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**Collaborative Learning
and
Peer-tutoring
in Mathematics**

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Abstract

This study sought to promote learning by enhancing the level of higher order cognitive talk among collaborative groups engaged on mathematical tasks. An intervention, designed to utilise structures such as listening, multiple retelling, questioning, elaboration, and justification to promote high-level discourse, was trialled and refined using an action research classroom study.

The collaborative skills training programme was based on Medcalf's peer-tutoring model (1997) and adapted to incorporate features of Lyman's Think-Pair-Share collaborative model (1992). The teacher's role was seen as crucial to the development of collaborative group practices which establish the structures for high-level discourse. Collaborative group practices were reinforced in follow-up class discussions where the teacher facilitated student reflection on the mathematical strategies and the collaborative group strategies. It was also seen as important for the teacher to select appropriately levelled tasks which maintained the learner in his/her Zone of Proximal Development.

Findings indicated that the structured intervention enhanced the level of higher order discourse between students and that it was an effective procedure to mediate learning. Several patterns of discourse were also identified that could provide useful indicators of higher level discourse to teachers during daily classroom observations.

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Chapter 1. Introduction

“Cognito, ergo sum” - Descartes

1.1 Background

The end of the twentieth century was characterised by the development of computer technology enabling vast quantities of information to be readily accessed by the world population.

The information explosion will continue to expand...Increasingly we will move away from defining educational success in terms of the quantity of information mastered. Instead, to a large extent, we will define educational success as the ability among students to generate, question, combine, categorise, re-categorise, evaluate, and apply information. Secondary will be the content of the information; primary will be thinking skills—thus, the need for thinking skills structures in our classrooms.

(Kagan, 1994, p. 111)

As a response to the global ‘information highway’ and changing work-place structures, the Ministry of Education developed the *New Zealand Curriculum Framework* (Ministry of Education, 1993) that identified eight generic essential skills: physical skills, self-management and competitive skills, communication skills, problem-solving skills, social and co-operative skills, information skills, numeracy skills, and work and study skills.

Curriculum documents were revised during the 1990s to apply the *New Zealand Curriculum Framework*’s essential skills to particular curriculum contexts. The first of these revised curriculum documents was *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992). This separated the mathematics curriculum into six key strands:

- mathematical processes,
- number,
- measurement,
- geometry,
- algebra, and
- statistics.

This core document and supporting documents such as *Implementing Mathematical Processes* (Ministry of Education, 1995) and *Development Band Mathematics* (Ministry of Education, 1996) were also influenced by the international mathematics community's emphasis on mathematical problem-solving which had arisen during the 1980s in the United Kingdom with the Cockcroft Report (1982) and the Agenda for Action Report in the United States (1980) and recognition of the importance of discourse and the value of collaborative learning practices in mathematics (National Council of the Teachers of Mathematics, 1989). For example, the *Implementing Mathematical Processes* document explicitly linked the need for 'new essential skills' with mathematical processes:

The abilities—to think flexibly, be adaptable, be a creative problem solver, and be able to communicate and work co-operatively—have been highlighted as essential skills for life in the world tomorrow. Mathematics teaching can play an important part in equipping our students with these fundamental skills. This is one of the reasons why the mathematics curriculum emphasises the importance of developing the students' abilities to reason, think flexibly, communicate, solve problems and collaborate in a mathematical context.

(Ministry of Education, 1995, p. 7)

Mathematics in the New Zealand Curriculum required teachers to “design courses to provide their students with mathematical experiences which will enable the students to

achieve the broader aims and achievement objectives for the curriculum” (Ministry of Education, 1992, p. 18). This included developing courses to emphasise the essential skills of logic and reasoning, communicating ideas, problem-solving and collaborative group work.

The processes of logic and reasoning and problem-solving were readily accepted by mathematics teachers as integral to mathematics learning (Holton, Anderson, & Thomas, 1996). From the era of Platonic deduction and proofs mathematical thinking has emphasised logic and reasoning. Moreover, many of the established traits of effective thinkers—with cross-disciplinary characteristics of persistence, creativity, questioning, precision, metacognition, flexibility, listening, and the ability to restrain impulsiveness (Costa & O’Leary, 1989)—parallel those traits of good problem-solvers within *Mathematics in the New Zealand Curriculum*, namely flexibility, creativity, the ability to reflect, experiment, improve, and use divergent and convergent approaches.

Parallel to the focus on processes the *Mathematics in the New Zealand Curriculum* encouraged New Zealand teachers to change their instructional practice of transmitting specific content in mathematics to interpreting and implementing mathematics through the essential skills base. This social constructivist approach suggested that classroom practices support students’ development of mathematical understanding through social interaction involving conjecture, dialogue and critical examination. While teacher-to-learner transmission of information, facts and ideas retains a place in learning, it is only when we test, confirm, assimilate and accommodate ideas into our conceptual frameworks that knowledge takes on meaning (Hill, 1992).

The new expectation within the socio-constructivist approach was that individuals would develop better mathematical thinking through discussions with peers when they gave more coherent explanations, responded to questions and challenges, listened to and made sense of others' explanations, and asked for clarification of ideas. The use of such conceptually orientated explanations, involving alternative solution strategies, assists in building relationships between forms of representation strengthening the students' mathematical achievement (Fuchs, Fuchs, Karns, Hamlett, Dutka, & Katzaroff, 1996).

Based on findings from collaborative research, Cobb and Wood (1990) contend that when children engage in this type of mathematical 'talk', it can result in "learning opportunities that rarely arise in traditional instructional settings" (p. 34). Students learn to reason analytically and to communicate by explaining and justifying their mathematical ideas (Wood, 1999).

1.2 Teaching Reforms and Collaborative Learning

The effective use of mathematical talk to foster learning in collaborative groups required teachers' perceptions of their role to change whereby they became facilitators of learning (Von Glaserfeld, 1990, cited in Mayers & Britt, 1995) and in the 1990s teachers were seen to implement collaborative group work in the mathematics classroom in a range of ways (Thomas, 1994b) from putting students in groups and expecting them to work together, to a formal structured collaborative model with specific group and learning outcomes. Junior classrooms which used a *Beginning School Mathematics* (1993) model often had groups of children collectively engaged on activities that were designed to consolidate a core concept that had been introduced by the teacher. Examples of children using co-operative/collaborative practices in later primary years included working in groups to

complete a one-off investigation or discussing a solution with a peer simply because of their physical proximity rather than because of collaborative engagement on a mutual task (Medcalf, 1995).

Thomas' study (1994a) into discourse in the New Zealand junior mathematics classroom noted that though most talk was task-related, more was related to the social aspects than the cognitive aspects of the task. The challenge then was to consider and develop processes that would encourage 'learning enhancing' discourse.

Thomas found that the social and cognitive nature of the task impacted significantly on the kind of talk which occurred and that problem-solving tasks were more likely to engage the children in explanations and abstract discussion compared to games or production tasks (making a model or collecting data). In order to reduce the social talk associated with organization within the groups most groups were reduced to pairs. Thomas also suggested that the ability to engage in collaborative abstract thought may be maturational and that "children in the first two years of school are not able to engage in substantial 'abstract' talk with their peers" (Thomas, 1994b, p. 327).

While the intention of *Mathematics in the New Zealand Curriculum* (1992) was to support improved learning outcomes utilising the social-constructivist approach to learning including cooperative group contexts, two international studies in mathematics achievement appeared to provide evidence to the contrary. The dual studies—the *Third International Mathematics and Science Study* (1994) and the *Third International Mathematics and Science Study - Repeat* (1998), showed no significant improvements in mathematics achievement by New Zealand students. A recent Education Review Office (ERO) report

(2000) criticised the curriculum documents and queried “ whether or not the structure and content of the curriculum documents provide a clear and practical guide for teachers, especially those who are not educated specialists in mathematics” (p. 104). The prevalence and effectiveness of teaching and learning in the New Zealand classroom by using small [co-operative/ collaborative] groups was questioned when other countries which seldom used small groups achieved more highly:

Inside the classroom, New Zealand primary school teachers typically use small group organization for teaching and learning mathematics. This assumes that students will learn better if they are taught in small groups for part of the time and then left to work individually or with groups not directly supervised by the teacher.

(Education Review Office, 2000, p. 101)

The Education Review Office report urged for a closer examination of small group teaching practices in New Zealand claiming that, “it is not self-evident, for instance, that the conventional New Zealand method is necessarily more child-centred, especially where it is the more or less exclusive arrangement” (p. 101). Specifically, ERO suggests we need further research information on group teaching “in terms of its contribution to developing mathematical understanding. This despite widespread claims as to its effectiveness, and the complexity of the process for teachers” (p. 101). Increasing research on teaching strategies and disseminating the findings of effective practice gives teachers better information on which to base their decisions.

The thesis contributes to the body of research investigating group learning practices by investigating the use of specific discourse structures to enhance the level of higher order cognitive talk within the context of collaborative groups in the mathematics classroom.

1.3 Definition of Terms

Definition of the terms associated with group learning practices are complex and often interpreted differently in different education settings internationally—by teachers, researchers and policy makers. For the purposes of this research, the following definitions are provided as starting positions to guide the reader. A fuller discussion of their origin in relation to literature is provided in Chapter 2.

- **Collaborative learning** is structured and organised small group work designed to actively engage students in “the learning process through inquiry and discussion with their peers” (Davidson & Worsham, 1992).
- **Peer-tutoring** is a form of co-operative learning in which students are trained to use specific instructional strategies to promote the learning of peers (Medcalf, 1992)

1.4 Research Questions

The aim of the study was to implement and investigate the effectiveness of a collaborative skills programme. The intervention programme was designed to increase students’ ‘learning enhancing’ discourse, that is, to increase the amount of cognitive talk students engaged in while working in collaborative groups. Using an action research framework, information gathered from students’ conversations as they worked in collaborative groups, anecdotal observations from their teachers, students’ questionnaire responses and a diagnostic test formed the basis of subsequent modifications to the intervention.

The following questions were investigated in the main study to evaluate the effectiveness of adaptations made to the structure of the programme and to examine resulting discourse patterns seen in the pilot study:

1. Does participation in the programme increase the frequency of 'higher order' cognitive interactions between children in the mathematics classroom?
2. Are there 'oral flags' which identify higher order cognitive thinking?

The collaborative skills programme was designed to be a practical classroom intervention which promoted peer scaffolding as an effective support for academic gains in mathematics. Effectiveness was addressed in relation to both teachers' and students' perceptions about the programme and through achievement gains in the Numeracy Development Project (Ministry of Education, 2002):

3. How did the students think that participation in the programme affected their academic or perceived academic achievement?
4. What changes did the teacher notice in individual/class interactions or attitudes when mathematical problem-solving?

1.5 Overview

Chapter 2 provides a theoretical framework for the role of discourse in learning and a rationale for the inclusion of specific techniques in the collaborative training programme. This is supported by a review of literature from America, Europe, Australia, and New Zealand, including discussion and evaluation of peer-tutoring and collaborative learning programmes.

Chapter 3 describes the action research methodology. This includes a description of the pilot study and the results which led to modifications of the collaborative training programme for the main study. The data collection and analysis processes are described.

Chapter 4 presents and summarises the data collected for the results from the transcriptions, the teachers' anecdotal comments, the students' questionnaires, and the academic data recorded from the diagnostic interviews.

In Chapter 5 the key findings related to the research questions are discussed along with the limitations of the study and concluding thoughts. Questions are raised for consideration in future research.

Chapter 2 Literature Review

2.1 Introduction

The literature review is divided into three sections. The first section considers the role of discourse within a social constructivist theory of learning whereby interactions are seen to promote learning by bringing about changes in the cognitive structures of those engaged in the interaction. The discussion of the socio-constructivist approach to teaching and learning, which underpins the current New Zealand mathematics curriculum, will include consideration of two theories of foundational relevance: Vygotsky's social learning theory (1978) and Piaget's developmental theory (1962).

The second section reviews peer-directed programmes and research studies which structure students' interactions to promote their engagement in higher order discourse and thinking. The section focuses on peer-tutoring and cooperative/collaborative learning models, and highlights specific structures which enhance mathematical skills in reasoning and thinking, problem-solving, and communication—all of which are key process strands in New Zealand and overseas mathematics curricula documents. Though the role of discourse in the home influences thinking skills, for the purposes of this project the review will be confined to research which has occurred in the classroom.

The third section investigates tools for the analysis of discourse in the classroom and provides a background and rationale for the inclusion and modification of the analysis tool used in this project.

2.2 Learning Theories and Discourse

Within the socio-constructivist approach to learning, the process of learning is seen as the active construction of knowledge within a social community (Simon & Schifter, 1991, p. 130). According to this approach learning is a by-product of interaction. Discussing new ideas with others transforms how we think about the ideas. Accordingly the process of accommodation:

Allows us to organise and reconstruct the new material for ourselves and integrate it into our existing knowledge base, thereby allowing us to understand and remember it better... During such interaction with another, we clarify ideas, negotiate meaning, develop new skills, and construct new knowledge.

(King, 1997, p. 221)

Therefore although “thinking is the personal process individuals use to create their own understanding” co-operative exchanges are seen as extremely beneficial because the resulting “shared visions and understandings enlarge the process spheres that individuals may explore, thus making enhancement of individual thought as boundless as the visions shared” (Davidson & Worsham, 1992, p. xx).

The socio-constructivist theory of learning, and its promotion of the role of discourse, is underpinned by two major theorists’ work: Piaget’s cognitive developmental approach to learning (1967) and Vygotsky’s social learning theory (1978).

2.2.1 Individual Development of Thought

Piaget (1967) believed that learning was acquired through the process of *assimilation* where the ideas or concepts, which he termed *schemata*, were developed in response to stimuli. Subsequent information or stimulus allowed for the development of a schema through a

process called *accommodation*. If new information compared favourably with existing schema it was accommodated into the schema, enlarging its cognitive reference. If the incoming stimulus compared unfavourably it was rejected, or a period of disequilibrium resulted while the individual searched for more information causing the reorganisation and reconceptualisation of the schema. Within this theory, discourse facilitates the exposure of one individual's schemata or concepts causing individuals to clarify, develop, expand and elaborate their thinking in defence of their particular schemata.

Piaget's theory favours a classical psychological approach to the measurement of the child's cognitive development. The child's independent activity and achievements are observed, passing through cognitive stages of maturity using stimuli that progress from concrete to semi-concrete to abstract. The first stage, is that of the sensorimotor period, (approximately 0-2 years) where the child responds from reflex action, then to the preoperational stage (2-7 years) with increasing imitation and experimentation, to the concrete operational stage (7-11 years), and then to the formal operational stage where the individual is able to visualise actions abstractly and perform them without physical experimentation.

According to this cognitive theory a child's mathematical cognitive development progresses from intuition to formal operations, from exploration through play to structured games, to representation through pictures and symbols and then formal operations, from a social experience of number to the learned abstractions and formal practices of mathematics. As such, cognitive development is influenced by the child's physical and maturational ability to imitate. It begins as a 'tentative exploratory activity' (Dienes, 1959) initially determined largely by spatial experiences. With maturity the development of language takes on an

increasing role; “physical actions become internalised and generalised into concepts and relations to which may be attached, either words or mathematical symbols” (Dickson, Brown & Gibson, 1984, p. 12).

For children, their schemata are qualitatively different to that of adults and commonly include partial understandings or misinterpretations. The development of their schemata is influenced by the linguistic and cognitive practices in their environment and are adjusted using an egocentric logic with subsequent experiences and social interactions. Thus in learning mathematics, individuals construct ideas, processes and unique understandings for themselves.

2.2.2 Social Development of Thought

In contrast to Piaget’s cognitive developmental theory where the independent activities of the child are measured, Vygotsky (1978) emphasises the social aspects of learning, claiming that each child has a higher level of cognitive development evidenced by his/her ability to imitate. In this way, Vygotsky focuses on the cognitive processes, which are still evolving, not those already mastered: “What the child can do in co-operation today he can do alone tomorrow” (Vygotsky, 1962, p. 104).

Vygotsky’s sociohistorical-cultural psychological theory of learning contends that the combination of speech and practical activity support the most significant intellectual development (Blanck, 1990). Vygotsky claims that because children’s higher mental functions originate in the activities and social dialogues in which they participate, those children involved in collective discourse, verbal and non-verbal are able to develop their thinking more effectively compared to children who are alone. However, Vygotsky

qualifies the nature of social interactions claiming that a child can imitate only that which is within his/her developmental level or *Zone of Proximal Development (ZPD)*—the “distance between the actual development level as determined by the independent problem-solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers” (Vygotsky, 1978, p. 86).

Thus, within the education context students need to be provided with experiences within their ZPD, just above their level of independent functioning, so that cognitive processes are developed. In order to keep the students in their ZPD the task and the environment must be appropriately structured so that the demands on the students are appropriately challenging. The amount of intervention provided by the adult or more capable peer needs to be adjusted in response to the students’ needs (Tharp & Gallimore, 1988, cited in Berk & Winster, 1995). Effective scaffolding occurs when:

The tutor or the aiding peer serves the learner as a vicarious form of consciousness until such a time as the learner is able to master his own action through his own consciousness and control. When the child achieves that conscious control over a new function or conceptual system then he is able to use it as a tool...the tutor in effect performs the critical function of scaffolding. (Bruner, 1985, p.215)

Discourse facilitates “the collaboration necessary between expert and novice to acquire the cognitive strategy” (Palinscar, 1986, p. 95). Without the use of scaffolding Gallimore claims that children will learn at a less than optimum rate (Gallimore, cited in Holton, Anderson & Thomas, 1996).

2.3 Structuring Discourse in the Classroom

2.3.1 Teachers and Students as a Mathematical Community.

The socio-constructivist approach to teaching and learning highlights the importance of providing social situations that allow children to construct and modify their mathematical knowledge through discourse (Yackel, Cobb, & Wood, 1991):

To understand what they learn, they must enact for themselves verbs that permeate the mathematics curriculum: “examine”, “represent”, “transform”, “solve”, “apply”, “prove”, “communicate”. This happens most readily when students work in groups, engage in discussions, make presentations, and in other ways take charge of their own learning. (National Research Council, 1989, p. 58, 59)

Reform instruction advocates that teachers and students within the classroom setting form a mathematical community in which they construct “their own knowledge individually and collectively, negotiating shared understandings and developing collaborative processes for validating ideas” (Neyland, 1994, p. 4). The teacher has a vital role as the expert who guides, models and interacts with students to provoke them to think mathematically, providing opportunities for students to verbalise their questions, explanations and strategies (Ministry of Education, 2002a). A teacher can be thought of as:

A discourse guide and each classroom as a discourse village, a small language outpost from which roads lead to larger communities of educated discourse... Teachers are expected to help their students develop ways of talking, writing and thinking which will enable them to travel on wider intellectual journeys. (Mercer, 1995, p. 83)

Peer discussions are seen as valuable not only if they lead to the correct answer but also because of the way they influence students’ mathematical behaviour. The way students

think about and solve problems is ‘influenced by discussions in which ideas are shared, challenged, and justified’ and is ‘related positively to mathematics achievement’ (Muth, 1997, p. 72). In particular co-operative learning puts students “in situations where they learn that reading, writing, listening and speaking in a co-operative manner are all-important components of successful problem-solving” (Muth, 1997, p. 72). Co-operative learning has many other advantages for supporting effective learning practices:

Students learn mathematical language from each other and are also able to learn, firsthand, the various problem-solving strategies that their peers use. Equally important, students learn that there are usually different ways to solve a problem. Finally, co-operative learning builds leadership, decision-making, and conflict management skills, and ideally, positive attitudes towards mathematics. (Muth, 1997, p. 72)

Within the mathematics classroom the main source of scaffolding is the teacher. The teacher’s scaffolding role is most commonly evidenced in classroom discourse when the teacher focuses, extends, clarifies and redirects the learner (Cambourne, 1988). Using an appropriate selection of tasks or problems the teacher builds on prior knowledge and maintains the student within the Zone of Proximal Development.

Another aspect of scaffolding by the teacher is the appropriate selection of tasks or problems to maintain the students within their ZPD (Diezmann, 1998). Stein, Grover, and Henningsen (1996) investigated the selection of mathematical tasks by teachers and concluded that appropriate task selection included having appropriate amounts of time to engage on the task. Inappropriate time limitations precipitated low-order recall. Sustained pressure for explanations and elaboration was important through successive multiple presentations of solutions and justifications to the wider group or class as this encouraged

the making of connections and hence higher order thinking.

2.3.2 Authority

Relationships and interactions within the classroom produce the construction of mathematical knowledge and identity, and are greatly influenced by the teacher's practices which establish social norms. Productive discourse practices in a classroom mathematical community not only support students' mathematical understandings but also give authority to their discourse, and value student authorship (Klein, 2002). The social goal in negotiating meaning is to resolve uncertainty and that in order to reach resolution students will appeal to the dominant authority. Prior experience, empirical evidence, a knowledgeable person or a text can represent this dominant authority (Clark & Helme, 1997). Brown (1994, cited in Brown & Renshaw, 1999) describes the process of 'Collective Argumentation' in which "authority is attained through discourse practices that privilege socio-mathematical norms such as meaningfulness, communicability and testability" (p. 114).

In the absence of appropriate authorities students find it difficult to make metacognitive judgements about errors, anomalous results and the appropriateness of their problem solving strategies (Clarke & Helme, 1997). The teacher's role in the process of learning to resolve uncertainty is important. If an autocratic teacher makes the majority of learning and organisational decisions in the classroom then students learn to rely on the teacher as their source of authority and to rescue them from uncertainty (Hill & Hill, 1990; Neyland, 1994). This can result in students developing poor decision making skills and exhibiting an unwillingness to attempt novel strategies or be open to other available authorities such as experience, empirical evidence, texts or knowledgeable group members (Davidson, 1985).

Teachers who consistently intervene when students are struggling take away their opportunities to make progress on their own (Stein, Grover, & Henningsen, 1996). Equally, a teacher who takes a laissez-faire approach and leaves the children to take responsibility for their own learning “does not provide for responsibility, cohesiveness, cooperative skills or academic achievement” (Hill & Hill, 1990, p. 18). In contrast, teachers who promote the model of positive interdependence required in collaborative learning and encourage an acceptance of peers’ ideas which are presented with logic and evidence, support cognitive development, and the movement of children’s beliefs from a position of dualism to ethical beliefs (Davidson, 1985).

Perry (1970, cited in Davidson, 1985) describes students’ beliefs about authority as cognitively developmental. At the initial level of development, Perry says students believe that all answers are right or wrong, and that only authority figures such as the teacher possess knowledge and answers. The next level of development shows students accepting some peers’ viewpoints and becoming less dependent on texts or the teacher as the authority. In the later stages of cognitive development the students are able to value the opinions of others when these opinions are presented logically and with evidence. The students in this stage are also able to reason in a wider range of contexts and treat their teacher as a fellow learner rather than a person of authority.

The teacher’s pedagogy (beliefs and practices) influences the learner’s sense of authority. Klein (2002) argues that the teacher can foster the learner’s ability to apply constructed mathematical knowledge in new contexts by celebrating “the student’s presence and ways of making sense of mathematics and experience” (p. 391) and by giving them space “ to

establish themselves in powerful ways in learning/teaching interactions as they construct mathematical ideas” (p. 392).

Discourse moulds not only the content knowledge but also the learner’s mathematical identity and perception of his/her competence as students learn its linguistic code and how to relate to a subject (Blackredge & Hunt, 1985). The learner develops a sense of power through his/her legitimate use of mathematical ideas and practices, and hence a sense of internal authority. For the learner to develop this power they must be “capable of, respected and valued in, speaking/writing the accepted ‘truths’ of a discourse, in enacting established ways ‘of being’ and in going beyond these to forge something new” (Davies, 1991, cited in Klein, 2002, p. 391). Teachers consider the relationships within discourse; of what is spoken and who it is spoken by, the power of the speaker’s social relationship or authority to develop socio norms within the mathematics classroom (Klein, 2002).

2.3.3 Transparency

One of the concerns for designing an intervention to enhance the level of higher order cognitive peer interactions is the degree to which the interaction amongst learners is structured. A balance needs to be achieved to enable the structure to promote both the exchange and challenging of ideas, and the freedom to pursue and develop individual thoughts (Cohen, 1994). The teacher has a major role in framing the language practices of the classroom as she acts as a translator for mathematical discourse “to help frame discussion, to pose questions, to suggest real life connections, to probe arguments, and to ask for evidence” (Adler, 1999, p. 51), and hence scaffold students’ participation and provide access to school mathematics.

Adler (1999) warns that too much focus on the practice of mathematics discussion makes the discussion become the “main object of attention instead of a means to the mathematics” (p. 50). The discourse practices should be used as learning tools.

Discourse as a social tool has a quality Lave and Wenger have labelled *transparency* (1991, cited in Adler, 1999). They describe transparency metaphorically as a window involving a complex interplay of visible and invisible characteristics:

A window's invisibility is what makes it a window. It is an object through which the outside world becomes visible. However, set in a wall, the window is simultaneously highly visible. In other words, that one can see through it is precisely what makes it highly visible.... [Windows are like] 'mediating technologies' in a practice...[that] need to be visible so they can be noticed and used, and they need to be simultaneously invisible so that attention is focussed on the subject matter, the object of the attention in the practice.

(Lave & Wenger, 1991, cited in Adler, 1999, p. 50)

The discourse strategies for this project were developed with *transparency* in mind and aimed to enhance cognitive mathematical learning. The students were made aware of the significance of their talk to learning, and their teacher and the researcher modelled specific kinds of mathematical discourse to them. They were encouraged to model the same practices within collaborative groups and engage the strategies as a successful method of learning the mathematics.

2.4 Collaborative Learning

The process of scaffolding can occur informally, or formally through the provision of structured co-operative and collaborative programmes that utilise discourse to enhance the teaching and learning of mathematics, enabling peers to construct robust knowledge structures as they clarify information and resolve discrepancies. Collaborative learning and peer-tutoring techniques provide scaffolding in the form of a peer who has authority because of their role (as tutor), their knowledge, or that they have the strategies to be able to utilise experience, evidence and texts in various combinations to provide authority.

Most New Zealand classroom teachers utilise peer discussion as a form of scaffolding (Ministry of Education, 1997) as peers negotiate meanings and ideas while involved in shared tasks. However, the enhancement of achievement through peer scaffolding is more complex than assigning children to a group to work together to solve a problem (Peter-Koop, 2002; Webb, 1989). Brown and Thomson (2000) make a firm distinction between what they call 'traditional group work' and learning together co-operatively to produce social or academic outcomes:

Working round a table on individual tasks with the opportunity for discussion is not co-operative learning. Nor is having a team discussion, where some students can dominate or "hitch-hike". The core notion of co-operative learning is that when we co-operate, we work together to accomplish shared goals.... While it helps to foster and develop interpersonal skills, co-operative learning is not a social skills programme. Rather it is an academic skills programme, a thinking skills approach that requires students to develop the necessary interpersonal and small group skills to enable them to work successfully together.

(Brown & Thomson, 2000, p. 38)

Cooperatively styled programmes that have an active, focused approach to learning where both the teacher and the student are aware of the importance of scaffolded practice are more likely to lead to more effective learning (Brown & Thomson, 2000, p. 26). There is a wealth of methods currently employed in the classroom to promote collaborative learning. They build on the momentum of the cooperative programme development of the 1970s and group learning methods such as Constructive Controversy (Johnson & Johnson, 1987), Team Games (DeVries & Edwards, 1974), Group Investigation (Sharan & Sharan, 1976), Jigsaw Procedure (Aronson, Blaney, Stephan, Sikes & Snapp, 1978) and Student Teams Achievement (Slavin, 1989, 1996). Work has continued into the 1980s and 90s with Cohen's Complex Instruction approach (1986), Kagan's structural PIES approach (1994), Brubacher, Payne and Rickett's collaborative approach (1990) and Brown's Collective Argumentation (1994, cited in Brown & Renshaw, 1999).

Johnson and Johnson define co-operative learning as "the instructional use of small groups so that students work together to maximise their own and other's learning" (1994, p. 1). However, literature shows that others employ the term 'co-operative learning' in a loose fashion to include all group work. To avoid this ambiguity there has been a move by researchers in mathematics education to differentiate organised and structured group learning under the term 'collaborative learning'.

Goos (2000) describes collaborative learning as those procedures "designed to engage students actively in the learning process through inquiry and discussion with their peers in small groups" (p. 39). It is a reciprocal process of *mutuality* where each other's reasoning and viewpoints are explored in order to construct a shared understanding of the task (Goos, 2000).

Davidson and Worsham (1992, p. xiii) identify four common attributes for all collaborative learning with the premise that the group work is organised and structured “to promote the participation and learning of all the group”:

1. There is student-to-student interaction in small groups.
2. There is individual responsibility and accountability, or positive interdependence.
3. There is structured cooperation.
4. A learning task/activity is selected for group work.

In the definitions of both Brown and Thomson (2000) and Johnson and Johnson (1994) the term ‘co-operative learning’ could be interchanged for ‘collaborative learning’ but to avoid confusion with other models of co-operative learning the term *collaborative learning* will be used in this study. While references will be made to co-operative learning research which conforms to the collaborative learning definition the study specifically differentiates *collaborative learning*, which involves the development of cognitive thinking through verbal negotiations of meaning with their peers, from that of involvement in cooperative group tasks. Cooperative group tasks are considered shared activities whereas collaborative learning is "a mutual task in which partners work together to produce something that neither could have produced alone" (Forman & Cazden, 1985, cited in Thomas, 1994, p. 25).

Working collaboratively in mathematics supports students’ learning when they resolve contradictions that arise and when:

...They attempt to make sense of a situation in terms of their current concepts and procedures, accounting for a surprise outcome (particularly when two alternative procedures lead to the same result), verbalising their mathematical

thinking, explaining or justifying a solution, resolving conflicting points of view, developing a framework that accommodates alternative solution methods, and formulating an explanation to clarify another child's solution attempt.

(Yackel, Cobb & Wood, 1991, p. 394)

Proponents of collaborative learning argue strongly that effective collaborative learning is not automatic. Students do not osmotically obtain effective learning strategies but rather they “gain them by exposure to capable models and from guided practice in their use” (Brown & Thomson, 2000, p. 33). This includes learning how to deal with disagreements, as collaborative learning is not about harmonising, as it often involves intellectual conflict (Hill & Hill, 1990). If students do not develop skills of positive interdependence whereby they work together to achieve the group’s learning goals through the synthesis of independent and collaborative contributions their learning is likely to be no more successful than competitive or individualistic learning models (Kneip & Grossman, 1979; Qin, Johnson, & Johnson, 1995).

2.4.1 Cooperative Model (Brown & Thomson, 2000; Johnson & Johnson, 1987)

Brown and Thomson (2000) are two researchers in Special Education who have contributed strongly to New Zealand work on co-operative learning and who differentiate it from traditional group work. They build on the extensive work of Roger Johnson and David Johnson who pioneered co-operative learning as a teaching procedure (Johnson & Johnson, 1994; Johnson, Johnson & Johnson-Holubec, 1993; Johnson, Johnson & Stanne, 2000; Johnson, Johnson & Zaidman, 1985; Yager, Johnson & Johnson, 1985).

According to Brown and Thomson’s model (2000), as students practise collaborative learning skills they move through four levels of functioning. These levels of functioning are

based on Johnson and Johnson's '4Fs': forming, functioning, formulating, and fermenting.

'Forming' skills are basic skills required for groups to function such as moving and talking quietly, using eye contact and group members' names, and encouraging all group members to participate. 'Functioning' skills are those skills, which allow greater self-management within the group. Individual members maintain their given roles, all group members are included and encouraged, and the interactions are both courteous and positive.

The next two levels of skills require more understanding of the task and the use of higher order thinking skills (Bloom, 1956). The third level of skills is 'formulating' and requires students to be able to apply and analyse ideas. Students engaged in a task ask for and listen to elaborations, justifications, and summaries from other group members. The fourth and final skill level is that of 'fermenting' when students are able to integrate ideas to form a concept or general principle. They are able to question, critique and evaluate peer ideas, and are able to develop and integrate other peer ideas into a new concept or application. This level aligns with synthesis and evaluation in Bloom's taxonomy (1956). At this level students are also able to handle controversy in a positive and constructive manner.

Collaborative group work encourages vocalisation amongst peers and as the students develop the higher level thinking skills of formulating and fermenting they become more effective learners.

Johnson, Johnson, Roy and Zaidman (1985) conducted research into the oral interactions of children (American Grade 4) and they found that a high proportion (90%) of talk was task-related. When they investigated the behaviours of high, medium and low achievers in the

groups they discovered that vocalisation had significantly more impact on achievement than listening. They noted that:

Vocalizing task-related information orally striving to obtain more facts and information about the subject under discussion, giving explanations, providing rationales, and relating what is being learned to previously learned information, and disagreeing with other members' task-related conclusions were all significantly related to achievement. (Johnson, Johnson, Roy, & Zaidman, 1985, p. 318-19)

They also found that medium achievers did not vocalize as much as high and low achievers in collaborative groups and concluded that medium achievers needed particular encouragement to elaborate in order to make more significant gains in achievement.

From their results, Johnson et al. (1985) recommended that teachers encourage collaborative groups to elaborate upon material by providing a structure that increased the level of on-task vocalisation and introduced controversies so that students had to negotiate meanings from other students' task-related conclusions.

2.4.3 Think-Pair-Share Model (Lyman, 1992)

Lyman (1981, 1992) developed a cooperative learning strategy called 'Think- Pair- Share': A question or problem is posed then the students are given time to think individually. The students cannot talk or raise their hands but may write down or draw a diagram to represent their individual ideas. At a designated time, indicated by the teacher, the students share ideas. The sharing of ideas results in a collective understanding, generated when they challenge and elaborate upon each other's thinking. Ideas may be subsequently shared with a larger group or the class. Using a variation of the Think-Pair-Share model in

mathematical collaborative learning investigations, Neyland (1994) found that use of individual think time at the start of an investigation resulted in more students contributing to the mathematics subsequently developed by the group. Students did not run with the first idea and they began to value the alternative thinking styles of their classmates as well as utilising a wider range of problem solving strategies.

The Think-Pair-Share model utilises the concept of 'wait-time'. Wait-time, a pause between the question and a student's response, provides an opportunity for a student to formulate a thoughtful reply, to recall relevant prior knowledge, or validate a possible solution. This skill can be developed by the students in the forming and functioning levels of collaborative group skills (Thomson & Brown, 2000). Various researchers and educationalists have investigated the use of wait-time to improve the quantity and quality of student verbal responses. Rowe (1974) and Tobin and Capie (1980) found that when teachers paused for three seconds or longer after posing their initial question, and again after the student's initial response, that the length of student responses increased, more frequent unsolicited contributions were made, explanations became more logical, students supported ideas with evidence, participation by less able class members increased, and the number of questions and speculative responses increased. The use of wait-time has also been shown to contribute to gains made by tutees in peer-tutoring reading programmes and written language programmes (Medcalf, 1999; Medcalf & Glynn, 1987).

2.5 Peer-tutoring

The initial collaborative discourse model considered for this study was a peer-tutoring programme. Peer tutoring is an instructional programme where students teach other students. The idea of student as teacher and learner is not new and has been documented as far back as Roman times. More recently, the pupil-teacher of the nineteenth century formed a structured part of classrooms in the United Kingdom and the Colonies (Limbrick & McNaughton, 1985). Teachers for large class numbers or diverse age groups in small rural schools have formerly used cross-age peer tutoring in New Zealand as a coping mechanism. Since the mid 1980s peer tutoring has gained in popularity because of the increased use of co-operative learning models (Medcalf, 1995) and individualised instruction, and because of perceived cost-effectiveness benefits (Jenkins & Jenkins, 1988).

Research has shown peer-tutoring programmes to produce positive academic gains in mathematics (Beirne-Smith, 1991; Fuchs, Fuchs, Hamlett, Phillips, Karns & Dutka, 1997; Greenwood, Sloane, & Baskin, 1974; Maheady, Sacca, & Harper, 1987); spelling (Garcia-Vasquez & Ehly, 1992; Mallette, Harper, Maheady, & Dempsey, 1991; Greenwood, Daiquiri & Carta, 1997); reading (Kreuger & Brown, 1999; Medcalf & Glynn, 1987; Robinson, Glynn, McNaughton & Quinn, 1979); written language (Huggard, 2002; Medcalf, 1994); and where students use English as a second language or are intellectually impaired (Bar-Eli & Raviv, 1982, cited in Garcia-Vazquez & Ehly, 1992).

Positive social gains have also been documented when the peer tutoring has been used directly to effect behavioural changes (Schunk, 1987) and to raise self-esteem as a by-product of increased academic success (Medcalf, 1995).

The term 'peer-tutoring' is used loosely and widely in literature to describe buddy systems, role-models, peer-testing, peer-monitoring and peer-teaching, but will be considered within this study as a programme in which students are trained to use specific instructional strategies to promote the learning of peers. The strategies used by the child in the tutor role include promoting higher order cognitive thinking by asking appropriate questions, waiting so his/her partner has time to think before responding, listening attentively, and giving positive feed-back and encouragement (King, Staffieri, & Adelgais, 1998). The central role of peer tutoring is to provide structured prompting of cognitively orientated talk in order to promote active construction through the analysis and integration of ideas. However peers talking while working on tasks independently of their teachers is not enough to "substantially further their mathematical understanding" (Thomas, 1994, p. iii). Effective training of peer tutors is imperative. Without training "peer tutors are likely to be less than helpful in supporting the learning of others" (Medcalf, in press).

Peer-tutoring in this study is not considered the same as peer-mentoring (the buddy-system), or peer-testing, which involves peers carrying out 'low order' tasks together to capitalise on the influence children have on each other and the comfort they have in working together. Some of the literature regarding peer tutoring encompasses these mentoring and testing programmes, particularly American literature. Peer tutoring is distinct from peer mentoring in that tutors are trained and monitored to carry out their role. For the purpose of this study peer tutoring will exclude 'buddy-support' or peer testing such as that used by Greenwood, Daquiri and Carta (1997). Though Greenwood et al.'s programme bears the name 'peer-tutoring' it is in fact a peer-testing programme which is used to provide immediate feedback to a partner rote-learning a set of spelling words and mathematics basic facts. It is a competitive programme, with the tutee receiving a series of points (for

correctly spelt words) to contribute to the team total. This type of programme does not promote “greater use of higher level reasoning strategies and increased critical reasoning competencies” (Medcalf, 1995, p. 11) which is inherent in the training of tutors for collaborative style learning, peer-tutoring programmes.

2.5.1 ‘Pause, Prompt and Praise’ Model (Medcalf, 1992, 1994)

Medcalf has developed a New Zealand based peer-tutoring programme for mathematics adapted from a research base associated with reading and written language. The mathematics peer-tutoring programme is “designed to help students identify errors in their thinking relating to their problem-solving efforts in maths, provide appropriate modelling and feedback and develop metacognitive skills” (Medcalf, p. 6, in press). In this programme peers are taught to perform four steps framed by the mnemonic WASP — Watch, Ask, Show, and Praise. The first step involves the tutor watching the tutee solve the problem. If the tutee has difficulties the tutor asks the tutee questions to redirect him/her to the next step or a previous error. If the tutee cannot proceed alone the tutor demonstrates (or shows) the next procedure in the problem or a similar example and then encourages the tutee to complete a similar problem independently. The last step is giving praise for completing the problem, or prompting the tutee as he/she works independently to complete the task with specific feedback on the issue the tutee was having difficulty with.

Grant has trialled this approach in two pilot studies: one involving measurement with a class of nine and ten year olds, and the second (Grant, 2000, cited in Medcalf, in press) with twelve 9/10 year olds using mathematics games to develop computational skills. Both studies revealed increased self-esteem in tutors/tutees, increased on-task behaviour in all mathematics lessons, social benefits to the class and a higher average performance in the

post-unit test by the tutor group compared with the control group. In each study, the tutors were trained to respond within a particular mathematics unit (i.e., measurement and computational skills) and the tutors had specific mathematical knowledge beyond the level of the tutees.

In contrast to Medcalf's programme, the tutor training in this study was developed so the instructional skills of the tutor could be applied generically to any strand of the mathematics curriculum. In this way the child in the role of tutor (or coach) did not have to be more mathematically knowledgeable about a specific task than the child in the role of tutee (see 2.6.3). In the study this was achieved by introducing a collaborative component in which both students in a pair shared responsibility for the functional group skills and completion of the task. The programme and training is explained in detail in Chapter 3.

2.5.2 'ASK to THINK-TEL WHY' Model (King, 1997)

Sometimes peers, especially younger or less able students, are unable to verbalise their ideas so there can be no negotiation of meaning but by providing "rubrics, or key phrases, to explicitly encourage them to engage with each other's thinking" (Goos, 2000, p. 43) their ideas can be heard and evaluated.

The 'ASK to THINK-TEL-WHY' programme was developed by King (1990, 1997) as an inquiry-based model of mutual peer-tutoring. Peer interactions were enhanced through the use of reciprocal peer-questioning strategies with generic stem questions. In King's model (1990) the question patterns created socio-cognitive conflict and forced the students to restructure their knowledge in order to respond to a particular pattern. They generated new examples and elaborated upon existing relationships between ideas in order to respond.

King found that students tended to interact at a "basic, concrete knowledge, retelling level" if left to their own devices, but by structuring the interaction with the provision of generic stem questions the students engaged "in the mutual exchange of ideas, explanations, justifications, speculations, inferences, hypotheses, conclusions, and other high level discourse known to promote peer-learning "(King, 1997, p. 222).

Different kinds of questioning promote different levels of cognitive thinking. Asking, "What happened?" promotes lower order descriptive responses that recall facts. The information is restated within an integral context—reporting what happened during the concrete experience of the task. Asking "Why?" or "How?" type questions promote 'higher order' responses involving justifications, explanations and inferences. This encourages the integration of complex knowledge through making new connections between the task and external contexts (King, 1998).

Similar models have been developed in Australia to assist peers, especially younger or less able students, to verbalise their ideas so there can be negotiation of meaning by providing "rubrics, or key phrases, to explicitly encourage them to engage with each other's thinking" (Goos, 2000, p. 43). These interactions engage students' higher order cognitive processes and assist them to build robust knowledge structures (King, 1997; King, Staffieri, & Adelgais, 1998).

2.5.3 Multiple Retelling Model (Cesar, 2001)

Another programme which focuses on the development of higher order cognition uses didactic peer interactions with the multiple retelling of solutions. This model was developed by Cesar (1998), who conducted a four-year project into social interactions of didactic pairs

and their mediating role in knowledge apprehension, cognitive development and skills acquisition in 5th to 11th graders. The didactic pairs were initially suspicious of one another and disinclined to work cooperatively. Problems were overcome by the implementation of rules, or a didactic contract, where each individual had to be able to explain both problem-solving strategies in order that either one of them could present their work during the class discussions. In this way they explained their own problem-solving strategy and asked questions and actively listened so that they would also be able to explain their partner's strategy. From this study Cesar(1998) argued that the interaction itself is enough to promote better performances. This contrasts with Vygotsky's belief that the social interactions of a *more competent* peer or adult best.

2.6 Nature and Analysis of Classroom Discourse

The complex interplay of content, context and relationships in the nature of discourse makes it difficult to select an adequate tool to measure and analyse classroom interactions. Three models were considered for this study: Thomas' model (1994); Johnson, Johnson, Roy, and Zaidman's model (1985); and King, Staffieri, and Adelgais' model (1998). The models and the implications suggested by the pilot research findings are discussed to provide a rationale for the intervention developed for this project.

King, Staffieri, and Adelgais' model (1998), which measured the interactions in three peer-tutoring conditions, was considered because it was developed to measure tutor responses in a pre-training and post-training situation. King et al. analysed the data using rates of interaction per minute and classified on-task interactions into three categories, those related to questioning, task statements, and supportive communication. This model required a question/statement or statement/statement interaction between tutor and tutee. After some

initial trial in the pilot study it was discarded because it was not able to be appropriately adapted to a multi-member group with equality of roles.

Johnson et al.'s model (1985) was considered because its development was supported by three decades of research by Johnson and Johnson in the area of co-operative learning. The particular model they used had been developed over three years as they observed co-operative groups and categorised the groups' oral interactions. The five-factor classification system resulted from factor analysis and input from Roy and Lyons' observational model of oral rehearsal and cognitive processing (1982, cited in Johnson et al., 1985). The five categorisations are:

- interactions exchanging task related information;
- interactions elaborating information;
- interactions encouraging each other to learn;
- interactions disagreeing with each other's conclusions; and
- non-task comments and sharing feelings.

This model, which was trialled during the pilot study, was extremely simple to administer with instances of talk able to be placed clearly in one of the five-factor categories.

The third model considered was Thomas' (1994) as it had been developed and used in New Zealand mathematics classrooms specifically to investigate the interactions of children as they worked in small groups independently of the teacher. Thomas classified transcriptions of children's small group interactions, which had been videoed, using a computer programme to encode and identify patterns in verbal data.

When Thomas (1994) conducted her study on the nature of talk in the New Zealand junior

mathematics classroom, teachers were surprised that 91% of the talk was task-related which is consistent with Johnson et al.'s findings (1985). Thomas also found that the usual pattern for non-task related talk was that it was "brief, intermittent and interspersed with task-related talk" (p. 113). In most cases the children continued to work on the activity while they engaged in non-task related talk. However of the task-related talk, 53% was to do with social management aspects of the task and only 37% with cognitive aspects of the task.

Thomas identified two further subcategories of cognitive talk (see Figure 1), that of reflection, when children considered their own thinking, and that of action talk, when they verbalised what they were doing, read aloud (reported on their writing), asked questions, gave help or commented to their peers about what they were doing with the expectation of a response.

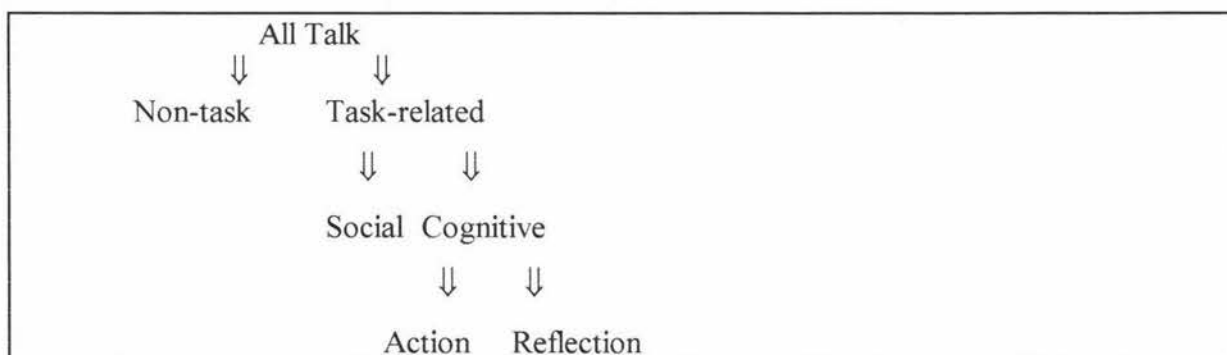


Figure 1: Thomas' Model of Talk (1994, p. 119).

Thomas found that amongst young children there was very little reflective talk (1%) unless the children were prompted by a teacher's question. This observation is also borne out by Higgins (1994) who found that the mathematical focus of an activity could be overrun by social and procedural problems in junior mathematics classrooms in New Zealand. Higgins also found that where teachers modelled mathematical language instead of everyday

language, and modelled discussion of an activity, that the children could apply these cooperative skills in later independent activities, by listening to explanations from other group members, asking questions of each other and explaining their thinking. Young-Loveridge (1993) also found that younger children involved in junior mathematics classrooms required more monitoring, encouraging and modelling by teachers to remain on task and develop effective cognitive skills.

In Thomas' study (1994), the majority of social talk was talk about the organization of materials for the task (29%) and about the organization of other children (16%), and whose turn it was (20%). Social talk included a degree of social bargaining to gain acceptability and access to groups, articulation of the rules, disputes and comments about progress, such as who was winning a game/ who had the most/ who had completed their worksheet first. The amount of social talk increased as group numbers increased in size.

2.7 Literature Review Summary

This literature review summarises the positive social and academic outcomes attributed to peer-tutoring and collaborative group-work. It emphasises the importance of the teacher's role to select 'rich' engaging tasks at an appropriate level, model discourse patterns, and utilise effective discourse strategies in collaborative programmes.

The review highlights the need for collaborative group skills to be taught in order to move students from basic group forming and functioning levels to effectively and consistently implementing higher order cognitive thinking skills at fermenting and formulating levels.

Chapter 3 Methodology

3.1 Introduction

The aim of this action research study was to develop, implement, and refine a collaborative programme which has practical benefits to classroom teachers who wish to develop their students' higher order cognitive thinking in mathematics. The intervention was designed to support effective discourse structures of small groups working collaboratively on problem solving in the mathematics classroom. Scaffolding was provided by the teachers in the form of modelling and programme structure, and reinforced by competent peers during group work, in order to increase students' involvement in higher order cognitive interactions. The following questions formed the basis for the research as to the effectiveness of the intervention:

1. Does participation in the programme increase the frequency of higher order cognitive interactions between children in the mathematics classroom?
2. Are there 'oral flags' which identify higher order cognitive thinking?
3. How did the students think that participation in the programme affected their academic achievement?
4. What changes did the teacher notice in individual/class interactions or attitudes when mathematical problem-solving?

3.2 Action Research Design

The research design is based on a classroom-based action research model involving three cycles (Kemmis & Taggart, 1981). This report focuses on Cycle 3 (Main Study) of the action research cycle, provides an overview of Cycle 1 and Cycle 2 (Pilot Study), and shows their relationship to the third cycle.

Lewin (1948) and Collier (1945, cited in Ebbutt, 1985) independently pioneered the concept of action research. The action research model “proceeds in a spiral of steps each of which is composed of a circle of planning, action and fact-finding about the result of the action.” (Lewin, 1948, p. 205). Action research was initially used to deal with community issues in social planning but has subsequently been employed in the education arena.

Kemmis and Taggart (1981) developed Lewin’s spiral action research model in Australia as the Action Research Planner and their draft was furthered almost immediately by Elliott (1981) in Britain as the Action Research Framework for Self-Evaluation in Schools. Elliott (1991) defines action research as “the study of a social situation with a view to improving the quality of action within it” (p. 61).

Kemmis and Taggart (1981) summarised action research as a cycle (Figure 2), which involved planning, acting, observing, and evaluating:

- “to develop a *plan* of action to improve what is already happening;
- to *act* to implement the plan;
- to *observe* the effects of action in the context in which it occurs; and
- to *reflect* on those effects as a basis for further planning, subsequent action and so on, through a succession of cycles” (p. 7).

This study design incorporates Kemmis and Taggart’s spiral model within a social situation involving small groups of students engaged in collaborative mathematical problem solving. The action research model follows a cycle of planning, action, observation, and reflection, with adaptations made to the next level of the cycle after each phase. The main study focuses on the third cycle of the action research model.

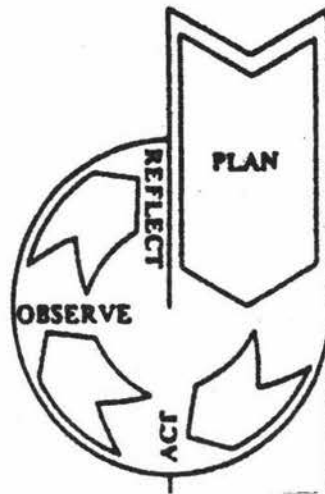


Figure 2: Kemmis and McTaggart's Action Research Planner (1981)

During the 1990s there was a rise in the amount of research undertaken within the context of the classroom; whereas before, the subject (s) undertook controlled, observed tasks and interviews outside the classroom. The action research model provided an ideal vehicle for increased teacher involvement in research projects (Ebbutt, 1985).

However, the idea of teacher as researcher alarmed some members of the academic research community who believed a teacher's research would not be academically rigorous (Carnine & Gersten, 2000). This argument was given support by those who did not differentiate 'normal' teaching practices from research, and who believed action research was an integral part of 'good teaching' and that it was inherent in the daily cycle of planning, assessment and evaluation in the classroom (Sharples, 1983, cited in Kelly, 1985). Others believed that by allowing teachers to maintain a dual role as researcher and teacher, 'coal-face' questions about learning and teaching would be more likely to be pursued in the microcosm of the classroom and that this in turn would provide impetus for further global research (Cochran-Smith, & Lytle, 1993).

Ebbutt (1985) worked with Elliott using the educational action research model and created a broad classification system to differentiate daily teaching practices from those of 'teacher-researcher' (see Appendix 1). The role of teacher-researcher in the second and third cycles of this study falls within Ebbutt's 'classic' action research model due to the degree of systemization, the testing of hypotheses beyond one classroom, and the collaboration undertaken with other teachers working in the same field of interest. The principles of planning, action, observation, and reflection are used for the systematic collection of information, and to reflect upon and plan change while maintaining flexibility and responsiveness to the context (Poskitt, 1994).

3.3 The Setting

The research was undertaken at a three classroom rural primary school (Roll approximately 60, decile 4¹). Each of the three classrooms was multilevel: The junior classroom included all Year 1, 2 children; the middle classroom Year 3, 4, 5 children; and the senior classroom Year 6, 7, 8 children.

The pilot study took place in the middle classroom in 2001 (see summary in 3.6.1). The main study was undertaken over four weeks in Term 2, 2002, in the senior classroom.

3.4 Profile of the Sample Group

A composite class of 25 Year 6, 7 and 8 children were involved in the collaborative training programme. The class had two teachers (teaching principal and release teacher) who took

¹ Funding in New Zealand schools is currently allocated per capita according to a socio-economic decile rating from 1 to 10, with decile 1 being the lowest and receiving the greatest funding per capita. A school receives a decile number according to the average of the socio-economic location of the children's residential addresses.

joint responsibility for the implementation of the mathematics programme. Twenty children volunteered to be part of the research study but data analysis was completed for only 15 Year 6 and 8 children who were present for the entire training programme. The sample group consisted of four girls and two boys from Year 6, and three girls and six boys from Year 8. Nine identified themselves as Pakeha/European, three as Maori, two as Pakeha/Maori, and one as Maori/Pacific Islander. All the Year 6 children had been involved in the pilot study the year before. None of the Year 8 children had been involved in the pilot study.

The class of twenty-five children from which the sample group was taken was concurrently involved in the 2002 Ministry of Education Numeracy Project. The numeracy teaching, during the period of the study, involved addition and subtraction of whole numbers and fractions. The classroom teachers used the draft Diagnostic Interview for the Numeracy Project (Ministry of Education, 2002c) to arrange sub-groupings and assess academic gains.

3.5 Timeline

Previous Year	Pilot Study with researcher's class.
Term 1	Literature Review.
Term 2 Week 1	<ul style="list-style-type: none"> • Classroom teacher administers Numeracy Project Diagnostic Interview. • Classroom teacher organises learning groups. • Researcher audiotapes samples of each learning group's pre-training discourse while engaged in problem solving.
Term 2 Week 2-5	Collaborative training programme commences as an integrated component of the students' mathematics class.

Term 2 Week 6	Post-training audiotaping of each learning group's discourse while involved in collaborative problem solving.
Term 3 Week 5	Post-unit Numeracy Project Diagnostic Interview.

Table 1: Daily group rotation during mathematics.

Activity during Mathematics Class	First 20 Minutes	Second 20 Minutes	Third 20 Minutes
Researcher - collaborative training in library.	Group 1	Group 3	Group 2
Independent- Problem-solving in groups in classroom.	Group 2	Group 1	Group 3
Teacher- instruction in classroom	Group 3	Group 2	Group 1

3.6 Developing the Instructional Strategies

3.6.1 Development of the Training Programme through Cycles 1 and 2

The first cycle of the action research model (Figure 3) was used when the researcher worked (as a collaborative teacher) with a Resource Teacher for Learning and Behaviour (RTLB) on a reading intervention plan for a student in 2000. The intervention plan was based on training pupil tutors using Medcalf and Glynn's (1987) 'Pause, Prompt and Praise' Model. In this model, tutors are specifically taught to praise tutees for self-corrections, correctly reading sentences, and the implementation of reading strategies. They are trained to pause to give the tutee time to think and to prompt if necessary with a series of reading strategies. These tutor-tutee interactions are designed to promote the high order cognitive skills of explanations, justifications, speculations, hypotheses, inferences, conclusions, and questioning.

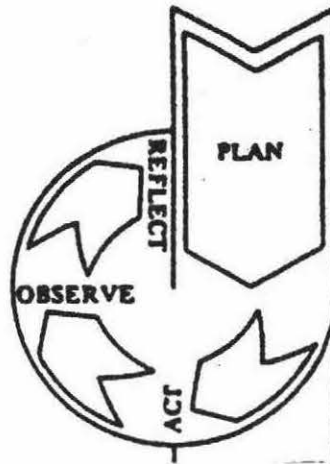
Reflect

Classroom Teacher and RTLB discuss the Peer-tutoring Programme and its high level of success in the development of the tutors' self-esteem, reading skills and reading achievement.

The Classroom Teacher and the RTLB raise the question as to whether the principles of the programme can be generalised to other curriculum areas such as writing and mathematics.

Observe

RTLB makes Running Records of the tutors' reading behaviour and levels, before and after the programme.



Revised Plan

RTLB plans adaptation for written English and Classroom teacher plans adaptation for mathematics.

Plan

Use "Pause, Prompt and Praise" method of peer-tutoring as an intervention for a child with a low reading level. Aim is to develop self-esteem, reading skills and reading achievement for this child.

Act

RTLB implements a peer-tutoring programme with four children as tutors. Classroom Teacher is instructed in the principles of the programme and schedules daily tutor/tutee meetings. Children act as tutors for four members of junior class.

Figure 3: Cycle 1 of this Action Research Study: The classroom teacher becomes involved in research at Ebbutt's Self-Evaluation Action Research Mode. (Refer to Appendix 1).

The action phase of Cycle 1 involved the RTLB implementing the peer-tutoring programme with four children. The classroom teacher was instructed in the principles of the programme and scheduled daily tutor/tutee meetings in which the four students tutored junior class members in reading. The observation phase involved the RTLB recording the tutors' reading levels and behaviours using the Running Record system (Clay, 1985) before and after the peer-tutor training. The classroom teacher and the RTLB reflected on the success of the programme as an intervention and discussed whether the principles of the programme could be generalised and implemented as an intervention in other curriculum areas.

In the second rotation of the action research cycle the classroom teacher, now in the role of researcher, developed an intervention based on Medcalf's peer-tutoring procedure (WASP) for mathematics (Medcalf, in press). Medcalf had adapted this model from the 'Pause, Prompt and Praise' for reading (Glynn & Medcalf, 1987). The following steps outline this procedure:

W = Watch

Watch as your friend shows you how they work out the problem.

A= Ask

Ask questions to check out your friend's understanding

E.g. "How did you get that answer?",

"What do you think the next step is?",

"Can you see where you went wrong?".

Sometimes asking the right question will be enough for the tutor to realise their error and be able to self-correct or complete the problem independently. If so the tutor may skip the next step and proceed to prompt and praise.

S=Show

Show your friend how to complete the problem. This will often involve modelling a particular step in the process that a tutee is experiencing difficulty with and/or completing incorrectly. The tutor can assist the tutee to complete a similar problem with their help and encourage the student to then attempt another example independently.

P=Prompt and Praise

Prompt your friend to complete a similar problem on their own and praise their efforts. The tutor prompts the tutee to attempt a similar problem independently. Before doing this the tutee is asked to tell the tutor what they will do to solve

the problem and complete the task. The tutor watches and provides specific positive feedback with special emphasis on the step or issue that the tutee was having difficulty with.

This pilot study, Cycle 2, aimed to test Medcalf's findings of social and academic benefits through replication, but also to evaluate and refine the type of questioning format in the 'Ask' phase. Questions were specifically designed to increase higher order thinking and scaffolding. The pilot study, undertaken within the teacher-researcher's own class of (twenty-four) Year 4 and 5 children, involved making anecdotal observations and audio recordings of classroom peer interactions during the collaborative group work in mathematics. The Year 4/5 class was concurrently involved with a pilot study for written language with the RTLB (Huggard, 2002). This reinforced the establishment of a classroom community in which students learned cooperatively and questioned each other's thinking.

Reflect

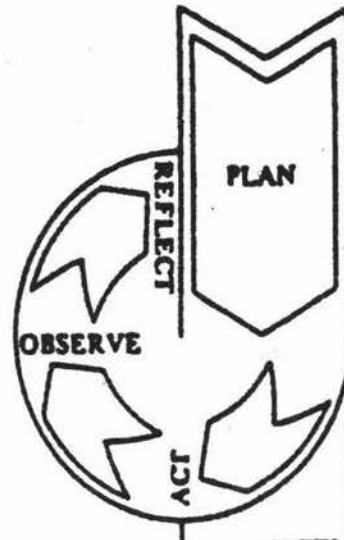
Think-Pair-Share model best promoted mathematical learning with peer support by requiring individual accountability, idea construction through negotiation of meaning and justification by presenting ideas to partners and the wider group.

Thomas' model has NZ research base and easy to use initial categories of on task/off task, and social/cognitive interactions. Need to redefine subcategories of cognitive reflection and cognitive action.

Observe

Transcribe and analyse interactions of children engaged in mathematical problem solving in small groups.

Make anecdotal observations of interactions that appear to promote mathematical learning amongst peers.

**Plan**

Review literature for:

1. Examples of peer-tutoring, cooperative/ collaborative learning programmes.
2. Analysis tools for discourse in the mathematics classroom.
3. The use of cognitive thinking skills to promote learning.

Use these literature ideas to develop and trial a pilot intervention which promotes peer learning in the mathematics classroom.

Act

1. Use WASP and Think-Pair-Share learning models.
2. Trial ways to quantify qualitative data in the analysis of classroom interactions: Thomas' Model (1994); Johnson et al.'s Model (1985); and King et al.'s Model (1998).

Revised Plan

Refinement of pilot programme to promote "wait-time", specific feedback, multiple presentations of solutions with explanations and justifications. Reduce off-task and social interactions by careful selection of rich tasks by teacher and reinforcement of cooperative group skills with teacher modelling. Define collaborative learning cf. cooperative learning. Redefine cognitive subcategories in terms of Bloom's taxonomy, as lower or higher order cognitive thinking. Identify more verbal flags that indicate cognitive thinking.

Figure 4: Cycle 2 the Pilot Study: The Teacher as Researcher in Ebbutt's Classic Action Research Mode. (Refer to Appendix 1).

The data from the second cycle was quantified and analysed using three models of discourse analysis: Thomas (1994), Johnson, Johnson, Roy, and Zaidman (1985), and King, Staffieri, and Adalgais (1998).

Johnson et al.'s model (1985) enabled children's talk to be clearly classified into either one category or another. Though it was the easiest of the three to administer for collaborative

groups, it did not give the researcher enough information about the type of information exchanged.

Thomas' model (1994) was both hierarchical and linear which suited discussion that modelled a hierarchical model of cognitive thinking such as Bloom's taxonomy (Bloom, 1956). The model also defined the concept of a 'turn' as the utterance made by one child before another child speaks. A turn could be a single word, a phrase, or one or more sentences (Sharan & Sachar, 1988, cited in Thomas, 1994) and this was used to define instances of data. Though the Thomas' model distinguished the classification of cognitive talk into subcategories of action and reflection a child's turn could be enmeshed with elements of both.

King et al.'s model (1998) required a question/ statement or statement/statement interaction between tutor and tutee and was found to be inappropriate for analysing collaborative group discourse.

These considerations alerted the researcher to the need to clearly match the method used to analyse the children's interactions with either the collaborative group model or peer-tutoring, dependent upon the research questions to be considered in the main study. For this reason, Lyman's Think-Pair-Share method of cooperative learning (1992) was trialled as an alternative to Medcalf's WASP technique (in press). The Think-Pair-Share method promoted mathematical learning with peer support by requiring individual accountability, idea construction through negotiation of meaning, and the development of higher order cognitive thinking when students elaborated and justified their ideas and their solutions to partners and the wider group.

The pilot study, conducted over twenty weeks for eighty hours, provided insights into factors which supported effective mathematical learning by children in collaborative groups.

These factors included:

1. Careful task selection at an appropriate level to reduce social/procedural task-related talk and increase cognitive task-related talk; and to maintain students in their *zone of proximal development* (Vygotsky, 1978).
2. Modelling and the provision of feedback by the teacher (and other groups) to develop and encourage collaborative strategies which reflect positive interdependence including:
 - listening to explanations from other group members;
 - asking questions to clarify own/other's thinking;
 - providing evidence to justify conclusions/solutions ("What's your evidence?");
 - praising and encouraging with positive specific feedback; and
 - giving and receiving help.
3. Development of mathematical strategies by teaching modelling of mathematical language.
4. Provision of individual thinking time to allow for the development of multiple strategies as a base for negotiating meaning and solutions.
5. Group size effects, particularly with reference to maturational age. Pairs usually resulted in each partner sharing the talk equally (50% of all talk). Groups of three or four could be dominated by a particular member.

In preparation for Cycle 3, which was to comprise the main study, the researcher used the information gained in the second cycle to:

- Select the Think-Pair-Share technique as the basis for collaborative learning in small groups.
- Select rich, appropriately levelled tasks to reduce task-related social interactions.
- Reinforce and model cooperative group skills by the teacher to reduce off-task social interactions.
- Select Thomas' model to analyse interactions but with the modification that the subcategories of cognitive interactions were replaced with lower cognitive interactions and higher cognitive interactions. The lower cognitive interactions were defined as those involving recall and recognition (knowledge) and translating symbols and words (comprehension). The higher cognitive interactions were defined as those, which involved elements of application, analysis, synthesis and evaluation based on Bloom's taxonomy (1956).
- Define and use the term 'collaborative' instead of 'cooperative'.
- Be aware of phenomena such as children using 'verbal flags' (e.g., 'so'), to identify the linking of ideas and hence higher cognitive thinking.

The next cycle of the action research study was to investigate the use of collaborative training to promote increased higher-level cognitive interactions between children.

3.6.2 The Main Study - Cycle Three

The main study, Cycle 3 (see Figure 5), involved the initiation of the collaborative/peer-tutoring training programme based on Lyman's Think-Pair-Share model (1994). The aim of the intervention was to develop the interpersonal skills necessary for effective collaborative work and to develop the higher order thinking skills of the students.

Reflect

Verbal flags were identified, *the link* (conjunctions), *the run-up* (rehearsal), *wait-time* (demand for individual cognitive thinking time). Are there more?

Was task selection to keep children motivated and in ZPD the primary reason the number of cognitive interactions increased?

Did the number of *higher* cognitive interactions increase significantly?

What limitations faced the programme?

Did the children have enough forming and functioning skills to reach fermenting and formulating levels?

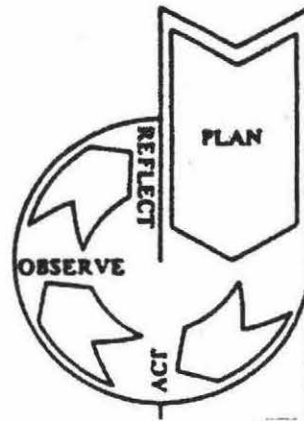
Observe

Interactions of children recorded as transcripts before and after the training programme.

Transcripts analysed.

Response questionnaire completed by children after the programme.

Verbal response and anecdotal comments given by classroom teachers.



Revised Plan
Future research

Act

25 children work with researcher in 3 collaborative groups to develop interpersonal group skills and thinking skills for collaborative learning.

Programme reinforced by teacher modelling and small group work in the classroom.

Plan

Use Lyman's Think-Pair-Share method as an intervention for a mathematics class. Aim is to develop interpersonal skills necessary for effective collaborative work and higher order thinking skills.

Figure 5: Cycle 3 The Main Study.

The collaborative-peer training was developed so that the students would advance through Johnson and Johnson's '4Fs' group skills: forming, functioning, formulating, and fermenting. In the initial group discussion between the researcher and students, basic self-

management skills were discussed at the group 'forming' level. These included principles of courtesy such as orientating the body to the speaker/listener and one person speaking at a time, as well as principles of encouragement such as no 'put downs', including all group members in discussion, and praising attempts or ideas.

Four key strategies were discussed in subsequent sessions to develop the students' higher-level group skills of functioning, formulating and fermenting. These strategies included:

- wait and give individual's time to think for his/herself;
- be specific with feedback;
- give help when asked in the form of a specific strategy, idea or question rather than an answer; and
- support agreement or disagreement with evidence.

The process involved two key ideas for the teacher: modelling and maintaining the '4Fs' in class discussions and group work, and selecting rich tasks which maintained the students in their ZPD.

Initially during the training, the students worked in pairs using Medcalf's (1992) peer-tutoring model. One member of the pair adopted the authority role of tutor or coach and was responsible for the use of the instructional prompts while his/her partner solved a mathematical task. When the tutee had solved his/her problem (or developed a solution to the best of his/her ability with the aid of the tutor) the pair swapped roles and the other partner completed a different problem with the same mathematics concept. The tutor/tutee role was alternated so that individuals would develop a greater awareness of the effects of their feedback. Pairs were encouraged to reflect and comment on the implementation of the

collaborative group skills and their own learning.

The positive interdependence of Lyman's Think-Pair-Share model (1992) of collaborative learning was incorporated into the next stage of training. Individuals worked on problems by themselves for a specified time and then they shared their solution and the evidence to support their conclusion with a partner. If a problem had a finite solution the partners compared answers and evidence. An identical answer meant they were either right or both wrong, and two different answers meant they needed to review their evidence to convince their partner their solution was correct. When the partners reached an agreed solution they presented the solution and evidence to another pair. Multiple solution paths were viewed and compared. Any answer could be challenged as long as the challenge was supported by reasoned argument.

Open-ended questions with more than one solution were approached in a similar manner. The students had individual thinking time and then presented their solution to a partner. If a partner was not convinced of the alternate solution he/she again challenged the partner for more corroborative evidence and the partner was required to justify the solution with more information. Alternative solutions and evidence were presented to the entire group. The children were encouraged to compare their solution and reasoning to those of other presentations. In this way each different solution was considered, and flawed solutions were challenged and discarded. Variations of this model occurred when a pair worked together and presented their solution to another pair and then the wider group; or individuals worked on the same problem and presented their solution and reasoning to a partner and then combined their solution and evidence or selected what they considered the strongest solution and most convincing evidence to present to the wider group.

The following extract is a recording of a cooperative group work training session, which shows the Think- Pair- Share pattern used by two boys during problem solving and the reinforcement of the model by the teacher.

Teacher: [Speaking to whole class] *I want you to work on problem four on page ten.² You'll have about ten minutes to work out your own answer. You can have more time if you need it. I'll check in about ten minutes. Remember this is your own time for ten minutes. You can write stuff down, draw pictures but don't talk to your partner. You'll have a chance to compare ideas after. Have you found page four Jeffery? That's right...the question at the bottom.* [The teacher reads the problem aloud both to focus the group and because several children have reading difficulties]. *Jan needs exactly 5 litres of water. She has only these two containers. How can Jan use her containers to measure exactly 5 litres of water? ...Okay? ...go for it.* [Teacher withdraws to her desk and observes the class. After four minutes she notices an early finisher and comments aloud.] *If you have an answer to the problem you might like to write it down or draw a picture so it's easy for your partner to understand what you've done.* [Ten minutes passes]. *Does anyone want more time? ...Okay Cherie, why doesn't your partner check her solution until you're ready. The rest of you can show your ideas to your partner. Remember if you see something you like or realise there's something you want to change you can change it. I want to hear people backing up their ideas with evidence.* [Teacher moves about the room interacting with groups then focuses on the two boys]. *How did you get on?*

Jeffery: *Good.*

Teacher: *I like your diagrams, that makes it easy for me to see how you worked out your answers and you've written something too. Did you both come up with the same ideas?*

Hamish: *Sort of.*

Teacher: *How do you mean?*

Hamish: *Well he wrote that you fill up the seven litre with the three litre three times and two litres comes out of the seven then you tip the two in the seven, fill up the three litre and tip it in the seven.*

Teacher: *What did Hamish say Jeffery?*

Jeffery: *He said you use the seven litre and fill it up three times with the three litre and two litres falls out of the container and you put it in the three litre then you tip it back in the seven litre and then you fill up the three litre and tip it in the seven litre and that's it.*

Teacher: *So do the answers say the same thing?*

Hamish: *He forgot to say you have to tip out the seven litre or you can't get the five in.*

Teacher: *Is that an important bit to add? What do you think Jeffery?*

Jeffery: [Doesn't speak but writes at the bottom of his notes "P.S. tip the rest out of

² Problem Four: Jan needs exactly 5 litres of water. She has only these two containers [diagram of two buckets labelled 7 litres and 3 litres respectively]. How can Jan use her containers to measure exactly 5 litres of water? (Ministry of Education, 2000, p. 10).

the 7 leter.”].

Teacher: *Are you happy with what you've both got there?*

Hamish: *Yep.*

Jeffery: *Yep.*

Teacher: *Your pictures and instructions make good evidence. You can go and share your ideas with Cherie's group. See what they came up with.*

3.7 Data Collection

In order to increase the validity of the research and to provide a deeper understanding of the collaborative interactions, data was triangulated through the use of multiple research instruments (Poskitt, 1994). Four kinds of data were collected: audio recordings, which were transcribed, questionnaires, anecdotal observations, and diagnostic interviews.

Diagnostic Interview

The Year 6, 7, 8 class concurrently participated in the Numeracy Development Project. Each child was interviewed by the classroom teacher using the Draft Diagnostic Interview (Ministry of Education, 2002c) and was assigned a level of knowledge and understanding within the Numeracy Framework (Ministry of Education, 2002d). Pre and post levels for the student's addition and subtraction mental strategies and fractional knowledge provided an assessment of academic progress over the course of the project.

The two participating teachers organised the class into three groups according to the diagnostic interview levels. These groups formed the basis for the collaborative groups. The teachers modified the sub-groups during the study. A student was moved to a 'higher' or 'lower' group dependent upon the rate at which they mastered the mathematical content for the level. To be able to compare the baseline and post-programme collaborative interactions the 15 children were recorded in the same initial sub-groups. These children were operating in either two groups of two, one of three, or two groups of four children.

Anecdotal and Observational Data from Classroom Teachers

An informal summary of the class's social and cognitive performance in mathematics was to be made by the class teachers based on anecdotal observations they had recorded during the fractions unit. This was to include evidence as to whether students were asking 'better' questions, reflecting on their own learning, and internalising the collaborative/peer-tutoring skills as part of their regular classroom learning behaviour. However, due to the classroom teachers' commitment to the Numeracy Project neither teacher recorded anecdotal data during the programme. Instead the teachers were interviewed about their perceptions of the programme and its impact in the classroom.

Student Questionnaire

The students completed a questionnaire about their participation in the training programme. The questionnaire (Appendix 5) was read to the children and then time was allowed for written responses. Not all of the questions were answered by every child. If the researcher was unclear about a particular child's meaning, the child's response was clarified individually through conferencing.

The questionnaire contained three statements for comment and four questions. The first three queries asked the student to consider why someone would prefer working in a group or alone to solve mathematics problems, and the student was required to give their personal preference as alone, in a group, or both ways. Questions four and five were designed to determine the extent to which the collaborative skills model had been internalised by the children and to find out their source of authority. Question 6 queried whether the child perceived the collaborative skills model was enacted in their classroom. The last query gave the child the option to make any comments on mathematics in general.

Audio-recording collaborative group discussion

Audiotapes of the talk between group members as they solved mathematics problems were recorded and transcribed. The students were acclimated to the recording equipment being present during collaborative tasks. They were initially very aware of the equipment but subsequently ignored it as they focussed on the mathematical task. Recordings were made prior to training to obtain baseline data and again four weeks later at the completion of the training period (see 3.5). Each group was audiotaped during the independent phase of the mathematics class as they worked in their collaborative group to solve a particular problem assigned by the teacher while she taught another group. Two problem solving sessions, one pre-training and one post-training, were randomly selected for transcription for each group. In one case a portion of the selected tape was inaudible, due to background classroom noise and the quiet talk of a particular group member, so another session was selected for that group. Allowance had also been made for the teacher to reject a tape if she believed a group member was behaving inconsistently to her usual observations but this proved unnecessary. The individuals in the groups were the same in both the pre and post training audio tapings. The tasks were selected by the teacher from the *Figure It Out* series (see Appendix 8) at an appropriate level for the particular group. Each taping continued until the group considered they had solved the task. No time limitations were given to the students but all groups completed the problem solving session within a 15-20 minute period which was consistent with the teachers expectation of the groups engagement on collaborative group problem solving within the classroom context.

3.8 Data Analysis

Recordings of group interactions during a problem solving episode were transcribed and analysed into categories.

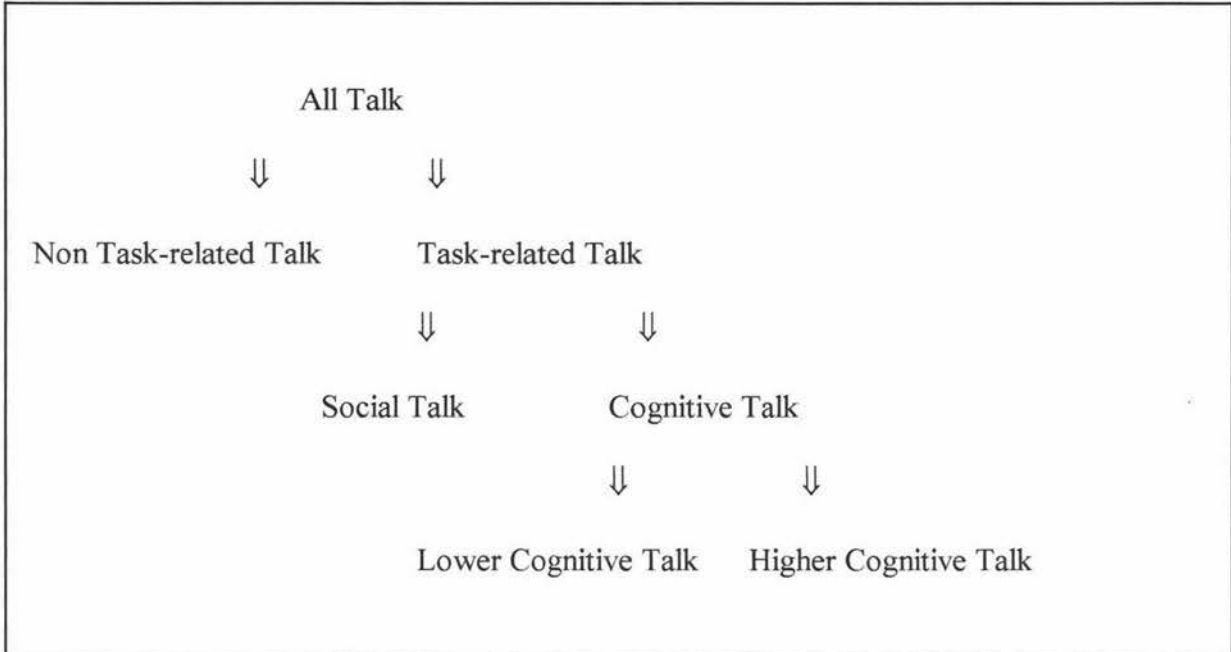


Figure 6: Classifications of collaborative group talk

The initial categories were non-task related talk and task related talk (cf. Figure 1). The non-task related talk included any non-task related statements or personal negative statements that were hostile/ridiculing/rejecting, and statements that could not be applied to any other category. The task related talk was divided into social and cognitive subcategories. The task related social talk included oral communication related to the social management of the group and materials, including discussion about rules, taking turns, the sharing and use of materials, and the functioning of the group.

E.g. *Shall we go to problem two?*

I said that ages ago.

We have to do problem one.

Lyle you're not listening.

Task-related social talk also included encouragement, praise, agreement and disagreement

with each other's conclusions, opinions or answers if it was non-specific.

E.g. *Yeah the answer's three days so we're done.*

Yeah we're done.

If the agreement/disagreement was linked to further reasoning it was classified as cognitive.

E.g. *You're right because half of a half is a quarter, and half of 120 is sixty so a quarter is 30. You did a good job. I can see you used Paul's half of a half idea.*

Task-related cognitive talk was subcategorised as lower cognitive talk and higher cognitive talk based on Bloom's Taxonomy (1956). These subcategories are distinct from Thomas' model in which she subcategorised cognitive talk as action-based: involving reading, writing, giving help, questioning and commenting; or reflective: involving requesting help, explaining, and thinking aloud. In this study lower order cognitive talk included the recall, repetition or rehearsal of previous information. Higher order cognitive talk included application, analysing, evaluation and synthesising. This involved genuine argument or queries, with the opinion backed by evidence and other supporting statements (see Appendix 7 for an example of a coded transcript). These supporting statements were often linked by conjunctions such as *so, if, because...*

E.g. *That's \$5 and five and five is ten so where's the room for the fries?*

Oh yeah so you have to add everything together and round them up, like take them up to the same place and then you just add them altogether.

See cos five and six is eleven, and five and five and six and six is twenty-two so it's got to have something to do with elevens.

The transcriptions were also analysed for the appearance of patterns in the students' interactions. These patterns were considered using an Interpretative Classroom Research

Approach (Maier & Voigt, 1991, cited in Peter-Koop, 2002) whereby the teacher-researcher looks for phenomena and theorises about their occurrence. A particular phenomenon that had occurred in the analysis of transcripts in Cycle 2 of the action research study had been the students' use of conjunctions such as "so" to link two group members' ideas. This phenomenon or pattern was labelled an 'oral flag' as it seemed to indicate progress in cognitive thinking in the students' interactions and was examined further to see if any other similar patterns occurred.

3.9 Ethical Considerations

The study was conducted under the ethical guidelines and principles of educational research developed by the New Zealand Association for Research in Education and the Code of Ethical Conduct developed by the Massey University Ethics Committee.

Permission was granted by the principal and Board of Trustees for the training programme and the research to take place. Both teachers involved in the classroom gave written consent to be involved in the research.

In order to preserve the equality of learning, all 25 students participated in the collaborative training programme but only 15 students in Year 6 and 8, who had volunteered to be involved in the research, were audio-recorded. Caregivers were well informed through an information sheet and personal contact (see Appendix 2).

Ethical consideration was needed to preserve the anonymity of the student participants because of the small sample size and the nature of the rural school. All participants were given pseudonyms and numbers, and the school is not identified.

Ethical requirements meant that collaborative group work was only permitted to be audiotaped and not videotaped. This meant that non-verbal information was available only through supplementary anecdotal field notes made while the groups were engaged in the task.

Chapter 4 Results

4.1 Introduction

The aim of the study was to ascertain whether the intervention programme enhanced the level of students' higher order cognitive talk. The collaborative skills training focussed on students developing the interpersonal group skills which were necessary for their effective involvement in collaborative group-work. In addition the training incorporated Lyman's Think-Pair-Share model (1991) to structure thinking skills, especially those involving questioning, justification and elaboration.

Information related to students' higher order talk was obtained through an analysis of the talk contributed by individual group members, questionnaire responses, and the teachers' anecdotal comments. Five collaborative groups were individually audiotaped during engagement on a problem-solving task one week before and one week after participation in the collaborative/cooperative training programme. The tapes were transcribed and analysed into categories of talk (see Figure 7). Overall, the results showed a mean gain in the amount of task related talk and a mean gain in the amount of cognitive talk and higher cognitive talk after the intervention.

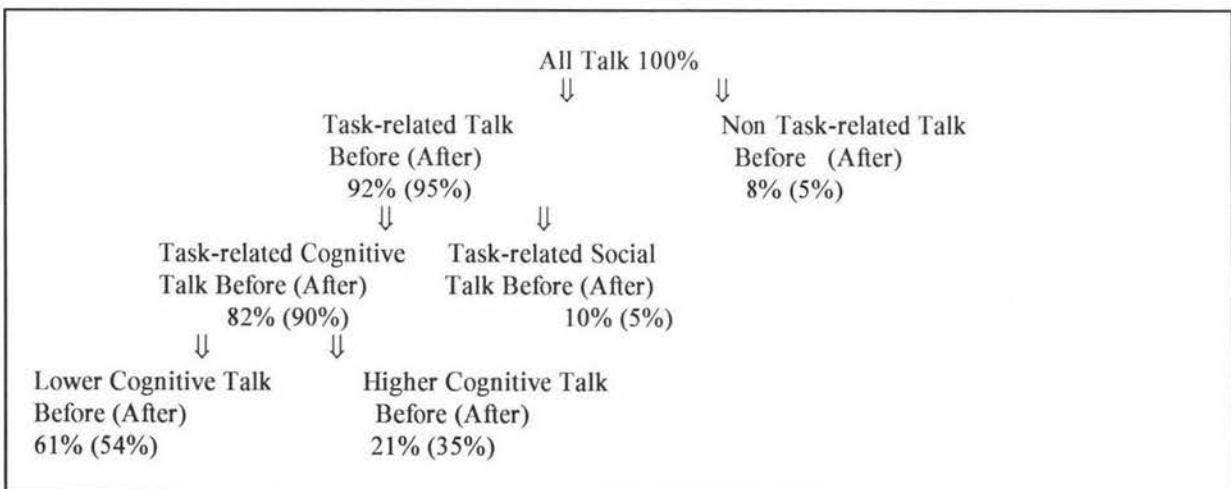


Figure 7: Summary of Mean Percentage of Talk for the Sample Group Before and After the Intervention.

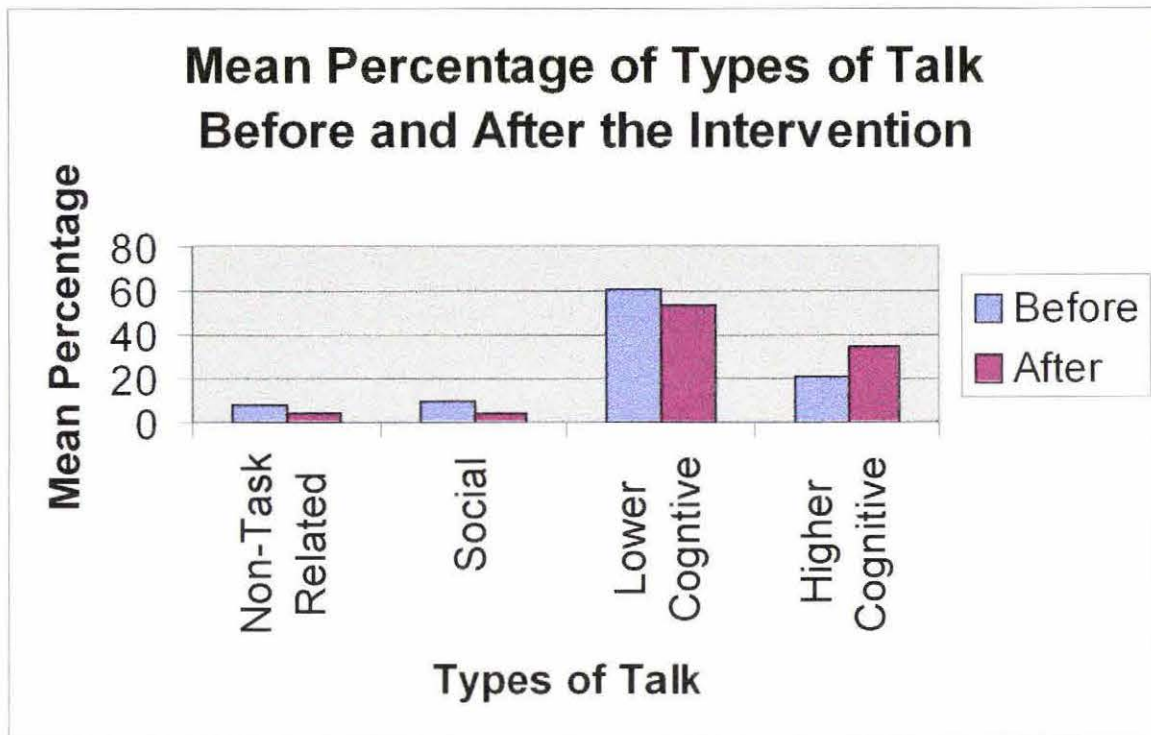


Figure 8: Mean percentage of types of talk before and after the intervention.

The tables and figures are summarised as mean percentages of all talk. The relationship between the types of talk is shown in Figures 7, and 8, and summarised again in Figure 9.

All Talk = Task-related Talk + Non-task Related Talk

Task-related Talk = Cognitive Talk + Social Talk

Cognitive Talk = Higher Cognitive Talk + Lower Cognitive Talk

e.g., Before the intervention All Talk (100%) = 8% + 10% + 21% + 61%

After the intervention All Talk (100%) \approx 5% + 5% + 35% + 54%

Figure 9: Summary of the Relationship Between the Mean Percentages of Talk.

4.2 Transcript Data

Table 2 shows the amount of task-related talk and non-task-related talk as a percentage of all talk. Samples of talk were recorded before the collaborative training programme (intervention) and at the end of the programme. The mean amount of task-related talk before training was 92% and the mean after the intervention 95%. Nine students increased their percentage of task-related talk with increases ranging from 3 to 34%. Two students remained the same and four students decreased their percentage of task-related talk with decreases ranging from 8 to 25%. An analysis of results using a paired t-test ($t = 0.08$, with 14 d.f.) showed no significant difference in the percentage of task-related talk before and after the intervention.

Table 2: Percentages of Task -related and Non-task-related Talk Before and After the Intervention.
* Pilot Study Children

Student	Task-Related Talk % of all talk		Non-Task-Related Talk % of all talk	
	Before	After	Before	After
1*	96	96	4	4
2*	91	100	9	0
3 ⁺	88	97	12	3
4*	66	100	33	0
5*	100	80	0	20
6*	90	100	10	0
7	98	90	2	10
8	97	100	3	0
9	100	92	0	8
10	90	100	10	0
11	86	100	14	0
12	96	100	4	0
13	100	75	0	25
14	100	100	0	0
15	87	100	13	0
Mean for sample group	92	95	8	5

Table 3 shows the mean percentage of task-related talk before and after the intervention within specific sub-grouping of Year and Gender. The Year 6 students, who had been involved in the pilot study, made more gains in the amount of task-related talk than the Year 8 students, who had not been involved in the pilot study. The Girls made more gains in task-related talk compared to the Boys. However, these gains may be relative as both groups that made the greatest gains also had the lowest initial scores.

Table 3: Mean Percentage of Task-related Talk Before and After the Intervention according to Grouping.

Group	Task-related Talk (Before Intervention)	Task-related Talk (After Intervention)
Year 6 (Pilot Study)	88.5%	95.5%
Year 8	94.9%	95.2%
Girls	90.9%	96.4%
Boys	93.6%	94.4%

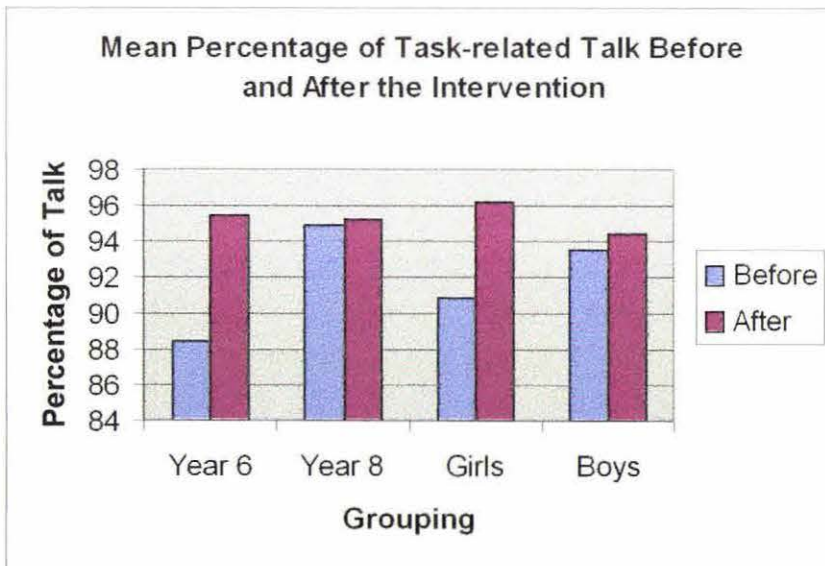


Figure 10: Mean Percentage of Task-related Talk Before and After the Intervention According to Grouping.

Table 4 shows the percentage of talk contributed to a particular group before and after the intervention. The mean expectation of talk for a group if all members contributed equally was 50% for paired groups, for three members 33% each, and for four members 25% each. A paired t-test ($t = 0.26$, with 14 d.f.) was used to analyse the proportion of talk contributed to the group by individuals before and after the intervention and there was found to be no significant difference.

Table 4: Percentage of talk contributed to the group
* Pilot Study Children

Student	Contributing Talk % of all group talk		Contributing Talk Compared to a mean expectation of equal talk	
	Before	After	Before	After
1*	43	42	(25) +1	(25) +17
2*	31	6	(33) +2	(33) -27
3*	24	51	(33) -9	(33) +18
4*	6	3	(25) -19	(25) -22
5*	9	27	(25) -16	(25) + 2
6*	13	14	(25) -12	(25) -11
7	41	27	(25) +16	(25) +2
8	45	43	(33) +12	(33) +10
9	42	49	(25) +17	(25) +24
10	51	48	(50) +1	(50) -2
11	8	11	(25) -17	(25) -14
12	38	39	(25) +13	(25) +14
13	47	53	(50) -3	(50) +3
14	53	47	(50) +3	(50) -3
15	49	52	(50) -1	(50) +2

Table 5 shows how the distribution of talk between group members varied from the expectation of equal contribution to talk. This can be seen as an indicator of group functioning, with groups closest to a mean deviation of zero as exhibiting the greatest positive independence where all members contribute equally. Initially the groups with two members shared the percentage of talk almost equally and the groups with four members showed the greatest deviations from the expected mean of equal talk. After the intervention, one group decreased their level of positive interdependence and one group increased their level of positive interdependence. Differences were visible in Group 3 because a dominant group member decreased her contribution to a more equitable level and another individual then contributed more. The reciprocal was also seen in Group 2 where a member began to dominate discussion and another member offered less talk. In both groups other group members continued to contribute almost equal percentages of talk.

Table 5: Mean deviation from equal talk before and after the intervention.

Group Size	Mean Deviation from Equal Talk Before	Mean Deviation from Equal Talk After
1 Four members	15.5	16
2 Three members	7.7	18
3 Four members	16.5	10.5
4 Two members	1.5	2
5 Two members	3	3

Overall, the intervention programme appears to have made little difference to the proportion of talk contributed by group members.

Table 6 shows the distribution of the cognitive and social talk as subcategories of task-related talk. The mean percentage of cognitive and social task-related talk is shown for the sample students before and after the intervention. The mean amount of task-related cognitive talk increased from 82% to 90%. Ten students increased their percentage of cognitive talk in collaborative group-work increases ranged from 5-34%. Five students

decreased their percentage of task-related cognitive talk – decreases ranged from 6-32%.

The mean amount of task-related social talk decreased from 10% to 5% after the intervention. Eleven students decreased their level of task-related social talk by 2-20%. One student remained the same, contributing no task-related social talk before or after the intervention. Three students increased their percentage of task-related social talk with an increase that ranged from 2-32%. The results for cognitive talk were analysed using a paired t-test. Students showed no significant gain in the percentage of cognitive talk after the intervention ($t = 1.64$, with 14 d.f.).

Table 6: Percentages of Task-related Talk Subcategorised as Cognitive and Social Talk Before and After the Intervention.

* Pilot Study Children

Student	%Task-related Talk		% Cognitive Talk		% Social Talk	
	Before	After	Before	After	Before	After
1*	96	96	86	93	10	3
2*	91	100	81	100	10	0
3*	88	97	68	97	20	0
4*	66	100	66	100	0	0
5*	100	80	87	80	13	0
6*	90	100	74	100	16	0
7	98	90	96	90	2	0
8	97	100	82	87	15	13
9	100	92	98	85	2	7
10	90	100	75	96	15	4
11	86	100	72	100	14	0
12	96	100	84	96	12	4
13	100	75	88	75	12	0
14	100	100	89	57	11	43
15	87	100	77	88	10	12
Mean of Sample	92	95	82	90	10	5

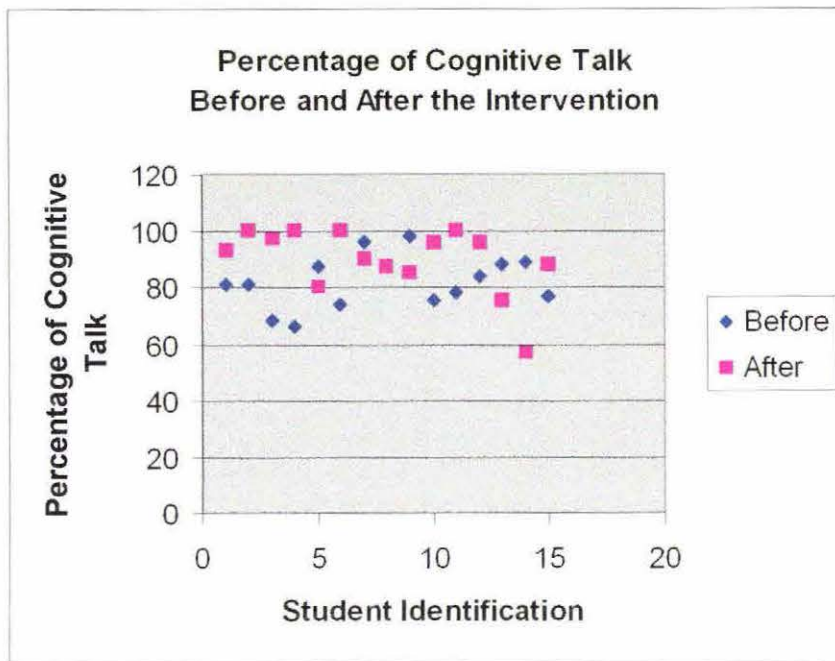


Figure 11: Percentage of Cognitive Talk Before and After the Intervention

Table 7 shows the mean percentage of task-related cognitive talk before and after the intervention when the results are analysed by Year and Gender. The Year 6 students who had been involved in the pilot study made more gains in task-related cognitive talk than the Year 8 students. The Girls made a greater increase in their percentage of cognitive talk compared to the Boys (see Figure 11 also).

Group	Cognitive Talk (Before the Intervention)	Cognitive Talk (After the Intervention)
Year 6 (Pilot Study)	76.5%	95%
Year 8	84.6%	86%
Girls	82%	93.1%
Boys	80.8%	86.5%

Table 7: Mean Percentage of Cognitive Talk Before and After the Intervention According to Grouping.

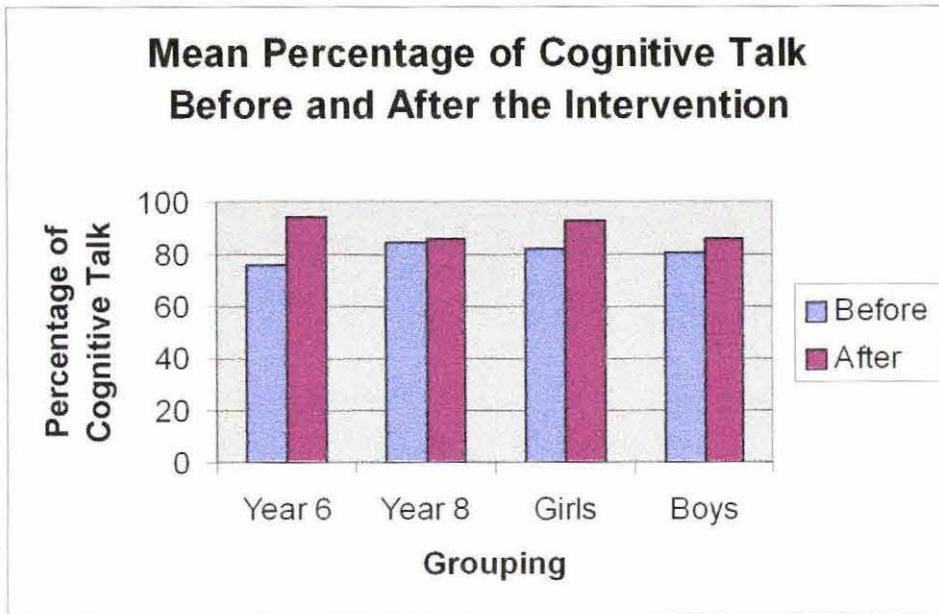


Figure 12: Mean Percentage of Cognitive Talk Before and After the Intervention According to Grouping.

Table 8 shows the percentages of task-related social talk when analysed by Year and Gender. In accord with increases in cognitive talk Year 6 decreased their level of task-related social talk by 11% to almost zero while Year 8 remained approximately the same. Both the Boys and the Girls decreased their level of task-related social talk with the Girls percentage of task-related social talk remaining lower than the Boys.

Table 8: Mean Percentage of Task-related Social Talk Before and After the Intervention According to Grouping

Grouping	Task-related Social Talk (Before Intervention)	Task-related Social Talk (After Intervention)
Year 6 (Pilot Study)	11.5	0.5
Year 8	9.9	9.7
Girls	7.0	3.3
Boys	12.1	7.4

Table 9 shows an analysis of the cognitive talk (lower and higher) as a percentage of all talk. Despite an overall percentage increase of cognitive talk from 82% to 90% there were

different trends in changes in higher and lower cognitive talk. Higher order cognitive talk increased from 21% to 35% and the percentage in lower order cognitive talk decreased from 61% to 54%. However the overall percentage of lower order cognitive talk, 61% and 54%, remained greater than higher order cognitive talk, 21% and 35%, both before and after the intervention. Results when analysed using a paired t-test ($t = 2.40$, with 14 d.f., $p < 0.05$) showed a significant gain in the amount of higher cognitive talk.

Table 9: Cognitive Talk Subcategorised as Higher and Lower Order Cognitive Talk Before and After the Intervention.

* Pilot Study Students

Student	Cognitive Talk (% of all talk)		Low Order (% of all talk)		High Order (% all talk)	
	Before	After	Before	After	Before	After
1*	86	93	61	66	25	27
2*	81	100	62	50	19	50
3†	68	97	58	69	10	28
4†	66	100	22	50	44	50
5*	87	80	50	30	37	50
6*	74	100	50	44	24	56
7	96	90	77	70	19	20
8	82	87	60	60	22	27
9	98	85	74	47	24	38
10	75	96	61	87	14	9
11	72	100	33	50	39	50
12	84	96	71	60	13	36
13	88	75	79	25	9	50
14	89	57	80	29	9	28
15	77	88	64	76	13	12
Mean	82	90	61	54	21	35

Table 10 shows that the percentage of higher order cognitive talk increased for all groupings. Both the Boys and the Girls showed equal levels of higher order cognitive talk before the intervention (22.3% of all talk). After the intervention the Girls and Boys percentage increased by 10.6% and 14.1% respectively. The Year 6 students who had been involved in the Pilot Study began with the highest percentage of higher order cognitive talk at 29.5% and recorded the largest increase of by 22.3%. Interestingly, Year 6 students engaged in slightly more higher order cognitive talk than lower order cognitive talk at the end of the intervention. The Year 8 students increased their amount of higher order cognitive talk by 13.5%, which was a similar gain to that of the Boys (14.1% gain) and the Girls (10.6% gain).

Table 10: Mean Percentage of Higher Order Cognitive Talk Before and After the Intervention.

Grouping	Higher Order Cognitive Talk (Before Intervention)	Higher Order Cognitive Talk (After Intervention)
Year 6 (Pilot Study)	29.5%	51.8%
Year 8	19.8%	33.4%
Boys	22.3%	36.4%
Girls	22.3%	32.9%

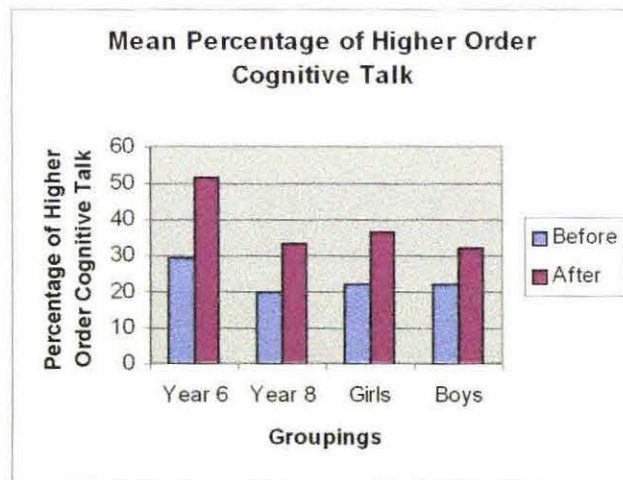


Figure 13: Mean Percentage of Higher Cognitive Talk Before and After the Intervention According to Grouping.

4.3 Academic Data

The academic gains can not be directly correlated to student participation in the training programme. Gains in learning involves more than increases in higher cognitive talk and also relates to the teaching in their numeracy programme. In support of this, a simple comparison of academic gains in fraction knowledge and gains in higher order cognitive talk revealed no significant relationship between the two variables ($r = 0.049$, 15 d.f.).

Table 11 shows the academic outcomes after the fractions unit measured by fraction knowledge, and subtraction and addition strategies in Number (see Appendix 9). Academic progress was determined as a function of progress on the stages of the Numeracy Development Project Diagnostic Interview. Twelve students made academic gains of one or more stages in their fractional knowledge. Three students remained at the same initial level. Six students made gains in their addition and subtraction strategies and nine remained at the same initial level.

Table 11: NUMP Diagnostic Interview levels to show Academic Outcomes Before and After the Intervention.

Student	Addition/Subtract		Fraction Knowledge Level	
	Before	After	Before	After
1*	5	5	5	6
2*	4	4	2-3	6
3*	5	5	4	5
4*	5	5	2-3	5
5*	4	5	2-3	5
6*	5	5	4	5
7	4	5	6	6
8	4	4	4	6
9	4	5	2-3	5
10	5	6	6	8
11	4	5	5	5
12	5	5	5	6
13	5	5	5	6
14	5	5	5	5
15	5	6	4	6

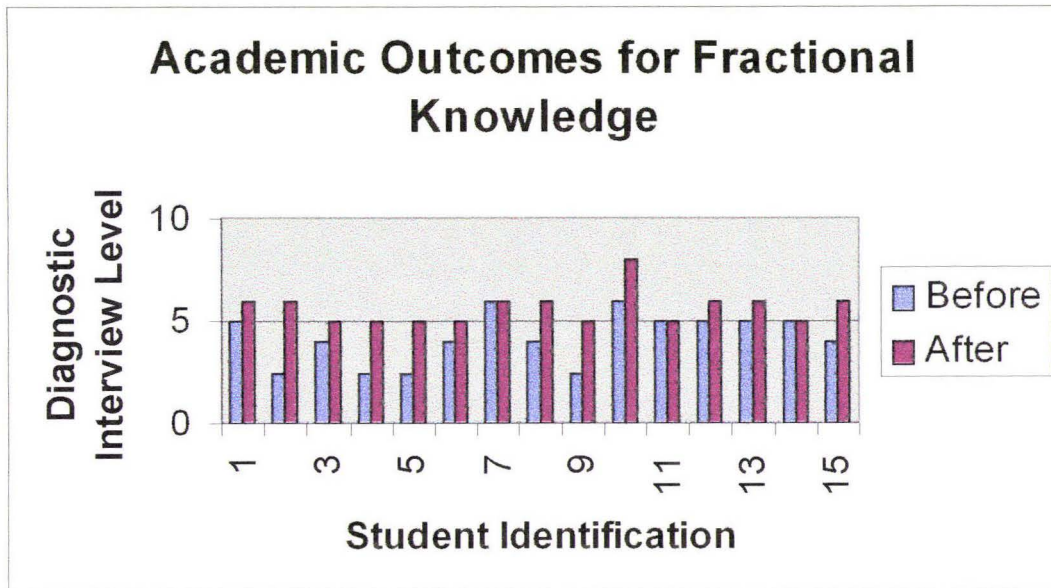


Figure 14: Academic Outcomes for Fractional Knowledge Before and After the Intervention.

Table 12 shows that overall the Year 6 children made greater gains than the Year 8 children in addition and subtraction strategies. The Year 8 children made greater gains in knowledge of fractions than the Year 6 children. The Girls made a greater average gain in both addition and subtraction strategies and in knowledge of fractions when compared to the Boys.

Table 12: Mean Academic Gains Before and After the Intervention Using the NUMP Diagnostic Interview to the Nearest Level.

Grouping	Add/Sub Before	Add/Sub After	Fractions Before	Fractions After
Year 6(Pilot)	5	6	4	5
Year 8	5	5	4	6
Boys	5	5	5	6
Girls	4	5	4	6

4.4 Identification of oral 'flags'.

Certain patterns of speech were associated with likely instances of higher order cognitive thinking. Three patterns were identified in the analysis of transcripts for this study—the 'run-up', the 'link', and 'wait-time'. The *run-up* is a rehearsal pattern like that of a high-jumper running up to the mark several times before finally lifting over the pole. Just as the high-jumper rehearses the speed and take-off position, and makes alterations in response to the conditions before the actual jump, the children sometimes rehearsed strategic solutions to a problem by repeating the known information several times. Each rehearsal was linked or tested against known schemata in the form of alternative mental strategies. An example of this is when Kahurangi added thirty-two and sixteen. He repeated the information several times as he sought a mental strategy by which to find the solution. He attempted to use a part/whole strategy (*Count Me In Too*, New South Wales Department of Education, 2000) by combining the thirty and the ten to make forty then grappled with the addition of six and two. He finally resolved the problem through the use of a written algorithm. Rachel interrupted him with her solution part way through the rehearsal but Kahurangi had enough information about the combination of forty and six and two to respond to tell her that her solution was incorrect because the solution had to be an even number.

K: *Thirty-two...thirty and sixteen...thirty-two and sixteen...thirty-two and sixteen...that's forty...er*

R: *Forty-five.*

K: *No that's not an even number...forty...forty...I'll write it here...It's forty-eight, so that's forty-eight Rachel.*

Another pattern commonly observed in higher order thinking sequences was the use of a conjunction such as 'so' or 'but' often with exaggerated intonation. These conjunctions were commonly used to link evidence and solutions or paths of reasoning, in order to justify

one's thinking or elaborate upon someone else's. O'Connor and Michaels (1996, cited in Brown & Renshaw, 1999) have labelled the use of 'so' as evidence of a marker for "revoicing", which they say is summarising and paraphrasing another's ideas. This pattern was labelled the *link* as it often occurred in a sequence between peers effectively bridging their insights and arguments.

An example of this is when Mitchell and Megan had solved a problem which involved rounding bags of gold to the nearest ten by adding a quantity from a broken bag (which contained an unspecified amount). Megan wanted to apply rounding practices that meant she could round up or down to the nearest ten but the problem required that she only round up. Mitchell and Megan use 'so' and 'but' to link their reasoning to evidence which confirms or rebuts each other's argument.

- M: *And say like there's eighteen in there you need to take some of it out of there [the broken bag] to make it up.*
 Me: *So I can take it away?*
 M: *So you just need two out of there to make it up to the even number.*
 Me: *But it says to fifty or eighty.*
 M: *It can be anything.*
 Me: *Oh yeah?*
 M: *So you just add two to that to make it twenty.*
 Me: *So we work with these numbers?*
 M: *Yeah to the nearest ten.*
 Me: *Okay.*

The pilot study and the analysis of children's interactions prior to the training programme both revealed instances of children spontaneously demanding *wait-time* for themselves. Children provided clear indications to other group members that they required and expected some uninterrupted, individual thinking time in order to clarify their ideas or perform calculations before making themselves and their ideas available to the group again. An

example of this is shown by Anna, Lucy and Leihana who were all involved in the pilot study. Both Lucy and Leihana indicated that they wanted *wait-time* in order to pursue individual calculations. Both “hold on” and “hang on” were expressions commonly used in the study by children to request individual withdrawal or wait-time to engage in their own cognitive thinking.

Anna: *Three dollars seventy-five.*

Lucy: *Hold on, hold on, it can't be. I know this.* (Lucy starts writing calculations on paper)

Anna: *It's three dollars seventy-five.*

Lei: *Anna be serious. We've only got six dollars to get two burgers and fries.*

Anna: (raises voice) *I'm talking about how much one hamburger is! One hamburger!*

Lei: *Well I'm going to scribble in the back of my book too.* (Everyone opens books and begins various calculations, talking to themselves. Only some of this is audible)

Lei: *Six.*

Lucy: *That works out. If you go five dollars twenty-five and add seventy-five cents, the seven and the two are nine and the five and the five are ten cents so that's six dollars.*

Anna: *That's one hamburger. It can't be.*

Lei: *Look Lucy, "At the burger place two hamburgers and one packet of fries cost six dollars altogether". That's two hamburgers Lucy, that'd be over ten dollars. It can't be.*

Lucy: *Oh...hold on...hold on.*

Lei: (Snaps her fingers) *Seventy-five and seventy-five.*

Lucy: *I've worked that out. It's one hundred and fifty.*

Anna: *One dollar and fifty cents.*

Lucy: *Yeah, one hundred is a dollar so it's a dollar fifty.*

Lei: *Hang on I've got to do something private* (She snatches up her book and writes so Anna and Lucy can't see)...*that so does not work out.*

Some children were more direct and verbalised exactly why they needed *wait-time*. In the following example Octavia pressurized Shannon to write all their problem-solving ideas on a sheet of paper as a group record. Shannon was still considering the problem and their calculations, and initially ignored her. Octavia attempted to strengthen her authority by aligning herself with the teacher's instructions about the group record.

Octavia: *This goes to Mrs Smith like a sample thing.*

Shannon: *Yeah I know.*

Octavia: *Shannon you've got to write down what I've done now.*

Shannon: *I'm thinking first.*

(Shannon orientates herself away from her. Octavia sits back in her seat and begins to drum her pencil on the desk).

Kieran and Dreyfus (1989, cited in Peter-Koop, 2002) label such a demand for 'wait-time' as an 'anti-reaction' whereby children deliberately refuse to interact with other group members for a period of time while pursuing individual thought. They noted, as in this study, that after a period of withdrawal the students rejoined the group discussion with their own position or information clarified.

4.5 Questionnaire Data

Following the intervention programme the questionnaire was completed by all the students in the class ($n = 24$) as a response to their involvement in the training programme. The questionnaire was designed to survey the students' perceptions about their involvement in the intervention including their knowledge of group processes, and their personal preferences and reasons for working alone or in a group.

4.5.1 Student Preferences for Group Work

The first three items³ in the questionnaire were designed to reveal the students' individual preferences for group work and to explain why a student would choose to work alone or in a group. None of the students involved in the training preferred working alone (see Table 13) but nearly half showed an expectation of being able to invoke wait-time where they

³ Question 1: Give a reason(s) why someone would prefer working on their own to solve a maths problem.
 Question 2: Give a reason (s) why someone would prefer working in a group to solve maths problems.
 Question 3: How do you prefer to work? On my own, in a group, both ways.

could sometimes withdraw from the group and work alone for a period of time to clarify their own thoughts, proceed at their own pace, and not be confused or distracted by having to stop and listen to others.

A quarter of the students believed that working in a group made it more likely they would solve the task or problem and that there were more opportunities of receiving help in a group context. Though students believed there were more ideas available in a group some also thought that this was a problem remarking that sometimes social difficulties arose in solving disagreements or some group members would not 'do their share'.

Table 13 : Percentage of class who preferred working alone, in a group or both ways.

Individual Preference	Percentage of class
Working alone	0%
Working in a collaborative group	52%
Both ways	48%

4.5.2 Authority

All the responses to Question 4⁴ said that Tom and Lisa should have the same answer to be correct. Authority and resolution of uncertainty was based on the solutions being the same. 91% of the class suggested that because Lisa and Tom had different solutions the pair should repeat the problem solving process until the same solution was obtained. However this suggestion had two variations: one was for Tom and Lisa to repeat the problem together and share ideas (44%), and the other was to repeat the problem-solving process alone and compare ideas (47%). The remaining students ($\approx 10\%$) suggested that an

⁴ Question 4: Lisa and Tom came up with different answers to the same maths problem. Both argued that they were right. How could they come up with an answer they both agreed on?

authority outside the group (e.g., that of the teacher (2%), manipulative equipment (1%) or a calculator (6%)) should be consulted to determine the correct solution.

4.5.3 Group Processes

Question 5⁵ indicated how well the children had internalised the collaborative group skills. The majority of responses related to the group forming level (see 2.4.1) and concerned issues about participation such as: listening to one another, having turns, using quiet voices, eliminating negatives such as “put-downs”, and no calling out. 40% of responses were at the group functioning level. These responses talked about skills of positive interdependence such as all members of the group contributing, cooperating, working together and supplying their best effort. Three children gave responses at the group fermenting level, specifically mentioning the higher order skill of justifying ideas with evidence. One of these students was a high achieving boy and had been part of the pilot study. Two of the students were girls whom the teacher considered to be poor and average achievers but had made strong academic progress during the study and exhibited the highest level of collaborative group skills in the class.

Table 14: Group Skills Level Indicated by Questionnaire Response.

Group Skills Level	Percentage of responses at this level
Group forming	48%
Group functioning	40%
Group fermenting	12%
Group formulating	0%

⁵ Question 5: If you were in charge of a group that was solving maths problems what rules would you have for your group?

From their responses to Question 6⁶, 74% of the children had not perceived any use of the collaborative group skills at work in their classroom. 26% of children felt they had used the collaborative group skills while working in small groups in their classroom but when they cited examples of skills they referred to mathematical knowledge (content) gained from specific mathematical problems which had been used in collaborative tasks as part of the training process.

4.5.4 Mathematical Identity

Children were asked to comment on their perceptions of mathematics⁷. Of the 19 comments received all involved indicators of interest and enjoyment (33% positive, 33% negative and 33% neutral). The positive descriptors used were “like”, “hard/challenge”, “easy”, and “fun/cool”. The negative descriptors used were “don’t like”, “boring”, and “easy”. All the negative statements were linked to an awareness of a reason for still “having to do” mathematics, such as a necessary requirement at school now and in the future at high school, and that mathematics was handy in real-life when dealing with money or using maths in a job. No links to the intervention were apparent.

4.6 Teacher Anecdotal Comments

Both teachers found their focus on the Numeracy Development Project precluded any recording of anecdotal observations on a daily basis within the classroom. Instead an informal interview was conducted with both teachers at the end of the main study in which

⁶ Question 6: Did you use any of the ideas you talked about in the library back in your class to help with your maths? (Please give an example with your yes or no answer).

⁷ Question 7: This is your space to make any other comments you’d like to about maths.

they gave their overall observations of the collaborative group work in the classroom, some remembered comments overheard between group members, and their opinion about the changes perceived in individual and class behaviour. The teachers both supplied supplementary information to specific questions that arose later during data analysis. These teacher perceptions are incorporated in the following discussion (see Chapter 5).

Chapter 5 Discussion and Conclusions

5.1. Effects of Peer-tutoring

The aim of the main study was to use an intervention training programme to foster particular kinds of cognitive talk to increase the level of higher order cognitive talk: application, analysis, synthesis, and evaluation—compared to lower cognitive talk: recall, recognition, and comprehension. In addition the principle of reducing the amount of social task-related talk and increasing the level of cognitive talk by selection of ‘rich’ quality tasks which were appropriately challenging for the students was incorporated into the Main Study from the Pilot Study.

In order to ensure students’ effective involvement in collaborative group-work the intervention developed students’ interpersonal group skills and students’ thinking skills. These involved questioning, justification and elaboration which were developed through the implementation of Lyman’s Think-Pair-Share structure (1991). While the internalisation and use of the collaborative group skills by the students was not systematically measured through classroom observation, an analysis of the amount of talk contributed by individual group members, questionnaire responses and the teachers’ anecdotal comments was used to measure changes in talk patterns.

At the start of the project it was considered optimistic that any clear indication of changes in behaviour should occur in the short duration allowed for the study. However the data analysis showed significant mean gains in the percentage of higher order cognitive talk and mean gains in the amount of task-related talk and overall cognitive talk for most students. It was encouraging that all students made positive gains in mathematical knowledge but the

effect of the intervention is difficult to determine and a causal outcome cannot be determined within the scope of this study. However, it is of note that the Year 6 students who had been involved in the Pilot Study made greater mean gains than the Year 8 students in the percentage of cognitive talk and the percentage of higher level cognitive talk. Both the Year 6 and Year 8 groups made equal mean gains overall in academic knowledge. The Girls made greater mean gains than the Boys in every area but no reason can be surmised to account for this within the study.

The principles of quality task selection to optimise students' learning (Stein, Grover, & Henningsen, 1996) were implemented in the programme prior to the recording of baseline data. This was done deliberately to encourage a high level of cognitive task-related talk and to provide an optimal learning environment for the students pre and post test measurements. This high level of task-related talk (92% before and 95% of all talk after) is consistent with other research both in New Zealand (Thomas, 1994; Higgins, 1993; Young-Loveridge, 1989) and overseas (Johnson, Johnson, Roy, & Zaidman, 1985).

The importance of task selection at an appropriate level can be seen in the results of three individual students (I.D. # 7, 9, 13) who were considered by the teacher as "hard-working, with excellent study habits" and who made academic gains during the study. The three students' overall percentage of task-related talk and cognitive talk decreased in the post-intervention data (Table 2), but all three increased their proportion of higher order cognitive talk (Table 9) and solved the task more quickly. A possible explanation is that as their overall competency and knowledge grew they required less talk to solve the task but their talk was more effective as evidenced by the increased proportion of higher order cognitive talk. Likewise, inappropriately levelled tasks could also be the reason two boys (I.D. #10,

15), seen by the teacher as “high academic achievers” were two of only three students to decrease their percentage of higher order cognitive talk. The task selected for analysis in the post-intervention may have been too easy to require much higher order cognitive thinking due to their academic progress during the fractions unit. This was further evidenced by the short time in which they solved the task (10 minutes) compared to the baseline data task (30 minutes), and the reduced overall vocalisation during the task.

There was one unexplained exception amongst the sample group. A boy (I.D.#5), who had been involved in the Pilot Study, made excellent academic gains but showed a decrease in his percentage of task-related talk and a decrease in the percentage of cognitive talk and higher order cognitive talk. His academic gains may suggest that his listening and cognitive processing was more effective but there was no observable evidence from talking. This highlights the complexity of the learning task in relation to observable variables.

The data obtained from the analysis of the percentage of contributing talk compared to the mean expectation of equal talk suggests that the students had not mastered the ‘forming’ and ‘functioning’ skills of positive interdependence, that every member should contribute. Without these skills it appeared that the group size affected contribution to talk, with groups of two being the most likely to contribute talk equally. Response data from the questionnaire also suggested that the students need to spend more time mastering basic group skills. 48% of responses indicated internalisation of the group skills at the ‘forming’ level and 40% indicated group skills at the ‘functioning’ level. One teacher saw evidence of the higher order thinking skill of application “reasonably frequently” in the collaborative groups in the classroom. This is supported by the independent transcript analysis which showed application was the most frequently exhibited higher order cognitive talk.

5.2 Oral 'flags' which identify higher order cognitive thinking.

Several oral flags were identified in the study—the 'link', the 'run-up' and the individual demand for 'wait-time'. The *link* is identified by the use of conjunctions such as 'so' and shows the connection of two ideas in a dyad. This pattern in oral interactions aligns with the use of the conjunction in written language to join two complete sentences and form one sentence with linked ideas.

These oral flags have practical application for teachers who can use them to readily identify higher order cognitive thinking in student interactions by listening for key words such as the conjunctions —*so, but, because*— and words that request 'wait-time' such as *hold on, hang on*, and the repetition of known information or a question in the form of a rehearsal.

The recognition of these oral flags has implications for the feedback teachers give to students as they interact with classroom groups. One of these implications is to teach the student to recognise and reflect on the process of their own thinking perhaps by making a comment such as: "*When I listen to what you're saying I can hear that you're joining two ideas, Paula's idea and your idea, because you said, 'so we need to put these together', you're building on Paula's idea with the next step. That's great.*" Teachers could provide feedback that encourages the students to give each other 'wait-time' such as: "*Just a moment Michael, I heard Angus say 'Hold on', let's give him a minute or so to think out his idea then he can tell us.*", or provide the time for individuals to work towards their own solutions such as, "*Andrea I can hear you're working out your answer. We'll give you some time to work that out then we can all compare the different ways we all worked them out*". In this way teachers could not only support the content knowledge base of students' reasoning but encourage them to think more deeply about the underlying strategies they

were using and the problems they were solving (Anthony & Walshaw, 2002).

5.3 Students' Perceptions

The results of the survey showed that 74% of the students believed they had not used the ideas gained in the collaborative programme in the classroom, whereas their teacher gave evidence that every individual had been engaged in collaborative work within the classroom. A clue to the students' perception is provided by the responses of the 26% of students who felt they had used the collaborative group skills in class. In each of their responses they referred to mathematical knowledge gained from the problems the researcher used and the mathematical strategies they witnessed other groups use in the 'Share' phase of Think-Pair-Share collaboration. In order to clarify this further, the researcher randomly asked three children (of the 74%) to give an example of an idea they had gained during collaborative skills training that they hadn't used in class. Each child gave an example to do with the mathematical content. This seems to indicate that all the children perceived the collaborative skills training sessions were not about the collaborative skills they were practising but about having extra time to rehearse problem solving and gain mathematical content knowledge.

This raises issues of the transparency of talk in small groups (Adler, 1999) and development of metacognitive knowledge (Artzt & Armour-Thomas, 1992). Would children engage in more 'higher order' cognitive talk when problem-solving if they are made actively aware of the desirability of this kind of talk, and that recognising and engaging in higher order cognitive talk is a desirable, reinforced socio-norm within the classroom? Or is it better to leave the process 'transparent'? The researcher had implemented the skills training programme in such a way as to make the collaborative framework transparent. Each

problem-solving session used the Think-Pair-Share model and feedback was given to groups for their use of praise, listening, 'wait-time', questions, elaboration and justification. However, the focus remained on the mathematical task and the solution strategies not on the group process of collaboration. Thomson and Brown (2000) stress the need for the students to be involved in active reflection about the process and the task and suggest that the first thing to be omitted in the busy teaching schedule is the group and individual reflection time.

The unequal contributions of talk amongst group members and the general lack of positive interdependence in the groups suggests that the collaborative intervention could be improved by group reflection and sharing not only of task solutions but of the collaborative group skills used in the process. This could be done with the teacher leading the reflection in small groups or by providing opportunities for groups to model successful practices to the whole class at the end of a collaborative session.

5.4 Teacher Perceptions

The teachers felt that the students were more focussed when working in small groups but were unsure whether this was an effect of the intervention programme or the greater attention to quality task selection, or a combination of both aspects. One teacher felt that the structure had provided a vehicle for the quieter, less confident members of class to make a contribution and 'be heard'. The other teacher thought that the collaborative skills programme had given confidence to students who had strong interpersonal skills but were weak academically in mathematics, such students they felt had taken more risks and experienced more academic success as a result.

The teachers themselves reported increased awareness of the benefits of the collaborative

interpersonal skills and thinking skills as a result of their participation in the research study and through subsequent discussions with the researcher. Since the conclusion of the study the collaborative group skills of 'forming' and 'functioning' have been introduced to a school-wide programme in cooperative learning. The interpersonal skills used to function successfully in a collaborative group are taught in every classroom with all teachers receiving professional development to introduce the collaborative group-work. The middle and senior classes are also both focussing on the use of the higher order thinking skills found at the 'fermenting' and 'formulating' levels of collaborative group-work. These skills are applied generically in literacy, numeracy, and integrated curricula units.

5.5 Limitations of the Study

As expected, in any classroom based study involving innovation and teacher commitment, timing and time, or lack of it, was a concern. The research release time period awarded by the Ministry of Education of four weeks during Term 2 determined the scheduling of the training programme. Participants in the programme had to 'fit' with their teachers' existing programme and this meant some changes in the original planning due to their existing commitment to the Numeracy Project 2002. Originally the students were to receive ten hours collaborative training over the four week period. In practice the students received only three hours collaborative training over the four week period of the study.

The gathering of more transcript data at a later period during the class year would have been desirable to determine the residual effects of the collaborative training and the degree of reinforcement required by the classroom teacher to maintain or increase the percentage of higher order cognitive talk within groups. This data could have been compared to that of

the pilot study students who responded to their renewed involvement in the programme with the greatest percentage gains in higher order cognitive talk.

The provision of teacher modelling of collaborative skills was expected to provide reinforcement and scaffolding opportunities for the participants of the collaborative skills programme. However, despite the best intentions of the classroom teachers, their involvement in the Numeracy Project was regarded as a priority in terms of their own professional development and classroom focus. Thus, in reality, classroom focus on the collaborative model was less than initially anticipated.

Thomas' model (1994) allowed all talk to be categorised as cognitive or social talk. Her model was adapted for this study and subcategories of 'lower' or 'higher' cognitive talk replaced action and reflective talk. As with Thomas' study, the data analysis within these subcategories proved difficult. Within one 'turn' there was sometimes a range of talk incorporating features of 'higher' and 'lower' cognitive talk. In order to remain consistent within the analysis, cognitive talk which contained features of both 'higher' and 'lower' talk was deemed as 'higher' talk, and the 'lower' cognitive talk was seen in a supporting role.

5.6 Concluding Thoughts and Implications for Further Research

Within the research study the links between learning and talk are implied but not substantiated by measurement of specific learning outcomes. Gains in learning were inferred because of the improvement in the levels of cognitive (and higher order cognitive) talk engaged in by the collaborative groups after the intervention and through the general academic gains in number concepts made by all the students involved.

The action research nature of the intervention allowed the researcher to reflect more clearly on her own teaching practices within the classroom. As she witnessed the implementation of the collaborative programme within another classroom she began to question the implicit practices and style of the mathematical community within her own classroom and tried to re-define those practices that explicitly helped to create a 'collaborative atmosphere'. Reflection led to the belief that the intervention is significantly dependent upon the role of the teacher in establishing the normative practices of the mathematical community; making explicit through teaching and modelling the interpersonal skills necessary for successful collaborative group work and the practices which lead to scaffolding peers' learning; and the selection of tasks to maintain individuals within their *Zone of Proximal Development*.

To be effective, the teacher also needs to have developed supportive facilitation skills so she/he is able to listen to and question collaborative groups, and assess and recognise the functioning and thinking that is occurring between the students.

Specific questions that could be considered in a subsequent action research cycle include:

- Students, after they had mastered the mathematical knowledge and understanding taught during the topic, appeared to engage in less external talk when problem-solving. What discourse tool can be used to measure and analyse internal talk?
- Would the intervention be more successful if students were made explicitly aware of the group functioning and thinking skills as Brown and Thomson (2000) suggest or is it better to leave the process transparent so there is more focus on the mathematics instead of the 'practice of mathematics discussion' (Adler, 1999), or is there a balance to be found between both perspectives? The students recognised their own need to withdraw

at times from the group to engage in uninterrupted thought (see 4.5.1) and this process could be supported by explicit student awareness of wait-time, when the wait-time concept was practised and reflected upon as a norm within the class mathematical community.

With the increasing recognition given to classroom discourse and communities of inquiry within the mathematical classroom, teachers need additional mathematical skills to listen to and recognise the validity of students' mathematical thinking to enable them to interact in such a way as to extend the students' thinking rather than imposing mathematical content. Teacher knowledge about collaborative group practises that must be taught in order for mathematical learning to be effective would assist in the implementation of the ideals of critical reflection stated in the *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) document. It is imperative that teachers of mathematics do not implicitly expect from students what is not explicitly taught with regard to collaborative group-work and communication of mathematical ideas.

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Appendix 1:

Ebbutt's (1985) Table of Broad Classification of a Range of Insider Activity Currently Occurring in School.

TEACHERS				
Works in isolation in own classroom	Works in isolation in own classroom	Works in isolation in own classroom	Works in isolation in own classroom as part of a coherent group that meets regularly	Works in isolation of own classroom
and	and	and	and	and
Reflects on own practice from time to time, may implement action steps.	Regularly reflects on own practice. May implement actions steps.	Regularly reflects on own practice. Systematically implements action steps.	Systematically reflects about own practice and systematically implements action steps.	Reflects about aspects of practice. Selects hypotheses from formal theory.
but	and	and	and	and
No use of external consultant.	May use external consultant or critical friend.	May request help from consultant or critical friend.	Almost certainly uses consultant or critical friend.	May request help from consultant or supervisor.
and	and	and	and	and
No systematic data collection	Informally collects some data Informally analyses data	Systematically collects data Systematically analyses data and generates hypotheses	Systematically collects data Systematically (perhaps as a group) analyses data and generates hypotheses	Systematically collects data Systematically analyses data to verify or falsify hypotheses
and	but	and	and	and
No written report	No written report	Writes report open to public critique	Writes separate and joint reports open to public critique.	Written report, open to critique
but	but	and	and	and
Incorporates reflections into practice from time to time	Tries to incorporate reflections regularly into practice.	Systematically incorporates reflections and subsequently changes practice.	Systematically incorporates reflections and changes practice. Also works towards improvement by testing hypotheses at institutional level.	Hopes to contribute to development of formal theory
Usual Teaching Mode	Teacher Self-Monitoring	Teacher-researcher Self-evaluation: action mode research	Teacher-researcher 'Classic' action research mode	Teacher-researcher Traditional research mode
[A]	[B]	[C]	[D]	[E]

Appendix 2: Information sheet for students and caregivers**Mathematics Research Study in Year 6 and 8**

April 2002

Welcome to Term 2

As many of you are aware I am studying for my masters degree in Educational Studies in Mathematics. This year is my thesis year. I will be undertaking a research study in mathematics at school. I have received a Teachers' Study Award from the Ministry of Education which will give me four weeks release from class to undertake the study in Term 2.

My study looks at the interactions and thinking processes of children in the mathematics classroom and how they can be more effective in the acquisition of mathematical knowledge, understanding and skills by using more effective questioning and thinking strategies.

There are two parts to the project. One part involves strategies training and one part involves research. All children in Year 6 and 8 will be involved in the strategies training as part of their usual mathematics time. Their usual teacher will be working with small groups to teach the mathematical knowledge and I will be working with small groups to teach them how to apply the strategies which they will use in subsequent class work.

The second part of the project involves me analysing the children's mathematical conversations for my research. In order to do this I need your permission to audiotape the children's conversations as they work in small groups to solve mathematics problems. This will help me evaluate the effectiveness of the training and to see if the children changed the kinds of questions they asked and the discussions they had after being involved in the training. The audio tapes will be transcribed as conversations and the participants given pseudonyms so that any particular individual's contribution can not be identified. The tapes will then be erased.

If you have any questions about the study or participation please don't hesitate to contact me.

Your sincerely

Kathryn Rowe

Contacts

Researcher:
Kathryn Rowe
[Contact Address]

Supervisor:
Dr Glenda Anthony
[Contact Address]

Appendix 3: Child and Caregiver Consent Form

Mathematics Research Study in Year 6 and 8

1. I have read the information sheet and have had the study explained to me satisfactorily.
2. I agree to my conversations being audio-taped and analysed for the research. I understand that the audiotapes will be destroyed after they have been transcribed and that my identity will be protected by the use of a pseudonym in the written transcriptions.
3. I agree to provide information to the researcher, including the completion of a response sheet on the understanding that I will not be identified by name, and that this information will be used only for this research and publications arising from this research project.

Please retain the top portion of this form for your information. Sign and return the lower portion of this form to your classroom teacher in the envelope provided

I volunteer to participate in the research project as outlined in the information sheet and consent form.

Signature Student.....

I agree for my child to participate in the research project as outlined in the information sheet and consent form.

Signature Parent/Guardian:

Date:/2002

Appendix 4: Teacher Consent Form

Mathematics Research Study in Year 6 and 8

1. I have read the research proposal and have had the study explained to me satisfactorily. I agree to participate by supporting the mathematical strategies training programme in my classroom practice.

2. I agree to discuss with the researcher anecdotal notes I make about children's mathematical conversations in the classroom and their academic gains during the course of the study and to provide time for the children to complete the response sheet at the end of the study.

3. I understand that my class and I are participating in the research and training with the full support of the principal and Board of Trustees.

Signature Teacher

Date:/2002

Appendix 5: Student Response Questionnaire

1. Give a reason (s) why someone would prefer working on their own to solve maths problems.

2. Give a reason (s) why someone would prefer working in a group to solve maths problems.

3. How do you prefer to work in maths? On my own In a group Both ways

4. Lisa and Tom came up with different answers to the same maths problem. Both argued that they were right. How could they come up with an answer they both agreed on?

5. If you were in charge of a group that was solving maths problems what rules would you have for your group?

6. Did you use any of the ideas you talked about in the library back in your class to help with your maths?
(Please give an example with your yes or no answer)

7. This space is for you to make any other comments you'd like to about maths.

Appendix 6:

Examples of Instructional Strategies Employed to Elicit, Support and Extend Children’s Mathematical Thinking (Fravillig, Murphy & Fuson, 1999, p.155)

Instructional components of ACT framework

Eliciting	Supporting	Extending
Facilitates students’ responding	Supports describers’ thinking	Maintains high standards and expectations for all students
Elicits many solution methods for one problem from the entire class	Reminds students of conceptually similar problem situations	Asks all students to attempt to solve difficult problems and to try various solution methods
Waits for and listens to students’ descriptions of solution methods	Provides background knowledge	Encourages mathematical reflection
Encourages elaboration of students’ responses	Directs group help for an individual student	Encourages students to analyse, compare, and generalise mathematical concepts
Conveys accepting attitude towards students’ errors and problem solving efforts	Assists individual students in clarifying their own solution methods	Encourages students to consider and discuss interrelationships among the concepts
Promotes collaborative problem solving	Supports listeners’ thinking	Lists all solution methods on the chalkboard to promote reflection
Orchestrates classroom discussions	Provides teacher led replays	Goes beyond initial solution methods
Uses students’ explanations for lesson’s content	Demonstrates teacher selected solution methods without endorsing the adoption of a particular method	Pushes individual students to try alternative solution methods for one problem situation
Monitors students’ levels of engagement	Supports describers’ and listeners’ thinking	Promotes use of more efficient solution methods for all students
Decides which students need opportunities to speak publicly or which methods should be discussed	Records symbolic representation of each solution method on the chalk board	Uses students’ responses, questions, and problems as core lesson
	Asks a different student to explain a peer’s method	Cultivates love of challenge
	Supports individuals in private help sessions	
	Encourages students to request assistance (only when needed)	

Appendix 7: Example of a coded transcript.

Coding used:

LC= lower order cognition, involving recall or recognition of previously known facts, or comprehension of words and symbols.

HC= higher order cognition, this is indicated by application of ideas such as estimating and linking to other group members ideas .

S= social on-task comments linked to the management of the task.

(LC)H: 550 kilograms. One elephant can pull 550 kilograms. What is the smallest number of elephants that would be needed to pull a 3 tonne log? I'd say...

(HC)J: A 1000 kilograms...

(LC)J/H (in unison): ...is one tonne.

(HC)H: So umm...

(HC)J: That's..

(HC)H: One tonne is two elephants...no...one tonne is...

(HC)J: No, one tonne is two elephants.

(HC)H: No two and a bit.

(LC)J: One thousand, one hundred.

(HC)H: Yeah but with that? One tonne and 100 kilograms.

(HC)J: 100 kilos...okay so they'd be 3...no...hmm (pause) ...two elephants...three elephants...four...

(HC)H: Four elephants is 3 tonne and 200 kilograms.

(HC)J: 3 tonnes.

(HC)H: Hmm...six.

(HC)J: Six, six elephants.

(LC)H: Six elephants and they're pulling...oh but...

(HC)J: 3 tonne, 300 kilograms.

(HC)H: Yeah but that's over.

(HC)J: Look, what's the *smallest* number of elephants? The smallest number is six.

(S)H: Yeah. (Hayden records their answer on paper) Okay let's do the next problem.

Appendix 8: Sample of a Problem-solving Task from *Figure It Out: Number (A), Level 2, Draft* (Ministry of Education, p. 3, 2002d).

Sacks of Gold

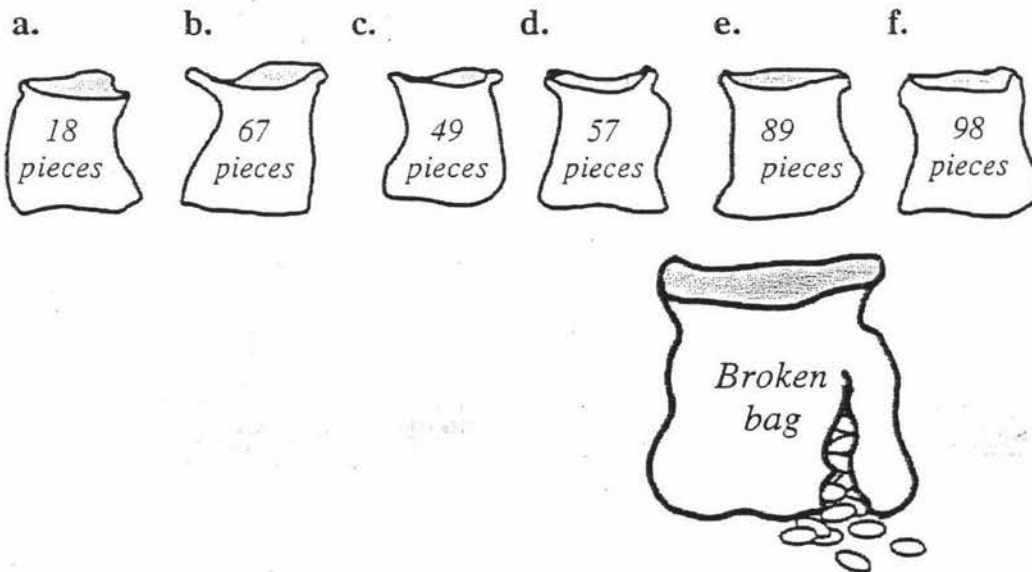
Activity

Pete the Pirate has sacks of gold in his treasure chest. He decides to bank his gold so it is safe.



The bank manager says you can only bank the gold in lots of 10, such as 50 or 80.

- How many pieces of gold from the broken bag will Pete the Pirate need to use so each bag can be banked?



- Pete adds gold pieces from the broken bag to the bags a, b, c, d, e, and f so that they can be banked. Then he banks the bags a, b, c, d, e, and f. How many gold pieces is Pete the Pirate going to bank altogether?
- If Pete the Pirate is only allowed to bank 150 gold pieces a day, how many days will it take to bank all the gold?

Appendix 9: NUMP stage and behaviour indicators for operational strategies for addition and subtraction and fraction knowledge in the *Draft Diagnostic Interview* (Ministry of Education, 2002c).

Operational Strategies for Addition and Subtraction	
Stage and Behaviour	
0 Emergent	The student has no reliable strategy to count an unstructured collection of items.
1 One-to-one Counting	The student has a reliable strategy to count an unstructured collection of items.
2 Counting-on from One on Materials	The student's most advanced counting strategy is counting from one on materials to solve addition problems.
3 Counting-on from One by Imaging	The student's most advanced strategy is counting on from one without the use of materials to solve addition problems.
4 Advanced Counting	The student's most advanced strategy is counting on or counting back to solve addition or subtraction tasks.
5 Early Additive Part-Whole Thinking	The student shows any part-whole strategy to solve addition or subtraction problems mentally by reasoning the answer from basic facts and/or place value knowledge.
6 Advanced Part-Whole Thinking	The student is able to use a broad range of mental strategies to solve addition or subtraction problems with whole numbers. The student is able to explain the method clearly.

Fractional Numbers	
Stage and Behaviour Indicators	
2-3 Non-fractions of regions	The student cannot assign symbols to fractions of regions and/or cannot name the symbols.
4 Assigned Unit Fractions	The student matches unit fraction symbols to regions and can name the symbols, e.g., $\frac{1}{4}$, $\frac{1}{2}$.
5 Ordered Fractions	Orders unit fractions, e.g., $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$.
6 Co-ordinated Numerators and Denominators	Describes the size of fractions with reference to both the numerator and denominator, e.g., $\frac{8}{6}$ is one whole and two sixths.