

PATTERN FORMATION IN ELECTRICALLY COUPLED PACEMAKER CELLS

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Excitable cells such as neurons, smooth and skeletal muscles, and endocrine glands can generate an electrical signal (action potential) when stimulated by external stimulus. The signal helps in the coordination of different physiological activities including transfer of information among neurons, vasomotion in muscle cells and secretion of hormones. Experimental and theoretical studies have shown that under certain conditions, the electrical signal can be produced spontaneously in excitable cells. Such behaviour is referred to as *pacemaker dynamics*. This research aims to study the pacemaker electrical activity in a population of electrically coupled smooth muscles.

In the thesis, we study the electrical activity in smooth muscle cells in the absence of external stimulation. The main goal is to analyse a reaction-diffusion system that models the dynamical behaviour where adjacent cells are coupled through passive electrical coupling. We first analyse the dynamics of an isolated muscle cell for which the model consists of three first-order ordinary differential equations. The cell is either excitable, nonexcitable or oscillatory depending on the model parameters. To understand this, we reduce the model to two equations, nondimensionalise, then perform a detailed numerical bifurcation analysis of the nondimensionalised model. One-parameter bifurcation diagrams reveal that even though there is no external stimulus, the cell can exhibit two fundamentally distinct types of excitability. By computing two-parameter bifurcation diagrams, we are able to explain how the cell transitions between the two types of excitability as parameters are varied [2].

We then study the full reaction-diffusion system through numerical integration. We show that the system is capable of exhibiting a wide variety of spatiotemporal behaviours such as travelling pulses, travelling fronts and spatiotemporal chaos.

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Through a linear stability analysis, we are able to show that the spatiotemporal patterns are not due to diffusion-driven instability as is often the case for reaction-diffusion systems [3]. It is as a consequence of the nonlinear dynamics of the reaction terms and coupling effect of diffusion. The precise mechanism is not yet well understood; this will be the subject of future work. We then examine travelling wave solutions in detail. In particular, we show how they relate to homoclinic and heteroclinic solutions in travelling wave coordinates. Finally, we review the spectral stability analysis for travelling waves and compute the essential spectrum of travelling waves in our system [1].

References

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