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Price formation in parimutuel markets

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Abstract

Two types of betting are common in sports betting: *fixed odds* betting and *parimutuel* betting. In fixed odds betting, the payout conditional on winning is fixed once the bet is placed and is not affected by the placing of subsequent bets. By contrast, winning bettors in a parimutuel contest share pro-rata in the total betting pool. This means that the payout to winning bettors in a parimutuel contest depends not only on selecting the winning outcome, but also on the amounts bet by other bettors (which cannot be observed at the time a bet is placed). Therefore a parimutuel contest can be viewed as a game at the level of individual bettors. Existing models in the parimutuel literature explain the data by either assuming a single, representative bettor with certain risk preferences or by assuming that a number of risk neutral bettors compete strategically within a game theoretic framework. Our contribution is to construct a novel theoretical framework of parimutuel markets in which we model *both* strategic interaction *and* risk preferences at the level of individual insiders, in the presence of exogenous outsiders. We solve this model analytically for the optimal insider betting amount in a static symmetric Nash equilibrium. Using a new dataset of 1.6 million individual horse race bets in New Zealand from 2006 to 2009, we document a strong inverse linear relationship between our model-implied insider risk preferences and the strength of insider beliefs relative to outsiders. That is, as the strength of insiders' beliefs relative to that of outsiders decrease, implied risk sensitivity moves from risk averse to risk loving. At a level of insider beliefs congruent with actual performance in the data, average implied risk preferences are close to zero, that is, insiders are effectively risk neutral. While risk neutrality is a standard assumption in strategic interaction models of parimutuel betting, our study is the first to provide empirical support for this assumption. Finally, we document a strong relationship (not previously reported in the literature) between the average bet size and the average payout ratio, suggesting that bettors with inside information self-select by placing larger bets.

Contents

1 Summary **1**

2 Literature review **6**

2.1 Introduction 6

2.2 Why are betting markets important? 7

2.3 Parimutuel betting markets 8

2.4 Empirical regularities 9

2.4.1 Market efficiency 9

2.4.2 The favourite longshot bias 10

2.4.3 Factors correlated with the favourite longshot bias 12

2.4.4 Early betting 14

2.4.5 Late betting is smart betting 15

2.4.6 Rounding of odds 15

2.5 Theories of price formation 16

2.5.1 Overview 16

2.5.2 Representative agent models 16

2.5.3 Market structure 19

2.5.4 Strategic interaction 22

2.5.5 The current state of theory 28

2.6 Areas for further work 28

2.6.1 Extending strategic interaction models to encompass risk sensitive bettors 28

2.6.2 Endogenising the distinction between inside and outside bettors 28

2.6.3 Deriving additional testable implications from existing theoretical frameworks . . . 29
2.6.4 Creating a richer set of empirical stylised facts . . . 29
2.7 Next steps . . . 29

3 Strategic interaction with risk preferences 31
3.1 Introduction . . . 31
3.1.1 Motivation . . . 31
3.1.2 Notation . . . 33
3.1.3 Risk neutral model . . . 33
3.2 Model . . . 37
3.2.1 Payoff . . . 37
3.2.2 Expectation . . . 38
3.2.3 Variance . . . 38
3.2.4 Utility . . . 38
3.3 Equilibrium . . . 39
3.3.1 Best response function (first order conditions) . . . 39
3.3.2 Nash equilibrium . . . 40
3.3.3 Admissibility and second order conditions . . . 41
3.4 Discussion . . . 43
3.4.1 Symmetry . . . 43
3.4.2 No equilibrium zones . . . 44
3.4.3 Insider beliefs . . . 45
3.4.4 Equilibrium final market probabilities . . . 46
3.4.5 Special case 1: Risk neutral insiders . . . 48
3.4.6 Special case 2: Risk sensitive representative insider . . . 49
3.4.7 Special case 3: A simplified model . . . 49
3.4.8 Empirical statics . . . 51

4 Empirical tests **52**

4.1 Introduction 52

4.2 Data description and analysis 52

4.3 Insiders vs outsiders 53

4.3.1 Cut-off point 53

4.3.2 Payout ratio and average bet size 53

4.4 Race level data 55

4.5 Operational definition of model parameters 57

4.6 Empirical tests and results 58

4.6.1 Overview 58

4.6.2 Direct estimation of risk sensitivity 59

4.6.3 Estimation conditional on specified insider beliefs 60

4.6.4 Model vs empirical final market probabilities 64

5 Conclusion **68**

6 Appendix **73**

6.1 Maple code to verify derivation of equilibrium insider betting amounts 73

6.2 Average payout rate and average bet size by percentile 74

6.3 Statics based on an empirical approximation 76

6.4 Stata code for estimating reduced form final market probability ($\hat{m}$) . 78

6.5 Stata code for direct empirical testing of predicted final market probability 80

6.6 Stata code for indirect empirical testing of predicted final market probability 81

List of Figures

2.5.1 Final market probability m for $p = 0.6$ and $p = 0.8$ as a function of N 25

3.1.1 Amount bet by individual insiders for given q and N 35

3.1.2 Net payout of individual insiders for given q and N 36

3.4.1 Critical values of α as a function of q, assuming different values of b_0 44

3.4.2 Risk loving insider betting amounts on either horse as a function of q, with associated utility (utility scaled 20x) 45

3.4.3 Risk averse insider betting amounts on either horse as a function of q, with associated utility (utility scaled 20x) 46

3.4.4 Final market probabilities where $N \to \infty$ and $\tau = 0.2$ 47

3.4.5 Insider betting with $N = 4$, $\tau = 0$ and $q = \frac{1}{2}$ 50

4.3.1 Payout ratio (per percentile) against average bet (per percentile) . . . 55

4.6.1 Histogram of implied α, at an insider/outsider cut-off point of $c = 100$ (Model compliant observations only) 59

4.6.2 Mean implied risk sensitivity α_c vs insider/outsider cut-off point c . . 60

4.6.3 Mean of model implied risk sensitivity α_q as a function of Δq 63

4.6.4 Estimated risk sensitivity α_q against Δq for different insider bet cut-off points 64

4.6.5 Regression coefficient γ against risk sensitivity α for different q . . . 66

List of Tables

4.1 Data summary – horse racing bets from 1 Aug 2006 to 31 July 2009 52

4.2 Data summary – insiders vs outsiders 54

4.3 Race level data summary 56

4.4 Implied risk sensitivity $\alpha_{\Delta q}$ for a range of Δq 62

4.5 Best fit regression results for α and q 67

6.1 Bets and payouts by percentile 75

6.2 Empirical approximation of model final market probabilities 77