

Copyright is owned by the Author of the thesis. Permission is given for a copy to be downloaded by an individual for the purpose of research and private study only. The thesis may not be reproduced elsewhere without the permission of the Author.

**MODELLING OF THE DRYING SECTION OF  
A CONTINUOUS PAPER MACHINE**

**A thesis presented in partial fulfilment of the requirement  
for the degree of Master in Production Technology at  
Massey University**

**HUITING MA, B.E. (East China Chem. Tech.)**

## SUMMARY

This thesis sought to develop a suitable dynamical model of the PM3 paper machine drying section at Tasman Pulp and Paper Ltd. which could be used for subsequent control system simulations.

An extensive literature search uncovered only two suitable mathematical models, that of Lemaitre *et al.* (1980) and Knight and Kirk (1975). Both models were simulated but only the Lemaitre's model was capable of reaching the observed data by suitable choice of parameters, and the Knight and Kirk's model was abandoned.

The Lemaitre's model was fitted to steady state data from the PM3 machine by finding empirical or theoretical values for some parameters and optimizing the remaining ones.

The model gave reasonable responses in simulations under varying conditions. It predicted:

1. an insensitivity to inlet air conditions between -4 and 23°C and 30 and 80% RH,
2. a relative insensitivity to steam temperature in the first section as compared with the final section,
3. that the cylinder (drum) speed could be increased if matched by an increase in steam temperature.

## References

Knight, R. L. and Kirk, L. A., 1975, "Simulation of the Papermachine Drying Section," *International Water Removal Symposium*, British Paper and Board Industries Federation - London (March 1975), pp. 188-226.

Lemaitre, A., Veyre, J., Lebeau, B. and Foulard, C., 1980, "Case Study: Method for Systematic Analysis of Paper Machine Multicylinder Drying Section," *4th IFAC-PRP 4 Conference*, Ghent-Belgium (3-5 June 1980), pp. 261-270.

## **ACKNOWLEDGMENTS**

The author would like to acknowledge the following persons for advice and assistance during the course of this project:

Dr. Huub H. C. Bakker, Department of Production Technology, Massey University, Chief Supervisor;

Dr. Robert I. Chaplin, Department of Production Technology, Massey University, Supervisor.

The author would like to acknowledge the Pulp and Paper Research Organization of New Zealand for the financial support in carrying out this project.

The author would like to acknowledge the Tasman Pulp and Paper Ltd. for providing the data of PM3 paper dryer machine.

The author would like to thank family and friends for their continual support and helpfulness.

The author would also like to thank Zhang for his moral support and for proof reading.

## TABLE OF CONTENTS

<b>SUMMARY .....</b>	<b>ii</b>
<b>ACKNOWLEDGMENTS .....</b>	<b>iii</b>
<b>TABLE OF CONTENTS.....</b>	<b>iv</b>
<b>LIST OF FIGURES.....</b>	<b>viii</b>
<b>LIST OF TABLES.....</b>	<b>x</b>
<b>1. INTRODUCTION.....</b>	<b>1</b>
1.1 Introduction .....	1
1.2 Research Objectives .....	2
1.3 References .....	2
<b>2. LITERATURE REVIEW .....</b>	<b>4</b>
2.1 Physical Description of Paper Drying Section .....	4
2.2 Development of Mathematical Models.....	7
2.3 References .....	22
<b>3. MODEL SELECTION .....</b>	<b>25</b>
3.1 Guideline for Model Selection .....	25
3.2 Model Selection .....	25
3.3 References .....	26
<b>4. FITTING OF MODEL NON-OPTIMISED PARAMETERS .....</b>	<b>29</b>
4.1 Selection of Non-optimised Parameters .....	29
4.2 Values of Non-optimised Parameters.....	29
4.2.1 Calculation of $t_I$ , $t_{II}$ , $t_{III}$ and $t_{IV}$ .....	31
4.2.2 Formula for Saturated Vapour Pressure.....	32
4.2.3 Formula for Specific Heat of Water .....	34
4.2.4 Formula for Latent Heat of Vaporisation.....	35
4.3 References .....	36
<b>5. OPTIMISATION OF MODEL PARAMETERS .....</b>	<b>37</b>

5.1	Choice of Performance Indices .....	37
5.1.1	Performance Index for Paper Temperature .....	37
5.1.2	Performance Index for Paper Moisture Content .....	38
5.2	Optimisation of Parameters .....	39
5.2.1	Programming of the Models .....	41
5.2.1.1	Programming of the Knight and Kirk Model .....	42
5.2.1.2	Programming of the Lemaitre Model .....	44
5.2.2	Program for Evaluating Performance Indices .....	45
5.2.3	Program for Optimizing Parameters.....	45
5.2.4	Main Program .....	46
5.2.5	Results and Discussion .....	47
5.3	References .....	54
<b>6.</b>	<b>SENSITIVITY TESTS OF THE MODEL .....</b>	<b>56</b>
6.1	Programming of Simulation Model .....	56
6.2	Change of Steam Temperature .....	56
6.3	Change of Drum Speed .....	57
6.4	Change of Paper Initial Condition.....	58
6.5	Change of Ambient Air Conditions.....	59
6.6	Discussion.....	61
6.7	Reference.....	62
<b>7.</b>	<b>CONCLUSIONS.....</b>	<b>63</b>
<b>8.</b>	<b>RECOMMENDATIONS .....</b>	<b>65</b>
	<b>NOMENCLATURE.....</b>	<b>66</b>
	<b>APPENDIX 4.2-1: CALCULATION OF <math>t_I</math>, <math>t_{II}</math>, <math>t_{III}</math> AND <math>t_{IV}</math>.....</b>	<b>I</b>
	<b>APPENDIX 4.2-2: PROGRAM OF THE FORMULA FOR SATURATED VAPOUR.....</b>	<b>V</b>
	<b>APPENDIX 4.2-3: PROGRAM OF THE FORMULA FOR SPECIFIC HEAT OF WATER.....</b>	<b>VI</b>

**APPENDIX 4.2-4: PROGRAM OF THE FORMULA FOR LATENT HEAT OF VAPORISATION..... VII**

**APPENDIX 5.2-1: PROGRAM AND PROGRAM BLOCK DIAGRAM OF THE KNIGHT AND KIRK'S MODEL..... VIII**

**APPENDIX 5.2-2: PROGRAM AND PROGRAM BLOCK DIAGRAM OF THE LEMAITRE *ET AL.*'S MODEL.....XIII**

**APPENDIX 5.2-3: PROGRAM FOR EVALUATING PERFORMANCE INDICES ..... XVI**

**APPENDIX 5.2-4: PROGRAM FOR OPTIMIZING PARAMETERS .....XVII**

**APPENDIX 5.2-5: MAIN PROGRAM..... XIX**

**APPENDIX 5.2-6: PROGRAM OF THE LEMAITRE'S MODEL AND MAIN PROGRAM FOR THE OPTIMIZED PARAMETERS OF THE 49 CYLINDERS..... XXI**

**APPENDIX 5.2-7: THE NUMERICAL RESULTS FOR THE OPTIMIZED PARAMETERS OF THE 49 CYLINDERS ..... XXIII**

**APPENDIX 6.1-1: PROGRAM OF THE LEMAITRE *ET AL.*'S MODEL FOR DRUM 1-49 ..... XXX**

**APPENDIX 6.1-2: PROGRAM FOR RUNNING AJ3.M (PROGRAM OF THE LEMAITRE *ET AL.*'S MODEL FOR DRUM 1-49)..... XXXV**

**APPENDIX 6.2-1: TABLES FOR CHAPTER 6.....XXXVI**

**LIST OF FIGURES**

Figure 2.1-1	A typical dryer section (Polat and Mujumdar 1987)	4
Figure 2.1-2	Condensate behaviour in a dryer (Chance 1989)	5
Figure 2.1-3	Resistances to heat transfer on a cylinder dryer (Polat and Mujumdar 1987)	6
Figure 2.1-4	Conventional drying curve (Sharma 1989)	7
Figure 2.2-1	Nissan's nomenclature for four phase of drying (Kirk 1984)	8
Figure 4.2-1	Saturated vapour pressure against temperature	33
Figure 4.2-2	Specific heat of water against temperature	34
Figure 4.2-3	Latent heat of vaporisation against temperature	35
Figure 5.1-1	Expected Profile for Paper Temperature	37
Figure 5.1-2	Expected Profile for Paper Moisture Content	39
Figure 5.2-1	Model Parameters Optimization Procedure	40
Figure 5.2-2	Schematic Block Diagram of the Knight and Kirk's Model	42
Figure 5.2-3	Paper Temperature and Moisture Content at First Cylinder for the Knight and Kirk's Model	44
Figure 5.2-4	Schematic Block Diagram of the Lemaitre's model	45
Figure 5.2-5	Heat and Mass Transfer Coefficients in Drum Phase, Critical Moisture Content and Affinity Coefficient Against Iteration Number	47
Figure 5.2-6	Heat and Mass Transfer Coefficients in Draw Phase and Performance Indices for Temperature and Moisture Against Iteration Number	48

Figure 5.2-7	Paper Temperature and Moisture Content in First Cylinder	48
Figure 5.2-8	Heat Transfer Coefficient in Drum Phase Against Cylinder Number	49
Figure 5.2-9	Mass Transfer Coefficient in Drum Phase Against Cylinder Number	50
Figure 5.2-10	Critical Moisture Content Against Cylinder Number	50
Figure 5.2-11	Affinity Coefficient of the Water for the Sheet Against Cylinder Number	51
Figure 5.2-12	Heat Transfer Coefficient in Draw Phase Against Cylinder Number	51
Figure 5.2-13	Mass Transfer Coefficient in Draw Phase Against Cylinder Number	52
Figure 5.2-14	Performance Indices for Paper Temperature and Moisture Content Against Cylinder Number	53
Figure 5.2-15	Paper Temperature and Moisture Content Against Cylinder Number	54
Figure 6.2-1	Final Paper Moisture Content Against Different Steam Temperature	57
Figure 6.3-1	Final Paper Moisture Content Against Different Drum Speed	58
Figure 6.3-2	Steam Temperature Against Different Dryer Section	58
Figure 6.4-1	Final Paper Moisture Content Against Initial Paper Temperature and Moisture Content	59
Figure 6.5-1	Final Paper Moisture Content Against Air Conditions Over the Cloth	60
Figure 6.5-2	Final Paper Moisture Content Against Air Conditions in the Pockets	60
Figure 2.2-1	Typical dryer arrangement (Drums 1-13)	I
Figure 4.2-2	Typical dryer arrangement (Drums 14-49)	III

**LIST OF TABLES**

Table 4.2-1	Temp. and Vapour Pressure of Pocket Air for Each Cylinder	30
Table 4.2-2	Length and Time for Each Phase of Drying	32
Table 6.2-1	Final Paper Moisture Content Against Different Steam Temperature	XXXVI
Table 6.3-1	Final Paper Moisture Content Against Different Drum Speed	XXXVII
Table 6.3-2	Final Paper Moisture Content Against Drum Speed and Steam Temp.	XXXVII
Table 6.4-1	Final Paper Moisture Content Against Initial Paper Temperature and Moisture Content	XXXVII
Table 6.5-1	Final Paper Moisture Content Against Partial Vapour Pressure Over the Cloth	XXXVIII
Table 6.5-2	Temp. and Vapour Pressure of Pocket Air for Each Cylinder	XXXVIII
Table 6.5-3	Final Paper Moisture Content Against Temp. and Vapour Pressure of Pocket Air	XLII

## 1. INTRODUCTION

### 1.1 Introduction

The invention of paper in 105 A.D. was a milestone in the history of civilization and demand for paper has been increasing steadily ever since. Although it has become more and more popular to store, process and transfer information in electronic forms, paper is to date still the most common means for recording information.

According to Storat (1993), production in the last twenty years has increased by more than 60 percent, while capital expenditures in the industry have grown to almost 12 percent of sales, or double the average expenditures of other manufacturing industries. This capital investment has gone towards capacity expansion and extensive rebuilds of existing mills - almost 60 percent of the existing capacity comes from modern facilities containing machines either newly installed or rebuilt in the past ten years. As a result, fossil fuel and energy consumption in this industry fell by 46 percent in the last two decades.

However, even with modern technology, the pulp and paper industry remains extremely energy intensive. It ranks fourth in total energy consumption by U.S. manufacturers, behind chemicals, steel and petroleum. The paper industry is the leading industry in total energy used for drying operations (Salama *et al.* 1987). Within the paper industry the drying process is the major consumer of energy. The dryer section of a typical paper machine removes water, via evaporation, to less than 0.4 percent of that originally present in the fiber/water suspension, and requires 7.5-14.5 MJ/kg of energy per ton of paper produced. This accounts for nearly one-third of the total energy requirement of an integrated mill and an even greater fraction of the energy used in a nonintegrated mill (Chiogioji 1979). Therefore, significant scope exists to reduce total mill energy consumption through reductions in the energy expended within the paper machine dryer sections.

Moreover the drying section has a strong influence on the quality of the paper. Paper drying is the last part of the water removal phase and the one in which most of its mechanical properties are conferred to the paper sheet. Reducing the moisture content variations of the sheet at the end of the machine will improve the quality of the paper.

## 1.2 Research Objectives

Although both the energy consumption and investment cost required by the paper drying process are high, the design of the drying section has so far largely been based on empirical rules and the optimum performances, as well as the corresponding working conditions, are not very well known. Improving the drying section running condition is of very great interest and this subject has been much studied (Seyed-Yagoobi *et al.* 1992b), but few studies have been done to develop methods for systematic dynamic analysis of paper machine drying sections. The present research has been performed in this area.

The conventional multicylinder (drum) dryer is by far the most common in the U.S. paper industry, representing 95 percent of the dryers in paper and board applications and over 82 percent of all pulp and paper dryers (McConnell 1980). It is also the most common in the New Zealand paper industry, and it provides the basis for this research.

The overall objectives of this thesis are to:

- (1) perform a literature search to find dynamical, analytical models of conventional multicylinder drying section paper machine,
- (2) select one or more of these models,
- (3) optimize the parameters of the model based on plant data,
- (4) perform a sensitivity analysis to gain insight into the paper machine drying section operation.

The research was carried out on a paper machine at Tasman Pulp and Paper, and is supported by the Pulp and Paper Research Organisation of New Zealand (PAPRONZ).

## 1.3 References

Chiogioji, M. H., 1979, *Industrial Energy Conservation*, Marcel Dekker, New York, NY.

Salama, S. Y., Minsker, B. S., and Olsen, K. G. 1987, "Competitive Position of Natural Gas: Industrial Solids Drying," Arlington, Va.: Energy and Environmental Analysis, Inc.

Storat, R. E., 1993, "The U.S. Pulp, Paper, and Paperboard Industry: A Profile," *TAPPI Journal*, Vol. 76, No. 3, pp. 53-57.

Seyed-Yagoobi, J., Ng, K.H., and Fletcher, L. S. 1992b, "Thermal Conductance of a Bone-Dry Paper Handsheet/Metal Interface," *ASME Journal of Heat Transfer*, Vol. 114, pp. 326-330.

McConnell, R. R. 1980, "A Literature Review of Drying Research in the Pulp and Paper Industry," *Drying '80*, Vol. 2, ed. A. Mujumdar, Washington, D.C.: Hemisphere, pp. 330-337.

*China Encyclopedia* (Chapter: Paper-Making Industry. Vol. Light Industries), China Encyclopedia Publisher, Beijing and Shanghai, 1991, pp. 586

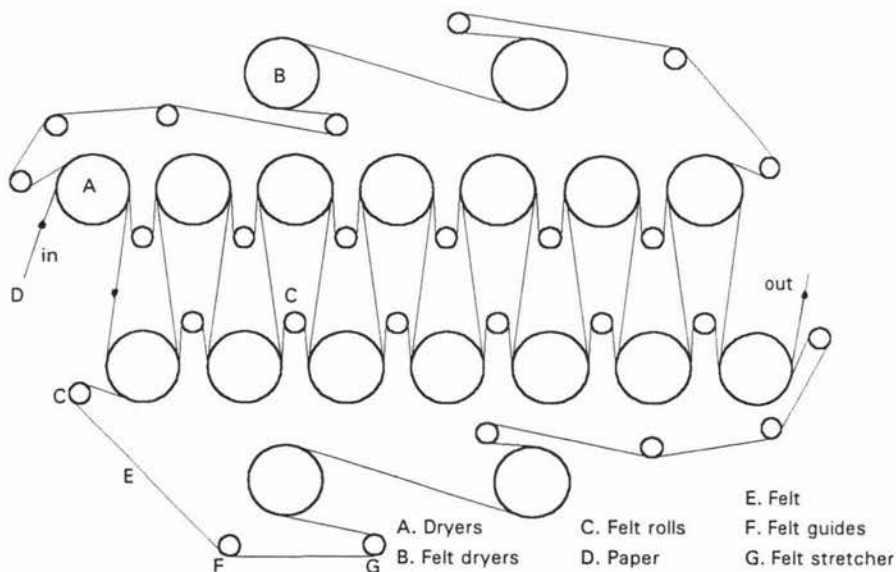
## 2. LITERATURE REVIEW

### 2.1 Physical Description of Paper Drying Section

Paper is most commonly dried in industry by threading a continuous wet web of paper around each of a series of dryer drums which successively dry the paper to an acceptable moisture content. At the entrance to the dryer section, the paper sheet moisture content is about 60% and ranges from 4 to 8% at the exit, depending on the type of drying system and grade of paper (Bell *et al.*, 1992).

The individual dryer drums consist of large, hollow, cast iron cylinders, essentially pressure vessels. The cylinders range from about 1 to 2 m in diameter and up to 10 m in length, with a shell thickness of about 25 mm. The outer surface of the cylinders must be highly finished and free of imperfections to avoid marking the paper. The typical number of cylinders in a dryer section ranges from 40-70, depending on the machine speed and the type of paper produced (Polat and Mujumdar 1987; Bell *et al.*, 1992). Fewer cylinders will be used for light grades of paper such as tissue.

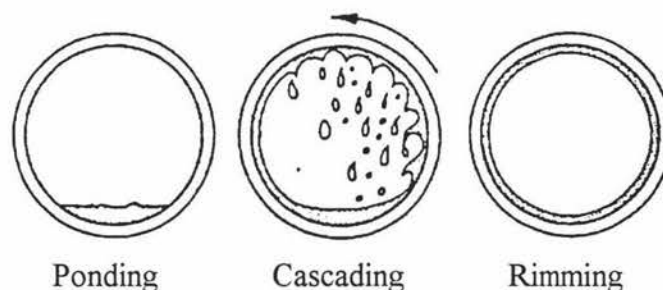
Part of a typical dryer section is shown in Fig. 2.1-1. The dryer felt (clothing) is a highly porous material whose main purpose is to hold the paper sheet in close contact with the dryer shell to increase heat transfer between the paper and the dryer and to help prevent shrinkage and deformation of the paper sheet.



**Figure 2.1-1** A typical dryer section (Polat and Mujumdar 1987)

The energy for the drying process comes from saturated steam injected under pressure into the cylinders. Steam pressures in drum dryers can reach up to 1035kPa (Bell *et al.*, 1992). As the steam condenses on the inside of the dryer shell, the latent heat of vaporization is released. The heat is transferred through the condensate layer and dryer shell to the paper on the outside surface.

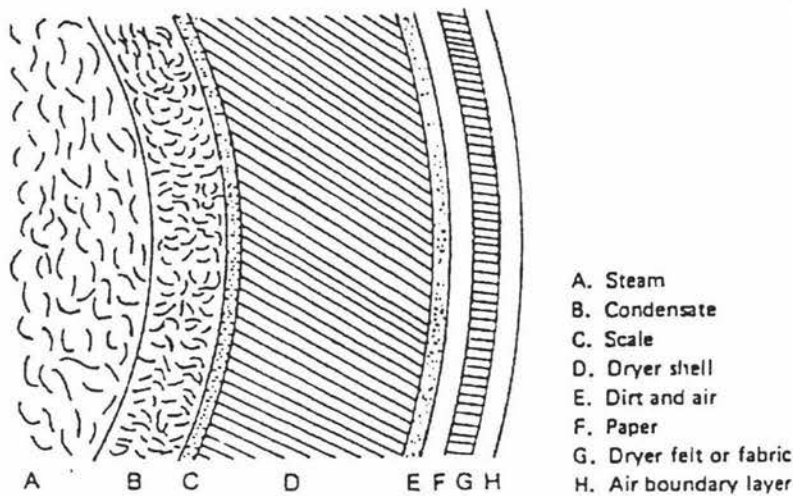
Fig. 2.1-2 shows that the heat transfer through the condensate is dependent on the behaviour of the condensate. At low dryer speeds, the condensate forms a puddle at the bottom of the cylinder, and the steam condenses directly on the inner wall. As the speed increases, a thin film of water is carried up the dryer wall. The bulk of the water remains in a puddle at the bottom of the dryer. The thickness of this film increases with increasing dryer speed. As the thick layer of condensate approaches the top, some of the condensate layer falls back to the bottom of the dryer in a motion called cascading, accompanied by large-scale turbulence and improved heat transfer. As the dryer speed increases, the centrifugal force pushes all the condensate into a relatively uniform layer around the inside of the dryer. This condition, called rimming, has decreased turbulence and heat transfer. At low rimming speeds, the condensate may still "slosh" slightly with respect to the cylinder wall, creating better heat transfer. As the rimming speeds increase, this sloshing effect virtually ceases, and the heat transfer takes place through an almost stagnant water film (Chance 1989).



**Figure 2.1-2** Condensate behaviour in a dryer (Chance 1989)

As shown in Fig. 2.1-3, the heat released by the condensing steam travels through several resistances before reaching the outside air. The heat first travels through the condensate layer, which creates high heat transfer in the ponding or cascading conditions, but low heat transfer in the rimming condition. Rust or scale on the inside dryer wall is the next resistance, followed by the dryer shell. Chemicals from the wet end form a coating buildup on the dryer's outer surface, which with the entrained air between the shell and paper sheet causes a thermal resistance between the dryer and paper. This

thermal or contact resistance is also affected by the finish on the outer surface of the dryer cylinder. A smooth finish gives the paper a greater area of contact with the shell, increasing the heat transfer. The paper sheet creates a resistance to heat transfer that is difficult to calculate because the conditions of the paper such as moisture content, temperature, and thickness constantly change as the paper is dried. The felt and the air boundary layer over the felt are the final resistances to heat transfer.

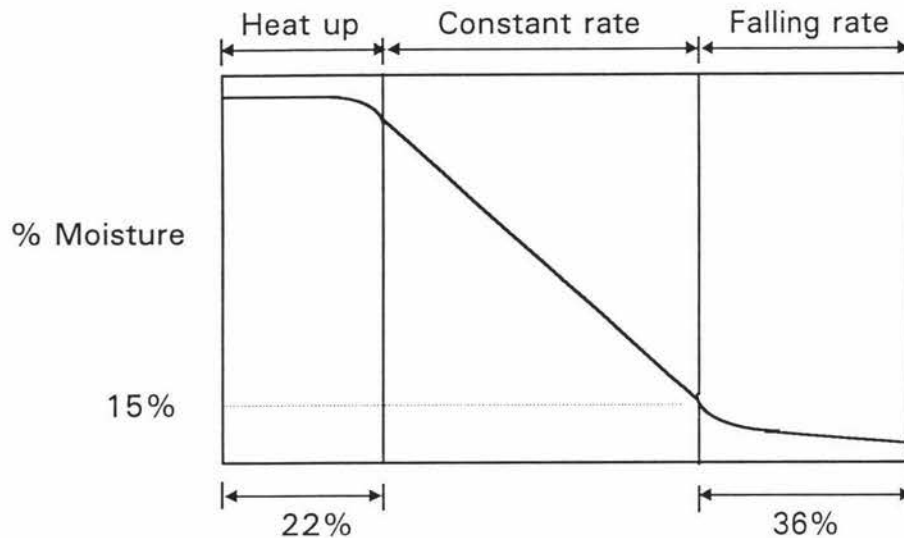


**Figure 2.1-3** Resistances to heat transfer on a cylinder dryer (Polat and Mujumdar 1987)

Most paper drying machines have three to five independently felted sections, each with variable speed control to maintain sheet tension between sections and adjust for any sheet shrinkage that occurs. Usually, three to five sections are also grouped for independent steam pressure control; these may be the same or different from the felt groupings (Smook, 1982). The whole drying section is set in a hood so that the atmosphere in which the evaporation takes place can be controlled by action on the inlet and extracted air flows (Lemaitre *et al.*, 1980).

The paper sheet, with a known solids and water content per unit area and of known temperature, approaches a cylinder of known surface temperature. One side of the sheet begins to heat up while the other side, being still open to the atmosphere, cools by evaporation, radiation and conduction. Thus, immediately temperature and moisture gradients are set up in the direction of the thickness of the sheet. Similarly, gradients are set up along the length of the sheet. After a short time the paper reaches the felt. It covers the outer side of the sheet and a new system of temperature and water content dynamics is established: this, in its turn changes as the felt leaves the cylinder and is free to dry on both sides in air.

Fig. 2.1-4 shows the whole drying process. Paper was dried through three periods; a heatup period, a constant-rate period and a falling-rate period.



**Figure 2.1-4** Conventional drying curve (Sharma 1989)

The drying operation can be divided into two areas: (1) heat transfer from the steam cylinders through the paper into the air, (2) mass transfer of the water evaporating from the paper sheet. Heat and mass transfer through the paper sheet cause the properties of the paper to vary continuously. Formulae can be derived from the physical laws governing the heat and mass transfer through the paper sheet to determine the evaporation rate. Temperature and moisture content are two of the most important variables, but others are: paper thickness, density, porosity, permeability, thermal conductivity, specific heat capacity, contact time, ambient air conditions, air motion, felt and paper tension, etc.

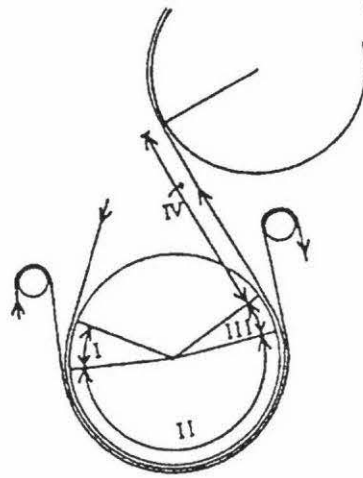
Including all these parameters as variables in the mathematical formulae would result in a set of partial differential equations very difficult to solve analytically or numerically. Approximations and assumptions, along with empirical relations, are necessary to reduce these equations to a solvable form (Bell *et al.*, 1992).

## 2.2 Development of Mathematical Models

Several researchers have attempted to develop detailed mathematical models of the drying process which could be applied to industrial paper machines. The mathematical

models could be divided in to two categories. In the first the major assumption for the mathematical models is that there is no temperature or moisture variation through the paper sheet thickness. In the second, mathematical models take into account heat and mass transfer within the paper sheet. Representative of researchers of the former method are Nissan and Kaye (1955) and the latter one is Han (1970).

One of the earliest mathematical simulations of paper dryers was by Nissan and Kaye (1955). In this model the dryer cylinder be divided into cycles of four phases (Fig. 2.2-1), which are repeated on each dryer cylinder.



**Figure 2.2-1** Nissan's nomenclature for four phase of drying (Kirk 1984)

Phase I: Begins when one side of the sheet first touches a cylinder and ends when the felt first touches the "outer" side of the sheet;

Phase II: Begins when the felt first touches the sheet and ends when the felt leaves the sheet;

Phase III: Begins when the felt leaves the sheet and ends when the sheet leaves the cylinder;

Phase IV: Begins when the sheet leaves the cylinder and ends when it next touches a cylinder.

Certain parameters were assumed constant throughout the entire drying operation (Bell *et al.* 1992):

1. Moisture-free weight of the solid per unit area
2. Speed of the paper sheet
3. Air conditions
4. Steam temperature

5. Length of each phase
6. Thermal conductivity of the paper sheet
7. Specific heat capacity
8. Overall heat transfer coefficient

Other arbitrary assumptions in their model were:

9. No temperature or moisture variation through the paper thickness
10. Arbitrary value of the heat transfer coefficient
11. Zero effect from cross-machine air currents
12. Zero pressure gradient across the sheet thickness
13. Small enough heat and moisture losses in phases I, II, and III, so that all the heat input raises the paper temperature
14. Constant conditions through each phase or subphase
15. Negligible radiation heat losses in phases I, II, and III
16. Conduction heat transfer only in the paper sheet
17. Mechanical pressure from the felt in phase II increases the heat transfer coefficient, but takes no part in water removal

The initial conditions at phase I of drum 1 are the known conditions of the paper at the entrance of the paper drying section. The conditions at the entrance to each phase are assumed to be the same as the conditions leaving the previous phase.

The mathematical formulae for the model can be summarized as follows:

For phases I, II, and III:

$$T_{\Delta t} = u - \left[ (u - T_0) \exp \left( \frac{h_{vp} \Delta t}{W(c_f + c_w M)} \right) \right] \quad (2.2-1)$$

where,

- $T_{\Delta t}$ : paper temperature change;
- $u$ : steam temperature;
- $T_0$ : initial sheet temperature;
- $h_{vp}$ : overall heat transfer coefficient;
- $\Delta t$ : time interval in phase or in subphase;
- $W$ : weight of moisture-free solids;
- $c_f$ : fiber specific heat capacity;
- $c_w$ : water specific heat capacity;
- $M$ : moisture content.

$$\frac{dm}{dt} = -2.12 \times 10^{-7} L^{0.77} b (P_f - P_a) (1 + 0.121 V^{0.85}) \quad (2.2-2)$$

(Powell and Griffiths 1935)

where,

- m: mass of water in sheet;
- t: time;
- L: length;
- b: width;
- $P_f$ : partial pressure of water vapour at the surface of web;
- $P_a$ : partial pressure of water vapour in the air;
- V: velocity.

The calculations of paper temperature and the rate of evaporation will be made for each phase separately since conditions change abruptly as the sheet enters the different phases. But, it is sometimes found that even within one phase, conditions at the beginning are substantially different from those at the end. In such a case, it is necessary to divide one phase into several subphases so that the approximation is made as close as is reasonably acceptable.

Since the over-all evaporation of water in these phases is small, it is neglected when calculating temperature, but it must be included to calculate the moisture loss.

For phase IV:

$$\frac{dT}{dt} = 2q \left[ W \left( \frac{(\rho_f c_f + \rho_w M)}{(1 + M)} \right) \left( \frac{1}{\rho_f} + \frac{M}{\rho_w} \right) \right]^{-1} \quad (2.2-3)$$

where,

- T: paper temperature;
- q: heat;
- $\rho_f$ : fiber density;
- $\rho_w$ : water density.

In Eq. (2.2-3) q is the sum of the heat due to convection, radiation, and evaporation. The rate of evaporation of phase IV is calculated from Eq. (2.2-2) and is doubled to account for the evaporation from both surfaces.

The Nissan and Kaye's method was used to evaluate a specific paper machine dryer section consisting of 37 total drying cylinders. The only predicted value compared with observed values is moisture content (the ratio of the weight of water to the weight of the dry solid). Despite the many assumptions and approximations, the overall calculated results compared reasonably with the observed values. The calculated predictions are adequate for the entrance to the drying section but deteriorate as the process continues.

Depoy (1972) followed up the work by Nissan and obtained a numerical solution for a dryer configuration with one felted cylinder and one unfelted cylinder. He deemed the most important observation offered by the simulation is that applying air directly to the dryer is more efficient than applying it to the paper web in the draw between dryers. He divided two steam dryer drums into 5 repeating boundary configurations which are similar to but not quite the same as those of Nissan. He reduced the Fourier one-dimensional heat transfer equation to the finite difference equation.

Han (1970) compiled the work of several researchers to develop a model for cylinder dryers based on partial differential equations representing the heat and mass transfer in the paper sheet.

Major assumptions of his model were (Bell *et al.* 1992):

1. Four phase system developed by Nissan and Kaye
2. Two dimensions (machine direction and paper thickness)
3. Moving coordinate system
4. Temperature at the end of draw equal to ambient air temperature
5. Steady state heat conduction through the cylinder shell
6. Negligible radiation heat losses
7. Liquid flow important only in the early drying stages
8. Contact resistance modeled by a stagnant film of air
9. Condensate thickness varies with position around the inside of cylinder
11. Diffusivity found by an experimental correlation
12. Linear reduction of paper thickness

The mass balance equation based on mass fluxes within the paper is:

$$\frac{\partial m}{\partial t} = - \frac{\partial J_w}{\partial z} - \frac{\partial J_v}{\partial z} \quad (2.2-4)$$

where,

- $J_w$ : water mass flux;
- $J_v$ : vapour mass flux;
- $z$ : paper thickness direction.

The heat balance equation is:

$$[c_w \rho_w s \varepsilon + c_f \rho_f (1-\varepsilon)] \frac{\partial T}{\partial t} = - \frac{\partial Q}{\partial z} - c_w (J_w + J_v) \frac{\partial T}{\partial z} - \lambda \frac{\partial J_v}{\partial z} \quad (2.2-5)$$

where,

- $s$ : saturation (volume of liquid/volume of voids);
- $\varepsilon$ : porosity (volume of voids / total volume);
- $Q$ : heat flux.

This equation represents the rate of heat accumulation per unit volume in the element resulting from the net heat input by conduction, associated with the flow of water and vapour, and heat loss due to evaporation ( on the left and right hand sides respectively).

For heat conduction, the Fourier law applies:

$$Q = - k \frac{\partial T}{\partial z} \quad (2.2-6)$$

where  $k$  is thermal conductivity

While the capillary flow of water follows Darcy's law:

$$J_w = \left( \frac{K}{v_w} \right) \frac{\partial P_{ca}}{\partial z} \quad (2.2-7)$$

where,

- $K$ : permeability;
- $v_w$ : water kinematic viscosity;
- $P_{ca}$ : capillary pressure

For vapour transport, Fick's law applies:

$$J_v = - \left[ \frac{D_a w_v}{(1-y_v)} \right] \frac{\partial C_v}{\partial z} \quad (2.2-8)$$

where,

- $D_a$ : air diffusivity;

- $w_v$ : vapour molecular weight;
- $y_v$ : vapour mole fraction;
- $C_v$ : vapour molar concentration.

Han gave a preliminary analysis of the energy balance for the first cylinder, the constant-rate drying period and the falling-rate drying period. He suggested the analysis may be further simplified or refined, depending on the purpose and information on hand. He gave no method of solution or any results based on his analysis.

Donner and Renk (1982) developed a system of partial differential equations which describe the paper drying process for pinpointing the source of poor dryer performance. The equations were similar to Han's with a few notable exceptions to further simplify the model and allow a numerical solution.

The assumptions that are different from Han's are:

1. The presence of felt was ignored
2. A fixed coordinate system was used
3. The temperature at the end of the draw was found by setting heat flux at the surface equal to the convective losses
4. A uniform condensate heat transfer coefficient was used
5. Contact resistance was neglected
6. A constant paper thickness was used and the liquid flow was neglected

A finite difference technique was used to solve the transfer equation. Web moisture and temperature distributions were calculated at virtually every point in the dryer section. The Technical Association of Pulp and Paper Industry (TAPPI) drying and heat transfer rate were computed for each dryer cylinder. The simulation was designed mainly as a diagnostic tool, and most of the results presented dealt with ways to improve dryer performance. Simulation results fitted the expected behaviour.

Seyed-Yagoobi *et al.* (1992a) gave the cylinder drying system a theoretical analysis which is partially based on Han's model, though the governing differential equations were improved and a set of realistic boundary conditions were derived.

They adopted Han's mass and heat balance equations and defined the porosity as:

$$\varepsilon = 1 - \frac{W}{Z\rho_f} \tag{2.2-9}$$

where Z is web thickness

and sheet caliper as follows:

$$Z = Z_{fin} \left( 1 + \frac{M \rho_{p,fin}}{\rho_w} \right) \tag{2.2-10}$$

where,

$Z_{fin}$ : web thickness at exit of dryer section;

$\rho_{p,fin}$ : paper density at exit of dryer section.

The finite difference technique was used to obtain the numerical solution. The surface nodes in the model were assumed to be located slightly (one-half step size in the z direction) inside the paper sheet, and not exactly on the surfaces. This approximation simplifies the numerical solution technique. Average sheet moisture content and temperature along the length of the dryer section as well as moisture content and temperature profiles within the sheet at cylinders 1, 30, and 65 are predicted by the model for certain test conditions.

Asensio and Seyed-Yagoobi (1992) followed Seyed-Yagoobi *et al.*'s (1992a) work to develop a theoretical model for simulation of conventional steam-heated cylinder dryers. An empirical correlation for the thermal contact conductance between the cast iron dryer surface and paper web is incorporated into the drying simulation model to reflect reductions in heat input to the sheet during drying. Expressions for the reduction in sheet porosity were developed for inclusion in the model.

The dryer-paper web interface contact conductance was given by:

$$h_i = - 20.13 + 19.87\ln(P) + \frac{20999.78}{W} + 97.70\omega^2\ln(P) - \frac{8543.43\omega^4}{W} + 188.65\omega^{\frac{3}{2}} \tag{2.2-11}$$

when,

$$0.68 < P \leq 328.81 \text{ kPa}, 0.084 \leq W \leq 0.3129 \text{ kg m}^{-2}, 4.8\% \leq \omega \leq 60\%,$$

$$T=85^\circ\text{C}, \sigma = 16.62 \text{ W m}^{-2}\text{K}^{-1} (4.6\%), \text{ and } R^2 = 0.98$$

where,

$h_i$ : dryer-paper web interface contact conductance;

- P: pressure;  
 $\omega$ : moisture content, wet basis (weight of water / (weight of water + weight of dry fibers));  
 $\sigma$ : standard deviation;  
 $R^2$ : adjusted coefficient of determination.

Porosity:

$$\varepsilon_{fin} = 1 - \frac{\rho_{p,fin}(1 - M_{fin}) - \rho_a}{\rho_f - \rho_a} \quad (2.2-12)$$

- $\varepsilon_{fin}$ : porosity (volume of voids / total volume) at exit of dryer section;  
 $M_{fin}$ : moisture content at exit of dryer section;  
 $\rho_a$ : air density.

$$\varepsilon = 1 - \frac{1 - \varepsilon_{fin}}{1 + \frac{MW}{\rho_w Z_{fin}}} \quad (2.2-13)$$

Average sheet moisture content and temperature along the length of the dryer section as well as average evaporation rates per cylinder are predicted by the model. The three stages of drying, *i.e.*, initial sheet warm up, a relatively constant rate and a falling rate, were all predicted by the model without any external correction factors. Consideration of the internal dynamics of the drying process allows profiles of sheet moisture content, temperature, liquid flux and vapour flux through the sheet thickness to be predicted by the simulation model throughout the dryer section.

Asensio and Seyed-Yagoobi (1993) presented a theoretical model which was similar to their previous work (1992). Liquid and vapour transport, evaporation from a porous surface, sheet shrinkage and porosity variation were all considered during model development. They studied the effect of drum/paper thermal contact conductance on the average sheet moisture content and evaporation and found that doubling the contact conductance decreases the required number of dryers from 95 to 58 by increasing the average evaporation rate from 13.7 kg m<sup>-2</sup>h<sup>-1</sup> to 22.4 kg m<sup>-2</sup>h<sup>-1</sup> for certain operating conditions. This produced a 64 percent improvement in the drying rate (see Table 1 in their paper). Hence, the drying efficiency is highly dependent on the contact conductance between the dryer drum and the paper web.

Baines (1973) developed a mathematical model of paper drying by considering paper sheet finite thickness and neglecting capillary transport of liquid and convection of vapour through the sheet.

For a sheet in contact with a cylinder (phases I, II, III):

$$\frac{\Delta T}{\Delta T_1} = z' + \frac{2}{\Pi} \sum_{\eta=1}^{\infty} \frac{(-1)^\eta}{\eta} \exp(-\eta^2 \Pi^2 t') \sin \eta \Pi z' \quad (2.2-14)$$

where,

$$\begin{aligned} \Delta T: &= T - T_o; \\ T_o: &\text{ temperature of wet-dry interface;} \\ \Delta T_1: &= T_1 - T_o; \\ T_1: &\text{ temperature of dry side;} \\ t': &= \frac{(\alpha_p + \alpha_v)t}{Z^2}; \\ \alpha_p: &\text{ paper thermal diffusivity;} \\ \alpha_v: &\text{ vapour thermal diffusivity;} \\ z': &= \frac{z}{Z}. \end{aligned}$$

$$E = \frac{D\varepsilon A \rho_v \Delta T_1}{Z} \left(1 + 2 \sum_{\eta=1}^{\infty} (-1)^\eta \exp(-\eta^2 \Pi^2 t')\right) \quad (2.2-15)$$

where,

$$\begin{aligned} E: &\text{ evaporation rate;} \\ D: &\text{ diffusivity;} \\ A: &\text{ a constant;} \\ \rho_v: &\text{ vapour density.} \end{aligned}$$

For a sheet in the draw phase (phase IV):

$$\frac{\Delta T}{\Delta T_1} = \frac{2}{\Pi} \sum_{\eta=1}^{\infty} \frac{(-1)^\eta}{\eta} \exp(-\eta^2 \Pi^2 t') \sin \eta \Pi z' \quad (2.2-16)$$

$$E = \frac{D\varepsilon \rho_v \Delta T_1}{Z} \sum_{\eta=1}^{\infty} \exp(-(2\eta - 1)^2 \Pi^2 t') \quad (2.2-17)$$

Baines considered this model complex enough to account for all aspects of sheet drying but, if it is used for the design and operation of a paper dryer, it must be extended in several ways:

1. More complex but realistic boundary conditions used in obtaining solutions
2. Numerical values for the physical properties of wet paper obtained
3. Experimental results used to verify solutions.

Powell and Strong (1974) presented a mathematical model reliant on several main assumptions as follows:

1. Two phases (drum phase which begins when the sheet first touches a cylinder and ends when the sheet leaves the cylinder and draw phase which begins when the sheet leaves the cylinder and ends when it next touches a cylinder)
2. Two dimensions (machine direction and paper thickness)
3. Values of the thermal conductivity and the contact conductance are simply estimated from the available experimental data
4. No evaporation takes place from the sheet while it is on the drum
5. Temperature at the end of draw equal to ambient air temperature

In addition, dimensionless variables are introduced by them to build the model.

For the drum phase:

$$T_{p \text{ fin}} = T_a + \theta(T_c - T_a) \quad (2.2-18)$$

where,

- $T_{p \text{ fin}}$ : temperature of the paper sheet as it leaves a dryer drum phase;
- $T_a$ : temperature of the air in the dryer section;
- $\theta$ : dimensionless temperature;
- $T_c$ : cylinder temperature.

For the draw phase:

$$\frac{dM}{dt} = \frac{3.6C_{\text{mpa}}}{WRT_a} (P_s - P_a) \quad \text{if } M \geq M_c \quad (2.2-19)$$

where,

- $C_{\text{mpa}}$ : mass transfer coefficient between paper and air;
- $R$ : gas constant;
- $P_s$ : saturated pressure;

$M_c$ : critical moisture.

$$\frac{dM}{dt} = \frac{3.6C_{mpa}}{WRT_a} \left( \frac{M - M_e}{M_c} \right) (P_s - P_a) \quad \text{if } M < M_c \quad (2.2-20)$$

where  $M_e$  is equilibrium moisture.

$$\frac{dT}{dt} = \frac{3.6fC_{mpa}(P_s - P_a)}{RT_a c_p (W + MW)} \quad (2.2-21)$$

where,

$$f: \quad = 1, \text{ if } M \geq M_c;$$

$$f: \quad = \left( \frac{M - M_e}{M_c} \right) \text{ if } M < M_c;$$

$c_p$ : paper specific heat capacity.

Powell and Strong admitted that the model was simplified to a very large extent, but the analysis was cast in a form where many of their assumptions can be removed in a systematic way as more detailed experimental information about the process becomes available.

Knight and Kirk (1975) derived a theoretical analysis of the operation of a conventional drying section which emphasised the operation of the drying section itself rather than the heat and mass transfer within the web. The model shows that an analysis of drying in the machine direction cannot be complete without consideration of cross direction effects. A basic assumption for the model is no temperature or moisture variation through the paper thickness.

Surface-air heat transfer coefficients:

$$h = (6.48 - 0.0098T_a)L^{-0.2}V^{0.8} \quad (2.2-22)$$

Mass transfer coefficients:

$$C_m = 1.09h \quad (2.2-23)$$

$$\frac{1}{C_{mpa}} = \frac{1}{C_{mpfe}} + \frac{1}{C_{mfea}} \quad (2.2-24)$$

where,

$C_{mpfe}$ : mass transfer coefficient between paper and felt;

$C_{mfea}$ : mass transfer coefficient between felt and air.

$$C_{mpfe} = \frac{\epsilon D}{Z_{fe}} \quad (2.2-25)$$

where  $Z_{fe}$  is felt thickness.

Mass transfer:

$$E = C_m(P_s - P_a) \frac{M}{M_c} \quad \text{if } M < M_c \quad (2.2-26)$$

$$E = C_m(P_s - P_a) \quad \text{if } M \geq M_c \quad (2.2-27)$$

For phases I, II, III:

$$\frac{dT}{dt} = \frac{1}{Wc_f + 0.01MWc_w} \{h_{cp}(T_c - T) - h_{pa}(T - T_a) - \lambda E\} \quad (2.2-28)$$

where,

$h_{cp}$ : heat transfer coefficient between cylinder and paper;

$h_{pa}$ : heat transfer coefficient between paper and air;

$T_a$ : the air temperature over the clothing;

$\lambda$ : latent heat of evaporation of water.

For phase IV:

$$\frac{dT}{dt} = \frac{-1}{Wc_f + 0.01MWc_w} \{h_{pa}[(T - T_{a1}) + (T - T_{a2})] + \lambda(E_1 + E_2)\} \quad (2.2-29)$$

where,

$T_{a1}$  or  $T_{a2}$ : air temperature of each side of the paper;

$E_1$  or  $E_2$ : evaporation rate of each side of the paper.

The results of comparison of the modeling with practical data have been reasonably good which suggests that the simplifications that have been made in the analyses do not detract from their ability to describe dryer section operations. Knight and Kirk deem the main limitations are due to errors introduced by the lack of theoretical or empirical knowledge of the drying process and these limitations will decrease with greater

understanding and can be overcome to a certain extent by the fitting of experimental data to provide values for unknown parameters.

Soininen (1980) derived two simultaneous differential equations from the heat and mass balance to model the contact drying process which can be applied especially for the study of the drying intensity in a conventional paper machine. The paper thickness was considered. He also discussed the nature and boundary conditions of the through-drying process. The model of the contact drying process cannot be solved analytically. He developed a program for the numerical solution, but didn't give any detail about the program. He showed a couple of worked examples for main parameters, but gives no comparison with practical data.

Lee and Hinds (1981) developed a laboratory technique for measuring the transport phenomena which occurs within moist sheets of paper or board dried under laboratory controlled conditions. Generalised constitutive relationships describing the internal movement of liquid, vapour, and conducted heat can be deduced from these laboratory data. The empirical expressions for the internal transport coefficients of liquid, vapour, and heat are specific to the composition and structure of the sheet used in these experiments. These empirical expressions will vary depending upon the furnish and formation of the sheet tested. It is important that the appropriate empirical expressions for the particular sheet of interest be determined in order to formulate a reliable model. They derived a semiempirical mathematical analogue for the thermodynamic behaviour of the moist sheet based on these empirical expressions and on elemental heat and mass balances. The model is capable of predicting the movement of moisture and heat throughout the sheet exposed to a variety of boundary conditions.

Hinds and Neogi (1983) developed a mathematical model of the drying process that took into account the heat and mass transfer both within and outside the paper sheet. The equations governing the elemental mass and energy balance are highly nonlinear, second-order, partial differential equations. Therefore a VAX computer with a VMS operating system was used to solve the equations using a finite difference technique. The results of computer simulation were compared with machine data for several paper types.

Lemaitre *et al.* (1980) described a method giving systematic analysis of industrial drying sections during their run and based on a mathematical model of paper cylinder drying.

The main simplifying hypotheses used to construct this mathematical model are as follows:

1. Two typical positions of the paper web are distinguished: web on the cylinder and web in the draw
2. Temperature and moisture content are supposed to be uniform through the web
3. All the internal phenomena (vapour diffusion, capillary transport and *etc.*) are considered only with global relationships
4. The drying section is in a steady state

For the web on a cylinder:

$$W(c_f + Mc_w) \frac{dT}{dt} - h_{vp}(T_v - T) - h_{pfe}(T_{fe} - T) + E\lambda(T) = 0 \quad (2.2-30)$$

where,

- $h_{vp}$ : heat transfer coefficient between steam vapour and paper;
- $T_v$ : steam temperature;
- $h_{pfe}$ : heat transfer coefficient between paper and felt;
- $T_{fe}$ : felt temperature;

$$E = W \frac{dM}{dt} = - \frac{C_{mpfe}}{R(273 + T)} P \log\left(\frac{P - P_a}{P - p_f}\right) \quad (2.2-31)$$

where  $P_a$  is vapour pressure in the air over the felt.

For the web in the draw phase:

$$W(c_f + Mc_w) \frac{dT}{dt} - 2h_{pa}(T_a - T) + E\lambda(T) = 0 \quad (2.2-32)$$

$$E = W \frac{dM}{dt} = - \frac{2C_{mpa}}{R(273 + T)} P \log\left(\frac{P - P_a}{P - p_f}\right) \quad (2.2-33)$$

where  $P_a$  is vapour pressure in the pocket air.

For the web on a cylinder and in draw phase:

$$P_f = P_s(T) \quad \text{if } M < M_c \quad (2.2-34)$$

$$P_f = P_s(T) e^{-k_f \left(\frac{1}{M} - \frac{1}{M_c}\right)} \quad \text{if } M < M_c \quad (2.2-35)$$

where  $k_f$  is (affinity) coefficient of the water for the sheet

The author applied a parameter estimation method to evaluate the heat and mass transfer coefficients of industrial processes to make the mathematical model workable. The model has been used for an industrial printing paper machine. Computer simulations allowed them to find the optimal size-press location in the drying section and the machine speed variations obtained after an increase in the number of cylinders and all this for three different basis weights.

Videau and Lemaitre (1982) described a mathematical model based on a theoretical analysis of the physical phenomena that occur in a multicylinder paper machine drying section. This model takes into account the main design and working parameters: the different evaporation phases, the heat and mass transfer conditions, the main ventilation variables and also the steam system constitution. The major approximations for the model of the sheet are the same as Lemaitre *et al.* (1980), such as, no temperature or moisture variation through the sheet thickness. The equations of the model of the sheet are almost similar to Lemaitre *et al.* (1980). The heat transfer equation (Eq. (2.2-36)) for web on the cylinder is considered only from hot cylinder surface.

$$W(c_p + Mc_w) \frac{dT}{dt} = h_{cp}(T_c - T) + h_{pfe}(T_{fe} - T) - E\lambda(T) \quad (2.2-36)$$

The heat and mass coefficients, as well as some unknown coefficients for the sheet model, the cylinder model, the steam system and ventilation system models, are determined for a given operating point by using an identification method. This identification method was tested on a pilot paper machine and on different industrial drying sections. The authors studied the influence of some working conditions like basis weight and machine speed and obtain results such that it is possible to keep constant the numerical values of the estimated parameters, for simulations around the selected operating point. But, the authors give no detail about the identification method.

### 2.3 References

Asensio, M. C. and Seyed-Yagoobi, J., 1992, "Further Analysis of Heat and Mass Transfer in a Paper Sheet During Drying," *Fundamentals of Heat Transfer in Porous Media*, ASME HTD-Vol. 193, pp. 123-130.

Asensio, M. C. and Seyed-Yagoobi, J., 1993, "Simulation of Paper-Drying System With Incorporation of an Experimental Drum/Paper Thermal Contact Conductance Relationship," *ASME Journal of Energy Resources Technology*, Vol. 115, pp. 291-300.

Baines, W. D., 1973, "Analysis of Transient Effects in Drying of Paper," *Pulp and Paper Mag. Can.*, Vol. 74 (2): T34, pp. 58-64.

Bell, D. O., Seyed-Yagoobi, J., and Fletcher, L. S., 1992 "Recent Developments in Paper Drying," *Advances in Drying*, Vol. 5, ed. A. Mujumdar, Hemisphere, Washington, D.C., pp. 203-261.

Chance, J. L., 1989, "Overview of Dryer Section," *TAPPI Seminar Notes: 1989 Practical Aspects of Pressing and Drying*, Atlanta, Ga, pp. 121-131.

Depoy, J. A., 1972, "Analog Computer Simulation of Paper Drying: A Workable Model," *Pulp and Paper Mag. Can.*, Vol. 73(5): pp. 67-74.

Donner, B. C. and Renk, F. J., 1982, "Evaluation and Improvement of Individual Dryer Performance," *TAPPI Proceedings of the 1982 Papermakers Conference*, pp. 227-236.

Han, S. T., 1970, "Drying of Paper," *TAPPI Journal*, Vol. 53, No. 6, pp. 1034-1046.

Hinds, J. A. and Neogi, A. N., 1983, "The Dynamic Computer Simulation of a Paper Machine Dryer," *TAPPI Journal*, Vol. 66, No. 6, pp. 79-82.

Kirk, L. A., 1984, "A Literature Review of Computer Simulation and Paper Drying," *Advances in Drying*, Vol. 3, ed. A. Mujumdar, Hemisphere, Washington, D.C., pp. 1-37.

Knight, R. L. and Kirk, L. A., 1975, "Simulation of the Papermachine Drying Section," *International Water Removal Symposium*, British Paper and Board Industries Federation - London (March 1975), pp. 188-226.

Lee, P. F. and Hinds, J. A., 1981, "Optimizing Dryer Performance: Modeling Heat and Mass Transfer within a Moist Sheet of Paper or Board," *TAPPI Journal*, Vol. 64, No. 12, pp. 39-44.

Lemaitre, A., Veyre, J., Lebeau, B. and Foulard, C., 1980, "Case Study: Method for Systematic Analysis of Paper Machine Multicylinder Drying Section," *4th IFAC-PRP 4 Conference*, Ghent-Belgium (3-5 June 1980), pp. 261-270.

Nissan, A. H. and Kaye, W. G., 1955, "An Analytical Approach to the Problem of Drying of Thin Fibrous Sheets on Multicylinder Machines," *TAPPI Journal* Vol. 38, No. 7, pp. 385-398.

Polat O., and Mujumdar, A. S., 1987, "Drying of pulp and paper," *Handbook of Industry Drying*, ed. A. Mujumdar, Hemisphere, Washington, D.C., pp. 643-682

Powell, R. W., and Griffiths, E., 1935, "The Evaporation of Water From Plane and Cylindrical Surfaces," *Trans. Inst. Chem. Eng.*, Vol. 13: pp. 175-198.

Powell, T. and Strong, A. B., 1974, "An Analysis of Drying on Conventional Paper Machines," *Pulp and Paper Mag. Can.*, Vol. 75(3), T77, pp. 71-78.

Seyed-Yagoobi, J., Bell, D. O., and Asensio, M. C., 1992a, "Heat and Mass Transfer in a Paper Sheet During Drying," *ASME Journal of Heat Transfer*, Vol. 114, pp. 538-541.

Sharma, R., 1989, "Using Infrared Selectively for Improving Paper Quality and Production," *Practical aspects of pressing and drying seminar*, Atlanta, Ga, pp. 85-92.

Smook, G. A., 1982, "Paper Manufacture dry end operations," *Handbook for Pulp & Paper Technologists*, Technical ed. M. J. Kocurek, Atlanta, Ga, pp. 246.

Soininen, M., 1980, "A Mathematical Model of the Contact Drying Process," *Drying '80*, ed., A. Mujumdar, Hemisphere Publishing Corp., New York, NY, Vol. 2, pp. 315-321.

Videau, J. L. and Lemaitre, A., 1982, "An Improved Model of a Paper Machine Multicylinder Drying Section," *Drying '82*, ed., A. Mujumdar, Hemisphere Publishing Corp., New York, NY, pp. 129-138.

### **3. MODEL SELECTION**

#### **3.1 Guidelines for Model Selection**

As shown in the literature review of Chapter 2, modeling of the drying section of a continuous paper machine has been studied for over 40 years. However, there are not many models because the process of paper drying is so complicated. The selection of models from those reviewed has been carried out according to the following guidelines:

1. Assumptions: For the paper drying process, a mathematical model is not available without realistic assumptions. Models with reasonable assumptions should be select to accurate results.
2. Solvability: Some mathematical models are highly nonlinear, second order and partial differential equations; they cannot be solved analytically. These models are not suitable for this project because of time limitations.
3. Complete drying system: The more input and output variables the models include, the better. Input variables are steam temperature or steam pressure, the drum speed of the machine and environmental air conditions; and output variables are paper moisture content and paper temperature. It will be easy to derive different results of output variables from changing the value of the input variables.
4. Coefficients: The coefficients of mathematical expressions are the most important factors for making models workable. No matter whether the coefficients of mathematical models are taken from published data or from empirical formulae, the key is whether they can be obtained within reasonable time and cost limits.

#### **3.2 Model Selection**

As described in the literature review of Chapter 2, one of the earliest mathematical models for a paper drying machine was developed by Nissan and Kaye (1955). The major problem with this model is that it neglects water evaporation when calculating the temperature in phases I, II, and III. This effect is significant and cannot be ignored.

The model developed by Han (1970) was recently improved by Asensio and Seyed-Yagoobi (1992, 1993) and Seyed-Yagoobi *et al.* (1992a). A set of realistic boundary conditions, expressions for sheet shrinkage as a function of mass of water removed and for reductions in sheet porosity, and an empirical correlation representing the thermal contact conductance between the cast iron dryer surface and paper, were all incorporated into the latter versions. But the models still cannot be solved analytically.

Other models such as Depoy's (1972), Donner and Renk's (1982), Soininen's (1980), Lee and Hinds's (1981), and Hinds and Neogi's (1983), cannot be solved analytically and are too computationally intensive to solve numerically.

If Baines's (1972) model is used in the design and operation of a paper dryer, it must be extended in several ways as mentioned in the literature review. The scope of this research did not include making such extensions.

Powell and Strong's (1974) model is based on the assumption that no evaporation takes place from the sheet while it is on the drum phase. Again this effect is significant and cannot be ignored.

Videau and Lemaitre's (1982) model of the sheet is almost the same as the model of Lemaitre *et al.* The difference is that the heat transfer equation for the web on the cylinder is calculated from the hot cylinder surface or from steam. Both of these models can therefore be considered together.

Having eliminated all of these models using the guidelines set out earlier we are left with only two models as possibilities, that of Knight and Kirk (1975) and Lemaitre *et al.* (1980). These models will now be studied in further depth.

### 3.3 References

Asensio, M. C. and Seyed-Yagoobi, J., 1992, "Further Analysis of Heat and Mass Transfer in a Paper Sheet During Drying," *Fundamentals of Heat Transfer in Porous Media*, ASME HTD-Vol. 193, pp. 123-130.

Asensio, M. C. and Seyed-Yagoobi, J., 1993, "Simulation of Paper-Drying System With Incorporation of an Experimental Drum/Paper Thermal Contact Conductance Relationship," *ASME Journal of Energy Resources Technology*, Vol. 115, pp. 291-300.

Baines, W. D., 1973, "Analysis of Transient Effects in Drying of Paper," *Pulp and Paper Mag. Can.*, Vol. 74 (2): T34, pp. 58-64.

Depoy, J. A., 1972, "Analog Computer Simulation of Paper Drying: A Workable Model," *Pulp and Paper Mag. Can.*, Vol. 73(5): pp. 67-74.

Donner, B. C. and Renk, F. J., 1982, "Evaluation and Improvement of Individual Dryer Performance," *TAPPI Proceedings of the 1982 Papermakers Conference*, pp. 227-236.

Han, S. T., 1970, "Drying of Paper," *TAPPI Journal*, Vol. 53, No. 6, pp. 1034-1046.

Hinds, J. A. and Neogi, A. N., 1983, "The Dynamic Computer Simulation of a Paper Machine Dryer," *TAPPI Journal*, Vol. 66, No. 6, pp. 79-82.

Knight, R. L. and Kirk, L. A., 1975, "Simulation of the Papermachine Drying Section," *International Water Removal Symposium*, British Paper and Board Industries Federation - London (March 1975), pp. 188-226.

Lee, P. F. and Hinds, J. A., 1981, "Optimizing Dryer Performance: Modeling Heat and Mass Transfer within a Moist Sheet of Paper or Board," *TAPPI Journal*, Vol. 64, No. 12, pp. 39-44.

Lemaitre, A., Veyre, J., Lebeau, B. and Foulard, C., 1980, "Case Study: Method for Systematic Analysis of Paper Machine Multicylinder Drying Section," *4th IFAC-PRP 4 Conference*, Ghent-Belgium (3-5 June 1980), pp. 261-270.

Nissan, A. H. and Kaye, W. G., 1955, "An Analytical Approach to the Problem of Drying of Thin Fibrous Sheets on Multicylinder Machines," *TAPPI Journal* Vol. 38, No. 7, pp. 385-398.

Powell, R. W., and Griffiths, E., 1935, "The Evaporation of Water From Plane and Cylindrical Surfaces," *Trans. Inst. Chem. Eng.*, Vol. 13: pp. 175-198.

Seyed-Yagoobi, J., Bell, D. O., and Asensio, M. C., 1992a, "Heat and Mass Transfer in a Paper Sheet During Drying," *ASME Journal of Heat Transfer*, Vol. 114, pp. 538-541.

Soininen, M., 1980, "A Mathematical Model of the Contact Drying Process," *Drying '80*, ed., A. Mujumdar, Hemisphere Publishing Corp., New York, NY, Vol. 2, pp. 315-321.

Videau, J. L. and Lemaitre, A., 1982, "An Improved Model of a Paper Machine Multicylinder Drying Section," *Drying '82*, ed., A. Mujumdar, Hemisphere Publishing Corp., New York, NY, pp. 129-138.

## **4. FITTING OF MODEL NON-OPTIMISED PARAMETERS**

### **4.1 Selection of Non-optimised Parameters**

In equations (2.2-26, 27, 28, 29, 30, 31, 32, 33, 34), the parameters such as: partial pressure of water vapour at the sheet surface,  $P_s$ , specific heat of water,  $c_w$ , latent heat of vaporisation,  $\lambda$ , are all functions of temperature,  $T$ . It is possible to derive their mathematical formulas from published data within certain temperature ranges. These parameters do not need to be optimised. Other parameters such as times spent by the paper in each phase ( $t_I$ ,  $t_{II}$ ,  $t_{III}$ , and  $t_{IV}$ ) can be calculated according to the length of each phase and to the drum speed.

### **4.2 Values of Non-optimised Parameters**

The static data of the PM3 paper dryer machine of Tasman Pulp and Paper Ltd were used to simulate the mathematical model. The calculations which follow are on the basis of these data.

There are 49 cylinders in PM3 paper dryer machine. The 49 cylinders were divided into four sections which are wet end section, cylinders 1-3, intermediate section, cylinders 4-13, main section, cylinders 14-41, and after section, cylinders 42-49. The steam pressure for each section is 25, 100, 110 and 100 KPa respectively. There are three hoods to cover all the cylinders, hood 1 is for cylinders 1-13, hood 2 is for cylinders 14-41, and hood 3 is for cylinders 42-49. The air temperature inside the hood 1, 2 and 3 is 83 °C, 91 °C and 89 °C respectively. The felt thickness is 1.8mm. The drum speed is 17.305 m s<sup>-1</sup>.

According to the steam pressure of each section, corresponding steam temperature is 105 °C, 120 °C, 122 °C and 120 °C respectively.

For the vapour partial pressure of the inside hoods, heating the fresh air of which the temperature is 12 °C and relative humidity is 40% (normal weather of New Zealand) to 83 °C, 91 °C and 89 °C, from the Psychrometric Chart, the relative humidity of dry air are obtained and the vapour partial pressure are calculated as 697.63 Pa, 730.03 Pa and 744.35 Pa.

The specific heat of paper was taken from the paper of Asensio and Seyed-Yagoobi (1993) as  $1340.0 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$ .

From the static data of the PM3 paper dryer machine, the temperature and relative humidity of the pocket air are known and the vapour pressure in pocket air can be calculated. The temperature and vapour pressure of the pocket air for all drums are shown as Table 4.2-1.

**Table 4.2-1** Temp. and Vapour Pressure of Pocket Air for Each Cylinder

Cylinder Number	Temp. of Pocket Air( $^\circ\text{C}$ )	Vapour Pressure in Pocket Air(Pa)
1	51.2	4489.230
2	47.1	6162.510
3	48.4	7197.970
4	48.4	8192.160
5	50.6	8864.520
6	49.9	10488.30
7	50.6	10522.85
8	49.3	10628.75
9	53.1	11373.62
10	53.6	11848.47
11	55.0	11663.51
12	54.6	9658.770
13	59.5	11388.14
14	70.3	15989.56
15	69.1	19325.21
16	73.6	19185.22
17	73.4	24782.21
18	78.0	32045.06
19	74.5	24382.96
20	76.3	29565.52
21	74.6	25464.94
22	79.5	34445.52
23	75.1	27296.22
24	79.2	33409.72

...continued

Table 4.2-1 continued...

Cylinder Number	Temp. of Pocket Air(°C)	Vapour Pressure in Pocket Air(Pa)
25	70.9	26378.70
26	75.9	32866.67
27	73.0	26655.14
28	76.5	30974.43
29	74.7	28916.71
30	76.9	28740.32
31	72.3	26710.08
32	75.9	23275.54
33	74.8	28263.89
34	77.3	28074.46
35	72.2	25976.60
36	76.3	22582.50
37	72.0	24423.27
38	76.5	22118.71
39	71.7	24173.36
40	76.3	21929.12
41	53.0	9293.780
42	59.1	8490.550
43	61.4	11824.12
44	64.4	12588.89
45	61.1	13422.10
46	64.7	10819.68
47	60.7	12048.27
48	65.6	9937.850
49	41.4	5411.060

#### 4.2.1 Calculation of $t_I$ , $t_{II}$ , $t_{III}$ , and $t_{IV}$

The calculation of the times paper spending in each phase  $t_I$ ,  $t_{II}$ ,  $t_{III}$ , and  $t_{IV}$  was based on the blueprints for first, second and third dryer sections arrangement by courtesy of Tasman Pulp and Paper Limited.

From the blueprints, the first and second dryer sections arrangement are different to the third dryer section. From the drawing of the roll positions of PM3 paper dryer machine, the arrangement of the fourth and fifth dryer sections is the same as the third dryer section.

The detail of the calculation of the  $t_I$ ,  $t_{II}$ ,  $t_{III}$ , and  $t_{IV}$  is attached at Appendix 4.2-1. The results of the calculation for  $t_I$ ,  $t_{II}$ ,  $t_{III}$ , and  $t_{IV}$  are shown as follow Table 4.2-2.

**Table 4.2-2** Length and Time for Each Phase of Drying

	Cylinder 1-13		Cylinder 14-49	
	Length (m)	Time (s)	Length (m)	Time (s)
Phase I	0.486	0.0281	0.247	0.0143
Phase II	1.966	0.1136	2.475	0.1429
Phase III	0.486	0.0281	0.247	0.0143
Phase I + II + III (Drum Phase)	2.938	0.1698	2.969	0.1715
Phase IV (Draw Phase)	0.922	0.0533	1.159	0.0670
Drum + Draw Phase	3.860	0.2231	4.128	0.2385

## 4.2.2 Formula for Saturated Vapour Pressure

The relationship between saturated vapour pressure and temperature is nonlinear. There is not a formula for this relationship. However there are some tables to show this relationship under certain conditions (within the different ranges of pressure or temperature).

The paper temperature continuously changes while it is drying. For model simulation, a formula for saturated vapour pressure is required.

The temperature of paper on the drying process is no less than 40 °C. and no higher than 133.2 °C. In this temperature range, the relationship between saturated vapour

pressure and temperature is deduced according to the table of saturated vapour pressure by using the MATLAB command "polyfit".

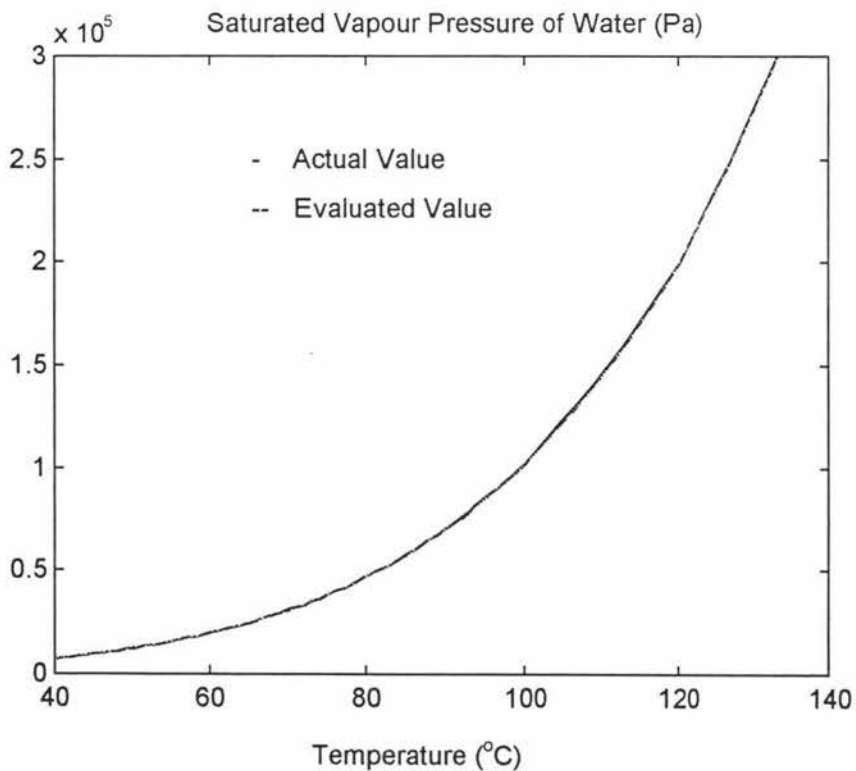
The "polyfit" command in MATLAB finds the coefficients of a polynomial  $p(x)$  of degree  $n$  that fits the data,  $p(x(i))$  to  $y(i)$ , in a least-squares sense. The result  $p$  is a row vector of length  $n+1$  containing the polynomial coefficients in descending powers:  $p(x) = p_1x^n + p_2x^{n-1} + \dots + p_nx + p_{n+1}$ .

The program for deducing formula for saturated vapour pressure is as Appendix 4.2-2.

From the program in Appendix 4.2-2, the formula for saturated vapour pressure was obtained as:

$$P_s = 0.0012T^4 - 0.081T^3 + 6.4811T^2 - 27.2847T + 129.8744$$

The actual and evaluated saturated vapour pressure are plotted against temperature in Figure 4.2-1.



**Figure 4.2-1** Saturated vapour pressure against temperature

Figure 4.2-1 shows that evaluated data for saturated vapour pressure tally with actual ones very well. Using a formula for saturated vapour pressure will be more accurate than using difference equation when temperature is not at tabulated point.

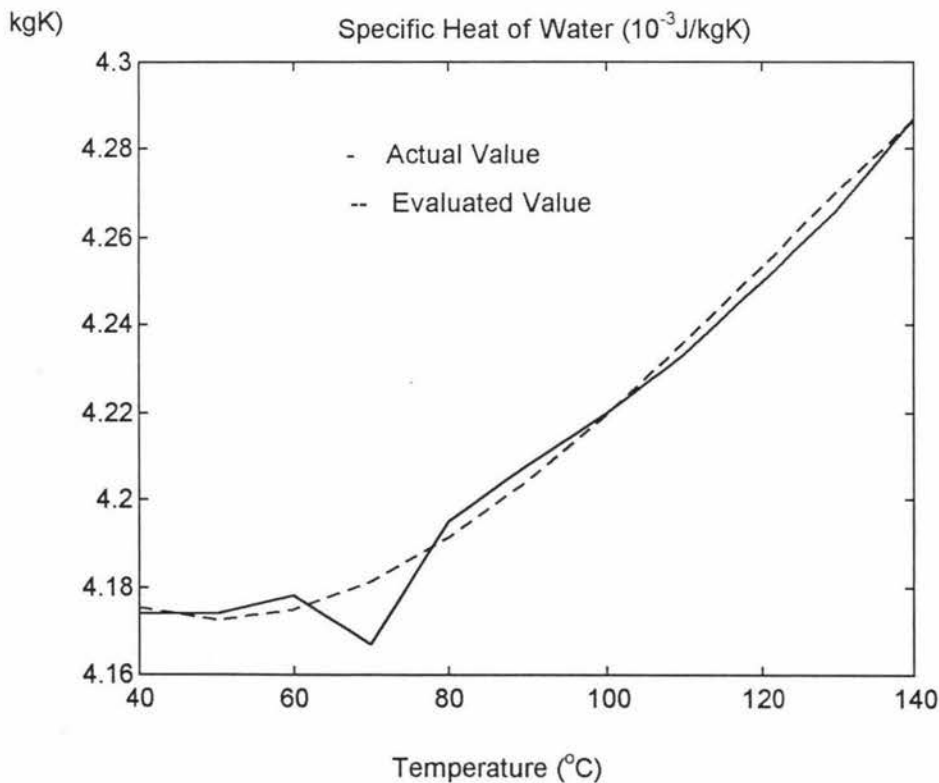
### 4.2.3 Formula for Specific Heat of Water

As for the saturated vapour pressure in section 4.2-2 a polynomial was used to calculate the specific heat of water. The program for deducing formula for specific heat of water is attached at Appendix 4.2-3.

From program in Appendix 4.2-3, the formula for specific heat of water was obtained as:

$$c_w = - 1.1927e-007T^3 + 4.2832e-005T^2 - 0.0034T + 4.2503$$

The actual and evaluated specific heats of water are plotted against temperature in Figure 4.2-2.



**Figure 4.2-2** Specific heat of water against temperature

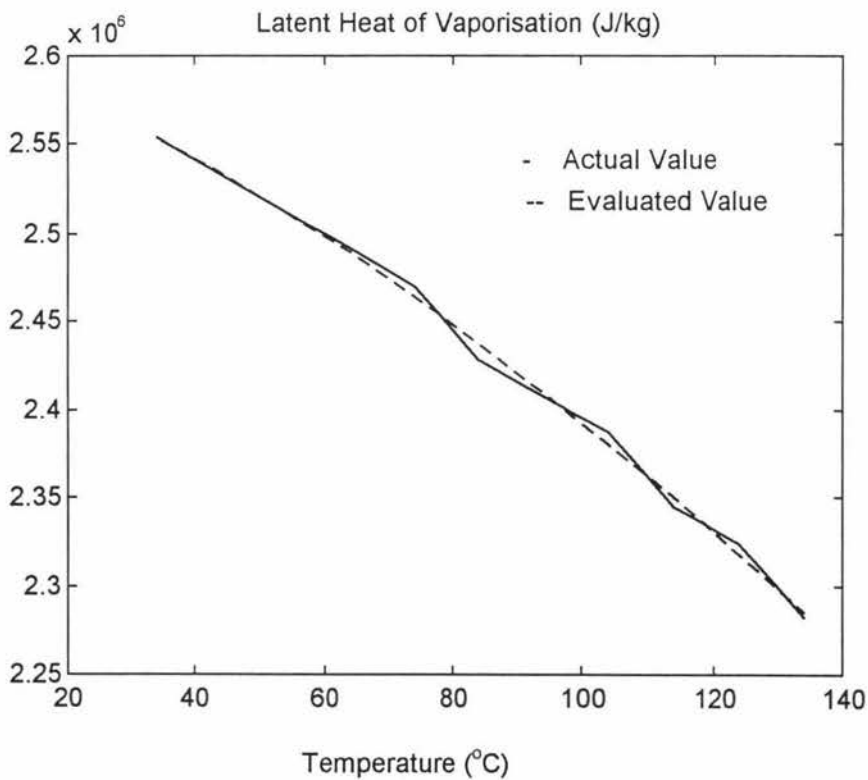
### 4.2.4 Formula for Latent Heat of Vaporisation

As for the saturated vapour pressure in section 4.2-2 and specific heat of water in section 4.2-3 a polynomial was used to calculate the latent heat of vaporisation. The program for deducing formula for latent heat of vaporisation is attached at Appendix 4.2-4.

From program in Appendix 4.2-4, the formula for latent heats of vaporisation was obtained as:

$$\lambda = 0.0122T^3 - 10.3943T^2 - 1236.2T + 2607600$$

The actual and evaluated latent heats of vaporisation are plotted against temperature in Figure 4.2-3.



**Figure 4.2-3** Latent heat of vaporisation against temperature

### 4.3 References

3rd Section - Dryer (blueprint), by courtesy of Tasman Pulp and Paper Limited.

1st & 2nd Dryer Section (blueprint), by courtesy of Tasman Pulp and Paper Limited.

Asensio, M. C. and Seyed-Yagoobi, J., 1993, "Simulation of Paper-Drying System With Incorporation of an Experimental Drum/Paper Thermal Contact Conductance Relationship," *ASME Journal of Energy Resources Technology*, Vol. 115, pp. 291-300.

*Chemical Engineering (Volume 1)*. Chemical Engineering Publisher, Beijing, 1980, pp. 347, 349-351, 359-360.

*Matlab For Microsoft Windows Reference Guide (1992)*, The MathWorks, Inc..

*Matlab For Microsoft Windows User's Guide (1993)*, The MathWorks, Inc..

*Tasman PM3*, 1994, Albany International.

## 5. OPTIMIZATION OF MODEL PARAMETERS

### 5.1 Choice of Performance Indices

Temperature and moisture content are two major parameters for measuring mass and thermal transfer of paper. The paper moisture content variations at the end of the paper dryer machine have a strong influence on the final paper quality. It is very important to take account of paper moisture content and temperature in the entire paper drying process. So, paper temperature and paper moisture are selected as performance indices for the paper dryer machine model.

Given that the two most important parameters are paper temperature and moisture content, experimental data is required to fit the chosen analytical model. This requires paper temperature and moisture content at all of the 49 drums of the PM3 machine at Tasman Pulp and Paper limited.

#### 5.1.1 Performance Index for Paper Temperature

Steady state paper temperature data was available for the PM3 paper dryer machine from previous work (Albany International, 1994) which was used as the expected profile for paper temperature. The expected profile for paper temperature is shown in Figure 5.1-1.

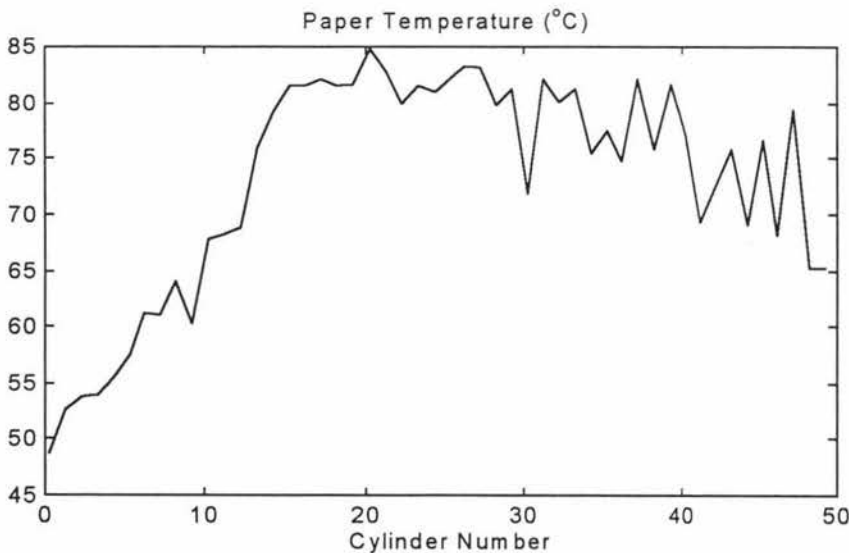


Figure 5.1-1 Expected Profile for Paper Temperature

The temperatures shown in Figure 5.1-1 are assumed to be the sheet entrance temperature for each drum rather than the exit temperature from the previous drum. The sheet temperature at the exit of the draw phase for a drum will be the sheet temperature at the entrance to the drum phase of the next drum.

### 5.1.2 Performance Index for Paper Moisture Content

Steady state paper moisture content data was not available for each drum of the PM3 paper dryer machine in Tasman Pulp and Paper Ltd. Entrance and exit values for the whole dryer section were however available, that is the initial paper moisture content into the drum and sheet moisture content exiting the last drum draw phase. These are 1.5 and 0.086 kg/kg dry basis (60% and 8% kg/kg wet basis) respectively.

Difficulties inherent in measuring the missing moisture content were beyond the scope of this project so an alternative method for obtaining this information was required.

As shown in Figure 2.1-4, the paper was dried through three periods; a heatup period, a constant-rate period and a falling-rate period. The evaporating rate is higher during the constant-rate period than in the heatup and falling-rate periods (Sharma 1989). According to Figure 2.1-4, the expected profile for paper moisture content of PM3 paper dryer machine is shown as the line of dashes in Figure 5.1-2. The computer experimental runs show it is impossible to find proper coefficients for the model under reasonable performance indices during the constant-rate period.

From considerations of heating, constant rate drying and falling rate drying and from computer experimental runs, the solid line profile in Figure 5.1-2 was selected as the expected profile for paper moisture content.

The sheet moisture content data presented in Figure 5.1-2 is for the paper as it exits each drum draw phase.

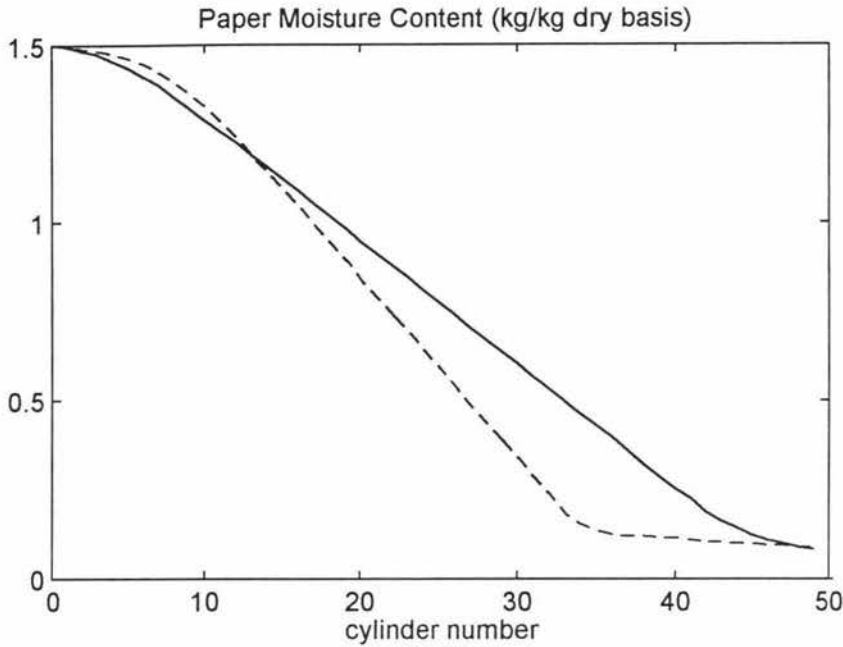


Figure 5.1-2 Expected Profile for Paper Moisture Content

## 5.2 Optimization of Parameters

From equations 2.2-30, 2.2-31, 2.2-32, 2.2-33 and 2.2-34, it can be seen that a significant number of parameters are required. The heat and mass transfer coefficients required are those; between the sheet and the felt ( $h_{pfe}$  and  $C_{mpfe}$ ), between the steam and the sheet ( $h_{vp}$ ), between the sheet and the air ( $h_{pa}$  and  $C_{mpa}$ ). Also required are the affinity coefficient of the water for the sheet ( $k_f$ ) and critical moisture content of the sheet ( $M_c$ ). These parameters are difficult to measure even in controlled environments.

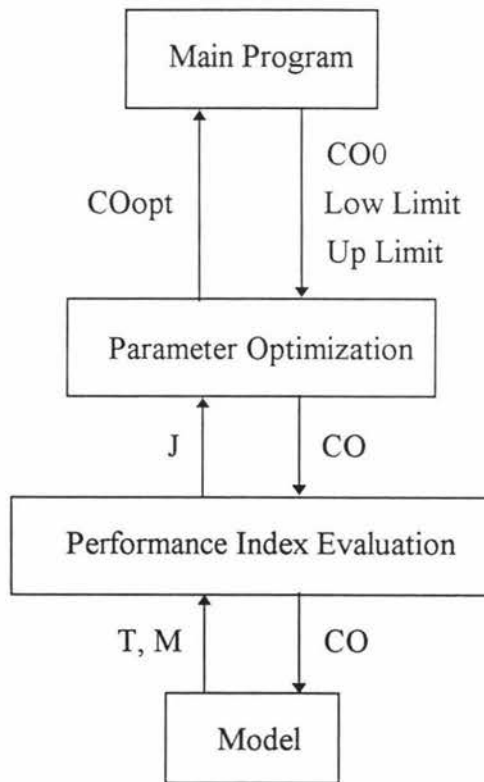
Lemaitre *et al.* (1980) pointed out that it is possible to ignore the heat transfer coefficient between the sheet and the cloth,  $h_{pfe}$ , because the temperatures of the cloth and of the sheet are almost equal and the heat transfer is low relative to the heat coming from the steam. Computer simulations show that the value of this coefficient can vary over a wide range and still have a negligible effect on the drying process.

To make the mathematical model workable, Lemaitre *et al.* (1980) used an identification algorithm to estimate the  $h_{vp}$ ,  $C_{mpfe}$ ,  $h_{pa}$  and  $C_{mpa}$ , but gave no details. They did not report any values for  $k_f$  and  $M_c$ .

In this thesis, a method of optimizing the parameters will be introduced to make the mathematical model for paper drying of Lemaitre *et al.* (1980) useable. The parameters;  $h_{vp}$ ,  $C_{mpfe}$ ,  $h_{pa}$ ,  $C_{mpa}$ ,  $k_f$  and  $M_c$  are those to be determined in order that the model will reasonably represent a particular dryer.

MATLAB makes it possible to optimize the six parameters above by using the Optimization toolbox. Its purpose is to find the best possible solution to a given problem (which may also include a number of limits and constraints). SIMULINK is a tool for modeling, analyzing, and simulating an extraordinarily wide variety of physical and mathematical systems, including those with nonlinear elements and those which make use of continuous and discrete time.

The parameter optimization procedure which was used is shown in Figure 5.2-1.



**Figure 5.2-1** Model Parameters Optimization Procedure

All blocks were written as M-files which are text files that contain MATLAB statements and have a file type of “.m” as the last part of the filename (hence the name

“M-file”). An M-file consists of a sequence of MATLAB statements, possibly including references to other M-files. An M-file can call itself recursively.

For each time step the main program sends vectors of initial coefficients,  $CO_0$ , of the model, coefficient lower and upper limits, initial states (paper temperature and moisture content) in addition to the expected values of paper temperature and moisture content to the parameter optimization file. The file of optimizing parameters sends a vector of model coefficients, initial states and expected values of states to the performance index evaluation file. The performance index evaluation file sends the vector of coefficients to the model file and calls the model at each time step to evaluate a set of ordinary differential equations returning values of paper temperature,  $T$ , and paper moisture,  $M$ . It then produces performance index  $J$ . The file of optimizing parameters uses this performance index to choose new coefficients,  $CO_{opt}$ .

The computer experiments showed that the model parameter optimization procedure took about ten time steps to obtain the optimized model coefficients,  $CO_{opt}$  (see results in section 5.2.5).

### 5.2.1 Programming of the Models

The formulae from 2.2-28 to 2.2-35 are nonlinear ordinary differential equations. By using SIMULINK, the equations of the models can be organized as an S-Function which is also an M-file.

This S-Function defines the dynamics of the model. It is used by the integration, linearization and trim routines to evaluate the dynamics of the system. It has the calling syntax:

```
sys = model(t, x, u, flag)
```

where ‘model’ is the model name and ‘flag’ is used to indicate the type of output to be returned in ‘sys’. For instance when flag is set to 1 “model” returns the state derivatives in the variable,  $sys$ , at the operating point defined by the time,  $t$ , state vector,  $x$ , and input vector,  $u$ . Other values of flag can return the discrete states, system outputs, the values of its root-functions, *etc.*

The detail of organizing the mathematical formulas of the models as an S-Function are shown in appendix 5.2-1.

### 5.2.1.1 Programming of the Knight and Kirk Model

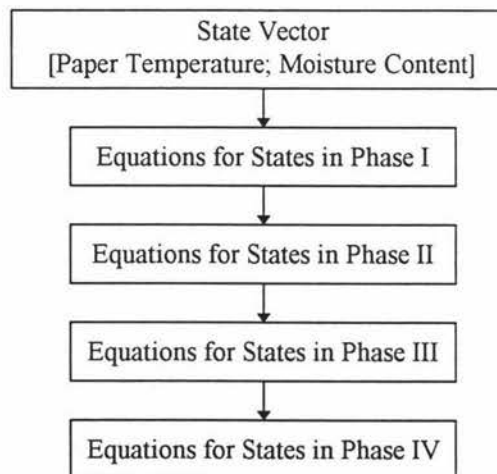
Knight and Kirk (1975) reported that the heat transfer coefficient between the cylinder and paper,  $h_{cp}$ , is  $800 \text{ Wm}^{-2}\text{K}^{-1}$ . This was found to give the minimum average deviation between the theoretical and actual profiles. The heat transfer coefficient between the paper and outside air in phases I and III,  $h_{pa}$ , can be solved by using equation 2.2-22. The mass transfer coefficient between paper and felt is given by equation 2.2-25. In equation 2.2-25,  $\varepsilon$  was obtained from experimental data and the derivation of  $D$  was not explained.

$\varepsilon$  was calculated by using equations 2.2-12, 2.2-13, and  $D$  was obtained by using the formulae from ASHRAE handbook Fundamentals (1993, page 5.2) as follows:

$$D = \frac{0.926 \times 10^{-6} T^{2.5}}{P(T + 245)} \quad (5.2-1)$$

where  $P$  is in kPa.

Figure 5.2-2 shows a schematic block diagram of the Knight and Kirk's model.



**Figure 5.2-2** Schematic Block Diagram of the Knight and Kirk's model

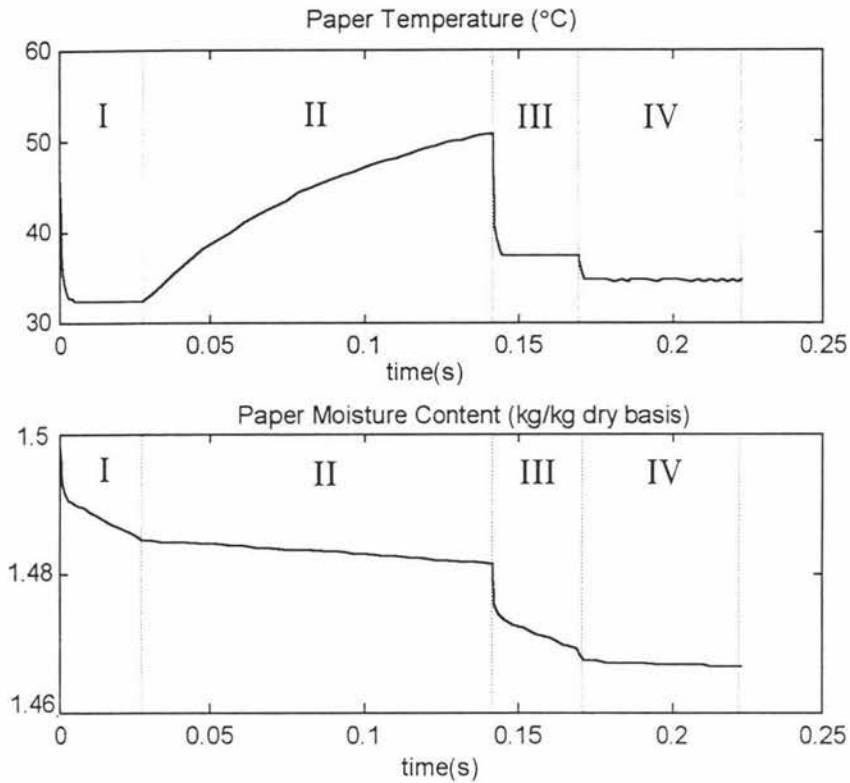
The program block diagram and code can be found in appendix 5.2-1. The numerical values of the heat transfer coefficients between paper and outside air are calculated from equations 2.2-12, 2.2-13, 2.2-22 to 2.2-25 and 5.2-1.

The “RK45” command in MATLAB is used to simulate the model. The “RK45” integrates a system of ordinary differential equations using the Runge-Kutta fifth order method. The syntax for “RK45” is as follows:

```
[T,X,Y]=RK45('SYSTEM',TF)
```

RK45('SYSTEM',TF) integrates the system of ODEs described in the SIMULINK system (or other S-functions) SYSTEM from 0 to TF 'time' units (typically but not necessarily seconds) and returns, without plotting, the integration time points, T, the output trajectories, Y, and the state trajectories, X.

The simulation results for first drum (Figure 5.2-3) show that the model of Knight and Kirk works very well at phase II; as expected the paper temperature rises considerably with little moisture loss. It also shows that the model produces unreasonable predictions for phase I, III and IV. It is not expected that the paper temperature and moisture content will drop so much during phase I and III, because the paper spent only a short time in both phase I and phase III and one side of the paper was still in contact with the cylinder so the paper temperature and moisture content change should have been smaller. For phase IV, the paper sheet loses both water and heat to the surrounding air and the paper temperature and moisture content should change considerably.



**Figure 5.2-3** Paper Temperature and Moisture Content at First Cylinder for the Knight and Kirk's Model

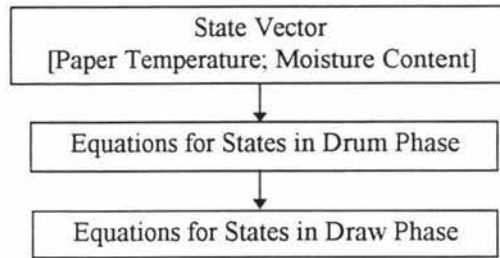
Computer experiments also showed that an order of magnitude change in any of the coefficients had little affect on the paper draw phase exit temperature. This indicates that it was impossible for the model to reach the expected values by adjusting the model coefficients.

For this reason, the model of Knight and Kirk was not explored further.

### 5.2.1.2 Programming of the Lemaitre Model

Figure 5.2-4 shows the schematic block diagram of the Lemaitre's model. The program block diagram and program code is listed in appendix 5.2-2.

For numerical reasons the MATLAB optimization algorithm requires all coefficients to be of the same order of magnitude. Thus the coefficients of the Lemaitre's model had to be scaled in order to use this algorithm to obtain optimal coefficient values.



**Figure 5.2-4** Schematic Block Diagram of the Lemaitre's model

## 5.2.2 Program for Evaluating Performance Indices

The program for evaluating performance indices is in appendix 5.2-3.

The performance index evaluation file accepts a vector of model coefficients, initial states and expected values of states from the file of optimizing parameters, produces performance index  $J$  for the file of optimizing parameters and quotes the command "GLOBAL" to send the vector of coefficients to the model file and performance index  $J$ ,  $y_j$  and  $x_j$  to the main program file. The command "GLOBAL" defines global variables. Ordinarily, each MATLAB function, defined by an M-file, has its own local variables, which are separate from those of other functions, and from those of the base workspace and non-function scripts. However, if several functions, and possibly the base workspace, all declare a particular name as GLOBAL, then they all share a single copy of that variable. Any assignment to that variable, in any function, is available to all the other functions declaring it GLOBAL.

## 5.2.3 Program for Optimizing Parameters

A listing of the program for optimizing parameters is attached at appendix 5.2-4.

The optimizing parameters file uses the "CONSTR" command in MATLAB to find the best possible model coefficients which make the predicted values of paper temperature and moisture content match the expected values (minimum performance index  $J$ ) whilst constraining the coefficients to be within user supplied upper and lower limits.

The syntax for "CONSTR" is:

`X=CONSTR('FUN',X0,OPTIONS,VLB,VUB)`

`X=CONSTR('FUN',X0,OPTIONS,VLB,VUB)` starts at `X0` and finds a constrained minimum to the function which is described in `FUN` (usually an M-file: `FUN.M` which should return two arguments; a scalar value of the function to be minimized, `F`, and a matrix of constraints, `G`), allows a vector of optional parameters (used by the optimization routines) to be defined and defines a set of lower and upper bounds on the design variables, `X`, so that the solution is always in the range  $VLB < X < VUB$ .

For the optimizing parameters file, the `FUN` is the function `CPERFORM` (evaluate performance indices), `X` is a vector of model coefficients, `F` is performance index `J` and `G` is an empty matrix because there is no constrained function for the model coefficients.

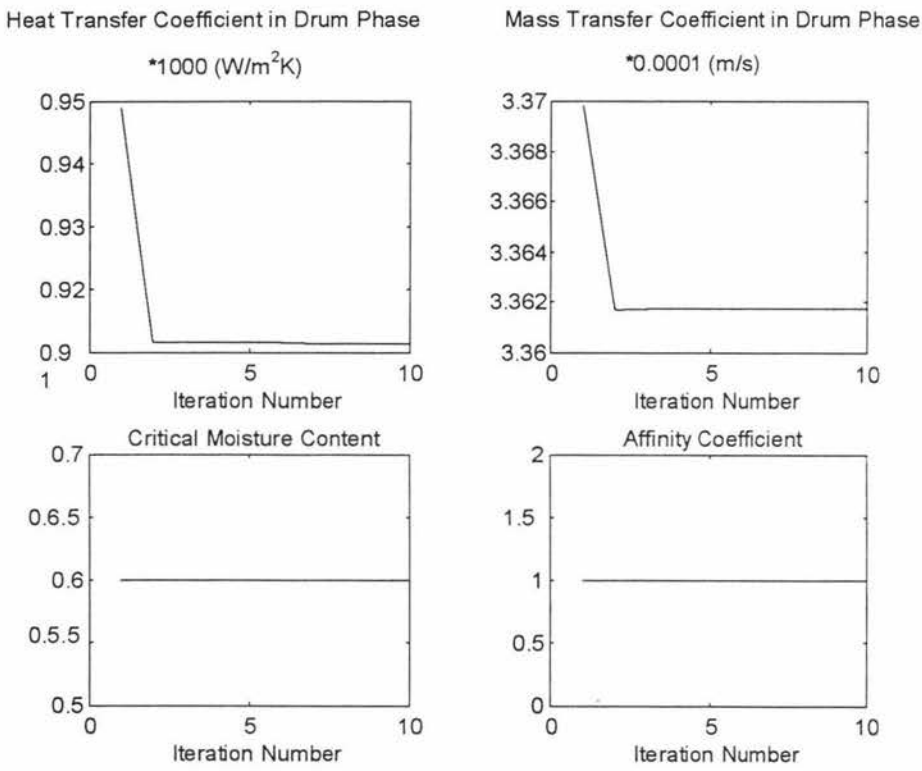
#### 5.2.4 Main Program

The main program sets a vector of initial coefficients `CO0`, a vector of low limit coefficients, a vector of high limit coefficients, initial states (paper temperature and moisture content), expected values of paper temperature and moisture content and the number of iterations the optimization process is going to use. It calls the function `MCP` (model coefficients predictor) and prints out the results. A listing of the main program is in Appendix 5.2-5.

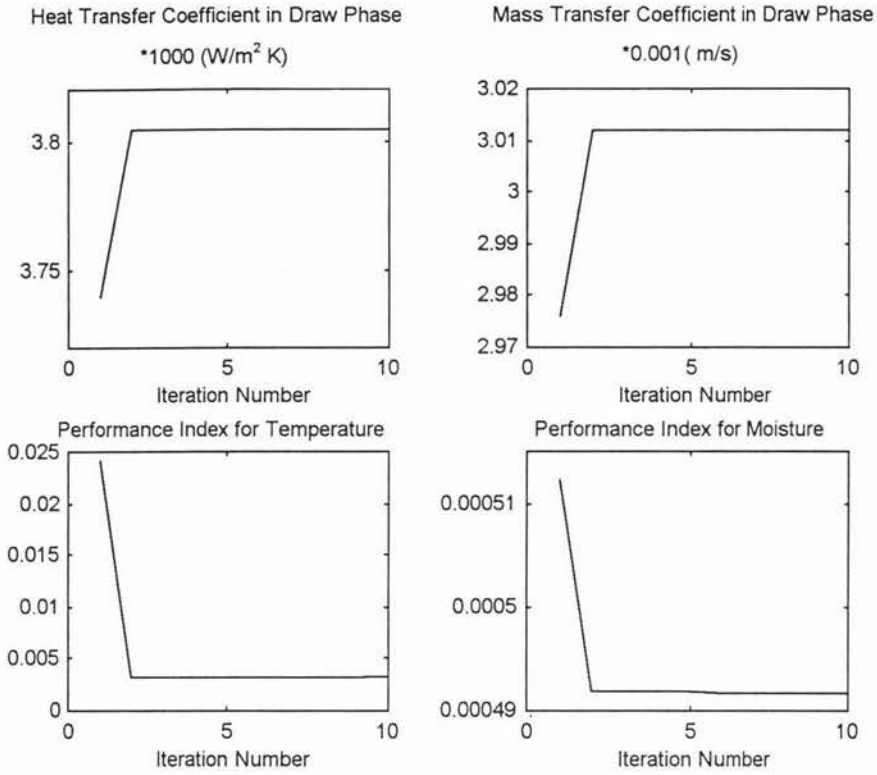
In order to optimize the parameters for the 49 cylinders, the program of the Lemaitre's model and the main program were changed as shown in Appendix 5.2-6. When the parameters of the 14th to 49th cylinders are optimized the time the paper spends in the drum phase and the time for one cycle (drum phase and draw phase) in the programs (program of the Lemaitre's model, program for evaluating performance indices and main program) must be changed from 0.1698s and 0.2231s to 0.1715s and 0.2385s. This was to accommodate the different geometry of drums 14 to 49 which had large drum and draw phases. During optimization the low and high limits and initial values of coefficients of the model were altered to eliminate the occurrence of complex valued solutions to the equations which, clearly, did not represent real phenomena.

**5.2.5 Results and Discussion**

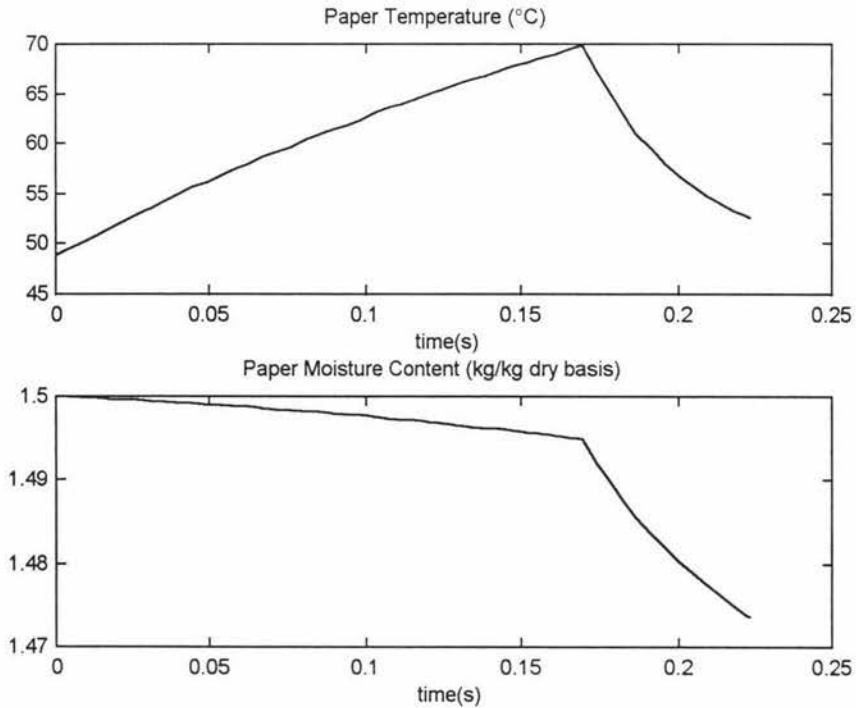
The results of optimizing the parameters for the first cylinder (Figures 5.2-5 and 5.2-6) show that convergence is rapid for all parameters and that 10 iterations are sufficient. The reason for critical moisture content and affinity coefficient are not change in Figure 5.2-5 is that the paper moisture content in first cylinder is larger than critical moisture content and affinity coefficient can't be used (see Equations 2.2-34 and 2.2-35).



**Figure 5.2-5** Heat and Mass Transfer Coefficients in Drum Phase, Critical Moisture Content and Affinity Coefficient Against Iteration Number



**Figure 5.2-6** Heat and Mass Transfer Coefficients in Draw Phase and Performance Indices for Temperature and Moisture Against Iteration Number

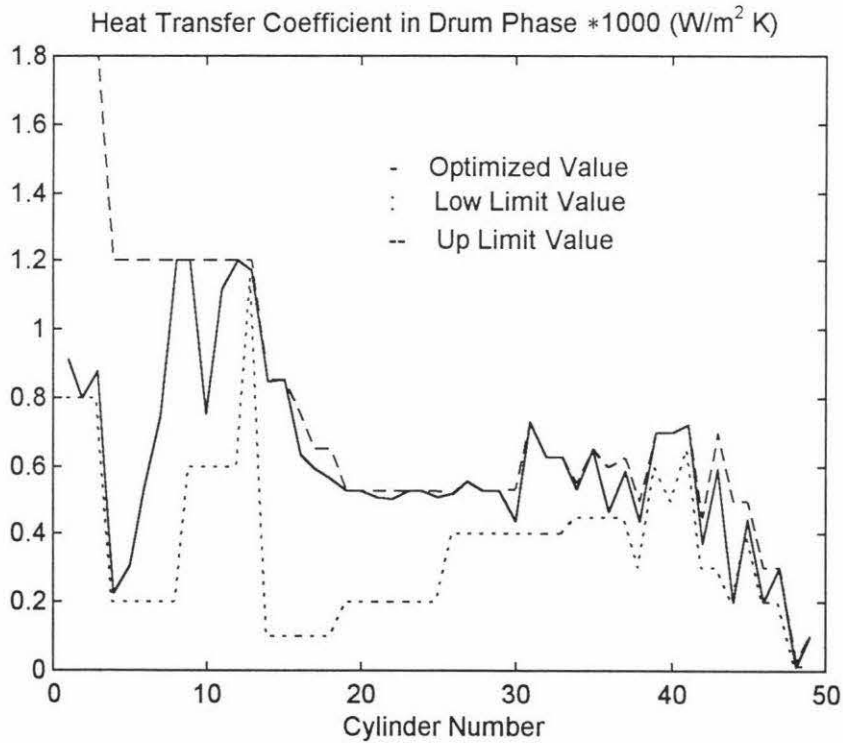


**Figure 5.2-7** Paper Temperature and Moisture Content in First Cylinder

Figure 5.2-7 shows the results of simulating the paper temperature and moisture content behavior in the first cylinder using the newly optimized parameters in the Lemaitre’s model. As expected in paper drying in the drum phase the paper temperature rises markedly whilst the moisture content decreases slightly. In the draw phase the paper temperature drops and the moisture content decreases.

The numerical results for the optimized parameters of the 49 cylinders are shown in Appendix 5.2-7.

The optimized parameter values for each cylinder with their upper and lower limits are shown in Figure 5.2-8 ~13.



**Figure 5.2-8** Heat Transfer Coefficient in Drum Phase Against Cylinder Number

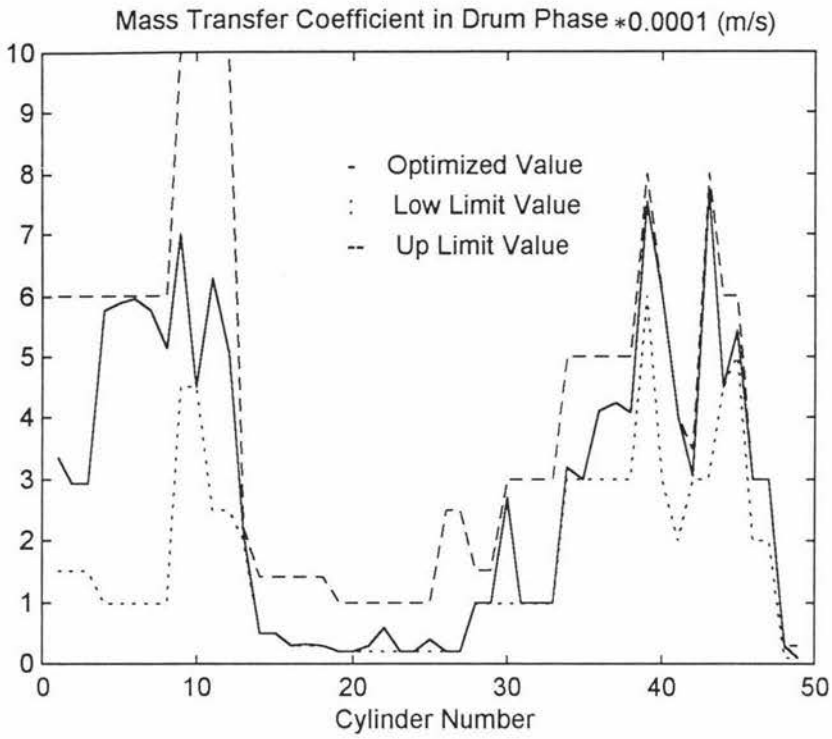


Figure 5.2-9 Mass Transfer Coefficient in Drum Phase Against Cylinder Number

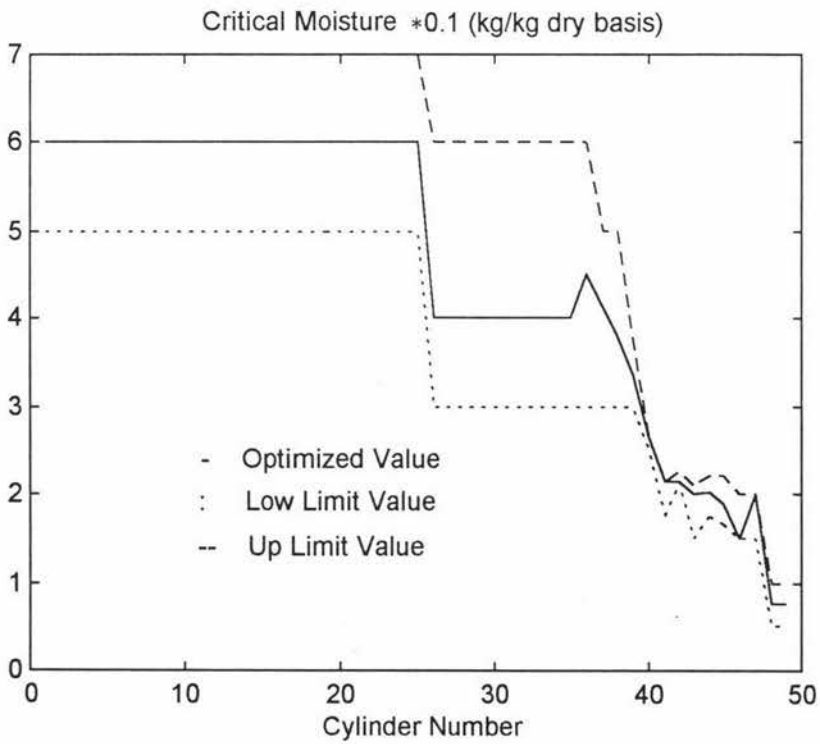


Figure 5.2-10 Critical Moisture Content Against Cylinder Number

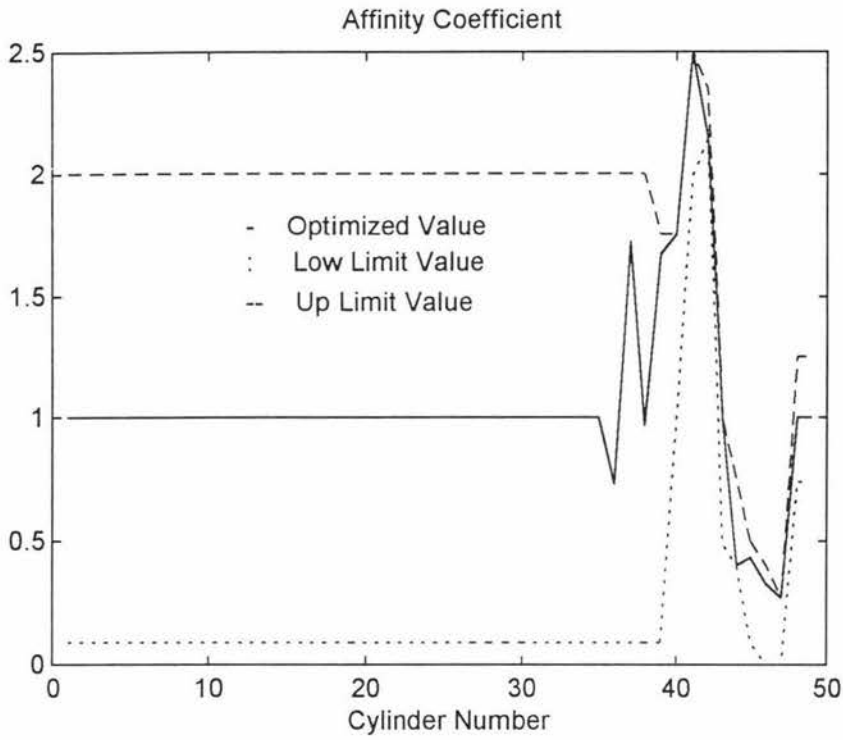


Figure 5.2-11 Affinity Coefficient of the Water for the Sheet Against Cylinder Number

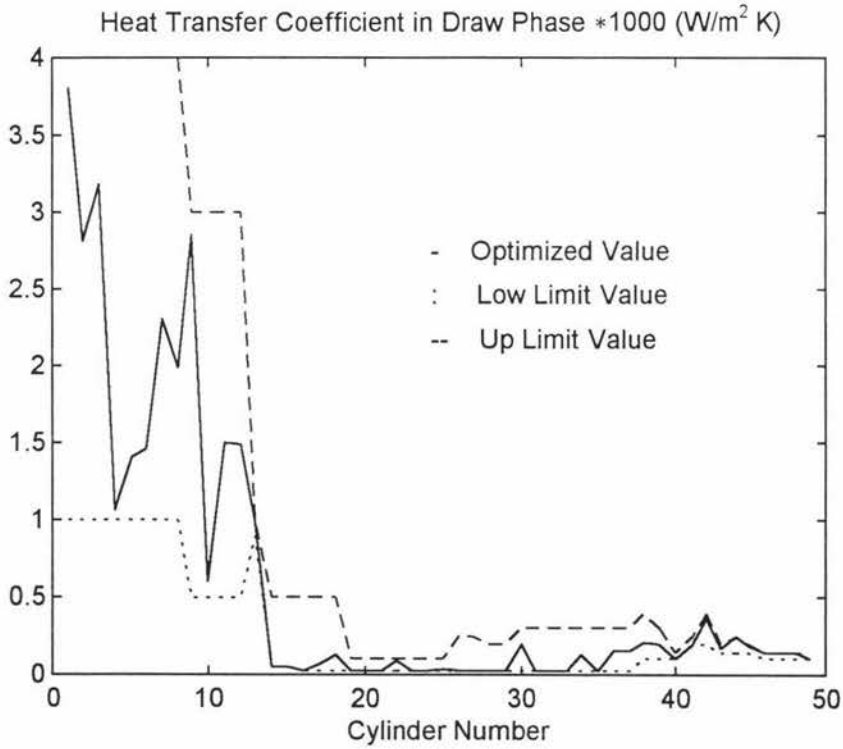
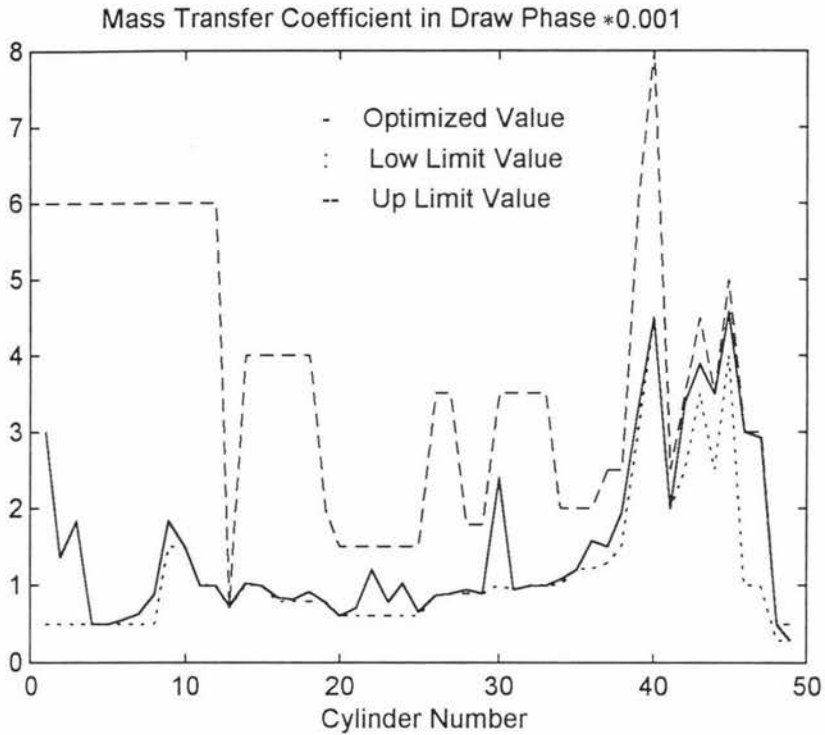


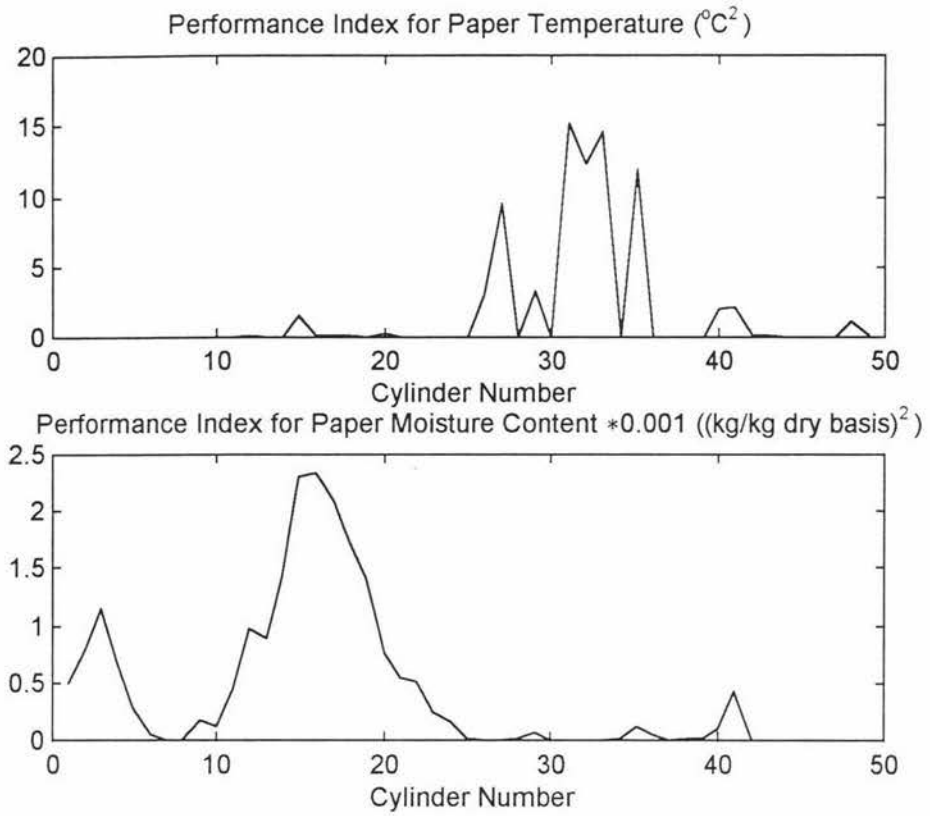
Figure 5.2-12 Heat Transfer Coefficient in Draw Phase Against Cylinder Number



**Figure 5.2-13** Mass Transfer Coefficient in Draw Phase Against Cylinder Number

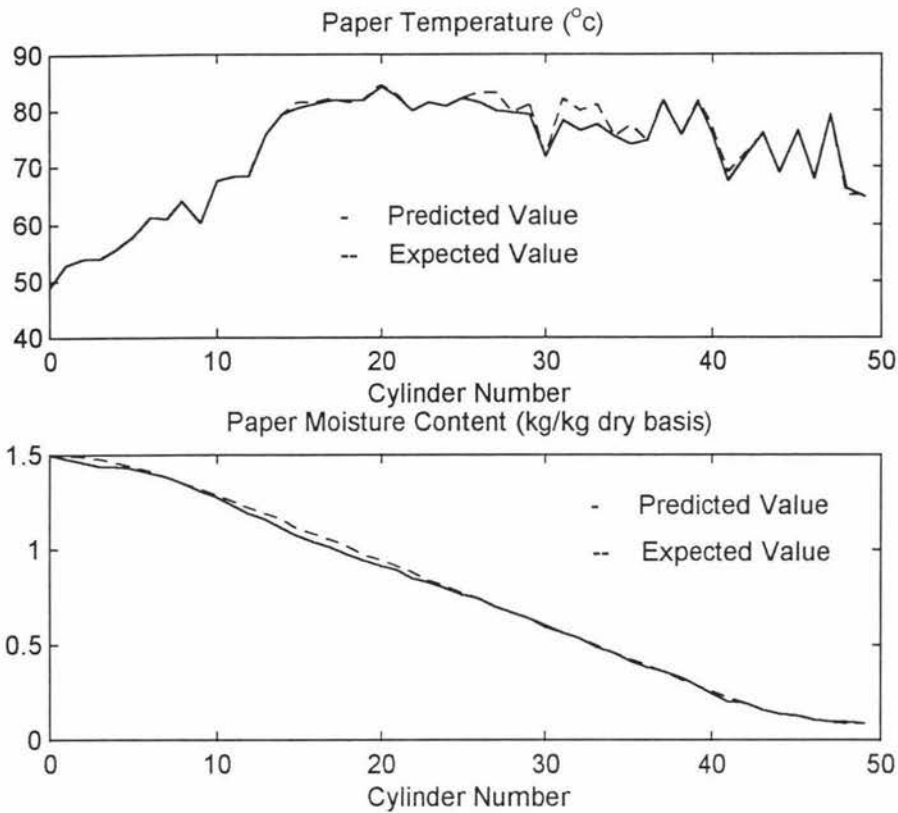
In order to ensure that complex number results were not generated the coefficient limits had often to be set very tight, usually but not always in the higher numbered cylinders. It can be seen from Figures 5.2-8 ~ 13 the parameters vary widely from cylinder to cylinder which could indicate that there is significant noise in the supplied drying section data.

The performance indices for paper temperature and moisture content against every cylinder are shown in Figure 5.2-14.



**Figure 5.2-14** Performance Indices for Paper Temperature and Moisture Content Against Cylinder Number

The paper temperature and moisture content in every cylinder are shown in Figure 5.2-15.



**Figure 5.2-15** Paper Temperature and Moisture Content Against Cylinder Number

From Figure 5.2-14 and Figure 5.2-15, it can be seen that the predicted values of paper temperature and moisture content of most cylinders coincides with the expected values. The average relative error for paper temperature is 0.81% and for paper moisture content is 1.86%. The maximum error for paper temperature is 3.89°C and for paper moisture content is 0.0484 kg/kg dry basis.

By taking different vectors of initial values, lower limit values and upper limit values for model coefficients, it may be possible to find different optimized coefficients. It is important that the optimized coefficients can make the model predict the values of paper temperature and moisture content correctly.

### 5.3 References

Knight, R. L. and Kirk, L. A., 1975, "Simulation of the Papermachine Drying Section," *International Water Removal Symposium*, British Paper and Board Industries Federation - London (March 1975), pp. 188-226.

Lemaitre, A., Veyre, J., Lebeau, B. and Foulard, C., 1980, "Case Study: Method for Systematic Analysis of Paper Machine Multicylinder Drying Section," *4th IFAC-PRP 4 Conference*, Ghent-Belgium (3-5 June 1980), pp. 261-270.

*Matlab For Microsoft Windows Reference Guide (1992)*, The MathWorks, Inc..

*Matlab For Microsoft Windows User's Guide (1993)*, The MathWorks, Inc..

Sharma, R., 1989, "Using Infrared Selectively for Improving Paper Quality and Production," *Practical aspects of pressing and drying seminar*, Atlanta, Ga, pp. 85-92.

*Tasman PM3*, 1994, Albany International.

*1993 ASHRAE handbook Fundamentals*, SI Edition, American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc., N. E., Atlanta, GA, Chapter 5.

## 6. SENSITIVITY TESTS OF THE MODEL

### 6.1 Programming of Simulation Model

As a result of the model fitting of Chapter 5, a workable mathematical model of Lemaitre *et al.* was obtained. For simulating the whole paper drying section which contains 49 cylinders, the 49 sets of optimized parameters were inserted into the model. The simulation program of the model of Lemaitre for the 49 cylinders is attached in Appendix 6.1-1 (as AJ3.m).

In Appendix 6.1-1,  $k$  corresponds to cylinder number and  $td$  corresponds to the time the paper spends in the drum phase. The values of  $k$  and  $td$  are defined by the program which runs AJ3.m (AJ3RK.m in Appendix 6.1-2). The parameter  $t$  is the time that the paper spends in the drum and draw phases.

The model was used to simulate a number of changes to the base conditions, namely, steam temperature, drum speed, initial paper conditions and ambient air conditions. These are described individually in the following sections.

### 6.2 Change of Steam Temperature

Studying the effect of steam temperature on the final paper quality is one of the most important topics for improving energy efficiency.

Varying the steam temperature to the simulation model for the 49 cylinders (fourth column of matrix "data" in Appendix 6.1-1) produced corresponding changes in final paper moisture content (Figure 6.2-1). The simulation was run with the steam temperature of one section varied while the other three sections were held constant (4 runs). Another run was carried out with variations to the steam temperatures of all four sections. For the former runs the steam temperature of one section (drums 1-3, 4-13 and 42-49) was varied by  $\pm 10\%$  and (drum 14-41) by  $\pm 2\%$ . For the latter run the steam temperature of all four sections was varied by  $\pm 2\%$ . The computer simulation results are shown in Table 6.2-1 (Appendix 6.2-1).

During the simulations complex-number results were sometimes obtained. In the program for running AJ3.m of Appendix 6.1-2, the MATLAB command 'RK45' had to

be changed to ‘RK23’ when this occurred the fifth order Runge-Kutta integration method, which uses a fourth order method for step size control, was replaced by the third order Runge-Kutta method, which uses a second order method for step size control. The discontinuity in the paper temperature and moisture content may not have been severe enough to have caused the RK45 algorithm to have identified the existence of a discontinuity. Thus result after the discontinuity could be in error and might well then have produced complex results. The RK23 algorithm would use a shorter step length to achieve the same error tolerances as RK45 and so the errors beyond the discontinuity would have been smaller.

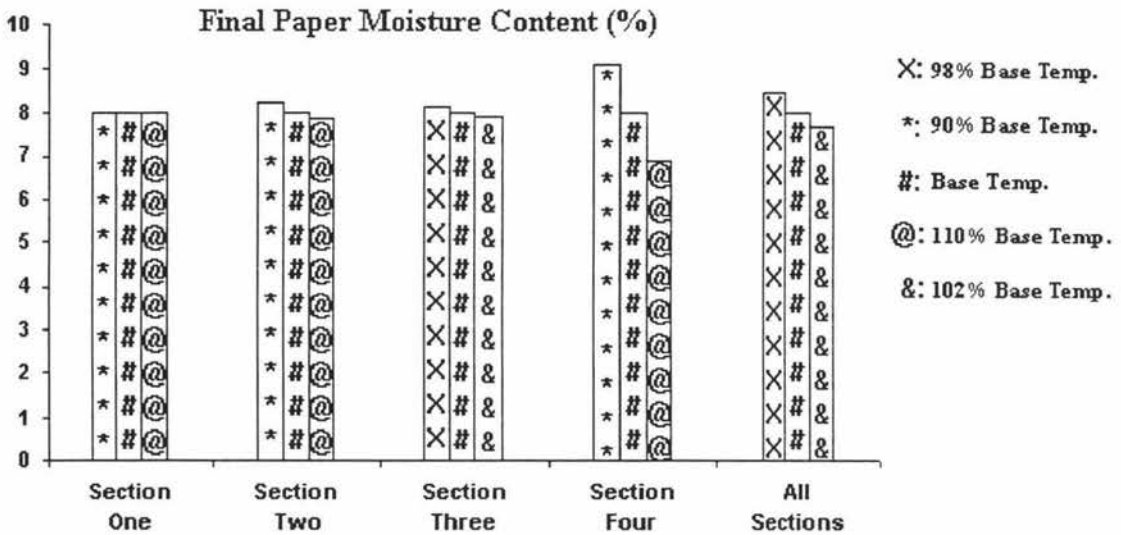


Figure 6.2-1 Final Paper Moisture Content Against Different Steam Temperature

### 6.3 Change of Drum Speed

Speeding up the drum speed means increasing the output of the paper and, presumably the moisture content. Slowing down the drum speed can ensure the quality of paper, i.e. that paper is dried enough. The simulation was run with variations of speed of  $\pm 5\%$  and  $\pm 7.5\%$ . This was done by varying  $td$  and  $t$  in the simulation program (Appendix 6.1-2). The simulation results are shown in Figure 6.3-1.

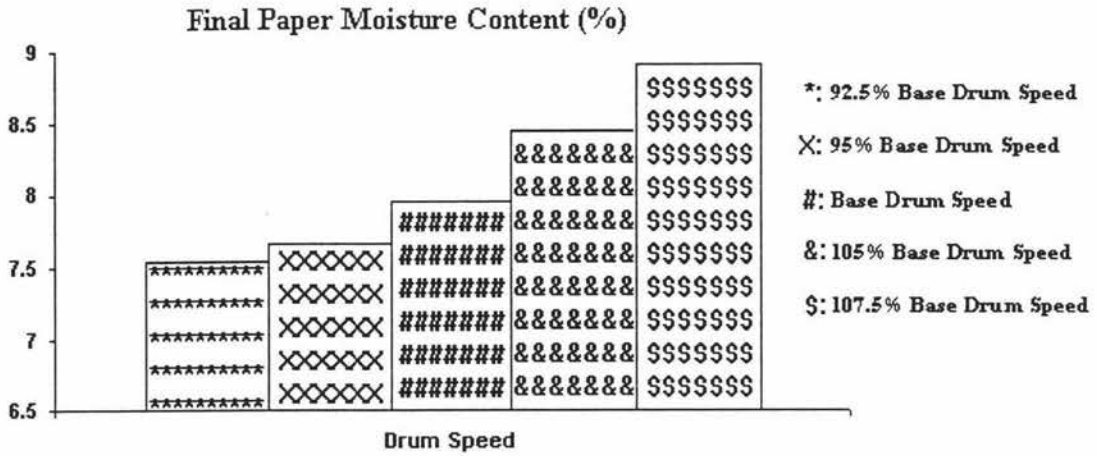


Figure 6.3-1 Final Paper Moisture Content Against Different Drum Speed

The simulation results are also listed in Table 6.3-1 (Appendix 6.2-1).

It is very important to pursue maximum paper production and to ensure the quality of paper simultaneously. A simulation was run to increase the output of paper 10% by speeding up drum speed 10% while maintaining the final paper moisture content at about 8% by adjusting the steam temperature. The result is shown in Table 6.3-2 (Appendix 6.2-1). The required steam temperatures were as Figure 6.3-2.

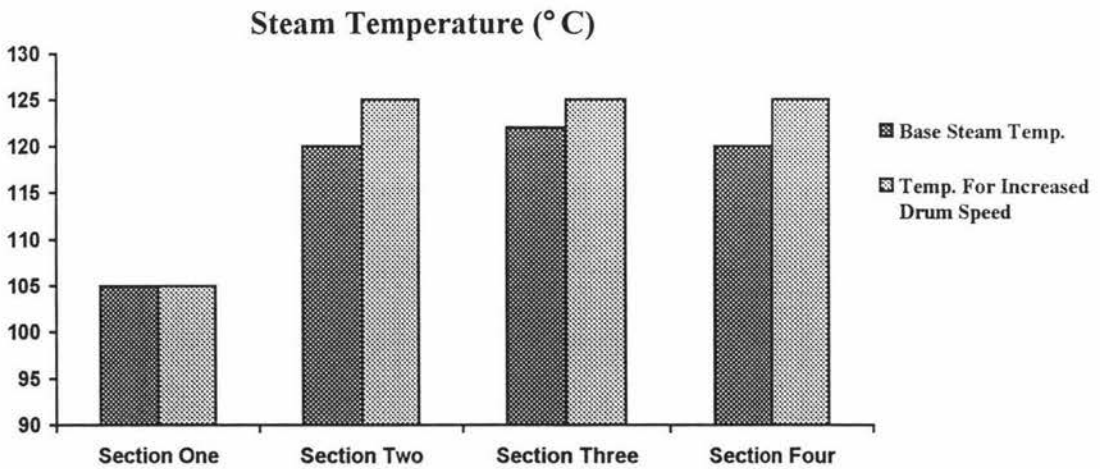
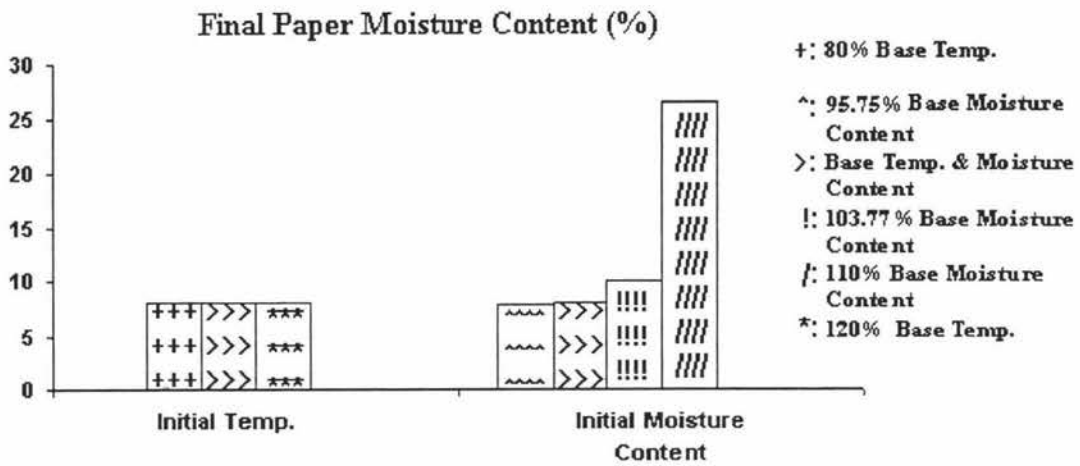


Figure 6.3-2 Steam Temperature Against Different Dryer Section

### 6.4 Change of Paper Initial Condition

When the initial conditions of the paper alter (temperature and moisture content) it is important to be able to predict the resulting paper condition at the exit of the dryer

section. For the simulations, the paper initial condition is changed by modifying x0 in the simulation program (Appendix 6.1-2). The initial paper temperature was varied by  $\pm 20\%$  while the initial moisture content was kept constant, and initial paper moisture content was varied by +10% (+0.4412 kg/kg dry basis), +3.77% (+0.15 kg/kg dry basis) and -4.25% (-0.15 kg/kg dry basis) while the initial paper temperature was kept unchanged. The simulation results are shown in Table 6.4-1 (Appendix 6.2-1) and Figure 6.4-1.



**Figure 6.4-1** Final Paper Moisture Content Against Initial Paper Temperature and Moisture Content

### 6.5 Change of Ambient Air Condition

The ambient air conditions for the paper are the temperature and relative humidity of the air over the cloth and the pocket. Varying the temperature and relative humidity of air causes the partial vapour pressure of the ambient air to change.

The temperatures of the air over the cloth in three hoods are 83°C, 91°C and 89°C (Albany International, 1994) which were held unchanged and their humidity values were varied in the simulation runs. It was assumed that fresh air at -4°C, humidity 30% and 23°C, humidity 80% respectively were heated to the temperatures of the air in the three hoods. From the Psychrometric Chart, the new relative humidities of the hot air were obtained and the partial vapour pressures were calculated as 536.64 Pa, 511.02 Pa and 541.35 Pa (Air Conditions I) for the former and 2361.21 Pa, 2409.11 Pa and 2436.06 Pa (Air Conditions II) for the latter fresh air conditions. For the simulations, the air conditions over the cloth were changed by modifying the first column of matrix “data” in

the simulation model (Appendix 6.1-1). The computer simulation results are shown in Table 6.5-1 (Appendix 6.2-1) and Figure 6.5-1.

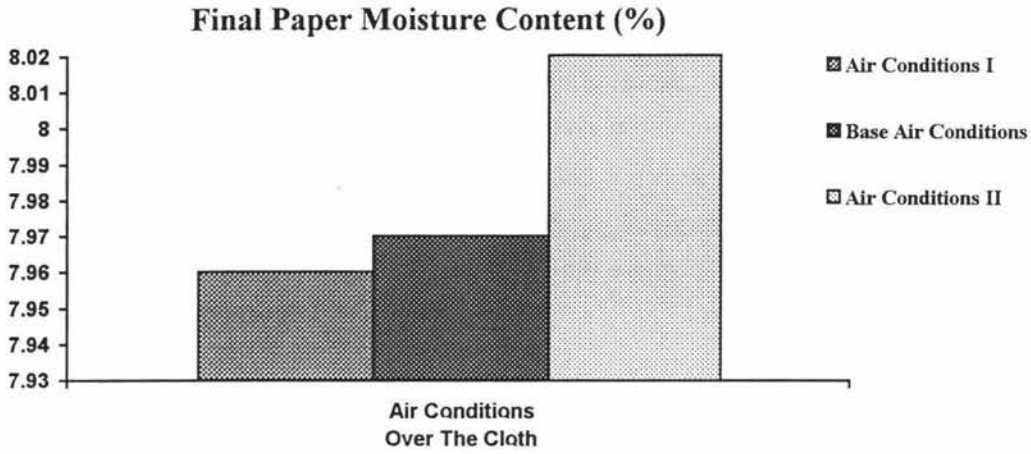


Figure 6.5-1 Final Paper Moisture Content Against Air Conditions Over the Cloth

For the air in the pockets, it was assumed that the temperature of the air varied by  $\pm 5\%$  while the relative humidity was kept unchanged and that the relative humidity of air varied by  $\pm 10\%$  while the temperature of air was kept unchanged. The temperature and vapour pressure of air in the pockets for each drum are listed in Table 6.5-2 (Appendix 6.2-1). For the simulations, the air conditions in the pocket were changed by modifying the second column (for partial vapour pressure) and third column (for air temperature) of matrix “data” in the simulation model (Appendix 6.1-1). The computer simulation results are shown in Table 6.5-3 (Appendix 6.2-1) and Figure 6.5-2.

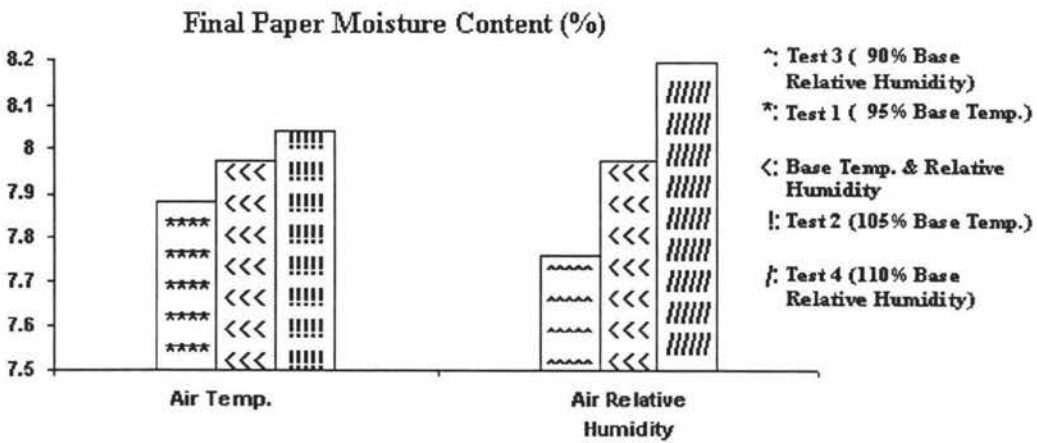


Figure 6.5-2 Final Paper Moisture Content Against Air Conditions in the Pockets

## 6.6 Discussion

The preceding sections in this chapter have outlined the design of the computer simulations and the simulation results. A number of variables such as steam temperature, drum speed, initial paper temperature, initial paper moisture content, ambient air temperature and relative humidity were covered to investigate their affect on paper drying. The results are based upon the Lemaitre's model as fitted to experimental data for the PM3 machine at Tasman Pulp and Paper Limited. Since the validity of this data and fitting are to some extent questionable the results presented here should be viewed with caution.

From Figure 6.2-1, the computer simulations show that the steam temperature for drums 1-3 affect the final paper moisture content the least and the steam temperature for drums 42-49 affects the final paper moisture content the most. It also answers the question – why the drum temperature of the first section is kept lower – since its effect is small and it would waste steam to raise the drum temperature.

Figure 6.3-1 and Figure 6.3-2 show that the drum speed influences the final paper moisture content and it should be possible to increase the output by adjusting the steam temperature. From the simulation results we would expect the drum speed increase to increase the final paper moisture content. Whilst increasing the paper output, it is also very important to consider economic factors, *e.g.* raising steam temperature may need more equipment and energy to produce the higher pressure steam.

The initial paper temperature affects the final paper moisture content less than the initial paper moisture content (Figure 6.4-1). According to the steady state data of PM3 paper dryer machine (Albany International, 1994) and simulations results the initial paper temperature could be changed  $\pm 20\%$  and has little influence in the final paper moisture content. It is very important to control the initial paper moisture content which is the one of the key factors affecting the paper quality.

The vapour partial pressure over the cloth influence the final paper moisture content less then were shown in Figure 6.5-1 and the temperature and relative humidity of the pocket air should influence the finial paper moisture more then were shown in Figure 6.5-2. These simulation results also show that the efficiency of the dry pocket ventilation is very important.

The simulation results show the correct trends and are therefore basically reasonable. The fitted Lemaitre's model could be used to determine the possibilities of improving paper dryer performance and to answer questions on the operation of the dryer. More accurate results would require better initial data including dynamic measurements across drums to help determine heat and mass transfer coefficients.

## 6.7 Reference

*Tasman PM3*, 1994, Albany International.

## 7. CONCLUSIONS

In this thesis a number of mathematical models of a paper machine drying section were investigated as candidates for fitting to empirical data. Following the guidelines in Chapter 3 the models of Knight and Kirk (1975) and Lemaitre *et al.* (1980) were chosen. Both models were implemented in SIMULINK as S-Functions using either the "RK45" or "RK23" Runge-Kutta integration algorithms. The model of Knight and Kirk (1975) was not explored in depth because the simulation showed that it was impossible for the model to reach observed values by adjustment of the model coefficients. The mathematical formulae for partial pressure of water vapour at the sheet surface, specific heat of water and latent heat of vaporization against temperature work well in the Lemaitre's model.

Fitting of the parameters in the Lemaitre's model to steady state data from the PM3 paper machine at Tasman Pulp and Paper Ltd. produced an optimally fitted model. Difficulty was encountered in the fitting process with the stability of the simulation. The upper and lower limits of the optimization function were used to ensure stability. The predictions of this model appeared reasonable in the physical sense.

The sensitivity tests of the model show that the output responses of the paper machine drying section (final paper temperature and moisture content) were reasonable. The final fitted Lemaitre's model was stable and workable.

The accuracy of the simulations, with regard to the PM3 machine, are somewhat questionable in view of the poor quality of the original data, the instability of the equations during the fitting process and the arbitrary moisture content profile during the drying section.

## References

Knight, R. L. and Kirk, L. A., 1975, "Simulation of the Papermachine Drying Section," *International Water Removal Symposium*, British Paper and Board Industries Federation - London (March 1975), pp. 188-226.

Lemaitre, A., Veyre, J., Lebeau, B. and Foulard, C.,1980, "Case Study: Method for Systematic Analysis of Paper Machine Multicylinder Drying Section," *4th IFAC-PRP 4 Conference*, Ghent-Belgium (3-5 June 1980), pp. 261-270.

*Matlab For Microsoft Windows Reference Guide (1992)*, The MathWorks, Inc..

*Matlab For Microsoft Windows User's Guide (1993)*, The MathWorks, Inc..

*Tasman PM3*, 1994, Albany International.

## **8. RECOMMENDATIONS**

The method of optimising parameters is not only used in the model of a paper dryer of Lemaitre *et al.* but other mathematical models of ordinary different equations also. In the future, when more dynamical, analytical models are available, the method of optimising parameters could be used again to make them workable and to choose the best one.

Felt tension is another important factor affecting the paper drying but is not included in the Lemaitre's model. The thermal contact conductance increases with increasing felt tension (Asensio and Seyed-Yagoobi 1993). When the formula for the heat transfer coefficient between steam and paper sheet are available then equation 2.2-11 should be introduced to study the felt tension action for the paper drying.

The fitted Lemaitre's model can be used to:

1. determine the possibilities of improving paper dryer performance without altering the real plant;
2. better understand the dynamics of the machine from a sensitivity analysis;
3. answer 'what if' questions easily and quickly.

The model might also be used in areas such as control system design or studies.

## **References**

Asensio, M. C. and Seyed-Yagoobi, J., 1993, "Simulation of Paper-Drying System With Incorporation of an Experimental Drum/Paper Thermal Contact Conductance Relationship," *ASME Journal of Energy Resources Technology*, Vol. 115, pp. 291-300.

NOMENCLATURE

A	a constant
b	width (m)
c	specific heat capacity ( $\text{J kg}^{-1} \text{K}^{-1}$ )
C	molar concentration ( $\text{mol m}^{-3}$ )
$C_m$	mass transfer coefficient ( $\text{m s}^{-1}$ )
D	diffusivity ( $\text{m}^2 \text{s}^{-1}$ )
E	evaporation rate ( $\text{kg m}^{-2} \text{s}^{-1}$ )
h	heat transfer coefficient ( $\text{W m}^{-2} \text{K}^{-1}$ )
$h_i$	interface thermal contact conductance ( $\text{W m}^{-2} \text{K}^{-1}$ )
i	thickness direction
$i = I$	external surface
$i = 0$	internal surface
J	mass flux ( $\text{kg m}^{-2} \text{s}^{-1}$ )
k	thermal conductivity ( $\text{W m}^{-1} \text{K}^{-1}$ )
$k_f$	(affinity) coefficient of the water for the sheet
K	permeability ( $\text{m}^2$ )
L	length (m)
m	mass of water in sheet ( $\text{kg m}^{-3}$ )
M	moisture content, dry basis (weight of water/weight of dry fibers)
P	pressure ( $P_a$ )
$P_f$	partial pressure of water vapour at sheet surface ( $P_a$ )
$P_a$	partial pressure of water vapour in the air ( $P_a$ )
Q	heat flux ( $\text{W m}^{-2}$ )
q	heat (W)
R	gas constant ( $\text{JK}^{-1} \text{mol}^{-1}$ )
$R^2$	adjusted coefficient of determination
r	moisture content / critical moisture content
t	time (s)
$\Delta t$	time interval in phase or in subphase (s)
s	saturation (volume of liquid/volume of voids)
S	surface ( $\text{m}^2$ )
T	paper temperature (K)
$T_o$	temperature of wet-dry interface (K)
$T_1$	temperature of dry side (K)
$\Delta T$	$= T - T_o$
$\Delta T_1$	$= T_1 - T_o$
$T_e$	temperature of paper as it exits drum (K)
u	steam temperature (K, °C)
V	velocity ( $\text{m s}^{-1}$ )
w	molecular weight ( $\text{kg kg}^{-1} \text{mol}^{-1}$ )
W	weight of moisture-free solids (basic weight) ( $\text{kg m}^{-2}$ )
y	mole fraction
z	paper thickness direction (from cylinder surface to air) (m)
Z	web thickness; spacing (m)

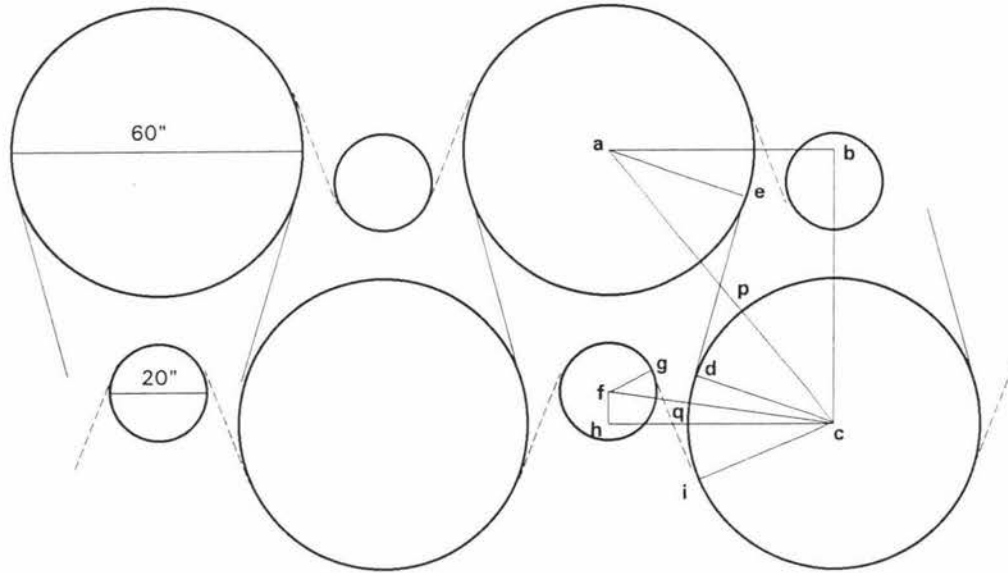
$Z_{fe}$	felt thickness (m)
$\alpha$	thermal diffusivity ( $W m^{-2} K^{-1}$ )
$\epsilon$	porosity (volume of voids / total volume)
$\lambda$	latent heat of evaporation of water ( $J kg^{-2}$ )
$\nu$	kinematic viscosity ( $m^2 s^{-1}$ )
$\theta$	dimensionless temperature
$\rho$	density ( $kg m^{-3}$ )
$\sigma$	standard deviation
$\omega$	moisture content, wet basis (weight of water / (weight of water + weight of dry fibers))

### Subscripts

A	constant
a	air
c	critical, cylinder
ca	capillary
e	equilibrium
E	water film
f	fiber
fe	felt
fin	exit of dryer section
g	gas
o	exit drum phase
p	paper
s	saturated
su	surface
$\Delta t$	time interval in phase or in subphase
v	vapour
w	water
0	initial

**Appendix 4.2-1: Calculation of  $t_I$ ,  $t_{II}$ ,  $t_{III}$  and  $t_{IV}$**

The layout of cylinders and felt rolls for cylinders 1-13 is shown in Figure 4.2-1.



**Figure 2.2-1** Typical dryer arrangement (Drums 1-13)

The relevant parameters of the PM3 paper machine are given below (the error bounds for the parameters are not exactly known, but they are estimated to be about  $\pm 0.01$  inch in length or about  $\pm 0.001 \text{ m s}^{-1}$  in paper speed):

Paper Speed =  $1038.3 \text{ m min}^{-1} = 17.305 \pm 0.001 \text{ m s}^{-1}$

Drum radius:

$$r_1 = \frac{60}{2} = 30 \pm 0.01 \text{ inches}$$

Felt roll radius:

$$r_2 = \frac{20}{2} = 10 \pm 0.01 \text{ inches}$$

Horizontal distance between the centres of circles of the drum and the felt roll:

$$ab = hc = 43.5 \pm 0.01 \text{ inches}$$

Vertical distance between the centres of circles of the two drums:

$$bc = 55 \pm 0.01 \text{ inches}$$

Vertical distance between the centres of circles of the drum and the left or the right felt roll:

$$fh = 6.5 \pm 0.01 \text{ inches}$$

There are six triangles which are of interest. They are defined as follows: two points of tangency of the paper with two drums and centres of the two drums form two equal right-angled triangles:  $\Delta_{aep}$  and  $\Delta_{cdp}$ ; similarly for the felt roll and drum there are two more right-angled triangles of similar shape:  $\Delta_{fgq}$  and  $\Delta_{ciq}$ ; two vertical lines extended to pass through the centres of the two felt rolls and two horizontal lines extended to pass through the centres of the two drums form two right-angled triangles:  $\Delta_{abc}$  and  $\Delta_{chf}$ .

For triangle  $\Delta_{abc}$ :

$$ac = 70.12 \pm \begin{matrix} 0.02 \\ 0.01 \end{matrix} \text{ inches}$$

Because triangle  $\Delta_{aep}$  and triangle  $\Delta_{cdp}$  are two equal right-angled triangles, the line  $pc$  equals half the line  $ac$ . From triangle  $\Delta_{abc}$  and triangle  $\Delta_{cdp}$ , it follows that angle  $\angle acb$  equals  $0.6692 \pm \begin{matrix} 0.0000 \\ 0.0001 \end{matrix}$  radian, angle  $\angle dcp$  equals  $0.5440 \mp \begin{matrix} 0.0001 \\ 0.0005 \end{matrix}$  radian and  $pd$  equals  $18.15 \mp \begin{matrix} 0.00 \\ 0.01 \end{matrix}$  inches; these are derived by using trigonometric relationships.

For triangle  $\Delta_{chf}$ :

$$\angle fch = \text{atan}(fh/ab) = 0.1483 \pm 0.0002 \text{ radian}$$

Because triangle  $\Delta_{fgq}$  and triangle  $\Delta_{ciq}$  are two right-angled triangles of similar shape, we obtain proportionate relationships as follows:

$$\frac{fg}{ic} = \frac{r_2}{r_1} = \frac{fq}{qc} = \frac{(fc-qc)}{qc}$$

So  $qc$  equals  $32.98 + \begin{matrix} 0.01 \\ 0.00 \end{matrix}$  inches. By using the trigonometric relationships again, angle  $\angle fci$  equals  $0.4286 \mp \begin{matrix} 0.0003 \\ 0.0005 \end{matrix}$  radian.

Because a quarter angle of a circle equals  $\frac{\pi}{2}$  radian, angle  $\angle dci$  meets the following equation:

$$\angle dci = \angle fci + \left( \frac{\pi}{2} - \angle acb - \angle dcp - \angle fch \right) = 0.6379 \mp \begin{matrix} 0.0004 \\ 0.0003 \end{matrix} \text{ radian}$$

According to the formula for circumference, the lengths of phases I, II, and III can be obtained as follows:

$$L_I = L_{III} = 19.14 \mp \begin{matrix} 0.01 \\ 0.00 \end{matrix} \text{ inches} = 0.486 \text{ m}$$

$$L_{II} = 77.430 \pm \begin{matrix} 0.06 \\ 0.07 \end{matrix} \text{ inches} = 1.967 \pm 0.001 \text{ m}$$

For phase IV:

$$L_{IV} = 2 \times pd = 36.29 + \frac{0.01}{0.03} \text{ inches} = 0.922 + \frac{0.000}{0.001} \text{ m}$$

The length of each phase and paper speed are known, therefore the times which the paper spend at each phase are as follows:

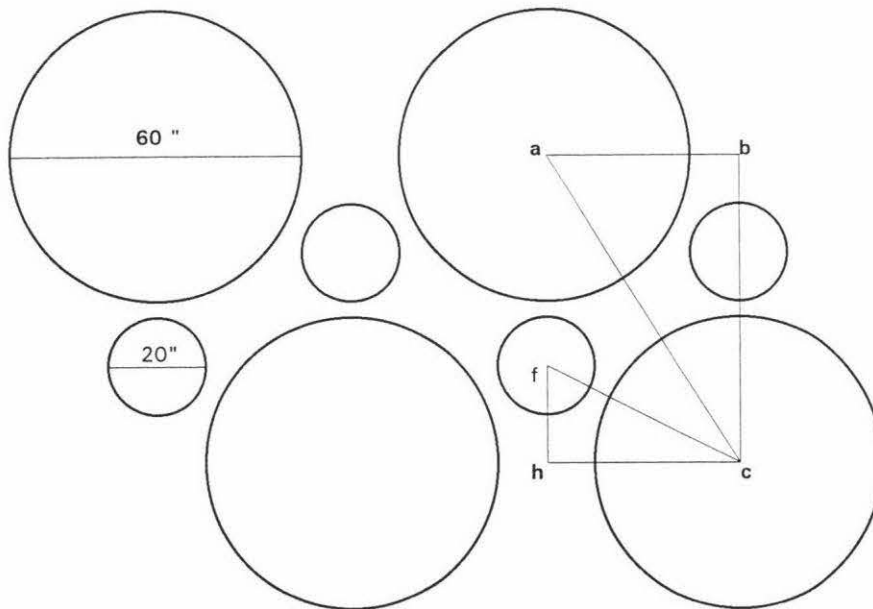
$$t_I = t_{III} = 0.0281 \text{ s}$$

$$t_{II} = 0.1136 \text{ s}$$

$$t_{IV} = t_{draw} = 0.0533 \text{ s}$$

$$t_{drum} = t_I + t_{II} + t_{III} = 0.1698 \text{ s}$$

For drums 14-49 part of the layout of dryers and felt rolls is shown in Figure 4.2-2.



**Figure 4.2-2** Typical dryer arrangement (Drums 14-49)

Horizontal distance between the centres of circles of the drum and the felt roll:

$$ab = hc = 39 \pm 0.01 \text{ inches}$$

Vertical distance between the centres of circles of the two drums:

$$bc = 64.5 \pm 0.01 \text{ inches}$$

Vertical distance between the centres of circles of the drum and the left or the right felt roll:

$$fh = 20 \pm 0.01 \text{ inches}$$

Using the same calculation as for drums 1-13, the results are as follows:

$$L_I = L_{III} = 0.247 \mp_{0.000}^{0.001} \text{ m}$$

$$L_{II} = 2.475 \pm_{0.001}^{0.002} \text{ m}$$

$$L_{IV} = 1.159 +_{0.001}^{0.000} \text{ m}$$

$$t_I = t_{III} = 0.0143 \text{ s}$$

$$t_{II} = 0.1429 \text{ s}$$

$$t_{IV} = t_{\text{draw}} = 0.0670 \text{ s}$$

$$t_{\text{drum}} = t_I + t_{II} + t_{III} = 0.1715 \text{ s}$$

### Appendix 4.2-2: Program of the formula for saturated vapour pressure

% x is the saturated vapour temperature (degC) and y is the saturated vapour pressure % (Pa). The values of x and y are found in Table 5 and 7 respectively, Appendix, % *Chemical Engineering (Volume 1)*. Chemical Engineering Publisher, Beijing, 1980, % pp. 349 & 350.

```
x = [ 40; 41; 42; 43; 44; 45; 46; 47; 48; 49;
      50; 51; 52; 53; 54; 55; 56; 57; 58; 59;
      60; 61; 62; 63; 64; 65; 66; 67; 68; 69;
      70; 71; 72; 73; 74; 75; 76; 77; 78; 79;
      80; 81; 82; 83; 84; 85; 86; 87; 88; 89;
      90; 91; 92; 93; 94; 95; 96; 97; 98; 99;
      100;104.5;109.2;113;116.6;120.2;127.2;133.3];

y = [ 7375.26; 7777.89; 8199.18; 8639.14; 9100.42;
      9583.04; 10085.66; 10612.27; 11160.22; 11734.83;
      12333.43; 12958.70; 13611.97; 14291.90; 14998.50;
      15731.76; 16505.02; 17304.94; 18144.85; 19011.43;
      19910.00; 20851.25; 21837.82; 22851.05; 23904.28;
      24997.50; 26144.05; 27330.60; 28557.14; 29823.68;
      31156.88; 32516.75; 33943.27; 35423.12; 36956.30;
      38542.81; 40182.65; 41875.81; 43635.64; 45462.12;
      47341.93; 49288.40; 51314.87; 53407.99; 55567.78;
      57807.55; 60113.99; 62220.44; 64940.17; 67473.25;
      70099.66; 72806.05; 75592.44; 78472.15; 81445.19;
      84511.55; 87671.23; 90937.57; 94297.24; 97750.22;
      101325.00; 120000; 140000; 160000; 180000;
      200000; 250000; 300000];

p = polyfit(x,y,4)

% p = [0.0012; -0.081; 6.4811; -27.2847; 129.8744] is the result of polyfit(x,y,4)
% y1 is vector of the evaluated value for saturated vapour pressure

y1 = ones(68,1)*NaN;
for r = 1:1:68
y1(r,1) = 0.0012*x(r)^4 - 0.081*x(r)^3 + 6.4811*x(r)^2 - 27.2847*x(r) + 129.8744;
end;

clg
plot (x,y,'-',x,y1,'--'),title ('Saturated Vapour of Water (Pa)'),
xlabel ('Temperature (degC)'),
gtext ('- Actual Value'); gtext ('-- Evaluated Value')
```

**Appendix 4.2-3: Program of the formula for specific heat of water**

```

% x is the water temperature (degC) and y is the specific heat of water (10(-3)J/(kgK)).
% The values of x and y are found in Table 3, Appendix, Chemical Engineering
% (Volume 1). Chemical Engineering Publisher, Beijing, 1980, pp. 347.

x = [40; 50; 60; 70; 80; 90; 100; 110; 120; 130; 140];

y = [4.174; 4.174; 4.178; 4.167; 4.195; 4.208; 4.220; 4.233; 4.250; 4.266; 4.287];

p = polyfit(x,y,3)

% p = [-1.1927e-007; 4.2832e-005; -0.0034; 4.2503] is the result of polyfit(x,y,3)
% y1 is vector of the evaluated value for specific heat of water

y1 = ones(11,1)*NaN;

for r = 1:1:11
y1(r,1) = - 1.1927e-007*x(r)^3 + 4.2832e-005*x(r)^2 - 0.0034*x(r) + 4.2503;
end;

clg
plot (x,y,'-',x,y1,'--'),title ('Specific Heat of Water *10(-3) (J/kgK)'),
xlabel ('Temperature (degC)'),
gtext ('- Actual Value')
gtext ('-- Evaluated Value')

```

#### Appendix 4.2-4: Program of the formula for latent heat of vaporisation

% x is the water temperature (degC) and y is the latent heat of vaporisation (J/kg)  
 % The values of x and y are found in Figure 12, Appendix, *Chemical Engineering*  
 % (*Volume 1*). Chemical Engineering Publisher, Beijing, 1980, pp. 359.

```
x = [ 34;    44;    54;    64;    74;    84;
      94;   104;   114;   124;   134];
```

```
y = [2554070; 2533135; 2512200; 2491265; 2470330; 2428460;
      2407525; 2386590; 2344720; 2323785; 2281915];
```

```
p=polyfit(x,y,3);
```

% p = [0.0122; -10.3943; -1236.2; 2607600] is the result of polyfit(x,y,3)  
 % y1 is vector of the evaluated value for latent heat of vaporisation

```
y1=ones(11,1)*NaN;
```

```
for r=1:1:11
```

```
  y1(r,1)=0.0122*x(r)^3-10.3943*x(r)^2-1236.2*x(r)+2607600;
end;
```

```
clg
```

```
plot (x,y,'-',x,y1,'--'),title ('Latent Heat of Vaporisation (J/kg)'),
xlabel ('Temperature (degC)'),
gtext ('- Actual Value')
gtext ('-- Evaluated Value')
```

## Appendix 5.2-1: Program and Program Block Diagram of the Knight and Kirk's Model

### Program of the Knight and Kirk's Model

% **KK.m**

```
function [y,x0]=KK(t,x,u,flag)
```

% y is the paper sheet temperature

% x=[x(1) x(2)]', x(1) is the sheet temperature and x(2) is the sheet moisture content

% u is the cylinder temperature

```
if nargin==0, flag=0; end
```

```
if abs(flag)==1
```

```
W=40;           % basis weight (g/m^2)
cf=1.34;        % paper specific heat (J/(gK))
pa1=0.69763;    % partial vapour pressure in the air over the clothing (KPa)
pa2=4.48923;    % partial vapour pressure in first drum pocket air
pa3=6.16251;    % partial vapour pressure in second drum pocket
Ta1=83;         % air temp. over the clothing (degC)
Ta2=51.2;       % pocket air temp. in first drum
Ta3=47.1;       % pocket air temp. in second drum
Mc=0.4;         % critical moisture content
%ps;           % saturation pressure of water corresponding to the web temp.
%cw;           % water specific heat at paper sheet temp.
%lda;          % latent heat of vaporization at drum paper sheet temp. (J/g)
```

```
u=Tc=59.3;      % cylinder temperature
```

```
%cw=-1.1927e-007*x(1)^3+4.2832e-005*x(1)^2-0.0034*x(1)+4.2503;
%lda=0.0000122*x(1)^3-0.0103943*x(1)^2-1.2362*x(1)+2607.6;
%ps=(0.0012*x(1)^4-0.081*x(1)^3+6.4811*x(1)^2-27.2847*x(1)+129.8744)*0.001;
```

```
hcp=800;        % heat transfer coefficient between cylinder surface and paper
                 (W/(m^2degC))
```

```
hpa1=67.5717;   % heat transfer coefficient between paper and air for phase I of the 1st
                 % drum
```

```
%hpa2;         % heat transfer coefficient between paper and air for phase II of the 1st
                 % drum
```

```
hpa3=68.0258;   % heat transfer coefficient between paper and air for phase III of the 1st
                 % drum
```

```

hpa4a=59.4534; % heat transfer coefficient between inner side of paper and air for phase
                % IV of the 1st drum

hpa4b=59.8530; % heat transfer coefficient between outer side of paper and air for phase
                % IV of the 1st drum

if t<=0.0281

    if x(2)>=Mc

r=1;

        elseif x(2)<Mc

r=x(2)/Mc;

        end;
A=[W*cf+0.01*W*(-1.1927e-007*x(1)^3+4.2832e-005*x(1)^2-0.0034*x(1)+4.2503)*
    x(2) -W*(0.0000122*x(1)^3-0.0103943*x(1)^2-1.2362*x(1)+2607.6);0 W];

y(1)=hcp*(Tc-x(1))-hpa1*(x(1)-Ta2);

y(2)=-1.09*hpa1*((0.0012*x(1)^4-0.081*x(1)^3+6.4812*x(1)^2-27.2847*x(1)+
    129.8744)*0.001-pa2)*r;

y=A\y(:);

elseif t<=0.1417

    if x(2)>= Mc

r=1;

        elseif x(2)< Mc;

r=x(2)/Mc

        end;

hpa2=0.472*(273+x(1))^2.5*(0.5474+0.7442*x(2))/[0.01*(273+x(1))^2.5*(0.5474+
    0.7442*x(2))+0.0012*x(1)^4-0.081*x(1)^3+6.4811*x(1)^2-27.2847*x(1)+
    129.8744)*(x(1)+518)*(1+0.7442*x(2))];

A=[W*cf+0.01*W*(-1.1927e-007*x(1)^3+4.2832e-005*x(1)^2-0.0034*x(1)+4.2503)*
    x(2) -W*(0.0000122*x(1)^3-0.0103943*x(1)^2-1.2362*x(1)+2607.6);0 W];

y(1)=hcp*(Tc-x(1))-hpa2*(x(1)-Ta1);

```

$$y(2) = -1.09 * hpa2 * ((0.0012 * x(1)^4 - 0.081 * x(1)^3 + 6.4812 * x(1)^2 - 27.2847 * x(1) + 129.8744) * 0.001 - pa1) * r;$$

$$y = A \setminus y(:);$$

$$\text{elseif } t \leq 0.1698$$

$$\text{if } x(2) \geq M_C$$

$$r = 1;$$

$$\text{elseif } x(2) < M_C$$

$$r = x(2) / M_C;$$

end;

$$A = [W * cf + 0.01 * W * (-1.1927e-007 * x(1)^3 + 4.2832e-005 * x(1)^2 - 0.0034 * x(1) + 4.2503) * x(2) - W * (0.0000122 * x(1)^3 - 0.0103943 * x(1)^2 - 1.2362 * x(1) + 2607.6); 0 \ W];$$

$$y(1) = hcp * (T_C - x(1)) - hpa3 * (x(1) - Ta3);$$

$$y(2) = -1.09 * hpa3 * ((0.0012 * x(1)^4 - 0.081 * x(1)^3 + 6.4812 * x(1)^2 - 27.2847 * x(1) + 129.8744) * 0.001 - pa3) * r;$$

$$y = A \setminus y(:);$$

$$\text{elseif } t > 0.1698$$

$$\text{if } x(2) \geq M_C$$

$$r = 1;$$

$$\text{elseif } x(2) < M_C$$

$$r = x(2) / M_C;$$

end;

$$A = [W * cf + 0.01 * W * (-1.1927e-007 * x(1)^3 + 4.2832e-005 * x(1)^2 - 0.0034 * x(1) + 4.2503) * x(2) - W * (0.0000122 * x(1)^3 - 0.0103943 * x(1)^2 - 1.2362 * x(1) + 2607.6); 0 \ W];$$

$$y(1) = -hpa4a * (x(1) - Ta2) - hpa4b * (x(1) - Ta3);$$

$$y(2)=r*[-1.09*hpa4a*((0.0012*x(1)^4-0.081*x(1)^3+6.4811*x(1)^2-27.2847*x(1)+129.8744)*0.001-pa2)-1.09*hpa4b*((0.0012*x(1)^4-0.081*x(1)^3+6.4811*x(1)^2-27.2847*x(1)+129.8744)*0.001-pa3)]$$

y=A\y(:);

end;

elseif flag==3

y=x(1);

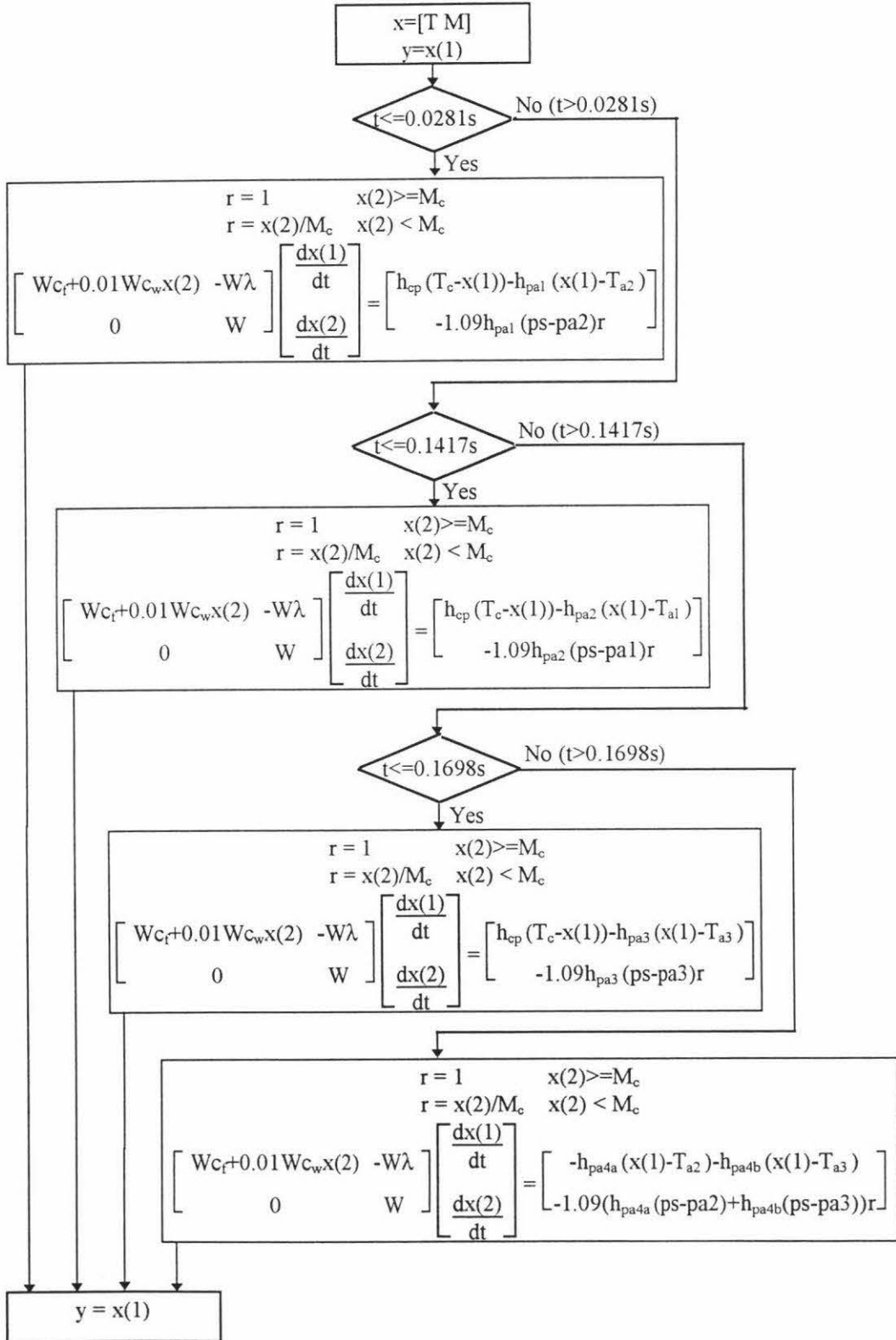
elseif flag==0

y=[2 0 1 0 0 0];

x0=[48.8 1.5]';

end

**Program Block Diagram of the Knight and Kirk's Model**



## Appendix 5.2-2: Program and Program Block Diagram of the Lemaitre *et al.*'s Model

### Program of the Lemaitre *et al.*'s Model

% AJ2.m

function [y,x0]=AJ2(t,x,u,flag,CO1)

% y is the paper sheet temperature

% x=[x(1) x(2)]', x(1) is the sheet temperature and x(2) is the sheet moisture content

% u is the steam temperature

% CO1=[h<sub>vp</sub> C<sub>mpf</sub> k<sub>f</sub> M<sub>c</sub> h<sub>pa</sub> C<sub>mpa</sub>]

% CO1(1)=h<sub>vp</sub> is a heat transfer coefficient between steam and sheet (W/(m<sup>2</sup>K))

% CO1(2)=C<sub>mpf</sub> is a mass transfer coefficient between sheet and clothing (m/s)

% CO1(3)=M<sub>c</sub> is critical moisture content

% CO1(4)=k<sub>f</sub> is a (affinity) coefficient of the water for the sheet

% CO1(5)=h<sub>pa</sub> is a heat transfer coefficient between sheet and air

% CO1(6)=C<sub>mpa</sub> is a mass transfer coefficient between sheet and air

if nargin==0, flag=0; end

if abs(flag)==1

W=0.04; % basis weight (kg/m<sup>2</sup>)

c<sub>p</sub>=1340; % paper specific heat (J/(kgK))

R=8.314; % gas constant (J/(kmol))

P=98066; % total surrounding pressure(Pa)

pa1=697.63; % partial vapour pressure in the air over the clothing for first drum

pa2=4489.23; % partial vapour pressure in the pocket air for first drum

Ta=51.2; % pocket air temp. for first drum (degC)

u=105; % steam temperature of first drum

%ps; % saturation pressure of water against the web temp.

%c<sub>w</sub>; % specific heat of water against web temp.

%lda; % latent heat of vaporization against web temp. (J/kg)

%c<sub>w</sub> = (-1.1927e-007\*x(1)<sup>3</sup>+4.2832e-005\*x(1)<sup>2</sup>-0.0034\*x(1)+4.2503)\*1000;

%lda = 0.0122\*x(1)<sup>3</sup>-10.3943\*x(1)<sup>2</sup>-1236.2\*x(1)+2607600;

%ps = 0.0012\*x(1)<sup>4</sup>-0.081\*x(1)<sup>3</sup>+6.4811\*x(1)<sup>2</sup>-27.2847\*x(1)+129.8744;

global CO1

%disp('CO1=');

%disp(CO1);

if t<=0.1698

```

if x(2)>=CO1(3)*0.1
pf=0.0012*x(1)^4-0.081*x(1)^3+6.4811*x(1)^2-27.2847*x(1)+129.8744;
elseif x(2)<CO1(3)*0.1
pf=(0.0012*x(1)^4-0.081*x(1)^3+6.4811*x(1)^2-27.2847*x(1)+129.8744)*exp(-
CO1(4)*(1/x(2)-1/(CO1(3)*0.1)));
end;
A=[W*cp+W*(-1.1927e-007*x(1)^3+4.2832e-005*x(1)^2-0.0034*x(1)+4.2503)*1000*
x(2)-W*(0.0122*x(1)^3-10.3943*x(1)^2-1236.2*x(1)+2607600);0 W];
y(1)=CO1(1)*1000*u-CO1(1)*1000*x(1);
y(2)=-CO1(2)*0.0001/(R*273+R*x(1))*P*log10((P-pa1)/(P-pf));
y=A\y(:);
elseif t>0.1698
if x(2)>=CO1(3)*0.1
pf=0.0012*x(1)^4-0.081*x(1)^3+6.4811*x(1)^2-27.2847*x(1)+129.8744;
elseif x(2)<CO1(3)*0.1
pf=(0.0012*x(1)^4-0.081*x(1)^3+6.4811*x(1)^2-27.2847*x(1)+129.8744)*exp(-
CO1(4)*(1/x(2)-1/(CO1(3)*0.1)));
end;
A=[W*cp+W*(-1.1927e-007*x(1)^3+4.2832e-005*x(1)^2-0.0034*x(1)+4.2503)*1000*
x(2)-W*(0.0122*x(1)^3-10.3943*x(1)^2-1236.2*x(1)+2607600);0 W];
y(1)=-2*CO1(5)*1000*x(1)+CO1(5)*2*1000*Ta;
y(2)=-2*CO1(6)*0.001/(R*273+R*x(1))*P*log10((P-pa2)/(P-pf));
y=A\y(:);
end;
elseif flag==3

```

```

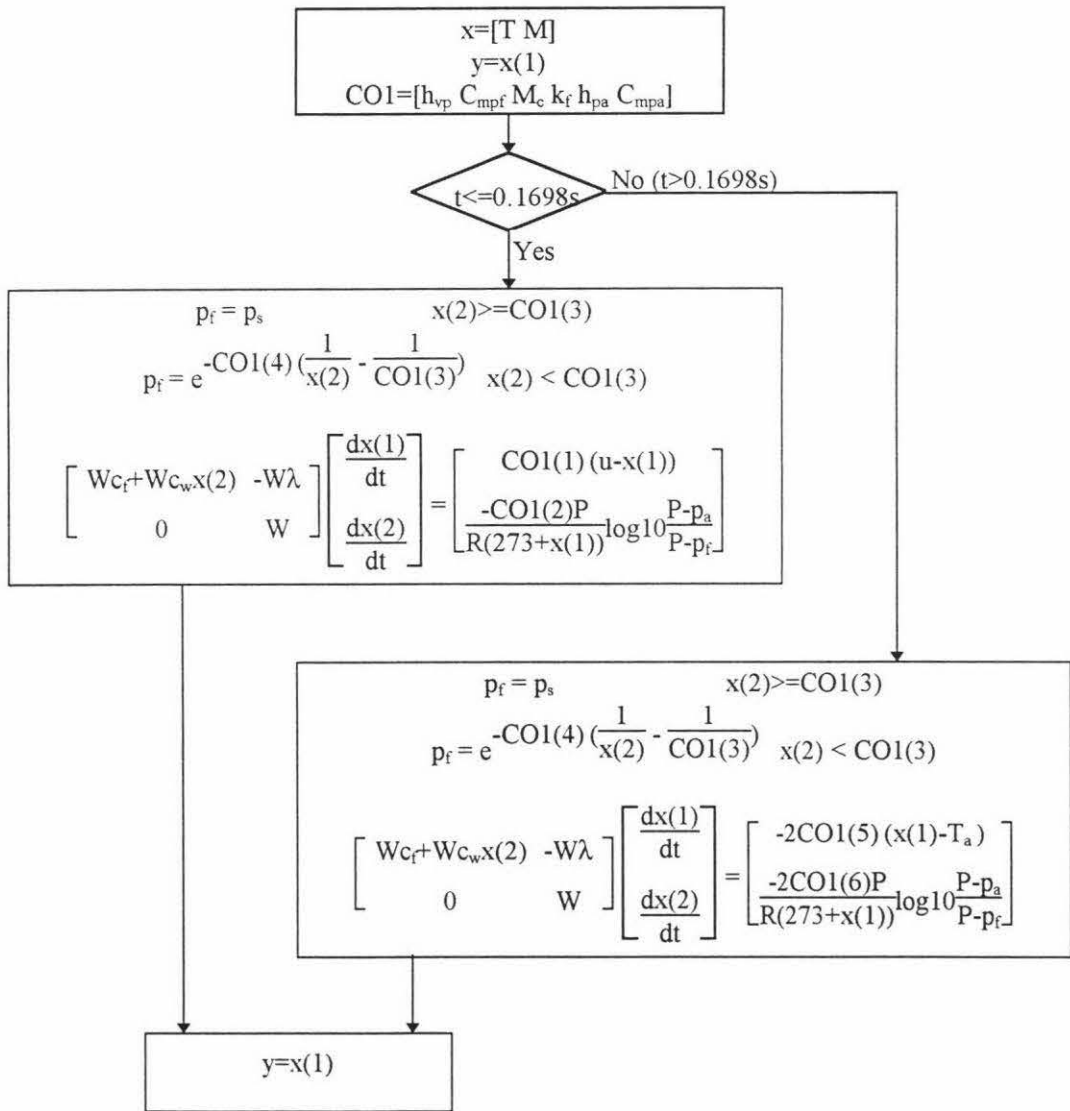
y=x(1);

elseif flag==0

y=[2 0 1 0 0 0];
x0=[48.8 1.5]';

end
    
```

**Program Block Diagram of the Lemaitre *et al.*'s Model**



### Appendix 5.2-3: Program for evaluating performance indices

```
function[J,g]=cperform(CO,model,CO0,x0,Yed,Xed);

% cperform evaluate performance indices

% [J,G]=CPERFORM(CO,'model',CO0, x0,Yed,Xed) returns the performance index for
% the model 'model' under the given model coefficients. 'model' is a string giving the
% name of the m-file containing the model. CO0 is the coefficient at CO(0).
% The performance index is given by

%  $J = \Delta Y * \Delta Y' + 1000 * \Delta X * \Delta X'$ 

% where delta Y is y-Yed (y is the model predictive temperature value, Yed is a
% expecting value of paper temperature), X is x-Xed (x is the model predictive
% moisture content value, Xed is a designed expecting value of moisture content).
% The model should return two arguments; a scalar of the function to be minimised,
% J, and a matrix of constraints, G (see the CONSTR function),
% i.e. [J,G]=model(CO,x0,Yed,Xed).

global CO1 J yj xj

CO1=CO;

[t,x,y]=rk45('aj2',0.2231,x0);

y=y(length(y));
Y=y-Yed;
yj=(diag(Y*Y'));

x=x(length(t),2);
X=x-Xed;
xj=1000*(diag(X*X'));

J=(yj+xj);
```

**Appendix 5.2-4: Program for Optimizing Parameters**

```

function[COopt]=mcp(Model,CO,FOpt,Low,Up,CO0,P1,P2,P3,P4);

% MCP Model Coefficients Predictor

% [COopt]=mcp('Model',CO,FOpt,Low,Up,CO0,P1,P2,P3,P4) returns optimal
% model coefficients, COopt, as estimated from the predictive model, 'model', using
% the performance index:  $J=\Delta Y*\Delta Y'+1000*\Delta X*\Delta X'$ . CO0 is the starting
% point for the optimisation. 'Model' is the name of an m-file containing the model. The
% model should return two arguments; a scalar of the function to be minimised, J, and a
% matrix of constraints, G(see the CONSTR function), i.e
% [J,G]=model(CO,P1,P2,P3,P4). The arguments P1, P2, P3 and P4 are passed directly
% to the function. Low and Up are row vectors of lower and upper bounds, respectively,
% on possible model coefficients. They are set to zeros and ones, respectively, if not
% given. FOpt, if given, is an options vector which is passed to the CONSTR function.

% Check for minimum number of arguments
if nargin<6,
    error('Too few arguments')
    return;
end

% Define the name of the performance index function
PerformFun='cperform';

% Define options for optimisation if not already specified
if length(FOpt)==0,
    FOpt=foptions;
    FOpt(16)=1;    % minimum change in variables for finite difference gradients
                  % calculation
    FOpt(2)=10;   % termination tolerance for x.(default: 1e-4).
    FOpt(3)=1;    % termination tolerance for f.(default: 1e-4).
    FOpt(14)=100; % maximum number of iterations.(default 100*no. of variables)
    FOpt(18)=1;   % step length.(default 1 or less).
end

% Initialise lower bounds on CO if no already done
if length(Low)==0, Low=zeros(1,NCoefs); end

% Initialise upper bounds on CO if no already done
if length(Up)==0, Up=ones(1,NCoefs); end

% Construct a string to evaluate the CONSTR function
evalstr=['constr(PerformFun,CO,FOpt,Low,Up,[],Model,CO0)'];
for i=1:nargin-6
    evalstr=[evalstr,'P',int2str(i)];

```

```
end;  
evalstr=[evalstr,')'];  
  
% Call the CONSTR function with the required arguments  
COopt=eval(evalstr);
```

**Appendix 5.2-5: Main Program**

```

% MCPTEST

% An m-file to test model coefficients predictor function

global J yj xj

x0=[48.8 1.5]';

% initialise the model coefficients matrix

CO0=[1 3.5 6 1 2 3.5];
COopt=CO0;

% initialise bounds on coefficients
Low=[0.8 1.5 5 0.1 1 0.5];
Up=[1.8 6 7 2 4 6];

% initialise the optimisation options vector
FOpt=foptions;
FOpt(16)=1; % minimum change in variables for finite difference gradients
FOpt(2)=10; % termination tolerance for x.(default: 1e-4).
FOpt(3)=1; % termination tolerance for f.(default: 1e-4).
FOpt(14)=100; % maximum number of iterations.(default 100*no. of variables)
FOpt(18)=1; % step length.(default 1 or less).

T=0.2231:0.2231:2.231';
Js=ones(length(T),1)*NaN;
yjs=ones(length(T),1)*NaN;
xjs=ones(length(T),1)*NaN;

% This is main loop
for g=1:1:10
COopt=mcp('aj2',COopt,FOpt,Low,Up,CO0);
CO0=COopt;
COs(g,:)=CO0;
yjs(g,1)=yj;
xjs(g,1)=xj;
Js(g,1)=J;
end

clg

subplot(221),plot(1:10,COs(:,1)),title('Heat Transfer Coefficient in Drum Phase'),
xlabel('Optimisation Sequence'),ylabel('*1000 W/m^2K');

```

```
subplot(222),plot(1:10,COs(:,2)),title('Mass Transfer Coefficient in Drum Phase'),  
xlabel('Optimisation Sequence'),ylabel('*0.0001 m/s');
```

```
subplot(223),plot(1:10,COs(:,3)),title('Critical Moisture'),  
xlabel('Optimisation Sequence');
```

```
subplot(224),plot(1:10,COs(:,4)),title('Affinity Coefficient'),  
xlabel('Optimisation Sequence');
```

```
pause  
clg
```

```
subplot(211),plot(1:10,COs(:,5)),title('Heat Transfer Coefficient in Draw Phase'),  
xlabel('Optimisation Sequence'),ylabel('*1000 W/m^2K');
```

```
subplot(212),plot(1:10,COs(:,6)),title('Mass Transfer Coefficient in Draw Phase'),  
xlabel('Optimisation Sequence'),ylabel('*0.001 m/s');
```

```
pause  
clg
```

```
subplot(211),plot(1:10,yjs(:,1)),title('Performance Index for Temperature'),  
xlabel('Optimisation Sequence');
```

```
subplot(212),plot(1:10,xjs(:,1)),title('Performance Index for Moisture Content'),  
xlabel('Optimisation Sequence'),ylabel('*0.001');
```

### Appendix 5.2-6: Program of the Lemaitre's model and Main program for the optimized parameters of the 49 cylinders

% AJ2.m

function [y,x0]=AJ2(t,x,u,flag,CO1)

% y is the paper sheet temperature

% x=[x(1) x(2)], x(1) is the sheet temperature and x(2) is the sheet moisture content

% u is the steam temperature

% CO1=[h<sub>vp</sub> C<sub>mpf</sub> k<sub>f</sub> M<sub>c</sub> h<sub>pa</sub> C<sub>mpa</sub>]

% CO1(1)=h<sub>vp</sub> is a heat transfer coefficient between steam and sheet (W/(m<sup>2</sup>K))

% CO1(2)=C<sub>mpf</sub> is a mass transfer coefficient between sheet and clothing (m/s)

% CO1(3)=M<sub>c</sub> is critical moisture content

% CO1(4)=k<sub>f</sub> is a (affinity) coefficient of the water for the sheet

% CO1(5)=h<sub>pa</sub> is a heat transfer coefficient between sheet and air

% CO1(6)=C<sub>mpa</sub> is a mass transfer coefficient between sheet and air

if nargin==0, flag=0; end

if abs(flag)==1

```
data=[697.63 4489.230 51.2 105;
      697.63 6162.510 47.1 105;
      697.63 7197.970 48.4 105;
      697.63 8192.160 48.4 120;
      697.63 8864.520 50.6 120;
      697.63 10488.30 49.9 120;
      697.63 10522.85 50.6 120;
      697.63 10628.75 49.3 120;
      697.63 11373.62 53.1 120;
      697.63 11848.47 53.6 120;
      697.63 11663.51 55.0 120;
      697.63 9658.770 54.6 120;
      697.63 11388.14 59.5 120;
      730.03 15989.56 70.3 122;
      730.03 19325.21 69.1 122;
      730.03 19185.22 73.6 122;
      730.03 24782.21 73.4 122;
      730.03 32045.06 78.0 122;
      730.03 24382.96 74.5 122;
      730.03 29565.52 76.3 122;
      730.03 25464.94 74.6 122;
      730.03 34445.52 79.5 122;
      730.03 27296.22 75.1 122;
```

```

730.03 33409.72 79.2 122;
730.03 26378.70 70.9 122;
730.03 32866.67 75.9 122;
730.03 26655.14 73.0 122;
730.03 30974.43 76.5 122;
730.03 28916.71 74.7 122;
730.03 28740.32 76.9 122;
730.03 26710.08 72.3 122;
730.03 23275.54 75.9 122;
730.03 28263.89 74.8 122;
730.03 28074.46 77.3 122;
730.03 25976.60 72.2 122;
730.03 22582.50 76.3 122;
730.03 24423.27 72.0 122;
730.03 22118.71 76.5 122;
730.03 24173.36 71.7 122;
730.03 21929.12 76.3 122;
730.03 9293.780 53.0 122;
744.35 8490.550 59.1 120;
744.35 11824.12 61.4 120;
744.35 12588.89 64.4 120;
744.35 13422.10 61.1 120;
744.35 10819.68 64.7 120;
744.35 12048.27 60.7 120;
744.35 9937.850 65.6 120;
744.35 5411.060 41.4 120];

```

```

global CO1 p
%disp('CO1=');
%disp(CO1);

```

```

W=0.04;           % basis weight (kg/m^2)
cp=1340;         % paper specific heat (J/kgK)
R=8.314;         % gas constant (J/kmol)
P=98066;         % total surrounding pressure(Pa)
pa1=data(p,1);   % partial vapoure pressure in the air over the clothing
pa2= data(p,2);  % partial vapour pressure in the pocket air
Ta= data(p,3);   % pocket air temperature (degC)

```

```

%ps;             % saturation pressure of water against the web temp.
%cw;             % specific heat of water against web temp.
%lda;           % latent heat of vaporization against web temp. (J/kg)

```

```

u= data(p,4);    % steam temperature

```

```

%cw = (-1.1927e-007*x(1)^3+4.2832e-005*x(1)^2-0.0034*x(1)+4.2503)*1000;
%lda = 0.0122*x(1)^3-10.3943*x(1)^2-1236.2*x(1)+2607600;

```

```
%ps = 0.0012*x(1)^4-0.081*x(1)^3+6.4811*x(1)^2-27.2847*x(1)+129.8744;
```

```
if t<=0.1698
```

```
    if x(2)>=CO1(3)*0.1
```

```
        pf=0.0012*x(1)^4-0.081*x(1)^3+6.4811*x(1)^2-27.2847*x(1)+129.8744;
```

```
    elseif x(2)<CO1(3)*0.1
```

```
        pf=(0.0012*x(1)^4-0.081*x(1)^3+6.4811*x(1)^2-27.2847*x(1)+129.8744)*exp(-
            CO1(4)*(1/x(2)-1/(CO1(3)*0.1)));
```

```
    end;
```

```
A=[W*cp+W*(-1.1927e-007*x(1)^3+4.2832e-005*x(1)^2-0.0034*x(1)+4.2503)*1000*
    x(2) -W*(0.0122*x(1)^3-10.3943*x(1)^2-1236.2*x(1)+2607600);0 W];
```

```
y(1)=CO1(1)*1000*u-CO1(1)*1000*x(1);
```

```
y(2)=-CO1(2)*0.0001/(R*273+R*x(1))*P*log10((P-pa1)/(P-pf));
```

```
y=A\y(:);
```

```
elseif t>0.1698
```

```
    if x(2)>=CO1(3)*0.1
```

```
        pf=0.0012*x(1)^4-0.081*x(1)^3+6.4811*x(1)^2-27.2847*x(1)+129.8744;
```

```
    elseif x(2)<CO1(3)*0.1
```

```
        pf=(0.0012*x(1)^4-0.081*x(1)^3+6.4811*x(1)^2-27.2847*x(1)+129.8744)*exp(-
            CO1(4)*(1/x(2)-1/(CO1(3)*0.1)));
```

```
    end;
```

```
A=[W*cp+W*(-1.1927e-007*x(1)^3+4.2832e-005*x(1)^2-0.0034*x(1)+4.2503)*1000*
    x(2) -W*(0.0122*x(1)^3-10.3943*x(1)^2-1236.2*x(1)+2607600);0 W];
```

```
y(1)=-2*CO1(5)*1000*x(1)+CO1(5)*2*1000*Ta;
```

```
y(2)=-2*CO1(6)*0.001/(R*273+R*x(1))*P*log10((P-pa2)/(P-pf));
```

```
y=A\y(:);
```

```
end;
```

```
elseif flag==3
```

```
y=x(1);
```

```
elseif flag==0
```

```
y=[2 0 1 0 0 0];
```

```
x0=[48.8 1.5]';
```

```
end
```

```
% MCPTEST
```

```
% An m-file to test model coefficients predictor function
```

```
global J yj xj p
```

```
y1=[52.7 53.8 53.9 55.5 57.6 61.2 61.0 64.1 60.3 67.8  
68.3 68.8 75.9 79.2 81.5 81.5 82.1 81.5 81.7 84.8  
82.9 80.0 81.6 80.9 82.2 83.3 83.1 79.8 81.2 71.9  
82.1 80.1 81.3 75.4 77.5 74.7 82.1 75.7 81.7 77.1  
69.2 72.6 75.8 69.1 76.6 68.1 79.4 65.2 65.2];
```

```
x1=[1.4958 1.4874 1.4748 1.4580 1.4370 1.4118 1.3845 1.3530 1.3205 1.2880  
1.2555 1.2230 1.1905 1.1558 1.1211 1.0864 1.0517 1.0170 0.9823 0.9476  
0.9129 0.8782 0.8435 0.8088 0.7741 0.7394 0.7047 0.6700 0.6353 0.6006  
0.5659 0.5312 0.4965 0.4618 0.4271 0.3924 0.3577 0.3230 0.2883 0.2536  
0.2200 0.1888 0.1630 0.1415 0.1243 0.1093 0.0986 0.0899 0.0856];
```

```
x0=[48.8 1.5]';
```

```
% initialise the model coefficients matrix
```

```
CO0=[1 3.5 6 1 2 3.5];
```

```
COopt=CO0;
```

```
% initialise bounds on coefficients
```

```
Low=[0.8 1.5 5 0.1 1 0.5];
```

```
Up=[1.8 6 7 2 4 6];
```

```
% initialise the optimisation options vector
```

```
FOpt=foptions;
```

```
FOpt(16)=1; % minimum change in variables for finite difference gradients
```

```
Fopt(2)=10; % termination tolerance for x.(default: 1e-4).
```

```
FOpt(3)=1; % termination tolerance for f.(default: 1e-4).
```

```
FOpt(14)=100; % maximum number of iterations.(default 100*no. of variables)
FOpt(18)=1; % step length.(default 1 or less).
```

```
xs=ones(49,2)*NaN;
COss=ones(49,6)*NaN;
Jss=ones(49,1)*NaN;
xjss=ones(49,1)*NaN;
yjss=ones(49,1)*NaN;
Lows=ones(49,6)*NaN;
Ups= ones(49,6)*NaN;
```

```
T=0.2231:0.2231:2.231';
COs=ones(length(T),length(CO0))*NaN;
Js=ones(length(T),1)*NaN;
yjs=ones(length(T),1)*NaN;
xjs=ones(length(T),1)*NaN;
```

```
for p=1:1:49
```

```
Yed=y1(p);
Xed=x1(p);
```

```
for g=1:1:10
COopt=mcp1('aj2-1',COopt,FOpt,Low,Up,CO0,x0,Yed,Xed);
CO0=COopt;
COs(g,:)=CO0;
yjs(g,1)=yj;
xjs(g,1)=xj;
Js(g,1)=J;
end;
```

```
COss(p,:)=COopt
Lows(p,:)=Low;
Ups(p,:)=Up;
Jss(p,1)=Js(length(fs));
xjss(p,1)=xjs(length(xfs));
yjss(p,1)=yjs(length(yfs));
[t,x,y]=rk45('aj2-1',0.2231,x0);
x0=[x(length(t),:)]';
xs(p,:)=x0';
end;
```

```
xs=[48.8 1.5;xs];
y1=[48.8 y1];
x1=[1.5 x1];
```

```
clg
```

```
plot(1:49,COss(:,1),'-',1:49,Lows(:,1),'!',1:49,Ups(:,1),'--'),
title('Heat Transfer Coefficient in Drum Phase *1000 (W/m^2K)'),
xlabel('Cylinder Number');
gtext ('- Optimizing Value')
gtext ('! Low Limit Value')
gtext ('-- Up Limit Value')
```

```
pause
clg
plot(1:49,COss(:,2),'-',1:49,Lows(:,2),'!',1:49,Ups(:,2),'--'),
title('Mass Transfer Coefficient in Drum Phase *0.0001 (m/s)'),
xlabel('Cylinder Number');
gtext ('- Optimizing Value')
gtext ('! Low Limit Value')
gtext ('-- Up Limit Value')
```

```
pause
clg
plot(1:49,COss(:,3),'-',1:49,Lows(:,3),'!',1:49,Ups(:,3),'--'),
title('Critical Moisture *0.1'),xlabel('Cylinder Number');
gtext ('- Optimizing Value')
gtext ('! Low Limit Value')
gtext ('-- Up Limit Value')
```

```
pause
clg
plot(1:49,COss(:,4),'-',1:49,Lows(:,4),'!',1:49,Ups(:,4),'--'),
title('Affinity Coefficient'),xlabel('Cylinder Number');
gtext ('- Optimizing Value')
gtext ('! Low Limit Value')
gtext ('-- Up Limit Value')
```

```
pause
clg
plot(1:49,COss(:,5),'-',1:49,Lows(:,5),'!',1:49,Ups(:,5),'--'),
title('Heat Transfer Coefficient in Draw Phase *1000 (W/m^2K)'),
xlabel('Cylinder Number');
gtext ('- Optimizing Value')
gtext ('! Low Limit Value')
gtext ('-- Up Limit Value')
```

```
pause
clg
plot(1:49,COss(:,6),'-',1:49,Lows(:,6),'!',1:49,Ups(:,6),'--'),
title('Mass Transfer Coefficient in Draw Phase *0.001 (m/s)'),
xlabel('Cylinder Number');
gtext ('- Optimizing Value')
```

```
gtext (':  Low Limit Value')
gtext ('-- Up Limit Value')

pause
clg
subplot(211),plot(1:49,yjss(:,1)),
title('Performance Index for Paper Temperature (degC^2)'),
xlabel('Cylinder Number');

subplot(212),plot(1:49,xjss(:,1)),
title('Performance Index for Paper Moisture Content*0.001 ((kg/kg dry basis)^2)'),
xlabel('Cylinder Number');

pause
clg
subplot(211),plot(0:49,xs(:,1),'-',0:49,y1(1,:),'--'),title('Paper Temperature (degC)'),
xlabel('Cylinder Number');
gtext ('-  Predict Value')
gtext ('-- Expecting Value')

subplot(212),plot(0:49,xs(:,2),'-',0:49,x1(1,:),'--'),
title('Paper Moisture Content (kg/kg dry basis)'),
xlabel('Cylinder Number');
gtext ('-  Predict Value')
gtext ('-- Expecting Value')
```

**Appendix 5.2-7: The numerical results for the optimized parameters of the 49 cylinders**

CO<sub>ss</sub> =

0.9113	3.3618	6.0000	1.0000	3.8045	3.0120
0.7999	2.9138	6.0000	1.0000	2.8205	1.3746
0.8753	2.9388	6.0000	1.0000	3.1854	1.8462
0.2270	5.7492	6.0000	1.0000	1.0625	0.5000
0.3102	5.8808	6.0000	1.0000	1.4088	0.5004
0.5323	5.9444	6.0000	1.0000	1.4576	0.5477
0.7455	5.7505	6.0000	1.0000	2.2982	0.6347
1.2000	5.1473	6.0000	1.0000	1.9874	0.8974
1.2000	7.0188	6.0000	1.0000	2.8580	1.8386
0.7517	4.5000	6.0000	1.0000	0.6080	1.5001
1.1180	6.2771	6.0000	1.0000	1.4966	1.0000
1.2000	5.0670	6.0000	1.0000	1.4810	1.0000
1.1750	2.1251	6.0000	1.0000	0.9912	0.7250
0.8488	0.5000	6.0000	1.0000	0.0500	1.0390
0.8500	0.5000	6.0000	1.0000	0.0500	1.0000
0.6340	0.3000	6.0000	1.0000	0.0300	0.8393
0.5945	0.3248	6.0000	1.0000	0.0630	0.8134
0.5604	0.3000	6.0000	1.0000	0.1355	0.9084
0.5250	0.2000	6.0000	1.0000	0.0200	0.8000
0.5250	0.2000	6.0000	1.0000	0.0200	0.6000
0.5082	0.3001	6.0000	1.0000	0.0300	0.7120
0.5056	0.5779	6.0000	1.0000	0.0986	1.2176
0.5249	0.2004	6.0000	1.0000	0.0201	0.7777
0.5250	0.2000	6.0000	1.0000	0.0200	1.0331
0.5064	0.3931	6.0000	1.0000	0.0405	0.6591
0.5220	0.2000	4.0000	1.0000	0.0200	0.8800
0.5550	0.2000	4.0000	1.0000	0.0200	0.9000
0.5250	1.0000	4.0000	1.0000	0.0200	0.9472
0.5250	1.0000	4.0000	1.0000	0.0200	0.9000
0.4362	2.6830	4.0000	1.0000	0.1909	2.4003
0.7270	1.0000	4.0000	1.0000	0.0200	0.9500
0.6300	1.0000	4.0000	1.0000	0.0200	1.0000

0.6300	1.0000	4.0000	1.0000	0.0200	1.0000
0.5355	3.1752	4.0000	1.0000	0.1323	1.0775
0.6500	3.0000	4.0000	1.0000	0.0200	1.2000
0.4651	4.1166	4.5101	0.7278	0.1598	1.5861
0.5833	4.2444	4.1224	1.7166	0.1528	1.4932
0.4388	4.0786	3.8018	0.9666	0.2119	1.9465
0.7000	7.5249	3.3604	1.6675	0.2002	3.2908
0.7000	6.0000	2.6500	1.7500	0.1000	4.5000
0.7200	4.0000	2.1500	2.5000	0.2000	2.0000
0.3754	3.0573	2.1525	2.1586	0.3656	3.3901
0.5920	7.7430	1.9985	1.0000	0.1770	3.8862
0.2000	4.5000	2.0360	0.4000	0.2500	3.5000
0.4448	5.4271	1.8847	0.4360	0.1777	4.5880
0.2000	2.9999	1.5000	0.3285	0.1500	3.0000
0.3000	3.0000	2.0000	0.2750	0.1500	2.9102
0.0100	0.3000	0.7500	1.0000	0.1500	0.5000
0.1000	0.1000	0.7500	1.0000	0.1000	0.3000

where the first column is heat transfer coefficient in drum phase; the second is mass transfer coefficient in drum phase, the third is critical moisture content; the fourth is affinity coefficient of the water for the sheet; the fifth is heat transfer coefficient in draw phase and the sixth is mass transfer coefficient in draw phase. The 1-49 rows are corresponding to 1-49 cylinders.

**Appendix 6.1-1: Program of the Lemaitre *et al.*'s model for drum 1-49**

**% AJ3.m**

function [y,x0]=AJ3(t,x,u,flag,CO1)

% y is the paper sheet temperature

% x=[x(1) x(2)], x(1) is the sheet temperature and x(2) is the sheet moisture content

% u is the steam temperature

if nargin==0, flag=0; end

if abs(flag)==1

global k td

W=0.04; % basis weight (kg/m<sup>2</sup>)

c<sub>p</sub>=1340; % paper specific heat (J/kgK)

R=8.314; % gas constant (J/kmol)

P=98066; % total surrounding pressure(Pa)

```
data=[697.63 4489.230 51.2 105;
      697.63 6162.510 47.1 105;
      697.63 7197.970 48.4 105;
      697.63 8192.160 48.4 120;
      697.63 8864.520 50.6 120;
      697.63 10488.30 49.9 120;
      697.63 10522.85 50.6 120;
      697.63 10628.75 49.3 120;
      697.63 11373.62 53.1 120;
      697.63 11848.47 53.6 120;
      697.63 11663.51 55.0 120;
      697.63 9658.770 54.6 120;
      697.63 11388.14 59.5 120;
      730.03 15989.56 70.3 122;
      730.03 19325.21 69.1 122;
      730.03 19185.22 73.6 122;
      730.03 24782.21 73.4 122;
      730.03 32045.06 78.0 122;
      730.03 24382.96 74.5 122;
      730.03 29565.52 76.3 122;
      730.03 25464.94 74.6 122;
      730.03 34445.52 79.5 122;
      730.03 27296.22 75.1 122;
      730.03 33409.72 79.2 122;
      730.03 26378.70 70.9 122;
```

```

730.03 32866.67 75.9 122;
730.03 26655.14 73.0 122;
730.03 30974.43 76.5 122;
730.03 28916.71 74.7 122;
730.03 28740.32 76.9 122;
730.03 26710.08 72.3 122;
730.03 23275.54 75.9 122;
730.03 28263.89 74.8 122;
730.03 28074.46 77.3 122;
730.03 25976.60 72.2 122;
730.03 22582.50 76.3 122;
730.03 24423.27 72.0 122;
730.03 22118.71 76.5 122;
730.03 24173.36 71.7 122;
730.03 21929.12 76.3 122;
730.03 9293.780 53.0 122;
625.93 8490.550 59.1 120;
625.93 11824.12 61.4 120;
625.93 12588.89 64.4 120;
625.93 13422.10 61.1 120;
625.93 10819.68 64.7 120;
625.93 12048.27 60.7 120;
625.93 9937.850 65.6 120;
625.93 5411.060 41.4 120];
    
```

```

pa1=data(k,1);    % partial vapour pressure in the air over the clothing
pa2= data(k,2);  % partial vapour pressure in the pocket
Ta= data(k,3);   % pocket air temperature (degC)
    
```

```

u= data(,4);     % steam temperature
    
```

```

COss =[0.9113  3.3618  6.0000  1.0000  3.8045  3.0120;
       0.7999  2.9138  6.0000  1.0000  2.8205  1.3746;
       0.8753  2.9388  6.0000  1.0000  3.1854  1.8462;
       0.2270  5.7492  6.0000  1.0000  1.0625  0.5000;
       0.3102  5.8808  6.0000  1.0000  1.4088  0.5004;
       0.5323  5.9444  6.0000  1.0000  1.4576  0.5477;
       0.7455  5.7505  6.0000  1.0000  2.2982  0.6347;
       1.2000  5.1473  6.0000  1.0000  1.9874  0.8974;
       1.2000  7.0188  6.0000  1.0000  2.8580  1.8386;
       0.7517  4.5000  6.0000  1.0000  0.6080  1.5001;
       1.1180  6.2771  6.0000  1.0000  1.4966  1.0000;
       1.2000  5.0670  6.0000  1.0000  1.4810  1.0000;
       1.1750  2.1251  6.0000  1.0000  0.9912  0.7250;
       0.8488  0.5000  6.0000  1.0000  0.0500  1.0390;
       0.8500  0.5000  6.0000  1.0000  0.0500  1.0000;
       0.6340  0.3000  6.0000  1.0000  0.0300  0.8393;
    
```

0.5945	0.3248	6.0000	1.0000	0.0630	0.8134;
0.5604	0.3000	6.0000	1.0000	0.1355	0.9084;
0.5250	0.2000	6.0000	1.0000	0.0200	0.8000;
0.5250	0.2000	6.0000	1.0000	0.0200	0.6000;
0.5082	0.3001	6.0000	1.0000	0.0300	0.7120;
0.5056	0.5779	6.0000	1.0000	0.0986	1.2176;
0.5249	0.2004	6.0000	1.0000	0.0201	0.7777;
0.5250	0.2000	6.0000	1.0000	0.0200	1.0331;
0.5064	0.3931	6.0000	1.0000	0.0405	0.6591;
0.5220	0.2000	4.0000	1.0000	0.0200	0.8800;
0.5550	0.2000	4.0000	1.0000	0.0200	0.9000;
0.5250	1.0000	4.0000	1.0000	0.0200	0.9472;
0.5250	1.0000	4.0000	1.0000	0.0200	0.9000;
0.4362	2.6830	4.0000	1.0000	0.1909	2.4003;
0.7270	1.0000	4.0000	1.0000	0.0200	0.9500;
0.6300	1.0000	4.0000	1.0000	0.0200	1.0000;
0.6300	1.0000	4.0000	1.0000	0.0200	1.0000;
0.5355	3.1752	4.0000	1.0000	0.1323	1.0775;
0.6500	3.0000	4.0000	1.0000	0.0200	1.2000;
0.4651	4.1166	4.5101	0.7278	0.1598	1.5861;
0.5833	4.2444	4.1224	1.7166	0.1528	1.4932;
0.4388	4.0786	3.8018	0.9666	0.2119	1.9465;
0.7000	7.5249	3.3604	1.6675	0.2002	3.2908;
0.7000	6.0000	2.6500	1.7500	0.1000	4.5000;
0.7200	4.0000	2.1500	2.5000	0.2000	2.0000;
0.3754	3.0573	2.1525	2.1586	0.3656	3.3901;
0.5920	7.7430	1.9985	1.0000	0.1770	3.8862;
0.2000	4.5000	2.0360	0.4000	0.2500	3.5000;
0.4448	5.4271	1.8847	0.4360	0.1777	4.5880;
0.2000	2.9999	1.5000	0.3285	0.1500	3.0000;
0.3000	3.0000	2.0000	0.2750	0.1500	2.9102;
0.0100	0.3000	0.7500	1.0000	0.1500	0.5000;
0.1000	0.1000	0.7500	1.0000	0.1000	0.3000];

$h_{vp} = CO_{ss}(k,1)$ ; % heat transfer coefficient between steam and sheet ( $W/m^2K$ )  
 $C_{mpf} = CO_{ss}(k,2)$ ; % mass transfer coefficient of the sheet and clothing (m/s)  
 $M_c = CO_{ss}(k,3)$ ; % critical moisture content  
 $k_f = CO_{ss}(k,4)$ ; % (affinity) coefficient of the water for the sheet  
 $h_{pa} = CO_{ss}(k,5)$ ; % heat transfer coefficient between sheet and air  
 $C_{mpa} = CO_{ss}(k,6)$ ; % mass transfer coefficient between sheet and air

$\%ps$ ; % saturation pressure of water against the web temp.  
 $\%c_w$ ; % specific heat of water against web temp.  
 $\%lda$ ; % latent heat of vaporization against web temp. (J/kg)

$\%c_w = (-1.1927e-007 * x(1)^3 + 4.2832e-005 * x(1)^2 - 0.0034 * x(1) + 4.2503) * 1000$ ;

```

%lda = 0.0122*x(1)^3-10.3943*x(1)^2-1236.2*x(1)+2607600;
%pps = 0.0012*x(1)^4-0.081*x(1)^3+6.4811*x(1)^2-27.2847*x(1)+129.8744;

if t<= td

    if x(2)>= Mc*0.1

        pf=0.0012*x(1)^4-0.081*x(1)^3+6.4811*x(1)^2-27.2847*x(1)+129.8744;

    elseif x(2)< Mc*0.1

        pf=(0.0012*x(1)^4-0.081*x(1)^3+6.4811*x(1)^2-27.2847*x(1)+129.8744)*exp(-
            kf*(1/x(2)-1/(Mc*0.1)));

    end;

A=[W* cp +W*(-1.1927e-007*x(1)^3+4.2832e-005*x(1)^2-
0.0034*x(1)+4.2503)*1000*
    x(2) -W*(0.0122*x(1)^3-10.3943*x(1)^2-1236.2*x(1)+2607600);0 W];

y(1)= hvp*1000*u- hvp*1000*x(1);

y(2)=- Cmpf*0.0001/(R*273+R*x(1))*P*log10((P-pa1)/(P-pf));

y=A\y(:);

elseif t> td

    if x(2)>= Mc*0.1

        pf=0.0012*x(1)^4-0.081*x(1)^3+6.4811*x(1)^2-27.2847*x(1)+129.8744;

    elseif x(2)< Mc*0.1

        pf=(0.0012*x(1)^4-0.081*x(1)^3+6.4811*x(1)^2-27.2847*x(1)+129.8744)*exp(-
            kf*(1/x(2)-1/(Mc*0.1)));

    end;

A=[W* cp +W*(-1.1927e-007*x(1)^3+4.2832e-005*x(1)^2-
0.0034*x(1)+4.2503)*1000*
    x(2) -W*(0.0122*x(1)^3-10.3943*x(1)^2-1236.2*x(1)+2607600);0 W];

y(1)=-2* hpa*1000*x(1)+ hpa*2*1000*Ta;

y(2)=-2* Cmpa*0.001/(R*273+R*x(1))*P*log10((P-pa2)/(P-pf));

```

```
y=A\y(:);  
end;  
elseif flag==3  
y=x(1);  
elseif flag==0  
y=[2 0 1 0 0 0];  
x0=[48.8 1.5];  
end
```

**Appendix 6.1-2: Program for running AJ3.m (program of the Lemaitre *et al.*'s model for drum 1-49)**

```
% aj3rk.m
```

```
global k td
```

```
x0=[48.8 1.5]';
```

```
xs=ones(49,2)*NaN;
```

```
for k=1:1:13
```

```
    td=0.1698;
```

```
    t =0.2231;
```

```
    [t,x,y]=rk45('aj3',t,x0);
```

```
    x0=[x(length(t),:)]';
```

```
    xs(k,:)=x0';
```

```
end;
```

```
for k=14:1:49
```

```
    td=0.1715;
```

```
    t =0.2385;
```

```
    [t,x,y]=rk45('aj3',t,x0);
```

```
    x0=[x(length(t),:)]';
```

```
    xs(k,:)=x0';
```

```
end;
```

```
xs=[48.8 1.5;xs];
```

```
clg
```

```
subplot(211),plot(0:49,xs(:,1)),title('Paper Temperature (degC)'),xlabel('Drum Number')
```

```
subplot (212),plot(0:49,xs(:,2)),title('Paper Moisture Content (kg/kg dry basis)'),
```

```
xlabel('Drum Number')
```

### Appendix 6.2-1: Tables for Chapter 6

**Table 6.2-1** Final Paper Moisture Content Against Different Steam Temperature

Cylinder Number	1-3			
Steam Temperature (°C)	94.5	105	115.5	
Final Paper Moisture Content (%)	7.99	7.97	7.96	
Cylinder Number	4-13			
Steam Temperature (°C)	108	120	132	
Final Paper Moisture Content (%)	8.20	7.97	7.86	
Cylinder Number	14-41			
Steam Temperature (°C)	119.56	122	124.44	
Final Paper Moisture Content (%)	8.14	7.97	7.87	
Cylinder Number	42-49			
Steam Temperature (°C)	108	120	132	
Final Paper Moisture Content (%)	9.11	7.97	6.89	
Cylinder Number	1-3	4-13	14-41	42-49
Steam Temperature (°C)	102.9	117.6	119.56	117.6
Final Paper Moisture Content (%)	8.47			
Cylinder Number	1-3	4-13	14-41	42-49
Steam Temperature (°C)	107.1	122.4	124.44	122.4
Final Paper Moisture Content (%)	7.63			

**Table 6.3-1** Final Paper Moisture Content Against Different Drum Speed

Drum Speed (m/s)	16.440		18.170	
Cylinder Number	1-13	14-49	1-13	14-49
<i>td</i> (s)	0.1788	0.1805	0.1617	0.1633
<i>t</i> (s)	0.2349	0.2510	0.2124	0.2271
Final Paper Moisture Content (%)	7.66		8.45	
Drum Speed (m/s)	16.007		18.603	
Cylinder Number	1-13	14-49	1-13	14-49
<i>td</i> (s)	0.1836	0.1854	0.1580	0.1595
<i>t</i> (s)	0.2412	0.2578	0.2075	0.2218
Final Paper Moisture Content (%)	7.53		8.92	

**Table 6.3-2** Final Paper Moisture Content Against Drum Speed and Steam Temp.

Drum Speed (m/s)	19.036			
Cylinder Number	1-3	4-13	14-41	42-49
Steam Temperature(°c)	105.00	125.05	125.05	125.05
<i>td</i> (s)	0.1544		0.1559	
<i>t</i> (s)	0.2028		0.2168	
Final Paper Moisture Content (%)	8.04			

**Table 6.4-1** Final Paper Moisture Content Against Initial Paper Temperature and Moisture Content

Initial Paper Temperature (°C)	39.04		58.56	
Initial Paper Moisture Content (%)	60			
Final Paper Moisture Content (%)	7.98		7.97	
Initial Paper Temperature (°C)	48.8			
Initial Paper Moisture Content (%)	57.45	62.26	66	
Final Paper Moisture Content (%)	7.80	10.15	26.63	

**Table 6.5-1** Final Paper Moisture Content Against Partial Vapour Pressure Over the Cloth

Cylinder Number	1-13	14-41	42-49
Partial Vapour Pressure Over the Clothing (Pa)	536.64	511.02	541.35
Final Paper Moisture Content (%)	7.96		
Cylinder Number	1-13	14-41	42-49
Partial Vapour Pressure Over the Clothing (Pa)	2361.21	2409.11	2436.06
Final Paper Moisture Content (%)	8.02		

**Table 6.5-2** Temp. and Vapour Pressure of Pocket Air for Each Cylinder

Cylinder Number	Test 1		Test 2	
	Temp. of Pocket Air(°c)	Vapour Pressure in Pocket Air(P <sub>a</sub> )	Temp. of Pocket Air(°c)	Vapour Pressure in Pocket Air(P <sub>a</sub> )
1	48.64	3413.377	53.76	4390.425
2	44.745	4908.14	49.455	6246.637
3	45.98	5466.425	50.82	6980.1
4	45.98	6221.456	50.82	7944.202
5	48.07	6588.233	53.13	8460.67
6	47.405	7636.647	52.395	9789.107
7	48.07	7820.725	53.13	10043.45
8	46.835	7859.051	51.765	10058.51
9	50.445	9287.418	55.755	12006.45
10	50.92	9856.595	56.28	12759.37
11	52.25	10216.64	57.75	13275.49
12	51.87	8337.241	57.33	10821.69
13	56.525	9879.346	62.475	12994.82
14	66.785	13641.42	73.815	18457.33
15	65.645	16510.78	72.555	22272.81
16	69.92	16253.21	77.28	22167.88
17	69.73	20993.52	77.07	28619.66
18	74.1	26986.3	81.9	37179.23

...continued

Table 6.5-2 continue...

Cylinder Number	Test 1		Test 2	
	Temp. of Pocket Air(°c)	Vapour Pressure in Pocket Air(P <sub>a</sub> )	Temp. of Pocket Air(°c)	Vapour Pressure in Pocket Air(P <sub>a</sub> )
19	70.775	20676.47	78.225	28260.58
20	72.485	24965.35	80.115	34264.36
21	70.87	21597.72	78.33	29526.63
22	75.525	29036.92	83.475	40135.16
23	71.345	23158.08	78.855	31696.63
24	75.24	28151.6	83.16	38886.33
25	67.355	22445.08	74.445	30414.05
26	72.105	27788.84	79.695	38104.81
27	69.35	22581.21	76.65	30754.76
28	72.675	26140.7	80.325	35893.77
29	70.965	24529.84	78.435	33543
30	73.055	24233.29	80.745	33304.72
31	68.685	22642.73	75.915	30786.75
32	72.105	19679.52	79.695	26985.09
33	71.06	23980.82	78.54	32799.89
34	73.435	23656.29	81.165	32540.87
35	68.59	22024.41	75.81	29938.8
36	72.485	19068.84	80.115	26171.54
37	68.4	20714.9	75.6	28145.1
38	72.675	18666.96	80.325	25631.59
39	68.115	20516.61	75.285	27855.37
40	72.485	18517.12	80.115	25414.31
41	50.35	7560.862	55.65	9771.795
42	56.145	7361.116	62.055	9671.978
43	58.33	10196.76	64.47	13481.28
44	61.18	10827.02	67.62	14429.43
45	58.045	11588.31	64.155	15308.71
46	61.465	9310.3	67.935	12417.87
47	57.665	10421.34	63.735	13752.25

...continued

Table 6.5-2 continue...

		Test 1		Test 2	
Cylinder Number	Temp. of Pocket Air( $^{\circ}$ C)	Vapour Pressure in Pocket Air( $P_a$ )	Temp. of Pocket Air( $^{\circ}$ C)	Vapour Pressure in Pocket Air( $P_a$ )	
48	62.32	8529.122	68.88	11402.81	
49	39.33	4756.261	43.47	5972.557	
		Test 3		Test 4	
Cylinder Number	Temp. of Pocket Air( $^{\circ}$ C)	Vapour Pressure in Pocket Air( $P_a$ )	Vapour Pressure in Pocket Air( $P_a$ )	Vapour Pressure in Pocket Air( $P_a$ )	
1	51.2	4040.307		4938.153	
2	47.1	5546.259		6778.761	
3	48.4	6478.173		7917.767	
4	48.4	7372.944		9011.376	
5	50.6	7978.068		9750.972	
6	49.9	9439.47		11537.13	
7	50.6	9470.565		11575.14	
8	49.3	9565.875		11691.63	
9	53.1	10236.26		12510.98	
10	53.6	10663.62		13033.32	
11	55.0	10497.16		12829.86	
12	54.6	8692.893		10624.65	
13	59.5	10249.33		12526.95	
14	70.3	14390.6		17588.52	
15	69.1	17392.69		21257.73	
16	73.6	17266.7		21103.74	
17	73.4	22303.99		27260.43	
18	78.0	28840.55		35249.57	
19	74.5	21944.66		26821.26	
20	76.3	26608.97		32522.07	
21	74.6	22918.45		28011.43	
22	79.5	31000.97		37890.07	
23	75.1	24566.6		30025.84	
24	79.2	30068.75		36750.69	

...continued

Table 6.5-2 continue...

		Test 3	Test 4
Cylinder Number	Temp. of Pocket Air(°c)	Vapour Pressure in Pocket Air(P <sub>a</sub> )	Vapour Pressure in Pocket Air(P <sub>a</sub> )
25	70.9	23740.83	29016.57
26	75.9	29580	36153.34
27	73.0	23989.63	29320.65
28	76.5	27876.99	34071.87
29	74.7	26025.04	31808.38
30	76.9	25866.29	31614.35
31	72.3	24039.07	29381.09
32	75.9	20947.99	25603.09
33	74.8	25437.5	31090.28
34	77.3	25267.01	30881.91
35	72.2	23378.94	28574.26
36	76.3	20324.25	24840.75
37	72.0	21980.94	26865.6
38	76.5	19906.84	24330.58
39	71.7	21756.02	26590.7
40	76.3	19736.21	24122.03
41	53.0	8364.402	10223.16
42	59.1	7641.495	9339.605
43	61.4	10641.71	13006.53
44	64.4	11330	13847.78
45	61.1	12079.89	14764.31
46	64.7	9737.712	11901.65
47	60.7	10843.44	13253.1
48	65.6	8944.065	10931.64
49	41.4	4869.954	5952.166

**Table 6.5-3** Final Paper Moisture Content Against Temp. and Vapour Pressure of Pocket Air

	Test 1	Test 2
Final Paper Moisture Content (%)	7.88	8.04
	Test 3	Test 4
Final Paper Moisture Content (%)	7.76	8.19