Making it Count: Teacher Actions to Support the Development of Multiplicative Reasoning Through the Use of Choral Counting Conceptual Starters

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Abstract

Learning to think multiplicatively is one of the most challenging transitions primary aged students must make in their journey of mathematical learning. This study explores the development of multiplicative reasoning among Year Five to Eight students in Aotearoa New Zealand through a conceptual starter activity involving choral counting. Additionally, it examines the pedagogical actions used by teachers to support students in engaging in collaborative discourse and advance students' multiplicative reasoning through the enactment of mathematical practices.

Relevant literature is examined to illustrate how students develop multiplicative reasoning and what key mathematical concepts they must first understand. The literature review provides evidence of the important role of teacher pedagogical actions that support students' multiplicative reasoning through facilitating their participation in investigating, noticing, conjecturing, explaining, justifying, and generalising.

An interpretivist approach was taken for this qualitative classroom-based case study. A collaborative planning and reflective partnership was established with two teacher participants to support the lesson development. Teacher interviews, video recorded observations, and classroom artefacts made up the data collection. On-going and retrospective data analysis was used to form the findings of this study.

The findings revealed important advances in students' multiplicative reasoning as the teachers facilitated collaborative discourse. Students were provided with many opportunities to share and clarify their thinking while building on the ideas of others. This occurred as explicit counting, place value, and multiplicative concepts were explored. The research findings provide insights into ways teachers can support students to develop conjectures, explanations, justification, and generalisations as they engage with the choral count starter activities. The results of this study suggest that student participation in choral count starter activities where teachers emphasise the enactment of mathematical practices through collaborative discussion support their development of multiplicative thinking.

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Chapter One: Introduction

1.1 Introduction

This chapter presents the background, rationale, and context for this study. Section 1.2 outlines current mathematical achievement challenges that students in Years Five to Eight are facing in Aotearoa New Zealand primary schools. This section also highlights how educators can support students to progress towards multiplicative reasoning strategies. Section 1.3 describes the rationale for this study, and section 1.4 provides the research questions. Section 1.5 provides an overview of the thesis.

1.2 Background to the Study

International research has highlighted that one of the most difficult transitions for students to make in their mathematics education is from additive to multiplicative reasoning (e.g. Askew, 2018; Callingham & Siemon, 2021; Hurst & Hurrell, 2016; Proulx, 2024; Siemon, 2013). In the Aotearoa New Zealand context, the current mathematics curriculum expects students in Years Three and Four to use additive strategies when working with whole numbers. This shifts in Years Five and Six to students being expected to know and use a range of multiplication strategies when working with whole numbers (Ministry of Education, 2007a). There is a considerable difference in the application of expected solution strategies for students in Year Five, signalling a significant area of new learning that is required, in order for students to be able to confidently work with larger numbers and more complex problems. Without first mastering a trust in counting, number sequences, and place value concepts when working additively, it may be difficult for students to progress to confidently using multiplicative strategies (Siemon et al., 2012).

In 2022, The National Monitoring Study of Student Achievement (NMSSA) reported that 42% of students in their final year of primary school in Aotearoa New Zealand were working at the appropriate curriculum level (NMSSA, 2022). As a part of current education reforms, the New Zealand Curriculum is in a state of refresh and is expected to be implemented in 2025. A 2023 NMSSA report highlighted that only 22% of students in Year Eight, their final year of primary school, were working at or above the expected curriculum level of the draft refreshed New Zealand Curriculum, with 66% of Year Eight students behind by more than one year (NMSSA, 2023). While students who took part in this study had not been formally taught from the refreshed curriculum, it highlights that there will continue to be areas where students require deeper knowledge and conceptual understanding if they are to successfully meet these new curriculum achievement standards.

1.3 Rationale

Government policy requires all students in Aotearoa New Zealand to participate in one hour of mathematics daily. While warm-up activities such as games with a focus on basic facts are common practice in many mathematics classrooms, a conceptual starter provides an opportunity for classroom teachers to develop and deepen mathematical understanding of their students, fully utilising the time allocated for mathematics instruction. For the purpose of the current study, a conceptual starter is defined as a planned starter activity that focusses on unpacking identified student misconceptions and scaffolding new learning of a mathematical concept through collaborative discussion.

When carefully planned and enacted, conceptual starter activities can provide opportunities for students to reason specifically with multiplicative concepts, making conjectures, developing explanations and justifications, and generalising multiplicative patterns. Multiplicative reasoning supports students to develop fact fluency, going beyond practicing and reciting memorised basic facts. Fact fluency is developed as students acquire a deep understanding of number patterns and place value and apply basic facts flexibly to larger numbers and problems (Siemon et al., 2012). Through collaborative discussion, all students are afforded the opportunity to develop their reasoning skills, grow their mathematical vocabulary, and learn to see solution strategies from multiple perspectives. As well as supporting individual learning, this collaborative approach also cultivates a community of learners where students feel empowered as their ideas and contributions are valued (Kazemi & Hintz, 2023).

The current study utilises choral counting as a conceptual starter activity. While collective counting activities are usually reserved for younger students, choral counting is a collaborative activity that allows students of all ages to collectively articulate their thought processes as specific number patterns are recorded and explored through facilitated discourse that seeks to understand why, and how these patterns occur (Franke et al., 2023). In the current study, lesson plans were carefully constructed to ensure that specific number patterns and multiplicative concepts would be explored with planned pauses and questions to guide teachers to facilitate deep discussion.

1.4 Research Objectives

The purpose of the current study is to explore the effectiveness of planned choral counting conceptual starter activities as a means to develop multiplicative reasoning in Year Five to Eight students. The study seeks to examine the specific teacher actions that contribute to the advancement of student reasoning. Two research questions are addressed:

1. How do planned choral counting starter activities support students to develop mathematical practices?
2. What teacher actions during a conceptual starter activity support the development of students' multiplicative reasoning?

1.5 Overview

Chapter Two provides a review of relevant literature from both Aotearoa New Zealand and international studies focussed on how students develop mathematical reasoning skills. The review highlights how students develop multiplicative reasoning skills and why these are necessary for students to advance their mathematical understanding. The review identifies specific teacher pedagogical actions that support the development of mathematical reasoning skills and promote participation in collaborative discourse. The review also examines traditional starter activities and basic facts practice in relation to how they support students to develop multiplicative thinking. Choral counting is examined as an example of a starter activity that can be used with students in Years Five to Eight to explore conceptual understanding of number patterns with a focus on collaborative discourse.

Chapter Three outlines the methodology used in the current study and the research design. Data collection and analysis methods are explained and the research setting, and participants are introduced. An outline of the research timeline is provided. Ethical considerations are addressed, and the role of the researcher is described. The validity and reliability of the current study is described.

Chapter Four presents the results of the study through an integrated analysis and discussion of the findings. The chapter examines the specific teacher pedagogical actions that supported students to make shifts in their multiplicative conceptual understanding. Specific pedagogical actions are analysed and discussed.

Finally, Chapter Five provides the conclusions of the study. The research questions are addressed and a summary of key findings and suggestions for practice are presented. Limitations of the study are outlined and suggestions for further research are provided.

Chapter Two: Literature Review

2.1 Introduction

The previous chapter introduced the background to the study and explained the rationale and the context of the study. The purpose of this chapter is to synthesize existing literature relevant to the context of this study. First, section 2.2 will explore the definition of mathematical reasoning and how this is positioned in the context of the New Zealand Curriculum. Section 2.3 examines specific pedagogical actions that teachers employ to support mathematical reasoning and sense making. Section 2.4 reviews relevant literature describing multiplicative thinking and how this is developed in students. Section 2.5 examines earlier research from both Aotearoa New Zealand and international settings focused on student participation in starter or warm-up activities and the impact of this on student learning and engagement. The final section 2.6 concludes with an exploration of choral counting and how this activity can be used to provide opportunities for students to develop both mathematical reasoning and multiplicative thinking.

2.2 Mathematical Reasoning

Changing perspectives of what it means for students to be knowers and doers of mathematics has seen a growing focus on how students' reason and make sense of mathematical concepts. As explained by Lannin et al. (2011), mathematical reasoning supports students to move from knowledge of limited examples towards recognition and understanding of relationships. To recognise these relationships and make sense of how and why they exist, students need to be afforded rich opportunities to investigate why particular mathematical concepts may be true or false, make and examine both valid and invalid conjectures, develop mathematical explanations, justify, and argue their own and others reasoning and make generalisations that extends the reasoning beyond the example from which it originated. Ball (2003) explains that in order to fully understand the how and the why of mathematics, it is the effective use of such mathematical practices as explaining, justifying, representing, clarifying and generalising that students and teachers become confident knowers and doers of mathematics. This enables the flexible use of mathematical knowledge when problem solving.

In educational settings, mathematical reasoning and practices are a key component of international mathematic curriculum documents. In Australia for example, mathematical reasoning is one of the four key mathematical proficiencies of the mathematics curriculum

(McCluskey et al., 2016). In the United States of America, mathematical reasoning is woven throughout the Standards for Mathematics. It is an important feature of the Common Core State Standards for Mathematical Practice, which provides detail in the ways in which students should be engaging with and learning mathematics (National Governors Association Center for Best Practices, Council of Cheif State School Officers, 2010). In the Aotearoa New Zealand context, the mathematics curriculum for primary schools is in a current state of redesign. A 2024 draft curriculum highlights a prominent focus on mathematical reasoning and how students "do" mathematics, with explicit mathematical practice expectations and progressions defined for each learning phase (Ministry of Education, 2024). This is a significant shift from typical mathematical classroom practices in Aotearoa New Zealand, where strategy and knowledge of mathematical concepts has been the focus of current and previous curriculum documents and mathematics programmes (Hunter & Hunter, 2018). As emphasised by Ball (2003), mathematical practices are not taught or learned in isolation, but rather within learning focused on student engagement in reasoning and sense-making.

The following sections explore in more detail the specific mathematical practices of investigating and conjecturing, developing explanations, justification and argumentation, as well as generalisation. These practices support students in reasoning and developing a deeper conceptual understanding of mathematics.

2.2.1 Investigating and Conjecturing

The Investigation of mathematical concepts and the formulation of conjectures regarding why or how various solution strategies work or do not work is fundamental to the process of mathematical reasoning. Such activities frequently serve as access to mathematical tasks and contribute to sense making (Lannin et al., 2011). Notably, what students notice as they interact with a task can serve as the foundation for developing a conjecture. This initial noticing or centre of focus has consequences for mathematical reasoning and understanding (Lobato et al., 2013). Lobato and colleagues, provide an example of two classes of seventh grade students, aged 12-to-13-years-old, which shows how the focus of their noticing dictated what conjectures students made and how well they reasoned with new mathematical concepts. In one of the tasks, students who only noticed one variable when looking at rates of change, were unable to conjecture as to how both rates were changing in relationship with each other when developing possible functional equations. Similarly, in her study with students aged 7-to-8-years-old, Hunter (2015) detailed more closely how a Year Three teacher used student noticing's, both planned and spontaneous, to develop conjectures which then lead to justifications and generalisations. It was through the investigation of student noticing of algebraic patterns, that they were able to form conjectures which then lead them to being able to prove or disprove their thinking as to why or how patterns were changing and growing. As highlighted by these two studies, the quality of the investigation of initial noticing led to the development of conjectures, which are the mathematical practices that support students to begin making sense of new mathematical ideas.

2.2.2 Developing Explanations

Developing explanations are a key component of students' construction of mathematical arguments. Explanations provide students with an opportunity to demonstrate their understanding, to communicate mathematical reasoning and to help someone else understand (Ingram et al., 2019). Moschkovich (2015a) claims that mathematical explanations support linguistic objectives, as students are provided opportunities to develop mathematical communication skills using specific mathematical language. As this technical language is learned and used, mathematical explanations can deepen. Students often require support to offer and construct their explanations. A UK study by Ingram et al (2019) involving students aged 7-to-18-years-old, and 17 teachers ranging in experience from three to 30 years, found that in large group discussions students rarely offered an explanation without a teacher prompt asking for a 'how' or 'why' response to an initial idea or solution raised. In contrast, a comprehensive study conducted by Hunter et al. (2018) in Aotearoa New Zealand explored teacher practices that foster collaborative enquiry in mathematics classrooms across thirty-two primary schools, involving students aged 5-to-13-years-old. The findings indicate that as students become more actively responsible for their own mathematical understanding and developed the confidence to ask and answer questions to clarify their thinking, the reliance on teacher-led orchestration of these interactions diminished.

2.2.3 Justification and Argumentation

Developing explanations is the basis for students progressing towards justification and argumentation. Being able to justify why a mathematical conjecture is true or false supports students ability to demonstrate their understanding of mathematical concepts (Bieda et al., 2022). Staples (2014) examined the pedagogical actions of two Middle school teachers in the USA that supported students to make justifications. By examining discourse of students aged 12-to-13-years-old, they concluded that justifications were used by students to agree, disagree, build, and refute mathematical ideas, as they made contributions to large group discussions. Building on ideas included students asking questions to query ideas or advance their own understanding. While this particular study demonstrates effective pedagogy that led to student justification, both Bieda (2010), and Jacobs et al. (2006), found when observing students aged 11-to-13-years-old in USA middle schools, student justifications were rare, and while opportunities to justify presented themselves, they did not often lead to the development of ideas. Clearly, supporting student justifications and arguments that lead to the development of conceptual understanding can be challenging for teachers, and students need appropriate scaffolding to achieve it.

2.2.4 Generalisation

Making generalisations occurs when students have been able to explain, justify, and examine their reasoning. It requires their active engagement in examining multiple patterns and testing possible solutions. Mathematical generalisation is the act of testing the properties of a mathematical claim to see if it holds true in an infinite number of cases (Carraher et al., 2008). In order to generalise, students are required to apply their understanding of a concept to an unknown situation and identify commonalities across the various cases, extending reasoning beyond the example from which it originated (Lannin et al., 2011). Generalisation is an important component of mathematical reasoning as it allows students to develop flexibility in their thinking as they "adapt, adjust and reorganise previous experiences" (Pinto & Cañadas, 2021, p. 114). Gibbs (2020) describes the ways in which a group of diverse students in Aotearoa New Zealand aged 10-to-12-years-old used a combination of both natural language and symbolic language when making generalisations about algebraic functions as a way of explaining how patterns were growing. This use of natural language helped students develop symbolic notation of their generalisations that referred to rules for the patterns. Gibbs (2020) highlighted the importance of known contexts, language and dialogue, including speakers of other languages, as a way to support students to express their understanding of generalised algebraic rules.

This section has discussed mathematical reasoning and explored the specific mathematical practices of investigating and conjecturing, explaining, justifying, and generalising. The following section explores, and outlines research related to teacher pedagogical actions that support students to reason with mathematical concepts.

2.3 Teacher Pedagogical Actions to Support Mathematical Reasoning

This section explores in detail the pedagogies of inquiry classrooms, collaborative discussion and teacher content knowledge.

2.3.1 Inquiry Classrooms

Inquiry classrooms in mathematics education focus on problem-solving, investigation and collaboration as the key strategies to foster deep mathematical understanding in students (Goos, 2004). In the Aotearoa New Zealand context, a research study by Hunter (2010) focused on how teachers can support the development of mathematical inquiry with students aged 5-to-13-years-old. In this classroom with Pacific and Māori students, an important aspect was mathematics being taught and learnt as a collaborative activity, where students worked as whānau or family to solve mathematics problems collectively. In inquiry communities such as described in Hunter (2010), students work in mixed-ability groups to grapple with a challenging task, and then discuss and share their thinking with the larger group to come to a collective understanding of mathematical concepts. Inquiry communities are informed by and contribute to international research that emphasises the development of key mathematical practices such as conjecturing, representing, explaining, justifying, and generalising as central to the progress of student mathematical capabilities (Hunter et al., 2018).

For students to participate equitably and successfully in inquiry classrooms, a respectful and safe environment must be created to ensure that learners feel comfortable taking risks and engaging in discussions with their peers, where their ideas can be questioned, clarified and explored (Boaler, 2006; Hunter et al., 2018; Johnson et al., 2022). The first step towards students feeling supported to collaborate with one another in pro-social ways is through establishing and promoting classroom norms (Kazemi & Hintz, 2023; Mueller et al., 2014). Students begin to form identities of themselves as mathematicians from an early age. These mathematical identities are based on whether students think they are good at maths or not, and are constructed from the classroom setting and ideals of what constitutes being good at mathematics (Darragh, 2015). An Aotearoa New Zealand based study of 22 students aged 12-to-13-years-old by Darragh (2015) highlighted that these students viewed concepts of putting their hand up and answering questions correctly, being fast and getting high marks as indicators of being good at maths. This study highlights the potential pre-existing beliefs that students bring to the mathematics classroom that may impact the outcomes of collaborative approaches to mathematics learning. The following section discusses in detail the fundamental aspect of inquiry classrooms of collaborative discourse.

2.3.2 Collaborative Discourse

For students to engage in mathematical reasoning, they must be given space to discuss their thinking and reasoning with each other with support from a teacher facilitating the discussion (Askew, 2018; Boaler, 2006; Chapin & O'Connor, 2007; Johnson et al., 2022; Moschkovich, 2015b; Selling, 2016). Hunter (2010) explains that providing rich opportunities for discussion is a complex task, and something that many teachers will grapple with due to their own educational experiences, fundamental beliefs about student-teacher participation roles, the views of the wider school communities and also the beliefs of students themselves. Ball et al. (2005) further explain that student understanding of mathematical concepts can often be incomplete and difficult for them to communicate, therefore difficult for others to interpret and understand. Productive discussions that support mathematical reasoning to develop require specific teacher actions to ensure new learning is taking place.

Chapin & O'Connor (2007) explained that there are certain "talk moves" that teachers can use to initiate discussion among all students. Prompting questions that ask students to revoice or repeat what another student has shared, and agree or disagree with an idea being presented, all provide an opportunity for students to clarify understanding, reason with mathematical thinking and help to advance the thinking of the larger group. Kazemi & Hintz (2023) added to these talk moves by suggesting a "turn and talk" (p. 21) move, where students share and clarify with a neighbour, followed by the teacher then asking if anyone has revised or changed their thinking, and a "revise" move where students are given the opportunity to reflect on their original ideas and decide if they have changed their thinking or not. Numerous research studies (e.g. Jacobs et al., 2022; O'Connor & Michaels, 2019; Walla, 2023) have highlighted how successful use of these talk moves have been shown to promote equitable participation for all students in various year groups as teachers shift their discussion focus from the transmission of information to instead information that is shared and accessible to all students.

Sustained conversations that lead to deeper reasoning require thoughtful questioning moves by teachers. A study by Franke and colleagues (Franke et al., 2015) involving students aged

4-to-12-years-old, highlighted the importance of teachers supporting students to deepen their understanding of their own ideas in relation to those of their peers. This was achieved by teachers intentionally pressing students for deeper reasoning through the use of thoughtful questioning, scaffolding, and positioning. Franke and colleagues concluded that misconceptions can be brought to the surface for further exploration through this deepened discussion as teachers support and sustain discourse. As Mueller and colleagues (Mueller et al., 2014) highlight, the teacher's role in these discussions is to understand from where the confusion stems from and what big idea needs clarification, rather than to simply correct students. They suggest this can be achieved by teachers ensuring that all students have the opportunity to safely make their ideas public and by amplifying contributions made and facilitating discussion that clarifies and supports students to attend to the ideas of their peers. Similar to Stein et al (2008), in relation to teacher monitoring, Franke et al. (2015) highlighted the importance of teachers noticing and monitoring the level of student engagement in collaborative discussions. While surface-level participation, where students merely agree or disagree with ideas shared, can serve as a starting point, participation can be extended when students are encouraged to reason further. Teachers can prompt students to build on each other’s ideas, propose alternative solutions, or collaborate to co-construct new ideas with their peers.

Traditional notions of direct teacher telling have been critiqued in recent reform education narratives. However, Lobato et al. (2005) argued that rather than minimising student opportunities to cognitively engage in mathematical exploration and reasoning, telling can be re-framed in ways that support student reasoning. Firstly, rather than declarative statements, teachers can employ a series of questions that may steer students towards the correct answer. In this situation, teachers may still make an assertive and substantive contribution to the discussion while maintaining a facilitating role, requiring students to justify or refute introduced ideas or information. The second attribute of supportive telling is when the focus of the teacher contributions is conceptual as well procedural. This involves supporting students to understand why procedures work and what connections there are to the students’ thoughts and understandings. Similarly, research by Moschkovich, (2007) examined specific teacher actions that scaffolded students towards new mathematical understanding. Her USA study of bilingual students aged 13-to-14-years-old noted that it is through a focus on mathematical practices, such as justifying and generalising that teacher contributions of new information support students to reason with mathematics and develop conceptual understanding. As summarised by Choy and Lai (2024), it is the intention of the telling that is important as this is what changes a traditional scenario of direct teacher instruction into an activity of mathematical reasoning for students. Research by Selling (2016) explains the importance of teachers explicitly naming and highlighting the use of mathematical practices students enact while participating in mathematical discussion. Selling's USA based study examined the mathematical discussions of 14-to-16-year-olds. Analysis showed that teacher actions that specifically identified the enactment of mathematical practices both within the lesson and across time, supported students to see when and how they were thinking and working mathematically and emphasised the relevance of how this contributed to overall understanding of the larger group.

It is important to note that teacher attempts to facilitate meaningful mathematical discourse amongst students may also produce counterproductive results. Zoest et al. (2023) examined mathematical discourse in both middle and high school classrooms with students aged 11-to-18-years-old and analysed whether teacher actions of collecting information, asking clarifying questions, and asking students to revoice were effective for supporting students to engage in high level sense-making. While these teacher actions often did result in advancing the learning of the students, there were also times where these same moves detracted from the overall development of student understanding. These researchers claim that when teachers over emphasise the contributions of as many students as possible that the opportunity to discuss already raised claims and questions becomes diluted. An example of this is Hunter's (2015) study of early algebraic reasoning with a class of Year Three students (aged 8-years-old) investigating the commutative property. In this case, students were initially limited in their opportunity to develop deep generalised understanding as the teacher herself provided a reason, without positioning students to investigate the claim further and explain why this example of the commutative property had worked.

2.3.3 Teacher Content Knowledge

Many researchers have concluded that the quality of mathematics teaching is directly impacted by the content knowledge of the teacher (e.g., Campbell et al., 2014; Ma, 1999). Further explanation by Ball et al. (2005) highlights the importance of pedagogical content knowledge of teachers, this moves beyond teacher understanding of mathematics to also encompass how and what their students may think about mathematics. Ball and colleagues (2005) define this as "knowing mathematics for teaching" meaning that teachers must not only understand why procedures and rules work, but also why a student might get something wrong, and also how to respond when a student solves something correctly but in an unanticipated way. For example, they describe how students when carrying out an algorithm for multiplication may misunderstand place value. They state that while being able to carry out this procedure correctly is essential for being able to teach the procedure, teachers must be able to examine student attempts and understand the source of any errors and be able to explain what specifically students understand and why these algorithms work. This affords students opportunities to reason with mathematics so that they can learn how and why solutions may or may not be correct.

A seminal research study by Stein and colleagues (2008), developed a framework of specific practices that teachers can use to respond to student solutions and develop rich discussion and greater conceptual understanding for students. The five practices documented by Stein et al. (2008) are anticipating, monitoring, selecting, sequencing, and making connections between student responses. Anticipating is the act of teachers planning for a range of student generated solutions and misconceptions. Monitoring involves teachers paying careful attention to how students are attempting to reason with mathematical concepts and understanding how and why students have come to this idea. Next, teachers carefully select and sequence student solutions for a large group discussion that allows students to understand the thinking of others, unpacking misconceptions and build deeper conceptual understanding (Meikle, 2016). Facilitating student discussion beyond sharing correct solutions can be challenging for teachers (Franke et al., 2015; Kazemi & Hintz, 2023). However, with the fifth practice of connecting teachers intentionally support all students to discuss and justify why results may or may not be correct, and to then generalise if this solution is also true in other situations or when using different numbers.

2.4 Multiplicative Thinking

Thinking multiplicatively goes beyond students simply knowing their times tables. Multiplicative thinking "is indicated by a capacity to work flexibly with the concepts, strategies and representations of multiplication (and division) as they occur in a wide range of contexts" (Siemon et al., 2005, p. 2). While being able to think multiplicatively is an indicator that upper primary students in Aotearoa New Zealand are meeting appropriate curriculum achievement levels, how students transition from additive to multiplicative thinking is not obvious nor is it consistent and linier (Proulx, 2024). This transition has been identified as one of the core obstacles preventing students from successfully learning mathematics concepts in the middle years of school (Siemon et al., 2005).

Askew (2018) examined how introducing young students to multiplicative concepts impacted primary school students' understanding of multiplicative reasoning. This study provides examples of 6-year-old students reasoning with multiplication concepts when engaging with functional relationship tasks and suggests that multiplicative reasoning can be developed as students informally produce models and communicate their solution pathways. Similarly, research by Cheeseman et al (2023) examined the responses of 21 students aged 5-to-6-years-old when they were given equal sharing tasks. This study highlighted that these young students were able to imagine and communicate concepts of two composite units. A key finding of this study is that this type of thinking can be seen as the initial stages of multiplicative thinking that leads to more sophisticated reasoning with multiplicative concepts later.

Other studies from Australia have focused on the development of big ideas in number. Siemon and colleagues (2012) identified a developmental hierarchy of distinct stages that are the pre-conditions to students being able to think multiplicatively, which in turn advances students towards proportional and algebraic reasoning. They suggest that trusting the count and place value knowledge are pre-requisites for multiplicative thinking. Trusting the count, the first phase of the developmental hierarchy, is explained by Hurst and Hurrell (2014) as students having deep understanding of the quantities associated with each number and how groups can be grouped and ungrouped in multiple ways while still giving the same result. It is this second point, they claim that informs the ideas of flexible partitioning and the distributive property of multiplication. Siemon et al. (2012) further add that being able to subitize, and having the ability to recognise the amount in a group without having to count is the foundation skill that supports students to work with larger numbers. The second stage in the developmental hierarchy, place value, supports students to understand number properties related to position, base ten, and number patterns, in the way we read and say numbers. By recognizing that numbers can be partitioned flexibly, applying both standard and non-standard groupings, students develop an understanding of the distributive property when working with factors and multiples (Hurst & Hurrell, 2014).

2.4.1 Fluency of Basic Facts

Students require strong foundational understanding of counting and partitioning properties before they can effectively think multiplicatively. When it comes to knowing and using basic facts such as times tables, it is important to understand the concept of fact fluency. Knowing basics facts is the ability to give the result of a computation automatically, rather than using a procedure to calculate. Having basic facts committed to memory is important as it eases the cognitive load on the brain when calculating more complex problems. Fact fluency goes beyond just knowing basic facts, it is also the ability to conceptually understand them (Small, 2019).

Fact fluency required learners to be flexible, accurate, efficient, and appropriate at using facts, and supports students to think multiplicatively as they move beyond additive and sharing strategies and learn to apply knowledge of factors and products when working with larger and more complex numbers (Siemon et al., 2012). In the Aotearoa New Zealand context, Tait-McCutcheon et al. (2011), explain that teachers' pedagogical actions may be at odds with their desired student outcomes. In this Aotearoa New Zealand based study in one classroom of students aged 10-to-11-years-old, the teacher identified how using timed basic facts tests to assess student knowledge of basic facts was an isolated event and did not provide students with opportunities to make connections to the maths they were doing in their daily lessons. There was also no affordance for students to make connections between sets of basic facts despite 'number knowledge' being the intention, along with no opportunity for students to receive feedback. The teacher also recognised that these assessments only test the speed of correct answers and give no indication of how students may be coming to their solution, either through recall or having to use strategies. In comparison, an Aotearoa New Zealand based study by Sleeman et al. (2021), showed that students aged 7-to-9-years-old made significant gains in their multiplication fluency when teachers used a combined approach of both explicit instructional teaching episodes of multiplicative concepts and strategies and independent self-regulated practice, compared to students who did not receive the explicit teaching episodes.

2.5 Mathematic Lesson Starters

Starter activities, also known as warm-ups, are activities teachers use at the beginning of a mathematics lesson to engage students in mathematical thinking from the outset of the lesson.

Graiser (2014) explains that the purpose of a warm-up can be two-fold – the first purpose is for providing students with a focus as they begin the lesson. The second purpose of a warm-up is to assist teachers in maximizing instructional time if the warm-up serves to activate student prior knowledge before the main lesson or review previously taught content. In addition, Johanning et al. (2011) explain that warm-ups provide opportunities for teachers to ensure that every minute of a lesson with time constraints is maximised, and that warm-up activities provide opportunities for students to reason with concepts and develop ways of thinking over time, particularly with hard to grasp concepts that may have only been covered in one or two lessons.

2.5.1 Mathematical Games

Mathematical games are an established activity used in mathematics classrooms. In a study of 248 Australian primary teachers, Bragg et al. (2021) found that while 75% of teachers surveyed (n=248) used maths games as warm-up activities several times per week, surprisingly only 45% of the same group of surveyed teachers used maths games to support rich mathematical investigations. Implications from this study suggest that maths games may be often used superficially for supporting student engagement and on-task behaviour management, but without mathematical purpose. While many games are designed to allow students to practise their basic facts, in an Australian study by Bragg (2012) working with students aged 10-to-12-years-old, highlighted that the playing of games that required students to draw on basic facts knowledge did not lead to increased competency in multiplication and division conceptual understanding.

Attard (2012) cautioned that while some students say they enjoy maths games, it may be the competition element of the game that some students enjoy rather than the mathematics itself. In her review of some commonly played mathematics games in Aotearoa New Zealand primary school classrooms, Darragh (2021) further explains that games involving competition portray to students mathematics as being about having one correct answer, speed, and an individualistic pursuit. This affords the winner's status of being ‘good at maths’, with the ‘not good at maths’ students also having their status as losers at mathematics publicly affirmed. One such game commonly used in classrooms in Aotearoa is ‘Around the world’ where students are pitted against each other to answer basic facts as quickly as possible. While it may seem that this may encourage students to learn their basic facts, Darragh (2021) challenges teachers to carefully consider the messages that students are receiving from such games, particularly if it is the same children who are always winning or losing, and to thoughtfully plan and choose games that allow for multiple solutions, collaboration and even the possibility of the winner being by chance. These types of considerations afford all students the opportunity to have correct answers and be winners at mathematics.

2.5.2 Counting Together as a Starter Activity

As previously highlighted, trusting the count is a foundational step towards multiplicative thinking (Siemon et al., 2012). In many junior primary classrooms, all-together counting routines are performed to support students to recognise and learn number sequences. The Numeracy Project Framework (Ministry of Education, 2007b) a key mathematics teaching resource in Aotearoa New Zealand for over 15 years, highlights counting together as an activity that helps students recognise multiples when skip counting forwards and backwards in sequences. This activity, aimed primarily at junior developmental stages and referred to as "advanced counting" (Ministry of Education, 2008, p. 3), encourages teachers to focus on the counting sequences of two, five and 10. However, because these activities rely heavily on rehearsal and rote learning, they may provide limited opportunities for students to reason about the number and counting patterns or multiplicative concepts beyond the fixed multiples being counted.

2.6 Choral Counting

Choral counting, in contrast to traditional all-together counting, is a complex counting activity that extends students beyond learning and memorizing counting sequences. The aim of choral counting is to support students to recognise and understand the underlying structure and patterns within a number sequence, as teachers guide students towards this understanding through the facilitation of collaborative discussion (Franke et al., 2023). In a choral count, teachers carefully design a forward or backward counting sequence that prompts students to think about specific patterns within the numbers. The sequence can become more challenging by starting at an unexpected number that breaks from familiar multiples – such as counting by 10's but beginning at 14. Teachers record the numbers and invite students to observe and discuss any patterns that they notice. According to Turrou et al. (2017), this process of recording and facilitating discussion represents an innovative approach to counting activities that promotes student reasoning and sense making of number patterns and sequences.

While counting together activities are typically used in junior classrooms, research by McMillan and Sagun (2020) demonstrates that choral counting can also be effectively applied in middle and high school mathematics lessons. Their study demonstrated how sophisticated choral counts can support the exploration of number concepts and patterns involving fractions with students aged 11-12-years-old, and suggest that integers, decimals, ratios and proportions and algebraic expressions can also be explored with older students. McMillan and Sagun's research highlights that group counting activities should not be confined to junior mathematics learning or simply for memorization of multiples, instead choral counting can serve as a powerful tool for fostering collaborative discourse around key number concepts and mathematical practices. Furthermore, a study of novice teachers by Anthony et al. (2015) emphasises that teachers need support and guidance on how to facilitate productive discussions with students when undertaking a choral counts. In this study, novice teachers were coached through rehearsal choral counts. Through these rehearsals, the novice teachers were able to discuss the specific pedagogical actions needed to connect student thinking to the big mathematical ideas being elicited through the count.

2.7 Summary

This chapter reviewed relevant literature related to how students develop mathematical reasoning skills and multiplicative thinking. The review highlighted specific teacher pedagogical actions teachers can enact to promote the development of mathematical practices, and how facilitating collaborative discussion supports students to reason with mathematical concepts. Traditional starter activities were reviewed in relation to how students learn and use basic facts to support multiplicative thinking. Finally, choral counting was reviewed as an example of a counting activity that can be extended to upper primary students, with a focus on conceptual understanding of number patterns and collaborative discussion and sense making. In the next chapter the research design and methodology will be discussed.

Chapter Three: Methodology

3.1 Introduction

The previous chapter discussed the literature that informs the current study. This chapter provides an overview of the research design and methods used in the study.

Section 3.2 provides a justification for the selection of a case study research design using qualitative methodology. Section 3.3 describes the role of the researcher. Section 3.4 presents the data collection instruments and methods. Section 3.5 introduces the research participants and research setting. Section 3.6 outlines the research schedule. Section 3.7 describes the data analysis and discusses the validity and reliability of the study's findings. Section 3.8 elaborates on considerations for ethical procedures. Finally, section 3.9 provides a summary of the chapter.

3.2 Justification of Methodology

A qualitative research approach was selected for the current study. Qualitative research seeks to encapsulate a comprehensive and holistic understanding of social phenomena, situations, and processes by investigating people's perceptions, actions, beliefs, and behaviours in their natural context (Merriam & Tisdell, 2015; Yin, 2016). Qualitative research involves gathering data usually in the form of words, images and artifacts to capture the multiple viewpoints of the study participants (Punch & Oancea, 2014). The aim of this study was to provide insight into how students reason with multiplicative number patterns while engaging in collaborative mathematical discussions, and which specific teacher actions support this. This aim influenced the choice of qualitative methodology. A qualitative approach was appropriate for the current study to provide the researcher an insight into student and teacher interactions within a mathematics classroom.

3.2.1 Interpretive Approach

For this research, an interpretivist approach was taken. The interpretive approach to qualitative research is embedded in the belief that reality is subjective therefore interpretive researchers must seek to understand how the research participants make sense of their experiences and actions as they interact with the world around them (Schwandt, 2014; Wahyuni, 2012). Interpretive research understands that individual participants bring their own experiences, history and culture to social contexts, as does the researcher (see also Section

3.3 researcher positioning) and these experiences influence the meaning and perception each person has of any given situation (Creswell & Poth, 2016).

3.2.2 Case Study Methodology

For the current study, an exploratory case study was selected to allow observation and analysis of teacher actions that facilitated productive mathematical discussions among students, and the diverse viewpoints and interpretations students made as they argued and reasoned with multiplicative concepts. A case study is a detailed in-depth analysis of a particular phenomenon within a real-world context, for the purpose of exploring and understanding the complexities of a situation. Case studies draw on a variety of data sources and methods of collection, bound by time, location and setting, personnel, and circumstance (Barth & Thomas, 2012; Punch & Oancea, 2014). According to Barth and Thomas (2012), there are three primary types of case studies; an explorative case study examines lesser-known phenomenon, an explanatory case study aims to inform or educate a particular audience, and an evaluative case study assesses the effectiveness of a phenomenon. The explorative case study design used for the current study, allowed for multiple perspectives to be captured over a period of time (Yin, 2016). In the current study, data was gathered from observations of two teachers and their students participating in six choral count activities in their mathematics classrooms.

3.3 Researcher Positioning

One feature of qualitative research is that the central role the researcher plays in data gathering and analysis (Punch & Oancea, 2014). Merriam and Tisdell (2015) describe how this allows the researcher to maintain the quality and consistency of the data collected. Accordingly, in the current study, the researcher was the sole collector and analyst of the data. To ensure quality data collection, the researcher used actions such as responding to or adapting questions during an interview or lesson planning sessions and clarifying the researcher's interpretations of events with participants.

In the current study, the researcher had a pre-established professional relationship with the participants as a mathematics mentor facilitating professional development within the school setting. While the premise of qualitative research is to conduct research with a reduced distance between researcher and participants (Pihama, 2015), in the current study, the researcher was viewed as an expert in the perception of the participants. For this reason, during classroom observations, a decision was made to use video recorded observations from the teacher only. Teacher participants maintained the sole responsibility of teaching the lessons, as the current study aimed not to change the teacher's actions during the lesson by their presence. However, when participants are aware they are being observed, they may behave in ways that differ from their typical conduct, behaving in a way they believe aligns with the researcher's expectations or attempting to present themselves in a more favourable light (Merriam & Tisdell, 2015; Spindler & Spindler, 2000; Yin, 2016). To mitigate these potential complications, the researcher emphasised the importance of a collaborative and reciprocal relationship, aided by the co-construction of lesson planning discussions to reflect on and discuss lesson outcomes.

The researcher is experienced in implementing an inquiry approach to teaching mathematics and is familiar with expected classroom practices and potential lesson outcomes. This strength may however also be seen as a weakness. There is potential for critical data being missed due to an over-familiarity with the research context and unconscious bias towards research participants (Merriam & Tisdell, 2015). Due to the researcher being the sole collector of data and analyst, it is important to be aware of the influential role the researcher may have had within the research setting, and how biases and assumptions might impact the study. Careful monitoring of interpretations of data is necessary (Merriam & Tisdell, 2015). As an interpretive researcher, it was important to mitigate any potential researcher bias and to balance potential teacher researcher positions of power. Open, unbiased questions were asked (see Appendix A1 and A2), teacher responses and contributions were considered, and reflections were made when creating lesson plans based on teacher feedback and suggestions. Discussions within the lesson planning sessions were confidential. Throughout the study, teacher and student participants were given opportunities to ask questions, seek clarification and opt out of the research if they desired to do so.

3.4 Participants and Research Setting

The study was conducted in two classrooms at an urban full primary school with a roll of approximately 530 students aged 5-to-13-years-old, in a low socio-economic suburb in Aotearoa New Zealand. The school was selected because of the existing relationship the researcher has as a mentor within the school’s mathematics professional development programme. Ethnicities of the students in the school are 75% Māori, 32% Pasifika, 9% European/Pākehā (note that ethnicity is a multiple response, and students may affiliate with more than one ethnic group).

Student participants consisted of 12 Year Five and Year Six students (9-to-11-years-old) from class one, and 12 Year Seven and Year Eight students (11-to-13-years-old) from class two. The two teacher participants were experienced educators who had been teaching for more than five years. Both teachers expressed an interest in being involved in the study as they regarded their participation as further professional development and a means to strengthen their instructional practices in developing collaborative discussion amongst their students. To ensure confidentiality, teachers were given the pseudonyms of Sarah and Alice.

3.5 Data Collection

In order to collect data that is reflective of the human experience, qualitative research draws on multiple sources of written, spoken, and visual data (Punch & Oancea, 2014). The use of multiple methods of data collection generate varied and detailed information, and adds to the validity of the research findings (Merriam & Tisdell, 2015). Data collection tools used in the current study were semi-structured interviews, lesson planning and reflection sessions, video recorded classroom observations, and photographs of classroom artefacts, including collective work on the classroom whiteboard and copies of teacher planning notes.

3.5.1 Semi-structured Interviews

In the current study, two semi-structured interviews were undertaken with individual teacher participants prior to and at the conclusion of lesson observations. Interviewing is an effective and commonly used technique in qualitative research (Punch & Oancea, 2014). Interviews are used to uncover phenomena such as thoughts, feelings, and intentions which cannot be directly observed (Merriam & Tisdell, 2015), and provide researchers with an in-depth view of participants' perceptions and understandings (Punch & Oancea, 2014). Semi-structured interviews follow a set question format but employ open-ended questions that can be flexibly worded (Merriam & Tisdell, 2015). The initial interview questions explored participants' current practices and encouraged participants to share their views and experiences when implementing warm-ups in their mathematics lessons (See Appendix A1). The intention of the concluding interview was to capture teacher participants thoughts on the use of planned choral count conceptual starters and what the perceived differences were between these and what they had been using prior to the study (See Appendix A2). Prior to being interviewed, participants were given an explanation of the purpose of the study and the role that interviews would play in the data collection, as well as the role confidentiality and consent played in their participation in the current study. Semi-structured interviews allowed for reciprocity in gathering information, as both the researcher and participant could respond in the moment to contributions and seek elaboration or clarification if required (Punch & Oancea, 2014). Each interview lasted approximately fifteen minutes and took place in either a quiet meeting room or the participant's classrooms after school in person, or the teacher participants own home via zoom. All interviews were video recorded and transcribed in their entirety for coding and analysis.

3.5.2 Collaborative Lesson Planning and Reflection Sessions.

Collaborative planning and reflection sessions were held before each pair of lessons were taught and observed. These sessions aimed to provide teacher participants with an opportunity to share their experiences and reflect on each other's contributions while developing the lesson plans for the study. Facilitated and supported by the researcher, participants discussed mathematical content, and collaborative discussion prompts and reviewed outcomes from previous lessons, which supported them to identify learning outcomes and anticipate future outcomes. Berryman (2015) describes these learning conversations as korero whakawhitiwhiti, "where people are comfortable to exchange and build on the ideas of others and where all can contribute to and through the emergence of spiralling discourses, in which new learning in constructed" (p.55).

3.5.3 Video recorded observations

In the current study, video recordings were the primary mode of data collection of the twelve classroom lessons. Video recording is a valuable qualitative data instrument, that captures a permanent record of interactions between participants and their environment (Wang & Lien, 2013). The use of video recording of observations allows researchers to selectively review and reflect upon field events. Opportunities to observe non-verbal cues, reactions and gestures of participants are also afforded (Wang & Lien, 2013; Yin, 2016). These recordings were undertaken by the teacher participants as the researcher was not present for these lessons. The reason for this was to allow teacher participants flexibility in determining when they would carry out the lessons at times that suited their schedules. As this research focuses on using a conceptual starter, only the opening phases of the mathematics lessons were recorded in this study. The camera was focussed on the large group of students, with the purpose being to investigate how students engaged with the number pattern, what conceptual understanding was being developed and how collaborative discourse was being facilitated by the teacher. Each lesson was filmed on an iPad using IRIS Connect software. Videos were uploaded and stored on a password-protected cloud hosted by IRIS Connect, and teacher participants were familiar with the use of this technology. All video recordings of the lessons were downloaded at the conclusion of the lesson and transcribed in their entirety to support interpretation of video recordings during data analysis phase.

3.5.4 Classroom Artefacts

Classroom artefacts offer supplementary evidence to the observation notes gathered by the researcher (Punch & Oancea, 2014). The teacher's planning of the choral count conceptual starters further highlighted the pedagogical beliefs, intentions, and motivations that influenced each lesson. Digital photographs of the recording of the choral counts and student representations of number patterns were taken at the end of each lesson by the teachers. This provided further evidence of how students were engaging with the number patterns and provides additional data that could be analysed in relation to the quality of contributions that students made during each lesson.

3.6 Research Schedule

In this section each phase of the data collection is discussed in detail. This is followed by a table outlining the research schedule.

3.6.1 Phase One

The first phase of the current study began with individual semi-structured interviews with teacher participants. The interviews followed a set of questions, designed to explore current practices regarding warm-ups in mathematics lessons (see Appendix A1). Additionally, an initial collaborative planning session was conducted to allow the two teacher participants to plan alongside the researcher the first two choral counts and draft the additional four counts (See Appendix B1). The initial choral counts were designed to build on current student knowledge and understanding of multiples of four.

3.6.2 Phase Two

The second phase of the current study involved teaching the series of choral count lessons and two collaborative planning and reflective sessions. The instructional sequence that formed the basis of the research was comprised of six choral counts, taught over six twenty-minute sessions at the beginning of regular classroom mathematics lessons. The lessons were taught over six weeks across Terms Three and Four (September – November) of 2023. The number of lessons was intended to provide students enough opportunities to engage with the number patterning, and to gain confidence in participating in large group discussion, and to provide teacher participants with opportunities to gain traction toward facilitating productive mathematical discussion with students.

Two collaborative lesson planning and reflective sessions were also carried out in this phase of the research. After each series of two choral counts were completed, the teachers and researcher reviewed the planned lesson objectives, analysed student responses, and modified the draft lesson plans to create a detailed lesson plan for the next series of two choral counts. The subsequent counts increased in complexity over the sequence of the lessons, working with multiples of eight and then 12, for the Year Seven and Eight class, and multiples of eight and three, for the Year Five and Six class (see Appendices B2 - B4).

3.6.3 Phase Three

In the final phase of the data collection, concluding individual semi-structured interviews were conducted between the researcher and teacher participants (See Appendix A2).

The schedule and summary of research activity and data collection for each phase of the current study is presented in Table 1.

Table 1

Summary of research activities and data gathering strategies implemented during each phase of the current study

Phase	**Research activity**	**Data Gathering strategy**
Phase 1 August – September 2023	Initial individual semi-structured interviews	Semi-structured interviews video recorded and transcribed
	First collaborative planning session of first two choral counts	Collaborative planning session video recorded and transcribed Lesson plans collected
Phase 2 September – November 2023	Choral Count lessons undertaken in each classroom (6 lessons in each)	Lessons video recorded and transcribed Photographs of whiteboard
	Two collaborative planning and reflection sessions, one after each series of two choral count lessons undertaken – reflective discussion and collaborative planning for next set of counts	Collaborative planning and reflection sessions video recorded and transcribed Lesson plans collected Researcher notes
Phase 3 December 2023	Final individual semi-structured interviews	Semi-structured interviews video recorded and transcribed

3.7 Data Analysis

Qualitative data analysis involves the researcher interpreting and describing the social phenomena they have observed (Punch & Oancea, 2014). Data analysis linked to the research questions in the current study required sense making of teacher actions, resulting student discourse and enactment of mathematical practices during collaborative mathematical sense-making episodes.

In qualitative research data analysis is carried out in two phases, ongoing and retrospectively (Merriam & Tisdell, 2015). In the current study, ongoing analysis was a collaborative process replicated throughout phase one and two of the study. For instance, after the initial interviews, the researcher worked with the teachers to develop the first series of lesson plans. Then after each series of lessons, the collaboration between researcher and the two teachers analysed the effectiveness of the lesson plan and informed the design of the next choral count. This analysis continued in iterations throughout the series of choral count lessons. Retrospectively, interviews and lessons were transcribed analysed and coded to identify correlations and connections (Punch & Oancea, 2014).

3.7.1 Coding and Developing Themes

Data retrospectively analysed involved the reducing the data collected from transcriptions of interviews and lessons by looking for emerging themes and patterns (Yin, 2016). Transcripts were read and re-read to develop initial broad themes. These broad themes developed from analyses of the data. These broad themes were refined with the support of Hunter's Communication and Participation Framework (2007) to identify specific teacher actions that supported students to engage in mathematical practices, and are presented on Table 2.

Table 2

Initial themes and coding categories of teacher actions

Initial Themes	Coding of Teacher Actions
Planning	Planning for multiplicative reasoning Planning for mathematical discourse
Investigating multiplicative Concepts	Questions that elicit student noticing's Questions that elicit conjectures Setting expectations Encouraging participation (repeat, turn and talk, positioning) Encouraging risk taking
Developing Conceptual Explanations	Supporting student to student discourse Supporting use of mathematical language Teacher modelling of an explanation Revoicing of idea by teacher Revoicing of idea by students Supporting students to ask questions to clarify thinking Questions that support conceptual explanations (can you prove it)
Justifying and arguing multiplicative concepts	Supporting student to student discourse Questions that elicit justifications (why does that happen) Encourage use of: because, if, so, when Encouraging students to agree or disagree and why
Generalisation of multiplicative Concepts	Encouraging predictions about what comes next Questions to support generalisations (does that always work, does that work in other places, or with other numbers, will this number be part of this pattern?) Encouraging students to agree or disagree and why

Coding was further refined to develop sub-categories that focussed on three stages of student engagement in mathematical practices that lead to students developing new understanding of multiplicative concepts (see Table 3).

Table 3

Sub-categories of teacher actions to support sense making through discourse

Level of Student reasoning	**Description of Teacher Action**	**Example**
Generate Reasoning	Generate actions teacher questions that prompt or encourage students to make a statement/conjecture based on either their own ideas or idea of another	Questions such as what do you notice, what do you think, can you make a prediction about…?
Support Reasoning	Support actions require students to add more detail, think deeper	Questions such as why do you think, can you add more detail, who agrees or disagrees with that and why…?
Advance Reasoning	Advance actions encourage students to summarise new thinking	Why is that the same or different, how else could you say that, does this always happen…?

Sub-categories of teacher actions to support sense making emerged from deep analysis of the types of questions teachers used as prompts or encouragements for students.

3.7.2 Validity and Reliability

In order for qualitative data to produce valid conclusions, analysis of data must be undertaken in a reliable manner (McArthur, 2022). Validity of qualitative research focusses on the accuracy and authenticity of the representation and interpretation of the studied phenomena (Yin, 2016). In the current study, validity was achieved by the extended iterations of classroom observations, video recordings of interviews and lessons and collaborative reflective discussions with teachers. The provision of rich reflective discussions with the teachers provided on-going opportunities to clarify and define observed phenomena.

Reliability in qualitative research refers to the consistency and dependability of the research analysis and findings. Yin (2016) explains that case studies can present challenges with reliability due to the complex nature of the contexts and environments in which they occur, and the dynamic nature of human behaviour Findings are highly dependent on the particular context and conditions in which they are observed and can also be influenced by researcher interpretations. These challenges in case study reliability can be addressed by ensuring there is trustworthiness of the findings in how well they represent reality (Merriam & Tisdell, 2015). In the current study, the trustworthiness of the findings was achieved through the triangulation of multiple data sources. Multiple perspectives of teachers were gathered during interviews and reflective discussions. Data was collected across two different classrooms with students ranging in age from 9-to-13-years-old, providing deeper and wider evidence than if data were collected from a single sample. In the current study, data collection methods, analysis processes, ethical considerations and researcher positioning have been detailed to provide further support to the trustworthiness of the research findings (Merriam & Tisdell, 2015).

3.8 Ethical Considerations

The current study was designed and conducted in accordance with the Massey University Code of Ethical Conduct for Research, Teaching and Evaluations Involving Human Participants (Massey University, 2017). The research proposal was reviewed, and approval was granted by the Massey University Human Ethics Committee prior to the commencement of data collection. Ethical considerations for the current study were guided by Titiriti o Waitangi principals as outlined in the Te Ara Tika: Guidelines for Māori Research Ethics framework (Hudson et al., 2010**.** Te Ara Tika prioritises positive outcomes for participants and their communities in the research design process and is guided by four principals: Whakapapa (purpose and relationships), Tika (benefice), Manākitanga (cultural and social responsibility) and Mana (justice and equity).

3.8.1 Whakapapa / Relationships

Establishing trusting relationship dynamics between the participants and the researcher is crucial for the success of the research. Tuhiwai Smith (2012) emphasises the significance of the three r's of research involving people; relationships, reciprocity and respect. In the current study, the researcher had pre-established professional relationships with the participants, which can be viewed as advantageous as trust already exists (Tuhiwai Smith, 2012). To continue to build trust in the unfamiliar process of research, participants were presented with information sheets that clearly outlined the intent and aims of the research (see Appendix C1). This was also provided to the school principal, board of trustees and student families (see Appendix C2 and C3). Time was given to address any questions or concerns that arose. Classroom observations were conducted in the second half of the year, giving teacher participants sufficient time to establish classroom norms and routines. Respect of school and teacher workload was observed, and interviews were held at times that were suitable for the teacher participants. The lesson observations formed part of the daily classroom programme so as to not impede on valuable classroom learning time. Reciprocity is demonstrated by the researcher acknowledging the time, energy and knowledge given by the participants (Pihama, 2015). In the current study, both teacher participants were presented with a koha (gift of appreciation) upon completion of the research.

3.8.2 Tika / Beneficence

The principal of tika emphasised the need for the current study to positively benefit teachers, students and the wider school community without any potential burdens being placed on participants (Macfarlane & Macfarlane, 2018). Participants were informed of the research's intent, which was to enhance teacher pedagogy and capabilities in the specific area of lesson starters and student multiplicative reasoning. It was made clear to all involved that participation would benefit both the teacher participants and their students through a focused examination of their practice and student outcomes. The researcher was transparent about who else may benefit from the study, including senior management and the broader teaching staff. Participants were assured that the intent of the study was not to judge their practice, but to document current practices and take reflective steps to develop teacher pedagogy.

3.8.3 Manākitanga / Cultural and Social Responsibility

Participant autonomy, or the right to make free and informed decisions about their involvement in a research study underpins manākitanga (Massey University, 2017). In the current study, participants and their whānau (families) were fully informed of the research intent, purpose and what their inclusion entailed. This avoids any potential deception, prejudice or perceived conflicts of interest from the researcher (Hudson et al., 2010). The researcher ensured there was no coercion to participate, and it was made clear that the research was voluntary and separate from the current professional development that participants were also involved in with the researcher. All participants retained the right to withdraw from the study at any time throughout the project without this having any effect on their work or learning opportunities.

To ensure privacy and confidentiality, location of interviews and planning sessions was considered and took place in various settings that were suitable for the participants, including classrooms, private interview rooms at school and their own homes via zoom. Interviews took place both in person and via zoom to allow participants to undertake the interviews at a time and place that was suitable for them. This ensured privacy and confidentiality was maintained to the participants. Participants maintained the right to view, discuss and clarify aspects of interview responses, video recordings, transcripts and classroom artefacts throughout the current study. All personal details such as names, including that of the school, participants including students in the class, were recorded using pseudonyms to ensure confidentiality and anonymity. All digital data including lesson and interview recordings were stored on an online password protected platform.

3.8.4 Mana / Justice and Equity

Ethical consideration of power dynamics was required for this study. There was potential for participants to perceive the researcher as more knowledgeable due to the existing relationship of mentor, creating potential power imbalances. The participants were provided with an explicit explanation of the different intentions of participating in the current study compared to being mentored. As previously mentioned, the researcher was not present during the classroom lessons to ensure that the teacher maintained authority and ownership of the direction and outcomes of the lessons.

3.9 Summary

This chapter has outlined the research design and methods used in the study. It has described the rationale for selecting a case study approach and defined qualitative methods of data collection and analysis. Data was collected using a variety of methods, including interviews, video recorded observations and artefacts. Data was analysed through the identification of themes and generation of codes. Reliability and validity of findings was supported by the triangulation of data. The Te Ara Tika Māori ethical model (Hudson et al., 2010) provided a culturally appropriate ethical framework to conduct research with the current study's participants. The findings and discussion of the study are presented in Chapter Four.

Chapter Four: Findings and Discussion

4.1 Introduction

The previous chapter provided an overview of the research design and methods used in this study. This chapter presents the findings and discussion. Specifically, the teacher actions that supported students to engage in mathematical discourse while reasoning with multiplicative concepts during a conceptual starter activity are presented. In addition, how students were supported to enact mathematical practices is discussed. Section 4.2 outlines the existing teacher practices and noticing's the teachers shared before the intervention began. Section 4.3 discusses the specific planning that was developed to support both conceptual understanding of multiplication and collaborative discourse. The remaining sections identify how students reasoned with multiplicative concepts by using mathematical practices. The teacher actions that supported these mathematical practices are identified in three stages: actions that generated reasoning, actions that supported reasoning, and actions that advanced reasoning. Section 4.4 identifies the specific teacher actions that provided opportunities for students to investigate and form conjectures about the number patterns and multiplicative concepts presented in the series of planned choral count activities. Section 4.5 identifies teacher actions that supported students to develop conceptual explanations. Section 4.6 draws on the series of six lessons in each classroom to describe student justifications and a deeper analysis of teacher moves that contributed to the depth of engagement and understanding students were able to reach. Section 4.7 presents teacher actions that supported students to generalise number patterns to numbers beyond those given in the task. Finally, section 4.8 provides teacher reflections from the final interviews and a concluding summary.

4.2 Existing Teacher Practices

In this section, the teacher participants' prior and existing practices regarding starter activities in mathematics lessons are analysed and discussed. Initial semi-structured interviews with both teachers highlighted that neither teacher was using planned starter activities with a conceptual focus before the research began.

Both teachers stated that some starter activities were sporadically used, and time constraints were often a reason why starter activities were not regularly implemented in their mathematics lessons. For example, Sarah (a pseudonym) stated that when she did use a starter activity in her lessons, she tended to draw from a personal bank of familiar activities such as facts revision or “quick 10” tests. She was aware that some of her students had enjoyed this type of starter activity that tests students' recall of facts, whereas others genuinely disengaged from them and displayed avoidance strategies to the task or physically expressed dissatisfaction from having to participate. These reflections are consistent with the findings of Tait-McCutcheon et al. (2011), showing teachers intentions behind their selected starter activities may not produce the desired results in student outcomes of learning basic facts. The dissatisfaction displayed by some students also mirrors the findings of several researchers (Attard, 2012; Darragh, 2015, 2021), showing students are likely to disengage from an activity when there is a chance they may not be seen as good at mathematics compared to their peers. Sarah sometimes used her starter activity to address student misconceptions or key concepts for the day's mathematical activity based on what she had noticed from previous lessons, wanting students to be “set up to succeed”. However, she also noticed that this often did not automatically transfer to future mathematical learning tasks. Sarah noted that her class struggled to share their thinking in larger groups during the daily lessons and many students often waited for her to provide them with an answer to questions she posed. She recognised that her current starter activities did not allow students to engage in collaborative discussion, and that this was a missed experience she could be providing her students in the hope that they would, over time, become more willing to participate during the main lesson with this extra practice.

The second teacher participant, Alice (a pseudonym), also noted that some students enjoyed the competitive aspect of mathematics games, however, she was aware that this was not a productive method for all students to engage in mathematical learning. Alice wanted fun and enjoyment to be the main outcome of her starter activities and felt that this set the mood for the lesson and helped students to see that mathematics can be fun. She explained that she had recently felt there was some success with changing students’ attitudes towards a mathematics lesson, when students were provided with opportunities to discuss and justify their thinking, when implementing a starter activity that focussed on place value. During the initial interview, Alice shared how this positive experience inspired her to learn more about conceptual starters that focus on productive discourse:

> I really think that the kids are enjoying it because I’ve made an effort. I want my warm-ups to get the kids excited about maths. I’m curious about how they go working together. The warm-up sets the scene really. I try to make them fun and varied, based on a need, but also you don’t know what your kids will enjoy until you try it. I really want that fun and enjoyment because some of these tasks really stretch the kids.

This attention to enjoyment as a pre-requisite for engagement in mathematical learning is similar to Attard’s (2012) findings that engagement occurs when mathematics is a subject that students enjoy. This enjoyment or motivation to participate in the mathematics learning can be affected by student confidence, influence of peers and teacher pedagogy, suggesting that simply providing students with fun activities may not always lead to engagement in mathematical learning.

Both teachers expressed that less than 25% of their classes (n=28 total students in each class) had their times tables committed to memory and felt that this was a barrier to students being able to work confidently with appropriately levelled mathematics for their age groups. Students were given opportunities to practice basic facts using online tools during independent work time. Times table practice was set as homework, however, both teachers expressed that this was rarely completed by their students. These findings mirror those of Tait-McCutcheon et al. (2011), who found that many teachers believe that memorisation of basic facts is achieved through the learning of multiplication facts as independent practice and homework activities. In contrast, Sleeman et al. (2021) found that students taking part in a specific and consistent self-directed basic facts fluency programme combined with precision teaching both increased their basic facts knowledge and also altered behaviour to practice these facts outside of school. In the current study, both teachers expressed a desire for support with developing and planning a conceptual starter that covered both mathematics and how to successfully deliver content so that students gained new understanding.

The analysis of teacher participants existing practices regarding starter activities in mathematics lessons revealed that neither teacher implemented planned activities with a conceptual focus prior to the research, often relying on familiar tasks due to time constraints. The planning sessions as described in the following section outline how teachers were supported to plan and deliver the choral count lessons that enhanced student participation and understanding of multiplication concepts.

4.3 Purposeful Planning

This section describes how opportunities for students to reason with multiplication and division concepts through the choral count starter activities were purposefully planned. The first section describes the goals that the teachers felt were important for their conceptual starters. The next section explains the specific mathematical goals for each lesson, and the last section describes the teacher prompts and questions that were planned to support mathematical reasoning through collaborative discourse.

4.3.1 Teacher Goals for the Conceptual Starters

At the first planning session the researcher worked alongside both teacher participants to support them to formulate and define goals for the conceptual starters. Both teachers expressed that they wanted the choral counts to provide opportunities for students to:

- learn and practice basic facts.
- participate in collaborative discourse to help build confidence and increase participation.
- investigate and make their own conjectures and justifications to support more active participation from students in answering questions and confidence in the correctness of answers.
- have fun and be engaged in an activity they want to participate in.

These goals highlight that while both teachers were aware that their starter activities needed to provide an opportunity for students to engage in mathematical learning inclusively and collaboratively, neither were regularly putting this into practice in their classrooms. Both teachers expressed that they were unsure of how to do this and felt that their own knowledge regarding how to learn multiplication was limited to simply rote learning facts. A discussion took place where the researcher supported the teachers to think about the big ideas involved in multiplication. This resulted in the concepts of repeated addition and the distributive property of multiplication being selected as the key ideas. The rationale for selecting these key ideas were to support teachers in their own understanding of multiplication and to provide an opportunity for students to build fact fluency and to develop multiplicative thinking strategies to support them to work with larger numbers appropriate for their curriculum levels. Participation was indicated as a high priority for building both confidence and mathematical understanding in the students and was a teacher focus during the collaborative discussion.

4.3.2 Planning for a Mathematical Goal

To begin with, the lesson plans were developed to meet the multiplication and division curriculum objectives from Phase Two and Three of the draft curriculum refresh that was current at the time of the study (Ministry of Education, 2024). These objectives form part of the "Know" section under number operations (See Appendices B1- B4 for full lesson plans). The teachers and researcher agreed on six choral counts being planned for each class, with a focus on three different number patterns counting both forwards and backwards. Each choral count plan contained key mathematical ideas teachers had anticipated that would develop within the count. These ideas supported students with both noticing counting patterns and place value knowledge, which describes as essential prerequisites to multiplicative thinking. The key mathematical ideas identified in the planning for the choral count lessons also correspond with the mathematical ideas of recognising patterns, additive ideas, multiplicative or grouping ideas, and equivalence that McMillan and Sagun (2020) highlighted in student thinking while participating in a sophisticated choral count in fractions.

Careful consideration was given to how the number patterns might develop through the count, with a focus on where to stop and start each line on the count so that specific patterns would emerge. For example, skip counting in fours grouped in lines of five numbers so that numbers increased by 20 in each column (see Table 4). This process aligns with Franke et al.'s (2023) suggestion to be intentional in order to highlight explicit patterns within the numbers being recorded. Planning focused on drawing student attention to the patterns through the use of specific pauses and deliberate teacher questioning to scaffold multiplicative reasoning.

The number patterns explored in each choral count and key mathematical ideas the counts aimed to address are shown in Table 4.

Table 4

Summary of choral count sequences and key mathematical ideas to be explored

<table>
<tr><th>Planned Choral Count</th><th>Key Mathematical Ideas to be Explored</th></tr>
<tr><td>Count forwards by four from 20
<table>
<tr><td>20</td><td>24</td><td>28</td><td>32</td><td>36</td></tr>
<tr><td>40</td><td>44</td><td>48</td><td>52</td><td>56</td></tr>
<tr><td>60</td><td>64</td><td>68</td><td>72</td><td>76</td></tr>
<tr><td>80</td><td>84</td><td>88</td><td>92</td><td>96</td></tr>
</table>
<i>Both Classes</i></td>
<td rowspan="8">Noticing Patterns

Odd and even numbers
The value of “odd numbers” in the Tens and Hundreds column
Numbers in Ones place (may stay the same or alternate)
Identifying known patterns in larger numbers (e.g. exploring the pattern of two and the pattern of 20 and how this relates to the power of 10 (e.g. 180, 160, 140 vs 18, 16, 14)
Noticing the pattern of two and four inside the patterns of four, eight, 12

Additive Ideas

Repeated addition and subtraction to count on or back
Noticing doubling of the pattern of two and four
Each column goes down by a specific number (e.g. 20)
Diagonal patterns add or subtract a specific amount
Using base 10 to support adding and subtracting with other numbers (e.g. counting in eight can be seen as + 10 – 2, or counting in 12 can be seen as + 10 + 2)

Groupings or Distributive Ideas

Links to multiplication from noticing of number increases (e.g. 4 x 5 = 20 and how this relates to each vertical pattern increasing by 20)
Noticing the power of 10 multiplication (e.g. 3 x 8 = 24 therefore 30 x 8 = 240 and how this helps us find how many groups of 8 in 264)
Noticing the way multiplication can be distributed e.g. 23 x 12 can be the same as 20 x 12 + 3 x 12
</td></tr>
<tr><td>Count backwards by four from 196
<table>
<tr><td>196</td><td>192</td><td>188</td><td>184</td><td>180</td></tr>
<tr><td>176</td><td>172</td><td>168</td><td>164</td><td>160</td></tr>
<tr><td>156</td><td>152</td><td>148</td><td>144</td><td>140</td></tr>
<tr><td>136</td><td>132</td><td>128</td><td>124</td><td>120</td></tr>
</table>
<i>Both Classes</i></td></tr>
<tr><td>Count forwards by eight from zero
<table>
<tr><td>0</td><td>8</td><td>16</td><td>24</td><td>32</td></tr>
<tr><td>40</td><td>48</td><td>56</td><td>64</td><td>72</td></tr>
<tr><td>80</td><td>88</td><td>96</td><td>104</td><td>112</td></tr>
<tr><td>120</td><td>128</td><td>136</td><td>144</td><td>152</td></tr>
</table>
<i>Both Classes</i></td></tr>
<tr><td>Count backwards by eight from 272
<table>
<tr><td>272</td><td>264</td><td>256</td><td>248</td><td>240</td></tr>
<tr><td>232</td><td>224</td><td>216</td><td>208</td><td>200</td></tr>
<tr><td>192</td><td>184</td><td>176</td><td>168</td><td>160</td></tr>
<tr><td>152</td><td>144</td><td>136</td><td>128</td><td>120</td></tr>
</table>
<i>Both classes</i></td></tr>
<tr><td>Counting forwards by 12 from 60
<table>
<tr><td>60</td><td>72</td><td>84</td><td>96</td><td>108</td></tr>
<tr><td>120</td><td>132</td><td>144</td><td>156</td><td>168</td></tr>
<tr><td>180</td><td>192</td><td>204</td><td>216</td><td>228</td></tr>
</table>
<i>Class Two (Year Seven and Eight)</i></td></tr>
<tr><td>Counting backwards by 12 from 420
<table>
<tr><td>420</td><td>408</td><td>396</td><td>384</td><td>372</td></tr>
<tr><td>360</td><td>348</td><td>336</td><td>324</td><td>312</td></tr>
<tr><td>300</td><td>288</td><td>276</td><td>264</td><td>252</td></tr>
<tr><td>240</td><td>228</td><td>216</td><td>204</td><td>192</td></tr>
</table>
<i>Class Two (Year Seven and Eight)</i></td></tr>
<tr><td>Counting forward by three from zero
<table>
<tr><td>0</td><td>3</td><td>6</td><td>9</td><td>12</td></tr>
<tr><td>15</td><td>18</td><td>21</td><td>24</td><td>27</td></tr>
<tr><td>30</td><td>33</td><td>36</td><td>39</td><td>42</td></tr>
<tr><td>45</td><td>48</td><td>51</td><td>54</td><td>57</td></tr>
</table>
<i>Class One (Year Five and Six)</i></td></tr>
<tr><td>Counting backwards by three from 150
<table>
<tr><td>150</td><td>147</td><td>144</td><td>141</td><td>138</td></tr>
<tr><td>135</td><td>132</td><td>129</td><td>126</td><td>123</td></tr>
<tr><td>120</td><td>117</td><td>114</td><td>111</td><td>108</td></tr>
<tr><td>105</td><td>102</td><td>99</td><td>96</td><td>93</td></tr>
</table>
<i>Class One (Year Five and Six)</i></td></tr>
</table>

The progression from counts of four to eight were designed with the intention that students would see the relationship between the two sets of number patterns. The third set of counts varied between the two classrooms with the older students in class two (11-to-13-years-old) exploring multiples of 12 in their final set of counts and the younger students in class one (9-to-11-years-old) exploring multiples of three. The count of three did not build on the previous two counts, however, the teacher goal in using a lower number than 12 was to support those students who had not yet learned their three times tables.

Planning included incorporating mathematical practices from the "Do" section in the draft curriculum (Ministry of Education, 2024), into the lesson plans. Planned pauses were included along with prompts for teachers to support students to investigate and make conjectures about the number patterns as well as specific teacher questions to support the exploration of repeated addition or subtraction and with links to specific examples of the distributive property. To ensure that multiplicative reasoning was developed, generalisation prompts were included in the teacher questions, for example, questions asking students to predict further terms and justify this. The planning of such prompts aimed to support student generated explanation and justification throughout the lessons.

This section has described in detail how the lesson plans aimed to support the development of multiplicative reasoning within the choral counts through the exploration and identification of number patterns, place value, additive and grouping strategies. The following section describes how mathematical discourse was planned to support the development of multiplicative thinking and mathematical practices within a large group discussion.

4.3.3 Planning for Mathematical Discourse

Lesson planning included planned pauses designed to provide student generated discussion. The use of the talk move "turn and talk" was identified by the teachers as the first strategy they would employ to generate discussion. As detailed by Kazemi and Hintz (2023), this talk move provides a low stakes environment for students to share their initial thinking with a peer and to also hear the ideas of others. To address individual participation, the teachers identified which students normally display confidence in large group discussions in the classroom setting, and which students tended to under participate. Teachers planned who they might draw on at different points of the lesson to support balanced student participation. Both teachers were experienced with using talk moves (Chapin & O'Connor, 2007), and planned to emphasise the use of repeating and revoicing to support students who were less confident with sharing their own ideas. As the lessons took place in term three of the school year, classroom norms and expectations were already established, however, both teachers expressed that they wanted to focus on reminding students of the expectations to participate and to take risks with their thinking and sharing of ideas when undertaking the choral count lessons. This aligns with research by Hunter and Hunter (2018) describing how teachers who were continually successful at promoting participation in collaborative discourse made social norms and expectations explicit and held all students accountable for their contributions to the development of mathematical thinking of the wider group.

This section discussed how teacher goals for the lessons and lesson plans were developed to support teachers when implementing the choral counts in the current study. Mathematical concepts to be explored and how teachers planned for mathematical discussions were highlighted. The following sections describe the findings of the enactments of the choral count activities and how the teachers used the planned lessons to support the development of mathematical reasoning and multiplicative thinking.

4.4 Investigating Multiplicative Concepts

In this section, two examples of how the teachers supported students to investigate number patterns that evolved from the choral counts in a large group discussion are examined. The actions of the teacher and the corresponding student responses are analysed in relation to how the discussion supported the development of multiplicative thinking through the mathematical practices of noticing and conjecturing.

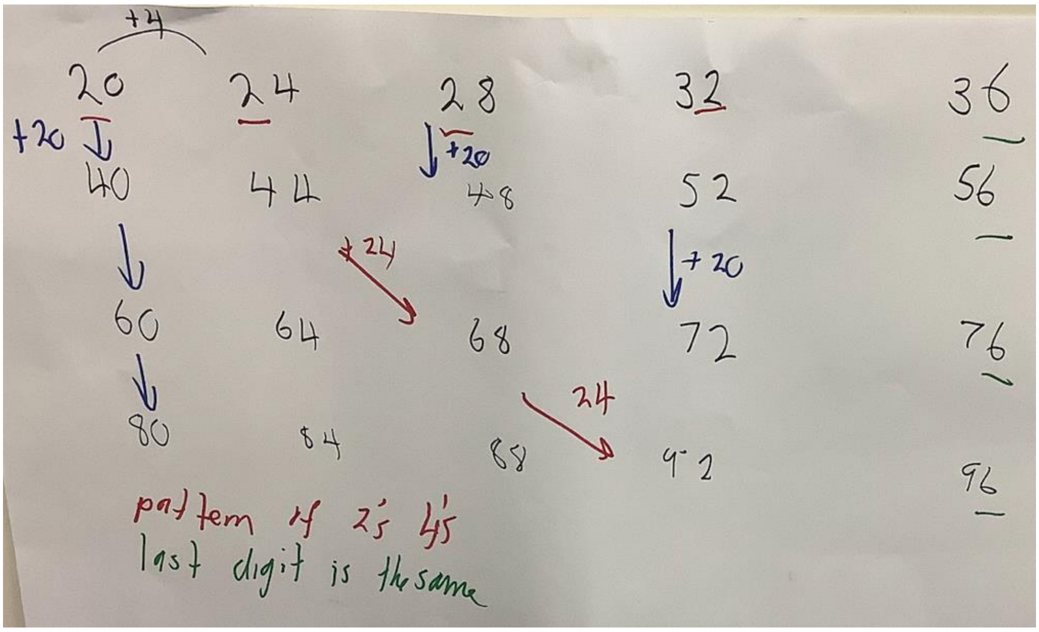
During the lesson planning sessions, both teachers anticipated that identifying number patterns such as counting in twos, all numbers being even, and groupings of 10’s would be the most accessible point for all students to be able to contribute to the large group discussion. This links to the concept of trusting the count as explained by Siemon and colleagues (2005), as the first step towards multiplicative thinking. The following vignette demonstrates how students in classroom one were supported to identify number patterns in their first choral count starter activity, counting forwards by four.

Class One – Counting forward by four		
Students: (altogether)	Forty, 44, 48, 52	
Teacher:	Ok and the next one?	
Student 1:	Fifty four.	
Student 2:	Fifty six.	
Teacher:	Okay let’s pause, is it 54 or 56? I’m going to ask you to turn and talk and I’m going to ask you what your buddy sees. Beautiful. I like the way you turned and looked at your buddy. Ok so Gina, what did Tiana notice?	*Generates reasoning*
Gina:	Tina said its 56.	
Teacher:	Is that right? Do you guys agree?	*Supports reasoning*
Students: (altogether)	Yes.	
Teacher:	Gina so what did Tiara notice? How did she know it was going to be 56?	*Supports reasoning and presses for justification*
Gina:	Because it goes two, two, two and then six, six, six.	
Teacher:	Do you mean here? *(points to last two columns and identifies Ones column in each row)*	*Supports reasoning*
Tiana:	Yeah, they are all the same.	
Teacher:	*(underlines the six in each number in the column)* And does anyone know why it goes two and then six, and not two and then four?	*Advances reasoning*
Student 3:	Because two plus four is six so you miss out the four?	
Teacher:	Yes, you are right, we are adding four, so it goes 52, 56 and misses out 54.	

Evident in the vignette is how the teacher utilised the turn and talk move to generate the discussion to predict the next number in the count when two different numbers were offered. She emphasised the expectation of students needing to listen to and then revoice what their buddy had said to promote participation. To support students to reflect on the conjecture offered by Gina, the teacher used the talk move of agree or disagree to engage the wider group and to press for justification. When it was widely agreed that the conjecture was correct, the teacher then asked Gina to think about how Tiana knew the next number was going to be 56. This required Gina to add her own reasoning to her buddy's initial idea. Contributing to the ideas of another is described by Franke et al., 2015) as high level engagement in the thinking of others. The teacher clarified what Gina was saying by using the visual representation of the count (see Figure 1) to support her noticing. Gina's reasoning of the numbers in the One's column all being the same digit relied solely on the visual representation of the count and did not include a mathematical reason for the next number in the count being 56.

Figure 1

Visual recording of choral count: Counting forwards by four



To advance the conjecture towards a connection to a mathematical concept, the teacher opened the discussion to the wider group by asking if anyone could offer a reason why the next number would not end in a four, linking back to the original incorrect response that the next number would be 54. This required students to reason with their knowledge of counting in twos, and why this choral count did not follow the same number pattern of two. When a student offered the correct reason, the teacher clarified why this new conjecture was correct. The episode presented in the above vignette shows that these students were engaging in additive thinking to continue the count. This is consistent with Siemon et al.'s (2005) findings that suggest that when students can make simple observations of numbers, they then progress

to recognising the difference between two numbers. Trusting the count of counting in twos allowed the students to apply this knowledge to a more sophisticated count in fours.
The teacher's pedagogical moves in the discussion scaffolded the wider group towards a mathematical reason by linking the ideas offered by the students and affirming a correct conjecture. These moves of pressing for reasoning align with the invitation and support moves described by Franke et al. (2015), and the initiate and sustain teacher talk turns identified by Selling (2016). These specific teacher pedagogical moves supported students to deepen their reasoning beyond simple observations.

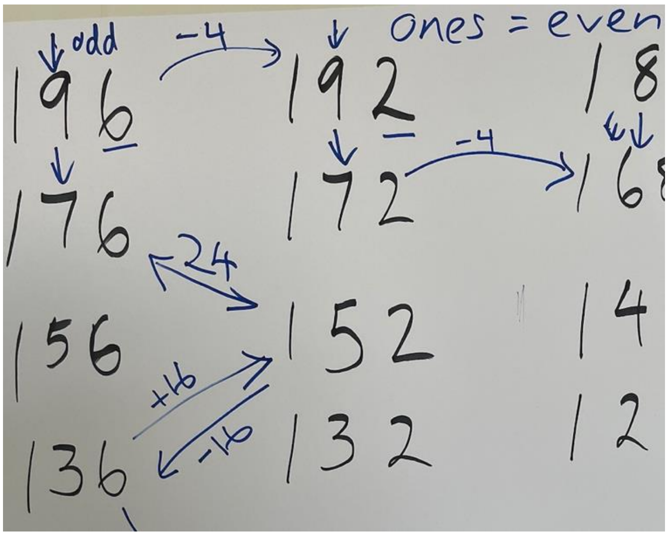
Based on curriculum expectations for the students, it was anticipated that they would be able to notice simple number patterns within larger numbers. The following vignette demonstrates one such instance, during lesson two (backwards count in fours) in class two with the Year Seven and Eight students, discussing observed number patterns.

Class Two – Counting forwards by eight		
Teacher:	Yazmine has just told us that all of the numbers are even. Does anyone disagree with that?	*Generates reasoning by asking students to think about a peer's idea.*
Student 1:	Some numbers are odd	
Teacher:	Where can you see odd numbers?	
Student 1	Nine, seven, five, three (Points to numbers in first column).	
Teacher:	Are nine, seven, five, three all odd numbers? What do we all think about that?	*Supports reasoning by asking all students to think about the statement.*
Student 2:	Well yes and no	
Teacher:	Can you elaborate on that? What do you mean by yes and no?	
Student 2:	Well technically nine, seven, five, and three are all odd numbers but they are tens. It is going in 20s	
Teacher:	So, what does that mean then? If these numbers are going in 20s, are they odd or even numbers? Turn and talk.	*Advances reasoning*

In this vignette, place value concepts were evident, and a misconception arose regarding how place value affects numbers being odd or even. Student 1 had identified within the Ten's column, a pattern of odd digits (nine, seven, five, three) in the first two columns, as illustrated in Figure 2.

Figure 2

Visual recording of choral count: Counting backwards by four



In the above vignette, the teacher provided space for students to discuss the idea of odd numbers by asking for clarification of where the student saw odd numbers. This move positioned the contribution as valid and worthy of discussion and provided an opportunity for the wider group to engage with the conjecture without it being immediately labelled as incorrect. Johnson et al. (2022) describes this type of teacher action as supporting emerging ideas and explain that it promotes participation of students who are not initially clear about the validity of their reasoning. Asking the second student to elaborate on their thinking in relation to numbers being both odd and even, provided the beginning of an explanation of parity of numbers and the difference between digits being odd in relation to place value and numbers being odd as determined by the digit in the Ones place. By revoicing the second student's conjecture and inviting the wider group to discuss concepts of parity after hearing different contributions, all students were provided an opportunity to explore the relationship between place value and parity and to engage in reasoning mathematically. This sequence of teacher moves to explore a misconception is similar to the findings in a study by Mueller and colleagues (2014), whereby teachers specifically focused students to listen to and consider the ideas of others.

The teachers reflected on how the first two planned choral count lessons provided opportunities for investigation and conjecture. Sarah provided the following reflection*:*

> Having the plan right next to me helped me to make sure I stopped and started at really crucial times. When I stopped and asked the questions, I could just see that it got them thinking. They wouldn't have thought about these things if I didn't stop and ask. It would have been easy for this to just be about counting, but knowing what I needed to unpack made it more challenging and gave it a purpose.

This reflection demonstrates the importance of teachers anticipating student responses during the planning phase of teaching mathematics. By anticipating, Sarah was able to notice and respond to the conjectures made by students. Comprehensive planning supported the teachers to press for reasoning with specific questions and prompts to facilitate productive mathematical discussion.

This section has demonstrated how teacher actions can be used to promote the investigation of number patterns through large group discussions. This includes how student understanding of multiplicative concepts can be enhanced when teachers respond to student thinking. Specific teacher actions included questioning and facilitating discussion to focus on misconceptions as well as supporting student engagement and reasoning.

The following section examines teacher actions that supported the development of conceptual explanations.

4.5 Developing Conceptual Explanations

In this section, two examples of how the teachers supported students to develop conceptual explanations that evolved from contributions in large group discussions are examined. The actions of the teacher and the corresponding student responses are analysed in relation to how the discussion supported the development of multiplicative thinking through explanations provided by students.

In the following vignette, the teacher of the Year Seven and Eight students (class two) initiates a discussion about common number properties of groupings of four and eight before the choral count begins.

Class Two – Counting forward by eight		
Teacher:	I'm wondering if counting in eight and four have any similarities. Turn and talk about if you think there might be anything that connects the fours to the eights. *Students discuss with peers.*	*Generates reasoning*
Teacher:	Who would like to share?	
Student 1:	Every second four would be an eight	
Teacher:	Ok who can explain that a bit more to us?	*Supports reasoning*
Student 2:	Every second four will be the same as one of the eights because four plus four equals eight	
Teacher:	I like the way you added on because four plus four equals eight. How else could we say that?	*Supports reasoning*
Student 3:	Four times two is eight	

In this example, to generate reasoning, the teacher posed a specific question requiring students to draw on their prior knowledge. The question linked to the anticipation in the lesson plan that students may notice or identify patterns when counting in twos, fours, and eights. She then employed the talk move of turn and talk to begin student discussion. This action aligns with findings of Kazemi and Hintz's (2023) research, who concluded that specifically positioning students to talk to each other facilitates student discussion.

Next, the teacher opened the discussion to the wider group by asking who would like to share, rather than selecting a specific student to share their thinking. Hunter (2010) explains that by opening the discussion in this way, teachers provide space for student participation to develop. After the first student provided a conjecture connecting the groupings of four and eight, the teacher then asked the wider group for further, detailed explanation. In this pedagogical action, the teacher explicitly named the mathematical practice of explaining, which aligns with what Selling (2016) concluded was necessary in supporting the development of mathematical practices. The students then provided a more detailed explanation outlining an additive strategy.

To further this reasoning, the teacher pressed again, first by revoicing the second student contribution and highlighting how the use of ‘because’ in the explanation was a valuable contribution. These actions align with those advocated by Selling (2016) to enhance the enactment of mathematical practices. After further prompting from the teacher, the third student contribution provided a multiplicative answer. It is evident that the teacher facilitated students to build onto the ideas of their peers. This supports what Franke and colleagues (2015) describe as a way teachers can support students to develop more sophisticated mathematical explanations.

The episode concluded when a multiplicative response was provided, however the opportunity to make connections to the distributive property was missed. This reflects Kazemi and Hintz (2023) suggestion that moving discussion beyond correct answers can present a challenge for teachers when facilitating collaborative discussion. Similar to the findings of Ingram et al. (2019), who noted that student initiated explanations from primary students were rare, throughout this episode, the student explanation was prompted by a teacher question rather than a spontaneous contribution by a student.

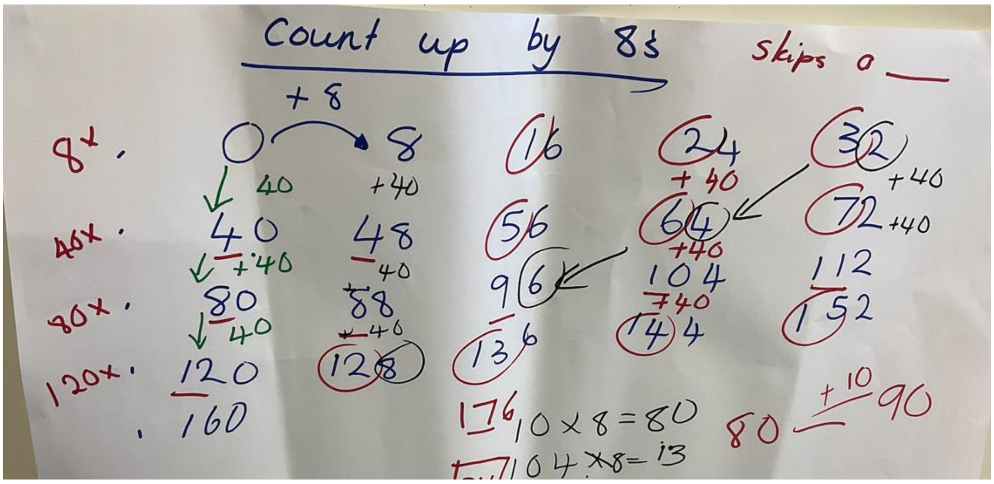
In the following vignette, an example is provided of how the teacher in class one supported students to develop a multiplicative explanation using grouping strategies after they identified that each column in the count was increasing by 40, or five groups of eight when completing a choral count forward by eight.

Class One – Counting forward by eight		
Teacher:	So, if we jump down two lines, how many groups of eight are we adding on now? *Points to the board and signals a jump from zero to 80*	*Generates reasoning*
Students:	Ten! Its 10!	
Student 1:	Ten times eight is 80. *Teacher records 10 x 8 = 80*	
Teacher:	Okay, yes, 10 times eight is 80. So, let's use what we know, and can you work out how many groups of eight there are in 104?	*Advances reasoning*
Student 2:	Thirteen.	
Teacher:	Thirteen? And how did you know it was 13? What did you do?	*Supports reasoning*
Student 2:	I just looked and its three more from 80	
Teacher:	Three more what?	*Supports reasoning*
Student 3:	Three more eights.	
Teacher:	Yes, it is. Everybody look up here. How many groups of eight are there in 80? *Points to 80*	*Supports reasoning*
Students:	Ten.	
Teacher:	Ok so how many more jumps do we do to get to 104? *Points to 80 and counts one, two, three… as she points to 88, 96 and 104*	*Supports reasoning*
Students:	Three.	
Teacher:	So how many groups of eight was that altogether?	*Advances reasoning*
Student 2:	Thirteen! 10 groups of eight, 80, and then three more makes 104.	

In the above vignette, the teacher responded to a student noticing that each line was increasing by 40 and pressed for further exploration by asking students to think about what had been stated and apply this thinking to the number 104. To support students to do this, she emphasised the visual recording of the count as a guide, as illustrated in Figure 3.

Figure 3

Visual recording of choral count: Counting forwards by eight



The teacher directed all students to the visual representation so students could see how many groups of eight there were in 80. After the student quickly claimed that it was 13 groups of eight, she specifically directed all students to look at the board and then counted how many numbers they had counted on from 80. Students used these three extra “jumps” to surmise that 104 is 13 groups of eight. In this example, the visual representation of mathematical ideas supported the students to develop their mathematical reasoning. Turrou and colleagues also highlighted visual representation as a feature of choral counting activities that enhanced student reasoning (Turrou et al., 2017).

Additionally, in the above episode, the teacher connected the student response of three “jumps” to how many groups of eight there are all together, modelling mathematical language for the students. Scaffolding the students in this way prompted the provision of a more detailed multiplicative explanation of how there were 13 groups altogether. While one student provided both explanations in this episode, the teacher facilitated participation from all students to work through the response together. In this instance, the teacher was required to enact several support moves before a mathematical explanation was developed by the students. This aligns with research by Mueller and colleagues (2014), who suggest that as the level of challenge increases for students, the need for teacher actions that encourage explanations also increases.

This section has explored two key episodes illustrating how teachers fostered explanations of multiplicative thinking through guided discussions. In the first example, the teacher facilitated a conversation about the relationship between counting in fours and eights, prompting students to articulate their thinking and develop an explanation that connected additive and multiplicative concepts. In the second vignette, the teacher supported students in analysing the choral count of eights, leading them to develop a multiplicative explanation for finding groups of eight within a larger number. Both episodes highlight the teachers' use of strategic questioning and the use of visual aids to support reasoning and encourage deeper mathematical explanations.

The next session provides an example of how teacher actions supported students to progress from providing explanations to making justifications.

4.6 Justifying Multiplicative Concepts

As the teachers and students progressed through the series of choral counts, it was noted that student explanations developed further into more detailed justifications. This section examines one vignette from class two as they explored the distributive property as a solution that emerges after discussing five groups of 12 and then 10 groups of 12.

Class Two – Counting forward by twelve		
Teacher:	How many groups of 12 are there in 240?	*Generates reasoning*
Student 1:	Twenty.	
Teacher:	How many groups of 12? Can you repeat that?	
	Points to another student	
Student 2:	Twenty.	
	Teacher points to another student	
Student 3:	Twenty.	
Teacher:	Okay, so you all say 20 groups of 12. Now you need to be able to justify and prove it to us. Turn and talk and see if you can justify how you know that 20 times 12 is 240.	*Supports reasoning*
Students:	*Talk to peers. Teacher listens in.*	
Kimi:	*(to peer)* Five times 12 is 60 and there are four 60s in 240	
Student 4:	*(to student 3)* What do you mean?	
Teacher:	*(to student 4)* Would it be helpful if she wrote that that down for you? I love the way you asked that. Kimi, can you please go up and write this on the board. Okay, let's all watch this. I'm going to also get Jake to share next because he had a different way too. Okay, talk us through Kimi.	*Supports reasoning*
Kimi:	In the first line, there's five 12s which is 60, and there's four 60s in 240, and so four times five is 20.	
	Kimi points across the rows on the recorded count on the board as she talks.	
Teacher:	Thanks Kimi. Do we want to agree with the first step? Yes? There are five lots of 12 in 60? Okay so then she says, well how many lots of 60 are there in 240? Which is?... Four lots. Lovely. Now let's just explain that last bit. How does four	*Advances reasoning*

	times five equals 20 help us find out how many 12s are in 240?	
Student 5:	I saw five 12s in each line and there's four lines so that's 20 12s. It seems easier *(student becomes unsure and starts shaking head)*	
Teacher:	Tell us why you think your way is easier. *(student shakes head).* It isn't wrong, it's just a different way of explaining. It's the same answer. Instead of 20 times 12 equals 240, Kimi showed us 240 divided by 60 equals four, and 60 divided by 12 equals five. It works the same. It's just different ways of people looking at things. Coming to the same solution which is good. Do you see how it's the same? (*Points to equations on board, student nods)* Okay? Who else saw it like this? Does that help us understand why four times five helps us to solve this? *(Students nod and agree).* Okay good. Jake, can you please come up and share your way because someone else's brain might work like yours did.	*Advances reasoning*

In this vignette, the teacher noticed the opportunity to press for justification when several students quickly answered the question. She then specifically named the practice of justifying or proving how they knew the answer was 20. The talk moves of repeat and turn and talk were employed to generate reasoning amongst students. These actions support those found in previous studies (e.g., Chapin et al., 2009; Kazemi & Hintz, 2023). In this instance, the turn and talk move was employed to ensure students understood the reason why 20 was the answer, rather than simply repeating what someone else had said as the answer. Walla (2023) analysed the enactment of talk moves and their effectiveness on supporting students to progress from participation in talk to participation in understanding and mathematical reasoning. The study found that it was difficult for students to participate in equitable discussion beyond repeating another student's idea if they did not first understand the content being discussed, and that by being given the opportunity to reason with each other’s thinking, students began to participate more effectively.

In the above vignette, the teacher noticed, during peer discussion, that Kimi had used groupings of five and then groupings of 60 to justify her answer of 20 groups of 12. Her peer appeared confused by her solution and asked a clarifying question. The teacher noticed this interaction and publicly praised her for seeking clarification of Kimi’s solution. This action, validating her question as a worthwhile mathematical contribution, aligns with findings of Hunter’s (2010) study where marginalised students were supported to be valued as mathematically competent through validation of contributions that supported the development of understanding within the wider group. The teacher then invited Kimi to record her thinking and to share with the wider group so that all students could engage with this solution strategy.

To support the understanding of the wider group, the teacher revoiced each step of the solution strategy. This is similar to what Lobato and colleagues (2005) describe as supportive teacher telling, where the teacher carefully scaffolds students towards justification with substantive and assertive questions. The teacher continued to ask the wider group to provide justification to support the last step of the solution of using four times five and to explain how this connected to the overall solution strategy. These teacher actions supported students to deeply engage in the thinking of others by adding on to and co-constructing a new idea. This aligns with Franke et al. (2015), who assert that specific teacher actions are required to position students to build on each other’s ideas.

In the next part of the discussion within this episode, Student 5 offered a valid explanation as to how four times five aligned within Kimi’s shared solution, however, the student was unsure and not confident that their solution was correct. This could indicate that the student, while understanding their own solution strategy, was unable to see how it connected to the previously shared solution strategy. The teacher’s next move supported the students to make links between both responses and highlighted that while different, both provided a valid and correct explanation. Johnson and colleagues (2022) explain that supporting students to see their contributions as valid is a necessary teacher action that publicly assigns competence to students. In this episode the teacher also used mathematical language and made reference to the inverse relationship between multiplication and division. These actions connect to findings by Moschkovich (2015a) that suggests that teacher contributions of new information support students to develop their conceptual understanding when justifying.

In this section, an example of how justification of multiplicative concepts was examined as the teacher facilitated student understanding through strategic questioning and collaborative discussion. By validating diverse approaches and highlighting interconnectedness of different mathematical strategies, the teacher created an environment where students' competence was emphasised, and students were supported in their reasoning processes.

The following section explores an example of teacher actions that supported students to generalise multiplicative patterns.

4.7 Generalising Multiplicative Patterns

One of the intentions of the choral count lesson plans was to provide opportunities to generalise multiplicative patterns. Specific questions pertaining to unidentified future numbers of the choral count were planned to support generalisations. Figure 4 shows an example from the lesson plan (see Appendix B4) of counting backwards by three and the teacher prompts to support students to identify unknown numerals and to reason as to whether specific numbers would appear in the count.

Figure 4

Example of lesson plan generalisation prompts: Counting backwards by three

150	147	144	141	138
135	132	129	126	123
120	117	114	111	108
105	102	99	96	93
a			b	
	c			d
Questions to develop reasoning: How many groups of 3 are in 30? How many groups of 3 are in 45? Will 86 be part of this count?	Will 71 be part of this count? Will 60 be part of this count? What numbers do a, b, c and d represent?			

The following vignette provides an example of how these specific prompts supported students in class one to make generalisations about multiplicative patterns that were generated when counting backwards in threes. Prior to this specific episode, the class had worked out that 72 was the number represented by the letter C on the lesson plan (see Figure 4) and had also noticed and discussed that there were horizontal groupings of 15 or five groups of three on every row. The following episode provides an example of the teacher supporting students to generalise beyond the current count to find the number represented by letter D on the lesson plan (see Figure 4).

Class One – Counting backwards by three		
Teacher:	Using this knowledge (groupings of three), let’s do a prediction of what number would be two numbers underneath 93.	*Generates reasoning*
	(Points to board to represent where the unknown number would sit on the recorded count)	
	Remember you need to prove why.	
Student 1:	Something that ends with a three.	
Teacher:	Who agrees with that? *Pauses and notices students nodding in agreement*	*Supports reasoning*
	Right, turn and talk to your buddy and see if you can work it out. Use the patterns to make your prediction.	*Supports reasoning*
	Students turn and talk and discuss with their peers. Teacher listens and notices.	
	Okay. Let’s see. Who agrees that the number will end with a three?	*Supports reasoning*
Student 2:	I agree, because each number is 15 less (vertically) so it will end in a three because the pattern goes three, eight, three, eight	
Teacher:	Yes, the pattern in this column is going three, eight, three eight in the Ones column *(points to fifth column on board).* Why does the pattern do this?	*Advances reasoning*
Student 3:	Because we take off five groups of three which is 15 and a five (grouping) always uses the same two numbers.	
Teacher:	Okay yes. Who can say that in their own words?	*Supports reasoning*

The above vignette illustrated how the teacher generated multiplicative reasoning by specifically telling students that they were going to make a prediction (generalisation) and that they needed to be able to prove (justify) how they got their answer. This is another example of the teacher specifically naming the mathematical practices that students will be engaging in to support reasoning, which aligns with the work of Selling (2016).

To support generalisation, the teacher enacted a series of talk moves, specifically agree or disagree and turn and talk to continue the collaborative discussion and provide students with opportunities to discuss their thinking. This also connects to Kazemi and Hintz (2023) who concluded that talk moves facilitate students to engage in collaborative reasoning and discourse. In this episode, the teacher advanced the students towards multiplicative reasoning by asking students to mathematically justify why there was a repeating pattern of three and eight in the Ones column. This deliberate action supported Student 3 to extend Student 2’s explanation of each number being 15 less, by adding on that 15 was represented by five groupings of three in the choral count.

Again, the teacher actions supported participation amongst students as they engaged in other students’ thinking by adding on and extending the original idea posed. Students demonstrated their developing multiplicative reasoning by identifying and utilising patterns and additive strategies. The teacher actions provided opportunities for students to further expand their explanations by showing a growing trust in the count and incorporating grouping strategies in their counting methods. These actions support those highlighted in studies by Hurst & Hurrell (2016), and Siemon et al. (2005), where examples of activities that explicitly addressed concepts of counting and place value were shown to advance the development of numeracy capabilities.

The vignette examined in this section provided an example of how the teacher supported students to make generalisations by using specific questioning prompts and talk moves. It has also demonstrated how important mathematical practices can be developed during a conceptual starter lesson. In addition, the teacher continually pressed the students to explain and justify their thinking in order to effectively generalise and test initial conjectures and ideas.

4.8 Final Interview Teacher Reflections and Summary

This section provides teacher reflections from the final interviews and a summary of the findings from the current study.

4.8.1 Final Interview Teacher Reflections

Final interviews were conducted with teacher participants to capture their reflections on their experience with the planned choral counts. Teachers were asked to reflect on how they felt they were supported by the lesson plans, and how they felt the overall development of their students' multiplicative reasoning was supported by their actions (see Appendix A2).

Of note, Sarah reflected on how her own growth in knowledge of supporting students how to learn to think multiplicatively:

> This whole experience made me think about different ways to learn times tables. It's always been just practice them and remember them. They really had to think about how many groups there were and why, and then how you can use those easy ones (times tables) to figure out harder ones. And even the students who weren't sure got a chance to see how the same patterns work in big numbers.

Alice reflected on the goal of supporting students to make their own conjectures rather than waiting for the teacher to tell them the answer and how her own actions supported this:

> It really all comes down to those talk moves doesn't it. Turn and talk, tell your buddy, don't tell me…. I think we just all got better the more we did, me and the kids. It's a real skill knowing what I should be getting them to think about and making sure I don't just skip over things.

These reflections provide valuable insight into how teacher participants' views of developing multiplicative reasoning shifted over the course of the study. Evident is the teacher's reflections on how the use of collaborative discussion supports both participation and conceptual understanding for all students. Teachers reflected that the goals they set at the beginning of the study were largely met over the course of the study, with development of skills over time, for both teachers and students, being a reflection both teachers shared.

4.8.2 Summary

This chapter has presented the findings and discussion with links to the relevant literature of how teacher actions facilitated student collaborative discourse around multiplicative concepts when implementing choral counting as a conceptual starter activity. The chapter outlined existing teacher practices and planning strategies. The vignettes provided evidence of how students were able to reason with multiplicative concepts, namely by noticing and conjecturing, explaining, justifying and generalising and how teacher actions were critical to the development of these mathematical practices. Importantly, the vignettes highlighted that it was a series of teacher moves that pressed students to not only provide correct answers, but to collaboratively unpack misconceptions and construct new learning. Evident were specific teacher actions across several stages of generating, supporting, and advancing student reasoning, that encouraged collaboration and conceptual understanding. This chapter concluded with analysis of two reflections from the teacher participant final interviews, providing examples of how teacher perceptions of multiplicative reasoning have changed.

The next chapter will present a summary of the research questions, themes identified in the data analysis and provide key findings for educators to draw on to support the development of multiplicative reasoning in upper primary students when enacting choral count activities.

Chapter Five: Conclusion

5.1 Introduction

This chapter concludes the research study by reviewing the main findings in relation to the research questions. Section 5.2 summarises the research questions. Section 5.3 presents key themes and recommendations of the research findings. Section 5.4 addresses limitations of the study. Finally, section 5.5 outlines suggested areas for future research and concluding thoughts.

5.2 Summary of Research Questions

The overall aim of the study was to investigate how students reasoned with multiplicative concepts while engaging in collaborative discussion. To examine the role a planned conceptual starter activity of choral counting can play in developing multiplicative reasoning in students, two research questions were developed.

1. How do planned choral counting conceptual starter activities support students to develop mathematical practices?
2. What teacher actions during the conceptual starter activity support the development of students' multiplicative reasoning?

5.3 Key Findings

This section summarises the five key findings of this study. Recommendations for how choral counting can be implemented as a conceptual starter activity that advances multiplicative reasoning are presented.

5.3.1 Lesson planning

Careful planning was undertaken for each conceptual starter activity. Planning focussed on both mathematical concepts and mathematical discourse. The teachers collaborated with the researcher to create a series of six choral counts that focussed on specific multiplication sequences, designed to support students to recognise and understand number patterns, place value concepts, additive strategies, and to explore grouping ideas. Careful consideration was given to how to record and pause during the count to provide opportunities for deeper thinking. For example, the planned counting sequence and how it was recorded not only demonstrated relationships between the numbers but also linked to broader multiplicative concepts such as the distributive property. The importance and effectiveness of implementing structured lesson plans for conceptual starter activities focussed on developing multiplicative reasoning is emphasised in the current study. The implementation of strategic pauses where planned questions were asked to support reasoning during the lesson allowed for meaningful student discourse. Recommendations for other educators include establishing and reinforcing social and group norms, and expectations that promote active participation for all students as well as ensuring all students feel empowered and supported to share their thinking.

5.3.2 Supporting investigations and conjectures

Through the investigation of multiplicative number sequences, students were afforded opportunities to explore concepts and misconceptions regarding place value and odd and even numbers. Teachers supported student reasoning by using strategic open-ended questioning to prompt discussion. Student reasoning was developed when the teachers facilitated peer discussion and prompted elaboration on shared ideas. Overall, these teacher actions effectively guided students in making conjectures and deepening their understanding of number patterns. One key recommendation from the current study to enhance student multiplicative reasoning, is educators should incorporate questioning and collaborative discussion strategies such as talk moves into their choral counting lessons, to focus student noticing on number patterns. This will create an environment where students have opportunities to investigate counting patterns and place value concepts, which are pre-requisites for developing multiplicative thinking.

5.3.3 Developing explanations

When undertaking the choral count starter activities, teachers used planned pauses and open-ended questioning techniques to support students to generate explanations. Drawing on students' prior knowledge allowed teachers to facilitate discussion within the larger group, supporting students to draw on each other's ideas to build a more sophisticated explanation of the multiplication patterns represented in the count. As highlighted in the vignettes in the previous section, multiple teacher support moves that pressed students to add more detail and clarify their ideas were used in order to advance students towards developing conceptual explanations that were linked to multiplicative concepts. It is recommended that educators make use of the visual representations that are generated during a choral count activity to draw students' attention to multiplicative patterns within the choral count. To support students to make conceptual explanations that link additive thinking to multiplicative reasoning, educators must persist with guiding questions that press students to provide mathematical reasons that go beyond a correct answer.

5.3.4 Justifying multiplicative concepts

It was evident that students' explanations developed into more detailed justifications as the study progressed. Through specific questioning and implementation of talk moves, teachers were able to facilitate collaborative discourse that supported students to develop their own reasoning of multiplicative concepts by building on the ideas of other students. By supporting students to revoice ideas in their own words, students had the opportunity to develop reasoning of how or why particular number patterns were linked to multiplication facts. It is recommended that educators support students to share a variety of solution pathways and provide space for students to make sense of ideas that are different to their own. Teacher validation of student contributions and teacher actions that used these contributions to support the overall learning of the larger group facilitated all students to engage in mathematical reasoning while recognising both their own and others' competence.

5.3.5 Generalising multiplicative patterns

By planning opportunities for students to make generalisations, teachers in the current study were able to use specific questions to prompt students to make reasoned generalisations during the choral count activities. By supporting students to identify and utilise number patterns and additive strategies, teachers were able to advance students to use grouping strategies to generalise to numbers beyond those recorded in the choral count. It is recommended that educators use specific language to support students to identify the mathematical practices they are engaging in, such as predicting and proving. Also of importance, is encouraging students to question and argue in relation to the mathematical ideas presented, including their own, to support deeper reasoning related to the links between known additive strategies and grouping strategies to progress multiplicative reasoning.

5.4 Limitations of the Study

As in any research study, there are limitations in this study. The current study took place in two classrooms within one school, where teachers were currently undertaking professional development in developing mathematical inquiry classrooms. For the teachers and students in the current study, collaborative discourse was a familiar way of engaging in mathematical reasoning. Teachers had some experience facilitating collaborative discussion and for students, expectations regarding participation were an ongoing feature of their daily mathematics lessons. Consequently, the ability to generalise the findings to teachers and students in classrooms which have not engaged in similar learning contexts is limited. Given the relatively small number of participants and the short timeframe, the interpretation of the results can only provide an emerging understanding of the ways in which multiplicative reasoning developed through the enactment of choral count starter activities. Working with a larger number of classroom teachers may have provided a broader range of outcomes of the lessons and teacher perspectives. The study also covered a wide area of research, investigating teacher practices that supported both mathematical practices and multiplicative reasoning, while students participated in collaborative discussion during a choral count starter activity. A narrower focus on one mathematical practice such as the enactment of justification, may have provided a deeper understanding of how reasoning can be supported by specific teacher actions during a choral count activity.

5.5 Suggested Areas for Future Research and Concluding Thoughts

In this section, a summary of suggested areas for future research is presented.
Given that the development of multiplicative thinking in primary school students is complex and develops over time, a longitudinal study investigating how planned conceptual starter activities such as choral counting can support this development would be warranted. Furthermore, a larger scale study involving a greater number of participants would help ascertain if the findings are relevant in other settings. The findings of this study highlight the need for further research to investigate how teachers can be supported in planning and implementing starter lesson activities with a conceptual focus. These activities should focus on incorporating mathematical practices and participation in collaborative discourse for all students, to support students to develop sense making of mathematical concepts. The enactment of teacher actions that press for deeper reasoning and sense making, not just correct answers is critical to the success of such activities.

In conclusion, to address the disparities in mathematics education achievement in Aotearoa New Zealand, it is important for educators to challenge current practices regarding the ways in which students are learning and practicing multiplication concepts, and the types of starter activities used in daily lessons. It is worthwhile for educators to investigate the implementation of teacher guided conceptual starter activities which promote the development of multiplicative reasoning and mathematical practices through collaborative discussion. Educators also need to ensure that students have a positive disposition towards risk taking and sense making in mathematics and that they feel valued in their contributions. When students are enabled to see a purpose in their participation in collective sensemaking, engagement of all members of the classroom will increase beyond the starter activities and across all mathematics learning areas.

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Appendices

Appendix A1: Semi-structured Initial Interview Questions

1. What mathematical warm-ups do you currently use in your class?

2. How regularly do you do each warm-up?

3. What is the main goal of each warm-up?

4. How do you select warm-ups to use with your class?

5. Do you notice transference of knowledge and skills from warm-ups when problem-solving / to other mathematics tasks / assessments?

6. What opportunities do students have to reason with each other's ideas during each warm-up?

7. What opportunities are there in your mathematics programme for basic facts learning and practice?

8. Do you notice a transference of these basic facts when problem-solving / to other mathematics tasks/ assessments?

Appendix A2: Semi-structured Final Interview Questions

1. What opportunities did you see students having to reason with multiplicative concepts during the choral counts?
2. Did you see any changes in engagement and or confidence in students as you did the choral counts?
3. If yes, what were the factors that you felt made the biggest difference in student engagement?
4. What specific strategies did you use to increase student participation?
5. Did you notice transference of knowledge and skills from the choral counts when problem-solving / undertaking mathematics tasks/ assessments?
6. What are your thoughts on using a warm-up with a planned outcomes/concept?
7. What would be more useful/helpful for implementing conceptual starters in your everyday lessons?

Appendix B1: Choral Counting Planning – Count One and Two: Counting Forwards and Backwards by Four

<table>
<tr><th>Understand</th><th>Know</th><th>Do (Ministry of Education, 2024)</th></tr>
<tr>
<td>• The world is full of patterns and structures that we use mathematics and statistics to understand.</td>
<td><i>Phase 2</i>
• find factors of numbers up to 100
• recall multiplication facts to 10 × 10 and corresponding division facts

<i>Phase 3</i>
• identify and describe the properties of composite numbers</td>
<td><i>Generalising:</i>
• recognise and explore patterns, and make conjectures and draw conclusions about them
• identify relationships, including similarities, differences, and new connections
• look for patterns and regularities that might be applied in another situation or always be true
• make and test conjectures, using reasoning and counterexamples to decide if they are true or not

<i>Explaining and Justifying:</i>
• make statements and give explanations inductively based on observations or data
• make statements and give explanations deductively based on knowledge, definitions, and rules
• critically reflect on others' thinking, evaluating their logic and asking questions to clarify and understand
• use evidence, reasoning, and proofs to explain why I agree or disagree with statements
• develop collective understandings by sharing, comparing, contrasting, critiquing, and building on ideas with others
• present reasoned explanations and arguments for an idea, solution, or process</td>
</tr>
<tr>
<th colspan="2">Task
Count forwards by 4 from 20</th>
<th>Patterns expected to be noticed</th>
</tr>
<tr>
<td colspan="2">Key Mathematical Ideas:
Noticing Patterns
• The pattern of 4 and how this works in the base 10 number system
• Noticing pattern of 4 in 2-digit numbers (0, 4, 8, 2, 6)
• The value of "odd numbers" in the tens and hundreds column
• Noticing the power of 10 in 20, 40, 60, 80 counts
Additive Ideas
• Noticing doubling of the pattern of 2
• Noticing each column going up by 20
• Noticing doubling of the pattern of 2</td>
<td>
<table>
<tr><td>20</td><td>24</td><td>28</td><td>32</td><td>36</td></tr>
<tr><td>40</td><td>44</td><td>48</td><td>52</td><td>56</td></tr>
<tr><td>60</td><td>64</td><td>68</td><td>72</td><td>76</td></tr>
<tr><td>80</td><td>84</td><td>88</td><td>92</td><td>96</td></tr>
<tr><td></td><td>a</td><td></td><td>b</td><td></td></tr>
<tr><td>c</td><td></td><td></td><td></td><td>d</td></tr>
</table>

Planned Pauses:
• Stop after first 4 lines and ask students to notice patterns.
• Ask some questions about what number will be here? (a, b, c and d) How do you know, turn and talk and justify.</td>
</tr>
</table>

<table>
<tr>
<td>Grouping or Distributive Ideas
• 4 x 5 = 20 and how this relates to each vertical pattern increasing by 20
• 4 x 10 = 40 and how this relates to every 2nd vertical pattern increasing by 40
• Noticing doubling of the pattern of 2 and the pattern of 20 and how this relates (20, 40, 60, 80 vs 2, 4, 6…) x 10
• Noticing that ? x 4 = 54 can be distributed as 10 x 4 + 4 x 4
• Use of inverse relationship x/÷</td>
<td>Questions to develop reasoning:
• Will 130 be part of this count?
• How many groups of 4 are in 40?
• How many groups of 4 are in 56?
• How many groups of 4 are in 84?</td>
</tr>
<tr>
<td>Task
Count backwards by 4 from 196</td>
<td>Patterns expected to be noticed</td>
</tr>
<tr>
<td>Key Mathematical Ideas:
Noticing Patterns
• The pattern of 4 and how this works in the base 10 number system
• Noticing pattern of 4 in 3-digit numbers (0, 4, 8, 2, 6)
• The value of “odd numbers” in the tens and hundreds column
• Noticing the power of 10 in 20, 40, 60, 80 counts in 3-digit numbers
Additive Ideas
• Use of repeated subtraction
• Noticing each column going down by 20
• Noticing that to subtract 4 you can still count backwards by 2
Grouping or Distributive Ideas
• 4 x 5 = 20 and how this relates to each vertical pattern decreasing by 20
• 4 x 10 = 40 and how this relates to every 2nd vertical pattern decreasing by 40
• Noticing doubling of the pattern of 2 and the pattern of 20 and how this relates (180, 160, 140 vs 18, 16, 14) x 10
• Noticing that ? x 4 = 180 can be distributed as 40 x 4 + 5 x 4 (or 10 x 4 + 10 x 4 + 10 x 4 + 10 x 4 + 5 x 4 etc)
• Use of inverse relationship x/÷</td>
<td>
<table>
<tr><td>196</td><td>192</td><td>188</td><td>184</td><td>180</td></tr>
<tr><td>176</td><td>172</td><td>168</td><td>164</td><td>160</td></tr>
<tr><td>156</td><td>152</td><td>148</td><td>144</td><td>140</td></tr>
<tr><td>136</td><td>132</td><td>128</td><td>124</td><td>120</td></tr>
<tr><td>a</td><td></td><td>b</td><td></td><td>c</td></tr>
<tr><td></td><td></td><td></td><td></td><td>d</td></tr>
</table>
Planned pauses:
• Stop after first 4 lines and ask students to notice patterns.
• Ask some questions about what number will be here? (a, b, c and d) How do you know, turn and talk and justify.

Questions to develop reasoning:
• What would come before 196?
• Will 118 be part of this count?
• How many groups of 4 are in 180?
• How many groups of 4 are in 148?
• How many groups of 4 are in 124?
</td>
</tr>
</table>

Appendix B2: Choral Counting Planning – Count Three and Four: Counting Forwards and Backwards by Eight

<table>
<tr><th>Understand</th><th>Know</th><th>Do (Ministry of Education, 2024)</th></tr>
<tr>
<td>• The world is full of patterns and structures that we use mathematics and statistics to understand.</td>
<td>Phase 2
• find factors of numbers up to 100
• recall multiplication facts to 10 × 10 and corresponding division facts

Phase 3
• identify and describe the properties of composite numbers</td>
<td>Generalising:
• recognise and explore patterns, and make conjectures and draw conclusions about them
• identify relationships, including similarities, differences, and new connections
• look for patterns and regularities that might be applied in another situation or always be true
• make and test conjectures, using reasoning and counterexamples to decide if they are true or not

Explaining and Justifying:
• make statements and give explanations inductively based on observations or data
• make statements and give explanations deductively based on knowledge, definitions, and rules
• critically reflect on others' thinking, evaluating their logic and asking questions to clarify and understand
• use evidence, reasoning, and proofs to explain why I agree or disagree with statements
• develop collective understandings by sharing, comparing, contrasting, critiquing, and building on ideas with others
• present reasoned explanations and arguments for an idea, solution, or process</td>
</tr>
<tr>
<th colspan="2">Task
Count forwards by 8 from 0</th>
<th>Patterns expected to be noticed</th>
</tr>
<tr>
<td colspan="2">Key Mathematical Ideas
Noticing Patterns
• The pattern of 8 and how this works in 2-and-3-digit numbers
• Noticing doubling of the pattern of 4 – links to pattern of 2
• Noticing the power of 10 in 8 counts
• Noticing all numbers are even
• Noticing the value of "odd numbers" in 10s and 100s column
Additive Ideas
• Pattern of 4 – links to previous count of four (doubling)
• Each column changes by 40</td>
<td>
<table>
<tr><td>0</td><td>8</td><td>16</td><td>24</td><td>32</td></tr>
<tr><td>40</td><td>48</td><td>56</td><td>64</td><td>72</td></tr>
<tr><td>80</td><td>88</td><td>96</td><td>104</td><td>112</td></tr>
<tr><td>120</td><td>128</td><td>136</td><td>144</td><td>152</td></tr>
<tr><td></td><td>a</td><td></td><td></td><td></td></tr>
<tr><td></td><td>b</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>c</td><td></td><td>d</td></tr>
</table>
Planned Pauses:
• Stop after first 4 lines and ask students to notice patterns
• Ask some questions about what number will be here? (a, b, c and d) How do you know, turn and talk and justify
</td>
</tr>
</table>

<table>
<tr>
<td>• Diagonal patterns add or subtract a specific amount
• Using base 10 to support adding and subtracting with other numbers (e.g. counting in 8 can be seen as + 10 – 2, or counting in 12 can be seen as + 10 + 2)
Grouping and Distributive Ideas
• Links to multiplication from noticing of number increases (e.g. 4 x 5 = 20 and how this relates to each vertical pattern increasing by 20)
• Use of the distributive property to work out how many groups of 8 are in 104</td>
<td>Questions to develop reasoning:
• How many groups of 8 are in 40?
• How many groups of 8 are in 80?
• How many groups of 8 are in 104?

• Why are there only even numbers in this count? (8 is an even number and when adding 2 even numbers you always get an even number)
• How does the pattern of 8 grow? (e.g. add 10 and take off 2 - that’s why there is only 0, 2, 4, 6, 8)
• Can anyone see the pattern of 4? (0, 40, 80, 120 as pattern of 4 to the power of 10)</td>
</tr>
<tr>
<td>Task
Count backwards by 8 from 272</td>
<td>Patterns expected to be noticed</td>
</tr>
<tr>
<td>Key Mathematical Ideas
Noticing Patterns
• The pattern of 8 and how this works in 2- and 3-digit numbers
• Noticing doubling of the pattern of 4 – links to pattern of 2
• Noticing the power of 10 in 8 counts
• Noticing all numbers are even
• Noticing the value of “odd numbers” in 10s and 100s column
Additive Ideas
• Pattern of 4 – links to previous count of four (doubling)
• Each column changes by 40
• Diagonal patterns add or subtract a specific amount
• Using base 10 to support adding and subtracting with other numbers (e.g. counting in 8 can be seen as + 10 – 2, or counting in 12 can be seen as +10 +2)</td>
<td>
<table>
<tr><td>272</td><td>264</td><td>256</td><td>248</td><td>240</td></tr>
<tr><td>232</td><td>224</td><td>216</td><td>208</td><td>200</td></tr>
<tr><td>192</td><td>184</td><td>176</td><td>168</td><td>160</td></tr>
<tr><td>152</td><td>144</td><td>136</td><td>128</td><td>120</td></tr>
<tr><td></td><td></td><td>a</td><td></td><td>b</td></tr>
<tr><td>c</td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>d</td><td></td></tr>
</table>

Planned Pauses:
• Keep starting from the beginning when it gets a bit tricky
• Stop after first 4 lines and ask students to notice patterns
• Ask some questions about what number will be here? (a, b, c and d) How do you know, turn and talk and justify

Questions to develop reasoning:
• How many groups of 8 are in 160? (can they link 8 x 2)
• How many groups of 8 are in 240? (can they link 8 x 3)</td>
</tr>
</table>

Grouping and Distributive Ideas • Noticing the power of 10 multiplication (3 x 8 = 24 therefore 30 x 8 = 240 and how this helps us find how many groups of 8 are in 264) (distributive property)	• How groups of 8 are in 264? (do they count on from 240 is 30 groups so 3 more groups?) • Why are there only even numbers in this count? (8 is an even number and when adding 2 even numbers you always get an even number) • How does the pattern of 8 decrease (e.g. subtract 10 and add 2) that's why there is only 0, 2, 4, 6, 8 • Can anyone see the pattern of 4? (0, 40, 80, 120 as pattern of 4 to the power of 10)

Appendix B3: Choral Counting Planning – Count Five and Six (class two): Counting Forwards and Backwards by 12

<table>
<tr><th>Understand</th><th>Know</th><th>Do (Ministry of Education, 2024)</th></tr>
<tr>
<td>• The world is full of patterns and structures that we use mathematics and statistics to understand.</td>
<td><i>Phase 2</i>
• find factors of numbers up to 100
• recall multiplication facts to 10 × 10 and corresponding division facts

<i>Phase 3</i>
• identify and describe the properties of composite numbers</td>
<td><i>Generalising:</i>
• recognise and explore patterns, and make conjectures and draw conclusions about them
• identify relationships, including similarities, differences, and new connections
• look for patterns and regularities that might be applied in another situation or always be true
• make and test conjectures, using reasoning and counterexamples to decide if they are true or not

<i>Explaining and Justifying:</i>
• make statements and give explanations inductively based on observations or data
• make statements and give explanations deductively based on knowledge, definitions, and rules
• critically reflect on others' thinking, evaluating their logic and asking questions to clarify and understand
• use evidence, reasoning, and proofs to explain why I agree or disagree with statements
• develop collective understandings by sharing, comparing, contrasting, critiquing, and building on ideas with others
• present reasoned explanations and arguments for an idea, solution, or process.</td>
</tr>
<tr><th colspan="2">Task
Count forwards by 12 from 60</th><th>Patterns expected to be noticed</th></tr>
<tr>
<td colspan="2">Key Mathematical Ideas
Noticing patterns
• The pattern of 12 and how this works in the base 10 number system
• Noticing pattern of 12 in 2- and 3-digit numbers
• Noticing each column goes down by 60 (links to 5 x 12)
Additive Ideas
• Repeated addition and subtraction to count on or back
Grouping and Distributive Ideas
• Noticing the way multiplication can be distributed (23 x 12 can be the same as 20 x 12 + 3 x 12)</td>
<td>
<table>
<tr><td>60</td><td>72</td><td>84</td><td>96</td><td>108</td></tr>
<tr><td>120</td><td>132</td><td>144</td><td>156</td><td>168</td></tr>
<tr><td>180</td><td>192</td><td>204</td><td>216</td><td>228</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>a</td><td></td><td></td><td>b</td><td></td></tr>
<tr><td></td><td>c</td><td></td><td></td><td>d</td></tr>
</table>

Planned Pauses:
• Keep restarting if the count becomes tricky
• Stop after first 3 lines and ask students to notice patterns</td>
</tr>
</table>

<table>
<tr>
<td>• Noticing the power of 10 in 12counts (e.g. if 5 x 12 is 60, 50 x 12 = 600</td>
<td>• Continue for two more lines. Keep revisiting previous lines if counting is becoming tricky
• Ask some questions about what number will be here? (a, b, c and d) How do you know, turn and talk and justify

Questions to develop reasoning:
• How many groups of 12 are there in 60?
• How many groups of 12 are in 120?
• How many groups of 12 are in 240?
• How could you work out what 15 groups of 12 would be? (Test strategies)
• How could you work out what 23 groups of 12 would be? (Test strategies)
• What will 51 groups of 12 be?

• Allow for a range of different strategies to be discussed with a focus on the distributive property.
• Check in with students who may not know what the answer is but may be able to repeat ideas or offer their own.

• Generalise power of 10 if necessary – 5 x 12 = 60, 50 x 12 = 600

• Why are there only even numbers in this count? (12 is an even number and when adding 2 even numbers you always get an even number)
• How does the pattern of 12 grow (e.g. add 10 and then add 2) that’s why there is only 0, 2, 4, 6, 8</td>
</tr>
<tr>
<td>Task
Count backwards by 12 from 420</td>
<td>Patterns expected to be noticed</td>
</tr>
<tr>
<td>Key Mathematical Ideas
Noticing patterns
• The pattern of 12 and how this works in the base 10 number system
• Noticing pattern of 12 in 2- and 3-digit numbers
• Noticing each column goes down by 60 (links to 5 x 12)
Additive Ideas
• Repeated addition and subtraction to count on or back
Grouping and Distributive Ideas
• Noticing the way multiplication can be distributed</td>
<td>
<table>
<tr><td>420</td><td>408</td><td>396</td><td>384</td><td>372</td></tr>
<tr><td>360</td><td>348</td><td>336</td><td>324</td><td>312</td></tr>
<tr><td>300</td><td>288</td><td>276</td><td>264</td><td>252</td></tr>
<tr><td>240</td><td>228</td><td>216</td><td>204</td><td>192</td></tr>
<tr><td>a</td><td></td><td>b</td><td></td><td></td></tr>
<tr><td></td><td>c</td><td></td><td>d</td><td></td></tr>
</table>

Planned Pauses:
• Keep starting from the beginning when it gets a bit tricky
• Stop after first 3 lines and ask students to notice patterns</td>
</tr>
</table>

(23 x 12 can be the same as 20 x 12 + 3 x 12) • Noticing the power of 10 in 12counts (e.g. if 5 x 12 is 60, 50 x 12 = 600	• Ask some questions about what number will be here? (a, b, c and d) How do you know, turn and talk and justify **Questions to develop reasoning:** • How many groups of 12 are in 360? (can they link 3 x 12 : 30 x 12) • How many groups of 12 are in 240? (can they link 12 x 2) • How many groups of 12 are in 300? (do they count on groups: 240 is 20 groups, so 5 more groups is 60?) • Generalise so how many groups are there in 396? • Does anyone notice the repeat pattern of 12, 24, 36, 48, 60 in the first 2 lines? Where else in the pattern of 12 might this occur and how do you know?

Appendix B4: Choral Counting Planning – Count Five and Six (class one): Counting Forwards and Backwards by Three

<table>
<tr><th>Understand</th><th>Know</th><th>Do (Ministry of Education, 2024)</th></tr>
<tr>
<td>• The world is full of patterns and structures that we use mathematics and statistics to understand.</td>
<td><i>Phase 2</i>
• find factors of numbers up to 100
• recall multiplication facts to 10 × 10 and corresponding division facts</td>
<td><i>Generalising:</i>
• recognise and explore patterns, and make conjectures and draw conclusions about them
• identify relationships, including similarities, differences, and new connections
• look for patterns and regularities that might be applied in another situation or always be true
• make and test conjectures, using reasoning and counterexamples to decide if they are true or not

Explaining and Justifying:
• make statements and give explanations inductively based on observations or data
• make statements and give explanations deductively based on knowledge, definitions, and rules
• critically reflect on others’ thinking, evaluating their logic and asking questions to clarify and understand
• use evidence, reasoning, and proofs to explain why I agree or disagree with statements
• develop collective understandings by sharing, comparing, contrasting, critiquing, and building on ideas with others
• present reasoned explanations and arguments for an idea, solution, or process.</td>
</tr>
<tr>
<td colspan="2">Task
Count forwards by 3 from 0</td>
<td>Patterns expected to be noticed</td>
</tr>
<tr>
<td colspan="2">Key Mathematical Ideas
• The pattern of 3 and how this works in the base 10 number system
• Noticing pattern of 3 in 2-digit numbers
• Noticing the growth of 15 and that this is 5 groups of 3
• Use of the distributive property to work out how many groups of 3 are in numbers beyond 30</td>
<td>
<table>
<tr><td>0</td><td>3</td><td>6</td><td>9</td><td>12</td></tr>
<tr><td>15</td><td>18</td><td>21</td><td>24</td><td>27</td></tr>
<tr><td>30</td><td>33</td><td>36</td><td>39</td><td>42</td></tr>
<tr><td>45</td><td>48</td><td>51</td><td>54</td><td>57</td></tr>
<tr><td>a</td><td></td><td></td><td>b</td><td></td></tr>
<tr><td></td><td>c</td><td></td><td>d</td><td></td></tr>
</table>
Planned Pauses:
• Pause after first two lines. What do students notice so far?
• Pause after third line. What do students notice now? (can anyone see the 0, 3, 6, 9 repeated yet?)
• Pause after 4th line. What do students notice now?
</td>
</tr>
</table>

- Does anyone notice the 5, 8, 1, 4, 7 repeat in the 1s column? What is the link to 3s here? 5+3= 8 etc)

Questions to develop reasoning:
- Will 61 be part of this count?
- Will 72 be part of this count?
- Will 95 be part of this count?
- What number does a, b, c and d represent?

Task
Count backwards by 3 from 150

Patterns expected to be noticed

Key Mathematical Ideas
- The pattern of 3 and how this works in the base 10 number system
- Noticing pattern of 3 in 2-digit numbers
- Noticing the growth of 15 and that this is 5 groups of 3
- Use of the distributive property to work out how many groups of 3 are in numbers beyond 30
- Noticing power of 10 and that 15 x 10 is 150 so if 5 groups of 3 are 15 how many groups of 3 are in 150?

150	147	144	141	138
135	132	129	126	123
120	117	114	111	108
105	102	99	96	93
a			b	
	c			d

Questions to develop reasoning:
- How many groups of 3 are in 30?
- How many groups of 3 are in 45?
- Will 86 be part of this count?
- Will 71 be part of this count?
- Will 60 be part of this count?
- What numbers do a, b, c and d represent?

- Before you start today, tell students you will be counting down in 3s from 150. Ask them if they think 136 will be in our count today. Why/why not.
- Justifications could include yes because 36 is a multiple of 3 or no because 100 does not divide evenly by 3.

Planned pauses:
- Questions to develop reasoning:
- Pause after first two lines. What do students notice so far?
- Pause after third line. What do students notice now? (can anyone see the 0, 3, 6, 9 repeated yet?)
- Pause after 4th line. What do students notice now?
- Does anyone notice the 5, 8, 1, 4, 7 repeat in the 1s column? What is the link to 3s here? 5 + 3 = 8 etc)
- Pause after 4th line. What do students notice now?
- Does anyone notice the 5, 8, 1, 4, 7 repeat in the 1s column? What is the link to 3s here? 5 + 3 = 8 etc)

	Questions to develop reasoning: • 15 x 10 is 150 so if 5 groups of 3 are 15 how many groups of 3 are in 150? • How many groups of 3 are in 90? • How many groups of 3 are in 99? • What numbers will a, b, c and d be?

Appendix C1: Teacher Participant Information and Consent Forms

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Using A Conceptual Starter to Facilitate Productive Mathematical Discussion and Support Mathematical Sense Making

INFORMATION SHEET – TEACHER PARTICIPANTS

Dear XXXXXXXX

My name is Lauren Frazerhurst, and I am a master's student at Massey University. My master's Thesis study is titled Using a conceptual starter to facilitate productive mathematical discussion and support mathematical sense making and will be conducted within the context of primary school mathematics classrooms being places where all students can successfully learn mathematics. The main purpose of this study is to explore how teachers can use a starter activity called 'Choral Counting' to support students to engage in mathematical discussions. The exploration will focus on teachers' specific pedagogical actions to support all students in mathematical discussions. The study will also specifically monitor how student interactions lead to sense-making and reasoning of number properties and patterns.

I am formally inviting you to be a part of this research as I examine how teachers can facilitate purposeful mathematical discussions. Your role in this project will be as the mathematics teacher of the student participants.

Permission to participate in the study will be sought from both the parents/caregivers of the students in your class and the students themselves. The students and their parents/caregivers will be given full information, and consent will be requested in due course. Consent will be sought for permission to be filmed during mathematics lessons.

I will initially interview you about your current warm-up routines. The time involved for individual interviews will be no more than 20 minutes and will take place over Zoom at a time that suits you. Three focus group hui to plan and discuss the implementation of the choral counts will also take place at mutually agreeable times outside of normal teaching times. A final reflection interview will then take place. The interviews and focus group sessions with you will be video recorded.

The duration of this project will be over term three and four. During this project, six mathematics warm-up lessons will be filmed. Whiteboard photographs from each lesson will also be collected. The filming will take place in the classroom and be part of your normal mathematics programme.

The time involved in the complete study for you will be no more than 12 hours over the period of terms three and four.

All project data collected during individual interviews and filming will be stored in a secure location, with no public access and used only for this research and any publication arising from this research. After the completion of five years, all data pertaining to this study will be destroyed in a secure manner. All efforts will be taken to maximize confidentiality and anonymity for participants. Names of all participants and the school will not be used once information has been gathered and only pseudonyms and non-identifying information will be used in reporting.

Please note that you are under no obligation to accept this invitation. If you decide to participate you have the right to:

- Decline to answer any particular question.
- Withdraw from the study at any time.
- Ask any questions about the study at any time during participation.
- Provide any information on the understanding that your name will not be used unless you give permission to the researcher
- To ask for the audio or video recorder to be turned off at any time during the interviews and any comments you have made be deleted.
- Be given access to a summary of the project findings when it is concluded.

If you have any further questions about this project, you are welcome to discuss them with me personally: Lauren Frazerhurst. Phone [redacted]. Email L.Frazerhurst@massey.ac.nz

Or contact my supervisors at Massey University

- Professor Jodie Hunter (09) 4140800 ext.43518. Email. J.Hunter1@massey.ac.nz
  Institute of Education, Private Bag 102 904, North Shore, Auckland 0745

- Dr Generosa Leach Email: G.Leach@massey.ac.nz
  Institute of Education, Private Bag 102 904, North Shore, Auckland 0745

This project has been evaluated by peer review and judged to be low risk. Consequently, it has not been reviewed by one of the University's Human Ethics Committees. The researcher(s) named above are responsible for the ethical conduct of this research.

If you have any concerns about the conduct of this research that you wish to raise with someone other than the researcher, please contact:

Dr Gerald Harrison, Ethicist, School of Humanities
Phone: +64 6 356 9099 extension 83570
Email: humanethicsouthb@massey.ac.nz

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Using A Conceptual Starter to Facilitate Productive Mathematical Discussion and Support Mathematical Sense Making

CONSENT FORM: TEACHER PARTICIPANT

THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

I have read the Information Sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I agree/do not agree to the interviews being sound recorded.

I agree/do not agree to the interviews being image recorded.

I agree/do not agree to mathematics lessons being sound recorded.

I agree/do not agree to mathematics lessons being image recorded.

I agree to participate in this study under the conditions set out in the Information Sheet.

Signature: .... **Date:** ....

Full Name - printed ....

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Using A Conceptual Starter to Facilitate Productive Mathematical Discussion and Support Mathematical Sense Making

FOCUS GROUP PARTICIPANT CONSENT FORM

THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

I have read the Information Sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I understand that I have an obligation to respect the privacy of the other members of the group by not disclosing any personal information that they share during our discussion.

I understand that all information I give will be kept confidential to the extent permitted by law, and the names of all people in the study will be kept confidential by the researcher.

Note: There are limits on confidentiality as there are no formal sanctions on other group participants from disclosing your involvement, identity or what you say to others in the focus group. There are risks in taking part in focus group research and taking part assumes that you are willing to assume those risks.

I agree to participate in the focus group under the conditions set out in the Information Sheet.

Signature: .... **Date:** ....

Full Name - printed ....

Appendix C2: Board of Trustees and Principal Information and Consent Form

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Using A Conceptual Starter to Facilitate Productive Mathematical Discussion and Support Mathematical Sense Making

INFORMATION SHEET

Dear XXXXXX

My name is Lauren Frazerhurst, and I am a master's student at Massey University. My master's Thesis study is titled Using a conceptual starter to facilitate productive mathematical discussion and support mathematical sense making and will be conducted within the context of primary school mathematics classrooms being places where all students can successfully learn mathematics. The main purpose of this study is to explore how teachers can use a starter activity called 'Choral Counting' to support students to engage in mathematical discussions. The exploration will focus on specific pedagogical actions teachers take to support all students to engage in mathematical discussions. The study will also specifically monitor how student interactions lead to sense-making and reasoning of number properties and patterns.

Two teachers from Y4-6 and Yr 7-8 have agreed to participate in this study as the mathematics teachers of the students involved in this project. The duration of this study will be over two school terms. During this study, the teachers will film six mathematics lessons during the normal classroom schedule. The teachers will also be invited to participate in an initial and reflection interview at the end of the study. These two interviews will take place at mutually agreed-upon times outside of the normal teaching times and will take no longer than 20 minutes each. In addition, three focus group sessions will be held at mutually agreed upon times outside of the normal teaching times, where the lesson material and resources will be created and discussed. The time taken for these sessions will be no longer than half an hour.

The teachers, the students and their parents/caregivers will be given full information and consent will be requested. Specifically, permission to allow the students to be filmed will be sought from both the parents of the students and the students within each class. The interviews with the teachers will be audio recorded. Work samples from each lesson may also be collected and photocopied.

The time involved in the complete study for the teacher will be no more than 12 hours over a period of two school terms. There is no expectation that the usual classroom programme will be disrupted in any way.

All project data collected during individual interviews and filming will be stored in a secure location, with no public access and used only for this research and any publication arising from this research. After completion of five years, all data pertaining to this study will be destroyed in a secure manner. All efforts will be taken to maximize confidentiality and anonymity for participants. Names of all participants and the school will not be used once information has been gathered and only pseudonyms and non-identifying information will be used in reporting.

Please note that the Board of Trustees is under no obligation to accept this invitation. If you decide to participate you have the right to:

- Decline to answer any particular question.
- Withdraw from the study at any time.
- Ask any questions about the study at any time during participation.
- Provide any information on the understanding that your name will not be used unless you give permission to the researcher
- Be given access to a summary of the project findings when it is concluded.

If you have any further questions about this project, you are welcome to discuss them with me personally: Lauren Frazerhurst. Phone [redacted]. Email L.Frazerhurst@massey.ac.nz

Or contact my supervisors at Massey University

- Professor Jodie Hunter Email. J.Hunter1@massey.ac.nz
  Institute of Education, Private Bag 102 904, North Shore, Auckland 0745

- Dr Generosa Leach Email: G.Leach@massey.ac.nz
  Institute of Education, Private Bag 102 904, North Shore, Auckland 0745

This project has been evaluated by peer review and judged to be low risk. Consequently, it has not been reviewed by one of the University's Human Ethics Committees. The researcher(s) named above are responsible for the ethical conduct of this research.

If you have any concerns about the conduct of this research that you wish to raise with someone other than the researcher, please contact:

Dr Gerald Harrison, Ethicist, School of Humanities

Phone: +64 6 356 9099 extension 83570
Email: humanethicsouthb@massey.ac.nz

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Using A Conceptual Starter to Facilitate Productive Mathematical Discussion and Support Mathematical Sense Making

CONSENT FORM: BOARD OF TRUSTEES AND PRINCIPAL
THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

We have read the Information Sheet and have had details of the study explained. Our questions have been answered to our satisfaction, and we understand that we may ask further questions at any time.

We agree / do not agree (circle one) for XXXXXXXXX and Room XX students to participate in this study under the conditions set out in the information sheet.

Date: ___________

Board of Trustees Signature: ____

Full Name – printed ____

Principal Signature: ____

Full Name – printed ____

Appendix C3: Parent’s and Student Participant Information and Consent Forms

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Using A Conceptual Starter to Facilitate Productive Mathematical Discussion and Support Mathematical Sense Making

INFORMATION SHEET – STUDENT PARTICIPANTS

Dear Students and Parents/Caregivers,

My name is Lauren Frazerhurst, and I am a master’s student at Massey University. My master’s thesis title is Using A Conceptual Starter to Facilitate Productive Mathematical Discussion and Support Mathematical Sense Making. The main purpose of my study is to explore how teachers can use a starter activity called ‘Choral Counting’ to support students to engage in mathematical discussions. The focus of the exploration will be on specific actions teachers take to support all students to engage in mathematical discussions. The study will also specifically monitor how student interactions lead to sense making and reasoning of number properties and patterns.

I would like to invite you with your parent’s permission to be involved in this study. Your classroom teacher has also agreed to participate in this study. XXXX XXXX, the school principal has also given her approval for me to invite you to participate, and for me to do this research in your classroom.

I will be observing you participating in six mathematics lessons in your classroom in term 3 and 4. Your classroom teacher will be teaching you at this time and these lessons will be part of your normal mathematics programme, whether you agree to be in the study or not. These lessons will be filmed, and you may at any time ask that the camera be turned off and any comments you have made deleted. With your permission I might sometimes collect copies of written work or charts you make to support your mathematical thinking. You have the right to refuse to allow the copies to be taken.

Taking part in this research will not in any way affect your learning, but rather may help you clarify what you know about number properties and patterns and how to clarify your thinking through discussing your ideas with your classmates.

All the information gathered will be stored in a secure location and used only for this research. After completion of the research the information will be destroyed. All efforts will be taken to maximize your confidentiality and anonymity which means that your name will not be used in this study and only non-identifying information will be used in reporting.

I ask that you discuss all the information in this letter fully with your parents before you give your consent to participate.

Please note that you have the following rights:

To say that you do not want to participate in the study

To withdraw from the study at any time

To ask for the audio or video recorder to be turned off at any time during the lessons and any comments you have made be deleted

To refuse to allow copies of your written work to be taken

To ask questions about the study at any time

To participate knowing that you will not be identified at any time

To be given a summary of what is found at the end of the study

If you have any further questions about this project, you are welcome to discuss them with me personally:

Lauren Frazerhurst. Phone: [redacted]. Email: L.Frazerhurst@massey.ac.nz

Or contact my supervisors at Massey University

- Professor Jodie Hunter (09) 4140800 ext.43518. Email. J.Hunter1@massey.ac.nz
  Institute of Education, Private Bag 102 904, North Shore, Auckland 0745

- Dr Generosa Leach Email: G.Leach@massey.ac.nz
  Institute of Education, Private Bag 102 904, North Shore, Auckland 0745

This project has been evaluated by peer review and judged to be low risk. Consequently, it has not been reviewed by one of the University's Human Ethics Committees. The researcher(s) named above are responsible for the ethical conduct of this research.

If you have any concerns about the conduct of this research that you wish to raise with someone other than the researcher, please contact:

Dr Gerald Harrison, Ethicist, School of Humanities
Phone: +64 6 356 9099 extension 83570
Email: humanethicsouthb@massey.ac.nz

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Using A Conceptual Starter to Facilitate Productive Mathematical Discussion and Support Mathematical Sense Making

CONSENT FORM: STUDENT PARTICIPANTS
THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

I have read the Information Sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.
I agree/do not agree to being sound recorded.
I agree/do not agree to being image recorded.
I agree to participate in this study under the conditions set out in the Information Sheet.

Child's Signature: .... **Date:** ....

Full Name - printed ....

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MATĀURANGA

Using A Conceptual Starter to Facilitate Productive Mathematical Discussion and Support Mathematical Sense Making

CONSENT FORM: PARENTS/CAREGIVERS OF STUDENT PARTICIPANTS
THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

I have read the Information Sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I agree/do not agree to ________________________________ being sound recorded.

I agree/do not agree to _________________________________being image recorded.

I agree to ____________________________________ participating in this study under the conditions set out in the Information Sheet.

Parent's Signature: .... **Date:** ....

Full Name - printed ....