent between the portfolio of risky assets and a risk free investment. By the dual solution, the value of risk free assets is also maximised, thus the optimal solution ensures the investment budget is invested with maximum caution⁷.

In practice, the duality theorem allows an optimal solution to be found where $\phi(x)$ equals the rate of interest on government bonds, by the now common assumption that government securities approximate a risk free investment.

Homogeneous programming thus overcomes a limitation of the original Mean-Variance approach by allowing an optimal solution to be found rather than requiring the portfolio choice to be made particular to each investor. This definition of optimality coincides with the more frequently quoted definition of Lintner (1965)⁸.

International Portfolio Selection.

The decrease in portfolio variance enabled by low correlation between investment assets led to the consideration of internationally diversified portfolios. Levy and Sarnat (1970)⁹ argue that there is a strong tendency for stock returns within a country to move together in unison with general economic activity. They also suggest that internationally diversified portfolios may overcome the problem of high correlation of stocks within an economy. The Mean-Variance approach is used, with stock indices and exchange rates from various countries providing the estimates of returns and the covariance matrix. It is concluded that potential gains from international diversification are significant, especially when developing or emerging countries' markets are included. The low correlation of these developing countries to other countries makes them particularly useful in reducing portfolio variance.

⁶ Note that in the extreme risk averse case, as $m \rightarrow \infty$, the homogeneous programme is undefined.

⁷ Young, Martin., "Portfolio Selection by Homogeneous Programming.", Unpublished Thesis, Massey University, (1985), p. 4.

⁸ Lintner, J., "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets.", Review of Economics and Statistics, (February, 1965).

⁹ *op. cit.*, Levy and Sarnat (1970). p. 668.

2.3 CRITICISMS OF THE USE OF THE MEAN-VARIANCE MODEL.

Major criticisms of the use of the Mean-Variance model for selecting portfolios stem from problems in estimating the inputs to the model, namely expected return, variance and covariance of securities. It was typical of early research into portfolio selection to use estimates based on historical data to select portfolios. The value of these estimates depends on the stability of returns and correlations over time. Any error in the estimates results in sub-optimal portfolio performance. The relative effect of these errors also provides some guidance as to where improvements can be made.

The Stability of International Correlations.

The use of estimates of correlation based on past data depends on the necessary condition, that correlations are less than one, and the sufficient condition that correlations are stable over time. Clearly, the second condition provides the more difficult barrier to successful portfolio selection. In an extensive review Philippatos et. al. (1983) consider this problem¹⁰. While many previous researchers had found support for stability in international correlations, there was some conflict. Philippatos et. al. address the conflict and come to the following conclusion:

"Having employed a wide variety of standard statistical methods on a large sample of industrialised countries we must agree with the inferences drawn in earlier studies... There indeed exist both structure and stability of such structure in the inter-temporal relationships among the national stock market indices of the industrialised world."¹¹

The stability is found in consecutive medium term sub-periods of four to five years, over the period 1959-1978. Subsequent to this, Meric and Meric (1989) consider the more recent period of 1973-1987¹². For sub-periods up to three years there is a significant difference in at least one half of the consecutive periods. However, for periods of five years and more, the hypothesis that the correlation matrices are the same cannot be rejected.

¹⁰ Philippatos, G.C., Christofi, A., and Christofi, P., "The Inter-Temporal Stability of International Stock Market Relationships: Another View.", Financial Management, (Winter, 1983).

¹¹ *Ibid.*, p.69

¹²Meric, Ilhan, and Meric, Gulser, "Potential Gains From International Portfolio Diversification and Inter-Temporal Stability and Seasonality in International Stock Market Relationships.", Journal of Banking and Finance, (1989), pp. 627-640.

The Stability of Expected Returns on Stock Markets.

The use of expected returns based on historical data has also come under question. Using monthly stock market returns, Jorion (1985) tested the predictive ability of both one and five year averages¹³. A least squares regression was performed to test past averages as a predictor of future performance. The coefficient of determination, R^2 , is very low for both one and five year averages, indicating poor predictive power¹⁴. Additionally, the mean return of the series is very small relative to the standard deviation. Therefore, it is to be expected that wide fluctuations in sample means will occur, and negative returns will occur for some periods. Clearly this indicates that the Mean-Variance model will suffer by using historical estimates of the means. The importance of this is outlined in the next section.

The Effect of Errors in Means, Variances and Covariances on Optimal Portfolio Choice (Chopra and Ziemba, 1993)¹⁵.

This paper is concerned with the relative effect of errors in the estimates, rather than the usefulness of historical data. The findings indicate that errors in the mean estimates have the most significant effect on portfolio optimality. Errors in variance are the next most important, followed by errors in the covariance estimates. With a typical risk tolerance, they find errors in means are around eleven times as important as errors in variance and greater than twenty times as important as errors in covariances. Thus, they conclude:

"The primary emphasis should be on obtaining superior estimates of means, followed by good estimates of variances. Estimates of covariances are the least important in terms of their influence on the optimal portfolio."¹⁶

2.4 SHRINKAGE ESTIMATES OF EXPECTED RETURNS.

There is a clear indication that an improvement in estimates of expected return is necessary due to the instability of returns based on historical data, and the far greater effect errors in the means have on portfolio performance. The following articles attempt to rectify this problem.

¹³ *op. cit.*, Jorion (1985).

¹⁴ The R^2 ranges from .0010 to .0658 with one year averages and from .0023 to .0451 with five year averages.

¹⁵ *op. cit.*, Chopra and Ziemba (1993).

¹⁶ *Ibid.*, p. 7.

*International Portfolio Diversification with Estimation Risk (Jorion, 1985)*¹⁷

Jorion suggests that a possible improvement on the sample mean may be found by pooling the data for each country and "shrinking" the sample means towards a common value. This approach is an application of Stein like estimators to the portfolio context¹⁸, and has also been referred to as "empirical Bayes", or "Bayes-Stein" estimation.

Stein showed that the sample mean was inadmissible in a multivariate problem with three or more unknown means. That is, a class of estimators was given that has lower estimation risk for all values of the population means. Estimation risk is defined as the average loss of investor utility in repeated samples. The general form of Stein estimators is:

$$E_{BS}[r|y] = (1-w)Y + wY_0\mathbf{1}; \quad 0 \leq w \leq 1 \quad (2)$$

where Y is the vector of sample means

Y_0 is an estimate of aggregate expected returns.

w is the weight applied to limit the strength of belief in Y , against Y_0 .

Y_0 and w are estimated from the data

$\mathbf{1}$ is a vector of ones

The Stein estimates have uniformly lower risk than the sample mean, whatever the value of Y_0 ¹⁹. However, Y_0 is an estimate of aggregate expected returns and utility loss is less if the estimate is close to the actual grand mean. Furthermore, the goal is not necessarily the accurate estimation of the individual means, but the reduction in utility lost due to errors in the estimates.

In the particular case, developed by Jorion, the prior estimate of the grand mean, Y_0 , is the mean of the minimum variance portfolio. Frost and Savarino (1986) state;

"As with any Bayesian decision rule, there is no definitive approach to specification of a particular informative prior."²⁰

It is possible that other prior estimates may exist that improve on this, or any other suggested prior.

¹⁷ *op. cit.*, Jorion, 1985.

¹⁸ Stein, C. "Inadmissibility of the Usual Estimator for the Mean of a Multivariate Normal Distribution.", Proceedings of the Third Berkeley Symposium on Probability and Statistics I, (1955), Berkeley: University of California Press.

¹⁹ Jorion, Phillippe, "Bayes-Stein Estimation for Portfolio Analysis.", Journal of Financial and Quantitative Analysis, (September, 1986), p. 284.

²⁰ Frost, Peter A. and Savarino, James E., "An Empirical Bayes Approach to Efficient Portfolio Selection.", Journal of Financial and Quantitative Analysis, (September, 1986), p. 299.

The amount of shrinkage, w , depends on the dispersion of the sample averages, Y , around Y_0 and of the sample size, T , as follows:

$$w = \frac{\Lambda}{(T + \Lambda)} \quad (3)$$

$$\Lambda = \frac{(N + 2)(T - 1)}{[(Y - Y_0 \mathbf{1})' S^{-1} (Y - Y_0 \mathbf{1}) (T - N - 2)]} \quad (4)$$

where S is the sample covariance matrix

$\mathbf{1}$ is a vector of ones

N is the number of assets

Shrinkage can also be done on the estimates of variance and covariance. However, as has been shown in section 2.3, these are typically estimated with relative precision and the adjustment is likely to have little effect on portfolio composition and performance. Thus it can be assumed that the covariance matrix is known and shrinkage of the covariance matrix ignored²¹.

The Effect of Shrinkage Estimates on the Portfolio Optimisation Process.

It can be seen from equation (2) that as w increases towards 1, less emphasis is put on the sample mean. From equations (3) and (4), this will occur as sample size decreases. Conversely, the closer the prior represents the sample estimates, w is lower and the less shrinkage will occur.

Of particular importance, in the portfolio context, is the weight of the various securities in the optimal portfolio. Estimation of the individual means is considerably less important, as it is only their effects on the overall portfolio mean and composition that is of concern to the investor. The ranking of the individual assets remains the same, in terms of the means, however the assets with sample mean furthest away from Y_0 get adjusted more towards the middle. This effect is appealing, as it is these estimates that are likely to contain the most error.

The effect on the efficient frontier is shown in Figure 1.

²¹ Grauer, Robert R., and Hakansson, Nils. H., "Stein and CAPM Estimators of the Means in Asset Allocation: A Case of Mixed Success", Unpublished Manuscript, (1992), Haas School of Business, University of California, Berkeley, pp. 28-29.

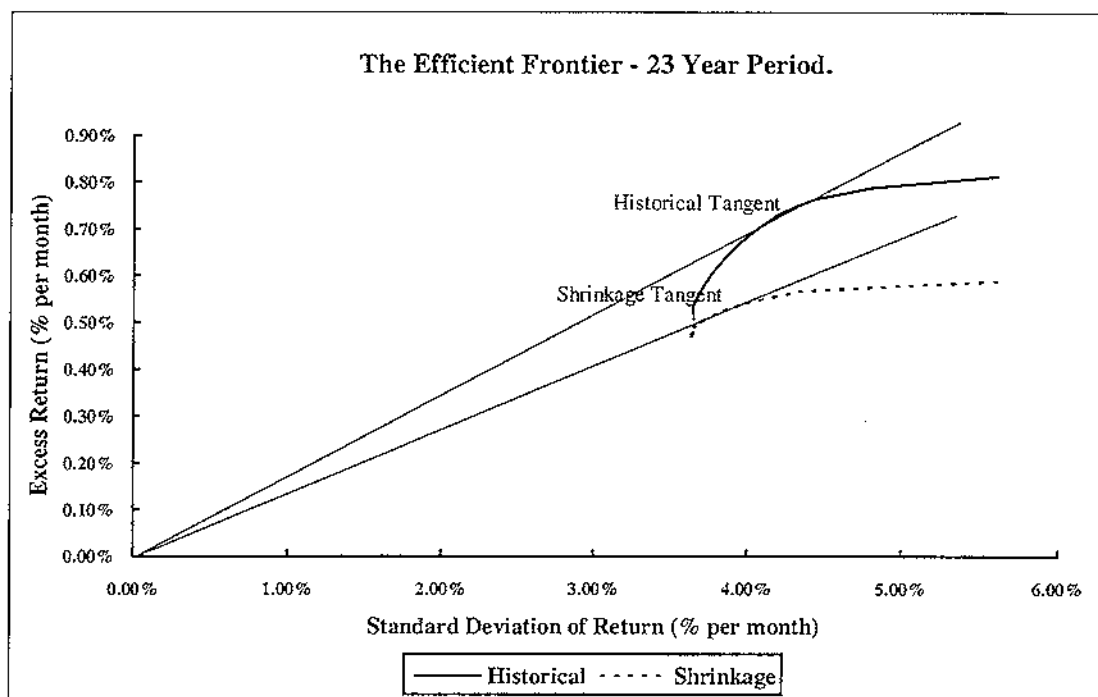


Figure 1: Comparison of Historical and Shrinkage Estimates.²²

The lines historical and shrinkage represent the efficient frontiers, namely the entire set of efficient portfolios, for historical and shrinkage estimates respectively. Expected returns are defined as returns above the risk free rate, commonly known as excess returns. It can be seen that the shrinkage estimates flatten the efficient frontier. The optimal portfolios are represented by the point of tangency on each of the curves. It can be seen that the shrinkage estimates result in a less risky *ex-post* portfolio being chosen. The greater the amount of shrinkage, the less emphasis is put on increasing returns and the more emphasis is put on risk reduction. For example, the efficient frontier based on historical data is equivalent to $w=0$, and the frontier for shrinkage, as shown, represents $w=0.66$. The other extreme, $w=1$, places no reliance on the sample estimates of the means and invests solely in the minimum variance portfolio²³. In this case, the shrinkage estimators show no preference with respect to expected returns, and the optimisation can only result in risk reduction.

The efficient frontiers of Figure 1 may also be represented in a graph of m versus λ , when homogeneous programming is used. This provides additional insight into the effects of the shrinkage estimates. Figure 2 illustrates this by an example.

²² Source: *op. cit.*, Jorion (1985), p269. Present data substituted.

²³ This effect would occur regardless of the prior Y_0 chosen. It is simply a result of the Mean-Variance optimisation when all means are equal.

m versus lambda - 23 Year Period.

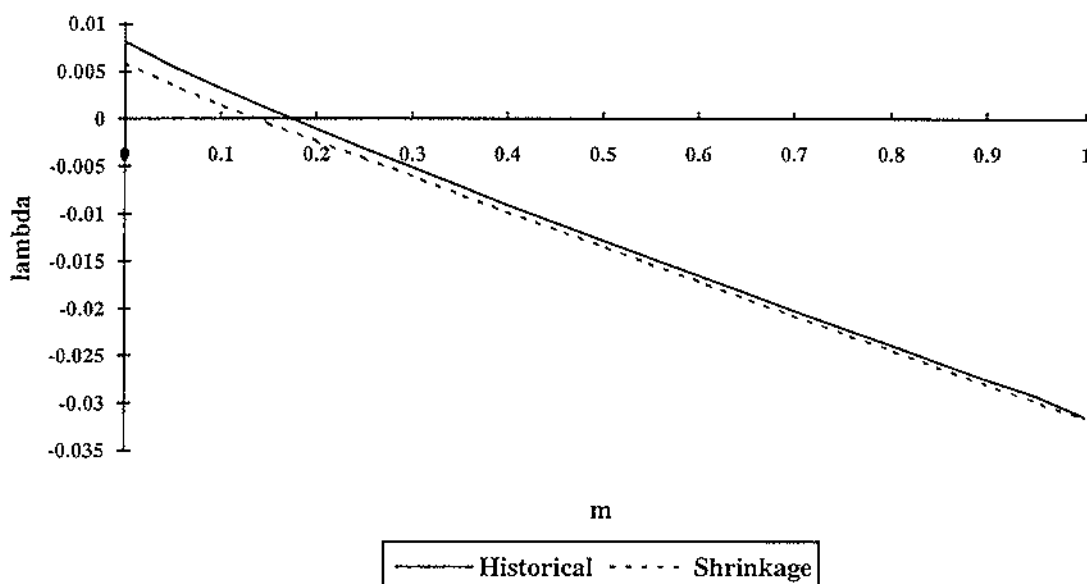


Figure 2: Efficient Frontiers in m versus λ space.

It can be seen from Figure 2 that the efficient frontiers are furthest apart while the risk aversion is low. As $m \rightarrow \infty$, the frontiers converge. The optimal portfolio is defined as the efficient portfolio where $\phi(x) = \lambda$. In Figure 2, a horizontal line should be drawn from the point on the y-axis where λ equals the monthly risk free rate. The portfolio where this line cuts the efficient frontier is the optimal portfolio.

The shrinkage estimates imply a less risk averse investor operates in the market, as m is lower. However, the optimal portfolio has lower standard deviation, as shown in Figure 1. The recognition of estimation error has reduced the ability of investors to trade off additional risk for additional return. More emphasis is put on risk reduction through diversification.

With short sales unrestrained and a sixty month sample, Jorion found that the historical estimates were significantly outperformed by shrinkage estimates, and also the minimum variance portfolio. The minimum variance, or complete shrinkage, portfolio apparently performed better than the shrinkage tangent portfolio, although this was not statistically significant. This lends some support to the notion that the historical data contained little, if any, information on expected returns.

*Stein and CAPM Estimators of the Means in Asset Allocation: A Case of Mixed Success (Grauer and Hakansson, 1992)*²⁴

Grauer and Hakansson applied various methods to the estimation of portfolio means. Included were estimates based solely on historical data, various forms of shrinkage and equilibrium, or CAPM, based estimates. They consider both international and local markets in a short sales restricted environment. In contrast to Jorion, they use a multi-period investment model, instead of the more common single period model. The results are based on quarterly revision of the investment portfolio over a twenty-one year period. The sample period is eight years.

Considerably different results to those of Jorion occur when international diversification is included. In this instance, shrinkage estimates perform significantly worse than the historical estimates. CAPM estimates performed significantly worse, in both international and local markets, than any of the other estimates. Reasons for the differences with Jorion are most likely to be due to differences in the data set and methodology. In particular, the short sales restriction, the longer estimation period and quarterly versus monthly portfolio revision are believed to have had the most effect²⁵.

2.5 SUMMARY

The two articles that are of most significance to this thesis are those of Jorion (1985) and Grauer and Hakansson (1992). Both have considered international portfolio selection using historical and shrinkage estimates of the expected returns. The fact that they get opposite results is of concern, and indicates that there is a need for further research into this topic.

The objective of this thesis is to use an intermediate methodology to those of Jorion and Grauer and Hakansson. The ability to do this is supplied by homogeneous programming, as it applies the single period model to the case of short sale restrictions. Other differences to the previous research, including the data used, are outlined in the next chapter.

²⁴ *op. cit.*, Grauer and Hakansson (1992).

²⁵ *ibid.*, pp. 25-26.

Jorion, Phillippe, "Bayesian and CAPM estimators of the Means: Implications for Portfolio Selection.", Journal of Banking and Finance, (June, 1991), p. 726.

CHAPTER 3

Data and Methodology

3.1 INTRODUCTION

To test the effectiveness of the various portfolio selection alternatives a database of thirteen industrialised countries has been compiled, including data for a twenty-three year period. Due to the weak power of the statistical test available to test portfolio performance, as large a database as possible is desirable. This chapter describes the database, and the methodology used to create portfolios over this period. The test of significance of the relative portfolio performance is also detailed. Finally, two alternative strategies, namely the minimum variance and equally weighted portfolios, are introduced for further comparative performance.

3.2 DATA

To formulate the historical returns, information on stock indices, exchange rates and average dividend yields has been collected. Stock indices have been used as a proxy for the investment opportunities available in each country. The indices for each country have been adjusted to \$NZ values as follows:

$$\text{Index}(\$NZ)_t = \text{Index}_t \div \text{Exchange Rate}(\$NZ - \text{Foreign})_t \quad (5)$$

where the exchange rate expresses the New Zealand dollar as a single unit, for example, \$NZ1 \approx \$US0.55 at present, and the index measure is taken at the end of the period. Excess return for period t , R_t , is then calculated from the following equation:

$$R_t = \frac{\text{Index}(\$NZ)_t - \text{Index}(\$NZ)_{t-1}}{\text{Index}(\$NZ)_{t-1}} + \frac{1}{2} Y_t - \frac{1}{2} I_t \quad (6)$$

where: Y_t = the average dividend yield on the foreign country index.

I_t = the long term New Zealand government bond yield.

Returns are defined in excess of the interest rate, I_t to enable the Jobson and Korkie test of significance to be used¹. This also dampens the impact of inflation on returns. No adjustment for tax is made due to the ever changing tax laws worldwide, and the differing tax treatment of various investors. As returns are defined in excess of the

¹ Jobson, J.D. and Korkie, M., "Performance Hypothesis Testing with the Sharpe and Treynor Measures.", Journal of Finance, (September, 1981), pp. 891-892.

interest rate, the optimal portfolio will be found where $\lambda = 0$. This ensures that the definition of returns has not affected m and the optimal portfolio chosen.

The data used covers the period from January 1970 to December 1992. The main limiting factor in collection of the data is the availability of dividend yields. In particular, dividend yields for New Zealand before 1970 are not published. Additionally, dividend yields for many countries have proved exceedingly difficult to obtain over a long period. Alternatively, gross indices that adjust for dividend reinvestment could have been used, and would have provided slightly better estimates of return. However, these are even more difficult to acquire for an extended period.

To test the performance of the return estimates, increasing the time period is of more importance than increasing the number of countries included, so long as a reasonable number of countries are under consideration. It is desirable to include the home country, New Zealand, due to the effect that exchange rates have in increasing the apparent risk level and correlation of the foreign countries. This is caused by a falling or rising New Zealand dollar making the foreign countries experience gains and losses in New Zealand dollar terms. The home country returns will not experience this. The effect also increases the apparent correlation of the other markets. Thus the twenty-three year period becomes the maximum length of time that can be considered.

It has been possible to collect data for twelve other countries for this period. A summary of sources used follows (Table 1). Full references for published information are contained in the bibliography. The main sources of data are the OECD "Main Economic Indicators" and "Interest Rates" publications. There remained some short gaps in the dividend yield data, and these were filled by interpolation of the dividend value².

The countries that are included are all OECD countries. The following (Table 2) is a complete list of countries included. The sources of the indices are detailed in Appendix I. Omissions that have been necessary include Hong Kong, as dividend yields were unavailable for the majority of the period, and many small or developing countries, where yields were also unavailable for most of the period. The developing countries may have provided some particular benefit as they are regarded as high risk and high

² Dividend values were interpolated as these are likely to be more stable over time than dividend yields. This is due to the variability of yields induced by the volatility of the share markets.

return markets. The composition of optimal portfolios chosen when these are included could be quite different³

Stock Indices	OECD, "Main Economic Indicators" IMF, "International Financial Statistics" The Financial Times
Exchange Rates	OECD, "Main Economic Indicators"
Dividend Yields	OECD, "Interest Rates" New Zealand Department of Statistics, "Monthly Abstract of Statistics" Canadian Statistical Review The Australian Stock Exchange CS First Boston New Zealand
New Zealand Interest Rate	OECD "Main Economic Indicators"

Table 1: Sources of Information.

Australia	Germany	New Zealand
Belgium	Great Britain	Switzerland
Canada	Italy	United States of America
Finland	Japan	
France	Netherlands	

Table 2: Countries Included.

The main criticism that could be made of the data is the approximation that has been made by using indices and yields rather than individual stock returns. However, this has been a common method of considering international portfolio selection. The additional information and, more importantly, the additional computational power necessary to consider large numbers of securities provide a limiting factor. However, this approximation is a more realistic approach for most investors, who do not have the resources to study individual securities. Investment in unit trusts that replicate the stock indices is possible and allows the investor to ignore the individual shares.

³ It would be expected that over some periods of time, the return of the developing markets will be so great as to base the optimal portfolio around them when using historical data. This is less likely using shrinkage estimates.

Additionally, the consideration from a New Zealand perspective could affect the result. The New Zealand economy went through stages of major increases in regulation and deregulation over the period, with resulting instability in the financial markets. As the markets will have been more volatile, and less predicatable over significant periods, there will be a tendency for the shrinkage estimates to perform better as they rely less on the estimates of the means. A more important concern would arise from instability of the correlation of returns over time. As the portfolios chosen using both historical and shrinkage estimates of the means rely on the stability of the correlation structure, considerable changes in correlations from one period to the next can frustrate the selection.

3.3 METHODOLOGY.

Optimisation of the Portfolios.

The homogeneous programming algorithm as developed by van Moeseke has been used to create a spreadsheet macro-language program for portfolio optimisation. Readers are referred to van Moeseke for the algorithm⁴. As returns are also defined as excess returns, the optimal portfolio is found where the efficient frontier passes through the x axis. For any level of m greater than that where $\phi(x)=0$, investment will not take place. As van Moeseke notes, if a person is too cautious for a particular investment, they will not engage in it⁵. With nominal returns, this implies that these cautious investors would prefer to hold cash than to invest in the particular market. With returns defined above the risk free interest rate, the interpretation changes, and the more cautious investors would prefer to be invested in risk free securities.

The macro is detailed in Appendix II. It allows the algorithm to be solved to any desired level of accuracy in λ , up to the limits of the spreadsheet. Calculations in this study were to at least seven decimal places, or five places in percentages. The shrinkage estimates are also calculated by the macro, using equations (2), (3) and (4). The necessary inputs to the macro are therefore an expected return vector and a covariance matrix.

As the mean of the minimum variance portfolio is an input to the shrinkage estimates, this must also be calculated. Construction of the minimum variance portfolio using homogeneous programming can be done by temporarily setting all the means equal, and m positive. Solving this gives the weights of the assets in the minimum

⁴ *op. cit.*, van Moeseke (1965), pp. 232-252.

⁵ *op. cit.* van Moeseke (1965), p. 250.

variance portfolio. Substituting the sample means for the temporary equal means allows the mean of the minimum variance portfolio to be found.

Periods Considered

This thesis uses two sample sizes, namely four and eight years of monthly data. This allows a comparison to both Jorion (1985) and Grauer and Hakansson (1992), as they consider similar time periods respectively. The theoretical benefit of shrinkage estimates is lower with the eight year period, and could provide the reason for them being outperformed in Grauer and Hakansson.

The portfolios are recalculated quarterly, as in Grauer and Hakansson (1992)⁶, the first quarter starting March 1970, for both sample sizes. The performance of each portfolio is traced over the following period of the same length as shown below (Figure 3). Thus a "sliding window" of periods is considered. This allows for sixty four year periods to be considered, and twenty-eight eight year periods.

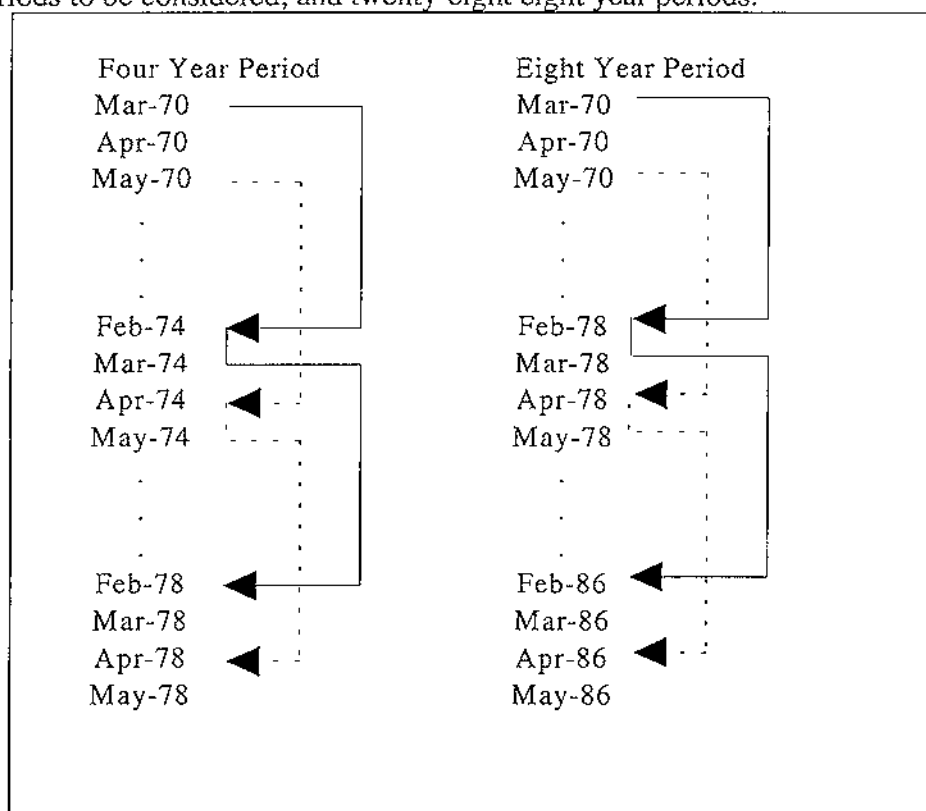


Figure 3: Sample and Comparison Periods.

Test of Performance.

Jobson and Korkie (1981), develop test statistics for portfolio performance hypothesis testing⁷. The test to be used here is the equality of Sharpe measures. When

⁶ *op. cit.*, Grauer and Hakansson (1992).

⁷ *op. cit.*, Jobson and Korkie (1981).

returns are defined in excess of the risk free rate, the Sharpe measure is defined as follows:

$$Sh_i = \frac{\mu_i}{\sigma_i} \quad (7)$$

where: μ = excess return
 σ = standard deviation of return

This measure is appropriate for testing performance in this case as the optimisation process is attempting to maximise this ratio.

An improvement in the paired difference test is found by transforming the difference to:

$$Sh_{in} = s_n \bar{r}_i - s_i r_n \quad (8)$$

where s_i , s_n , r_i and r_n are the sample standard deviation and mean for i and n respectively.

The asymptotic distributions of the transformed distribution are normal, with mean Sh_{in} and variance, θ , as follow:

$$\theta = \frac{1}{T} \left[2\sigma_i^2 \sigma_n^2 - 2\sigma_i \sigma_n \sigma_{in} + \frac{1}{2} \mu_i^2 \sigma_n^2 + \frac{1}{2} \mu_n^2 \sigma_i^2 - \mu_i \mu_n / 2\sigma_i \sigma_n (\sigma_{in}^2 + \sigma_i^2 \sigma_n^2) \right] \quad (9)$$

where: σ_{in} = covariance of i with n .

The estimators of the means, variances and covariances are substituted for their true values.

The performance hypothesis is thus:

$$H_0: Sh_i = Sh_n$$

and the test statistic is given in equation 10 below.

$$z_{Sh_{in}} = \frac{Sh_{in}}{\sqrt{\theta}} \quad (10)$$

The test statistic is quite weak, especially when the co-efficients of variation are high⁸. That is, it does not show a strong ability to distinguish between differences in the performance parameters.

The estimates for monthly variance and covariance can be found by pooling the sample variance and covariance of realised returns over the entire period.

Alternative Strategies

Further to the Mean-Variance optimisation using shrinkage and historical estimates, two more strategies are considered. These are the minimum variance and equally

⁸ *op. cit.*, Jobson and Korkie (1981), pp. 900-901.

weighted portfolios. These four portfolios can be considered as the portfolio choice under four various states of knowledge as follows (Table 3).

Portfolio	Implied Knowledge
Equally Weighted Portfolio	Positive expected returns, $\rho < 1$.
Minimum Variance Portfolio	Above, plus correlation structure known and stable
Shrinkage Estimates of Return	Above, plus limited information on expected return
Historical Estimates	Full knowledge of risk and expected return

Table 3: Knowledge Implied by Various Portfolios.

Any number of other portfolio selection strategies with varying merit could be included, however those given are all that are considered here.

3.4 SUMMARY.

In outlining the data and methodology used in this study, the differences between this and previous studies can be assessed. An attempt has been made to use a method between the two most similar studies. Thus, a single period model is used, as in Jorion (1985), but with short sales restricted as in Grauer and Hakansson. The use of quarterly revision of the portfolios is aligned with the latter study. Using both four and eight year periods should also shed light on whether the sample size affects the result.

CHAPTER 4

Analysis and Results

4.1 INTRODUCTION.

This chapter is split into two main sections. The first analyses the behaviour of portfolios under the various methods used. There is also an analysis of the risk preference term over time. The other section is concerned with the actual performance of the portfolios. Thus the realised returns, standard deviations and Sharpe measures for the portfolios are considered. This leads on to the test of equality of Sharpe measures.

4.2 PORTFOLIO ANALYSIS.*Portfolio Composition.*

Figure 4 shows the average performance of the individual markets over the whole period. Individually, Japan and the United States appear to dominate the other countries, particularly Japan as it has the highest level of excess return for risk. The return for the United States is somewhat lower, but it also less risky. Clearly Italy has averaged the worst performance.

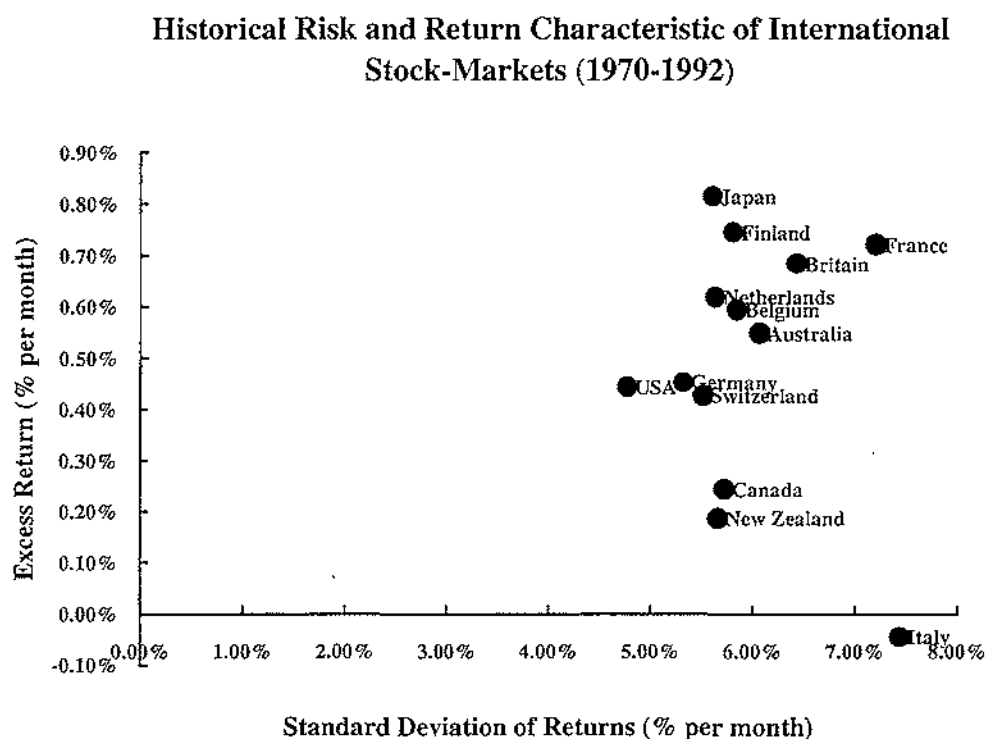


Figure 4: Historical Market Performance.

The average proportion of each country in the various portfolios is shown in Appendix I. The New Zealand stock market makes up a considerable proportion of portfolios in all cases, except obviously the equally weighted portfolio. As the New Zealand market's performance has been lacklustre, the lower correlation it has to other markets has ensured its inclusion¹. As would be expected, Japan makes up a substantial amount of the portfolios, although the United States does not play much of a role. However, Finland is highly represented in the average portfolios. Italy and Belgium are almost entirely ignored. This is not surprising in the case of Italy, but Belgium's exclusion is perhaps more surprising. High correlation to the other European markets is likely to be the cause.

Portfolio Stability.

The averages do not show up the stability of the portfolio composition over time. The proportion that individual countries make up of the optimal portfolios varies considerably over time. In addition, a country that is highly represented in one quarter can easily disappear in the next. The diagram below shows this (Figure 5).

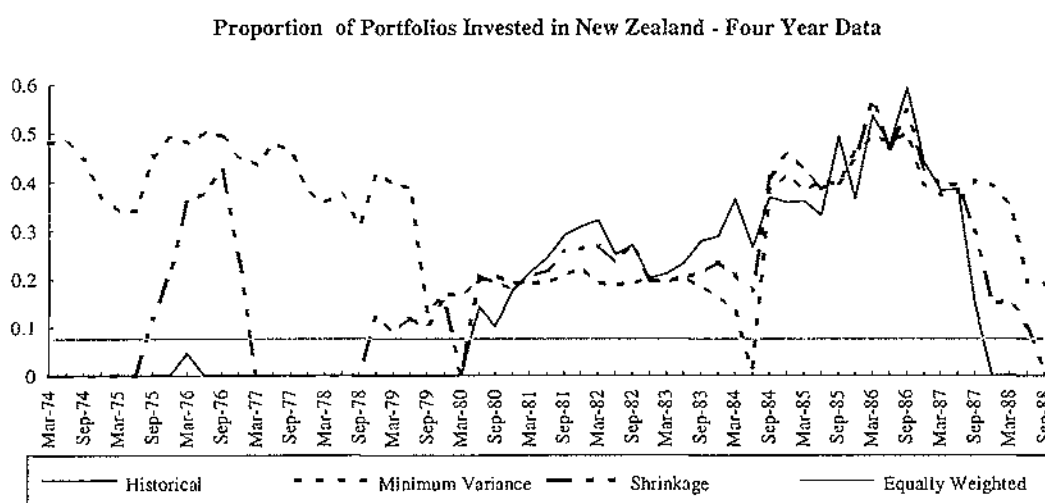


Figure 5: The Weight of New Zealand in Portfolios Over Time.

As a major event happens, the Mean-Variance approach using historical data, will select future portfolios based on this. In Figure 5, the New Zealand market disappears immediately after the world wide stock market crash, after making up around forty

¹ The low correlation is enhanced by the exchange rate effect of being the home market. For example, if the New Zealand currency drops in value, all offshore markets rise *ceteris paribus*. This makes offshore markets appear to be highly correlated to each other, due entirely to New Zealand exchange rate movements.

percent in the previous months. The stability of portfolio weights that is added using shrinkage estimates, as found by Jorion² is not as prevalent in this case. This may negate much of the benefit of the shrinkage estimates. The eight year estimates do lead to more stable portfolios.

Another interesting result to come from the study is the movement of m over time (Figure 6). Using the historical data, m has averaged 0.45. However, it can be seen that it has varied considerably over time.

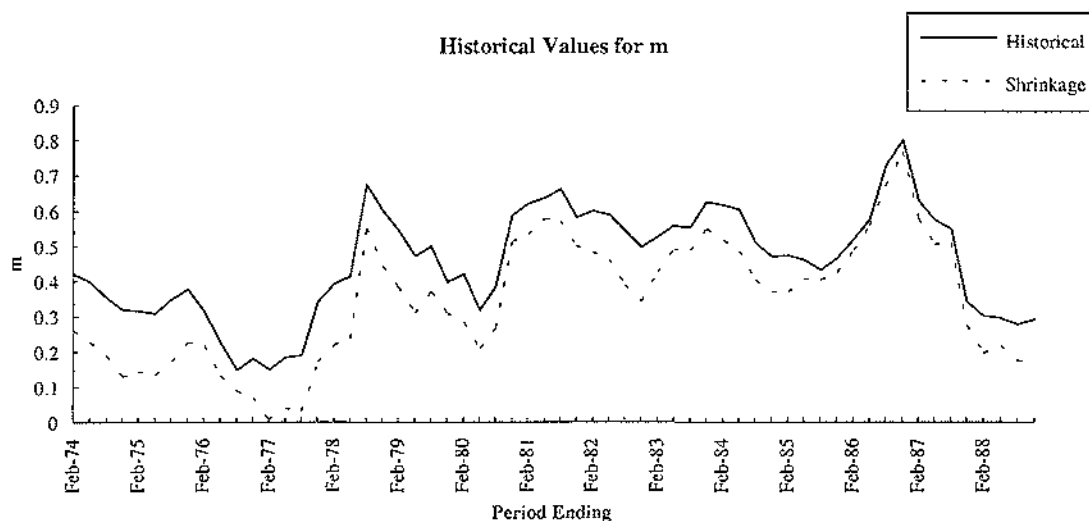


Figure 6: The Movement of the Risk Preference Term Over Time.

Investors may be able to use the variation of m as an indication of the relative value of the market. However, the viability of this as a market timing approach to investment is beyond the scope of this study.

4.3 RESULTS.

The performance of the four year portfolios is shown in the following tables (Tables 4 and 5), and compared to the expected values, based on the historical data.

² *op. cit.*, Jorion (1985), pp. 272-273.

Strategy	Mean		Standard Deviation		Sharpe
	<i>Ex-post</i>	<i>Ex-ante</i>	<i>Ex-post</i>	<i>Ex-ante</i>	<i>Ex-ante</i>
Historical	1.427%	.290%	3.479%	4.437%	.0654
Shrinkage	0.984%	.329%	3.219%	4.363%	.0754
Minimum Variance	0.728%	.548%	2.692%	4.079%	.1344
Equally Weighted	0.748%	.717%	3.647%	4.024%	.1781

Table 4: Performance of Four Year Portfolios.

Measurements of means and standard deviation are in percent per month. The difference between *ex-post* and *ex-ante* performance is substantial. Notably the mean of the historical portfolio is highest *ex-post* but drops to the lowest in the following period. The shrinkage estimates also lead to a considerable drop in means. The ranking of standard deviations is as expected, except that the equally weighted portfolio manages to just beat the historical minimum variance portfolio. The corresponding Sharpe measures show that the more information we assume is contained in the data, the worse the portfolio performance. Four year samples of expected returns have provided disinformation. As the equally weighted portfolio performs best, it seems that even the variance and co-variance estimates have been poor predictors. The significance of these results is shown in Table 5, with negative values indicating the portfolios in the column are outperformed by those listed horizontally.

	Shrinkage	Minimum Variance	Equally Weighted
Historical	-0.36	-0.93	-1.34
Shrinkage		-0.94	-1.26
Minimum Variance			-0.57

Table 5: Pairwise Test of Equal Performance, Four Year Data.

As can be seen, the significance of the tests is low. The highest z-score of 1.34 is only significant at the ten percent level.

The equivalent analysis for the eight year data is shown below (Tables 6 and 7).

Strategy	Mean		Standard Deviation		Sharpe
	<i>Ex-post</i>	<i>Ex-ante</i>	<i>Ex-post</i>	<i>Ex-ante</i>	<i>Ex-ante</i>
Historical	.889%	.912%	3.096%	4.697%	.1941
Shrinkage	.593%	.891%	2.787%	4.503%	.1979
Minimum Variance	.450%	.955%	2.418%	4.220%	.2263
Equally Weighted	.505%	.919%	3.193%	4.510%	.2037

Table 6: Performance of Eight Year Portfolios.

	Shrinkage	Minimum Variance	Equally Weighted
Historical	-0.09	-0.28	-0.01
Shrinkage		-0.30	-0.06
Minimum Variance			0.24

Table 7: Pairwise Test of Equal Performance, Eight Year Data.

With the eight year samples the performance of the historical shrinkage and minimum variance portfolios is much improved relative to the performance of the equally weighted portfolio. There is insufficient evidence to suggest any material difference in the performance of the portfolios. The minimum variance portfolio has produced both the highest return and lowest standard deviation, and therefore the best Sharpe measure. Clearly it would be desirable to get a longer overall time period to see if this similarity in performance continues. As the strength of the test does not improve markedly with sample size, it seems that proving any difference in performance is unlikely.

4.4 SUMMARY.

It seems from the above result that the historical data provides little benefit, if any, in selecting *ex-ante* optimal portfolios using the Mean-Variance method. With eight year data, the model does manage to overcome the problem of errors in the data leading to bad selection. The roughly equal performance is achieved with less diversified portfolios than the equally weighted portfolio, indicating a small amount of selective ability. It may be that even longer sample periods are necessary for historically based estimates to give a fair indication of the risk and return relationship of markets as a whole. To get a statistically significant outcome would require a substantially longer time span of data. At present this is not available here.

CHAPTER 5

Conclusions

From the results of the study, historical data for periods of four and eight years is of little use to the investor. No significant gains can be made over investing in an equally weighted portfolio of countries. It may be that even longer periods of time are necessary to get reasonable estimates of expected returns. In particular, it appears that investors should be especially wary of any portfolios based on data for periods of around four years.

It may be that the volatility of the New Zealand market has distorted the results, and a repeat of the analysis using a different base currency could return higher estimation accuracy. The applicability of this to New Zealand investors would then have to be questioned. However, the potential benefit of shrinkage estimates should be greater with a volatile market, as the wide fluctuations in observed returns are not given as much emphasis as with the historical approach. In addition, the more unpredictable the market is, the more shrinkage will occur and risk reduction becomes the main benefit of the optimisation. In this instance, this effect has been of little benefit.

The fall in the value of the New Zealand currency could be pointed to as a possible problem. However, as returns are defined above the interest rate, New Zealand's higher interest rates will have reduced the impact. Over a long period, it can be expected that the falling currency is largely due to relatively higher inflation, and interest rates will include a premium for inflation. Thus, while the volatility of the overseas markets is increased, the exchange rate should have a lesser effect on returns¹.

Business cycles may have played a part in reducing the accuracy of estimates based on past data. The Mean-Variance model using this data increases the weights of securities that have experienced higher returns. The investment may have missed the major increases in the market due to an economic upturn, and also remained strongly invested in the market until significant losses are incurred. Again, portfolios based on shrinkage estimates should not be affected as severely. This also illustrates the point that the Mean-Variance model is not based on market timing, but rather on the long run risk and return characteristics of the securities. If economic cycles are affecting the result, it can be concluded that the four year samples do not provide a long enough time

¹ The poor returns on the New Zealand market compared to the other markets may be a result of high inflation, rather than the depreciating currency. The impact of long run inflation on sharemarkets is outside the scope of this study.

period to cover an entire cycle, and get a true indication of the longer run characteristics. Again, the recommendation is to consider longer time periods.

The failure of the shrinkage estimates to provide a significant improvement on the historical estimates in the short sale restricted case could be due to the composition of the minimum variance portfolio. With short sales restricted, several of the countries are usually left out of the historical minimum variance portfolio. If short sales are allowed, some countries may not be included, but typically all countries will appear in the portfolio in either positive or negative quantities. The lower diversification of the short sale restricted portfolio places a higher need for the covariance structure to remain stable over time. However, the reasonably good performance of the minimum variance portfolio suggests that instability in correlations cannot be entirely to blame.

In comparison to other studies, there is no definitive conclusion that can be made. The estimates improve with longer sample periods, as the effect of errors evens out. However, the correction made by the shrinkage estimates has been insufficient to achieve a reasonable result with the shorter sample period. Neither of the previous studies, by Jorion and Grauer and Hakansson, have provided a general answer to the estimation of means in the international context.

A change in methodology appears necessary for the Mean-Variance framework to provide reasonable results with historical data. As the equally weighted portfolio performed quite well, it may be desirable to shrink the weights of the portfolio towards this, or perhaps some other standard, such as shrinkage towards market weights. However, the equally weighted and market weighted portfolios are likely to not appear on the *ex-post* efficient frontier. The desired effect cannot be achieved by shrinkage of the means if the portfolio we want to adjust towards is not on the efficient frontier. Thus an adjustment would need to be made to the weights themselves.

The overall result is discouraging for investors wishing to get an indication of market characteristics using historical data. Good estimates of the means are needed for the Mean-Variance approach to be of use, and these have not been found, even after shrinkage of the means, in an international context. It may be that the judgement of informed investors, as recommended in Markowitz' original article², provides a better solution than can be found by manipulation of historical data.

² *op. cit.*, Markowitz (1952), p 91.

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APPENDIX I

Summary of Data

SHARE INDICES USED (AS FOUND IN THE OECD "MAIN ECONOMIC INDICATORS" SERIES.)

Country	Source
Australia	Australian Stock Exchange Industrials
Britain	FT-SE Actuaries "500 Shares"
Belgium	Industrials
Canada	Toronto Stock Exchange
Finland	Helsinki Stock Exchange Industrials
France	I.N.S.E.E. Industrials
Germany (West Germany only)	Federal Statistical Office Industrials
Italy	Milan Stock Exchange
Japan	Tokyo Stock Exchange
Netherlands	Amsterdam Stock Exchange All Share Index
New Zealand	Reserve Bank Index (December 69-June 86) NZSE All Ordinaries Index (July 86-December 92)
Switzerland	All Shares
United States	Standard and Poor's Index

SUMMARY STATISTICS OF NEW ZEALAND DOLLAR ADJUSTED EXCESS RETURNS.

	<i>Australia</i>	<i>Belgium</i>	<i>Britain</i>	<i>Canada</i>	
Mean	0.0055	0.0059	0.0068	0.0024	
Standard Error	0.0037	0.0035	0.0039	0.0035	
Median	0.0022	0.0012	0.0036	-0.0015	
Standard Deviation	0.0608	0.0586	0.0644	0.0575	
Variance	0.0037	0.0034	0.0042	0.0033	
Kurtosis	6.3318	2.4843	9.0117	1.3410	
Skewness	-0.1528	0.7225	1.0414	0.2002	
Range	0.6317	0.4443	0.7170	0.4378	
Minimum	-0.3471	-0.1579	-0.2553	-0.2000	
Maximum	0.2846	0.2864	0.4617	0.2379	
	<i>Finland</i>	<i>France</i>	<i>Germany</i>	<i>Italy</i>	
Mean	0.0074	0.0072	0.0045	-0.0005	
Standard Error	0.0035	0.0043	0.0032	0.0045	
Median	0.0003	0.0057	0.0012	-0.0066	
Standard Deviation	0.0583	0.0722	0.0534	0.0746	
Variance	0.0034	0.0052	0.0028	0.0056	
Kurtosis	1.8787	2.0678	2.3969	0.9078	
Skewness	0.5424	0.2764	-0.1195	0.4886	
Range	0.4196	0.5556	0.4368	0.4775	
Minimum	-0.1827	-0.2550	-0.2505	-0.2253	
Maximum	0.2369	0.3006	0.1862	0.2522	
	<i>Japan</i>	<i>Netherlands</i>	<i>New Zealand</i>	<i>Switzerland</i>	<i>United States</i>
Mean	0.0081	0.0062	0.0019	0.0043	0.0044
Standard Error	0.0034	0.0034	0.0034	0.0033	0.0029
Median	0.0064	0.0017	-0.0011	0.0008	0.0027
Standard Deviation	0.0562	0.0565	0.0568	0.0553	0.0479
Variance	0.0032	0.0032	0.0032	0.0031	0.0023
Kurtosis	1.0294	2.6315	4.0307	1.6854	4.6901
Skewness	0.1474	0.5669	-0.1702	0.3958	0.2616
Range	0.3658	0.4275	0.5431	0.3629	0.4824
Minimum	-0.1485	-0.1755	-0.2976	-0.1403	-0.2197
Maximum	0.2173	0.2520	0.2455	0.2226	0.2627

APPENDIX II

Spreadsheet Macro for Portfolio Selection

The following pages contain a print out of a Portfolio Selection program written in Quattro Pro ("Quattro") macro language¹. It has been tested on Quattro versions 3 and 4, using both MS-DOS 3.3 and 5.0². A description of the program and instructions for those familiar with the basics of Quattro follow.

SPECIFICATION

The macros allow the solving of the portfolio problem using homogeneous programming. The necessary inputs are a covariance matrix and a vector of means. If the vector of means is based on a sample of historical data, then shrinkage estimates of the means may be used as inputs to the calculation.

Outputs of the program are the weights of the securities in the portfolios, the expected return, standard deviation of return, m and λ , for each m value used. If shrinkage estimates are used, the minimum variance portfolio is also shown, as well as the shrinkage estimates of the means and the amount of shrinkage, w .

The solution can be found to any desired level of accuracy in λ , up to the limits of the computer arithmetic. The program has not been tested on problems requiring very large matrices. Quattro's matrix handling limits the size handled to a 90×90 matrix. The macro places no further restriction on this.

As matrix size increases, the time to solve the problem increases at a more than 1 to 1 ratio. Using a 25Mhz 486sx based computer, the time to solve the homogeneous programme for a single value of m is around two minutes for the 13×13 matrix. The calculation is substantially slower if the WYSIWYG display mode is used.

¹ Quattro Pro is copyright of Borland International Limited.

² MS-DOS is copyright of Microsoft Corporation.

INSTALLATION

- Each heading (in bold) is the name of the macro that is immediately below it. Only one of these needs to be run by the user. The first line of each macro must be in a block named with the macro name. This can be achieved by using the `{/Name|Labels|Down}` command if the macro name is typed above the cell where the macro starts, as is done below.
- All lines of a macro must follow in cells directly below the first. No empty cells may occur within a macro, as this causes the macro to halt.
- Each line must be entered as a label and for many lines this requires typing an apostrophe (') so that is not treated as a command or a number. This is necessary wherever a line starts with a forward slash (/), a mathematical symbol or a number. Alternatively, the commands could be entered into a text editor and imported into Quattro from a text file. A scanner with optical character recognition software would avoid having to type in the code.
- Lines that are too wide to be printed in the width available here are shown indented on consecutive lines. They may not be split over several cells.
- Borders are shown around the two menus that are used. Each section of the menu must start in the cell immediately to the right of the first line of the previous section.
- The listing is for the macro on a disk in the root directory of the "a drive". If the macro is in any other directory all appearances of "a:\" must be replaced with the drive and path to where the macro is stored. This may be done using the `{/Edit|Search & Replace}` command.
- The listing also requires that the file is named as "macroa.wq1". If the file is named otherwise `{/Edit|Search & Replace}` should be used to replace "macroa" with the new file name. The extension is not necessary.
- All lines of code should appear in the first 200 rows of the spreadsheet. Rows below this are reserved for calculation.

RUNNING THE MACRO

Users should have the macro sheet and a sheet containing the mean vector both open. To start the macro, switch to the macros window. Use the command `{/Tools|Macro|Execute}`. A prompt for the block of macros to use is displayed. Type:

zpfselection

Then press the Enter, or Return, key. From now on this will be shown as {enter}.

The macro will start running and prompt for the name of the sheet containing the inputs. Type the name of the sheet and press {enter}. Next the user is asked for the the number of securities. Type this is and press {enter}.

After a while the user is presented with a menu. To select an option, either type the first letter of the option's name, or use the mouse or cursor (arrow) keys and {enter}. The use of this menu follows.

Covariance Matrix

After selecting this option, the user should select the range of cells containing the covariance matrix. This can be done by name, cell references, mouse or the cursor keys. Then press {enter}. Do not include the titles of the row and columns. The matrix must be filled, thus both sides of the matrix must be entered.

Mean Vector

After selecting this option, the user should select the range of cells containing the mean vector. This can be done by name, cell references, mouse or the cursor keys. Then press {enter}.

Type

The user is presented with another menu. If the portfolio is to be selected using the mean vector unadjusted, choose Historical {enter}. If shrinkage estimates of historical estimates contained in the mean vector are required, choose Shrinkage {enter}. After selecting Shrinkage, a prompt for the sample size is displayed. Type the number of observations used to get the average return. Press {enter}.

Frontier Size

The program requests the user for a starting value for m, a finishing value, and a size of steps between portfolios. Type in the appropriate values and press {enter}.

Display Area

A prompt is made for the place where output will be sent. Select the block and press enter. Please note that no check is made that the output will not overwrite existing data. It is advisable to make the output area below all used cells.

Go

Used to start calculations of efficient portfolios as indicated by the *Frontier Size* option.

Optimise

Used instead of the *Frontier Size* and *Go* combination. An optimal portfolio is found for any level of λ . The user is first requested for the risk free rate. This is the value for λ that is solved for. Type the value and press {enter}. A lower and higher estimate of m is then requested. Type the values and press {enter}. The program will handle m being outside the range estimated, but the solution may take longer, as the estimates of m are adjusted in steps of .2. The desired level of accuracy, in decimal places, is then requested. Type the number of places (a positive whole number), and press {enter}.

Quit

Stops the macro execution. Alternatively, the {Ctrl}{Break} key combination will stop the macro execution at any time.

PROBLEMS

The following are some problems that may arise when running the program.

1. An error message such as "Invalid Input Block" is displayed. There are three known reasons why this may occur.

First, if m is set to 0 or negative, this error may occur. Do not make m negative. The risk neutral solution may approximated by setting m to a very small positive number. The exact solution is quite simply found by observation of the security with the highest expected return. Negative values of m may occur with the *Optimise* option if the lower estimate of m is below 0.2 but is above the optimal value. In this instance, run the macro again, with m set to a very small positive value.

Second, if the *Optimise* option is used, and all estimates of expected returns are negative, the optimal value of m is negative, and the program will not be solved.

Finally, when the size of covariance matrix used is changed. It may be remedied by clearing the block A200 to IV8192 in the macro sheet using the {/Edit|Erase} command. To prevent this problem, users are recommended not to save changes to the macros file when closing it. Additionally, the MS-DOS command ATTRIB can be used to make the file read only. The command, if the file is in the root directory of the "a drive", saved as "macroa.wq1" is

ATTRIB +r a:macroa.wq1 {enter}

For help with this command type:

HELP ATTRIB

while at the DOS prompt.

2. Very small negative values appear in the portfolio weights. This is a result of the inaccuracy of computer arithmetic, where marginal securities are mistakenly included. Remove the affected security from the covariance matrix and mean vector and run the macro again.

3. The program seems to be in an endless loop with the *Optimise* option. This sometimes occurs when errors are made in entering the estimates of *m*. Stop the program using {Ctrl}{Break} and start again.

4. Numbers appear in cells, that are different to those entered. This is a problem that has occurred with older computers with MS-DOS 3.3. The exact cause is presently unknown. It may be caused by MS-DOS, the memory manager being used, or the way Quattro interacts with them. Users experiencing this problem should refer it to their software supplier.

zPFselection

```

/wpm
{windowsoff}
{paneloff}
{zdataarea1}
{zdataarea2}
{getlabel "Name of Sheet Containing Inputs:",yname}
{zsetwindow}
{getnumber "Number of Securities to be Analysed:",ystocks}
{menucall zPFmenu}

```

zPFmenuCovariance Matrix

Inputs the Covariance Matrix of Security Returns

```
/endycovmatrix~{goto}yw~{down 3}
```

```
/encycovmatrix~~
```

```
{zdatawin}
```

```
{windowson}{panelon}
```

```
/ev
```

```
{?}~[a:\macro.s.wq1]ycovmatrix~
```

```
{windowsoff}{paneloff}
```

```
/wpm
```

```
{menubranh zpfmenu}
```

Mean Vector

Inputs the Means of Security Returns

```
{zdatawin}.
{windowson}{panelon}
/ev
{?}~[a:\macro.s.wq1]ymeanvector~
{windowsoff}{paneloff}
/wpm
{goto}ymeanvector~
/endymeanvector~
/encymeanvector~.{right ystocks-1}~
{menubran ch zPFmenu}
```

Type

Choose Between Historical or Shrinkage Mean Estimates

```
{menucall zhist/shrink}
{menubran ch zPFmenu}
```

Frontier Size

The Range of Risk Preferences to Compute Optimal Portfolios For

```
{getnumber "Starting m value:",ymstart}
{getnumber "Finishing m Value:",ymfinish}
{getnumber "Size of Steps Between Portfolios:",ymstep}
{menubran ch zPFMenu}
```

Display Area

Locates the Place to Output Results

```
{zdatawin}
{windowson}{panelon}
/encyoutputarea~
{?}~
/wpm
{windowsoff}{paneloff}
{menubran ch zPFmenu}
```

Go

Compute the Efficient Frontier

```
{zProcess}.
{zdatawin}
```

Optimise

Find the Optimal Portfolio

```
{getnumber "Risk Free Rate:",ylambdastar}
{getnumber "Lower Estimate of m:",ym1}
{getnumber "Higher Estimate of m:",ym2}
{getnumber "Number of Decimal Places Accuracy:",yaccuracy}
{zSearch}
{zdatawin}
```

Quit

Return to the Worksheet

```
{QUIT}
```

zhist/shrink

Historical

Uses Historical Means as the Estimate of Security Returns

```
{let ymeantype,"HISTORICAL"}
{RETURN}
```

Shrinkage

Shrinks the Historical Means Towards a Common Value

```
{let ymeantype,"SHRINKAGE"}
{getnumber "Sample Size :",ysamplesize}
{return}
```

zDataareal

```
{stepoff}
{goto}[a:\macro.sa.wq1]a200~
/ee.{down 500}{right 100}~
{goto}[a:\macro.sa.wq1]a200~
```

ymeantype~{down}
ymstart~{down}
ymfinish~{down}
ymstep~{down}
ystocks~{down}
ym~{down}
ycount~{down}
yrepeats~{down}
ymax~{down}
ycountphi~{down}
ycountxbar~{down}
yphimax~{down}
yguq~{down}
yquq~{down}
ygug~{down}
ylambdaplus~{down}
ylambdaminus~{down}
ylambda~{down}
yw1~{down}
yw2~{down}
ycountxt~{down}
ycountf~{down}
yincluded~{down}
yvcount1~{down}
yvcount2~{down}
ycvp~{down}
ycvc~{down}
ypvp~{down}
ystotal~{down}
ypfmean~{down}
ypfsd~{down}
ypfxvx~{down}
ylastcount~{down}
ydisplaycount~{down}
ycounter~{down}
yxvx~{down}

yrootvx~{down}
ycovblock~{down}
ycovmatrix~{down}
ymeanvector~{down}
yvfinalrow~{down}
yc~{down}
yphi0~{down}
yx~{down}
ysigma~{down}
yphixi~{down}
yxbar~{down}
yxv~{down}
yxbarv~{down}
yw1x~{down}
yw2xbar~{down}
yxt1~{down}
ychoriz~{down}
ycvert~{down}
yxt~{down}
yv~{down}
yxbart~{down}
ycfinal~{down}
yp~{down}
ycinvfinal~{down}
ypinvfinal~{down}
ylambdap~{down}
yclambdap~{down}
yproportt~{down}
yvfinal~{down}
yvinvfinal~{down}
ypt~{down}
ycfinalt~{down}
yclambdapt~{down}
ys~{down}
yproportions~{down}
yname~{down}

```

ygamma0~{down}
ygamma1~{down}
ylambdaq~{down}
ym1~{down}
ym2~{down}
ylambda1~{down}
ylambda2~{down}
ylambdastar~{down}
yaccuracy~{down}
yu~{down}
yuinverse~{down}
ylambdashrink~{down}
ysamplesize~{down}
yY0~{down}
yY01~{down}
yY01t~{down}
yY01V~{down}
yY01Vy01~{down}
yweight~{down}
{goto}[a:\macrosa.wq1]a200~
/enr~
lr.{down 201}~
{;stepon}
{return}

```

zDataarea2

```

{stepoff}
/endygamma~
{goto}[a:\macrosa.wq1]a200~
{down 100}
/enc
ygamma~{esc}
.{right}~
/endygammat~
{goto}ygamma~
{down 2}

```

```

/enc
ygammat~{esc}
.{down}~
/endygamma0~{goto}ygamma~
/encygamma0~~
/endygamma1~
{goto}ygamma0~{right}
/encygamma1~~
/endyu~
{goto}ygammat~
{down 3}
/encyu~{esc}
.{down}{right}~
/endyuinverse~
{goto}yu~
{down 3}
/encyuinverse~{esc}
.{down}{right}~
/endyq~
{goto}yuinverse~
{down 3}
/encyq~{esc}
.{right}~
{goto}yq~1~{right}1~
/endyqt~
{goto}yq~
{down 2}
/encyqt~{esc}
.{down}~
{goto}yqt~1~{down}1~
/endygammauinv~
{goto}yqt~
{down 3}
/encygammauinv~{esc}
.{right}~
/endyquinv~

```

```

{goto}ygammauin~
{down 2}
/encyquinv~{esc}
.{right}~
/endyglq~
{goto}yquinv~
{down 2}
/encyglq~{esc}
.{down}~
/endylambdaq~
{goto}yglq~
{down 3}
/encylambdaq~
.{down}~
/endyw~
{goto}ylambdaq~
{down 3}
/encyw~{esc}
.{down}~
{;stepon}
{return}

```

zProcess

```

{zdataarea3}
{}
{}
{}
{}
{stepoff}
{let yrepeats,0}
{;Mean Calculation}
{if @rows(ymeanvector)>1}{zmeantranspose}
{if @rows(ymeanvector)=1}{zmeanmove}
{if ymeantype="historical"}{zHistorical}
{if ymeantype="shrinkage"}{zShrinkage}
{zTitles}

```

```
{;Main loop}
{for ym,ymstart,ymfinish,ymstep,zProgram}
{return}
```

zDataarea3

```
{stepoff}
{goto}[a:\macro.sa.wq1]ycovmatrix~
/encycovmatrix~/encycovmatrix~.{down ystocks-1}{right ystocks-1}~
/encyvinverse~
{goto}ycovmatrix~
{down ystocks+1}
/encyvinverse~{esc}
.{down ystocks-1}{right ystocks-1}~
/tai
ycovmatrix~
yvinverse~
/endyc~
{goto}yvinverse~
{down ystocks+1}
/encyc~{esc}
.{right ystocks-1}~
/andyphi0~
{goto}yc~
{down 2}
/encyphi0~{esc}
.{right ystocks-1}~
/andyx~
{goto}yphi0~
{down 2}
/encyx~{esc}
.{right ystocks-1}~
/endysigma~
{goto}yx~
{down 2}
/encysigma~{esc}
.{right ystocks-1}~
```

```

/endiylx~
{goto}ysigma~
{down 2}
/encyphixi~{esc}
.{right ystocks-1}~
/endiylbar~
{goto}yphixi~
{down 2}
/encyxbar~{esc}
.{right ystocks-1}~
/endiylV~
{goto}yxbar~
{down 2}
/encyxV~{esc}
.{right ystocks-1}~
/endiylbarV~
{goto}yxV~
{down 2}
/encyxbarV~{esc}
.{right ystocks-1}~
/endiylW1x~
{goto}yxbarV~
{down 2}
/encyW1x~{esc}
.{right ystocks-1}~
/endiylW2xbar~
{goto}yW1x~
{down 2}
/encyw2xbar~{esc}
.{right ystocks-1}~
/endiylxt1~
{goto}yw2xbar~
{down 2}
/encyxt1~{esc}
.{right ystocks-1}~
/endiylchoriz~

```

```

{goto}yxt1~
{down 2}
/encychoriz~{esc}
.{right ystocks-1}~
/endyY01~
{goto}ychoriz~
{down 2}
/encyY01~{esc}
.{right ystocks-1}~
/endyY01V~
{goto}yY01~
{down 2}
/encyY01V~{esc}
.{right ystocks-1}~
/endycvert~
{goto}yY01v~
{down 2}
/encycvert~{esc}
.{down ystocks-1}~
/endyxt~
{goto}ycvert~
{right 2}
/encyxt~{esc}
.{down ystocks-1}~
/endyVx~
{goto}yxt~
{right 2}
/encyVx~{esc}
.{down ystocks-1}~
/endyxbart~
{goto}yVx~
{right 2}
/encyxbart~{esc}
.{down ystocks-1}~
/endyY01t~
{goto}yxbart~

```

```
{right 2}
/encyY0lt~{esc}
.{down ystocks-1}~
{;stepon}
{return}
```

zMeantranspose

```
/evymeanvector~
ycvert~
/etycvert~
ychoriz~
{return}
```

zMeanMove

```
/ev
ymeanvector~
ychoriz~
{return}
```

zTitles

```
{zdatawin}
{goto}youtputarea~{down yrepeats}
{for ydisplaycount,1,ystocks,1,zwritex}
m~{right}^lambda~{right}^mean~{right}
^Std.Dev.~{right}
/wpm
{return}
```

zWriteX

```
@string([a:\macro.sa.wq l]ydisplaycount,0)~
/ev~~
{right}
{return}
```

zHistorical

```
/ev
```

ychoriz~

yc~

{return} ..

zShrinkage

/efyc~1~0~~

{zdatawin}

{goto}youtputarea~

{down yrepeats}

Minimum Variance Portfolio~

/wpm

{let yrepeats,yrepeats+1}{let ym,0.5}

{zprogram}

/wpm{goto}ycfinal~

{for ycountf,1,ystocks,1,zfillshrink}

/tamycfinal~

yproportions~

ypfmean~

{zdatawin}

{goto}youtputarea~{down yrepeats}

{right ystocks}

Undefined~{right}

Undefined~{right}

+{a:\macro.sa.wq1}ypfmean~/ev~~

/wpm{let yY0,ypfmean}{for ycount,1,ystocks,1,zfillly01}

/et

yY01~

yY01t~

/tam

yY01~

yVinverse~

yY01V~

/tam

yY01v~

yY01t~

yY01Vy01~

```

{let ylambdashrink,(((ystocks+2)*(ysamplesize-1))/(yY01Vy01*(ysamplesize-ystocks-
  2)))}
{let yWeight,ylambdashrink/(ylambdashrink+ysamplesize)}
{for ycount,1,ystocks,1,zshrinkcfill}
{let yrepeats,yrepeats+1}
{zdatawin}
{goto}youtputarea~
{down yrepeats}
Shrinkage Estimates~
{down}
{let yrepeats,yrepeats+1}
{for ycount,1,ystocks,1,zdisplayc}
{right}{up}
Weight~
{down}
@cell("contents",[a:\macro.sa.wq1]yweight)~
/ev~~
{let yrepeats,yrepeats+2}
/wpm
{return}

```

zFillsrink

```

{if @cellindex("contents",yx,ycountf-1,0)>0}{zPutinsrink}
{return}

```

zPutinsrink

```

@cellindex("contents",ymeanvector,ycountf-1,0)~
/ev~~
{right}
{return}

```

zFilly01

```

{put yY01,ycount-1,0,@cellindex("contents",ymeanvector,ycount-1,0)-yY0}
{return}

```

zShrinkcfill

```
{put yc,ycount-1,0,(@cellindex("contents",ymeanvector,ycount-1,0)*(1-
  @CELL("contents",yweight)))+( @CELL("contents",yweight)*@CELL("contents",y
  Y0))}
{return}
```

zDisplayc

```
@cellindex("contents",[a:\macro.sa.wq1]yc,[a:\macro.sa.wq1]ycount-1,0)~
/ev~~
{right}
{return}
```

zProgram

```
{stepoff}{zclear}
{let yrepeats,yrepeats+1}
{let ycount,0}{for ycount,1,ystocks,1,zphi0calc}
{let ycount,0}
{for ycount,1,ystocks,1,zCreatex}
/et
yx~
yxt~
/evyx~yxbar~
/evyxt~yxbart~
{branch zIteration2}
```

zPhi0calc

```
{put yphi0,ycount-1,0,@cellindex("contents",yc,ycount-1,0)-
  ym*sqrt(@cellindex("contents",ycovmatrix,ycount-1,ycount-1))}
{}
{}
{}
{}
{if ycount=1}{zfirst}
{if (@CELLINDEX("contents",yphi0,ymax-1,0))<(@cellindex("contents",yphi0,ycount-
  1,0))}{zfirst}
{return}
```

zFirst

```
{let ymax,ycount}
{return}
```

zCreatex

```
{put yx,ycount-1,0,@if(ycount=ymax,1,0)}
{return}
```

zIteration2

```
/et
yx~
yxt~
{branch zdifferent}
```

zDifferent

```
/tam
ycovmatrix~
yxt~
yvx~
/tam
yx~
yvx~
yxvx~
{let yrootvx,@sqrt(yxvx)}
/tam
yx~
ycovmatrix~
ysigma~
{goto}yphixi~
{for ycountphi,1,ystocks,1,zcalcphixi}
{for ycountphi,1,ystocks,1,zphicompare}{;stepon}
{if @cellindex("contents",yx,yphimax-1,0)<>0}{Branch zFinIter}
{for ycountxbar,1,ystocks,1,zcreatexbar}
/etyxbar~yxbart~{;stepoff}
{branch zDual}
```

zCalcPhixi

```
{put yphixi,ycountphi-1,0,@cellindex("contents",yc,ycountphi-1,0)-
  (ym*@cellindex("contents",ysigma,ycountphi-1,0))/yrootxvx}
{return}
```

zPhiCompare

```
{if ycountphi=1}{zMaxphi}
{if (@cellindex("contents",yphixi,ycountphi-
  1,0))>(@cellindex("contents",yphixi,yphimax-1,0))}{zMaxphi}
{return}
```

zMaxphi

```
{if (@cellindex("contents",yphixi,countphi-
  1,0))>(@cellindex("contents",yphixi,yphimax-1,0))}{zMaxph
{let yphimax,ycountphi}
{return}
```

zCreatexbar

```
{put yxbar,ycountxbar-1,0,@if(ycountxbar-1=yphimax-1,1,0)}
{return}
```

zDual

```
/tam
yc~
yxt~
ygamma0~
/tam
yc~
yxbart~
ygamma1~
/etygamma~
ygammat~
/tam
yx~
ycovmatrix~
yxV~
```

/tam
 yxbar~
 ycovmatrix~
 yxbarv~
 /tam
 yxv~
 yxt~
 yu~
 /tamyxV~
 yxbart~
 {esc}{down}~
 /tam
 yxv~
 yxbart~
 {;esc}
 {right}
 {up}~
 /tam
 yxbarv~
 yxbart~
 {;esc}
 {down}~
 /tai
 yu~
 yuinverse~
 /tam
 ygamma~
 yuinverse~
 ygammaUinv~
 /tam
 ygammaUinv~
 yqt~
 yguq~
 /tam
 ygammaUinv~
 ygammat~

```

ygug~
/tam
yq~
yuinverse~
yquinv~
/tam
yquinv~
yqt~
yquq~
{Let ylambdaplus, ((yguq+(yguq^2-yquq*(ygug-ym^2))^.5)/yquq)}
{Let ylambdaminus, ((yguq-(yguq^2-yquq*(ygug-ym^2))^.5)/yquq)}
{let ylambda, @min(ylambdaplus,ylambdaminus)}
{put ylambdaq,0,0,ylambda}
{put ylambdaq,0,1,ylambda}
{put yglq,0,0, @cellindex("contents",ygamma,0,0)-ylambda}
{put yglq,0,1, @cellindex("contents",ygamma,1,0)-ylambda}
/tam
yuinverse~
yglq~
yw~
{let
  yw1,(@cellindex("contents",yw,0,0))/(@cellindex("contents",yw,0,0)+@cellindex("c
  ontents",yw,0,1))}
{let
  yw2,(@cellindex("contents",yw,0,1))/(@cellindex("contents",yw,0,0)+@cellindex("c
  ontents",yw,0,1))}
/eeyw1x~
/eeyw2xbar~
{for ycountxt,1,ystocks,1,zcreateyw1x}
{for ycountxt,1,ystocks,1,zcreateyw2x}
{}
{}
{}
{}
{for ycountxt,1,ystocks,1,zfillxt1}
/ev

```

```

yxt1~
yx~
{branch ziteration2}

```

zFillxt1

```

{put yxt1,ycountxt-1,0,@cellindex("contents",yw1x,ycountxt-
1,0)+@cellindex("contents",yw2xbar,ycountxt-1,0)}
{return}

```

zCreateyw1x

```

{put yw1x,ycountxt-1,0,@cellindex("contents",yx,ycountxt-1,0)*yw1}
{return}

```

zCreateyw2x

```

{put yw2xbar,ycountxt-1,0,@cellindex("contents",yxbar,ycountxt-1,0)*yw2}
{return}

```

zFinIter

```

{;stepon}
{let yincluded,0}
{for ycountf,1,ystocks,1,zcreatecfinal}
{zdataarea4}
{goto}ycfinal~
{for ycountf,1,ystocks,1,zfillc}
/ev
ycfinal~
ycfinal~
/et
ycfinal~
ycfinalt~
{let yvfinalrow,0}
{goto}yvfinal~
{for yvcount1,1,ystocks,1,zVloop}
/ev
yvfinal~
yvfinal~

```

```

/tai
yvfinal~
yvinvfinal~
/tam
ycfinal~
yvinvfinal~
ycinVfinal~
/tam
yp~
yvinvfinal~
ypinVfinal~
/tam
ycinVfinal~
ycfinalt~
ycvc~
/tam
ycinvfinal~
ypt~
ycvp~
/tam
ypinVfinal~
ypt~
ypvp~
{let ylambdaplus,(ycvp+((ycvp)^2-(ypvp)*(ycvc-ym^2))^0.5)/(ypvp)}
{let ylambdaminus,(ycvp-((ycvp)^2-(ypvp)*(ycvc-ym^2))^0.5)/(ypvp)}
{let ylambda,@min(ylambdaplus,ylambdaminus)}
/ef
ylambdap~
ylambda~
0~10^99~
{for ycountf,1,yincluded,1,znormala}
/ev
yclambdap~
yclambdap~
/et
yclambdap~

```

```

yclambdapt~
/tam
yvinvfinal~
yclambdapt~
ys~
{let ystotal,@sum(ys)}/evys~ys~
{for ycountf,1,yincluded,1,zRatio}
/tam
ycfinal~
yproportions~
ypfmean~
/tam
yvfinal~
yproportions~
yvproport~
/et
yproportions~
yproportt~
/tam
yproportt~
yvproport~
yPFxvx~
{let ypfsd,@sqrt(yPFxvx)}
{let ycounter,1}
{zdatawin}
{goto}youtputarea~
{down yrepeats}
{for ylastcount,1,ystocks,1,zxoutput}
+[a:\macrosa.wq1]ym~/ev~~
{right}
+[a:\macrosa.wq1]ylambda~/ev~~
{right}
+[a:\macrosa.wq1]ypfmean~/ev~~
{right}
+[a:\macrosa.wq1]ypfsd~/ev~~
/wpm

```

```
{return}
```

zCreatecfinal

```
{if @cellindex("contents",yx,ycountf-1,0)<>0}{zportnos}
{return}
```

zPortnos

```
{let yincluded,yincluded+1}
{return}
```

zFillc

```
{if @cellindex("contents",yx,ycountf-1,0)>0}{zPutin}
{return}
```

zPutin

```
@cellindex("contents",yc,ycountf-1,0)~
/ev~~
{right}
{return}
```

zVloop

```
{for yvcount2,1,ystocks,1,zVfill}
{if @cellindex("contents",yx,yvcounti-1,0)<>0}{let yvfinalrow,yvfinalrow+1}
{goto}yvfinal~
{down yvfinalrow}
{return}
```

zDataarea4

```
{stepoff}
/endycfinal~
{goto}ycvert~
{down ystocks+1}
/encycfinal~{esc}
.{right yincluded-1}~
/endyp~
{goto}ycfinal~
{down 2}
```

```

/encyp~{esc}
.{right yincluded-1}~
/endycinvfinal~
{goto}yp~
{down 2}
/encycinvfinal~{esc}
.{right yincluded-1}~
/endypinvfinal~
{goto}ycinvfinal~
{down 2}
/encypinvfinal~{esc}
.{right yincluded-1}~
/endylambdap~
{goto}ypinvfinal~
{down 2}
/encylambdap~{esc}
.{right yincluded-1}~
/endyclambdap~
{goto}ylambdap~
{down 2}
/encyclambdap~{esc}
.{right yincluded-1}~
/endyproportt~
{goto}yclambdap~
{down 2}
/encyproportt~{esc}
.{right yincluded-1}~
/endyVfinal~
{goto}yproportt~
{down 2}
/encyVfinal~{esc}
.{down yincluded-1}{right yincluded-1}~
/endyvinvfinal~
{goto}yvfinal~
{down yincluded+1}
/encyVinvfinal~{esc}

```

```

.{down yincluded-1}{right yincluded-1}~
/andypt~
{goto}yvinyfinal~
{down yincluded+1}
/encypt~{esc}
.{down yincluded-1}~
/andycfinalt~
{goto}ypt~
{right 2}
/encycfinalt~{esc}
.{down yincluded-1}~
/andyclambdapt~
{goto}ycfinalt~
{right 2}
/encyclambdapt~{esc}
.{down yincluded-1}~
/andys~
{goto}yclambdapt~
{right 2}
/encys~{esc}
.{down yincluded-1}~
/andyproportions~
{goto}ys~
{right 2}
/encyproportions~{esc}
.{down yincluded-1}~
/andyvproport~
{goto}yproportions~
{right 2}
/enc yvproport~{esc}
.{down yincluded-1}~
/efyp~1~0~~
/etyp~ypt~
{;stepon}
{return}

```

zSetwindow

```
{let zdatawin,/wp}
{return}
```

zDatawin

```
/wp
@left(yname,1)      *** Note - this is entered as a formula, not a label.***
```

zClear

```
/eeygamma~
/eeygammat~
/eeyu~
/eeyuinverse~
/eeygammauinv~
/eeyquinv~
/eeyglq~
/eeylambdaq~
/eeyw~
/eeyphi0~
/eeyx~
/eeysigma~
/eeyphixi~
/eeyxbar~
/eeyxv~
/eeyxbarv~
/eeyw1x~
/eeyw2xbar~
/eeyxt1~
/eeychoriz~
/eeycvert~
/eeyxt~
/eeyvx~
/eeyxbart~
/eeycfinal~
/eeyp~
/eeycinvfinal~
```

```

/eeypinvfinal~
/eeylambdap~
/eeyclambdap~
/eeyproportt~
/eeyvfinal~
/eeyvinvfinal~
/eeypt~
/eeycfinalt~
/eeyclambdapt~
/eeys~
/eeyproportions~
{return}

```

zFind

```

{if @round(ylambda,yaccuracy)=@round(ylambdastar,yaccuracy)}{zdatawin}{quit}
{if
  @round(ylambda1,yaccuracy)<@round(ylambdastar,yaccuracy)#and#@ROUND(yla
  mbda2,yaccuracy)<@ROUND(ylambdastar,yaccuracy)}{zdecn}
{if
  @round(ylambda1,yaccuracy)>@round(ylambdastar,yaccuracy)#and#@ROUND(yla
  mbda2,yaccuracy)>@ROUND(ylambdastar,yaccuracy)}{zincn}
{if
  @round(ylambda1,yaccuracy)>@round(ylambdastar,yaccuracy)#and#@ROUND(yla
  mbda2,yaccuracy)<@ROUND(ylambdastar,yaccuracy)}{zinterpolate}
{zprogram}
{zFind}

```

zIncn

```

{let ylambda1,ylambda2}{let ym1,ym2}
{let ym,ym2+.2}
{zprogram}
{let ylambda2,ylambda}{let ym2,ym}
{let ylambda,ylambda1}{let ym,ym1}{branch zfind}

```

zDecn

```

{let ylambda2,ylambda1}{let ym2,ym1}

```

```
{let ym,ym1-.2}
{zprogram}
{let ylambda1,ylambda}{let ym1,ym}
{let ylambda,ylambda2}{let ym,ym2}{branch zfind}
```

zInterpolate

```
{if ylambda>ylambdastar}{zadjustup}
{if ylambda<ylambdastar}{zadjustdown}
{let ym,ym1+((ylambda1-ylambdastar)/(ylambda1-ylambda2))*(ym2-ym1)}
{return}
```

zAdjustup

```
{let ylambda1,ylambda}
{let ym1,ym}
{return}
```

zAdjustdown

```
{let ylambda2,ylambda}
{let ym2,ym}
{return}
```

zVFill

```
{if @cellindex("contents",yx,yvcount1-
  1,0)<>0#and#@cellindex("contents",yx,yvcount2-1,0)<>0}{zputinV}
{return}
```

zPutinV

```
@cellindex("contents",ycovmatrix,yvcount1-1,yvcount2-1)~
/ev~~
{right}
{return}
```

zNormalA

```
{Put yclambdap,ycountf-1,0,((@cellindex("contents",ycfinal,ycountf-1,0))-
  (@cellindex("contents",ylambdap,ycountf-1,0))):value}
{return}
```

zRatio

```
{put yproportions,0,ycountf-1,@cellindex("contents",ys,0,ycountf-1)/ystotal}
{return}
```

zXoutput

```
{if @cellindex("contents",yx,ylastcount-1,0)<>0}{zdisplayval}
{if @cellindex("contents",yx,ylastcount-1,0)=0}{zdisplay0}
```

zDisplayval

```
@cellindex("contents",[a:\macro.sa.wq1]yproportions,0,[a:\macro.sa.wq1]ycounter-1)~
/ev~~~{right}
{let ycounter,ycounter+1}
{return}
```

zDisplay0

```
0~
{right}
{return}
```

zSearch

```
{zdataarea3}
{let yrepeats,0}
{;Mean Calculation}
{if @rows(ymeanvector)>1}{zmeantranspose}
{if @rows(ymeanvector)=1}{zmeanmove}
{if ymeantype="historical"}{zHistorical}
{if ymeantype="shrinkage"}{zShrinkage}
{;let yrepeats,0}
{zTitles}
{;Main Loop}
{let ym,ym1}
{zProgram}
{let ylambda1,ylambda}
{if @round(ylambda,yaccuracy)=@round(ylambda1,yaccuracy)}{zdatawin}{quit}
{let ym,ym2}
```

```
{zProgram}  
{let ylambda2,ylambda}  
{if @round(ylambda,yaccuracy)=@round(ylambdastar,yaccuracy)}{zdatawin}{quit}  
{branch zFind}
```