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# The Writing of Explanations and Justifications in Mathematics

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## ABSTRACT

This study reports on the writing of explanations and justifications in mathematics. A variety of approaches including a document analysis, teacher survey, students' responses to problem solving tasks, and student interviews were used to examine the complexities and interpretations of writing explanations and justifications in mathematics. The study involved six teachers and 36 Year 11 students from a provincial co-educational secondary school; 14 of the students were interviewed.

An analysis of the Year 11 national mathematics examination, School Certificate, revealed a significant increase in emphasis on the writing of explanations; from 2.7% of the total marks in 1992, to 16% of the total marks in 1997. It was not until 1997 that students were specifically asked to write justifications. In this study students experienced some difficulties writing explanations and had concerns about whether their explanations were satisfactory; a variety of modes of representation were used by students. Most students surveyed were unable to write justifications; they lacked knowledge and confidence in justifying their solutions. The teachers believed that the writing of explanations and justifications was an important process but expressed a number of concerns. These concerns were the class time needed, and the lack of resources and professional development. Both students and teachers were concerned about not knowing what makes a quality response.

The writing of explanations and justification should be a valued and regular part of the mathematics programme so that students are able to develop and evaluate mathematical arguments and proofs and effectively communicate their findings to others. The study suggests that students and teachers need to work together in negotiating an understanding of what is meant by an explanation, and a justification, and what makes a quality response.

## PREFACE AND ACKNOWLEDGEMENTS

This thesis is essentially about the interface of mathematics, language, and the communication process. This study began to form in my mind when I became aware of the impact of a new mathematics syllabus which specifies mathematical processes as a separate and identifiable strand. Teachers and students were expressing interest in, and concern about the writing process in mathematics and so I decided to take the opportunity, in the final stages of my degree, to explore a topic that appeared important and timely. It also gave me an opportunity to mesh two personal interest areas, language and mathematics, and more specifically to try and answer key questions about the writing of explanations and justifications in mathematics. I felt confident that a qualitative methodology was appropriate in order to answer these questions and so my journey began...

I would like to acknowledge and thank the many people who made this study possible. Firstly I would like to acknowledge Dr. Glenda Anthony, my main supervisor, who provided continuing interest and invaluable professional support. My thanks are extended to Brian Finch, my second supervisor who gave such positive and encouraging feedback throughout the study.

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Lastly, I must acknowledge my family for their patience and tolerance in having a mother and wife whose attention at times seemed more focused on this study than on family interests and commitments. I have returned!

*Good company in a journey makes  
the way to seem the shorter.*

Izaak Walton, *The Compleat Angler*

## TABLE OF CONTENTS

|   |       |
|---|-------|
| <b>ABSTRACT</b>   | (ii)  |
| <b>PREFACE AND ACKNOWLEDGEMENTS</b>   | (iii) |
| <b>TABLE OF CONTENTS</b>  | (iv)  |
| <br>  |       |
| <b>CHAPTER 1: INTRODUCTION</b>  |       |
| 1.1 Background to the Study   | 7     |
| 1.2 Research Objectives   | 9     |
| 1.3 Definition of Terms   | 9     |
| 1.4 Overview  | 10    |
| <br>  |       |
| <b>CHAPTER 2: LITERATURE REVIEW</b>   |       |
| 2.1 Introduction  | 11    |
| 2.2 Language and Mathematics  | 12    |
| 2.3 Communication   | 15    |
| 2.4 Problem Solving   | 26    |
| 2.5 Assessment  | 31    |
| 2.6 Summary   | 42    |
| <br>  |       |
| <b>CHAPTER 3: RESEARCH DESIGN</b>   |       |
| 3.1 Introduction  | 43    |
| 3.2 Data Collection Methods   | 45    |
| 3.3 The Project: Setting, Sample, and Schedule                                | 51    |
| 3.4 Quality Criteria  | 54    |
| 3.5 Summary   | 59    |
| <br>  |       |
| <b>CHAPTER 4: SCHOOL CERTIFICATE ANALYSIS</b>                                 |       |
| 4.1 Introduction  | 60    |
| 4.2 Analysis Criteria   | 61    |
| 4.3 What Emphasis is Given to the Writing of Explanations and Justifications? | 62    |
| 4.4 In Which Content Strands Do the Questions Appear?                         | 65    |
| 4.5 What Prompts Are Used and How Well Did Students Respond to Them?          | 66    |
| 4.6 Summary   | 74    |

|  |     |
|--|-----|
| <b>CHAPTER 5: STUDENT RESPONSES</b>  |     |
| 5.1 Introduction   | 76  |
| 5.2 Problem Analysis   | 76  |
| 5.3 Summary  | 94  |
| <b>CHAPTER 6: STUDENT VIEWS</b>  |     |
| 6.1 Introduction   | 95  |
| 6.2 Students' Feelings About the Writing of<br>Explanations and Justifications | 96  |
| 6.3 Classroom Practice   | 101 |
| 6.4 Knowing What Makes a Quality Response                                      | 102 |
| 6.5 How Does Writing in Mathematics Differ<br>From Writing in English?         | 103 |
| 6.6 Summary  | 104 |
| <b>CHAPTER 7: TEACHER RESPONSES</b>  |     |
| 7.1 Introduction   | 106 |
| 7.2 The Importance of Writing Explanations and<br>Justifications               | 107 |
| 7.3 Using 'Mathematics in the New Zealand Curriculum'                          | 108 |
| 7.4 Problems Identified By the Teachers  | 110 |
| 7.5 Summary  | 113 |
| <b>CHAPTER 8: DISCUSSION AND CONCLUSION</b>                                    |     |
| 8.1 Introduction   | 115 |
| 8.2 The Role of Writing Explanations and<br>Justifications                     | 115 |
| 8.3 Issues of Concern  | 116 |
| 8.4 Implications   | 120 |
| 8.5 Further Research   | 123 |
| 8.6 Conclusion   | 125 |
| <b>BIBLIOGRAPHY</b>  | 127 |
| <b>APPENDICES:</b>   |     |
| Appendix A: Information Sheet  | 138 |
| Appendix B: Problem Solving Tasks  | 139 |
| Appendix C: Student Interview Schedule   | 142 |
| Appendix D: Teacher Questionnaire  | 143 |

**LIST OF TABLES:**

|     |   |     |
|-----|---|-----|
| 4.1 | Explanations and Justifications in School Certificate Mathematics   | 62  |
| 4.2 | Marks Allocated to Explanations and Justifications in School Certificate Mathematics by Content Strand    | 65  |
| 4.3 | Writing Prompts Used in School Certificate Mathematics for the Writing of Explanations and Justifications | 67  |
| 5.1 | Students' Mathematical Arguments for Problem 1  | 78  |
| 5.2 | Students' Modes of Representation for Problem 3   | 85  |
| 5.3 | Students' Mathematical Arguments for Problem 5  | 91  |
| 7.1 | Years of Teaching Experience  | 106 |
| 7.2 | Suggested Year for Introducing Writing Explanations and Justifications                                    | 109 |

**LIST OF FIGURES:**

|      |  |    |
|------|--|----|
| 2.1  | Elaborations in student expository writing (Shield & Galbraith, 1998, p. 42) | 22 |
| 4.1  | Percentage of total marks for each content strand, 1992-1997                 | 65 |
| 4.2  | School Certificate, 1994, Question Two                                       | 68 |
| 4.3  | School Certificate, 1994, Question Eleven                                    | 69 |
| 4.4  | School Certificate, 1995, Question Eleven                                    | 71 |
| 4.5  | School Certificate, 1995, Question Eight                                     | 72 |
| 4.6  | School Certificate, 1997, Question One                                       | 74 |
| 5.1  | The measurement problem  | 77 |
| 5.2  | An example of an explanation using symbols                                   | 79 |
| 5.3  | An example of an explanation using symbols and words                         | 79 |
| 5.4  | An example of an explanation which includes a diagram                        | 80 |
| 5.5  | The number problem   | 82 |
| 5.6  | The statistics problem   | 84 |
| 5.7  | An example of a verbal explanation   | 86 |
| 5.8  | An example of an explanation combining words and symbols                     | 86 |
| 5.9  | An example of an incomplete verbal explanation                               | 87 |
| 5.10 | The algebra problem  | 88 |
| 5.11 | The geometry problem   | 90 |
| 5.12 | An example of an explanation using symbols                                   | 92 |
| 5.13 | An example of an explanation using symbols, words, and a diagram             | 92 |