Optimal Cow Replacement On New Zealand Seasonal Supply Dairy Farms.

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Bevin Lyal Harris

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Abstract.

An intraherd best linear unbiased prediction (BLUP) model for predicting the future milkfat production of individual cows was developed. A major advantage of the BLUP technique was to enable prediction of the future milkat production of freshening heifers, since relationships between animals were included in the model. These predictions of future performance were incorporated, along with various costs and revenues of production in New Zealand and calving date, into a model to arrive at an expected net revenue for each individual cow.

Three models to rank cows on future profitability were developed and evaluated. Two models utilised dynamic programming procedures. One model estimated the annualised present value of the net returns of each cow and her replacement up to a predetermined planning horizon. The second model used the same criterion, but also allowed optimal replacement to occur in future seasons. The third model utilised replacement model evaluation techniques and estimated the annualised present value of the net returns based on the remaining economic lifespan of individual cows.

The models were tested over a large number of different situations. The effects of changes in the different economic parameters are discussed and the behaviour of each model is documented. The parameters directly associated with the cost of replacement had the greatest effect on the annual present value's (APV) of individual cows. The optimal rankings were affected by the price of the heifer replacement and the price of manufacturing beef, whereas milkfat price played an insignificant role. Varying the price of manufacturing beef and the price of the heifer replacement simultaneously had only a small effect on the ranking of the cows. The parameters such as interest rate and planning horizon also affected the APVs produced by the dynamic models. Increasing the planning horizon past 10 years caused a reduction in the variation between the

APVs.

It was concluded that the dynamic programming model which allowed optimal replacement in future seasons provided the best system for ranking cows on expected future income.

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Table of contents.

List of Tables.

Number **Page**

List of Figures. vii

Chapter 1

1.1 Introduction.

The study of animal replacement should draw on not only the theory of animal breeding but also the theory of economic replacement. The theory of economic replacement depends on knowledge of operations research and investment theory. The purpose of this study is to combine these areas of knowledge to develop a ranking system which can be used as a management guide to aid the dairy cow replacement decision process.

Decisions to replace dairy cows are either voluntary or involuntary. Involuntary decisions are, predominantly, not in the farmer's control and include death, barrenness and mastitis. Voluntary replacement is within the farmers control and is commonly based on a combination of subjective and objective information about the cow. To aid the farmer in voluntary replacement decisions, the Livestock Improvement Division of the New Zealand Dairy Board produces production indices and breeding values for each cow for which the required information is available. The production index is a measure of a cow's milkfat production free of the influence of age, herd environment, stage of lactation and herd genetic level (Wickham et al 1980). The breeding value is a measure of a cows genetic value, half of this genetic value is transmitted to the next generation. These guides may excel in selecting those cows which could be used to improve the genetic quality of the herd. However, on commercial farms where income is derived from the sale of milk and meat, voluntary replacement decisions should be based on comparison between the anticipated income from the present cow and that of a replacement heifer. In a number of studies, techniques involving essentially this comparison have been used to provide general replacement guidelines and policies however, little attention has so far been paid to the development of culling guides for individual cows. The aim of this study is to explore, develop and evalute methods which

could be used to rank cows on future profitability.

1.2 Outline of the work.

The literature pertaining to the theory and application of replacement models and dynamic programming with markov processes are reviewed in chapters 2 and 3.

In chapter 4 the future production of individual cows are predicted using best linear unbiased prediction (BLUP) techniques and a model to predict the preformance, revenues and costs of dairy cows on a New Zealand dairy farm is developed. Two dynamic models and an annuity model, based on replacement model theory which rank cows on the basis of expected future earnings are presented in chapter 5. In chapter 6, these 3 models are tested using average prices for the 1985 season. In chapter 7, to test the robustness of solutions from the models, the economic parameters are varied to test the robustness of the solutions from the models. Finally, in chapter 8, the overall performance of the 3 models are evaluated and the problems associated with their implementation in the dairy industry are discussed.

Chapter 2.

2.1 Replacement models

In livestock production systems, farmers commonly have as their major objectives :

i) achieving economic profit from their animals, and

ii) maintaining or improving the aesthetic quality of their animals by type ratings or other assessments of their physical attributes.

The importance of, and criterion for, the second objective reflect a varying mixture of motives and intangible factors which may vary between farmers. It will be assumed throughout this study that the first objective is of primary interest and the second objective will be ignored due to the difficulties of measuring and defining it, even though it may an important objective to many farmers. This action is consistent with the observation that farmers are becoming increasingly business oriented following a downturn in the agricultural economy and increased urbanisation (MacArthur 1975). In this study, the cow will be considered as an asset owned and operated for profit. Hence, replacement will take place for economic reasons. Several studies have considered the problem of when to replace an asset, but few have addressed the problem of which asset to replace.

Before discussion of the replacement models an outline of the economic theory which is the basis of the several economic replacement models will be presented.

When considering the net returns (net costs, profit or utility) generated by an asset there are three measures of interest; total net return, average net return and marginal net return. The concepts of total and average net return should be well known. The concept of marginal net return refers to the

change in the net return occuring from a unit change in time. Therefore, the marginal net return can be measured as the slope of the total net return curve at any point in time.

When an asset generates a number of net returns in future years, time preference for consumption must be taken into account. The time preference for consumption assumes that consumption in the present year is prefered to consumption in future years. Therefore, a net return of a given amount is worth more if it is produced in the current year than in some future year. To calculate the decrease in value of the future net returns the concept of discounting is used. A discount factor β is introduced which discounts the future revenues to their present value in the current year. The discount factor is :

$$\beta = 1/(1+i)$$

where: i is the interest rate.

To calculate the present value from a stream of net returns from the next n years equation 2.1 is used.

$$PV = \beta^0 R_1 + \beta^1 R_2 + \ldots + \beta^{n-1} R_n \qquad 2.1$$

where: PV is the present value, and

R_j is the revenue from year j.

The annualised present value (APV) of a stream of net returns can be calculated from the present value. To enable the comparison of income streams for varing numbers of years, the concept of annualised present value (APV) can be used. The annualised present value can be considered a weighted average of the net returns, where the weights take into account time preference for consumption. The annualised present value (or annuity) is

given by :

$$APV = PV \times AMF,$$

where: $$AMF = \frac{i}{1-(1+i)^{-n}}, \qquad ...2.2$$

AMF is the amortisation factor, and

n is the number of years over which the returns are received.

The amortisation factor converts the present value to a current annual value.

Several replacement models will be reviewed and evaluated in the following section. Before discussing these replacement models, the problems which exist in considering cow replacement deserve mention :

i) age effects on cashflows - cashflows from younger cows tend to rise whereas cashflows from older cows tend to decrease,

ii) cows in a herd have different economic lifespans, and

iii) non-identical replacement. Identical replacement assumes that the cashflow consists of a number of identical cycles. In this study, cows are replaced by heifers. These have different cash flows ; hence, identical replacement does not occur.

2.2 Literature review.

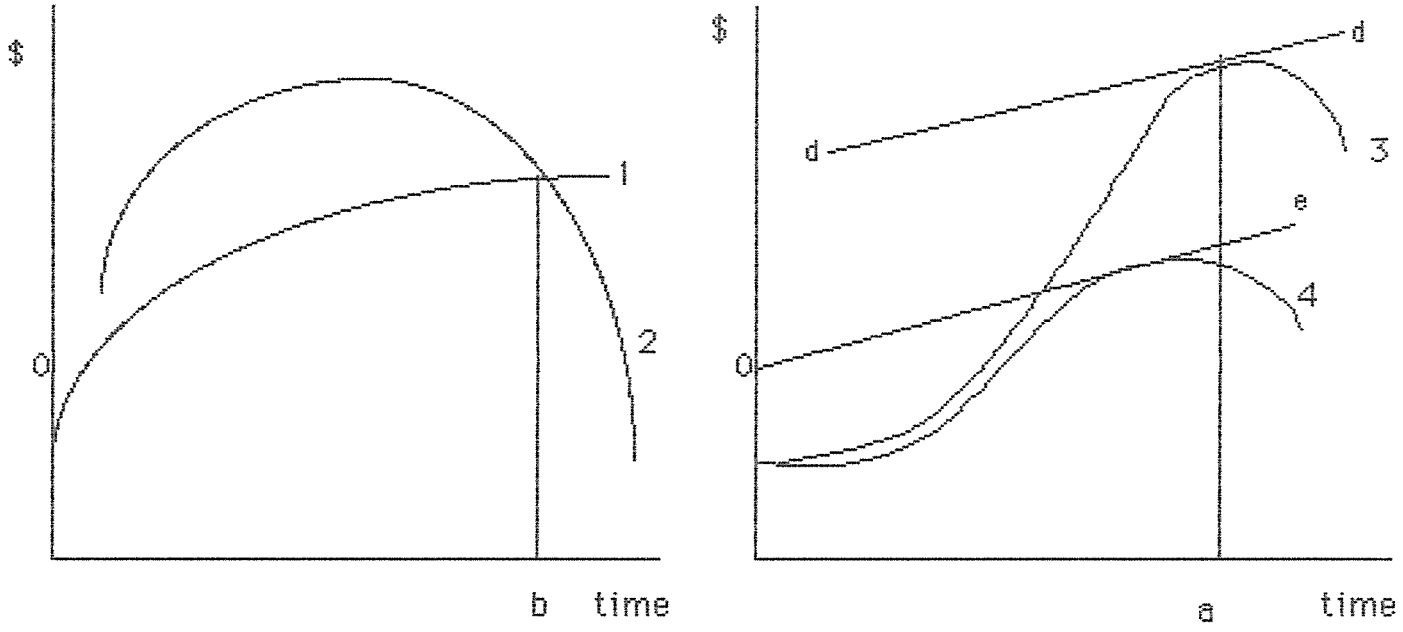
Faris(1960) developed replacement models for cattle fattening, forestry and orchard enterprises. For the cattle fattening unit, the objective was to

maximise the average net revenue over time . The time unit was a ten day feeding period and, because of this short period, discounting was not used. This procedure is known as the asset replacement model. The objective for this model is to maximise the average net revenue or minimise average net cost over a period of time, usually the life of the asset. This procedure assumes that the period of time or cycle analysed is representative of all periods of time. This method is inappropriate for the cow replacement problem since the average net revenues from one cow's lifetime is not representative of the replacement's average net revenues .The cattle fattening example assumed the net revenue curve was constant for successive cattle fattening lots. The situation where this assumption was invalid was also considered and from the preceding analysis Faris(1960) derived the rule : " the present lot should only be carried to the point where the marginal net revenue anticapted from the net revenue equals the maximum average net revenue anticapted from the subsequent lot". Faris(1960) further illustrated this concept for an orchard replacement problem. However, such a principle is invalid over long time periods because the value of a unit in the future may be different for the same unit at present. To overcome this, Faris(1960) introduced present value techniques using a discounting procedure. Faris(1960) then restated the decision criterion as : " The optimum time to replace is when the marginal net revenue from the present enterprise is equal to the highest annualised present value of the anticapted net returns from the enterprise immediately following".A graphical representation of the marginal net revenue model is given in figure 2.1.

Burt(1965) extended the marginal net revenue replacement model to cases where involuntary and voluntary replacement occured. Net revenues for each period were discounted and weighted by the probability of each event occuring. The optimum time for replacement depended on the discount rate , shape of the marginal net revenue curve and whether or not involuntary

Figure 2.1

The optimum replacement time for the undiscounted(A) and discounted(B) case, assuming identical replacement.



A B

Curve 1 : marginal net revenue curve of the existing enterprise
Curve 2 : Average net revenue curve of the subsquent enterprise
Curve 3 : Total net revenue curve of the existing enterprise
Curve 4 : discounted total net revenue curve of the subsquent enterprise

The optimum time for replacement for the non-discounted case is at point b where the average net revenue of the subsquent enterprise equals the marginal net revenue of the existing enterprise. The optimum time for replacement for discounted case is at point a, where the marginal net revenue (slope d-d) of the present asset equals the maximum average discounted return (slope 0-e) of the subsquent asset. Noting that the maximum average discounted return is the slope of the line drawn as a tangent to the discounted total net revenue curve, and passes through the origin.

replacement took place. The objective of this model was to maximise the net revenues of the current and all subsquent assets, which implied an infinite planning horizon. The probability of involuntary replacement is an important component of the dairy cow replacement problem, whereas, the probability of voluntary replacement is not a component of the dairy cow replacement problem. A dairy cow replacement model's objective would to be to aid the decision associated with voluntary replacement rather than including the probability of the event occurring.

Renkema and Stelwagen(1978) used these principles to determine the optimal age of replacement, and quantified the economic consequences of changes in the involuntary replacement rate for a dairy herd. They concluded that " a cow of a particular age should be kept as long as her expected marginal profit is higher than the expected average profit during a replacing young cow's life". They also concluded that an increase in average herd life from 3.3 to 5.3 lactations would increase the income from the herd by approximately 20% each year. Increasing the average herd life above 5.3 lactations introduced the law of deminishing returns on any further increases in income. Future revenues were not discounted and identical replacement was assumed. Hence, the same cashflows were used for the present and replacement cow. Using the same cashflow for present and replacement cows cannot be justified since cows are replaced by heifers which have different cash flows. Not discounting future returns resulted in the increased income being overestimated.

The marginal net revenue model is not appropiate for cow replacement since the model looks only at one cycle of the process and assumes that this cycle is representative of all cycles. This model does not have the ability to overcome problems caused by different cashflows for each cycle and different economic lifespans; for example the economic lifespan for a 6 year old cow would be considerably shorter than her heifer replacement. The model also assumes a infinite planning horizon which is inappriopate for the

cow replacement problem where the objective is to maximise the net return for the farmers planning horizion, which could be between 1 to 40 years.

Korver and Renkema(1979) derived a production criterion, below which first lactation cows should be replaced from an economic viewpoint. The production criterion was derived from the comparison of the average annual income of the present cow and an average replacement .The production criterion was defined as the sum of expected differences between the production of the cow in question and an average replacement heifer's production. When this value became negative it was considered optimal for replacement to take place. This production criterion is of no value for comparing cow's in different lactactions, since the model assumes that the average annual income of the present cow is representative of the average annual income from future generations. This would not be the case for cows not in their first lactation because cash flows from their remaining economic life would not be the same as the cash flows from cow's in future generations. Whereas, the cash flows of a cows in their first lactation would representative of all future cows assuming identical replacement.

The cow replacement problem involves the comparison of cash flows spaced over different production periods. For comparison, alternatives must cover the same time period. Aplin et al (1977) suggested using annualised present values to overcome the problem of varing lifespans. Annualised present value techniques were used to overcome the problem associated with comparing income from time spans of different length. Comparing two assets by their annualised present values assumes identical replacement , since the annualised present value of an asset is considered representative of a replacement asset in future cycles. Kuipers (1982) suggested that the use of discounting would reduce the impact of the differences in revenue between the present asset and a non-identical future replacement.

One study has used the the concept of annualised present values for the cow relacement problem. Hlubik(1979) designed a model to provide an

economic basis for making culling decisions in a dairy herd. The model calculated the present value and annualised present value of each cow. The model allowed the user to specify the planning horizon for each cow.In this study, the time to replace was when the marginal revenue of the defender was less than the annualised present value of the challenger. This was the same criteria as used in the marginal net revenue model. However, results from Hlubik's(1979) model indicated that the decision to replace was made by comparison of the annualised present value of the cows, rather than replacing the defender when its marginal revenue became less than the challengers annualised present value. The defending cows with the lowest annualised present value were considered candidates for replacement. Specifing a planning horizon overcame the problem of non-identical replacement in the future, since each cow was compared only on their remaining economic life.

Kuipers(1982) described a cow replacement model based on Terbough's (1949) model for machinery replacement. The technique resulted in the same optimum time for replacement as the previous models and had the same limitations(van Arendonk, 1984). It did, however, allow depreciation and obsolescence of the current asset to be accounted for. Kuipers(1982) also developed a replacement guide based on Terbough's (1949) replacement model. The guide predicted monthly returns for the current lactation and the first six months of the next lactation. Individuals with the lowest values were considered candidates for replacement. This guide would underestimate the potential of younger animals and overestimate the potential of older animals due to the short time period during which comparsion of the net returns took place. This was because the short time period did not take into account first, the increase in cashflows for younger cows as they reached maturity and secondly, the reduction in cashflows caused by replacement of older cows by heifers.

Most of the above models have been concerned with the optimum time to

replace and have ignored the problem of which animal to replace and with which replacement. Replacement models developed by Faris(1960) and extended by Burt(1965) are not applicable to cow replacement as they do not adequately cope with cashflows of varing lengths and are unable to overcome the problem of non-identical replacement. Nor do they answer the question of which cow to replace. Kuiper's(1982) replacement guide provides an answer to the question of which cow to replace, but the guide does not take into account all future revenues of the animal concerned. Using annualised present values to rank cows as indicated by Hublik(1979) also answers the question of which cow to replace. The impact of the departure from the identical replacement assumption in this approach is lessened by using discounting techniques and defining a planning horizon for each cow, and hence, only considering the economically interesting prediction period. The limitation of using such a planning horizon is that the model is not maximising the future returns over the farmer's planning horizon. Because of the above problems, most of the replacement models are unsuitable for ranking cows on future profitability, and no reports of their application to the dairy industry at large have been published.

There is a need to develop and test a model for ranking cows in a seasonal supply dairy herd under grassland conditions on future profitability , based on the annualised present value method with a defined planning horizon for each cow. Dynamic programming enables most of the problems encountered above to be overcome. The theory and usage to date of this technique will be presented in the following chapter.

Chapter3

Dynamic programming.

Dynamic programming is concerned with a process which involves a sequence of decisions made over some time period (Bellman 1957). Replacement decisions are made over time and may be considered as a sequential decision process. Dynamic programming with markov processes will be used to consider the problem of cow replacement. The important concepts and theory of dynamic programming with markov processes will be discussed in the next section, which will be followed by a review of literature on the applications of dynamic programming to cow replacement.

3.1 An introduction to dynamic programming with markov processes.

A model of some given system (e.g. a cow) can be established by considering a subclass of stochastic processes,called the markovian processes. A stochastic process is a sequence where the outcome at the current stage (e.g. a lactation) is dependent on the outcomes of previous stages in a probabilistic sense , such that the probability distribution at any particular stage is known when the actual outcomes of all previous stages are known. Where a stage is defined as a subdivision of a system for example, the subdivision may be in time or in position. A markov process is a stochastic process where the probability distribution of outcomes at any given stage depends on the actual outcome only in the proceeding stage rather than the outcomes of all previous stages. Howard (1960) was the first to formulate the replacement problem as a markov process and showed how dynamic programming could be used to obtain solutions. The basic concepts of a markov process are those of the 'state' (e.g. age,calving status , ...) of the system and 'transition' from one state to another.

When a system is completely described by the values of the variables

which define the state, then the system is said to be in that state. The system makes state transitions when the variables change to those specified by another state. The transitions are indexed in time thus, the system can be considered as a discrete-time process. The probabilistic nature of the state transitions are specified as the probability of transition from state i to state j in the next period. The transition is a function of state i and state j, not any previous state the system may have been in. Since the system must be in one state after the next period then :

$$\sum_{j=1}^{m} \pi_{ij}=1 \text{ and } 0 \leq \pi_{ij} \leq 1$$

where : m is the number of states and

π_{ij} is the transition probability from state i to state j.

When several processes (e.g. several lactations) are considered, the question of optimality arises. Bellman's(1957) principle of optimality is a concise description of the phenomenon which enables problems amenable to solving by dynamic programming solutions to be viewed as a sequence of smaller problems. For example the net revenues from a cow over the next ten years may be divided into 10, one-year periods. Bellman (1957) stated the principle of optimality as : " an optimal policy has the property that whatever the initial state and decision are, the remaining decisions must constitute an optimal policy with regard to the decision", ie ., past events will not influence the future decision at any stage. Moreover , any portion of a time sequence of optimal state values is also optimal.

From the above principle of optimality it is possible to use dynamic programming to formulate a recursive relationship between each of the smaller problems (stages) of the sequence. The problem can be specified when a system is in one of m states and a decision k is chosen from the set

of s decisions . The return (for example the net present value) from the first stage is R_{ij}^{k}, if decision k is made enabling the system to move from state i to state j.The objective is to maximise the expected returns over the next n stages, so a discount factor β is introduced .

The expected return from the system in moving from state i to state j can be represented by the equation (the proof is provided in appendix one):

$$\Phi_{(i)} = \max_{s}[R_{ij}^{s} + \beta\sum_{j=1}^{m}(\pi_{ij}^{s}\Phi_{(j)})]$$

where : $\Phi_{(i)}$ is the expected return , when starting in state i, and

s is a set of all decisions k_1 to k_t.

If $\Phi_{n(i)}$ is the expected return over the next n periods of time using the optimal policy and begining at state i then, if n>1 :

$$\Phi_{n(i)} = \max_{s}[R_{ij}^{s} + \beta\sum_{j=1}^{m}(\pi_{ij}^{s}\Phi_{n-1(j)})] \qquad \ldots\ldots 3.1$$

and thus :

$$\Phi_{1(i)} = \max_{s}[R_{ij}^{s} + \beta\sum_{j=1}^{m}(\pi_{ij}^{s}\Phi_{0(j)})]$$

Equation 3.1 is known as Howard's functional equation, where the expected return from the final stage for state j ($\Phi_{0(j)}$) is assumed to be known (Howard ,1960). The expected return in the final stage has subscript 0 since the optimization starts at the final stage and moves towards the present stage. The method used to solve these equations works backwards for any finite n (or any finite planning horizon). This series of steps are known as the value iteration method (Howard ,1960). The steps are as follows ;

1. Define R_{ij}^{s} and $\phi_{O(j)}$

2. Use the recursive formula (equation 3.1) for n=1 to N
   to find the optimal policy s(n) at each stage.Where N is the
   planning horizon.

3. When the optimal policy has stabilised in a constant pattern, stop.
   For example when s(n)=s(n-1) occurs.

When solving dynamic programming problems using the value iteration method, there are two conditions which must be upheld to ensure a valid solution. These are the separability and optimality conditions. It is possible to show that these conditions are upheld for the formulation given in equation 3.1 (see appendix 1).

In this study, a finite planning horizon is assumed to enable the maximisation of profit over the period of time the farmer is operating the farm. For the case where an infinite planning horizon is appropriate, several methods can be used to solve the recursive formula. These include, the policy iteration routine (Howard,1960), linear programming (Ross ,1983) and policy value iteration (Hastings,1973). As the time period approaches infinity the optimal policy stablises. The above methods use the stability of the optimal policy as a basis for solving the dynamic programming problem. Thus, these methods are not suitable for solving a dynamic programming problem with a finite planning horizon since stability of the optimal policy cannot be assumed.

3.2 The properties of dynamic programming

Dynamic programming has the advantage of placing no restrictions on the

nature of any of the functions describing the system (Throsby ,1968). It is well known that cost , return and production functions which describe agricultural systems tend to be non-linear and discontinuous due to the discrete nature of the input and output patterns. Whereas , dynamic programming can handle extremes of non-linearity and discontinuity, linear programming for example requires linear functions and constraints. Futhermore, the nature of the system specified by the various states makes it possible to include stochastic elements in the transition probabilities. This enables the model to account for variation in milk production and other variables, and to include involuntary replacement.

For the case of a finite planning horizon, not all the elements of the model need to remain stationary to determine the optimal policy using the value iteration method. Thus ,it is possible to increase revenues over time (Burt and Allison ,1965), and hence account for say, genetic improvement.

3.3 Applications of dynamic programming to cow replacement.

In a number of cases, dynamic programming has been used to determine optimal replacement policies for dairy herds. These are summarised in Table 3.1.

Jenkins and Halter (1963) were the first to use dynamic programming for the cow replacement problem. They formulated the problem as a maximisation of net returns, their methodology closely followed Bellman's(1957) work on dynamic programming. Two different stochastic elements were introduced into the model. First, it was assumed that for each production period there was a given probability of failure, this was expressed as a function of age. Where failure was defined as both death and involuntary removal. Secondly, milkfat production was assumed to be a function of age. The study was an illustration of given principles, rather than an attempt to solve a realistic problem. The second major study was that of Giaever's(1965), which used Howard's(1960) methology as the basic model.

Table 3.1

A summary of previous dynamic programing studies

Dynamic programming models.	Total number of states	state variables. Age or lactation number	stage of lactation	Calving interval	Milk or milkfat	Fat%	Body weight	Stage length (months)	genetic improvement	Planning Horzion (years)	Country
Jenkins and Halter (1963)	12	yes						12	no	12	USA
Giaever (1965)	106	yes		yes	yes			a	no	∞	USA
Smith (1971)	15138	yes		yes	yes			a	yes	15	USA
Stewart et al (1977)	2695	yes			yes	yes	yes	12	yes	5/10	USA
Stewart et al (1978)	2695	yes			yes	yes	yes	12	yes	10	USA
van Arendonk (1985a,b,1986a)	29880	yes	yes	yes	yes			a	yes	20	Holland
Killen et al (1978)	9	yes						12	no	20	Ireland
Mac Arthur (1973)	560	yes			yes			12	no	20	N Z
Mac Arthur (1975)	560	yes			yes			12	yes	20	N Z
Mac Arthur (1985)	------	No details available						---	---	----	N Z

a : In these studies the stage length varied due to variation in calving interval.

This study was important in defining the methodology which has been used in most of the later reports. In order to define state variables for milk production in such a way that the resulting stochastic process was a markovian process, Giaever(1965) defined a set of linear combinations of 305 day milk production for all lactations up the present lactation. The resultant sequence of variables gave a markov process. Using this method, Giaever(1965) derived seven production classes(ie, seven discrete intervals which described milk production, each production class had an approximate range of 490 litres of fat corrected milk), the transition probabilities were computed from a stochastic model using a multivariate normal distribution. The model also included a stochastic variable to account for variation in calving intervals.

Smith(1971) elaborated on Giaever's(1965) study, by not only including genetic improvement but also markedly increasing the number of state variables describing the system. Smith(1971) predicted production in the next lactation using production in the previous two lactations, or the previous lactation when only one lactation was available. The model did not predict the production for heifers with no previous records. He considered Giaever's(1965) method of defining the production class too complex for a large model with a great number of production states and ignored the stochastic relationships between previous and current lactations. Smith(1971) considered the extra cost of computation of the transition probabilities to account for the stochastic relationships between current and previous lactations did not outweigh the loss in precision which occurred from breaking the optimality principle. No evidence was presented to support this assumption.

MacArthur (1973) designed a stochastic model to study culling decisions in New Zealand dairy herds. This model was used in conjunction with a Monte Carlo simulation to compare differences between using an optimal

replacement policy (derived from dynamic programming procedures) and a replacement rate of 20% (based on genetic value). The dynamic programme did not include genetic improvement. MacArthur (1973) found that it took about 15 years to reach a stable policy at the present stage. A stable policy is defined by step 3 of Howard's (1960) policy iteration method (see page 15).

MacArthur(1975) extended his previous model to include genetic improvement and to overcome the problem of inclusion of prior information in the prediction of the current lactation yield. An iterative heuristic procedure was used to cope with the change in genetic level of replacement heifers. The procedure involved calculating the genetic structure of the herd and the genetic values of the future heifers. A stable policy was found by alternating between the derivation of genetic level of replacement heifers and the estimation of the optimal policy. This method was complex; it increased the computation time and made it difficult to study the effect that changes in the rate of genetic improvement had on the optimal policies. To overcome the problem of including prior information concerning previous lactation yields, MacArthur(1975) specified the production class as the average of the previous mature equivalent records. But using average values reduces the effect of any extremes and furthermore, the expected value of the current production yield is estimated without giving any regard to the distribution around the average.

In both of MacArthur's models, an average salvage value for all age groups is used .This overestimated the returns generated by the replacement heifers when compared with older cows who had reached their mature weights. Neither model took account of variation in calving date, which can have an important effect on both milk and calf revenue.

In the model designed by Stewart et al (1977) cows stayed in the same production and weight class for their entire lifetime. The absolute production for each class was changed by consideration of the repeatibilities

of previous lactation records. They found the optimal polices were sensitive to changes in the prices received for the milk constituents and the interest rate, but not beef prices. They also found that more intense culling occured when the planning horizon was shortened. Their model included variation in heifer milk production and weight, inclusion of both these sources of variation increased the replacement rate. They concluded that the use of an average heifer underestimated the value of replacement. However, Van Arendonk(1984) considered that this effect was overestimated, because there was no variation in the milk production variables describing the cow, since the cows remained the same production class.

Stewart et al (1978) used the same model as Stewart et al(1977) to compare the 10 year discounted revenues for heifers of four different breeds. In this way the profit of each breed was determined using the corresponding optimal replacement policy .

Killen et al (1978) used a simple dynamic programming model, which only included state variables for age, to compare optimal replacement rates with actual replacement rates in Ireland from 1957 to 1976.

Mac Arthur(1985) used dynamic programming and simulation to examine the economic consequences of alternative culling policies. No information is available on the dynamic programming model. The optimal replacement decisions were used to economically justify herd testing.

Van Arendonk(1985a,b and 1986a) developed a dynamic programming model to determine the optimal replacement policies for dairy cattle in Holland. The model included variation in conception time. In this study the milk production state variables were expressed as percentages of the mature equivalent. Future production was estimated from the current and last lactation yield. The optimal policies were found for both replacement rates and the insemination dates for cows.

Dynamic programming overcomes the problems associated with fitting the replacement models to cow replacement. However, to ensure valid

results the system must be specified in such a way that the markov requirement of independence is met. Specifying future milk production as a function of previous records breaks this requirement. Futhermore, if the expected net present values are not only dependant on the present states but also on previous states ,then the dynamic programming equation (equation 3.1) is no longer valid (Bellman 1957). The effects of such departures on optimality of the solutions are not known.

Kuipers(1982) considered the use of state variables to specify the cow system rather than using the cow itself as a major disadvantage. This disadvantage is only proportional to the size of the model. Obviously, there must be a trade off between the size of the model and the cost of computation. The size of model will depend on the description of the cow system.

Dynamic programming has been used in several studies. Most of these studies have been developed for the non-grassland situation where milk is produced throughout the entire year. The studies which have concentrated on the grassland, seasonal, milk production situation have not included calving date in the models nor have they considered the variation in weight of the cow according to age. There is a need to develop a model which includes such variables.

Few studies have addressed the problem of markov independence. The markov requirement is a problem which is conceptually difficult. To ensure a valid use of the markovian dynamic programming framework as a basis for the dairy cow replacement problem, a markov process must be defined such that the probability distribution of the outcome at any stage is completely determined when the outcome at the proceeding stage is given.

In the present study variation in cows weight and a state variable for calving date will be included to closely model the cow system in a New Zealand seasonal supply dairy farm. The problem of markov independence will be overcome by using a measure for production which is constant over time.

The future milkfat production will be measured as the sum of the additive genetic and permanent environmental effects, this measurement will overcome the markov independence requirement since, the level of milkfat production for the current stage will only be dependant on the level of milkfat production in the previous stage.

The studies to date have generated optimal replacement policies. However, their policies have been constrained by factors outside the model's scope , such as numbers of offspring reared or purchased to become replacements. None of these approaches have been applied to the dairy industry at large. Thus, it may be more appropiate to utilise dynamic programming for providing management guides which identify candidates for replacement. In this study dynamic programming will be used to rank cows on future profitability. This will enable the farmer to choose the number of cows to be retained or replaced. This approach will not be constrained by the number of stock reared or purchased nor by increasing or decreasing herd size. This approach should be applicable to seasonal supply dairy industries at large.

Chapter 4.

A model to estimate the performance , revenues and costs of dairy cows in New Zealand.

4.1 Introduction

In the previous two chapters several techniques which enable optimal replacement decisions to be determined have been discussed. To utilise these techniques , information about the expected revenues and costs during the productive life of the present and replacement cows has to be obtained. Futhermore, information is required about the probability of, and the financial loss associated with, involuntary removal and death.

An economic index based on predicted monthly revenues has been developed by Kuipers(1982). Gartner(1981) constructed a model to evaluate different replacement rates on a grassland dairy farm. The model estimated total herd profitability rather than considering individual cows. Van Arendonk (1984) developed a model for Dutch dairy farms which considered seasonal effects on prices and costs. None of these models addressed the problem of seasonal grassland production.

In this chapter, a model is constructed to estimate the future performance, revenues and costs of individual cows with different levels of milkfat production.

4.2 General concepts

In New Zealand, dairy production is based on the conversion of grass to milk by grazing cows. Compared to other countries, the quantities of silage , hay and meal fed per cow are small. This reliance on pasture production dictates the seasonal nature of the production of the milkfat. To ensure that feed requirements of the cows are supplied by grass production, the calving interval has to remain equal or close to 365 days so that the start of calving coincides with the start of the spring grass

growth. The major components contributing to the productivity of any cow are represented in figure 4.1

There are numerous economic components affecting the financial return achieved by the dairy cow. However, costs and returns which can be assumed to be the same for all cows can be ignored, e.g. equipment, buildings and labour.

4.3 Prediction of future production.

4.3.1 Introduction.

The usefulness of a dynamic programming or any economic model to predict the future profitability of a cow is dependent on the prediction of the cow's future milkfat production.

The milkfat production of a cow will be affected by management, environment and the genetic makeup of that cow. Environmental effects, other than those for which correction can be made, including management must be assumed random for all cows .The records of close relatives will be useful for predicting the expected production of a heifer. Once a cow has records of her own ,the information from relatives should be weighted accordingly.

Most of the reports summarised in chapters 2 and 3 have based their prediction of future production on a regression equation including previous lactation yields and in some cases calving interval and age at first calving. The limitation of using regressions based on prior production records to predict future production is the reliance on having at least one previous record. Thus, it is not possible to estimate the future production of freshening heifers and consquently, it is not possible to compare incoming heifers with existing cows on the basis of future profitability. For this reason, it is important to have a prediction method which predicts

the future production of all animals which could be potentially in the herd in the coming season, since all these animals could contribute to the income generated from the herd in that season.

In the following sections best linear unbiased prediction (BLUP) is described and a BLUP model which allows the estimation of the future production of all animals in or entering the herd is discussed .

4.3.2 Best Linear Unbiased Prediction.

BLUP is a statistical technique which has been utilised for predicting sire and dam breeding values. The method requires the definition of a linear model, usually containing both random and fixed effects, and all related assumptions. The details of BLUP have appeared in several publications, for example Henderson (1973). The purpose of this section will be to briefly summarise the theory of BLUP .The following section will describe the application of BLUP techiques to the estimation of future production of dairy cows.

To describe the theory of BLUP the following reports will be utilised : Henderson(1973,1975 and 1977) , Searle(1967,1971) ,Schaeffer(1975) and Thompson(1979).

Suppose there is a record y_1 from which a prediction of an unknown record , y_2 , is required. Assume that $E(y_1) = \mu_1$, $E(y_2) = \mu_2$ and , y_1 and y_2, are identically and independently distributed with variance-covariance matrix $V\sigma^2$. The objective is to obtain a predictor $\hat{y}_2$ whose expected value is y_2. A logical criterion for such a predictor is one which minimises the mean square error (MSE) of the prediction , ie:

$$\text{minimise} \quad E(\hat{y}_2 - y_2)^2 \qquad \ldots 4.1$$

For any known, joint, distribution of y_1 and y_2 the predictor which minimises MSE is $E(y_2|y_1)$. This predictor is known as the best predictor (BP) and is also unbiased. Unfortunately the joint distribution of y_1 and y_2 is usually unknown or complex; thus in practice, BP is not often used. If the class of predictions is limited to those that are linear then y_2 can be predicted as :

$$\hat{y}_2 = \mu_2 + \sigma_{12}/\sigma^2_1 (y_1-\mu_1) \qquad \ldots 4.2$$

where : σ_{12} is the covariance between y_1 and y_2, and

σ^2_1 is the variance of y_1.

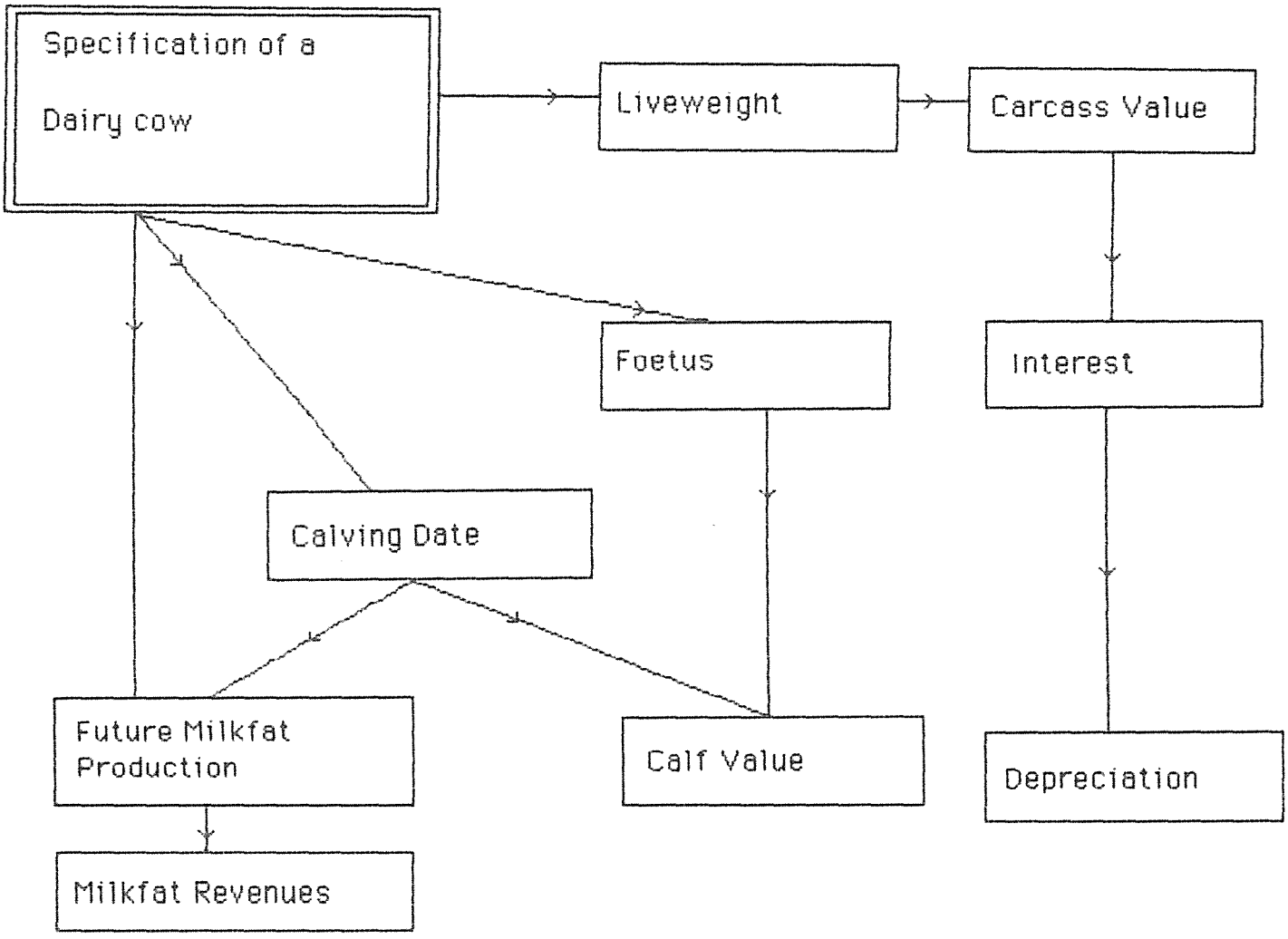
The predictor of y_2 in equation 4.2 is called the 'the best linear predictor' (BLP) and is also unbiased.

If y_1 and y_2 are assumed to be jointly normally distributed, then BLP = BP. With BLP the first and second moments are assumed to be known without error. There may be some justification to assume that the variances and covariances for traits such as milk and fat yield are essentially known 'without error', given the large number of such estimates. But this is not the case for the means, especially when the means represent the means of future records. To overcome this limitation a futher predictor for y_2 can be derived such that $E(\hat{y}_2)=E(y_2)$ and it can be shown that:

$$\hat{y}_2 = \hat{\mu} + \sigma_{12}/\sigma^2_1 (y_1-\hat{\mu}) \quad ,$$

Figure 4.1

The cow system diagram



where $\hat{\mu}$ is the generalised least square (GLS) estimator of μ. The quantity $\hat{y}_2$ is then the BLUP of y_2. Only GLS estimators provide linear unbiased predictors with the smallest MSE. When the distribution is multivariate normal, $\hat{y}_2$ is a best linear unbiased estimator and maximum likelihood estimator of the conditional mean of y_2. Futhermore, if the observations and random variables have a multivariate normal distribution the probability of correctly ranking pairs of random variables is maximised.

The usual case is to have a vector of records or observations, say vector $\underline{y}$, so that the estimation of each predictor would require the estimation of a linear function of $\underline{y}$,whose expected value is the predictor and which will also minimise the MSE. In these cases there can be a large number of predictants. This would require minimisation of a function for each predictant. Fortunately, Henderson(1973) has proved several invariance properties associated with BLUP solutions, which eliminates the need to minimise a function for every predictant .

Henderson (1949) derived a set of equations named the 'mixed model equations' which gave BLUP solutions. These equations have made it computationally feasible to consider large data sets. If $\underline{y}$ is the vector of observations then the mixed model is :

$$\underline{y} = X\underline{b} + Z\underline{u} + \underline{e}$$

where: $\underline{b}$ is a vector of known fixed effects,

$\underline{u}$ is a vector of random effects,

$\underline{e}$ is a vector of random non-observable effects associated with each record,

X is a matrix which describes the fixed effects for each record and

Z is a matrix which describes the random effects for each record.

The assumptions for the model are: the vector of random effects and the vector of random non-observable effects are independently distributed with means zero; the vector of random effects and the vector of random non-observable effects are assumed uncorrelated and:

$$E\begin{bmatrix}\underline{y}\\ \underline{u}\\ \underline{e}\end{bmatrix}=\begin{bmatrix}X\underline{b}\\ 0\\ 0\end{bmatrix} \qquad \text{and} \quad \text{var}\begin{bmatrix}\underline{y}\\ \underline{u}\\ \underline{e}\end{bmatrix}=\begin{bmatrix}V & ZG & R\\ GZ' & G & 0\\ R & 0 & R\end{bmatrix}$$

where: ′ denotes the transpose ,

$V = ZGZ'+R$, and

G and R are known nonsingular matrices.

R is an identity matrix in most animal breeding applications .

The mixed model equations based on the above model and assumptions are:

$$\begin{bmatrix}X'R^{-1}X & X'R^{-1}Z\\ Z'R^{-1}X & Z'R^{-1}Z+G^{-1}\end{bmatrix}\begin{bmatrix}\tilde{\underline{b}}\\ \tilde{\underline{u}}\end{bmatrix}=\begin{bmatrix}X'R^{-1}Y\\ Z'R^{-1}Y\end{bmatrix}$$

where: $\tilde{\underline{b}}$ is the BLUP of $\underline{b}$ and

$\tilde{\underline{u}}$ is the BLUE of $\underline{u}$.

Henderson has proved that $\tilde{\underline{b}}$ and $\tilde{\underline{u}}$ are both GLS solutions for $\underline{b}$ and $\underline{u}$ respectively.

4.3.3 Application of BLUP to the prediction of future production.

Henderson(1974, 1976) developed models for intraherd prediction of breeding values and real producing abilities.

A model for records within a herd is :

$$\underline{y} = X\underline{b} + Z\underline{a} + Z\underline{p} + \underline{e} \qquad \ldots 4.4$$

where: $\underline{y}$ is a vector of records,

$\underline{b}$ is a vector of fixed unknowns,

X is a known matrix,

Z is a matrix which relates the elements of $\underline{a}$ to the elements of $\underline{y}$,

$\underline{a}$ is a vector of additive genetic values ,

$\underline{p}$ is the vector of nonadditive genetic values and the permanent environmental effects , and

$\underline{e}$ is the vector of temporary environmental effects.

The vectors $\underline{a}$, $\underline{p}$ and $\underline{e}$ have means zero. The variance of $\underline{a}$ is :

$$\text{var } \underline{a} = Ah^2\sigma^2_y$$

where: A is the relationship matrix,

h^2 is heritability of the trait, and

σ^2_y is variance of records in a noninbred population.

The variance of $\underline{p}$ is :

$$\text{var } \underline{p} = I(r-h^2)\sigma^2_y$$

where: r is the repeatability between records, and

I is an identity matrix.

The variance of $\underline{e}$ is :

$$\text{var } \underline{e} = I(1-r)\sigma^2_y$$

The model assumes that $\underline{a}$, $\underline{p}$ and $\underline{e}$ are mutually uncorrelated and that the additive genetic variance is the only cause of correlation between records on different animals. The variance of animals with an inbreeding coefficient f is $(1+h^2f)\sigma^2_y$. The covariance between two records from animal i and animal j , is $h^2a_{ij}\sigma^2_y$, where a_{ij} corresponds to an element of A . The covariance between two records on the same animal is $(r+h^2f)\sigma^2_y$.

The equations for finding the best linear unbiased predictors of $\underline{a}$ and $\underline{p}$ and the best linear unbiased estimates of $\underline{b}$ are :

$$\begin{bmatrix} X'X & X'Z & X'Z \\ Z'X & Z'Z+tA^{-1} & Z'Z \\ Z'X & Z'Z & Z'Z+kI \end{bmatrix} \begin{bmatrix} \tilde{\underline{b}} \\ \tilde{\underline{a}} \\ \tilde{\underline{p}} \end{bmatrix} = \begin{bmatrix} X'\underline{y} \\ Z'\underline{y} \\ Z'\underline{y} \end{bmatrix} \qquad \ldots 4.5$$

where: t is $(1-r)/h^2$, ie [var $\underline{e}$/var $\underline{a}$] ,and

k is $(1-r)/(r-h^2)$, ie [var $\underline{e}$/var $\underline{p}$].

The prediction of future production of the ith animal is $a_i + p_i$.

Henderson (1974) reported that the expectations of $\underline{b}$, $\underline{p}$ and $\underline{a}$ were unaffected when selection and culling were present providing the relationship matrix was included in the mixed model equations and all animals are represented in the same equations, including those culled. Hence, culling and selection do not bias the prediction of $\underline{b}$, $\underline{p}$ and $\underline{a}$. Futhermore, Henderson (1980) has shown that, if selection is based on linear functions of $\underline{y}$ which have been adjusted for the fixed effects using estimators which are unbiased in the non-selection model, the BLUP will be optimum in the selection model.

It is possible to eliminate the rows of equation (4.5) corresponding to the prediction of $\underline{p}$ by absorption since the lower submatrix is diagonal. Henderson (1974) presented an algorithm which enabled the computation of the absorbed equations:

$$\begin{bmatrix} S11 & S21 \\ S21 & S22 \end{bmatrix} \begin{bmatrix} \tilde{\underline{b}} \\ \tilde{\underline{a}} \end{bmatrix} = \begin{bmatrix} \underline{q}1 \\ \underline{q}2 \end{bmatrix} \qquad \ldots 4.6$$

The algorithm is as follows :

1) The diagonal coefficients of the S_{11} are computed from $\Sigma_i(n_i + k-1)/(n_i + k)$ where the summation is over animals in that season and n_i is the number of records for the ith cow .

2) The off diagonals of S_{11} are computed from $-\Sigma_i\ 1/(n_i + k)$ where the summation is over animals in the particular pair of seasons.

3) The elements of S_{12} and S_{21} which equal 1 become $k/(n_i + k)$ where the i corresponds to animal i.

4) The diagonal coefficients of S_{22} become $c_{ij} - n_i^2/(n_i + k)$, i=j and the off diagonals equal c_{ij} , i≠j where c_{ij} are the elements of $Z'Z + tA^{-1}$

5) The elements of g_1 become $w_i - \Sigma_i\ y_i/(n_i + k)$ where the summation is over cows having records in that season and w_i is the ith element of $X'\underline{y}$.

6) The elements of g_2 are computed from $ky_i/(n_i + k)$.

After solving equation 4.6 $\underline{p}$ can be computed from :

$$\underline{p} = tA^{-1}\underline{a}/k \qquad \ldots 4.7$$

This relationship can proved by considering the following two equations derived from equation 4.4 :

$$X'X\underline{b} + (Z'Z+tA^{-1})\underline{a} + Z'Z\underline{p} = X'Y$$

$$X'X\underline{b} + Z'Z\underline{a} + (Z'Z+kI)\underline{p} = X'Y$$

therefore

$(Z'Z+tA^{-1})\underline{a} + Z'Z\underline{p} = Z'Z\underline{a} + (Z'Z+kI)\underline{p}$

and

$(Z'Z+tA^{-1})\underline{a} - Z'Z\underline{a} = (Z'Z+kI)\underline{p} - Z'Z\underline{p}$

thus

$tA^{-1}\underline{a} = k\underline{p}$

which is the relationship given in equation 4.7.

Henderson(1974) discussed a method which allowed the inclusion of information from AI sire evaluation. The method required the assumption that the records of daughters in the sire proof were perfectly adjusted for any fixed effects. Although this assumption may be invalid, it could be a good approximation since procedures in sire evalution do include adjustment for fixed effects (Van Vleck, 1982). The method involves adjusting the diagonal of the matrix tA^{-1} for each sire such that:

tA^{-1} becomes $tA^{-1} + \rho(1-r)(4-h^2)$

where ρ is a measure of the accuracy of the sire evaluation. The respective elements in the vector $Z'y$ become :

$[2(1-r)(4+(\rho-1)h^2)s]/h^2(4-h^2)$

where s is the sire evaluation which is half the additive genetic merit of the sire.If ρ is large then $a_i = 2s_i$. Van Vleck(1982) has reported the derivation of the above adjustments.

4.3.4 A numerical example.

The data in table 4.1 will be used to illustrate the intraherd BLUP model. The objective will be to predict the future production of the dams.

Table 4.1

Data for numerical example

number	sex	dam	sire	season 1	2	3	total
1	dam	--	--	200	210	205	615
2	dam	--	--	185	190	170	545
3	sire	--	--				
4	dam	1	3		187	194	381
5	dam	2	--		194	198	392
6	sire	--	--				
7	dam	1	6			200	200
8	dam	1	--			204	204
total				385	781	1171	2337

Assuming the relationships between the animals given in table 4.1 then A is

$$\begin{bmatrix} 1 & 0 & 0 & 1/2 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1/2 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 1 & 0 & 0 & 1/4 & 1/4 \\ 0 & 1/2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1/2 & 0 \\ 1/2 & 0 & 0 & 1/4 & 0 & 1/2 & 1 & 1/4 \\ 1/2 & 0 & 0 & 1/4 & 0 & 0 & 1/4 & 1 \end{bmatrix}$$

and A^{-1} is

$$\begin{bmatrix} 7/3 & 0 & 1/2 & -1 & 0 & 1/2 & -1 & -2/3 \\ 0 & 4/3 & 0 & 0 & -2/3 & 0 & 0 & 0 \\ 1/2 & 0 & 3/2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & -2/3 & 0 & 0 & 4/3 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 & 3/2 & -1 & 0 \\ -1 & 0 & 0 & 0 & 0 & -1 & 2 & 0 \\ -2/3 & 0 & 0 & 0 & 0 & 0 & 0 & 4/3 \end{bmatrix}$$

Assuming $h^2=0.25$ and $r = 0.5$, then $k=2$ and $t=2$. Assuming the model given in equation 4.4 then;

$$X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Z = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Y = \begin{bmatrix} 200 \\ 210 \\ 205 \\ 185 \\ 190 \\ 170 \\ 187 \\ 194 \\ 194 \\ 198 \\ 200 \\ 204 \end{bmatrix}$$

Using the blup equations given in equation 4.5 then ;

$$\begin{bmatrix}
2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 4 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 6 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 23/3 & 0 & 1 & -2 & 0 & 1 & -2 & -4/3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 17/3 & 0 & 0 & -4/3 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 3 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & -2 & 0 & -2 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & -4/3 & 0 & 0 & 14/3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 3 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -2 & 0 & 0 & 0 & 0 & -2 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & -4/3 & 0 & 0 & 0 & 0 & 0 & 0 & 11/3 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 1 & 1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} \tilde{b} \\ \tilde{a} \\ \hat{p} \end{bmatrix}
=
\begin{bmatrix}
385 \\ 781 \\ 1171 \\ 615 \\ 545 \\ 0 \\ 381 \\ 392 \\ 0 \\ 200 \\ 204 \\ 615 \\ 545 \\ 0 \\ 381 \\ 392 \\ 0 \\ 200 \\ 204
\end{bmatrix}$$

The solutions to the blup equations are;

[193.56 196.54 194.52 3.92 -4.36 -1.27 0.05 -1.56 0.47 2.67 3.47 3.72 -5.14 0.00 -2.54 1.01 0.00 0.94 2.01]

If the permanent environmental effects are absorbed the blup equations become ;

$$\begin{bmatrix}
1.6 & -.4 & -.4 & .4 & .4 & 0 & 0 & 0 & 0 & 0 & 0 \\
-.4 & 3.1 & -.9 & .4 & .4 & 0 & .5 & .5 & 0 & 0 & 0 \\
-.4 & -.9 & 4.43 & .4 & .4 & 0 & .5 & .5 & 0 & 2/3 & 2/3 \\
.4 & .4 & .4 & 5.86 & 0 & 1 & -2 & 0 & 1 & -2 & -4/3 \\
.4 & .4 & .4 & 0 & 3.86 & 0 & 0 & -4/3 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 3 & -2 & 0 & 0 & 0 & 0 \\
0 & .5 & .5 & -2 & 0 & -2 & 5 & 0 & 0 & 0 & 0 \\
0 & .5 & .5 & 0 & -4/3 & 0 & 0 & 8/3 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 3 & -2 & 0 \\
0 & 2/3 & 2/3 & -2 & 0 & 0 & 0 & 0 & -2 & 14/3 & 0 \\
0 & 2/3 & 2/3 & -4/3 & 0 & 0 & 0 & 0 & 0 & 0 & 11/3
\end{bmatrix}
\begin{bmatrix} \tilde{b} \\ \tilde{a} \end{bmatrix}
=
\begin{bmatrix}
153 \\ 357.75 \\ 611.08 \\ 246 \\ 218 \\ 0 \\ 190.5 \\ 196 \\ 0 \\ 133.33 \\ 136
\end{bmatrix}$$

The solutions to equations are identical to the previous solutions for $\underline{b}$ and $\underline{a}$. In this case the solutions for $\underline{p}$ can be generated from equation 4.7.

4.3.5 The computer program.

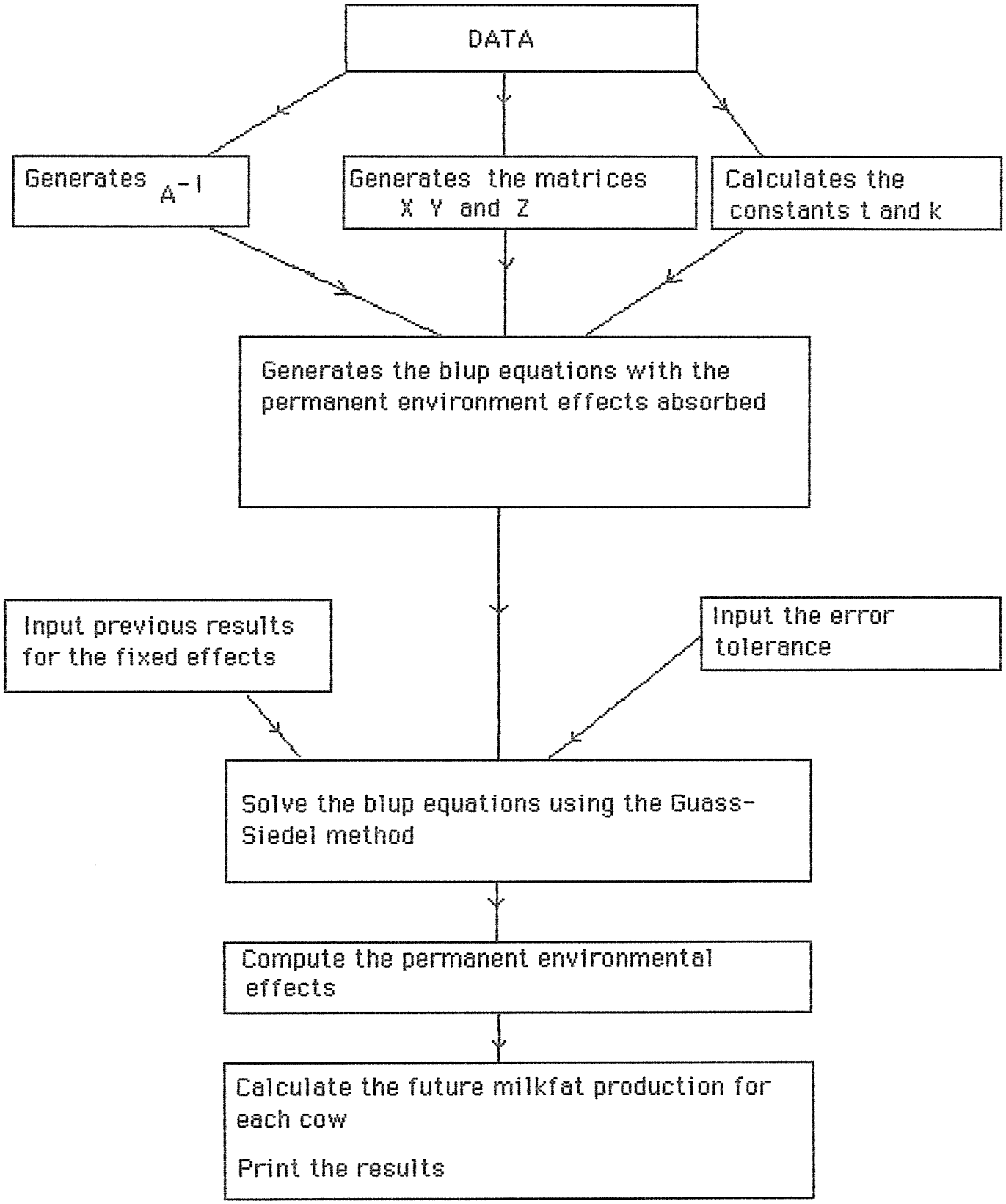
The intraherd BLUP model outlined in the previous section has been programmed in Fortran77. This program has been applied to the prediction of future milk fat production for dams in five dairy herds with records available for 1982-1983, 1983-1984 and 1984-1985. Figure 4.2 shows the flow of data through the program's operations. Sires were included when they had more than one daughter. Dams culled before the 1984 - 1985 season were not included if they had no daughters in the herd since they contribute little information. The sires and dams of the cows in the herd are identified by their number in the data set. The data set is arranged from the oldest to the youngest animal. The data for each animal include five descriptive parameters and their records. These parameters are the identification number, sex (0=dam, 1=sire), dam number (default is 0) , sire number (default is 0) and the number of lactation records.

The records were corrected for age and length of lactation using the correction factors given in table 4.2 to give the 305 mature equivalent lactation record. The program computes A^{-1} using Henderson's(1976) simplified method for calculating the inverse of the relationship matrix . To minimise the memory storage required by the program the permanent environmental effects are asborbed.

The Gauss-Siedel iteration procedure is used to solve the equations . Solutions for the seasonal effects from previous runs can be used to speed up convergence. The differences between the each element of the right hand side (RHS) in equation 4.6 the elements of the RHS generated from the current solutions in each interation were calculated. Convergence was assumed to have occurred when the greatest difference between the original RHS and the generated RHS was less than an error tolerance. The number of iterations required for convergence markedly increased as the error tolerance increased from 0.001 to 0.0001. In the present study the error

Figure 4.2

A diagram of the flow of data through the intraherd blup model.



tolerance was set at 0.001. On average, the number of iterations required for convergence varied between 30 and 95.

Table 4.2

Age and stage of lactation adjustment factors.

Days in milk	Adjustment [a] factors	Age	Age correction factors [b]
Up to 29	15.88	2... years	1.341
30-59	5.39		
60-89	3.33	3... years	1.176
90-119	2.46		
120-149	1.96	4... years	1.068
150-179	1.64		
180-209	1.43		
210-239	1.27	5+... years	1.000
240-269	1.14		
270-304	1.05		

a Source Batra and Lee (1981)

b Source Castle and Searle (1959)

4.3.6. Variation in the within-herd repeatability

Castle and Searle (1959) published within herd repeatabilities for 30 herds, calculated over 4 years. With the year/season effects eliminated, the estimates varied from 0.16 to 0.75. However, since 1959 there has been a marked change in the composition of the national herd, away from Jersey towards Holstein and Freisian or their crosses. It could be possible that this parameter has shifted over the last 25 years. Anon(1984) published an estimate of 0.6 , this estimate was based on work done by Castle and Searle.

To examine the effect of the magnitude of repeatability on within herd cow ranking, estimates of the future milkfat production of over 1000 cows from 5 herds with records in 1982-1983,1983-1984 and1984-1985 were calculated. Five estimates of within-herd repeatability were used : 0.5 , 0.55, 0.60 , 0.65 and 0.70. Within each herd, changing the within-herd repeatability had little effect on the ranking of individual cows according to future milkfat production. For one herd of 168 cows each cow was ranked on their future milkfat production for each of the five within-herd repeatabilities . A Spearmans rank correlation was calculated for each the combinations of within-herd repeatabilities. These results are given in table 4.3. There was little difference between these results and results for the other herds

Table 4.3

Spearmans rank correlation within-herd for the rankings of individual cows according to future milkfat production for 5 values of within-herd repeatability

Within-herd repeatability	Within-herd repeatability 0.50	0.55	0.60	0.65	0.70
0.50	1.000	0.999	0.998	0.997	0.996
0.55	——	1.000	0.999	0.998	0.996
0.60	——	——	1.000	0.999	0.998
0.65	——	——	——	1.000	0.999

Although a change in within-herd repeatability of 0.2 did change the estimate of actual future milkfat production, the relative rankings of the individuals is more important than the absolute estimate, for a

replacement model. As shown above, within-herd ranking of cows is relatively robust to changes in repeatability. In the present study an estimate 0.6 will be used.

4.4 The value of milkfat production.

The future milkfat production of an individual cow is given as the 305 day mature equivalent which is computed by the intraherd BLUP model presented in the previous section. In New Zealand, the payout for dairy production is based on fat content. Regional price differences can occur, these being partly due to variation in processing costs between different units and the cooperative structure of the dairy industry. Since the measure of production is expressed as the mature equivalent, the estimate must be divided by the appropriate age correction factor when calculating the value of production in the current year. The value of the milkfat production is calculated from equation 4.8 :

$$MFR = (FMFP \times 1/ACF) \times MFP \qquad ...4.8$$

where: MFR is the milkfat production revenues in dollars,

FMFP is the estimate of the future milkfat production in kgs,

ACF is the appropriate age correction factor, and

MFP is the price per kg of milkfat in dollars.

4.5 Value of the new born calf.

The value of the calf born will be dependent on the probability of survival of the new born calf and the value of the bobby calves. The value of a calf is calculated from equation 4.9 :

$$CV = (0.95 \times BCV) \qquad ...4.9$$

where: CV is the calf value,

0.95 is the probability of survival (Holmes and Wilson 1984) ,

BCV is the bobby calf value.

4.6 The effect of calving date

4.6.1 The effects of variation in calving date on seasonal milk production and management practices.

The herd's calving date can influence the level of feeding available to a cow in early lactation and also her lactation length, both of which will affect her milkfat production.

A concentrated calving , provided it occurs at the correct time, will allow the herd's feed requirements to match the expected grass production more closely. It will also reduce the number of late calvers and increase the average number of days in milk for the herd (Holmes and Wilson 1984).

Calving date variation is the major factor influencing lactation length among cows within a herd (MacMillan et al 1984a). Calving dates for cows within a herd are not normally distributed about the mean. MacMillan et al (1984) using data from approximately 3700 cows in 35 herds in the Matamata area reported that 50% of the cows calved within the first 18.3 days , the next 25% over the interval of 17.5 days and the last 25% over an interval of 36.3 days. On average, the the optimal calving spread is between 5 to 8 weeks (MacMillan 1975 and Holmes and Wilson 1984). If there was an unlimited supply of feed, the maximum herd and per cow milkfat production would occur if all the cows were to calve on day 1 of the planned start of calving. Under New Zealand conditions the supply of feed is constrained by the spring grass growth. Thus, a system where all cows calve on day 1 of the planned start of calving would cause a severe feed shortage in early lactation. Rather than nutritionally stressing the

cows under this hypothetical situation, it is currently more desireable and practical to simply aim for a compact calving. Variation in the desired length is caused by seasonal effects on grass supply in early lactation.

MacMillan(1979a) found that comparison of production from groups of identical twins calving at different time intervals, in similar conditions to commericial herds, resulted in significant production differences. These differences were attributed to differences which arose in early lactation yields. He concluded that each day's increase in lactation length could increase the milkfat yield by 0.7-0.9 kg milkfat/cow/day.

Inducing late calvers is a well established management practice used by farmers to concentrate the calving period . A survey by MacMillan et al (1984a) found that 72 out of 79 farmers in the Waikato induced late calvers. The proportion of each herd induced ranged from 1.2% to 26.1%. Most induced cows calved after the first 6 weeks of the planned calving date. Welch et al's (1979) results showed, for the Wellington -Hawke Bay region ,that a large proportion of the cows calving 7 weeks after the planned start of calving were induced .

The procedure of induction, consists of an injection of slowly absorbed corticosteriod followed, if necessary, by an injection of rapidly absorbed corticosteriod or prostaglandin 7-12 days after the first injection. The first injection is administered 3-6 weeks before the expected calving date. MacDiarmid and Moller (1981) found that 56-58% of cows require two injections, and on average they estimated that 1.7 injections were required per cow to induce premature calving.

Calves from induced cows have a mortality rate of 20-36% (Welch et al 1979, Holmes 1984) compared with 5% for calves from noninduced cows (Holmes and Wilson 1984).

There is no evidence to show that induced cows have signficantly lower milkfat production or poorer subsequent reproductive ability, than cows which calve normally (Welch et al 1979, Holmes and Wilson 1984).

4.6.2 Defining the value of calving date

A concentrated calving period ,provided that it occurs at the correct time of the year, will allow the feed requirements of the herd to more closely match the expected grass growth rates of the pasture. When including calving date in a model, the optimum time to calve is not the planned start of calving. Although late calvers should be penalised, it must be recognised that some optimal calving spread exists for each herd. Obviously , the optimal time to calve for any given cow is the planned start of calving, but the herd should be considered as a whole and the effect of a concentrated calving on management assessed .For this reason five categories have been proposed to account for calving in the present model. Table 4.4 summarises the features of the five categories.

Table 4.4

A summary of the features of the five calving date categories.

	Calving date category.				
	1	2	3	4	5
Period	0 -6 weeks	7-9 weeks	10-12 weeks	13-15 weeks	> 15 weeks[b]
% Calf Mortality	0.05	0.28	0.28	0.28	------
Milkfat[a] Penalty kg	nil	33.6	50.4	67.2	------
Inducing Cost	nil	1.7* vc[c]	1.7*vc	1.7* vc	------

a number of days at start of period multipied by 0.8kg milkfat per day.

b cows culled for non-pregnancy or excessive lateness.

c vc = veterinary cost

In most cases it is desirable that cows calve within the first 6 weeks after the planned start of calving . This time period defines the first category. All cows in this category are assumed to have full term pregnancies; hence the percentage of calf mortality is assumed to be 5%. Cows in this category will not be penalised for the variation of their individual lactation lengths. This is to ensure that the 6 week period remains the optimum time to calve.

It is assumed that inducing late calvers is a widespread management practice which is used to take full advantage of the potential benefits of a concentrated calving period . Any cows calving outside the 6 week period will be considered as candidates for induction. Thus, the model must take into account the cost of induction, and the higher calf mortality.

The second, third and forth categories correspond to the time periods , 7-9 , 10-12 and 13-15 weeks after the planned start of calving. It will be assumed that cows in any one of these categories will have been induced to calve early. Thus, they will be assumed to have a higher rate of calf mortality (28%). They will also incur the cost of induction and be penalised for their shorter lactation length since they calved outside the optimum period. This penalty will be calculated from the start of the period defined by each category. The penalty will equal the number of days from the planned start of calving to the beginning of the given period multiplied by the loss in milkfat yield per day.The reason for using the beginning of each period is that it is assumed cows in these categories will be induced and, thus, would be expected to calve 7-14 days early. This may over-penalise cows with a calving date near the start of each period and similiarly under-penalise cows with calving dates toward the end of the categories. But it is not possible to predict the exact calving date for each animal, since it is not possible to know at what stage of pregnancy the cow will be induced.

The fifth category is for cows who calve later than 15 weeks after the

planned start of calving and cows who have not conceived. Cows calving after the first 15 weeks would have little or no chance of being bred in the next season. Hence, their calving interval would be greater than 365 days, which is unsuitable for seasonal dairy farming. Thus, all cows in this category are assumed unsuitable for dairy production and therefore culled. These cows will have no milk or calf production in the next or subsequent seasons .

As well as defining the 5 calving date categories, it is also necessary to define the probability of a cow being in a given calving date category in the next and subsquent seasons. To date, no New Zealand data has been published which identifies the variables which effect the calving interval of individual cows over successive seasons , nor indeed any data which quantifies the changes in calving intervals of individual cows over successive seasons.

Johansson(1961) cited several estimates of repeatability (0.076,0.135 and 0.133) and heritability (0.0,0.03,0.148 and 0.0)for calving interval, suggesting that most of the variation in calving interval was due to temporary environmental effects. Further investigation into the causes of variation in the length of calving interval is beyond the scope of this investigation. Consequently, the probability of a cow being in a given calving date category in a subsquent season will be assumed independent of age and level of production and dependant only on the previous calving date.

To calculate the probabilities associated with calving date the following assumptions were made :

i) the subsequent planned start of calving is 365 days after the present planned start of calving,

ii) the probability of conception is 0.68 (Anon,1984),

iii) the gestation length is 283 days for noninduced cows,

iv) the calving to first oestrus period is 36 days, and

v) the breeding period is 12 weeks long.

To calculate the probabilities , the number of breeding chances cows (from a given calving date category) have to change categories or stay in the same category has to be calculated; these results are presented in table 4.5 along with date of calving and date of the first breeding for each category.

Table 4.5

The number of breeding chances that cows in a given category have to stay in the same category or change to different categories in the subsequent season.

Calving date Category	a	b	Subsequent calving date category			
			1	2	3	4
1	21	57	2	1	1	1
2	42	78	2	1	1	1
3	63	99	1	1	1	1
4	84	120	-	1	1	1
5	–	–	–	–	–	–

a The average calving date for each category expressed as days after the planned start of calving.

b The average start breeding date expressed as days after the planned start of calving.

The probability that a cow in category 1 remains in category 1 in the subsquent season is the sum of the probability she conceives in the first breeding plus the product of the probability she conceives in the second mating times the probability she does not conceive in the first breeding , for example :

$0.68 + (0.68 \times 0.32) = 0.8976$.

The probability that the same cow moves to the second calving date category is :

$0.32 \times 0.32 \times 0.68 = 0.0696$.

The probability that the same cow moves to the third calving date category is :

$0.32 \times 0.32 \times 0.32 \times 0.68 = 0.0223$.

The probability that the same cow moves to the forth calving date category is :

$0.32 \times 0.32 \times 0.32 \times 0.32 \times 0.68 = 0.0071$.

Finally, the probability that the same cow moves to the fifth calving date category is :

$1 - (0.8976 + 0.0696 + 0.0223 + 0.0071) = 0.0034 = 0.32^5$.

Table 4.6 gives the probabilites of movement from one calving category to the same or different categories in the subsquent season.

Table 4.6

The probability of movement from the present calving category to a category in the next season

Present calving category	Subsequent calving category 1	2	3	4	5
1	0.8976	0.0696	0.0223	0.0071	0.0034
2	0.8976	0.0696	0.0223	0.0071	0.0034
3	0.6800	0.2176	0.0696	0.0223	0.0105
4	0.0000	0.6800	0.2176	0.0696	0.0328
5	0.0000	0.0000	0.0000	0.0000	1.0000

To calculate the probability that an individual cow is in one of the five calving categories in year n, given she is in one of the five categories in year one ,a decision tree approach was used. For example, the probability a cow is in category 1 in year 3 given she was in category 1 in year 1 can be calculated from the decision tree given in figure 4.3. Thus, the probability of the cow being in category one in year 3 ,using the probabilities given in table 4.6 , is :

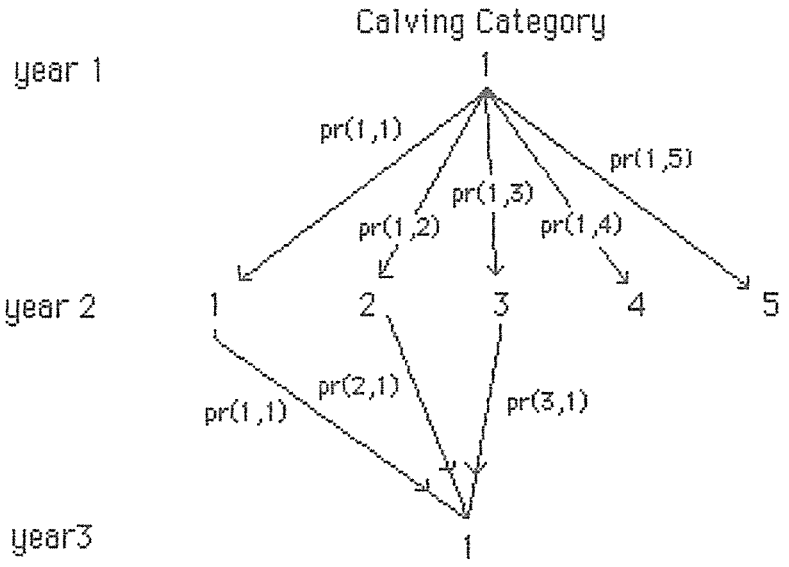
$$pr(1,1)_3 = (0.8976 \times 0.8976) + (0.0696 \times 0.8976) + (0.0223 \times 0.6800)$$

$$= 0.8833.$$

In the present investigation cows will be evaluated up to and including their tenth lactation . The probabilities of movement from one calving category to the same or another calving category in a future year are presented in appendix 2.

Figure 4.3

The decision tree for calculating the probability that a cow is in category 1 in year 3, given that she was in category 1 in year 1.



where pr(i,j) is the probability of movement from the ith calving category to the jth calving category in the next year.

4.6.3 Incorporation of the costs and probabilities associated with calving date category.

Variation in calving date will affect both the value of milkfat production and the value of the new born calf, therefore equations 4.8 and 4.9 have been modified to account for these affects. Equations 4.10 and 4.11 are the modified equations for milkfat value and newborn calf value respectively.

$$MFR_{j1} = (\{\sum_{j=1}^{5}(pr(i,j)_1) \times (FMFP \times 1/ACF)\} - [(pr(2,j)_1 \times 33.6] + [pr(3,j)_1 \times 50.4] + [pr(4,j)_1 \times 67.2)) \times MFP] \quad \ldots 4.10$$

$$CV_{j1} = pr(1,j)_1 \times (0.95 \times BCV) + \sum_{j=2}^{4}(pr(i,j)_1 \times (0.72 \times BCV)) \quad ...4.11$$

where : MFR_{j1} is the milkfat revenue in the 1th year for a cow who was in the jth calving date category in year 1,

$pr(i,j)_1$ is the probability that a cow is the ith calving date category in the 1th year given she was in the jth calving date category in year 1,

CV_{j1} is the calf value in the 1th year for a cow who was in the jth calving date category in year 1, and

0.72 is the probability of survival of an induced calf .

As well as the effects on milkfat revenue and calf value, there is also the cost of inducing. This is given in equation 4.12.

$$ID_{j1} = \sum_{j=2}^{4}(pr(i,j)_1 \times (1.7 \times VC)) \quad ...4.12$$

where : ID_{j1} is the inducing cost in year 1 for a cow who was in the jth calving date category in year 1, and

VC is the veterinary cost.

4.7 Salvage value of the cow.

The salvage value of the cow is a function of the cow's weight and the value for manufacturing beef. The choice of the beef value for salvage value is disputable as this assumes that the removal of cows is only for slaughter. Where cows are sold for dairy purposes, a better procedure

would be to use the market value of a cow. However, no data is available on the disposal of cull cows, so this study assumes cows are only removed for slaughter purposes. Quarterman and Carter(1969) and MacMillan et al (1984b) presented liveweights of Friesian and Jersey cows by age. The liveweights used in this study (see table 4.7) are based on this data. Table 4.7 also gives the carcass weights derived from assuming a dressing out percentage of 0.56. The salvage value is calculated from equation 4.13.

$$SV_k = PMFP \times CW_k \qquad \ldots 4.13$$

where: SV_k is the salvage value of cow of age k,

PMFP is the price per kg manufactering beef,and

CW_k is the carcass weight of a cow of age k.

Table 4.7

Liveweight and carcass weight data used in the present study

Liveweight (kg)			Carcass weight (kg)		
Age	Friesian	Jersey	Age	Friesian	Jersey
2	351	343	2	197	192
3	379	358	3	212	200
4	410	393	4	230	220
5>	435	411	5>	244	230

4.8 Appreciaton on a cow.

Cows gain weight during the years before they reach maturity . This weight gain expressed as dollars ,results in appreciation of the value of the cow. This may also be referred to as capital gain. There is virtually no

change in liveweight in later years hence, the differences in salvage values only affect cows in their first three lactations. Equation 4.14 calculates the appreciation for a given cow.

$$A_k = (SV_{k+1} - SV_k) \times PMFB \qquad ...4.14$$

where: A_k is appreciation for a cow of age k.

4.9 Interest on the capital value of the cow.

In this study a cow is considered an asset. Interest on the capital value of an asset is the opportunity cost of that capital. The capital value of a cow is assumed to be the cow's salvage value at the beginning of the current lactation. The forgone interest is calculated from equation 4.15

$$IC_k = SV_k \times IR \qquad ...4.15$$

where: IC_k is the interest on the capital value of a cow of age k, and

IR is the interest rate

4.10 Genetic Improvement.

Improvements in the genetic capacity of cows to produce milkfat must be taken into account. As logic would suggest, the greater the rate of genetic improvement, the shorter the replacement cycle. With artifical insemination it could be theoretically possible to get up to 2% annual genetic change (Van Vleck, 1977) . In New Zealand between 0.5 - 1.0% has been achieved (Anon, 1981). To incorporate genetic improvement, equation 4.10 has been modified, the modification is given in equation 4.16 :

$$MFR_{jkl} = (\{\sum_{j=1}^{6}(pr(i,j)_l) \times (FMFP \times 1/ACF)\} + (l-1) \times RGI - [(pr(2,j)_l \times 33.6] + [pr(3,j)_l \times 50.4] + [pr(4,j)_l \times 67.2)) \times MFP] \quad ...4.16$$

where: MFP_{jkl} is the milkfat revenue in lth year for a cow of age k in the jth calving date category in year l, and

RGI is the rate of genetic improvement expressed as kilograms of milkfat per year.

4.11 The probability of involuntary removal and death.

There is no New Zealand data which classifies wastage rates according to level of production and age. In this study it will be assumed that wastage rates are dependent on age but are independent of level of production. Glaever (1965) analysed about 12000 records from about 4000 cows in the U.S.A and concluded that involuntary replacement was dependent on age , but not production. Anon (1957) published wastage rates according to age; these rates are given in Table 4.8. It is with some reservations that this information is used. It is likely that the frequencies of the causes of wastage have changed considerably with advances made in animal health since 1957. This would be especially true for the incidences of bloat and metabolic disorders, as on-farm treatment of these problems is now commonplace.

The wastage rates have been classified into two categories :

i) those likely to lead to involuntary replacement, and

ii) those likely to result in death

The wastage rates for sold for dairying, old age , sterility and abortion

and mastitis were aggregated as those likely to lead to involuntary replacement. The wastage rates for accident and injury , calving trouble, bloat, metabolic disorders and death and sundry were aggregated as those likely to result in death.

Table 4.8

Wastage rates for dairy cows according to age.

Cause of wastage or culling	Percentage of age group culled								
	Age of cow at beginning of season culled								
	2 yrs.	3 yrs.	4 yrs.	5 yrs.	6 yrs.	7 yrs.	8 yrs.	9 yrs.	10 years or over
Sold for dairying	2.51	3.20	2.80	2.59	2.53	2.26	1.96	2.44	2.43
Low production	6.44	7.51	6.11	5.13	4.58	4.46	4.10	4.66	4.15
Accident and injury	0.66	0.50	0.43	0.46	0.51	0.59	0.84	0.96	1.32
Old age and sundry	1.17	1.32	1.21	1.27	1.15	1.25	1.41	2.38	10.06
Disease—									
Sterility and abortion	2.29	3.09	3.37	3.50	3.93	4.41	5.33	6.05	8.02
Calving trouble	0.10	0.09	0.13	0.08	0.12	0.12	0.12	0.19	0.20
Mastitis	0.37	0.57	0.68	0.85	1.13	1.01	1.50	2.06	1.73
Tb.	0.25	0.28	0.41	0.36	0.44	0.32	0.49	0.54	0.38
Bloat	1.18	0.60	0.56	0.47	0.62	0.49	0.52	0.38	0.38
Metabolic disorders	0.07	0.07	0.14	0.35	0.40	0.50	0.58	0.90	0.51
Death and sundry	0.80	0.94	0.94	1.10	1.15	1.25	1.31	1.65	1.96
Total disease wastage	5.06	5.64	6.23	6.71	7.79	8.10	9.85	11.77	13.18
Total wastage	15.84	18.17	16.78	16.16	16.56	16.66	18.16	22.21	31.16
Total number of cows culled	3,438	3,222	2,354	1,918	1,681	1,359	1,195	1,157	3,392
Total number cows in each each age group in herds in survey	21,700	17,727	14,032	11,866	10,151	8,155	6,580	5,209	10,885

Reprinted from 33rd New Zealand Dairy Board Report 1957.

The data in table 4.8 were submitted to a regression analysis to enable prediction equations for the probability of involuntary replacement (π_{ir}) and for the probability of death (π_d), using linear, quadratic or cubed terms of lactation number as the independent variables. The involuntary replacement analysis (see table 4.9) showed linear and quadratic terms were significant whereas linear, quadratic and cubed terms were

signficant for death (see table 4.10) The estimated probabilities of

Table 4.9

Analysis of variance tables for the rate of involuntary replacement.

Source of variance	Sum of squares	df	Mean squares	F statistic
Regression (x^2,x)	4.60	2	2.30	35.00
Linear(x)	3.55	1	3.55	F(.990,1,7) = 12.20
Curvature $(x^2\|x)$	1.05	1	1.05	
Sum of squares errors	0.22	7	0.03	
Total	4.82	9		

Table 4.10

Analysis of variance tables for the rate of death.

Source of variance	Sum of squares	df	Mean squares	F statistic
Regression (x^3,x^2,x)	250.96	3	83.65	17.26
Linear(x)	143.67	1	143.67	F(.990,1,6) = 13.7
Curvature $(x^2\|x)$	70.21	1	70.21	
Curvature $(x^3\|x)$	37.05	1	37.05	
Sum of squares errors	12.88	6	2.15	
Total	263.84	9		

involuntary replacement (π_{ir}) and death (π_d) are computed from the equations :

$$\pi_{ir} = 2.84 - .32x + .06x^2 \qquad \ldots 4.17$$

$$\pi_{d} = -1.0 + 7.89x - 2.13x^2 + 0.17x^3 \qquad \ldots 4.18$$

where: x is the lactation number .

Table 4.11 gives the observed and estimated rates for death and involuntary replacement.

Table 4.11.

The observed and estimated rates for death and involuntary replacement (%).

Lactation number	Rates of involuntary replacement		Rates of death	
	Estimated	Observed	Estimated	Observed
1	2.58	2.81	5.83	6.34
2	2.44	2.20	8.52	8.18
3	2.42	2.20	8.98	8.06
4	2.52	2.46	8.25	8.21
5	2.74	2.80	7.34	8.54
6	3.08	2.95	7.26	8.93
7	3.64	3.37	9.05	10.10
8	4.12	4.08	13.71	12.20
9	4.82	4.37	22.28	25.36
10	5.64	---	35.70	---

4.12 Milkfat production by the replacement heifer.

In the previous section the probability of death and involuntary replacement was discussed. When a cow is removed from the herd due to either of these two reasons, she will be replaced by a heifer in the models to be discussed in chapter 5. To avoid overly complicating the model, a fixed level of milkfat production for the replacement heifers is assumed. This level will be set to the mean value of lactation yields of 2 year old heifers in the current year. The mean expectation of a 2 year old heifer increases with time to account for continous genetic improvement. The replacement heifer is assumed to calve in the first

calving date category.

4.13 The choice of interest rate.

The choice of and the interpretation of, the interest rate for use in discounting procedures is a traditional point of dispute among economists. Anderson et al (1977) suggest three possible choices:

i) the borrowing rate . i.e the compound rate of interest of the cost of capital actually prevailing if funds were borrowed,

ii) the oppununity cost. i.e the rate of interest which could be earned in the most attractive investment, and

iii) the subjective rate of time preference. i.e the rate that the decision maker considers appropriate for discounting net flows to their net current value.

Smith (1978) has criticised earlier uses of discounting for failing to account for inflation . Instead of using the actual rate of interest he derived the following formula to calculate the effective annual rate of interest (q) :

$$q = (i-f)/(1+f)$$

where: f is the annual rate of inflation, and
i is the annual rate of interest.

There is the problem of choice of the real annual rate of interest. Bird and Mitchell (1980) discuss two distinct approachs, which are the social time preference and the social opportunity cost. The social time

preference revolves around the concept that consumption today is preferable to consumption in the future. The rate of social time peference is almost impossible to measure in the real world but usually the rate is set at about 2-3% per year. The social opportunity cost can be viewed as the tool for achieving the proper balance between the private and public sectors. This rate can be measured and in practice is the average inflation-adjusted rate of return for the private sector. Bird and Mitchell (1980) concluded that the social time preference rate should be used when evaluating national schemes , whereas individual enterprises should use the social opportunity cost.

Table 4.12

A summary of various interest rates for the 1984 - 1985 season.

	Average private sector investment [a] rate	Average private sector borrowing [a] rate	Average public sector borrowing [a] rate	Dairy farm cost index [b] rate
Real annual percentage	14 -17.5	13 -19.5	13 - 13.5	14.2
Inflation adjusted annual percentage	4.2 -7.4	3.3 - 9.2	3.3 - 3.7	4.4

a Source Anon.(1985)

b Source Anon.(1986)

c The annual inflation rate for the 1984 - 1985 season was 9.4%[a]

In this study the interest rate used will be the inflation adjusted private sector interest rate. This rate is the inflation adjusted opportunity cost or the social opportunity cost. Table 4.12 summarises the interest rate indicators for the 1984-1985 season.

Chapter 5.

Three models to rank cows on future profitability.

In this chapter , three methods of ranking cows based on future profitability projections are developed. In all three cases, the decision to keep or replace is based on comparisons of the expected average returns from future income streams. The first method calculates the annuity from the future incomes of individual cows while the second and third methods utilise a dynamic programming approach. The following two sections will discuss the specifications of the models.

5.1 Annuity Model.

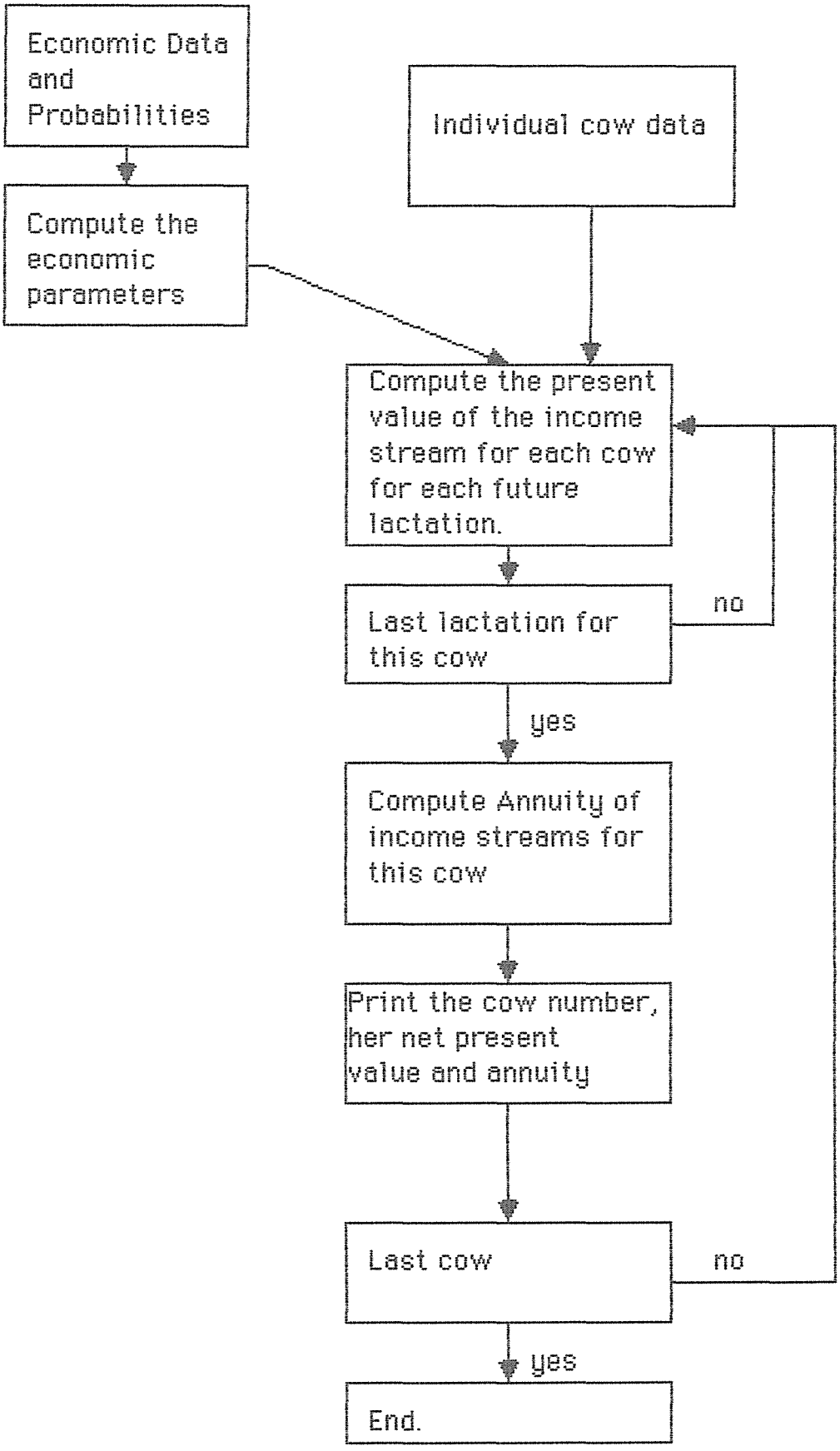
This model predicts the future net income streams for individuals for their remaining economic life. The remaining economic life is assumed to be up to and including the tenth lactation. Thus , a cow starting her fifth lactation would have 6 remaining future income streams, whereas a cow starting her tenth lactation would have 1 future income stream. The principle of discounting is used and thus, each future income stream is discounted to the net present value. The total net present value from the income streams is converted to the annuity (or the annualised present value). Hence, the model's objective is to rank cows on their annuity computed from the net present value of individual cow's future incomes, cows with the lowest values being candidates for removal from the herd in the coming season.

A computer program has been developed to compute the annuity for individual cows and figure 5.1 illustrates the flow of data through the program.

To predict the net incomes the following assumptions have been made :

Figure 5.1

Flow diagram for the annuity model program.



(i) the future milkfat production has been predicted from the BLUP model presented in the previous chapter,

(ii) the calving date for the coming season is known, and

(iii) all cows which are candidates for the herd in the coming season are known.

To calculate the net revenues for a given season, revenues and costs are weighted by the probability of the occurrence. In the case of cow failure and death, the model assumes the cow is replaced by a heifer replacement with an average milkfat yield (see section 4.10). Futhermore, the model assumes that the failed cow produces at its predicted level of milkfat production for the coming season, whereas in the case of death no production is expected . If a cow fails, income from the salvage value is earned but not so in the case of death. Due to assumption (iii), the probability of failure is not a component of the equation to calculate the net returns in the coming season (year 1) since all the candidates available for selection into the herd in the coming season are known. Thus, the equation for calculating the net returns for season one is :

$$NR_{jk1} = \{[\, MFR_{jk1} - ID_{j1} + CV_{j1} + A_k - IC_k \,] \times [\, 1 - \pi_{d\,(k',1)} \,]\} + \{\pi_{d\,(k',1)} \times [NR_1^{Hfr} - CHfr\,]\} \qquad ...5.1$$

where NR_{jk1} is the net returns of a given cow of age k in calving category j in season 1,

MFR_{jk1} is the value of milkfat production for a cow of age k in calving category j in season1 ,

CV_{j1} is the calf value for a cow in calving category j in season1

A_k is the appreciation for a cow of age k,

IC_k is the interest on capital value for a cow of age k,

CHfr is the cost of the heifer replacement,

NR_1^{Hfr} is the net returns of a heifer replacement in season 1,

ID_{j1} is the cost of inducing a cow in calving category j in season 1, and

$\pi_{d(k',1)}$ is the probability of death for a cow of age k' in season 1 in season 1.(These values have been calculated from the probabilities given in section 4.12; the values are presented in appendix 4.).

For subsequent seasons where probability of involuntary replacement also has to be accounted for the the equation for the net returns is :

$$NR_{jkl} = \{[\, MFR_{jkl} - ID_{j1} + CV_{j1} + A_k - IC_k \,] \times [\, 1-\pi_{d\,(k',1)} \,] [1-\pi_{ir\,(k',1)}\,]\} + \{\pi_{d(k',1)} \times [\, NR_1^{Hfr} - CHfr \,]\} + \{\pi_{ir\,(k',1)} \times [\, SV_k + NR_1^{Hfr} - CHfr \,]\}$$

$l \neq 1$ ...5.2

where: $\pi_{ir(k',1)}$ is the probability of involuntary replacement for a cow of age k' in season 1, in season 1.(These values have been calculated from the probabilities given in section 4.12; the values are presented in appendix 4.).

The net present value of the income streams is computed from the following equation :

$$NPV = \beta.NR_1 + \beta^2.NR_2 + \beta^3.NR_3 + \ldots\ldots\ldots \quad ...5.3$$

where: NPV is the net present value, and

β is the discount rate.

The annuity is calculated from the net present value using the amortisation factor given in equation 2.2.

5.2 The dynamic programming models.

All the studies summarised in chapter 3 with the exception of Killen et al (1978) have been concerned with, at some stage ,finding the optimal replacement policy. This study is concerned with the ranking of cows on future profitability. Dynamic programming can be used to predict the future profitability of income streams. The future profitability equals the difference between the present value of future revenues obtained from the optimal decisions and the present value of immediate replacement (Smith 1971). In this situation, the present value of immediate replacement would be the present value of the net returns for an incoming heifer with an average milkfat production (see section 4.10).

Using dynamic programming, optimization starts at the end of a given planning horizon , and proceeds backwards in time stage by stage. The maximum net present value of all cash flows anticipated from each cow is determined at each preceeding stage. This process is continued until the present stage is reached.

For both the dynamic models, a stage is defined as 12 months which corresponds to one season. Cows are described by three state variables namely, lactation number, mature equivalent milkfat production and calving date. There are 10 possible lactation number states. The model assumes cows who have completed their tenth lactation are replaced by 2 year old heifers . As previously described, there are 5 calving date states. There are 52 states describing mature equivalent milkfat production

(ME MFP). These states are defined in the following arrangement:

State	1	2	3	 51	52
ME MFP (kg)	0 -<100	100-<105	105-<110	345-<350	350-∞

The values of the mid-points of states 2 to 51 are used to calculate the value of the net present value. For states 1 and 52 the values 97.5 and 352.5(kg) are used, respectively. These three state variables resulted in a total of (52 x 5 x 10) 2600 possible states that any cow might fall into.

The first dynamic programming model, which will be called the dynamic1 model, computes the annualised present value (APV), or annuity, for each state in the present stage. This assumes that each cow in the present stage is retained, and that optimal replacement has occurred in the future stages. This model calculates the expected net present value of the cash flows for each state up to a fixed planning horizon rather than calculating the expected net present value of the cash flows for each cows remaining economic lifetime, as was undertaken with the annuity model. The expected net present value of the cash flow during the remainder of the planning horizon, given the initial state of the cow at the beginning of stage i, is :

$$\phi_i(j,k,l) = \max [\, keep(j,k,l), replace(j,k,l)] \quad for\ i \neq 1, \text{ and} \qquad ...5.4$$

$$\phi_i(j,k,l) = \max [\, keep(j,k,l)] \quad for\ i=1.$$

$$keep(j,k,l)=\beta\{[\, NR_{jkl}+[1-\pi_{ir}]\sum_{J=1}^{52}\sum_{l=1}^{5}\sum_{k=1}^{10} T_{jkl}\phi_{i-1}(j,l,k)+ \pi_{ir}[\, SV_k + NR_i^{Hfr} - CHfr\,]]\,[1-\pi_d]+\pi_d[\, NR_i^{Hfr} - CHfr\,]\}, \qquad ...5.5$$

$$replace(j,k,l)= \beta\{[\,[\, SV_k - CHfr + NR_i^{Hfr}]]\,[1-\pi_d]+\pi_d[\, NR_{i-1}^{Hfr} - CHfr\,]\}, \qquad ...5.6$$

$$NR_{jkl} = MFR_{jkl} - ID_{kl} + CV_l + A_k - IC_k, \qquad ...5.7$$

where: MFR_{jkl} is the milkfat production value for a cow of age k in the jth milkfat production state and the lth calving date state ,

CV_l is calf value for a cow in the lth calving date state,

A_k is the appreciation for a cow of age k,

IC_k is the interest on capital value for a cow of age k,

CHfr is the cost of a replacement heifer,

$NR_i{}^{Hfr}$ is the net present value for a replacement heifer in stage i,

ID_{kl} is the cost of inducing a cow in calving date state k in season l,

T_{ijk} is the transition probability matrix,

π_{ir} is the probability of involuntary replacement,

π_d is the probability of involuntary death, and

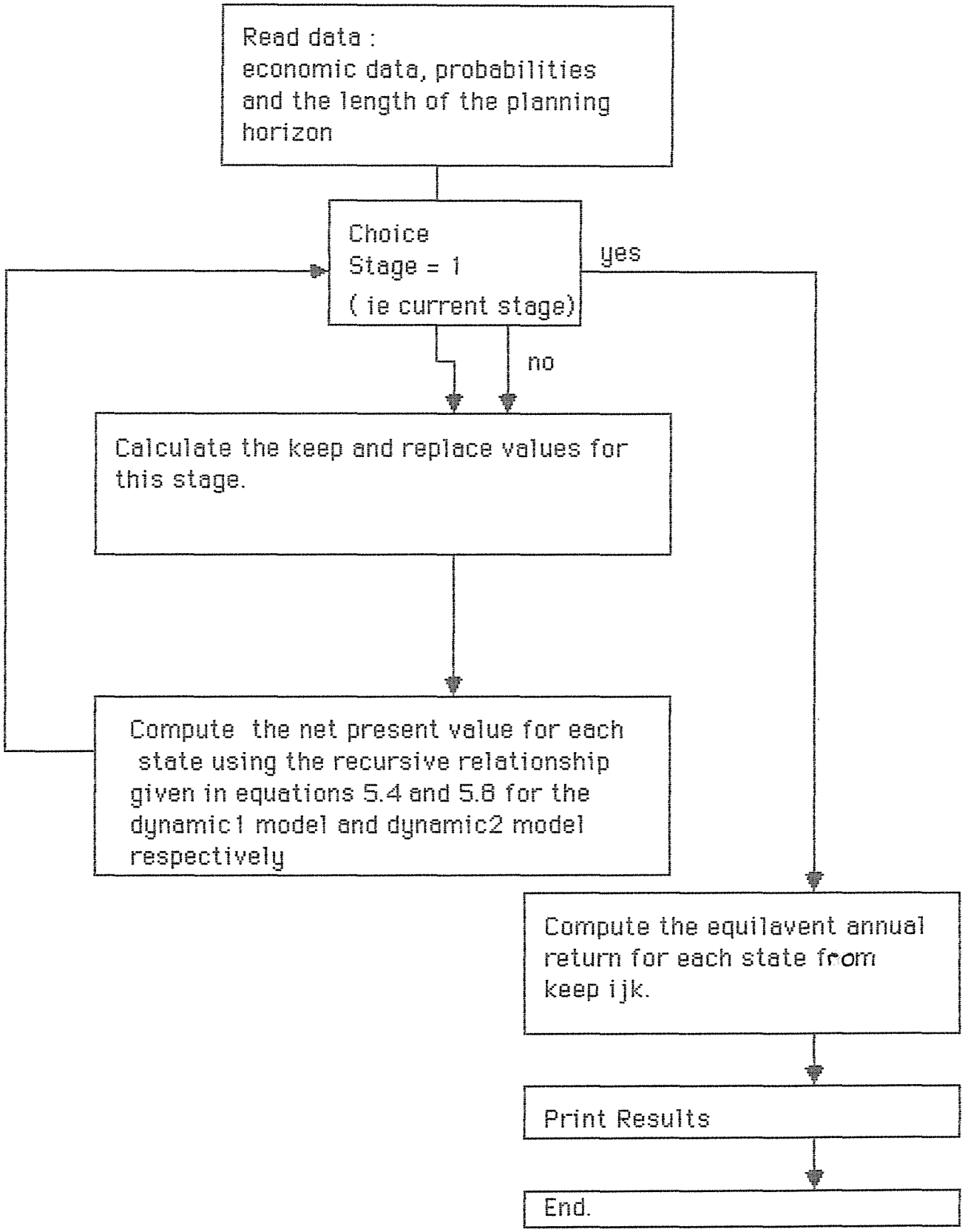
β is the discount rate.

The APV for a cow in state j,k,l in the present stage is $\phi_1(j,k,l)$ multiplied by the amortisation factor.

The second dynamic programming model , the dynamic2 model, assumes a cow is kept until she completes her tenth lactation and is then replaced by a 2 year old heifer with an average milkfat yield adjusted for the rate of genetic improvement. Thus, the model computes the net present value of future income streams for that cow and her replacement. The difference between this model and the dynamic1 model is that replacement only occurs because of old age , death or failure. As for the dynamic1 model this model calculates the expected net present value of the cash flows for each state up to a fixed planning horizon, which differs from the annuity model which calculates the expected net present value of the cash flows for each cows remaining economic lifetime. For this model, the expected net present value of the cash flow during the remainder of the planning

Figure 5.2

Flow diagram for the dynamic models programs.



horizon given the initial state of the cow at the beginning of stage i is :

$$\Phi_i(j,k,1) = \max [\ keep(j,k,1)] \qquad ...5.8$$

The calculation of the APV is the same as for the dynamic1 model . In both cases, the APV is used to rank the cows in a herd. The cows with the lowest values are considered candidates for culling.

For both the dynamic models, the transition probabilities are computed from the probabilites of transition for each state variable. The probabilities for lactation number are given in table 5.1. Because of the definition of milkfat production , which has the same expectation in all future lactations for a given cow , the probability of transition from the present milkfat production state to another is 0. Conversely the probability of transition to the same state is 1.0

Table 5.1

Transition probabilities for lactation number

Lactation number in present stage	Lactation number in next stage									
	1	2	3	4	5	6	7	8	9	10
1		1								
2			1							
3				1						
4					1					
5						1				
6							1			
7								1		
8									1	
9										1
10	1									

The probabilities for calving date states have been defined for the corresponding calving date categories. These probabilities are presented in table 4.6. Combining all the probabilities the following relationship exists:

$Pr[(j,k,l) \text{ to } (j',k',l')] = Pr(l,l')$ where $j'=j+1$ and $k'= k$

$Pr[(j,k,l) \text{ to } (j',k',l')] = 0.0$ where $j' \neq j+1$ and $k' \neq k$

where : $Pr[(j,k,l) \text{ to } (j',k',l')]$ is the probability of transition from state j,k,l to j',k',l', and

$Pr(l,l')$ is the probability of transition from calving date state l to l'.

Hence the structure of the transition matrix is in essence the same as in table 5.1 where the 1's are replaced by indentical submatrices which correspond to table 4.6 . Thus, defining the state variable for milk production in such a way that the expectation is the same in future lactations, drastically reduces the amount of computer memory required to store the transition matrix. In this model, it is only necessary to store the 5x5 submatrix which corresponds to the calving date probabilities. This reduction in computing requirements is a desirable property of this model. Two computer programs in have been written to run the dynamic models. Figure 5.2 presents a schematic diagram which illustrates the flow of data through the programs.

5.3 Parameters required for the 3 models.

The costs, prices and production characteristics used in the above models are given in table 5.2.The prices are related to the average situation during the 1985 season. (See over for table 5.2.)

Table 5.2

Costs, prices and production characteristics used in the present study.

Milkfat in wholemilk for manufacture (c/kg) [1]	364.2
Bobby calfs ($/head) [1]	25.3
Manufacturing beef (c/kg) [1]	144.3
Vet inducing costs ($) [1]	5.00
Rate of genetic improvement(%)	0.66
Replacement heifer price ($) [1]	550
Replacement heifer production (kg) [2]	200
Interest rate (%)	5.8

1 Source Anon(1986)

2 Mature Equivalent

Chapter 6

Results

Figures 6.1, 6.2 and 6.3 illustrate the results from the annuity , dynamic1 and dynamic2 models for three age groups respectively. These figures illustrate the change in APV for cows in the five calving date categories as the milkfat production increases from 100 to 300 kilograms per lactation. Also shown is the impact of the calving date categories on the APV within an age and milkfat production level. Three lactations are illustrated for each model to indicate possible trends with increasing age.

There are three main interactions to discuss:

(i) production level within calving date category and age,

(ii) age within calving date category and production level, and

(iii) calving date category within production level and age.

(i) For both the annuity model and dynamic2 model, the increase in APV within a given calving date category and age group, is directly proportional to the increase in milkfat production. This is because the probabilities of involuntary removal, death, and movement to other calving date categories are constant, irrespective of milkfat production. For the dynamic1 model however, the increase in the APV within age and calving date category is curvilinear with respect to milkfat production (see figure 6.2). Except for the tenth lactation where the increase in APV is directly proportional to the increase in milkfat production and the 5th calving date catergory where there is no increase in APV with increasing milkfat production. To illustrate the curvilinear nature of the change in APV, consider the 2nd

lactation and the 1st calving date state where increasing the milkfat production from 100 kg to 150 kg increases the APV by $20 ($530 to $550) whereas increasing the milkfat from 250 kg to 300 kg increases the APV by $120 ($600 to $820). The difference in the size of the APV increase is due to the increasing probability that a cow will not be replaced by heifer (as the milkfat production increases replacement is no longer optimal). The curvilinear effect decreases as age increases since the time difference between optimal and planned replacement decreases. There is no difference between the time of planned and optimal replacement for cows in the tenth lactation since all cows of this age will be replaced in the next lactation. Therefore, there is a constant change in the APV with increasing milkfat production, for this lactation.

(ii) Increasing the age within a given calving date category and production level decreases the APV for the annuity model. This effect may be explained by the increase in the probabilities of death and involuntary removal associated with increasing age and the interaction between these probabilities and discounting. Cows in older age groups encounter the higher probabilities of death and involuntary removal earlier in the future. Thus, the total effect of these probabilities is greater for older cows since it is discounted less.

Increasing the age in both dynamic models reduces the extremes of the APV. For both dynamic models, as age increases, the APV diminishes in high milkfat production states and increases for cows in low milkfat production states. These effects may be explained by first, the increase in the probability of death and involuntary removal with age and secondly, that planned replacement due to old age occurs at an earlier date. The decrease in the APV for high producing older cows is related to the reduction in the number of years their superiority in milkfat production is expressed before planned replacement. For example, a high producing cow

in her second lactation will express her superiority in milkfat production for the next nine lactations, whereas, a high producing cow in her tenth lactation will express this superiority for one lactation. The size of this effect is dependent on the discount rate. As the discount rate increases, later lactations become less important and the effect will be smaller.

A similar trend occurs for cows of low production within a given calving date category in the dynamic2 model. An increase in the APV occurs as the age increases and planned replacement is nearer. This is explained by the decrease in the number of years the low production is expressed. The size of the effect is also dependent on the discount rate.

For cows with low production within a calving date category in the dynamic1 model there is a gradual increase in the APV as age increases. This is due to the higher probabilities of death and involuntary removal associated with increasing age. Planned replacement does not occur for cows with low production in this category since it is optimal to replace these cows in the next lactation and hence, the low production is expressed for one lactation regardless of age groups.

(iii) A decrease in the APV with movement from calving date categories one to four within an age and milkfat production group is due to the reduction in lactation length and probability of calf survival. The decrease in APV with movement from these calving date categories to category five is due to no milkfat and calf production for the economic lifetimes of the cows in this category (annuity and dynamic2 models only). The size of the decrease between the fifth calving date category and the other categories is markedly reduced as age increases for the dynamic2 model. This is because as age increases planned replacement occurs earlier. In the dynamic1 model the reduction in APV between the fifth calving date category and the other categories is much less than for the other models since it is optimal to replace cows in this category in

the next lactation.

The three models were applied to a herd of 168 Jersey dairy cows. The BLUP model predictions for the future milkfat production for each cow are given in appendix 3. These estimates ranged from 136.2 to 337.9 kilograms of milkfat with the herd average of 214.5 kilograms of milkfat and stardard deviation of 29.50 kilograms of milkfat These cows were all candidates for the herd in the 1985-1986 season. 82% of the cows were expected to calve in the time interval defined by the first calving date category and all cows were expected to calve within the first 15 weeks. On average, most herds in New Zealand would expect 70 to 85% of cows to calve in the first 6 weeks and 95 to 100% of the cows to calve in the first 15 weeks.It is assumed that the herd size remains at the 1984/85 level of 147 cows , hence 21 cows are to be culled. Table 6.1 gives the total APV and average APV per cow for the culled and unculled herd. The values for each cow for each model are given in Appendix 3. Not surprisingly culling on APV increased the per cow and herd average APV. The results from the annuity model exhibited the greatest range in APV from $445 to $1071 with a standard deviation of $77.60.This is because the extremes in milkfat production were not offset by replacement due to old age in future lactations.The dynamic1 model produced results with the least range, with values from $548 to $879 and had a standard deviation of $46.10 dollars. This can be explained by the the reduction in the extremes in milkfat production through replacement due to old age of high producing cows with heifers and optimal replacement of cows with low net present values. The dynamic2 model had a range between the other two models ($485 to $874) with a standard deviation of $48.00. Replacement due to old age in future lactations reduced the extremes in milkfat production but not by the same extent as the dynamic1 model for the low extremes. The data on the 168 cows will also serve as an illustration for

the sensitivity analysis presented in the next chapter.

Table 6.1

The average per cow APV for the unculled and culled herd.

	Unculled cows (n=168)	culled cows (n=147)
Annuity.Model	700.11	717.46
Dynamic1 Model.	661.70	671.26
Dynamic2 Model.	656.40	666.78

Figure 6.1
An illustration of the results for the annuity model from 2nd, 6th and 10th lactation.

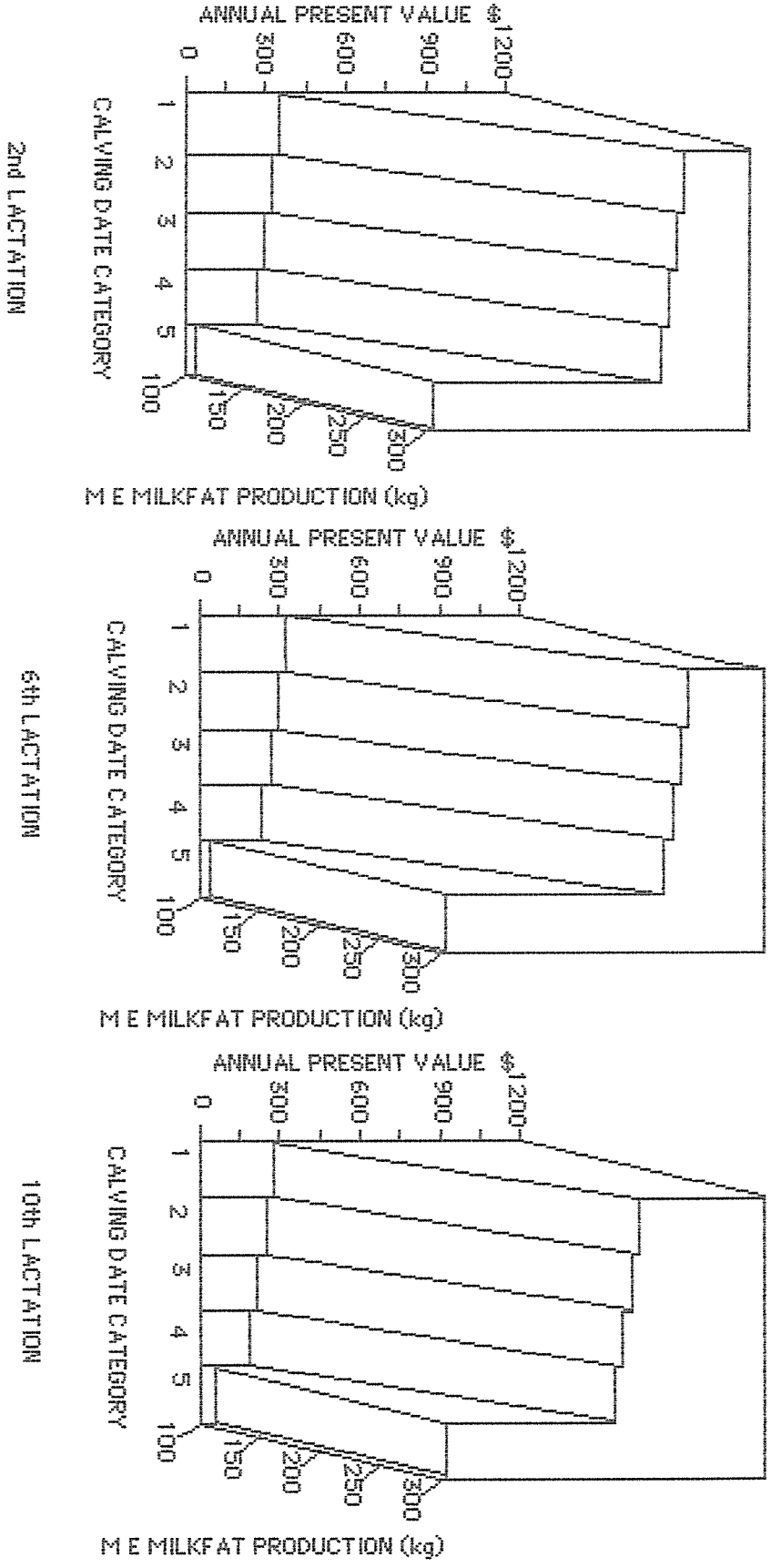


Figure 6.2

An illstration of the results for the dynamic1 model from 2nd, 6th and 10th lactations.

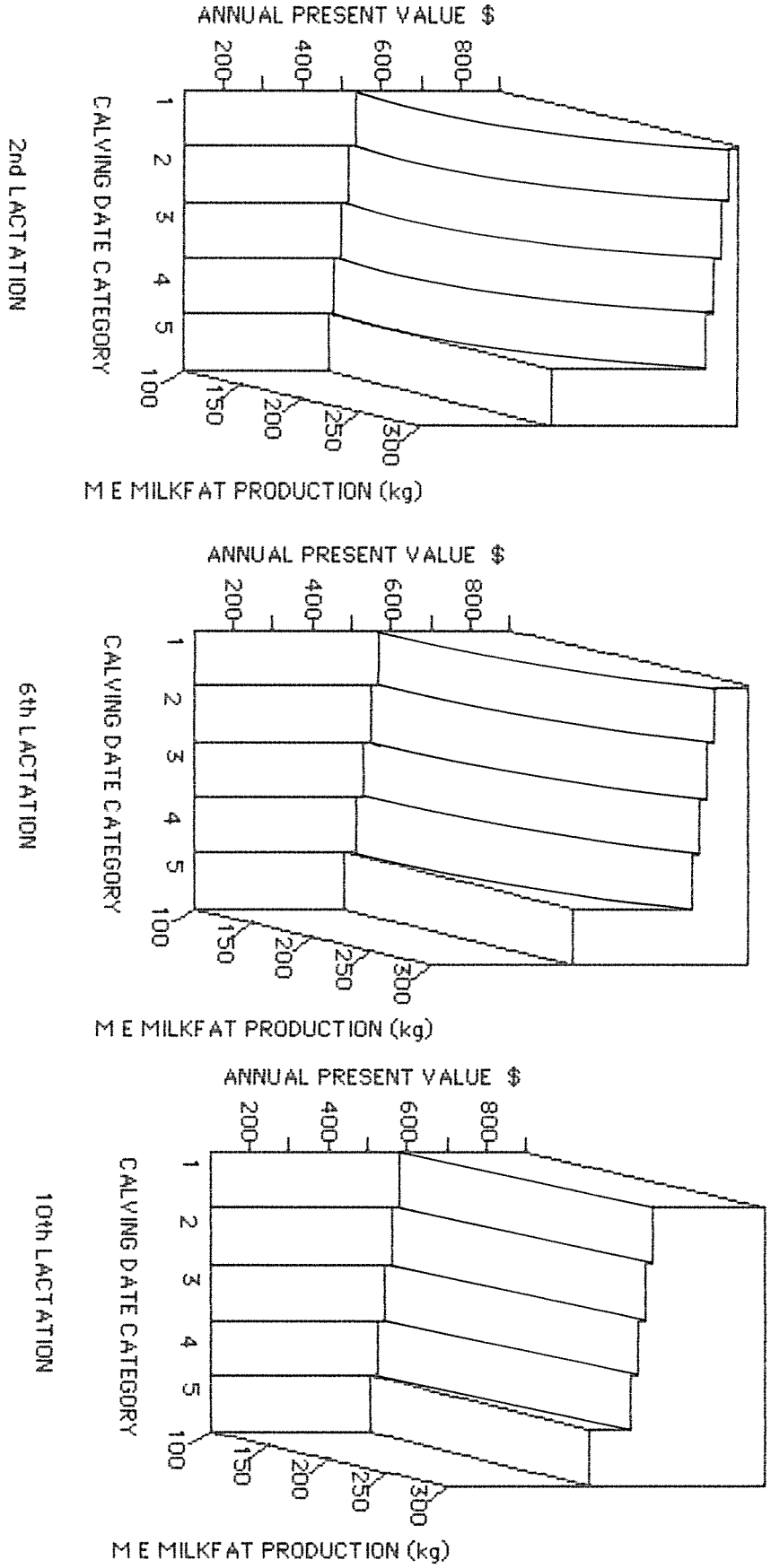
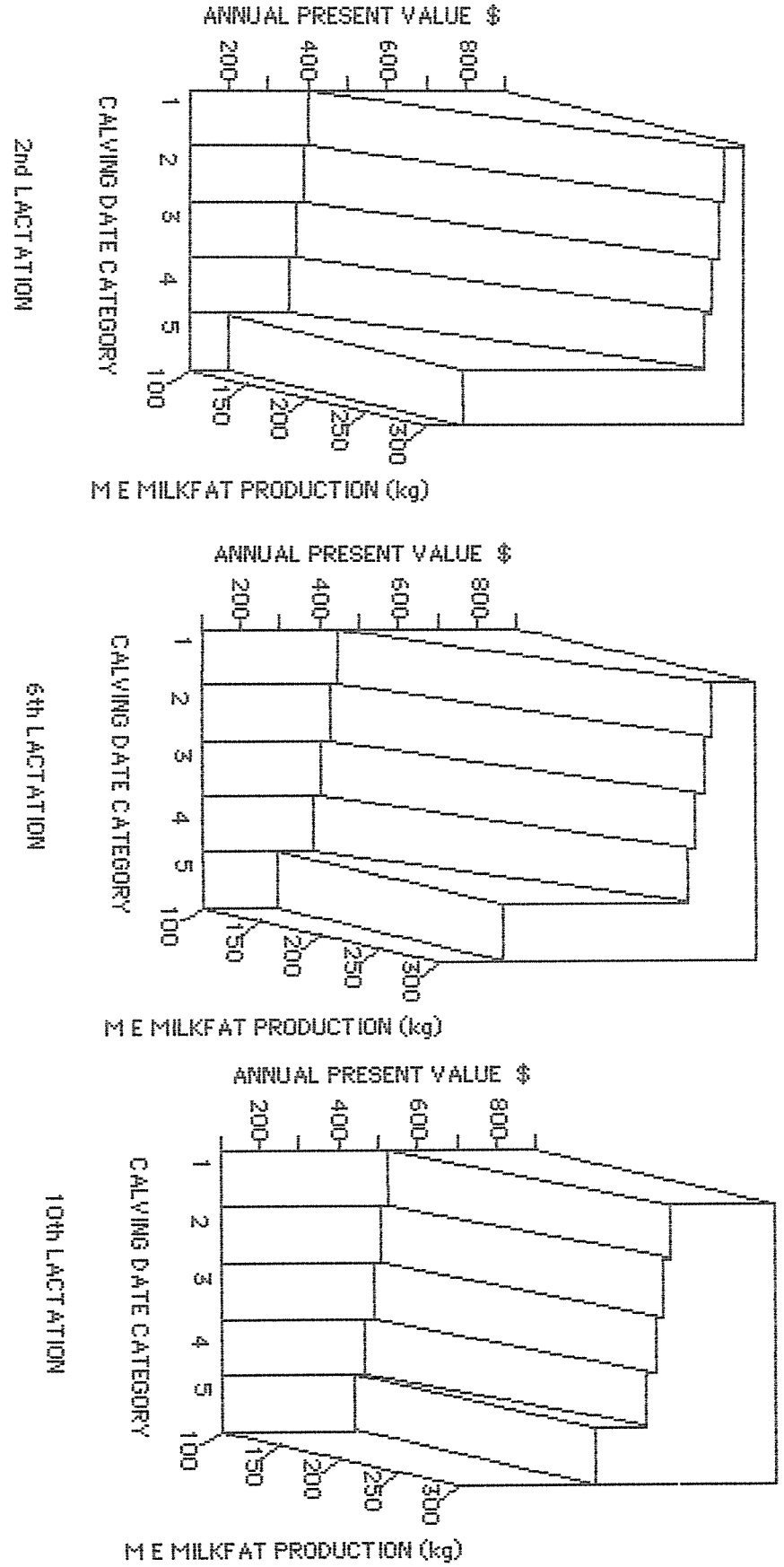


Figure 6.3

An illustration of the results for the dynamic2 model from 2nd, 6th and 10th lactation.



Chapter 7.

Sensitivity Analysis.

The farmer expects a higher profit when cows are replaced (Kuipers 1982). Hence, decisions to replace cows are expected to be based mainly on economic rather than biological considerations (such as genotype). Thus, as well as construction of methods to rank cows on future profitability, it is also important to examine the effect of changing the parameters (cost and prices) that the farmers base their decisions on. The results in chapter 6 were produced using only one set of parameters. A criticism of these results is that the choice of parameters may have influenced the cow rankings.In this chapter, the objective is to determine to what extent the results of the three models depend on the parameters used.

In the following sections the effect of changes to individual parameters and a scenario of high and low beef and milkfat prices are discussed. The Jersey herd described in the previous chapter is used as an example.

7.1 Changes in the price per kilogram milkfat.

The price per kilogram milkfat was varied from $2.10 to $5.10 in $0.50 steps. The effects on average per cow APV are given in table 7.1 and illustrated for the prices $2.60 and $ 4.60 in figure 7.1. Since milkfat price is an important component of dairy cow profitability, the substantial effect on the magnitude of average per cow APV shown in table 7.1 is not surprising. However, the changes in milkfat price did not alter the rankings of the cows. Within each model, a rank correlation (Kendall et al 1969) was calculated between the individual cows APV for the following combinations of prices : $2.10 and $3.10, $2.10 and $ 5.10 and $3.10 and $5.10. All correlations had a value of 1.00. The changes only increased the

variance between the rankings, which can be observed from the increase in the gradients of the graphs in figure 7.1 for the higher milkfat price compared with the lower milkfat price.

Table 7.1

The effect of changes in the price per kilogram milkfat on average per cow APV.

Price per kilogram milkfat	Average per cow APV $/cow		
	Annuity	Dynamic1	Dynamic2
2.10	385.5	384.5	381.7
2.60	487.5	474.4	470.7
3.10	589.5	564.1	559.7
3.60	691.5	654.1	648.8
4.10	793.6	743.9	737.8
4.60	895.6	833.8	826.8
5.10	997.6	923.7	915.9

7.2 Changes in the price per kilogram of manufacturing beef.

The price per kilogram of manufacturing beef (MB) was varied from $0.50 to $2.50 in $0.50 steps. The effects of this variation on average per cow APV are given in table 7.2.

Varing the value of MB had a substanially greater effect on the average per cow average APV for the dynamic models compared with the annuity model. This is due to the important role salvage value plays in offsetting the cost of replacement in the dynamic models.

The annuity model shows a smaller response to the variation in the price of MB because it does not consider planned replacement due to old age. An increase in the price of MB causes a marginal increase in the cows APV .This effect increased with age. The trend with age can be attributed to older cows having higher probabilities of failure in the near future . Therefore the degree to which the salvage value reduces the cost of replacement is not discounted to the same extent as would be the case for

younger cows .

The effect of increases in the price of MB in of the dynamic2 model is to cause an increase in the APV. This effect increases with age. The increase in the effect with age is due to the reduction in the time of planned replacement; hence the effect of the salvage value in offsetting the cost of replacement occurs sooner and is discounted less.

Table 7.2

The effects of changes in the price per kilogram of manufactering beef on the per cow APV.

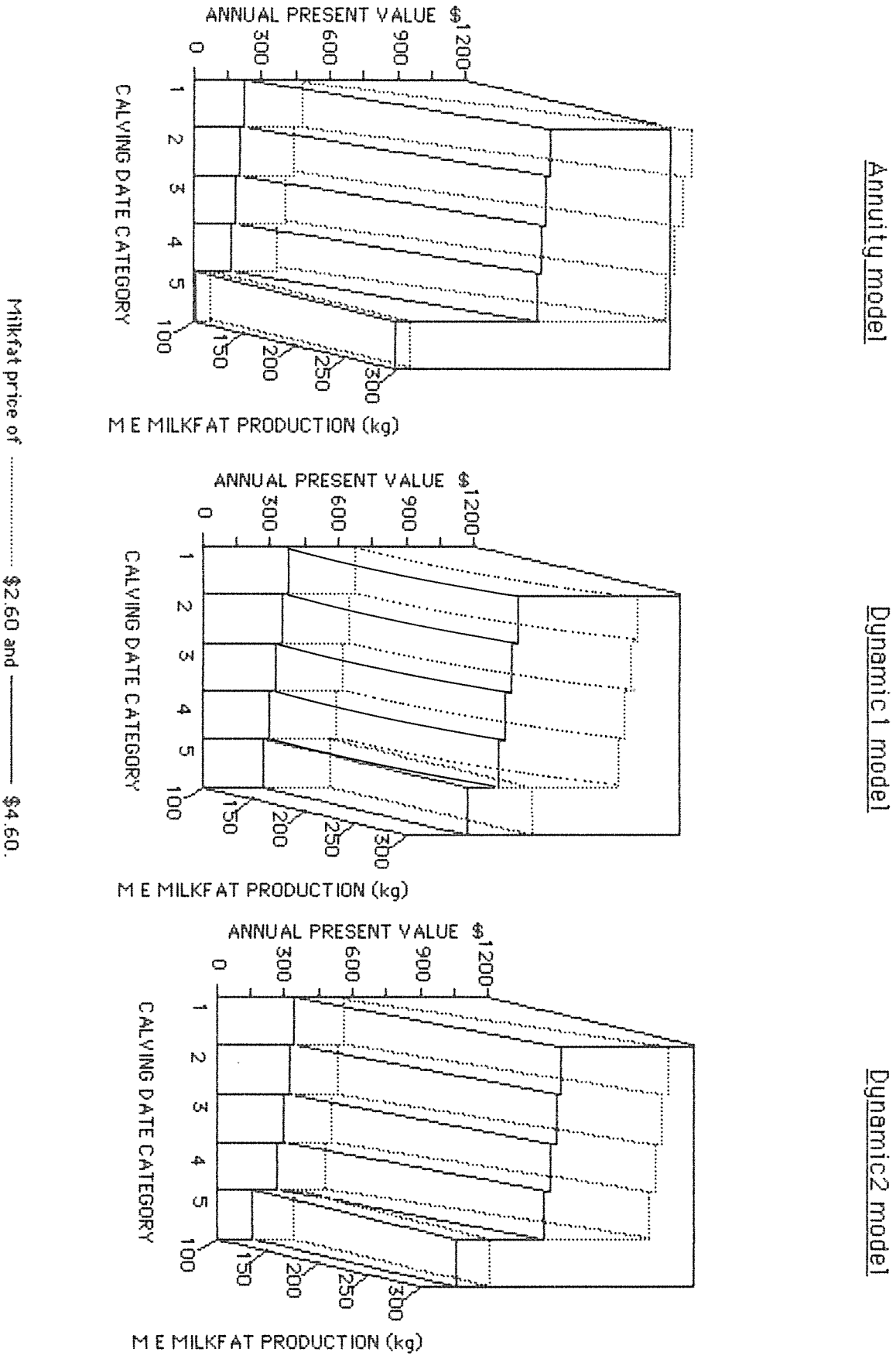
The price per kilogram MB ($)	Average per cow APV $/cow		
	Annuity	Dynamic1	Dynamic2
0.50	690.7	618.9	614.6
1.00	695.7	641.4	636.7
1.50	700.7	664.1	658.8
2.00	705.7	687.3	680.8
2.50	710.6	711.4	702.9

The same trends occur in cow APV for the dynamic1 model as occurred for the dynamic2 model. A second trend produced by the dynamic1 model is associated with optimal replacement. Since replacement may occur in the subsequent season, an increase in the price of MB will increase the cow's (for whom the model considers optimal to replace) APV by the greatest amount compared to cows the model does not consider optimal to replace. The gain in the APV for cows in the above category increases as cow age increases but only up to the fourth lactation. This is due to the interaction of weight with age (see section 4.11) .Increasing the price of MB also increases the rate of optimal replacement in subsequent stages since the cost of replacement becomes cheaper.

For a $2.00 change in MB price ($0.50 to $2.50) the rank correlation

Figure 7.1

An illustration of the effect of changing the milkfat price from $2.60 to $4.60/kg on the annualised present value for cows in their 6th lactation.



between individual cow APV's for each price within each model was calculated. The correlation between the cow rankings for the annuity model was 1.00. Whereas, the correlations between the cow rankings for the dynamic models were both 0.98 for the MB prices of $0.50 and $2.50. This indicates that changes in the of price of MB did alter some of the cow rankings, although the magnitude of alteration appears to be small.

Changes in the price of MB have a great effect on the cost of replacement. The effect is greatest when replacement occurs in the near future, compared with replacement in the distant future, due to the interaction with discounting.

7.3 Changes in heifer price.

The heifer price was varied from $450 to $750 in steps of $100. The resultant changes in average per cow APV are given in table 7.3. An increase in the heifer price decreased the average per cow APV. This is because the price of the heifer replacement is an important component in the cost of replacement. The decrease in the APV was small for the annuity model, although this decrease in APV increased with age. This effect can be attributed to the increasing probabilites of death and involuntary removal associated with increasing age. For the annuity model, changes in heifer price did not alter the rankings of individual cows . For a change in heifer price from $450 to $750 the rank correlation between individual cow APV's for each price was 1.00.

The decrease in APV resulting from increases in the heifer price also increased with age for both the dynamic models. This increase is due to the reduction in time of planned replacement. Hence, older cows incur the cost of replacement at an earlier date and this cost is discounted less. A second trend exists in the dynamic1 model is again associated with optimal replacement. The cost of replacement is greatest for the cows the dynamic1 model optimally replaces in the next season; thus, change in the

heifer price has the most effect on these cows.

Table 7.3

The effect of changes in heifer replacement price on average per the cow APV

Heifer Price ($)	Average per cow APV $/cow		
	Annuity	Dynamic1	Dynamic2
450	705.7	674.1	668.1
550	700.7	661.5	656.3
650	695.7	649.2	644.4
750	690.7	637.0	632.6

The rank correlation between individual cow APV's for the heifer prices of $450 and $750 were 0.98 and 0.99 for the dynamic1 and dynamic2 models, respectively. As for changes for the price of MB , changes in the heifer replacement also causes some changes to the optimal cow rankings.

7.4 Changes in interest rate.

The interest rate was increased from 0.0% to 15% in 5% steps. The effects of these changes on the total and average APV are a given in table 7.4 and illustrated in figure 7.2.

Changing the interest rate had little effect on the cow APV's produced by the annuity model. Increases in the interest rate resulted in the higher probability of death and involuntary replacement associated with old age becoming less important for younger cows because the effects are reduced by the greater discounting. The increases in interest rate caused a uniform drop in the total and average per cow APV. The rank correlation between individual cow APV's for the change in interest rates of 0.01% and 0.15% were 1.00 for the annuity model.

Increases in interest rate in the dynamic models increased the

extremes in the APV's . In both models, the change at the extremes became less with age, except for cows which were optimally replaced in the subsequent season by the dynamic1 model. The reduction in the effect with increased age is due to planned replacement occuring earlier in the future. For cows with low milkfat production, the increase in interest rate reduced the effects of the returns generated by planned replacement through the increased effect of the discount rate. The same effect increases the APV for high producing cows. The effect of the higher discount rate is least on cows which are optimally replaced in the subsequent season since the returns generated from planned replacement are proportionately greater and reduced less by the higher discount rate. The rank correlation between individual cow APV's for the interest rates of 0.01% and 15% were 0.97 and 0.98 for the dynamic1 and dynamic2 models, respectively.

Table 7.4

The effect of changes in interest rate on the average per cow APV .

interest rate (%)	Average per cow APV $/cow		
	Annuity	Dynamic1	Dynamic2
0.001	710.9	651.9	646.2
0.05	701.9	660.2	654.9
0.10	690.5	666.8	661.8
0.15	679.0	669.5	664.9

Rank correlation for interest rates of 5% and 10% , and 10% and 15%, were 0.99 for both dynamic models and both sets of interest rates. Thus, changes in the rate of interest do alter the optimal cow rankings. The degree of the change in the optimal rankings is dependent on the size of the difference between the two interest rates used.

7.5 Changes in the rate of genetic improvement.

The rate of genetic improvement was varied from 0.0% to 1.5% in 0.5% steps. The changes to average per cow APV are given in table 7.5.

Increasing the rate of genetic improvement increased the APV's produced by the annuity model . The magnitude of the increase was greater for younger cows since the number of opportunities for replacement by a genetically improved heifer is greater because the younger cows are evaluated over a greater number of seasons.

Increasing the rate of genetic improvement also increased the APV's produced by the dynamic models. The magnitude of increase was almost constant across all age and milkfat production groups.This is because the differences between the genetic values of replacement heifers entering the herd in the near future and distant future are offset by interaction with the level discounting chosen for this study. A different discount rate would have resulted in different levels of change in the APV across age and milkfat production groups. The levels of change would also be dependent on the probabilities of death and involuntary replacement and the time of both optimal(in the dynamic1 model) and planned replacement.

The absolute magnitude of APV change would be small and unlikely to cause change in the optimal cow rankings with the current levels of genetic gain attainable in the dairy industry.

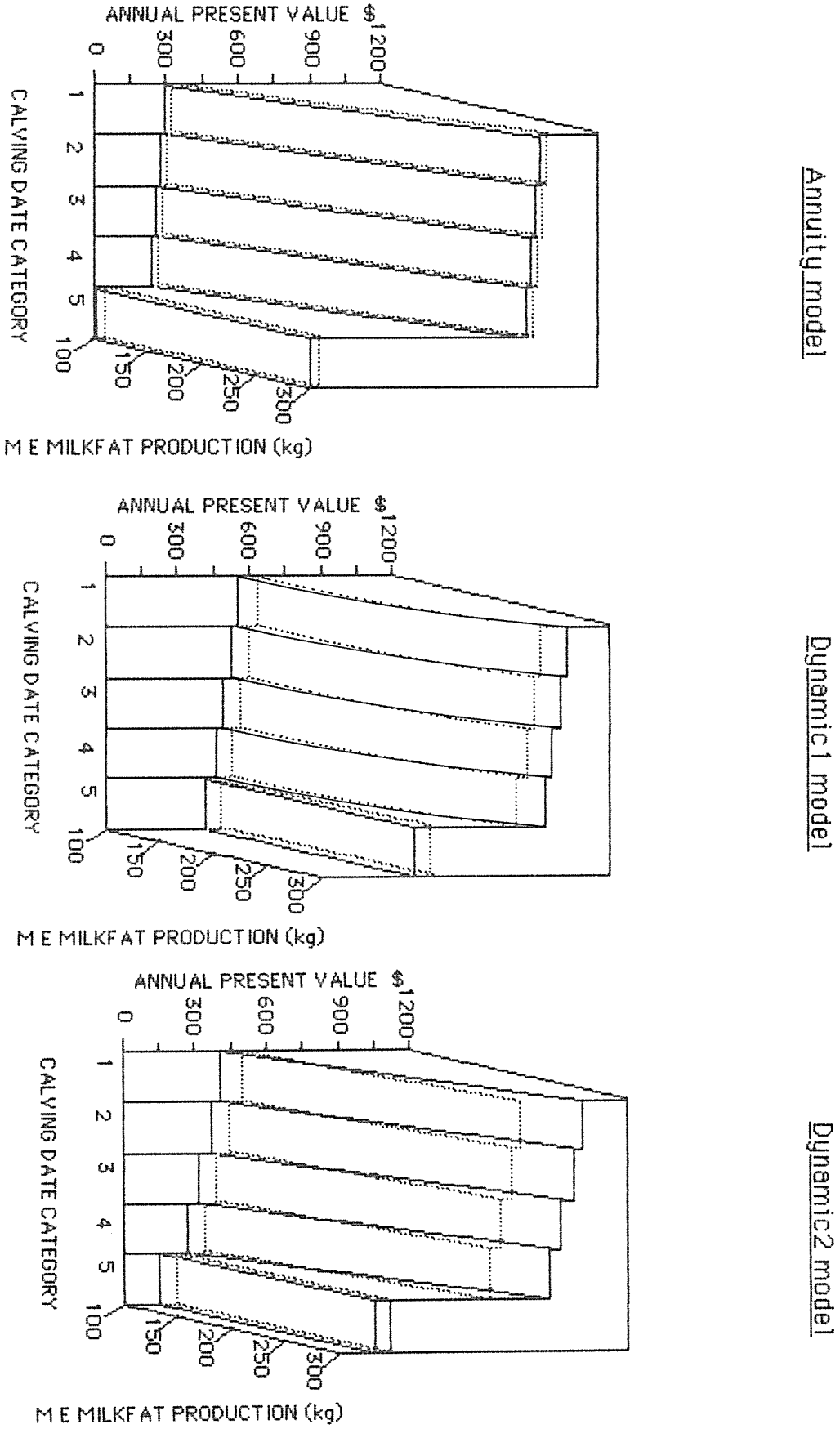
Table 7.5

The effect of changes in the rate of genetic improvement on the average per cow APV .

Rate of genetic improvement	Average per cow APV $/cow		
	Annuity	Dynamic1	Dynamic2
0.0	698.6	659.8	654.5
0.5	699.7	661.1	655.8
1.0	700.9	662.4	657.1
1.5	702.0	663.7	658.4

For all 3 models changes in the rate of genetic improvement did not alter

Figure 7.2
An illustration of the effect of changing the interest rate from 0.0001% to 15% on the annualised present value for cows in their 6th lactation.



the optimal cow rankings. The rank correlations between individual cow APV's for the rates of genetic improvement of 0.0% and 1.5% were 1.00 for all models.

7.6 High and low milkfat and manufacturing beef prices.

A scenario of high and low milkfat and manufacturing beef prices was used for the three models. The different prices used for the four runs are given in table 7.6. The results for each situation and model are given in table 7.7 and illustrated for the dynamic1 model in figure 7.3.

Table 7.6

The parameters used in the testing of the response of the models to changes in milkfat and manufacturing beef prices .

	H^d/H^b	H^d/L^b	L^d/H^b	L^d/L^b
Milkfat price per kg. ($/kg)	4.60	4.60	2.60	2.60
Manufacturing beef ($/kg) price per kg.	2.00	1.00	2.00	1.00
Heifer price. ($)	650	550	600	450

b =beef , d =dairy

H =high prices , L =low prices

From the results it can be observed that changes in the milkfat prices have the largest effect on the average per cow APV.

Analysing the APV's produced by the three models for the options H^d/H^b and H^d/L^b , and also the options L^d/H^b and L^d/L^b, it is possible to consider the effect of changes to both the price of manufacturing beef and the heifer replacement price in the same run. The change in the APVs have the same nature as those caused by changes in the individual parameters , but the magnitude of change was considerably less. Thus, changes in the price of manufacturing beef were offset by changes in the price of the

heifer replacement. Such a pattern is what one would expect in the market place, so although changes in each of these parameters individually has considerable bearing on the final APVs, when considered together their overall effect is much smaller.

Examination of the APV's for H^d/H^b and L^d/H^b clearly shows the effect of changes in milkfat production. The magnitude of difference between the APV's of low and high producers is increased. For both the dynamic models this increase in difference decreases with age, which is caused by the interaction between heifer price and earlier planned replacement.

The rank correlations for the four price combinations were 1.00 for the annuity model and 0.99 for both dynamic models. This indicates the changes to the optimal rankings caused by changes in either the heifer price or the price of MB are lessened when the two parameters are considered together, due to their interaction.

Table 7.7

The effect of changes to manufacturing beef and milkfat prices on the average per cow APV.

	Average per cow APV $/cow		
	Annuity	Dynamic1	Dynamic2
h^d/h^b	889.0	846.8	839.6
h^d/l^b	891.2	813.5	807.3
l^d/h^b	487.0	493.5	489.4
l^d/l^b	495.3	466.5	463.0

7.7 Ignoring the probability of failure and death.

For each of the models, the probabilities of failure and death were ignored. This was undertaken to examine whether the inclusion of these parameters in the model was justified. The resulting changes in the average APV are given in table 7.8 and are illustrated in figure 7.4.

The effect of ignoring the probability of failure and death in the

annuity model was to lower the APV's for cows with low milkfat production and to increase the APV's for cows with higher milkfat production. This is because death and failure cause replacement by a heifer which may have a higher or lower producing ability than the cow which is replaced, depending on the cow which is replaced. The magnitude of the change increases with age. This is because the effect of the higher probabilities of, especially, death associated with increasing age are not discounted to the same effect for older cows compared with the younger cows.

The dynamic2 model reacts in a similar way as the annuity model except the magnitude of change decreased with age. This is because planned replacement occured sooner for older cows and thus, the effect of ignoring failure and death only occured over a small period of time.

Ignoring failure and death in the dynamic1 model, increases the APV's for all cows, the magnitude is greatest for high producing younger cows. The magnitude of the effect is least for cows optimally replaced in the subsequent season and cows who are replaced in the subsequent season because of old age.

Table 7.8

The effect of ingoring the probability of failure and death on the average per cow APV.

	Average per cow APV $/cow		
	Annuity	Dynamic1	Dynamic2
ingoring the probability of failure and death	755.3	716.0	708.1
accounting for the probability of failure and death	700.1	661.5	656.3

The rank correlation between individual cow APV's for including and ignoring failure and death were 0.98 for all models. Ignoring failure and

death caused small changes in the optimal cow and state rankings. Not including the probabilities of failure and death would underestimate lower producing cows APV and would overestimate higher producing cows APV, resulting in small changes to the optimal cow rankings.

7.8 Changes to the length of planning horizon.

For both the dynamic programming models the planning horizon was varied from 5 to 20 years. The annuity model has no fixed planning horizon, the average planning horizon for this model is dependent on the age structure of the herd, the longest possible planning horizon for an individual cow is 10 years. The changes to the average per cow APV are given in table 7.9 and illustrated in figure 7.5. The cow average APV increases with increasing planning horizon , although the magnitude of increase decreases as the planning horizon increases. As the planning horizon increases towards an unbound horizon the asymptotic nature of the APV would be displayed.

Table 7.9

The changes to the average per cow APV due changes in the planning horizon

Planning Horizon (yr)	Average per cow APV $/cow		
	Dynamic1	Dynamic2	
5	629.9	628.0	
10	661.5	656.3	
15	671.0	665.0	
20	678.8	672.2	

Increasing the planning horizon increases the APV's of lower producing

Figure 7.3

An illustration of the effects of changes in the beef and dairy prices on the annualised present value for cows in their 6th lactation and the dynamic1 model.

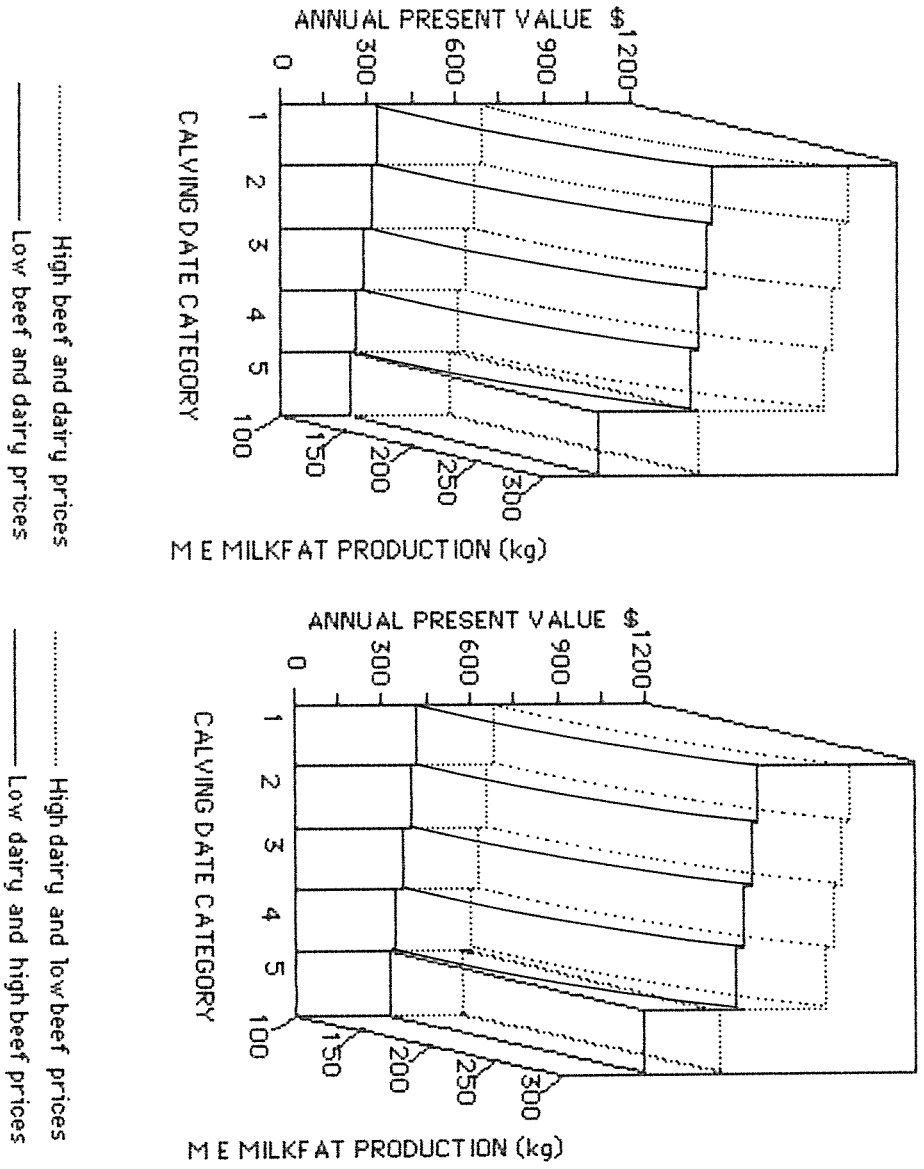
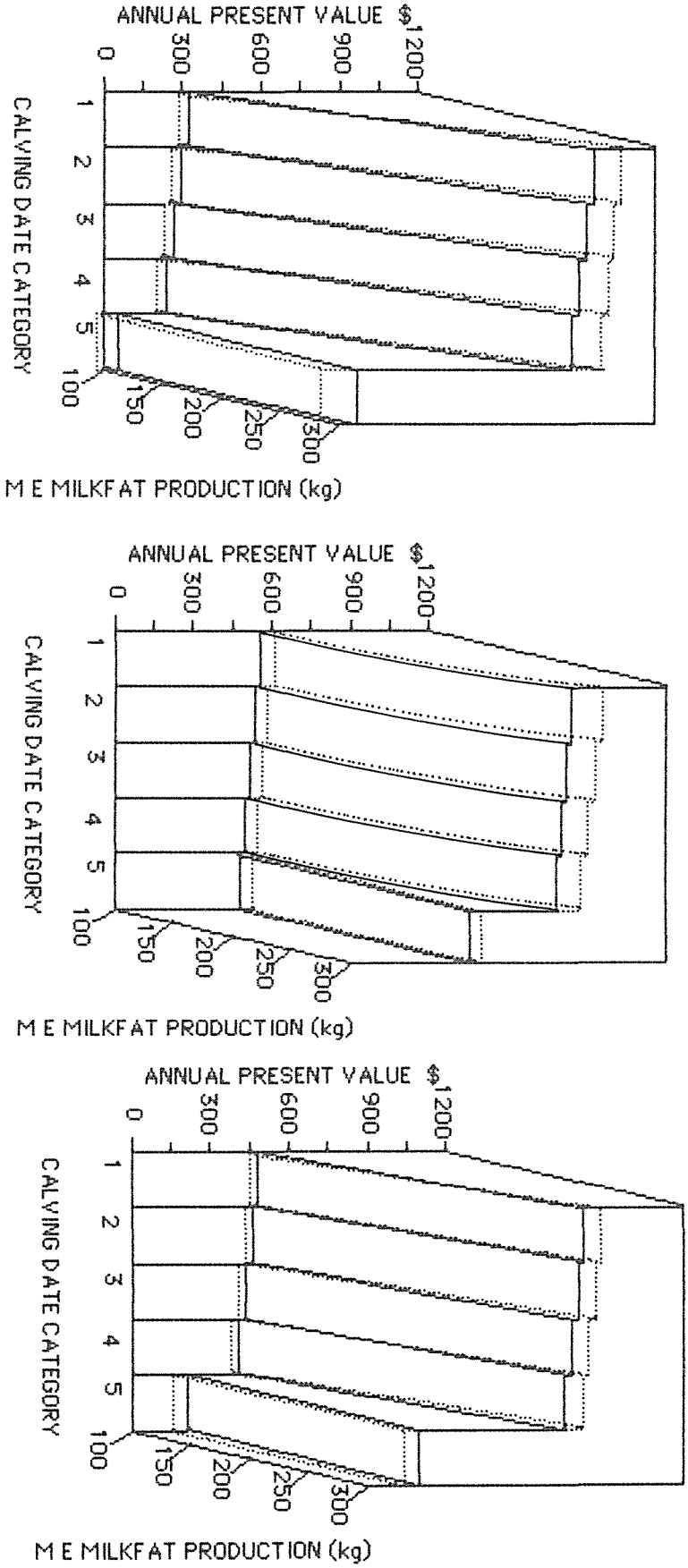


Figure 7.4
An illustration of the effect of ignoring the probability of death and involuntary replacement on the annualised present value for cows in their 6th lactation.



——— the 1984-85 results and the ..... results when the probabilities of death and involuntary replacement are set to zero

Figure 7.5

An illustration of the effect of changing the planning horizon from 5 to 15 years on the annualised present value for both dynamic models for cows in their 6th lactation.

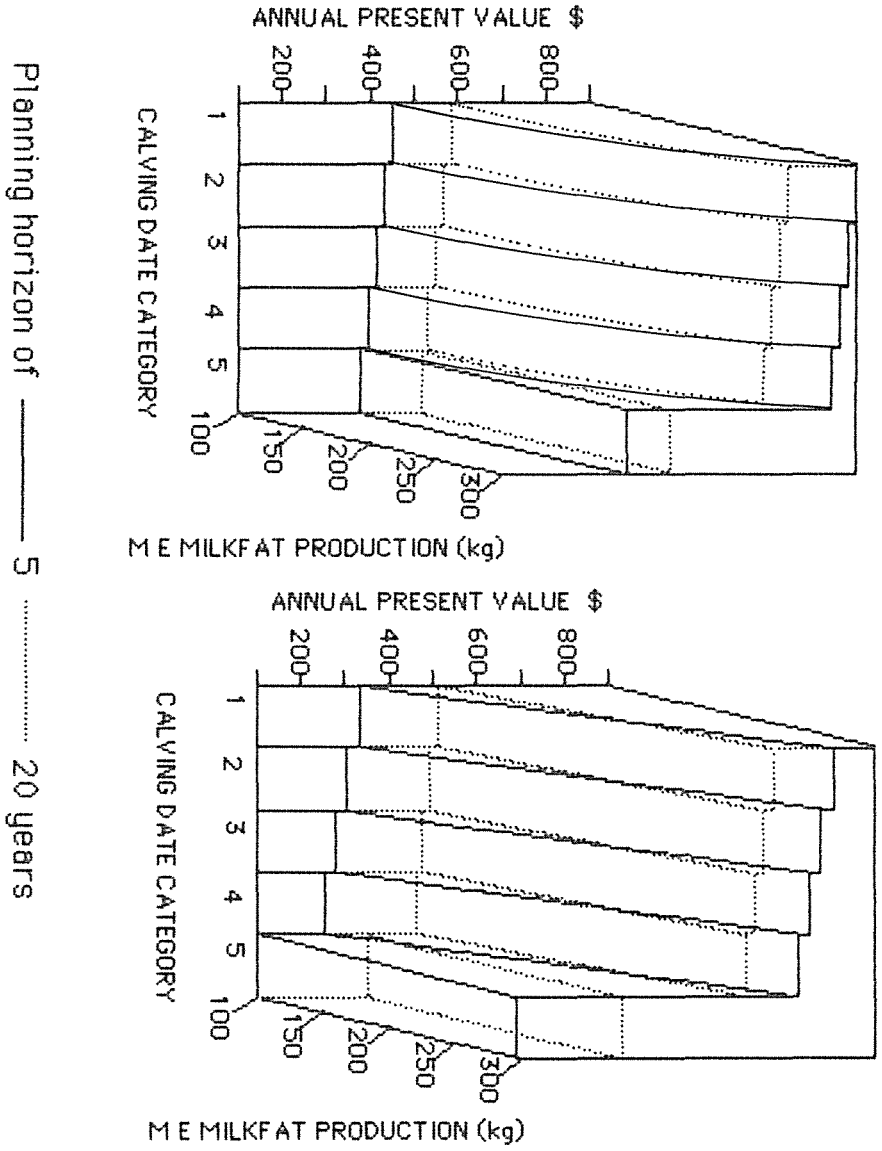
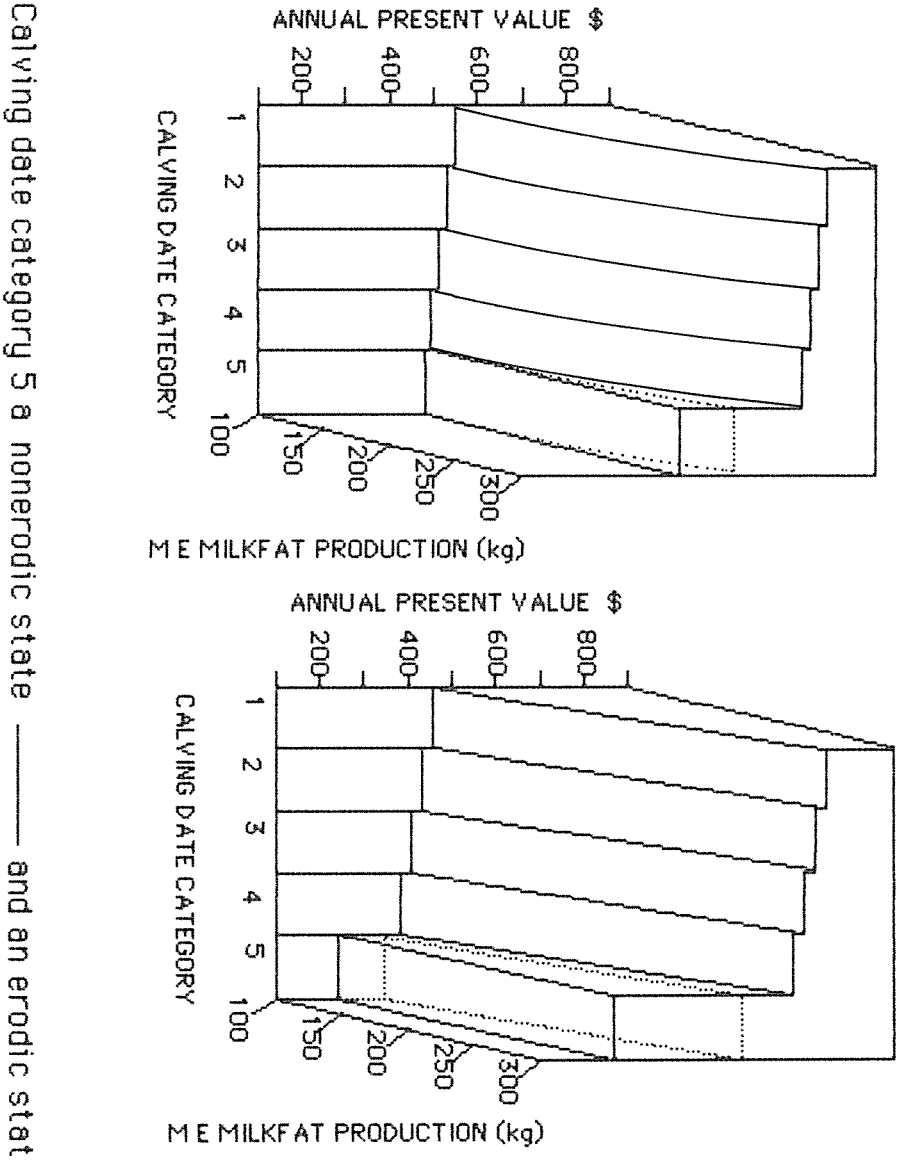


Figure 7.6
An illustration of the effect of changing the definition of the fifth calving date category form an erodic state to an nonerodic state on the annualised present value for cows in their 6th lactation.



is cows and decreases the APV's of higher producing cows. This is because the high or low incomes from cows are offset by the increasing production length from the average replacement cows. The magnitude of change is dependent on the interest rate. The greater the discount rate the smaller the effect of the planned replacements.

The rank correlations between individual cow APV's for the planning horizons of 5 and 20 years were 0.96 for both models. However, rank correlations between individual cow APV's for the length of planning horizons of 10 and 20 years were 0.99 for both models. These rank correlations in combination with the information in table 7.9, indicate that as the planning horizon increases, the APV's for individual cows stablise. As the planning horizon increases past 10 years all cow's will have been replaced by an average heifer which will cause a reduction in the variation in the APV's. Any change in the expected net revenues generated after 10 years will be due to the probabilities of involuntary removal, death and shifting between calving date categories and to the genetic level of the replacement heifer. However, the rate of reduction is dependant on the discount rate, the greater the discount rate the less the reduction in the variance of the APV's. With an interest rate of 5.8% the reduction in the variance only had a minor effect on the indivdual cow rankings as the planning horizon increased towards infinity.

7.9 Changing the definition of the fifth calving date state.

For both dynamic programming models, the assumption that the fifth calving date state is a non-erodic state was relaxed. Cows in the fifth calving date state were given equal chance to enter all the calving date states in subsequent seasons.

The results are illustated in figure 7.6 and given in table 7.10. Since

none of the cows in the herd examined were initially calving date state 5 the effect on the APV was minimal.The effect on the APV's would be only significant for cows in the fifth state.

Table 7.10

The effect of changing the definition of the calving date state 5 to an erodic state from an non-erodic state on the average per cow APV.

	Average per cow APV $/cow		
	Dynamic1	Dynamic2	
Calving date state 5 non-erodic	661.5	656.3	
Calving date state 5 erodic	661.5	659.4	

The effect of the change in definition is greatest on the APVs produced by the dynamic2 model, since in the dynamic2 model, state 5 cows remain until their tenth lactation whereas, in the dynamic1 model they are replaced in the subsequent season. The effect of relaxing the definition of the fifth calving date category will be dependant on the nature of the herd and the probabilities of reentry to the other calving date categories.

To better estimate the effect of changing the definition of the fifth calving date state more work is needed in defining the probabilities of movement from the different calving date states and also more work is needed with herds with cows in the fifth calving date state.

7.10 Discussion.

The parameters directly associated with the cost of replacement had the greatest effect on the APV's of individual cows .The parameters such as interest rate and planning horizon also affected the APV produced by the dynamic models, which was expected since both of these models explore the situation of future replacement.

The optimal rankings in both dynamic models were affected by the price of the heifer replacement and the price of manufacturing beef, whereas milkfat price played an insignificant role. These results are in agreement with Stewart et al (1978), Allaire and Cunningham (1980) and van Arendonk (1986a,b).

The planning horizon for an individual farm should be known. Thus, for any given analysis, there will be no choice involved in assigning the length of the planning horizon for the dynamic models. The interaction between the length of the planning horizon and the interest rate used will affect the amount of variation between the APV's. It is unlikely that the usefulness of the dynamic programming models would be lessened by the reduction in the variation when the planning horizon is greater than 10 years duration. Since the further expected net returns are generated in the future the less their effect is on the APV (through the actions of discounting).Thus, the total effect of the reduction in the variation of expected net returns is likely to be small. This reduction of variation is associated with the prediction of the unknown characteristics of future individuals and thus, would affect any model which has a planning horizon of greater than 10 years duration. This suggests there may be little benefit in predicting further than 10-15 years in the future.

The choice of interest rate is a sensitive area. Smaller changes (≤5%) in the interest rate resulted in a higher correlation between the optimal rankings compared with larger changes (>5%). If the inflation rate adjusted measurement (Smith 1978) is used, the variation between the choices is likely to be smaller than using market value interest rates, hence the affect on cow ranking will be kept to a minimum.

The magnitude of difference between the price of the heifer replacement and the price of manufacturing beef is the most important component of the cost of replacement. In reality, movement in the price of manufacturing beef will result in an opposite movement in the price of the

heifer replacement. As a consequence of the combined movement, varying the price of manufacturing beef was then found to have only a small effect on the ranking of the cows.

From table 6.8 it can be observed that assuming no involuntary removal or death increased the per cow APV by approximately $54, depending on the model used. From an economic point of view, this reinforces that management and breeding policies should be concerned with reducing the amount of involuntary removal or death .

When the 5th calving date state was changed to an erodic state, the recomputed rankings suggest that it may more profitable to keep high producing cows who were not in calf and bring them back into the herd in the subsequent season. Such a conclusion may be erroneous, depending on the reason for the cow entering into the 5th calving date state. From the management point of view, it would important to ascertain the cause of the reproductive problem. Where a permanent reproductive problem exists, obviously the cow should be replaced. If the cause is due to human error (eg, poor heat detection) the farmer may consider keeping the cow. In doing so, the farmer must take into account the added cost of managing an extra class of stock.

Chapter 8.

Conclusions.

Dynamic programming has been used to determine optimal replacement policies for dairy herds in grassland and non-grassland situations. Three dynamic programming models had been developed for the New Zealand dairy production situation by Mac Arthur (1973, 1975 and 1985). These models did not include variables for calving date, nor did they vary live weight according to age. Replacement models have been used in agriculture to determine the optimal time of replacement for various enterprises. One such model based on annualised present value concepts, has been developed to rank cows on future profitability in a non-grassland situation. None of the above approaches provided a viable system which could be applied to the dairy industry as a whole.

The objective of this study was to evaluate the use of replacement and dynamic programming models by developing a model which ranked cows on future profitability under New Zealand seasonal dairy farming conditions. To achieve this objective, a model was developed to estimate the performance and calculate the revenues and costs of dairy cows in New Zealand. The results of this model were then used in the development of two dynamic programming models and one replacement model to rank cows based on future profitability.

The implementation of the three models developed in this study was hindered by the lack of adequate information with respect to many of the economic and biological components. Some factors which warrant further investigation are listed below :

i) the relationships between age, milkfat production, breeding index, production index and weight are not well defined,

ii) a more detailed investigation of the market values would allow calculation of salvage values, appreciation and replacement costs to be

more objective,

iii) there is a need for an up-to-date quantification of wastage rates according to age. The last complete analysis for New Zealand conditions was undertaken in 1957 (Anon 1957). With the changes in animal health practices it may be that these rates bear little resemblance to the rates occurring in the industry at present, and

iv) quantification of the reproductive behaviour of individual cows across several seasons is needed so that the probability of movement to future calving date categories can be estimated more accurately.

The 3 models were tested over various cost and price situations. From these analyses it was possible to draw several conclusions.

The annuity model's behaviour was more stable in the sensitivity analysis than either of the dynamic models. This was caused by the limitations of the model rather than by a better ability to cope with the varying situations. Three major limitations of the annuity model are :

i) no definite planning horizon,
ii) the consequences of planned replacement are not considered, and
iii) there is no appropriate method for handling salvage value.

The planning horizon of the model is dependent on the age structure of the herd. At most, the model only predicts the future profitability up to the tenth year. The consequences of not considering planned replacement are that the APV.'s of cows with high milk fat production are overestimated and conversely the APV's of cows with low milk fat production are underestimated. To overcome all the limitations of the annuity model, future incomes for individual cows and their replacements could be predicted up to a fixed horizon. This would result in a more complicated form of the dynamic2 model with no addded advantages, thus such an extension to the annuity model is not warranted.

The behaviour of the two dynamic models was similar under most circumstances. The only difference in behaviour was associated with utilising optimal replacement in the future. In the dynamic2 model, no optimal replacement took place since the objective was to measure the future profitability of a cow up to the end of her tenth lactation, and then to measure the future profitability of her replacement. Using this criterion one limitation was apparent, cows in the fifth calving date state had a ranking which was dependent on the time of planned replacement. In the New Zealand dairy farming situation it would be expected that cows in the 5th calving date category would be replaced in the next season, rather than at the time of planned replacement due to old age. This was not a problem for the dynamic1 model. The dynamic1 model replaces cows with low APV's which includes cows in the fifth calving date state in future lactations. This would be the expected situation on dairy farms.

A potential criticism of the dynamic1 model is that the replacement rates could reach unreasonable levels. It is not possible to include constraints to ensure the replacement rate in the future remains at a realistic level, because the concept of states rather than individual cows is used. The rankings produced by the dynamic1 model (even with high replacement rates) are still valid since the states are ranked relative to each other under the conditions of the system. Hence, all the states within the system would be subjected to the high replacement rates.

In conclusion, the dynamic1 model best meets the objective of ranking cows on future profitability. Two major advantages of this method for providing management guides, are first, that the method provides rankings for all age classes including freshening heifers. Secondly, the rate of replacement can be determined by the farmer rather than being part of the models which determine the optimal policy. The dynamic1 model's advantages outweighs the increased complexity of the the model. The increased complexity of the first model compared with the second model

relates only to the inclusion of maximisation function at the end of each stage in the dynamic1 model formulation. To enhance the annuity model so that it would operate under a fixed planning horizon would result in this model becoming more complex than either of the dynamic models with no added advantage. Therefore, it appears that this type of replacement problem is best solved by the use of dynamic programming.

Finally it should be emphasised that the rankings must be considered as a management guide used in conjunction with other information the farmer has available about individual cows.

The New Zealand Dairy Board currently provides a production index for individual cows. This production index is used in conjunction with the the expected calving date in the coming season to indicate which cows should be considered as potential candidates for replacement (Anon1970). Cows with low production indices and/or who will be potential late calvers are identified as replacement candidates. Using this procedure the likely criteria for replacement is maximisation of milkfat production in the short term. The shortcomings of the above culling guide are that it does not look far enough into the future and it does not incorporate any economic variables.

The replacement guides presented in this study overcome the above limitations. The future net returns for individual cows are estimated, these returns are weighted by the probability of events occuring in the future (e.g. death or involuntary removal). The cow's expected calving dates are incorporated into the model, which allow the economic effects of calving date to be accounted for. These culling guides allow the farmer to cull on the expected monetary return from any one cow.

None of the studies reviewed in chapters 2 or 3 have been applied to the industry as a whole. This is a reflection of the complexity of the models and/or the type of replacement information the models produced. The

replacement guides outlined in this study compute the expected annualised present value of future incomes for individual cows. These culling guides can be used with any other information the farmer has about individual cows at the time the replacement decisions are being made to aid the decision process. The usefulness of these culling guides is not limited by the number of stock reared or the nature of the herd size (e.g. increasing, decreasing or static) which makes this system workable and applicable to the New Zealand seasonal supply dairy industry as a whole.

8.3 Future Developments.

This study is by no means exhaustive and there are several developments which could be incorporated into the dynamic l programming model:

i) the number of states could be increased, thus reducing the distance between each state. The most obvious state variable to increase is calving date, instead of using three week intervals, it may be more desirable to use weekly intervals. As more information becomes available or the payout structure changes, it may be desirable to include more state variables such as milk protein yields and cow weight.

ii) extensions to the BLUP model are forseeable. If payout becomes based on the price per kilogram of protein and milkfat, a shift to a multitrait model would be an obvious move. It may also be desirable to use a multitrait model which uses test day records so that the prediction can be updated after every herd test. Modification of the BLUP model to include age and lactation length into the fixed effects would also be a useful extension.

iii) to predict accurately the value of the newborn calf it would be desirable to include the breeding index as a state variable. Then it would be possible to regard female calves from high breeding index cows as possible candidates for rearing. Similarly it may be possible to regard

male calves from very high breeding index cows as possible candidates for sire breeding schemes.

iv) in this study the models have been used to rank cows in seasonal supply herds. Versions of a similar model with the time of calving as a state variable could be useful for town supply herds. These models could also have applications in other species such as poultry, swine, deer, goats and sheep.

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Appendix 1

To prove Howard's functional equation $f_n(i,\$_n)$ is defined as the expected return over the next period of time , starting in state i and using policy $\$_n$. Under the assumption of additivity then :

$$f_n(i,\$_n) = R_n^{S}(i) + \beta\hat{\Sigma}_{j=}(\pi_{ij}^{S} f_n(j, \$_{n-1}))$$

where $R_n^{S}(i)$ is the return generated in stage i,

β is the discount rate ,

π_{ij}^{S} is probability of transition from state i to state j and

$\$_{n-1}$ is a forward contraction of $\$_n$ to the remaining n-1 periods (a forward contraction is a subpath beginning at stage s and ending at n-1).

If $\Phi_n(i) = \max_{s(n)} [f_n(i,\$_n)]$ and since $\pi_{ij}^{S} \geq 0$ then

$$\Phi_n(i) = \max_{s(n),s(n-1)}[R_n^{S}(i) + \beta\hat{\Sigma}_{j=}(\pi_{ij}^{S} f_n(j, \$_{n-1}))$$

$$= \max_{s(n)}[R_n^{S}(i) + \beta\hat{\Sigma}_{j=}(\pi_{ij}^{S} \max_{s(n-1)} f_n(j, \$_{n-1}))]$$

$$= \max_{s(n)} [R_n^{S}(i) + \beta\hat{\Sigma}_{j=}(\pi_{ij}^{S}\Phi_{n-1}(j))] \qquad \ldots A_1$$

Equation A_1 is Howard's functional equation , which is used to discribe the system.

Several authors have discussed the two optimality conditions which are sufficient for the validity of the value iteration method, for example Bellman(1957),Bellman and Drefus(1962) and Hastings(1973).

The first condition is the separability condition, which states "for every plan the value of every state must be expressed as a function of the immediate return ,and the value of the succeeding stage" (Hastings 1973).

Consider a process at stage n, the process moves from state i_n to i_{n-1} under decision s^n which generates a return $R_n^{s}(i_n)$. Let $\$_n$ be the sequence of decisions $s^n,s^{n-1},s^{n-2},\ldots\ldots\ldots,s^2,s^1$. Assume the process has m stages, hence the objective is to maximise some function, δ_m, of the stage returns. The value of state i_m is denoted by $f_m(i_m,\$_m)$ under policy $\$_m$ at stage m. Hence:

$$f_m(i_m,\$_m) = \delta_m[R_m^{s}(i_m) + R_{m-1}^{s}(i_{m-1}) +,\ldots\ldots, +R_1^{s}(i_1)] \qquad \ldots A_2$$

If the separability condition holds then :

$$f_m(i_m,\$_m) = \delta_m[R_m^{s}(i_m)+f_{m-1}(i_{m-1},\$_{m-1}) \qquad \ldots A_3$$

Now if the function is, say, the discount factor, β, then :

$$f_m(i_m,\$_m) = [R_m^{s}(i_m) + \beta R_{m-1}^{s}(i_{m-1})+,\ldots,+\beta^{m-n}R_n^{s}(i_n)+,\ldots,+\beta^{m-1}R_1^{s}(i_1)]$$

and

$$f_m(i_m,\$_m) = R_m^{s}(i_m) + \beta\, f_{m-1}(i_{m-1},\$_{m-1}) \qquad \ldots A_4$$

It can be seen that equation A_4 has the same form as equation 1 on page ,thus the separability condition holds for equation 1.

The second condition is the optimality condition, which states " for every state,within a stage and a policy, the optimal policy is to consist of the given immediate decision followed by that policy which is optimal with

respect to the successor states" (Bellman 1957).

Consider equation A_2 :

$$f_m(i_m,\$_m) = \delta_m[r_m^{s}(i_m)+f_{m-1}(i_{m-1},\$_{m-1})$$

now if $\$_m = s_m + \$_{m-1}$ then :

$$f_m(i_m,\$_m) = f_m(i_m,s_m + \$_{m-1})$$

Assuming we wish to maximise the process, then the optimality condition requires that :

$$f_m(i_m,s_m + \$^*_{m-1}) \geq f_m(i_m,s_m + \$_{m-1})$$

where $\$^*_{m-1}$ is the optimal policy up to stage m-1.

So for the case where discounted returns are used , then :

$$f_m(i_m,\$_m) = r_m^{s}(i_m) + \beta\, f_{m-1}(i_{m-1},\$_{m-1})$$

and hence :

$$r_m^{s}(i_m) + \beta\, f_{m-1}(i_{m-1},\$^*_{m-1}) \geq r_m^{s}(i_m) + \beta\, f_{m-1}(i_{m-1},\$_{m-1}).$$

This condition will always hold when β is positive because of the definition of $\$^*_{m-1}$

Appendix 2.

The probability of movement to the 5 calving date categories in future years, given the initial calving category in year 1.

The probability of movement to the 5 calving date categories in future years, given the cow was in calving category 1 in year 1.

Future year	Calving date category 1	2	3	4	5
1	1.0000	0.0000	0.0000	0.0000	0.0000
2	0.8976	0.0696	0.0223	0.0071	0.0034
3	0.8833	0.0769	0.0247	0.0079	0.0072
4	0.8788	0.0775	0.0248	0.0079	0.0109
5	0.8753	0.0774	0.0248	0.0079	0.0147
6	0.8719	0.0771	0.0247	0.0079	0.0185
7	0.8686	0.0768	0.0246	0.0078	0.0222
8	0.8563	0.0765	0.0245	0.0078	0.0259
9	0.8629	0.0762	0.0244	0.0078	0.0297
10	0.8587	0.0759	0.0243	0.0077	0.0334

The probability of movement to the 5 calving date categories in future years, given the cow was in calving category 2 in year 1.

Future year	Calving date category 1	2	3	4	5
1	0.0000	1.0000	0.0000	0.0000	0.0000
2	0.8976	0.0696	0.0223	0.0071	0.0034
3	0.8833	0.0769	0.0247	0.0079	0.0072
4	0.8788	0.0775	0.0248	0.0079	0.0109
5	0.8753	0.0774	0.0248	0.0079	0.0147
6	0.8719	0.0771	0.0247	0.0079	0.0185
7	0.8686	0.0768	0.0246	0.0078	0.0222
8	0.8563	0.0765	0.0245	0.0078	0.0259
9	0.8629	0.0762	0.0244	0.0078	0.0297
10	0.8587	0.0759	0.0243	0.0077	0.0334

The probability of movement to the 5 calving date categories in future years, given the cow was in calving category 3 in year 1.

Future year	Calving date category 1	2	3	4	5
1	0.0000	0.0000	1.0000	0.0000	0.0000
2	0.6800	0.2176	0.0696	0.0223	0.0105
3	0.8530	0.0929	0.0297	0.0095	0.0150
4	0.8692	0.0787	0.0252	0.0080	0.0189
5	0.8678	0.0767	0.0246	0.0079	0.0226
6	0.8649	0.0765	0.0245	0.0078	0.0263
7	0.8616	0.0762	0.0244	0.0078	0.0300
8	0.8583	0.0759	0.0243	0.0077	0.0337
9	0.8551	0.0756	0.0242	0.0077	0.0374
10	0.8518	0.0753	0.0241	0.0077	0.0411

The probability of movement to the 5 calving date categories in future years, given the cow was in calving category 4 in year 1.

Future year	Calving date category 1	2	3	4	5
1	0.0000	0.0000	0.0000	1.0000	0.0000
2	0.0000	0.6800	0.2176	0.0696	0.0328
3	0.7583	0.1420	0.0456	0.0145	0.0397
4	0.8391	0.0824	0.0264	0.0084	0.0437
5	0.8451	0.0756	0.0242	0.0077	0.0474
6	0.8429	0.0746	0.0239	0.0076	0.0510
7	0.8398	0.0742	0.0238	0.0076	0.0547
8	0.8366	0.0739	0.0237	0.0075	0.0582
9	0.8334	0.0737	0.0236	0.0075	0.0618
10	0.8302	0.0734	0.0235	0.0075	0.0654

The probability of movement to the 5 calving date categories in future years, given the cow was in calving category 5 in year 1.

Future year	Calving date category 1	2	3	4	5
1	0.0000	0.0000	0.0000	0.0000	1.0000
2	0.0000	0.0000	0.0000	0.0000	1.0000
3	0.0000	0.0000	0.0000	0.0000	1.0000
4	0.0000	0.0000	0.0000	0.0000	1.0000
5	0.0000	0.0000	0.0000	0.0000	1.0000
6	0.0000	0.0000	0.0000	0.0000	1.0000
7	0.0000	0.0000	0.0000	0.0000	1.0000
8	0.0000	0.0000	0.0000	0.0000	1.0000
9	0.0000	0.0000	0.0000	0.0000	1.0000
10	0.0000	0.0000	0.0000	0.0000	1.0000

Appendix 3

Cow Number	Calving Date Category	BLUP of Future Milkfat Production(kg)	Annualised Present Value ($)		
			Annuity	Dynamic1	Dynamic2
4.	2.00	204.32	651	620	617
24.	1.00	246.55	813	731	727
37.	1.00	228.77	754	674	669
39.	2.00	215.44	686	647	643
81.	1.00	213.00	704	659	655
6.	2.00	193.22	606	601	598
7.	1.00	192.99	632	622	618
8.	1.00	221.88	721	668	664
15.	2.00	211.32	662	632	629
22.	1.00	214.22	699	652	649
28.	2.00	177.65	567	579	573
29.	1.00	189.55	622	614	610
33.	2.00	183.11	586	586	582
47.	1.00	220.44	718	668	664
52.	1.00	171.99	579	592	585
57.	1.00	240.66	779	698	695
60.	1.00	268.88	857	747	743
61.	1.00	219.99	715	660	657
63.	1.00	216.11	713	667	664
65.	1.00	209.55	684	644	641
70.	2.00	182.00	583	586	582
72.	2.00	186.99	602	595	590
90.	1.00	209.55	684	644	641
91.	1.00	201.99	659	637	634
98.	1.00	206.44	675	644	641
103.	1.00	214.88	699	652	649
104.	1.00	197.88	647	629	626
105.	2.00	193.99	606	601	598
106.	1.00	242.33	785	711	708
107.	1.00	218.33	719	667	664
108.	1.00	222.99	732	676	673
109.	2.00	242.66	770	701	697
110.	1.00	193.22	645	622	618
111.	1.00	227.00	739	675	672
113.	1.00	185.66	610	614	610
114.	1.00	203.55	665	637	634
115.	1.00	198.99	650	629	626
116.	1.00	189.66	622	614	610
119.	1.00	213.11	696	652	649
120.	1.00	200.00	656	637	634
121.	1.00	207.44	678	644	641
122.	1.00	204.22	668	637	634
124.	1.00	218.44	712	660	657
125.	1.00	210.66	687	652	649
127.	2.00	203.88	637	616	613
132.	1.00	204.66	668	637	634
134.	1.00	171.99	567	593	587
135.	1.00	190.55	625	622	618
136.	2.00	211.44	662	638	634
138.	1.00	187.77	622	614	601
139.	1.00	252.99	816	714	711
140.	1.00	198.99	650	629	626
142.	1.00	230.33	757	694	690
86.	2.00	208.55	672	633	629
88.	1.00	183.66	612	603	597
96.	2.00	192.66	621	604	600
131.	1.00	201.99	659	637	634
3.	2.00	211.22	681	643	639
13.	1.00	199.55	664	634	630
18.	1.00	190.22	636	625	620
25.	1.00	176.99	591	599	591
31.	1.00	169.44	569	584	572
38.	2.00	203.11	656	623	619
55.	1.00	180.11	604	607	601
56.	1.00	230.33	762	702	698
58.	1.00	211.22	702	663	659
67.	1.00	206.88	686	653	649
73.	1.00	192.99	642	625	620
92.	1.00	191.33	638	625	620
49.	1.00	167.77	533	595	591
54.	1.00	172.22	575	592	584
82.	1.00	211.22	579	626	622
1.	1.00	147.66	498	573	526
2.	1.00	182.88	609	603	597

Cow Number	Calving Date Category	BLUP of Future Milkfat Production(kg)	Annualised Present Value ($)		
			Annuity	Dynamic1	Dynamic2
19.	2.0	136.2	445	548	485
16.	1.0	250.0	824	745	740
36.	1.0	235.9	777	714	710
40.	1.0	227.6	751	694	689
45.	1.0	232.1	767	704	699
69.	1.0	245.1	808	735	730
71.	2.0	237.6	746	694	689
75.	1.0	222.1	735	684	679
77.	1.0	228.6	754	694	689
80.	2.0	240.2	774	704	699
83.	1.0	230.0	761	704	699
84.	1.0	214.2	710	664	659
87.	2.0	205.6	663	632	628
89.	1.0	238.1	786	714	710
94.	1.0	223.1	739	684	679
100.	2.0	247.1	795	714	710
118.	1.0	229.1	758	694	689
123.	1.0	239.8	789	714	710
133.	1.0	221.3	732	684	679
85.	1.0	238.4	786	714	710
9.	1.0	175.4	508	579	571
10.	2.0	206.8	662	627	622
12.	2.0	226.6	724	669	664
14.	1.0	243.1	794	721	716
21.	1.0	224.8	735	679	674
26.	1.0	231.4	756	700	695
30.	2.0	203.2	652	616	611
34.	1.0	206.0	678	637	632
35.	1.0	233.2	763	700	695
42.	1.0	240.2	785	721	716
44.	1.0	199.7	656	627	621
46.	1.0	237.8	775	711	705
48.	1.0	199.7	656	627	621
50.	1.0	186.1	615	606	600
51.	1.0	235.3	769	711	705
59.	2.0	266.3	849	753	748
66.	1.0	236.2	772	711	705
74.	1.0	200.2	660	637	632
76.	1.0	259.5	844	753	747
78.	1.0	239.1	781	711	705
79.	2.0	214.1	687	637	623
93.	2.0	229.2	734	669	664
95.	2.0	237.2	759	690	685
99.	1.0	226.5	741	690	684
101.	1.0	248.5	810	732	726
102.	2.0	245.5	783	711	706
112.	1.0	227.5	744	690	684
117.	1.0	212.0	697	658	653
126.	1.0	237.9	775	711	705
128.	1.0	251.2	819	742	737
137.	1.0	251.9	827	742	737
143.	1.0	282.4	915	805	800
144.	1.0	202.6	666	637	632
145.	1.0	200.8	659	637	632
146.	1.0	218.4	716	669	663
148.	1.0	195.7	645	627	621
149.	1.0	202.6	666	637	632
150.	1.0	193.5	638	616	611
151.	1.0	219.0	719	669	663
152.	1.0	200.3	660	637	632
153.	1.0	195.7	645	627	621
27.	2.0	337.9	1071	900	895
20.	1.0	193.6	638	616	611
43.	1.0	189.5	625	606	600
305.	3.0	242.0	738	675	667
309.	1.0	235.1	749	694	689
311.	2.0	218.1	681	632	627
317.	1.0	222.7	709	663	658
320.	1.0	217.7	694	653	648
323.	1.0	227.3	724	674	668
332.	1.0	226.6	721	674	668
341.	2.0	234.4	729	663	658
343.	1.0	223.1	713	663	658
349.	1.0	228.2	728	674	668
351.	1.0	219.0	700	653	648
353.	1.0	218.6	697	653	648

Cow Number	Calving Date Category	BLUP of Future Milkfat Production (kg)	Annualised Present Value ($)		
			Annuity	Dynamic1	Dynamic2
354.	2.0	218.1	681	632	627
362.	1.0	241.2	768	705	700
364.	1.0	232.4	740	684	679
368.	1.0	231.4	737	684	679
382.	1.0	232.4	740	684	679
397.	1.0	215.1	688	653	648
429.	1.0	214.3	685	653	648
430.	3.0	235.0	716	665	657
441.	1.0	226.6	721	674	668
447.	1.0	216.0	691	653	648
454.	1.0	224.3	716	663	658
455.	1.0	225.8	719	674	668
456.	1.0	218.7	697	653	648
457.	1.0	218.1	697	653	648
458.	1.0	230.2	734	684	679
459.	1.0	234.4	746	684	679
460.	1.0	219.0	700	653	648
461.	1.0	219.0	700	653	648

Cow numbers >300 refer to cows entering the herd for the time.

Appendix 4

The probabilities of death and involuntary replacement in future years given the cows age in year one.

The probabilities of death.

Future year \ Age in year 1	2	3	4	5	6	7	8	9	10	11
1	0.0583	0.0852	0.0898	0.0825	0.0734	0.0726	0.0905	0.1371	0.2228	0.3570
2	0.0802	0.0821	0.0751	0.0673	0.0673	0.0839	0.1247	0.1935	0.2777	
3	0.0774	0.0687	0.0613	0.0617	0.0778	0.1156	0.1749	0.2398		
4	0.0647	0.0561	0.0562	0.0714	0.1072	0.1622	0.2181			
5	0.0528	0.0514	0.0649	0.0983	0.1503	0.2023				
6	0.0484	0.0594	0.0895	0.1379	0.1874					
7	0.0559	0.0819	0.1255	0.1720						
8	0.0771	0.1148	0.1565							
9	0.1081	0.1432								
10	0.1348									

The probabilities of involuntary replacement

Future years \ Age in year 1	2	3	4	5	6	7	8	9	10	11
1	0.0258	0.0244	0.0242	0.0252	0.0274	0.0308	0.0354	0.0412	0.0482	0.0564
2	0.0238	0.0236	0.0246	0.0267	0.0300	0.0343	0.0397	0.0462	0.0537	
3	0.0230	0.0240	0.0261	0.0292	0.0334	0.0385	0.0446	0.0515		
4	0.0234	0.0254	0.0285	0.0325	0.0375	0.0420	0.0496			
5	0.0248	0.0278	0.0317	0.0365	0.0420	0.0481				
6	0.0271	0.0310	0.0356	0.0410	0.0468					
7	0.0302	0.0348	0.0400	0.0456						
8	0.0339	0.0390	0.0454							
9	0.0380	0.0434								
10	0.0423									