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Mathematical Models for Dispersal of Aerosol Droplets in an Agricultural Setting

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Abstract

Agrichemical spray drift is an issue of concern for the orcharding industry. Shelterbelts surrounding orchard blocks can significantly reduce spray drift by intercepting droplets from the airflow. At present, there is little information available with which to predict drift deposits downwind, particularly in the case of a fully-sheltered orchard block.

In this thesis, we develop a simple mathematical model for the transport of airborne drifting spray droplets, including the effects of droplet evaporation and interception by a shelterbelt. The object is for the model to capture the major features of the droplet transport, yet be simple enough to determine an analytic solution, so that the deposit on the ground may be easily calculated and the effect of parameter variations observed.

We model the droplet transport using an advection-dispersion equation, with a trapping term added to represent the shelterbelt. In order to proceed analytically, we discretise the shelterbelt by dividing it into a three-dimensional array of blocks, with the trapping in each block concentrated to the point at its centre. First, we consider the more straightforward case where the droplets do not evaporate; solutions are presented in one, two and three dimensions, along with explicit expressions for the total amount trapped and the deposit on the ground. With evaporation, the model is more difficult to solve analytically, and the solutions obtained are nestled in integral equations which are evaluated numerically. In both cases, examples are presented to show the deposition profile on the ground downwind of the shelterbelt, and the corresponding reduction in deposit from the same scenario without the shelterbelt.

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Notation

Upper Case

C	Droplet number concentration [$\# \text{ m}^{-3}$]
D_L	Alongwind (longitudinal) dispersion coefficient [$\text{m}^2 \text{ s}^{-1}$]
D_T	Crosswind (transverse) dispersion coefficient [$\text{m}^2 \text{ s}^{-1}$]
D_V	Vertical dispersion coefficient [$\text{m}^2 \text{ s}^{-1}$]
D_W	Diffusivity of water vapour in air [$\text{m}^2 \text{ s}^{-1}$]
H	Release height [m]
L_L	Dominant alongwind (longitudinal) turbulence length scale [m]
L_T	Dominant crosswind (transverse) turbulence length scale [m]
L_V	Dominant vertical turbulence length scale [m]
L_v	Latent heat of vaporisation [J kg^{-1}]
M_W	Molecular mass of water [kg mol^{-1}]
Q	Mass release [kg]
R	Dimensionless function non-zero only within a region of trapping [-]
R_g	Universal gas constant [$\text{J mol}^{-1} \text{ K}^{-1}$]
S	Droplet settling speed [m s^{-1}]
T	Absolute temperature [K]
X_0	release x coordinate [m]
Y_0	release y coordinate [m]

Lower Case

a	Droplet radius [m]
c	Droplet mass concentration [kg m^{-3}]
c_p	Specific heat of water [$\text{J kg}^{-1} \text{ K}^{-1}$]
d	Droplet diameter [m]
f_h	Ventilation coefficient for heat [-]
f_w	Ventilation coefficient for water vapour [-]
g	Gravitational acceleration [m s^{-2}]
k_a	Thermal conductivity of moist air [$\text{W m}^{-1} \text{ K}^{-1}$]
k_b	Background trapping rate [s^{-1}]
k	Effective trapping rate [$\text{m}^3 \text{ s}^{-1}$]
m	Droplet mass [kg]
p	Laplace transform variable [s^{-1}]
p_{sat}	Saturation pressure [Pa]
t	Time [s]
u	Mean wind speed (positive x direction) [m s^{-1}]
x	$\left. \begin{array}{l} x \\ y \\ z \end{array} \right\}$ Cartesian coordinate system [m]
y	
z	

Bold

<u>D</u>	Dispersion tensor [$\text{m}^2 \text{s}^{-1}$]
i	Unit vector in the positive x direction [-]
j	Unit vector in the positive y direction [-]
k	Unit vector in the positive z direction [-]
u	Mean wind velocity [m s^{-1}]
v	Droplet velocity relative to the origin [m s^{-1}]

Greek

μ_a	Dynamic viscosity of air [$\text{kg m}^{-1} \text{s}^{-1}$]
ρ_a	Air density [kg m^{-3}]
ρ_w	Droplet density (water) [kg m^{-3}]
ϕ	Relative humidity (expressed as a fraction) [-]
ψ	Spatial Fourier transform variable [m^{-1}]
ω	Spatial Fourier transform variable [m^{-1}]