

Copyright is owned by the Author of the thesis. Permission is given for a copy to be downloaded by an individual for the purpose of research and private study only. The thesis may not be reproduced elsewhere without the permission of the Author.

**A SYSTEMATIC INVESTIGATION OF THE ESTIMATION
OF THE DIRICHLET MODEL**

A thesis presented in partial fulfilment
of the requirements for the degree of Doctor of Philosophy
in Marketing at Massey University

Zane Kearns

1999

ABSTRACT

The NBD/Dirichlet is a stochastic model of purchase incidence and brand choice which parsimoniously integrates a wide range of well-established empirical regularities in fast moving consumer goods markets. More recently this work has been extended into other areas such as the prescribing of pharmaceuticals, (Stern 1994); airline aviation fuel contracts, (Uncles and Ehrenberg 1990a); and the visiting of retail store chains, (Uncles and Ehrenberg 1990b).

By combining the stochastic assumptions of the model, namely Poisson purchasing of products, with mean rate distributed gamma across the population, and brand choice represented by multinomial probabilities distributed Dirichlet across consumers; a number of aspects of the aggregate behaviour of consumers can be successfully predicted successfully.

This thesis examines the estimation issues in the Dirichlet model, specifically, the central Dirichlet parameter S used to represent heterogeneity in brand choice.

ACKNOWLEDGEMENTS

In the first instance I would like to thank Professors A S C Ehrenberg, G J Goodhardt, and C Chatfield for their development of what has become known as the Dirichlet model. It is difficult to discern the unique contributions of these researchers, but overall their efforts represent a serious but often unrecognised contribution to marketing and the study of consumer behaviour.

I would also like to thank Professor Ehrenberg more personally for the time taken to answer my queries about the model, and being overly kind in not pointing out how silly some of my questions were, or that I should really have known the answer.

Of course the greatest burden of gratitude goes to my supervisor Associate Professor A C Lewis. His guidance is evident throughout this thesis, and without it, the thesis would be undeniably poorer. Thanks must also go to Dr Greg Arnold who lent valuable advice.

TABLE OF CONTENTS

Chapter One	Introduction	
1.1	Background	1
1.2	Objectives of the Thesis	2
1.3	Outline of the Chapters	4
 Chapter Two	 Stochastic Modelling of Consumer Behaviour	
2.1	Introduction	5
2.2	Loyalty	5
2.3	Stochastic Modelling of Consumer Behaviour	8
2.4	The Dirichlet model of Consumer Behaviour	10
	2.4.1 Aggregative Considerations	10
2.5	Dirichlet Findings	12
	2.5.1 The Fit of the Dirichlet model	15
	2.5.2 Known Systematic Discrepancies	24
2.6	Summary	25
 Chapter Three	 Assumptions of the Dirichlet model	
3.1	Introduction	26
3.2	The Purchase Incidence Model	27
	3.2.1 The Poisson Process	27
	3.2.2 Gamma Heterogeneity	32
	3.2.3 Negative Binomial Distribution of Purchases	35
3.3	The Brand Choice Model	39
	3.3.1 Multinomial Process	39
	3.3.2 Dirichlet Heterogeneity of Choice Probabilities	44
3.4	Summary	46

Chapter Four	Estimation of the Dirichlet model	
4.1	Introduction	48
4.2	Background	49
4.3	Product Class Purchasing	50
4.3.1	Estimation	50
4.3.2	Comparisons	53
4.4	Brand Choice	54
4.4.1	Estimation	55
4.4.2	Alternatives and Comparisons	55
4.5	Summary	58
Chapter Five	Methodology	
5.1	Introduction	60
5.2	Method	60
Chapter Six	Results	
6.1	Introduction	63
6.2	Parameters of the Sampling Distributions of the Simulated Panels	63
6.3	Estimates of the Dirichlet S Parameter	70
6.4	Effects of Sampling Error in the Dirichlet Predictions	75
Chapter Seven	Discussion	
7.1	Introduction	80
7.2	Background	80
7.3	Results	81
References		83

LIST OF TABLES

Table

2.1	Brand performance measures for Laundry Detergent	15
2.2	Conditions under which the Dirichlet patterns are known to occur	17
2.3	Annual performance measures for the eight leading brands	19
2.4	Predictions of purchase frequency and share of requirements	20
2.5	Market share and share of requirements	23
3.1	A stochastic representation of consumer purchases (x)	27
3.2	Purchase incidence models and their properties	38
3.3	A stochastic representation of brand choice	39
3.4	Summary studies on the order of the brand choice process	41
4.1	“Constant D” approximation predictions of duplication and Bemmaor’s estimator	57
5.1	Simulation parameters	62
6.1	Recovery of simulation parameters	64
6.2	Comparison of the standard error of m obtained from the simulated samples and theoretical value.	65
6.3	Standard Deviations of the Estimates of Penetration and Frequency from the Simulated Samples	67
6.4	Kurtosis, Skewness and Kolmogorov-Smirnov Statistics from the Simulated Samples	68
6.5	Mean and Range of Estimates of S	71
6.6	Ratio of Range in S(n) to S(1000)	72
6.7	Ratio of range in estimates of S to the standard error of the mean rate of buying	74
6.8	Ratios of the Standard error of the mean rate of buying to range of Dirichlet Predictions: Simulation A	76
6.9	Ratios of the Standard error of the mean rate of buying to range of Dirichlet Predictions: Simulation B	77
6.10	Ratios of the Standard error of the mean rate of buying to range of Dirichlet Predictions: Simulation C	78

LIST OF FIGURES

6.1	Histograms and Normal Probability Plots for Brand C Parameters	69
6.2	Simulated values of penetration and frequency and resulting estimates of S	73