Using reciprocal teaching and learning methods to enhance comprehension in mathematics word problems.

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Abstract

This study reports on a classroom design experiment focused on the use of reciprocal teaching methods when solving mathematical word problems. The design experiment was set in a Year 5 classroom where the teacher and students used a newly designed model to assist when solving word problems. The intervention was implemented in a regular classroom environment and used Figure It Out resource books (Ministry of Education, 1999-2008).

The intervention was developed from reciprocal approaches used in a number of previous studies. Reciprocal reading was originally designed and implemented by Palincsar and Brown (1984). The objective of this study was to adapt this model and incorporate it into a mathematics programme for solving word problems. Students worked in groups while using the model and were explicitly taught procedures at each stage. Discussions within groups were an important component of the design experiment. Throughout the study, students discussed thoughts and strategies that could be used to solve the word problems. They reflected on their answers, ensuring they were providing relevant information and then articulated any errors so they could make changes.

Findings indicated that students were able to use the model to assist them when solving mathematical word problems. They followed the process and were able to identify the key aspects of word problems and answer specific questions correctly, providing sufficient detail. Both the teacher and students found the model to be a useful tool when working with the Figure It Out resource books. Through participating in this design experiment students gained confidence when solving word problems.

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Introduction

1.1 Background

Why are students able to solve mathematical equations but unable to solve the same problem when put into a word problem context? Students seem to get confused about what they are expected to do and what answer they need to provide. Having a logical, sequential process to aid in the solving of word problems would be beneficial in assisting students to experience success with this type of problem.

Understanding and comprehending word problems in mathematics is a well-documented issue (Chapman, 2006; Roth, 2009). "Comprehension involves understanding text and context as exhibited by the ability to act on what is read" (Adams & Lowery, 2007, p.161). This is understood in the area of reading but not so clear when working with word problems in mathematics. There might be a link between understanding in a reading lesson and understanding in a mathematics lesson. The idea of comprehension is consistent within reading and mathematics but what students are asked to do with this contextual information is different.

Word problems are written to enable students to apply their mathematical knowledge to a given situation. Originally word problems were simple statements that resulted in a question, for example, asking for the sum of a series of numbers or dividing a number into even shares. While originally developed to test basic comprehension and operational methods, word problems have gradually adopted greater complexity, requiring analysis of data and synthesis of solution methods (Swetz, 2009). As questions become increasingly complex, students need to decode questions then ensure they provide exactly what has been asked of them.

Research has shown that students have difficulties when dealing with word problems in mathematics (Chapman, 2006; Galbraith, 2006; Roth, 2009). Roth argues that language used in these problems is one of the contributing factors. These problems often use a great deal of 'school language' that students are expected to know and understand. Gerofsky (2009) notes that because of the language used the problems can also be rather ambiguous and open to interpretation. Based on the ambiguity of such tasks there is the opportunity for explicit teaching and learning if teachers and students are willing to embrace it. Often, however, the challenge of doing this is seen as too great. This study examines how we might improve students' ability to solve word problems by explicitly teaching students to understand and make use of the information provided within word problems.

1.2 Reciprocal teaching

Through my teaching experience I have successfully used reciprocal reading as developed by Palincsar and Brown (1984), as an instructional strategy to enhance comprehension and critical thinking when students are capable of reading the text but not so clear on knowing what it means. In reciprocal reading students work in small groups using collaborative discussions in examining and interpreting the texts they are asked to read. Questioning is a large part of the reciprocal reading process and students develop skills to ask questions of each other to broaden their own understanding and that of the other members of their group (Ministry of Education (MOE), 2003). This questioning also assists students when they meet unfamiliar words. Research clearly demonstrates that this approach has the potential to increase students' comprehension. As mathematical word problems are a specific form of text, this process may assist students to comprehend what they have been asked to do when engaged in word problems. A resource that focuses on the solving of such problems are the Figure It Out (FIO) resource books (MOE, 1999 - 2008).

During the reciprocal process, students are able to discuss their thinking with their group members. Although students often ask the teacher for assistance, their first preference is to discuss problems with their peers and ask questions of themselves and each other (Holton & Thomas, 2001; Rawlins, 2007). That is to say that, students are supported both by their peers and the teacher in the skills of questioning, argumentation and validation (Artzt & Yaloz-Femia, 1999). By using a reciprocal approach in mathematics instruction, students may develop confidence in their abilities to solve word problems and build on existing knowledge by listening to the ideas and knowledge of other students in their groups.

1.3 New Zealand National Standards

The recent National Standards (MOE, 2009) use activities based on the FIO books as exemplars for achievement in mathematics. The use of these types of activities will increase in classrooms if students are to reach the desired standards. Through experience and discussions I have observed that many teachers see FIO resource books with their focus on mathematics in realistic situations as “too hard”. Dees (1990) recognises that teachers have difficulty teaching ‘word or story problems’ and therefore believes that these activities or relevant sections in textbooks are often avoided. She also believes that the principal reason for student difficulties with word problems is that they do not understand the problem when they read it. They want to start calculating the problem straight away. One of the goals in the Mathematics Standards is for “the curriculum and the standards playing a vital role in the development of students’ ability and inclination to use mathematics effectively – at home, at work, and in the community” (MOE, 2009, p. 6). If students are not able to comprehend word problems, they are going to struggle solving mathematics problems set in real life situations.

To ensure students gain confidence when solving word problems, they need to be explicitly taught how to solve them. Lack of exposure to this style of activity hinders student learning, and consequently, the strategies required to solve these problems effectively are not learned. My decision to focus on the FIO resource was prompted by my concerns that this resource is currently not being used to its full potential. FIO activities are listed on the NZ maths website (see www.nzmaths.co.nz), as suitable activities for students to be using once they have worked with the teacher on a particular knowledge component or strategy. My concern is what is going to happen if the standards are based on this type of activity if teachers are not using them. Will students be unable to reach standards through no fault of their own?

1.4 Rationale behind research

I believe a reciprocal style approach when learning to solve word problems would be beneficial to enhancing students' comprehension and eventual solutions of problems. A reciprocal style approach provides students with the opportunity to work in a social setting where they are comfortable asking questions and are planning to reach the end result. Gabriele (2007) advocates that students focus on learning goals. He believes that "students who adopt learning goals exhibit high levels of intrinsic motivation, prefer challenging work and persist in the face of difficulties, report more active cognitive engagement, engage in adaptive help-seeking and tend to utilize higher quality learning strategies" (p. 124). He contrasts this perspective by looking at students who have a performance focus and notes that students are less likely to converse with higher achieving peers due to the fear of looking incompetent. Students who work in groups are able to experience the success of solving word problems. Teachers who encourage students to work through challenges together create a sharing environment where students are comfortable asking each other questions to assist with the given task.

A resource set that clearly demonstrates developments towards word problem understanding in mathematics education are the Figure It Out (FIO) resource books (Ministry of Education, 2001). These resources have rich mathematical content but, in my observation as a teacher, students and teachers often have difficulty sorting through background information before finding relevant details. A concern of mine that is raised from the National Standards reporting system is that students may become more achievement driven rather than focused on deep learning. This observation prompted me into thinking of a way to enable resources to become rich tools for enhancing learning experiences. If students are able to comprehend the questions within these activities, it seemed mathematics would be more appealing to students and likely to maintain their mathematical interest when solving problems.

The introduction of a similar system in England attracted concerns from teachers that the standards are excessively detailed and over-prescriptive, comprising a list of items that must be ticked-off once achieved (Askew, 2001). At times tests covered the entire mathematics curriculum and tests contained tasks that were situated in realistic contexts where students had to decipher what they are being asked to do (Cooper & Harries, 2009). Similarly, the Numeracy Development Project (2007) in New Zealand places emphasis on the learning involved with tasks.

This study uses a reciprocal teaching base to enhance the learning process. It does this by guiding students through mathematical word problems. A design experiment approach is used to work with a class of students and their teacher in implementing a five-stage process that explicitly guides them through FIO activities. It also seeks to build student confidence. Students work in groups to enable them to ask questions in relation to the word problem and to highlight what they are required to do in order to complete the activity and answer the questions. This process was designed to be incorporated in an existing classroom programme, which in this case focussed on Statistics.

Through the process of working with another teacher and class of students I was guided by the notion of 'community of practice' (Lave and Wenger (1991, cited in Anthony & Walshaw, 2007). This theory is based on collective knowledge where members of a group are working together to support each other in the understanding of how this new knowledge could be used in different situations. Both individual and collective knowledge is developed through the process of working with new ideas and trialling adaptations of these ideas. Contributions from all participants are valued. In this research the participants are considered to be the researcher, teacher and students.

1.5 Research objectives

My interest in word problems began when using the FIO books with my own class and noticing the difficulties that some of the students were having. Following on from this, I discussed the underuse of FIO books with other teachers and realised they had similar experiences. All of this began well before the National Standards were introduced to

New Zealand teachers. Initially I wanted to investigate how these FIO resources could be used to best effect in a classroom situation. In my view, it was both the complexity of the language and the amount of text on the page that were the major hindering factors. I wanted to introduce a process that would assist students in completing these activities. I also wanted teachers to use the activities to enhance the teaching and learning in the application of mathematics concepts, as this resource was designed to do.

With the introduction of National Standards, the use of FIO activities has now become a high interest topic around schools. Hence, research that aims to investigate ways that we can improve the teaching and learning of word problems using FIO activities is both timely and significant.

The key objective of this research is to:

- Design and implement a process, adapted from the reciprocal reading approach, which will assist students when solving word problems.

From this key objective, a number of research questions were posed.

1. How could the reciprocal reading process be adapted and incorporated into the mathematics programme?
2. How do students use the phases within the model when solving word problems?
3. What are the effects of the model on students' learning outcomes?

1.6 Thesis Overview

Chapter Two is a review of the literature concerning the use of word problems in teaching mathematics. It also investigates reciprocal teaching as an approach to enhance comprehension. A summary of previous studies that have investigated reciprocal style approaches in the teaching of mathematics is included as these studies form the basis of the process that is tested in the classroom during the design experiment.

In Chapter Three, the methodology used in this design experiment research is described. Also included in this chapter are details of the participant background and the setting where the design experiment took place. Data collection and analytical methods are discussed, along with the timeframe. Included in this section, are the diagrams used to guide students through the new process. Limitations of the study are also noted.

Chapters Four concentrates on the findings of this design experiment. There are many examples of students' work samples using the intervention model. I have included information from discussions with students and the teacher including how the teacher explicitly taught the model and guided the students through the process before allowing them to work with greater independence.

Chapter Five discusses significant factors and draws conclusions based on the findings of the design experiment relating to the initial goals of the research. In this chapter, I have used information from discussions with the teacher to suggest refinements of the process and future research.

Chapter Two

Literature Review

2.1 Introduction

When teaching mathematics, teachers have a series of large goals that they aim for their students to achieve. Artz and Armour-Thomas (1998) list five general goals from the Curriculum and Evaluation Standards (NCTM, 1989) that summarise these larger goals. These are: valuing mathematics, becoming confident in their ability to do mathematics, becoming mathematical problem solvers, learning to communicate mathematically, and learning to reason mathematically. These goals are similar to the range of outcomes specified in The New Zealand Curriculum. In justifying the importance of teaching mathematics the document states that:

> *by studying mathematics and statistics, students develop the ability to think creatively, critically, strategically, and logically. They learn to structure and to organise, to carry out procedures flexibly and accurately, to process and communicate information, and to enjoy intellectual challenge* (MOE, 2007b, p. 26).

To achieve these learning outcomes, students need to have access to effective and equitable learning opportunities as advocated in reform practices in mathematics education (Walshaw & Anthony, 2008). This literature review focuses on those aspects of teaching and learning that allow students to build understanding through communication with their peers and other students. Within classroom situations, the grouping of students and questions that they are asked and/or asking contribute to their understanding and comprehending of mathematical processes, including the understanding of word problems (Gabriele, 2007; Meloth & Deering, 1992). Importantly, it also contributes to their ability to solve these correctly and effectively.

The focus of this review is on studies that have involved comprehension of word problems and systems that have been established to guide students through a process leading to an answer of a question.

2.2 Classroom environment

Traditionally, mathematics classrooms have involved independent task completion where answers are relatively quick to calculate and where very little discussion or communication with others in the same environment is involved (Schoenfeld, 2002; Walls, 2007). When this was the norm, there was a likelihood that those students who found tasks challenging would give up too quickly without developing the strategies to make sense of what they were being asked to do (Doerr, 2006). Recent reforms in mathematics (see Walshaw and Anthony, 2008) advocate a shift towards mathematics teaching, focused on making meaning. Students are encouraged to discuss and debate their thinking and their interpretations (Angier & Povey, 1999; Doyle, 2007).

In the New Zealand classroom, the mathematics curriculum signals a move away from computational procedures towards a focus on understanding ideas, relationships and concepts. Furthermore, successful teachers are “connectivist”. They make connections between domains of mathematics for their students. Students are also encouraged to make their own connections between what they are doing in mathematics and the world around them (Askew, Brown, Rhodes, William, & Johnson, 1997). Stein, Grover, and Henningsen (1996) separated tasks into two groups, low and high demand. Low demand tasks required the recall of and memorisation of learned facts, definitions or standard procedures whereas high demand tasks used procedures in a way that builds connections to mathematical meaning and require complex thinking (cited in Anthony & Walshaw, 2007).

Students making sense of what they are asked to solve is a key feature of reform mathematics classrooms (Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver, & Human, 1997). This requires that students have opportunities to make connections to previous learning, explain their responses and at times justify their interpretation (Anthony & Hunter, 2005; Doerr, 2006; Doyle, 2007). Doyle names this as mathematical literacy and cites Romberg (2001) in describing it "as having knowledge of the intricacies of mathematical language in order to gather and understand information on concepts and procedures" (p. 246).

Teachers have an important role to play in teaching mathematical literacy in mathematics lessons. Mau and D'Ambrosio (2003) argue that thinking mathematically involves altering perception of the nature of mathematical thought, developing self-confidence in order to persevere, building problem solving strategies, developing self-monitoring techniques and learning to justify their thinking. By developing these practices, teachers are providing students the opportunity to operate effectively in a world that requires mathematical reasoning in an increasingly technologically advancing society (Doyle, 2006). Such skills are critical for the success of students when working on tasks (English, 2006).

2.3 Scaffolding

Teachers' pedagogical content knowledge plays an important part in their teaching of mathematics. Before teaching students, teachers need to identify the fundamental critical mathematical components within particular concepts. They need to have an awareness of the relationships between topics, procedures and concepts in order to support the assimilation of these ideas by their students (Baker & Chick, 2006). As word problems require interpretation, effective teachers have to be prepared to support the variety of directions that students may take through their process towards solving a problem (Doerr, 2006). Teachers also have to be prepared to let the students attempt to solve a problem with the support of other students. When working with mathematical problems, teachers have a tendency to fracture the problems for the student (Angier & Povey, 1999). This does not allow the students to develop their ability to understand, decipher and solve word problems. They then become reliant on the teacher when trying to solve these problems as they know that when they are not making progress in their own attempts to solve a problem the teacher will break it down for them into smaller, more manageable segments.

Vygotsky (1978) believed that learning takes place in social contexts and is built from a level where students are comfortably working to a potential development level through support from teachers or peers. This level was called the Zone of Proximal Development (ZPD). When students are providing this support, the role of teachers then changes to one of facilitation. As facilitators, an important aspect of planning a programme is to provide effective tasks within a student's ZPD so they are able to assimilate new knowledge with previous understanding with the assistance of others within that social context (Hodge, Visnovska, Zhao, & Cobb, 2007).

The term 'scaffolding' is typically used in conjunction with Vygotsky's ZPD. Scaffolding is described as "assistance that allows students to complete tasks they are not able to complete independently" (Eggen & Kauchak, 1999, p. 51). Usually the teacher plays the role of 'scaffolder' or facilitator assisting students to develop cognitive and metacognitive strategies through supporting and guiding them as they see necessary (Anthony & Hunter, 2005; Hunter, 2009). Effective scaffolding involves being responsive to the needs of the student and withdrawing support when the student is able to continue independently. Rather than showing and telling students, which was the traditional role of the teacher, they guide the development of student's own thinking (Anghileri, 2006).

2.4 Peers, grouping and group work

Kutnick, Blatchford, and Baines (2002) describe using a range of groupings as a useful strategy to guide students during lessons. The learning environment and opportunities provided by group work can differ from whole class learning in a variety of ways. For example, whole-class discussions can be demanding and daunting to some students (Doyle, 1983, cited in Walshaw and Anthony, 2006). When sharing with a large group, students are conscious of wanting to be correct and being able to explain clearly on their first attempt. Moreover, Baxter, Woodward, and Olsen (2001) noted cases where highly articulate students tended to dominate class discussions while lower academic achievers remained passive. In contrast, small group work is often an effective way for students to share their initial thinking. Students often have greater confidence to share ideas with a small group rather than sharing these ideas to the whole class. Having discussions with peers provides an opportunity to develop mathematical thinking and become cognitively engaged in the task they are completing.

Peers are also an important resource when developing mathematical thinking (Walshaw & Anthony, 2006). However Anghileri (2006) found that assistance in goal directed activities provided by more capable peers is largely absent in classrooms. Sharing with only a few people enables the communication to be more conducive to questioning and open discussion. Holton and Clarke (2006) describe this type of interaction as ‘reciprocal scaffolding’. This takes place when “two or more people are involved in working collaboratively on a common task” (p. 136). The main scaffolding role varies in this situation depending on the previous experience and knowledge of members within the group. There are times when different students take on the role of ‘scaffolder’ depending on the task and who is most confident with particular concepts.

When working in small groups students are benefiting in two different ways. The first involves learning the cognitive perspective of structures, concepts and procedures and is often completed independently although this increasingly involves other class members. The second involves the social aspect, where they can learn the intricacies of mathematical discourse and how to participate in a supportive learning community (Anderson, Greeno, Reader, & Simon, 2000, cited in Doyle 2007; Davidson, 1990). The collaboration and social interaction required when working in groups provides support for all students in their construction of personal understanding of concepts (Borasi & Siegel, 2000; Slavin, 1996).

Wood and Yackel (1990) discussed benefits for students working with others including extending their own framework of thinking. Through listening to others students were able to make sense of what they were saying and coordinate with their own thoughts on solutions. Students who are motivated to solve problems tend to “be more persistent when they find work challenging, spend more time on tasks, use more complex processing and monitor their comprehension, select more challenging tasks, employ greater creativity and take risks” (Middleton & Spanias, 1999, cited in Allen & Johnson-Wilder, 2004, p. 238). Allen and Johnson-Wilder claim that students who lacked personal authority over their learning were focused on performance goals rather than learning goals. Whereas in traditional mathematics classrooms, students worked individually, focusing on speed and accuracy and avoided collaborative work situations and consequent negotiation of meaning, teachers are encouraging students to think with greater complexity and learn from those around them. Previously, students' ideas of mathematics were related to answering questions quickly and getting subsequent praise. There was a desire for certainty, and satisfaction resulted when students were able to complete a task correctly (Morgan, Watson, & Tikly, 2004). Within mathematics educational reforms students are encouraged to work with their peers and use them as a valuable resource.

For groups to be successful, many aspects must be considered. Doyle (1983, cited in Anthony & Walshaw, 2007) identifies the following as being important: spatial configurations and interdependencies among participants, familiarity with activity, established rules, students' inclinations to participate, and their competencies. When students are held accountable for their work as a group there will be strong forces between group members to prevent some students drifting off task (Cohen, 1986). Cohen also recognises that groups of four to five tend to be the most effective and that groups should have a mix of abilities. Diversity in prior knowledge amongst students can be beneficial as each student within the group is able to contribute to the discussion when constructing new ideas (Schoenfeld, 1987, in Mevarech & Kramarski, 1997). Groups need to have the space to work away from distractions and space that allows for easy interactions. The Numeracy Professional Development support material (MOE, 2007a), argues that within a balanced numeracy programme there should be a variety of grouping situations to suit the needs of the students including whole-class instruction, ability grouping, mixed ability grouping and individual work. Ability grouping allows students to work on problems that are aligned with their own learning progressions whereas mixed ability grouping allows students to discuss and learn from students of differing levels.

Groups also have to be clear in regards to what they are trying to achieve during the task. Higgins (2000) demonstrated that teachers are often unclear about what their role entails during student group work and students' explanations of what they were doing were also unclear. However, when students were clear of the intent of the mathematical activity, the students appeared to engage more actively with the mathematics. Teachers, therefore, must ensure that the intention of the mathematical activity is clearly articulated at the beginning and during the feedback session. Timperley and Parr (2009) emphasise the need for intentions or goals and feedback to be clearly linked and related to the task or else students are likely to get confused with what they are aiming to achieve. This ensures that both parties are clear of the purpose of the task and all members of groups are working towards to same goal (Absolum, 2006).

When participating in group discussions, it is important that students develop skills which enable them to work effectively, communicate with others and give feedback that centres on mathematical explanations and justification rather than just on the answers (Alton-Lee, 2003). The discussions that take place benefit all members of the group. The person doing the explaining is clarifying their idea and the others are given the opportunity to deny, criticise, and justify concepts. Boaler (2008) also states that students begin to appreciate the contributions of others and develop a heightened awareness of different ways which students may contribute mathematically. After this process has occurred the initial speaker may re-look at their idea and build a stronger argument (Mevarech & Kramarski, 1997; Rojas-Drummond & Zapata, 2004; Whitenack & Yackel, 2002, both cited in Hunter 2006).

To allow this to happen successfully, teachers have to ensure that there is a climate of shared respect, openness, and willingness to share ideas within the classroom (Boaler, 2008). There should be a set of pre-discussed rules and procedures for students and teachers to follow. This may be as simple as the teacher starting each session with a presentation explaining any new material or posing questions for investigation (Davidson, 1990). Students have to be able to participate in mathematical content discussions as well as the social discourse process of knowing when and how to explain, question, agree, disagree or challenge ideas (Hunter, 2006). Students need assistance and support in the beginning stages to know how to question each other in an open and supportive way. By asking these questions and collectively solving mathematics problems students develop a greater ability to interpret, analyse, reason, seek relationships and patterns within elements, explain, justify and predict solutions through the assistance of their peers (Lesh & Doer, 2003).

Conversely, the need to work independently is also seen as important within classrooms. Sfard and Kieran (2001) discuss the idea that articulating mathematical thinking to oneself can be beneficial. Students, when working independently, do not need to concern themselves with the many demands of their attention that can occur when they are working with the group. Some students may even become reliant on working with the group and may not develop their own mathematical disposition or sense of mathematical identity (Walshaw & Anthony, 2006). When students are working independently, it is useful for them to have a process to assist them think in a systematic and logical way.

In much earlier research, Polya (1957) stated his belief that students should acquire as much experience of working independently as possible. Although Polya's work was completed quite some time ago, the main ideas are still relevant in the teaching of mathematics today. He saw that there was a fine balance between leaving the student with insufficient help, therefore making no progress, and helping them too much and leaving no work for the student to complete. The aim of the teacher was two-fold in Polya's work. The first aim was to help the students solve the problem that they are currently working on and the second is to develop ability of students so that they can solve similar problems in the future.

There are many situations in which students are expected to work independently. This is often true in regards to summative assessment, where students have to be able to complete tasks independently to clearly demonstrate their own learning. By asking students to demonstrate their ability to solve a task, teachers are able to recognise any areas of weakness and they can also see which students are relying on others during group work. Johnson and Johnson (1990) use the term "hitchhiking" when students are getting the credit for work completed in a group situation without actually contributing themselves. There are situations that are best suited to either group or individual approaches to problem solving. If the goal of the Numeracy Project (MOE, 2007a) is to enable students to use mathematics in their everyday lives, then teachers have to provide the contexts that help them to achieve this.

When students enter the workforce they are going to be put in situations where they are working with others in a group, but there may also many situations where students would have to solve problems by themselves including earning and spending their own money. Sfard (2003) notes that three models of learning are incorporated into regular teaching as suggested by NCTM. These include "group and individual assignments, discussion between the teacher and students and among students, and exposition by the teacher" (NCTM, 1989, p. 10, cited in Sfard, 2003). The selected arrangement, whether individually or in groups, has to be continually monitored by the teacher to ensure that students are participating and achieving in which ever situation they are asked to work (Walshaw & Anthony, 2006).

2.5 Word Problems

Students need to be taught how to understand word problems and learn to identify key points, phrases or words within the problems that will assist them after initially reading the question (Moreau & Coquin-Viennot, 2003). Chapman (2006) recognises that there is often no focus on how the context of word problems is dealt with. She suggests providing strategies such as deleting unnecessary sentences to assist students to focus on what is relevant to solving the problem.

One of the purposes of giving students word problem activities in mathematics is to assist them in gaining skills to apply mathematical knowledge in 'real' contexts. As noted by Chapman (2006) word problems can be used "as a basis of integrating the real world in mathematics education" (p. 212). Their importance as a tool for learning mathematics, according to Chapman includes, providing practice with real life problem situations, motivating students to understand the importance of mathematics concepts, and developing creative, critical problems solving abilities. Hyde (2007) also builds on this in his research by making explicit connections between reading and mathematics. He claims that when students are reading they are making connections to their previous knowledge and understandings. However, Hyde also acknowledges that making meaning and comprehending mathematics requires "deep conceptual understanding of abstract ideas" (p. 44). To assist with the links for students, Hyde used a chart with the headings Know, What, Conditions, (KWC) (Figure 2.1). This chart supports students to link their previous knowledge to what they are intending to find out in a simple and effective format. In this chart students are recording what they know for sure, what they need to do, and any special conditions that have been mentioned in the word problem. By identifying these key aspects, students are able to plan how they are going to solve the problem and important points that need to be considered.

What do I know for sure?	What do I want to do, figure out, find out?	Are there any special conditions, rules or tricks that I have to watch out for?
Know	What,	Conditions

Figure 2.1 KWC Chart

Many authors (e.g., Hyde, 2006; Mevarech & Kramarski, 1997; Nunokawa, 2005) have noted the importance of explicitly introducing new concepts to students and linking these new ideas to previous learning. Through explicit introduction of these ideas, students are able to make connections to their current understandings and assimilate new concepts without thinking it is entirely unrelated and something completely new to learn. In his theories, Piaget (1973, in Leder & Forgasz, 1992) claimed that learning was based on intellectual development and occurs when students have the available cognitive structures necessary for assimilating new information. Students are then able to accommodate this information by changing or altering pre-existing ideas (Papalia & Olds, 1998).

The language used in word problems is recognised as one of the major reasons why students find them difficult. This factor is likely to influence success in mathematics (Martiniello, 2008; Pierce & Fontaine, 2009). Language can make understanding very difficult. When students concentrate on the meaning of each individual word they are less likely to comprehend the broader intention of the problem (Martiniello, 2008). Many of the words used in mathematical word problems involve unusual mathematics-specific words or ambiguous words where the context is vital when deciphering the meaning of a word or phrase (Pierce & Fontaine, 2009).

Another contributing factor in the perceived difficulty of word problems are the priorities of students when they are given these tasks. Schoenfeld (1991) notes that executing calculations is seen as more important than considering real life meaningfulness that can be gained from this style of problem. There is much detail to sort through when reading word problems. Students often distract themselves with the non-mathematical features present in a word problem rather than identifying and locating information which is considered indispensible (Moreau & Conquin-Vinnot, 2003; Silver & Smith, 1980).

Chapman (2006) also provides evidence that students had to eliminate irrelevant material from the word problems. Mevarech and Kramarski (1997) and Hyde (2006) include this area as part of their problem solving processes. By asking metacognitive questions and determining the importance of information provided in word problems, students are thinking about what they are being asked to solve and what information is purely there to add to the 'story'. Importantly, Chapman notes that there has been little attention given to the explicit teaching of word problems in mathematics and no focus on how the context is dealt with from the perspective of the teacher or the class. These are all aspects of word problems that make them challenging for students to solve. Word problems are given to students to enhance their creative, critical and problem solving abilities. They provide an opportunity for students to apply mathematical concepts to real life situations.

In her thesis, Lawerence (2007) summarises a variety of processes involved in solving word problems. She based these around the translation and solution phases of Mayer (1992) and the emphasis on the need to read and understand the problem which is highlighted by Reed (1999). Comprehension, translation and solution link with Polya's (1957) four stage model and other models for solving word problems that will be discussed later in this chapter.

Polya (1957) uses "understanding the problem" as the first phase in his problem solving process. He notes that this is a vital component when students are solving a word problem because if they are not able to identify the key features then they are not going to be able to complete any further work on it as they have no knowledge to work from to devise a plan to solve the problem effectively. Moreau and Coqui-Viennot (2003) partition word problems into different parts. They discuss solving information, considered indispensible for solving the problems, and situational information, which specifies extra details including actions and events in everyday concepts. When reading tasks, students have to be aware of these differing parts and ask themselves ‘What is the information that is vitally important and what do I not need to worry about?’

2.6 Metacognition

Metacognitive questioning and thinking requires targeted modeling by the teacher to enable students to understand what terms means and how to articulate what they are thinking during the solving of mathematical word problems. Wilson (2001, cited in Wilson & Clarke, 2004) refers to metacognition as “the awareness individuals have of their own thinking; their evaluation of that thinking; and their regulation of that thinking” (p. 26). This depth of thinking can be very challenging for students to develop. Yevdokimov and Passmore (2008) worked with high school students and concluded that some students, even at this level, found explaining their understanding of a concept or how they completed a problem-solving task very difficult to do clearly.

By modeling this type of questioning and thinking, teachers are enabling students to develop skills that will enhance their questioning ability and thought processes when working with others to solve mathematical word problems. When targeted questioning occurs students remain on task and concentrate on the mathematical reasoning involved (Doyle, 2007). One example of how teachers can explicitly support the development of metacognitive skills is provided by Wilson and Clarke (2004) in their study. The teachers in the study used a range of cards to help students to reflect on their thinking process. They divided the cards up into three categories: awareness, evaluation, and regulation. These categories allowed students to think back on the whole process they had just completed and reflect on how they started the process through to how they checked that what they did culminated in a realistic answer. Did they link the current problem to one that they had met before? Did they think about the direction they were heading in while completing the question? Did they think about what they would do differently next time?

Artzt and Armour-Thomas (1999) note that by working with others during this process, students are encouraged to listen to, respond, and question each other so they can evaluate, discard (if necessary) or revise ideas before taking full responsibility for the mathematical conjectures or conclusions that they eventually arrive at. Mevarech and Kramarski (2003) found that when students worked in collaborative settings where they received elaborated explanations the achievement levels were higher than when they were given final answers or no answers at all. The researchers also noted that when students used metacognitive questioning when working on word problems they gradually internalised the process and thus were able to use these with greater regularity in future word problem solving situations.

However, having students report on metacognitive strategies, and develop an awareness of how they can help their learning is challenging. Wilson and Clarke (2004) noted that as tasks become more challenging the need for metacognitive thought processes increases but when students are working it can be very difficult to record these processes. These authors attempted to use a 'think aloud' technique where students explained what they were thinking as they worked. They identified that although this worked at the beginning of their time working with students it was challenging to maintain as automatic responses became more common with familiarity with tasks.

2.7 Enhancing comprehension through reciprocal teaching

Palincsar and Brown completed the initial work on reciprocal reading in 1984. They believed that "reciprocal teaching, with an adult guiding the student to interact with the text in more sophisticated ways, led to a significant improvement in the quality of the summaries and questions" (1984, p. 117). Their four stage teaching method was designed to foster comprehension skills and monitoring activities. The four areas covered in their reciprocal teaching approach were: summarising, questioning, clarifying and predicting. This instructional strategy has been successfully used to enhance the comprehension of readers who are very good at decoding words but are not very good at remembering or relating to what is in the text (Kelly & Moore, 1994). It is a co-operative approach where members within the group take on the responsibility of being the leader and working with the rest of the group in the way a teacher would. This approach can also assist students to understand how to read from differing perspectives: for example from a scientist, mathematician, or historian perspective (Davis, 2007).

Reciprocal teaching is an instructional approach where students are guided towards greater autonomy within their learning groups. This approach explicitly teaches students to think metacognitively with the goal of deriving meaning from text (Davis, 2007; Palincsar & Brown, 1984). Students are taught each stage of the reciprocal teaching process: summarising, questioning, clarifying and predicting (Palincsar& Brown). Then they are expertly scaffolded to eventually manage the process with minimal supervision (Davis). When the teacher is introducing these stages to the students, students are beginning to imitate the structure that is expected. By using this form of modeling students are attempting a variety of ways to communicate effectively and then get the opportunity to imitate in a more appropriate form if necessary (Anghileri, 2006).

Greenway (2002) sees other benefits of the use of reciprocal reading in classrooms. She contends that this approach promotes autonomy. Following and gradually adopting the teacher's questioning skills within group situations enables students to take on the role of leader or teacher of their own group. This independence allows students to discuss ideas in a set way and to monitor their own cognitive processes. It also allows for students to discover a variety of approaches from others within their group. Davis acknowledges that this instructional approach is not just suited to reading lessons. This process of learning could also be applied to mathematics comprehension. The underlying principles of reciprocal teaching are based on Vygotsky's work (1978). In connection with his ZPD (mentioned in relation to scaffolding) students are able to assist each other in reaching new levels of learning. He believed that activities should be contextualised and not broken down to isolated components.

Solving mathematical word problems relies on the action taken based on what is read (Adams & Lowery, 2007). As students progress through their education, word problems become increasing difficult and demand greater reading skills (van Garderen, 2004). To support students in this greater complexity, many authors have used reciprocal teaching as an instructional approach in mathematics classrooms (Adams & Lowery, 2007; Erktin & Akyel, 2005; Hyde, 2006; van Garderen, 2004, Vilenius-Tuohimaa, Aunola & Nurmi, 2008). They have identified that there is a strong link between reading comprehension and mathematical word problem solving skills. Erktin and Akyel found that supporting students to develop their comprehension skills led to higher performance when solving word problems. Vilenius et al. qualified this by showing that the better the children's' reading comprehension skills, the better they were at solving word problems.

Through involvement in a group based approach, students are able to develop awareness of their own strategies when reading with the assistance of other group members. Students gradually increase their active engagement in their learning as they internalise the process that they have seen modeled (Davis, 2007; Palincsar & Brown, 1984). Linking this process to mathematics, would allow students to have access to a variety of strategies and to actively construct their own mathematical knowledge in unique ways. Students are encouraged by their peers and through discussions are able to transfer, evaluate and modify pre-existing strategies (van Keer & Verhaeghe, 2005).

2.8 Previous studies linking mathematics to reciprocal teaching

Critics of traditional ways of teaching mathematics, in which concepts and skills are often taught in isolation or students are told which strategy they are to use for a particular problem, argue that students are not supported to source information from their prior learning and put this knowledge to use in different contexts (Chapman, 2006; Hyde, 2006). Active learning requires intellectual challenge and curiosity, which are best aroused by having discussions with other students (Johnson & Johnson, 1990). Burns (1990) argues that learning mathematics requires students to create and then recreate mathematical relationships in their own minds. Carpenter and Lehrer (1999) build on this concept by saying that "tasks must be engaged in for the purpose of fostering understanding not simply for the purpose of completing the task" (p. 216).

Throughout the mathematical learning process ideas are combined and altered. Hearing other viewpoints gives students the opportunity to incorporate a range of interrelated concepts into their own developing knowledge pool.

This study investigates the use of reciprocal teaching as one way to organise instruction in the mathematics classroom. A number of earlier studies have focused on comprehension and these have guided this study. In those previous studies students were working in groups to discuss the concepts and language used to solve mathematical problems. Van Garderen (2004) worked with a teacher to implement reciprocal teaching into her classroom as she was concerned that students were unable to read and comprehend mathematical word problems and therefore were not achieving as well as they could be in state assessment tasks. To assist the students in the task, mathematical dictionaries were made available and charts were provided to record various aspects of the problem including information that they did know and information that they still required.

Hyde (2006) uses a similar approach with a KWC chart (see earlier figure 2.1) as part of The Braid Model for Problem solving (Appendix 1). He used this chart in the understanding the problem/reading the story section while working with young children who were in the second grade (American levels). He used this approach when first reading a problem to eliminate irrelevant information and help students see clearly what they were supposed to be finding.

The Braid Model for Problem solving (Appendix 1) is based on Polya's model of problem solving. Polya's model (1957) consisted of four phases, understanding the problem, devising a plan, carrying out the plan, and looking back. Hyde has developed these headings further in his model by including reading comprehension strategies to support each of the phases. He includes many questions in his model targeting both students and teachers in relation to what they are thinking at various stages of the process. Within this model there are a number of prompts to assist students and teachers including a range of problem solving strategies that could be used to help in a variety of situations.

Mevarech and Kramarski (1997) developed a similar approach to the comprehension of mathematical problems. They call their process IMPROVE which has seven steps. These are:

Introducing new concepts,
Metacognitive questioning,
Practicing,
Reviewing and reducing difficulties,
Obtaining mastery,
Verification, and
Enrichment.

The authors have devised this model based on “three interdependent components: metacognitive questioning, cooperative learning, and the systematic provision of feedback-corrective-enrichment” (pp.374-374).

Metacognitive questioning is seen as a vital aspect and therefore Mevarech and Kramarski have split this area further into three types of question: comprehension questions, connection questions, and strategic questions. Comprehension questions are designed so students can reflect on the question before they start solving it. During this time, students discuss what the question is actually asking them to do. They identify the crucial aspects required to answer the questions without concentrating on the details. Connection questions require the students to focus on this task and think about the similarities and differences relating to tasks that they have completed previously, thus helping them to differentiate between the surface and deep mathematical structures of a problem. Finally, strategic questions prompt students to think about how they are going to solve a particular question and what strategies they should use.

These three types of questions are well suited to mathematics, as they guide students to think about what the problem involves rather than using the overarching term of comprehension questions which is usually the case with reciprocal reading. By responding to these questions students are able to identify what they are being asked to solve and can apply their previous knowledge to the question and therefore figure out the best approach in that particular instance. By being able to work in a team, students are given the opportunity to share their thoughts and processes. They are able to discuss their reasoning and articulate how they came to a particular answer and others are able question the validity of what they are saying. Webb (1989, cited in Mevarech & Kramarski, 1997) says that shared activities also enhance the cognitive resources available to students with differing levels of ability and previous knowledge.

Students need to have a balance between being explicitly taught specific strategies, and having a problem where they have to apply the correct strategy and justify why that particular strategy is the best suited to that particular question. Mevarech and Kramarski (1997) recognised through their research the students gradually began to distinguish between surface and deep mathematical structures within a problem. When students are beginning to recognise these aspects they should also be beginning to apply previously used and taught strategies to problems. This is in contrast to traditional instructional settings as Schoenfeld (1991, cited in Chapman 2006) suggests, where classroom conditioning makes executing mechanical calculations a greater priority rather than considering the real life meaningfulness of what they are solving. Due to this conditioning, students are reliant on the teacher to guide them to solve the problem rather than trusting their own knowledge and ability to solve a problem in a group situation.

In another classroom study, Yevdokimov and Passmore (2008) used a conceptual framework for problem solving that involved four dimensions. Their four phases of the problem solving process are: first step of the solution, main information, completion of solution, and possible generalization. They see these four aspects as dynamic and interrelated, overlapping each other and influencing the factors involved with the students' implementation of the problem solving process. They believe that some of the factors that influence students' implementation of the problem solving process are: students' collaborative and individual work on a problem, time constraints, and teacher and student dispositions.

These factors need to be considered by teachers especially when using reciprocal processes and when they are setting tasks for students to complete. Teachers often do not allow students to work on a word problem for a period of time without success and hence students do not develop thinking beyond the basic level (van Keer & Verhaeghe; 2005). In reciprocal groups, students are expected to discuss their thoughts with other group members and this can be time consuming. They often assist the students and give them hints to reach the next step, which is not making them think beyond the obvious. Mau and D'Ambrosio (2003) looked into the ways that teachers perceive the learning of students in their classes and realised that the discussions taking place between students which was seen as rather straightforward thinking could have been developed into rich mathematical learning if teachers took the time to model it through formal mathematical lessons. Once problems have been resolved, the solver is then able to assimilate the information used during that particular problem either to build on previous knowledge of how to solve a problem to add another strategy to their existing knowledge bank that can be used in future problem solving situations (Lester & Kehle, 2003, cited in Nunokawa, 2005). But as is often the case, time pressures do not allow teachers to set enough time for students to work their way through the whole question. Teachers have a set amount of time for mathematics teaching and this often does not include time for intensive problem solving where students are asked to assimilate new understandings and make connections between the current task and what they have covered before.

Within the range of intervention methods investigated above there are many similarities in the sequencing of phases and what each phase involves. The four processes listed in Figure 2.2, target the same goals but they each use slightly different terminology to categorise each phase. The reciprocal reading process is mainly used for enhancing reading comprehension (Palincsar & Brown, 1984). This is a simple process but has a number of stages that are specifically targeted to reading and are harder to link to mathematical questioning as mathematics involves process thinking rather than some of the predictive skills that are required for reading comprehension. The other three methods, (Hyde, 2006; Mevarech & Kramarski, 1997; Polya, 1957) target mathematics. It is interesting to note similarities in terminology when they are put next to each other (Table 2.1).

Reciprocal reading	IMPROVE	Arthur Hyde	Problem solving process
Palincsar & Brown (1984)	Mevarech & Kramarski (1997)	Hyde (2006). Reading comprehension strategies	Polya (1957)
Predicting	Introducing new concepts	Making connections	
Reading	Metacognitive questioning	Asking questions	Understanding the problem
Clarifying	Practicing	Visualising	
Questioning	Reviewing and Reducing difficulties	Inferring and predicting	Devising a plan
Summarising	Obtaining Mastery	Determining importance	Carrying out the plan
	Verification	Synthesising	Looking back
	Enrichment	Metacognitive monitoring	

Table 2.1 Summary of Teaching Approaches

Hyde built on each of Polya's phases and linked in reading comprehension strategies to form The Braid Model of Problem solving (Appendix 1). Borasi and Siegel (2000) also identified striking parallels between inquiry-oriented views on mathematics knowing and learning and the view that reading is a transactional process of 'meaning-making'. These similarities can be a focus for teachers when introducing new concepts to students during mathematics lessons, as they are able to make connections to the strategies that students use when reading to assist with difficulties in mathematics, in particular word problems.

2.9 Summary

In summary, the methods used to teach mathematics are constantly changing as teachers develop skills to teach with greater effectiveness. Teachers are integrating effective methods to enhance the learning opportunities for their students. There is a move towards inquiry based group learning where the teacher is becoming a facilitator of learning rather than telling students what they need to be doing. Students, therefore, are spending their time discussing ideas and working with other students to share thoughts and possible methods to solve mathematical questions.

Word problems are often seen as challenging by students, but many researchers have reported on processes that assist students solve them. As students get older there is a greater link between reading comprehension and mathematical word problems. If students know some key strategies to assist them when solving these problems, then their confidence will increase and in turn the results in test situations should also increase. By using their peers, students are able to draw on a wide range of knowledge and are developing reasoning and articulation skills to assist others.

There have been many previous studies that have attempted to assist students in the comprehension of word problems. Many of these have common themes and have been adapted to suit particular situations. As Figure 2.2 illustrates, many of the processes have similar focus areas but are given a different title to suit the groups of students that they are intended for. It is interesting to note Polya's research that is over 50 years old, as this forms the basis of many of the newer versions. His model is very simple with only four stages but it remains an effective model to use when teaching students how to read, understand and solve word problems successfully.

The following research uses Polya's model as a base but simplifies the terminology to assist students to memorise the phases. Whichever process is being used, the desired outcome is the same. Teachers are aiming to teach students the skills required to understand what they are being asked when they are presented with word problems. They want students to identify the key ideas within the word problem and ignore information provided to set the scene. Comprehending word problems is an ongoing issue in mathematics teaching but is also vital to ensure success for students through their mathematical learning journey.

Chapter Three

Methodology

3.1 Introduction

When working in classroom settings, many researchers, (see Cobb, 2000; Collins, Joseph and Bielaczyu, 2004; Gorard, Roberts and Taylor, 2004; Steffe, Thompson & von Glasersfeld, 2000) have recognised the benefits of design experiments. Within design experiments or teaching experiments, as they are also known, researchers and teachers are able to make changes to interventions based on how the various strategies are working in a regular classroom environment (Cobb, 2000). Gorard et al. recognise that educational processes operate in complex social situations where there are many uncontrollable factors contributing to results and generalisations. Therefore the use of this approach encourages researchers to build on and make alterations to previous design experiments rather than dismissing them because they may not have achieved the desired results.

In relation to the thesis topic, Staples and Truxaw (2010) found that students were hindered in solving word problems because of their poor understanding of content and their inability to read and comprehend what they were given. This research aims to build comprehension skills to enhance the understanding of students when asked to complete work problems in mathematics by means of a design experiment to enhance comprehension in a reading context.

3.2 Justification of Chosen Methodology

The key objective of this research was to:

- Design and implement a process, adapted from the reciprocal reading approach, which will assist students when solving word problems.

Hence a design experiment approach was adopted as best suited collecting relevant data and answering these targeted questions:

1. How could the reciprocal reading process be adapted and incorporated into the mathematics programme?
2. How do students use the phases within the model when solving word problems?
3. What are the effects of the model on students' learning outcomes?

This methodology allowed me to work closely with one teacher to develop and implement a new teaching model in the classroom during a regular mathematics programme. I was able to note progress made by the students and could discuss with the teacher how he felt about using the model with his students.

Brown (1992) conducted design experiment research in regular classroom environments and noted that "central to the enterprise is that the classroom must function smoothly as a learning environment before we can study anything other than the myriad possible ways that things can go wrong" (p. 141). She also recognised that since there are many complex components within a classroom, that it would be too difficult to study one aspect and alter components without taking other parts into consideration. Design experiments allow researchers to study learning in context rather than using traditional experimental approaches where students are removed from the classroom and taught in isolation (Collins et al, 2004; Gorard et al., 2004). Using this approach allowed me to see how the intervention could be implemented in an operational classroom and what the effects were.

As this type of research is based on interventions the main purpose is to inform practice and allow teachers to trial and make changes to their pedagogy to suit the students in their classes (Brown, 1992; Cobb, 2000). Gorard et al. (2004) believed that design experiments could lead to classroom interventions being used independently by teachers in their own classrooms as teachers and researchers were able to make "in situ changes to the intervention allowing them to determine which were critical and non-critical elements" (p. 579) thus establishing how well the strategy worked.

An important aspect of design studies is that the teacher and researcher work collaboratively (Cobb, 2000; Gorard et al., 2004). The methodology allows the expertise of teachers to be incorporated within the production of interventions, through the collection of artifacts, for use in classroom situations. Collaboration is improved when the researcher is present everyday while the teaching experiment is taking place and is most effective, according to Cobb, when there is a short debriefing session collaborating with the teacher immediately after each teaching session. Post lesson debriefings enable the researcher and teachers to discuss future directions for the design experiment, therefore ensuring the experiment is beneficial to all parties, researcher, teacher and students.

Design or teaching experiments usually begin with a loosely articulated structure (Steffe et al, 2000). From there the progression of activities is based on the needs of the students, informed by information from the ongoing analysis of classroom events. Cobb (2000) describes two aspects to design experiments. These are (i) instructional development and planning guided by evolving instructional theory, and (ii) ongoing analysis of classroom activities and events. He saw two different levels to the experimentation process; the first being the micro level which occurs on a day-to-day basis, and the second at a broader level where the entire instructional sequence is put together and revised continually.

There are also some limitations to this research method. In her research on reciprocal reading Brown (1992) reflected on previous design experiments and one of the limitations presented was called the Hawthorne Effect. The Hawthorne Effect "refers to the fact that any intervention tends to have positive effects merely because of the attention of the experimental team to the subjects' welfare" (Brown, 1992, p. 163).

With reference to her own research, Brown countered this by explaining that if her research was truly showing Hawthorne Effect then she would have been able to identify which students would make the most progress based on the time given to parts of the intervention. Since this was not the case, the outcomes could not be directly linked to the time and priority on particular parts.

Collins et al. (2004) have identified another limitation to design research. They note that generally, this type of research is carried out in the "messy situations of actual learning environments" (p. 19) and many of the variables in these environments cannot be controlled. Due to the complex situations in which design experiments take place, Cobb (2000) notes that it is not always possible to account for differences between groups even though they have supposedly been given the same instructional programme. Moreover, as classroom environments are complex and unique it is also hard to replicate situations in future design experiments.

There are further replication issues that result from continual modifications to the programme as the design experiment progresses (Collins et al., 2004). Each design experiment will be slightly different based on the needs of all of those involved. Furthermore, participants respond differently to the design experiment.

When research involves the observation of people, these people must be ethically considered and researchers have to be aware of the 'observer effect'. This effect occurs when participants become conscious of being observed and alter their behaviour. To mitigate this effect, Denscombe (2007) advises researchers to spend time on site so that they become "part of the furniture" (p. 53). He also advises researchers have minimal interaction with those being observed.

The advantages of design research that had the greatest influence on my choice for the methodology for this research are the flexibility to make alterations to the research design during the research period and the collaborative nature of it. During the process I had the opportunity to make alterations to suit the needs of the students after having discussions with the teacher. We were also able to use an existing classroom where other factors are taken into consideration but are not required to be noted as this methodology allows for the wide range of factors that are already present in classroom environments.

3.3 Design of Intervention

This research involves the design and trial of a new teaching and learning process in relation to mathematics word problems. This situation is ideal for design experiment methodology as it gives the researcher the opportunity to use the environment as a 'test-bed' for experimenting with change to a specific factor. It also allows the researcher to trial a new idea or process in a real life setting that is pre-existing rather than instigating a new situation where the teacher involved in the research has to set up new routines.

This research approach also has clear links into action research. Action research encourages researchers to experiment through intervention and reflect on what they have encountered (Avison, Lau, Myers, & Nielsen, 1999). McNiff (1992) maintains that action research is particularly useful in teacher professional education as researchers are considering the needs of the students and are making alterations to programmes to suit these needs. However, a point of difference is that action research requires a full cycle to be completed before the research team reflects on what takes place, whereas design experiments allow alterations to be made within the cycle of intervention. For instance, in this research, discussions were held with the teacher participant after each lesson in relation to what was successful and how we might progress to maximise student development. Alterations were made to the activities given to students based on previous task success and what best suited their learning needs. Importantly, whilst the process model used by the students was not altered, how it was implemented and supported was varied as a result of discussions between the teacher and researcher.

Data gathering used a range of approaches including: observations, interviews, a questionnaire, and collecting student work samples. These enabled me to gather evidence of students' views and their work and allowed participants to elaborate on their thought processes. It also allowed me to cross check between what I was observing with what students were saying and producing in their work samples. By completing this triangulation, I was able to question particular students on their strategies or techniques without disrupting the flow of the work within their groups. The various approaches are matched to my research questions in table 3.1.

	Question	Method used to gather data
Design Experiment	How could the reciprocal reading process be adapted and incorporated into the mathematics programme?	Observations Documentary evidence, including researcher observations Audio recordings of group work Interviews
	How do students use the phases within the model when solving word problems?	Observation Interviews Work Samples Voice recordings Questionnaire
	What are the effects of the model on students' learning outcome?	Work samples Voice recordings Questionnaire Interviews

Table 3.1 Data gathering methods used in this research

Working in a class with one teacher best suited the research objective as it involved trialing a new process. As this new approach linked reading and mathematics comprehension strategies it was essential that the teacher was involved in learning about the elements in the process. Since the research investigated the effects of a new process on a particular group of students' comprehension of word problems, a large sample of students was not required.

3.4 Implementation of Intervention

The teaching resources used throughout this research were chosen from a selection of books printed by the Ministry of Education under the series title Figure It Out. These books contain contextualised word problems based on either a strand, for example statistics, or a situation. The situational texts, such as exemplified in Gala (MOE, 2001), have a variety of activities or problems based on that particular context that draw on a variety of mathematical strands. The combination of computation and analytical questions require students to use previously taught concepts. It also supports the teaching and learning of new concepts.

Each student resource book is accompanied with answers and teachers notes books. Books (see chapter 4 and in appendix 2 for examples) are often targeted at a particular level of the curriculum ranging between level 2 and 4. Some use a combination, such as level 2 to 3.

> The series is an important component of the Literacy and Numeracy strategy that focuses on clarifying the expectations for learners' progress and achievement, and developing teacher professional capability. (www.tki.org.nz/r/maths/curriculum/figure/index_e.php)

3.4.1 *Classroom context*

The research class was a Year Five class of 25 students within a large decile nine suburban school. The class composition was16 girls and 9 boys. About half the class had used the Figure It Out resource books in their previous classes.

Based on the New Zealand Curriculum these students should be at the end of level 2 and beginning of level 3 although, as with all classes there was great variation in their abilities (MOE, 2007b). The intervention took place early in the school year so students were still becoming familiar with classroom norms and the expectations of their teacher.

Typically mathematics classes are organised around 3-4 groups working with other students at the same stage of learning (MOE, 2007a). The group works with the teacher and then completes an activity to develop understanding. There is also the opportunity to practise previous skills or knowledge and often there are game based activities to reinforce concepts. How the teacher in the design experiment introduced the word problem-solving model is not usually how teachers would introduce a mathematics lesson.

In a regular mathematics class, students would be ability grouped and would probably have 3-4 sessions with the teacher during the week (MOE, 2007a). They will also probably have 3-4 follow up activities of which only 1-2 of them would be using Figure It Out resources. Figure It Out activities are often used at the end of a unit of work to carry out investigations to assess how well the students have learned new concepts. The ability grouping allows students to work at a level that tightly matches the next step in their learning trajectory (MOE, 2007a).

3.4.2 *Phases of intervention*

I allocated four weeks to work in the classroom with the teacher. This allowed the teacher to introduce the process, work on activities in their groups, and finally evaluate the process. This process was named the 'Figuring It Out' process as we were working with the Figure It Out resource. I thought this would make the clear link to students. This would not be able to be used for commercial purposes as the name is too closely linked to the name of the actual resource.

The process was introduced to students in a sequential and deliberate way, and modeled to the whole class. Each phase was explicitly taught. Initially the teacher used a scanned version of the Figure it Out activity projected onto a whiteboard. By doing this, all students in the class were able to see what was written as part of the activity and were able to make suggestions about what would be the most relevant information. The teacher used Statistics examples to begin with as this was the focus on the school long term plan (see Appendix 3). Later some number knowledge activities were used to assist with addition, subtraction and place value understanding.

The teacher went over the process many times, making the order of the steps very clear. He also referred each time to the questions at the bottom of the sheet to assist the students with their thinking (Figures 3.2, 3.3). Throughout the process, the teacher explained the purpose of reading the questions carefully and explained the plan that was required.

Throughout the introduction process, the teacher made links to real life situations. For example, during one lesson he made connections to the television show ‘The Apprentice’ (Mark Burnett Productions) which had just started showing on television again. He drew connections to not completing tasks correctly and the boss not getting what he or she asked for. If this happened too often the person concerned was ‘fired’. The students connected with this analogy and re-checked their work before the final check stage to ensure that they were not going to be ‘fired’.

The process was modeled in whole class situations many times with the expectation that students should have been able to follow the model when they had the opportunity to work in their smaller groups. Moreover, when the students were working in smaller groups, the teacher reminded them of what they had to do and used the terminology within the diagrams. This reinforced the process. At first it was noted that students needed constant reminders about checking their answers. However, as time went on, these reminders were used less frequently.

In the third week of the design experiment, the teacher and I decided to incorporate the problem-solving model into a regular class group teaching situation. The six groups were put into two larger groups (three of the smaller groups in each). The teacher then taught each group in relation to the activity they were doing and the level at which they were working. This is common practice in a mathematics lesson. Students still completed activities within their smaller groups but by teaching two groups rather than the whole class, the teacher was able to differentiate the learning to suit the needs of the students.

3.4.3 *Teacher Participant*

The teacher is an experienced teacher, teaching both in New Zealand and overseas. This teacher indicated that he would be interested in participating in this research and verbal permission was given after he had read the information sheet (see Appendix 4). In agreeing to participate in the experiment he expressed interest in finding a way to use the Figure It Out resource in an effective way. He noted that he had previously discovered that the students found the language confusing and therefore were not attaining the mathematical learning that is intended.

Before this research the teacher used the Numeracy Development Project (2007a) as the main focus of his teaching during number focused units of work. He grouped students according to ability into three groups and taught the groups on a rotational system. During other units of work based on statistics, measurement and geometry, he mainly used whole class teaching approaches, interspersed with group work. Having three groups within the class meant that each group size is about 7-10. Once the students had finished working with him in a group situation they were required to do independent tasks to support the teaching that had been the focus during the group lesson.

3.4.4 *Student Participants*

Students were invited to participate in the research. An information sheet (see Appendix 4) was sent home for both them and their parents to read so they understood what was involved. Permission slips (see Appendix 5) were also sent home and collected on return. Only two students elected not to be interviewed as part of the research.

During this study, grouping of students was based loosely on ability. The teacher assigned lower achieving students with mid-level and higher achieving students also with some mid-level students. The intention was to ensure that groups were able to work together without some members struggling to understand explanations and to avoid students becoming reliant on the higher achieving students to complete the tasks for them. This type of mixed ability grouping allowed for students of differing abilities to learn from each other through questioning and explaining (Slavin, 1996).

Groups of four were selected as it has been shown that such groups allow for discussions and sharing of ideas amongst all students (Kutnick, Blatchford, & Baines, 2002). In the second half of the study, the plan was to have students first work in larger groups and then work in their smaller groups to answer and check their responses. This replicated what would usually happen in the classroom where many groups are taught separately by the teacher and then asked to complete reinforcing activities while the teacher concentrates on another group. This process also allowed task differentiation so students were working on tasks that best suited their learning needs.

The groups were scaffolded in their learning. They went over tasks one step at a time to ensure that they knew about what they were supposed to do. Gradually they were invited to work within their groups and to highlight important information themselves. The teacher remained very deliberate in his explanations of tasks so that students were not left pondering how to solve questions by themselves.

The teaching of skills that are required when completing mathematics activities is an important component of mathematics lessons. During these lessons the teacher had to teach graphing skills and the correct formatting when presenting this information. It is important for the students to connect these new concepts with their previous learning. They also need to become familiar with how questions are worded and identify features that they need to be aware of when reading word problems. The language of mathematics can be ambiguous or involve specific mathematical vocabulary (Pierce & Fontaine, 2009). Words can mean different things within different contexts. Students can find this challenging if they have never read a particular word within mathematical contexts. It can be even more challenging when they have never seen the word before. For example, in statistics, both of these difficulties are evident with the words mean, mode and median. Students may have come across ‘mean’ in a different context but the other two terms are specific to a particular context.

3.4.5 *My role as researcher*

As the researcher, I was in the classroom to observe the teacher and students throughout the intervention period. When observing, I had to be very aware of the 'observer effect' (Denscombe, 2007) but the students became very comfortable with me in the classroom and were very open to sharing their ideas and thoughts when asked. They also accepted that they were being recorded at times and did not appear to let this influence what they were saying in their groups.

A vital component of the design experiment approach is the discussion between members involved in the research. I had meetings after each session with the teacher where we discussed what we observed and what we thought would benefit students in future sessions to enable them to use the new process effectively. During this time, I was also able to share my vision of what may help the students and the focus on each of the sections in the diagram. I worked with the teacher in deciding which activities were going to be used and how they could be introduced. The teacher provided the background teaching material and used diagrams and links that he felt would be most useful to support his teaching of new concepts.

3.5 Background information that influenced the design of the process

Table 3.2 summarises aspects of approaches noted in the literature review that I regarded relevant to learning mathematics (less relevant aspects are in light grey). As mathematics is conceptually based, the second two approaches have a focus on making connections and introducing these new concepts whereas the reciprocal reading approach does not need to have such a focus. In Polya's (1957) process there is an emphasis on understanding the problem to ensure the other phases can be completed. The processes with mathematical focus also have sections that highlight the importance of getting mathematics questions correct. Even though students are completing a process to solve a problem, accuracy is still important.

Reciprocal reading	IMPROVE	Arthur Hyde	Problem solving process
Palincsar & Brown (1984)	Mevarech & Kramarski (1997)	Hyde (2006) Reading comprehension strategies	Polya (1957)
Predicting		Inferring and predicting	
	Introducing new concepts	Making connections	
Reading Clarifying Questioning	Metacognitive questioning	Asking questions Determining importance	Understanding the problem Planning how to solve problem
	Practicing	Visualising	Solving the problem
Summarising	Reviewing and Reducing difficulties	Synthesising	
	Obtaining Mastery		
	Verification	Metacognitive monitoring	Checking back
	Enrichment		

Table 3.2 Summary of relevant aspects to mathematics of four main approaches

Each of these approaches emphasises the importance of questioning. By asking questions students are able to clarify what they have been asked to do. They also have the opportunity to make connections to prior learning. There are many similarities between strategies used to enhance reading comprehension and those used for the comprehension of mathematics word problems.

Each of these approaches has been used with a range of age groups and adaptations have been made to suit younger learners. The earlier students are exposed to this type of thinking, the better their comprehension levels will be as they progress through their schooling (Hyde, 2006). Yevdokimov and Passmore (2008) would support this, as they worked with high school students who were still unfamiliar with strategies to solve word problems.

Based on approaches listed in Table 3.2 I drafted my own process. This was trialed within my own class in the previous year (2009) using FIO activities. The sequence of stages were:

- Introduce new concept
- Question
- Practise
- Reflect
- Verify.

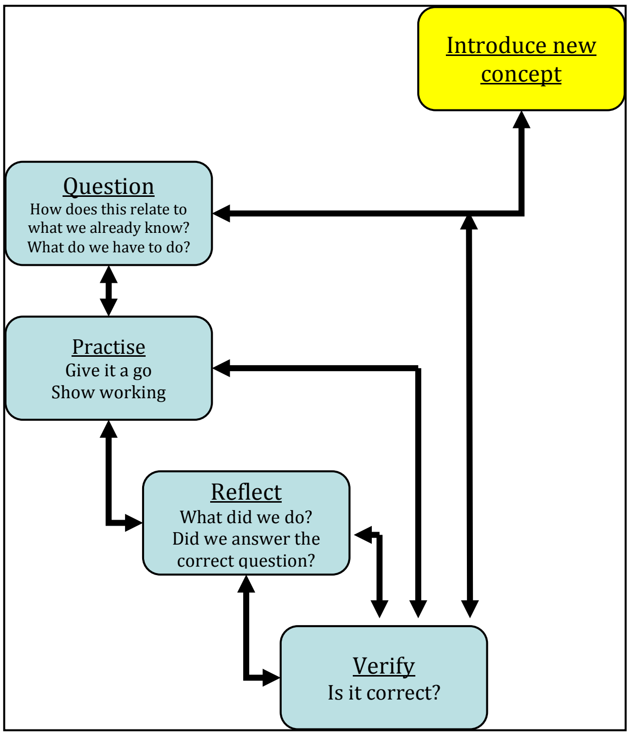


Figure 3.1 Pilot programme prompt sheet

They were presented in diagrammatic form where the questions or commands linked to each phase were put directly under the heading (Figure 3.1).

Having worked with the reciprocal reading process before, I was keen to keep it as closely linked with that process as possible. This would allow students to work with the new process in mathematics and make connections to what they had already used as part of their reading programme. Having already been introduced to reciprocal reading the students already had some strategies to question what they were reading and had developed some comprehension strategies.

I introduced this programme after using reciprocal reading in term 2 of 2009 to give students an idea of what questioning is required during this process. Students were following the complete reciprocal reading process (Palincsar & Brown, 1984). They were reading non-fiction texts and were focused on the questioning stage of the process where they checked understanding of meaning with others in their groups to ensure they had read it correctly and could clarify meanings before asking each other questions and summarising it.

I was aware that students already found mathematics word problems daunting. I also hoped that once they had comprehended the questions students would be able to resort to other previously known strategies.

I introduced this process during a fractions and decimals unit in term 3 of 2009. I thought this would be best suited to this process as there are often confusions around what is being asked for in word questions especially involving fractions. Students usually do not demonstrate patience when they attempt to solve problems (Dees, 1990; Doerr, 2006). They expect to find out everything they need to know after the first time they read the problem. They also do not take time to read what the question is actually asking them. Each phase of the process was introduced sequentially and explicitly taught so the students were aware of what is involved in each stage of the process.

Overall the process was useful to the students, but the language was complex and they were not confident when using it during lessons. Based on this pilot experience a simplified version (see Figure 3.2) was developed that included the following prompts:

- Make connections
- Read It
- Plan It
- Solve It
- Check It.

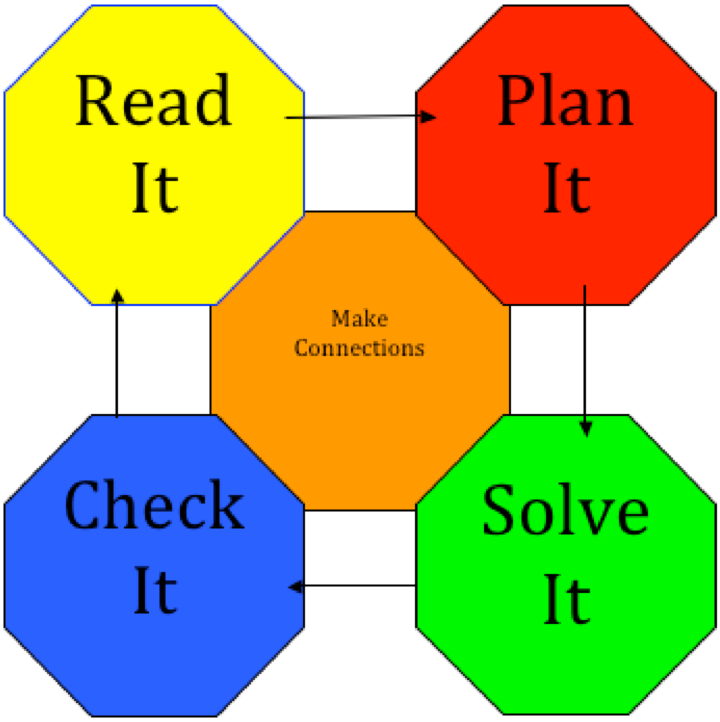


Figure 3.2 'Figuring it out' model

The simple terminology was designed to help students to remember the order in which the stages normally occur even though they intertwine at times. This process is based on the work of Polya (1957) with an added emphasis on making connections added and the simplification of language. Putting it in a diagrammatic form also assisted students to remember the process when working with it in the future.

To assist with the phases a supporting questions diagram was put directly underneath the previous diagram (Figure 3.3). On the sheet handed to the students, each of the hexagons was a different colour. These were:

- Read it – Yellow,
- Plan it – Red,
- Solve it – Green,
- Check it – Blue, and
- Making Connections – Orange.

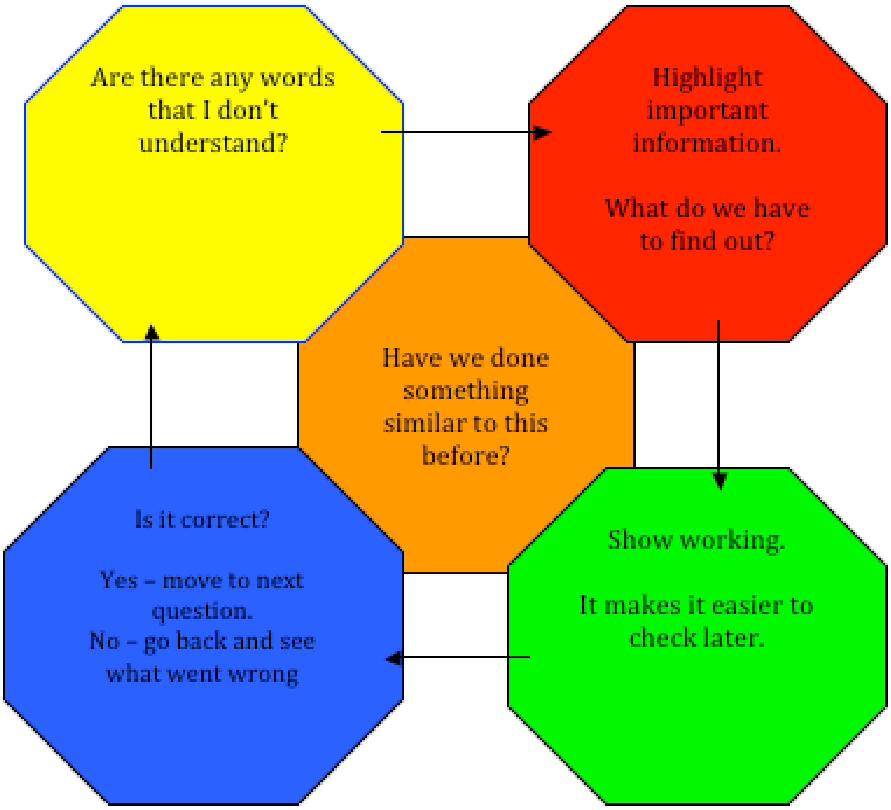


Figure 3.3 'Figuring it out' model supporting questions

The associated questions used the same colours to demonstrate the relationship between the two diagrams. The questions in this diagram were used as prompts when students were unsure of what they should be looking to do next, especially when they were being introduced to the process.

- Yellow – Are there any words that I don't understand?
- Red – Highlight important information. What do we have to find out?
- Green – Show working. It makes it easier to check later.
- Blue – Is it correct? Yes – move to next question. No – go back and see what went wrong.
- Orange – Have we done something similar to this before?

3.6 Ethical considerations

The school I completed the research in is the school in which I am currently a staff member. Ethical considerations have been taken very seriously to ensure that there was no conflict of interest between, the teacher, the students, and myself. During the research process I was on study leave so was not interacting with the students at any other time apart from the observations. As this research occurred at the beginning of the year, the students in the selected class had no previous contact with me as they were in a completely different area of the school to where I had been.

Clearance to progress with this research was obtained from Massey University Human Ethics Committee. Students and parents signed permission forms after reading an information sheet outlining the intention of the research (see Appendices 4 & 5). Students were given the opportunity to withdraw from the research at any stage and were given a clear outline of what the research was going to involve. Students' conversations were recorded and activities were completed in separate books so that the learning development could be tracked carefully. There were some slight alterations made to the process as the needs of the students were identified. This is common in a design experiment research approach.

All efforts have been made to ensure anonymity of participants. Pseudonyms have been used in reporting of the research findings. Care was taken when planning tasks to follow the existing school's long term plan so that participating or being part of the research class did not disadvantage students.

3.7 Trustworthiness and Limitations

Throughout the 15 observed lessons, I recorded the conversations students were having within their groups. The recordings allowed me to hear how the students were questioned each other, as they often do not share these conversations with adults standing around them. Recording these conversations minimised any 'observer effect', which may have occurred with me standing or sitting nearby.

The activities were selected based on the learning needs of the students and requirements of the long-term plan that the school teaches to (see Appendix 3). Copies of the daily plans can be found in Appendix 6. Time was a major issue in this research. The teacher and I were not completely sure of how long activities were going to take students to complete. Activities often lasted longer than expected, some spanning two days to complete the tasks. Flexibility and teacher judgement became very important as the days progressed. Students became quicker at completing tasks and were less reliant on the teacher to guide them through each of the steps as time passed.

For recording their answers, groups were given a scrapbook in which all their working was to be completed. They also used felt pens with an assigned colour so that both the teacher and I could see who was contributing and who was letting other members of the group do work for them. By working in a scrapbook, the students were less concerned with how they were laying out work and tended to concentrate on showing how they reached particular answers. They could also glue in the photocopy of the activity to show what information they had highlighted. The teacher was then able to monitor which groups needed greater assistance.

Students became familiar with the sequence on the 'Figuring It Out' model and throughout the lessons repetition of the process was encouraged. Students worked together to ensure that they were all following the process in the correct sequence and were able to direct each other to the correct stage within their groups.

When open-ended questions are asked in a questionnaire there is an opportunity for deeper exploration of key research themes (Glanz, 2003; Scott & Usher, 1999). The questionnaire used during this design experiment involved a combination of open and closed questions (see Appendix 7). The open questions often elaborated on the closed response asking students why they responded a particular way. This provided me with a clear indication of what thought processes students were using to reach a particular answer. It also gave them the opportunity to elaborate without anyone else hearing or seeing their responses.

I am cautious of the trustworthiness of the data as some students may want to give answers to please me as the researcher rather than give an actual picture (Mitchell, 2000). Students generally want to say what they think will please the teacher or other adults. Triangulating the data, provided an opportunity to see which students were thinking clearly about the process. That is, the questionnaire data would support interview information, which in turn, would be reflected in their work in scrapbooks and on recordings.

As in all design experiments the findings relate specifically to the context in which the research took place. However, the hope is that the process might be useful to and generalized to other classroom contexts.

3.8 Summary

This research used a variety of approaches to ensure the data gathered are a fair representation of thoughts and actions of all involved. A range of previous studies informed the process used to support students throughout this research. These studies have shown that interactions increase students' confidence when solving word problems. The new diagrammatic format developed ('Figuring It Out' model) provided a visually appealing and memorable base for students to work from. It also supported the explicit teaching of the teacher who was able to refer to the colours in the diagram and the sequence when he was teaching each part.

The design experiment methodology allowed me to work in an existing classroom while collaborating with the teacher in relation to what process would best suit the needs of the students. The data were collected using a variety of tools to ensure that data were reliable and fairly represented what was happening during the design experiment process.

By conducting a pilot process, I was able to see areas that could be improved before trialing the process with another teacher and a class of students. This pilot process was developed over time and resulted in the current model used as the core of this research.

Chapter Four

Classroom Findings

4.1 Introduction

This design experiment research was conducted to explore the effects of an instructional strategy. Multiple data collection strategies were used including: observations of groups' interactions during the mathematics lessons of one classroom, interviews and questionnaires. Interviews were conducted with students and the teacher to gain insight into their thoughts and opinions towards the instructional approach being used during the lessons. Students' work was documented so the progression of their learning could be easily monitored over the four weeks of lessons (Weekly planning, Appendix 6). Groups were recorded so that conversations within groups could be monitored and replayed to demonstrate the interactions and deliberation between group members without teacher assistance.

The purpose of the research was to determine if a new process (Figuring It Out model), based on previous research and linking to the reciprocal reading instructional strategy, was effective when teaching students to work through word problems. The teacher explicitly described and reinforced a process model that would assist students to solve word problems within their groups. The aim was that students would eventually use the same process when working on similar problems independently. This chapter discusses how the use of explicit modeling and teaching of the process assisted students in gaining confidence to work on more challenging activities throughout the research process.

In this research, I wanted to observe the implementation of the process and the effects it had on both the students and the teacher. Putting students into groups was initially an organisational tool to help support the introduction of the process but the groups had greater influence on the findings than I thought they would. Throughout this chapter the findings have included some factors influencing the work of groups although the main focus is principally on the process and how this was incorporated effectively.

4.2 Teaching and grouping

The teacher made deliberate choices relating to teaching techniques throughout the research. He began teaching the process to the whole class before dividing them into groups to complete set tasks. The grouping of the students was loosely based on ability, using Progressive Achievement Test data (New Zealand Council for Educational Research (NZCER), 2006). In this test, students are tested on a range of mathematics strands and are given an overall stanine score. The given score ranges from one (the lowest) to nine (the highest). This is a nationally standardised test and gives a generalised idea of student achievement in relation to others in the class, school and across the country. Based on these scores, lower to middle students were put together and middle and higher students formed other groups. These groups each had four members (with one group of five) as this number is considered to be effective (Cohen, 1986). As this research took place at the beginning of the year, the teacher was still learning the dynamics of the class and who would work well together. By supporting lower students with middle ability students, there was a feeling that they would be supported and be able to develop within their Zones of Proximal Development (Vygotsky, 1978).

Groups were assigned colours to help in the collation of data and along with this each student was given a pseudonym. The six groups were purple (P), blue (B), green (G), red (R), orange (O), and yellow (Y). Purple and Blue contained the lowest achieving students in the class and orange and yellow the highest. Throughout the observations it was informative to see how each of the groups dealt with new situations and how they worked together to complete activities.

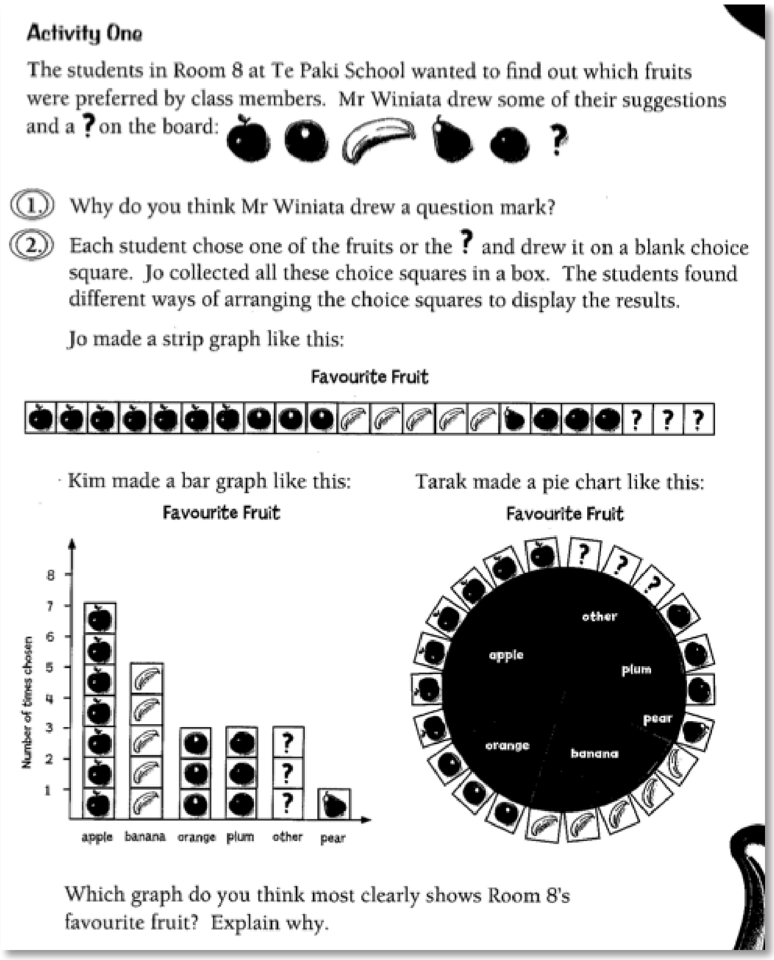
4.3 Introducing the model

As discussed in Chapter Three, the new intervention model (Figure 3.2) involved five phases. Four of the phases were positioned around the outside of the model showing the likely sequence of stages involved in this word problem solving process. Making connections was located in the middle showing that this was to be considered at each of the other phases. It was also used by the teacher when introducing new concepts within activities. Since this process was new to all the students in the class, the teacher spent two lessons introducing the stages to the students and explaining what each of the stages involved. Given that the teacher viewed the 'Plan It' stage as the most challenging and important of the stages, he spent the majority of the time, during the first two lessons, explaining what was involved at this point of the process.

> "This is the bit where you need to think about what the question is asking. You need to work with your group to decide what is important and what isn't important. This is the part that is a bit different. We are very good at reading it and then solving it. This is the bit where your reading skills come in." (Teacher recording, Lesson 1).

Anthony and Walshaw (2007) argue that opportunities to learn are influenced by what is made available to students. While explaining the process to the students, the teacher continually linked mathematics to real life situations to reinforce that the details of questions are of great importance. He was explicit when explaining what the students should be looking for and kept redirecting them to the questions in the bottom half of their process sheet (see Figure 3.3). He modeled how he would highlight relevant information and kept asking students if they thought the same information was important. He repeated instruction on all phases many times to ensure the students were familiar with the sequence of these and what was involved in the hope that they were able to make use of this model to support their understanding. Chapman (2006) emphasises the need to eliminate unnecessary information when reading word problems as a strategy to assist with understanding them. This skill is one that needs to be taught so students are confident in locating all relevant details.

At the beginning the students were not very confident when locating important information. This was evident when one of the students (P1) was asked to give an answer based on a paired discussion. When working on a task from the resource Figure It Out – Statistics, Revised Edition, Levels 2-3 (MOE, 2008a), the question asked: "Which graph do you think most clearly shows Room 8's favourite fruit? Explain why" (p. 1, see Figure 4.1). His answer was "apples". The teacher mentioned that it could be the answer to a question but another student (R4) then said, "It's not the question he was asked." This demonstrated to the students the importance of identifying the specific question being asked to ensure they are answering correctly. The teacher then linked this back to the "Figuring It Out" model chart identifying that the student did not plan the question very well as he had not identified all the relevant information.



Activity One

The students in Room 8 at Te Paki School wanted to find out which fruits were preferred by class members. Mr Winiata drew some of their suggestions and a ? on the board:

1. Why do you think Mr Winiata drew a question mark?
2. Each student chose one of the fruits or the ? and drew it on a blank choice square. Jo collected all these choice squares in a box. The students found different ways of arranging the choice squares to display the results.

Jo made a strip graph like this:

Favourite Fruit

Kim made a bar graph like this:

Favourite Fruit

Tarak made a pie chart like this:

Favourite Fruit

Which graph do you think most clearly shows Room 8's favourite fruit? Explain why.

Figure 4.1 Figure It Out activity (MOE, 2008a)

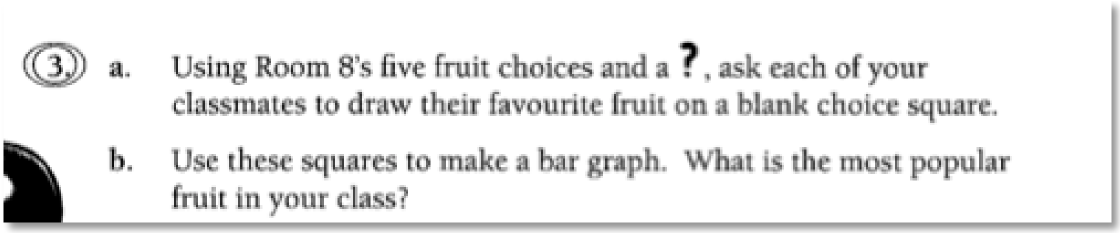
On the second day of introducing the new process to the students, the teacher reinforced what he had gone over previously and drew a grid on the board that would assist the students during the ‘Plan It’ phase. Figure 4.2 shows what the students considered to be important when reading word problems and planning what they had to do. By completing this chart, students had a guide showing what they should be looking for when reading word problems and what information was largely irrelevant. During this phase the teacher was expertly scaffolding the students with the expectation that students would eventually be able to manage the process with minimal supervision as Davis (1990) suggests happens during the teaching of the reciprocal reading process. Anghileri (2006) also notes that by using this technique students are likely to imitate what the teacher has said or done when they are working independently.

Important	Not important
Operation + - x / (Steps) How Numbers What	Names Who Where When

Figure 4.2 Chart that students and teacher developed during Lesson 2

Students have to read the question before they are able to sort through information. Hyde (2006) explains that students could not possibly determine the importance of information until they have read the question and thought about the nature of the problem. By completing a chart (Figure 4.2) the teacher was supporting students to generalise what they are looking for after the initial reading of the problem. This generalisation gave a starting point for highlighting relevant information as they had a list of key points to support them if they were uncertain. As time progressed, students became familiar with common elements in word problems and were able to locate relevant information without the support of the chart.

When students had the opportunity to trial highlighting the activity for themselves they managed to locate all the relevant information. This question clearly demonstrated where the students were making errors when answering word problems.



(3) a. Using Room 8's five fruit choices and a ?, ask each of your classmates to draw their favourite fruit on a blank choice square.

b. Use these squares to make a bar graph. What is the most popular fruit in your class?

Figure 4.3 Question from MOE, 2008a

In Figure 4.3 (see question 3b) the students were asked to make a bar graph and then answer a question. When they thought they had answered the question, they checked it. Not one group had written the answer that matched the information they had highlighted. They had completed the graphing task but they had not answered the question asked. The activity required students to create a bar graph using squares to show favourite fruits of members of their class.

The teacher took this opportunity to demonstrate the importance of answering the question. Using a current analogy, as mentioned in Chapter Three, he related this task to the television show “The Apprentice” (Mark Burnett Productions, New Zealand version on TVNZ, originally on NBC in America). In this show, contestants are asked to complete tasks to demonstrate who is best suited to be employed by the large business director who is the judge on the show. The teacher asked the students if they would be the strongest competitor if they did not provide the answer that ‘the boss’ had requested. Unanimously, the students said no and that the consequence for this would be being “fired” which is the phrase always used at the end of the show.

This link worked to stimulate students’ interest, and from that point on, they could be heard talking to each other before checking answers and coming back with “we’re not fired!” if they answered the question correctly. This analogy was one that the students were familiar with and served as motivation in a fun and non-threatening manner. Eggan and Kauchak (1999) identify challenge as a form of motivation for students. They state that “to promote motivation, success should occur in moderately difficult or challenging tasks, and it should be explained in terms of personal effort, increasing ability and well-chosen strategies” (p. 429). This form of motivation inspired the students to check their work and put in greater effort to ensure that they had met set criteria.

At the end of the sessions the teacher brought the students back together to reflect on what they had done. As he had been working with groups throughout the session he was formatively assessing them. This formative observation formed the basis of discussions during the lessons and at the end of the allocated time. Previous research has shown co-operative settings in which groups received elaborated explanations enhanced achievement whereas receiving final answers or no answers had no effect on achievement (Mevarech & Kramarski, 2003).

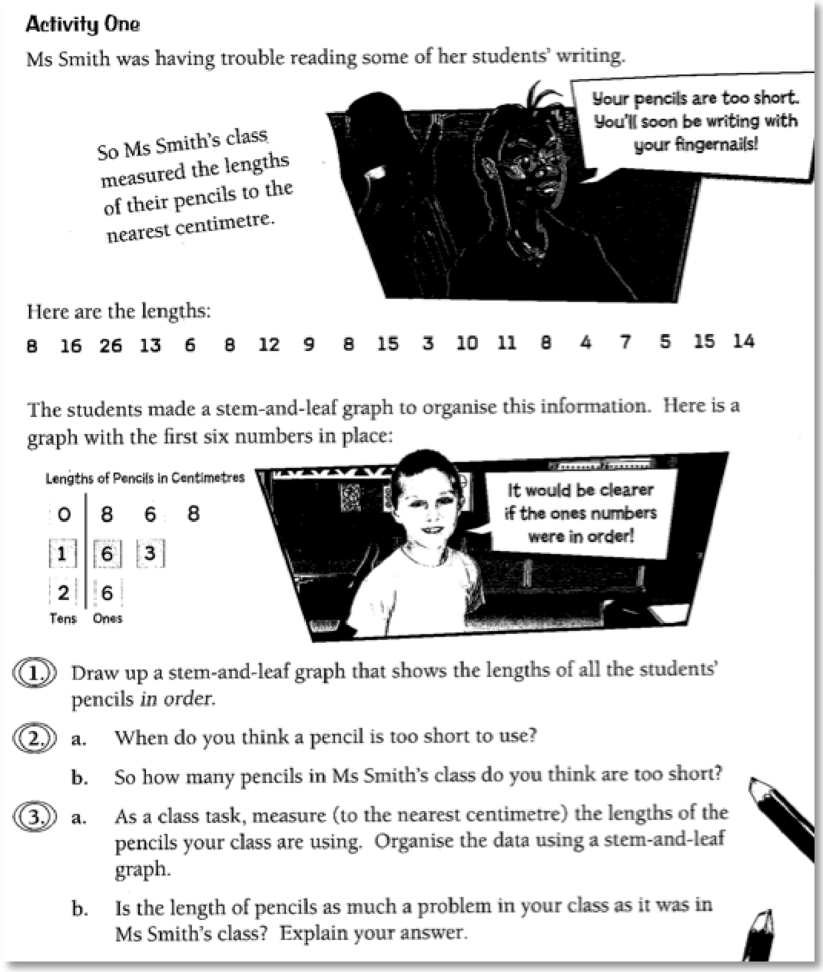
This formative assessment also influenced the future planning of tasks. The use of formative assessment to identify the needs of students is seen as critical to quality teaching in the Numeracy Development Project material (MOE, 2007a). At the end of each lesson the teacher and researcher reflected on the task completed during the day and how well the students had performed. This was an important component of design experiment research. From this the teacher and researcher identified future teaching points and based the next lesson objectives on these. The task selection was based on students' needs and the teacher had to ensure that he had checked the entire task to ensure the students had the knowledge required to complete the task. If he thought they may require extra knowledge of concepts, he included these in the introductory teaching sessions. Absolum (2006) has shown that formative assessment is used to gather dependable information about the group or individual's learning progress and the implications for current learning and what could be learnt next.

4.4 Development of groups over time

Initially when observations were being undertaken there was a lack of organisation within groups. This was especially evident when students tried to decide on techniques and strategies that would make tasks easier to complete. For example, when students were asked to create a tally chart based on given data, they lacked strategies to complete this in a logical and systematic way that ensured all data were recorded.

The “Petty Differences” activity (MOE, 2008b, pp. 2-3, see appendix 2) used in lesson 3 highlighted the fact that prior knowledge is important when solving word problems. This task asked for the pets to be put into categories. Red group did not recognise that budgies and canaries could go into the bird category. Very few students knew what an axolotl was and many groups had to be told that a ‘rotty cross’ was a type of dog. There were a number of discussions within groups to determine particular categories. When nobody in the group is familiar with a certain piece of information, the group can miss important information (Hunter, 2006; Anthony & Walshaw, 2007).

At the beginning students were very reliant on the teacher. They often waited for the teacher to tell them what to do. Progress, in terms of less reliance, was evident by the end of the observation period when doing one of the final activities, all groups completed the same activity, albeit at different times (MOE, 2008a, pp. 8-9, Figure 4.4). Groups went about completing this activity in different ways. Students demonstrated less reliance on the teacher and interacted in a style that Holton and Clarke (2006) would describe as ‘reciprocal scaffolding’. In these interactions students were assisting each other in smaller group situations rather than getting assistance from the teacher or waiting for the whole class group to discuss the problem that they were working through.



Activity One

Ms Smith was having trouble reading some of her students' writing.

So Ms Smith's class measured the lengths of their pencils to the nearest centimetre.

Here are the lengths:

8 16 26 13 6 8 12 9 8 15 3 10 11 8 4 7 5 15 14

The students made a stem-and-leaf graph to organise this information. Here is a graph with the first six numbers in place:

Lengths of Pencils in Centimetres

Tens	Ones
0	8 6 8
1	6 3
2	6

1. Draw up a stem-and-leaf graph that shows the lengths of all the students' pencils *in order*.
2. a. When do you think a pencil is too short to use?
   b. So how many pencils in Ms Smith's class do you think are too short?
3. a. As a class task, measure (to the nearest centimetre) the lengths of the pencils your class are using. Organise the data using a stem-and-leaf graph.
   b. Is the length of pencils as much a problem in your class as it was in Ms Smith's class? Explain your answer.

Figure 4.4 Copy of stem and leaf graph activity from MOE, 2008a

The groups that included the perceived lesser mathematicians actually completed the task with greater accuracy than 'higher' groups. They highlighted all the relevant information and sequenced the data as they were asked to do in the question. The teacher explained what was required in a stem and leaf graph in very similar ways with each group emphasising that a stem and leaf graph required data to be sequenced on each of the branches.

Lengths of Pencils in Centimetres	
0	3, 4, 5, 6, 7, 8, 8, 8, 8
1	0, 1, 2, 3, 4, 5, 5, 6
2	6
Tens	Ones

Table 4.1 Purple group stem and leaf graph

Stem and leaf graph	
0	8, 6, 8, 9, 8, 3, 8, 4, 7, 5
1	6, 3, 2,5, 0, 1, 5, 4
2	6

Table 4.2 Orange group stem and leaf graph

In this case the outputs did not match the 'ability groups'. These two charts (Tables 4.1 & 4.2) demonstrate the different approaches to the task of completing a word problem. Purple group had an accurate title and key included in their graph along with sequenced data. Orange group did not make the title specific and did not include a key or sequence data. Within this group were some of the higher achieving mathematicians in the class. Hyde (2006) emphasises that when working with word problems, identifying and acting on the relevant information is an important component and teaching all students skills to assist with this identification is intended to help with success when dealing with this type of problem.

The observations over the four weeks revealed that there was enhanced understanding of the process within all groups, although some groups were more successful than others. I have chosen to report the findings of each group individually although many of the points are linked and could be used as examples for any or all of the groups. Groups used the new process in different ways, demonstrating that perceived ability did not necessarily make students the best or most successful when completing tasks.

4.4.1 Purple Group

This group consisted of two low achieving students (P1, P2) and two middle level students (P3, P4). One of the students (P4) was quite a good mathematician but seemed to lack confidence to start tasks and did not check he was right to begin with. He may have been better suited to work with higher achieving students who would support his thoughts and boost his confidence to take risks.

In the follow up interview P3 said that using the new process had gotten easier over time and that they were probably going to continue using the process when they came across a difficult task in the future. One of the most difficult aspects in their group was actually reading the tasks. P1 and P2 were also having difficulty with reading. "Sometimes you get stuck on words when reading and you don't know what to highlight." (P3) P4 supported this view in a separate interview by saying that "sometimes you miss some of the words that are really important."

Students in this group gained confidence in following the process with the assistance of group members. This was evident in the recording taken during lesson 11.

> "We need to move onto number 2."(P3)
> "We better check number 1." (P4)
> "This is the answer – skip counting in 100s or adding 100 to the previous number." (P4)
> "I just wrote the yellow pipe is adding 100 to the number each time the machine goes round." (P3)
> "Yeah!! Tick it." (P2)
> "Onto number two." (P3)

After this period of time, the students were confident in interacting with each other and were trusting their interpretations. It was pleasing to hear the enthusiasm when they got the question correct.

An analysis of the final independent work samples for this group revealed that they were highlighting information. Two of the group identified the question they needed to answer but only one managed to record that. This demonstrates that the full process could not be retrieved quickly but with time and further explicit teaching it could be developed further and built into their long-term memory (Eggen & Kauchak, 1999).

4.4.2 Blue Group

This group consisted of two very low achieving students who were reluctant to participate in conversations (B1, B2) and two students who were not confident mathematicians but were willing to try new things (B3, B4).

In one session near the beginning, Blue group skipped the first question in ‘Petty Differences’ activity (MOE, 2008b, pp. 2-3, Appendix 2) until the teacher came along and went over each part of the question. He asked them to read the question and then asked what the important information was. He had to prompt this group to see what was relevant information. When they started following the question, they went to the teacher and asked him to tell them what they needed to do. The teacher responded with “Don’t ask me – the information is in the book, read the book. What do you need to highlight?” When they had highlighted the relevant information he directed them to the next stage. “Now you need to solve it.” (Group recording, lesson 3).

By the end of the teaching sessions (lesson 12) the Blue group members were asking each other questions as they were reading the questions. They all read the question out loud together and then were clarifying ideas. For example, the activity involved presenting data on a stem and leaf graph (FIO, 2008a, pp. 8-9).

> “Draw up a stem and leaf graph that shows the lengths of all the students’ pencils in order.” (All students reading together)
> “Does that mean we have to go like, 3 then 6 then 6, 6 then 8?” (B4)

These two examples demonstrated that even though the teacher may have sounded a little ‘mean’ in telling them not to ask him what they had to do, he was supporting them and letting them know that he had confidence in their abilities to make connections and solve problems without his assistance. By the end of the sessions, they were displaying confidence that may not have developed had the teacher explained each question to them at the beginning. Angier and Povey (1999) suggest that teachers often fragment problems for students to make it easier for them but in doing this, they are actually hindering the students' ability to work through the problems. Rather than lower the task demands, the teacher in this case was prepared to let the students attempt the problem independently.

When interviewed, B3 explained that she found solving the questions the hardest part, but she had attempted the new process when given an independent task to do at the end of the four weeks. This process allowed her to attempt problems that she may not have had confidence to attempt previously by providing a scaffold that she could trust and follow as she worked through the problem. She also mentioned that one member in particular was not very productive and never joined in with conversations. This made it difficult for the group at some stages as they were trying to include this person but were not getting the opportunity to learn from them.

This was an interesting group to observe and analyse as they were able to articulate what they were supposed to do, generally with prompting, but they did not transfer knowledge to independent work. Not one member of the group managed to provide an answer on the final independent activity. Two managed to identify and highlight the relevant parts of the text. This group will need much more explicit teaching and scaffolding to enable them to use the process consistently in either independent or group situations. An influencing factor in the achievement of this group may have been the lack of diversity between each group member. They were of similar ability and therefore lacked a range of knowledge that could have contributed to further discussions. Schoenfeld (1987, in Mevarech & Kramarski, 1997) has argued that diversity in prior knowledge amongst group members can be beneficial to the group as each student has something different to contribute.

4.4.3 Green Group

Green group consisted of middle achievers within this class. They were able to make connections to previous knowledge and contribute evenly within the group. They listened to each other and reached an agreement when there were any disagreements about how to continue with a particular question. They were also confident to start questions and make alterations when the teacher came and worked with them.

Early in the teaching sessions, this group was able to complete activities by following the new process. They checked with each other to make sure that they were carrying out the process correctly.

> "We've read it, we've planned it and now we need to solve it." (G4)
> "Yes, but does this make sense? Divide coloured parts, strip graph, show favourite pets." (G2)
> "It doesn't need to make sense – it is just the important parts that we need to do." (G4) (Recording of group work, lesson 5)

In a later session the group was asked to add numbers together including some numbers over 100. They provided an incorrect answer on the first question and the teacher then came and demonstrated a different strategy that would make it easier to solve the problems. This strategy involved using place value knowledge to expand the numbers before adding them together so tidy numbers could be added first.

> "Spread them all out and just work through them and change them like that as you are going." (T)
> "Thank you." (G4)
> (Recording, lesson 10)

Example of Green group's working from scrapbook after discussion with teacher		
210 + 6 + 9 + 64 =		
200 +10 60 + 4		
4 + 9 + 6 = 19	10 + 60 = 70	200 + 0 = 200
4 + 6 = 10 10 + 9 = 19	70 + 10 = 80	
9		200 + 80 = 280
280 + 9 = 289 ✓		

Figure 4.5 Example of solving equation using place value knowledge

The students were able to see where they had made an error in the first question and used the strategy that the teacher had shown them to complete the next question in less time and with greater accuracy. By looking at the working shown in the first question the teacher was able to build on the knowledge that the students already had in relation to place value and adding (Figure 4.5). He made connections to their previous working to ensure students included all the numbers as that was what caused the error in the first example. By providing a process that assisted the students to complete future questions, the teacher was acting as a 'scaffolder'. Anghileri (2006) contends that effective scaffolders are responsive to the needs of the students and are aware of when this support can be withdrawn.

Three of the four members of this group followed the full process to complete the final independent task. The fourth member (G2) managed to highlight the question but was not able to answer it. As observed the students were able to use the process to complete other tasks and they were using it to ask each other questions to clarify responses.

4.4.4 Red Group

This was another group of middle achievers. They listened to the suggestions that their group members made and were prepared to discuss which ideas would be the most effective before continuing with a given task.

At the beginning this group acted the same as all others in not identifying and answering specific questions that had been asked within the word problem. Over time they became used to following the new process, and wrote down clear answers to questions. This was evident throughout their scrapbook. They answered questions using full sentences and showed working to explain how they reached specific answers.

By writing down their working out processes they were able to check their work easily to see where they went wrong if they provided an incorrect answer. For example, they wrote, “the most popular choice is books for the library” (Written recording, lesson 6). When working with numbers this group wrote their thinking in full sentences. “Half of a thousand is five hundred and Room 7 think half will play throw the gumboot.” They then started to incorporate numbers within the sentences “¾ of 1000 is 750 and room 9 thinks ¾ of 1000 will go to the food stall” (Written recording, lesson 9).

Given that this group were so successful at answering the specific questions it was interesting to see that only R3 used the process to reach the required answer when working independently. None of the others managed to identify the question that they were asked to respond to. When filling in the questionnaire all members of the group were able to explain what the phases were and what parts of the process were useful. It is interesting to note that they did not transfer this knowledge to independent tasks.

4.4.5 Orange Group

Orange consisted of middle and high achievers. Based on interviews and questionnaire data, they were used to getting questions correct without having to plan what they were going to do. There were times when this group looked like they were not focused on what they were supposed to be doing.

This group was similar to Red group in relation to the clarity of recorded answers in their scrapbook. They answered questions using full sentences and clearly justified answers when they were asked for. In their answers they also showed the strategies they had used. For example “2 is 1/5 of 10 so 20 is 1/5 of 100 and that means 200 is 1/5 of 1000” (Written recording, lesson 9). By showing this thought process they were able to explain to others how they reached a particular answer and were able to check where any errors had been made during the checking phase.

Two group members, when interviewed (12/3/10), were extremely positive towards the Figuring It Out model. They said that initially they didn't understand what they needed to do, but at the time of the interview were able to grasp the key ideas in their heads and identify what they had to do. They were used to being able to complete tasks without having to physically highlight any specific information and now they knew exactly what they would do when faced with a challenging word problem. They identified that there was often more than one correct response so when they were checking it was hard at times because the answers in the book were quite long and contained many variations.

There is an interesting division in the way students in this group went about solving a word problem independently. O3 used the new process to reach the answer. O1 located and highlighted the question but did not provide the answer and O2 answered the question but provided no sign of highlighting or locating the question. This student may have felt sufficiently confident to answer without needing to highlight.

4.4.6 Yellow Group

This group contained middle to high achieving students. As a whole they formed the 'highest' achieving group. They did not appear to make connections to their previous knowledge. They spent a lot of their time worrying about superficial, organisational aspects of solving problems, for example, who was drawing the lines for the graphs and making tables.

An interview (12/3/10) with two group members provided insights as to why this group did not complete as much as other groups. Y1 preferred to work on activities such as these independently but, along with Y2, agreed that they could find the process useful, but only if they were working with other people. They were all good mathematicians and were used to getting questions correct. They may not have been familiar with the challenging tasks provided that required some thought and discussion. Doerr (2006) has noted that this is common in traditional mathematics classrooms where students were used to calculating answers quickly and not having to use a variety of strategies to solve challenging problems. Students in the Yellow group noted that there were many steps involved in these word problems and they had to understand them before working out the equation or task they had to complete. This was a new experience for them. In a separate interview Y3 agreed with the points that the other two had raised. In my observations of these students, the group dynamics appeared to hinder the work produced by them.

Throughout the observation period this group was making mistakes due to lack of attention to detail and to a desire to complete work quickly. They made assumptions about what the question was asking them to do without highlighting parts to make certain. Moreau and Conquin-Vinnot (2003) note that students who are used to being given equations to solve often get distracted when completing word problems as there are many non-mathematical features included and students find locating the relevant information difficult. During the activity where they had to categorise fish (MOE, 1999, p 23), this group required much scaffolding to guide them towards a solution. They were not confident at creating their own categories and answering the questions.

All students in this group responded with a correct answer when given an independent word problem. Two completed the highlighting process while the other two did not. These two may have felt confident to continue without highlighting as they are capable mathematicians.

4.5 Teaching groups – sorting groups into two bigger teaching groups

Once the students had practised using the model for nine sessions as a whole class, the teacher felt that he would be able to work with two separate groups. This was moving towards how mathematics classes are usually run with differentiated tasks and with the teacher spending time teaching new concepts or strategies with a smaller group to greater suit their learning needs. The six existing groups were put into two teaching groups. Purple, blue and green were in one group and red, orange and yellow in the other. The groups continued to work on tasks in their smaller group but the teacher worked with half the class at one time rather than all together.

The teacher was then able to alter activities to suit the needs of specific groups and target teaching points to their needs. During this teaching time, he was also able to go over the activity with the students to ensure they understood the terminology being used and explain what words meant if they were not sure. He could also advise the groups on a range of strategies that could be useful to complete tasks and make connections between tasks that they had already completed and the current one.

The next step for the teacher in this class would be to incorporate this new process into his regular mathematics programme, linking to the Numeracy Development Project that the school uses to teach the number components in their long term plan. The next unit of work was based on place value knowledge so the teacher chose to use the Numeracy teaching books (MOE, 2007a) in conjunction with FIO books as supporting material and a range of other activities to enhance the learning of students within his class.

4.6 Summary of achievement

By the end of the teaching sessions many more students were taking note of what they were asked to answer. In an independent task given at the end of the observation period, 11 out of 23 answered the given question. Eight of these had used the full process and had highlighted the question to demonstrate their understanding. The other three were quite capable mathematicians who may have just made a mental note rather than physically highlighting.

As none of the groups managed to answer the question in the first activity they were given, I see this as significant improvement. Considering this was a focus in the classroom for only four weeks, the students had assimilated the new process and were willingly using it to assist when solving word problems. With more time and continued links to previous knowledge, the number who did so would hopefully increase.

Lower achieving groups showed less uptake of new process. Is this because they are less able mathematically so they are not as confident with what the question is actually asking them to do, or does it take them longer to gain new knowledge and apply it? There may also be a link to the language used in word problems and how some students are concentrating on decoding the text rather than concentrating on what it is asking them to do with information.

When listening to group work from voice recordings there was an increase in confidence throughout the research period. The students were comfortable working together to read the question and start answering questions. They started work quickly but some groups were distracted during the middle of sessions. The students were able to articulate their thinking when the teacher was working with particular groups and they were also beginning to go back to questions they had checked and were incorrect to see where they had made errors. They also were beginning to seek assistance from the teacher when they were unsure of next steps but instead of asking a general question to get him to break down the question, they were asking specific questions relating to the question.

4.7 Teacher Observations

At the beginning and end of lessons the teacher made explicit connections to previous knowledge and skills that would be required in sessions to follow. Absolum (2006) argues that sharing this information with students enables all group members to have a clear purpose and know why they are working towards the shared goal. At the beginning of lesson 4 the teacher demonstrated explicitly the way he would collate data in the form of a tally chart. He went through the process of putting the first section of data on a tally chart and articulated his thought processes in a think aloud technique (MOE, 2003). At the end of the session he made connections to tally charts and then introduced pie graphs as a way of displaying data. This was an introduction to the following day’s activity where students were taking data from a tally chart and displaying it in other graph forms.

By introducing this concept earlier, the teacher felt that he was able to spend more time facilitating groups and less time setting up at the beginning of the session. He simply had to refresh their memory in the next session before allowing them to work in their groups. During this activity (MOE, 2008a, p 2-3) all of the students were engaged in the task for quite a substantial period of time. They enjoyed the flow when starting with a strip graph, turning it into a pie graph and then cutting it up to make a bar graph.

As most of the groups were able to continue the task without teacher intervention there was the opportunity for the teacher to scaffold the learning of some of the lesser able groups. Anghileri (2006) has noted that the role of the teacher is changing "from showing and telling to responsive guidance in developing the pupils' own thinking" (p. 33). The teacher was able to help the other groups with the planning part of the process and ensure they were able to get started. He could also move around the groups to ensure they were completing the graphs correctly and point out areas that they could develop, for example, titles, keys and labeling. Borasi and Siegel (2000) also noted in their research that teachers in inquiry-oriented classrooms spent less time transmitting information and spend more time supporting students' inquiry. In this environment students are encouraged to take risks and have a greater degree of responsibility as they participate actively in their groups. An example of this was seen in lesson 5 when the teacher was able to visit each group as they were working and was available when groups had questions to ask. He gave prompts to assist the students in completing a pie graph and linked back to the introduction with the success criteria of what made an effective graph, such as a key and title.

One of the challenging aspects for the teacher during the introduction of the process was guiding students towards eliminating irrelevant information. The teacher prompted the students when working in a whole class or larger group situation and also prompted when moving around the groups when they were working independently. To eliminate information, students had to be able to identify what the most important information was which was difficult for some students who were not used to this style of questioning.

The central part of the 'Figuring It Out' process diagram (Figure 3.2) is making connections. This includes making connections to previous work as well as wider general knowledge. The teacher linked lessons throughout the observational period of time. He also made connections to the outside world and used poor examples of graphs so the students could identify what was wrong with them and transfer this knowledge when checking their own work to see if they met criteria relating to the presentation of particular graphs. Baker and Chick (2006) along with Doerr (2006) highlight the importance of teachers being aware of relationships between topics, procedural, and conceptual knowledge. They argue that teachers have to be prepared to support the many directions that student may take through their investigations and application of strategies to solve word problems.

The use of scrapbooks to collect examples of written work assisted when planning future lessons. By including the photocopied pages from the FIO books, in the scrapbooks, the teacher was able to see what information the students were highlighting as important. Students were not always highlighting but were reading and discussing as evident on voice recordings. Ideally, this is the direction students should be heading towards where they are confident in discussing information without having to highlight it. In future lessons when they are confident with using the process they should be able to "highlight in their head" (O1). O1 and O3 recognised that the photocopied pages were only there to support them at the beginning of the process as they would not be able to have them available all the time and they could not highlight the books. They also noted that this was the reason they were discussing highlighting in their heads and making notes if necessary.

4.8 Instructional Activities and Resource Use

Throughout the data gathering period I had discussions with the teacher in relation to the relevance of FIO activities. Overall, the consensus was that they had a place in a mathematics programme but would possibly only be used once a week as a supporting activity within the regular programme. The teacher noted that the resource would be better suited once the students had learned the range of skills required to complete the task so that the students could apply knowledge. He also acknowledged that there were some excellent teaching points within the activities that could be used as a base from which to begin teaching.

The teacher felt that some of the tasks included in the FIO books were quite complex and required many steps before an answer could be reached. As a result of this complexity, tasks often took quite some time to complete, in some cases over an hour. Many of the activities required a large amount of data analysis. This was a new experience for many students and because of that, explaining their answers and recording responses logically was yet another skill they required. The students also had to interpret answer books. Many questions asked for their opinion, and this did not necessarily match exactly the suggestions given in the answer books. They had to interpret the answers given to see if their responses were similar. To assist with the checking process, the teacher had to be aware of possible students' answers and be clear what was acceptable so that he could clarify any points that the students were unsure of.

Asking each other "Does the solution make sense?" assists students before reading the answers (Mevarech & Kramarski, 2003). If students are clear in their thoughts and trust what they have done, then they are able to go and check the written answers. Part of the 'check it' phase is to ask each other does our answer make sense in relation to what is asked for in the book? Should we go and check it now? Have we answered what we are supposed to? Students learned to re-read the question when their answer was incorrect, to see where they went wrong. Some students made small errors that contributed to mistakes while others went through the whole process again.

Throughout the course of the lessons, group dynamics started to influence success of various groups when completing tasks. The Red and Yellow groups were host to interesting debates surrounding various activities. At some points through the process group members were left out of discussions or lost interest due to the lack of support within their groups.

4.8.1 National Standards at Year 4/5

The National Standards document states that by the end of Year 4,
"In contexts that require them to solve problems or model situations, students will be able to:

- Investigate questions by using the statistical enquiry cycle independently:
    - gather and display category and simple whole number data
    - interpret displays in context”

(MOE, 2009, p. 27)

This is the level that all students should be at in the design experiment class as they are at the beginning of Year 5. Most of the students are able to do this, although a few are not meeting the ‘independent’ aspect.

These students are now working towards the next standard that differs slightly from the previous one. It states that by the end of Year 5,
“In contexts that require them to solve problems or model situations, students will be able to:

- Investigate summary and comparison questions by using the statistical enquiry cycle:
    - gather, display and identify patterns in category and whole number data
    - interpret displays in context”

(MOE, 2009, p. 31)

The activities within FIO books introduced students to summary and comparison questions and asked them to identify patterns. Most students found generalising the patterns a challenging task. They were either very broad in their statements or were not clear in what they were explaining. This skill would be developed through more experience. The examples in the National Standards in the ‘by the end of Year 5’ section were a stem and leaf graph and dot plot to show grouping of data. Students used both of these graphs during observed lessons and were gaining confidence when displaying data using them and interpreting data from pre-existing graphs.

4.9 Linking to the curriculum plan

This process had to link to the long-term plan of the school where the research took place (Appendix 3). Through our early discussions, the teacher and I chose to use statistics activities as this was scheduled for the period during which the research took place. We also started to include some number knowledge activities as students were moving onto place value in the next phase of their mathematics programme. The teacher felt that the process would be able to be used in any area of mathematics. He thought he would now include the FIO books into his regular programme with greater frequency.

4.10 Student Perspectives

The opinions of some of the students that were gained through interviews have already been mentioned in the analysis of the groups within this chapter. The students also completed a questionnaire (see Appendix 7), which asked questions in relation to the new process and the group environment that they were working in.

The questionnaire results supported the focus put on particular phases during the teaching sessions. Students overall agreed that 'plan it' was the most important phase followed by 'read it'. Nobody thought that 'solving' or 'checking' were the most important. During the teaching sessions, the teacher had emphasised the planning phase, as this was new to the students and a phase that most of them would not have been encouraged to use previously. All responses to the questionnaire can be seen in Appendix 8. From these responses it is clear that students understood the reasoning behind each of the phases in the process. They are able to articulate the purpose of each of the phases and know why it is an important component when solving word problems.

Students were asked to elaborate on why 'plan it' and 'check it' were important phases. They came up with a variety of reasons including:

- check it is important so we learn from mistakes (P2)
- I think planning is good because you might miss read something and get it wrong (B3)
- Plan it so you know what your important information is and you know what you're doing. Check it so you know if you are wrong or right and so you know what you did wrong (O2)

- Plan it is important because it helps us to understand it. Check it is important because you get to know what you did wrong so you know for next time (Y2).

When asked if the process had made solving word problems easier 21 of the 23 students stated that it had. They again had a variety of reasons supporting this:

- because planning helped me think about the important parts (P4)
- it helped me practice my different graphs. It helped me if I'm ever stuck on something (R1)
- it gave me a new way to think about the answer, I thought plan it helped the most (R4)
- well it helps when you do one bit at a time instead of rushing ahead and going onto the next question (O2)
- because it helped you to know what the important parts are (Y1).

One of the intentions of this intervention was to build student confidence when solving word problems, particularly in working with activities in FIO books. When asked their opinion of this 21 said that they felt greater confidence now.

Group dynamics was an interesting factor throughout this research. When asked if solving word problems was easier in a group 14 said it was and 9 did not find it easier. This was not as positive as the feedback towards the process. Students were able to identify reasons that the groups were not so successful. Some did not listen to other group members and there were some students who wanted to do everything and therefore other group members felt left out. This caused some disagreements that took time to solve. There was also a level of distraction within some groups, as they were not always concentrating on the task.

The suggestions given by the students in favour of working in groups matched reasons stated in Angier and Povey's research (1999). These included:

- It's better in a small group… you make up for each other's faults (James, p 154)
- You get it explained more clearly [to each other] (Dean, p. 154)
- Talking to other people helped me understand (Louise, p. 154).

Students in this design experiment responded in similar ways. For example:

- because you can discuss your answer with them (Y1)
- because I can't work this out myself so when I have somebody else we can do more than one thing at a time (R1)
- everyone has ideas so it is easier to solve it (P3)
- because everyone shares ideas so it's easier for everyone (P1)

However, having a range of opinion seemed to be both a help and hindrance in groups. When some groups had more than one idea they lacked the group skills to discuss variations and agree on a common answer.

- everyone fights over the answers (O2)
- it is hard when too many answers come at you, it's easier to get on with your work (Y3).

As the students were gaining confidence in using the process, the disputes started to decrease as they were referring to the book and the actual question they were asked to complete.

4.11 Teacher Perspective

Throughout the process the teacher and myself discussed various aspects of the design experiment. He appreciated the step-by-step nature of the process and therefore was able to explicitly teach each phase before letting the students use it independently. At the end of the observational period, he acknowledged that he had built his own knowledge of how to incorporate this process into his regular programme and would be using this type of activity with greater regularity.

The teacher commented that he would have liked to divide the focus time with two weeks to introduce the process and then have a break and re-visit it for another two weeks to see how the students had retained the information. He also thought that altering the groups might have been beneficial as students spent four weeks working intensively with the same group. For some groups this was an advantage but for others it contributed to some disagreements and it would have been interesting to see how different combinations dealt with the word problems.

When looking at the scrapbooks, the teacher recognised the improvement made by the students over the period of four weeks and was pleased with the increased level of independence demonstrated by the class. He enjoyed being able to move around the groups with the understanding that other groups knew what they had to do in order to solve the word problems.

4.12 Summary

The teacher and students found the newly introduced process beneficial. By explicitly teaching each phase of the process, the teacher felt that students were comfortable following the diagram and were using the supporting questions when working with word problems. By having this supporting document, students were able to attempt word problems that they had not been confident doing previously.

Although the focus of the research was the implementation of a new teaching and learning process, the ways in which the students worked together towards the goal seemed to be one of the major influencing factors. Most students learned how to work successfully in groups and were beginning to have faith in the knowledge of others. The teacher therefore had the freedom to work with individual groups while trusting the other groups to continue with the task they had been given. This also allowed the teacher to target teaching to individual or small group needs rather than trying to cover many points in one lesson.

This research highlighted the base of effective teaching required for any new process to be successfully introduced. These include: grouping students effectively, explicitly teaching the new process to ensure understanding, and using formative assessment to guide future lessons to best suit the needs of the individual student or groups of students. Through a combination of these factors the achievement of the students over the four weeks in which the research took place was significant. Half of the students went from not writing an answer that was asked for in the first session to answering correctly by the end of the unit. Many of the other students showed signs of implementing part of the process and with a little more explicit teaching would be able to provide a final answer as well.

The 'Figuring It Out' process gave students a framework to work through with each word problem. The simple and clear stages meant that students knew what was required at each stage and were able to identify the features that were vital within each word problem. This support allowed students to confidently attempt word problems and rework sections of the problem that were not correctly completed the first time. Overall I feel that this is a successful model that would be useful and easy to implement in mathematics programmes in primary schools.

Chapter Five

Discussion and Conclusions

5.1 Introduction

The purpose of this design experiment was to implement a new process relating to reciprocal learning and to see how students would use a new process to assist them when solving word problems, in particular those in Figure It Out resource books (MOE, 2001). This study focused on one class and the various groups within the class along with the teacher in implementing a new process. Whenever a new process is trialed, issues and questions are raised. While observing this class many questions were raised by both the focus teacher and myself.

Overall, the introduction of this new process was perceived to be a success since both students and teacher found it useful. The process was introduced in a way that suited the teacher and the students in the class. In fitting with the design experiment methodology, the teacher had a major input into how the process would be used and taught to students. Teachers all have their own interpretation of what works best for them. Accordingly, together we developed this intervention to suit this particular class and the teacher. Therefore, the approach taken here is not the only way the process could be used.

This chapter discusses the issues, complications, and strengths of the 'Figuring It Out' process that are highlighted in relation to the key sections in the findings. The final section of this chapter offers conclusions in direct relation to the key objective and key questions that have been the central focus throughout this research.

5.2 Teaching and grouping

As the design experiment progressed we noticed that students' reading level turned out to be one of the limiting factors when reading and answering word problems. Students P3 and B3 identified this as an issue in their interviews with me (12/03/10). Students struggle with words makes it difficult to solve mathematics word questions. This is a factor that teachers need to consider when giving students word problems and when grouping students. Pierce and Fontaine (2009) take the issue further by drawing attention to the ambiguity of some of the language used in mathematical word problems and the use of unusual mathematics specific words that are also used. By focusing on the 'read it' phase, in the research students identified words with which they are unfamiliar. However, this phase was found beneficial when one or more of the students within the group were able to clarify the meanings but it was not as useful if all members were unsure of specific meanings. To preempt this difficulty, the teacher, on many occasions, discussed terminology during teaching sessions that he had recognised may need clarification to enable students to identify the important and relevant information.

In considering the apparent correlation between lower reading levels and being unable to complete word problems, the study prompted us (the researcher and teacher) to wonder whether it was due to the lower achiever being unable to read the words or is it a comprehension problem where they are not making connections between what they read and what they have to do? To assess this, we would have to look at simple computation work and compare this to word problem examples while at the same time looking at the reading comprehension levels of students.

In relation to grouping we wondered about the optimum size of the students groups. Was four too many (Kutnick et al., 2002)? Would two or three have been more effective? There were times during the observations where it may have been more effective for students to work in pairs or threes to ensure that all group members were engaged in the task. There was a danger when there were four in the group that one or two within each group were not contributing to discussions or they were relying on the other group members, or as Johnson and Johnson (1990) would put it, they were "hitchhiking".

This research was conducted early in the year and the students had not had many opportunities to work with other members of the class. It may have been more productive if the students had greater familiarity with their group members. There were some issues that they had to work through in ensuring that all students were able to voice their ideas. Throughout the process they were developing group skills along with mathematical skills and knowledge.

5.3 Setting up the new process

At the beginning of the discussion of the implementation section in the previous chapter I noted that the teacher saw the 'plan it' phase as the most important phase in the process. This phase was given greater time allocation than the other phases. The teacher emphasis was reflected when students completed the questionnaire at the end of the research as the majority put 'plan it' as the most important phase. If I were replicate the research I would emphasise that each of the phases should have equal importance and show that without each of the other phases students would not easily solve questions. Students should also see that solving the question correctly is just as important as recognising what they have to do. Polya (1957) identified, in his similar process, that each of the phases has its importance and mistakes can be avoided if sufficient time is taken at each phase.

As time progressed, one aspect that I felt needed extra reinforcement was the separation between reading and planning. Often students highlighted important parts of a question as they read it for the first time. This did not allow them to register what the question was actually about. They tended to make assumptions about which parts of the question were important. By reading through the question first they gained an overall understanding of what is being asked before locating the specific important information.

The teacher was very effective when linking the mathematics activities to current or real life events. Making it relevant was noted by students and proved to be a highly motivational component of the problem-solving process. The teacher recognised a weakness and found a way to make it relevant to the students.

5.4 Development of groups over time

The group development over time was one of the positive outcomes of this research. Students were listening to each other and working together to solve each of the word problems. However, it would have been interesting to see how students interactions would have changed if we had altered the groups throughout this period of time. Although some students mentioned that they had some issues with the group they were working with, the progress the groups made as a whole when working through the process and solving word problems was clearly noticeable in both written work and in the recordings.

Using colour-coded pens seemed at the outset to be a good idea but in practice students were often more concerned about getting everyone's colour on the page than getting down all their thoughts. Even when all colours were on the page there was no guarantee that what was written were the thoughts of the person whose colour it was. Analysis of the audio recordings provided evidence that some student had been told what to write by others in the group to ensure that everyone, but not everyone's ideas, were represented.

One of the benefits of photocopying the activities each day was to show the progress of students in the 'plan it' phase. Students' records showing what information they were highlighting could be used to guide teacher planning for the following lesson. The process also assisted when the students had checked their answers and found they had made a mistake. They were able to go back and see if they had identified all the relevant information and make slight alterations in order to reach a correct answer.

The next step would be to get the students to work straight from the resource books. This would have included teaching students to make notes of the important information. This is where the inclusion of a chart similar to Hyde's KWC chart (Figure 2.1) would have assisted the groups. As O1 identified in his interview with me (12/3/10) there was a need to move away from photocopying each activity move towards and "highlighting in your head." That was a skill that required development.

5.5 Achievement

Why did only 11 out of 23 students answer the question when working independently? The answer possibly relates back to the reasons why students were not providing answers at the very beginning. Students were not looking carefully at the information they were given and were making assumptions in relation to what they had to do based on previous activities. As this research was completed over a short period of time, students were still learning the process and may not have been ready to use it independently. Teachers also are used to giving information to students and breaking questions down for them, in effect, doing the thinking for them, which hinders their word problem solving ability. As Angier and Povey (1999) call it, 'fracturing' the task. Therefore students were unfamiliar with having to plan their work independently.

By introducing an easy to follow process, students are more likely to think about what they had to do throughout the question while keeping the main point in their minds. During this intervention, students allocated a larger proportion of their time to planning what they intend to do to solve the identified question. This phase was intended to disrupt students' previous patterns of rushing to complete problems without taking time to plan and check their answers. With initial guidance, this intervention has demonstrated that, students were for the most part capable of solving word problems and providing thoughtful answers. They had to trust in their abilities and the teacher had to give them space to work together rather than them being reliant on him.

At times we (the teacher and researcher) had some issues with the answer books for the FIO resources. The answers in them could be quite confusing and contained ‘continued’ answers. That is to say that if one was wrong, the rest would be too. This was also the case with interpretations. During the “Petty Differences” activity (MOE, 2008b) there were some questions that needed to be clarified and if students grouped animals in a particular way their analytical questions were incorrect in a later part of the activity, as the answer book did not allow for these variations in opinions. If the teacher was marking these activities then they would be able to use discretion to decide if the answers given answered the initial question. This was a difficult skill for the students to master. At times the teacher had read the answers previously and used himself as the ‘check it’ phase to save the students sorting through the long explanations given with the answers.

When analysing the final independent activity, which was completed in a post test situation (see Appendix 9), many of the students were part way towards reaching the final answer. Some students had highlighted the information but not provided the answer at the end. This demonstrated that students had started to apply the process and may just require more time and activities to ensure that they have assimilated this new information and how to apply it. All students made some progress when comparing work samples from the beginning observations. They are now confident when locating important information and are willing to attempt word problems within their groups rather than waiting for the teacher to tell them what to do. From these observations it is clear that the teacher is also aware of which students still require support when working through word problems and which ones need reminding of various phases of the process.

5.6 Instructional activities and resource use

By the end of this research, the teacher and students had greater confidence when using FIO resource material. The teacher could see the benefits of setting word problems and said that he would have greater confidence to use them in other areas of mathematics. The students also had greater confidence and now have a starting point when they are given a similar word problem to solve.

However, both the teacher and I remained concerned as to the time it takes to complete the tasks properly. Was time taken in relation to activities a signal that the activities were too hard for the students, or an indication that the process was indeed more time consuming that we anticipated, or and indication of the students' poor time management skills? We came to the conclusion that the answer varied depending on the group. We placed a greater emphasis on the detailed completion of tasks rather than just the completion. Time is a major issue as teachers have a perception of how quickly they need to move on in order to cover the content within particular units (discussion with teacher, 17/3/10). Due to this we wondered about the depth of learning that is taking place. Are teachers skimming over the surface or are they allowing for investigative thinking where students are required work through a problem involving a series of components, both computational and analytical?

5.7 Formative assessment

This research has highlighted the significant role of formative assessment for effective pedagogy. The teacher was using the information he gathered through observation and work samples to adapt instruction towards students' learning progressions. He identified areas that needed development and included these in future lessons. Absolum (2006) argues that this is one of the crucial aspects of formative assessment. By collecting this information the teacher was able to "skillfully interpret and evaluate information for individuals and groups of students in order to decide on what might be done next to support learning" (p. 23). Arguably, this information is continually changing as some students grasp new concepts and others still require support.

The design experiment approach incorporated the formative assessment to guide which direction the teaching would take. The teacher and I would sit together at the end of each lesson and see where there were areas that still needed development or consolidation. We were able to use the data gathered in that particular lesson through either of our observations and work completed by the students. Through our discussions we worked towards a plan for future lessons to suit the needs of the students.

5.8 Conclusions in Relation to Key Objective and Questions

- Design and implement a process, adapted from the reciprocal reading approach, which will assist students when solving word problems.

The Figuring It Out process achieved this key objective. The reciprocal approach to solving mathematical word problems aided students who were a little unsure how to initially interpret the word problem. Additionally, the support of peers provided a useful scaffold when students may not have been able to complete the task independently (Anghileri, 2006; Hodge et al., 2007). As the process was introduced systematically and broken down into the individual components, students were able to build on each session in order to use the process in their groups without so much assistance from the teacher.

The colour coding of the diagram made it visually appealing and memorable for the students. In future implementations, I would revisit the questions section of the diagram after two weeks to see if the students had anything they thought could be added to enhance the model. There also could have been a large scale model displayed somewhere in the room where new supporting questions could be added.

1. How could the reciprocal reading process to be adapted and incorporated into the mathematics programme?

The reciprocal reading process designed and implemented by Palincsar and Brown (1982) used similar stages to the process that Polya (1957) used to develop understanding of mathematical word problems many years before. This therefore, made creating an adaptation of the reciprocal reading process relatively straightforward. Drawing on the findings of earlier studies, a new process was designed based on the needs of the students and observations made in a pilot study. The key phases of the process were incorporated to ensure there was opportunity for students to read the questions carefully and make connections to what they already knew. There also had to be equal weighting on solving and checking the answers to demonstrate that these aspects are equally important as reading and making connections.

Students were able to freely use the new process to assist them when completing word problems and were able to articulate the stages and how they used them when working together.

2. How do students use the phases within the model when solving word problems?

Explicit teaching of each of the phases supported the students' focus on one aspect at a time to begin with before using the process in its entirety. In the "Setting up the process," however, many of the students blended the first two stages of planning and reading. This did not allow them to base their plan on the whole problem. This aspect would need to be a focus when implementing this process again.

3. What are the effects of the model on students' learning outcomes?

Students achieved their learning outcomes by being able to work through word problems in Figure It Out resource books using the process to assist them. They worked successfully in groups and many were able to transfer this learning to independent tasks. Students had greater success with word problems at the end of the research period. It is difficult to tell if it was entirely due to the introduced process or just because there was an increased focus in class on solving these problems. As the students answered the questions correctly they were definitely using aspects of the process, as they knew they had to read the question carefully to ensure they were producing the answer that was asked for within the question. The research only focused on one unit of work over a four week period. It would be interesting to see if the students continued to use this process when working through word problems in future units of work.

5.9 Possible future steps

This project was trialed in one classroom, and our experience suggests many further avenues of potential development of the model and aligned research into ways to support effective use of word problems:

- Varying the grouping arrangements

  i. Having a combination of groups of four, pairs and independent activities would give the researcher and teacher a clearer idea of who is applying the process and which students still require some assistance. In this research an independent activity was only used at the end.

  ii. Mixing up groups regularly would allow students to work with a range of other students and discuss a variety of ideas. Students may also pick up on strategies used in some other groups, as they would have greater opportunities to see a variety of ideas.

- Selecting word problems across a wider range of mathematical and statistical content strands. We were limited by the long term plan of the school. The statistics context worked well as it enabled the groups to discuss their thoughts on graphs and data and the levels were not as diverse as they could have been with a number focus. Any other strand would also be interesting to investigate, in particularly measurement investigations.

- Post intervention monitoring of students' word problem solution processes. Go into classroom at a later date to see if students are still using process as a strategy when completing word problems.

- Block teaching rather than continuous teaching of the word problem process. A suggestion made by the teacher was to spread out the teaching of the new process into two or three blocks so that the students had time to consolidate their learning. If this time was available to the researcher it could be a consideration to assess students over a longer period of time and revisit students throughout the process.

5.10 Summary

Overall this research has been successful in its primary objective. The teacher and students in the class gained confidence when using the Figure It Out resource material. The students were able to work together to solve word problems and provide the correct answer. At the beginning of the research period they were not reading the question carefully and, as a result, were not providing exactly what was being asked for. At the end of the time, many students were taking more care to read the question and discuss what was asked for.

The diagrammatic form of the process (Figure 3.2, 3.3) aided the students in solving problems, as they were bright and vibrant along with using simple language that was easy for the students to remember and understand. The teacher explicitly taught each section so the students were clear on what each one entailed. As the time progressed the students were using the terminology when working together to solve the problem. The diagram gave equal weighting to each phases showing that checking work is just as important as solving the problem.

Throughout this research the initial questions have been focused on and answered but they also raised more questions that would be a good starting point for future research or be taken into consideration if this process was being replicated in another classroom. As with any new approach or strategy, there are components that teachers would replicate exactly as they were done during this research and there would be others that they would change to suit their students taking into consideration the age and achievement levels of the students within their classrooms.

This research demonstrated the advantages of a ‘community of practice’ where the researcher, teacher and students were all working together to gain knowledge of a new process. Discussions between parties assisted when planning, and assessing the effectiveness of the process and identifying problems that we may have discovered along the way.

By having a systematic model, students were able to approach word problems in a consistent way. Students worked with other students and discussed various approaches to solving mathematical word problems and together came up with ways to answer the questions correctly. Teachers using effective pedagogy are taking a greater facilitation role in classrooms. By having a reliable and easy to follow model, students are able to work independently allowing teachers to work with individuals or groups to enhance knowledge and develop skills rather than supporting them to solve one particular word problem. As a result students gain greater autonomy in their learning and take an active role in the solving of mathematical word problems. Through the use of the 'Figuring It Out' model, students are assisted in the comprehension of word problems and teachers are able to concentrate on mathematical skills and concepts that are inherent within word problems.

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Appendix 1 – Braid Model of Problem Solving

The Braid Model of Problem Solving
New entries from Chapter 4 are in italics.

Understanding the problem/Reading the story
Visualization
Do I see pictures in my mind? How do they help me understand the situation?
Imagine the SITUATION
What is going on here?
Asking Questions (and Discussing the problem in small groups)
K: What do I know for sure?
W: What do I want to know, try to figure out, find out, or do?
C: Are there any special conditions, rules, or tricks I have to watch out for?
Making Connections
This reminds me of . . . Math to Self; Math to World; Math to Math
Infer
What inferences have I made? For each connection, what is its significance?
Look back at notes on K and C. Which are facts and which are inferences?
Are my inferences accurate?

Planning how to solve the problem
What REPRESENTATIONS can I use to help me solve the problem?
Which problem-solving strategy will help me the most in this situation?

Make a model	Draw a picture	Make an organized list
Act it out	Make a table	Write an equation
Find a pattern	Use logical reasoning	Draw a diagram
Work backward	Solve a simpler problem	Predict and test

Carrying out the plan/Solving the problem
Work on the problem using a strategy.
Does this strategy show me something I didn't see before now?
Should I try another strategy?
Am I able to ***infer*** *any* ***PATTERNS****?*
Am I able to ***predict*** *based on this inferred pattern?*

Looking back/Checking
Does my answer make sense for the problem?
Is there a pattern that makes the answer reasonable?
What **CONNECTIONS** link this problem and answer to the big ideas of mathematics?
Is there another way to do this? Have I made an assumption?

Appendix 2 – Example of Figure It Out activity

Petty Differences

You need ★ classmates

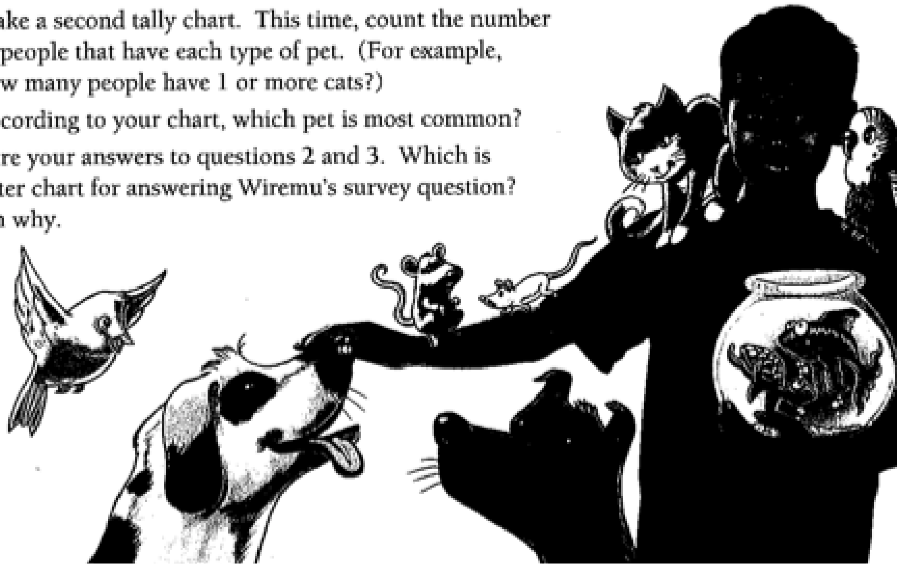
Activity One

Wiremu is allowed to choose a pet for his birthday. He decides to find out what pets his classmates have. He gives each classmate a blank data card, asks them to list any pets they have, and collects the cards.

What Pets Do You Have?

1 cat	None, unless you count 2 goldfish in a bowl	2 turtles and a rotty cross	2 cats and 1 dog
Nothing	1 dog	0	Nothing
None	1 dog, 2 cats, 5 fish	8 dogs, 7 cats, 3 horses	My sister
A canary	2 cats	1 cat, 12 fish	1 axolotl
1 cat, but she's my mum's.	7 fish, a budgie, a poodle – and a cat in Aussie	1 cat	1 dog. I had 3, but I gave 1 to my cousin and 1 died.
No pets	3 birds. 1 is sick and being treated.	3 female mice	3 female mice

1. With a classmate, discuss Wiremu's data. Which information is unnecessary? Which data is difficult to interpret?
2. a. Using a tally chart, count the number of each type of pet.
   b. According to your chart, which pet is most common?
3. a. Make a second tally chart. This time, count the number of people that have each type of pet. (For example, how many people have 1 or more cats?)
   b. According to your chart, which pet is most common?
4. Compare your answers to questions 2 and 3. Which is the better chart for answering Wiremu's survey question? Explain why.



2

Activity Two

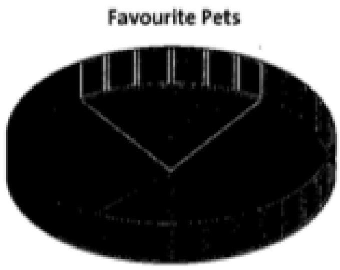
1. The class next door did a similar survey, but they were asked to name their favourite pet. The results are shown on this tally chart:

Cats	卌 卌 ||	12
Dogs	卌 |	6
Horses	卌 |||	8
Guinea pigs	||||	4

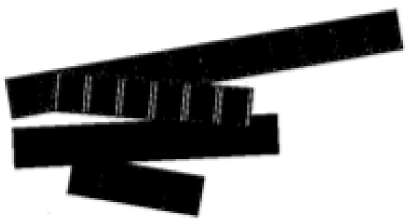
a. What is the total number of students in the class next door?

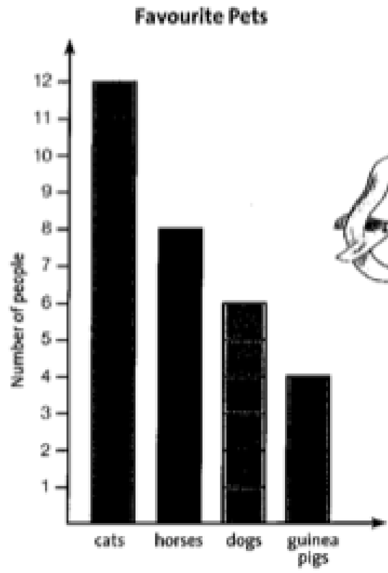
b. Divide a cardboard strip into equal sections to match the number of students in the class next door. Using a different colour for each kind of pet, colour the sections of your cardboard strip to make a strip graph of the favourite pets for that class.

c. Bend it into a circle and use it to make a pie chart like this:



d. Cut it into pieces to make a bar graph.





e. What information do the graphs show?

2. Discuss with a classmate which graph you would prefer to use to show the information and why.

Focus Cleaning and collating data and communicating findings

Appendix 3 – School Long Term Plan

	Term 1	Term 2	Term 3	Term 4
Even Year	**NUMBER: Strategies and Knowledge** Addition and Subtraction	**NUMBER: Strategies and Knowledge** Multi & Division	**NUMBER: Strategies and Knowledge** Decimals and Fractions	
Even Year	**STATISTICS** Graphing Investigations and Literacy	**GEOMETRY & MEASUREMENT** Measuring : Area, Volume, Mass (+ 2D/3D Shape)	**GEOMETRY & MEASUREMENT** Transformation Geometry Position, Orientation, transformation	**NUMBER: Strategies and Knowledge** Algebra **NUMBER & ALGEBRA** Equations and Expressions Year Review
Odd Year	**NUMBER: Strategies and Knowledge** Addition and Subtraction	**NUMBER: Strategies and Knowledge** Multi & Division	**NUMBER: Strategies and Knowledge** Decimals and Fractions	
Odd Year	**STATISTICS** Probability	**GEOMETRY & MEASUREMENT** Measuring Units: Time, Timetables and	**GEOMETRY & MEASUREMENT** 2D and 3D shape (+ lines and angles)	**NUMBER: Strategies and Knowledge** Algebra **NUMBER & ALGEBRA** Patterns and Relationships Year Review

AsTTle: *Number Knowledge and Operations Level 2/3 3/4 or 4. Pretest at beginning of year, Term 1 Week 2-4 for class placement and post-test Term 4 Week 2-4 for reports and to see movement during the year. Geometry/Measurement test as required Term 2/3.*

Appendix 4 – Information Sheet for Participants

Using reciprocal teaching and learning methods to enhance comprehension in mathematics word problems.

INFORMATION SHEET

Researcher(s) Introduction
This research is part of my Master of Education degree and will look at using a simple cycle to help when you are reading maths problems. It will be similar to what you may already do in reading.

Project Description and Invitation
This project will focus on how you as students follow a set process to solve mathematical word problems. You will work in small groups to begin with. Then work in pairs and eventually by yourself if you want to. The teacher will introduce maths concepts to you and then you will have to read a word problem which uses that concept. You will work with others to decide which information is relevant and decide on a plan you are going to use to solve it. Finally you will check your answer to make sure you are right.

You and your teacher are working together with me, the researcher, during normal maths lessons. As a student in this class, you are invited to participate in this project with your class. There may be times when I will ask you to explain your thinking.

Participant Identification and Recruitment
All members of this class have been invited to participate in this project. It would be great for the whole class to participate as there will be a range of opinions and everyone will have different ideas about the best way to solve word problems.

Project Procedures
The project will last for four weeks where I will be working with your teacher and watching how you use the cycle to help when you are learning maths. I will ask you questions about how you are using the cycle and ways that we could make it better for others to use in the future. I will be interested to see how you get your answers and how you work together to solve maths problems. I hope you will be part of this study so that I can hear your thoughts and see how you work with others.

Data Management
I will collect the information for my study purposes only. I will not be using your names in the published work.

Participant's Rights

You do not have to accept this invitation.
If you decide to participate, you have the right to:

- decline to answer any particular question;
- withdraw from the study (12 February 2010);
- ask any questions about the study at any time during participation;
- ask for the recorder to be turned off at any time during the interview.

Project Contacts

Researcher:
Phillipa Quirk
Student of Massey University

Supervisor:
Margaret Walshaw
School of Curriculum and Pedagogy
Massey University College of Education
Palmerston North

If you have any questions about this research at any stage during the process, please feel free to contact me. I will be happy to discuss any queries with you.

This project has been evaluated by peer review and judged to be low risk. Consequently, it has not been reviewed by one of the University's Human Ethics Committees. The researcher(s) named above are responsible for the ethical conduct of this research.
If you have any concerns about the conduct of this research that you wish to raise with someone other than the researcher(s), please contact Professor Sylvia Rumball, Assistant to the Vice-Chancellor (Research Ethics), telephone 06 350 5249, email humanethics@massey.ac.nz.

Appendix 5 – Participant Consent Form

Using reciprocal teaching and learning methods to enhance comprehension in mathematics word problems.

PARTICIPANT CONSENT FORM

I have read the Information Sheet and have had the details of the study explained to me. I had an opportunity to ask any questions that I had and they were answered. I understand that I may ask further questions at any time.

- I agree to any interviews and group work being sound recorded.
- I agree not to talk about anything discussed in the Focus Group.
- I agree to participate in this study under the conditions set out in the Information Sheet.

Participant Consent

Signature: .... **Date:** ....

Full Name - printed ....

Parent Consent

Signature: .... **Date:** ....

Full Name - printed ....

Appendix 6 – Weekly Classroom Plan

Term 1 Week 3	Monday	Tuesday	Wednesday	Thursday	Friday
Whole class lesson Making connections	Introduction to new learning sequence. Whole class modelling of each phase	Recap process from yesterday. Set up own class data then set class off to complete task following the flow diagram.	Analysis of data – collecting and presenting raw data	Analysis of data. Drawing tally charts. Making a pie graph lesson for use tomorrow	Go over page and remind students of what a pie graph needs.
Purple Blue Green Red Orange Yellow	Activity for all groups. FIO Statistics, Level 2-3 pg 1-3 LI – use graphs to communicate findings and tell stories	Activity for all groups. FIO Statistics, Level 2-3 pg 1-3 LI – use graphs to communicate findings and tell stories Whole class modelling to begin then move onto independent groups.	Activity for all groups FIO Statistics level 3 pg 2	Activity for all groups Continue with pg 2 from yesterday, set up for tomorrow FIO statistics level 3 Pg 3	Activity for all groups. Continue same pages from yesterday working on pie graphs today
Observations	Teacher modelling today – move onto groups tomorrow	All groups left out the last part of the question which asked which was the favourite fruit. Blue group need extra help on graph layout	Found the data challenging to read –a little frustrating at times. Groups were answering and writing this down	Completed this task much better today. Gave students more guidance – fine line between not enough and too much	Most groups got close to completing the two graphs. Working with greater independence. Blue not working so fast

Term 1 Week 4	Monday	Tuesday	Wednesday	Thursday	Friday
Whole class lesson Making connections	Analysis of data and will have to complete own class vote for them to continue with task 3.	Creating own graph and asking questions about it	Tally Ho! Interpreting data and stating opinions about data	Number activity 1:30	Teacher at planning day
Purple Blue Green Red Orange Yellow	FIO Gala level 2-3 pg 2 Holding a vote. Students have to add up data in a table then add their own class information. Have to complete simple analysis of data	FIO Under the sea level 2-3 pg 23 Students categorise the objects and create own graph. Then they ask other people questions about their own graph before re-graphing someone else's data groups.	FIO Statistics level 2-3 pg 20 Tally charts – interpreting data Stating relevant opinions	FIO Gala Level 2-3 pg 3 How many people? Fractions of 1000 Applying problem solving process to number problems	
Observations	Still need work on depth of answers when adding opinions to responses	Still have to go over instructions with many of the groups – need to trust each other and think about what would make most sense	Questions not very clear to help with getting detail in answers.`	Most groups (not blue) were able to solve questions involving fractions or 1000. Do still have to read carefully	

Term 1 Week 5	Monday	Tuesday	Wednesday	Thursday	Friday
Whole class lesson Making connections	Group teaching – then students go off to complete the tasks in their existing groups	Groups still working through the same problem solving process – deliberate teaching at the beginning of the lesson	Teacher at Softball tournament	Go over FIO task with them – identify when they need key bits of knowledge eg. draw stem and leaf graph	
Purple Blue Green	T1 – FIO Number level 2-3 pg 11 Alien addition Go over addition strategies, any examples can be written in their own group big book. Use the activity as a base for teaching. Complete task.	T1 – FIO Number level 2 book 2 pg 6 It's a magic mishmash Using skip counting and number knowledge work out the machines Do some other examples Complete task		T1 – FIO Statistics Level 2-3 Stem and leaf graphs pg 8-9 Act 1 part of 2 & 3 Teaching point – what is a stem and leaf graph and how do we draw one Complete task	Continue task from yesterday
Red Orange Yellow	Complete FIO number pg 10 activity 2 – addition triangles T2 – FIO statistics Level 2-3 pg 8-9 activities 1, part of 2 and 3 Teaching point – what is a stem and leaf graph and how do we draw one	Continue task from yesterday		Continue task from yesterday T2 – FIO Stats level 3 pg 8, Activity 1 Interpreting stem and leaf graphs. Use last section of work as example of analysis Start task	Continue task from yesterday
Observations	Group 1 worked well once they were set off. Nico and Jack lacked perseverance when they couldn't get the answer straight away	Group 1 task was a lot easier today although some still not reading question carefully. Group 2 did stem and leaf graph well, some did not sequence the data		Impressed by how both groups worked. Group 1 managed to sequence data in stem and leaf graph. Most now thinking carefully about relevant information.	Groups continued to work on tasks from yesterday. Giving reasons is still an area that groups need to concentrate on.

Appendix 7 - Questionnaire

Thank you for participating in this research. It has been very enjoyable working with you all during your maths time and I hope it has helped with your learning. I have a few questions to ask, please be honest as I would like to know how successful this process has been.

1. Which phase helps your group the most when completing tasks?
Rank from most helpful to least helpful. (Read it, plan it, solve it, check it)

Most helpful Least helpful

2. Why is it important to plan and check each question?

3. Does checking your answers as you go help with your learning?

Yes No

4. What do you do when you check and answer and it is incorrect?

5. Do you think you will use this process in the future?

Yes No

6. Has this process made word problems easier to solve?

Yes No

Why?

7. What is the hardest part of doing word problems for you?

8. Do you feel more confident now than four weeks ago when getting a Figure it Out task to complete?

Yes No

9. How well do you think your group worked together?

1	2	3	4	5
Not well		OK		Very well

10. Was is easier solving word problems in a group?

Yes No

Why?

11. What is the hardest part about working in a group?

12. Do you think other students should be taught this process when working with Figure It Out books?

Yes No

Why?

13. General comments

Thank you for your thoughts.

Appendix 8 - Questionnaire results

Question 1 – Ranking of tasks

Where most helpful was given 1 and least helpful 4

Read It - 50 8 thought it was most important

Plan it – 40 17 thought it was most important

Solve it – 72 8 thought it was second most important

Check it – 89

Most emphasis was put on reading and planning during the teaching process. Nobody thought solving or checking was the most important.

Question 2 – why important to plan and check?

- So you understand what you are doing. Check it is important because it shows what you got wrong and right. (P4)
- Check it is important because we learn from mistakes. Plan it is important because we think about our questions. (P2)
- plan it is important because you wouldn't know what to do. Check it important (P1)
- plan it important because then if you didn't have it your group might not know what to do. Check it is important so it you get something wrong you can solve it. (P3)
- I think planning is good because you might miss read something and get it wrong. Check it is important because you won't know the right answer (B3)
- plan it is important because it gives you a lot of information. Check it is important so you know what you've done (B1)
- plan it is important because if you don't plan the answer you can get it wrong (B2)
- plan it is important because it tells you what you need to know. You don't know what your activity is. Check it so you know if you have got it right. So you know if you're learning the right thing (R1)
- plan it is important because if you don't plan it you can't figure it out (B4)
- plan it is important because you will answer the question. Check it is important because it can help further on in the question. Check it is important because it tells you if you are right or wrong (R4)
- plan it; highlighting the most important detail and talking about it with your group. Check it: checking if you got it correct in the check it book (R3)
- plan it – to find out the question's answer easily. Check it – to find out if your answer is correct and to go to the next question (O3)
- plan it is important because you will know what to do. Check it – you don't know what's wrong (O1)

- plan it so you know what you're important information and know what you are doing. Check it so you know if you are wrong or right and so you know what you did wrong (O2)
- plan it is important because you have to find the keywords for the problem. Check it is important because you can find out what you did wrong (R2)
- it is good to plan it because you find out all the important stuff in the text. It is good to check it so you can learn from your wrong answers (G3)
- planning is important because then you know what to solve about it. Checking is important just in case if you get it wrong (G1)
- It is the most because it helps you solve it (G2)
- The plan it is important so you can set it out. Check it is important so you know you got it right. (G4)
- plan it is important so you know what you should use to help yourself. Check it is important so you know if you're right or not (Y4)
- Check it is important because if people need to know if they got it right and if people did not check they would think it's right and think that's the answer when it might be wrong. Plan it is important because if you don't plan it you won't be able to what to listen to (Y3)
- plan it – because you've got to know what's important. Check it – because you've got to know if you've got it right or wrong so you can learn it (Y1)
- plan it is important because it helps us understand it. Check it is important because you get to know what you did wrong so you know for next time (Y2)

Question 3 – does checking answers help?

Yes – 18
No – 5

Question 4 – What do you do when you check and it is incorrect?

- you do it all again (P4)
- go back and correct it (P2)
- you go back a bit (P1)
- try and figure it out again and see if you read it properly (P3)
- you write the correct answer next to it (B3)
- you go back and try it again solve it (B1)
- you go back and do it all again (B2)
- you can fix it up or write in underneath (R1)
- you go back and work it out (B4)
- go back and try again. Try and see where you were and find where your mistake is (R4)
- you put the correct answer next to the incorrect one (R3)
- go back and read it, plan it, solve it (O3)
- I correct the answer (O1)
- I go back and see what I did wrong and start to solve it again (O2)
- I read it again and fix the answer (R2)
- We go back and see where we messed up, or just move on (G3)
- You give it a cross and write the right answer next to it (G1)
- You try it again (G2)

- You go back to it and do it again (G4)
- I write the right answer and put a cross by the right one (Y4)
- Leave it and try to figure out how the answer became that then write the correct one next to it (Y3)
- Put a cross and write the correct answer (Y1)
- You go back and see what you've done wrong and cross it (Y2)

Question 5 – Use process in the future

Yes – 19
No – 1
Maybe - 3

Question 6 – has process made word problems easier to solve?

Yes – 21
No – 2

Why?
- because planning helped me this about all the important parts (P4)
- because it makes me think it over (P2)
- because – (P1)
- instead of quitting you can do the problem (P3)
- because I can use the read it, plan it, solve it, check it phase (B3)
- because you can go back and check it, plan it, solve it (B1)
- because (B2)
- it helped me practice my different graphs. It helped me if I'm ever stuck on something (R1)
- because it is important to learn but not always right (B4)
- it gave me a new way to think about the answer, I thought plan it helped the most (R4)
- because it was easier to make charts than just answering without it (R3)
- it doesn't make a difference to the usual way (O3)
- because it is easier to solve the problems (O1)
- well it helps when you do one bit at a time instead of rushing ahead and going onto the next question (O2)
- because I will remember what I did in my group (R2)
- when you use the text you know what bits are important, and if you want to find a bit in it that is important you don't have to read it over (G3)
- the solve it helped me the most (G1)
- because you read it, plan it, solve it, check it, and it's easier to use than solve it straight away (G2)
- it was easier because you had to put it into groups and find other ways to work it out (G4)
- because I didn't used to plan my problem (Y4)
- it was the same way I used to do it (except for the planning, that did not help me)(Y3)
- because it helped you to know what the important parts are (Y1)
- because I didn't always plan it (Y2)

Question 7 – hardest part of doing word problems

- reading them (P4)
- understanding the problem (P2)
- planning it and solving it (P1)
- plan it and solve it (P3)
- was finding the answer (B3)
- solving it (B1)
- when it takes a really long time (B2)
- that hardest part is doing the solving and working out who's is who's (R1)
- understanding the problems (B4)
- to understand it (R4)
- counting, if it's on a chart because it is hard to tell which is which (R3)
- co-operating with my team and not getting any work done (O3)
- the plan it because you do the most thinking at that time (O1)
- well the solving because sometimes you get confused (O2)
- solving the answer (R2)
- the highlighting because the group fights over what bits are important (G3)
- solving them (G1)
- plan it (G2)
- solve it (G4)
- solving it (Y4)
- finding out what numbers we are using or what the question is focused on (Y3)
- understanding what they mean (Y1)
- planning it (Y2)

Question 8 – more confident now

Yes – 21
No - 2

Question 9 – rating how well group worked together

1- 3 Not well
2- 4
3- 39 OK
4- 16
5- 5 Very well

Average 2.9

Look at totals for individual groups

Purple – average = 3
Blue – average = 4
Green – average 2.25
Red – average 3.25
Orange – 2.6
Yellow - 2

Question 10 – Easier solving word problems in a group

Yes – 14
No – 9

Why?

- because they never concentrate (Y2)
- because you can discuss your answer with them (Y1)
- it was hard when too many answers come at you and its easier to get on with work (Y3)
- because someone missed something so the other people told them (Y4)
- so you can work it out together (G4)
- because we always get in a fight (G2)
- because then you don't have so many ideas in your head (G1)
- because you would not have to fight the whole way through it (G3)
- because sometimes they didn't listen to me (R2)
- well because everyone fights over the answer (O2)
- you get more ideas (O1)
- one person kept the book all to herself and the other three had nothing to do (O3)
- because you can talk it through with the group (R3)
- because you have more people's answers to consider (R4)
- since they were helpful (B4)
- because I can't work this out myself so when I have somebody else we can do more than one thing at a time (R1)
- you can get it right (B2)
- because you know what you are doing (B1)
- people might have the wrong answer, but it might give you a clue what the answer is (B3)
- everyone has ideas and so it is easier to solve it (P3)
- because everyone shares ideas so it's easier for everyone (P1)
- because we played too much (P4)

Question 11 – hardest part of group

- you have too many ideas (P4)
- because we had different ideas and could not decide (P2)
- some people think their ideas are better (P1)
- some people think they're right and want to get it on the paper (P3)
- when others don't bother joining in and the other people do all the work (B3)
- because you don't know if the answer is wrong or right (B1)
- when we have fights (B2)
- tow people are always fussing around and being silly while one or two do most (R1)
- that you should concentrate on talking (B4)
- sometimes people don't listen to you (R4)
- we all have different perspectives (R3)
- co-operating with our group and getting the work done (O3)
- everybody wants to do the work (O1)
- probably the communication and co-operation (O2)
- see above (R2)
- it would be to agree on the same answer (G3)
- everybody has different ideas and you don't know which one to do (G1)

- sometimes we get into a fight (G2)
- you might not listen to other ideas and you might fight (G4)
- two kept fighting (Y4)
- trying to get on with work and not being distracted while trying to figure it out(Y3)
- deciding on the answer (Y1)
- they won't listen (Y2)

Question 12 – teaching others this process

Yes – 21
No – 2

Why?

- because they might not do one of the steps (Y2)
- because it's making you think harder (Y1)
- it's hard for us to keep up (Y3)
- because it was fun and I learned lots (Y4)
- so it will be easier (G4)
- because you're good at teaching maths (G2)
- because they would have more strategies (G1)
- because it helps a lot (G3)
- because you can know how to do word problems (R2)
- because then everyone can work it out by themselves (O2)
- more ways (O1)
- it will be difficult for my group (O3)
- because it helped me with word problems it should help other students (R3)
- it helps with word problems (R4)
- because it will help them a lot (B4)
- it will help them practice their graphs and help them figure out something (R1)
- it's quicker (B2)
- so other people understand (B1)
- so others can learn how to work out math word problems (B3)
- it is easier than quitting so if lots of people know it that's great (P3)
- because (P1)
- because it's a great way of learning (P2)
- so everyone can learn this learning (P4)

General comments

- it was brilliant (P2)
- I really liked doing it, I've really learned lots (B3)
- I think you should make them more harder so we learn more things (R1)
- I think it was a good process (R4)
- Thank you for helping me (R3)
- More work (O1)
- Thank you (G3)
- You did very well (G2)
- Never put those two in a group together again (Y3)

Appendix 9 – Summative Assessment Task

You have just got a new job at a book manufacturer. Your boss wants to sell more books to schools. Your first job is to find out what are the most popular sizes of books.

You complete a survey. Here are your results.

1a8 1b5 2b5 1b4 1e5 2b4 1b5 2b5 1e5 2b4 1b5 1a8 1b5

1e5 2b4 1b5 1b5 1a8 2b4 1b5 1b5 1e5 2b4 1b5 1b5 2b5

1e5 2b4 1b5 1b5 1e5 2b4 1b5 1b5 1a8 2b4 1b5 1b5 2b5

1b4 1e5 2b4 1a8 1b5 2b5 1b4 2b5 1b4 1b4 2b5 1b4 1b4

1b5 1a8 2b4 1b4 2b5 1b4 1b4 2b5 1b4 1b5 1a8 2b4 1b4

Decide how you are going to collect and display your results. Remember to do the job that your boss has asked.

Also provided is a sheet of grid paper.