LEARNING STRATEGIES IN MATHEMATICS EDUCATION

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Abstract

Interest in learning strategies is particularly relevant to current curriculum reforms in mathematics education. The body of literature concerning the constructivist perspective of learning characterises the learner as being cognitively, metacognitively and affectively active in the learning process. The learner must appropriately control his or her learning processes by selecting and organising relevant information and building connections from existing knowledge.

In order to assist students in becoming more active, and self-regulated, it is timely that we learnt more about learning strategies, and their relation to knowledge construction and effective performance. This ethnographic study examines sixth form students' use and awareness of learning strategies. Data was obtained from observations, questionnaires, and stimulated recall interviews. Case studies of four students provided descriptive learning profiles of strategic behaviours in context.

Learning strategies are classified according to cognitive, metacognitive, affective, and resource management goals. Examples of students' specific use of learning strategies indicates that a wide range of strategies are employed. However, the use of learning strategies *per se* is not inherently indicative of purposive, intentional learning behaviour. There is a strong indication that the appropriateness and effectiveness of strategies relate to the learning goal and the task demands.

Learning behaviours that contribute to successful learning include rehearsal, elaboration, organisation, planning, monitoring and, self-evaluation. In addition, more successful students modify their learning tasks, know when it is appropriate to seek help, and are able to adapt their physical and social learning environment to optimise their learning opportunities.

Contributing factors of low achievement include: lack of relevant prior knowledge; lack of orientation towards mastery learning and an associated confusion about task goals; and inappropriate use of learning strategies related to monitoring understanding. Less successful students provide infrequent reports of metacognitive behaviours to control learning and employ ineffective use of help seeking and resources.

The study provides ample evidence of passive learning behaviours. Students sample selectively from the flow of instructional stimuli according to their needs and interests, but seldom take action to adapt the lesson to their individual requirements. Specific instructional factors which appear to contribute toward passive learning behaviours are highlighted in this study.

The present study provides evidence to support the proposed *Interactive Model of Learning Mathematics*. The influence of presage and product factors on strategic learning behaviours is clearly demonstrated in reports of the students' classroom and home learning environments.

Success of new curriculum developments in mathematics is critically linked to creating a suitable learning environment. To promote higher-order thinking in the mathematics class we may require a less instrumental approach - one that transfers some of the burden for teaching and learning from the teacher to the student, creating greater student autonomy and independence in the learning process.

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Chapter 1

Introduction

We know less about the ways learners approach their individual acts of learning than we do about how we, as teachers, would like them to approach learning.

(Galloway & Labarca, 1990: 127)

1.1 Background

For many years learning was viewed as something that happened to the individual: a process of absorbing knowledge transmitted by the teacher. In the recent past, when the accepted learning theory was a behaviourist one, emphasis in the mathematics classroom was placed more on the teacher 'covering' a well-defined set of content topics than on the processes needed to ensure that students learnt the presented material. Recent researchers agree that "it is crystal clear that the former does not guarantee the latter" (Shuell, 1988: 276).

Increasingly, classroom research (Marland & Edwards, 1986; Marx & Walsh, 1988; Peterson, Swing, Stark, & Waas, 1984; Winne & Marx, 1982), focusing on the mediating role of the learner, has acknowledged that the student plays a crucial role in determining what and how much is learnt. The **use of learning strategies** has emerged as a critical variable in the learning process (Nolen, 1988; Wang, Haertel, & Walberg, 1993). Learning strategies are behaviours and thoughts affecting the learners' motivation or affective state, or the way in which the learner selects, acquires, organises and integrates new knowledge (Weinstein & Mayer, 1986).

Three predominant factors signify the importance of learning strategy research in mathematics education. Firstly, given the present drive for educational excellence and the consequent targeting of high-level learning, thinking and problem-solving skills in the *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992), there is a desire to identify learning strategies in order to help students acquire the required knowledge and skills. Specific learning strategies are seen as one way to improve learning outcomes. "Strategies to enhance vocabulary learning, reading, comprehension, and mathematical problem solving have potential for directly improving students' achievement in reading and mathematics" (Peterson & Swing, 1983:283).

Secondly, in an ever increasing technological society, in which one may be expected to have many jobs in a life time and constantly adapt to an increasing knowledge base, educationalists are looking for learning environments that foster the development of life-long learning skills. In view of the explosion of mathematical knowledge, and its importance for future education and employment, competence in the flexible handling of this knowledge is essential.

> *In an increasingly technological age, the need for innovation, and problem-solving and decision-making skills, has been stressed in many reports on the necessary outcomes for education in New Zealand. Mathematics education provides the opportunity for students to develop these skills, and encourages them to become innovative and flexible problem solvers.* (Ministry of Education, 1992:7)

Thirdly, from a constructivist learning perspective, learners are seen as responsible for attending to instruction and engaging in strategic learning behaviours. What learners do to select, organise and relate new information to what they already know is an important determinant of whether the information will be learned and remembered (Weinstein & Mayer, 1986).

Learning strategy research in varied domains, such as reading (Palincsar & Brown, 1984), mathematical problem solving (Schoenfeld, 1985) and languages (White, 1993) has shown that the ability to select and use appropriate learning strategies and the ability to monitor and control the learning process are characteristics of successful students. In contrast, less able students have been characterised as either not having effective strategies in their repertoire, or not employing them at appropriate times.

A key question in the study of strategic learning is what aspects of strategic behaviour are most relevant for academic work, and how can these be taught to students who might benefit from them? (Ames & Archer, 1988; Corno, 1989; Wang *et al.*, 1993). The present research study focuses on mathematics students' use and awareness of learning strategies in the classroom environment. Additionally, an examination of contextual factors affecting strategy development and deployment will go some way to providing answers to these concerns.

Before outlining the more specific problem to be addressed by this study the central concept of 'learning strategy' and terms associated with the classification of strategic learning behaviours related to mathematics learning are briefly introduced.

1.2 Learning Strategies

The term strategy was originally a military term that referred to procedures for implementing the plan of a large-scale military operation. The more specific steps in implementation of the plan were called tactics. It has since been applied to non-adversarial situations where it has come to refer to "the implementation of a set of procedures (tactics) for accomplishing something" (Schmeck, 1988:5). The logical consequence of this definition is that learning strategies are a sequence of procedures for accomplishing learning.

Researchers have referred to learning strategies in a variety of ways: "thinking skills" (Swing, Stoiber, & Peterson, 1988); "strategic behaviour" (Bachor, 1991); "higher-order thinking processes" (Resnick, 1987); "self-regulated behaviour" (Pressley, Borkowski, & Schneider, 1987); cognitive and metacognitive skills (Collins, Brown, & Newman, 1989); "learning skills" (Levin, 1986); "learning tactics" (Derry, 1990b); "cognitive processes" (Peterson *et al.*, 1984); "metastrategies" Dansereau (1985); and "mediating processes" (Marland, Patching, & Putt, 1992a). With such an array of terms it is not surprising that a precise definition of learning strategies is lacking. "The concept of learning strategies appears to be fuzzy, not unlike metacognition" (McKeachie, Pintrich, & Lin 1985:153). Brown, Bransford, Ferrara, and Campione (1983:85) comment "some systematic activities that learners use are referred to as strategies, although what is strategic and what is not has not been made particularly clear in the literature".

Some researchers limit the concept of learning strategies to "mental processing techniques" (Derry & Murphy, 1986). However, it is clear that most researchers now include learning behaviours that are used to control and regulate the learning process (Galloway & Labarca, 1990). Weinstein and Mayer (1986) provide a more broad definition of learning strategies to include **cognitions** or **behaviours** that the learner engages in during learning, that are intended to influence the encoding process, and facilitate acquisition and retrieval of new knowledge.

Learning strategies can be categorised according to their specific goal: **Cognitive strategies**, such as elaboration or rehearsal, are related to individual learning tasks, operating directly on incoming information, manipulating it in ways to make cognitive progress (O'Malley & Chamot, 1990). **Metacognitive strategies**, such as planning and evaluation, are invoked to control and monitor the learning process. **Affective strategies**, such as self-talk are employed to enhance one's concentration. **Resource management strategies** (Pokay & Blumenfeld, 1990), such as help seeking or modifying the task, are employed to operate on the learning environment so as to indirectly enhance learning performance.

Detailed taxonomies of learning strategies in domains of reading (Lorch, Lorch, & Klusewitz, 1993) and foreign language learning (White, 1993) have been proposed, as well as more general learning strategies inventories (Weinstein & Mayer, 1986; Zimmerman & Martinez-Pons, 1986), but no specific taxonomies of learning strategies for mathematics learning are widely available.

Pressley's (1986:140) description of strategy as a "broad term and, in fact, almost synonymous with the term 'procedural knowledge' " in which "mathematical algorithms and problem-solving routines qualify as strategies" alludes to a need to clarify the distinction between learning and problem-solving strategies in mathematics education. Problem-solving strategies such as reflection, monitoring understanding, and evaluating processes affect problem-solving performance (Schoenfeld, 1985; Garofalo & Lester, 1985). Thus the use of these strategies affects the learning performance and are pertinent to this study. But, whether a student employs a specific algorithmic strategy, such as using the quadratic formula, or factorising when solving a quadratic problem, will not be a focus in this study. The following example clarifies the distinction between research on **teaching, problem solving, learning** and **learning strategies**:

> *Suppose the mathematics task is to find 15% of 200. Research on **teaching** will focus on the use of exposition, concrete materials, and group discussion; the emphasis is on activities organised and managed by the teacher. Studies on **problem solving** may identify the strategies used by students to solve this task; for example, direct multiplication, or finding 10% of 200 and than adding its half. Research on **learning** may examine the misconceptions students hold about percentages. In contrast to these studies, research on **strategy** will consider what students **do** with regards to the teaching approach, how they develop the problem solving strategies that have been identified, and what cognitive and metacognitive processes they engage in to develop an understanding of percentages. For example, a common strategy to monitor one's understanding is to ask oneself questions such as 'Do I understand this?'* (Wong & Herrington, 1992:129)

The spontaneous employment of learning strategies is not at all automatic, but rather intentional, deliberate and goal directed (Garner, 1990a, 1990b). Many contextual and learner factors affect the use and effectiveness of learning strategies in the classroom and homework situation. The nature of these factors and the role of learning strategies in mathematics learning will be discussed fully in the literature review (Chapters 2 and 3).

1.3 The Specific Problem

The following statements from recent curriculum documents all reflect the importance given to the development of effective learning strategies for students:

- "*Learning how to learn* is an essential outcome of school programs." (*The Curriculum Review*, Department of Education, 1987:10);

- "The curriculum should *enable* students to *take increasing responsibility* for their learning. With their teachers they should be involved in setting goals, planning their activities, organising their studies to gain skills and understanding, and evaluating their progress." (*Draft National Curriculum Statement*, Department of Education, 1988:7); and

- "We need a learning environment which enables students to attain high standards and develop appropriate personal qualities. As we move towards the twenty-first century, with all the rapid technological change which is taking place, we need a work-force which is increasingly highly skilled and adaptable." (*The New Zealand Curriculum Framework*, Ministry of Education, 1993:1).

More specifically, *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) acknowledges the importance of complex learning strategies such as planning, monitoring, checking and reflection. In reference to the mathematics curriculum, Nightingale (Ministry of Education, 1994:2) states that changes are necessary in

approaches to thinking by teachers and students - "thinking that involves self-regulation of the thinking process". Collins *et al.* (1989: 460) go so far as to propose that cognitive and metacognitive strategies are particularly important in mathematics: mathematics unlike school subjects such as chemistry or history "rests on relatively sparse conceptual and factual underpinnings, turning instead on students' robust and efficient execution of a set of cognitive and metacognitive skills".

The present problem is that students' knowledge about these strategies is rarely considered as an integral curriculum component of school programs, in instructional planning, or in the monitoring and assessment of student learning progress (Wang & Peverly, 1986). If one were to ask a student who is having difficulty in the classroom, what he or she does to learn mathematics, one might hear the response: "I study." Likewise if one asked the student what could be done to improve his or her performance the reply might be: "Study more." A plausible assertion is that many mathematics students have not developed the ability to identify and use appropriate learning strategies. Too little attention in the mathematics classroom is given to the 'how to learn' - it is not enough to repeat the same explanation or to offer a different representation of the concept; teachers and learners need to be more aware of the learning strategies involved in learning mathematics if effective life-long learning is the goal. The active role of the student signifies that a significant improvement in student learning depends "on a fundamental shift from teacher to student in responsibility for, and control of learning" (Baird, 1986:263).

Of further concern is the fact that the demand for autonomous learning behaviours is of increasing importance as students progress through the academic system. By the time students reach tertiary level "students are increasingly called upon to shoulder responsibility for their own learning and for the management of learning related resources" (Thomas & Rohwer, 1987:382). The use of metacognitive strategies, enabling one to control and monitor the learning process, is seen to be an important indicator of learning success at tertiary level (Anthony, 1991). Even at elementary school, data from studies indicate that American students spend approximately 65-75% of their time in independent seatwork (Wang & Peverly, 1986).

Although analysis and description of the role of the student in the learning process have been the focus of recent research, much of the current information comes from laboratory related experimental studies. Despite research on learning strategies in many domains there is a lack of corresponding classroom research (Marland & Edwards, 1986) and in particular mathematics classroom research (Briars, 1983; Wong & Herrington, 1992).

1. 4 The Research Objective

To make a real difference in students' alibility to learn and foster self-regulated autonomous learning we need to further our understanding of learning processes engaged in actual classrooms. Teachers and researchers have all observed that students approach mathematics learning in different ways. For example students' behaviours vary in such things as questions asking, on-task behaviour, homework completion and setting out work. These overt learning behaviours are easily observed, but little is known of the covert learning strategies students employ, such as comprehension of teacher explanations, self-testing for understanding, evaluation of performance and recognising the need to revise. Pressley, Woloshyn, Lysynchuk, Martin, Wood, & Willoughby (1990) recommend that researchers first determine what strategies students use in classroom environments before implementing various strategy instruction programmes. They argue that the teaching of strategies can be improved only if it is known what students do, and fail to do, in the absence of instruction. Kardash and Amlund (1991) support this notion, suggesting that spontaneous strategy use is especially important at the secondary school level because of research evidence suggesting that students adopt preferred strategies (often ineffective) which lessen the likelihood that they will be amenable to strategy training. A first step is to determine what strategies learners use on their own, how these strategies relate to one another, and which strategies are related to enhanced learning outcomes.

Thus the principal objective of the present research study is to examine students' use of learning strategies in an authentic learning situation. The study will provide a description of 6th form mathematics students' use and awareness of learning strategies both during classroom learning and learning at home. Furthermore, because learning strategies are not applied in a vacuum, but are influence by a multitude of contextual variables such as students' prior knowledge, availability of resources, demands of the task, and the classroom instructional context, analysis of students' use of learning strategies will provide evidence of factors affecting strategy use.

As will be discussed further in the literature review, the outcome of learning depends on the learning behaviour that the student engage in. In turn, learning strategies that students engage in depend on the context in which a learning activity takes place (Thomas & Rohwer, 1993). A secondary objective of the research is to examine students' reported or observed learning strategies in relation to learning outcomes. Documenting students' use of learning strategies in the natural classroom setting may provide a possible explanation as to student differences in products of learning given the same educational instruction.

1.5 Summary

By adopting a constructivist perspective of learning, one accepts that the knowledge and skills that students bring to the learning situation, and the cognitive activities that they pursue, are the major determinants of their learning outcome. There is a growing interest in defining the learning process, and encouraging students to take charge of their own learning. What the student does is more important than what the teacher does in determining the effectiveness of the teaching/learning process. An important factor of what the student does is the employment of learning strategies (Shuell, 1988).

"A major direction in current cognitive research is to attempt to formulate explicitly the strategies and skills underlying expert practice, to make them a legitimate focus of teaching in schools and other learning environments" (Collins *et al.*, 1989: 480). To increase our understanding of the learning process in mathematics it is essential that we have qualitative information of students' employment of learning strategies in authentic learning environments. Through our understanding of the variation of students' approaches to learning in the mathematics classroom, instructional intervention can be designed to directly address and adapt to learners' existing strategic knowledge. Moreover, knowledge of students' learning processes will put us in a more favourable position to interpret a student's failure. Such research will hold particular relevance for increasing the effectiveness of schools in providing improved chances for students with poor academic prognosis (Leinhardt & Putnam, 1987).

The following literature review (Chapters 2 and 3) situates the current research study in terms of existing strategy research. Chapter 2 establishes the importance of strategic knowledge and behaviours in the constructivist learning paradigm. An *Interactive Model of Learning Mathematics* is proposed as the basis for examining learning strategies in the classroom context. Chapter 3 examines the nature of learning strategies as they relate to mathematics learning. A review of research in mathematics education relating to strategy use and instruction provides further support for the interactive nature of learning mathematics.

Chapter 2

Towards a Model of Learning Mathematics

Learning is an active, constructive, cumulative, goal-oriented process (See Shuell, 1986). It is ***active*** *in that a student must do certain things while processing incoming information in order to learn the material in a meaningful manner. It is* ***constructive*** *in that the new information must be elaborated and related to other information it order for the student to retain simple information and understand complex material. It is* ***cumulative*** *in that all new learning builds upon and/or utilizes the learner's prior knowledge in ways that determine what and how much is learned. It is* ***goal oriented*** *in that learning is most likely to be successful if the learner is aware of the goal (at least in the general sense) towards which he or she is working ...*

(Shuell, 1988: 277-8)

2.1 Introduction

Over the course of this century, the view of learning has changed in ways that have affected educational practice and research. In particular, changing views of the role of 'domain knowledge' and 'strategic knowledge' and the role of the learner in the construction of knowledge have greatly influenced research on learning strategies.

The early behaviourist views of the learner as a passive being, whose repertoire of behaviours is determined by rewards and punishment, focused research on learning outcomes more than processes. Cognitive theories of learning in the 1950s and 1960s emphasised learning as knowledge acquisition. Information-processing models of the 1970s and 1980s recognised learning as a constructive process; that is, learning involves selecting relevant information and interpreting it through one's existing knowledge.

In a recent landmark review of variables affecting school learning Wang *et al.* (1993: 266) proposed that "one of the most significant educational findings of the last decade has been the documentation of metacognitive processes that serve to guide students through tasks".

This chapter discusses the importance of these changing perspectives on learning and learning strategies. In line with current constructivist views of mathematics learning the role of learning strategies, and in particular metacognitive behaviour, is incorporated into an *Interactive Model of Learning Mathematics.*

2.2 Domain Knowledge versus Strategic Knowledge

Educationalists and psychologists have long asked the question, 'which kind of knowledge counts most - general strategic knowledge or specific knowledge about a domain?' Collins *et al.*(1989: 477) describe strategic knowledge as:

> *the usually tacit knowledge that underlies an expert's ability to make use of concepts, facts, and procedures as necessary to solve problems and carry out tasks. This kind of expert problem-solving knowledge involves problem-solving strategies and heuristics, and the strategies that control the problem-solving process at its various levels of decomposition. Another type of strategic knowledge, often overlooked, includes the learning strategies that experts have about how to acquire new concepts, facts, and procedures in their own or another field.*

In contrast, domain knowledge is the conceptual and factual knowledge and procedures explicitly identified with a particular subject matter; these are generally elucidated in school textbooks, class discussions, and teacher explanations.

Strategic knowledge position

In answer to the question of which kind of knowledge is the most important, the oldest theory of expertise and intelligence maintained that a student builds up his or her intellect by mastering formal disciplines. The study of subjects likes mathematics, logic and Greek was intended to train the mind's forms as opposed to training to impart knowledge. It was assumed that "these subjects build minds as barbells build muscles" (Bruer, 1993: 52). Early cognitive research assumed that general skills and reasoning abilities were at the heart of skilled performance: "True ability resided in the general strategies, with the database an incidental necessity" (Perkins & Salomon, 1989:17). As a consequence of these assumptions learning research focused on abilities such as memorisation and problem solving, using tasks upon which the possible effects of pre-existing knowledge had been carefully controlled. Support for the 'general strategies' perspective came from Polya's (1957) analysis of mathematical problem solving. Polya argued that problem-solving success depended on students having knowledge of a repertoire of heuristics, such as breaking a problem into sub-problems, solving a simpler problem, using a diagram or examining a special case.

In the 1950s and 1960s initial success in Artificial Intelligence research added further support to the strategic knowledge position. Artificial Intelligence programs demonstrated the ability to solve simple puzzles and logic problems using such strategies as 'means-ends analysis' and 'hill-climbing'. It was argued that Artificial Intelligence showed that successful performance depended upon a repertoire of heuristic knowledge and general mental strategies. Domain knowledge, although acknowledged, was not accorded a central position in the role of expertise.

Domain-based knowledge position

However, by the mid 1970s gathering evidence from cognitive research suggested that general domain independent skills could not adequately account for expertise. Firstly, support for the demise of the 'general strategies' position came from investigations of expertise in domains such as chess, mathematical problem solving, and physics. Evidence

from Chi and Bassok (1989) revealed that experts possessed a large knowledge base of domain-specific patterns that are organised differently to novices. Experts are likely to organise their knowledge on the basis of concepts, principles and abstractions that reflect a deep understanding of the domain. This enables rapid recognition of situations where these patterns apply and reasoning then moves from such recognition directly to a solution.

In contrast "novices tended not to see the relevant patterns, because they did not know them or lacked rapid recognition-like access to them" (Perkins & Salomon, 1989:18). Novices often based their reasoning on superficial problem content such as literal objects, and relationships explicitly mentioned in the problem. Problem solution involved focussing first on the unknown and seeking equations or rules that bridged back from the unknown towards the givens (means-ends). Perkins & Salomon (1989: 18) noted:

> *the broad heuristic structure of expert as contrasted to novice problem solving - the reasoning forward rather than the reasoning backward - seemed attributed not to any heuristic sophistication on the part of experts, but to the driving influence of the experts' rich database.*

This concurs with Glaser's (1984:99) earlier interpretation that the problem-solving difficulty of novices "can be attributed largely to the inadequacies of their knowledge bases and not to limitations in their processing capabilities such as the inability to use problem-solving heuristics".

Secondly, it was argued that weak-general strategies account for little of the variance in learning performance and are in fact a derivative of domain knowledge. Chi (1987), a leading exponent of the 'knowledge position', argued that strategic knowledge, such as the ability to accurately monitor one's understanding, judge the difficulty of problems and checking procedures, are a derivative of domain knowledge:

> *...younger children's inability to accurately monitor their current state of knowledge (such as preparedness for recall), as well as their inadequate allocation of attention, is attributed to an inadequacy in part of their domain knowledge related to the stimulus items, rather than strictly undeveloped monitoring processes.* (Chi, 1987: 260)

Chi contends that checking in mathematics is totally an outcome of the presence of the relevant domain knowledge in memory and not a meta-strategy that some individuals have and some do not. Thus it is conjectured that the reason that children may not check their solutions as readily as adults, reflects not so much deficits in their control or monitoring process, but rather, the lack of a relevant schema in the declarative knowledge base to tell them that the answer was inappropriate.

Further research with Artificial Intelligence found that there were difficulties designing generic programs to deal with complex problem solving in information rich domains such as mathematics and physics.

> *When new to a domain, all a computer or human could do was deploy weak methods that turned out weak results. Real power in problem solving emerged over time, as application of weak methods created the opportunity to learn and store up the ramifications of particular moves in the domain and build the rich database. This database would become the real power behind good problem solving, leaving the weak methods behind.* (Perkins & Salomon, 1989:18-19)

As a consequence, researchers successfully turned their attention from programming a system with powerful search heuristics to prógramming a system to possess a large quantity of organised knowledge.

Further support for the 'knowledge position' came from research in mathematical problem solving. Schoenfeld (1985, 1987) found that attempts to teach Polya's heuristics as an isolated unit met with little success. Students understood the heuristics in broad terms but didn't seem to understand the mathematics well enough to apply them in the complex and context sensitive ways required. Domain knowledge, more than general problem-solving heuristics, appeared to be the major stumbling block to successful

performance. Moreover, evidence from Owen and Sweller (1985) found that encouraging 10th grade students to use goal-orientated problem-solving strategies (means-ends) during work on trigonometry problems retarded schemata acquisition. They suggested that these findings were possibly because the students were investing more effort to solving the problems than to becoming familiar with the underlying schemata.

Using stimulated recall, Peterson, Swing, Braverman, and Buss (1982) obtained data on 5th and 6th grade students' self-generated mental strategies during mathematics lessons. Students' achievement scores were found to correlate positively with their use of task-specific mental strategies, but negatively with the frequency of general, global strategies. A pattern was noted in which the high-ability students used specific strategies but low-ability students tended to report the use of weak-global strategies.

Thus while it was agreed that generalised thinking and problem-solving skills are of value where existing knowledge is minimal, the skilful problem solver within a given domain rapidly moves away from applying generalised mental strategies to develop domain specific pattern-recognition skills. "These critical encoding skills enable stored knowledge to be brought to bear on new problems to enable quality solutions to be reached" (Yates & Chandler, 1991:139).

Thirdly, according to the 'general strategies' theories, much of the knowledge acquired in a particular domain is inherently general and should lead to transfer to other areas. It was assumed that the study of mathematics would improve one's ability to reason and to solve problems confronted in the real world. Grube (1974:18) claims for Plato that "those who are by nature good at calculation are, as one might say, naturally sharp in every other study, and ... those who are slow at it, if they are educated and exercised in this study, nevertheless improve and become sharper than they were". But increasingly so, research has shown that training in mathematics has no measurable influence on other cognitive functions (Stanic and Kilpatrick, 1989). Overall, research on transfer suggests the same conclusions as the arguments from expertise and weak methods:

> *Thinking at its most effective depends on specific, context-bound skills and units of knowledge that have little application to other domains. To the extent that transfer does take place, it is highly specific and must be cued, primed, and guided; it seldom occurs spontaneously. The case for generalizable, context-independent skills and strategies that can be trained in one context and transferred to other domains has proven to be more a matter of wishful thinking than hard empirical evidence.* (Perkins & Salomon, 1989:19)

These collective research findings on the pervasive influence of domain based knowledge convinced many to take the view that it is knowledge, not strategies, that is the central issue in the development of competence (Chi, 1987; Glaser, 1984). Emphasis in research and instructional development was now directed at the representation of knowledge. Questions about the knowledge base changed from a consideration of the accumulation of facts and their reinforcement, to consideration of the organisation and coherence of information along with the compatibility of new information to prior experience (Brown *et al.*, 1983).

In mathematics, this focus was reflected in research on schemata for solving addition and subtraction word problems and in the argument that successful problem solving involves being fluent with a repertoire of representation systems (Putnam, Lampert, & Peterson, 1990). Blais (1988) discussed the implication of experts being able to recognise the "essence" as support for the hypothesis that experts construct different mental representations of problems than do novices. For example, experts perceived the essence of 2/7 + 3/7 as roughly two things plus three things, which are five things. In contrast, novices preferred to use nineteen dots. Similarly, Silver (1979) found that those who were unsuccessful at solving mathematical word problems were more likely to rely on surface features when categorising word problems than those who were successful.

Synthesis Position

While still acknowledging the centrality of the knowledge base, some theorists advocated a shift to a "two factor" (Peverly, 1991), or a "synthesis" (Perkins & Salomon, 1989; Prawat, 1989) knowledge and strategies theory. Support for the synthesis position came from many researchers (Alexander, 1992; Alexander and Judy, 1988; Pressley *et al.*, 1987; Resnick, 1987; Schoenfeld, 1985; Sternberg, 1985; Yates & Chandler, 1991).

> *Intuitively, it would seem that the effective and efficient learning in the classroom is dependent upon the continual orchestration of one's content and strategy knowledge. We might hypothesize, for example, that competent learners weigh their content knowledge against the demands of the task and then bring the appropriate form of strategic knowledge to bear on the task. As the learners' knowledge of the content relative to the task increases, then it is likely that the need for strategic behaviour decreases.* (Alexander & Judy, 1989: 375)

It was argued that "much of the research used to support the knowledge-based position is methodologically problematic" (Peverly, 1991:74). Most of the research on expertise had examined experts addressing standard single-level tasks in a domain. These problems have often been too difficult for the novice who, without a suitable domain knowledge base, has had to resort to backwards processing. The same problem has been too easy for the expert, who has retrieved the solution set from schemata and thus not truly solved a problem. Clements (1982, cited Perkins & Salomon, 1989) demonstrated that experts solving atypical physics problems applied general strategies such as analogies, intuitive mental models, and the construction of a simpler problem. He suggested that a number of general heuristics, not apparent when experts face typical problems, may play a prominent role when experts face atypical problems. These general heuristics do not substitute for domain knowledge, rather they operate in a highly contextualized way, accessing, and utilising the extensive domain knowledge (Alexander & Judy, 1988). These results challenged the picture of expert performance as driven solely by a rich knowledge base of highly context-specific schemata.

Moreover, researchers of expert performance "noticed that there were intelligent novices: people who learned new fields and solved novel problems more expertly than most, regardless of how much domain-specific knowledge they possessed" (Bruer, 1993). Intelligent novices controlled and monitored their thought processes and made use of general domain-independent strategies and skills where appropriate. Peverly (1991:75) suggested that "strategies independent of the knowledge base (especially metacognitive strategies) are important to memory and development". Students without a rich repertoire of strategic knowledge upon which to draw are likely to accumulate inert knowledge (Bransford, Sherwood, Vye, & Rieser, 1986; Collins *et al.*, 1989), that is, knowledge accessed only in constrained routine contexts.

Cognitive researchers now looked to the role of learning strategies and higher-order thinking processes in expert performance. Prawat (1989:22) suggested that "the expert has available a more general, flexible set of strategies than the novice, whose skills are much more welded to particular contexts". For example, Gavelek & Raphael (1985) found that experts are better at asking and answering questions, independent of background knowledge. Thomas & Rohwer (1993) also note that successful students are distinguished by the extent to which strategies of selective allocation, generative processing and monitoring are employed.

In mathematics (Garofalo & Lester, 1985; Lawson & Chinnappan, 1994; Peterson, 1988; Schoenfeld, 1987; Swing *et al.*, 1988) the research focus shifted to higher-order learning and problem solving. Swing *et al.* (1988) worked with fourth-grade teachers to enable them to instruct students in the use of certain problem-solving strategies in mathematics, including the pictorial representation of problem solving. This intervention was particularly effective for low-ability students, apparently because they do not spontaneously engage in processes like this during problem solving. Schoenfeld (1987) also noted that the use of strategic modes of processing combined with active manipulation of information, was characteristic of the superior problem-solving performance of experts.

However, the failure of many initial training studies to effect major changes in the intelligent use of strategies promoted further studies investigating the role of strategic learning and metacognition in domain-based contexts. For example, Palincsar and Brown's (1984) reciprocal teaching indicated the effectiveness of strategy training in the reading context. Similarly, Schoenfeld (1985, 1987) emphasised that success in mathematical problem-solving instruction requires teaching heuristics in a contextualized way, so as to make good contact with students' domain knowledge base. For example, a counter example in mathematics requires different criteria to a counter example in a legal claim; checking a mathematical solution by substitution is different to checking a science experiment by repetition. As an alternative to teaching the general heuristics suggested by Polya (1957), Schoenfeld's instruction focused on specifying the strategies at a level of detail that included more of the mathematics knowledge involved. These studies demonstrated that certain learning strategies improve learning performance, and advocated the teaching of these strategies as a routine component of content-based instruction.

Increasingly, research recognised the importance of metacognitive knowledge and beliefs about the domain of study (examples in mathematics research include: Cardelle-Elawar, 1992; Garofalo & Lester, 1985; Herrington, 1992; Schoenfeld, 1985). "If children are to learn how to take charge of their own problem solving, it is important to give direct attention in instruction at every level to metacognitive aspects of the learning of mathematical ideas" (Lester, 1988:119). This aspect of students' metacognitive behaviours will be further discussed in section 2.3 and Chapter 3.

In further support of the "synthesis" position Borkowski, Schneider, and Pressley (1989) and Peverly (1991) argued that domain-specific knowledge and strategies, and domain-general knowledge and strategies, interact to produce competent problem-solving performance. For example, the possession and activation of relevant prior knowledge enables a learner to encode new experiences with a high level of efficiency. This is immediately apparent with respect to chunking, elaboration, and monitoring strategies.

But, on the more subtle level, prior knowledge and familiarity allow a learner to free up necessary resources for the selection, execution, and coordination of strategic processing. Additionally, in the classroom context, prior knowledge may serve to render a student less dependent for success upon available instruction, and more able to cope with independent learning.

Knowledge can also directly prompt a learner to become strategic in an almost automatic or stimulus-driven manner. For example, much of early education in language, reading, and mathematics is aimed at making the child's mental operations more automatic, less onerous, and more enjoyable (Yates & Chandler, 1991). In the initial stages of skill acquisition, a high level of practice is used to build up procedural knowledge to the point where attention processes become available in the service of higher mental goals. If for example, children can quickly access the basic facts used in more complex computation, their attentional resources can be devoted to remembering and performing more complex procedures, or working out new problem solutions (Resnick, 1989a).

Thus research from varied disciplines has found that differences in student success cannot be attributed solely to differences in domain content knowledge. In particular, beliefs and intuitions, strategic knowledge, self-awareness and self-regulation were found to be important determinants of mathematical behaviour. "*Knowing* a lot of mathematics may not do the students much good if their beliefs keep them from using it. Moreover, students who lack good self-regulation skills still may go off on wild goose chases and never have the opportunity to exploit what they have learned" (Schoenfeld, 1987:198).

Summary

Early advocacy of general cognitive skills overlooked the importance of a rich knowledge base, assuming that general heuristics would make ready contact with a person's knowledge base, and that transfer would happen more or less spontaneously. Developments in the psychology of expertise, and Artificial Intelligence systems, highlighted the role of a well structured domain knowledge base as a dominant factor in development. However, more recent expert/novice research has demonstrated that the amount of knowledge is not the sole determinant or predictor of learning performance. Successful students seem to differ from less successful students on the basis of the number and nature of the strategies they bring to bear on a task and on the basis of their facility at selecting and monitoring strategies in task appropriate ways (Thomas & Rohwer, 1993).

Strategic knowledge, including domain-specific strategies and domain-general strategies, combined with metacognitive behaviours to regulate and control learning, play a major role in the learning process. It is apparent from the review of research that the development of competence involves the interaction of domain with strategic knowledge rather than the predominance of knowledge or strategies alone.

In the next section these research findings supporting the 'synthesis position' are related to componential models of learning. As well as incorporating the interactive role of domain and strategic knowledge, the influence of contextual variables is also an indispensable feature of the proposed model for learning mathematics.

2.3 Interactive Model of Learning Mathematics

The purpose of this section is to develop a suitable model of mathematics learning in which the role of strategic knowledge, and related learning strategies and metacognitive behaviours, are related to other variables affecting learning performance.

To situate the model in the cognitive psychology research over the last decade one needs to review the changing beliefs about learning in general, and mathematics learning in particular. Currently many educational theorists conceive of learners as 'architects building their own knowledge structures' (Wang *et al.*, 1993). The view of the learner has changed from that of a recipient of knowledge to that of a constructor of knowledge with metacognitive skills for controlling his or her cognitive processes (Candy, 1989; Confrey, 1990; Fennema, 1989).

Three important assumptions (Biggs, 1989; Resnick, 1989b; Shuell, 1986) related to this view have a direct impact on the role of learning strategies and the development of our model of learning. Firstly, learning is a process of knowledge construction, not of knowledge recording or absorption. Secondly, learning is knowledge-dependent; people use current knowledge to construct new knowledge. Thirdly, the learner is aware of the processes of cognition and can control and regulate them; this self awareness, or metacognition (Flavell, 1976) significantly influences the course of learning.

Constructivism

The above assumptions are important tenets of the widely accepted theory of knowledge construction - 'constructivism'. Constructivism is concerned with the constant dialectical interplay between construing and constructing: how learners construe (or interpret) events and ideas, and how they construct (build or assemble) structures of meaning (Candy, 1989). Wheatley (1991:10) states the two main principles underlying constructivism as follows:

1. "knowledge is not passively received, but is actively built up by the cognizing subject"; and

2. "...the function of cognition is adaptive and serves the organization of the experiential work, not the discovery of ontological reality..."

These principles (especially the first) have been widely embraced in mathematics education (Leder & Gunstone, 1990; Putnam *et al.*, 1990; Schoenfeld, 1992). The most important implication of the constructivist learning theory is that learning is an idiosyncratic, active and evolving process: each of us make sense of our world by synthesising new experiences into what we previously have come to understand.

> *Rather than passively receiving and recording information, the learner actively interprets and imposes meaning through the lenses of his or her existing knowledge structures, working to make sense of the world. At the same time, learning or development takes place, not by the simple reception of information from the environment, but through the modification and building up of the individual's knowledge structures.* (Putnam *et al.*, 1990:87-8)

Central to constructivism is the role of existing knowledge: prior domain knowledge influences what information is selected and attended to, and what meaning is given to that information.

> *Students' prior conceptual knowledge influences all aspects of students' processing of information from their perception of the cues in the environment, to their selective attention to these cues, to their encoding and levels of processing of the information, to their search for retrieval of information and comprehension, to their thinking and problem solving.* (Pintrich, Marx, & Boyle, 1993:167).

Not only does the amount of prior knowledge influence current learning, but also the way that knowledge is structured. "Prior knowledge that is well understood influences learning differently than prior knowledge that is less understood" (Hiebert & Carpenter, 1992:80). Additionally, Alexander & Judy (1988) stress the importance of domain-specific knowledge for the efficient and effective utilisation of strategic knowledge. For example, a student is unable to check an algebraic solution for a simultaneous equation by sketching a graph if he or she has limited knowledge of graphing procedures.

However, as discussed in section 2.2, metacognitive knowledge and beliefs also impact on the learning process. "Constructivism not only emphasizes the essential role of the constructive processes, it also allows one to emphasize that we are at least partially able to be aware of those constructions and then to modify them through our conscious reflection on that constructive process" (Confrey, 1990:109). In this sense metacognition is seen as the key to developing autonomous learning behaviours necessary for constructive learning activity.

In addition to endorsing the importance of learning strategies to effect meaningful learning, constructivism acknowledges that the social context, particularly the teacher, contribute to the construction of meaning. Learning is influenced by the "social and cultural context in which learning takes place, including the physical structure, the purpose of the activity, the existence of collaborative partners and the social milieu in which the problem is embedded" (Hennessy, 1998:1). Social interactions, whether they be self-dialogue or discussion with peers or teacher, in which students attempt to explain their interpretation and listen to others' understanding are important features of the knowledge construction process (Garrison, 1993).

Metacognition

The term metacognition was introduced by two developmental psychologists, John Flavell and Ann Brown, in the mid 1970s, to describe the understanding individuals have of their thinking and learning activities. Metacognition, which literally means 'transcending knowledge', was defined by Flavell (1976: 232) as:

> *knowledge concerning one's own cognitive processes and products or anything related to them, e.g., the learning-relevant properties of information or data...Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects on which they bear, usually in the service of some concrete goal or objective.*

In later literature Flavell (1987:21) suggested the concept of metacognition be "broadened to include anything psychological, rather than just anything cognitive...Any kind of monitoring might also be considered a form of metacognition". This interpretation of metacognition was expanded into more functional categories:

1. the regulation and control of cognition;
2. knowledge and beliefs about cognition; and
3. metacognitive experiences and their effects on performance.

The regulation and control of cognition: Early metacognitive research in mathematics (Garofalo & Lester, 1985; Silver, 1985) concentrated primarily on the regulatory and control aspects of metacognition in problem solving. To monitor and control one's learning; metacognitive strategies of planning, self-questioning, assessing progress, and evaluating learning are employed. Specific research findings regarding the use and role of metacognitive strategies in mathematics learning will be further discussed in Chapter 3. It is sufficient for the purpose of developing our model of mathematics learning to note that the employment of metacognitive strategies was seen as an important determinant of students' learning (Schoenfeld, 1985, 1987).

Metacognitive knowledge: Metacognitive knowledge is concerned with what a person knows about cognitive abilities, processes, and resources in relation to the performance of specific cognitive tasks; knowledge in this context also includes beliefs. Researchers (Brown *et al.*, 1983; Garner, 1990b) regard metacognitive knowledge as stable, thus it is retrievable for use with learning tasks, and can be reflected upon and used as the topic of discussion with others. However, metacognitive knowledge may be fallible, so that what one believes about one's cognitive processes may be inaccurate, such as the belief that simple rote repetition is the key that underlies all learning.

Students use metacognitive knowledge to generate self-appraisals and personal reflections about their knowledge strategies and abilities (Paris & Winograd, 1990). Metacognitions of this sort can answer such questions as, "Do I know how to factorise this expression? Can I do this calculation without a calculator? Can I derive the formula to find the volume of a sphere? In Flavell's terms (1987), these questions are judgements

about one's cognitive abilities, task factors, or strategies that may impede or facilitate performance.

Flavell categorises metacognitive knowledge into three categories: **person**; **task**; and **strategy** knowledge, which usually interact in any learning situation. In mathematics, person knowledge includes self-assessment of one's own capabilities and limitation with respect to mathematics in general and also with respect to a particular mathematical topic or task. Also included are one's beliefs concerning the nature of mathematical ability and the effects of affective variables such as motivation, anxiety and perseverance (Garofalo & Lester, 1985). Garner (1992) suggests that learners' beliefs about their ability to perform a task are more potent than personal skills in determining their willingness to attack, and persevere at, that task. If they have learned that they are unlikely to succeed or if they think success comes only with ability (in which they presume themselves to be deficient) rather than effort, then not pursuing an activity is an adaptive response.

Task knowledge includes knowledge about the scope and requirements of the task as well as knowledge about the factors and conditions that make some tasks more difficult than others. One's beliefs about the nature of mathematical tasks and mathematical thinking is extremely influential. Schoenfeld (1985) and Silver (1985) have each outlined issues related to the role that beliefs might play in mathematical problem solving. For example, the commonly observed phenomena that students tend to think that problems should be solved rapidly, that solutions should depend on recently taught techniques, and that every problem should conform to some model they have been taught, all represent potentially serious impediments to successful mathematics learning.

Strategy knowledge is knowledge of general and specific strategies along with awareness of their potential usefulness. Each strategy in a students' repertoire is qualified by detailed information about appropriate goals and objects, appropriate tasks, range of applicability, expected performance gains, effort required and enjoyment value (Palmer & Goetz, 1988). In mathematics strategy knowledge includes knowledge of algorithms and heuristics, but also includes a student's awareness of strategies to aid in comprehending

problems, organising information, planning solution attempts, executing plans, and checking results - that is knowledge of how to effectively learn mathematics.

With regard to strategy knowledge, Garner (1992: 238) warns that "it is important to note that *knowledge* is not *use*. A learner can know all the components of an effective strategy but still not use any of them in real-world situations where employing the routine would assist learning". Paris, Lipson, & Wixson (1983) introduced the term "conditional knowledge" to capture the dimension of knowing when to apply various strategies. They proposed that skilled learners should know when and where each strategy may be useful (conditional knowledge), as well as the cost associated with each strategy, such as the amount of cognitive effort it requires.

Metacognitive experiences and the role of affect: Flavell (1987) defines metacognitive experiences as "conscious experiences" that are both cognitive and affective. Examples of metacognitive experiences would be if one suddenly has an anxious feeling that one is not understanding something, or that something is hard to solve or remember, or conversely, that one feels one has just about understood something or that the material is getting easier to comprehend. Suddenly noticing that a problem is similar to another considered recently, for which a certain strategy was relevant, is a common metacognitive experience in mathematics learning. Metacognitive experiences play an important role in the learning process in that they may redirect cognitive actions or reaffirm cognitive actions as appropriate, or contribute to the development of metacognitive knowledge to enhance learning.

Metacognitive experiences are related to individuals' goals, prior knowledge and affects. For example, tolerance for feelings of failure to understand or remember is related to students' expectations or goals of learning. "Sometimes we are aware that we are not 'getting it' but we do not care enough to expend extra energy to remedy the situation. Sometimes we are aware of cognitive confusion, but our metacognitive knowledge base is not rich enough to provide us with appropriate remedial strategies" (Garner, 1992: 242). In other instances students may ~~not~~ ~~be~~ not possess adequate prior knowledge to be aware of cognitive failure.

The interactive nature of these components of metacognition are captured in Flavell's (1981) model of cognitive monitoring - Figure 1. The model learner is assumed to select cognitive actions (e.g., repeating a formula aloud) in pursuit of certain learning goals (e.g., memorising a formula), which lead to metacognitive experiences (e.g., "I didn't learn this very well"), that in turn refine the student's metacognitive knowledge about learning (e.g., "Rehearsal isn't as good as practicing with problem exercises for this type of task").

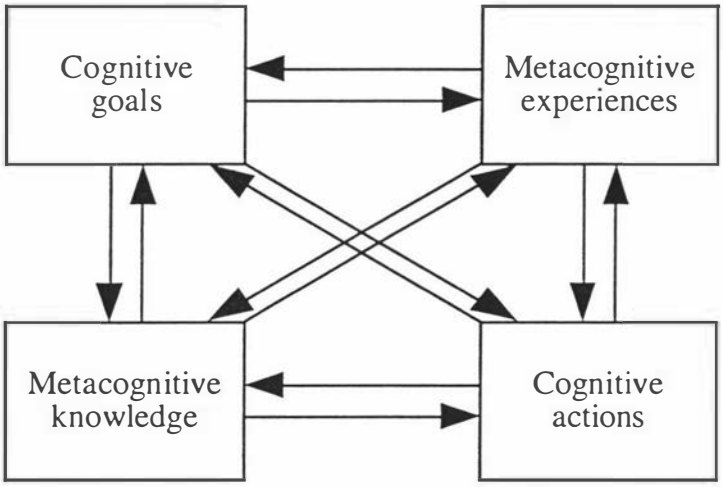

Figure 1. A Model of Cognitive Monitoring

In summary, the metacognitive knowledge that students construct interacts with metacognitive experiences to achieve the cognitive goal of learning. Consistent with constructivist accounts of learning, metacognition promotes positive self-perceptions, affects, and motivations among students. When learners ask questions, reread difficult material, or select learning activities appropriate to a given task, they are active participants in their own performance and learning rather than passive recipients of instruction and imposed experiences.

Model of Learning Mathematics

As we can see from the above discussion, current learning theories recognise learning to be a multifaceted complex phenomenon, involving the dynamic interaction of domain-specific knowledge and strategic knowledge, supported by appropriate metacognitive and affective variables. Generic models of learning such as the Tetrahedral Model, (Brown *et al.*, 1983); the Self-Instructive Processes Model, (Wang & Peverly, 1986); the Good Information Processing Model (Pressley, Borkowski & Schneider, 1989); and the 3P Model (Biggs, 1991, 1993) are all based on a common interactive component base of student variables, contextual variables, learning process variables and learner outcomes. Concurrent factors included in all models are the "entering characteristics of students, the cognitive and self-management activities that students engage in while studying, the proximate aspects of the study task, including materials and directions, and the more distal aspects of setting, including the nature of the criteria and other features of the course of instruction" (Thomas & Rohwer, 1993:2).

Recent models of learning mathematics are more likely to incorporate strategic knowledge and beliefs. For example, Fennema's Model of Autonomous Learning (Fennema, 1989; Fennema & Peterson, 1985) acknowledges the role of autonomous learning behaviours in mathematics. Also, Wong and Herrington (1992) provide a comprehensive interactive model of learning which incorporates strategic knowledge, and the interactive nature of learning strategies, beliefs and mathematical outcomes.

In view of the importance of metacognitive behaviours in the constructivist theory of learning mathematics, it is proposed here to incorporate Flavell's (1981) interaction of metacognitive components and Biggs' (1993) 3P ('Presage', 'Process', 'Product') model of learning to develop an *Interactive Model of Learning Mathematics* as shown in Figure 2. The crucial feature of this model is that the learning process involves the ability to access knowledge, skills and strategies, and to evaluate and regulate these relative to the learning task. Students' availability, selection and employment of learning strategies are perceived as central to the learning process and metacognition is the key variable in monitoring and regulating the learning process. Task processing illustrates the interaction

of learning strategies with the cognitive goal of the learning task and metacognitive experiences and metacognitive behaviours as proposed by Flavell (1981).

Presage factors are important in a constructivist based model as they result in qualitatively different ways of experiencing the learning situation which are unique to individual learners. A common assumption adopted in studies of learning in the educational context, is that different learners read the same text, solve the same problems, listen to the same class discussion and then - as they are equipped differently - do different things with the text, problem, discussion they have somehow internalised. Marton and Neuman (1992:1) argue that this assumption is invalid:

> *The conclusion we arrived at was that learners do not really read the same text, solve that same problems or listen to the same lecture...We found that regardless of what situation or phenomenon people encounter, a limited number of qualitatively different ways of experiencing or understanding that situation or phenomenon can be identified.*

The proposed model incorporates the multiple factors influencing the use of learning strategies. The presage factors of student variables, including preferred learning styles, perceptions of mathematics, prior knowledge and experiences, age and motivation, will affect the range of learning strategies available and the tendency to employ them at appropriate times. Contextual factors, including the nature and difficulty of the task, course assessment, nature of instruction, and climate of the classroom, will also affect students' use of learning strategies and consequent learning outcomes.

The model indicates the two-way interactive nature of learning: learning outcomes provide feedback (dotted lines) to the student and teacher. For example, success in a test may supply valuable information concerning learning strategies which may be added to the student's metacognitive knowledge base, or student failure may result in changes in the teacher's instructional method.

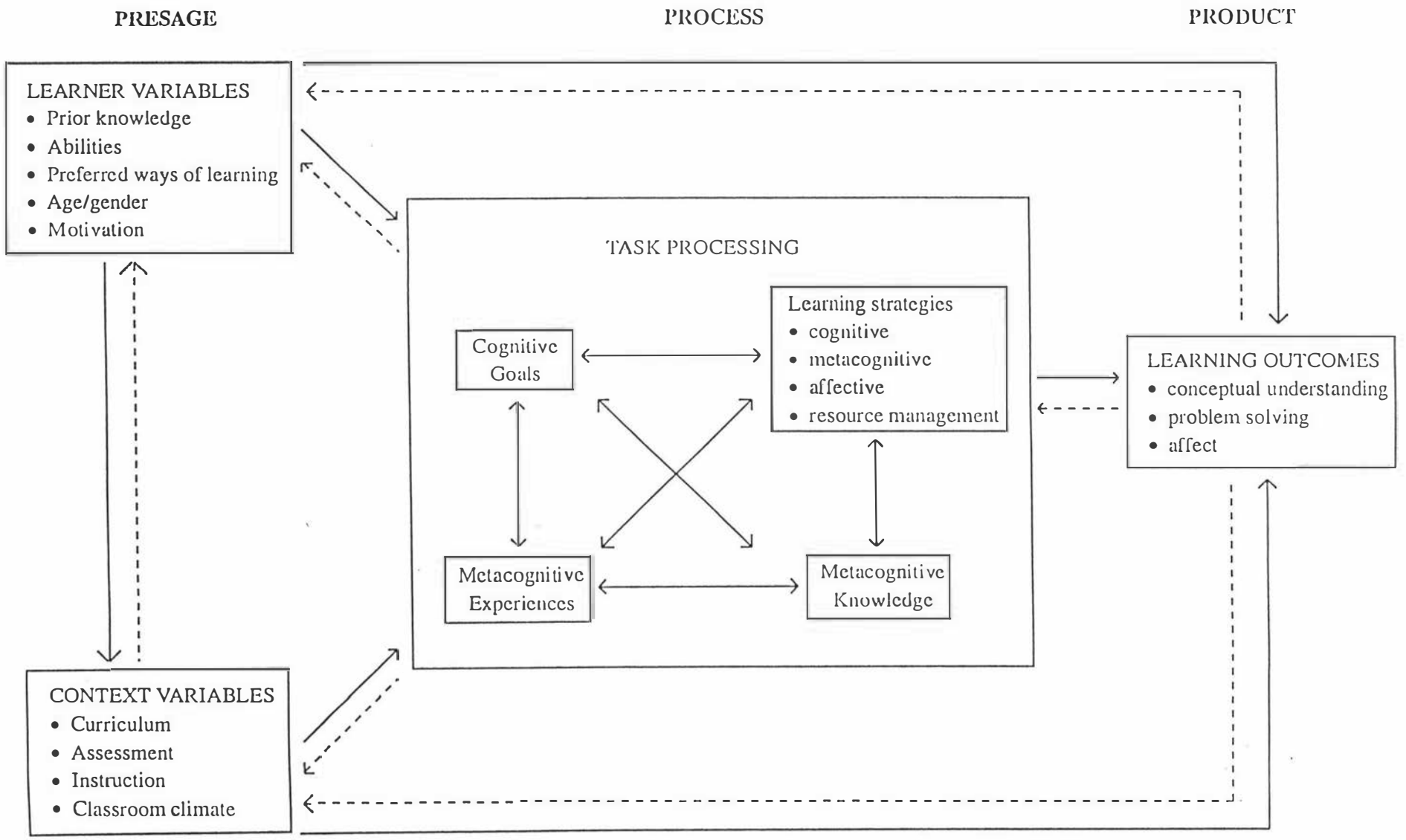

Figure 2: **AN INTERACTIVE MODEL OF MATHEMATICS LEARNING**

2.4 Summary

The constructivist perspective has had a profound influence on the way mathematics educators think about understanding and learning (Leder & Gunstone, 1990). The learner is no longer conceived as a passive information storage system, but as a self-determining agent who actively selects information from the perceived environment, and who constructs new knowledge in the light of what the individual already knows (Shuell, 1986).

The proposed *Interactive Model of Learning Mathematics* explicitly acknowledges the role of prior knowledge and experiences both of the domain and metacognitive nature. Central to the model is the use of learning strategies directed and controlled by the student: "by using various *learning strategies*, people can intentionally influence the form and quality of the knowledge they do acquire" (Derry, 1990b:348).

The **increased attention** given to learning strategies is supportive of the current constructivist learning perspective prevalent in mathematics education. Effective learners are characterised in the research literature as being cognitively, metacognitively and affectively active in the learning process. "The self-regulated learner must appropriately control his or her learning processes by selecting and organizing relevant information and building connections from relevant existing knowledge" (Mayer, 1992:409). They are capable of learning independently and deliberately through identification, formulation and restructuring of goals; use of planning, development and execution of plans; and engagement of self monitoring.

Accordingly, a primary goal of education is to help students develop expertise in how to learn and to use that expertise to construct useful knowledge. Many researchers (Alexander & Judy, 1988; Ames & Archer, 1988; Garner, 1990a) urge for further research concerning the interaction of domain and strategic knowledge and the role of contextual factors affecting the students use of learning strategies in the classroom situation.

> *To make real differences in students' skill, we need both to understand the nature of expert practice and to devise methods appropriate to learning that practice. To do this, we must first recognize that cognitive and metacognitive strategies and processes are more central than either low-level subskills or abstract conceptual and factual knowledge. They are the organizing principles of expertise, particularly in such domains as reading, writing, and mathematics.* (Collins *et al.*, 1989: 455)

The following chapter will discuss the nature of learning strategies, identifying those which are of particular importance to mathematics learning. The interactive nature of the learning model will be further explored in a discussion of research literature related to factors affecting strategy use in the mathematics classroom.

Chapter 3

Learning Strategies in Mathematics

Knowledge construct generation involves the child in a series of cognitive processes: obtaining information, creating associative links, elaborating the content, evaluating the truth and consistency of information, and developing metacognitive awareness.

(Alton-Lee, Nuthall, & Patrick, 1993: 61)

3.1 Introduction

Students acquire knowledge by generating specific knowledge constructs as they engage in the process of making meaning out of the curriculum content. This process of making meaning involves the use of learning strategies. When children learn mathematics they often engage in activities to enhance their understanding, and to help remember the rules and procedures. When reading a mathematical example, one might stop to inquire, "Do I understand where this line comes from?" If not, one might reread the information in the text, or try to rework the example on paper. The student may try to think how the example relates to an earlier example or ask, "What if this value was negative instead of positive?" Also students may engage in activities to facilitate performance of a task. For example, when asked to add 5 and 3, young children may use their fingers. All of these activities represent examples of strategic behaviour - the children are engaging in processes aimed at learning mathematics.

In sections 3.2 and 3.3 the nature and classification of these learning strategies is examined. In section 3.4 recent research is reviewed to identify what is known about students' use of learning strategies in mathematics. The *Interactive Model of Learning Mathematics* (section 2.3) emphasised the numerous factors affecting strategic learning behaviours. Findings from current research, related to factors influencing strategy use and development in the mathematics classroom, are examined in section 3.5.

3.2 The Nature of Learning Strategies

An examination of the nature of learning strategies needs first to establish what characteristics define cognitions or behaviours as being strategic. Garner (1990b) proposes four defining characteristics of effective strategies: **goal orientation**; **intentionality**; **effortfulness**; and **performance enhancement**.

Learning strategies have learning facilitation as a **goal**. The goal of a learning strategy is to "affect the learner's motivational or affective state, or the way in which the learner selects, acquires, organises, or integrates new knowledge" (Weinstein & Mayer, 1986: 315). For example, question answering, paraphrasing, summarising and imagery all increase elaborative encoding and improve recall and transfer. When revising for a test, a learner may use positive self-talk to reduce feelings of anxiety and thus effect changes in his or her affective state.

In contrast to the internally oriented cognitive and metacognitive strategies, external strategies are used to manage the environment and available resources. Pressley, Goodchild, Fleet, Zajchowski, & Evans (1989) discuss "setting the environment" as a strategy goal. For example, good strategy users find a quiet setting for study, arrange the lighting so that their eyes do not tire easily, and timetable their study so that they have the time to accomplish tasks. They make choices about which tasks to do first and what resources to use, and whether to work alone or with others.

Because strategies are goal oriented, the nature and demands of the learning task will influence the choice and effectiveness of strategies. The cognitive demands of the task will determine whether a student needs to recall specific procedures elicited by cues, recall specific knowledge, apply conceptual knowledge to a problem-solving task, or use higher-order procedures involving interpretation, transfer of rules or unfamiliar materials. Doyle (1988) argues that as much of school mathematics appears to consist of memorisation and practice of routine tasks (through the application of formulae and the matching of salient aspects of exercises with known procedures) commonly used learning strategies may be incongruent with those needed for higher-order learning.

Resnick (1987:49) proposes that reorienting basic instruction in mathematics to "focus on intentional, self-managed learning and strategies for meaning construction, rather than on routinized performances", will provide a strong base for higher-order skill development.

Garner (1990b) argues that strategy use is **intentional**. Pressley and colleagues (Pressley, 1986; Pressley *et al.*, 1987) qualify the intentionality aspect of strategy deployment. They claim that strategies, although not always conscious, are almost always potentially controllable behaviours that could be deployed deliberately. Intentional activity implies selection: from a repertoire of possible activities one selects those strategies that seem most likely to enhance performance (Paris *et al.*, 1983). For example, if a proficient learner meets a new situation that is not so obviously congruous with prior knowledge, he or she will:

> *analyze the situation and select specific strategies for it on the basis of matches between problem attributes and the attributes coded in specific strategy knowledge that define when particular strategies are called for. If the strategy requires some world knowledge, assessment of the situation includes whether the learner has relevant non strategic knowledge stored away.* (Pressley, 1986: 144)

Bisanz and Lefevre (1990) further qualify the concept of intentionality with the suggestion that the student's behaviour must involve flexible selection from alternative strategies. That is, a student who has only one way of memorising a set formula is not acting strategically when he employs that method.

Strategies require effort by the student and the effort required may be a determinant in its selection. Paris *et al.* (1983) suggest that students may weigh the value of a strategy in terms of its utility and efficiency against the effort required. Moreover, Garner (1990b:248) argues that, "given the frenetic pace of most classrooms, students are unlikely to slow down their activity flow to incorporate unpractised cognitive and metacognitive strategies". For example, when asked to summarise a reading, some students may feel that it is quicker for them to copy some sentences and delete others, than to work at a reduced, coherent summary that integrates important ideas.

Furthermore, effort is rarely expended on activities that are perceived as meaningless, futile or unrewarding. Strategies that are not yet routinised to some degree, nor actively promoted as achieving desired learning goals, are likely to be abandoned in the classroom. This poses questions as to which strategies are more efficient for classroom tasks, and whether these strategies are helpful in promoting knowledge construction, understanding and autonomous learning behaviours?

Effortfulness is also critical for successful studying outside the classroom. Homework and self-instigated study are frequently performed in situations where alternative activities are somewhat more alluring (Thomas & Rohwer, 1986). Moreover, learning at home is often isolated and unrewarding. In the absence of external direction or incentives students require volition, the disposition to exert effort, to persist, and most importantly they must supply their own feedback about their success - a metacognitive activity.

Garner's last criterion is that learning strategies may **enhance learning performance** in some instances and not in others. Researchers offer varied definitions of effective strategies as follows: Dansereau (1985:210) defines an effective learning strategy as a "set of processes or steps that can facilitate the acquisition, storage, and/or utilization of information". Pressley's (1986:140) definition includes the notion of efficiency as well as effectiveness: "Good strategies are composed of the sufficient and necessary processes for accomplishing their intended goal, consuming as few intellectual resources as are necessary to do so." Paris *et al.* (1983:296) incorporate the influence of contextual appropriateness: "(success) depends on the contextual appropriateness of the action, intentions and capabilities of the agent, available alternatives, and the 'costs' to the individual...thus learners can vary greatly in their perception of useful actions and their applications of the actions to different situations."

Paris *et al.*'s definition indicates that using strategies does not always result in enhanced performance. In particular, ill informed or unintelligent use of strategies can be detrimental to learning (Alexander & Judy, 1988). For example, students are often impeded in their development by the existence of serviceable, well used, inferior strategies that result in partial success. An example of these "primitive routines that get the job done" (Garner 1990a:519) was noted in research on the use of worked examples (Anthony, 1991; Chi & Bassok, 1989). It was found that weak students' learning was characterised by a lack of elaborations; they learnt only the sequence of actions, thus acquiring an algorithmic procedure which was not readily transferable to a problem application. Because students meet with initial success, it is difficult to get them to use more complex strategies directed at understanding worked examples and structuring them according to conceptual schemata rather than individual instances.

Due to variation of resources, task demands and learner factors, strategies that may be useful in some instances, may not always be particularly useful in other instances. For example, if a mathematics text has no worked solutions, a student may initially find it more profitable to spend time working through the text worked examples rather than trying to do exercises. Domain knowledge can also be important in making a strategy appropriate, or not, in a particular context. For example, if a student already knows a great deal about a topic, then strategies are likely to play a major role in determining the quality of understanding and recall performance (Pressley *et al.*, 1987). In other instances, knowledge of certain facts may render strategic processing unnecessary. For example, the child who has memorised basic number facts does not need to employ a strategy to solve the problem $5 + x = 9$. On the other hand, there are many strategies that simply cannot be executed without a well developed knowledge base (Garner, 1990a).

In this review of the characteristics of learning strategies it is evident that strategy knowledge is not sufficient to ensure that students use them in appropriate situations. The deployment of effective learning strategies is related to learner, instructional, and context variables. The following section considers the classification of specific learning strategies and their role in the learning process.

3.3 The Classification of Learning Strategies

Introduction

The examination of mathematics learning strategies requires classification of specific strategies. However, Weinstein (1988) comments that given the relatively young and somewhat disorganised nature of the field, there is not yet one organisational scheme that is generally accepted as a way of classifying learning strategies. More recently, Nunan (1991:168) reported that the major problem for learning strategy theorists was "the development of a coherent taxonomy of learning strategy types".

Several approaches to classification have been used. Strategies may be classified according to their goal as cognitive, metacognitive and affective strategies (Weinstein & Mayer, 1986; O'Malley & Chamot, 1990). Alternatively, strategies may be classified according to their relationship with the learning task. Dansereau (1985: 209) uses the latter in his discussion of an interactive learning strategy system: "This system is composed of both *primary* strategies, which are used to operate on the text material directly (e.g., comprehension and memory strategies) and *support* strategies, which are used to maintain a suitable state of mind for learning." Support strategies of planning and scheduling, concentration management, and monitoring have an indirect impact by generally improving the level of the learner's cognitive functioning. Oxford (1990) classifies language learning strategies as "direct and indirect". Direct strategies involve direct learning and use of the subject matter (memory and cognitive strategies). Indirect strategies, including metacognitive, affective and social strategies, contribute indirectly but powerfully to the learning. Other researchers have defined more specific classifications for behaviours which afford students the opportunity to learn and indirectly contribute to the learning goal. Pokay and Blumenfeld, (1990:42) for example, define "resource management strategies" as those behaviours related to effort, time use, help seeking and the establishment of a study environment. A 'social strategy' classification is also proposed by Galloway and Labarca (1990) to represent strategies involving interaction with other persons to assist learning.

While there exists no widely used taxonomy of learning strategies for mathematics learning in general, several researchers have identified and coded strategy use in problem-solving episodes. An example of a categorisation developed by Artzt and Armour-Thomas (1992) of strategies employed during students' problem-solving episodes is presented in Figure 3.

Figure 3: Framework Episodes Classified by Predominant Cognitive Level

Episode	Predominant Cognitive Level
Read	Cognitive
Understand	Metacognitive
Analyze	Cognitive
Explore	Cognitive and Metacognitive
Plan	Metacognitive
Implement	Cognitive and Metacognitive
Verify	Cognitive and Metacognitive
Watch and listen	Level not assigned

For the purposes of this study the finer classifications of cognitive, metacognitive, affective and resource management strategies are preferred. However, it is noted that although conceptually one can distinguish between strategy types, operationally the distinction is often blurred. The possible separability of metacognition and cognition does not preclude their constant interaction. For example, "cognition is implicit in any metacognitive activity, and metacognition may be present during a cognitive act, although perhaps not apparent" (Artzt & Armour-Thomas, 1992:141). The need for strategies to be referenced to learning episodes to assist in identification will be further discussed in Chapters 5 and 6.

Cognitive strategies

Cognitive strategies are necessary to encode new concepts and make them understandable. They relate to individual learning tasks by operating directly on incoming information and manipulating it in ways to enhance learning (O'Malley & Chamot, 1990). Galloway and Labarca's (1990:145) definition of cognitive strategies

encapsulates the characteristics of learning strategies discussed previously in section 3.2. Cognitive strategies are characterised by:

1. *the active mental engagement of the learner in the purposeful establishment of new functional knowledge through contextualized practice, and*
2. *the formation of stable and meaningful connections between prior knowledge and new information.*

Weinstein and Mayer (1986) suggest that cognitive strategies can be subsumed under three broad groups: rehearsal, elaboration and organisation. Dansereau's (1985:219) primary strategies, which include "strategies for acquiring and storing information (comprehension/retention strategies), and strategies for subsequently retrieving and using this stored information (retrieval/utilization strategies)" are of similar nature to the cognitive strategies proposed by Weinstein and Mayer (1986).

Rehearsal strategies help students to store and retrieve information and include basic learning tasks such as repetition and practice. In mathematics imitation and practice of exercises is seen as a major learning activity - it is necessary for both pattern-recognition and action-sequence productions (Derry, 1990a). While rehearsal strategies are regarded as both necessary and important Weinstein and Mayer (1986) suggest that there is little evidence that reliance on these strategies will help learners to construct internal connections, or integrate the information with prior knowledge. Rather, Weinstein (1988) notes that rehearsal strategies are effective when they provide further opportunities for more meaningful processing to take place via elaboration, organisation, or comprehension monitoring. Likewise, both Thomas and Rohwer (1986) and Gage and Berliner (1992) point out that repetition and over-learning may be necessary for real understanding of complex material and for learning how to solve problems.

The goal of elaboration strategies includes "integration of presented information with prior knowledge - i.e., transferring knowledge from long-term memory into working memory and integrating the incoming information with this knowledge" (Weinstein & Mayer, 1986:320). Appropriate and specific elaborations, such as paraphrasing, summarising, imagery, linking with prior knowledge, use of metaphor and answering questions, form helpful connections to ideas in the existing schema. Hiebert and

Carpenter (1992) hypothesise that this well-connected information is better remembered, and more easily retrieved, for two reasons. Firstly, a network of knowledge is less likely to deteriorate than an isolated piece of information, and secondly, retrieval of information is enhanced if it is connected to a larger network.

The significance of the use of elaboration in the constructivist paradigm is related to the active generation of meaning which is personally relevant to the individual student's prior knowledge and experiences (Weinstein, 1988). Elaborations occur when the student thinks about new ideas and prior knowledge together so that this thinking stimulates the generation of additional ideas about how new information and prior knowledge are related. Moreover, it is proposed that elaborative information enhances learning even when information is not actually present in the network. By providing more information for logical reasoning processes to use, elaborative information may help students construct appropriate responses (Derry, 1990b).

Research also indicates that we need to be concerned with the quality and appropriateness of elaborations as well as the frequency. Bereiter (1992) proposes that learners elaborate information in qualitatively different ways. Active or intentional learners, are likely to link new information with a highly elaborated structure of "persisting problems of understanding". Less skilled learners, who have no persisting problems in memory to attach new information to, attach new information to "referents". The result will be "referent centred knowledge", which may be recalled on the appropriate cue, but which has no function in making sense of the world. The nature of students' self-explanations, especially while processing worked examples or complex material, have been found to differentiate successful learners from less successful learners (Anthony, 1991; Chi, Bassok, Lewis, Reimann, & Glaser, 1989). In accord with Bereiter's characterisation of active, intentional learners, Chi and her colleagues refer to self-explanations as the "students' contribution to learning".

The development of schemata requires the learner to combine, integrate, and synthesise discrepant information from a variety of sources. Organisational strategies will assist in organising information in a form unique to the individual's requirements: the focus is on translation of information into another form that makes it easier to understand. Summarising, grouping, and highlighting the material are examples of organisational strategies that assist in the integration and retrieval process by separating salient information from non-salient information. Organisational strategies, like elaborative strategies, require a more active role from the learner than do rehearsal strategies.

Metacognitive strategies

Regulation of learning, as distinguished from knowledge about learning, uses metacognitive strategies (sometimes referred to as higher-order or executive strategies). Metacognitive strategies involve thinking about the learning process, planning for learning, monitoring comprehension or production while it is taking place, and self-evaluation after the learning activity has been completed.

The use of metacognitive strategies is often seen as a major factor distinguishing active or intentional learners from passive learners (Anthony, 1991; Biggs, 1987; Galloway & Labarca, 1990; White, 1993). Students who have not yet learned how to plan, direct and assess their learning, often equate learning with 'being taught'; they are content to do what the teacher and teaching materials say to do. In contrast, students who use metacognitive strategies effectively are able to self-regulate their learning by diagnosing their learning needs, formulate goals, identify resources necessary for learning, choose and implement appropriate learning strategies, and evaluate learning outcomes.

Planning activities include goal setting, previewing problems or a text, generating questions before reading a text, and doing a task analysis of the problem (Pintrich & Schrauben, 1992). For example, before doing seatwork exercises the student might write a formula at the top of the page. Planning activities help the learner plan their use of cognitive strategies and also activate relevant aspects of prior knowledge, making the organisation and comprehension of material much easier (Pressley, 1986). It is of

concern that although planning is identified as optimal by educational theorists; it is perceived as being the least helpful by younger students. (Rohrkemper & Corno, 1988)

Cognitive monitoring requires the student to "establish learning goals for an instructional unit or activity, to assess the degree to which these goals are being met, and, if necessary, to modify the strategies being used to meet the goals" (Weinstein & Mayer, 1986:323). For monitoring to be effective students must be able to detect when their behaviour is not sufficient to meet task demands so that they can make appropriate adjustments. Van Haneghan and Baker (1989:216) describe students' cognitive monitoring in mathematics as "attempts to determine whether they have given a correct answer, chosen a correct strategy for solving a problem, or understood a problem or concept". If monitoring strategies are effective they should lead to either diagnosis or directly to remedial actions (Collins *et al.*, 1989).

Effective monitoring is largely dependent on access to relevant prior domain knowledge (Garner, 1990a). Furthermore, monitoring is unlikely when a task is viewed as unimportant, or if a learner is not devoting conscious attention to it. In research with distance-education mathematics students, Anthony (1991) observed that surface learners may ignore information in worked examples that they do not understand by treating it as irrelevant to the problem solution, or study only those examples and problems that they can manage.

In addition to monitoring understanding, monitoring of strategy effectiveness enables students to form accurate metacognitive strategy knowledge; knowledge that forms the basis for successful strategy maintenance and transfer. Evaluating and reflecting on one's learning processes and progress are also very important metacognitive strategies. Recognising that performance does not always go as well as planned and that failure can be a signal to change strategies, is necessary for good information processing (Pressley *et al.*, 1989). Feedback that results from assessment of previously studied materials also adds to strategy efficacy: "Items associated with effective strategies are typically remembered much better than items associated with ineffective strategies, and students come to realise this" (Levin, 1988:197).

Affective strategies

Affective factors play a central role in mathematics learning. A major source of affect is from metacognitive experiences when solving problems or trying to comprehend new information. Since interruptions and blockages are an inevitable part of learning mathematics students will experience both positive and negative emotions. These metacognitive experiences are more noticeable when the tasks are novel (McLeod, 1991). Additionally, students will develop positive and negative attitudes towards different topics in mathematics, as they move through the secondary school. The purpose of affective strategies is to change or control the students' attitudes and orientation towards learning. They can be used to motivate, encourage and reward the learning, to reduce or counter anxiety, frustration and fatigue, to focus attention and maintain concentration, and to manage time effectively. For example, the exercise of 'self-talk', or the redirecting of negative thoughts about one's capability to perform a task with assurances that the task performance is within reach, will reduce anxiety about a task.

Resource management strategies

Resource management strategies are those which students use to promote learning indirectly, such as task management and controlling the learning environment (Pokay & Blumenfeld, 1990). Rohrkemper and Corno (1988) argue strongly that resource management strategies are particularly important elements of "adaptive learning". To perform tasks efficiently students need to see both the approach they take, and the task itself, as malleable. For example, to reduce excessive task demands a learner may simplify or streamline a task, seek assistance from a book, or remove distractions from the environment.

Social interactions with other people (e.g., cooperative learning, asking questions for clarification from the teacher or peer, or eliciting additional explanation) are important resource management strategies. Traditionally, help-seeking was "viewed as a manifestation of dependence, immaturity, and even incompetence" (Newman &

Schwager, 1992:123). More recently, help-seeking has been considered as a characteristic of self-regulated learning and high achievement (Zimmerman & Martinez-Pons, 1986). However, like many of the other strategies, not all help-seeking activities are desirable. Newman (1991) contrasts desirable adaptive help-seeking behaviours, strategic posing of direct questions for the purpose of acquiring information for learning or mastering a task, with dependency-based help-seeking behaviours.

Summary

It is evident that the learning process involves the coordination of strategies. As Slife, Weiss, & Bell (1985:438) noted, "metacognition requires something to plan, monitor and regulate, and cognition requires control processes to guide its functioning". The 'Good Strategy User' model, developed by Pressley and his colleagues (Pressley, 1986; Pressley *et al.*, 1987; 1989), states that the competent learner analyses task situations to determine the appropriate strategies. A plan is then formed for executing the strategies, and progress during strategy execution is monitored. In the face of difficulties, ineffective strategies are abandoned in favour of more appropriate ones. In contrast low achievers are likely to react affectively to problems, viewing them as confirmation of failure expectation, rather than as cues to appropriate strategic activity.

Additionally, it is important to remember that strategic behaviour involves an awareness of oneself as a learner - of one's patterns, needs, approaches, and goals - as well as some personal philosophy of what mathematics is, how it works, and how it is learned. These implicit beliefs influence both the variety of strategies a learner uses, and his or her ability to use them flexibly.

The following section reviews the research on students' use of strategies in the mathematics classroom, including intervention studies to assess the effectiveness of specific strategies and strategy instruction.

3.4 Learning Strategy Research in Mathematics Education

While there has been a large amount of learning strategy research in the domains of reading and language learning, research related specifically to strategies associated with the learning of mathematics is relatively scarce (Wong & Herrington, 1992). There is, however, a large amount of related research about problem solving in mathematics education. Problem-solving research studies provide valuable insights into students' metacognitive behaviours and beliefs, which are central factors in strategic learning (see *Interactive Model of Learning Mathematics*, section 2.3). Research studies will be reviewed in the following groups:

1. studies to define and classify strategies;
2. classroom research studies to determine strategy use and effectiveness;
3. studies to validate the influence of strategic processing on learning through either correlational or experimental work on the effectiveness of strategy training; and
4. studies related to problem solving.

Studies to define and classify strategies

Unlike the domains of reading and language, where research has provided comprehensive taxonomies of learning strategies, no widely accepted learning strategy classification has been located for mathematics learning. In the past, most of the instruments available for assessing learning strategies focused on study skills with little emphasis on 'active' learning processes (Weinstein, Zimmerman & Palmer, 1988). The *Learning and Study Strategy Inventory-High School* (LASSI-HS) developed by Weinstein and Palmer (1990a) is an example of a more recent questionnaire designed to find out how students learn and study. LASSI-HS places increased emphasis on the use of cognitive and metacognitive strategies for knowledge acquisition, but like most commercially available questionnaires, it is not specific to particular a domain of learning.

However, there are several recent studies in mathematics education designed to elicit students' self reports of learning strategies. Wong (1990) developed a questionnaire '*Study Behaviours in Mathematics*' which covers aspects of organisation, use of notes, problem solving, memorisation, reading and preparation for tests. The Likert-type questionnaire was administered to a large sample of both Australian and Singapore secondary students. Memorisation was reported as a very frequent study activity, but Australian students were more likely to use notes rather than rely on their memory for doing homework. Most students believed that they memorised through practice, rather than mental imagery, mnemonics, reciting the formula orally, or writing something down several times. Other commonly reported study behaviours included paying attention in class, attending to hints about tests, handing homework in on time and learning from mistakes. On the negative side, Wong (1990:570) reported:

> *45% (of students) did not read the relevant section of the text book after lessons and 60% did not redo the examples from class in their own way. 60% did not revise their work or read ahead before coming to class. Surprisingly, 87% did not borrow any mathematics books from the school library. Obviously, the students need to pay more attention to what they ought to do before and after each lesson.*

In Herrington's (1990) study involving primary-grade children, Grade 7 students reported that mathematics learning involved rehearsing rules by practice, asking others to quiz you, self-testing and using mnemonics. Grade 6 students perceived practice and copying from the blackboard as prime strategies for learning mathematics. Another recent Australian study by Southwell and Khamis (1991), using questionnaires, found that primary and secondary students reported that learning mathematics consists mostly of memorising facts and procedures.

These studies all reinforce the Second IEA Study of Mathematics (Robitaille & Garden, 1989) finding that memorising rules and formulae was considered a very important learning activity by about 85% of the students and 80% of the teachers. While it appears that rehearsal methods for learning mathematics are common to students across all age groups, it should be noted that the IEA reports by students and teachers did not endorse the idea that learning mathematics involves mostly memorising.

Classroom Research Studies

Early research examining students' use of learning strategies was conducted mainly in laboratory settings with learning outcome measures that were narrowly defined and related directly to the learning task. A shift to educational research studies investigating students' cognitive processes as mediators between teacher behaviour and student achievement in the early 1980s was reflected in the mathematics educational research of Peterson and colleagues (Peterson *et al.*, 1982, 1984).

Peterson *et al.*'s (1982) research used students' reports of cognitive processes from stimulated-recall of a mathematics lesson, and a questionnaire to investigate strategy use in the classroom. Interview responses were coded into five categories: attending; understanding; reasons for not understanding; cognitive strategies; and teaching processes. Students' reported cognitive processes were related to mathematics ability and achievement. When compared to lower ability students, higher ability students were more likely to report:

- attending to the lesson;
- understanding the lesson;
- either employing a variety of specific cognitive strategies or engaging in these processes more frequently;
- engaging in processes that involved problem-solving steps or showed insights into the material; and
- using the specific strategy of relating new information to prior knowledge.

These findings suggest cognitive processes that define ability and produce student achievement. In particular, students' ability to diagnose and monitor their own understanding was seen as an important predictor of mathematics achievement. Furthermore, analysis of on-task behaviour indicated that students' attention to the task might not be as important as the thought processes that students report engaging in while attending the lesson.

In a related classroom study, Peterson *et al.* (1984) again found that observations of student engagement in mathematics was unrelated to mathematics achievement. However student ability and achievement were significantly related to reports of active

cognitive engagement. Additionally, this study foreshadowed the interaction of students' affective thoughts and beliefs with strategic learning behaviours. Results suggested that the students' reported affect, as well as cognitions, mediated the relationship between instructional stimuli and student achievement and attitudes.

Thomas & Rohwer (1987) reported similar findings from large scale research studies, across domain and grade levels, of students' study activities.

> *Neither the total time spent doing routine studying nor the time reportedly spent preparing for tests was related to achievement at any grade level...the present results cast doubt on the currently popular proposition that academic achievement can be elevated simply and directly by increasing the time students are required to spend on homework. Instead it appears that achievement depends on the kinds of study activities students deploy during this time and the congruence between these activities and the instructional demands and supports of their courses.* (p. 384-5)

Although these studies showed a relationship between cognitive processes and achievement, Swing *et al.* (1988:124) reflected that "we consistently found that the elementary school students do not spontaneously use the sophisticated kinds of strategies that have been identified by instructional psychology as effective" - for example, defining and describing, comparing, thinking of reasons and summarisation.

A study by Zimmerman and Martinez-Pons (1986) adds to the discussion concerning the relationship between strategies and mathematics achievement. Zimmerman and Martinez-Pons administered a 'self-regulated' learning strategies interview to Grade 10 students of both high and low achievement. Interviews used open-ended questions about 'methods for preparing' that focused on: classroom situations; completing mathematics assignments outside class; preparing for and taking tests; and times when poorly motivated. They found that 93% of the students could be correctly classified into their appropriate achievement group through knowledge of their self-regulation practices. High achieving students reported using significantly more learning strategies in all contexts, and a heavier reliance on social sources of assistance than lower achievers.

An example of experimental based studies aimed at identifying strategy use and possible instructional effects is Van Haneghan and Baker's (1989) research on cognitive monitoring by mathematics students. They gave 3rd and 5th grade students word problems to check. Whenever students identified an error they were asked to explain what was wrong with the problem; this step was taken to determine whether they actually noticed the intended errors. To determine whether giving students specific information about the kinds of errors that they were likely to find would have an effect, one group was told that some of the answers to the problems were wrong and that some of the stories did not make sense. A second group was told specifically about the nature of the errors they were to find, and were given examples of each. It was found that the nature of the errors affected detection probability; calculational errors were most likely to be found and unanswerable problems were least likely.

These studies highlight individual differences in strategy use and assist in identifying which strategies are used more frequently by high achievers. The suggestion that 'what the student does when attending' may be a more significant indicator of achievement than 'attending itself' - leads naturally to intervention studies designed to promote students' strategic behaviour.

Intervention Studies

Some researchers (Hiebert & Carpenter, 1992; Leinhardt, 1988) argue that mathematical understanding involves making meaningful connections among the multiple types of knowledge, such as symbols, quantities, concrete representations, concept terms and procedures. Thus when information is interconnected and relations among the information are specified, memory of the information should be enhanced. To test the effectiveness of such elaborative procedures Swing and Peterson (1988) designed an intervention study that involved students completing mathematics seatwork that required them to engage in elaborative and integrative processing. The seatwork problems of the intervention group of Grade 5 students included 'pre-questions' that required them to analyse, compare and define problem information before answering a computational or conceptual exercise or story problem. These questions were designed to facilitate interconnection of the measurement knowledge being learned. Results from completed achievement and retention tests of memory and understanding provided evidence for the usefulness of elaborative and integrative processing.

The importance of elaborative processing is noted by other researchers. Anthony (1994) found that when adult mathematics students learned from textual material the more successful students elaborated steps in the worked examples. In order to optimise learning from worked examples the student must actively construct an interpretation of each action in the example, in the context of the principles introduced in the text. Chi and Bassok (1989) refer to these elaborations as self-explanations and hypothesised that differences in problem-solving success, from students with similar declarative knowledge, may result from differences in how students studied examples.

In a more extensive intervention study, Swing *et al.* (1988) instructed elementary school teachers on how to teach students to use thinking skills in their mathematics learning. Their aim was to improve students' mathematical understanding and problem solving by teaching students to use strategies of defining and describing (operationalised as analysis, conceptual and pictorial representation, and generation of alternative representation), comparing, thinking of reasons (justifying an answer or procedure), and summarising. With the exception of summarising, the included skills had been associated with improved performance in earlier studies (Swing & Peterson, 1988). As hypothesised, student performance on achievement tests and reported use of strategic processes increased, although differential effects were noted for individual students. The effects of mathematical ability on students' achievement was decreased when both lower and higher ability students gained increased proficiency in using instructed strategies. By including a parallel 'time-on-task' intervention study, Swing *et al.* (1988) concluded that it is unrealistic to expect students of lower ability to make progress by simply increasing time-on-task, without providing cognitive strategy instruction to remediate for low ability.

Lester (1988) reports an intervention study which incorporates aspects of Palincsar & Brown's (1984) reciprocal teaching method. The teaching approach had three components: (a) teacher as an external monitor; (b) teacher as a facilitator of students' development of metacognitive awareness; and (c) teacher as a model of a metacognitively-aware problem solver. An interesting method of encouraging students to reflect on their own thought processes and analyse their own performance was to

have students view a videotape of someone else solving a problem and to discuss with them the good and not so good behaviours they see. Lester concludes that this teaching method provided growth in students' problem-solving abilities with respect to comprehension, planning and execution strategies.

In a more recent intervention study Herrington (1992) designed an instructional program, involving concept mapping, Think Board, self-questioning and writing; to teach elaboration, organisational and metacognitive strategies. The program's concurrent focus on developing appropriate beliefs about learning mathematics reflected the interactive nature of strategy deployment and metacognitive knowledge. Herrington reported a positive, if only slight, improvement in students' attitudes and achievements.

There have also been several recent classroom intervention studies by teachers. Gray (1991) observed changes in her students as a result of increased emphasis on metacognitive instructional and learning activities. Learning activities included paired problem solving, writing out descriptive explanations of solution steps, class discussions of problem-solving attempts, writing assignments, and self marking and error diagnosis. Qualitative changes in student behaviours included increased motivation, willingness to attempt more challenging problems, increased awareness and flexible use of strategies, improved communication of thinking and strategies, and increased planning.

Cardelle-Elawar (1992) also used a metacognitive instructional intervention with low-ability sixth-grade students. Treatment combined components of metacognitive instruction, problem-solving instruction, and diagnostic feedback of individual student's performance. As a result of the instructional intervention low mathematics ability students progressed as problem solvers.

Toumasis (1993) describes a 3 year teaching experiment at senior high school level in which the focus is on developing students' ability to take more responsibility for learning from resources. Prepared reading organiser worksheets, group work, critical reading tuition, and vocabulary instruction resulted in significant improvements in students' attitudes towards mathematics learning.

Many other studies (Artzt & Armour-Thomas, 1992; King, 1991; Schoenfeld, 1985) have found that encouragement of metacognitive activities such as planning, metacognitive questioning and monitoring were associated with the increased use of problem-solving heuristics and improved levels of performance. All of these teacher-researchers noted qualitative improvements in student learning behaviours and attitudes when instruction focused on strategic processing and metacognition. In agreement with Swing *et al.* (1988) their studies suggest that teachers need to focus, not on labelling students (e.g., low performers), but on student learning behaviours and on the encouragement of active student involvement in the learning process.

Research on Strategic Behaviours during Problem Solving

Problem solving often involves constructing an appropriate sequence of cognitive activities. Thus, process-oriented aspects of metacognition, such as strategy selection, monitoring, and evaluation of progress, planning ahead, efficiently apportioning cognitive resources and time, are seen as major determinants of success (Artzt & Armour-Thomas, 1992; Briars, 1983; Schoenfeld, 1987). Examples of students monitoring and directing their own learning are the asking of questions such as, "*Is it getting me anywhere*", "*What else could I be doing instead*?", or drawing a sketch to test one's understanding of a problem statement. In recent years, problem solving has received considerable research attention, both in laboratory situations (Garofalo & Lester, 1985; Swanson, 1990) and classrooms (Schoenfeld, 1985; Siemon, 1992b).

Like the classroom research studies, research findings related to problem solving revealed limited use of elaborative and metacognitive strategies that have been identified as most instrumental in producing meaningful learning (Swing *et al.*, 1988). For example, research by Resnick (1988) involving collaborative problem solving by elementary school children, found that only a small number of the utterances made while working on a problem were coded as arguments or elaborations.

Strategy deficits have also been noted in the problem-solving efforts of older students. Schoenfeld (1987), in comparing problem-solving performance on non-routine problems, of college mathematics students and mathematicians, found that college students had very poor managerial strategies. Students rarely, if ever, assessed the potential utility of a plan of action, they gave inadequate consideration to the utility of potential alternative methods, and in general did not monitor or assess their progress, so had little way to end an unproductive line of reasoning. In contrast, the mathematicians initially concentrated on analysing the problem, devoted attention to planning and monitoring efforts throughout the process, and were successful at reaching a correct solution. Since, in Schoenfeld's study, the students had recently studied the requisite information, while the mathematicians had not, he concluded that the planning and self-monitoring behaviours employed by the mathematicians contributed to successful problem solving.

More recently, Evans (1991a) conducted think-aloud interviews with secondary students. After students worked an exercise they reflected on what they had done by saying what the question meant, and how they had gone about working it out. Like Schoenfeld (1987), Evans found that monitoring strategies of checking interpretation, checking the appropriateness of procedures, and checking the consistency or plausibility of results were often omitted. Students frequently claimed that, in tackling an exercise, they were mainly concerned to follow the procedure they had been taught. Tasks similar to those set in school appeared to be accomplished by a majority of the students in an 'associative' rather than 'constructive' way. Novel tasks requiring minimal reconstruction of known procedures proved very difficult.

As well as strategy deficits further research studies indicated qualitative differences in strategy application between high and low achieving students. Anthony (1991) noted differences in tertiary mathematics students' monitoring behaviours when reviewing worked examples. The realisation of comprehension failure triggered episodes of self-explanations from high achieving students. In contrast, low achieving students, either chose to ignore information that was difficult to understand by treating it as irrelevant to the problem solution, or reread the example, or attended only those examples and problems that they 'found easy'.

Lawson and Chinnappan (1994) compared high and low achieving secondary students during solution of geometry problems using a think-aloud procedure. Analysis indicated that high-achieving students not only accessed a greater body of geometric knowledge but also used that knowledge more effectively. In particular, on the more difficult problems, high-achieving students "showed a greater tendency to notice errors and to engage in some form of management following identification of an error. This increased the likelihood that errors would be corrected more frequently by those students" (p. 86).

Swanson (1990) asked 4th and 5th grade students to solve pendulum and combinatorial problems after first completing a self-report interview designed to measure metacognitive knowledge. Swanson demonstrated that high metacognitive students outperformed lower metacognitive students in problem solving regardless of their overall aptitude level. He concluded that high metacognitive skills can compensate for overall ability by providing a certain knowledge about cognition.

A study (Siemon, 1992a) of primary school children's problem-solving performance clearly illustrates the relationship between cognitive goals and monitoring. Siemon's interview data suggests that differences in approach to problem solving are related to differences in the way and extent to which cognitive goals and cognitive actions are generated, retrieved and monitored. Individual children appear to attend to, and value, qualitatively different aspects of the problem-solving experience, in relation to their knowledge and beliefs about learning mathematics. Some children were more likely to monitor the implementation and accuracy of their cognitive actions, but not the relevance of these actions to the problem condition. They seemed concerned with procedures and algorithms (procedural knowledge of a cognitive and metacognitive kind). Their cognitive goals appeared to be concerned with producing an answer, in what was perceived to be a socially acceptable and locally valued way, rather than whether or not the actions made any sense in terms of the original problem. Other children were more likely to monitor their cognitive goals (what they decided was needed in relation to the problem conditions) and their cognitive and metacognitive knowledge. They were more concerned with understanding and representing the problem meaning (conceptual knowledge of a cognitive and metacognitive kind).

Siemon's (1992a, 1992b) research into children's problem solving in mathematics concluded that there was a complex interaction between the knowledge and control aspects of metacognition, and between metacognition and cognition. The metacognitive knowledge that the child brought to the problem solving situation, appeared to be extremely robust and resistant to change. For example, the belief that school mathematics was about "doing sums to get answers", usually in the shortest possible time with a minimal amount of thought, seemed to play a much more important role in determining a child's problem-solving performance than the managerial, control decisions that many researchers have concentrated on. Siemon (1992b:2) noted that while such executive actions are both necessary and important, "it is the solver's store of *metacognitive knowledge* which primarily determines the formation, access and operation of these actions". These findings reinforce Schoenfeld's (1985) earlier findings related to teaching problem-solving heuristics to college students in which the student's belief system, including beliefs about oneself, about the world and about mathematics was found to be a critical factor for problem solving success.

The reviewed studies in this chapter all suggest that the learners' actions appear to be largely determined by their beliefs about the nature and purpose of school mathematics. The central role accredited to metacognitive knowledge and its impact upon one's cognitive goals and subsequent cognitive actions adds support to the view of metacognition proposed by Flavell (1981) and the proposed *Interactive Model of Learning Mathematics* (section 2.3).

Additionally the research has clearly demonstrated differential strategic behaviours between low and high achieving students. Analysis of on-task behaviour indicated that students' attention to the task might not be as important as the thought processes that students report engaging in while attending the lesson. Metacognitive behaviour, including problem diagnosis and comprehension monitoring, was seen as an important predictor of mathematics achievement. However, in a recent review of research related to problem solving and metacognition in mathematics education, Schoenfeld (1992) stated that we need more knowledge of control mechanisms and their relationship to the domain and more knowledge of the interaction of the cognitive and affective factors in mathematics learning.

3.5 Factors Affecting Strategic Learning

Strategy use is embedded in the context in which it is used. When the context varies, the nature of strategic activity often varies as well (Ames & Archer, 1988; Biggs, 1991; Dweck, 1986; Garner, 1990a; McLeod, 1991; Ramsden, 1988; Zimmerman & Martinez-Pons, 1988).

> *One need not look outside the school or classroom for evidence that social and cultural conditions play an important role in what is learned. It is clear that the sorts of interactions students have among themselves and with their teachers, as well as the beliefs, values, and expectations that are nurtured in school contexts, shape not only what mathematics is learned, but also how it is learned.* (Lester, Garofalo, & Kroll, 1989:78)

A feature of the *Interactive Model of Learning Mathematics* (section 2.3) is the numerous factors which interact with strategy development, strategy deployment, and strategy effectiveness. Chapter 2 discussed the interactive role of prior domain and strategic knowledge in the learning process. This section examines factors relating to the classroom context and affective issues, such as metacognitive knowledge, beliefs, and motivation.

FACTORS RELATING TO THE CLASSROOM CONTEXT

Social Factors

The social setting of the classroom provides occasions for modelling effective thinking and learning strategies. Verbalisation and modelling of appropriate strategies by both the teacher and other students seems to be helpful to students' efficacy and development of higher-order thinking and learning (Gavelek & Raphael, 1985; Resnick, 1987).

Constructivists view mathematical learning as an interactive as well as constructive activity (Cobb, Wood & Yackel, 1990). Social interactions are essential to the ongoing process of negotiation of mathematical understandings (Voigt, 1994). In communicating with others about the problems that they engage in, students develop the power to reflect on, evaluate, and clarify their own thinking.

In addition to providing support for learning ('scaffolding'), social interaction may also generate cognitive conflict which, in turn, can promote learning. By expressing ideas publicly, by defending them, and by questioning the ideas of others, students are forced to deal with incongruities and are encouraged to elaborate, clarify, and reorganise their own thinking. Hiebert (1992) and Gabrys, Weiner and Lesgold (1993) all suggest that peers may be especially effective in social interaction roles because the differences in thinking, and the ideas expressed, are likely to be within a range that will generate real conflict. Peer advice is most likely to have shared meanings or lead to negotiated meanings.

In a classroom study of younger children, Alton-Lee (1984) found that behaviours which involved opportunity to attend to peer activity, verbal and non verbal, appeared to be highly related to student learning. In fact, direct individual contact with peers appeared to be more consistently related to student learning than individual contact with the teacher. However, while social interactions are generally thought to encourage strategic behaviour, research by Fennema and Peterson (1985) found that girls' engagement in social activities, one-to-one interaction with the teacher, and receiving help from the teacher, were negatively related to high-level achievement. Their study showed that 4th grade girls depended more on the teacher and peers for assistance on high-level mathematics problems, and that such assistance tended to be provided at this grade in lieu of a press for autonomous work. Thus, Fennema and Peterson suggest that these social classroom processes reflect a dependence on others and may in fact be counterproductive for developing autonomous learning behaviours.

Instructional Factors

Classroom instruction will affect the acquisition and maintenance of task-related strategies. Teachers' statements about the purpose of learning influence the student's goal and learning behaviours. However, according to Campione, Brown, & Connell (1989) much of the instruction the students receive in school is "blind instruction" in which students are rarely told about why they practice the activities they do. As a consequence, there is a resulting lack of transfer of strategies to related problems. If flexible use of instructed strategies is the goal, students need to be informed of the purpose of the skills they are taught, and given instruction in the monitoring and regulation of those resources.

Because teachers have the final say on students' academic success, students seek information and form opinions about 'what the teacher wants' and tailor their strategies to suit. Teachers who state clearly and early what they expect of students in their course, not only provide their students with information about what, how, and how much to study, but they "tend to reduce the amount of floundering and defensiveness that can occur when students do not know what to expect" (Thomas, 1988:269). However, there needs to be a balance, as many tasks are so directive as to provide limited opportunities for students to alter their approaches to the task itself (Rohrkemper & Corno, 1988). When students are given responsibility for the management of their own learning they are more likely to use strategic behaviours.

Although mathematics curricula emphasise the importance of higher-order cognitive processes, mathematics teachers often neglect interpretive analysis and strategic decisions when presenting lessons or structuring tasks. For example, a model answer may suppress all evidence of mathematical thinking and present only the abstract application of algebraic tools (Burton, 1984). Instruction that emphasises the acquisition of essential mathematical facts and algorithmic skills in isolation from problem-solving situations is detrimental to the development of appropriate of high-level metacognitive behaviours. Campione *et al.* (1989) note that this practice is more common with lower ability students who in fact need more explicit instruction of high-level skills.

Teachers' desire to enhance and recognise students' success may be expressed in instructional strategies that inhibit rather than enable the development of adaptive strategic learning behaviours. Cullen (1985) argues that many everyday teaching practices provide a learning situation that encourage passivity or non strategic responses, inhibiting the use of self-directed strategies to cope with failure experiences. Of particular relevance to the mathematics classes are such counterproductive practices as:

- **Erase example**: Children are permitted to erase incorrect work and recommence without checking or identifying the error.
- **Teacher response failure**: the teacher fails to follow up incorrect written or verbal responses.
- **Presentation of work**: there is an over-emphasis on appearance of the work, to the detriment of content mastery.
- **Attitude to errors**: the practice of marking examples at the end of the mathematics lesson, when little time is available for practicing correction skills.

Demands of the Task

Directly related to instruction are the task demands. According to Thomas and Rohwer (1993:12) demands should "prompt students to engage in demand-responsive autonomous learning activities". However, Burton (1992:348) suggests that present classrooms are particularly prone to rushing through content without allowing or expecting students to actively question their understanding and reflect on their learning: "expectations of finding answers in the absence of personal questions has, for a long time, been integral to many mathematics classrooms operating on a model of 'delivery'." In classrooms where the focus is on computational procedures and accuracy, and students in general know in advance which computational procedures are needed to solve problems, the criterial tasks are unlikely to be sufficiently challenging to prompt students to engage in strategic learning.

Doyle (1988:177) contends that "by providing a large amount of prompting to keep production rates high, [mathematics] teachers limit students' opportunities to develop autonomous learning capabilities and reinforce their dependency on the teacher for task accomplishment". These 'compensatory' teacher behaviours decrease the cognitive

demands of a task (Rohwer & Thomas, 1989). For example, teachers may compensate for problem-solving demands by providing a 'rule of thumb' for dealing with a problem - such as a keyword strategy. The student may complete the problem, but without understanding what the problem describes, without modelling the problem mathematically, and without acquiring the intended procedural knowledge.

Additionally, Bereiter (1992) claims that the demands of problem-solving tasks in many mathematics classes limits the development of problem-centred knowledge and the consequent need for active or intentional leaming. Bereiter reasons that the development of problem-centred knowledge depends on problems that persist so that they become organising points for knowledge .

> *Problem solving as it typically appears in mathematics and science curricula is the antithesis of the kind of activity that could be expected to lead to problem-centred knowledge of high-level concepts. It consists of strings of problems that are forgotten as soon as the assignment is completed. The recurrent elements, which therefore become the organizing points for knowledge, are of a low-level procedural kind often quite unrelated to the mathematical or scientific concepts supposedly being taught.* (Bereiter, 1992:346-7)

Use of Resources

Another important requirement for strategic learning behaviour is the use of resources such as summaries and textbooks. A study by Shapiro (cited in Corno, 1989) examined the use of mathematics textbooks as an aid to enhancing strategic learning. Using a specially prepared algebra text, Shapiro produced supportive results to demonstrate that strategy use in mathematics can be learned through textbook instruction alone. However, Confrey (1992) suggests that American mathematics textbook publishers typically present the material with as low a reading level as can be tolerated, providing step-by-step examples as models that allow students to complete the exercises by imitation, and avoid the need for verbal discussion. In general, many text-presented problems can be solved without thinking about the underlying mathematics, and by

blindly applying the procedures that have been studied in the current lesson. Thus, "workbook mathematics gives students little reason to connect ideas of "today's" lesson with those of past lessons or with the real world" (Romberg, 1992:48).

Nature of the Learning Goal

Goals are cognitive representations of the different purposes students may adopt in different achievement situations. Pintrich *et al.* (1993) state that from the variety of conceptualisations of academic achievement goals there are two distinct orientations - intrinsic, mastery and task orientation; and an extrinsic, performance and ego-involved orientation. Students who adopt a mastery orientation focus on the process of learning, understanding, and mastering the task; while those who adopt a performance orientation focus on obtaining a good grade and outperforming others.

There have been a number of studies which show that students exhibiting differing goal orientations exhibit different patterns of cognitive engagements (Ames & Archer, 1988; Pintrich & De Groot, 1990; Pintrich & Schrauben, 1992). For example, Pintrich and De Groot (1990) showed that college students, who adopted a mastery goal focussed on understanding, were more likely to report using deep processing strategies (Biggs, 1991) such as elaboration, in addition to metacognitive and self-regulatory strategies. Nolen (1988), in a laboratory study of text comprehension, found that junior high-school students who adopted a mastery orientation were more likely to use both deep and surface processing strategies, while those with a performance orientation were more likely to use surface processing strategies.

Volet and Lawrence's (1989) research illustrates how mathematics learning can be influenced by individual goals. They interviewed undergraduate statistics students to determine relationships between their goals and their academic achievement, background knowledge, and age. Mature aged students reported using more adaptive and independent learning strategies than did recent school leavers. These strategies were found to be more closely related to the students' goal and their age than to their entering knowledge. Similar findings with adult distance education mathematics students were found by Anthony (1991).

Bereiter and Scardamalia (1989) describe the goal component of learning as "intentional learning". For example, when children in mathematics manipulate blocks to solve an arithmetic problem, some children appear to put effort into, not only solving the problem, but also to understanding the underlying mathematical concept. The learning that results is not an incidental consequence of solving mathematics problems, but rather a goal to which the children's problem solving efforts were directed. This intentional learning is influenced by the:

- availability and use of appropriate learning strategies;
- the student's conceptions or theories of learning and knowledge; and
- the instructional situation as it relates to students efforts to learn.

How do classroom contexts affect the nature of the learning goals? In the classroom students' learning is guided by an understanding of what they ought to be able to do when their studying is completed. Proficient learners are conscious of, and will seek out, cues concerning what is most important in a course. They use information about the criterion to select processing strategies and review strategies that are most appropriate for this criterion performance (Thomas, 1988). However, many students in mathematics classes misunderstand the goal of early mathematics education; they come to believe that mathematics consists only of running off well practiced routines that have been supplied by the teacher. Neyland (1994a:3) provides the following scenario of a junior mathematics classroom learning environment:

> *Debbie likes addition; she knows lots of different ways to combine numbers and starts to explore some new ideas. But the teacher says, no Debbie, I want you to do it this way. So Debbie learns to add the teacher's way. The next day the new teacher comes again and Debbie waits, this time, to be told how she is to do things. Debbie is learning to be passive, to accept that the teacher's way is better than her own, and that rule following is more important than inventing.*

Furthermore, research studies report that for many mathematics students the goal of problem-solving exercises is to complete the problem; that is, to get the same answer as the back of the book. For example, Peterson (1988:7) reported that elementary students

"tend not to focus on the meaning of the content to be learned. Rather, they report that their goal is to get the task finished or completed". Ames (1992) contends that the mathematics student's focus is highly product orientated and that the high visibility of these products is likely to orientate the student away from the task of learning. Ames suggests that performance orientation may be the more adaptive approach for a student to adopt, given the common organisation of mathematics instruction and students' perception that learning mathematics is a reproduction of seemingly unrelated facts and rules.

A concern of the present study is that to aid task completion, low-achieving students may develop and use strategies such as frequent help-seeking, copying from peers, checking answers with book rather than self-checking, and copying procedures from worked examples. Learning may occur, but it would be an incidental by-product of task completion rather than an intentional goal. The variance in 'intentional learning' by individual students may go some way to explain some students' failure to succeed in mathematics learning, despite the investment of long hours of studying.

The single most significant influence on students' perception of learning is their perception of assessment (Ramsden, 1988). Ames (1992) argues that a product orientation may shift to a performance orientation when correctness, absence of errors and normative success are emphasised through assessment. Performance orientated or competitively orientated environments encourage an ability focus that does not support the use of strategies that require sustained effort over time.

In a discussion of assessment practices in New Zealand mathematics education, Ritchie and Carr (1992:197) argue that instead of encouraging mastery orientation, current mastery assessment practices "may make pupils over-concerned with grades and marks, reduce their level of risk taking, and be ineffective at developing pupils' own knowledge of their understanding (metacognition)". In accord with Pressley *et al.*'s (1987) 'Good Strategy User' framework, Prawat (1989) notes that while both performance and mastery dispositions have their place; it is important for students to be able to access either one when appropriate.

MOTIVATIONAL FACTORS

As strategic behaviour involves the deliberate application of skills and knowledge in a goal oriented context, "motivation is thus a necessary component of strategic behaviour and a precursor of strategy use" (McCombs, 1988:153). Strategies are differentially effective according to the students' motivational pattern. How a student is motivated will determine: what strategies the student selects and how effectively the student utilises these strategies.

Biggs (1991) found that more effective learners are better able to align their strategic thinking with their motivational orientation; the strategies they select are more consistent with what it is they are trying to accomplish. The surface learner who is extrinsically motivated, uses predominantly rote learning strategies, perceives learning mathematics as a rule-based discipline, and lacks metacognitive skills and knowledge. Anthony (1991) noted that those distance education mathematics students who used surface-learning processes, perceived homework assignments in terms of content coverage rather than content mastery. They used strategies that contributed to task completion, but which did not necessarily contribute to the mastery of the content, or to the acquisition or maintenance of adaptive 'learning to learn' skills. In contrast, deep-learning approach students were intrinsically motivated, used assignments to extend their learning of the topic, had well organised study methods, and used metacognitive knowledge and strategies to monitor their learning.

Metacognition influences students' orientations to learning tasks and their beliefs in their personal abilities. "Students' motivational investment flows from these personal beliefs about learning. That is precisely why metacognition is essential for the development of self regulated learning" (Paris & Winograd, 1990:26). Good strategy users are motivated to be strategic, believing performance can be enhanced by procedures well matched to learning challenges. They do not believe achievement is due to effort alone or to factors outside their control, such as luck, innate ability or task difficulty (Pressley *et al.*, 1989).

Metacognitive Knowledge

Metacognitive knowledge: an awareness of the nature and process of learning, personal learning styles and deficiencies, is critical for the students' development of self-control. Students make judgements about (a) the personal significance of the goals within a task, (b) the perceived utility, value, and efficiency of alternative actions and (c) the self-management of effort, time and knowledge. These decisions, made during learning tasks, are based on learners' values and beliefs and can promote or deter continued motivation and learning (Paris *et al.*, 1983). Proficient learners will have a repertoire of strategies for maintaining concentration, getting themselves started on learning tasks, and a good sense of the time needed for completing learning tasks.

However, Blumenfeld and Meece's (1988) research findings suggest that metacognitive knowledge leads to spontaneous strategy use only when combined with an interest in understanding (i.e., intentional learning). Hidi's (1990) summary of the research on interest concludes that both personal interest and situational interest have a profound effect on cognitive functioning and the facilitation of learning. Hidi notes that interest may not necessarily result in more time spent processing information, rather the differences lies in the quality of the processing, not the quantity of the processing, or time spent on task. Similarly, Schiefele (1991) has shown interest to be positively related to college students' self-reported use of elaborating strategies, the seeking of information, and their engagement in reflective thinking, and negatively related to the use of rehearsal strategies.

While some researchers focus on 'interest', most mathematics researchers have focused on beliefs. Beliefs are vital because they "energise strategic behaviour" (McCombs, 1988). Beliefs about mathematics and mathematical problem solving, including the nature of mathematics, its difficulties, and its usefulness, can influence how one organises content knowledge in memory, and what one determines is important. Lampert (1990: 32) provides the following summary of mathematics students' beliefs:

> *Commonly, mathematics is associated with certainty: knowing it, with being able to get the right answers, quickly...These cultural assumptions are shaped by*

school experience, in which ***doing*** *mathematics means following the rules laid down by the teacher;* ***knowing*** *mathematics means remembering and applying the correct rule when the teacher asks a question; and mathematical* ***truth*** *is* ***determined*** *when the answer is ratified by the teacher.*

These beliefs clearly will shape mathematical learning behaviours "in ways that have extraordinarily powerful (and often negative) consequences" (Schoenfeld, 1992: 359).

A second category of beliefs deals with students' beliefs about themselves and their relationship to mathematics (metacognitive person knowledge). This category has a strong affective component, and includes beliefs related to self-confidence, self-concept and causal attribution of success and failure. Beliefs about competency for particular tasks (e.g., I am good at maths), or about ability in general (e.g., I am a capable learner), will affect the learner's motivation to perform strategically and to acquire new procedures. For instance, a student may come to believe that effort is a determinant of success and will continue to apply effort in a learning situation. A strategic learner may believe that success on task X depends on the use of a strategy appropriate to X, and attribute failure to inappropriate strategy selection.

Pintrich and De Groot (1990) found that junior high school students' use of cognitive and metacognitive strategies was positively correlated with self-efficacy judgements. More specifically, research on self-concept in learning mathematics (McLeod, 1991) indicates that there are substantial differences between males and females on this dimension. In general males tend to be more confident than females - even when females may have had better reason, based on their performances, to feel confident. Students having a low sense of self-efficacy may avoid studying. They may put in less than the amount of study time actually needed, or they may decline to invest the quality of mental effort required to construct, select, and employ effective strategies.

Attributions of success or failure appear to have a significant influence on metacognitive processes. Negative views of themselves as learners can inhibit the development of strategic thinking in some students (Rohrkemper & Corno, 1988). Moreover, when

students meet some cognitive failure, those who attribute failure to strategic effort are more likely to ask "*What must I do differently to succeed?*", thereby inducing strategic behaviour. Those who attribute failure to lack of ability are more likely to 'give up'. Fennema's (1989) research on gender-related attributions shows that males are more likely, than females, to attribute their success in mathematics to ability. Females are more likely, than males, to attribute their failure to lack of ability. Additionally, females tend to attribute their success to extra effort, more than males do, and males tend to attribute their failures to lack of effort, more than females do. The resulting male attributions are hypothesised to have a more positive influence on achievement.

Cullen (1985) notes that there are problems with attributing performance success to effort alone. There is a danger that continued exhortation to try harder may serve to increase helplessness, rather than the desired strategic behaviour, if the student does not possess the personal resources for coping with the task. These concerns are also reflected in Swing *et al.*'s (1988) findings (section 3.4). Effort attribution may need to be associated with either past successes or with specific strategies for coping with the task (Ames & Archer, 1988; McCombs, 1988).

As well as attributions for success, students in mathematics classes need to cope with 'getting problems wrong' and failure to achieve in what is traditionally a competitive climate. Negative affect associated with failure may impede both students' metacognitive development and their efficient use of available metacognitive strategies. Cullen (1985) suggests that anxiety may interfere with the effective use of existing metacognitive strategies, in particular cognitive monitoring. High levels of anxiety could be manifest in under-achievement, thereby reducing the possibility of failure by lowering aspirations, or by adopting an overcautious style of learning. Students, who are labelled as indifferent to learning, lazy or unmotivated may well be engaging in self-protective behaviours that emerge because of their feeling of failure in competitive settings (Dweck, 1986). Cullen's research with primary school children's ability to cope with failure concluded that effective achievement behaviours are facilitated by the availability of a range of both metacognitive and affective strategies for coping with ambiguity and error.

It is clear that all the discussed factors interact in numerous ways: for example, contextual factors directly influence the formation of beliefs, as well as the extent to which a student is willing or able to engage in regulatory learning behaviours. It is apparent that the outcome of learning depends on the kind of learning in which students engage. In turn, the kind of learning students engage in depends on features of the contexts in which their learning activity takes place. Many educational researchers (Collins *et al.*, 1989; Mitchell, 1992b; Thomas & Rohwer, 1993) argue that contexts must provide and require autonomous learning behaviours, and must value strategic learning, if students are to engage in forms of learning that result in productive knowledge construction.

3.6 Summary

It is clear that the effective use of learning strategies enhances learning outcomes and performance. For learning to be effective students' learning behaviours should include:

- strategies for selectively attending to the most informative aspects of instructional stimulus;
- strategies for effective encoding of new material so that it can be easily retrieved;
- knowledge of the conditions under which a given strategy is effective; and
- monitoring of the effectiveness of one's strategies.

Strategy research in mathematics education has found that students report engaging in a wide range of cognitive processes and strategies during mathematics instruction. These strategies may be simple or complex, appropriate or inappropriate, intelligent or unwise. In a review of learning strategy research in mathematics education Peterson (1988:14) notes that:

(T)hose processes and strategies that students report most often were not those that are frequently proposed and researched by educational psychologists as facilitative of learning and achievement. For example, students seldom reported spontaneously using sophisticated kinds of learning strategies such as memory strategies, strategies for relating new information to prior knowledge, for discriminating, and for comparing information. To the extent that students' reports maybe viewed as evidence of what elementary students do naturally to help themselves learn during mathematics, students' reports may be viewed as indicative of their knowledge about the learning process...If, however, students' reports are viewed as indicating how little they know about cognitive processes and strategies that facilitate learning, then such lack of knowledge of cognitive strategies and metacognitive processes may, in fact, be limiting their potential for mathematics learning, and particularly for higher-order mathematics learning.

Students' use of learning strategies will vary according to preference, perception of mathematics learning, teacher demands, nature and difficulty of the task, prior knowledge and experience, stages of learning, perceived purpose, and degree of self-investment. Recent research studies have stressed the importance of students' idiosyncratic metacognitive knowledge as the basis for selecting and activating learning strategies. There is ample evidence (Campione *et al.*, 1988; Collins *et al.*, 1989; Schoenfeld, 1985, 1987) that students who know more than enough subject matter often fail to solve problems because they do not use their knowledge appropriately.

Additionally, researchers argue that although students of all abilities use learning strategies, many students are low achievers simply because they rely on infrequent or inappropriate use of a narrow, limited repertoire of strategies such as keyword, rote memorisation, and matching problem procedures with worked examples. Rohwer & Thomas (1989:115) in reviewing a range of domains, including mathematics, note that "few demands are made for the kinds of knowledge structures and procedures that research has shown to be characteristic of expert problem solving". Strategies often promoted by certain instructional approaches, to surmount short-term obstacles such as a test, may not lead to more long-term knowledge construction.

Chapter 4

The Present Position

Before we decide that students do not have the interest or intellectual ability to learn something, we need to be sure that students know ***how*** *to learn what it is we are trying to teach them.*

(Gage & Berliner, 1992: 301)

4.1 Introduction

There have been significant influences since the 1980s that have caused mathematics educators to be greatly concerned with learning strategies and related research findings. International studies in the 1980s highlighted students' lack of mathematical performance in basic skills and problem solving. Additionally, research findings (section 3.4) indicated students' lack of appropriate strategic learning behaviours, and the different use of learning strategies by high achievers when compared with low achievers. Of particular concern is students' limited use of metacognitive strategies to direct and control their learning and the implication of inhibiting instructional factors. Together, these factors prompted mathematics educators and researchers to look for instructional remedies and instigate curriculum reforms.

To counteract student and instructional deficiencies in strategic learning, both cognitive theory and empirical research studies have provided evidence for the possible benefits of specific classroom-based instruction in learning strategies (Herrington, 1992; Schoenfeld, 1985; Swing *et al.*, 1988). However, it is argued here that there is still much to learn from further research involving mathematics students' present use and development of existing learning strategies in the classroom environment. In the light of

recent mathematics curriculum developments, both in New Zealand and overseas, which promote a constructivist learning environment, there is a need to focus on students' use and development of strategies that will enable them to actively construct their own knowledge and self-regulate their learning.

4.2 The Present Focus of Learning Strategies in Mathematics Education

Findings from learning strategy research studies have prompted a number of mathematics educators to promote the teaching of specific strategic behaviours (Kilpatrick, 1985; Herrington, Wong & Kershaw, 1992). Peterson (1988) suggests that strategy instruction is indeed a challenge for the 1990s. She argues for that there is a need for an increased instructional focus on teaching higher-level skills in mathematics to all students. "Such an increased focus might be particularly important for lower-achieving students, who have more difficulty than their peers in learning these higher-order skills on their own" (p.2). In addition to traditional concerns with content, mathematics instruction should place a greater focus on the teaching of strategic methods that contribute to capable thinking, with students taking a more active role in managing their learning (Cobb, 1988; Pressley, 1986; Pressley *et al.*, 1989; Schoenfeld, 1988).

From research studies Peterson (1988) identifies specific strategies related to mathematics learning as follows: checking answers, reworking problems, rereading directions or problems, relating new information to prior information, asking for help, using aids, using memory strategies, decision-making, trying to understand the lesson, and doing a mathematics problem using a specific operation.

Other educators focus on a smaller group of strategies which are promoted as necessary for constructivist learning, autonomous learning, or development of problem-solving skills. An overriding metacognitive behaviour that is repeatedly emphasised in mathematics education is reflection. "In mathematics the *reflective process*, wherein a construct becomes the *object* of scrutiny itself, is essential" (Confrey, 1990:109). In a discussion on the constructivist learning paradigm, Ritchie & Carr (1992:198) claim that:

> *When a learner, actively involved in making sense of new information, is encouraged to reflect upon what has been learned, then in this process critical modes of thinking are brought into play...*

Hiebert (1992:442) provides three reasons to support arguments that encourage reflection for learning mathematics. Firstly, "reflection yields products of value. Reflecting on mathematical experiences, activities, and procedures transforms them into mathematical objects ... that can be manipulated on mentally". A second reason is that reflection "provides a way of gaining control over one's thoughts". Planning and monitoring are critical control processes that involve reflection. The third reason is that "reflection engenders an appropriate frame of mind": an orientation to the subject that is essential for thinking mathematically and doing mathematics.

> *Rather than perceiving mathematics as a practice of recalling and executing memorised rules, mathematics becomes a subject in which the most natural experience is to mull a problem over in one's mind, to think about it from different perspectives, to search for relationships to other things. Reflection influences what one thinks mathematics is.* (Hiebert, 1992:443)

Hiebert's second reason explicitly acknowledges the metacognitive aspect of reflection. Kilpatrick (1985) also claims that reflection is a metacognitive process that induces an awareness of one's cognitive processes which ultimately leads to their regulation. The self-regulatory aspect of reflection is further expanded upon by McLeod (1989) who suggests that reflection should enable students to bring to consciousness an awareness of

their emotional reaction. An increased awareness of these emotional influences should give students greater control over their cognitive processes.

Wheatley (1992) also argues that reflection plays a critical role in mathematics learning: just completing a mathematical task is insufficient. It is one thing to solve a problem; it is quite another to take one's own action as an object of reflection. Students who reflect have greater control over their thinking so that in the midst of a lesson students can be reminded or informed of alternative ways of responding to the situation. They can decide which of several paths to take, rather than simply following a given procedure.

In relation to problem solving, Polya (1957) stressed the importance of reflection and evaluation of the process and solution. Gabrys *et al.*, (1993) suggested that successful learning by problem solving depends largely on **intentional** learning strategies such as hypothesis testing, reflection, and planning. Similarly, the process of reflection during problem solving is endorsed in *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992).

Communication is another strategy which is widely promoted as a means of developing mathematical understanding (Begg, 1993a; Hiebert, 1992; Neyland, 1994b). Begg (1993a:216) suggests that "communication is emphasised not only because it is part of what mathematicians do but also because, from a constructivist perspective, it enhances learning". Communication can promote and guide reflection and reflection can enrich what is shared through communication.

> *Learning to communicate about and through mathematics is part of learning to become a mathematical problem solver and learning to think mathematically. Critical reflection may be developed by encouraging students to share ideas, to use their own words to explain their ideas, and to record their thinking in a variety of ways, for example, through words, symbols, diagrams, and models.* (*Mathematics in the New Zealand Curriculum,* Ministry of Education, 1992: 11)

However, while communication and reflection, "seem to capture the cognitive heart of the reform" in mathematics education, they are not the only important strategies (Hiebert, 1992: 440).

Conjecture, operationalised as hypothesising and questioning, is important in learning mathematics. "The process of sorting out ideas, whether initiated by a text or by working on a question, involves making conjectures ... mathematical thinking is best supported by adopting a conjecturing attitude" (Mason, 1988:8). Similarly, Romberg (1992) supports the central role of conjecture in the mathematics learning process.

> *As long as students are making conjectures, their mathematical knowledge will always be structured, consciously or unconsciously, because conjecture cannot be created from nothing. Clearly, the work of students in such an environment is no longer a matter of acting within somebody else's structure, answering somebody else's questions, and waiting for the teacher to check the response. In the creation of knowledge, there is only that which fits the structure of mathematical knowledge already created by the student and that which does not and should, therefore, prompt conjecture.* (p. 48)

Visualisation, the process of forming images (mentally, or with pencil and paper, or with the aid of technology), is promoted by mathematics educators as a means of effecting mathematical discovery and understanding (see Zimmerman & Cunningham, 1991). While visualisation usually refers to graphic or pictorial images, metaphoric images and analogies are also important elaborative processes. Knight (1992) proposes that a more deliberate exploitation of metaphor in mathematics education might aid the development of metacognitive or thinking skills, aid memory, and change attitudes to mathematics.

Generalisation, more typically associated with problem solving, is also important in the learning process. The recognition of pattern or regularity appear to be "the building blocks used by learners to create order and meaning out of an overwhelming quantity of sense data" (Burton, 1984:38).

An important, but rarely referenced, learning strategy for senior mathematics students is the ability to use the text as a source of information. Reading from a mathematics text is not an easy task for students:

> *They must make mental connections between concepts, symbols, pictures, examples and diagrams, that require a high level of mental activity...We do not assume that students learn to read prose by 'picking it up along the way', and we cannot assume that they will learn to read symbolic material that way either. It is one of the most significant teacher's duties to get the students more directly involved in their learning and teach them how to read a mathematics textbook by themselves.* (Toumasis, 1993:558)

Curriculum initiatives are typically a response to educational research (and political whims). Many of the sentiments expressed above are reflected in recent mathematics curriculum developments of several countries. The following aims are included in *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992: 9-10):

- to develop the ability to reflect critically on the methods they have chosen;
- to develop the skills of critical appraisal of a mathematical argument or calculation, use mathematics to explore and conjecture, and learn from mistakes as well as successes;
- to develop the ability to estimate and to make approximation, and to be alert to the reasonableness of results and measurements;
- to express ideas, and to listen and respond to the ideas of others;
- to develop the knowledge and skills to interpret written presentations of mathematics; and
- to recognise patterns and relationships in mathematics and the real world, and be able to generalise from these.

The *National Statement on Mathematics for Australian Schools* states that all students should "continue to learn mathematics independently and collaboratively" (Australian Education Council, 1991:15). Similarly, in America, metacognition as a component of mathematics instruction is also emphasised in the *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989). Metacognitive objectives include: the ability to "reflect upon and clarify (their) thinking about mathematical relationships" (p. 40); "the inclination to monitor and reflect on (their) thinking and performance" (p. 233); the ability to "judge the relative merits of alternative procedures" (p. 232); and the ability to "give reasons for the steps in the procedures" (p. 232).

In summary, a wide range of learning strategies are actively promoted by mathematics educators and curriculum documents. Cognitive strategies of elaboration, including summarising, questioning, and imagery, are seen as key strategies for encoding information. Communication, whether it be an inner dialogue (self-regulatory), self-explanation of the text, peer interaction, or involvement in classroom discussion, is regarded as an essential processes in negotiation of meaning. The process of reflection, incorporating metacognitive strategies of planning, monitoring, and evaluating, is seen as especially important for enhancing problem-solving performance and learning. These strategies are not promoted in isolation. The interactive nature of cognitive and metacognitive strategies is acknowledged, as is the role of social and cultural aspects of the classroom learning environment.

Many of these strategies are backed by research findings from classroom-intervention studies. However, evidence of improved performance is usually related to problem-solving performance rather than to the development of mathematical understanding and mathematical learning processes. While mathematic educators and curriculum documents may suggest which strategies are regarded as essential to classroom learning, this in itself does not ensure that these strategies are presently **valued** and **used** by students in the classroom learning environment. In order to realise the possibilities that a constructivist pedagogy offers we need to take a closer, and more respectful look, at the learner.

4.3 Active Learning and Constructivism

Although helping students to become life-long learners has been an accepted long-term goal of education, the short-term goal of obtaining basic academic skills is often translated into actual schooling practice. Currently, instruction in many mathematics classes is oriented towards helping students become proficient at computation, algorithmic skills and practicing symbol manipulation rules. Cognitive skills tend to be driven out altogether by a demand for teaching even larger bodies of knowledge, with the idea that their application to reasoning and problem solving can be delayed (Resnick, 1987). Reform documents however, both overseas and in New Zealand (*Mathematics in the New Zealand Curriculum,* Ministry of Education, 1992), call for radical changes in emphasis from computational practices to problem-solving experiences (Hiebert, 1992).

> *Students need frequent opportunities to work with open-ended problems...Closed problems, which follow a well-known pattern of solution, develop only a limited range of skills, They encourage memorisation of routine methods rather than consideration and experimentation.* (Ministry of Education, 1992: 11)

As discussed in Chapter 1, part of this change is a result of changing views on what constitutes mathematics competency in our ever changing technological society. As technology is increasingly available to execute symbol manipulation procedures and algorithms, so the nature of the skills which are viewed as desirable outcomes of education are changing.

Change in the emphases, found in new curriculum documents, signals an equally dramatic change in how mathematics is learnt (Neyland, 1994b, Ritchie & Carr, 1992). Hiebert (1992) argues that there are significant changes in the cognitive processes that should be engaged during mathematics classes: "the purpose of studying procedures and algorithms shifts from proficient execution to reflective analysis of mathematical patterns and relationships"(p. 448). Moreover, in the future students will be faced more often with the problem of managing resources and using them effectively to solve problems.

For many mathematics educators constructivism captures the essence of the proposed learning changes (Leder & Gunstone, 1990). Constructivism, as promoted in recent curriculum documents suggests that the automation of skills and passive learning should be replaced by active learning processes. Learning is understood as a self-regulated process of resolving inner conflicts that often become apparent thorough concrete experience, collaborative discourse and reflection: "As new experiences cause students to refine their existing knowledge and ideas, so they construct new knowledge" (Ministry of Education, 1992:12).

While constructivism is essentially a theory of learning, rather than teaching, it does imply a new set of goals for the classroom. "Teaching mathematics should be understood as providing students with the opportunity and the stimulation to construct powerful mathematical ideas for themselves and come to know their own power as mathematical thinkers and learners" (Begg, 1993b:18). It is however, a concern of the researcher that emphasis focused on creating a suitable learning environment will be of little value if students are unaware of, or unwilling to employ, learning strategies to enable them to cope with the learning demands of such a constructivist approach.

While active learning in secondary school mathematics classrooms is now widely advocated, there is concern that some teachers may be lulled into a false sense of security by providing students with numerous investigations, open-ended problem-solving experiences, and hands on activities with the expectation that students are successfully constructing knowledge from these experiences.

> *The major problem facing the marked increase in the use of active learning activities in secondary schools is the tendency by some teachers to believe that active learning activities always promotes active mental experiences.* (Kyriacou & Marshall, 1989: 4)

In examining this issue further, Kyriacou and Marshall (1989) make an important distinction between the two major uses of the term 'active learning'. The first usage is to regard active learning as denoting learning activities in which students are given

considerable autonomy and control of the direction of the learning activities. Learning activities commonly identified in this manner in the *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) include investigational work, problem solving, small group work, collaborative learning and experiential learning. In contrast, 'passive learning' activities, in which the students are passive receivers of information, include listening to the teacher's exposition, being asked a series of closed questions and practice and application of information already presented.

Kyriacou and Marshall argue that a second usage of the term 'active learning' is equally important. In this instance, 'active learning' denotes "a quality of the *pupils' mental experience* in which there is active intellectual involvement in the learning experience characterised by increased insight" (p. 2). It is an attitude of active intellectual inquiry. This concept of 'active learning' encompasses the notions of mental effort or intentional learning (Bereiter & Scardamalia, 1989), meaningful learning, and metacognitive learning strategies. As with the first definition this form of 'active learning' may be contrasted with 'passive learning' which is characterised by an emphasis on assimilating new knowledge through memorisation and practice.

Kyriacou and Marshall's contention that these two dimension of 'active learning' are relatively independent of each other is in accord with Noddings' (1990) suggestion that all mental activity is constructive, but that some acts involve "weak constructions" rather than "strong constructions". As such, an active learning activity can foster either an active mental experience or a passive mental experience, just as a passive learning activity can foster either an active mental experience or a passive mental experience. The crucial point in terms of the present study is that strategic learning behaviours need to be aligned with 'active' mental experiences, which result in strong acts of construction if students are to learn the desired mathematical understandings (Herrington, 1990).

If constructivist learning is conceived as a self-regulated process of resolving inner conflicts, students need to be equipped with the learning strategies to cope with these demands. Perkins (1991) however, raises concerns about the high level of demands

placed on the learner in a constructivist learning environment. Perkins suggests that high demands are a result of the following three factors:

- The 'conflict faced' path of constructivist instruction has a very high cognitive demand;
- Learners are asked to play more of a task management role than in conventional instruction; and
- Learners need to have an appropriate attitude to learning in a constructivist classroom.

A concern of the present study is to determine whether, in fact, students are aware of a range of learning strategies and whether they utilise learning behaviours appropriate for the development of the "strong acts of construction" (Noddings, 1990). Prior experience with secondary school students in the PEEL project (Baird & Northfield, 1992) found that attempts to reflect on one's understanding, or to resolve discrepancies between what a student thinks some input means and their existing understanding, is not a common student behaviour. "It requires students to expose themselves as uncertain, to challenge the teacher, to use tentative language, to waste time, as the teacher is likely to say...students are inclined not to bother" (Mitchell, 1992c).

Bereiter (1992:354) is concerned that "one of the ironies of the present age of cognitive enlightenment is that the constructivist view of learning, although widely shared by educators, is kept hidden from the students". In support, Hennessy (1993) argues that few of today's classrooms encourage pupils to perceive what they are doing as the construction of knowledge. The literature review suggests that the present instructional demands and practices may do little to promote and encourage the development of appropriate learning behaviours (Peterson, 1988). There is also evidence that some students cope with school tasks by using strategies that actually have the effect of subverting learning. For example, copying methods step-by-step from worked examples may meet the short-term goal of completing an assignment but may fail to address the long-term goal of constructing knowledge.

To develop the ideal learning environment Collins *et al.*, (1989) suggest we need to pay due attention to students' development of:

a) domain knowledge;
b) problem strategies and heuristics;
c) control strategies with monitoring, diagnostic and remedial components for managing problem solving and learning (i.e. metacognitive strategies);
d) learning strategies; and
e) belief systems appropriate to learning (ie. metacognitive knowledge).

They argue that at present, most mathematical instruction focuses almost exclusively on domain knowledge, although recent emphasis on problem solving is evident in many classrooms, and certainly in *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992). Collins and colleagues suggest that the former instruction leaves metacognitive issues unaddressed, raising the likelihood of students developing maladaptive strategies and/or misunderstandings in categories (c), (d), and (e). In such an environment passive learning behaviours are common: most secondary school teachers are unaware of, or underestimate, the extent of the passive learning behaviours in their classrooms (Mitchell, 1992b).

Noddings (1993:38) warns that "turning students loose "to construct" will not in itself ensure progress toward genuinely mathematical results." Until we further understand the extent and nature of students' passive learning behaviours we cannot expect students to cope with the cognitive demands of constructivist teaching goals, nor can we expect active learning activities to automatically result in 'strong acts of construction'. Questions concerning learner behaviours that contribute to knowledge construction and teacher expectations of the learner are of great importance. A "crucial aspect missing from this current discussion on constructivism is [information about] the strategies used by students in constructing their own meanings" (Wong & Herrington, 1992:130).

4.4 The Classroom Setting

The literature review (Chapters 2 and 3) emphasised the importance of contextual factors in examining strategic learning behaviour. The *Interactive Model of Learning Mathematics* (section 2.3) suggests that learning behaviours can only be properly understood in the context of the learner variables, the learning task, and the learning environment. While the opportunity to learn in class is potentially similar for each student the presage factors and the learning behaviours (especially metacognition) interact and influence the effectiveness of the opportunity for each student.

Although some of the research studies discussed in the literature review have used the classroom setting, researchers (Evans, 1991a; Pressley, 1986; Peterson, 1988; Wong & Herrington, 1992) have identified a need to further examine strategic learning behaviour as it relates directly to the mathematics student's learning environment. Bachor (1991:145) reports that:

> *In nearly every study reviewed in which executive or content-specific strategies, that are used spontaneously by learners have been examined, the research was conducted within the context of a single subject area and incorporated a limited set of experimenter-provided stimulus materials. Moreover, in these studies, students' strategic learning behaviours typically have been sampled in only a single task or instructional sequence. Typically a single instrument or method for collecting data on students' cognitive processing has been incorporated into such studies, which limits the confidence that can be placed in the validity and reliability of their findings.*

Further support comes from Marland and Edward (1986) who argue that at this early stage of research into students' cognitive processes during classroom instruction there is considerable justification for pursuing purely descriptive studies in classroom settings:

Such studies should ensure that hypotheses and questions posed in subsequent correlational and experimental research, having been framed with an extensive and clear knowledge of the nature of covert learning in the classroom in mind, are relevant and sensible; that constructs and variables used in research have ecological validity; and that research designs take account of naturally occurring phenomena and other aspects of classroom life. (p.76)

Garner (1990a: 523) also contends learning strategies are not fruitfully studied without consideration of the setting

Given the apparently potent effect of context on learning in general and on strategy use in particular, it is discouraging to note that in most strategy research studies, context is treated as a footnote or, worse, as a nuisance.

Garner provides the following summary of contextual factors related to strategy use in the setting:

- strategies need to be applied conditionally by learner knowledge base and by domain appropriateness;
- certain situations are more likely to elicit cognitive monitoring than others;
- uninstructed strategies are often disguised by learners when they use them in instructional settings;
- meagre knowledge about task demands in a particular setting can inhibit application of strategies;
- students report using more strategies when they perceive that effortful activity is valued in the classroom setting; and
- some strategies are welded to the settings in which they were acquired.

A further aspect of learning strategies that makes them particularly context dependent is that they are goal driven. Thomas and Rowher (1986) offer four ways in which the use of learning strategies for studying in classroom contexts differs from the research context of the laboratory:

1. the clarity of the information students have about the criteria to be met;
2. the degree of congruence between the content learned and the content tested;
3. the amount of support provided for attaining the performance criteria; and
4. the conditions affecting spontaneous strategy use and maintenance.

These setting influences again support the need for the present research to be conducted in the classroom environment.

In summary, because learning depends on both situational and intrinsic factors it is necessary to focus on: (a) the opportunities provided by the learning environment; (b) students' actual use of learning strategies in the learning environment; (c) as well as the nature of the students' knowledge of learning strategies. Therefore to improve the theoretical arguments that influence reform recommendations we need to increase our understanding of relationships between students' learning and classroom learning that involves strategic activities, and increase our knowledge of the way in which strategic learning behaviours facilitate mathematical understanding.

4.5 The Research Objectives

The preceding analysis of the literature, current research, and curriculum reforms suggest that there is a need to further understand relationships between students' learning and classroom instruction. Research suggests that discrepancies exist between strategies students use and those that they should, or are expected to, use. In the event that learning strategies displayed by students in this study are limited, or defective, it would be helpful to gain more understanding about the learners' introspective knowledge of their strategic processes and beliefs about learning mathematics. Increased knowledge of the way in which learning strategies facilitate mathematical understanding and knowledge construction may be helpful in suggesting instructional remedies.

From these broad areas of need, more specific research objectives were framed.
The present study aims to:

- Examine and classify the present usage of learning strategies by students of a 6th form class.
- Explore the factors in the student's learning environment (both at home and at school) which either encourage or dissuade the development and appropriate use of learning strategies.

The interpretative study of learning strategies seeks not to establish decontextualised generalisations but to produce qualitative description of individual student's learning that will lead to a better understanding of the role of strategic behaviours in mathematics education. An implication of this ethnographic study is that a clearer understanding of mathematics students' use of learning strategies will contribute to the following areas:

- explaining individual differences in mathematical achievement;
- explaining contributing factors in failures to learn mathematics and help to remedy those failures;
- highlight instructional factors which contribute to, or impair strategy deployment and development; and
- increase our understanding of the role of learning strategies in mathematics learning.

4.6 Summary

A learning environment in which the teacher simply solves problems and the students simply watch is inadequate to provide an effective model for learning, particularly in mathematics, where many of the relevant processes and inferences are hidden (Burton, 1984; Collins *et al.*, 1989). As teachers we often expose students to a very narrow set of strategies; often taking for granted their effectiveness without analysing how and if they are working with students.

Mathematics curriculum reforms suggest that students need a learning environment which provides relevant experiences to enable students to construct meaningful interpretations and assimilate new understandings. While constructivism emphasises learner activity and the use of activities which relate personal experiences and prior knowledge "it is not useful for teachers to create tasks that increase the opportunities for cognitive conflict and then leave students entirely to their own devices to resolve the conflict" (Pintrich *et al.*, 1993: 187).

> *We must give students "tools to think with" - and these are not merely formulas and algorithms. They include concepts and powerful metaphors and heuristic procedures and understanding, including even a determination to acquire an even deeper understanding of oneself and one's own mode of learning and thinking* (Davis, Maher, & Noddings, 1990:188).

In order to assist students to become more active, self-regulated learners, and to design and implement instructional interventions, it is timely that we further our knowledge of learning strategies, and their relation to effective performance and knowledge construction. In addition to knowing about students' prior knowledge and experiences, and the nature of the mathematical task, teachers need to know more about their students' own strategic learning behaviours.

Chapter 5

Research Method

Qualitative data are sexy. They attract eager researchers who want to sniff the richness of the real world, see things in their contexts, track complex processes over time, and explain linkages among processes and their associated outcomes.

(Miles, 1990:37)

5.1 Introduction

Because of the exploratory nature of the research, specific hypotheses regarding strategic learning behaviours were not formulated prior to the study. The complexity of the Interactive Model of Learning Mathematics (section 2.2) implies that students' learning behaviours are so influenced by context and individual factors, that general principles will be well hidden, if they exist at all. Ethnographic research is "the process of providing scientific descriptions of educational systems, processes, and phenomena within their specific contexts" (Wiersma, 1991:218). Therefore the use of the natural classroom setting as a direct source of data is an appropriate forum in which to examine students' use of learning strategies. Additionally, the use of learning strategies in out-of-school contexts has also been built into the research design.

In terms of the methodology, no particular techniques are associated exclusively with ethnographic research, although several methodologies such as classroom observation, student case studies, and interviews support ethnographic data collection. Large scale use of quantitative approaches, such as multi-option format questionnaires and frequency counts of observed behaviours, are seen as inappropriate. They fail to recognise the importance of the appropriateness of a learning strategy to the learning task, individual

students' prior experiences, and specific learning contexts. In this respect data collection strategies have to be sympathetic to a qualitative, phenomenological approach, focusing on the respondents' view of their individual learning behaviours in the natural context.

> *Researchers should, as far as possible, seek to elicit from respondents, and to represent as faithfully as possible, the views of self-directed learners themselves about their interests, attitudes, intentions and understandings. Moreover, since these factors are likely to be situationally variable, a constructivist approach demands field-based enquires as far as possible.* (Candy, 1989: 101)

Because different types of data collection procedures may lead to different conclusions, multiple data collection strategies (triangulation) are essential. Triangulation provides a partial solution to understanding the complex reality. "Every method of data collection is only an approximation to knowledge. Each provides a different and usually valid glimpse of reality, and all are limited when used alone" (Warwick, 1973:190). The "reality" comes to be seen as located in the different perspectives and suppositions of the individual students (Denzin, 1988). However, Peshkin (1993:28) quite rightly reminds us that " 'reality,' a slippery notion at best, does not become clarified by any one person's construction or approach to inquiry".

While the students' perspective is central, a key research instrument in any ethnographic study is the researcher, who must ultimately reinterpret any data. Prior experience of teaching at the 6th form level ensured that the researcher had realistic expectations of the classroom organisation, management of students, and expected learning outcomes. While this experience facilitated informed interpretations of the data, it also meant that the researcher had some prior assumptions and expectations of students' learning behaviours - namely, the assumption that many mathematics students are passive learners, with relatively negative or neutral affective reactions towards learning mathematics.

Because of the naturalistic nature of the study the findings are context-bound generalisations, and it must be remembered that the learning behaviours of every student are unique. Thus, the ensuing interpretations that result are to some extent imperfect generalisations: simplifications of a more complex reality. Their purpose is to extend our understanding of students' strategic learning behaviours by providing detailed descriptions that enable the reader to understand similar situations. However, the findings can be extended in subsequent research, either with additional case studies, or with more structured designs.

Ethical Considerations

For the present study permission was obtained from the School Board of Trustees, Head of Mathematics, and class teacher. An information letter detailing the nature and objectives of the research was sent to the parents of the class members (Appendix 1). In addition the researcher conducted a preliminary information-sharing session with the selected class, in which the objectives of the study and the nature of student involvement were explained.

Students were invited to query any concerns about their expected level of involvement in the study. All students were asked to voluntarily complete written questionnaires related to study behaviours and participate in short interviews concerning specific learning issues. Four target students were asked to complete a maximum of three stimulated recall interviews during their study periods. At the end of term one there was another opportunity for class discussion about the progress of the research study. The video procedures (to be used in term two) and the timing and length of stimulated recall interviews were explained. Target students completed consent forms (Appendix 2).

Observational data was recorded in the form of field notes and interviews were taped and transcribed by the researcher. Security and confidentiality of records was maintained at all stages of the study. Students were assured of anonymity in any written research reports. Additionally it was emphasised that all interviews with the researcher were confidential and videos were only to be viewed by the researcher and student.

5.2 Pilot Study

A pilot research study was completed in the preceding academic year. The study, conducted over a four week period, involved students from a sixth form mathematics class at a local secondary school. The first week was spent observing students' strategic learning behaviours, peer interactions, and the class instructional methods related to strategy use. In the remaining weeks potential data collection strategies were trialled.

Episodes from several lessons were video-taped by the researcher, and target students were withdrawn from class to complete a short stimulated recall interview related to their learning behaviours. These interviews provided evidence of a range of reported learning strategies, but also revealed that some students had difficulty discussing their learning behaviours. The decision to use two cameras and a split screen image in the present study was an attempt to check the veridicality of student reports.

Students were requested to complete homework diaries over several nights. The usefulness, in terms of data, varied between respondents. Some students provided information about their learning processes, thoughts and beliefs, whereas other students provided a summary of the exercises completed. This may have been a consequence of the instrument itself, but may have also reflected the nature of the individual student's learning goal.

Questionnaires on revision strategies were trialed prior to an assessment test. While written responses were limited, students were keen to elaborate on written comments in an interview situation. Interviews provided evidence of a range of rehearsal, help-seeking, and reading strategies.

The pilot study raised issues related to the following areas:

- some students appeared to be on-task all lesson but made little progress;
- peer cooperation was successfully used by some of students, but not at all by others;
- some students relied greatly on the teacher for assistance in class;
- the availability of resources and help from home varied;

- students in the same learning environment behaved in uniquely individual ways; and
- classroom instruction related to content and task completion.

Methodological issues uncovered the need to improve video procedures to assist students' recall, the limited response from written questionnaires, and the necessity to use multiple data collection strategies to achieve more valid findings.

5.3 The Research Setting

Setting

The research is conducted in the natural learning environment thus preserving the 'ecological validity'. The selected school was a coeducational secondary school in a large provincial city. Teachers in the mathematics department were experienced and well informed of current curriculum developments. The Head of the Mathematics Department assigned a sixth form mathematics class to the research study. The selected class teacher was an experienced teacher, personally known by the researcher. The teacher's previous educational research experience meant that she was likely to be sympathetic and tolerant of any intrusive effects the research process might have. The teacher was aware of the overall objectives of the research study as outlined in the letter to the Board of Trustees. The researcher's requirements concerning access to class lessons and student availability for interviews was discussed and agreed upon.

Subjects

Initially there were sixteen students in the class - this number dropped to twelve students by the end of the year. Only reports relating to the twelve remaining students have been included in this study. Because of the subject option choices, students in this class represented a cross section of interests, motivation, achievement levels, gender and ethnicity. It soon became apparent that although the class size was small there were twelve different learning approaches being used, that is, given the same instructional material, each student was perceiving, interpreting and learning in a uniquely individual manner.

Table 1 provides a list of the students' names (pseudonyms), gender, and teacher rated achievement (as at the completion of the study).

STUDENT NAME	*GENDER*	*TARGET*	*ACHIEVEMENT*
ABE	M		D
ADAM	M	YES	A
BRENT	M		D
CRAIG	M		C
DEAN	M		C
FAYE	F		A
GARETH	M	YES	E
JAKE	M		D
JANE	F	YES	C
KAREN	F	YES	B
KANE	M		B
LUCY	F		B

Table 1 : Student Participants in the Study

Case Studies

In the first term four students were identified as target students for case studies. They were selected after classroom observations and trial interviews so as to represent a cross-section of the class in terms of achievement and gender. The purpose of the case studies was to provide a more detailed description of how learning strategies are used in the learning context and to examine the appropriateness and effectiveness of each of the target students' strategic learning behaviours. The target students participated in stimulated recall interviews during term two, completed all student interviews, and became the object of more intensive classroom observations as the study progressed.

The lesson context

The class followed a pre-determined sixth form syllabus and assessment structure. Only scheduled lessons were used, and the teacher was asked not to consciously vary her regular or planned approach. Lessons typically began with homework review, followed by whole class instruction through teacher exposition and teacher-student interaction. Discussion and questions were controlled by, and channelled through, the teacher. In all lessons, students were assigned exercises from a textbook. During seatwork the teacher moved about the classroom encouraging individual students and checking whether they had any difficulties. Most students worked independently, although in some groupings there was considerable peer interaction.

5.4 Data Collection Strategies

Methodologies such as classroom observation, student case studies, and interviews support ethnographic data collection. These data collection strategies aim to elicit the students' viewpoints, perceptions, and belief systems - the constructed realities of the student.

Timetable:

The research study was completed within an academic school year, beginning late in March and ending in October. The following is a timetable of data collection implementation.

Term One

- Discussion with teacher and students concerning the nature and purpose of research
- Classroom observations
- Trial videos and interviews
- General interviews
- Questionnaires concerning learning mathematics, homework, and test revision

- Discussion with teacher and students concerning the video procedures for stimulated recall interviews

Term Two

- Classroom observation
- Ten video lessons plus stimulated recall interviews
- Homework diaries
- Motivation survey

Term Three

- LASSI-HS survey
- Classroom observations
- Homework interviews
- Revision interviews
- Use of summaries and resources interviews

CLASSROOM OBSERVATIONS

During each teaching week several lessons were observed (52 lessons in total). In some weeks all lessons (4) were observed, in others it was two or three depending on the researcher's teaching commitments. Class tests were not observed.

Because of the small class size the researcher was able to sit at different places in the class and observe different groups or individuals throughout the study. Students were generally happy with this arrangement, as long as peer groupings were maintained. When sitting next to students the researcher was able to discuss aspects of the student's work such as note-taking, checking answers, possible links with topics, willingness to ask questions, or to capture the feelings of those being observed. However, one needed to avoid 'bothering' students and risk jeopardising the goodwill established between researcher and students.

Field notes were taken of all observed lessons. The focus of observations varied in response to research questions and data from interviews. For example, the focus may have been on peer cooperation, an individual student, use of questions or teacher cuing. Flexible observation schedules allowed for the recording of any relevant observations during the lesson. The field notes included observer reactions, and queries for later corroboration. Student work was occasionally photocopied. However students were generally reluctant to allow this as the location of the photocopier meant that students were without their books for some time.

Despite observations providing a richness of data there are inherent difficulties that need to be acknowledged in this methodology. Any observed strategic behaviour is only a sample of what might have been used. For this reason, observations were not intended to stand alone, rather they were both a precursor to interview questions and the means to corroborate student reports.

QUESTIONNAIRES AND STUDENT DIARIES

- **Questionnaires:** In the first term students completed open-ended questionnaires, related to test revision and learning mathematics (Appendix 3). They were used to provide some preliminary data for later interviews.

- **Homework Diaries:** Homework diaries were used to obtain data on learning behaviours at home. Students were given a record form to complete which required details of homework activities and the associated learning behaviours (Appendix 4) The format was discussed with the students and the emphasis was on description of factors related to planning, monitoring, help seeking and general work habits.

- **LASSI-HS Survey:** *Learning and Study Strategies Inventory-High School Version* (Weinstein & Palmer, 1990a) is an assessment tool designed to measure students' use of learning strategies and study methods. The focus is on both covert and overt thoughts and behaviours that relate to successful learning. The Likert-type

questionnaire was designed for high school students to provide a diagnostic measure to identify areas in which students could benefit from educational interventions.

After completing the questionnaire, students were able to self mark and compare their own scores within provided percentile placements. Time was allowed for a brief discussion of some of the implications of high or low scores on each of the LASSI-HS scales. A copy of the students' questionnaire was taken and reviewed for any inconsistencies against the data collected from students' self reports.

Motivation Survey: A survey (Appendix 5) adapted from *High School Science* (Nolen, 1988; Nolen & Haladyna, 1990) was administered in the second term to assess the students' perception of classroom orientation (mastery or performance).

INTERVIEWS

Overt strategies such as note-taking and help-seeking are relatively easy to observe, but the mental processes underlying these overt strategies may or may not entail such strategic modes of processing as self-monitoring and elaboration. Paris *et al.* (1983) and Garner (1988) contend that because strategies are consciously invoked they are available for introspection and or conscious report. Verbal-report data from both general interviews or stimulated recall interviews are particularly useful in that they provide a glimpse of the covert strategic activity that is not accessible except as described by strategy user.

Kardash and Amlund (1991) found that students' reports of covert strategies (rather than overt strategies), used to process information from an expository text, are associated with enhanced learning outcomes. This finding reinforces the conclusions of Peterson and colleagues (1982; 1984): that students' reports of strategic behaviours were a more reliable and valid indication of classroom learning than observations. Additionally, Zimmerman and Martinez-Pons (1986) concluded that interview procedures provided reliable evidence concerning students' self-regulation reports. Their findings were validated by a further study (Zimmerman & Martinez-Pons, 1988).

General Interviews

Semi-structured interviews were conducted in an attempt to discover what students were experiencing, how students interpreted their experiences, and how they structured and adapted their learning environment to enhance the learning process. Questions were related to a theme (e.g., homework behaviour, use of the textbook), but students were also encouraged to respond at length; and answers took unanticipated directions, such as discussion of instruction from previous years in relation to note-taking. These expansions on the topic were encouraged.

First term interviews focussed on students' perception of the learning processes for mathematics:

- how does one learn mathematics,
- what are their learning goals,
- what is the role of the teacher in the learning process; and
- what are the effects of classroom assessment on learning processes.

In the second term interview time was spent with stimulated recall interviews. During the third term interviews focussed more particularly on possible interpretations and verification of emergent findings. Specific contexts such as homework, test revision, summary writing and the use of textbook and other resources were explored so as to corroborate previous observations or responses.

Stimulated Recall Interviews

Stimulated recall interviews involve the recording of a lesson and subsequent replaying of the video to stimulate student recall of learning behaviours and thought processes. Stimulated recall procedures have previously been used to study the thinking of teachers and students during instruction (Marland & Edwards, 1986; Winne & Marx, 1982), and problem solving in mathematics (Peterson & Swing, 1982; Peterson, 1988).

The reliability of student-reported learning behaviours was verified by the use of stimulated recall interviews. In particular, stimulated recall interviews increase the likelihood of access to students' thoughts and covert learning behaviours. Although the stimulated recall interview procedure is not as likely to produce complete and accurate data as concurrent methods (e.g., think-aloud protocols) stimulated recall interviews do provide an important alternative when it is not possible to have students think aloud in the classroom. Additionally, the stimulated recall procedure avoids some of the problems associated with traditional interview techniques, in that it provides "non directive retrieval cues that serve to enhance the veridicality of the reports" (Peterson *et al.*, 1982: 546). Although verbal facility remains a potential confounding factor, memory failure, hypothetical questions and over-cuing are diminished in impact.

During term one several lessons were video-taped and students interviewed. These activities familiarised students and teacher to working with a camera in the classroom, watching lesson replays, and seeing oneself on video. In term two each of the four target students completed two or three stimulated recall interviews. Each week, for a total of ten weeks, lessons were recorded using two video cameras. One camera was focused on the teacher and the other on the target student creating a split-screen image of teacher and student on a single tape.

It is not necessary to have a split-screen image for stimulated recall interviews, but it was felt that the dual image of student and teacher enhanced the visual image of the classroom situation and thus the ability of the student to recall learning behaviours. For example, one was able to relate the puzzled look on a student's face to the content of the blackboard, or relate the willingness of a students to ask a question to the location of the teacher in the room. However, a disadvantage of the split-screen image is the need for a number of recording devices (some of a specialised nature), and a technician. In this study, the small class size meant it was possible for the equipment to be placed to one side of the room not normally occupied by students. Despite retaining, as far as possible, the natural class setting, on occasions the students' normal seating arrangements were disrupted, resulting in a different pattern of peer interaction.

On two occasions the videoing of two target students, sitting next to each other, was to be attempted, so as to increase the number of student interviews. On both occasions one of the target pupils was absent. As the cameras need to be positioned and focussed with a horizontal split **before** the commencement of the lesson, this resulted in a video of one student plus an empty chair!

After each lesson the researcher reviewed the videotape and selected a variety of teaching/learning episodes. Stimulated recall interviews were conducted the following day during the students' study period. Each student was requested to view the lesson segments and relive, as fully as possible, the classroom situation. The interview followed guidelines to facilitate full disclosure of class learning behaviour and thoughts, as suggested by Marland and Edwards (1986:77). Principal guidelines include:

- *create a relaxed, informal setting for the interview;*
- *encourage students to initiate self-reporting as much as possible;*
- *listen attentively to, and show interest in and respect for, the student;*
- *respond to student self-reporting with encouragement and invitation for further disclosure;*
- *request clarification or confirmation where necessary;*
- *avoid leading questions, making evaluative comments, sounding critical;*
- *initiate student self-reporting if and where necessary by asking, "W ere you thinking anything there?" or "What were you thinking there?"*

Student responses included thoughts that they had at the time of the lesson, as well as those they experienced as they watched the lesson. For example, a student discussed reasons for not having done her homework while she watched the class homework review session on video. Additionally, the researcher was able to further explore issues such as students' lack of help-seeking questions, in relation to specific episodes on the video. Stimulated recall interviews provided a rich source of data relating to student reports of strategies in use, their metacognitive knowledge and their perceptions of learning mathematics.

All stimulated recall interviews were audio-taped. The tapes were transcribed and then a second transcript was prepared incorporating descriptions of the lesson content, relevant teacher comments, and other students' comments and actions. This provided a more detailed record of contextual factors, and was available for later analysis.

Students' response to assurances of confidentiality and anonymity was evidenced in their willingness to openly discuss feelings about classroom instruction and peer interactions. Positive motivation was also established by instructing the students that their responses would not be evaluated for correctness (i.e., against criterion), and by encouraging them to respond as completely and honestly as possible.

However there are limitations for the use of stimulated recall reports to investigate learning strategies. Galloway and Labarca (1990) suggest that there is a need to be aware that:

- much of the learning may be unconscious and, therefore, inaccessible to mental probes;
- learners may forget their distinct combination of strategies once their learning goal or task is complete;
- learners may misclaim strategies, or describe what they think the interviewer wants to hear; and
- it is difficult to ascertain from self-reports which strategies contribute significantly to learning and which have only a marginal effect.

5.5 Data Analysis

The qualitative research approach assumes that nothing in the natural setting is trivial, that everything has the potential of being a clue that might unlock a more comprehensive understanding of what is being studied. Data are the constructions offered by the students and data analysis leads to a reconstruction of these constructions (Lincoln & Guba, 1985).

According to McMillan and Schumacher (1993) qualitative data analysis entails several cyclical phases:

- continuous discovery, especially in the field, but also throughout the entire study, so as to identify tentative patterns;
- categorising the data;
- qualitatively assessing the trustworthiness of the data, so as to refine one's understanding of the patterns;
- writing an abstract synthesis of the themes.

Data Preparation

The process of data analysis in an ethnographic study necessarily begins as the researcher mentally processes the numerous ideas and facts while collecting data. This analysis enables the focus of the study to be changed as new questions arise. To keep track of thoughts the researcher added memos to field notes; and specific impressions, questions, and incidents were recorded with respect to each of the target students at the end of observations and interviews. These notes marked the beginnings of the initial working conceptualisations and descriptions.

Broad classificatory schemes of cognitive, metacognitive and social strategies (including affective and resource management), as identified in the research literature (Weinstein & Mayer, 1986; White, 1993), were used to group strategic learning behaviours from the data. While this enabled an overview and the formation of tentative findings, in order to categorise the existing strategies used by students, a more rigorous inductive approach to forming a classification scheme was needed.

Data Reduction

A major objective of the research study was to report and analyse learning strategies **in use** rather than report on strategies that one **thinks are** or **should be** in use. To ensure that the learning strategy classifications were firmly grounded in the research, a data reduction process was necessary. Data reduction has been described by Miles and Huberman (1984: 21-2) as:

> *...the process of selecting, focussing, simplifying, abstracting and transforming the 'raw' data that appear in written up field notes. As data collection proceeds, there are further episodes of data reduction (doing summaries, coding, teasing out themes)...And the data reduction/transforming process continues after fieldwork, until a final report is complete.*

The first stage of the data analysis was to reduce all data from observations and interviews to a manageable list of learning behaviours. The data was coded into simple descriptions of learning behaviours such as:

- answers the teacher's question;
- mumbles answer to herself;
- skims the chapter; and
- copies worked examples from the board.

This resulted in a very large list of learning behaviours. Samples of the initial coding of learning behaviours are provided in the following Appendices:

- Appendix 6 is Jane's reported and observed learning behaviours from a single lesson (Source: stimulated recall interview 1).
- Appendix 7 provides examples of Karen's learning behaviours during seatwork (Source: observations during the year, stimulated recall interviews).
- Appendix 8 provides a list of Adam's learning behaviours relating to homework (Source: interview, diary and questionnaire).
- Appendix 9 provides a list of learning behaviours related to Gareth's test preparation (Source: interviews, questionnaires).

Overall Data analysis

The implication for data analysis is that it will be inductive, rather than deductive. Data was analysed in two phases. Firstly, data was analysed according to strategy types and a classificatory scheme established. These classifications, do not in themselves provide meaning to the learning process, rather they provide suitable descriptions to enable a discussion of the role of learning strategies in mathematics. Because this is a significant part of the research objective the classification of strategies is discussed separately in the following chapter (Chapter 6). The role of each of the learning strategies identified by the classification scheme is discussed in Chapter 7.

To present a more holistic view of the student's learning process in the classroom environment data was then analysed by considering strategy use in specific learning episodes such as homework review, class discussion and seatwork. Analysis focussed on learning strategies used by each of the target students. Findings from this analysis are discussed in case studies of the target students (Chapter 8). Specific data relating to learning outside of the classroom obtained from questionnaires, diaries, and general interviews is discussed in Chapter 9.

At all levels of analysis data was obtained from the multiple data collection strategies. Stimulated recall interview data provided the major part of the data for the more detailed strategy profiles of the target students. The following data sample from multiple data sources (Table 2) illustrates how triangulation of time and data sources assists in providing an analysis of Gareth's participation in questioning during class discussions. The starting point for understanding Gareth's behaviour was of course the observations of the actions of the teacher and Gareth as they interacted in the discussions. However, these initial observations did not immediately lead to understanding, rather they raised questions to be answered and explored in further interviews and observations.

Table 2: Triangulation of time and data source relating to Gareth's answering of questions in class			
Source	**Method**	**Information Added**	**Interpretation**
L4	Observation	G answers many questions	G is active participant and keen.
L5	Observation	G active, 3 out of 6 answers are correct	G appears to have some knowledge of the content.
	Test result	G scores 15%	Achievement well below average.
L6	Observation	G fails to answer questions directed to him by teacher.	Appears to choose which questions to call answers out for.
	Interview re maths learning	Sees learning maths as hard work involving a lot of memory.	Rote learning, reliance on worked examples means that he can call out the answers in class but not recall correct procedures in test.
L7	Observation	Response to teacher directed question: "I'm not on the right page."	Although working, G's fixation with wanting to complete all the calculation means he is often out-of-step with the teacher.
L15	Observation	Uses keyword strategy to answer question.	Memory learning
L16	Observation	Answers low level questions requiring one word answers.	Corresponding question in exam incorrectly answered.
L17	Observation	Answers calculation questions.	Selectively attends to calculations.
L24	Observation	Uses text to help answer question.	Uses text to compensate for prior knowledge.
L25	S-R interview	Muffles answer	Answers some questions to himself.
		Reports answering question in his head.	Doesn't answer more complex questions publicly.
		Answers calculation question correctly.	Attends to calculation rather than conceptual material.
	General Interview	"When you're discussing in class you sort of remember the lesson more."	Answers question to aid recall of lesson.
		"If you get an answer wrong she (teacher) usually goes into the example in more detail."	Answers questions as a form of help-seeking.
		"I answer lots of questions because it comes into a good report at the end of the year."	Seeks teacher's approval for assessment purposes.
		No help available at home	Importance of seeking help in class.
L34	S-R interview	Tries to answer other students' questions.	Monitoring production
		Answers teacher question then when prompted for and explanation of process is unable to respond.	G can answer what to do next in a procedure but not how or why.
		Teacher directed question	Gareth not given time to finish - needs to answer straightaway if wanting to participate. Teacher only expects Gareth to answer the easy part of the question.

5.6 Validity of Interpretations

There are four major factors in the research design proposed to ensure the "trustworthiness" (Glesne & Peshkin, 1992) of the research interpretations: time at the research site; triangulated findings; video and audio recordings; and low inference descriptions.

Data provides an estimate of what a person did, not what he or she might have done on a different occasion. In this research, the shift to obtain data over an extended period of time ensured that there were sufficient occasions for observations and interviews, so as to reflect the students' continuum of strengths and weaknesses in awareness, and use of strategies. Additionally, the length of the study enabled the researcher to build sound relationships with the students and teacher. The class felt comfortable with the researcher; their discussions, both in interviews and in class, appeared uninhibited to the extent that they revealed their likes and dislikes about specific teaching approaches and classroom organisation. There was also time for students to become familiar with the video equipment in the classroom, thus increasing the validity of the recorded lessons.

The video and audio recordings were referred to by the researcher on several occasions during the data analysis stage to assist in recall of an incident, to clarify some notes, or to corroborate student reports, or findings from observations.

Muralidhar (1993: 445) argues that "the need to interrelate and integrate data is inherent in studies involving fieldwork because field study is not a single method or a single technique seeking a single kind of information". The triangulation of data collection strategies used in this study is advantageous in two respects. Firstly, triangulation enables the researcher to cross check results initially obtained from one source by another source within the data collection phase. For example, rather than simply recording that a certain behaviour has occurred, the researcher attempted to understand what the behaviour meant to the learner by discussing the behaviour in an interview or by questioning the student directly (member checking).

Secondly, in the analysis phase, triangulation of observation, interview, students' work and questionnaire data provides corroborated evidence of researcher inference. The key aspect is not just the combination of data from different collection strategies but the attempt by the researcher to make sense of the phenomenon under study and to counteract the possible threat to validity. In doing this one needs to be aware that triangulation will not always result in a totally consistent picture - the value is in the interpretation of understandings of when and why data presents inconsistent findings (Mathison, 1988).

Limitations

In demonstrating the trustworthiness of data one needs to realise the limitations of the study (Glesne and Peshkin, 1992). The very nature of naturalistic classroom research brings certain limitations to the research process. Research dealing with respondents in their natural setting needs to balance the desire for ecological validity with the needs to create a setting suitable for data collection.

In general, the students were cooperative and a good relationship with the researcher was established. The length of the study enabled the researcher to be regarded as 'part of the setting', but it did have a limiting effect. Towards the end of the study some students got 'sick' of being interviewed. If students showed reluctance to be interviewed, either because of other work commitments, or negative feelings, it was usually possible to negotiate a more suitable time with the student.

To set up the video equipment the classroom had to be unoccupied before the lesson: This happened only on Wednesdays, where the lesson directly followed the lunch break. On occasions when the lesson was rescheduled because of sports trips, or the teacher was sick, the video lesson was cancelled. This reduced the planned number of video lessons available for data analysis. In retrospect, although video-taping took place only once a week, the continuation throughout term two was quite demanding of the class and teacher. While the number of interviews for each target students appears small, there is some doubt as to whether students would have been able to sustain interest in any more stimulated recall interviews.

Chapter 6

Learning Strategies: Classification and Distribution

Man looks at this world through transparent patterns or templets, which he creates and then attempts to fit over the realities of which the world is composed. The fit is not always very good. Yet, without such patterns the world appears to be such an undifferentiated homogeneity that man is unable to make any sense out of it. Even a poor fit is more helpful to him than nothing at all.

(Kelly, 1955: 9-10)

6.1 Classification Of Learning Strategies

For the purposes of classification, behaviours were first coded according to their goal (Garner, 1990b) as either cognitive, metacognitive, affective (Weinstein & Mayer, 1986) or resource management (Pokay & Blumenfeld, 1990). See Data Analysis (section 5.5) and of codings of learning behaviours (Appendices 6-9). Learning behaviours, in each of these four broad classifications, were then grouped into representative learning behaviours identified by the proposed strategy classifications.

Although the classifications arrived at here are grounded in research data, they are also consistent with descriptions commonly used in the literature (see Christopoulos, Rohwer, & Thomas, 1987; Corno, 1989; O'Malley & Chamot, 1990; Pokay & Blumenfeld, 1990; Swing *et al.*, 1988; Weinstein & Mayer, 1986; White, 1993; Zimmerman & Martinez-Pons, 1986; and also section 3.3 for a fuller review of existing classification systems).

COGNITIVE LEARNING STRATEGIES

Chamot and O'Malley (1987: 242) state that while using cognitive strategies the learner:

> *interacts with the material to be learned by manipulating it mentally (as in making mental images or relating new information to previously acquired concepts or skills) or physically (as in grouping items to be learned in meaningful categories, or taking notes on or making summaries of, important information to be remembered).*

Cognitive learning strategies are classified under the broad classifications suggested by Weinstein and Mayer (1986) of rehearsal, elaboration and organisation. Rehearsal strategies are used to enhance encoding and information retrieval; elaboration strategies add detail, explanation, or examples and other information from prior knowledge to present knowledge; and organisational strategies organise information in a form unique to the individual learner's requirements.

Rehearsal Strategies

- **Problem practice (C1).** Applying a new procedure with problems/exercises in class or for homework.
- **Writing out formulae or notes (C2).** Repetitive writing of material to aid memorisation.
- **Rereading (C3).** Reading or repeating aloud notes or formulae.
- **Revision practice (C4).** Doing problems of similar type for revision or consolidation of procedures.

Elaboration Strategies

- **Linking (E1).** Linking new information with prior academic or personal knowledge.
- **Imagery elaboration (E2).** Using mental pictures or diagrams to represent information.
- **Comparing (E3).** Comparing or contrasting one's own work with that from another source (worked example, another student's answer). Recognising similar patterns in problems concepts and operations (generalising, specialising and discriminating).

- **Questioning (E4).** Generating or answering questions related to the concept or problem process.
- **Self-questions or self-explanations (E5).** Explanations given to the self to clarify meaning. Specific statements concerning reasoning, or wondering why particular actions are (or are not) appropriate.
- **General (G1).** Unspecific statements concerning attempts to comprehend the information (e.g., *"I was trying to follow what the teacher was doing").*
- **Other (G2).** Attending, watching the teacher, listening, following teacher's instruction.

Organisational Strategies

- **Summarising (O1).** Purposeful recording of information selected on the criteria of difficulty or relevance. .
- **Note-taking (O2).** Keeping a record of given information, problems attempted, corrected solutions and teacher given summaries.
- **Layout (O3).** Recording information in different formats (highlighting, underlining, colour coding, and formatting). Organisation of workbook sections (date, page numbers, notes, exercises).

METACOGNITIVE LEARNING STRATEGIES

Metacognition includes knowledge of one's own cognitive processes, along with monitoring, evaluating, and regulating them, and beliefs about factors that affect cognitive activities. Metacognitive strategies of planning, monitoring and evaluation imply a measure of self-determination, or autonomy in learning and problem solving.

> *Whereas cognition refers to the 'what' of learning, metacognition refers to controlling the 'how' and the 'when, where and why' or in other words, the* ***procedural and conditional knowledge of learning.*** *Such knowledge is particularly important in carrying out cognitive activities.* (Biggs & Moore, 1993: 307)

- **Previewing (M1).** Reading or scanning ahead in the anticipated learning task, previewing homework in class, text inspection.
- **Planning (M2).** Consciously planning the timing or content of study to facilitate learning.
- **Predicting (M3).** Attempts to predict the results of the teacher's or one's own action.
- **Problem Identification (M4).** Trying to diagnose the cause of task failure or identify the central point needing resolution in a task.
- **Reflection (M5).** Self initiated reflection on a mathematics concept/problem over an extended period of time; evaluation of the teacher's methods, or alternative methods against an external criteria (e.g., efficiency).
- **Selective Attention (M6).** Evaluation of worthiness of an activity. Selectively attending to important information. Cue seeking about test content.
- **Self Monitoring (M7).** Checking or verifying one's comprehension or performance in the course of a learning task. Answering or asking questions for verification of understanding or performance. Also evidenced by changing strategies (e.g., rereading question, looking up answer during problem solving or seeking help).
- **Production evaluation (M8).** Checking specific task performance against internal criteria (e.g. "*I thought the answer was right because I could do them easily*") or external criteria (e.g. *"I thought they were right because I did them the same way as the other problems"* or textbook answers);
- **Self-evaluation (M9).** Evaluating one's overall learning progress by test results, or by time factor, or quantity or quality of work done in comparison with others. Evaluating one's strategy use or ability to perform the task.
- **Revision (M10).** Being aware of the need to review aspects of the task to aid learning. Using classroom and tutorial opportunities to review.
- **Metacognitive knowledge (MK).** Students' reports of knowledge about themselves as learners, the nature of the task, and their learning strategies.
- **Metacognitive experience (ME).** Feelings related to cognitive activity (e.g., anxiousness, difficulty in remembering, feeling that something 'clicked', or that the work seemed difficult or hard).

AFFECTIVE LEARNING STRATEGIES

Affective strategies are used to help the learner relax, gain confidence or maintain effort so that more profitable learning can take place.

- **Effort control (A1).** Acknowledging the need to attend (e.g., a student reports special effort towards understanding a lesson or completing a problem).
- **Self encouragement (A2).** Using mental redirection of thinking to assure oneself that a learning activity will be successful or to reduce anxiety about the task.
- **Self consequences (A3).** Arrangement or imagination of a reward at the completion of a learning task.
- **Attention Control (A4).** Vary routine, time out to reduce boredom or fatigue.

RESOURCE MANAGEMENT LEARNING STRATEGIES

Resource management strategies help students adapt to their environment as well as change the environment to fit their learning goals and needs. Tasks and environmental management strategies afford students the opportunity to learn, and indirectly contribute to the learning goal of the student.

- **Task management (R1).** Modifying the task so as to make it easier (e.g., skipping questions so as to keep up with the class), harder or more challenging (e.g., adding a time constraint). Selecting alternative or additional problems..
- **Determining the progress of the lesson (R2).** Students may attempt to speed up the pace of the lesson by answering teacher questions, or may attempt to slow down the pace by reporting difficulties or engaging in off-task behaviour.
- **Monitoring the teacher's movements and comments (R3).** Students attend to where the teacher is in the class in relation to the blackboard and other students. Learners seek to derive interaction opportunities with teacher.
- **Listening to comments between other students or between teacher and another student (R4).**
- **Seeking help from peers, teacher or adults (R5).**

- **Seeking help from resources (R6).** Use of text glossaries, index, worked examples, summaries, explanations or checking alternative texts.
- **Cooperation with peers (R7).** Working with peers to solve a problem, pool information, check notes, or exchange feedback. Getting another student to seek help.
- **Environmental control (R8).** Student-initiated efforts to select or arrange the physical setting on order to affect learning. For example, sitting in a particular group or sitting next to a particular student, clearing desk, arranging books or shutting windows.

6.2 Discussion of the Classification System

This classification system is not designed to be an exemplary, definitive classification for mathematics learning. It is important to re-emphasise that the learning strategy classifications represent only those learning behaviours that were evident in this research study. Because of the strong contextual influence it is quite conceivable that different students in a different class, of a different age, would have produced a different range of learning behaviours and consequent classification system. If the purpose was to prepare a typology of learning strategies for a questionnaire one could conceivably break these classifications into finer classification (see White, 1993), or employ different terms to represent classifications.

To ensure that the classifications are representative of the data, instances where there were only a few examples of learning behaviours have been grouped together under broader classifications. For example, 'evaluation of strategy in use' or 'evaluation of ability' have been grouped under 'self-evaluation'. Checking of answers, which was a common activity, in both the classroom and at home, could also be included as 'self evaluation' but because of the importance and frequency of this behaviour a separate classification was devised - 'production evaluation'.

In much of the research literature the term 'reflection' often encompasses a wide range of metacognitive behaviours (Biggs & Moore, 1993) related to problem diagnosis, monitoring, evaluating, mulling over a problem or concept and a general awareness of one's learning. However, the classification of the metacognitive strategy 'reflection' in the present study refers only to students' reflection about aspects of a mathematical procedure or concept. Although rarely reported, it is thought to be a distinctive and important learning behaviour illustrative of active learning, thus it was given a separate classification. The act of reflecting on a given method in terms of evaluating the efficiency or usefulness of the method is evidence of a student thinking about the nature of the task, rather than solely trying to complete the task. However if students merely commented that the given method *"seemed okay"* or *"easy"* this behaviour was included under 'production evaluation'.

'Summarising' is included as an organisational strategy in that it relates to forming networks of ideas and outlining as referenced by Weinstein and Mayer (1986). However it could have been categorised under elaborative strategies as the act of summarising requires students to actively paraphrase information; or under rehearsal, as active student input, "might help the learner remember mathematics information by highlighting important points and by requiring the learner to rehearse the mathematics information" (Swing *et al.*, 1988:129); or even as a metacognitive strategy, as making summaries provides a general test of one's comprehension.

'Reviewing' is also a strategy which is sometimes considered a cognitive strategy in some research studies, and in others a metacognitive strategy. In this study reviewing is classified as a metacognitive strategy that refers principally to students' awareness and plans for revision: the learner shows evidence of a conscious decision to engage in the learning process. With this decision there is also an element of goal setting, whether it be a specific goal of learning some particular formula or procedure, or a more global goal of wanting to do well in a test.

Problems with coding learning behaviours

There were several problems in coding the learning behaviours into a classificatory learning strategy scheme. There are difficulties deciding what behaviours are cognitive and what behaviours are metacognitive. Artzt and Armour-Thomas (1992) argue that although a conceptual distinction is possible, operationally the distinction is often blurred: cognition is implicit in any metacognitive activity and metacognition may be present during a cognitive act, although perhaps not apparent. For example the potential for confusion occurs when coding such behaviours as revision (a metacognitive learning strategy) which itself involves a number of cognitive strategies such as rehearsal and elaboration. Slife *et al.* (1985:442) make the valid point that "theoretically, some educators may cut the metacognition/cognition 'pie' in different portions".

Moreover, dividing behaviours into appropriate learning strategy classification is highly sensitive to interpretation within the learning context. For example, a student may make a copy of a worked example to provide a record of important information for test revision (summarising), or as result of teacher instruction (note-taking), or for practice of a problem type (practice). Behaviours while copying the worked example may include self-questioning to enhance understanding, colour coding to signify importance, and linking or comparing with previous examples to strengthen schema formation. Similarly a student may ask a question to seek help with a problem (resource management), or to verify an answer (evaluation), or to check his or her understanding (monitoring), or to a express a link with prior knowledge (elaboration).

Metacognitive strategies by their very nature involve the control and regulation of cognitive strategies thus there is considerable overlap with cognitive strategy classification (Weinstein, 1988). Additionally, there is considerable overlap between metacognitive control strategies and affective and resource management strategies. Thus, a single description given by the learner frequently represents the concurrent use of several strategies. For example, when students reported doing revision there was usually a critical interplay of cognitive, metacognitive, affective and resource

management learning strategies. The following examples demonstrate the interdependency between the strategy types during revision.

Katy: *"I worry if I know I'm going to fail it* (test) *and if I know it's going to be a hard test then I will do a lot of study."* (affective / metacognitive / cognitive)

Jane: "*We've got a list in our course outline of things that I go by when I study*." (resource management / metacognitive)

From these examples one can see that the categorical scheme is not meant to imply orthogonality among the classes of strategies; rather, an interaction is more the norm.

The coding of learning behaviours also included metacognitive knowledge and experiences. According to the *Interactive Model of Learning Mathematics* (section 2.3) metacognitive knowledge and experiences are closely related to students' metacognitive strategies. To the extent that students' metacognitive knowledge or metacognitive experience statements reflect previous self evaluations they are evidence of strategic learning behaviours. There is evidence of the interaction of metacognitive knowledge categories and metacognitive strategies definitions in several research studies. For example, O'Malley and Chamot's (1990: 137) definition of 'self-management' strategy in language learning clearly includes task and strategy metacognitive knowledge:

> *Understanding the conditions that help one successfully accomplish language tasks and arranging for the presence of those conditions; controlling one's language performance to maximize use of what is already known.*

Coping strategies are an important part of some students' behaviour (Corno, 1989). While there was evidence of such behaviours as copying answers, letting attention fade in and out, covering up for not understanding, or avoiding answering questions, it is not appropriate to classify these behaviours as learning strategies. Such behaviours which intentionally subvert the learning goal will be discussed later in relation to the use of specific learning strategies, especially resource management strategies.

Reliability of coding learning strategies

Learning episodes were coded with the purpose of verifying the proposed classificatory scheme and obtaining an indication of strategy distributions among students. The reliability of the coding process was checked by a research assistant by providing an interrater coding check. The research assistant and the writer jointly coded a stimulated recall interview script. Disagreements were discussed and, after negotiation, appropriate codings were agreed upon. The research assistant then independently coded three further stimulated recall interviews. Interrater reliability was assessed by dividing the identical categorical judgements of both coders by the total number of strategies initially identified. Over the three interviews reliability rating ranged from 86% to 95%. Total reliability over 234 items was 90%. Nearly all of the disagreements were within the categories of cognitive and metacognitive, rather than between categories. It is noted that the resource management classifications of 'determination of the progress of the lesson' (R2), and 'environmental control' (R8), were not reported in the stimulated recall interviews. In light of the research objectives the interrater reliability was considered sufficient.

Frequency counts and the distribution of occurrences of the various learning strategies provide only limited information: the goal is not to quantify the various strategies, nor to determine statistical relationships among the classifications. However, the following section will discuss the implications of trends indicated by the coding distributions of students' reported strategy use in stimulated recall interviews.

6.3 Quantitative Analysis of Strategy Use

For each of the stimulated recall interviews reported learning strategies can be counted and compared. However the value of such an exercise is limited in that each interview was based on a different lesson content, and interviews were selective in that students responded to selected episodes of the lesson. However, despite these limitations one can see from the Percentage Frequency of Reported Strategy Use (Table 3) some similarities in the distribution of strategy types seen between students.

Strategy	Gareth	Jane	Karen	Adam
Cognitive Strategies				
Rehearsal Elaboration Organisation	38%	29%	34%	35%
Metacognitive Strategies				
Control (M1, M2, M3, M4, M5, M6)	14%	9%	14%	18%
Awareness (M7, M8, M9, M10).	27%	36%	26%	21%
Metacognitive knowledge and experiences				
Metacognitive knowledge and experiences	10%	19%	16%	10%
Affective Strategies				
A1, A2, A3, A4	0%	1%	2%	3%
Resource Management Strategies				
Task and environmental management (R1, R2, R8, R9)	1%	2%	3%	7%
Social (R3, R4, R5, R6, R7)	10%	4%	5%	6%

Table 3: Percentage Frequency of Reported Strategy Use

The differences in the interview scenarios, combined with the problematic nature of the data coding, suggests that this data should only be used to highlight the following broad trends:

- For all students - reports of metacognitive learning strategies was considerably higher than cognitive learning strategies.
- For all students - the proportion of metacognitive learning behaviours related to awareness (metacognitive strategies M7 - M10 and metacognitive knowledge and experiences) was considerably higher than metacognitive learning strategies related to control and regulation of learning behaviours (M1-M6). That is, students reported mainly on their mental states in very general terms, and less frequently reported efforts to control or direct their thinking.
- Individual students reported differing use of resource management strategies.

I will address the first trend in this section. The high proportion of reported use of metacognitive strategies, when compared with cognitive strategies, is in direct contrast to studies involving students in areas of reading textual material; such as reported by Biggs (1987) and Marland *et al.* (1992a).

There are two possible contributing factors which would account for this result. Firstly the nature and timing of the stimulated recall experience would stimulate reports of overt learning strategies such as help seeking, asking questions, and checking, as well as strategies related to strong affective feelings (metacognitive experiences). Learning episodes that elicited the highest student response involved teacher explanation and review. Although it is hoped that students engage in elaborative strategies of self-explanations, imaging and self-questioning during these episodes, such strategies are more likely to be under estimated by student reports than the metacognitive strategies of production evaluation and self-monitoring. In particular, those learning episodes which involved difficulties, and thus invoked a range of metacognitive experiences and strategies, are more likely to be easily recalled than those involving cognitive strategies minus metacognitive experiences.

A second possibility is that the nature of mathematics learning and assessment requirements requires greater use of metacognitive learning strategies than is required in textual learning areas. Anthony (1991) found that mathematics distance education students reported greater use of metacognitive behaviours than did students in other areas of the social sciences. Certainly, in the present classroom context, there was a strong visible link between student accountability and comprehension monitoring. The nature of instruction actively encouraged students to constantly monitor their understanding or production with such comments as "*Does everyone follow that?*" and "*Are there any problems with that*?" Additionally, the problem-solving nature of seatwork demands that students pay a good deal of attention to metacognitive aspects such as production monitoring, monitoring understanding and goal attainment.

The high use of metacognitive strategies could be viewed as positive except for the fact that the proportion of metacognitive learning strategies related to control and regulation of learning behaviours was considerably lower than metacognitive learning strategies related to awareness. In many instances students seemed unwilling or unable to take strategic action as a result of their monitoring behaviours. A full discussion of the implications of this, as well as an examination of student's differential use of learning strategies, will be discussed in the following chapters (Chapters 7 and 8).

As well as frequency counts, one can also examine the range of reported strategies. All target students reported a similar wide range of strategies. To illustrate, the range Table 4 lists the learning strategies reported by Karen during a stimulated recall interview of a single lesson.

Learning Strategy	Examples of Learning Behaviour
	Cognitive Strategies
Problem practice	Trying problems from text.
Revision practice	Does some more problems to help understanding.
Linking	Thinking back to previous knowledge of quadratics from an earlier unit.
Imagery	Seeing a picture of the graph in her mind.
Comparing	Compares own problem with the one on the board.
Self question/explanation	Negotiates understanding through dialogue with self.
General	Thinking what it means.
Others	Attending to the teacher's board work.
Note taking	Copies down notes from the board.
Layout	Writes headings in different colours.
	Metacognitive Strategies
Previewing	Reading through question in homework project.
Planning	Thinking about how long the project will take.
Predicting	Anticipates lesson direction.
Problem identification	Trying to diagnose the specific area of confusion.
Selective attention	Concentrates on tyring to find out about a specific concern.
Self monitoring	Monitors understanding of teacher's explanation.
Production evaluation	Marks problem from answers in text.
Self evaluation	Evaluates teacher's answer to a student's question.
Revision	Writes note in her logbook to study quadratics at home.
	Affective Strategies
Effort control	Concentrating extra hard
Self encouragement	Assures herself she can still do the task in the allowed time.
	Resource Management Strategies
Task management	Selects only some of the notes to take down to make it easier.
Help from resources	Looks up a word in the glossary.
Monitoring the teacher	Monitors the teacher's movements around the class.
Listening to teacher's comments to others	Listens in on a teacher's explanation with another student.

Table 4: Karen's Reported Strategy Range from a Single Lesson

Although all target students reported a similar wide range of strategies there were a few notable differences which differentiated between students:

- Gareth reported no affective strategies (A1 - A4) and limited reports of 'self-questions or self explanations' (E5).
- Adam was the only student to report 'reflection' (M5) in all three stimulated recall interviews.
- The reported ratio of metacognitive control strategies to metacognitive awareness reports is greatest for Adam.
- No target students reported 'rereading' (C3), or 'determining the progress of the lesson' (R2), in stimulated recall interviews.
- Adam reported the largest number of task modification strategies.

6.4 LASSI-HS Questionnaire

The *Learning and Study Strategies Inventory - High School Version* (LASSI-HS) questionnaire (Weinstein & Palmer, 1990a) is an assessment tool designed to measure students' use of learning and study strategies at the secondary school level. Scores across ten learning and thought behaviours (Attitude, Motivation, Time management, Anxiety, Concentration, Information processing, Selecting main ideas, Study aids, Self-testing, and Test strategies) provide another form of student self-report data. However, the limitations of this quantified data are also significant. The test was designed to measure learning and study strategies across disciplines in an American context. However, the norms developed in research by Weinstein and Palmer (1990b) relate to a similar age group of grade 11 students.

Students' scores for each of the ten learning behaviours were converted to percentile scores from the provided percentile distributions. Box plots representing the class distribution for each of the ten learning behaviours are shown in Figure 4. The results suggest that the majority of students in this class reported limited use of appropriate learning strategies and study behaviours. In particular, low scoring categories are

Attitude, Time management, Self-testing, Selecting main ideas, and Information processing. It is of concern that all of these behaviours are directly related to autonomous learning behaviours valued by constructivist learning/teaching, and indeed, necessary for continuing mathematics study in tertiary education.

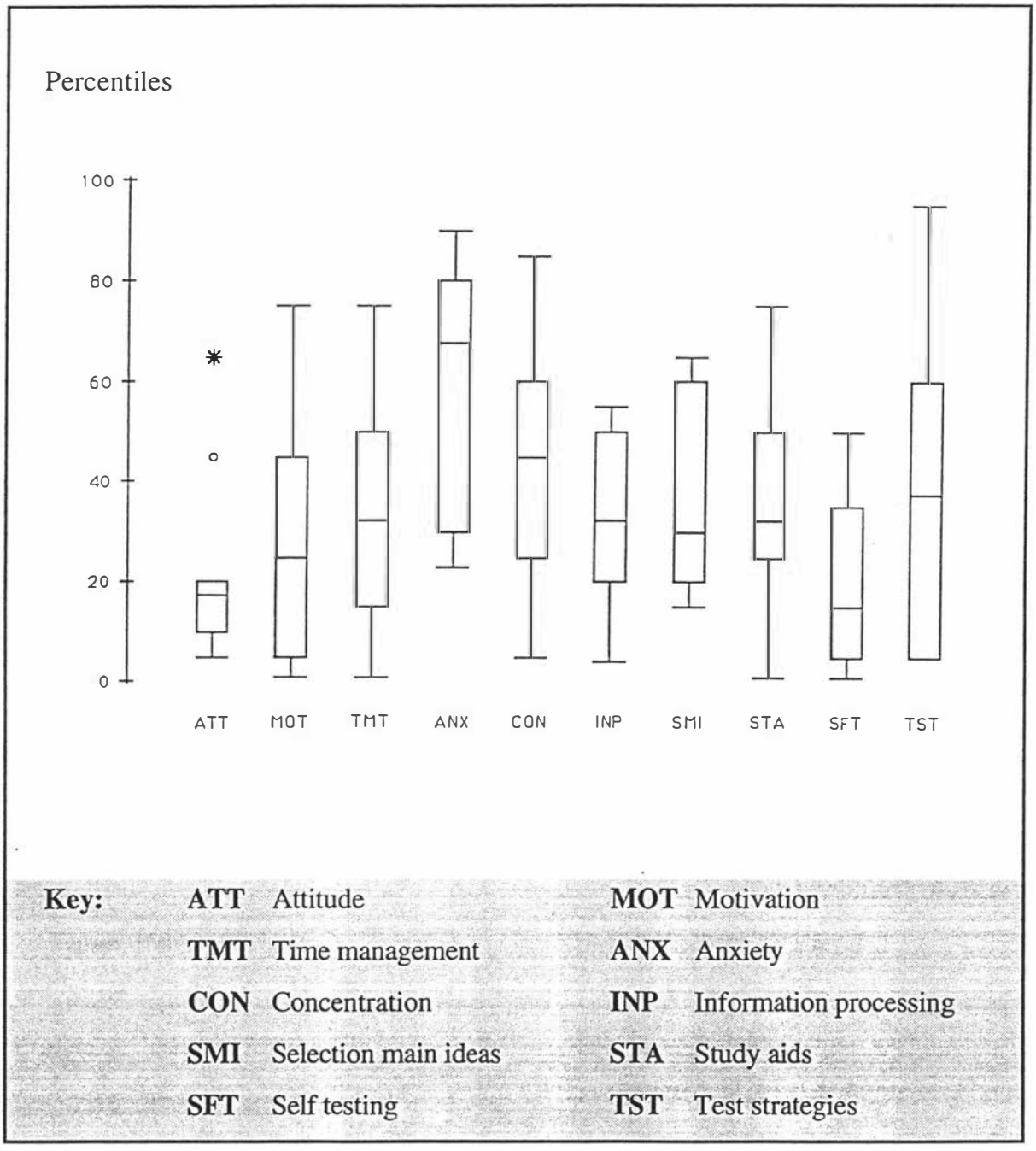

Figure 4: Distribution of Students' Scores from LASSI-HS Questionnaire

Diagnostic interpretations provided by Weinstein and Palmer (1990b) suggest that students who score low on attitudinal measure do not see school as relevant to life goals, and lack the necessary motivation to take responsibility for their own learning. Low scores on time management reflect a lack of ability to deal with distractions, competing goals and planning. Information processing scores relate to students' use and awareness of the cognitive strategies of elaboration and organisation. A students who does not have a repertoire of these strategies will find it difficult to incorporate new knowledge and understanding in such a way that acquisition and recall will be effective, often despite the large amount of time spent studying.

The variables of self-testing and selecting main ideas relate to metacognitive strategies. These low scores are very consistent with findings related to stimulated recall interviews. Only Adam reported being aware of actively self-testing his knowledge during class. During revision most students' self-testing was applied in a sporadic rather than controlled manner (see section 7.1 and 7.2).

On an individual level, Adam (an A grade student) was the only student to score in the 50th percentile, or better, for all ten categories. Even so, Adam also gained relatively low scores on information processing, self-testing, and selecting the main ideas categories. Possibly, Adam's low scores, along with the rest of the class, is a direct reflection of the learning demands of the mathematics instruction. It is argued in this thesis that a learning environment in which students are rarely expected to read for information, to provide their own summaries, to hypothesis, to plan for revision, to direct their own learning, and to reflect on their learning processes, does little to encourage and support the appropriate and effective use of learning strategies.

Moreover, the following chapters (Chapters 7 and 8) will demonstrate that the low scores on almost all of the behaviours of the LASSI-HS are consistent with the observed passive learning behaviours of most of the students in the class. Chapter 9 discusses factors related to instructional, student and context variables that appear to support and promote students' passive learning behaviours.

6.5 Summary

The following caution from Oxford (1990: 16-7), with regard to language learning strategy research, also appears timely for mathematics education research.

> *It is important to remember that* ***any*** *current understanding of language learning strategies is necessarily in it s infancy, and* ***any*** *existing system of strategies is only a proposal to be tested through practical classroom use and through research. At this stage in the short history of language learning strategy research, there is no complete agreement on exactly what strategies are; how many strategies exist; how they should be defined, demarcated, and categorized; and whether it is - or ever will be - possible to create a real, scientifically validated hierarchy of strategies.*

With this caution in mind the strategy classification appears to be sufficient for identifying those learning strategies reported by students of mathematics. While an examination of learning strategy frequency distributions indicates that students use a wide range of learning strategy, it also suggests that further investigation of issues regarding the nature of the metacognitive behaviours and the differential use cognitive, affective, and resource management strategies by individual students is needed.

Furthermore, the results of the LASSI-HS questionnaire (Weinstein & Palmer, 1990a) indicate an underlying tendency for students to report passive learning behaviours. Strategies such as elaboration, self-testing and information processing, which are necessary for the development of autonomous learning behaviours, appear limited in their use. The following chapters will examine the strategic learning behaviours in the classroom and home learning context, and discuss contextual factors influencing the development and use of strategic learning behaviours.

Chapter 7

The Role of Learning Strategies

Strategies are undoubtedly important in that they empower students to pursue cognitive goals of their own and thus be less dependent on school work procedures... A crucial issue is what goals the strategies are harnessed to.

(Bereiter & Scardamalia, 1989: 385-6)

The purpose of the learning strategy classification system is to provide a list of learning strategies as evidenced by the research data. This section reports examples of learning behaviours (strategies in use) from the classroom and home context that illustrate these cognitive, metacognitive, affective, and resource management strategies.

7.1 COGNITIVE STRATEGIES

Cognitive learning strategies concern behaviours that the student may use to acquire, retain, and retrieve information. Their purpose is to help students learn, remember, and understand the material.

Rehearsal

In mathematics the most common rehearsal strategies are imitation and practice. Practice is an important strategy for learning procedural knowledge; it aids both pattern recognition and action-sequence productions (Derry, 1990a). While class lessons provided limited opportunities for practice of procedures during seatwork, most students reported that homework gave them an opportunity to practice and consolidate class work.

Dean: *"I think it's* (homework) *important for the reconciliation* (sic) *of the work that you've done in class. Like if you don't do your homework you sort of forget what you've done for the next day."*

Craig: *"I think homework helps you to remember how to do the problems. It's good for practice."*

Kane: *"Homework is very important. Like when you do it in class you sort of understand it there, but when you try and do it the next day without doing homework it's really hard. But when I do homework afterwards and sort of study the stuff I've learnt in class it sort of sticks in my mind better."*

Int: *"What sort of homework is most useful for you?"*

Kane: *"Set exercises, just repetitively doing what you've learnt for practice."*

Adam, reports that he consciously uses class discussion time to enhance his learning via rehearsal of the content, and also to self-evaluate mastery.

Adam: *"I'm working through, finding the relationship. I'm sort of refreshing because I thought about it the night before the lesson, thinking it through again."*

In another lesson Adam reported:

Adam: *"I've finished all the questions so I'm just looking at things on the blackboard. I'm thinking, trying out the answers again even though I know the answers. "*

However, not all students appreciated the importance of practice as part of the learning process.

Abe: *"I don't think homework is essential. I think I could get 6th form certificate without doing homework."*

Gareth: *"I'll only do homework if it's hard. If it's easy I'll just whiz through the first line."*

In mathematics, concept learning, understanding, and problem solving require the learner to recall mathematics information (Swing *et al.*, 1988). The ability to memorise procedures and concepts enables the learner to use newly introduced procedures or concepts in both familiar and new situations. For this reason memory strategies, combined with practice, are particularly important for test revision. Most students reported reading their notes and trying some problems for revision. However, as the following report from Faye illustrates, some students relied solely on these rehearsal strategies with little thought to their effectiveness.

Int: *"What did you do for revision for your test?"*

Faye: *"I just wrote out the notes - that's it. I just rewrote my notes."*

Int: *"Why do you think that helped?"*

Faye: *"I don't know. I just wrote them down as they were."*

The value of rehearsal strategies is limited if they only involve duplicative processing or recycling the given information (Thomas, 1988). Without efforts to accompany rehearsal strategies with elaboration, organisation and monitoring, the students' knowledge is likely to be only useful for solving similar problems.

Despite the importance of memory strategies, and student reports that learning mathematics involves a lot of memory work, there were no specific reports of using mnemonics, check lists and diagrams for memorising formulae and rules. Most of the students appeared unable to discuss any further specific strategies for improving their revision sessions other than reciting formula, writing out formula, and rereading notes.

Gareth: *"If I have to memorise the formula that they won't give me or a graph I just write it out a few hundred times."*

Jake: *"Well I just revise over the notes, just go over the notes you've done and the ones in the book, just read them really I suppose."*

Gareth also reports relying on specific memory strategies to assist during tests.

Gareth: *"When I got stuck I wrote out some of the notes and stuff to help clear my mind...I think I get the work into my mind but it's hard to bring it out on paper."*

Furthermore, Gareth indicates that he equates practice principally with memorising; any learning that results from practice is incidental rather than intentional.

Gareth: *"As I work through it I might learn how to do it once and keep going with the same sort of ideas sort of thing."*

To summarise, many students appear to undervalue or be unaware of the benefits of a range of rehearsal strategies. Although rereading and repetitive writing were commonly employed, students reported little monitoring of the effectiveness of these strategies. Limited reports of memory strategies involving elaboration and organisation may mean that students do not use these memory strategies at this level, unless prompted by the teacher, or alternatively these strategies are automated, and students are unaware of their use. Additionally, student reports of memory strategies may be under represented because much of the research data relates to reports of classroom learning rather than home learning.

Adam was the only student to be aware of the value of repetition and over-learning. Perhaps an emphasis on the desirability to learn by understanding has created a dichotomy in student's minds: many students were more inclined to report that if they felt that they understood, or could do, problems they no longer needed to practice the exercises.

Elaboration Strategies

A basic tenet of constructivism is that, in order to learn new information students need to activate and utilise their prior knowledge, integrating it with new information in a coherent and logical manner. Elaboration strategies all involve making sense of the incoming information by adding details, explanations, examples and mental images that relate the information at hand to prior knowledge.

During the introduction of new concepts it was common for students to link new information to prior knowledge. However, most reported links were prompted by teacher directed comments. For example, in answer to the teacher's request Karen was prompted to recall her prior knowledge.

Mrs H: *"Let's brainstorm everything we know about quadratics."*

Karen: *"I was thinking like double bracket. I knew you had four operations."*

Forming mental images, and creating analogies and metaphors, are examples of metaphorical thinking which assist in integration and recall of new information (Marland *et al.*, 1992b). The teacher used metaphoric illustrations on several occasions. These included reference to mnemonics such as FOIL (First, Outside, Inside, Last) to aid recall of algebraic procedures and metaphoric images in graphing situations to aid recall of correct equation formats. For example, the outcome of the second derivative test was related to a positive ☺ or negative ☹ smiley face, which was then related to the corresponding minimum or maximum turning points on the curve. Another metaphor involved cubic graphs: the orientation of the cubic graph was related to the orientation of a snake (a positive coefficient meant that the snake's head was pointing upwards), and the shape of the $2x^3$ graph was contrasted with the x^3 graph: *"This one* ($2x^3$) *is older so it's taller and thinner."* Students were observed using these metaphors to aid recall on several occasions both in class discussion and during seatwork.

Int: *"In your test you had to know expansions like* $(a + b)^2$ *- how did you learn these?"*

Gareth: *"Well I just used FOIL or a smiley face like Mrs H showed us."*

Another elaboration strategy, the keyword strategy, was promoted by the teacher and reported by several students.

Mrs H: *"When you hear 'gradient' or 'tangent', what should you thinking about?*

This strategy, like mnemonics and metaphors, enhances linking and recall of information, but it s use has inherent dangers, in that students may actively seek the keyword without reference to the meaning of the problem. Some students' elaborations involved imagery, but reports of these were uncommon.

Karen: *"I was thinking back to functions and I was saying 'graph it', and I saw a picture in my mind of a function with a vertical line."*

While these elaboration strategies can facilitate learning it is important to note that the elaboration process, so critical for effective learning, was being performed by the teacher rather than the student. At no time was there any evidence that students invented or used any of their own metaphors. It is possible that students' acceptance of these strategies reflects the need to memorise complex procedures for mathematics tests.

Mrs H/Jane: *"Can you give me a key memory thing for this?"*

Jane: *"FOIL."*

There was some indication that the quality and frequency of student elaborations were related to the mood of the student. For example, Jane's report of a lesson on cubic graphs, which built on previous graph work, did not include any specific elaborative statements:

Jane: *"I'm just looking, not really thinking about anything."*

However, in a lesson on normal distribution, Jane appeared more interested and awake! She reported several examples of self initiated linking between parts of the lesson knowledge which resulted directly in learning:

Jane: *"I didn't realise that - you know how you find out .98 something. I didn't realise that it was a percentage and when she* (teacher) *said percentage and 98% I learnt something."*

There was also evidence of students attempting to make links with prior knowledge only to find that they couldn't remember, or didn't have the prior knowledge needed. This often resulted in frustration. Gareth attempted to compensate for his lack of prior knowledge by using resource management strategies such as skimming through the text book, and using the glossary and index to locate the current topics under discussion.

Reports of linking academic knowledge with personal knowledge were rare. In a lesson involving discrete and continuous variables, Karen reported:

Karen: *"I thought of the size of somebody's shirt but then someone else said that and I was looking at the light switch and I thought of light switches in classrooms."*

On another occasion Faye linked radians with work from another discipline.

Mrs H: *"What does radians remind you of?"*

Faye: *"One radius."*

Mrs H: *"How many radiuses in a circumference?"*

Faye: (very excited) *"Six - we did it in design drawing."*

Comparing or contrasting problem types and methods was a frequently used strategy. This was reinforced by the fact that students' initial seatwork problems usually related to examples on the board enabling students to match their working, either during or after problem solving, with the 'model' problem on the board. When problems were not on the board, students referred to the text worked examples. Additionally, when the teacher reviewed seatwork problems, or homework problems students reported comparing solution steps with their own, even when they knew that their answer was correct.

Karen: *"There are no more problems to do so I'm seeing if her explanation is sort of the same as what I get."*

Gareth: *" I checked to see if I'd done it - I counted the numbers like she did."*

Some students also reported noticing when seatwork problems were of a different type: this process of classifying problem types is important for effective problem-solving schema formation.

Gareth: *" I'm taking in that these are different sort of questions."*

However, more often than not, problems in a seatwork episode were of a similar structure providing little opportunity for students to discriminate between problem types.

Self-explanations are a large part of learning and engaging with mathematical information. Despite their importance, there were limited reports of self-explanations; possibly because of their covert nature they were more difficult to recall during stimulated recall interviews.

Karen: *"When she* (teacher) *did 'two to the what power' I wondered why these problems were in base two when all the rest were in base ten."*

Adam: *"I'm working through finding the relationship. I was thinking what is she* (teacher) *doing. I was thinking she's having problems drawing the graph. I was thinking about how to draw it, about how else to do it."*

Related to self-explanations are student attempts to ask and answer questions. Students query information, and form and test links between prior knowledge and new information in order to comprehend and extend their knowledge. This asking and answering of one's own question is perhaps the most powerful indicator of meaningful, active learning. However, the tendency to ask questions during the class discussion appeared to be more strongly related to the student, rather than the content. For example, Dean, Jake, Faye, and Lucy frequently queried the teacher about procedural steps in worked examples, and Dean and Gareth in particular, frequently tested their new knowledge by answering questions. Other students expressed concern about asking public questions but still reported private self-questions.

Karen: *"I don't really ask questions but if someone else in the class asks a question I'll listen."*

Abe suggested that the motive for participating in answering and asking questions was not always what it seemed:

Abe: *"I think a lot of it isn't genuine - um - most of the time they're* (students) *just hassling the teacher. I call out the answers to keep the discussion going."*

Overall, the quality of student elaborations was limited in forming strong conceptual links. While Adam gave several examples of more complex elaborative statements, researcher questioning during seatwork revealed instances when appropriate elaborations were not included. For example, when Adam completes a set of calculus problems involving the calculation of the gradients of tangents at specified points on the curve, he did not associate his answers with any visual images of the graphs involved.

During class discussion there was evidence of conceptual questions, especially by Jake, but these were often mumbled self-questions, or asked in a rhetorical manner which implied that a superficial answer was acceptable. Usually simplified answers were given, and students rarely challenged the teacher for further information.

There was a disturbing tendency for low-achieving students' elaborations to rely on recalling having done a similar problem rather than recall appropriate conceptual or procedural information.

Gareth: *"I can remember that we did it in term one. I can remember that it was quite hard."*

On occasions these were prompted by teacher comments such as: *"Think back to Form Three"* and *"Look back to Question 7 and do it the same way"* or *"Remember how we did it yesterday"*.

In summary, the academic elaborations required to form associative networks were generally very simple. Reported elaborations mostly involved only two discrete elements such as discrete procedural steps of a worked example or between parts of a problem, rather than between problem types or the overall conceptual issues of a topic. This simplistic elaboration attests to the view that students' thinking appeared focussed on detail, or pieces of discrete knowledge, rather than on interconnections between content.

Organisational Strategies

Making summaries and 'taking' notes has always been regarded as an important part of the mathematics lesson. It is common for many mathematics secondary school students to have a 'note-book' in addition to their practice book, and most student reported that reading their notes was a priority for test revision.

All students, regardless of mood (e.g., students may have been tired and not inclined to be very productive) or achievement, regularly took notes of worked examples, and copied teacher summaries from the board. All students copied notes that were explicitly provided by the teacher: some students automatically copied down any information from the board, and some checked with the teacher as to whether they should copy it down, especially on occasions when the teacher had not specifically cued note-taking (e.g., "*Take a new heading 'Logarithms' ").*

On one occasion when the teacher suggested that students should read through the notes first and then ask questions, most students immediately proceeded to copy the notes down; no student asked questions. Abe was heard to remark: "*Notes are like a get-out-of-jail card free."*

A few students reported making their own notes. These notes tended to be modified summaries of the teacher's notes, rather than self-generated summaries, or extra worked examples from board work resulting from class discussions. A comment from Jane illustrates selective note-taking:

Jane: "*I'm not writing these examples down because I understand it, but if she's* (teacher*) doing something and I understand it, but I know I won't remember it, I write it down. If it's something that I just don't understand altogether I won't write it down."*

Janes' cognitive learning strategy behaviour is affected as result of metacognitive monitoring combined with metacognitive knowledge. Jane's decision not to take notes if she does not understand the content may be unhelpful if no further action is taken.

In general however, students were reluctant to make notes or summaries unless prompted and directly assisted by the teacher. Early in the year Faye (an above average student) reported that she preferred the teaching style of the last year's teacher who handed out notes at the beginning of each new topic so that you didn't have to bother making your own notes. On one occasion during the third term I asked the teacher to set homework that required students to make a summary of a trigonometry section. The majority of these summaries consisted of a copied formula and sometimes a worked example from the text. Few pupils seemed able to determine what were the main concepts to be noted.

Occasionally the teacher annotated a worked example with a hierarchical task list, but on no occasion was any pupil observed to do this for themselves. The inability and reluctance to use strategies of paraphrasing and summarising are possibly linked with experiences in learning from the text. The texts used by the class were '*Form 6 Mathematics: Revision*' (Barrett, 1990) and '*Sixth Form Mathematics*' (McLaughlin, 1985). Both texts provided minimal explanatory material followed by a few worked examples and the content was divided into short discrete units of work. Texts presented in this manner provide little opportunity for students to construct outlines or reorganise the presented material. It is probable that the nature of the students' texts combined with teacher-provided summaries, not only this year but in previous years, directly influences students' ability and willingness to provide their own summaries.

A second possibility alluded to in Marland *et al.* (1992b:91), when discussing students' non-use of organisational learning strategies while studying from textual material, is the fact that students' "goals, in-text activities and assessment task, may have had an analytic rather than synthetic emphasis": thus reducing the need for students to employ organisational strategies. This is a strong possibility given the high level of specificity of learning objectives and the nature of assessment activities in mathematics. These factors will be discussed more fully in Chapter 9.

A third possibility is that the more complex organisational strategies like networking and concept mapping are not part of these students' repertoire of learning strategies. Although students valued the information from summaries, they appeared unwilling to participate in teacher prompts to jointly form concept maps and summaries. Students are apparently unaware, or undervalue, the potential learning value of these activities.

> *In a good summary, the learner extracts the key points that might serve as a conceptual framework or scaffold on which the learner can 'hang' details. Main ideas are easier to remember and, once recalled might be used by the learner to cue specifics.* (Swing *et al.*, 1988:129)

In interviews with all the students concerning note-taking, Lucy was the only student to comment on the benefit of making one's own notes:

Lucy: "*Writing your own notes makes you think more, you know, step-by-step sort of thing. When you write your own notes you tend to write more easier words and stuff but not maybe the proper terms, But when she* (teacher) *writes notes sometimes you don't really take any notice of them.*"

In considering Lucy's enlightening view, it is a great pity that students were not afforded more encouragement to prepare their own summaries. It appears that most students view note-taking and summary writing merely as a way to organise notes needed for test revision, rather than as a means of aiding comprehension. This inability of senior students to work independently to formulate summaries would be a major deficiency in learning skills necessary for tertiary study.

On a more positive note, all students had developed an effective organisational system for their work: notes were kept separate from their exercises and many students used headings and colour coding for emphasis, and coding signs for 'attention getters'.

Karen: "*If there's something I really don't understand I'll put a big red box around it and 'study this'.*"

Gareth: "*When I have to really know something, like an equation I have to know, I write it down in big letters or numbers.*"

7.2 Metacognitive Behaviour

Metacognition refers to students' awareness and knowledge of their individual learning process, as well as the ability and tendency to control this process during learning. Many recent studies have found that metacognitive behaviour has proved a vital component of expert mathematical performance and learning (Campione *et al.*, 1988; Swanson, 1990). Important metacognitive strategies for mathematics learning and performance are: planning for learning; reflection, or thinking about the learning process (Hiebert, 1992; Wheatley, 1992); monitoring the learning task (Anthony, 1991; Siemon, 1992a); and evaluating how well one has learned.

Planning and Previewing

Planning is directly related to one's goals. The influence of students' overall learning goals will be discussed more fully in Chapter 9; at this point only specific instances of goal setting will be discussed. On one occasion, when a major statistics assignment was given out, students sought clarification as to the nature of the task, and expressed concerns about the time allowed. The teacher listened and allayed concerns, thereby assisting students to negotiate a goal - an agreed contract was established between the student and the teacher as to expectations of the task. However, it is important to note that the establishment of the goal centred on task completion: students asked for clarification of time and length of the task. The teacher provided information regarding procedures for storing the data-tape and conferring with other students. There was little discussion as to the **learning outcomes** of the project - are they assumed to be incidental to the task completion?

To be fully effective in planning and controlling their own learning, students must be aware of their learning style, abilities, the strategies, and the **nature of the learning task**. With the exception of Adam, students' conscious use and reports of planning strategies were limited. Adam, concerned with identifying what ought to be learnt, often reported either having read ahead the night before, or in class. Although other students reported previewing the chapter at the beginning of a unit, they were more concerned to see if it looked hard, or interesting, or long.

In class students often flipped through the seatwork exercises in order to gauge the anticipated difficulty or length of the work. Also, most of the students previewed the homework during class time. As a result of this quick preview, combined with an evaluation of their understanding of the lesson and their consequent ability to complete the homework, several students reported making a decision on whether to take the homework home.

Because test and exam revision is in part self-initiated, evidence of planning would be expected. Homework sessions involved some limited planning by a few students - however, the basis for planning was usually related to time rather than any specific learning goal. Several students reported using their 6th form *Course Outline* to help plan and check topics for revision, but other students were unaware of the existence of the *6th Form Course Outline*, relying on the teacher to suggest topics for revision or using notes as a guide.

Gareth: *"I didn't need to think about the kind of questions- Mrs H gave us a list of things to learn and I knew there would be something on expanding."*

Those students who planned revision were more inclined to use strategies involving comprehension monitoring, rehearsal, elaboration and self-evaluation. In contrast, students who failed to plan tended to rely more on rehearsal strategies. Their study sessions, confined to the night before revision, involved re-reading of notes, writing out notes, and practice with a few examples. Time constraints, rather than evaluation of performance against a goal, determined the length of study time. The following extract from an interview with Jane illustrates the consequences of lack of planing.

Int: *"How do you decide what exercises to do for revision?"*

Jane: *"I don't know, just pick some of them I suppose."*

Int: *"How do you decide when to finish revising ?"*

Jane: *"I don't know, just when I get sick of it, when I've finished doing the stuff I need to."*

Predicting

Students reported anticipating the teacher's answers, other students' answers, or predicting the teacher's next example or lesson segment. As a direct consequence of Adam's predilection to preview content, Adam was often able to predict the direction of the lesson.

Adam: *"I'm thinking about the normal distribution. I know she's* (teacher) *going to talk about that because what we are doing she told us is about the normal distribution. I've looked ahead in my book to see what was coming."*

It was common for some students to race the teacher and try and work out the answers - this acts to monitor their understanding, and ensures that student are actively involved in the lesson. Students who are actively involved with the teacher's explanation are in a better position to elaborate. When answers do not match their own anticipated answers, they should receive sufficient stimulus to self-question or seek help. This contrasts with those students who reported a more passive approach: *"I'm just watching and waiting to see what she* (teacher) *does."* However, there appears to be fine balance between anticipating the teacher's answer and reflecting on the present process or explanation. For example, Gareth seems concerned with concentrating on the arithmetic calculations involved in a worked example, and his efforts to race the teacher meant that little time was spent reflecting on the overall conceptual structure of the worked example.

Selective attention

In a normal class environment there is a large amount of peripheral information in each lesson, whether it be mathematical, disciplinary, social or administrative in nature. Leinhardt and Putnam (1987:570) contrast the classroom setting to learning in an apprenticeship mode, for:

> *Unlike a person learning in an apprenticeship setting, a student in a classroom must be able to anticipate and respond to critical points in the lesson because they may only be repeated a few times...important concepts in a mathematics lesson may be mentioned only once or twice.*

The features of the learning task most likely to capture a student's attention are determined to a large extent by the student's expectations about the learning task and prior knowledge. If these expectations and/or prior knowledge are inconsistent with the desired learning then the student may focus on the wrong characteristic of the task. Failure to select and focus on the critical procedures may lead to 'buggy' procedures (Leinhardt & Putnam, 1987).

The following learning episode illustrates the necessity of selective attention to focus on the important ideas of a lesson. In a lesson on rationalising surds the teacher demonstrates rationalising $\sqrt{21}/\sqrt{2}$:

Mrs H: *"What would I have to do to get rid of* $\sqrt{2}$*? What would I do to the* $\sqrt{2}$ *- and I have to do it to the top and bottom?"*

Faye: *"Square it."*

Mrs H: *"If I square it I get* $\sqrt{21} \times \sqrt{2}$ *on the top and* $\sqrt{2} \times \sqrt{2}$ *on the bottom equals* $\sqrt{42}/2$."

Unfortunately the instruction to "**square it**" was an incomplete description of the process which in fact did not explain clearly that one needed to multiply the numerator and denominator by $\sqrt{2}$! Gareth focused on the squaring instruction and did not elaborate this instruction with the $\sqrt{21} \times \sqrt{2}$ - thus a 'buggy' algorithm resulted. Confident that he had the correct method, Gareth completed several exercises without checking the answers.

Gareth: *"It was making sense not having surds on the bottom because it wouldn't go into any other number...I think they are all right so far. I'm just following what's on the board."*

Gareth proceeded to solve the likes of $2/\sqrt{7} = 4/7$.

Constructivist learning theory suggests that students need to constantly compare and link new information with prior knowledge: selective attention is necessary if students are to challenge existing ideas by attending to discrepant information (Perkins, 1991; Pintrich *et al.*, 1993).

The following extract, from a stimulated recall interview with Karen, illustrates her efforts to attend to and discriminate important conceptual information. The teacher introduces an example application of logarithms. Returning to a problem introduced in the previous lesson, the number of bacteria (n) is given by, $n = 1000 \times 10^{(t/2)}$

Mrs H: *"Yesterday we did how many days until we had 10 million bacteria* (n=10000000*) and in that case it was quite easy. We found that if you put it all in and divided through it was 8 days?"* (raised intonation). At this point the teacher counted the digits from right to left in 10000000.

Karen: (Reported thinking) *"I thought back to yesterday. I knew it was 8 days. I didn't know quite why it was 8 - she was counting the zeros. I counted through and realised it was 8 numbers and she had done that yesterday. It's probably something to do with base 10."*

Unfortunately Karen's focus on the counting of digits has drawn her attention away from the focus on $10\,000 = 10^{(t/2)}$ which would enable her to deduce that $t = 8$. However, Karen is still feeling rather uneasy about her reasoning (metacognitive experience). When the teacher does another example, the anticipated counting is not used, and Karen is able to resolve the conflict and correctly focus her attention on the algebraic procedure.

When processing information in a classroom, students reported ignoring sizeable blocks of time, or data, either because there was an initial signal that the teacher is 'going off on a tangent', or because at some point the information seems irrelevant.

Jane: *"She's (teacher) on about all these z's and x's and all that stuff and it's hard to understand what she's going on about.*

Int: "*Did you make any effort to understand?*

Jane: "*Not after a while, I couldn't understand what's she's* (teacher) *going on about. She was saying that we didn't have to use it, it's off the topic, not really worth listening to."*

Cue-seeking behaviour is another form of selective attention. Before a test students must determine what it is that they need to be able to do as a result of studying. Some students are sensitive to teacher provided cues (both implicit and explicit), and some may seek out criterion information before a test. Teacher cues include verbal emphasis, reiteration of a point, teacher provided summaries, and practice test questions and hints.

Mrs H: *"Make sure you put it in your log book as something important you will have to learn."*

Mrs H: *"Read Chapter 41 - it's not long and it's extremely important, and it will be in the test."*

Student reports indicated that the extent to which students were aware of and/or used cues varied considerably. Some students, like Dean and Faye, were active cue-seekers, but other students, like Gareth, were either unaware, or ignored such cues.

Dean: *"We haven't done these - is it going to be in the test?"*

A report from Brent illustrates the interactive nature of metacognitive behaviours affecting selective attention. Brent's awareness of the need to revise is tempered by planning decisions to selectively attend to only those aspects of the content that he feels able to cope with, and which he feels are 'going to be worth' learning, in terms of allocated test marks.

Brent: *"I won't bother revising the things that I think I won't be able to have a chance at really. I concentrate on some things, like if something that's quite hard is only worth a few marks it's not a big deal."*

Reflection

As noted in the discussion of strategy classifications (section 6.2) this category is confined to behaviours related to students' reflections about the mathematical task, procedure and solution. Reports related to the reflection on one's learning process and progress are reported under self-evaluation.

A few students reported thinking about the nature of a procedure or problem, rather than concentrating solely on problem completion or 'seeing how a problem is done'. For example, Adam, Jane, and Karen all reported several instances of reflection on an approach/method presented by the teacher or a student. They used criteria of 'ease of computation', 'understanding versus rule following' or 'efficiency' to make judgements about the suitability of methods.

Adam: *"I was thinking about a quicker way - now I see hers* (teacher) *I think through it and see if it really works - I check through to see if it works, and if it's really faster, and it seems okay."*

Karen reported thinking about a blackboard example, mulling it over in her mind, rather than rushing to copy it down - her report is rich in metacognitive activities of planning, anticipation, reflection and metacognitive knowledge.

Int: *"You waited until she* (teacher) *got all the graph finished before you started to copy down the notes?"*

Karen: *"Yeah, I was wondering what she was going to put on it, because this is what she did earlier. I'm still thinking about what it means - I think this helps my learning."*

However, when completing seatwork problems the emphasis was on completion and most students relied on peer or teacher assistance when stuck rather than self-reflection.

Int: *"Do you ever try checking by another method?"*

Dean: *"No, I just let the answers do that."*

Both Adam and Karen reported thinking about mathematics problems for extended periods of time both in class and out of class.

Adam: *"Sometimes I think for a very long time, probably half an hour, an hour, or a whole day thinking and sometimes just come up with the thing"*(answer).

Karen: *"Sometimes I think about my maths for a while after I've finished my homework - I still consciously think about it and sometimes when I go to sleep at night I think about maths."*

Monitoring

The ability to correctly monitor one's understanding and performance has a direct bearing on students' subsequent cognitive actions (Peterson *et al.*, 1984). Firstly, monitoring strategies should lead either to diagnosis, or directly to remedial action, and secondly, monitoring provides new metacognitive knowledge about the effectiveness of one's problem-solving procedures and learning strategies.

Reported data reveals two main types of monitoring behaviour significant in the learning process: monitoring one's understanding of teacher presented concepts and worked examples, and monitoring one's performance when completing exercises. In both situations students monitored and controlled their learning processes in individual ways.

All students reported numerous monitoring statements related to their understanding of the lesson content. A count of the monitoring of understanding statements in Jane's two lessons found that approximately two-thirds were related to not understanding, or trying to understand: *"I didn't know what she* (teacher) *was on about";* and one-third related to understanding: *"This time when she goes over it again, I understood it."* As a result of this monitoring Jane makes decisions about the nature and seriousness of her difficulties, and whether or not to take some remedial action.

In contrast, Gareth more often reported understanding the teacher explanations. His monitoring statements, *"I was a bit confused until she put the numbers in"* and *"I was understanding why she put the numbers over there"* relate to calculations or procedures rather than concepts. However, when Gareth attempted problems he demonstrated an incomplete knowledge of the concept of standard deviation and related procedures. Gareth's inaccurate monitoring is likely to have serious consequences: if he does not notice that he is not understanding, then he is unlikely to engage in remedial strategic processing (Anthony, 1991). Gareth's inability to monitor understanding is further hindered by his attempts to provide reliable self-evaluations of performance.

When completing seatwork all students reported the influence of monitoring individual performance on their checking behaviour. Monitoring had a direct influence on the frequency, or need, to check work: if they felt they were getting the answers correct they would do several before marking; if they were stuck, or not sure, they would mark each problem on completion.

Dean: *"...if you know what you're doing you usually know it it's right, but if you don't know what method you're using; if I don't know what I'm doing I go straight to the answers and see if I can work back through the work."*

Evaluation

There are several components of students' self-evaluation: students may evaluate their performance on a task (production evaluation), or they may evaluate their learning process (overall progress in a lesson, or strategy evaluation), or their overall ability to cope with the learning task.

Checking one's work is the most common form of performance evaluation exhibited in the classroom. However, while most students reported checking calculations, few evaluated their work with respect to the operational choice or semantic sensibility. The criteria for evaluating the correctness of answers varies from student to student, as illustrated by the following two reports:

Int: *"Do you think you got that problem correct?"*

Jane: *"Yes, because I got the one before it correct."*

Int: *"Do you think this answer is right?"*

Adam: *"Yes, because I have done the steps correctly."*

Adam reports by far the most effective checking strategies. His combination of metacognitive knowledge of appropriate checking strategies, and his accurate monitoring, ensure effective and efficient checking procedures are used. The following is an example of a checking procedure invoked by a metacognitive experience:

Adam: *"I'm unsure of the answer so I'm thinking if its right or wrong. I know I'll get it right if I've put the correct things in my calculator - so I'm just checking that."*

None of the other students reported any self-checking of problems preferring to rely solely on the text, or on teacher verification.

All students reported evaluating other student's answers to class directed questions, but usually these evaluations were not accompanied by the explanations or justifications which one would expect with deep processing of the content. Criteria for judging the correctness of the answer was often based on knowledge of the person answering, teacher response, or a matching of the answer with student's own, rather than any critical examination of the content of the answer. For example, Gareth reported, *"I was thinking yes, he's right, his answer is the same as mine and if two people get the same answer it must be right"*. The fact that the teacher went on to negate the answer did not register with Gareth. This practice of evaluating answers by match is constantly reinforced by students' checking problem answers with textbook answers, or comments from the teacher such as, *"Did anyone else get 7.5 - fine, it will be right"*. The criteria for reasonableness of an answer, or appropriateness of method, was seldom modelled by the teacher.

There is also evidence of contrast between students in their evaluation of their overall performance or mastery. Gareth decides he understands and will be able to retain important information simply because he has done an exercise or 'understood' the teacher doing it:

Gareth: *"As I work through I might learn how to do it once and keep going with the same ideas sort of thing."*

Despite not being able to get any of the seatwork correct during a lesson on standard deviation, Gareth reported favourably about his progress:

Gareth: *"It was a good lesson. I learnt how to do the standard deviation. I learnt to keep drawing up the table. At the start it was difficult because I didn't know how to do standard deviation and plus that frequency column, but later on it got really easy. I'm good with standard deviation problems."*

In contrast, Adam makes a decision on understanding only after he tests himself (via problem completion), or otherwise generating feelings and information about his actual state of mastery:

Adam: *"The work on standard error is new. I take down the notes. I was wondering what standard error is. I thought I might ask my brother or father. I don't really understand why we do find the standard error that way. She (teacher) didn't really explain where it comes from."*

Lucy reported evaluating her learning progress at the end of each lesson, either as she packs up her work or waits for dismissal. A negative evaluation of understanding and progress results in a decision to 'follow up' with some further work at home.

Lucy: *"I sort of worry if I don't understand. I'll have a go through the work again, read my notes over."*

Revision

During stimulated recall lessons all students indicated that they were aware of the need to revise material. Karen made a note in her logbook; Jane added question marks against problems to be reviewed; Adam mentally noted to seek further help from family members; and Gareth noted details of exam revision tutorials and inquired about extra review work.

Adam made use of class time to revise. His intention was to check his understanding, over-learn, and manage idle time.

Adam: *"I just revise all these things in my head again, tried to remember them better."*

Metacognitive knowledge

An additional and critical component of metacognitive learning behaviour is metacognitive knowledge. Learners need a sound metacognitive knowledge base for effectively controlling and regulating their learning (Weinstein, 1988). Firstly, learners need to know something about themselves as learners - self-knowledge about one's skills, strengths and weaknesses. This helps learners to know how to schedule their study activities, and the kinds of resources or assistance they will need to perform efficiently and effectively. For example, Abe acknowledges that he is prone to make errors under test conditions and modifies his checking behaviour:

Abe: *"I get confused easily, like I'd have to check an answer about four times. In an exam I'll work out the answer and then I'd have to go back over and check it and often it's different so I'd have to go back over and check it again until I get the same answer twice."*

Secondly, learners need task knowledge - knowledge about the way in which the nature of the task influences performance and the anticipated or desired outcomes. For example, students need to know what is required when one revises for an open book test as opposed to a closed book test.

Karen: *"In an open book test you can go through the text and learn the formulas or find out from the text book how to solve it. So you don't have to do as much revision 'cause everything is in the text book."*

Brent: *"You don't need to learn all the formula and stuff for an open book test, but it seems harder 'cause the questions are not always the same as in the book."*

Unfortunately, the teacher's comment, *"Because you haven't had much time for revision this test will be open book"* does little to support formation of metacognitive knowledge about appropriate learning strategies for open book tests.

Thirdly, students need strategy knowledge - knowledge regarding the differential value of alternative strategies for enhancing performance. This knowledge needs to include actually being able to use these strategies, and knowing when it is appropriate to use certain strategies. However, as discussed earlier, many students appear relatively

unaware of their learning process. Moreover, they are unaware of alternative strategies and have little or no knowledge of how other students go about learning mathematics:

Adam: *"Revision time in class has reduced my nervousness..I don't know about other students, I don't know how other students prepare for tests."*

During interviews, and also during class lessons, students made statements about themselves as learners of mathematics, and expressed their feelings and attitudes about the content and learning, thereby revealing much of their metacognitive knowledge about mathematics learning. Figure 5 lists metacognitive knowledge statements from Karen's interview from a lesson on polynomials.

Karen's reported metacognitive knowledge during a single lesson

Self-knowledge

- I cringed, I hate algebra.
- I'm a slow writer.

Task knowledge

- I immediately thought quadratics - I can't do quadratics easily.
- $4x^2$ those are the ones I hate...I can't do these problems.
- The other one (text) doesn't get taken out of my locker. I don't like the way it's arranged.
- I knew that we had done these things before and that there are basic ones and harder type ones.

Strategy knowledge

- I don't tend to call out, I just sit there and listen.
- I prefer to work by myself at home if I don't get it.
- I don't spend at lot of time reading them (notes) at home. I think it's a better idea to try and read it now and try and understand it.
- I'm still thinking about what it means, I think this helps my learning.

Figure 5: Metacognitive Knowledge (Karen)

Metacognitive knowledge is not always clearly categorisable: often examples can be placed in more than one category, or involve the interaction of strategies. For example, Jane's metacognitive task and strategy knowledge were used to assist her in the following example in which Jane attempted an exercise not set by the teacher:

Jane: *"I wanted to do a whole question, not just finish off the one the teacher had started. I chose No. 8, because it looked big. I read it first and thought I could do - understand it. At the end of it I got stuck so I went back to No. 6. I thought it would be easier since it was back further, because they get harder at the end."*

In this case the strategy proved ineffective: the fact that Jane was conscious of her strategy selection indicated that the experience would then add to further metacognitive knowledge.

In summary, metacognitive activity usually occurred when students encountered difficulties with understanding a concept, following a procedure or completing an exercise. Although all students reported numerous instances of metacognitive behaviour, **qualitative differences** in their application of strategies were noted. Some students reported simply an awareness or knowledge of their thinking in general terms, such as; concentrating or not concentrating, thinking or not thinking, or 'easy' or 'hard. Although aware of their mental states, there was a disappointing lack of student attempts to analyse, evaluate, or direct thinking.

There was little evidence that low achieving students sought to explore alternative ways of resolving their problems in understanding. For example, apart from the knowledge that they were not understanding, or were stuck, several of the low achievers had little recourse but to depend upon the teacher 'going over' the material again. Of particular concern is students' limited use of those metacognitive strategies directly related to controlling learning, such as planning, previewing, reflection, selective attention and problem diagnosis. Without developing and using these strategies students are confined to 'passive' learning behaviours and are limited in their ability to behave autonomously.

7.3 Affective Strategies

Affective strategies result from affective responses or metacognitive experiences. Their purpose is to change learner attitudes and orientations towards learning. Stimulated recall interviews, general interviews, and observations revealed a high number of affective responses. Examples of affective statements in Table 5 illustrate the range of affective responses provided.

AFFECT	STUDENT RESPONSE
Pleasure	*"I like the look of No. 8 - it looks bigger."* (Jane) *"I felt happier when Gareth asked that question."* (Karen)
Anticipation	*"I thought great, I'll see how it's done."* (Gareth)
Relief	*"It was wrong..I was glad I didn't say any thing."* (Karen) *"Oh choice, I thought we were going to do graphs and functions."* (Gareth's response to new algebra topic)
Worry	*"I was worried about it."* (Karen)
Annoyance	*"There was no one around me...I couldn't talk to Karen or anyone - so I just had to sit there."* (Jane) *"You (teacher) tricked us."* (Faye)
Boredom	*"I don't do anything. I feel bored. I look at my book but for nothing in particular."* (Jane) *"I was getting bored so I thought I would do the next example."* (Karen)
Fear	*"I cringed, I hate algebra."* (Karen)
Disapproval	*"The noise does disturb my thought a little bit."* (Adam)
Frustration	*"I asked her* (teacher) *but I still don't understand."* (Jake) *"Can you show me how to use this stupid calculator?"* (Karen) *"Give us a chance. How long have you* (teacher) *been doing this stuff?"* (Dean)

Table 5: Affective Responses

Specific reports of affective strategies were limited. Adam reported an incident involving a puzzle type question used to introduce a lesson about sequences. When faced with difficulty he maintained his persistence and motivation by a positive self-talk dialogue: *"I just try to solve it, still solve, solve it. I think, sure I can solve it, and things like that."*

On other occasions students reported a boost of confidence and motivation when marking - evidenced by such comments as : *"Oh good it's right!"* and " *I knew that one, I felt really good about that."*

Metacognitive experiences such as the 'aha' - 'I see it now' type or feelings of confusion are also related strongly to an affective component of learning (Flavell, 1981).

Karen: *"When she put 13 - I just realised that you had to add 13 and change the equation."*

In the following example we can see how Karen's feeling of confusion with the teacher's demonstration of factorising forced her to confront her own conflicting knowledge and seek a resolution.

Karen: *"Now I was confused: you know how you've got to add to get to one and multiply to get to the other. I had it round the other way."*

In another instance Karen's feeling of confusion and uncertainty again led to extra effort:

Karen: *"I'd never heard of polynomials before, I was thinking it was probably something to do with one number. I was not sure what it had to do with quadratics, so I was concentrating extra hard, trying to find out what she* (teacher) *was talking about."*

At times some students (Karen, Jane and Adam) found the classroom routine and work boring - no such reports from Gareth. Each of these three students had individual and varied responses to boredom. For example, Adam reported thinking about science (his favourite subject) or 'filling in' the time with independent work (exercises or revision). Karen also reports that doing something is better than being bored:

Karen: *"I was getting so bored so I thought I would do the next example."*

Karen: *"I don't really revise unless I'm really bored and then I just read through my notes."*

For both Karen and Adam boredom was likely to be the result of 'knowing the work'. When boredom is a result of not being able to follow the lesson it was more likely to result in off-task behaviour.

Jane: *"I just didn't understand anything she* (teacher) *was talking about. I'm feeling bored. I don't bother to try for that one 'cause I don't understand graphs at all, so I wouldn't know what she's going on about."*

7.4 Resource Management Strategies

Resource management strategies are those activities learners engage in which afford the opportunity to learn: task control; setting control; and actively seeking help from resources, teacher, peers and other adults.

Modification of the task

Generally, teacher directed tasks were prescriptive, providing little opportunity for students to alter their approach to the task, or the task itself. Often the teacher set guidelines as to the expected time available to complete a task: *"By 9.50 I want you to be at Question 3a."* When the teacher set exercises to be completed in class, Adam skimmed through the exercises and evaluated the level of difficulty and time needed. He modified the task by adding a performance criteria of a time limit to complete seatwork, thus adding a challenge to a fairly routine task.

However, not all task management was beneficial to learning. For example, occasionally when Adam could find nothing better to do, he slowed down: *"I'm working slowly to fill in the time."* Gareth reduced the number of homework exercises when he felt that the homework was either too long or too easy.

Gareth: *"If it's easy, I'll just whiz through the first line."*

Int: *"What about trying some problems at the end of the exercise?"*

Gareth: *"No, I only do them if they are hard, because if I can work out how to do them, I just need to practice, to cruise through. If they are easy I'll just do like 1a, 2a, 3a."*

Despite knowing that "they get harder at the end" Gareth is satisfied that he only needs to do the first one of each exercise.

Only a few students reported occasionally looking, reading or attempting extra or alternative exercises when completing homework: most did not see it as **their role** to determine which exercises to attempt or explore.

Determining the progress of the lesson

By providing feedback to the teacher, students were able, or attempted, to change the pace of the lesson - and their consequent learning requirements and outcomes. For example, students sometimes attempted to slow down the lesson when they were experiencing difficulties by calling out comments such as:

Karen: *"We don't understand it if you go too fast."*

However, often complaints of going too fast were a way of covering for off-task behaviour. For example, Brent who had spent the seatwork time talking about weekend activities and had not attempted any work called out, *"You* (teacher) *don't give us enough time - then you do it on the board and we don't get to do anything."*

On other occasions students attempted to speed up the lesson by answering other students' questions, or by anticipating answers, or even answering two questions at once. In one instance Faye responded to the teacher's comment that the lesson was rushed by saying *"No, it's good"*. Faye also felt that homework review sessions were a "waste of time" and that her participation was largely to speed up the lesson:

Faye: *"It was a waste of time - I just sit there and talk or answer the questions if she wants some answers and no one else knows them."*

Monitoring the teacher

Monitoring the teacher was a very common student activity. Students were often able to split attention between what they were doing and teacher's questions or movement to the blackboard.

Karen: *"I'm reading the example. I'm sort of listening and reading at the same time."*

During seatwork, Karen and Jane were conscious of the teacher's movements around the class. They would 'save' a question until the teacher approached them or 'listen in' to a teacher explanation with another student. They used this strategy to gain help and to monitor their progress against other students.

Help seeking

Help seeking has recently been acknowledged by a number of researchers as representing an important aspect of school learning.

> *An independence-based view of help-seeking characterizes the help-seeker as acting maturely and purposely, alleviating a "real" difficulty, learning and mastering the task at hand, and in the end achieving autonomy* (Newman & Schwager, 1992:125).

Nisbet and Shucksmith (1986) suggest that although learning is largely intuitive, the learner should be able to move from the intuitive to the deliberate when some difficulty intervenes, stopping to consider the source of difficulty and selecting a strategy to deal with it. Students who know when (as a result of monitoring), how (strategy knowledge), and from whom to seek help, should be more successful than those students who do not seek help appropriately (Newman, 1991).

When having difficulty with a problem students reported a range of behaviours, some of which were effective and others ineffective. For example, Gareth reported that his keenness to answer questions was a form of help seeking:

Gareth: *"If I get a question wrong she'll* (teacher) *probably answer it in a more easier way to understand so I'll get it right. If you get something wrong she usually goes into the example in more detail."*

When Jane got a problem wrong she reported:

Jane: *"I've written the answer in red. I tried to work it out again, but I didn't get it, so I put a question mark next to it and went on to the next question."*

Int: *"What is the question mark for?"*

Jane: *"So that I'll know I didn't do it"*

Int: *"Will you come back to it later on?"*

Jane: *"I don't know, if I really wanted to know I could ask someone at home, but I probably won't bother."*

This illustrates an important facet of learning strategy behaviour: although Jane demonstrates a knowledge of help-seeking strategies and acknowledges the need to seek further assistance, strategy knowledge, in itself, is not enough to ensure that Jane will invoke the appropriate strategic behaviour.

Students gave various reasons for not seeking help. Jane reported a reluctance to seek help during class discussion as she feels that she is the only one that does not know what the teacher is talking about. Her belief is reinforced when other students answer questions:

Jane: *You notice other students answering. You think maybe you've missed something - here's another bit I didn't know."*

Similarly, Lucy's metacognitive knowledge of help-seeking strategies is a result of previous experience.

Lucy: "*She* (teacher) *sort of goes - oh no! I feel like it's a put down - it's not worth the hassle - it's easier to try and work it out yourself."*

Gareth, when he was stuck, turned to the textbook answer and tried to work backwards from the answer to his solution. He seems to believe that problem solving is simply a matter of trying all the operations and hoping one of them gives the same answer as the textbook. This belief appears to override any need to truly understand the problem. For example, when he had used π in the formula he divided the given answer by π to see if

he was out by a factor of π. When he was unable to locate the source of his error he asked for teacher assistance: *"I don't see how they got 69.8 for No. 2".*

More successful learners are likely to have available a network of help-seeking strategies from which to select appropriate actions related to the task problem. For example, a more active approach was used by Karen and Jane. They checked the answers for some prompting, but also referred back to worked examples in the textbook for assistance. Jane also reported the strategy of trying previous exercises (similar to a look-back strategy in reading) as a lead up to the more difficult problem. But, as is discussed more fully in the case studies (Chapter 8), Karen and Jane's help-seeking behaviours were not always adaptive and effective. Adam reports awareness of specific strategies to deal with a wrong answer:

Adam: *"My answer is wrong, I went back and reread the question and then I looked through my work again to see what I did wrong- I found the problem and crossed out the incorrect work and wrote it out again."*

Faye's help-seeking behaviours were highly visible and varied. She worked with a peer group by talking through the exercises out-loud and listening and responding to others' comments. Most problems were checked with a neighbour, and when difficulties arose help was sought and provided almost instantaneously from peers. When the group as a whole reached an impasse there was an immediate seeking of teacher help. This resulted in a very quick task completion rate. In contrast Gareth, a low achiever, spent a lot of time looking at notes, board work and answers, but made no attempt to communicate with his neighbour. He completed the exercises at a very slow rate.

Classroom observations provided further examples of help-seeking strategies such as copying a neighbour's answers; calling out statements indicating difficulty, such as *"this is dumb"* and *"how are we meant to do any of this"*; making eye contact with the teacher; frowning; and sitting in the proximity of a helpful student.

Cooperation between Peers

Despite the fact that peer interaction was neither required nor explicitly encouraged, interaction with neighbours was very common for some students. Students used peers to assist in monitoring their progress by checking where their neighbours were 'up to'; asked or assisted peers with explanations; and to ask for teacher assistance (that is, they jointly negotiated the need for help and agreed who should ask for help, or they both indicated the need for help thus increasing the likelihood that they would receive teacher attention). Jane's comment illustrates the importance attached to supportive peer interaction:

Jane: *"I think the others* (Faye, Lucy, and Karen) *got higher marks because they work together - it's helpful because they support each other. Last year I used to get help from a friend, but I don't sit with anyone in this class. Well sometimes I sit near Brent, but he doesn't want to talk about maths, usually we talk about other things."*

Collins *et al.*'s (1989) suggestion that peer help is often more effective than that of the teachers is reflected in some students' preference for peer help. Peers may have recently had the same or a similar difficulty themselves, and thus are often better able to assist other students to grasp the rationale of a new concept or skill.

Brent: *"When I'm stuck sometimes I'll have a think through myself and go to the back of the book and check the answer and I'll try to go through with the answer. If I can't get that, then maybe I'll ask someone next to me, like if they have it right, they can explain it, but if they don't know I'll ask Mrs H."*

Peers check others' progress in an attempt to monitor their own progress. In the following instance Lucy monitors Faye's work. The teacher had asked students not to copy the work from the board as she would make more formal notes later. Faye however, looks up the topic in the text, and with her text on her lap, under her desk, she proceeds to copy the text summary:

Lucy/Faye: *"What are you doing, what are you writing down?"*

Faye/Lucy: *"I've just copied the notes out from the book."*

Karen, Faye and Lucy often worked cooperatively, sometimes allowing other students into their 'group', but only with their approval. The following illustrates peer cooperation between Faye, Lucy and Karen in seeking help from the teacher:

Karen: *"I'm lost"* (directed at the teacher who approaches). *"Lucy you listen in, you are just as confused."*

Faye: *"I think I'm right now."*

In another instance Faye and Karen cooperated with help sharing, but it should be noted that it involved more than just simply one student giving the other the answer:

Karen: *"Could you do No. 8?"*

Faye: (Hands over her book to Karen) *"Learn how to do it, don't just copy it."*

Although Faye, Lucy and Karen's cooperative efforts were usually effective in enhancing the learning process, it was noted that their learning episodes were more often than not interspersed with off-task talk. Marland and Edwards (1986) suggest that controlled off-task behaviour may be effective in providing needed breaks from cognitive engagement (a form of task management). However, in general peer contact appeared most beneficial when closely associated with relevant task or content.

There was another grouping in the class (Brent, Abe and Dean) that regularly worked together, but often for social and psychological reasons rather than academic goals. Much of their cooperative behaviours involved only off-task behaviours. They developed coping strategies of asking for teacher help as she approached their group, so as to appear interested and involved, and managed to avoid the teacher scrutinising their lack of real work.

When peer cooperation was used to cover up for lack of understanding, or to provide answers, or jointly disrupt the lesson, peer interaction did not enhance the learning process. In many instances peers fell into a pattern of answering each others' questions. Some seemed especially adept at doing this when they knew that the targeted student had not done the required work. On other occasions students reinforced each others' comments concerning workload, difficulty, or speed of the lesson, not in a genuine attempt to direct the lesson but rather to cover for student off-task behaviour.

Environmental Control

Classroom learning demands control from the numerous distractions and attentional stimuli (Corno, 1989). Seating arrangements were the most common form of environmental control exercised by students. For example, Gareth, who liked to answer teacher questions, and preferred teacher help over peer help, always sat at the front of the room. Predictably, Brent, Abe, and Dean always sat towards the back of the room. Several students made conscious decisions as to which students they would sit with. For example, when beginning a new unit on calculus, Kane, a 7th form student, was asked by Faye if he was good at calculus (from last year), on replying that he was, he was invited to sit with the group for that unit.

On other occasions students would ward off personal attention so as to protect their learning opportunities: "*Why'd you ask me, ask the teacher.*" Dean indicated an awareness of the importance of controlling peer distractions:

Dean: "*I could improve my concentration if I sat separately. I know that I should be paying fuller concentration but I sort of get easily distracted, After I got kicked out of class I have been sitting separately for about the last five lessons. It's been a good way to concentrate. If I don't concentrate I can still comprehend the lesson but it doesn't stay there.*"

However, Dean only occasionally avoided the temptation of personal distractions by removing himself from his peers.

7.5 Summary

Do these reported and observed learning strategies promote higher-order thinking, knowledge construction, and appropriate metacognitive knowledge? The examples of students' specific use of learning strategies indicates that a wide range of strategies are employed. In some instances learning strategies appear to enhance the learning process. In other instances students' use of learning strategies appears to be either ineffective or inappropriately directed towards the goal of task completion.

Mathematics in the New Zealand Curriculum (Ministry of Education, 1992) suggests that students should develop the ability to reflect critically on the methods they have chosen, express ideas, listen and respond to the ideas of others, critically appraise mathematical arguments, explore and conject, learn from mistakes as well as successes, set learning goals, and access a broad range of resources. While examples of these strategic learning behaviours were in evidence, their use was limited.

Rehearsal strategies involved practicing exercises and frequent reports of rereading. Rereading of notes was sometimes accompanied by self-summarising, but often (even with high achievers) processing appeared to be duplicative rather than generative. Elaborative strategies involved linking between parts of the problems, limited use of imagery and no reports of student generated metaphors or analogies. Reports of self-questioning appeared effective, but use of in-class questions was determined largely by personal factors.

Although research shows that metacognitive strategies are important (Anthony, 1991; Campione *et al.*, 1989; Herrington, 1992), many students in the present study used these strategies with little awareness of their importance. Metacognitive strategies were reported more frequently than cognitive strategies, however they were limited in their range and effectiveness. Numerous monitoring and evaluation strategies were reported but resulting behaviours were not always appropriate or effective. For example, reports of not understanding or being stuck did not always lead to appropriate changes in the learners' behaviour.

Reports of numerous affective reactions are consistent with a constructivist learning perspective. However, metacognitive experiences and affective reactions need to be controlled or harnessed in an appropriate manner. More successful students appeared to deal with negative affect in a positive manner, whereas less successful students appeared somewhat unaware or unable to invoke appropriate or effective strategies to alleviate problems of confusion, boredom or frustration.

Resource management behaviours are particularly necessary in a social learning context. Students who invoked appropriate resource management strategies were able to adapt themselves and their learning environment to maximise their learning opportunities. The use of resources and appropriate help seeking was demonstrated as especially important, and it characterised the more successful students. Students who sought appropriate help gave evidence of not only monitoring their own cognitive processing, but also of an attempt to alleviate their difficulties and ensure success. In contrast, those students who were reluctant to seek help and who sat in class disengaged from the learning process appeared to have defaulted on their potential self-control. Additionally, behaviours which involve the opportunity to attend to peer activity, verbal or non verbal, are related to student learning. In some cases direct facilitative contact with peers appears to be more consistently related to student learning than direct contact with the teacher.

To answer the question posed at the beginning of this summary more fully it is necessary to consider students' individual use of learning strategies. It appears that the use of learning strategies *per se* is not inherently indicative of purposive, intentional learning behaviour. The implementation of an appropriate and effective strategy is more important than knowledge of strategies in general. The following chapter will discuss these issues with case studies of individual student's actual use of learning strategies.

Chapter 8

Case Studies

What a student learns depends to a great degree on how he or she has learned it.

(National Council of Teachers of Mathematics, 1989:5)

8.1 Introduction

In comparing students' strategy use one could simply list strategies. Table 6 compares the average (rounded) frequency of reported strategies per stimulated recall interview for each student. Where several categories achieve similar goals they have been grouped together (See section 6.1 for the classification system).

One can see that strategy frequencies are relatively similar and little information is gained about the appropriateness and effectiveness of each strategy application. As already noted, simply listing reported strategies used does not reveal the value of the strategies to the learning process. For example, it would be presumptuous to conclude from these statistics that the large number of elaborations reported by Gareth implies that he was more active in encoding and linking information than, say Adam. When we take the presage factors into account, it could be argued that Gareth's lack of prior knowledge meant that there were more conscious attempts (often accompanied by cognitive failure and metacognitive experiences) to elaborate. Similarly, one could argue that Adam's superior prior knowledge meant that accessing information was automatic in many instances, and thus not reported.

An alternative to examining frequency counts is to consider the effectiveness of any particular strategy in relation to the individual student, the learning goal, the task demands, and the learning context (Garner, 1990a; Pintrich & Schrauben, 1992). For

example, Adam's large number of task modification strategies may be related to the purpose which Adam expects the lesson to serve. As will be detailed in section 8.5, much of Adam's classroom learning is directed towards identifying what ought to be learned - he appears to use classroom time for consolidation and rehearsal, and often needs to employ task management strategies to avoid boredom.

Strategy	Gareth	Jane	Karen	Adam
Cognitive Strategies				
Rehearsal	5	5	4	4
Elaboration	22	14	20	14
Organisation	5	4	7	4
Metacognitive Strategies				
Planning (M1, M2, M3)	4	3	8	6
Attention (M4, M5, M6)	8	4	5	7
Monitoring (M7, M8, M9)	21	27	21	18
Revision (R10)	2	0	3	1
Metacognitive knowledge and experiences (MK, ME)	8	14	15	6
Affective Strategies				
A1, A2, A3, A4	0	1	2	2
Resource Management Strategies				
Task management (R1)	1	2	3	5
Pace of lesson(R2)	0	0	0	0
Help (R3, R4, R5, R6)	8	3	5	4
Peer Cooperation (R7)	0	0	1	0
Setting (R8)	0	0	0	0

Table 6: Frequency of Reported Learning Strategies

This chapter discusses strategy use in various classroom learning episodes, including homework and test revision, for each of the four target students. The case studies synthesise data from all of the data collection strategies to provide a comprehensive and reliable learning strategy profile for each of the target students. The chapter concludes with a discussion relating students' prior knowledge, learning goals, and use of learning strategies to passive and active learning behaviours.

8.2 Case Study 1: GARETH

PROFILE

Gareth is a seventeen year old student who feels he is not very good at mathematics. He views mathematics as important for his career but finds learning mathematics "*pretty hard as it involves mostly memory work*". Despite performing very poorly in assessment tests Gareth applies himself fully and is **ever hopeful** of improving his grade:

Gareth: "*I work very hard at maths. I have to work my hardest at this subject because I've always been bad at maths.*"

He reported that "*learning maths is done by just looking through the book and reading your notes and doing problems non-stop*". Gareth prefers to do mathematics in class where he can get 'on-line' help from the teacher.

Gareth's interviews and observed behaviour suggest that he strongly believes that doing a mathematics problem correctly, or having a record of how to do a problem, is what learning mathematics is all about:

Gareth: "*As I work through it* (Standard deviation table) *I might learn how to do it once and keep going with the same ideas sort of thing.*"

Int: "*How will you learn which columns to total up?*

Gareth: "*You only total up about three of the columns out of the whole graph* (sic). *When I come to revise, if I have to, I'll just look back in my notes and see which ones I totalled up.*"

LEARNING EPISODES

During class discussion Gareth is always attentive. Gareth's lack of prior domain knowledge severely limits his ability to use the necessary elaboration strategies such as linking, paraphrasing, imagery or self-explanation. Gareth does attempt to recall prior

knowledge in response to teacher prompts, but does not always achieve the teacher's intended outcome. For example, in the following episode the teacher requests that the class recall why one would square the ($x - \bar{x}$) column. Rather than recalling the previous lesson content, Gareth retrieved his earlier numbers framework.

Gareth: "*I was thinking in my head, how do you, why do you square it and stuff like that, and I thought of an answer, I wasn't really sure. I came up with an idea that you square it so you get a bigger number that's easier to work with.*"

Often Gareth tries to compensate for lack of prior knowledge by locating the text reference when the teacher introduces a new topic. This may enable him to answer the teacher's questions based on what he reads in front of him, but Gareth possibly deludes himself into thinking that he understands the topic.

During the discussion time Gareth copies all the information from the board. When attending to discussions on worked problems he attends to each procedural step separately, rather than the links between each step. Because of his focus on individual calculations he is often able to answer teacher questions such as "*What will the mean be?*" or "*Factoring will give you?*"

Gareth: "*I had the answer because I'd finished it off as she* (teacher) *was talking about the grids*" (standard deviation tables).

However, in selectively attending to the calculations, Gareth often ignored the teacher's explanation of conceptual material. In the following example Gareth misinterprets the teacher's comment about the **"next column"** as **"nx column"**, but later dismissed this concern when he sees how to do it with numbers!

Gareth: "*I had the mean right, but not the rest. I was a bit confused when she* (teacher) *started talking about* ***nx column*** *and stuff.*"

Int: "*You were a bit confused when she talked about ?*"

Gareth: "*The nx, the column, she was going on about some column. I was a bit confused until I saw how she put the numbers in on the board.*"

Gareth's focus on procedural knowledge, with few apparent connections between procedures and concepts, is evident in his handling of algebraic symbols: absent from the use of algebraic symbols is any link with conceptual concepts. In an episode involving the calculation of standard deviation Gareth has little idea of what each of the variables *f*, *fx*, and $(x - \bar{x})$ represent. He sees them merely as headings of the graph (sic) which is used to organise the calculation procedure:

Gareth: "*I could see why she put the frequency and the* f^2, *fx, thing up because we save a lot of space in the box.*"

Int: "*What do you think the 10 was in the total?*"

Gareth: "*The f one- the total of all the frequency data added up.*" (Gareth does not know where the frequency score comes from, thus he doesn't know that the total 10 relates to the total number of data items.)

Int: "*What do you think the fx column represents?*"

Gareth: "*Um, fx is the frequency plus x; oh no, it's times x, so it's just the frequency times the tally, the data next door to it. In the graph you've got the x and you've got the frequency so you just times f times x.*"

In light of Gareth's metacognitive knowledge (beliefs about learning), and focus on computation, it is not surprising that specific references to understanding such as: "*I was understanding why she put the numbers in over there*" and "*I understand why she put the 'f ' row* (sic) *there*", commonly refer to arithmetic or organisational features of an example.

Compared with the other target students Gareth is active in question answering, especially answering cued questions. For example, when studying calculus, questions such as "*What do we do first*?" always elicited the response "*Differentiate*" - a fairly safe answer! However, about 30 percent of his answers are incorrect. Gareth reports that guessing answers was an appropriate strategy. He reasons that if you are wrong the teacher will give you the correct answer, and it is good to answer lots of questions as it will go on your report at the end of the year - "*Gareth participates in class discussion or stuff like that.*" Thus it appears that Gareth employs the strategy of answering

questions to gain teacher approval and information, rather than to assist in monitoring understanding and elaboration.

As well as answering questions, Gareth evaluates other students' answers. However, the criterion for evaluating correctness is often based on knowledge of the person answering, teacher response, or matching of the answer with his own, rather than any critical examination of the content of the answer.

Gareth: *"I was thinking yes he's right, his answer is the same as mine, and if two people get the same answer it must be right."*

Gareth did not notice that the teacher went on to negate the answer. Gareth's practice of evaluating answers by match is constantly reinforced by student's checking problem answers with textbook answers, or comments from the teacher such as *"Did anyone else get 7.5 - fine, it will be right"*.

Gareth also reported evaluating teacher given methods, but again the criterion was based on ease of computation rather than understanding.

Mrs H: *"This is quite a complicated step."*

Gareth: *"It wasn't really as complicated as she* (teacher) *said. You just shift the tables from one column to the next."*

Seatwork

Gareth always works on-task during this period, being one of the first in the class to begin work, and one of the last to finish at the end of the lesson. Gareth usually goes straight to the first question. However in Video 1 Gareth skipped the assigned reading and flipped through the set questions to evaluate his ability to complete them.

Gareth: *"I'm looking at the questions we have to do - it looks quite a lot - it's all mainly the same thing, like finding the mean, mode and stuff."*

Gareth initially uses examples on the board to guide him through the first few problems. This strategy is reinforced by the fact that seatwork problems are always strongly related

to what the teacher has just done. The following sequence from a lesson on simplifying surds is typical of Gareth's approach.

Gareth: *"I'm feeling okay about starting (d), I'm trying to look up at the board to see how, follow the steps through. I got* $\sqrt{48} = \sqrt{3\times16}$ *, so I thought there's no square root of 3 so I swapped that around to* $\sqrt{16\times3}$ *so I had the same pattern as on the board."*

On the next problem, of a different format, Gareth has some difficulty. He looks at the board and raises his hand just as the teacher writes the answer up on the board for the class. Gareth copies it down and continues onto the next problem reporting:

Gareth: *"I was okay about these, all the steps were around and she* (teacher) *made it look easy all the time."*

Further into the lesson Gareth asks the teacher *"How do you do* $\frac{\sqrt{21}}{14}$ *when there are no factors?"* Gareth has copied the problem down incorrectly and is concerned that the answer in the text is $\frac{\sqrt{3}}{2}$

The teacher also misreads Gareth's problem and demonstrates $\frac{\sqrt{21}}{\sqrt{14}} = \frac{\sqrt{7\times3}}{\sqrt{7\times2}} = \frac{\sqrt{3}}{\sqrt{2}}$.

Int: *"Why did the teacher put the square root sign on the 14?"*

Gareth: *"I didn't ask because she knows what we are after."*

Int: *"How do you solve this type of problem?"*

Gareth: *"I don't know, oh you take the square root of 21 or something like that, she* (teacher) *came out with the right answer."*

This episode reveals the difficulty Gareth faces when problems are tackled as copies of examples. The first problem required Gareth to take out a perfect square factor, a procedure Gareth seems to have been able to transfer from the teacher's examples, but the last problem required cancellation of surd factors, a process which Gareth had no model exercise on which to base his solution. Moreover, Gareth seems unconcerned that the teacher has changed the problem, implying that the correct solution (although it wasn't) justifies the means.

When examples are not on the board Gareth will, where possible, complete seatwork problems with reference to worked examples in the text. Sometimes he will copy the worked example before starting a section of exercises. He includes the explanation statements as supplied by the text. Sometimes Gareth puts these explanations alongside his own working *"to help remember what to do"* but he provides no elaborative statements or self-explanations for the procedural steps during this process. In contrast, research concerning the use of worked examples (Chi and Bassok, 1989; Anthony, 1991) found that good students make frequent self-explanations of the procedural steps which assist elaborative encoding of the new material.

To complete the problems Gareth copies the step-by-step procedures used in the worked examples. This strategy can lead to incomplete or incorrect solutions. For example, when completing the exercise: "Find the turning point of the function $y = x^2 - 6x+11$, and the values for which the function is increasing or decreasing", Gareth copies the steps: "differentiate and solve for 0", to successfully find the turning point (3,2), but continues by following the given steps: "substitute x = -2" and "substitute x = 0". Gareth interpreted these two text explanations as generalised rather than specialised procedures and applied them literally to his problem! Gareth's misuse of the supplied text explanations reinforces the idiosyncratic, constructivist nature of learning. For a good student who can generate his or her explanations, the given explanations would be redundant. However for the weak student who has little understanding, such explanation may actually confuse rather than clarify, and perhaps limit learning (Anthony, 1991). Gareth's reliance on these explanations supports Blais' (1988) position that providing students with a maximum of explanation will often serve to perpetuate the "remedial processing" of novices.

To evaluate his learning Gareth relied on task completion, checking with text answers or teacher verification. The frequency, or need, to check problems was related to his monitoring activities; unfortunately he often equated doing a problem with the strong possibility that it was correct.

Gareth: "*I was putting a few ticks and crosses - it was some work on quadratic equations that I did last night. I knew which ones were right.*" (not referring to any answers)

Int: "*How did you know which ones were right?*"

Gareth: "Well if I couldn't do them, um, they must be wrong and the ones I could do were right."

The following episode relates to standard deviation problems:

Gareth: "*I felt I was getting them right. If I had felt worried I would have looked up the answers straight away. When I looked the answers up I found that I got them all wrong.*"

Int: "*Why did you get them wrong?*"

Gareth: "*I got them wrong because I didn't copy down the graph* (lines of the table) *because in the book it said you didn't have to copy down the graph.*"

Again we see the influence of procedures rather than reasonableness of the answer on Gareth's self evaluation of difficulties. In this problem tables were not seen as a procedural tool to organise the data, but rather they were regarded as a major determinant of the correctness of the solution.

Gareth devotes a lot of attention to peripheral aspects of the task such as ruling up pages, writing the date, and ruling up tables. He reported using coding strategies to assist in highlighting important content. However, the coding is to aid recall of the material rather than assist integration or organisation of new content knowledge.

Gareth: "*When I have to really know something, like an equation I have to know, I write it down in big letters or numbers.*"

Help-seeking

Gareth reported, "*I like doing problems in class because in class you can get the teacher to help you if you have problems. You learn maths when you work one-to-one with the teacher*". When Gareth asks for help he expects the teacher to show him how to do the problem:

Gareth: "*It's easy with a teacher there because if you've got problems you can go and see her* (teacher) *and she'll tell you the answers and go over the questions.*"

This is usually the case; when the teacher did help Gareth she directed Gareth's work towards presentation and production at the expense of conceptual validity. Often the teacher either writes all, or most of, the solution out for him.

Gareth: "*I liked her* (teacher) *writing down the step in my book, it's something I could refer to if I got stuck.*"

Clearly, getting the teacher to do the problem, or copying step for step from worked examples, are effective strategies for achieving Gareth's goal of task completion, but they offer little chance of any real knowledge being constructed. At best Gareth's learning strategy will result in acquisition of an algorithmic procedure that will not readily be transferable to related applications.

Gareth also reported help seeking by monitoring the teacher and class activities.

Gareth: "*I keep an ear out on what comments she* (teacher) *makes to other people so if I've got something wrong I might get help.*"

However, despite Gareth's stated preference for help seeking in class, he does not seek help often. Usually the teacher checks his work, during her walk around the class, and Gareth may put his hand up, or call out for assistance **once** during seatwork.

Gareth's tendency to complete problems incorrectly, without accurate monitoring, or regular checking, means that Gareth is often blissfully ignorant of the extent of his difficulties. Bereiter and Scardamalia (1989) suggest that knowledge of what one does not know is a vital part of intentional learning. Without it, the only kind of learning goal one can set is to learn more about the topic. When the researcher asked Gareth why he skipped an assigned reading, and went straight to the exercises the response "*I've done that reading before*" is typical of his focus on the goal of completion rather than understanding.

Review of homework or seatwork

Gareth finds homework review sessions provide a useful opportunity to correct work that he had been unable to do at home. If homework had not been marked he spends a lot of time putting ticks or crosses on work in class. For example, when reviewing homework on standard deviation Gareth ticked **every** data entry that had been correctly copied into the table.

He is keen to answer teacher questions, but needed to refer to his work to get the answer on many occasions. If he has a problem incorrect Gareth always copies the teacher's example into his book. Additionally, Gareth likes to do any calculations rather than just copy the teacher's answer. For example when Gareth is copying down a standard deviation problem he calculated the mean (9.2 + 1.2 +... 4.6) ÷ 10, rather than attending to the teacher's explanation on checking procedures. Metacognitive strategies of selectively attending to arithmetic procedures and monitoring understanding based on whether or not he can perform the calculations are both ineffective and inefficient in learning the desired content of the review session.

Homework

Gareth always attempts homework, except when deadlines from other subjects create time pressures. He sees homework as a time to consolidate what you did in class

Gareth: "*The more practice you get the more understanding you'll have.*" and

Gareth: "*Homework is heaps important, 'cause you can't get through everything you need to need in class, like all the examples she* (teacher) *might tell you, but you won't be able to do that many examples in class.*"

However, Gareth does express the view that doing exercises at home is different to doing them in class:

Gareth: "*It's different learning at home 'cause if your having trouble with something in class you've got the teacher there who can come and show you, but at home you have to do it by yourself - so it's harder.*"

When homework is assigned during the lesson he previews the exercises and makes some judgement as to the amount and difficulty. At home, after tidying his study area and arranging his books, he looks over all the exercises. He does not read any of the text explanation or worked examples but goes straight to the exercises.

Gareth: *"If I've got any problem I first go straight to the back of the book and see their answer and work backwards to the question. If I haven't got any problems I just sort of whiz through them ... I just whiz through the first line of each section."*

Gareth's evaluation is based on metacognitive experiences of whether the material is "easy or hard". An "easy" problem is one that can be completed; there appears to be little concern as to the reasonableness of the answer. Furthermore, Gareth's selective marking of only the hard problems *"because I've looked them up to help with working backward but if they are straightforward I won't always mark them."* means that opportunities to learn from errors are limited.

As with seatwork, Gareth's propensity to marking his work is sometimes based on inaccurate monitoring.

Int: *"In the review the teacher asked you the answer to No 13(c) and you gave the wrong answer. Had you marked it at home?"*

Gareth: *"I hadn't marked it 'cause I thought I had it right."*

In the Video 1 interview Gareth does refer to seeking some help from the text, but is again thwarted by lack of relevant prior knowledge.

Gareth; *"I did everything except the standard deviation column and frequency. I looked through the book to see how they did it but couldn't find it because it wasn't written down in big clear words that it was standard deviation - it was just under variance. I couldn't find a worked example the same.*

Gareth reported that he didn't know what frequency meant and an examination of the text revealed that this was assumed knowledge. The worked example had the frequency already in the table thus it was impossible for Gareth to infer where it had come from in terms of the given data.

Revision for a maths test

Unlike most other students in the class, Gareth plans revision both in terms of time and topics.

Gareth: *"We've got exams coming up so I've done other chapters; rereading through chapters and doing some examples from them."*

Most other students reported attending to teacher cues, whereas Gareth appeared relatively unaware of cues indicating which material would be in the test.

Gareth does several hours of revision in a quiet room, away from interruptions and distractions, trying some problems, reading notes over and over again, reading over worked examples in the book, and doing a few problems from last year's revision book. He explains that last year when he practiced examples the night before *"my brain just couldn't handle all the examples, and I kept bumming out, so now I don't go over the examples the night before because it doesn't really help me much, I just lose concentration."* This is an example of how one's metacognitive knowledge, determined by past learning experiences, affects strategy selection. When asked how Gareth thought he could improve his performance he replied, "*Go through the examples slower*". Gareth also makes reference to memory strategies such as rehearsal:

Gareth: *"If I have to memorise the formula that they won't give me or a graph, I just write it our a few hundred times."*

In response to interview questions Gareth was the only student who reported using specific motivation strategies at home.

Gareth: *"In the weekends I usually do about two hours of maths, and then I can play on the computer for about half-an-hour, and then I go and do another two hours."*

Summary

Regardless of Gareth's will and effort input, the combination of ineffective learning strategies and weak domain knowledge precludes successful learning. Much of Gareth's learning is of a "passive" (Mitchell, 1992a) nature, dependent on the teacher or text to tell him what to do and how to do it. To a large extent his cognitive strategies involve duplicative processing (Thomas, 1988), which involve unaltered encoding, or mental recycling of the given information.

Despite of the fact that Gareth's procedural rules are often incorrect they enable him for the most part to participate in classroom tasks and discussions. However, Gareth's test results show evidence of procedures which are combined with other subprocesses in inappropriate ways, or are only partially remembered. Hiebert and Lefevre (1986) suggest that procedures that lack connections with conceptual knowledge may deteriorate quickly and are not reconstructible.

Greatly influenced by his beliefs about learning mathematics, Gareth's monitoring and help-seeking strategies were directed to task completion rather than understanding. Gareth appears to overestimate his understanding; possibly he is confusing familiarity or recognition with understanding. In some cases Gareth's problem-solving efforts failed, not from a lack of monitoring *per se*, but because of a lack of relevant prior knowledge, and a lack of ability to apply monitoring strategies to what is known, as opposed to what is done.

For Gareth **doing** mathematics problems does not guarantee successful learning. Using learning strategies which have inappropriate learning goals, and which in many cases are inefficient, does not guarantee successful learning outcomes. The knowledge and skills that Gareth acquires tend to be inert, and available only when clearly marked by context.

8.3 Case Study 2: KAREN

PROFILE

Karen is a sixteen year old who likes mathematics and is reasonably happy with her progress in the sixth form: *"I'm getting in the seventies, that result is fine."* Her stated goal is *"just to pass"* and she wants to continue with mathematics in the seventh form. Karen says, *"I think maths is reasonably important. I know it will be of some use later on, but I'm not exactly sure what for."*

She has well defined views about mathematics and mathematics learning:

Karen: *"Learning maths involves a lot of memory - you've got to memorise a lot of formula and stuff like that, and it's hard work and logical thinking. You need to put effort into it. I like working through problems by myself - and I love it when I can do them."*

LEARNING EPISODES

Introduction of new content via teacher explanation and worked examples

During class discussion of a new topic Karen was attentive and followed the teacher's instructions to attend to certain pages and review work. She attempts to answer the teacher's questions, but notably these answers are rarely made public. Apart from occasional reports of boredom Karen appeared to be very involved in meaningful knowledge construction.

Karen's reports of learning can be linked to a number of elaborative strategies. For example, Karen reported trying to answer the teacher's questions:

Mrs H: *"What other ways can we display data?"*

Karen: *"I was actually thinking of a pictograph or a pictogram or something. I wasn't sure of the proper name."*

Similarly, Karen evaluates other students' answers against her own.

Karen: *"I didn't think Faye's answer of histogram was right. I think about other students' answers but I wait for the teacher to confirm it as well."*

Like Gareth, Karen reported some unusual procedural criteria for judging correctness.

Karen: *"I'm pretty sure Kane's right because she's* (teacher) *writing it on the board - I was thinking I've really bummed out because it was not what I had."*

Karen reported numerous linking statements in which new knowledge was related to prior knowledge. However, a noticeable difference from Gareth's elaborations is Karen's specific linking to content rather than just to a memory of having done something similar.

Karen: *"I was trying to remember what a histogram was like."*

Karen: *"I was thinking back to functions and I was saying 'graph it', and that functions are when you have a vertical line."*

Karen's elaborations involve more personal linking examples and reports of imagery, than any of the other target students.

Karen: *"At first I wondered what it was and when she said something about length of feet I almost burst out laughing. I had just pictured who's feet - Gareths?"*

Karen: *"I was thinking back to when we did parabolas at the beginning of the year. I saw a picture in my mind."*

During the introduction of new material Karen monitors her understanding, and when faced with confusion makes special efforts to attend and resolve conflicts. The following is an example in which Karen evaluates another student's answer, monitors her understanding, and then reacts with another self-question and answer.

Karen: *"I was confused. I thought you could use a line graph as well* (Faye's answer to teacher question), *but she* (teacher) *started saying you can't 'cause it's not a trend and I'm saying* (to herself) *well what's a trend...I thought at first, yeah you can use that - so now I know you can't."*

In another instance the teacher is discussing the meaning of the term 'quadratic'. Karen evaluates the teacher's answer and relates it to her own answer:

Karen: *"I thought it would have something to do with four terms...She's* (teacher) *sort of confusing me. I don't know what she means by the highest power of 2 because when we did it at the beginning of the year I thought we did things like* ax^3. *It didn't seem to fit what I thought. I didn't know why she* (teacher) *was talking about squares and squared area."*

By clearly identifying areas of conflict Karen is able to be active in resolving any issues. In the following example we can see how Karen's anticipation of the teacher's answer momentarily causes conflict which alerts her to a possible resolution.

Mrs H: *"So if we had* $x^2 - 5x - 7 =$ (pause) *-13"*

Karen: *"I thought she* (teacher) *was going to put it equal to zero. When she put -13 I just realised that you had to add 13 and change the equation."*

Another example, from a different lesson, shows Karen using links from a prior knowledge framework for solving algebraic equations. Again her explicit anticipation of the problem direction allows for the algebraic framework to be challenged and extended to accommodate solving exponential equations. The teacher had got the problem to the stage $20\,000 = 10^{(t/2)}$ by dividing both sides by 1000:

Karen: *"I knew she* (teacher) *would have to simplify it but I wondered what she would do after that. I thought she would have to get it down to t over two* (t/2) *on one side but I wasn't sure how she was going to do it."*

Further on in the lesson, Karen reports comparing problem types (elaborations between problems) encountered during the teacher's explanations.

Karen: *"I did not really understand when she said get them back to the same base whether it was log 2 or something like that. When she said, '2 to what power?' I wondered why these problems were in base 2 when all the rest were in base 10...I was thinking she might explain it some more - this log 2 bit..."*

In another lesson Karen determined that extra effort and selective attention is needed.

Karen: *"I'd never heard of polynomials before. I was thinking it's probably something to do with one number. I was not sure what it had to do with quadratics so I was concentrating extra hard, trying to find out what she* (teacher) *was talking about."*

Seatwork

During seatwork Karen often works cooperatively with a peer group (Faye, Lucy and Karen): helping each other; sharing problems and notes; and monitoring progress. For example, Karen reported that she is quite a slow writer and she can get the notes from Faye if she gets behind.

Peer interactions heard by the observer during a seatwork episode include:

Karen/Lucy: *"Do you understand this work?"*

Faye/Karen: *"Get the teacher."*

Karen/Mrs H: *"I'm lost."*

Karen/Lucy: *"Lucy, you listen in, you are just as confused."*

Karen/Lucy: *"What number are you doing?"*

Lucy/Karen: *"Is that the answer?"*

Karen/Faye *"How do you do that."*

Karen/Lucy: *"Have you finished?"*

Ocassionally Karen prefers to work seatwork problems alone: this was the case in the videoed lessons, as her peer group did not want to be included in the video. When she is stuck in these situations she sometimes writes a note in her log book to do some revision at home.

Karen: *"$4x^2$ - oh these are the ones I hate. I thought - oh God here we go. I can't do these problems. I'm thinking if I sat there and stared at it, it might go away. I wrote down the numbers but I'm stuck. I wrote down a note in my log book to do some at home. I was very confused."*

During seatwork Karen is aware of the teacher's movements, and comments to other students:

Karen: *"I'm half listening. I knew she was explaining something to Kane but I wasn't sure what she was talking about."*

Karen: *"I know she's talking to Faye. I have a quick listen. I know Faye was quite a bit ahead; she works quite a bit ahead in class so I didn't think what she was saying would be relevant."*

Help seeking

In class Karen is often aware of the need for help and anticipates helpful information. However, Karen is reluctant to seek help directly from the teacher. She doesn't ask or answer question (publicly) because:

- *"I'm not sure if it's correct."*
- *"I thought she would explain it anyway."*
- *'I didn't want to say anything in case it's so far wrong I embarrass myself."*
- *"I don't tend to call out; I just sit there and listen."*
- *"I sort of half sort it out, I thought I must have been wrong and I'd go along with it I suppose."*
- *"I don't really speak out in class; I feel uncomfortable in a class position."*

Her help-seeking approach is relatively passive in that she relies on other students in the class to ask questions.

Karen: *"I felt happier when Gareth asked that question."*

When Dean says, *"I'm stuck Miss"* Karen reported thinking, *"Good so am I; perhaps she will explain it now."* When she is experiencing difficulty Karen hopes that the teacher will review the material again and provide further explanations.

Karen: *"I made a note in my book to study quadratics, I was hoping I would find out what polynomials were in the lesson. I looked it up in the index and it wasn't there."*

Karen: *"I looked at page 125 and wondered what exercises we would need to do and hoped she would explain it some more."*

Although passive on the surface, this anticipatory behaviour has the benefit that Karen selectively attends and seeks teacher cues as to what information is going to be discussed next. The following is a reference to a problem which Karen could not solve during seatwork: rather than seek help directly she anticipates that the teacher will review the seatwork problems, and is ready to attend to the teacher's explanation:

Karen: *"Well this time I didn't really understand what I was doing and I was hoping that she was going to help when she went over it again on the board."*

However, if the teacher does not review Karen's problems or her confusion is unresolved Karen has back up strategies of self-questioning, review at home, and use of resources.

Karen: *"I was thinking that she* (teacher) *might explain it some more; this log 2 bit, and if she didn't I could read through the book at home."*

Karen: *"It didn't really make a lot of sense. I'd thought she* (teacher) *would write notes and we had some notes from the day before. I'll probably go back and try a couple of these in the holidays."*

Review of homework or seatwork

Karen reports that most of the homework review is boring: *"the teacher uses the same methods so there is not much new."* When Karen hasn't done her homework she tries to work out the answers during the review time. In two of the videoed lessons homework and seatwork reviews consisted of 'taking a turn around the class' sessions. Although Karen hadn't done the questions she read each question and tried to work out the answers, carefully watching the teacher to anticipate when her turn would be.

Karen: *"I was trying to think of an answer before she got to me. I thought of the size of somebody's shirt, then Abe said that ...and I was looking at the light switch and I thought of 'light switches in classes'...Kane said birthdays as continuous and I thought that's wrong. I'm checking the others' answers over in my mind."*

Karen is conscious that homework and seatwork review can provide help with problems that she was unable to complete. She gains help not by asking question but by anticipating and attending to the teacher's comments.

Homework

Karen reports that she thinks homework is a good thing to do. She usually does homework, especially if she has not understood the class work particularly well. The following example demonstrates Karen's sound criteria for assessing her need to do homework:

Karen: *"I do homework if I don't get what I've done in class. If don't understand what we've done in class I definitely do the homework. It sort of helps me understand it. I'll go through the book and read the notes if I don't get it. If everything's okay I don't usually do it. If I've got every single question right I don't worry about the homework."*

Furthermore, her approach to homework demonstrates that learning via self-instruction, as well as task completion, is indeed the goal:

Karen: "*I read through it all* (homework) *and decide which question is easiest and I do that one first. Then I go through it and as I finish each one I'd check the answer and if I got it wrong I'd go back to the chapter, read the notes on how to do that problem and try to do it again. I would see if I can work out where I went wrong."*

Karen reported modifying the homework task as a result of monitoring her progress:

Karen: *"I start off saying, "I'm going to do all of it" but if I go through and I've got everything wrong I'll just give up totally* (and hope the teacher goes over it the next day?). *If I'm getting about four or five out of every ten wrong I'll probably do it all, and if I get everything right I'll just think it's a bit pointless*

doing it. If I know how to do it I'll do the first two questions and the last two questions."

Additionally, Karen reports doing extra revision work during homework times. This is directly related to her awareness to review difficult or unclear work from the class lesson.

Karen: *"I was going to read through the whole chapter on algebra and do some problems if I didn't understand it. If I haven't got a lot of homework I'll just go over some of my own work. I prefer to work by myself if I don't get it."*

Also Karen reports previewing some of the current chapter and trying additional problems:

Karen: *"Sometimes, if I haven't got a lot of homework I'll just have a go at a few extra problems further on - on a scrap of paper and check the answer. If I get them right I usually think I'm pretty smart. Sometimes if it looks interesting I'll look through the rest of the chapter."*

Karen was one of two students (Adam) who reported reflecting on mathematics at a time other than during homework:

Karen: *"I sometimes think about my maths for a while after I've just finished my homework - I still consciously think about it and sometimes when I go to sleep at night I think about maths."*

There is a strong affective element in Karen's comments about learning at home. Learning at home contrasts the class situation in which she is not always comfortable about speaking out and keeping up with the pace of the lesson.

Karen: *"I like working at home; it adds to what you've done in class. I find it easier to learn working on my own. It's not hard learning in class but it's easier when I'm on my own - you're not on a time limit and you can work through it at a steady pace."*

Revision for a maths test

Like most of the students, Karen reports reading through class notes. Karen has strong organisational strategies to assist revision. She was observed, organising notes and corrections to aid later revision.

Karen: *"I write my headings in red and my marking in red and my corrections if I want them to stand out. If there's something I really don't understand I'll put a big red box around it - and 'study this'...If I've got an exam coming up I'll go through my book and see what I've got circled and read it."*

She also reports making extra notes *"if I think she's* (teacher) *missed an important point"*.

Most of her revision is done as a normal part of the learning process during homework sessions. She is not a big fan of last minute revision. In response to questions about proposed revision for a test the following day Karen reported:

Karen: *"I might do some, I sort of think for tests that if I know it I know it, and if I don't I'm not really going to learn it at the last minute. I'm not worried about this test too much 'cause it's an open book test - but I think I'm going to have to go through it and have a look at it tonight."*

Karen did not report any specific planning of topics to study, but did indicate an awareness of teacher cues about probable questions:

Karen: *"I'm not too worried about what's in the test. I was hoping somebody was going to tell me what was in it. I know graphs would be in the test; Mrs H said so. I have sort of got a basic knowledge of all the things she's gone through in class - I can do most of them - that's good enough.*"

Near the end of the year one can see the effect of Karen's metacognitive knowledge and self attributions on revision strategies.

Karen: *"I'm too lazy to do much revision for tests. I can usually fluke tests, I'd like to pass this test, but it doesn't really matter 'cause my average is quite high. "*

Although Karen was able to state the things she should be doing for test preparation she reported feeling too tired and fed up with tests to bother any more - this seemed an understatement considering that she had five assessment tests that week! One of the questions in the up-coming test (cued by the teacher) was to be the cosine proof. Karen reported that, *"I can't remember proofs. I'm not even going to try and remember it."* When I asked Karen what she would need to do if she was going to learn the proof for the test her proposed strategy was as follows:

Karen: *"I would read it over and over and over again until I've got it drilled in. I'd write it out, but I'd read it for about half and hour over and over."*

It was probably a sensible strategy that Karen decided against *learning* the proof for the test as this sounds indeed a most unpleasant task!

Summary

Again we see a unique learning strategy profile emerging. Karen employs a wide range of elaborative strategies which, for the most part, are effective and appropriate for knowledge construction. In particular, her linking with prior knowledge involved reorganisation and accommodation of existing frameworks rather than vague recollections of 'having done that before'. Organisational strategies were used to advantage in reviewing material. Karen was however reluctant to become to involved in writing her own summaries, preferring to rely mainly on teacher-given summaries.

Karen values practice, and is sometimes encouraged by her own efforts to try problems other than those set by the teacher. There is, however, an element of 'she's good enough' in Karen's overall goal which constrains the amount of practice and subsequent performance outcome.

Karen exhibits advanced metacognitive behaviour in that she accurately monitors her understanding; there were numerous reports of confusion, of not quite understanding well enough, and feelings of *"I've sort of got it*". Often, but not always, this awareness resulted in some positive action: Karen anticipated and attended to teacher explanations; used resources for help; or used peers to assist. What was lacking was Karen's direct seeking of help from the teacher; she preferred to rely on other students to seek clarification, or simply hope that the teacher would review the material. It is difficult to speculate, but it would not be unreasonable to suggest that had Karen been more direct in seeking help she may have enjoyed the learning process more, she may have made more efficient use of the valued resource - time, and she may have been better able to resolve learning conflicts.

8.4 Case Study 3: JANE

PROFILE

Jane, a sixteen year old student, likes mathematics: *"It's good compared to English, you don't have to write a lot of stuff - its just a right or wrong answer sort of thing. I think it's harder than say English 'cause you've got more stuff to remember, there's heaps of stuff you can get confused with."* Jane is achieving at a C- B grade and reports that she is *"middle, but I would prefer to be at the top but I can't do anything about it this year"*. This comment refers to her concern about a decrease in achievement (compared to Form five) which she relates to lack of peer support, a change in teaching style, and her present lack of understanding.

Jane: *"I really enjoyed it last year so I thought it might be the same this year but it's a lot harder. I got higher marks last year and I thought I would get about the same. I suppose it's lower because I don't understand half of the work - everyone else seems to do alright - so it must be me. I think it would be helpful to get support from others in the class. This year it's more like I've got to memorise things but last year I understood things."*

However, she does think she could improve her marks with more effort in writing her own notes and more study at home. Jane's learning goal is to get good marks and she thinks that *"mathematics is important to life in general - it makes it easier to know about everything"*.

In both general interviews, and stimulated recall interviews, Jane expressed some strong beliefs about the nature of mathematics learning (metacognitive knowledge) which relate to strategy selection during learning episodes. Jane reported that mathematics learning involves a lot of memorising formulas and applying them to the questions - but also adds that *"thinking comes into it but you need to memorise things first"*. In the stimulated recall interviews Jane's concern about not understanding the material is further amplified as she tries to resolve the conflict of memorising methods versus learning with understanding.

Jane: *"I like it when I understand what's actually happening, but some of the stuff when you just get given formulas, you don't understand - so you can't do much about it - you've just got to try and remember the formula. It's heaps easier if you know what you are actually doing."*

In examining Jane's work during actual lessons we will see how she deals with this conflict and what strategies, if any, she employs so as to make the material meaningful.

LEARNING EPISODES

Introduction of new content via explanation and worked examples

In both of the stimulated recall interviews Jane's behaviour during the introduction of new work is often passive. There are frequent reports of inactivity and learning behaviours were punctuated by shifts in attention. She reported not listening one minute and paying attention in the next.

Jane: *"I'm just waiting, and seeing if I can see where she gets the answer from. I was just listening, I wasn't actually thinking about it, I'm just listening."*

Similarly, she is not always involved with other students' questions and answers.

Jane: *"I heard it* (Lucy's question), *but I didn't know what she was on about. I'd forgotten about that x over x stuff. I didn't bother to listen to the answer."*

On several occasions Jane reports ignoring information which is incomprehensible.

Jane: *"I didn't understand all that stuff about the area under the graph, or what she's* (teacher) *going on about."*

Int: *"Did you pay special attention because you didn't understand it?"*

Jane: *"No, 'cause she says the same thing over and over again, so I'm not going to understand it. I was thinking that I don't understand this."*

Jane selectively decides to take notes based on monitoring her ability to understand or remember material.

Jane: *"I didn't take these examples down. I understand this, but if she's* (teacher) *doing something and I understand it but I know I won't remember it I write it down. If it's something that I just don't understand altogether I won't write it down."*

Jane also ignored information based on cues from the teacher that information may be off the topic.

Jane: *"I couldn't understand what she's going on about - she was saying that we didn't have to use it . It's off the topic, not really worth listening to."*

Although Jane is aware of her non-understanding in these instances she seems to have no effective strategies for dealing with it.

When Jane is prepared to be more active in the lesson, by participating in activities such as looking up tables or doing calculations, she appears more motivated to resolve difficulties. For example, Jane's attention to the solution process, as a result of monitoring her previous failure, leads to an elaboration between parts of the problem resulting in knowledge construction.

Jane: *"You know how you find out 0.98, I didn't realise that was a percentage and when she* (teacher) *said percentage and 98% I learnt something."*

Also Jane uses comparison of problems to construct knowledge :

Jane: *"I didn't know why she* (teacher) *used 1/2 in (2x -1), but when she used 4/3 in the next one* (3x - 4), *I could see where she'd got it from. I'm able to work it out now."*

However, several of Jane's reported elaborations appear to be general, and probably less effective, when compared to the specific elaborations used by Karen. For example, while the following report illustrates active thinking, it lacks specific links with relevant prior knowledge.

Jane: *"The first time she* (teacher) *did this I didn't really understand because I wasn't really listening - I wasn't really thinking about it, but this time I understood. Today I probably had heard it all before and now she's going over it again it's easier."*

In another instance Jane is aware of her lack of prior knowledge - her metacognitive self-knowledge that she can't 'do graphs' is stronger than her metacognitive strategy knowledge.

Jane: *"I just don't understand anything she was talking about. I'm feeling bored. I'm sort of trying, but I didn't bother for that one 'cause I don't understand graphs at all so I wouldn't know what's she's* (teacher) *going on about."*

Jane appears either to have little strategic knowledge, or be unwilling to use strategies such a help-seeking to overcome her lack of prior knowledge and to learn from the episode.

Later, in the same lesson, the teacher discusses cubic functions. Jane reported that she knew that x^3 graphs had two turning points but that she was unsure how one would find the turning points. Her prior knowledge would be typical of most students, yet Jane's self evaluation of ability and understanding remained low. This lack of confidence affects her willingness to be part of the class discussion or seek help.

Int: *"Why don't you ask the teacher about this?"*

Jane: *"'Cause you might be the only one who doesn't know what she's* (teacher) *going on about."*

Int: *"You think everyone else knew?"*

Jane: *"Yeah, other people seem to be answering questions."*

Int: *"Does that worry you?"*

Jane: *"You notice it. You think maybe you've missed something - here's another bit I don't know!"*

At other times during class discussion episodes Jane reported self-questions, but her reluctance to ask public questions, or selectively attend to discrepant information, meant that her self-questions went largely unresolved.

Jane: *"All of this stuff seems...I don't know how it relates, see I don't know why you would want to get g of 2, but there must be a reason why. I wondered what it had to do with anything."*

During Video 2 lesson there is a discussion of 'synthetic division'. This method was suggested by Dean as an alternative to the teacher's method of long division. Jane's metacognitive beliefs about effective learning involving understanding are an important influence on her evaluation of Dean's method and her consequent learning outcomes.

Jane: *"It* (synthetic division) *seems easier, but I don't really understand what is happening - it's just a method. It would work for people who don't understand the long division, but I couldn't see how it works. I was just taking it as you just times this and put this over here and so on."*

Although Jane tries out both methods of division, influenced by her metacognitive beliefs, she adopts 'long division' as the preferred alternative. Later, she appears so secure in her knowledge of this procedure that she actively evaluates the teacher's explanation.

Mrs H: *"...change the sign and add and you get 5x"*

in reference to

$$x-2 \;\overline{)4x^2 - 3x - 2}$$
$$\underline{4x^2 - 8x}$$
$$5x$$

Jane: *"I was thinking I don't know why she* (teacher) *doesn't just minus it. She sort of confuses you by saying change the sign and add; then you think that's what you've got to do - but really, you're just subtracting. It's better if you know what you're doing then you don't get yourself confused."*

Seatwork

Jane most often works independently at seatwork problems. She is usually on task, follows teacher's instructions, and reports valuing the opportunity for practice.

Jane: *"I understood it when she* (teacher) *was going through it on the board, but it's important to try some exercises."*

When the teacher assigns exercises Jane usually previews the exercises.

Jane's production evaluation strategy is based on sound criteria resulting in regularly marked work.

Jane: *"Well at the start I check the answers, like, for every question, like for these ones I'd check for each one and if I keep getting answers right then I do two or three before I check the answers. If I'm not sure I mark it straight away, 'cause if you leave it, then mark it, and find it's wrong, and you don't really know what you've done wrong, you've got to go through the whole question again."*

Like Gareth, Jane is sometimes over optimistic about her success, but unlike Gareth she is more likely to be able to self-correct her problems:

Jane: *"I thought I was going quite well. I thought it was easy, but I wasn't getting them all right, I forgot to subtract the things to the end."*

When Jane has an incorrect answer she puts the correct answer in red. Usually Jane refers to the answer to see if it adds any further information and often she tries the question again. Several times during the videoed lessons, when these strategies still did not help, she put a question mark against the problem.

Int: *"What's the question mark for?"*

Jane: *"So I'll know I didn't do it."*

Int: *"Will you come back to that later on?"*

Jane: *"I don't know. If I really wanted to know I could ask someone at home, but I probably won't bother."*

When Jane got No. 8 wrong, further on in the same lesson, after looking at the board examples and checking the working, she eventually wrote the answer in red.

Jane: *"Well I wasn't doing the questions that we were suppose to do. I did No. 8 and I thought well it's probably getting harder so I didn't worry."*

This response was affected by her metacognitive knowledge of the text structure and furthermore, when asked why she was doing No. 8, we see the strong influence of affect in Jane's decision to modify the task.

Jane: *"Cause she'd* (teacher) *done (a), (b), (c), and I just wanted to go through a whole question like 7 (a), (b), (c), (d), and not just (d) and (e) of a question. I choose No. 8 'cause it looked big. I read it first and thought I could do it, understand it ...I liked the look of No. 8."*

Jane's metacognitive knowledge of text structure further influences her 'next move'.

Jane: *"Well I got stuck near the end, so I went back to No. 6. I thought it would be easier since it was back further; 'cause they get harder at the end."*

Like the other target students, Jane is aware of the teacher's movements around the class and listens in on conversations to gain help.

Jane: *"Sometimes I listen if I'm sitting next to them and she's* (teacher) *going over something I don't understand - but I don't really ask her myself."*

It is of some concern that Jane fails to use more active help-seeking strategies in the classroom. She makes no reference to using textbook examples or extra reading, and like Karen, Jane is reluctant to seek help publicly both during seatwork and in class discussion.

Int: *"When you're stuck did you think about asking for help?"*

Jane: *No, I didn't even think about asking for help."*

Jane: *"I'd rather just sit there. I'd rather just do it by myself. I don't like talking in front of the class."*

As discussed earlier, Jane would have preferred to have some peers to work with: the other three girls in the class work in a strong peer grouping, which only occasionally included Jane.

Int: *"In class you don't ask many questions. What do you do if you need help?"*

Jane: *"Sometimes I ask, like if I'm sitting with Faye and that, otherwise I don't do anything."*

There were a few occasions when Jane cooperated with Jake, both in seeking, and giving help. On one occasion when Jake asked the teacher about how to find the y intercept Jane prompted Jake to ask again when the teacher did not reply.

Organisational strategies employed by Jane included writing out a summary of the question requirements in red, *"So I can see it, so I know what I have to find out"*, coding of problems with question marks, and copying notes.

Review of homework or seatwork

In the first videoed lesson Jane had not completed the homework. During the review she answered some of the teacher's questions to herself, but for much of the time reported that she was "*just watching*". On one occasion when Jane was observed to mouth a response to the teacher she reported: *"I thought 'draw a diagram', but it was wrong - I was glad I didn't say anything."* Unfortunately we see that the metacognitive experience of relief only adds to her belief that it is better not to answer publicly in class.

Like Karen, Jane to some extent, relies on the review of seatwork to gain help with difficult problems.

Jane: *"I left No. 7 until she explained it. I went on to No. 1..."*

Faye: (later) *"Mrs H how do you do No. 7?"*

Jane: *"I wanted to know this so I watched what the teacher was doing on the board."*

When the teacher does review this problem, Jane's conflict between wanting to understand, and being able to follow a method, surfaces. Jane followed the teacher's example even to the extent of pointing out an error in her calculations, but she still had reservations about 'learning' from this example.

Jane: *"This is just like a method thing, like when she* (teacher) *actually finished the question I could go through and do the same steps, like substitute and divide it by something. I don't know why she's doing it. I'm following what she's doing, but I don't know why she's doing it."*

Following on from this example Jane went on to do the next problem - a problem not set by the teacher.

Jane: *"I went on to No. 8 to see if I could do it. They are the same type of question and I wanted to see if I could do it. I needed to look at the board to see what Mrs H has done. I write No. 7 down first then try No. 8. I mark No. 8: it was right - and then I go back to Question 2."*

This reported learning episode is 'loaded' with strategic learning behaviour:

- task management (modifying teacher set task);
- self-evaluation ("...to see if I could do it.");
- elaboration (comparing problem types);
- help seeking (looking at the worked example on the board);
- organisational strategy (recording No. 7); and
- production evaluation (marking work).

In addition, Jane's success at this problem will reinforce her metacognitive knowledge about strategies and outcomes of learning mathematics. She reinforces that doing problems by comparing methods is a successful strategy, but at the same time she reinforces her belief that many problems need to be solved by following teacher-presented methods rather than using meaningful constructed knowledge.

In another instance, when the teacher is reviewing seatwork problems, Jane observes that the teacher is using a different method to the one that she used to complete the problems. This awareness of conflict causes Jane to selectively attend to effect a resolution.

Jane: *"I don't understand her way. I found it easier to do long division. If I write down the notes for this I should be able to do it. I think I'll listen and write it down after she's finished. I'm checking to see if her way* (Remainder Theorem) *gives the same answer."*

Homework

Like the other target students, Jane values the learning opportunities homework provides.

Jane: *"Homework is important so that you can just go over the stuff and get use to doing it all. Like you only had to do three question last night but after I did three I could do them all because you just get into the habit of doing the same thing in each question."*

Jane: *"I think maths homework is important, like any homework that you get. Like it's just going over examples and that's going to make you better, but it's not so important that you always have to get it done."*

Jane's decision to do homework is largely related to availability of time. She works for 19.5 hours outside of school time and complains of pressure of homework from competing subjects. Metacognitive decisions regarding her ability to complete the task and affective reactions also are also contributing factors. When Jane begins her homework she usually looks over the problems *"to see if I know what's happening"* and if she finds the first one easy she does the rest:

Jane: *"Sometimes I'll just look at the maths homework as I go through all the homework I've got through the day. I'll look at it, and if it looks too hard I'll put it aside and if I finish all my other homework I'll get back to it, but usually if it looks too hard I just won't do it. It just depends 'cause sometimes I'll feel like doing maths and sometimes I'll feel like doing English."*

When Jane is stuck with her homework she uses her notes and worked examples from the text to help, or *"if I can't find anything I usually leave a question mark or leave it out'*. When asked what happens with the question mark Jane's response indicated passive rather than adaptive help-seeking.

Int: *"Are you prepared to follow up your difficulties from homework in review time the next day?"*

Jane: *"If she* (teacher) *goes over it I'll just listen, but if she doesn't I'll just leave it."*

Thus, while Jane's coding of difficulties with question marks, both in class time and in homework, draws attention to her difficulties and signals anticipation of teacher help, Jane is largely reliant on other students and the teacher to determine if, and when, help is forthcoming.

Jane is also aware that answers can be helpful. She qualifies this metacognitive strategy knowledge with the view that this strategy is more useful with certain topics.

Jane: *"It's easy in topics like algebra where sometimes if you're just a couple of x's out or something, you can work back and see if you've minused something that you're suppose to add or something and it's usually enough. It's not usually easy for graphing and topics like that."*

Like Gareth, Jane's marking on homework appears to be based on rather shaky criteria.

Jane: *"If I want to know if they are right or not, I mark them. Well usually some of them I do - some I can't do, but if I feel good about them and think I've got a good answer I'll look it up and see if it's right."*

Other strategic behaviours discussed by Jane include task management: she occasionally tried some extra problems if the work was easy and she sometimes made notes in the form of jottings on her work about the solution method.

Test revision

Jane appears to be somewhat vague about learning for tests. She does no planning in terms of schedules for test revision, however she does report using her *Course Outline* to hep decide what to study.

Jane: *"I just go through my notes and if I don't understand anything...(*she stops unable to think what she would do)... *I don't know, just do some exercises."*

Int: *"There are a lot of exercises, how do you decide which ones to do?"*

Jane: *"I don't know, just pick some of them I suppose."*

When asked what she does with exercises that she marked in class she replied that, *"I might look at them. I just do some exercise from the book and if I get stuck I might look at how I did the ones in class."* One could understand Jane's reluctance to review marked problems as in many instances she had not resolved the difficulty with the problems during class time.

To improve her revision sessions and performance Jane suggested that she should study more exercises: *"I only really study the easy ones as harder ones take too long."* Earlier in the year Jane also seemed concerned about notes.

Jane: *"She doesn't really give us much notes and Faye and that, they just go to the book and get their own notes but I don't do that. I should write more notes so I've got them for later, like I've only got about two pages of notes so far."*

Jane sees the process of tests as helpful in the overall learning process: it's value is in terms of revision rather than new knowledge construction.

Jane: *"I think it's good, you can just summarise everything that you've learnt and you need to know - it gives you extra practice. You're just going over what you already know: it puts all the stuff back in my mind."*

Summary

Although Jane employs a wide range of learning strategies, her cognitive strategies appear to lack the depth of those used by Karen. Her reports of processing varied from a general state of awareness to a more focussed attentiveness. Generally, Jane seems less able to make full use of a range of elaborative strategies needed to integrate new content with previous knowledge. In particular, Jane's efforts to link previous knowledge are often hindered by negative self-evaluation of domain knowledge and lack of confidence.

Although Jane reported many monitoring statements concerning her understanding, or lack of understanding, subsequent control strategies were not always effective in resolving learning difficulties. Help seeking, cooperation with peers, and use of resources were absent, or ineffectually applied by Jane on many occasions.

Jane's behaviour, more than any of the other target students, illustrates how affective reactions to mathematics, and to learning mathematics, affect strategy use. Her classroom learning behaviours and moods fluctuate greatly from active and interested to passive, bored or frustrated, as she attempts to cope with the changing demands of learning the sixth form mathematics content, and the different classroom environment.

Her decrease in performance levels (as measured by test results) have negatively affected Jane's metacognitive knowledge. Although Jane expresses a will to 'understand' mathematics, her ability to adapt to her learning environment, and to overcome her difficulties, is limited. Her lack of peer involvement, and perceived lack of teacher support, combined with her metacognitive self-knowledge, appear to have resulted in a gradual acceptance of passive learning tendencies, and consequent decline in performance. Jane has made little if any progress in developing effective metacognitive strategies necessary to control and self-regulate learning behaviours needed for future study in mathematics.

8.5 Case Study 4: ADAM

PROFILE

Adam is a fifteen year old accelerated mathematics student. He rates himself as okay at mathematics: science is his favourite subject. Adam thinks that mathematics is important for science and his learning goal is directed towards understanding. He prefers learning by making his own notes and doing exercises; strategic memorising is not a feature of his learning style but rehearsal is definitely important.

Adam: *"Maths is a thinking and doing subject. It's not necessary for you to memorise. If you do lots of work then you just remember the maths - after you do the work a few times you just remember the whole thing."*

Adam feels good about his learning when he can do all the questions but adds *"and I check that I understand everything"*. Adam's intention to understand influences all facets of his learning behaviour as illustrated by the following episode in which students were asked to make a summary of their trigonometry unit (Sine and Cosine rules). Rather than copying the formulae and/or worked examples, as was the case with all other students, Adam not only made a summary but used this opportunity to enhance his learning. He saw the task as a means of learning rather than one of task completion.

Adam: *"I just revised all the proofs and wrote them down on paper and ran through how to prove the three formulas. I went through to check to see if I understood them. If I forgot how to do it I read through it and tried to work it out. I also looked in our other textbook to see if there was any other way of doing them."*

LEARNING EPISODES

Class Discussion

During class discussions Adam exhibits a wide range of learning strategies. He reports effective cognitive strategies to construct new knowledge from the day's lesson and he is

also very aware of the need to control his learning environment by using opportunities for rehearsal and task management activities.

Of the four target students, Adam is the only one who reports actively planning and previewing the material to be covered, both before the lesson and during the lesson.

Adam: *"When she said turn to Page 128 I looked and I actually read the question before she* (teacher) *said what she was going to do."*

Previewing material provides Adam with the opportunity to go over the material several times. Comments regarding the value of practice in automating procedures and revisiting the material to aid understanding are typical throughout the stimulated recall reports.

Adam: *"I think it's good to read ahead because - I don't know - I just finish everything so I just read the next one. If I don't understand something I'll read it again...I've read the work, but not really learnt it. It's easier for me, it helps you to understand, to remember, and it makes it faster."*

Also, during discussion of a new topic Adam will skim through the text, often looking for further information. For example, when the teacher introduces logarithms and exponents Adam looks for references in the text and index, trying to find out about natural logarithms. He is not very satisfied, but rather than ask the teacher, he reported that he would investigate it further at home. Where the book provides alternative explanations to those given by the teacher, such as was the case with proofs of the sine and cosine rules, Adam will study and evaluate the alternatives.

Adam quickly works through class discussion problems, thus is able to anticipate answers and closely monitor his progress.

Adam: *"I knew the answer before Mrs H worked it out. I checked all the calculations on my calculator, then went on to the next question in the book."*

During class discussions Adam employs the metacognitive strategy of selective attention to achieve three purposes. Firstly, he is conscious of selectively attending to important information.

Adam: *"When I hear important things I would listen and try and remember them; like 'limit', 'definition', and things like that. I actually hear what I know and what I don't know. If I hear something I know I'm not very good at I just listen."*

Notes are also selectively taken on the basis of importance to the learning process.

Adam: *"I always decide whether I'm going to put an example into my notebook or into my exercise book. I'm thinking what is she* (teacher) *going to do with it - I was thinking that she might do something new in using this example, I might learn something new so I put it in my notebook."*

Secondly, when the teacher is reviewing problems Adam selectively attends to the process and conceptual material rather than the calculations.

Adam: *"I don't worry about the calculations 'cause I know I can do that - I can program my calculator to do all of those."*

Thirdly, Adam uses selective attention, in combination with monitoring understanding and production, to attend only to those parts of the lesson relevant to his learning needs. The remainder of the time Adam manages his own learning tasks.

Adam: *"I just check with the board to see if my answer is right or look to see if there is anything important. Normally I pay attention when she asks a question, otherwise I work independently on my own problems."* and

Adam: *"I already know that so I continue on my own work. I'm listening to what she* (teacher) *is saying but I know it. Now she's doing some new work. I couldn't remember the work on standard error so I start to copy it down."*

Adam's reports of elaborative statements are not very explicit. This is not unexpected as many of the links with prior knowledge would be automatic, rather than as a result of confusion or uncertainty, as was the case with Karen. However, there were repeated instances of imagery, indicating a very active participation in the knowledge construction process. For example, when the teacher talks about the derivative Adam reported that, *"I just have a diagram, I think, just drawing a diagram in my head".*

In instances where the links with prior knowledge are not automatic, Adam reports intentional elaborative strategies of linking. Unlike instances with the other target students, Adam is not prepared to just watch and wait for the final answer, nor is he only concerned with the calculations involved. Rather, elaborative strategies are employed to further consolidate and integrate new information with existing knowledge.

Mrs H: *"Yesterday we came up with the idea that we added them up by doubling them."*

Adam: *"I made notes on this part for homework, I'm going back to look for the formula she's (teacher) talking about."* (Teacher continues with worked example) *"I'm putting the things, I'm finding out the relationship between the formulas and what she's using for the example...I'm working it through - finding the relationship."*

On another occasion, when Adam tries to link with previous knowledge, he exhibits strategic learning behaviours to deal with the uncertainty of the association with prior knowledge.

Adam: *"I'm looking at my calculator to see if I can calculate standard error. I'm wondering what this n-1 button is. I can remember that my brother told me that standard error is something to do with sampling so I thought this button, the sample standard deviation, might be something to do with it...When I used* σ_{n-1} *I came out with a very different answer so I think about it. I redo it. I'm still not sure what standard error is so I'm listening to see what she* (teacher) *might come up with."*

When involved with learning from class discussion and teacher explanations Adam is often more critical, than are the other students, of the teacher's explanation and methods. As he examines teacher-presented methods he often draws on prior knowledge of alternative strategies:

Adam: *"I was thinking, 'what's she* (teacher) *doing?'...I think she's having problems drawing the graph so I'm just watching what she's doing. I was thinking about how to draw it, about how else to do it. I'm thinking it looks a bit strange not like the normal distribution....I thought that she shouldn't be drawing the lines because it is discrete data. I'm thinking 'what is she doing and how am I*

going to draw it in my book?' I've just started to draw my graph. I thought it was a bit strange that she joins the lines."

Furthermore, when the class data results don't seem to provide a suitable distribution Adam is able to use his prior knowledge to resolve this conflict.

Adam: *'I looked back at the tree diagram. I was thinking that we probably didn't have enough data to make the 'mean' work. I wasn't worried too much because when I did it at home I took the sample mean of my ten trials and it was actually very close to the population mean which is 4·5. I found the sample mean was 4·1. I thought this data gives strange results but I didn't worry 'cause I knew what should happen."*

Unfortunately Adam is the only student in the class to have completed a graph at home so probably he will be the only student who has constructed the knowledge as intended by the teaching episode.

Adam's critical examination of teacher/class discussion is also driven by his desire to understand the content. However, unlike Jane who appeared to be stuck in an 'understand or not understand' mode, Adam is able to use metacognitive and resource management strategies to cope with any difficulties.

Adam: *"I was wondering what standard error is, I thought I might ask my brother or my father. I don't really understand why do we find the standard error that way. She didn't really explain where it comes from. I didn't really understand why do we do it that way."*

Like the other target students Adam rarely answers questions publicly: this appears to be an affective reaction rather than a concern for correctness.

Adam: *"I remembered that but I didn't answer. I don't usually answer the questions. I know the answers but I don't answer - it probably depends on the mood. Sometimes I feel good and I answer the questions - other times I don't answer the questions."*

He does, however, always answer questions privately, frequently with an accompanying metacognitive evaluation of understanding or production. Adam is also actively answering and evaluating other students' questions. In the following example Adam

employs elaborative strategies of *comparing* and *answering questions* combined with metacognitive *monitoring understanding.*

Adam: *"I was listening to what he* (Dean) *is saying and just thinking what sort of problem he is asking and see if I can do it."*

Adam's ability to 'keep up' with ease, combined with planning and previewing, increases the likelihood that the lesson will become boring. On several occasions Adam reports frustration with the slow pace but he has developed some affective strategies which, when combined with task management and rehearsal strategies, can successfully relieve the boredom factor.

Adam: *"I read from the blackboard from the other teacher's work - it was something to do."* and

Adam: *"I'm looking at the next part so it doesn't get too boring."*

Another strategy Adam used when the discussion is paced too slowly is to use this opportunity for rehearsal. His deliberate strategy of over-learning, or structured reviewing, means that the procedural content becomes natural and automatic - hence his earlier comment concerning his non use of memorisation.

Adam: *"I just revise all these things in my head again, tried to remember them better. We've finished the topic, we've learnt everything it's just revision."* and

Adam: *"I'm sort of refreshing because I thought about it the night before the lesson, thinking it through again because it would be boring not thinking."*

Seatwork

Adam is very prompt to begin seatwork; usually beginning with a flip through the exercises. He works at a fast pace, but rather than waiting for the rest of the class he manages the task by doing extra exercises*: "I actually did five when everyone did one."* The following comment, related to Adam's task management strategies, again illustrates the fact that Adam's learning is directed to understanding, and knowledge construction, rather than problem completion, and further illustrates Adam's ability to control his learning:

Adam: *"It's not the exercises that are important. I need to know what we are going to learn, the pages. I can find the exercises myself."*

Adam reports comparing problem types during seatwork episodes.

Adam: *"I'm thinking about No. 5 - if I can put that equation into my calculator or not - it's a bit different to the others."*

Unlike most of the other students Adam's monitoring is not reliant totally on the text answers - his prior knowledge and confidence allows Adam to evaluate the correctness.

Adam: *"I didn't look at the answers because I know it is right - I believe it is right. If I was not sure I would check each answer, but because here I've got the right formula and the right numbers I know it's right."*

His checking strategies are securely based on production monitoring and involve several stages of diagnoses when difficulties arise.

Adam: *"I'm unsure of the answer so I'm thinking if it's right or wrong. I'll know I'll get it right if I've put the correct things in my calculator so I'm just checking that first."*

When completing problems Adam uses a coding system based on his anticipated ability to complete the problems.

Adam: *"I used a pencil to write the answer because I didn't actually know if the answer was correct."* and

Adam: *"I write the question down - if I know the answer I just write it down with pen and if I'm not sure I just write it down with pencil."*

Like most other students, Adam is reluctant to ask for help during the class session. He mentally notes that he needs to further address the problem himself: he does this by self study of the text both in class and at home, seeking help from his father or brother at home, and by self-reflection.

Adam: *"I didn't really understand what that meant so I look in my textbook. I'm looking in my book for anything about standard error."* and

Adam: *"I was puzzled about standard error instead of standard deviation. I didn't really understand it. I decided to ask my brother."* and

Adam: *"I'm doing the next part of the question but I'm still thinking about the part I couldn't do."*

Adam's ability to self-help is evidenced in the lesson on variance calculations. Many students became frustrated trying to adapt the teacher's instructions to their individual calculator model:

Karen: *"Can you show me how to used this stupid calculator."*

Adam, on the other hand, gets out his manual and successfully self instructs himself in the variance calculation procedure.

There was only one occasion during all of the observations when Adam was observed to be experiencing considerable difficulty with a set of exercises. The exercises involved sequence completions of a puzzle type nature:

Complete the following sequences: No. 2 O, T, T, F, ...

No. 3 J, F, M, A, ...

Adam's behaviour during this episode (Video 2) illustrated that he has a range of learning strategies available for such a situation. Firstly, Adam reported prolonged engagement with the problem - he continually reflected, returning to the problem sequence in between successful attempts at later sequences.

Adam: *"I'm still thinking about the question 'cause I'm stuck. I'm trying to sort out what the answer might be. I was thinking what would the letters stand for. I'm wondering if they represented some kind of number like 'b' is the second and 'a to z' but it didn't work so I'm still thinking."*

Adam ignores the teacher's call to attend to No. 1 and continues to work on No. 2.

Adam: *"I knew I'd got it* (No. 1) *right so I didn't bother to listen. I work quite a lot on my own. I was thinking about my problems. I was trying to make a connection with the alphabet but it didn't work."*

At this stage Adam reported feeling very worried about still not being able to solve the sequence. He employs an affective strategy designed to assure himself of attainable success.

Adam: *"I'm just thinking try to solve it, still solve it, solve it* (nervous laugh). *I think sure I can solve it and things like that."*

Usually Adam is not concerned with other students' progress, but on this occasion he actively monitors the class activity.

Mrs H: *"How many people haven't finished No. 2?*" (Adam looks around the room).

Mrs H/Lucy: "*You are suppose to keep the answer to yourself."* (Adam looks over in Lucy's direction).

Adam: *"I heard someone had solved it. I heard someone say 'One, Two, Three', so I went straight onto the next question* (No. 3). *I made the connection and solved that too."*

Adam consistently follows the teacher's direction, unless there is a legitimate reason to modify the task. When students were asked to read worked examples or notes from the text before attempting the exercises, many of the students would go straight to the exercises preferring to refer to the readings only if stuck. Adam, on the other hand, always completed the recommended reading first.

Adam reported monitoring the teacher's movements and comments during seatwork time. Most of the time he knows what he needs to do and reports that he ignores the teacher's comments (selective attention), however he usually tries to answer class-directed questions.

Mrs H: *"If I asked you to write down in your own words what a limit is."*

Adam: *"I'm thinking in my head, I just go through in my head what she was asking. I just tried to give a definition in my head to see if I could. I was happy with my answer. I was trying it in my own words."* and

Adam: "*I looked up because I thought this might be important; it sounds a bit confusing, sounds a bit new."*

Review of Homework and Seatwork

Adam admits that most of the homework review sessions are a waste of time and that productivity during these sessions depends on his mood.

Adam: *"Going through with her, she* (teacher) *might link homework with something else. It depends on my mood. Sometimes I might be thinking about other subjects. Sometimes I might do some more maths, but often I can't really go ahead because I don't know where to look for what I want."*

On a particular occasion, when only Gareth and Adam had done the assigned homework, the teacher spent a large amount of class time letting the students take random samples to provide data.

Adam: *"I was thinking of something else, not maths. I'm looking around the class. I feel bored; I've done the work. I was just waiting. I'm looking at Gareth's work. I'm looking at Gareth's results. I wondered how long it would take."*

When the teacher displayed the results on a stem and leaf plot Adam evaluated the validity of the method rather than his own ability to complete the diagram.

Adam: *"I thought that's one way to do it - I suppose. It seemed a good way to do it."*

When the teacher continued with displaying the class results one could see how Adam's prior knowledge, gained from homework sessions, enabled him to anticipate the lesson direction.

Adam: *"I'm thinking about the normal distribution of the curve. I knew that she was going to talk about that 'cause she* (teacher) *told us that we are going to do something about random numbers, normal distribution and standard error, and things like that. I'd looked ahead in my book to see what was coming."*

Adam is conscious of opportunities to learn, and sometimes uses review time to work independently on further exercises: *"Well sometimes it's good, you can fit revisions into the lessons."* In the above episode, after expressing boredom, Adam decided to move on to some different work while the class caught up with the homework.

Adam: *"I'm getting bored. I'm reading the notes I took a week ago from the book. I remember that I forgot to do something - so I've looked up the book about surveys - there are two types and I hadn't finished it so I'm writing some more. I've decided to work by myself until the class finishes...I'm working slowly to fill in the time."*

At other times, when Adam attends the review, he tries to use the opportunity to positively effect a learning outcome.

Adam: *"I've finished all the question, so I'm just looking at things on the blackboard. I'm thinking, trying out the answers again even though I know the answers I answer her questions in my own mind."*

And in other instances Adam critically reviews the teacher's methods:

Mrs H: *"Can anyone tell me the quicker way?"*

Adam: *"I was thinking about a quicker way. I didn't really think about that method* (the teacher's quicker way*). Now I see hers I think it through and see if it really works. I check through to see if it works and if it's faster, and it seems okay."*

Again we see an instance where Adam evaluated the method against criterion of effectiveness and efficiency rather than solely against his ability to 'follow' or 'do' the method.

Homework

Adam reports starting homework in class. When the teacher sets the homework in class Adam records the homework in his logbook, previews the homework and decides if there is enough time to do some or all of it in class: *"I think, oh that will take about 10 minutes."* At home Adam sets a time limit and reports that he usually finishes it before or around the time set. It is important to set a schedule as Adam has homework from other subjects to complete, and homework completion is very much the norm for Adam.

Adam feels that homework is mostly just for practice. He reported that he learnt a bit about calculus and statistics at home but most of the learning was in class time. As well as consolidation, Adam uses the homework time to look ahead and prepare for future class work.

Adam: *"I look ahead a bit to see what's coming up, but I won't properly do all of the questions."*

However, Adam reported that efforts to read ahead were somewhat hampered by lack of knowledge of what was required.

Adam: *"I use to work ahead a lot last year because I had the form five curriculum and I could do the whole thing before the teacher teaches it but I can't this year because I haven't got the schedule. It was really good last year because I could go ahead and do things myself."*

All homework is marked, although not in the physical sense of ticks and crosses.

Int: *"Do you check your homework?"*

Adam: "*I use to; I use to tick them, tick them right, but now I just look at the answers and compare them to mine."*

Int: *"So you check them; so you know you got say nineteen out of twenty."*

Adam: (indignant) *"No, I don't do that. I don't count. If I got some wrong I redo the question."*

On one occasion Adam modified his homework by actually doing less than was required! However, there was a sensible explanation. When given a copy of last year's test to complete for homework, followed by a period of revision time, Adam carefully utilised his time by reading the test through at home, evaluating his learning requirements, and leaving the test to complete in class when he knew that he would have nothing else to do.

Adam: *"I read it through and worked it out mentally. I know I can do it all except the limit question which I've checked with Mrs H. Now I'm going to do the test and time myself - it should take about half-an-hour and I should get 100%."*

In summary, homework is an important feature of Adam's learning. He strategically uses the time to consolidate, to check for understanding, to seek help either by self-reflection or from the text or family, and also to prepare for the next lesson.

Revision

Because of the thoroughness of Adam's homework efforts he feels that he does not need to make a big effort when it comes to revising for tests.

Adam: *"I find if you have done all your homework you are pretty well prepared. I don't think it's necessary for you to revise; only if you don't understand something, 'cause you've just learnt the topic. I know the things already. Unless I'm not sure how to do something I won't do much revision."*

Adam reported some planning of revision but he felt it was more appropriate for exam revision than unit tests.

Adam: *"I go through in my mind some of the topics but I don't worry too much about scheduling revision for tests, but I set a revision schedule for exams."*

Like the other students Adam reported reading through his notes (the day before) and doing some exercises as test preparation. Adam also acknowledges the help from teacher cues, especially from copies of previous years' tests.

Adam: *"Well, we've already seen last year's and the year's before test so I don't think there is going to be much difference. They* (teachers) *don't change them much."*

Adam's goal of sitting test is *"to gain as high a mark as possible"* (where 95+% is acceptable) and he sees tests as a part of the classroom system.

Adam: *"Tests help me make sure I understand things. I think tests are just an extra to make sure you understand things - they don't make me work harder. I don't learn anything by sitting tests."*

Summary

We can see from the given examples that Adam uses a wide range of learning strategies which are both appropriate and effective in enhancing knowledge construction. In class Adam is concerned about identifying what ought to be learnt. He prefers to understands concepts on the spot, but is not unduly worried if difficulties arise. He is willing to reflect on his learning and seek appropriate help to allay any concerns.

Adam values practice, not just for it's own sake, but as an integral part of the learning process. He subscribes to the view that if one practices enough one avoids the need to rote learn formulae and procedures: practice provides Adam opportunities to review, monitor and extend his understanding.

Through "intentional learning" (Bereiter & Scardamalia, 1989) Adam develops a problem-solving approach to learning in which he is conscious of his learning goal, plans his learning processes through active strategy deployment, monitors his progress towards the learning goal, and is able to take remedial action where necessary. Adam's critical examination and use of the resources (text, teachers and family) is a result of his desire to understand and integrate new knowledge.

Adam's strong prior knowledge base is obviously a major factor in his success, but it needs to be acknowledged that this knowledge is not just domain based but also metacognitively oriented. Adam's beliefs about himself as a learner, his beliefs about mathematics learning, and his knowledge of a range of learning strategies and their applicability, contribute to his effective autonomous learning behaviours. Adam's effective self-regulatory strategies enable him to: monitor his comprehension; self-test by generating and working through similar problems to those being presented; to anticipate the answers to the teacher's questions; and to check his own answers against those of the teacher as well as the other students. It appears that it is only when one is able to simultaneously **monitor and control** one's learning, that the strategy of reflection can be brought to bear on the learning situation. In summary, given sufficient resources, Adam is fully able to self-direct his own learning both in class and at home.

8.6 Passive versus Active Learning

While it is acknowledged that much of the data reflects the students' ability to discuss their learning, student reports and research observations indicated that these four students engaged in a wide range of learning strategies with consequently wide ranging learning outcomes. In what ways are their learning strategies differentiated to produce these different learning outcomes? One could classify their learning styles on a continuous scale from passive through to active learning (Mitchell, 1992a). On the one extreme Gareth's learning is of a passive nature, and Adam's is of an active nature. Jane and Karen fit somewhere in the middle ground. An examination of some specific uses of learning strategies will illustrate these contrasting learning behaviours.

Firstly, these students all differ in their learning goals and beliefs about learning mathematics. Gareth, in particular, has little sense of learning, and sees mathematics in terms of problems to do. His learning strategies are directed at absorbing knowledge from the teacher; they are directly linked to his belief that, if he follows the steps used by the teacher, he has learnt the required mathematics. This is an extremely restrictive approach to learning which is epitomised by Gareth's comments during a test review: "*Oh, is that all that question wanted; if you had asked it the same way as you had in class I could have done that one!*" Karen and Jane also view learning mathematics as largely a matter of taking in information from the teacher and textbook and storing it in memory. While Jane and Karen do acknowledge the importance of understanding in the learning process, when understanding proves elusive, they lack specific strategies to remedy their difficulties. In contrast, Adam's learning goals are firmly directed towards understanding the content and constructing new knowledge. Bereiter and Scardamalia's (1989) concept of intentional learning appropriately captures Adam's learning process:

> *...(those) who were "**trying** to learn" were not simply investing extra effort in trying to solve the problems they were presented. Instead they were dividing their effort between solving those problems and solving other, unassigned problems, which were problems having to do with the state of their own understanding of the phenomena...The learning that resulted was not an incidental consequence of solving mathematics problems but rather a goal to which (their) problem-solving efforts were directed.* (p. 365-6)

Another major area of strategy differentiation is in the use of selective attention. Although all students used selective attention, we see that the focus of their attentions differs and thus the constructed knowledge differs. Gareth, and to some extent Jane and Karen, focused solely on the current work without attempting to look for connections with what was done previously. Each lesson, each problem, or each instruction was seen in isolation. Gareth, in particular, focused on the procedural and arithmetic steps in the problem. In contrast, Adam's attention critically focuses on the conceptual details, almost to the exclusion of the calculations.

Student evaluations of teacher-presented methods, or textual material also differed. Adam critically evaluated methods on the criteria of efficiency, ease, and completeness in terms of explanation and his own understanding. His self-questions led him to explore beyond the given data and construct new knowledge. Jane and Karen were also able to distinguish between those explanations which they understood and those which were 'just methods'. In contrast, Gareth accepted the teacher's answers uncritically, even in cases where they were incorrect, and did not expect to understand the explanations. His evaluations were based on whether he 'followed' the method - which resulted in his being able to supply the answer. Having a copy of the worked example was critical for Gareth's learning process as this provided the 'recipe' needed to do further similar examples.

Another contrast is found in students' ability to correctly monitor their learning process. Although all students reported numerous monitoring statements, they were related to different criteria of learning. Gareth's monitoring is related to task completion, and is accompanied by statements of the lesson being 'easy' or 'hard'. Gareth decides that he understands and will be able to retain important information simply because he has 'read it' or 'seen it being done' by the teacher. In contrast, Adam's approach is more active. He evaluates his learning only after testing himself, or otherwise generating feelings and information about his actual state of memory and understanding.

As well as differing strategies related to learning awareness, the students' ability to control their own learning was remarkably differentiated. Gareth's passive, dependent, uninformed approach to learning mathematics meant that he relied heavily on the teacher to provide the information and instruction. He saw the teacher as someone who told him what to do, how to do it, what examples are worth investigation and what questions are worth considering. This dependency extends to monitoring. Gareth is reliant on the text and the teacher to provide the authority for the correctness of his work, as evidenced by his acceptance of the teacher modifying his working: "*She knows what she is doing.*"

Peterson (1988) suggests that the reporting of a definitive or diagnostic reason for not understanding may reflect students' general tendencies to be involved actively in the mathematics task, regardless of their immediate understanding. We have seen examples where Karen has, as a result of her involvement, been able to construct sufficient mathematical knowledge and understanding to enable her to eliminate difficulties with a minimum of new information. Thus with an active learning approach cognitive failure can become a metacognitive success if appropriate action is taken such as redirecting attention, re-accessing information sources, or help seeking.

Both Karen and Jane provide evidence of episodes of self-regulated learning. Karen provided evidence of independent study to remediate conceptual difficulties, and Jane reported doing extra exercises and looking ahead on occasions. However, although aware of difficulties, the weakness of their self-regulatory strategies are evidenced by their reluctance to seek appropriate help when required. Both Karen and Jane are

unwilling to ask questions, or to answer questions, unless they are sure they are correct. Their reluctance to become involved in discussions means that they do not profit from feedback, and are reliant on other class members to prompt for help.

Rohrkemper and Corno (1988) suggest that in order to perform a task efficiently, be it learning or problem solving, students need to see both the approach they take and the task itself as malleable: these three students (Karen, Jane and Gareth) appear unwilling to control their learning environment to any great extent. Adam, however, is in complete control of his mathematics learning: he plans his learning, anticipates the lesson direction, and actively seeks help. Adam's reports of reflective thinking are also more frequent than any of the other students, and appear to be effective in resolving conflicts and constructing knowledge. Furthermore, Adam is able to adapt himself, the task, and the learning situation, to maximise the learning opportunities. He fully appreciates the difference between 'doing mathematics' and 'learning mathematics' as expressed by Leinhardt and Putnam (1987: 559):

> *...by learning mathematics we mean grasping the intentional content of the lesson, connecting and integrating that content with prior mathematics knowledge.*

Homework and revision sessions involve learning that is isolated and self-directed (Thomas & Rowher, 1993). It is in these learning episodes that one would expect learning strategies to play a crucial role. Thus, not surprisingly, we see a variation in use of learning strategies among our four target students. Gareth reports doing what he can, but is not able to make any progress in resolving conflicts from the lesson, or furthering his knowledge. Jane appears to be guided by affective reactions and metacognitive knowledge - some difficulties are resolved, other are left with a question mark. Karen selectively attends to problems experienced in the lesson and reports actively using the text as a resource. Like the others, Adam completes the exercises, but in addition he uses homework sessions to further his learning by reviewing and previewing material, self-reflecting on concepts, and seeking help from family members.

In summary, although clearly all these target students use a range of learning strategies, the appropriateness and effectiveness of these strategies are related to the learning goal and the demands of the task. With the exception of Adam, for the most part students' learning was of a passive nature: students sampled selectively from the flow of instructional stimuli according to their needs and interests, but seldom took action to adapt the lesson to their individual requirements. They accepted their learning environment as given, expected the teacher to provide explicit instruction and help, relied on the teacher or text to monitor their progress, and made little use of the available resources in independent study. Students' ability to control their learning and adapt themselves, the task, and the learning environment to maximise the learning opportunity, is largely a result of metacognitive knowledge and the use of metacognitive strategies.

The role of prior knowledge in enabling learning strategies, especially elaborative and monitoring strategies, to be successful must also be acknowledged. Other contributing factors differentiating strategic learning included the availability of help at home, the availability of resources, and the availability of study time. These and other factors affecting strategic learning will be discussed in the following chapter.

Chapter 9

Factors Affecting Learning Strategy Use

If you want to see a very faint star you should look a little to the side because your eye is more sensitive to faint light that way - and as soon as you look right at the star it disappears.

(Waldrop, 1992: 319)

As illustrated with the *Interactive Model of Learning Mathematics* (section 2.3) there are numerous person, instructional, and contextual factors affecting strategic learning. This chapter will discuss factors evident in this study which influence students' use and development of strategic learning behaviours. The discussion draws on the literature review, research observations, student reports, and the researcher's knowledge of the teaching-learning environment. Although these factors are considered under separate organisational headings, it is noted that in reality, strategic learning behaviours are influenced by a multiplicity of factors.

9.1 Person Factors

Prior Domain Knowledge

Relevant prior knowledge, both domain based and metacognitive, may be the student's most valuable resource in relation to learning: a resource which greatly affects strategy use. When learning, students need to activate and utilise their prior knowledge so as to integrate it with the new information in a coherent and logical manner (Weinstein & Mayer, 1986). In the present study students' prior knowledge, as measured by the previous year's examination, ranged from very weak (Gareth) to expert (Adam). The differential access to relevant knowledge affected the applicability and effectiveness of learning strategies.

Instruction frequently linked new content to prior learning and students responded with personal elaborations relevant to their prior knowledge and experiences. However, whether any elaboration (either teacher supplied or student initiated) can be used, or is appropriate, depends on the information at hand and the existing knowledge of the student. Elaborations from low achievers often involved trying to recall having worked some similar method, or remember past suggestions by the teacher, rather than recalling the conceptual or procedural information relevant to the problem. The strength of the urge to remember past experiences is unfortunately reinforced by teacher comments such as, "*Remember how we did it yesterday" and "Look back at No 8. to see how you did it"*. Consequently, instruction could assist elaborations which relate to conceptual and procedural recall by asking "What do we know about solving these type of equations?" ($x + 4 = 2x + 1$), rather than "*Remember what we did yesterday*". The open and conceptual nature of this question would be more conducive to appropriate student response.

In cases where students **have no** relevant prior knowledge each procedure must be learnt in isolation. For example, when learning surd manipulation the teacher illustrates the following steps in a worked example:

$$\sqrt{9x8} + \sqrt{8} = 3\sqrt{8} + \sqrt{8} = 4\sqrt{8} = 4\sqrt{4x2} = 8\sqrt{2}$$

Because of his weak algebraic knowledge Gareth does not make any elaborative connections with algebraic simplification: he sees these steps as a totally new rule-based procedure to be learnt in isolation and recalled when he is tested on surd questions.

Gareth: *"I'm going over all the procedures until it gets in my mind how to do it. I wrote down the procedures."*

Int: *"Could you have explained the steps in your own words?"*

Gareth: *"I could have if I had the working down."*

Later, when trying his first problem, Gareth reports swapping the factors of $\sqrt{3x16}$ around so as to get the same pattern as on the board (i.e., the square factor first). With a more relevant knowledge base, involving a recognition of commutativity of

multiplication, Gareth would not be in the position of having to mimic every step of the worked example. The availability of the assumed prior knowledge would enable him to make appropriate elaborative links and thus see this problem as an extension or variation of the given worked example.

Students' ability to perceive and carry out cognitive processing intended by the teacher sometimes depended on prior knowledge (Marx & Walsh, 1988). Gareth's lack of the assumed prior knowledge meant that mislearning was more likely to occur: an example of Gareth constructing a 'buggy' algorithm in relation to rationalising surds was discussed in section 7.2.

Those students who lack the necessary prior knowledge need to be adaptive: they may need to use alternative strategies to produce equivalent knowledge. For example, Gareth, who is aware of his weak prior knowledge (metacognitive person knowledge), consistently tries to minimise the effects of limited prior knowledge by applying resource management learning strategies. In class, whenever the teacher introduces a new topic, Gareth skims his text for definitions and formula in readiness for class questions. He sometimes reported previewing previous years' work in an attempt to 'make up' for a limited background knowledge on a topic.

Alexander and Judy (1988) suggest that the **nature** of strategy use changes as individuals become more knowledgeable in a domain. Those students with sufficient prior knowledge are more likely to access this knowledge and use elaborative strategies to integrate the new knowledge into existing schemas, or construct new schemas where necessary. Karen and Adam both reported more specific elaborations with previous content, rather than just elaborations involving recall of previous experiences with content. In particular, Adam's use of previewing strategies meant that his domain knowledge was sufficient to allow him to anticipate teacher directions, and plan which episodes of the lesson to attend to, according to his personal learning goal.

Students' ability and inclination to monitor their understanding and learning process is greatly dependent on the availability of prior knowledge. Access to appropriate prior knowledge assists students with monitoring their understanding. In contrast, if memory resources are strained monitoring is unlikely to occur. Examples of Gareth's learning illustrates that without adequate prior knowledge it is difficult to monitor the reasonableness of an answer to problems.

Beliefs and metacognitive knowledge

When students repeatedly do not understand their formal instruction, and written assignments do not make sense, they may come to conclude that mathematics is not supposed to make sense. Gareth, in particular, is influenced by his belief that mathematics does not need to make sense.

Int: *"Is it important to understand the concepts; like say to know what standard deviation is about?*

Gareth: *"No, it's more important that I can do the problems."*

Siemon's (1990a) data indicated that a student's belief that mathematics is not concerned with meaning was the driving force in determining his or her monitoring behaviour. It is probable that because of such a belief, students like Gareth are likely to stop monitoring their work thoughtfully. Several instances were observed in which students were not the least bit troubled by answers that were clearly unreasonable. For example, in a practical trigonometry exercise, students were quite happy to calculate the height of the building (six metres) as ranging between one and forty metres! Gareth also reported several instances, including the following response involving calculations of variances, in which he concerns himself totally with syntax rather than semantics:

Int: *"What is that last column* $(x-\bar{x})^2$ *?"* (Interviewer points to column in the table)

Gareth: *"I don't really know. I just know how she's* (teacher) *done it. All I know is that it helps you when you get the standard deviation."*

While some students demonstrated a commitment to monitoring calculations, few students monitored the formulation and evaluation of their cognitive goal in relation to the task. However, there were occasions when students, such as Adam and Jane, valued

and attended to the construction of meaning, They demonstrated a preparedness to analyse the problem statements before making any decision about what needed to be done. When completing exercises in a statistics unit Jane always wrote out a summary of each of the question requirements in red, *"So I can see it, so I know what I have to find out"*. In several instances Adam reported critically evaluating the teacher or his own method. The following instance is an example of Jane's evaluation of alternative methods against the cognitive goal of understanding rather than ease of computation:

Jane: *"I decided to try both methods* (synthetic division and long division) *with these exercises - to see which one is the easiest and which one I understand the most...When I use both methods I did one and got two different answers so I looked up the answers after each one to see which method gave the right answer."*

Self assessment of one's successes or failures is important in the formation of metacognitive knowledge. Gareth readily talks about his weakness in learning mathematics.

Gareth: *"My weak points are that I can't really do a whole lot of maths questions at one time because I get really impatient. I get bored doing maths questions over and over all the time, I find it hard to concentrate. My good points, um, I have none."*

However, when questioned further, Gareth did decide that he did some things that were helpful for learning.

Gareth: *"Well, if I don't understand something it helps if you read it over and over again...Taking part in discussions is good 'cause it helps you remember the period more. Like if you're just doing questions* (exercises) *all period you don't really take much in. So when you're really discussing it in a group it gets real, it's easier."*

Thus, although Gareth is able to reflect on his learning strategies (metacognitive strategy knowledge) one can again see that he believes that learning mathematics is about 'taking' in content and that success is measured by 'easiness'. His admission that doing exercises is in itself a limited learning experience is consistent with what one would expect from a passive learning experience (Kyriacou & Marshall, 1989; Mitchell, 1992a).

Nature of the learning goal

The nature of students' learning goals will affect the nature of their learning strategies (Garner, 1990a). Leinhardt and Putnam (1987) suggest that because of the difficulty in getting students to define learning (as opposed to task completion) as a goal it is unlikely that students effectively assess progress towards the goal of learning at higher levels. Some students in the present study did not see themselves primarily as learners; their metacognitive processes were directed at working out a comfortable, or at least acceptable way of coping with the school task. For these students the goal of understanding everything is unrealistic and consequently marginalised.

Brent: *"I don't really understand everything. I just try and get the basic idea. Like she'll* (teacher) *explain a topic before you start doing it and you should be able to do the starting stuff, later on it changes things 'round a bit and adds new things on - it gets a lot harder. Sometimes you cannot understand anything in a lesson, like yesterday's lesson on compound interest. I just leave it and try and concentrate on the basic stuff - like, it's not really worth trying to understand."*

Such students are not always aware of the purpose of seatwork, they lack a firm conceptual grasp of the goal of the task in which they are engaged. It is assumed that sets of exercises are to help students become aware of the general techniques - the theory being that if you do enough examples you will 'see through' the particular to the general. The value of promoting the generalisation (for example, being able to state the conditions when Cosine Rule is appropriate for finding the unknown side of a triangle) is that the generalisation is the student's own knowledge and they no longer need to rely on memorised formula. Some students, however, perform the necessary sub-skills, or algorithms, on demand, but do not grasp the significance of the learning activity. Instead, their goals were clearly directed towards task completion rather than intentional learning. In particular, low achievers used strategies such as copying problem solutions from peers or worked examples, checking answers from the book rather than self-checking, and frequent help seeking that contributed to content coverage rather than content mastery. For example, when these students sought help they asked questions

such as, *"Can you show me how to do this?"* and *"I can't get the same answer as the book does"*, so as to facilitate task completion.

In contrast, students whose goal is to learn with understanding were more inclined to ask questions directed to obtaining specific information. For example, on one occasion when Adam evaluates the teacher given summary, he queries a specific piece of information:

Adam: *"Where does the 0 come from in $f'(x) = 4 + 0$?"*

Students such as Adam see themselves primarily as learners; they select appropriate strategies for knowledge construction, rather than task completion, and monitor their understanding of it.

Affective Factors

Marland and Edwards (1986:79) suggest that "the 'private, inner-worlds' of moods, feelings, interests, self-images, previous experiences and fantasies" often direct students thinking, attention, involvement and learning processes. They found that interview protocols of secondary school students suggested that engaging in mental activities connected with these 'inner worlds' sometimes aided and sometimes impeded learning. Similarly, students in this study reported many affective reactions to learning mathematics, being in a classroom situation, revision, and 'taking' tests. For some students these affective factors were an important influence on their strategic learning behaviours. Interest and motivation factors were especially influential.

Garner (1988: 64-5) suggests that "metacognitively sophisticated learners know whether or not the criterion task to be completed warrants the costly expenditure of time and effort involved in strategic processing". If the task is viewed as unimportant, or if the learner is not devoting conscious attention to it, monitoring is unlikely. For example, if a student views understanding as unimportant, or at best an incidental consequence of doing problems, then his or her monitoring is based on completion criteria rather than understanding. The element of choice in strategy use was evidenced when students' reported that knowing **which** learning strategy (they should employ to improve their learning) was a different matter to actually **using** the strategy.

Jane: *"...if I really wanted to know I could ask someone at home, but I probably won't bother."*

Karen: *"To start with I couldn't be bothered to write down the notes so I just thought I was going to put down the occasional sentence of what she* (teacher) *wrote. I was going to wait and see what she wrote first."*

Both Karen and Jane had after-school employment: Karen worked 18 hours a week and Jane worked several hours before school each day and in the weekends. In class they showed day-to-day variations in their cognitive strategies, ranging from 'just looking' and 'not really thinking' to complex elaborations. Their obvious tiredness, and resulting lack of motivation, may go some way to explain why appropriate learning strategies were not always invoked.

Several students reported feelings of disinterest and boredom. Nickerson (1989:24) notes that "the tendency of students sometimes to balk at making the effort required to understand ideas, rather than simply acquiring surface or algorithmic knowledge, may be the reflection of a deep preference and not just laziness or lack of adequate instruction". The fact that learning mathematics is at times very difficult, requiring higher-order thinking, resolution of ambiguity, tolerance for uncertainty, self-criticism, and hard mental work, means that it is not surprising that students' interest may wane from time to time. For many students boredom, which was the result of the work being too hard or uninteresting, led to task avoidance, rather than a more adaptive strategic response. In contrast, Adam's response to boredom frequently involved affective control, or task management, resulting in performance success, or the creation of rehearsal opportunities.

Moreover, Adam's interest in understanding the content meant that he was more likely to employ elaborative and information seeking strategies. Clearly, learning strategies involving the relation of new material to prior knowledge, posing questions, searching for main ideas, looking for additional sources of information, and critical evaluation (which typified Adam's deep-level processing) would be more time consuming, require effort, and need to be sustained by an underlying interest. Adam's use of these deep-level strategies made it unnecessary for him to fall back on simply memorising material.

9.2 Instructional Factors

How does instruction affect the students' strategic learning behaviours? Campione and colleagues (1989) commentary on mathematics instruction suggests an air of foreboding. Emphasis on direct instruction, emphasis on sub-skills, emphasis on skills before understanding, lack of on-line diagnosis, absence of explicit strategic instruction, and assessment practices all contribute to students' distorted view of mathematics learning.

> *Students are not made aware of the reasons for the skills and procedures they are taught. They are seldom given explicit teaching regarding the orchestration, management, and opportunistic and appropriate use of those skills. And they are seldom required to reflect on their own learning activities. These factors help to induce in students a flawed understanding of themselves as learners and of the academic domains they are called upon to master.* (Campione *et al*, 1989: 111)

Mastery versus Performance Orientation.

Ames and Archer (1988) found that the goal orientation of classrooms, as perceived by the students, affected the use of learning strategies. Students constantly construct interpretations of their teacher's behaviour and expectations, and the nature and purpose of classroom activity. In classrooms that emphasise a mastery learning approach success is seen as dependent on effort and strategic behaviours. In classrooms that emphasise a performance orientation, students are socialised with the goal of getting good grades, being judged able, and feel success is dependent on ability (Newman & Schwager, 1992). A survey (Appendix 5), adapted from *High School Science* (Nolen, 1988; Nolen & Haladyna, 1990), showed that the students of this study perceived their mathematics instruction to be principally mastery orientated. In particular, students expressed positive perceptions about cooperative work, teaching for understanding, learning from mistakes, independent thinking, and questioning. In fact, no student disagreed with the statements:

- Students in this class often help each other;
- Most of the students in this class work well together;
- Our teacher thinks mistakes are okay as long as we learn from them;
- Our teacher tries to get us to think for ourselves; and
- Our teacher wants us to learn to solve problems on our own.

There were however, some indicators of performance orientation expressed simultaneously. The majority of students felt that they moved onto new topics before they had really understood the old one, that you had to compete to get good grades, that it was difficult to 'keep up', and that you had to memorise lots of material. Adam was the only student that disagreed with the statement, "To get good grades in this class, you have to memorise a lot of facts". Performance orientation would have been reinforced by the common instructional practice of indicating students' ranked performance within and between classes.

Perceptions of performance orientation were also reflected in students' negative views of public help-seeking. Both Karen and Jane reported relating the desire not to seek help publicly to feelings of personal inadequacy, and a wish to avoid comparison with other students.

Despite instances of performance orientation, overall it appears that students are well aware of the desirability of a mastery orientation and accept that the teacher would like to be able to encourage this approach. In reality however, there are still many pressures resulting from the 6th form assessment system and coverage of the course, which, when combined with a lack of prior knowledge, cause many students to adopt a performance orientated approach. Indeed the teacher, faced with keeping the class in parallel with other classes, sometimes looked for shortcuts, or side-stepped the more demanding parts of the course, to make up for missed periods. For example, when introducing differentiation concepts, the teacher spent time on the concept of the derivative, including the calculations from first principles. However, the learning goal became confused when the teacher stated that, *"Now we are going to forget about how it is found and just use the formula"*. While acknowledging that calculations from first principles are onerous for higher degree formulae, efforts to link the rule based approach with the first principles, rather than disregarding the introductory material, would have increased the focus on mastery as well as performance.

Despite the demands for course completion the students' overall perception of a mastery goal orientation matches the researcher's impression of the classroom instruction. Most students wanted to learn with understanding, and valued cooperation and the sharing of ideas. However, in reality, many students lack the knowledge and control of the range of learning strategies necessary to attain this goal. What is missing from this mastery orientation is the explicit valuing of appropriate learning strategies: instruction must not only focus on the need for understanding and learning from errors, but must also provide students with explicit **modelling and teaching** of appropriate learning strategies.

Demands of the task

The majority of the seatwork and homework time was spent on exercises which provided practice for the teacher-provided examples. Thus, the focus is on computational procedures and accuracy: students know in advance which computational procedures are required to solve the exercises. Students rarely worked on problems requiring the integration of information across several topics and assessment was mostly restricted to single topics. While practice is important for the learning of procedural skills, the reliance on this type of exercise will limit the need for active or intentional learning (Bereiter, 1992).

The teacher often reminded students that practice was the key to learning.

Mrs H: *"Try them all. People who have attempted lots of work tend to do better in exams.";* and

Mrs H: *"We are going to do lots of examples to make sure we know what we are doing."*

The concern is that without opportunities to do problems requiring higher-order skills, students may come to view practice as a way of memorising set examples and procedures that are to be tested in exams. Thus, for the low achiever the means to success is not to think through the problem and integrate information to form new ideas, but rather to recall how the teacher (or oneself) did a similar problem.

Mrs H: *"The reason some of you are not doing very well is that you are not doing your homework. You need to practice until you can say, 'I have met this question before'."*

Mrs H: *"The main thing is to ask yourself have I done anything like this before?"*

The implications of these beliefs were seen in the elaborative strategies of many students who relied on recalling a similar problem. For the low achieving student, the reliance on teacher-given examples is particularly disempowering. These passive learning behaviours meant that it was difficult for students to have any purposeful strategies for coping when stuck.

Jake: *"I didn't do any homework. I don't know how to do it in class, so I couldn't do it at home."*

Furthermore, when tasks become difficult, involving high-level cognitive processing, or when the answers are not readily available, there was a tendency for many students to resist task engagement. In such situations students either participate minimally, seek assistance, or give up totally.

Mrs H: *"Which of these* (Cosine or Sine Rule) *would be the easiest to prove?"*

Lucy: *"You're the teacher, you tell us."* and

Mrs H: *"What do you think makes it quadratic?"*

Faye: *"Because there is four letters."*

Mrs H: *"No."*

Faye: *"Well, just tell us."*

In some instances students invented strategies for producing answers in ways that circumvented the intended learning demands of the task. For example, students copied work from other students, they answered questions using prompts from other students or the textbook answers, or they offered provisional answers (guesswork) to indicate that they had been engaged in the task.

The students mostly expected the teacher to present 'official' algorithms for solving problems step-by-step, without their needing to reflect on the process. Thus the teacher's activities are constrained by obligations, and the "students are not only 'victims' of this classroom culture but also are the 'culprits' " (Voigt, 1994: 287).

Faye: *"She's a very good teacher, she writes down the answers for you."*

In the following example the teacher's request to have students answer a genuine problem, rather than recall information, met with substantial student resistance:

Mrs H: *"How do you think we could solve sin x = cos x?"*

Dean: *"If we haven't done it, how can we tell you?"*

Students' desire to have the information supplied rather than to be actively involved in the generation and integration of information was clearly demonstrated by their preference for teacher 'given' summaries.

Brent: *"Can you* (teacher) *write a glossary of terms we need to know?"*

When tasks were high in procedural complexity (e.g., practical trigonometric investigation) most students spent more time focusing on the procedures of measurement, locating a suitable building and recording the information, than on the content. This was reinforced by the teacher's instructions, which also focused on what students needed to do to complete the task, rather than the learning outcome. Further evidence of students' focus on products was to be seen in students' evaluations of lessons or homework sessions. Students rarely commented on the conceptual learning outcomes, preferring to note the length of time spent, or the amount of work done, or whether or not the work was 'easy or hard'.

Instruction generally favoured "linear learning" (Mitchell, 1992b:179) in which attempts are made to link successive ideas and events, but only in the order they are presented. Little attempt was made to form links between ideas or procedures learnt in different topics. For example when teaching differentiation techniques there was no attempt to link the product of differentiation with the gradient of the graph, nor were any links made between the evaluation of Cosine Rule and Pythagoras, nor were there any links between solving simultaneous equations and the graphing of these sets of equations.

Another major factor affecting strategic learning behaviours is the balance between task demands, instructional support, and compensatory behaviour (Thomas & Rohwer, 1993). Supports are teacher or text provided aids that serve to prompt or sustain student engagement in the learning activity, such as information aids, opportunity for practice, or psychological support. Compensations, on the other hand, reduce or eliminate the demands. For example, the teacher may reduce the demands of a test by providing an alternative pathway to achievement (make up test) thus reducing the need for students to engage in autonomous learning activities. Rohrkemper and Corno (1988) found that to reduce cognitive loads teachers subdivide tasks, set short term learning goals, and scale down test questions so that students can succeed. These findings were confirmed in the present study: the teacher provided informational products, no doubt with the intention of supporting the learner, such as a list of specific items in a test, graphs, tables, and summaries that students would otherwise need to generate. An alternative would be to provide orienting information (e.g., a list of content areas to be responsible for), a model for a process (e.g, a table to complete), or a concept map or flow diagram from which the student would form a summary.

Key Word

The teacher's instruction made frequent reference to keywords. For example, when the teacher quickly reviews the students' exam papers (with the emphasis on the teacher reviewing rather than the students) the following comments are included:

Mrs H: *"When you hear 'gradient' or 'tangent' what should you think about?*

Mrs H: *"What are the keywords, what should the words 'rate of change' tell you?"*

Mrs H: *"Look at the paper, the most important thing is to find the keywords."*

It is assumed that the recognition of keywords will help students recall the appropriate sequence of actions necessary to solve a problem. While the recognition of keywords may help some students to complete a problem it does little to help students construct meaningful mathematical knowledge. By compensating for problem-solving demands, keywords enable students to complete a problem without necessarily understanding the problem situation, without modelling the problem mathematically, and without acquiring

the intended procedural knowledge. It serves to reinforce the goal of performance rather than mastery.

Low achieving students, looking for ways to remember problem methods, are often the ones to pick up on the keywords. For example, Gareth called out correctly the answers to the first two of the above questions, but he was still unable to do either of the questions correctly. The low achiever is particularly vulnerable to misusing the keyword strategy. For example, when trying to solve the problem: *"Find the equation of the line, given m = 2 and the x intercept is 8."* Gareth first writes $8 = 2x + c$, looks puzzled, then refers to his text for a worked example. He then writes the answer as $y = 2x + 8$ using the keyword 'intercept' to identify the (incorrect) solution method!

Summaries

The teacher's use of summaries was intended to support the students' learning. On the several occasions when the teacher encouraged students to participate in providing summary statements she was met with a total reluctance by students to offer suggestions. The students, via this negative feedback, probably precipitated the teacher to eventually supply all the summary material.

Mrs H: *"I'll get you* (class) *a course outline and do a summary from that."*

Although sometimes encouraged to participate in summary writing, students were often given an option or way out of the process:

Mrs H: *"If you need notes on what we have done today and you don't trust your own, use Chapter 22.";* and

Mrs H: *"Because this work is new, I don't feel we can do examples and then you write notes. I feel I need to give you the notes first."*

Rohwer and Thomas (1989) argue that with no external requirement to read for meaning, or to be selective, and with no expectation that students will be responsible for demonstrating their knowledge of the main ideas in a lesson, students have little opportunity to develop the learning strategies of selective attention, paraphrasing and organisation that are needed for autonomous, self-regulated learning.

Worked examples

Students differed substantially in the use of worked examples. Some students (Dean, Brent, Craig, Gareth) attended to the computational procedures, and were able to provide answers to the teacher's step-by-step questions. Rarely did students ask questions related to the conceptual nature of the problem - preferring to direct their attention to the acquisition of specific information needed for the algorithmic activity. In effect, they sabotaged the instruction by selecting from it only the minimum necessary to achieve correct performance.

Brent: *"Where did the 2 come from in the last line?"*

Dean: *"Do we have to know all of them?"* (reference to trig ratios)

In contrast, more successful students directed their attention to the underlying structure of the worked examples. They used discussions as an opportunity to self-question and generate self-explanations, which are critical for effective learning (Chi & Bassok, 1989). These self-explanations have the characteristic of adding tacit knowledge about the actions of the example solution, thus inducing greater understanding of the principles involved. Students' use of worked examples from their text will be discussed more fully in section 9.3

Opportunity to think

To use learning strategies effectively instruction needs to provide students with time to clarify what has been happening in the lesson. Tobin and Imwold (1992:21) suggest that time is needed "so that students are able to engage in such processes as are required to evaluate the adequacy of specific knowledge, make connections, clarify, elaborate, build alternatives, and speculate". In reality this was not the case: a large amount of teacher prompting, self-answering of questions, and limited wait time was evidenced in most lessons. Doyle (1988) suggests that this drive to keep the production rate high, to keep the lesson moving, limits the opportunity to develop autonomous learning capabilities and reinforces students' dependency on the teacher for task accomplishment.

Students learn that non-answers quickly generate teacher prompting and many accept a passive role in class discussion. If the teacher regularly answers her own questions she abrogates the need for students to engage in cognitive processing and self-management.

Mrs H: *"What is our conclusion going to be?"*

Class: (no response)

Mrs H: *"Okay, the conclusion we can draw is..."*

Low achieving students in particular were given less time to respond. The teacher often interrupted with a prompt or the answer, rather than guidance when they responded incorrectly, and rarely praised their success.

Mrs H: *"What is the thing inside the square root called? Can anyone remember - it begins with v."*

Dean: (calls out) *"Velocity."*

Gareth: *"Variance."* (No acknowledgment from the teacher)

Mrs H: *"Variance, not velocity. You may be asked to find the variance in the test."*

Stimulated recall interviews did reveal however, that often students were answering questions, but privately; perhaps because of the expectation that others, or the teacher would answer.

Adam: *"I noticed that she* (teacher) *forgot to times by n/2 but I didn't really want to speak out because I feel like, because I thought someone else might pick it up as well."*

The teacher often used instructional stimuli such as questions related to comprehension and brainstorming to encourage students to make judgements about their knowledge. There was an unstated, but mainly unfulfilled, requirement that if the student judged that personal mastery was inadequate, the student would request help.

Teacher directed learning

Many of the instructional demands were very structured. While the intent may have been to support the students by guiding their learning, students were in fact given little encouragement to preview material, explore the text, or generally take any responsibility for directing their own learning. On one occasion, when Adam had completed the required homework and was working independently while the rest of the students completed the given task, the teacher checked his work. Rather than inquire about Adam's self-directed work she immediately set some alternative task.

Adam: *"She (*teacher*) said read estimation. I thought she would just come to see what I was doing. I didn't know she would tell me to read something else* (surprised tone). *It doesn't matter. I can do that at home sometime - it doesn't worry me. I've already done the work on estimation, but I didn't tell her, so it will be like revision anyway."*

For many students the teacher or the provided answers are the source for 'revealing correctness'. What is missing is regular prompting for *students to decide* on the reasonableness of their solutions, the *justification* of their procedures, the *verbalisation* of their processes, and *reflection* on the their thinking - *all* of the behaviours that lead to the development of mathematical thinking. If students are to be expected to behave autonomously in the tertiary sector, some preparation in the development of appropriate learning strategies is necessary.

Another influence is the instructional cues which enable students to anticipate learning activities. Marland and Edward (1986) found that secondary school students in a biology class reported committing the teacher's last question in a segment of classroom discourse to memory. They did this because a tactic commonly used by the teacher for securing and sustaining attention was to ask a student to recall the teacher's last question. In the present study, the teacher sometimes used an instructional technique of going around the class for answers to a set of problems. The intention was to encourage all students to participate. However, some students reported concentrating only on thinking of an answer for 'their turn' - this practice usually interfered with the process of evaluating other students' answers.

Homework Review

Homework reviews were often not linked to the needs of the students. For many individuals homework reviews were either unnecessary, as they had successfully completed the homework, or inappropriate as they had not attempted the homework.

Dean: *"It's* (homework review) *sort of a waste of time, she* (teacher) *should ask if anyone has any problems and then go over it from there."*

Some students expressed approval of the homework review as a means of getting help.

Craig: *"She* (teacher) *explains things if you don't know how to do it - this is good. I don't always get help at home."*

Jake: *"I don't bother to take the homework home if it's too hard - if you don't understand it there's no point, kind of thing. Most days she* (teacher) *goes over the homework. It's a good idea 'cause if you don't understand it at home you can make sense of it when she goes over it, and ask questions then. It's going to be easier when she's going over it 'cause it reminds you of how to do it and that."*

However, although many students appreciated the opportunity to receive help, they largely let the teacher determine the nature and extent of the help.

Lucy: *"You feel a bit dumb asking questions. I sometimes ask, but if I got one wrong and the rest right I wouldn't really worry."*

The predicability of the homework review suggested to some students that there was a limited need to check work, to complete homework, or to persevere when homework became difficult. For instance Abe rarely completed homework:

Abe: *"Homework is important, but for some reason I just don't do it!...She* (teacher) *goes over homework most days, I can pick up things there. I tune in, have the page ready. She'll probably ask me a question so it's best if I'm following."*

Lucy: *"I try to sort out the problem from the answer, but usually I just give up; we'll go through it in class anyway."*

Another viewpoint of some students was that homework review provided an incentive to complete the homework.

Kane: *"It's* (homework review) *a good idea. I supposed it's the sort of thing like, when you're at home you think, I've got to get this done because she'll* (teacher) *be going over it and sort of getting into trouble type of thing."*

However, rather than complete homework, some students developed coping strategies to disguise their lack of completion or complained that it was too difficult:

Abe: *"She* (teacher) *doesn't usually check your work. If she asks me a question I just open up my book, look at it and hope someone else will tell me the answer."*

Jake: *"I didn't do my homework 'cause I didn't know what to do."*

Assessment

In the classroom situation performance feedback is critical for shaping accurate metacognitive knowledge. Specifically, in the acquisition of procedural knowledge attention to feedback, when one has made a mistake, plays a crucial role in learning (Ames & Archer, 1988; Evans, 1991b). Evans (1991b:67) suggests that "feedback which is simply given to the learners without clear reference to the way they tackled the task is unlikely to lead to control". In the present study most students relied on the textbook answer, or the teacher, to provide feedback during seatwork and homework episodes. Such feedback is often limited to a 'right or wrong' judgement. Only Adam demonstrated a thorough self-diagnosis of his errors, and was truly able to learn from all his mistakes. As discussed earlier (section 7.4) students who were reluctant to fully investigate their errors, or seek help when needed, missed opportunities to learn from these situations.

Similarly, when tests were returned the focus of both instruction and the student was on the product, rather than the learning process. Short and Weissberg-Benchell (1989) suggest that teachers should explicitly teach students to recognise the multiple causes responsible for learning outcomes. "Success experiences would provide information regarding task-appropriate strategies, whereas failure would provide feedback regarding task-inappropriate strategies" (p.50).

In the present study there was limited explicit teacher references to checking procedures, and to the value of checking. A notable exception was with solving simultaneous equations, where checking by back-substitution was an integral part of the procedure. Indeed, Gareth expressed the view that this procedure was called the "substitution method" because you "substituted in at the end"! Unfortunately no connection with a graphical representation was discussed as an alternative way of checking the reasonableness of the solution.

The formal assessment used in this 6th form course was dominated by questions requiring repetition of teacher-given procedures. This practice can substantially reduce the task demand and the corresponding development and use of effective study strategies (Thomas & Rowher, 1993). Examples of questions which required a modicum of original thought either occurred at the end of the test paper, and were awarded few marks, or were accompanied by hints so as to effectively reduce the demand for high-level thinking. For example, the following question is the final question for a test on *Graphs and Functions*:

Sketch the graph of $y = \dfrac{x^2 - 9}{x + 3}$

[Hint ...factorise first]

Continuous exposure to this level of assessment influences students' learning strategies. Gareth demonstrates that he feels influenced by the type of questions in the tests:

Gareth: "*It's more of a concern to know how to get the right answers because you don't really get checked much on understanding, all you get is a list of problems in the test.*"

Int: "*Do you think you might change your learning if the tests had different kinds of problems?*"

Gareth: "*Yeah, yeah, like discuss what the mean is and definitions of the mean and formulas and stuff like that would make it heaps easier. It would make me change the way I learn.*"

The students were well aware of the structure and content of each test.

Adam: *"We've already seen last year's and the year's before test, so I don't think there is going to be much difference. They* (teachers) *don't change them much."*

Students gained information from teacher sought and teacher given cues, and from revision of previous years' papers. The following are examples of teacher provided cues:

Mrs H: *"These questions are going to be very similar to the ones in the exam." ;* and

Mrs H: *"If there is a question exactly the same as this, with the numbers changed - which is extremely possible - I will be extremely cross if you haven't achieved something."*

The following are examples of students seeking cues from the teacher:

Dean: *"So you reckon that this one will be in the test?";*

Faye: *"Will they give us these formula?"* ; and

Jane: *"Do we need to know all of those special angle things?"*

The predicability of the test content and structure would encourage a passive learning approach in which revision is reduced to a quick flip through the classroom examples and teacher-provided summaries.

Mrs H: *"On Monday I will give you an algebra summary of all the things you should be able to do."*

For the most part, students relied on these teacher given cues about the test content, teacher given summaries of topics to study, and class revision periods the day before (which often use copies of previous test) to direct their study activities.

Review sessions reduced the need for students to engage in memory augmentation activities on their own. What are intended by the teacher as supports for learning become compensations (Thomas & Rowher, 1993), disempowering students and denying their needs to self-regulate their own learning. Evidence from a class study session illustrates how low achieving students are cued to memorise examples to be recalled in test situations:

Mrs H: *"You need to be able to say in a test, I've done this before, this is how I go about it."*

In another review session what began as the modelling of appropriate study strategy was quickly reversed because of a class management decision.

Mrs H: *"I suggest strongly that you use those kinds of questions in your revision. You have ten minutes to identify what you can and cannot do and we will go over that."*

After just a few minutes the teacher interrupted the class and began to go over the paper starting at No. 1 and continuing on - without regard to the previous instruction which encouraged students to identify their own particular learning needs.

The students in this study generally lacked an awareness of the role of learning strategies for revision, nor were they aware of how other students study. Without opportunities to reflect on their own learning and an awareness of possible alternative strategies students are unable and uninterested in improving their learning performance. The most common reply when asking students how they could improve their grades was to do more of the same!

In summary, the present assessment encourages students to use learning strategies appropriate for rote memorisation, and recall of previously seen examples. The incentive to use metacognitive monitoring and control strategies to direct the learning process is limited. There is little need to plan or schedule revision as students know, or hope, that the teacher will direct their revision a day or so before each test. If one believes that learning requires independent thinking, assessment should include new tasks requiring new applications of general principles or procedures (Mitchell, 1992a).

9.3 Contextual Factors

Classroom Discussions

The social nature of the classroom situation lends itself to opportunities for developing and encouraging a range of learning strategies. Skilled thinkers (often the teacher, but sometimes more advanced students) can demonstrate desirable ways of tackling problems, analysing texts, and constructing arguments. "This process opens normally hidden mental activities to inspection" (Resnick, 1987:40). Specifically, in class discussions students can use elaboration strategies (asking and answering questions), and demonstrate metacognitive strategies involving evaluation of understanding and overall progress.

However, research has found that by the time students reach senior high school there may be a divergence in attitudes and behaviours regarding help-seeking and questioning (Newman & Schwager, 1992). Several of the students in the present study reported that they felt uncomfortable speaking publicly in class:

Brent: *"Well even if I do listen, it's still more for other people in the class. Like, I don't know, I just don't feel the class is directed at my learning capabilities, it's directed higher. Everyone else is more intelligent. The class just moves too fast for me."*

Although the sample size of students was small, female students were noticeably reluctant to participate in discussions - thus limiting opportunities to enhance their learning with effective and personal feedback.

Karen: *"I honestly thought it was called a pictograph. I don't want to say anything in case it is so far wrong I embarrass myself."*

Jane: *"Some of the time I don't understand the stuff enough in mathematics to answer questions 'cause I'll probably get it wrong. I only answer questions if I know the answers."*

Int: *"What about asking questions in class?"*

Jane: *"If I don't understand usually someone else asks her to slow down."*

Brooks and Brooks (1993:7) suggest that students' unwillingness to answer teacher's questions, unless they are confident that they already know the sought after response, is a direct consequence of teachers' use of questions: "When ~~students~~ teachers ask questions, most teachers seek not to enable students to think through intricate issues, but to discover whether students know the "right" answers." In this study when female students were unsure of the answer to a direct question they would provide an evasive answer so as to minimise the risk of exposing mistakes or lack of knowledge. This technique is also used by students who have not attended to the task - rather than admit to not trying, they suggest that the task is too difficult, or that they haven't quite finished it yet, and thus put the onus back on the teacher to do the work! One could view this tactic as a form of help seeking - but it is limited by its reliance on the teacher and the situation.

Lucy: *"I haven't a clue"*

Jane: *"I didn't know how to start it."*

Jake: *"I'm not sure how to do it."*

Female students however, **did** report being active in terms of self-dialogue, and on occasions when they were sure of an answer they participated in the discussion.

Jane: (Corrects the teacher's error) *"I had worked it out on the calculator. I knew she was wrong because I had checked it on the calculator. It was quite good to find a mistake."*

Peer Interaction

Although not explicitly encouraged by the teacher, many students cooperated with peers in help-seeking and help-giving learning activities. As discussed in section 7.4, cooperation enables students to share knowledge and skills, and provides students with additional opportunities to learn new concepts or procedures. When students articulate processes and concepts, they gain conscious access and control to cognitive and metacognitive processes. However, it was noted that much of this content-relevant peer discussion occurred outside the teacher's awareness, as did much non-relevant discussion!

Also, peer groupings were sympathetic and supportive of other students' behaviour. For example, students often covered for a peers' off-task behaviour by supplying the answer or prompting the student.

Mrs H: *"Abe the answer to 'b'?"*

Abe: *"I'm not up to part 'b' yet.* (Dean pushed his book in front of Abe) *Oh! 6."*

A modification of the above scenario is the collective effort of the students to pressure the teacher to present the information.

Mrs H: *"We took five numbers and took the mean."*

Faye: *"I don't remember that, does anyone else remember that?"*

Some students (Dean, Kane, Faye) provided feedback as to the level of detail that they wanted, especially in relation to proofs, and constantly cued the teacher for information regarding test questions and topics.

Dean: *"We haven't done these - is it going to be in the test?"* (a reference to limit questions in a previous year's test paper)

Brent: *"Do we have to know this?"* (a reference to trig ratios)

Some students reported a different perspective of the teacher and peers as helpers, and this view was reflected in their choice of helpers.

Jake: *"I ask her* (teacher) *but I still don't understand."*

Lucy: *"I sort of worry about it* (the lesson) *if I didn't understand and I don't really have time to ask her* (teacher). *She sort of gets a bit annoyed when you ask her 'cause she already knows so she expects it to be easy for you."*

The social setting also enabled students to collectively, or sometimes individually, attempt to determine the pace of the lesson. Feedback, often supported by peers, as to the difficulty or pace of the lesson was directed to the teacher in an attempt to speed up or slow the pace.

Faye: *"Yeah, yeah, we get that, we get that."*

In other instances Faye attempts to speed up the pace of the lesson by answering several questions at once, or by suggesting that the homework review be skipped.

Mrs H: *"Any problems with the homework?'*

Faye: *"We've marked them all!"*

Within the social setting some students also looked for peer or teacher approval. Dean in particular, makes public comments about his progress, his ability to answer questions and his homework completion.

Mrs H: *"Abe, see if you can see any pattern?*

Abe: *"Pass."* (teacher gives correct answer)

Dean: (calls out) *"I knew that, I told him that."*

Gareth also suggests that answering teacher questions will enhance the teacher's assessment of his ability and cooperativeness.

Classroom seating arrangements were also used by students as a method of environmental control. Students selected seating according to peer groupings: seating arrangements sometimes enhanced the learning process, but for some students arrangements provided the necessary distractions to avoid the learning task. Dean usually was involved with a lot of off-task talk with peers. When separated from peers by the teacher he reported, *"you get a lot more work done separated"*.

Use of resources

Students were supplied with two texts and a course summary. As discussed in section 7.1 the students' main textbook (*Form six mathematics*: *Revision,* Barrett, 1990) is divided into small discrete units which provides little incentive for students to connect topics. Karen was the only student (from interviews with each class member) who reported referring to another unit during the trigonometry section:

Karen: *"I went back to the other trig section when I was looking for proof of sine, cosine and tangent rule, and there was something else I didn't understand*

about a right angle triangle and I went back to the other trig chapters in the book."

Each chapter has a short introduction, followed by worked examples (with explanation steps), and exercises. Only Adam reported regularly reading the introduction: he used it to help with learning but found that it had insufficient depth of information. This resulted in Adam seeking further help from his family. A few students noted that they had looked at the introductory material if it was part of homework reading.

In contrast, all students reported referring to the worked examples. Concordant with a previous study (Anthony, 1994), involving distance education mathematics students, there were reported differences in the manner in which students processed worked examples.

Faye: *"I refer to worked examples to check formula I haven't memorised yet."*

Dean: *"I read worked examples - if I don't understand it I always refer back to them and think, oh that's how you do it."*

Lucy: *"I try and work them out and see what they have done. There are not enough worked examples."*

Low achieving students tended to use the worked example as a recipe; they matched the steps in the example with the problem. When the student reads the example, learning only the sequence of actions, they will at most acquire an algorithmic procedures to be recalled with similar problems.

Craig: *"When I'm stuck I look at a worked example and try to do the same thing."*

Gareth: *"I work through the worked example while I'm doing work, like if I've got trouble with a question I come back and see how they do it with a worked example. I see how they do it and work back from their answer."*

Brent: *"I always use the worked example during homework, that's the only way I understand how to do it. I just go through them and see if I understand them and if I don't I write them out, just go through the steps, read what they have in the side."*

Self-explanations are the process of developing meaning for the self and as such are also a vital part of learning from worked examples (Chi & Bassok, 1989). What gets explained (when one is explaining to oneself during worked examples) is how to work around the problem, how to connect a new piece of information, or how to restructure or rearrange existing information. Genuine self-explanations will only be initiated when an incongruity is noted or some integration is needed.

Lucy reports an active learning approach to processing the explanations provided by the text to supplement her own self-explanations:

Lucy: *"I don't usually read the explanation, unless I'm stuck. I can usually sort of see what they've done anyway."*

Jake uses the explanations as a check after first trying to work it out for himself.

Jake: *"I look at the worked examples to see what you're doing and look at the explanations to see what you're meant to be doing."*

However, low achieving students regard these explanations as 'recipe instructions' - the supply of explanations means that these students no longer need to apply elaborative learning strategies to construct meaning from the worked example. For example, despite Gareth reporting that, *"the explanations are pretty helpful in generally working it out, sort of seeing where they are going"*, he sometimes misuses these explanations when applying them to exercises (see example in section 8.2). These behaviours contribute to dependence, eliminate the need to think for oneself, and foster the growth of learned helplessness.

The text glossary is a particularly valuable resource at this level. It was frequently used by students.

Dean: *"If she* (teacher) *asks what something means I always turn to the back to say, 'um this is what it means'."*

Karen: *"I always look up the glossary when there is a word I don't understand."*

Several students had difficulties with mathematics terminology. For example, Gareth confuses the terms *derivative* and *deviation*, *x dash* and *x bar*, *table* and *graph*, and had trouble understanding the definition of *frequency* when the glossary referred to it as a

tally. Dean asked what the word *assumption* means - the word was *asymptote.* Brent asked, *"what does the c arrow thing mean?"* - referring to the < sign. More encouragement to use the glossary in homework assignments may be of some help. Students need **explicit instruction** in ways to help themselves to learn from textual resources, including learning the language requirements of mathematics.

While some students (Dean, Brent and Gareth) referred to previous years' notes to assist with revision, in general, students' learning was constrained by their lack of use of resources.

Brent: *"I don't use the other text* (McLaughlin, 1985). *It's in the wrong order - too hard to find the same topics. I go back to the third and fourth form on algebra and stuff. It's got really basic stuff and it just refreshes your memory."*

The majority of students used their texts only as directed by the teacher, confining themselves to a narrow set of exercises, and referring to worked examples and explanatory material only when stuck. With regard to their present text students generally felt that there were plenty of exercises. However, the more dependent, passive learners in the class expected the teacher to explain what the book says rather than make sense of it themselves. Their suggestions for an ideal text included: *"a good cover"*; *"more notes";* and *"questions like you would get in an actual exam - they should be worded the same."* In contrast, more active learners explored their text by trying further problems, seeking further information about the topic, and previewing material. While these students felt that there were enough exercises, they suggested that the ideal book should have more explanations, *"like where formulas come from"* and more worked examples.

The teacher did remark that reading a mathematics textbook is different to reading a novel. However, on the few occasions when students were expected to read some explanatory material, it was assumed that students had the necessary skills to effectively process the text.

Mrs H: *"Instead of doing examples for homework you can do some reading and summarising. I'm not just asking you to read like you would a novel - it's a concentrated read."*

Students who view texts solely as a source of exercises may become dependent on oral instruction from the teacher and have difficulty using the text effectively to overcome difficulties during homework sessions. Students need to be taught how to make effective uses of their text if they are to function as autonomous, self regulated learners in tertiary studies.

Another resource given to students was an outline of the 6th form topics and assessment plan (*Course summary).* While some students used this to check topics for each test, other students appeared unaware of its existence. Possibly when it was handed out at the beginning of the year these students either did not value its usefulness, or felt hopeful that the teacher would provide such information for them at appropriate times during the year.

Learning outside of the classroom

In an ideal learning environment learning mathematics should not cease as soon as the student leaves the classroom. Homework and revision sessions should provide opportunities for consolidating achievement and farther independent learning. However, students' opinions about homework were varied, ranging from "essential" and "helpful" to "hopeless"!

Lucy: *"I don't mind homework if it's not too long. I think it's quite important to practice what you've done or you'll forget it. Also it's important to do it by yourself because in class you usually see what your friends have done."*

Lucy's comments reflect metacognitive evaluations as well as the more usual rehearsal strategy referred to in the following comment from Dean.

Dean: *"I think it's* (homework) *important for the reconciliation* (sic) *of the work that you've done in class...Doing exercises is the most important thing."*

Brent however, doesn't like homework at all!

Brent: *"I don't like homework. I attempt it, but I just don't understand it so I give up."*

In the present study, students' tendency to do homework was influenced by monitoring of their understanding in class. They interpret this in an uniquely individual manner. For example, Jake supports the idea of homework as a form of revision but finds that his lack of understanding of the lesson inhibits his ability to complete homework.

Jake: *"I only attempt homework if I know what I'm doing. There's no point in going through it if you don't understand the days work 'cause you won't be able to do the homework. I'm not understanding the days work so that's why I haven't done much homework lately."*

This view contrasts to those of Kane and Abe who feel homework is beneficial when understanding is lacking in class.

Int: *"If you were having difficulty with the lesson would you take the homework home?"*

Kane: *"It would be more important to take the homework home because if you didn't learn anything then you're going to be behind in the next class."* and

Abe: *"If there's something I don't understand in class then I might just look at the book, but I won't get any paper out or anything. I just look through it - I look at the part of the book where it explains it right at the beginning. It doesn't happen very often!"*

Zimmerman and Martinez-Pons (1986) found that high school students use of self-regulated learning strategies in non-classroom situations displayed substantial correlation with academic achievement. As learning outside of the classroom is largely unsupervised one would expect that knowledge and use of learning strategies would be especially important (Thomas & Rowher, 1993). Evans (1991a), using both concurrent and retrospective interviews to examine secondary school mathematics students' homework behaviours, concluded that learning strategies used at home largely reflected those practiced by students in the classroom.

While there was a considerable and varied range of higher order procedures enacted in the classrooms, the major actor in this was the teacher, the students' main role being to answer questions posed by the teacher. The amount of time allocated to knowledge and understanding of mathematics structures was less, and again it was the teacher who took responsibility for this work. The procedures used by the students in their homework overall reflected these classroom emphases, with application of specific procedures being the most common processing activity...It is questionable whether the absence of active reflection on the part of the students, either in class or during homework, constitutes the most useful approach to mathematics education." (p. 141)

Findings in the present study support Evan's conclusion. There were disappointingly few reports of the metacognitive behaviours of planning and previewing for this level of senior mathematics students. The majority of the students' homework activities were teacher directed: they did not see it as necessary, nor important, to do any activity other than those specified by the teacher. Most students did only the set exercises and appeared somewhat surprised at being asked if they did any alternative problems, or further reading. Student reports reflected the teacher-dependent attitude evidenced in their class behaviours. They felt that they would not be able to understand anything that the teacher hadn't covered in class, and that it was irrelevant to their learning to read the text further:

Int: *"Do you ever do any more exercises or look ahead in the text?"*

Faye: *"No because I won't understand it."*

At home there were two major factors influencing strategic learning behaviours: availability of time, and availability of expert help and resources.

Pressure of Time

Only three of the twelve students (Adam, Craig and Lucy) did not have out of school employment, and seven of the twelve students worked ten hours or more. With these commitments, combined with extra-curricula activities, doing homework every night seemed an impossibility for many students. Revision also seemed vulnerable to pressures of time:

Karen: *"I'm not going to have time for revision, I haven't done my English assignment."*

Craig: *"I just do revision if I have time."*

Time was a certainly a major factor toward the end of the 6th form assessment period. Several students reported giving up under the strain of five for six tests and assignments due in one week.

Faye: *"If I'd done some study I could have done better* (in a mathematics test) *but I had too much pressure on me this week. I have had 10* (maybe an exaggeration?) *tests this week."*

Availability of help

Availability of help at home influences students' learning in an on-going way - rather than just as a presage entry variable. For many students the decision to take homework home, and the efforts that go into homework, are moderated by the availability of help outside the classroom.

Kane: *"My brother's at Massey. He's pretty good at maths and if I'm stuck I ask him. I normally try and work it out myself 'cause then I know what I've done wrong. I would go to my brother if I didn't understand it at all."*

Faye: *"I don't have any help at home, but if I'm really, really stuck I'll just ring someone else who'll probably know what they're doing and ask the teacher next day. I remember which one it was and ask."*

Int: *"Have you got anyone at home who can help with your maths?"*

Lucy: *"No, not really, I mean we go through it the next day in class anyway so I don't really need to."*

Availability of help at home also influences in-class help-seeking behaviours. For example, in the statistics lesson Adam reported that he made a decision in class to seek help at home rather than press the teacher to elaborate on the nature of the standard error. Dean's class behaviour also appears to be influenced by the availability of help from his tutor. His class behaviour is often disruptive and he appears to give up when the

'going gets tough' - *"Oh, I'll get my tutor to show me this"*, and puts the responsibility for learning onto his tutor.

Behaviours reported for revision were of a similar nature to homework strategies. The majority of students felt that revision did not actually provide opportunities for learning mathematics, rather revision enabled one to rehearse what was already known.

Jake: *"Revision just puts all the stuff back in my mind. I don't think tests help you learn at all. They just make you really nervous. You've just got to try and cram it in, a certain amount, and you're not really learning anything at all."*

Karen: *"I think tests are important, but they don't make a lot of difference. I worry if I know I'm going to fail it, and if I know it's going to be a hard test, then I will do a lot of study for it."*

Jane: *"I think tests are good. You get to summarise everything that you've learnt and you need to know. It gives you extra practice. You don't really learn anything more, it's just going over everything you know."*

Adam: *"I think the test is just an extra to make sure you understand things - they don't make me work hard."*

Dean however, expressed the view that tests were important for collecting marks:

Dean: *"I think tests are mainly so we can get end of year marks. 'Cause when are you going to use like the sort of trigonometry we do when you leave school? - there's probably a one-in-a-million chance!"*

One can see the impact of Dean's metacognitive knowledge on his revision strategies:

Dean: *"I'd do a couple of hours before I go to bed so I'm nice and fresh for the test in the morning."*

Int: *"Do you pay much attention to the teacher cues about the test?"*

Dean: *"Cues are heaps important 'cause it's not much point studying for something that's not going to be in the test. If you know what's sort of going to be in the test you can base your study around it."*

It appears that while many students view homework as important, its value is in providing practice opportunities. Very little planning, previewing, diagnosis of difficulties, or reflective learning activities were reported. Similarly, revision was important for consolidating ideas, but students made little use of higher-order organisational strategies and self monitoring strategies to enhance revision, tending to rely more on the rehearsal strategies of rereading and practice.

Moreover, homework provided little incentive for students to develop autonomous learning behaviours. Rather than support the development of higher order self-regulatory learning strategies through the use of resource management strategies, affective control and metacognitive behaviour, homework tended to reinforce the view that learning mathematics was all about completing problems from a textbook, especially problems set by the teacher.

9.4 Summary

The present study has provided much evidence to support the *Interactive Model of Learning Mathematics* (section 2.3). The influence of both presage and product factors on strategic learning behaviours was clearly demonstrated in both the students' classroom and home learning environments. In support of the constructivist learning paradigm, the influence of prior knowledge was seen to be especially important. Without access to relevant domain knowledge students were unable to provide appropriate elaborations and therefore the likelihood of forming generalisations from worked examples was limited. Moreover, without appropriate metacognitive knowledge, strategic behaviours focused on task completion rather than knowledge construction and understanding.

Positive instructional factors affecting students' strategic learning behaviours included:

- teacher proximity to students while moving about the room encouraged help seeking;
- teacher/student discussions encouraged elaborations;

- summarising on the blackboard prompted students to review their understanding; and
- teacher direction and task specification promoted student attention.

Additionally, this study highlights various instructional factors which appear to contribute toward passive learning behaviours. In particular, if classroom instruction and assessment revolve around the end product of the task, students will be encouraged to view learning in terms of 'doing' or 'completing' a task and gear their strategies to that end.

> *"The affective tone, characteristic goals, ways of approaching mathematical tasks, and attitudes to knowledge - whether constructed or acquired from others - that the learner observes in the classroom, may become what mathematics is for him or her, and may contribute to the kinds of thinking and learning and problem-solving strategies that the learner uses in undertaking mathematical tasks."* (Evans, 1991a: 126)

In the present study there were occasions when the teacher, in trying to support the student's learning, *effectively reduced the learning demands on the student by doing the student's thinking and processing for them.* Students were not disposed to seek and evaluate information on their own, preferring to rely on the teacher to automatically direct their learning. *As students convince the teacher to be more direct, and to lower the ambiguity and risk in classroom tasks, instruction may inadvertently mediate against the development of higher-order skills.*

To promote higher-order thinking in the mathematics class we may require a less direct instructional approach - one that transfers some of the burden for teaching and learning from the teacher to the student, and promotes greater student autonomy and independence in the learning process. Moreover, if mathematics is to be viewed as a social activity, more acknowledgment needs to be afforded to the social nature of classroom learning. Results from this study showed that not all students benefited from the opportunity to interact with peers and resources, and help-seeking behaviours were greatly influenced by contextual factors.

Chapter 10

Conclusions

An awareness of learning strategies and the ability to employ them can provide students with the potential to learn with understanding and the requirements to overcome failure.

(Herrington, 1990:333)

10.1 What Learning Strategies are Important in Mathematics?

This study has shown that students use a wide range of cognitive, metacognitive, affective and resource management learning strategies. From a constructivist learning perspective these learning strategies are seen as essential: how else can students be actively involved in constructing their own mathematical knowledge, if not by using learning strategies? However, the case studies clearly demonstrate that knowing about learning strategies *per se* is not the issue. It is the **knowledge and use** of appropriate learning strategies that differentiates between successful and unsuccessful learning outcomes.

Learning strategies are, by definition, planned and goal directed, therefore the appropriateness of any strategy is in part mediated by the students' metacognitive knowledge, in particular their beliefs about learning mathematics. It was evident from the present study that when students' learning was directed towards understanding and constructing new knowledge, appropriate learning strategies were more likely to be employed. Active or intentional learners viewed classroom and homework activities as a

means to learn and understand mathematics, rather than solely as tasks to be completed.

In contrast, less successful students, such as Gareth, who believe that mathematics is largely about recalling worked examples, employed learning strategies largely to meet this goal. As we have seen, obtaining a written record, either by copying from the teacher's worked examples or completing exercises, was a major focus of Gareth's classroom activities, and he consequently directed his learning strategies towards task completion. Any mathematical understanding and knowledge construction that did occur was largely incidental to task completion.

Appropriate beliefs regarding the nature of learning and the discipline of mathematics need to develop in concert with learning strategies in a mutually reinforcing way. Effective learners need to have knowledge of a wide range of learning strategies to enable selection of strategies appropriate to individual tasks. A "good strategy user" (Pressley *et al.*, 1987) needs to know the 'whens' and 'whys' of strategy use. That is, a student needs extensive metacognitive knowledge of both the general utility of the strategy and of the appropriate task conditions. For example, it is important that students can differentiate between strategies appropriate for revision of open-book tests and closed-book tests. While some students expressed a knowledge of differing task demands, few were able to effectively adapt their revision strategies to cope with the varying demands of different types of tests. Karen's strategy reflects a reliance on the text to provide the answers for her, although she does acknowledges that some familiarity with the text would be advantageous.

Karen: *"In an open-book test you can go through the text and learn the formulas or find out from the textbook how to solve it. So you don't have to do as much revision 'cause everything is in the textbook. I might just read the textbook through the night before."*

Cognitive strategies employed by students to encode and integrate new information with prior knowledge included rehearsal, elaboration, and organisational strategies. Seatwork and homework exercises were used as an opportunity to consolidate new procedures.

The more successful students also evaluated their own understanding, and sought assistance from peers, family or the teacher during these learning episodes. Students who were aware of the value of over-learning actively sought opportunities to rehearse information. This was in direct contrast with less successful students who frequently expressed the view that once a procedure is learnt or understood practice is no longer necessary. These students employed rehearsal strategies mainly to promote short-term memory of procedures for recall in tests.

Successful students employed elaborative strategies, such as linking between topics and problem types, imagery, self-explanations, and questioning to enhance their learning processes. Their elaboration strategies relied to a large extent on a requisite level of prior knowledge for successful implementation. All students were prompted by instructional cues to elaborate. However, prior knowledge determined not only what material was elaborated, but also influenced the nature of the elaboration process itself. Without relevant prior knowledge students may not always achieve the teacher-intended outcome. Insufficient domain knowledge, combined with surface learning goals, resulted in elaborations focussed on recall of previous experiences, rather than recall of conceptual information. Those students using recall were largely limited to attempting problems types similar to those seen before.

Brent: *"Is that the same as the limit question we had in the test the other day?"*

Gareth: *"Oh, is that all that question wanted. If you had asked it the same way as you had in class I could have done that one."*

In contrast, students with relevant prior knowledge were often able to resolve conflicts and construct new knowledge, based on a reconstruction of existing knowledge incorporated with new information.

The process of reflection is widely advocated by mathematics educators as an essential element of the constructivist learning process. Constructivism "allows one to emphasize that we are at least partially able to be aware of these constructions, and then to modify them through our conscious reflection on that constructive process" (Confrey, 1990:

109). Reflection, used in this sense, covers a wide range of metacognitive behaviours which were evident in the behaviour of successful students.

Metacognitive strategies enabled students to control and regulate their learning. Although all students reported using metacognitive strategies, the effectiveness and specificity of the strategies varied between individuals. Student reports and observations of monitoring understanding and production evaluation were widespread. However, the accuracy and subsequent action taken, as a result of monitoring and evaluations, differentiated the more successful students from the less successful students. For example, Gareth was unable to successfully evaluate his learning; more often than not he over estimated his ability to complete seatwork and expressed unfounded confidence in his understanding of the lesson.

Gareth: "*It wasn't really as complicated as she* (teacher) *said. You just shift the tables from one column to the next.*"

Gareth often had many, and sometimes all, of his seatwork exercises incorrect, but because he neither marked them, nor sought assistance, he profited little from his efforts.

In contrast, more successful students reported frequent episodes of monitoring related to not understanding. The recognition of conflicts and confusion enabled them to seek strategic remedies in the form of questioning, help seeking, anticipation, problem diagnosis, selective attention, and reflection.

Selective attention to features of the learning task was mediated by the students' expectations about the learning task and their prior knowledge. Students whose goal was to understand, focussed on procedural and conceptual issues. They reported elaborations of linking between topics, self-questions and self-explanations. They were aware of the need to focus attention on particular concerns and reported anticipating teacher explanations which would assist to resolve conflicts and difficulties. In contrast, when expectations and/or prior knowledge are inconsistent with the teacher-presented learning goals, the student may focus on inappropriate aspects of the task. For example, during homework review, Gareth focused on marking with ticks and completing calculations, rather than on specific difficulties to be resolved.

Reports of affective reactions were common from all students. However, there was no evidence that there were qualitative differences in affective strategies between less and more successful students. Whereas some students (like Adam) reported being on task for most of the lesson, other successful students (like Faye and Karen) interspersed episodes of work with off-task talk. Corno (1989) suggests that this may be a purposeful strategy to avoid overtaxing the information-processing system. Efforts to stay motivated and on-task appeared to be based largely on personal learning styles and somewhat unrelated to performance outcomes.

The appropriate employment of resource management strategies are closely linked to metacognitive behaviours. Resource management strategies enabled students to behave adaptively and control the learning environment. Successful students used resource management strategies to enhance their own learning needs by adapting their learning task, to seek help from other persons or resources, and to control the physical nature of their environment.

The use of appropriate help-seeking was found to be a critical factor in students' learning success. With the exception of Adam, those students who sought help cooperatively from peers appeared to benefit most from classroom seatwork. They were able to bridge gaps in procedural and conceptual knowledge and complete all the required exercises. Those students who preferred to 'wait and see' if help came *via* the teacher review were less likely to be successful. Willingness to seek help in homework sessions was important if homework was to be more that just a practice of what was already learnt in class. The fact that Adam rarely asked questions in class may have been viewed as inefficient, if it were not for the fact that Adam always sought help from family members at home. Although not explicitly encouraged by the instructional demands, knowledge of how best to use the textbook, including the contents, index, introduction and glossary, assisted students to optimise their learning outside the classroom.

Of all the students in the present study, Adam (A grade) consistently reported using the range of learning strategies which are widely promoted by mathematics educators (see section 4.2). However, although his learning was intentional, well planned and controlled by effective metacognitive behaviours, he preferred to rely on help from home rather than from peers and teacher. He rarely questioned the teacher, and his efforts to plan and work independently were hampered by lack of course information. Also, as a direct result of the teacher-provided summaries, Adam only occasionally made summaries himself, and he exhibited limited use of higher-order organisational strategies.

Examples of strategic behaviours which were characteristic of the more successful students in this study are provided in Figure 6. However, as noted earlier, having strategic knowledge, or even using a range of strategies, did not necessarily guarantee successful learning. Levin (1988:196) suggested that "in order for students to apply learning strategies effectively and independently two critical "cogs" (Levin, 1986) - as represented by cognitive and metacognitive components - must be in place and smoothly functioning". Appropriate metacognitive knowledge and behaviour empower students to pursue cognitive goals of their own and thus makes them less dependent of school-work procedures.

The present study concludes that not only must these 'critical cogs' be in place, but also students must be willing to be 'active' or 'intentional' learners (Bereiter & Scardamalia, 1989). Quality learning can only occur with the consent of the learner: "One cannot mandate high order intellectual activity such as reflecting on and contributing towards one's own ideas" (Mitchell, 1992a:80). Successful students in the present study appeared willing to accept a greater responsibility for their achievement outcomes by selecting, structuring and creating a learning environment which optimised their learning.

However, as discussed in the previous chapter, unless the learning environment values and encourages mastery learning, and explicitly encourages the development of autonomous learning behaviours necessary for constructivist learning, many learners will fail to optimise their strategic learning potential.

Strategic Learning Behaviours of Successful Students

Cognitive Strategies

- completes a lot of problems for practice/over-learning (Adam)
- elaborated new knowledge to personal knowledge and beliefs (Karen)
- elaborates new knowledge to prior knowledge, links information as well as recall (Karen, Adam)
- organises notes and exercises for later reference (Jane)
- codes exercises to indicate problems for later reference (Jane, Karen)
  uses colour coding, font changes for emphasis (Gareth)
- is attentive and on-task most of the time (Adam, Karen)

Metacognitive Behaviours

- plans and anticipates learning by reading ahead in the text (Adam)
- monitors personal progress in relation to other students (Karen)
- monitors understanding effectively (Adam, Jane, Karen)
- evaluates procedures and algorithms critically (Adam, Jane and Karen)
- reflects on the work over a period of time (Adam)
- is aware of the need for revision (Karen, Adam)
- uses a range of checking strategies (Adam)
- diagnoses reasons for comprehension and mastery failures (Jane, Adam)
- self-evaluates progress over a topic, assesses readiness for test (Adam)
- selectively attends to conceptual aspect of the material (Adam)
- selectively attends to teacher's comments based on interest and need (Adam, Karen)

Affective Strategies

- controls self motivation (Adam)
- varies routine to avoid boredom (Faye)

Resource management strategies

- seeks opportunities for rehearsal during teacher/class discussion (Adam)
- seeks appropriate assistance (Adam)
- proactively seeks out information from resources (Adam)
- cooperates with other peers (Karen), family (Adam)
- modifies tasks to increase learning potential (Adam, Faye)
- monitors the teacher's movements/comments during seatwork (Karen, Adam)
- selects seating arrangement to suit personal learning style (Adam, Faye, Karen)

Figure 6: Strategic Learning Behaviours

10.2 When Students Fail to Use Learning Strategies

While establishing that learning strategies, and in particular metacognitive strategies, are a significant factor in successful student learning, it is equally important to identify situations in which students do not behave strategically. Although many students do 'figure out' how to learn as a result of repeated exposure to tasks that require strategic planning, a large number of students fail to acquire adequate learning-to-learn skills. Some learners, despite considerable effort, both in class and at home, are unsuccessful in their learning endeavours. The present study has shown that an examination of students' strategic learning behaviours reveals: many students are either not using appropriate learning strategies; or there is a discrepancy between learning strategies that they should (or are expected) to use, and those that they do use; or those strategies that they do use are ineffective.

Specifically, learners may not benefit from strategic learning because:

- they fail to realise that a cognitive problem exists;
- their strategic knowledge is inadequate for the problem they have identified;
- they have the necessary content, or strategic knowledge to remedy the existing problem, but apply it ineffectively;
- they choose not to remediate the problem; or
- the learning environment is not supportive of strategy use.

Failure to Identify the problem

Strategies are either processes that are used to reach a learning goal, or they are brought into play when a problem in understanding or performance is encountered. Research studies (Anthony, 1991; Chi & Bassok, 1989) suggest that it is the less successful students who either monitor inaccurately or do not monitor at all. When learners (like Gareth) do not fully understand how to evaluate their learning, they do not necessarily detect this failure. Such students may make incorrect judgements about their understanding, based on criteria such as following the teachers' working, getting a certain percentage correct, or merely getting the first exercise correct.

Inappropriate metacognitive knowledge concerning learning goals, and beliefs about mathematics learning, are often related to inaccurate self-monitoring. In Gareth's case we saw that he equated 'completing' a problem with 'being able to understand' or 'having learnt' the problem. Sometimes the very act of **doing** a problem assured Gareth that it was correct. He did not reflect on the reasonableness of the answer and, on occasions when he failed to verify his answer, he missed the opportunity to learn from probable errors. In an example reported in section 8.2, Gareth felt assured that he was correctly squaring the numerator and denominator, so did not mark his work.

Gareth: *"I thought it was a good lesson. I thought it was pretty easy, some of it...I know when you have a surd at the bottom you've got to square it so you wouldn't have one at the bottom and I know why. It makes sense not having surds at the bottom because it wouldn't go into any other number."*

Other students also reported not marking, or even not doing, their homework based on personal judgement of how easy it was.

Int: *"Do you mark your homework?'*

Dean: *"If I'm having difficulty I do, but if I think I'm understanding it I just leave it."*

Alexander and Judy (1988) suggest that strategic processing is more effective and efficient when students possess at least a foundation of domain knowledge. For a student like Gareth, a lack of prior knowledge means that most of the content appeared relatively 'new' and there is little basis on which to assess present understanding. Furthermore, past experiences in learning mathematics had shaped Gareth's metacognitive knowledge to the extent that he expressed little, if any, expectation of understanding the presented content. If no problem in understanding is identified (even if one does exist), a student is unlikely to be strategic (Garner, 1990a). Because recognising that there is a problem is a precondition to employing any kind of remedial strategy, it is not surprising that Gareth rarely seeks explicit help from the teacher, nor expresses frustration in class.

Gareth: *"It wasn't really as complicated as she said. You just shift the tables from one column to the next."*

Strategic Knowledge

Sometimes learners fail to be strategic, not because they cannot recognise that a problem exists, but because they do not know how to remedy the problem that they have identified. For example, Jane recognised that she did not understand the teacher's explanation, but previous attempts at asking for teacher assistance rarely proved helpful, resulting in the following reported metacognitive knowledge:

Jane: *"If you ask the teacher a question she just explains it in the same way. If you didn't understand it the first time it's not much use."*

Moreover, Jane lacked the necessary strategy knowledge to remedy the situation. She occasionally cooperated with peers to gain extra assistance, but more often withdrew from the learning situation. Jake was another student who lacked help-seeking skills. It was not unusual for Jake to spend all of his seatwork time appearing to be working, but actually doing no work at all in his book. This pattern continued with homework:

Jake: *"I only attempt the homework if I know what I'm doing. There's no point in going through the homework - if you don't understand the day's work you won't be able to do the homework...I don't bother to take the homework home if it's too hard."*

An examination of students' revision strategies indicated another area in which some students have inadequate knowledge of appropriate learning strategies. While a few students reported that they thought about the topics to be studied, many failed to make use of the course summary, or report any planning of revision content. Moreover, some students failed to assess whether they had done adequate revision - it had not occurred to them to evaluate or monitor their revision strategies. Replies to the question "*How do you know whether you have done enough study?*" included:

Gareth: *"I don't know, when you start getting heaps of questions right I suppose."*

Jane: *"I don't know, just when I get sick of it."*

Dean: *"Well if you're getting it right with the answers in the back of the book."*

Most students were unaware of alternative methods of improving their revision performance. Their suggestions of general strategies, usually involving doing more of the same, reflected their reliance on increased effort, rather than specific knowledge of alternative strategies:

Faye: *"Do some examples, just the same sort of thing really."*

Jane: *"Spend longer."*

Gareth: *"Go through the examples more slowly."*

Jake: *"Try and understand the work more."*

Brent: *"Work longer and harder."*

Only Adam qualified his suggestion to do more: *"Do more exercises if possible, but as long as they are not all straightforward exercises."*

Only a few students previewed topic material. The majority preferred to let the teacher tell them what tasks and when to do them. It is not conclusive from this study whether students were not aware of the value of previewing, or whether they simply chose not to preview. Gareth however, did comment that he had tried reading ahead and doing some extra exercises, but because he had met with little success he had abandoned the strategy.

Gareth: *"Sometimes I do* (read ahead) *when it's straightforward, but usually everything else is too hard."*

On the few occasions when homework reading was assigned there was little indication as to the purpose or value of the activity and most students simply ignored this type of homework.

No students reported, or were observed, to be using organisational strategies such as concept mapping or flow diagrams. Moreover, as discussed more fully in section 9.2, students' summarising behaviours were almost exclusively teacher directed. It is of some concern whether students have sufficient expertise in locating the main ideas of a lesson to enable them to form their own summaries.

Applying knowledge appropriately

Students may possess a repertoire of learning strategies but not always employ the appropriate strategy. Nickerson (1989) refers to misapplication of a strategy as a "failure of commission". Garner (1988) suggests that a significant aspect of strategic learning is the need for flexible use. This implies that knowing **when** to use a learning strategy (conditional knowledge) is as important as knowledge **how** to use it. If the wrong strategy is used, or if an appropriate strategy is used ineffectively, then learning performance may be diminished. For example, when Gareth is stuck with a problem in class he is aware that he can ask for help, or try to help himself by comparing the problem to a worked example, reading the text, or working backwards from the answer. However, on several occasions Gareth was observed to unsuccessfully work backwards from an answer by trying various substitutions on his calculator; in one instance this was observed for a period of over five minutes. When asked why he didn't ask for help, Gareth replied, "*I only call her if I'm really in trouble, I learn more by figuring out how they got it myself, you know doing it myself*". Unfortunately, the instances when Gareth actually 'figured it out' were rare. Gareth's situation was compounded by the fact that when he did ask for assistance the teacher was likely to attend Gareth's presentation, or to do the problem for him, rather than assist him to 'figure it out'! Gareth's selective focus on computations, and indeed his persistence in self-checking each step during teacher explanations, is another example of an ineffective strategy application.

In other instances, well practiced routines, such as keyword strategies that produce a product, any product, can also inhibit the use of more effective learning enhancing strategies. Levin (1986) suggests that both more and less successful learners believe that an ineffective learning strategy is effective just because it is commonly used or prescribed. Unfortunately the tendency to persist in using procedures once they are well rehearsed, without reflection on them, or examining them further, has been widely noted (Hiebert & Carpenter, 1992).

Choosing to be strategic

Another explanation for students' failure to use strategic learning behaviours is that even when students have appropriate strategic knowledge they may choose not to use it (Borkowski & Muthukrishma, 1992; Palmer & Goetz, 1988; Pintrich & Schrauben, 1992). Because employing learning strategies requires effort, and does not always result in successful learning, students may merely avoid their use (Garner, 1990a). There are three interrelated reasons why students from this study sometimes chose not to employ strategies, despite having knowledge of appropriate strategies.

Firstly, if students do not judge the learning behaviour as significant or useful, it is unlikely that they will pursue it in the absence of external directives or incentives (Paris *et al.*, 1983). Without conditional knowledge regarding why a particular strategy is useful, it might only be executed in compliance with a teacher request. For example, some students were unaware of the value of using the textbook as a resource - preferring to rely on the teacher for information and instructions about which pages to study, and which exercises to complete.

Secondly, on many occasions students appeared to not want the bother of acting strategically. Passive learning behaviours exhibited by some students suggested that they are accustomed to being 'spoon fed' and told what to do. For example, when the teacher questioned students' understanding she assumed, but did not require, that students would respond accordingly. However, students who knew that general questions such as, *"Do you all follow that?"*, did not necessarily require a response, would be unlikely to monitor their cognitions rigorously. By assigning much of the responsibility for their learning to the teacher, students simply avoided employing strategic learning behaviours. Students who were accustomed to tasks that required minimal involvement resisted the teacher's attempt to engage them in more complex and ambiguous tasks by negotiating the task demands downwards.

Additionally, the cost-benefit trade-off and effort required by strategic learning behaviours may influence whether students choose to employ them. Students may regard an action to be relevant, meaningful, and useful for a particular goal, but they

may also perceive it to be cumbersome or demanding. Lucy, for example, expresses some concern about the personal cost of behaving strategically:

Int: *"Would you ask for help?"*

Lucy: *"I probably would, but you feel a bit dumb."*

The stress involved in expecting to look stupid (and avoiding situations where that is most likely to occur) "can lead ego-involved students to give up, and decide against invoking effortful strategies" (Garner & Alexander, 1989:153).

Moreover, students are unlikely to invoke strategies demanding time and effort if they believe that the strategies will not make any difference, and that they will fail to perform successfully. Gareth, for example, feels that last minute revision of exercises is not effective in improving performance.

Gareth: *"Last year I kept on bumming out because I did examples and my brain just couldn't handle all the examples."*

Thirdly, at times student choices of actions and goals are influence by individual factors such as mood, tiredness, learning styles, risk taking, achievement aspirations, self-concept, fear of failure, and fluctuating motivation. To succeed strategically the learner needs both the "skill and the will" (Pintrich & De Groot, 1990) - if the student is not motivated to achieve high grades, or not interested in mathematics, then the student is unlikely to invoke deep-processing strategies.

Self attributions are another significant factor. Garner (1990a:521) concludes that "without high self-esteem and the tendency to attribute success and failure to their level of effort, both children and adults are unlikely to initiate or persist at strategic activity". In light of the following self-assessment from Brent it is not surprising that he only occasionally attempts homework.

Brent: *"I think homework is to see if you can do it on my own. Like, I can't do it on my own - it reflects in my tests and stuff."*

In particular, students who attribute success to luck, or ability (which they haven't got) are unlikely to be motivated to be strategic.

Classroom learning environment

This study has found that although classroom instruction is designed to support strategic learning there are, on occasions, contextual and instructional factors which inhibit the development of such behaviours - especially strategies related to self-regulation and autonomous learning. Maintaining coverage of the course content while focussing on quality learning has been a persistent tension in mathematics instruction. Contextual influences on strategic learning behaviours, such as availability of peer support, involvement in class discussions, availability of resources and help; combined with instructional factors issuing from teacher-directed instruction, questioning techniques, use of homework review, and assessment, have been discussed fully in Chapter 9.

In summary, conventional instructional practices which emphasise routines for solving 'textbook problems' and domain content knowledge, place too little emphasis on the other aspects of mathematical knowledge (see Neyland, 1994b) - including metacognitive knowledge. As a consequence, appropriate metacognitive knowledge that is critical for mathematical thinking does not develop. Students who lack knowledge of learning strategies, including conditional strategy knowledge, are unable to apply appropriate learning strategies. Without the ability to monitor and control their learning, these students are limited to relying on teacher instruction and direction of their learning. In particular, passive learners will be out of their depth in a constructivist learning environment: without a range of appropriate learning-to-learn skills, to cope with the cognitive demands of constructing knowledge and evaluating their own learning, success will be limited.

10.3 Methodological Implications

This ethnographic research was conducted in the classroom setting with the emphasis on understanding the learning process from the students' perspective. Research findings relied heavily on interpretations of students' self-reports of strategic learning behaviours. Although these verbal reports are necessarily incomplete, the collection of verbal report

data was crucial, as it provided direct evidence about processes that would otherwise be invisible.

In particular, stimulated recall interviews proved to be very successful in providing a rich data source. Interview reports confirmed the limitation of relying solely on observation of classroom behaviours (Peterson *et al.*, 1982, 1984). Although students often appeared to be 'doing' the same things in class, stimulated recall interviews highlighted significant qualitative differences in the manner in which students approached their learning. Some students were able to go through the motions of displaying appropriate behavioural engagement in class with only minimal active learning occurring. Contributing factors, such as inaccurate assessment of understanding, inappropriate selective attention, and the nature of elaborations, where highlighted in students self-reports.

Adverse critics of verbal reporting suggest that problems may occur because more successful learners may be unaware of the complexity of their thinking, or that less successful learners may be unable to explain their thinking. The quality of student responses provided in Chapter 8 indicate that these reservations are unjustified. Moreover, data from the present study found that all target students reported relatively similar frequencies of learning strategy types (see section 8.1).

The stimulated recall interview method could profitably be further adapted by videoing several students simultaneously. Interviews would provide comparative data of students' reported strategies. However, while this would control for instructional variables, it would not control for the impact of beliefs and domain knowledge, which were found to be major determinants in students' selection and application of strategies.

Ethnographic research tends to reveal the complexity of educational phenomena. Wiersma (1991:243) suggests that "as more ethnographic research is done, the educational community should become better informed and become more sensitive to the importance of context in educational research". As we discover more about the learning strategies that students use, we will be better able to test hypotheses about the strategies that we predict as likely to produce the greatest success for given types of learners.

10.4 Implications for Classroom Instruction

The following recommendations are put forward based on the findings of the current study. They are tentative recommendations since the study did not encompass the dimension of strategy training, neither did it seek to develop specific guidelines for mathematics learners and teachers.

Few students learn to become active learners on their own. Collins *et al.* (1989) suggest that a model of cognitive apprenticeship is critical. While some students receive this modelling from their home environment, all should receive instruction in such skills throughout their schooling. The recommendations outlined below are proposed as important considerations if mathematics learners are to be self-regulated and able to learn how to do for themselves what teachers typically do for them in the classroom.

1. Teachers should be encouraged to model effective learning strategies and provide explicit strategy instruction within the context of their current mathematics program (see Herrington *et al.*, 1994). In particular, students should be informed of the value of metacognitive strategies such as planning, previewing, monitoring, and self evaluation.

2. Teachers need to be aware of the influence of the instructional context, including the task demands, learner support, and assessment methods, on their students' strategic learning behaviours:
    - The instructional environment must minimise the kind of compensations that reduce or eliminate demands for cognitive activities. The 'end product' should not be handed over without the engagement of the learner. Teacher-supplied summaries, too little wait time after a question, and the acceptance of answers from other students, may effectively deny students the opportunity to actively engage in the learning process for themselves.
    - Predictable classroom routines, such as homework reviews and questions around the class, may enable students to subvert the intended learning activity by

participating minimally. Such routines can be manipulated by the students to the detriment of their own learning.

- Assessment should be more collaborative, assess higher-order skills, and be as congruent as possible with the learning being asked for in class.

3. During class discussion there needs to be a greater tolerance for alternative responses. A slower pace would provide opportunities for students to reflect on procedures, and build connections between ideas and skills. Furthermore, in order to make effective use of worked examples, instruction needs to give more attention to the metacognitive behaviours and self explanations apparently needed in order to profit from such instruction.

4. Students, especially at senior level, need to be encouraged to take a more active role in their own learning. This will help students feel that they 'own' the mathematics they are learning, and will empower students to cope with any future mathematical study they may undertake:
    - Students need to be aware of the value of help-seeking strategies. Networking and ensuring peer help is available, is a possible redress for students who have no help available at home.
    - Students need to be taught to make effective use of printed resources.
    - The learning environment needs to provide opportunities for all students to be involved in collaborative learning. Communication amongst students is seen as essential, not only to support the social learning process but also to stimulate the process of reflection.
    - Students need to be able to effectively monitor their understanding, and be able to self-assess and accurately mark completed exercises.

5. Students need to build an appropriate metacognitive knowledge base:
    - Students need to become more aware of their own learning strengths and weakness.
    - Instruction needs to engender appropriate beliefs about mathematics and mathematics learning.

- Students should be provided with experiences which allow them to assess strategy use, and the comparative effectiveness of different strategies for different learning tasks.

6. Recognition should be given to the fact that the use of mathematics learning strategies is interrelated to numerous presage and product factors. Strategic learning behaviours cannot be imposed on students *en mass*, rather they must be selected and employed by the students on an individual basis.

In summary, the present study suggests that teachers need to be concerned about the extent of the presence of students' passive learning behaviours in the mathematics classroom. Instruction should be concerned with students' learning strategies and beliefs, as well as their content knowledge. Just as mathematics teachers in the 1980s devoted much instruction to the explicit teaching of problem solving strategies, we need in the 1990s to teach not just ***how to do*** mathematics but also ***how to learn*** to do mathematics. The learning environment must actively encourage the development and use of students' strategic learning behaviours by providing feedback on their use of strategies and demonstrating their improved performance.

10.5 Additional Research

This study has highlighted the passive nature of many students' learning behaviours, their lack of awareness of appropriate strategies, and inappropriate beliefs and learning goals. Additionally, aspects of classroom instruction are seen to support passive learning rather than promote active learning behaviours. In view of the fact that current curriculum documents support a constructivist learning paradigm, in which learners are expected to take more responsibility for actively constructing their own knowledge, it is important that further research explores instructional factors which will promote and support the development of self-regulated learning strategies necessary for autonomous learning.

Areas which warrant further attention are as follows:

- Research is needed into practical ways to encourage students to accept greater responsibility for their own learning. The apparent unwillingness, or inability, of many students to play a more dominant role in adapting instruction to suit their own learning goals is of concern. In particular, research needs to investigate the development of metacognitive behaviours related to control of learning.

- In a constructivist learning environment there is an increased need for peer interaction and communication. However, much of the peer cooperation and classroom discourse in this study involved only a small proportion of the class, suggesting that the potential of student tutoring and cooperative learning for facilitating understanding and enhanced performance is under-realised and should be further explored, particularly in senior mathematics classrooms.

- There is a need to focus on further classification and analysis of the nature of students' mental experiences within specific learning episodes; in particular, the nature of students' elaborations as used with worked examples. An exploration of activities that encourage appropriate student elaborations is also necessary.

- The effects of assessment on student learning behaviours are significant. Further research needs to examine ways in which assessment can support and promote desired learning behaviours. In particular, there is a need to investigate the development of more collaborative assessment methods.

- Students' experiences of learning mathematics outside the classroom need further study. In particular, the role of homework and student revision, and teacher/student expectations of independent learning, are largely unexplored issues.

- The development and encouragement of appropriate help-seeking behaviours, whether they involve self-help from problem diagnosis, use of textual resources, or help from other students and adults, needs further attention.

- Further research is needed to determine learning activities that will foster the development of appropriate metacognitive beliefs, and learning goals which will ensure that students' learning is appropriately directed toward understanding rather than task completion.

10.6 Summary: Major Outcomes

1. Students' learning of mathematics was found to be a highly idiosyncratic process. While commonalities in the range of learning strategies existed, the use of learning strategies in specific instances differentiated individual learning approaches and outcomes.

2. Simply having students engage in activities does not always result in the desired level of mathematical competence. The level of cognitive engagement that the student engages in is of utmost importance, rather than instructional time or time-on-task *per se*. In some instances students' learning strategies enhanced the learning process and in others it appeared to hinder learning. Thus, the implementation of appropriate strategy use is as important a measure as is students' time on-task and use of strategies in general.

3. A requisite level of prior knowledge (both domain and metacognitive) is necessary for effective strategic learning behaviour.

4. Students metacognitive knowledge (beliefs about learning mathematics, beliefs about themselves as learners, strategy and task knowledge) largely determines their learning goals. As strategies are goal directed students' metacognitive knowledge greatly influences the choice of learning strategies employed.

5. Students' learning goals were a major mediator in determining the appropriateness of strategic learning behaviours. Those students who intended to understand the mathematical content reported more appropriate elaborations, and self monitoring of understanding. Their awareness of lack of comprehension facilitated appropriate strategies to remediate problems. Those students who regarded learning mathematics as an activity, rather than as a goal, used learning strategies appropriate to accomplishing task completion rather than the cognitive objective for which the task was designed. Their learning behaviours focused on rehearsal strategies and elaborations related to recalling prior instances of problems rather than information related to problem types or concepts.

6. While students reported numerous accounts of metacognitive awareness of mental states, attempts to analyse, evaluate or direct their thinking (metacognitive control strategies) were less frequent.

7. There was ample evidence of passive learning behaviours. These behaviours reflect students' lack of awareness of appropriate learning strategies. Without such strategic knowledge students accepted their learning environments as given, expected the teacher to provide explicit instruction and help, relied on the teacher or text to monitor their progress, and made little use of available resources in independent study.

8. Instruction may have unwittingly contributed to passive learning behaviours. Passive learning was supported by compensatory instructional methods such as teacher-given summaries, teacher-directed revision and planning, external verification of production, and limited cognitive demands in discussion and assessment tasks. Teacher feedback was directed primarily at helping students to generate products. Furthermore, instruction provided little opportunity or incentive for students to develop and use autonomous, self-regulatory strategic learning behaviours.

9. Strategic learning behaviours contributing to successful learning included rehearsal, elaboration, organisation, self-instruction and self-evaluation at various stages during the learning process. More successful students planned their work, were able to self-instruct, self-assess and correct their work, they modified their learning tasks, and knew when it was appropriate to seek help. High achieving students relied heavily on social sources of assistance, whether it be family members, teacher or peers. Additionally, successful students were able to adapt their physical and social learning environment to optimise their learning opportunities.

10. Contributing factors of low achievement were found to be :
    - lack of relevant prior knowledge and experiences;
    - lack of orientation towards mastery learning and an associated confusion about task goals;
    - inappropriate use of learning strategies related to monitoring understanding;
    - infrequent reports of metacognitive behaviours to control learning;
    - ineffective use of help seeking and resources; and
    - low access to out-of class resources, including help from adults.

11. Stimulated recall interviews offered a valuable way of investigating students' awareness and use of learning strategies in the classroom.

Finally, results from this study hold promise for improving teaching by making instruction more adaptive to the needs, interests and learning strategies of the student - and for improving learning, by developing students' awareness of the necessary learning strategies for effective mathematics learning. Success of new curriculum developments in mathematics is critically linked to creating a suitable learning environment which fosters students' development of autonomous learning behaviours. To achieve this, both teacher and student need to work in a partnership aimed at developing appropriate metacognitive knowledge and behaviours. The learning environment must provide learning opportunities that require higher-order thinking and strategies, and provide feedback which enhances the development of knowledge and appropriate learning behaviours.

Appendix 1: Information Letter

Dear Parents,

As part of my research in Mathematics Education I am investigating the learning behaviours and strategies of senior mathematics students. I have received permission from the Principal, ----------, the Board of Trustees, and the classroom teacher, Mrs H....to conduct my research. During Term One I have been an observer in your daughter's/son's mathematics class: observing the interaction of instruction and students' learning behaviours and discussing with students their learning behaviours.

To increase our knowledge of which strategies are most effective and responsive to classroom instruction, mathematics teachers need more detailed knowledge of what strategies are presently used in the classroom environment. I am particularly interested in the students' awareness of ways that they go about learning, understanding and doing mathematics in the classroom and at home.

To collect data directly from the students, I wish to discuss learning mathematics strategies in more detail with some of the students in the class. I will use video segments of a class session to help students recall their learning behaviours. All students have been told about the nature of the study and have had any questions answered. Unfortunately there will not be time to interview all students. If your son or daughter is to be interviewed their written consent will be obtained. Interviews will take place in students' study periods with a maximum of 3 interviews. Additionally, I will ask all students to voluntarily complete occasional questionnaires and diaries of homework sessions.

As well as providing valuable information on actual use of learning strategies in the classroom, it is hoped that discussions with students during the research will increase students' awareness of their learning behaviours.

Confidentiality and anonymity of all data will be respected and a written summary of the study will be presented to the --------Board of Trustees at the completion of the project in Term 3. If you have any queries about this study please feel free to contact me at Massey University----------or at home ---------.

Sincerely

Glenda Anthony

Appendix 2: Consent Form

LEARNING STRATEGIES IN MATHEMATICS

STUDENT'S CONSENT FORM

I have been present at the researcher's discussion of the research study and have had the details of the study explained to me. My questions about the study have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I agree to the researcher making a video of me working in the classroom and I am willing to discuss my learning behaviours with the researcher in a subsequent interview,

I also understand that I am free to withdraw from the study at any time, or to decline to answer any particular questions in the study. I agree to provide information to the researcher on the understanding that it is completely confidential.

Signed: _____

Name: _____

Date: _____

Appendix 3 Questions from Questionnaires

STUDENTS' PERCEPTIONS ABOUT MATHEMATICS LEARNING questionnaire

1. What is your favourite subject this year?
2. What is your least favourite subject this year?
3. How do you feel about learning maths?
4. Why are you studying maths this year?
5. What goals/expectations do you have for your learning of maths?
6. How does learning maths differ from learning other subjects that you are taking this year?
7. How much time and effort do you put into learning maths?
8. Who do you think is mostly responsible for your learning of maths?
9. In what way does the teacher, and/ or classroom instruction affect your learning?
10. In what ways can you control you learning of maths?

LEARNING FOR A MATHS TEST (completed at the end of a calculus unit)

1. Have you learnt most of the material in class or will most of the learning be from your revision work?
2. What have you already done to prepare for the test (a) during class time
   (b) during homework times
3. How will you complete your revision/study for the test?
4. What resources (e.g., notes, tests, people) will you use?
5. Will you be mostly trying to understand the material or to remember the material?
6. How do you feel about learning for the test?
7. How well do you think you will do in the test?
8. Do you think your result will be better or worse than your usual maths results?
9. What things could you do differently, in either maths lesson or a home, that could result in increased performance in another test?
10. Did you do the review test in the weekend? If so what did you learn by doing it?

Appendix 4: Homework Diary (condensed format)

I am interested in how you go about learning mathematics away from the classroom.

Please complete this HOMEWORK DIARY for each session you spend learning/doing mathematics at home, over the next week.

I would like you to briefly record any activity you do associated with your mathematics. Some of these activities will be observable, such as: do Exercise 6.1; check answers for help; and clear my desk. However, other activities are not easily observable as they occur in your thought process. For example: questioning your understanding of a problem, trying to remember what the teacher said in class, and thinking of a "reward" (cup of coffee) when you finish.

Every five minutes, or a more suitable interval, during a study/homework session, please record all of your activities on the form provided:

NAME:_____ DATE_____

TIME	ACTIVITIES -Observable	ACTIVITIES - Thinking

Did you complete all the set homework for today?

Did you understand the homework?

What did you achieve by doing the homework/ study?

Did you do any thinking, studying, or exercises that were not set by the teacher? (explain)

What learning/thinking activities were most useful in this study session?

Appendix 5: ORIENTATION SURVEY (Condensed format)

Your opinions about this maths class

Please mark the response A, B, C, D, or E that best matches your opinion.

A	B	C	D	E
Strongly Agree	**Agree**	**Neutral**	**Disagree**	**Strongly Disagree**

1.	I think I am learning a lot in this class.	**A B C D E**
2.	Students in this class work together so everyone can learn.	**A B C D E**
3.	We often move on to a new topic before we really understand the old one.	**A B C D E**
4.	When students ask questions in this class it slows us down.	**A B C D E**
5.	You have to compete in this class to get good grades.	**A B C D E**
6.	I'm satisfied with how much we are learning in this class.	**A B C D E**
7.	In this class, it is OK to spend extra class time on a hard topic.	**A B C D E**
8.	In this class, only a few students can really succeed.	**A B C D E**
9.	Students in this class often help each other learn.	**A B C D E**
10	Students in this class often have trouble keeping up with the work.	**A B C D E**
11.	After class, I usually feel satisfied with what I have learned.	**A B C D E**
12.	Only the smartest students can get a good grade in this class.	**A B C D E**
13.	Most of the students in this class work well together.	**A B C D E**
14.	Our teacher wants us to work cooperatively with each other.	**A B C D E**
15.	Our teacher wants us to understand *why* things happen the way they do.	**A B C D E**
16.	Our teacher thinks mistakes are OK, as long as we learn from them.	**A B C D E**
17.	Our teacher has to cover *all* the material in this course, even if we don't understand it all.	**A B C D E**
18.	To get a good grade in this class, you have to memorise a lot of facts.	**A B C D E**
19.	Our teacher doesn't like students to make mistakes.	**A B C D E**
20.	Our teacher tries to get us to think for ourselves.	**A B C D E**
21.	In this class it's more important to get right answers than to understand *why* they're right.	**A B C D E**
22.	Our teacher helps us see how what we learn relates to the real world.	**A B C D E**
23.	Our teacher wants us to learn to ask questions about maths.	**A B C D E**
24.	Our teacher lets us know how we compare to other students.	**A B C D E**
25.	Our teacher wants us to learn how to solve problems on our own.	**A B C D E**
26.	Our teacher encourages us to ask questions when we don't understand something.	**A B C D E**

Appendix 6: Stimulated Recall Interview 1 : Jane

Reported and observed learning behaviours and metacognitive knowledge from a single lesson about Normal distribution and use of tables.

Reported/observed behaviour	Learning Behaviour	Strategy Type
"Yeah, I know the answer."	Monitoring understanding	Metacognitive
"...sometimes it's quite hard to understand what she's trying to ask."	Metacognitive knowledge	Metacognitive knowledge
Looks at book in response to teacher's reference to (b) and (c).	Refers to questions in the text.	Cognitive
"Most of this stuff I understood, it was easy."	Evaluation of understanding	Metacognitive
"The first time she did this I didn't really understand because I wasn't really listening, I wasn't really thinking about it, but this time I understood."	Monitoring understanding and metacognitive knowledge	Metacognitive/ metacognitive knowledge
"She said we had done this in class so I am looking back to see if I had it in my book."	Looking back in book for previously studied work	Cognitive
"I'm just waiting and seeing if I can see where she gets the answers from."	Trying to follow the procedure/anticipation	Cognitive/ metacognitive
"I'm just listening."	Listening to the teacher	Cognitive
Jane is watching the work on the board	Attending to the board work	Cognitive
"Now that she is going over it again its easier."	Linking with previous work/evaluation	Cognitive/ metacognitive
"Yeah, I know the answer."	Monitoring understanding	Metacognitive
"I heard it, (but I didn't know what she was on about)."	Listening to other student's question (monitoring understanding)	Metacognitive
"...well she's on about all those z's and x's and all that stuff and it's hard to understand what she's going on about."	Monitoring understanding /metacognitive knowledge	metacognitive
"She was saying that we didn't have to use it,. it's off the topic , not really worth listening to."	Selective attention	Metacognitive
"I wasn't listening to Dean, it's got nothing to do with me what he's going on about."	Selective attention	Metacognitive
"I'd rather sit there. I'd rather just do it by myself."	Metacognitive knowledge	Metacognitive knowledge
"I don't like talking in front of the class."	Metacognitive knowledge	Metacognitive knowledge

"I'm looking at Question 5 and 7 that we were supposed to do for homework."	Following teacher's instruction	Cognitive
"I thought, draw a diagram..."	Answering teacher's question to herself	Cognitive
"...but it was wrong, I'm glad I didn't say anything."	Metacognitive experience	Metacognitive experience
"I don't do anything, I feel bored. I look at the book but for nothing in particular."	Metacognitive experience	Metacognitive experience
"...this is easy, I understand this."	Monitoring understanding	Metacognitive
"When she used x's and all those letters and stuff it's too hard to understand. I wish she'd go over it more."	Metacognitive knowledge	Metacognitive knowledge
"I can remember the formula."	Recalling formula	Cognitive
I: "Why don't you answer some of the teacher's questions. Jane: "I've never done it."	Metacognitive knowledge	Metacognitive knowledge
"I looked up the table at the back of the book, I tried to work it out."	Trying out one of the problems	Cognitive
Looks over the seatwork problems.	Previewing set questions	Metacognitive
Looking at teacher's working on the board.	Attending to teacher explanation	Cognitive
"I'm understanding this."	Monitoring understanding	Metacognitive
"...if she's doing something and I understand it, but I know I wont remember it I write it down. If it's something that I just don't understand altogether, I wont write it down."	Selective note taking	Metacognitive/ cognitive
"...we've already done some of these in our books anyway."	Evaluating the need to take notes	Metacognitive
Reads book then attends to teacher's working on the board.	Reads question in text in conjunction with teacher's explanations	Cognitive
"I thought I better do some work."	Regulates behaviour	Affective
Gets calculator out. "I worked out (27-40)/6 and tried to find out what it was, I looked at the back of the book (tables)."	Calculates answer to step in teacher's worked example.	Cognitive
"I couldn't find it."	Evaluating performance	Metacognitive
"Your know how you find 0.98, I didn't realise that it was a percentage and when she said percentage and 98% ..."	Linking of teacher explanation with prior knowledge.	Cognitive

"...then I learnt something."	Metacognitive experience	Metacognitive experience
Read seatwork problems before getting books and pencils out.	Reading through seatwork problems.	Metacognitive
"I marked it after I had done (c)."	Marking exercises	Metacognitive
"I looked up the back of the book (tables) to see where they got that answer from."	Trying to diagnose error (working backwards)	Metacognitive
"I tried to work it out again..."	Redoing problem	Cognitive
"...but I didn't get it."	Evaluating performance	Metacognitive
"I put a question mark next to it."	Coding/organisation	Cognitive
"if I really wanted to know I could ask someone at home, but I probably wouldn't bother.	Help seeking/ metacognitive knowledge	Resource management/ metacognitive knowledge
"When I have a new question I write it out in a different colour."	Colour coding	Cognitive
I: "What you write it out in full? Jane: "No, like it is the probability that it's less than something so I can see it, so I know what I have to find out."	Selective processing/organisation	Cognitive
"I just go through my notes and if I don't understand anything I just go through some of the exercises."	Revision based on evaluating understanding	Cognitive/ Metacognitive
"If I get stuck on the questions I'm doing, I'll go back and have a look how I did them."	Help seeking	Resource management
"I'm finding these quite easy."	Evaluating performance	Metacognitive
"Well I wasn't doing the questions that we were suppose to do. I did No. 8 'cause she'd done (a) and (b) and I'd just wanted to go through a whole question like 7(a), (b), (c), (d), and not just (d) and (e) of a question...I like the look of No. 8; it looks bigger."	Selecting own problem	Resource management
"I thought well it's probably getting harder so I didn't worry."	Metacognitive knowledge / monitoring performance	Metacognitive knowledge / metacognitive
"I read it first and thought I could do it, understand it.	Previewing, assessing ability to complete the task	Metacognitive
"I don't always do the one's she puts on the board..."	Modifying the task	Resource management
"...but for homework I do the set ones, if I do the homework."	Doing homework	Cognitive

"Well I got stuck near the end so I went back to No. 6."	Help seeking	Resource management
"I thought it would be easier since it was back further, 'cause they get harder at the end."	Metacognitive knowledge	Metacognitive knowledge
"I can hear her."	Monitoring the teacher's action around the classroom	Resource management
"I sometimes listen, if I'm sitting next to them and she's going over something I don't understand I just listen but not often."	Help seeking	Resource management
"I'm drawing a diagram for this problem."	Drawing a diagram	Cognitive
"I thought I was going quite well, I thought it was easy, but I wasn't getting them all right ..."	Evaluating progress	Metacognitive
"...I forgot to subtract things at the end.	Diagnosing error	Metacognitive
"I didn't really learn anything new, but I got more practice."	Evaluating learning	Metacognitive
"It's important to try some exercises."	Metacognitive knowledge	Metacognitive knowledge
"I didn't understand all that stuff about the area under the graph, or what she's going on about."	Monitoring understanding	Metacognitive
I: "Did you pay special attention because you didn't understand it?: Jane: "No, 'cause she just says the same thing over and over again, so I'm not going to understand it."	Metacognitive knowledge	Metacognitive knowledge
"I was thinking that I don't understand this."	Monitoring understanding	Metacognitive
"I see what she's trying to get at ."	Trying to follow the teacher explanation.	Cognitive
"I understand some of it."	Monitoring understanding	Metacognitive
"For that question there - when she got to the end I found out how she did it - so I was just waiting until she got the final answer..."	Following the teacher's explanation	Cognitive
"...'cause at the end when she gets the final answer its easier to understand I think."	Metacognitive knowledge	Metacognitive knowledge
"I'm trying to find the answer in my tables."	Trying to answer the teacher's question	Cognitive
"I got lost. I couldn't understand what she's going on about - I couldn't find it."	Evaluating performance	Metacognitive
"I'm trying again, but I couldn't find it."	Redoing problem	Cognitive

Appendix 7: Seatwork Behaviour: Karen

Reported behaviours and metacognitive knowledge from classroom observations and three simulated recall interviews with Karen.

Reported behaviour	Learning Behaviour	Strategy Type
"I wasn't sure about the last problem."	Monitoring understanding	Metacognitive
"I'd started to click what logs were and I could do the problems."	Metacognitive experience/ evaluating progress.	metacognitive
"I do my marking in red and my corrections in red if I want them to stand out."	Colour coding work for emphasis	Cognitive
"If there's something I really don't understand I'll put a big box around it."	Selective organisation	Cognitive
"If I've got an exam coming up I'll go through my book and see what I've got circled in my book and read it."	Reading selected information from exercise book	Cognitive
"I was sort of listening, but I decided to finish the problem off."	Monitoring the teacher/ selective attention	Resource management/ metacognitive
"I don't like leaving unfinished problems."	Personal knowledge	Metacognitive knowledge
"Well, I know I got that one right."	Production evaluation	Metacognitive
"I was thinking I was pretty smart."	Person Knowledge	Metacognitive knowledge
"I wrote down the numbers but I'm stuck."	Production evaluation	Metacognitive
"I wrote a note in my log book to do some at home."	Aware of the need for revision	Metacognitive
"I was very confused."	Confusion	Metacognitive experience
"I went to the third one because I couldn't do the second one."	Production evaluation / task management	Metacognitive/ resource management
"I'm reading the example."	Previewing work	Metacognitive
Asks Jane if she has a ruler	Peer cooperation	Resource management

"I looked up to see what she (teacher) was going on about..."	Monitoring the teacher	Resource management
"...and I thought, 'I know that'."	Self evaluation	Metacognitive
"I was incredibly bored, wishing that maths was over."	Affective reaction	Metacognitive experience
Flips through the next few pages	Previewing work	Metacognitive
"I took a while to compare it with the answer"	Comparing own to model answer	Cognitive
"I've finished all my work so I'm just reading the next page."	Task management	Resource management
Marks a block of work and on finding it is all wrong then marks each one from there on.	Uses feedback of success to manage her checking behaviour	Metacognitive
Seeks help from Faye	Peer cooperation	Resource management
Reads Faye's work	Help from peers	Resource management
"Mrs H how do you do 4c?"	Seeks help form the teacher	Resource management
Uses textbook to seek information	Help seeking	Resource management

Appendix 8: Homework Interview: Adam

Reported behaviours and metacognitive knowledge from homework interview with Adam

Reported behaviour	Learning Behaviour	Strategy Type
"I always feel that I can do it in a certain time limit that I set for myself."	Sets time limit to complete homework	Metacognitive
"I always feel that I can finish it easily."	Metacognitive knowledge	Metacognitive knowledge
"I decided if there is time for me to get the questions done - I would think, oh that will take me ten minutes."	Planning time allocation	Metacognitive
"I get my books out, get my exercise book and pencil case out."	Organise study environment	Resource management
"If I haven't had a look in class I skim through to see what it's about."	Preview homework	Metacognitive
"I usually finish it before or around the time - unless there's a problem."	Metacognitive knowledge	Metacognitive knowledge
"I used to tick them right, but now I just look at the answers and compare them to mine."	Check homework	Metacognitive
"...if I got some wrong I redo the question."	Correct working	Cognitive
"I finish a whole block, sometimes I mark after a big question."	Check at the end of section rather than every part	Metacognitive
"...before an exam I would do some revision."	Revision exercise	Cognitive
"I do look ahead a bit now to see what's coming up."	Previewing class work	Metacognitive
"If I don't know how to do a question I would look back at the explanations."	Use text explanations for help	Resource management
"I usually think about science not maths, but sometimes, but not a lot."	Thinking about maths	Metacognitive
"I usually complete my homework."	Completes homework	Cognitive

"I do sometimes set some sort of schedule that I can follow, especially with revision. For homework I usually look at my log book and think how long it will take for each subject and the set a schedule."	Planning schedule	Metacognitive
"Last year I learnt a lot at home; this year not much, just a few bits about calculus and statistics.	Evaluate learning	Metacognitive /metacognitive knowledge
"I do a lot more (learning) in class this year."	Metacognitive knowledge	Metacognitive knowledge
"...she might link homework with something else."	Looking for links with prior knowledge	Cognitive
...it depends on my mood. Sometimes I might be thinking of other subjects, sometimes I might do some more maths..."	Attention control	Affective
"Last year we were given a detailed prescription of what to learn. I need to know what it is we are going to learn, the pages. I can find the exercises myself."	Need for learning goal	Metacognitive knowledge
"Well sometimes it's good you can fit revision into the lessons."	Revision during lesson	Cognitive

Appendix 9: Test Revision: Gareth

Gareth's reported behaviours and metacognitive knowledge from test revision interview and questionnaire.

Reported behaviour	Learning Behaviour	Strategy Type
"I bummed out as usual."	Metacognitive knowledge	Metacognitive
"I did some questions and read through my notes."	Rehearsal	Cognitive
"I split the trigonometry topic into different sections."	Planning topics to study	Metacognitive
"I don't do the examples the night before..."	Planning time schedule for study	Metacognitive
"...it doesn't really help me much because I just lose concentration."	Metacognitive knowledge	Metacognitive knowledge
"I just go over my notes the night before."	Rehearsal	Cognitive
"I just write it (formula) out a few hundred times."	Rehearsal	Cognitive
"I read my notes over and over again."	Rehearsal	Cognitive
"I sit in a quiet room with no interference."	Environmental management	Resource management
"I do some revision from 4th Form revision book."	Revision exercises	Cognitive
I: "What did you learn in your review session?" G: "Everything came back into my mind."	Recall	Cognitive

Bibliography

Alexander, P. A. (1992). Domain knowledge: Evolving themes and emerging concerns. *Educational Psychologist,* 27, 33-51.

Alexander, P. A. & Judy, J. E. (1988). The interaction of domain-specific and strategic knowledge in academic performance. *Review of Educational Research,* 58, 375-404.

Alton-Lee, A. G. (1984).*Understanding learning and teaching: An investigation of pupil experience of content in relation to immediate and long term learning.* Unpublished PhD thesis, University of Canterbury, Christchurch.

Alton-Lee, A., Nuthall, G., & Patrick, J. (1993). Reframing classroom research: A lesson from the private world of children. *Harvard Educational Review,* 63(1), 50-84.

Ames, C. (1992). Classrooms: Goals, structures, and student motivation, *Journal of Educational Psychology,* 84, 261-271.

Ames, C., & Archer, J. (1988). Achievement goals in the classroom: Students' learning strategies and motivational processes. *Journal of Educational Psychology,* 80, 260-267.

Anthony, G. (1991). *Learning approaches and study patterns of distance education students in mathematics.* Unpublished MPhil thesis, Massey University, Palmerston North.

Anthony, G. (1994). *The role of worked examples in learning mathematics.* In A. Jones *et al.* (Eds.), *SAMEpapers 1994* (pp. 129-143). Hamilton: Centre for Science and Mathematics Education Research, University of Waikato.

Artzt, A. F., & Armour-Thomas, E. (1992). Development of a cognitive metacognitive framework for protocol analysis of mathematical problem solving in small groups. *Cognition and Instruction,* 9(2), 137-175.

Australian Education Council (1991). *A national statement on mathematics for Australian schools.* Melbourne, Victoria: Curriculum Corporation.

Bachor, D. G. (1991). Thinking skills or strategic behaviour and arithmetic word problems: research, assessment and instruction. In R. F. Mulcahy, R. H. Short, & J. Andrews (Eds.), **Enhancing learning and thinking** (pp. 137-159). New York: Praeger.

Baird, J. R.(1986). Improving learning through enhanced metacognition: a classroom study. *European Journal of Science Education,* 8, 263-282.

Baird, J. R., & Northfield, J. R. (1992). *Learning from the PEEL experience.* Melbourne: Monash University Printing Services.

Barrett, J. R. (1990). *Form 6 mathematics: Revision.* Auckland: ESA Publications.

Begg, A. (1993a). Tomorrow's mathematics curriculum. In E. McKinley *et al.* (Eds.), *SAMEpapers 1993* (pp. 210-231). Hamilton: Centre for Science and Mathematics Education Research, University of Waikato.

Begg, A. J. C. (1993b) *Professional development of high school mathematics teachers.* (Report to the Research Division of the Ministry of Education). Hamilton: Centre for Science and Mathematics Education Research, University of Waikato.

Bereiter, C. (1992). Referent-centred and problem-centred knowledge: Elements of an educational epistemology. *Interchange,* 23(4), 337-361.

Bereiter, C., & Scardamalia, M. (1989). Intentional learning as a goal of instruction. In L. B. Resnick (Ed.), *Knowing, learning, and instruction: Essays in honor of Robert Glaser* (pp. 361-392). Hillsdale, NJ: Lawrence Erlbaum Associates.

Biggs, J. B. (1987). *Student approaches to learning and studying.* Melbourne: Australian Council for Educational Research.

Biggs, J. B. (1989). Approaches to enhancement of tertiary teaching. *Higher Education Research and Development,* 8(1), 7-25.

Biggs, J. B. (1991). A student learning in the context of school. In J. Biggs (Ed.), *Teaching for learning* (pp. 7-29). Hawthorn: Australian Council for Educational Research.

Biggs, J. B. (1993). From theory to practice: A cognitive systems approach. *Higher Education Research and Development,* 12(1), 73-85.

Biggs, J. B., & Moore, P. (1993) *The process of learning.* Sydney: Prentice Hall.

Bisanz, J. & Lefevre, J. (1990). Strategic and nonstrategic processing in the development of mathematical cognition. In D. F. Bjorklund (Ed.), *Children's strategies: Contemporary views of cognitive development* (pp. 213-244). Hillsdale, NJ: Lawrence Erlbaum Associates.

Blais, D. M. (1988). Constructivism - a theoretical revolution for algebra. *Mathematics Teacher*, 624-631.

Blumenfeld, P. C., & Meece, J. L. Task factors, teacher behaviour, and students' involvement and use of learning strategies in science. *The Elementary School Journal,* 88, 235-250.

Borkowski, J. G., & Muthukrishna, N. (1992). Moving metacognition into the classroom: 'Working Model' and effective strategy teaching. In M. Pressley, K. Harris, & J. Guthrie (Eds.), *Promoting academic competence and literacy in schools* (pp. 477-501). San Diego: Academic Press.

Borkowski, J. G., Schneider, W., & Pressley, M. (1989). The challenges of teaching good information processing to learning disabled students. *International Journal of Disability, Development and Education,* 36, 169-185.

Bransford, J. D., Sherwood, R., Vye, N., & Rieser, J. (1986). Teaching thinking and problem solving: Research foundations, *American Psychologist,* 41, 1078-1089.

Briars, D. J. (1983). An information-processing analysis of mathematical ability. In R. F. Dillon, & R. R. Schmeck (Eds.) *Individual differences in cognition* (Vol 1, pp. 181-204), New York: Academic Press.

Brooks J., & Brooks M. (1993). *In search of understanding: the case for the constructivist classroom.* Alexandria: Association for Supervision and Curriculum Development.

Brown, A. L., Bransford, J. D., Ferrara, R. A. & Campione, J. C. (1983). Learning, remembering, and understanding. In J. H. Flavell & E. H. Markman (Eds.), *Handbook of child psychology: Vol.3. Cognitive development* (pp. 77-166). New York: Wiley.

Bruer, J. T. (1993). *Schools for thought.* Cambridge: MIT Press.

Burton, L. (1984). Mathematical thinking: The struggle for meaning. **J***ournal for Research in Mathematics Education,* 15(1), 35-49.

Burton, L. (1992). Becoming a teacher of mathematics. *Cambridge Journal of Education,* 22(3), 377-386.

Campione, J. C., Brown, A. L., & Connell, M. L. (1989). Metacognition: On the importance of understanding what you are doing. In R. I. Charles & E. A. Silver (Eds.), *Research agenda in mathematics education: The teaching and assessing of mathematical problem solving* (pp. 93-114). Reston, VA: National Council of Teachers of Mathematics.

Candy, P. C. (1989). Constructivism and the study of self-direction in adult learning. *Studies in the Education of Adults,* 21(2), 95-116.

Cardelle-Elawar, M. (1992). Effects of teaching metacognitive skills to students with low mathematics ability. *Teaching and Teacher Education,* 8(2), 109-121.

Chamot, A. U., & O'Malley, J. M. (1987).The cognitive academic language learning approach: A bridge to the mainstream. *TESOL Quarterly*, 21(2), 227-249.

Chi, M. T. (1987). Representing knowledge and metaknowledge: Implications for interpreting metamemory research. In F. E. Weinert & R. H. Kluwe (Eds.), *Metacognition, motivation, and understanding* (pp. 239-266). Hillsdale, NJ: Erlbaum.

Chi, M. T., & Bassok, M. (1989). Learning from examples via self-explanations. In L. B. Resnick (Ed.), *Knowing, learning, and instruction: Essays in honor of Robert Glaser* (pp. 251-282). Hillsdale, NJ: Erlbaum.

Chi, M. T., Bassok, M., Lewis, M., Reimann, P., & Glaser, R. (1989).Self-explanations: How students study and use examples in learning to solve problems. *Cognitive Science, 1*3, 145-182.

Christopoulos, J. P., Rohwer, W. D., & Thomas, J. W. (1987). Grade level differences in students' study activities as a function of course characteristics. *Contemporary Educational Psychology,* 12, 303-323.

Cobb, P. (1988). The tension between theories of learning and instruction in mathematics education. *Educational Psychologist,* 23(2), 87-103.

Cobb, P., Wood, T., & Yackel, E. (1990). Classrooms as learning environments for teachers and researchers. In R. Davis, C. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics (*pp. 125-146). Reston: National Council of Teachers of Mathematics.

Collins, A., Brown, J. S., & Newman, S. E. (1989). Cognitive apprenticeship: Teaching the craft of reading, writing, and mathematics. In L. B. Resnick (Ed.), ***Knowing, learning, and instruction: Essays in honor of Robert Glaser*** (pp. 453-494). Hillsdale, NJ: Lawrence Erlbaum Associates.

Confrey, J. (1990). What constructivism implies for teaching. In R. Davis, C. Maher, & N. Noddings (Eds.), ***Constructivist views on the teaching and learning of mathematics*** *(*pp. 107-122). Reston: National Council of Teachers of Mathematics.

Confrey, J. (1992). *Learning to see children's mathematics: Crucial challenges in constructivist reform.* Paper presented at the Seventh International Congress of Mathematics Education, Quebec, Canada.

Corno, L. (1989). Self-regulated learning: A volitional analysis. In B. J. Zimmerman & D. Schunk (Eds.), *Self-regulated learning and academic achievement: Theory, research and practice* (pp. 111-141). New York: Springer-Verlag.

Cullen, J. L. (1985). Children's ability to cope with failure: Implications of a metacognitive approach for the classroom. In D. L. Forrest-Pressley, G. E. MacKinnon, & T. G. Waller (Eds.), *Metacognition, cognition and human performance* (Vol. 2, pp. 267-300). Orlando, FL: Academic Press.

Dansereau, D. F. (1985). Learning strategy research. In J. W. Segal, S. F. Chipman & R. Glaser (Eds.), *Thinking and learning skills: Relating instruction to research* (Vol. 1, pp. 209-239). Hillsdale, NJ: Erlbaum.

Davis, R. B., Maher, C. A., & Noddings, N. (1990). Suggestions for the improvement of mathematics education. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 187-191). Reston, VA: National Council of Teachers of Mathematics.

Denzin, N. K. (1988). Triangulation. In J. Keeves (Ed.), *Educational Research, methodology and measurement: An international handbook* (pp. 511-513). NY: Pergamon Press.

Department of Education (1987) *The Curriculum Review.* Wellington: Department of Education.

Department of Education (1988) *National Curriculum Statement (Draft).* Wellington: Department of Education.

Derry, S. J. (1990a). Remediating academic difficulties through strategy training: The acquisition of useful knowledge. *Remedial and Special Education,* 11(6), 19-31.

Derry, S. J. (1990b). Learning strategies for acquiring useful knowledge. In B. Jones & L. Idol (Eds.), *Dimensions of thinking and cognitive instruction* (pp. 347-379). Hillsdale, NJ: Lawrence Erlbaum.

Derry, S. J., & Murphy D. A. (1986). Designing systems that train learning ability: From theory to practice. *Review of Educational Research,* 56(1), 1-39.

Doyle, W. (1988). Work in mathematics classes: The context of students' thinking during instruction. *Educational Psychologist*, 23(2), 167-180.

Dweck, C. S. (1986). Motivational processes affecting learning. *American Psychologist*, 41, 1040-1048.

Evans, G. (1991a). Lesson cognitive demand and student processing in upper secondary mathematics. In G. Evans (Ed.), *Learning and teaching cognitive skills* (pp. 124-143). Melbourne: Australian Council for Educational Research.

Evans, G. (1991b). Student control over learning. In J. Biggs (Ed.), *Teaching for learning: The view from cognitive psychology* (pp. 51-70). Melbourne: Australian Council for Educational Research.

Fennema, E. (1989). The study of affect and mathematics: A proposed generic model for research. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem-solving: A new perspective* (pp. 205-219). New York: Springer-Verlag.

Fennema, E., & Peterson, P. L. (1985). Autonomous learning behaviour: A possible explanation of gender-related differences in mathematics. In L. C. Wilkinson & C. Marrett (Eds.), *Gender influences in classroom interaction* (pp. 17-35). Orlando, FL: Academic Press.

Flavell, J. H. (1976). Metacognitive aspects of problem solving. In L. B. Resnick (Ed.), *The nature of intelligence* (pp. 231-235). Hillsdale, NJ: Lawrence Erlbaum.

Flavell, J. H. (1981). Cognitive monitoring. In W. P. Dickson (Ed.), *Children's oral communication skills* (pp. 35-60). New York: Academic Press.

Flavell, J. H. (1987). Speculations about the nature and development of metacognition. In F. E. Weinert & R. H. Kluwe (Eds.), *Metacognition, motivation, and understanding* (pp. 21-29). Hillsdale, NJ: Lawrence Erlbaum Associates.

Gabrys, G., Weiner, A., & Lesgold, A. (1993). Learning by problem solving in a coached apprenticeship system. In M. Rabinowitz (Ed.), *Cognitive science foundation of instruction* (pp. 119-147). NJ: Lawrence Erlbaum Associates.

Gage, N. L., & Berliner, D. C. (1992). *Educational psychology*. Boston: Houghton Mifflin.

Galloway, V. & Labarca, A. (1990). From student to learner: Style, process, and strategy. In D. W. Birckbichler (Ed.), *New perspectives and new directions in foreign language education* (pp. 111-158). Illinios: National Textbook Company.

Garner, R. (1988). Verbal-report data on cognitive and metacognitive strategies. In C. E. Weinstein, E. T. Goetz, & P. A. Alexander (Eds.), *Learning and study strategies: Issues in assessment, instruction, and evaluation.* San Diego: Academic Press.

Garner, R. (1990a). When children and adults do not use learning strategies: Towards a theory of settings. *Review of Educational Research,* 60(4), 517-529.

Garner, R. (1990b). Children's use of strategies in reading. In D. F. Bjorklund (Ed.), *Children's strategies: Contemporary views of cognitive development* (pp. 245-268). Hillsdale, NJ: Lawrence Erlbaum Associates.

Garner, R. (1992). Metacognition and self-monitoring strategies, in S. J. Samuels & A. E. Farstrup (Eds.), *What research has to say about reading instruction* (pp. 236-252). Delaware: International Reading Association.

Garner, R., & Alexander, P. (1989) Metacognition: Answered and unanswered questions. *Educational Psychology*, 24(2), 143-158

Garofalo, J., & Lester, F. K. Jnr. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education,* 16, 163-176.

Garrison, D. R. (1993). A cognitive constructivist view of distance education: An analysis of teaching-learning assumptions. *Distance Education*, 14(2), 199-211.

Gavelek, J. R. & Raphael, T. E. (1985). Metacognition, instruction, and the role of questioning activities. In D. L. Forrest-Pressley, G. E. MacKinnon, & T. G. Waller (Eds.), *Metacognition, cognition, and human performance: Vol. 2. Instructional practices* (pp. 103-136). Orlando, FL: Academic Press.

Glaser, R. (1984). Education and thinking: The role of knowledge. *American Psychologist*, 39, 93-104.

Glesne, C., & Peshkin, A. (1992). *Becoming a qualitative researcher: An introduction.* New York: Longman.

Gray, S. (1991). Ideas in practice: metacognition and mathematical problem solving. *Journal of Developmental Education*, 14(3), 24-26, 28.

Grube, G. (Trans.). (1974). *Plato's Republic.* Indianapolis: Hackett.

Hennessy, S. (1993). Situated cognition and cognitive apprenticeship: Implications for classroom learning. *Studies in Science Education*, 22, 1-41.

Herrington, A. J. (1990). Strategies for developing mathematical understandings. In K. Milton & H. McCann (Eds.), *Mathematical turning points: Strategies for the 1990s* (pp. 325-340). Hobart, Tasmania: Australian Association of Mathematics Teachers.

Herrington, A. J. (1992). *Student beliefs and strategies for learning mathematics.* Unpublished PhD thesis. The University of Western Australia, Perth.

Herrington, T. Wong, K., & Kershaw, L. (1994). *Fostering mathematical thinking and learning.* Adelaide: Australian Association of Mathematics Teachers Inc.

Hidi, S. (1990). Interest and its contribution as a mental resource for learning. *Review of Educational Research*, 60, 549-571.

Hiebert, J. (1992). Reflection and communication: Cognitive considerations in school mathematics reform. *International Journal of Educational Research,* 17, 439-456.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65-97) New York: Macmillan.

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis, In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1-27). Hillsdale, NJ: Lawrence Erlbaum Associates.

Kardash, C. M. & Amlund, J. T. (1991). Self-reported learning strategies and learning from expository text. *Contemporary Educational Psychology*, 16, 117-138.

Kelly, G. A. (1955). *The psychology of personal constructs*. New York: Norton.

Kilpatrick, J. (1985). Reflection and recursion, *Educational studies in mathematics*, 16, 1-36.

King, A. (1991). Effects of training in strategic questioning on children's problem-solving performance, *Journal of Educational Psychology*, 83, 307-317.

Knight, G. (1992). "A formula is a leather armchair": metaphor and mathematics education. In B. Bell *et al.* (Eds.), *SAMEPapers 1992*. Hamilton: Centre for Science and Mathematics Education Research, University of Waikato.

Kyriacou, C., & Marshall, S. (1989). The nature of active learning in secondary schools. *Evaluation and Research in Education*, 3(1), 1-5.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowledge and teaching. *American Educational Research Journal*, 27, 29-63.

Lawson, M. J., & Chinnappan, M. (1994). Generative activity during geometry problem solving: Comparison of the performance of high-achieving and low-achieving high school students. *Cognition and Instruction*, 12(1), 61-93.

Leder, G. C., & Gunstone, R. F. (1990). Perspectives on mathematics learning. *International Journal of Educational Research*, 14(2), 105-120.

Leinhardt, G. (1988). Getting to know: Tracing students' mathematical knowledge from intuition to competence, *Educational Psychologist*, 23(2), 119-144.

Leinhardt, G., & Putnam, R. T. (1987). The skill of learning from classroom lessons. *American Educational Research Journal*, 24, 557-587.

Lester, F. K. (1988). Reflections about mathematical problem-solving research. In R. I. Charles & E. A. Silver (Eds.), *The teaching and assessing of mathematical problem solving* (pp. 115-124). Reston, VA: National Council of Teachers of Mathematics.

Lester, F. K., Garofalo, J., & Kroll, D. L. (1989). Self-confidence, interest, beliefs, and metacognition: Key influences on problem-solving behaviour. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem-solving* (pp. 75-88). New York: Springer-Verlag.

Levin, J. R. (1986). Four cognitive principles of learning-strategy instruction. *Educational Psychologist*, 21, 3-17.

Levin, J. R. (1988). Elaboration-based learning strategies: Powerful theory = Powerful application. *Contemporary Educational Psychology*, 13, 191-205.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Beverly Hills, California: Sage Publications.

Lorch, R. F., Jr., Lorch, E. P., & Klusewitz, M. A. (1993). College students' conditional knowledge about reading. *Journal of Educational Psychology*, 85(2), 239-252.

Marland, P. W. & Edwards, J. (1986). Students' in-class thinking. *Instructional Science*, 15, 75-88.

Marland, P., Patching, W., & Putt, I. (1992a). Thinking while studying: A process tracing studying of distance learners. *Distance Education*, 13(2), 193-217.

Marland, P. W., Patching, W., & Putt, I. (1992b). *Learning from text: Glimpses inside the minds of distance learners*. Townsville: James Cook University of North Queensland.

Marton, F., & Neuman, D. (1992). *Constructivism and phenomenography.* Paper presented at the International Congress of Mathematics Education-7, Quebec.

Marx, R. W., & Walsh, J. (1988). Learning from academic tasks. *The Elementary School Journal*, 88(3), 207-219.

Mason, J. (1988). Learning and doing mathematics. London: Macmillan.

Mathison, S. (1988). Why triangulate? *Educational Researcher*, 17(2), 13-17.

Mayer, R. E. (1992). Cognition and instruction: Their historic meeting within educational psychology. *Journal of Educational Psychology*, 84, 405-412.

McCombs, B. L. (1988). Motivational skills training: Combining metacognitive, cognitive, and affective learning strategies. In C. E. Weinstein, E. T. Goetz, & P. A. Alexander (Eds.), *Learning and study strategies: issues in assessment, instruction, and evaluation* (pp. 141-169). San Diego: Academic Press.

McKeachie, W. J., Pintrich, P. R., & Lin, Y. (1985). Teaching learning strategies. *Educational Psychologist*, 20, 153-160.

McLaughlin, J. W. (1985). *Sixth form mathematics: Notes and examples.* Auckland

McLeod, D. B. (1989). The role of affect in mathematical problem solving. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 20-36). NY: Springer-Verlag.

McLeod, D. B. (1991). Research on learning and instruction in mathematics: The role of affect. In E. Fennema, T. P. Carpenter, & S. Lamon (Eds,), *Integrating research on teaching and learning mathematics* (pp. 55-82). Madison, WI: Wisconsin Centre of Educational Research, University of Wisconsin.

McMillan, J. H., & Schumacher, S. (1993). *Research in education: a conceptual introduction.* New York: Harper Collins College Publishers.

Miles, M. B. (1990). New methods for qualitative data collection and analysis: vignettes and pre-structured cases. *Qualitative Studies in Education*, 3(1), 37-51.

Miles, M. B., & Huberman, A. M. (1984). *Qualitative data analysis: a sourcebook of new methods.* Beverly Hills: Sage Publications.

Ministry of Education (1992). *Mathematics in the New Zealand Curriculum.* Wellington: Learning Media.

Ministry of Education (1993). *The New Zealand Curriculum Framework.* Wellington: Learning Media.

Ministry of Education (1994). *The New Zealand Education Gazette*, 73(6). Wellington: Ministry of Education.

Mitchell, I (1992a). The class level. In J. R. Baird & Northfield (Eds.), *Learning from the PEEL experience* (pp. 61-104). Melbourne: Monash University Printing Services.

Mitchell, I. (1992b). A perspective on teaching and learning. In J. R. Baird & Northfield (Eds.), *Learning from the PEEL experience* (pp. 178-193). Melbourne: Monash University Printing Services.

Mitchell, I. (1992c). *Bringing constructivism into the classroom* (sound recording). Paper presented at the Annual meeting of the American Educational Research Association, San Francisco.

Muralidhar, S. (1993). The role of multiple data sources in interpretive science education research. *International Journal of Science Education*, 15(4), 445-455.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Newman, R. S. (1991). Goals and self-regulated learning: What motivates children to seek academic help? In M. L. Maehr & P. R. Pintrich (Eds.), *Advances in motivation and achievement, Vol, 7* (pp. 151-183). Greenwich, Connecticut: JAI Press.

Newman, R. S., Schwager, (1992). Student perceptions and academic help-seeking. In D. H. Schunk & J. L. Meece (Eds.), *Student perceptions in the classroom* (pp. 123-146). Hillsdale, NJ: Lawrence Erlbaum Associates.

Neyland J. (1994a). Introduction. In J. Neyland (Ed.), *Mathematics education: A handbook for teachers, Vol. 1* (pp. 1-5). Wellington: Wellington College of Education.

Neyland J. (1994b). Problems with the proposed unit standard framework for mathematics. *The New Zealand Mathematics Magazine*, 31(1), 1-9.

Nickerson, R. S. (1989). On improving thinking through instruction. *Review of Research in Education,* 15, 3-57.

Nisbet, J., & Shucksmith, J. (1986). *Learning strategies.* London: Routledge and Kegan Paul.

Noddings, N. (1990). Constructivism in mathematics education. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 7-18). Reston, VA: National Council of Teachers of Mathematics.

Noddings, N. (1993). Constructivism and caring. In R. Davis & C. Maher (Eds.), *Schools, mathematics, and the world of reality* (pp. 35-50). Boston: Allyn and Bacon.

Nolen, S. (1988). Reasons for studying: Motivational orientations and study strategies. *Cognition and Instruction*, 5, 269-287.

Nolen, S., & Haladyna, T. M. (1990). Motivational and studying in high school science. *Journal of research in Science Teaching*, 27(2), 115-126.

Nunan, D. (1991). *Language teaching methodology: A textbook for teachers.* Englewood Cliffs, NJ: Prentice-Hall.

O'Malley, J. M., & Chamot, A. M. (1990). *Learning strategies in second language acquisition.* Cambridge: Cambridge University Press.

Owen, E., & Sweller, J. (1985). What do students learn while solving mathematics problems? *Journal of Educational Psychology*, 77, 272-284.

Oxford, R. L. (1990). *Language learning strategies.* Boston: Heinle & Heinle Publishers.

Palincsar, A. S., & Brown A. L. (1984) Reciprocal teaching of comprehension-fostering and comprehension-monitoring activities. *Cognition and Instruction*, 1(2), 117-175.

Palmer, D. J., & Goetz, E. T. (1988). Selection and use of study strategies: The role of the studier's beliefs about self and strategies. In C. E. Weinstein, E. T. Goetz, & P. A. Alexander (Eds.), *Learning and study strategies: Issues in assessment, instruction, and evaluation* (pp. 41-61). San Diego: Academic Press.

Paris, S. G., Lipson, M. Y., & Wilson, K. K. (1983). Becoming a strategic reader, *Contemporary Educational Psychology*, 8, 293-316.

Paris, S. G., & Winograd, P. (1990).How metacognition can promote academic learning and instruction, In B. F. Jones & L. Idol (Eds.), *Dimensions of thinking and cognitive instruction.* (pp. 15-51). Hillsdale, NJ: Lawrence Erlbaum Associates.

Perkins, D. N. (1991). What constructivism demands of the learner. *Educational Technology*, 31(9), 19-21.

Perkins, D. N., & Salomon, G. (1989). Are cognitive skills context-bound? *Educational Researcher*, 18(1), 16-25.

Peshkin, A. (1993). The goodness of qualitative research. *Educational Researcher*, 22(2), 23-29.

Peterson, P. L. (1988). Teaching for higher-order thinking in mathematics: The challenge for the next decade. In D. A. Grouws & T. J. Cooney (Eds.), *Research agenda for mathematics education: Perspective on research on effective mathematics teaching* (pp. 2-26). Reston, VA: National Council of Teachers of Mathematics.

Peterson, P. L., & Swing, S. R. (1982). Beyond time on task: Students' reports of their thought processes during classroom instruction. *The Elementary School Journal*, 82, 481-491.

Peterson, P. L., & Swing, S. R. (1983). Problems in classroom implementation of cognitive strategy instruction. In G. M. Pressley & J. R. Levin (Eds.), *Cognitive strategy research: Educational applications* (pp. 267-287). New York: Springer-Verlag.

Peterson, P. L., Swing, S. R., Braverman, M. T., & Buss, R. (1982). Students' aptitudes and their reports of cognitive processes during direct instruction. *Journal of Educational Psychology*, 74, 535-547.

Peterson, P. L., Swing, S. R., Stark, K. D., & Waas, G. A. (1984). Students' cognitions and time on task during mathematics instruction. *American Educational Research Journal*, 21, 487-515.

Peverly, S.T. (1991). Problems with the knowledge-based explanation of memory and development. *Review of Educational Research*, 61(1), 71-93.

Pintrich, P. R., & De Groot, E. V. (1990). Motivational and self-regulated learning components of classroom academic performance. *Journal of Educational Psychology*, 82(1), 33-40.

Pintrich, P. R., Marx, R. W., & Boyle, R. A. (1993). Beyond cold conceptual change: The role of motivational beliefs and classroom contextual factors in the process of conceptual change. *Review of Educational Research*, 63(2), 167-199.

Pintrich, P. R., & Schrauben, B. (1992). Students' motivational beliefs and their cognitive engagement in classroom academic tasks. In D. Schunk & J. Meece (Eds.), *Student perceptions in the classroom: Causes and consequences* (pp. 149-183). Hillsdale, NJ: Erlbaum.

Pokay, P., & Blumenfeld, P. (1990). Predicting achievement early and late in the semester: The role of motivation and use of learning strategies. *Journal of Educational Psychology*, 82(1), 41-50.

Polya, G. (1957). *How to solve it* (2nd Ed.). Princeton, NJ: Princeton University Press.

Prawat, R. S. (1989). Promoting access to knowledge, strategy, and disposition in students: A research synthesis. *Review of educational Research*, 59(1), 1-41.

Pressley, M. (1986). The relevance of the good strategy user model to the teaching of mathematics. *Educational Psychologist*, 21(1 & 2), 139-161.

Pressley, M., Borkowski, J. G., & Schneider, W. (1987). Cognitive strategies: Good strategy users coordinate metacognition, and knowledge. In R. Vasta & G. Whitehurst (Eds.), *Annals of child development* (Vol. 4, pp. 89-129). New York: JAI.

Pressley, M., Borkowski, J. G., & Schneider, W. (1989). Good information processing: What it is and how education can promote it. *International Journal of Educational Research*, 13, 857-897.

Pressley, M., Goodchild, F., Fleet, J., Zajchowski, R., & Evans, E. (1989). The challenges of classroom strategy instruction. *The Elementary School Journal*, 89, 301-342.

Pressley, M., Woloshyn, V., Lysynchuck, L. M., Martin, V., Wood, E., & Willoughby, T. (1990). A primer of research on cognitive strategy instruction: The important issues and how to address them. *Educational Psychology Review*, 2(1), 1-58.

Putnam, R. T., Lampert, M., & Peterson, P. L. (1990). Alternative perspectives in knowing mathematics in elementary schools. *Review of Research in Education*, 16, 57-150.

Ramsden, P. (1988). Context and strategy: Situational influences on learning, In R. R. Scmeck (Ed.), *Learning strategies and learning styles* (pp. 159-184). New York: Plenum Press.

Resnick, L. B. (1987). *Education and learning to think.* Washington DC: National Academy Press.

Resnick, L. B. (1988). Treating mathematics as an ill-structured discipline. In R. I. Charles & E. A. Silver (Eds.), *The teaching and assessing of mathematical problem solving* (pp. 32-60). Reston, VA: NCTM.

Resnick, L. B. (1989a). Developing mathematical knowledge. *American Psychologist*, 44(2), 162-169.

Resnick, L. B. (1989b). Introduction. In L. B. Resnick (Ed.), *Knowing, learning, and instruction: Essays in honor of Robert Glaser* (pp. 1-24). Hillsdale, NJ: Lawrence Erlbaum Associates.

Ritchie, G., & Carr, K. (1992). A constructivist critique of mastery learning in mathematics. *New Zealand Journal of Educational Studies,* 27(2), 191-201.

Robitaille, D. F., & Garden, R. A. (1989). *The IEA Study of Mathematics II: Contexts and outcomes of school mathematics.* Oxford: Pergamon.

Rohrkemper, M., & Corno, L. (1988). Success and failure on classroom tasks: Adaptive learning and classroom teaching. *Elementary School Journal*, 88(3), 297-312.

Rohwer, W. D., & Thomas, J. W. (1989). Domain specific knowledge, metacognition, and the promise of instructional reform. In C. B. McCormick, G. E. Miller, & M. Pressley (Eds.), *Cognitive strategy research* (pp. 104-132). New York: Springer-Verlag.

Romberg, T. (1992). Mathematics learning and teaching: What we have learned in ten years, In C. Collins & J. N. Mangieri (Eds.), *Teaching thinking: An agenda for the twenty-first century* (pp. 42-64). Hillsdale, NJ: Lawrence Erlbaum Associates.

Schiefele, U. (1991). Interest, learning, and motivation. *Educational Psychologist,* 26(3 & 4), 299-323.

Schmeck, R. R. (1988). An introduction to strategies and styles of learning. In R. R. Schmeck (Ed.), *Learning strategies and learning styles* (pp. 3-19). NY: Plenum Press.

Schoenfeld, A. H. (1985). *Mathematical problem solving.* New York, FL: Academic Press.

Schoenfeld, A. H. (1987). What's all the fuss about metacognition? In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 189-215). Hillsdale, NJ: Lawrence Erlbaum Associates.

Schoenfeld, A. H. (1988). When good teaching leads to bad results: The disasters of "well-taught" mathematics courses. *Educational Psychologist*, 23(2), 145-166.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). NY: Macmillan.

Short, E. J., & Weissberg-Benchell, J. A. (1989). The triple alliance for learning: Cognition, metacognition, and motivation, In C. B. McCormick, G. Miller, & Pressley (Eds.), *Cognitive strategy research: From basic research to educational applications* (pp. 33-63). NY: Springer-Verlag.

Shuell, T. J. (1986). Cognitive conceptions of learning. *Review of Educational Research*, 56(4), 411-436.

Shuell, T. J. (1988). The role of the student in learning from instruction. *Contemporary Educational Psychology*, 13, 276-295.

Siemon, D. (1992a). *Children's approach to mathematical problem solving.* A paper presented at the Annual Conference of the Mathematics Education research Group of Australasia, Sydney, Australia.

Siemon, D. (1992b). *Cultural influences on children's approaches to school.* Paper presented at the International Congress of Mathematics Education-7, Quebec, Canada.

Silver, E. A. (1979). Student perceptions of relatedness among mathematical verbal problems. *Journal for Research in Mathematics Education*, 10, 195-210.

Silver, E. A. (1985). Research on teaching mathematical problem solving: Some underrepresented themes and needed directions. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 247-266). Hillsdale, NJ: Lawrence Erlbaum Associates.

Slife, B. D., Weiss, J., & Bell, T. (1985). Separability of metacognition and cognition: Problem solving in learning disabled and regular students. *Journal of Educational Psychology*, 77(4), 437-445.

Southwell, B., & Khamis, M. (1991). *Beliefs about mathematics and mathematics education.* Paper presented at the Annual conference of the Australian Association for Research in Education, Gold Coast, Queensland.

Stanic, G., & Kilpatrick, J. (1989). Historical perspectives on problem solving in the mathematics curriculum. In R. Charles & E. Silver (Eds.), *The teaching and assessing of mathematical problem solving* (pp. 1-22). Reston, VA: National Council of Teachers of Mathematics.

Sternberg, R. J. (1985). *Beyond IQ: A triarchic theory of human intelligence.* Cambridge, London: Cambridge University Press.

Swanson, H. (1990). Influences of metacognitive knowledge and aptitude on problem solving. *Journal of Educational Psychology*, 82, 306-314.

Swing, S. R., & Peterson, P. L. (1988). Elaborative and integrative thought processes in mathematics learning. *Journal of Educational Psychology*, 80, 54-66.

Swing, S. R., Stoiber, K. C., & Peterson, P. L. (1988). Thinking skills versus learning time: Effects of alternative classroom-based interventions on students' mathematics problem solving. *Cognition and Instruction*, 5, 123-191.

Thomas, J. W. (1988). Proficiency at academic studying. *Contemporary Educational Psychology*, 13, 265-275.

Thomas, J. W., & Rohwer, W. D. (1986). Academic studying: The role of learning strategies, *Educational Psychologist*, 21(1 & 2), 19-41.

Thomas, J., & Rohwer, W. D. (1987) Grade-level and course-specific differences in academic studying: Summary. *Contemporary Educational Psychology*, 12, 381-385.

Thomas, J. W., & Rohwer, W. D. (1993). Proficient autonomous learning: Problems and prospects. In M. Rabinowitz (Ed.), Cognitive science foundations of instruction (pp. 1-32). NJ: Lawrence Erlbaum Associates.

Tobin, K., & Imwold, D. (1993). The mediational role of constraints in the reform of mathematics curricula. In J. A. Malone & P. Taylor (Eds.), *Constructivist interpretations of teaching and learning mathematics* (pp. 15-34). Perth: Curtin University of Technology.

Toumasis, C. (1993). Fostering auto-learning abilities in mathematics: A Greek experience. *International Journal of Mathematics Education in Science and Technology,* 24(4), 553-567.

Van Haneghan, J. P., & Baker, L. (1989). Cognitive monitoring in mathematics. In C. B. McCormick, G. Miller, & M. Pressley (Eds.), *Cognitive strategy research: From basic research to educational applications* (pp. 215-238). NY: Springer-Verlag.

Voigt, J. (1994). Negotiation of mathematical meaning and learning mathematics. *Educational Studies in Mathematics,* 26, 275-298.

Volet, S. E., & Lawrence J. A. (1989). Goals in the adaptive learning of university students. In H. Mandl, E. De Corte, N. Bennett, & H. F. Friedrich (Eds.), *Learning and instruction:: European research in and international context* (Vol. 2-3, pp. 497-516).Oxford: Pergamon.

Waldrop, M. M. (1992). *Complexity: The emerging science at the edge of order and chaos.* New York: Simon & Schuster.

Wang, M. C., Haertel G. D., & Walberg, H. J. (1993). Toward a knowledge base for school learning. *Review of Educational Research*, 63, 249-294.

Wang, M. C., & Peverly, S. T. (1986). The self-instructive process in classroom learning contexts. *Contemporary Educational Psychology*, 11, 370-404.

Warwick, D. P. (1973). Survey research and participant observation: A benefit cost analysis. In D. P. Warwick & S. Osherson (Eds.), *Comparative research methods* (pp. 189-302). Englewood Cliffs, NJ: Prentice-Hall.

Weinstein, C. E. (1988). Assessment and training of student learning strategies, In R. Schmeck (Ed.), *Learning strategies and learning styles* (pp. 291-316). New York: Plenum Press.

Weinstein, C. E., & Mayer, R. E. (1986). The teaching of learning strategies. In M. E. Wittrock (Ed.), *Handbook of research on teaching* (pp. 315-327). NY: Macmillan.

Weinstein, C. E., & Palmer, D. R. (1990a). *LASSI-HS (Learning and Study Strategies Inventory-High School Version).* Clearwater, FL: H & L Publishers.

Weinstein, C. E., & Palmer, D. R. (1990b). *LASSI-HS User's Manual.* Clearwater, FL: H & L Publishers.

Weinstein, C. E., Zimmerman, S. A., & Palmer, D. R. (1988). Assessing learning strategies: The design and development of the LASSI. In C. E. Weinstein, E. T. Goetz, & P. A. Alexander (Eds.), *Learning and study strategies: Issues in assessment, instruction and evaluation* (pp. 25-40). San Diego: Academic Press.

Wheatley, G. H. (1991). Constructivist perspectives on science and mathematics learning. *Science Education*, 75(1), 9-21.

Wheatley, G. H.(1992). The role of reflection in mathematics learning. *Educational Studies in Mathematics*, 23, 529-541.

White, C. (1993). *Metacognitive, cognitive, social and affective strategy use in foreign language learning: a comparative study.* Unpublished PhD thesis, Massey University, New Zealand.

Wiersma, W. (1991). *Research methods in education.* Boston: Allyn and Bacon.

Winne, P. H., & Marx, R. W. (1982). Students' and teachers' views of thinking processes for classroom learning. *The Elementary School Journal*, 82(5), 492-518.

Wong, K. Y. (1990). How secondary school students learn mathematics. In K. Milton & H. McCann (Eds.), *Mathematical turning points: Strategies for the 1990s* (pp. 566-572). Hobart, Tasmania: Australian Association of Mathematics Teachers.

Wong, K. Y., & Herrington, A. (1992). Research in learning strategies in mathematics. In B. Atweh, & J. Watson (Eds.), *Mathematics education in Australasia* (pp. 129-142). Queensland University of Technology.

Yates, G. C., & Chandler, M. (1991). The cognitive psychology of knowledge: Basic research findings and educational implications. *Australian Journal of Education*, 35(2), 131-153.

Zimmerman, W. & Cunningham, S. (Eds.) (1991). *Visualization in teaching and learning mathematics.* The Mathematical Association of America.

Zimmerman, B., & Martinez-Pons, M. (1986). Development of a structured interview for assessing student use of self-regulated learning strategies. *American Educational Research Journal*, 23, 614-628.

Zimmerman, B., & Martinez-Pons, M. (1988).Construct validation of a strategy model of student self-regulated learning. *Journal of Educational Psychology*, 80, 284-290.