

Copyright is owned by the Author of the thesis. Permission is given for a copy to be downloaded by an individual for the purpose of research and private study only. The thesis may not be reproduced elsewhere without the permission of the Author.

BER Performance of MC-DS-CDMA Systems in the Presence of Timing Jitter

A thesis presented in partial fulfillment of the requirements for the
Degree of

Master of Engineering

in

Information and Telecommunications Engineering

at Massey University, Palmerston North,

New Zealand.



Massey University

Lindong Xu

2010

Acknowledgement

I would like to express my gratitude to all those who guided and assisted me to accomplish this thesis. First and foremost, I gratefully acknowledge the help of my supervisor Dr. Xiang Gui. I do appreciate his patience, encouragement, and professional instructions during my thesis writing. The completion of this thesis would be impossible without his guidance, supervision and assistance. Many thanks to my best friend Rui Li who always shares his research experience with me.

I also would like to take this opportunity to express my heartfelt gratitude to my beloved parents who have always been helping me out of difficulties and supporting and caring for me all of my life.

BER Performance of MC-DS-CDMA Systems in the Presence of Timing Jitter

Abstract

Multi-carrier direct-sequence code division multiple access (MC-DS-CDMA) technique, which is a combination of orthogonal frequency-division multiplexing (OFDM) and code division multiple accesses (CDMA), has been considered as an important technique for the future generation wireless systems due to its bandwidth efficiency, frequency diversity, and immunity to channel dispersion. OFDM has already been employed in many areas, such as digital audio and video broadcasting, wireless local/metropolitan area networks, and asynchronous digital subscriber lines (ADSL). Leveraging the multiple access capability of CDMA, the MC-DS-CDMA technique is an important enhancement to OFDM.

Nevertheless, a major disadvantage of the MC-DS-CDMA systems is their high sensitivity to timing errors between transmitter and receiver due to the use of a large number of carriers and the superposition of signals of multiple users. In this thesis, we study the bit error rate (BER) performance of MC-DS-CDMA system under the effects of timing jitter in additive white Gaussian noise (AWGN) channel and multi-path Rayleigh fading channel, respectively. In particular, we have derived the analytical BER expressions for the MC-DS-CDMA signals in presence of white or colored timing jitters and verified the results via computer simulations.

Contents

1. Introduction.....	1
1.0 General introduction	1
1.1 Thesis structure	2
2. Literature review	4
2.0 Introduction.....	4
2.1 OFDM.....	4
2.1.1 History of OFDM.....	4
2.1.2 Mathematical description of OFDM.....	8
2.1.3 Basic principles of OFDM.....	9
2.1.4 Advantages and disadvantages of OFDM.....	14
2.2 Spread spectrum technique	15
2.2.1 Direct sequence spread spectrum.....	16
2.2.2 Frequency hopping spread spectrum	16
2.2.3 Advantages of spread spectrum	17
2.3 Multi-carrier code division multiple access.....	18
2.3.1 MC-CDMA scheme	19
2.3.2 MC-DS-CDMA scheme.....	21
2.3.3 MT-DS-CDMA scheme.....	23
2.4 Summary	24

3. Implementation of the MC-DS-CDMA system..... 26

3.0 Introduction.....	26
3.1 Transmitter.....	27
3.2 Dual OVSF spreading codes.....	29
3.3 Channel simulation.....	35
3.4 Receiver.....	38
3.5 Simulation results and conclusions.....	39

4. BER performance analysis on the effect of timing jitter 42

4.0 Introduction.....	42
4.1 Problem formulation.....	42
4.2 ICI due to timing jitter.....	44
4.2.1 ICI due to white timing jitter.....	47
4.2.2 ICI due to colored timing jitter.....	53

5. BER performance test methods..... 59

5.0 Introduction.....	59
5.1 Testing method in AWGN channel.....	59
5.2 Testing methods in multi-path Rayleigh fading channel.....	59

6. Results and discussion 61

6.1 BER performance on the effect of timing jitter in AWGN channel.....	61
---	----

6.2 BER performance on the effect of timing jitters in multi-path Rayleigh fading channel	70
7. Conclusion	74
References	75
Appendix: Complete Matlab codes	81
A1 OVSF code	81
A2 Simulation code of the MC-DS-CDMA system in AWGN channel	83
A3 Simulation code of the MC-DS-CDMA system in multi-path Rayleigh fading channel	87

List of Figures

Figure 2.1 Comparison of the bandwidth utilization for FDM and OFDM.....	6
Figure 2.2 (A) Spectrum of an OFDM sub-carrier (B) Spectrum of an OFDM signal.....	6
Figure 2.3 Block diagram of an FFT-based OFDM system	10
Figure 2.4 Example of the guard interval	12
Figure 2.5 Time and frequency representation of OFDM using guard interval.	13
Figure 2.6 Direct sequence spread spectrum (DSSS) block diagram	16
Figure 2.7 Frequency hopping spread spectrum (FHSS) block diagram.....	17
Figure 2.8 The transmitter diagram of MC-CDMA scheme	19
Figure 2.9 The receiver diagram of MC-CDMA scheme	20
Figure 2.10 The transmitter diagram of MC-DS-CDMA scheme.	21
Figure 2.11 The receiver diagram of MC-DS-CDMA scheme.....	22
Figure 2.12 The transmitter diagram of MT-DS-CDMA scheme	23
Figure 2.13 The receiver diagram of MT-DS-CDMA scheme.....	24
Figure 3.1 Block diagram of four-user MC-DS-CDMA system	27
Figure 3.2 Block diagram of MC-DS-CDMA transmitter for a single-user.....	29
Figure 3.3 Multi-path demonstration.	36
Figure 3.4 Block diagram of MC-DS-CDMA channel model.....	38
Figure 3.5 Block diagram of MC-DS-CDMA receiver	39
Figure 3.6 BER curves of SUI Channels	40

List of Tables

Table 3.1 Comparison of peak correlation values: length-4 HOVSF and dual OVSF	35
Table 3.2 Comparison of peak correlation values: length-8 HOVSF and dual OVSF	35
Table 3.3 Parameters of SUI-1.....	37
Table 3.4 Parameters of SUI-2.....	37
Table 3.5 Parameters of SUI-3.....	37
Table 3.6 Parameters of SUI-4.....	37
Table 3.7 Parameters of SUI-5.....	37
Table 3.8 Parameters of SUI-6.....	38
Table 3.9 BER of SUI-1 Channel	40
Table 3.10 BER of SUI-2 Channel	40
Table 3.11 BER of SUI-3 Channel	41
Table 3.12 BER of SUI-4 Channel	41
Table 3.13 BER of SUI-5 Channel	41
Table 3.14 BER of SUI-6 Channel	41

List of Abbreviations

MC-DS-CDMA: multi-carrier direct-sequence code division multiple access

OFDM: orthogonal frequency-division multiplexing

CDMA: code division multiple access

ADSL: asynchronous digital subscriber lines

BER: bit error rate

AWGN: additive white Gaussian noise

MCM: multi-carrier modulation

SP: serial-to-parallel

SNR: signal-to-noise ratio

OVSF: orthogonal variable spreading factor

FFT: Fast Fourier Transform

DMT: discrete multi-tone

FDM: frequency division multiplexing

FDMA: frequency division multiplexing access

VLSI: very-large-scale integration

QAM: quadrature amplitude modulation

HDSL: high-bit-rate digital subscriber lines

VHDSL: very high-speed digital subscriber lines

DAB: digital audio broadcasting

FM: frequency modulation

IFFT: inverse Fast Fourier Transform

ISI: inter-symbol interference

D/A: digital to analog

LPF: low-pass filter

RF: radio frequency

ICI: inter-carrier interference

SS: spread spectrum

DSSS: direct sequence spread spectrum

FHSS: frequency hopping spread spectrum

PN: pseudo-noise

FFHSS: fast frequency-hopping spread-spectrum

SFHSS: slow frequency-hopping spread-spectrum

MC-CDMA: multi-carrier orthogonal frequency-division multiplexing

MT-DS-CDMA: multi-tone direct-sequence code division multiple access

FEC: forward error control

4G: fourth-generation

HOVSF: Hadamard orthogonal variable spreading factor

JOVSF: Jacket orthogonal variable spreading factor

SUI: Stanford University Interim

1. Introduction

1.0 General introduction

Multi-carrier modulation (MCM) that is known as orthogonal frequency-division multiplexing (OFDM) has drawn considerable attention in high speed mobile communications due to its bandwidth efficiency, frequency diversity, and immunity to channel dispersion during the last decade. Currently, MCM is being used in digital audio and video broadcasting, wireless local/metropolitan area networks, and asynchronous digital subscriber lines (ADSL) [1]. With recent advances in modulation and multiple access techniques, MCM becomes more and more popular.

Today, the enormous increase in demand for high-speed data transmission by many wireless multi-media services pushes the development of advanced modulation and multiple access techniques that can provide reliable and high data rate transmission with high bandwidth efficiency and strong immunity to multi-path distortion.

Recently, different combinations of OFDM and code division multiple accesses (CDMA) have been investigated and presented by various researchers. One of these combinations is the multi-carrier direct-sequence code division multiple access (MC-DS-CDMA) technique, which has been considered as an important technique for the fourth-generation wireless systems [2]. In the MC-DS-CDMA systems, the transmitter transmits the serial-to-parallel (SP) converted data stream using a given spreading sequence in the time domain, so that the resultant spectrum of each sub-carrier can remain orthogonal and keep the minimum required frequency separation.

Nevertheless, a major disadvantage of multi-carrier direct-sequence code division multiple access (MC-DS-CDMA) systems is their high sensitivity to timing errors between transmitter and receiver due to the use of a large number of carriers. The influence of timing errors on multi-carrier systems has been reported in [45, 46] for OFDM and in [39] for MC-DS-CDMA. In [45] and [46], it was shown that the OFDM

system suffers severely from a clock frequency offset between the transmitter and the receiver. In [39], the effect of timing jitter on MC-DS-CDMA was studied in terms of signal-to-noise ratio (SNR) degradation. While SNR is indicative of the performance of digital communication systems, the bit error rate (BER) is the direct performance measure. In this thesis, we extend the results of [39] and study the BER performance of MC-DS-CDMA system under the effects of timing jitter in additive white Gaussian noise (AWGN) channel and multi-path Rayleigh fading channel, respectively. In particular, we have derived the analytical BER expressions for the MC-DS-CDMA signals in presence of white or colored timing jitters and verified the results via computer simulations.

1.1 Thesis structure

This thesis is organized as seven chapters and one appendix.

Chapter 2 gives a literature review related to our research including orthogonal frequency division multiplexing technique, spreading spectrum technique, and multi-carrier code division multiple access technique.

Chapter 3 presents an MC-DS-CDMA system simulated in AWGN channel and multi-path Rayleigh fading channel using Simulink in Matlab. The simulation of transmitter, channel and receiver are described in this chapter. The orthogonal variable spreading factor (OVSF) codes used as spreading codes in our research are also presented.

Chapter 4 describes the analysis of BER performance of MC-DS-CDMA system due to timing jitter in AWGN channel and multi-path Rayleigh fading channel, respectively. The analytical BER expressions for MC-DS-CDMA in presence of white and colored timing jitters are formulated and discussed.

Chapter 5 introduces the algorithm we used to test the BER performance of MC-DS-CDMA system.

Chapter 6 shows the test results of the system. A discussion is also included.

Chapter 7 contains the conclusion of this thesis.

The appendix contains the simulation algorithm codes.

2. Literature review

2.0 Introduction

Our research is on MC-DS-CDMA and the simulation is based on Simulink and programming in Matlab. Related topics are OFDM, spread spectrum, and combinations of MCM and CDMA. We will present a literature review of these topics in the following sections.

2.1 OFDM

MCM is a method to data transmission that divides data stream into several sub-streams and deliver each of these sub-streams to modulate carrier signals. With recent advances in efficient implementation of Fast Fourier Transform (FFT), the MCM concept is becoming widely adopted in practical applications. It can be found in IEEE standards 802.11a, 802.15.3 and 802.16a, as well as standards for digital broadcasting of television and radio [3]. MCM has been referred to by different names such as discrete multi-tone (DMT) modulation in wire line communication and OFDM in wireless communication. We will use OFDM in this thesis.

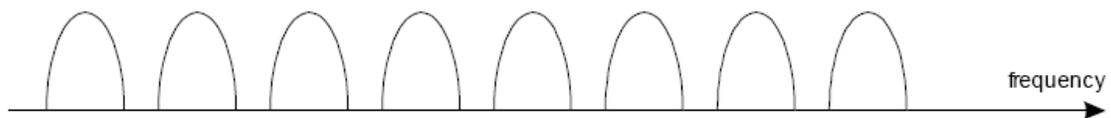
We next present the history, mathematical description and basic principles of OFDM.

2.1.1 History of OFDM

Frequency division multiplexing (FDM) is a technology that transmits multiple signals simultaneously over a single transmission path. In frequency division multiplexing access (FDMA) each user is typically allocated a single channel that is used to transmit all the user information, each signal travels within its own unique frequency range (carrier), which is modulated by the data [4].

OFDM is a transmission technique based on the idea of FDM. In OFDM, the transmitter transmits a single data stream over a number of lower rate orthogonal sub-carriers coupled with the use of advanced modulation techniques on each component, resulting in a signal with high resistance to interference. OFDM can be simply defined as multi-carrier modulation where its carrier spacing is carefully selected so that each sub-carrier is orthogonal to the other sub-carriers [5]. One main reason to use OFDM is to increase the robustness against frequency selective fading or narrowband interference. In a single carrier system, a single fade or interference can cause the entire link to fail, but in a multi-carrier system, only a small percentage of the sub-carriers will be affected. Then error correction coding can be used to correct for the few erroneous sub-carriers. The concept of using parallel data transmission and FDM was published in the mid-1960s [6, 7]. Some early development can be traced back in the 1950s [8]. A U.S. patent was filed and issued in January, 1970 [9].

In a classical parallel data system, the total signal frequency band is divided into non-overlapping frequency sub-channels. Each sub-channel is modulated with a separate symbol and then the sub-channels are frequency-multiplexed. It seems good to avoid spectral overlap of channels and eliminate inter-channel interference. However, this leads to inefficient use of the available spectrum. To cope with the inefficiency, the ideas proposed from the mid-1960s were to use parallel data and FDM with overlapping sub-channels to avoid the use of high speed equalization and to combat impulsive noise, and multi-path distortion as well as to fully use the available bandwidth [10].



Conventional FDM multi-carrier modulation technique

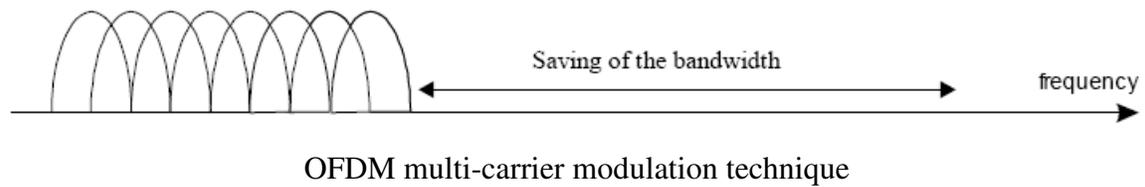


Figure 2.1 Comparison of the bandwidth utilization for FDM and OFDM [5]

Figure 2.1 illustrates the difference between the conventional FDM multi-carrier modulation technique and the OFDM multi-carrier modulation technique. As shown in Figure 2.1, over 50% of bandwidth can be saved by using OFDM multi-carrier modulation technique. However, crosstalk between sub-carriers needs to be reduced in OFDM technique, which means orthogonality between the different modulated carriers is necessary.

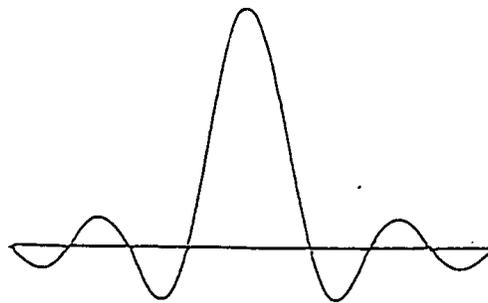


Figure 2.2(a) Spectrum of an OFDM sub-channel [5]

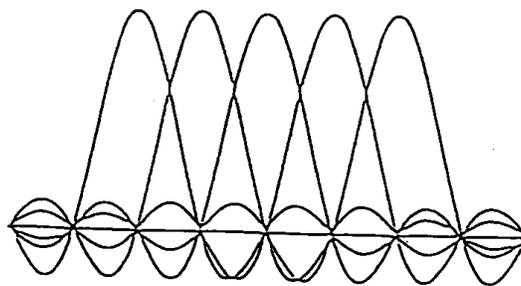


Figure 2.2(b) Spectrum of an OFDM signal [5]

Much of the research focuses on the highly efficient multi-carrier transmission scheme based on orthogonal frequency carriers. In 1971, Weinstein and Ebert applied the discrete Fourier transform (DFT) to parallel data transmission systems as part of the modulation and demodulation processes [11]. Figure 2.2(a) shows the spectrum of an OFDM sub-channel and Figure 2.2 (b) presents the spectrum of the OFDM signal. From Figure 2.2, it can be seen that at the central frequency of each sub-channel, there are no cross talks from other channels. Therefore, if we use DFT at the receiver and calculate correlation values with the center of frequency of each sub-carrier, the transmitted data could be recovered with no crosstalk.

Moreover, to eliminating the banks of sub-carrier oscillators and coherent demodulators required by FDM, a completely digital implementation could be built around special-purpose hardware performing the fast Fourier transform (FFT), which is an efficient implementation of the DFT. Recent advances in very-large-scale integration (VLSI) technology make high speed and large size FFT chips commercially affordable [12]. Consequently, both transmitter and receiver are implemented using efficient FFT techniques that reduce the number of operations from N^2 in DFT down to $N \log N$ where N is the FFT size [13].

In the 1960s, the OFDM technique was used in several high-frequency military systems such as KINEPLEX [8], ANDEFT [14] and KATHRYN [15, 16]. For example, the variable-rate data modem in KATHRYN was built for the high-frequency band. It used up to 34 parallel low-rate phase-modulated channels with a spacing of 82 Hz.

In the 1980s, OFDM was studied for high-speed modems, digital mobile communications, and high-density recording. One of the systems realized the OFDM techniques for multiplexed quadrature amplitude modulation (QAM) using DFT [17]. Furthermore, variable-speed modems were developed for telephone networks [19].

In the 1990s, OFDM was employed for wideband data communications over mobile

radio channels, high-bit-rate digital subscriber lines (HDSL, 1.6 Mbps), asymmetric digital subscriber lines (ADSL, up to 6 Mbps), very high-speed digital subscriber lines (VHDSL, 100 Mbps), digital audio broadcasting (DAB), and digital television and high-definition television (HDTV) terrestrial broadcasting [10]. Casas and Leung proposed OFDM/frequency modulation (FM) for data communication over mobile radio channels [20]. It claimed that OFDM/FM systems could be implemented simply and inexpensively by retrofitting existing FM radio systems. Chow et al, studied the multi-tone modulation with DFT in transceiver design and showed that it is an excellent method for delivering of high speed data to customers, both in terms of performance and cost, for ADSL (1.536 Mbps), HDSL (1.6 Mbps), and VHDSL (100 Mbps) [21, 22].

2.1.2 Mathematical description of OFDM

Following the history of OFDM, we discuss the mathematical definition of the OFDM system. This allows us to see how the signal is generated and how receiver must operate, and it gives us a tool to understand the effects of imperfections in the transmission channel. As presented above, OFDM transmits a large number of narrowband sub-carriers, closely spaced in the frequency domain. In order to avoid a large number of modulators and filters at the transmitter and complementary filters and demodulators at the receiver, it is desirable to be able to use modern digital signal processing techniques, such as FFT [5].

Mathematically, each modulated OFDM sub-carrier can be represented as the real part of the following complex signal

$$S_c(t) = A_c(t)e^{j[\omega_c t + \phi_c(t)]}, \quad (2.1)$$

where ω_c is the sub-carrier angular frequency; while $A_c(t)$ and $\phi_c(t)$ represent amplitude and phase modulation on the sub-carrier, respectively, and varies from symbol to symbol. OFDM consists of many sub-carriers. Therefore, the complex OFDM signal $S_s(t)$ with N sub-carriers can be written as

$$S_s(t) = \frac{1}{N} \sum_{n=0}^{N-1} A_n(t) e^{j[\omega_n t + \phi_n(t)]}, \quad (2.2)$$

where $\omega_n = \omega_0 + n\Delta\omega$ and $\Delta\omega$ is the sub-carrier spacing in angular frequency.

The amplitude and phase modulation do not change within an OFDM symbol, hence $A_n(t)$ and $\phi_n(t)$ can be rewritten as A_n and ϕ_n , respectively, when equation 2.2 is used to represent the waveform of a single OFDM symbol. If the symbol waveform is sampled at a sampling frequency of $\frac{1}{T}$, where $T = T_s/N$ and T_s is the duration of one OFDM

symbol, then the resulting signal samples are given by

$$S_s(kT) = \frac{1}{N} \sum_{n=0}^{N-1} A_n e^{j[(\omega_0 + n\Delta\omega)kT + \phi_n]}, \quad k = 0, 1, \dots, N-1. \quad (2.3)$$

Without a loss of generality by assuming $\omega_0 = 0$, equation 2.3 can be simplified as:

$$S_s(kT) = \frac{1}{N} \sum_{n=0}^{N-1} A_n e^{j\phi_n} e^{j(n\Delta\omega)kT}, \quad k = 0, 1, \dots, N-1. \quad (2.4)$$

Let

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{1}{NT} = \frac{1}{T_s}, \quad (2.5)$$

equation 2.4 becomes equation 2.6, which is the general form of the Inverse DFT (IDFT).

$$S_s(kT) = \frac{1}{N} \sum_{n=0}^{N-1} A_n e^{j\phi_n} e^{j2\pi nk/N}, \quad k = 0, 1, \dots, N-1. \quad (2.6)$$

Equation 2.5 is the same condition that is required for sub-carrier orthogonality. Thus, the OFDM signal can be defined by using Fourier Transform procedures [5].

2.1.3 Basic principles of OFDM

a. Generation of sub-carriers using IDFT

Consider a data sequence $d = (d_0, d_1, \dots, d_n, \dots, d_{N-1})$, where $d_n = A_n e^{j\phi_n} = a_n + jb_n$.

According to the definition of the N -point IDFT, which is

$$x[k] = \frac{1}{N} \sum_{n=0}^{N-1} X[n] e^{j(2\pi/N)kn}, \quad (2.7)$$

the data sequence d can be written as a vector $D = (D_0, D_1, \dots, D_m \dots, D_{N-1})$ after performing DFT.

$$D_m = \frac{1}{N} \sum_{n=0}^{N-1} d_n e^{j(2\pi nm/N)} = \frac{1}{N} \sum_{n=0}^{N-1} d_n e^{j2\pi f_n t_m}, \quad m=0, 1, \dots, N-1, \quad (2.8)$$

where $f_n = n/(NT) = n/T_s$, $t_m = mT$ and T is the time interval of d_n .

The real part of the vector D is

$$Y_m = \text{Re}\{D_m\} = \frac{1}{N} \sum_{n=0}^{N-1} [a_n \cos(2\pi f_n t_m) - b_n \sin(2\pi f_n t_m)], \quad m=0, 1, \dots, N-1. \quad (2.9)$$

If the samples Y_m is applied to a low-pass filter at time interval T , then the obtained signal is the continuous-time OFDM waveform.

$$y(t) = \frac{1}{N} \sum_{n=0}^{N-1} [a_n \cos(2\pi f_n t) - b_n \sin(2\pi f_n t)], \quad 0 \leq t \leq T_s. \quad (2.10)$$

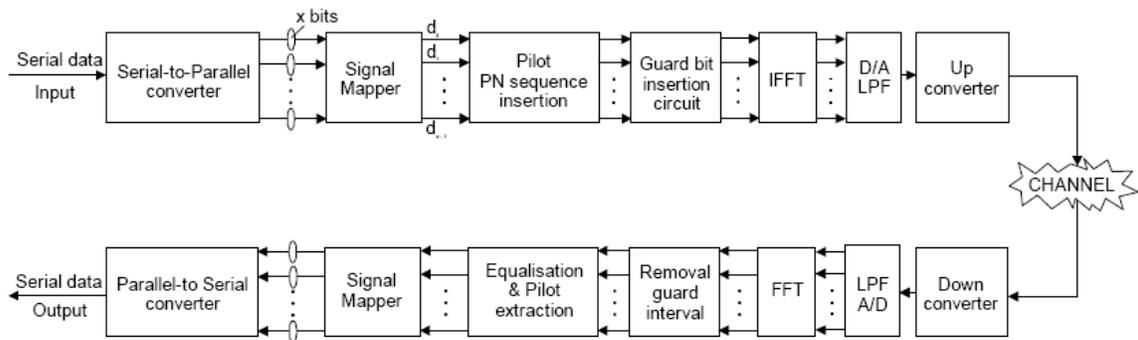


Figure 2.3: Block diagram of an FFT-based OFDM system [10]

Figure 2.3 presents the block diagram of a typical FFT-based OFDM system. At

transmitter side, the original serial data is first serial-to-parallel converted and grouped into x -bits and then mapped to complex numbers. Then the complex numbers are modulated in the baseband using the inverse FFT (IFFT) and converted back to serial data for transmission. In order to avoid inter-symbol interference (ISI) caused by multi-path distortion, a guard interval is inserted between symbols. Finally, the symbols are converted from digital to analog (D/A) and low-pass filtered (LPF) for radio frequency (RF) up-conversion. At receiver side, the inverse process of the transmitter is performed and one-tap equalizer is used to correct channel distortion. The tap-coefficients of the filter are calculated according to the channel information [10].

b. Guard interval and cyclic extension

Theoretically, the orthogonality of sub-channels in OFDM can be maintained and individual sub-channels can be completely separated by the FFT at the receiver. However, these conditions can not be obtained in practice due to inter-symbol interference (ISI) and inter-carrier interference (ICI) introduced by transmission channel distortion. Linear channel distortions such as multi-path can cause each sub-channel to spread energy into the adjacent channels and make crosstalk between different sub-carriers. Consequently, this results in ISI and ICI.

In order to prevent multi-path components interfering from one symbol with the next symbol, a cyclically extended guard interval, which is a periodic extension of each OFDM symbol itself, is applied for each OFDM symbol. The total symbol duration is $T_{id} = T_g + T_s$, where T_g is the duration of guard interval and T_s is the duration of useful symbol. If the guard interval is chosen longer than the channel impulse response or the multi-path delay, the ISI and ICI could be eliminated [10]. The ratio of the guard interval to useful symbol duration depends on different systems. Since the insertion of guard interval will reduce data throughput, T_g is usually less than $T_s/4$ [23].

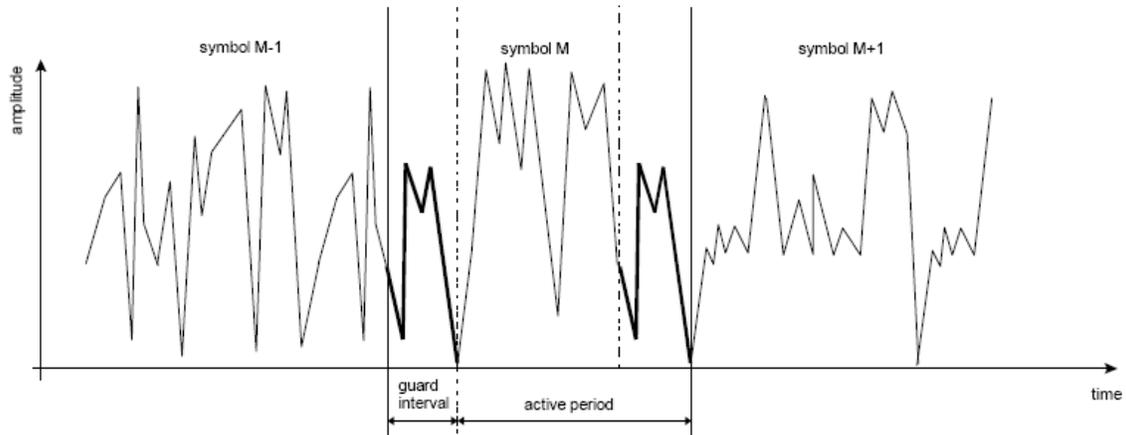


Figure 2.4: Example of the guard interval [5]

Figure 2.4 presents the example of the guard interval of the OFDM symbol. Each symbol consists of two parts: guard interval and active period. The useful symbol M is contained in the active period and the last part of active period is also repeated at the start of the symbol and is called the guard interval.

The reasons to use a cyclic prefix for the guard interval are [10]:

1. To maintain the receiver carrier synchronization. Some signal instead of a long silence must always be transmitted
2. Cyclic convolution can still be applied between the OFDM signal and the channel response to model the transmission system. Figure 2.5 shows the time and frequency representation of OFDM using guard interval. With the two-dimensional signal representation, the symbols are overlapped in the frequency domain and are separated by the guard interval in the time domain.

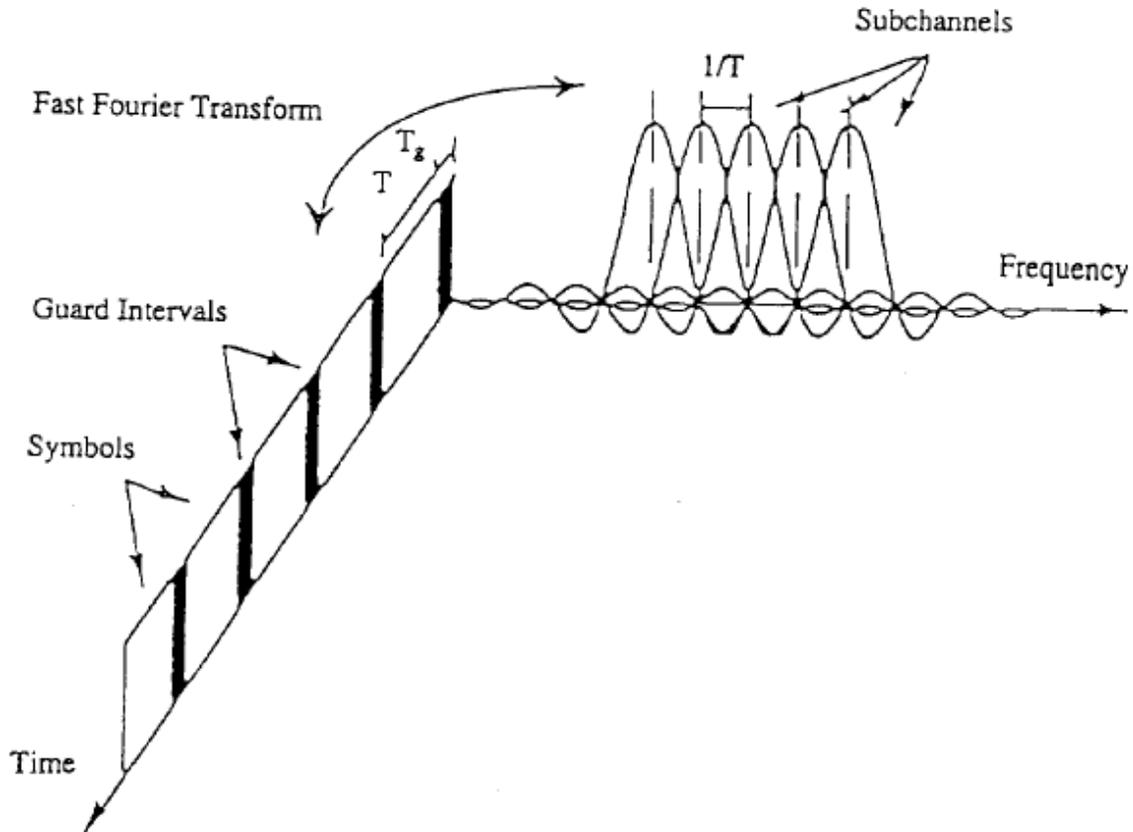


Figure 2.5: Time and frequency representation of OFDM using guard interval [10]

c. Choice of key elements

1. Useful symbol duration

The useful symbol duration can be fixed after the guard interval has been set. To minimize the signal-to-noise ratio (SNR) loss caused by the guard interval, it is desirable to have the useful symbol duration larger than the guard interval. However, it can not be arbitrarily large because larger useful symbol duration results in more sub-carriers with a small sub-carrier spacing, larger implementation complexity of FFT calculation and more sensitivity to phase noise and frequency offset [24], as well as an increase of peak-to-average power ratio [25]. Hence, a practical design choice is to make the useful symbol duration at least five times the guard interval [13] so that the channel is stable for the duration of a symbol.

2. Number of sub-carriers

After the symbol duration and guard interval are fixed, the number of sub-carriers can be determined by the required bit rate divided by the bit rate per sub-carrier. The bit rate per sub-carrier is defined based on the modulation type, coding rate and the useful symbol duration. What is more, the number of sub-carriers corresponds to the number of complex points being processed in FFT. In HDTV applications, the number of sub-carriers is in the range of several thousands so as to accommodate the data rate and guard interval requirement [5].

3. Modulation scheme

The modulation scheme in an OFDM system can be chosen based on the requirement of power or spectrum efficiency. The type of modulation can be specified by the complex number $d_n = a_n + jb_n$, which is defined in the section 2.1.3a. The symbols a_n and b_n can be set as $(\pm 1, \pm 3)$ for 16-QAM and ± 1 for QPSK [5]. In general, the selection of the modulation scheme applying to each sub-channel depends on the compromise between the data rate requirement and transmission robustness. Furthermore, another advantage of OFDM is that different modulation schemes can be used on different sub-channels for layered services [5].

2.1.4 Advantages and disadvantages of OFDM

The OFDM transmission scheme has the following key advantages [13]:

1. OFDM is an efficient way to deal with multi-path. The overall signal spectrum is divided into narrowband flat-fading sub-channels. As a result, channel equalization is accomplished through a simple bank of complex-valued multipliers, thereby avoiding the need for computationally demanding time domain equalizers.
2. OFDM significantly enhances the capability of interference suppression through the use of the cyclic prefix.
3. OFDM is robust against frequency selective fading and narrowband interference, because such interference affects only a small percentage of the sub-carriers.

-
4. OFDM has higher spectral efficiency. More than 50% of the bandwidth can be saved compared with FDM due to overlapping sub-carriers in the frequency domain.
 5. OFDM makes digital implementation simple by using DFT/IDFT operations.
 6. OFDM provides an opportunity of selecting the most appropriate coding and modulation scheme on each individual sub-carrier according to the measured channel quality. In practice, higher order constellations are normally used on less attenuated sub-carriers in order to increase the data throughput, while robust low-order modulations are employed over sub-carriers characterized by low SNR values.

On the other hand, OFDM suffers from the following drawbacks compared with single-carrier modulations:

1. OFDM is more sensitive to frequency synchronization errors and phase noise.
2. OFDM has a relatively large peak-to-average power ratio, which tends to reduce the power efficiency of the RF amplifier.
3. OFDM has an inherent loss in spectral efficiency related to the use of the cyclic prefix.

2.2 Spread spectrum technique

The term spread spectrum (SS) has been used in a wide variety of military and commercial communication systems. It was first described on paper by an actress and a musician in 1941 [26] and was developed by military due to the use of wideband signals which are difficult to detect. In recent years, SS has become increasingly popular for commercial applications, especially in local area wireless networks.

Spread spectrum is a signal structuring technique that each signal to be transmitted requires significantly more RF bandwidth than a conventional modulated signal would require. In SS system, a sequential noise-like signal structure is used to spread normal narrowband information signal over a large bandwidth in frequency domain and receiver correlates received signals to retrieve the original information signal. That is to say, the

information signal is spread as wide bandwidth as possible and as close to the background noise as possible in SS system. This makes SS communication very difficult to find in the frequency spectrum and can not be easily tracked and more difficult to jam [27]. There are two different types of spread spectrum techniques that have been extensively used. These are direct sequence spread spectrum (DSSS) and frequency hopping spread spectrum (FHSS). We will describe both techniques in the following sections.

2.2.1 Direct sequence spread spectrum

Direct sequence is one of the most popular types of spread spectrum techniques. In this method, a spreading signal which is a high-speed pseudo-noise (PN) code sequence created by a pseudo-random code generator is directly multiplied with narrow-band PSK modulated signal. Thus, desired transmission radio frequency (RF) bandwidth can be set directly by the spreading signal.

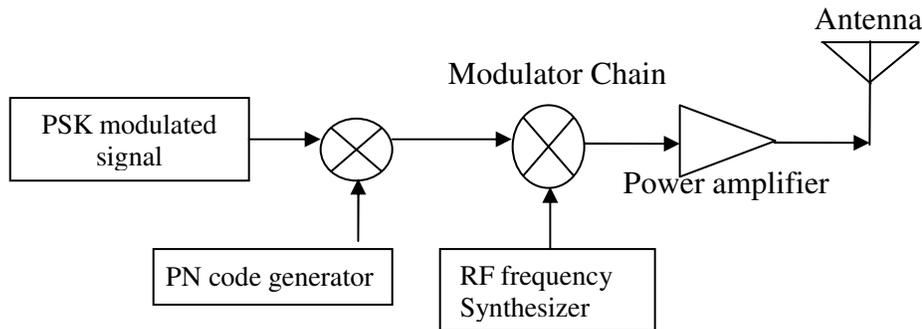


Figure 2.6 Direct sequence spread spectrum (DSSS) block diagram

2.2.2 Frequency hopping spread spectrum

Frequency hopping is another type of spread spectrum technique which has an

implementation concept similar to that of DSSS technique. In this method, spreading occurs by hopping frequency synthesizer to one of many available frequencies over a wide band according to a hopping table defined by the PN sequence generator. If the hopping rate, which is chip rate, is higher than the bit rate, then it is called fast frequency-hopping spread-spectrum (FFHSS). If the hopping rate is slower than the data rate, there are several or many bits per frequency hop, and then it is called slow frequency-hopping spread-spectrum (SFHSS).

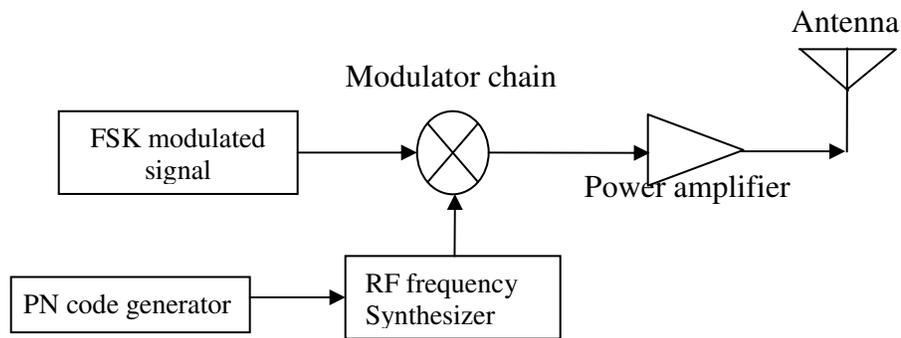


Figure 2.7 Frequency hopping spread spectrum (FHSS) block diagram

2.2.3 Advantages of spread spectrum technique

The spread spectrum technique has the following key advantages [28]:

1. Improved interference rejection.
2. Low-density power spectra for signal hiding.
3. High-resolution ranging.
4. Secure communications.
5. Anti-jamming capability.
6. Lower cost of implementation
7. Graceful degradation of performance as the number of simultaneous users of an RF channel increases.

2.3 Multi-carrier code division multiple access

Recently, a number of CDMA systems based on the combination of CDMA schemes and OFDM technique, which are referred to as multi-carrier CDMA systems, has received a lot of attention in the field of wireless communications due to their effectiveness in combating the effects of multi-path fading channels and various kinds of interference in high speed data transmission by spreading signals over several carriers. Among these systems, the signal can be efficiently modulated and demodulated using Fast Fourier Transform (FFT) device without substantially increasing the transmitter and receiver's complexity and these systems also exhibit the attractive features of high spectral efficiency and frequency diversity [29].

Depending on whether all the sub-carriers are activated on each transmission, multiple access schemes based on a combination of CDMA and OFDM technique can be classified as three types: multi-carrier CDMA (MC-CDMA), multi-carrier DS-CDMA (MC-DS-CDMA) and multi-tone DS-CDMA (MT-DS-CDMA), which were developed and presented by different researchers, namely, MC-CDMA by N.Yee, J-P.Linnartz and G.Fettweis, K.Fazel and L.Papke, and A.Chouly, A.Brajaj and S.Jourdan; MC-DS-CDMA by V.DaSilva and E.S.Sousa; and MT-DS-CDMA by L.Vandendorpe [30]. The following section will review the three types of multi-carrier CDMA schemes mentioned above.

2.3.1 MC-CDMA scheme

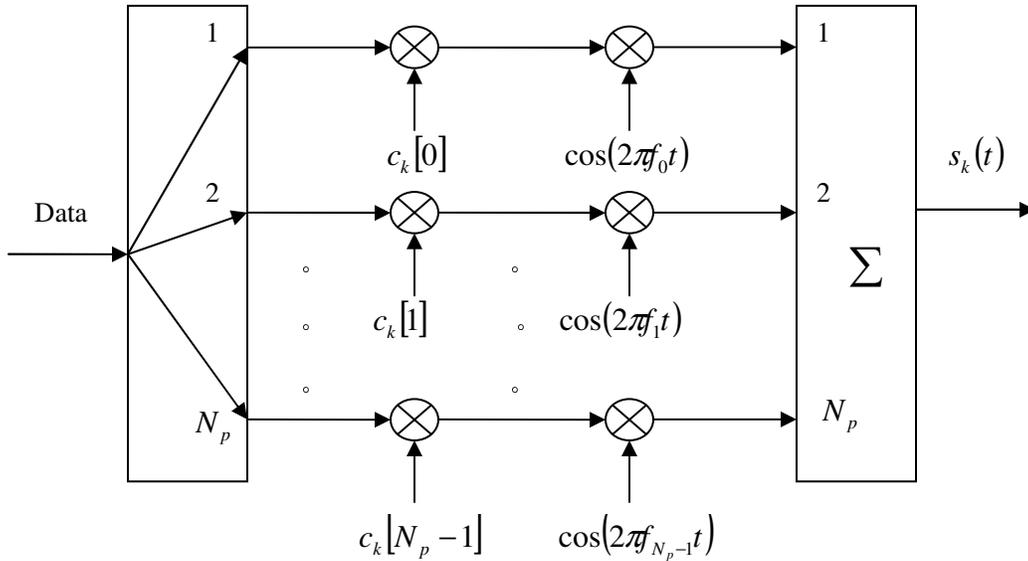


Figure 2.8 The transmitter diagram of MC-CDMA scheme

Figure 2.8 shows the transmitter diagram of the MC-CDMA scheme. The MC-CDMA transmitter spreads the original serial data stream over N_p , which is the number of sub-carriers as well as the spreading gain, sub-carriers by a given spreading code of $\{c_k[0], c_k[1], \dots, c_k[N_p - 1]\}$ in the frequency domain. In this scheme, it does not include serial-to-parallel data conversion and no spreading modulation is implemented on each sub-carrier. Therefore, the data rate on each of the N_p sub-carriers is the same as the input data rate and the fading effects of multi-path channels can be mitigated by spreading each data bit across all of the N_p sub-carriers [29].

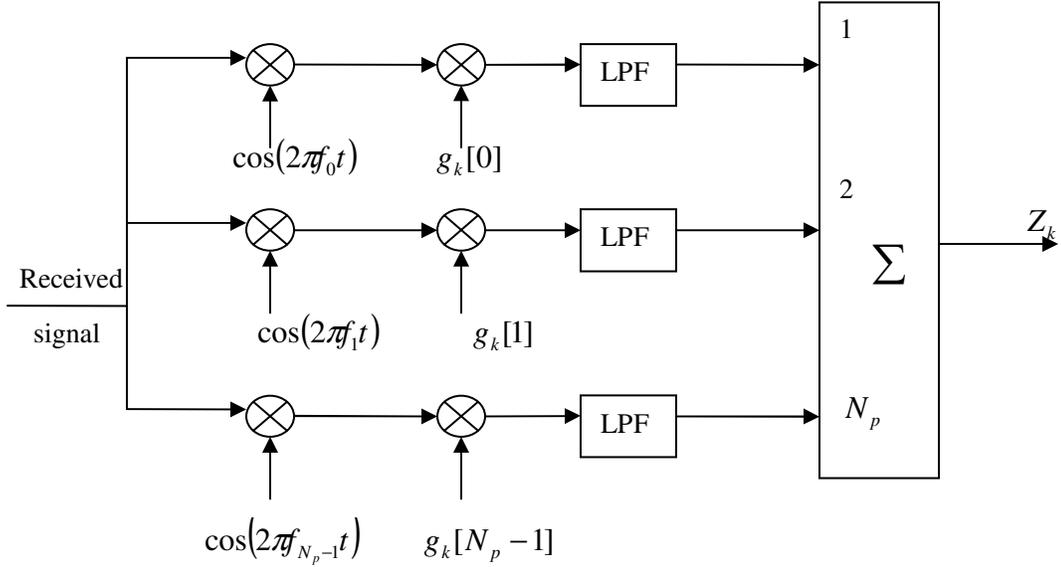


Figure 2.9 The receiver diagram of MC-CDMA scheme

Figure 2.9 shows the receiver diagram of the MC-CDMA scheme. In this receiver structure, the fading impaired signals of the sub-carrier from diverse users are first equalized and then separated according to their different spreading codes. What is more, the received signal is combined in the frequency domain. Therefore, the receiver can always make use of all the received signal energy scattered in the frequency domain. This is the main advantage of the MC-CDMA schemes over other schemes [30]. However, in a frequency selective fading channel, different sub-carriers may encounter different amplitude attenuations and phase shifts, which can consequently result in distortion of the orthogonality of the sub-carriers.

This MC-CDMA scheme can be applied in a multiple access system which allocates different spreading code for each user, and then the spreading codes can provide separation for different users. In addition, in MC-CDMA scheme the number of sub-carrier does not have to be the same as the processing gain. If the original symbol rate is high, the signal suffers frequency selective fading. Then the signal needs to be serial-to-parallel converted, mapping the data to a number of reduced-rate streams before spreading over the frequency domain [29].

2.3.2 MC-DS-CDMA scheme

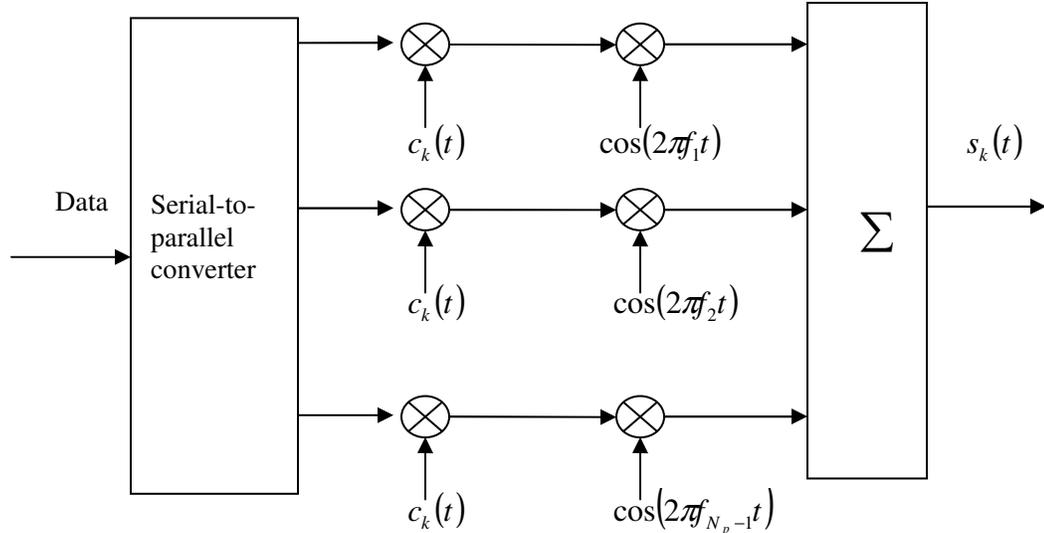


Figure 2.10 The transmitter diagram of MC-DS-CDMA scheme

The MC-DS-CDMA transmitter spreads the serial-to-parallel converted data streams using a given spreading code in the time domain, so that the resulting spectrum of each sub-carrier can satisfy the orthogonality condition with the minimum required frequency separation [29]. The transmitter diagram of the MC-DS-CDMA scheme is shown in Figure 2.10. In this scheme the initial data stream is serial-to-parallel converted to a number of lower-rate streams and each of these lower-rate streams is spread by the spreading code $c_k(t)$ with the same rate in the time domain, so that it is easier to synchronize the spreading sequences by a large chip time [30]. Furthermore, each of the lower-rate parallel streams is bit-interleaved and these streams modulate the orthogonal sub-carriers with a successively overlapping bandwidth [29].

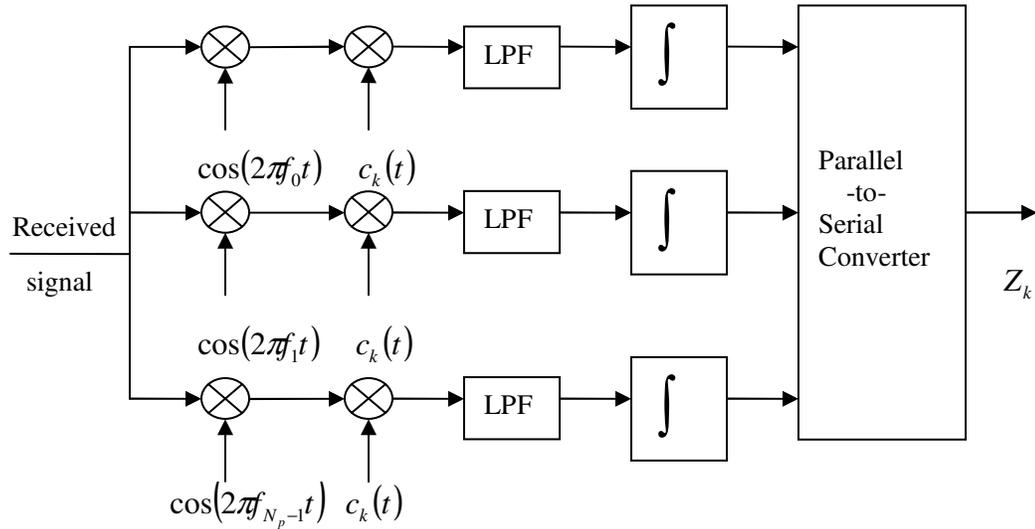


Figure 2.11 The receiver diagram of MC-DS-CDMA scheme

The receiver diagram of MC-DS-CDMA scheme is shown in Figure 2.11. The receiver contains a multiple correlator for each sub-carrier and each correlator is synchronized to a different resolvable path in the received composite signal. Then the serial data stream is recovered by a parallel-to-serial converter.

The MC-DS-CDMA scheme has the following advantages. First, the spreading processing gain is increased compared to the corresponding single-carrier DS-CDMA scheme. Second, the effect of multi-path interference is mitigated by implementing DS spreading. Finally, frequency/time diversity can be achieved [31].

In addition, each sub-carrier signal is subject to frequency selective fading in the MC-DS-CDMA scheme system. Consequently, a high complexity RAKE receiver can be implemented and forward error control (FEC) techniques can be utilized in order to enhance its associated performance [29].

2.3.3 MT-DS-CDMA scheme

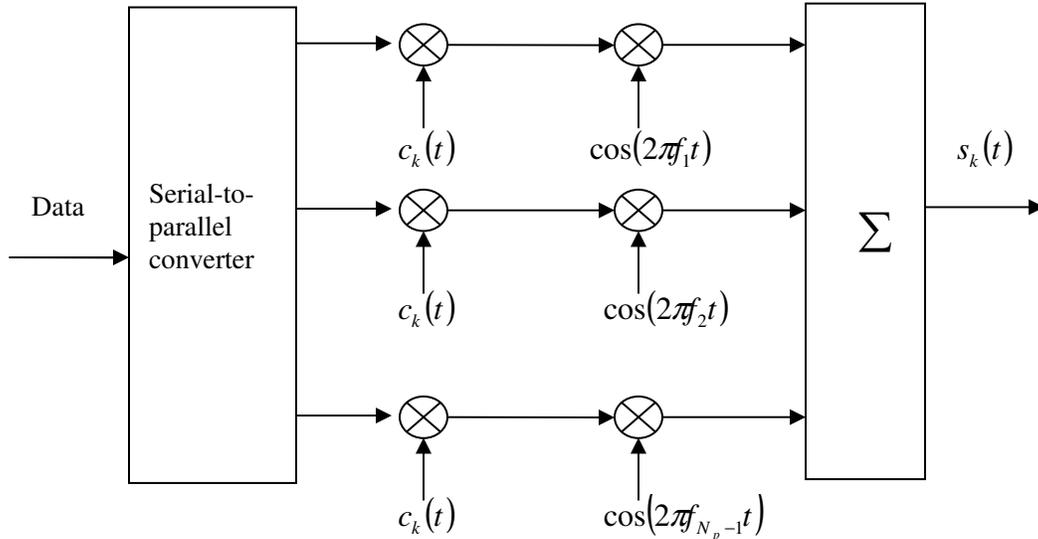


Figure 2.12 the transmitter diagram of MT-DS-CDMA scheme

The MT-DS-CDMA transmitter spreads the serial-to-parallel converted data streams using a given spreading code in the time domain so that the spectrum of each sub-carrier *before* spreading operation can satisfy the orthogonality condition with the minimum frequency separation [29]. Therefore, the resulting spectrum of each sub-carrier no longer satisfies the orthogonality condition and strong spectral overlap exists among the different sub-carrier signals *after* DS spreading [30]. Figure 2.12 shows the transmitter diagram of MT-DS-CDMA scheme. In this scheme, the original binary data stream is firstly serial-to-parallel converted to parallel sub-streams and then spectrum spreading occurs in multiplying parallel sub-streams with spreading code $c_k(t)$. Finally, the MT-DS-CDMA signal is generated by adding all of the different sub-carriers' signals.

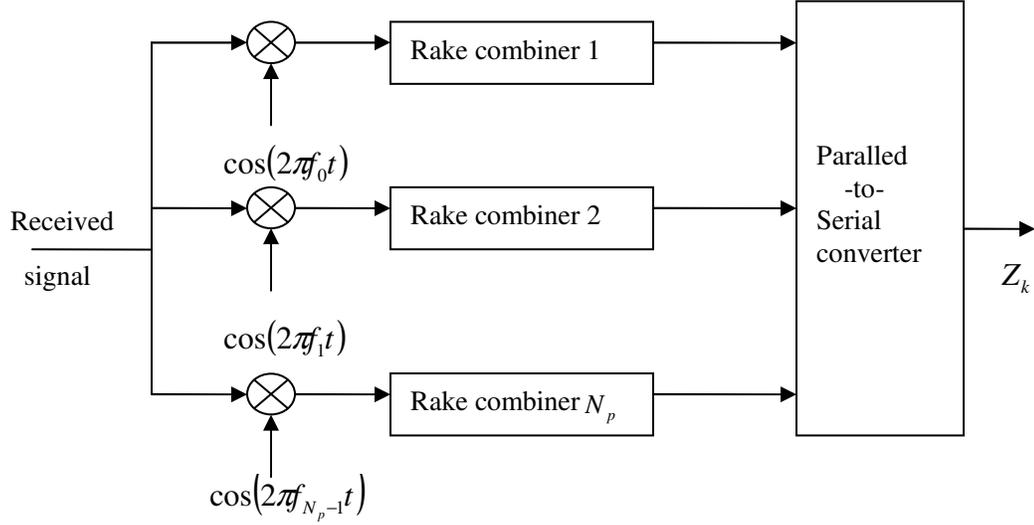


Figure 2.13 the receiver diagram of MT-DS-CDMA scheme

The receiver block diagram of the MT-DS-CDMA scheme is shown in Figure 2.13. The receiver consists of RAKE combiners and each of receivers has the same structure as the single-carrier DS-CDMA RAKE receiver. Unfortunately, the MT-DS-CDMA scheme suffers from inter-sub-carrier interference and requires a high-complexity RAKE-based receiver [29]. However, compared to the corresponding single-carrier DS-CDMA scheme, the capability to use longer spreading codes can reduce the self-interference and multiple access interference [30]. The MT-DS-CDMA scheme uses longer spreading codes than the normal single-carrier DS-CDMA scheme, where the relative code-length extension is in proportion to the number of sub-carriers. Therefore, the MT-DS-CDMA system can accommodate more users [29].

2.4 Summary

In this chapter, the main topic is to present a review on the literature related to our research area. OFDM technique, SS technique and some CDMA schemes have been discussed and advantages and disadvantages of these techniques and schemes have also been described. From the discussion and comparison, it is apparent that MC-DS-CDMA,

which is a combination of OFDM and SS technique, is among the very strong contenders for the fourth-generation (4G) wireless systems.

3. Implementation of the MC-DS-CDMA system

3.0 Introduction

MC-DS-CDMA that is a technique of combination of multi-carrier modulation and spread-spectrum has recently drawn widespread attention due to its potential for high speed transmission and its effectiveness in mitigating the effects of dispersive multi-path fading channels [32] and in combating various kinds of interferences. In this section, we simulate a four-user MC-DS-CDMA system using simulink in Matlab with the effect of AWGN channel and multi-path Rayleigh fading channel.

The block diagram of the MC-DS-CDMA system for four mobile users is shown in Figure 3.1. In this system, the transmitted downlink data for four different users is spread respectively using the orthogonal spreading code sequence assigned to each user and then summed together and transmitted simultaneously through the same channel. Then the data stream is recovered correctly by despreading using the spreading code sequence of each user at receiver side. More details are explained in the following sections.

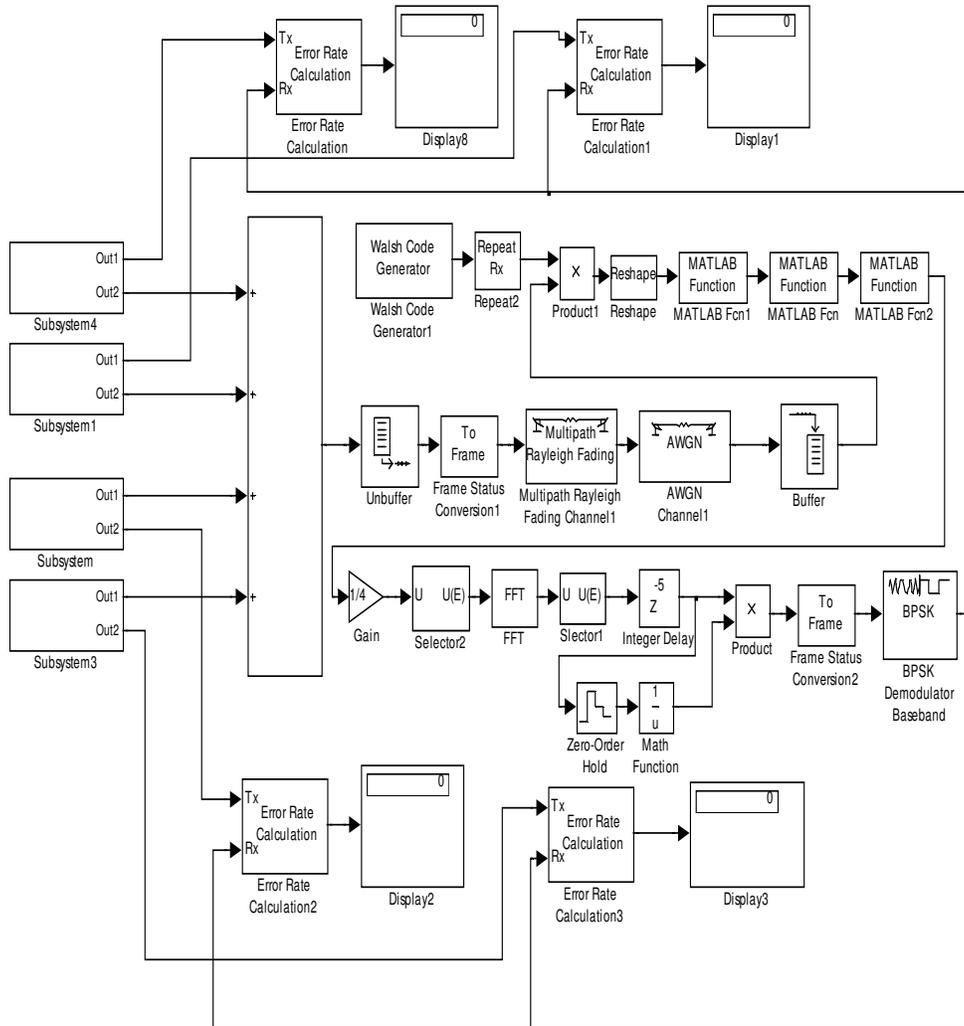


Figure 3.1 Block diagram of four-user MC-DS-CDMA system

3.1 Transmitter

The block diagram of the MC-DS-CDMA transmitter for a single user is shown in Figure 3.2. In MC-DS-CDMA system, the original input data is first converted from serial stream into parallel stream and then modulated by the binary phase shift keying (BPSK) modulator. In order to reduce aliasing in D/A conversion, sub-carriers at or near the Nyquist frequency need to be avoided by inserting zeros. Assuming 16 samples, [1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16], are selected to do IFFT calculation, they need to be arranged as [0 2 3 4 5 6 7 0 0 11 12 13 14 15 16] before IFFT calculation.

Next, the arranged data is processed by IFFT and converted back to serial data for transmission in time domain. Specifically, the IFFT converts the data in frequency domain into the corresponding representation of this data in time domain with the spectrum of each sub-carrier satisfying orthogonality conditions.

After performing IFFT calculation, a cyclically extended guard interval is applied between symbols to eliminate ISI caused by multi-path distortion. In more details, a periodic extension of each MC-DS-CDMA symbol is inserted and the duration of guard interval T_g is set as a quarter of the duration T of the useful symbol.

The extended data is then multiplied with each chip of a spreading sequence which is allocated uniquely to each user for the sake of separation of the different user signals at the receiver. In our research, we consider dual orthogonal variable spreading factors (OVSF) code as the spreading sequence. More details of dual OVSF are presented in the following section.

In addition, assuming multi-path channel is slow varying, a set of pilot signals is inserted periodically to estimate the multi-path channel state at transmitter. Consequently, an accurate estimation of the state of the multi-path channel is obtained by using the received pilot signals at the receiver.

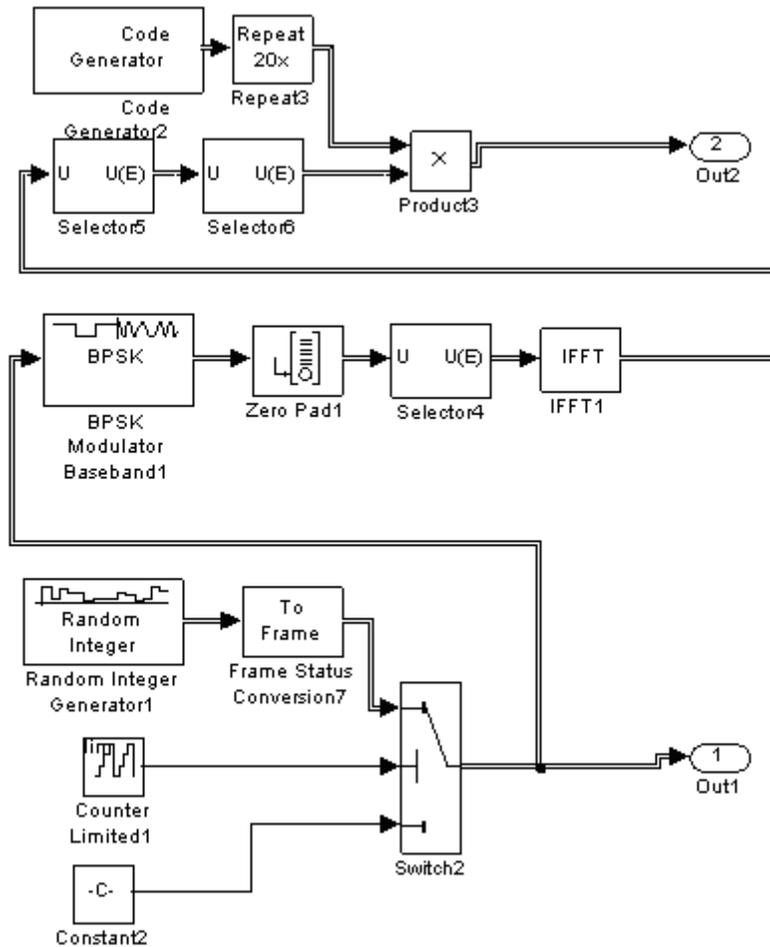


Figure 3.2 Block diagram of MC-DS-CDMA transmitter for a single user

3.2 Dual OVSF spreading codes

In order to preserve the orthogonality among different sub-carriers, a different OVSF code is assigned to each user. OVSF code can support higher and variable data rates for different users by using two different modulations without loss of the orthogonality and it can also reduce the correlations for asynchronous transmission [33]. In this section, we introduce a class of complex valued dual OVSF codes called JOVSF via quadric-phase orthogonal matrix [34] based on Jacket matrix.

The tree generation and periodic correlations of the dual OVSF codes are the same as that

of the conventional binary OVVSF codes based on Hadamard, which is referred to as HOVSF, but the seeds of those codes are different [33]. The dual OVVSF can be decomposed as two parts, one is conventional HOVSF, and the other is the JOVSF from Jacket matrix [33].

The typical HOVSF can be presented as

$$\begin{bmatrix} C_N(2l-1) \\ C_N(2l) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes C_{N/2}(l), \quad (3.1)$$

where \otimes denotes Kronecker product and C_N denotes the set of N spreading codes whose n th row vector $C_N(n)$ has $N = 2^k$ elements and k is a positive integer, $l = \{1, 2, \dots, N/2\}$ and $C_1 = 1$.

A periodic auto-correlation of HOVSF can be expressed as

$$R_i(\tau) = \sum_{i=0}^{L-1} a_i a_{(i+\tau) \bmod L}^* \quad (3.2)$$

and a periodic cross-correlation is

$$C_i(\tau) = \sum_{i=0}^{L-1} a_i b_{(i+\tau) \bmod L}^* \quad (3.3)$$

where a_i , b_i are the i th element in the sequence a and b respectively; L is the length of the sequence a and b , τ is the shift coefficient, $0 \leq \tau \leq L$.

From the correlations of HOVSF, it is easy to verify that

$$R_i(0) = \sum_{i=0}^{L-1} a_i a_i^* = L \quad (3.4)$$

$$C_i(0) = \sum_{i=0}^{L-1} a_i b_i^* = 0 \quad (3.5)$$

In order to provide a better correlation performance than the conventional HOVSF, a seed matrix generated via Jacket matrix is employed in JOVSF. A typical order four Jacket matrix is given by:

$$[J]_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -i & -1 \\ 1 & -i & i & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \quad (3.6)$$

and

$$\begin{aligned} [CJ]_4 &= [P_r]_4 [CJ]_4 [P_c]_4 \\ &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -i & -1 \\ 1 & -i & i & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ -1 & 1 & i & -i \\ -1 & 1 & -i & i \end{bmatrix} \end{aligned} \quad (3.7)$$

where $[P_r]_4$ is the row permutation matrix and $[P_c]_4$ presents column permutation matrix.

Thus a JOVSF sequence can be written as

$$CJ_4(1): 1 \ 1 \ 1 \ 1$$

$$CJ_4(2): 1 \ 1 \ -1 \ -1$$

$$CJ_4(3): -1 \ 1 \ i \ -i$$

$$CJ_4(4): -1 \ 1 \ -i \ i$$

and its orthogonality can be described by

$$[CJ]_4 [CJ]_4^H = 4[I]_4 \quad (3.8)$$

where $[CJ]_4^H$ denotes the Hermitian of matrix $[CJ]_4$.

From Equation 3.2, the auto-correlations of $[CJ]_4$ are

$CJ_4(1):$	$R_i(\tau): 4\ 4\ 4\ 4$
$CJ_4(2):$	$R_i(\tau): 4\ 0\ -4\ 0$
$CJ_4(3):$	$R_i(\tau): 4\ -2\ 0\ -2$
$CJ_4(4):$	$R_i(\tau): 4\ -2\ 0\ -2$

From Equation 3.3, the cross-correlations of $[CJ]_4$ are

$CJ_4(1), CJ_4(2):$	$C_i(\tau): 0\ 0\ 0\ 0$	$CJ_4(1), CJ_4(3):$	$C_i(\tau): 0\ 0\ 0\ 0$
$CJ_4(1), CJ_4(4):$	$C_i(\tau): 0\ 0\ 0\ 0$	$CJ_4(2), CJ_4(3):$	$C_i(\tau): 0\ 2\ -2i\ 0\ -2+2i$
$CJ_4(2), CJ_4(4):$	$C_i(\tau): 0\ 2+2i\ 0\ -2-2i$	$CJ_4(3), CJ_4(4):$	$C_i(\tau): 0\ 2i\ -4i\ 2i$

Starting from $[CJ]_4$, we can derive a similar formula from Equation 3.1,

$$\begin{bmatrix} CJ_8(2l-1) \\ CJ_8(2l) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes CJ_4(l) \quad (3.9)$$

Thus length-8 JOVSF sequence is

	$CJ_8(1)$
	1 1 1 1 1 1 1 1
	$CJ_8(2)$
	1 1 1 1 -1 -1 -1 -1
	$CJ_8(3)$
	1 1 -1 -1 1 1 -1 -1
	$CJ_8(4)$
	1 1 -1 -1 -1 -1 1 1
	$CJ_8(5)$
	-1 1 1 -1 -1 1 1 -1
	$CJ_8(6)$
	-1 1 1 -1 1 -1 -1 1
	$CJ_8(7)$
	-1 1 -1 1 -1 1 -1 1
	$CJ_8(8)$
	-1 1 -1 1 1 -1 1 -1

The auto-correlations of $[CJ]_8$ are

$CJ_8(1):$	$R_i(\tau): 8 8 8 8 8 8 8 8$	$CJ_8(2):$	$R_i(\tau): 8 4 0 -4 -8 -4 0 4$
$CJ_8(3):$	$R_i(\tau): 8 0 -8 0 8 0 -8 0$	$CJ_8(4):$	$R_i(\tau): 8 4 0 -4 -8 -4 0 4$
$CJ_8(5):$	$R_i(\tau): 8 -4 0 -4 8 -4 0 -4$		
$CJ_8(6):$	$R_i(\tau): 8 -4-4i 8i 4-4i -8 4+4i -8i -4+4i$		
$CJ_8(7):$	$R_i(\tau): 8 -4 0 -4 8 -4 0 -4$		
$CJ_8(8):$	$R_i(\tau): 8 -4+4i -8i 4+4i -8 4-4i 8i -4-4i$		

The cross-correlations of $[CJ]_8$ are

$CJ_8(1), CJ_8(2):$	$C_i(\tau): 0 0 0 0 0 0 0 0$	$CJ_8(1), CJ_8(3):$	$C_i(\tau): 0 0 0 0 0 0 0 0$
$CJ_8(1), CJ_8(4):$	$C_i(\tau): 0 0 0 0 0 0 0 0$	$CJ_8(1), CJ_8(5):$	$C_i(\tau): 0 0 0 0 0 0 0 0$
$CJ_8(1), CJ_8(6):$	$C_i(\tau): 0 0 0 0 0 0 0 0$	$CJ_8(1), CJ_8(7):$	$C_i(\tau): 0 0 0 0 0 0 0 0$
$CJ_8(1), CJ_8(8):$	$C_i(\tau): 0 0 0 0 0 0 0 0$	$CJ_8(2), CJ_8(3):$	$C_i(\tau): 0 0 0 0 0 0 0 0$

$CJ_8(2), CJ_8(4):$	$C_i(\tau): 0 \ -4 \ -8 \ -4 \ 0 \ 4 \ 8 \ 4$
$CJ_8(2), CJ_8(5):$	$C_i(\tau): 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$
$CJ_8(2), CJ_8(6):$	$C_i(\tau): 0 \ 4 \ 0 \ 4i \ 0 \ -4 \ 0 \ -4i$
$CJ_8(2), CJ_8(7):$	$C_i(\tau): 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$
$CJ_8(2), CJ_8(8):$	$C_i(\tau): 0 \ 4 \ 0 \ -4i \ 0 \ -4 \ 0 \ -4i$
$CJ_8(3), CJ_8(4):$	$C_i(\tau): 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$
$CJ_8(3), CJ_8(5):$	$C_i(\tau): 0 \ 4+4i \ 0 \ -4+4i \ 0 \ 4-4i \ 0 \ -4+4i$
$CJ_8(3), CJ_8(6):$	$C_i(\tau): 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$
$CJ_8(3), CJ_8(7):$	$C_i(\tau): 0 \ 4+4i \ 0 \ -4-4i \ 0 \ 4+4i \ 0 \ -4-4i$
$CJ_8(3), CJ_8(8):$	$C_i(\tau): 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$
$CJ_8(4), CJ_8(5):$	$C_i(\tau): 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$
$CJ_8(4), CJ_8(6):$	$C_i(\tau): 0 \ -4i \ 0 \ 4 \ 0 \ 4i \ 0 \ -4$
$CJ_8(4), CJ_8(7):$	$C_i(\tau): 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$
$CJ_8(4), CJ_8(8):$	$C_i(\tau): 0 \ 4i \ 0 \ 4 \ 0 \ -4i \ 0 \ -4$
$CJ_8(5), CJ_8(6):$	$C_i(\tau): 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$
$CJ_8(5), CJ_8(7):$	$C_i(\tau): 0 \ 4i \ -8i \ 4i \ 0 \ 4i \ -8i \ 4i$
$CJ_8(5), CJ_8(8):$	$C_i(\tau): 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$
$CJ_8(6), CJ_8(7):$	$C_i(\tau): 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$
$CJ_8(6), CJ_8(8):$	$C_i(\tau): 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$
$CJ_8(7), CJ_8(8):$	$C_i(\tau): 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$

As a result, the dual OVSF sequence can be generated recursively from Equation 3.1 with

any length L as a power of 2 and the periodic correlations of the dual OVSF can be derived from Equation 3.2 and 3.3. The advantage of dual OVSF is demonstrated in the following comparison of peak correlation values with HOVSF [33].

	Autocorrelation		Crosscorrelation	
	HOVSF	Dual OVSF	HOVSF	Dual OVSF
Peak value	4	4	4	4
Number of peak values	12	8	2	1

Table 3.1 Comparison of peak correlation values: length-4 HOVSF AND dual OVSF codes

	Autocorrelation		Crosscorrelation	
	HOVSF	Dual OVSF	HOVSF	Dual OVSF
Peak value	8	8	8	8
Number of peak values	32	28	8	4

Table 3.2 Comparison of peak correlation values: length-8 HOVSF AND dual OVSF codes

3.3 Channel simulation

A crucial requirement for the MC-DS-CDMA system is to describe characteristics of wireless channel accurately. Statistical characteristics of the wireless channel heavily depend on the physical multi-path propagation environment [35]. Multi-path is a salient feature in high-speed wireless communication. It results in transmitted radio signals' reflecting from terrain features such as trees or mountains, or obstructions such as people, vehicles or buildings and then reaching receiver at different times by two or more paths. Figure 3.3 shows demonstration of the multi-path. The same transmission is spread in all directions, but some signals reflect from mountains and buildings. The receiver obtains

more than one copy of the original signal. Since the indirect path takes more time to travel to the receiver, the delayed copies of the original signal interfere with the direct signal and the receiver can not decode the original signal correctly [36].

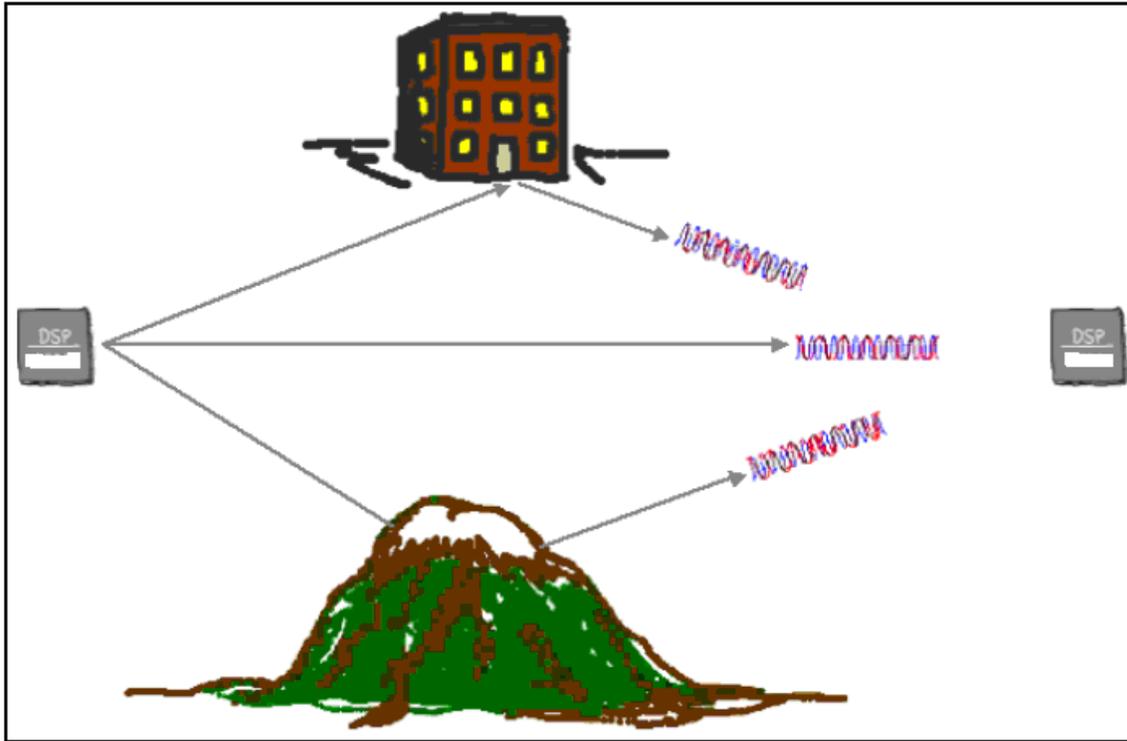


Figure 3.3: Multi-path Demonstration [36]

The channel simulation is characterized by AWGN channel simulation and multi-path Rayleigh fading channel simulation in our research. AWGN channel is simulated by adding random numbers with Gaussian distribution to the transmitted signal as background noise. Multi-path Rayleigh fading channel simulation involves multiplying the transmitted signal by channel gain vector and adding delayed copies of the transmitted signal in propagation. This simulates the problem in wireless communication when the signal propagates on many paths [36]. Furthermore, parameters of channel gain and delay time from a set of 6 typical channels called modified Stanford University Interim (SUI) Channel Models are applied to test the performance of MC-DS-CDMA system in this research and the parameters are presented as Table 3.3- Table 3.8 [37]:

SUI-1 Channel				
	Tap 1	Tap 2	Tap 3	Units
Delay	0	0.4	0.9	μs
Channel gain	0	-15	-20	dB

Table 3.3 parameters of SUI-1

SUI-2 Channel				
	Tap 1	Tap 2	Tap 3	Units
Delay	0	0.4	1.1	μs
Channel gain	0	-12	-15	dB

Table 3.4 parameters of SUI-2

SUI-3 Channel				
	Tap 1	Tap 2	Tap 3	Units
Delay	0	0.4	0.9	μs
Channel gain	0	-5	-10	dB

Table 3.5 parameters of SUI-3

SUI-4 Channel				
	Tap 1	Tap 2	Tap 3	Units
Delay	0	1.4	4	μs
Channel gain	0	-4	-8	dB

Table 3.6 parameters of SUI-4

SUI-5 Channel				
	Tap 1	Tap 2	Tap 3	Units
Delay	0	4	10	μs
Channel gain	0	-5	-10	dB

Table 3.7 parameters of SUI-5

SUI-6 Channel				
	Tap 1	Tap 2	Tap 3	Units
Delay	0	14	20	μs
Channel gain	0	-10	-14	dB

Table 3.8 parameters of SUI-6

Figure 3.4 show the channel model of the MC-DS-CDMA system. Multi-path Rayleigh fading channel implements the simulation of multi-path and fading effects and AWGN channel adds white Gaussian noise effect.



Figure 3.4 Block diagram of MC-DS-CDMA channel model

3.4 Receiver

The receiver implements the inverse process of the transmitter. First, the received signal is converted from serial into parallel stream and decoded using users' OVSF code index. Accordingly, the receiver can identify the data stream from different users. Next, the guard interval is removed. Then the FFT calculation is applied and the inserted zeros are removed from data stream. After performing the system delay, the channel gain of multi-path channel is calculated according to the received pilot data. The effect of multi-path fading channel is eliminated by multiplying received data by reciprocal of the channel gain. Finally, BPSK demodulation is applied. Thus the original data can be recovered. Figure 3.5 presents the receiver of the MC-DS-CDMA system.

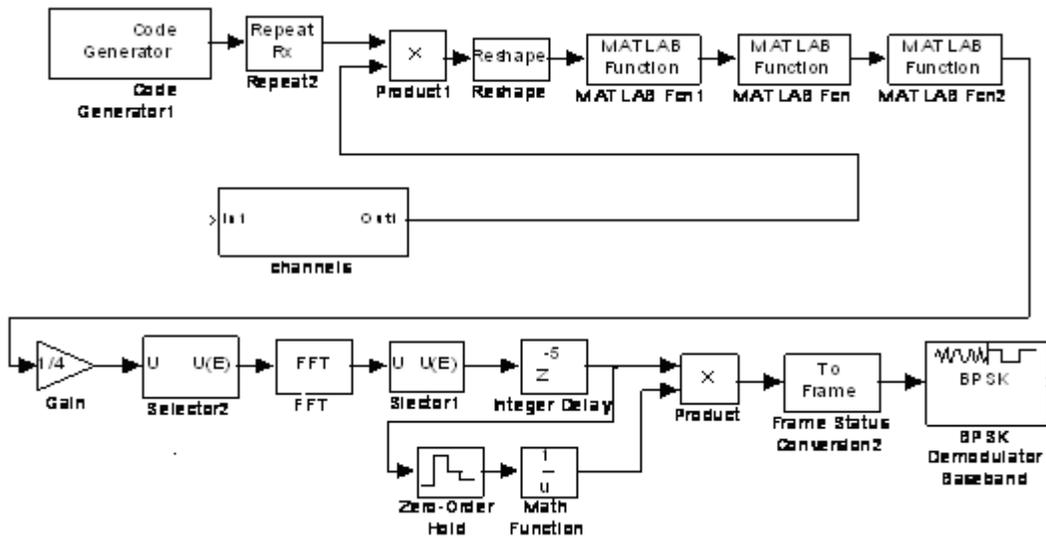


Figure 3.5 Block diagram of MC-DS-CDMA receiver

3.5 Simulation results and conclusions

The received data is compared with the original data to calculate the BER in different SUI Channel Models. In this research, 6×10^5 of simulations are run to obtain the bit error rate (BER) at each value of E_b/N_0 . Table 3.9- Table 3.14 presents the value of BER at different value of E_b/N_0 from 0 dB to 27 dB in different SUI Channel Models, respectively. Figure 3.6 shows the results using channel models SUI-1 to SUI-6 with different values of E_b/N_0 . The vertical axis is the BER and the horizontal axis is the signal to noise ratio (E_b/N_0). Blue curve denotes the theoretical BER performance of Rayleigh fading MC-DS-CDMA system; Red curve with x-mark, cyan curve with plus, magenta curve with star, yellow curve with square, black curve diamond and curve with circle shows BER performance from SUI-1 to SUI-6 Channel, respectively. It can be seen that MC-DS-CDMA system is robust in multipath fading channels. Due to the use of guard interval, the orthogonality between users is maintained and there is no multiuser interference.

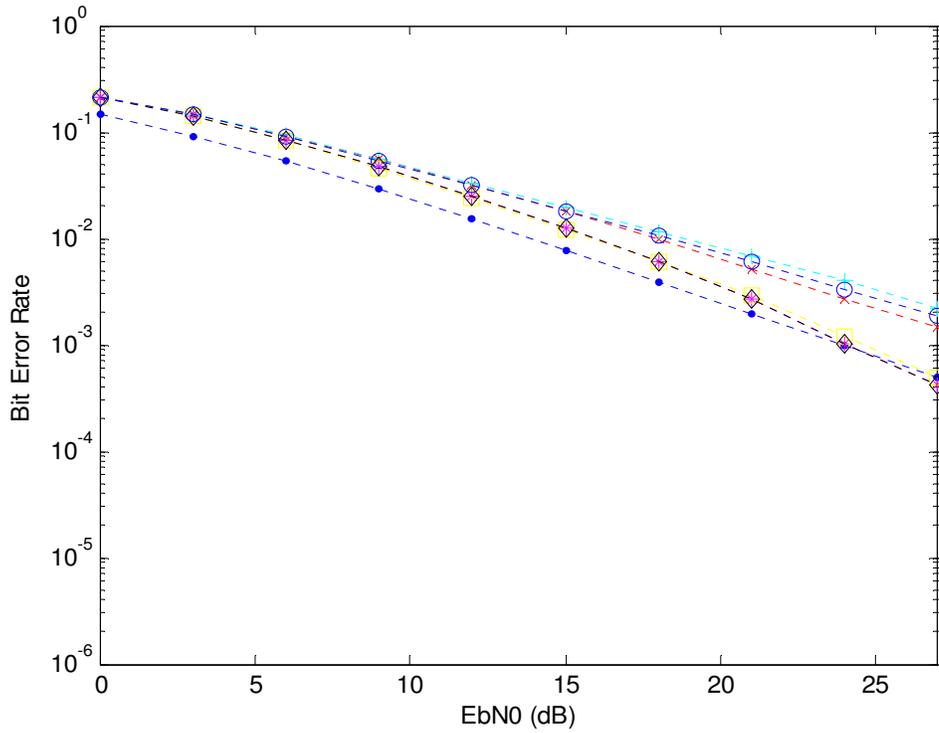


Figure 3.6 BER curves of SUI Channels

SUI-1 Channel					
$E_b/N_0 (dB)$	0	3	6	9	12
BER	0.2133	0.1466	0.09221	0.05469	0.03163
$E_b/N_0 (dB)$	15	18	21	24	27
BER	0.0179	0.009682	0.00507	0.002741	0.001467

Table 3.9 BER of SUI-1 Channel

SUI-2 Channel					
$E_b/N_0 (dB)$	0	3	6	9	12
BER	0.2137	0.1467	0.09253	0.05507	0.03231
$E_b/N_0 (dB)$	15	18	21	24	27
BER	0.01909	0.01138	0.006783	0.004006	0.002166

Table 3.10 BER of SUI-2 Channel

SUI-3 Channel					
$E_b/N_0 (dB)$	0	3	6	9	12
BER	0.2092	0.14	0.08425	0.04678	0.02462
$E_b/N_0 (dB)$	15	18	21	24	27
BER	0.01239	0.005925	0.002649	0.001004	0.0004251

Table 3.11 BER of SUI-3 Channel

SUI-4 Channel					
$E_b/N_0 (dB)$	0	3	6	9	12
BER	0.2081	0.1387	0.08264	0.04573	0.0239
$E_b/N_0 (dB)$	15	18	21	24	27
BER	0.01211	0.00614	0.002944	0.001212	0.0004901

Table 3.12 BER of SUI-4 Channel

SUI-5 Channel					
$E_b/N_0 (dB)$	0	3	6	9	12
BER	0.2092	0.14	0.06307	0.04678	0.02464
$E_b/N_0 (dB)$	15	18	21	24	27
BER	0.01239	0.005926	0.002651	0.001009	0.0004201

Table 3.13 BER of SUI-5 Channel

SUI-6 Channel					
$E_b/N_0 (dB)$	0	3	6	9	12
BER	0.2127	0.146	0.0914	0.0538	0.03116
$E_b/N_0 (dB)$	15	18	21	24	27
BER	0.01811	0.01064	0.006073	0.003276	0.0087

Table 3.14 BER of SUI-6 Channel

4. BER performance analysis on the effect of timing jitter

4.0 Introduction

The MC-DS-CDMA systems are more sensitive to errors in sampling time which is generally called timing jitter than single-carrier systems due to the use of large number of sub-carriers. Timing jitter caused by mismatch of sampling time between the transmitter and receiver degrades performance of the system seriously because timing jitter destroys orthogonality among sub-carriers and results in ICI. To avoid the degradation associated with a timing jitter, the timing jitter should be corrected by a proper timing synchronization algorithm. It has been shown that the performance degradation for the MC-DS-CDMA system caused by timing jitter is independent of the number of sub-carriers, of the spreading factor, and of the spectral contents of the jitter, but only depends on the timing jitter variance [38].

In this section, the investigation of effects of timing jitter on the MC-DS-CDMA system is introduced. In particular, we will pay more attention on the BER performance of the MC-DS-CDMA system due to the timing jitter both in additive white Gaussian noise (AWGN) channel and multi-path Rayleigh fading channel. We firstly formulate the analytical expressions for the MC-DS-CDMA signals in presence of the timing jitter and then compare BER performance of the ideal MC-DS-CDMA system with the BER performances affected by timing jitters when the timing jitters are independent and dependent, respectively.

4.1 Problem formulation

In MC-DS-CDMA communication systems, the complex data symbols to be transmitted are first split into lower rate symbol sequences, and then each of these lower rate symbol sequences modulates a different sub-carrier of the orthogonal multi-carrier system. The received MC-DS-CDMA signal over the useful symbol duration of T_s at

$(m-1)(T_s + T_g) \leq t \leq m(T_s + T_g) - T_g, m = 1, \dots, L$ can be expressed as

$$s(t) = \frac{1}{N} \sum_{k=0}^{N-1} \sum_{\ell=1}^{N_{user}} a_{k,\ell} C_{m,\ell} b_{k,\ell} e^{j2\pi f_k [t - (m-1)(T_s + T_g)]}, \quad (4.1)$$

where $f_k = k * f_s$ is the frequency of the k th sub-carrier and N is the number of sub-carriers, $f_s = 1/T_s$ is the sub-carrier spacing related to the symbol duration T_s , T_g is the guard interval, N_{user} is the number of users and $a_{k,\ell}$ is the data symbol transmitted by user ℓ on the k th sub-carrier. Without loss of generality, $a_{k,\ell}$ can be assumed to be zero-mean with variance σ_s^2 . $\{C_{m,\ell} | m=1, \dots, L\}$ denotes the m th chip of the sequence that spreads the data symbols from user ℓ , and L is the length of the spreading sequence. $b_{k,\ell}$ is the fading channel gain for k th sub-carrier of user ℓ and it is a complex Gaussian random process with zero-mean and variance $\sigma_{k,\ell}^2$. Note that the spreading sequence $C_{m,\ell}$ does not depend on the sub-carrier index k , i.e., all data symbols from user ℓ that are transmitted during the same symbol interval are spread with the same spreading sequence [39].

To demodulate the MC-DS-CDMA signal using the DFT at the receiver, the sampled $s(t)$ must be collected. Ideally, with perfect timing synchronization, the symbol $s(t)$ is sampled at every $T = T_s/N$ second and T is known as the sampling interval. However, random errors called timing jitters may occur at the receiver due to timing synchronization errors and limitations of sampling devices. This means that the sampling instants $t_{n,m}$ are not $nT + (m-1)(T_s + T_g)$ but

$$t_{n,m} = nT + (m-1)(T_s + T_g) + \xi_{n,m}, n = 0, 1, \dots, N-1, \quad (4.2)$$

where $\xi_{n,m} \equiv T\xi(n,m)$ is the timing jitter of the n th sampling instant for the MC-DS-CDMA symbol corresponding to the m th chip of the spreading sequence and $\xi(n,m)$ is the timing jitter normalized by the sampling interval T . The sequence $\xi_{n,m}$ is often modeled as a wide sense stationary Gaussian random process statistically

independent of the input signal with zero mean and variance σ_j^2 [40]. Therefore, without loss of generality, the actual sampled signal can be expressed as

$$\begin{aligned}
x_{n,m} &= s(nT + (m-1)(T_s + T_g) + \xi_{n,m}) = \frac{1}{N} \sum_{k=0}^{N-1} \sum_{\ell=1}^{N_{user}} a_{k,\ell} C_{m,\ell} b_{k,\ell} e^{j2\pi f_k (nT + \xi_{n,m})} \\
&= \frac{1}{N} \sum_{k=0}^{N-1} \sum_{\ell=1}^{N_{user}} a_{k,\ell} C_{m,\ell} b_{k,\ell} e^{j2\pi \frac{k}{TN} [nT + T\xi(n,m)]} \\
&= \frac{1}{N} \sum_{k=0}^{N-1} \sum_{\ell=1}^{N_{user}} a_{k,\ell} C_{m,\ell} b_{k,\ell} e^{j2\pi \frac{k}{N} [n + \xi(n,m)]}
\end{aligned} \tag{4.3}$$

With the increase of sampling rate, the timing jitters at adjacent samples may become more correlated. Correlation model of timing jitters is described as Gaussian-shaped or exponential in [40, 41]. To analyze the effects of different degrees of correlation between sampling timing errors on the performance of MC-DS-CDMA system, we adopt the following discrete autocorrelation function of timing jitter [41]

$$\rho(n) = \sigma_j^2 a^{|n|}, \quad |a| \leq 1, \tag{4.4}$$

where σ_j^2 is the variance of timing jitter with zero-mean; a can be described as the level of correlation between different timing jitters. For instance, $|a|=1$ corresponds to a constant offset on the sampling instants, while $a=0$ corresponds to uncorrelated timing jitter on different samples [41].

4.2 ICI due to timing jitter

The ICI caused by the timing jitter in the MC-DS-CDMA system based on the AWGN channel and the multi-path Rayleigh fading channel is analyzed in this section.

Using Equation 4.3, the decision variable Z_i for the data symbol carried by the i th sub-carrier of user ℓ' can be obtained from the received signal samples after despreading as

$$\begin{aligned}
Z_i &= b_{i,\ell}^* \frac{1}{N} \sum_{n=0}^{N-1} \left(\frac{1}{L} \sum_{m=1}^L x_{n,m} C_{m,\ell}^* \right) e^{-j2\pi f_i n T} \\
&= \frac{1}{NL} b_{i,\ell}^* \sum_{n=0}^{N-1} \sum_{k=0}^{N-1} \sum_{\ell=1}^{N_{user}} \sum_{m=1}^L a_{k,\ell} C_{m,\ell} C_{m,\ell}^* b_{k,\ell} e^{j2\pi \frac{k}{N} [n+\xi(n,m)]} e^{-j2\pi \frac{i}{N} n} \\
&= \frac{1}{NL} b_{i,\ell}^* \sum_{n=0}^{N-1} \sum_{k=0}^{N-1} \sum_{\ell=1}^{N_{user}} \sum_{m=1}^L a_{k,\ell} C_{m,\ell} C_{m,\ell}^* b_{k,\ell} e^{j\frac{2\pi}{N} [(k-i)n+k\xi(n,m)]} \\
&= \frac{1}{NL} b_{i,\ell}^* \sum_{\ell=1}^{N_{user}} \sum_{k=0}^{N-1} a_{k,\ell} b_{k,\ell} \sum_{m=1}^L C_{m,\ell} C_{m,\ell}^* \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N} [(k-i)n+k\xi(n,m)]}
\end{aligned} \tag{4.5}$$

For simplicity of further analyses, we break Equation 4.5 into five components $\eta_{i,\ell} a_{i,\ell}$, $\alpha_{i,\ell}$, $\beta_{i,\ell}$, $\mu_{i,\ell}$ and Γ_k using the decomposition

$$Z_i = \eta_{i,\ell} a_{i,\ell} + \alpha_{i,\ell} + \beta_{i,\ell} + \mu_{i,\ell} + \Gamma_k \tag{4.6}$$

$$\text{where } \eta_{i,\ell} = \frac{1}{NL} b_{i,\ell}^* b_{i,\ell} \sum_{m=1}^L \sum_{n=0}^{N-1} e^{j2\pi \frac{i}{N} \xi(n,m)}, \ell = \ell', k = i \tag{4.7}$$

$$\alpha_{i,\ell} = \frac{1}{NL} b_{i,\ell}^* \sum_{k=0, k \neq i}^{N-1} a_{k,\ell} b_{k,\ell} \sum_{m=1}^L \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N} [(k-i)n+k\xi(n,m)]}, \ell = \ell', k \neq i \tag{4.8}$$

$$\beta_{i,\ell} = \frac{1}{NL} b_{i,\ell}^* \sum_{\ell=1, \ell \neq \ell'}^{N_{user}} \sum_{k=0, k \neq i}^{N-1} a_{k,\ell} b_{k,\ell} \sum_{m=1}^L C_{m,\ell} C_{m,\ell}^* \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N} [(k-i)n+k\xi(n,m)]}, \ell \neq \ell', k \neq i \tag{4.9}$$

$$\mu_{i,\ell} = \frac{1}{NL} b_{i,\ell}^* \sum_{\ell=1, \ell \neq \ell'}^{N_{user}} a_{i,\ell} b_{i,\ell} \sum_{m=1}^L C_{m,\ell} C_{m,\ell}^* \sum_{n=0}^{N-1} e^{j2\pi \frac{i}{N} \xi(n,m)}, \ell \neq \ell', k = i \tag{4.10}$$

$$\Gamma_k = \sum_0^{N_{user}-1} \varphi_i C_{m,\ell}^* b_{k,\ell}^* \tag{4.11}$$

In more details, $\eta_{i,\ell}$ is the gain and phase rotation of the desired symbol $a_{i,\ell}$. $\alpha_{i,\ell}$ denotes interference introduced by symbols from the same user in other sub-carriers. $\beta_{i,\ell}$ can be described as interference from other users in other sub-carriers. $\mu_{i,\ell}$ represents the interference from other users but within the same sub-carriers. Γ_k , a complex Gaussian random variable with zero-mean and a variance of σ_φ^2 , is the

contribution from thermal noise and the real and the imaginary parts of Γ_k can be assumed independent and identically distributed; hence,

$$\text{var}(\Re[\Gamma_k]) = \text{var}(\Im[\Gamma_k]) = \frac{1}{2} \text{var}(\Gamma_k) = \frac{1}{2} \sigma_\varphi^2 \quad [42]. \quad \varphi_i \text{ is a sample of the thermal noise.}$$

Note that the symbols $a_{k,\ell}$'s are assumed to be zero-mean. Therefore,

$$E(a_{i,\ell'}) = 0$$

$$E(\beta_{i,\ell'}) = 0$$

$$E(\mu_{i,\ell'}) = 0$$

Thus the variance of $\alpha_{i,\ell'}$, $\beta_{i,\ell'}$, and $\mu_{i,\ell'}$ can be written as following from Equation 4.8-4.10, respectively.

$$\begin{aligned} \text{Var}(\alpha_{i,\ell'}) &= E(\alpha_{i,\ell'}^2) - [E(\alpha_{i,\ell'})]^2 = E(\alpha_{i,\ell'}^2) \\ &= E \left\{ \left| \frac{1}{N} \sum_{k=0, k \neq i}^{N-1} a_{k,\ell'} b_{k,\ell'} b_{k,\ell'}^* \sum_{m=1}^L \sum_{n=0}^{N-1} e^{j \frac{2\pi}{N} [(k-i)n + k\xi(n,m)]} * \frac{1}{N} \sum_{k'=0, k' \neq i}^{N-1} a_{k',\ell'} b_{k',\ell'} b_{k',\ell'}^* \sum_{m=1}^L \sum_{n=0}^{N-1} e^{-j \frac{2\pi}{N} [(k'-i)n' + k'\xi(n',m')] } \right| \right\} \end{aligned} \quad (4.12)$$

$$\begin{aligned} \text{Var}(\beta_{i,\ell'}) &= E(\beta_{i,\ell'}^2) - [E(\beta_{i,\ell'})]^2 = E(\beta_{i,\ell'}^2) \\ &= E \left\{ \left| \frac{1}{N} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} a_{k,\ell} \sum_{m=1}^L C_{m,\ell} C_{m,\ell}^* b_{k,\ell} b_{k,\ell}^* \sum_{n=0}^{N-1} e^{j \frac{2\pi}{N} [(k-i)n + k\xi(n,m)]} \right. \right. \\ &\quad \left. \left. * \frac{1}{N} \sum_{\ell'=1, \ell' \neq \ell'}^{Nuser} \sum_{k'=0, k' \neq i}^{N-1} a_{k',\ell'} \sum_{m=1}^L C_{m,\ell'} C_{m,\ell'}^* b_{k',\ell'} b_{k',\ell'}^* \sum_{n=0}^{N-1} e^{-j \frac{2\pi}{N} [(k'-i)n' + k'\xi(n',m')] } \right| \right\} \end{aligned} \quad (4.13)$$

$$\begin{aligned} \text{Var}(\mu_{i,\ell'}) &= E(\mu_{i,\ell'}^2) - [E(\mu_{i,\ell'})]^2 = E(\mu_{i,\ell'}^2) \\ &= E \left\{ \left| \frac{1}{N} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} a_{k,\ell} b_{k,\ell} b_{k,\ell}^* \sum_{m=1}^L C_{m,\ell} C_{m,\ell}^* \sum_{n=0}^{N-1} e^{j \frac{2\pi}{N} k\xi(n,m)} * \frac{1}{N} \sum_{\ell'=1, \ell' \neq \ell'}^{Nuser} a_{k',\ell'} b_{k',\ell'} b_{k',\ell'}^* \sum_{m=1}^L C_{m,\ell'} C_{m,\ell'}^* \sum_{n=0}^{N-1} e^{-j \frac{2\pi}{N} k'\xi(n',m')} \right| \right\} \end{aligned} \quad (4.14)$$

Therefore, conditional mean of demodulated signal Z_i can be expressed as:

$$\begin{aligned}
E(Z_i) &= E(\eta_{i,\ell} a_{i,\ell} + \alpha_{i,\ell} + \beta_{i,\ell} + \mu_{i,\ell} + \Gamma_k) \\
&= E(\eta_{i,\ell} a_{i,\ell}) + E(\alpha_{i,\ell}) + E(\beta_{i,\ell}) + E(\mu_{i,\ell}) + E(\Gamma_k) \\
&= E(\eta_{i,\ell} a_{i,\ell})
\end{aligned} \tag{4.15}$$

Conditional variance of demodulated signal Z_i can be expressed as:

$$\begin{aligned}
\text{Var}(Z_i) &= \text{Var}(\eta_{i,\ell} a_{i,\ell} + \alpha_{i,\ell} + \beta_{i,\ell} + \mu_{i,\ell} + \Gamma_k) \\
&= \text{Var}(\eta_{i,\ell} a_{i,\ell}) + \text{Var}(\alpha_{i,\ell}) + \text{Var}(\beta_{i,\ell}) + \text{Var}(\mu_{i,\ell}) + \text{Var}(\Gamma_k) \\
&= \text{Var}(\eta_{i,\ell} a_{i,\ell}) + \text{Var}(\alpha_{i,\ell}) + \text{Var}(\beta_{i,\ell}) + \text{Var}(\mu_{i,\ell}) + \sigma_\varphi^2
\end{aligned} \tag{4.16}$$

Finally, the bit error rate (BER) of the MC-DS-CDMA system can be calculated as [43]

$$\text{BER} = 1/2 * \text{erfc}\left(\frac{E(Z_i)}{\sqrt{2\text{Var}(Z_i)}}\right) \tag{4.17}$$

where the complimentary error function erfc is defined as

$$\text{erfc}(Z_i) = \frac{2}{\sqrt{\pi}} \int_{Z_i}^{\infty} e^{-t^2} dt$$

$\text{Var}(Z_i)$ is the variance of Z_i and $E(Z_i)$ is the mean value of Z_i

4.2.1 ICI due to white timing jitter

When timing jitter is white, the correlation coefficient is expressed as:

$$\rho(n) = \begin{cases} 1, n = 0 \\ 0, n \neq 0 \end{cases} \tag{4.18}$$

then the variance of $\alpha_{i,\ell}$ can be written as following from Equation 4.12,

$$\begin{aligned}
\text{Var}(\alpha_{i,\ell'}) &= E(\alpha_{i,\ell'}^2) - [E(\alpha_{i,\ell'})]^2 = E(\alpha_{i,\ell'}^2) \\
&= E \left\{ \frac{1}{N} \sum_{k=0, k \neq i}^{N-1} a_{k,\ell'} b_{k,\ell'} b_{k,\ell'}^* \sum_{m=1}^L \sum_{n=0}^{N-1} e^{j \frac{2\pi}{N} [(k-i)n + k\xi(n,m)]} * \frac{1}{N} \sum_{k'=0, k' \neq i}^{N-1} a_{k',\ell'} b_{k',\ell'} b_{k',\ell'}^* \sum_{m'=1}^L \sum_{n'=0}^{N-1} e^{-j \frac{2\pi}{N} [(k'-i)n' + k'\xi(n',m')]} \right\}
\end{aligned}$$

When $k = k', m = m', n = n', \ell = \ell''$

$$\begin{aligned}
\text{Var}(\alpha_{i,\ell'}) &= \frac{1}{N^2} E \left\{ \sum_{k=0}^{N-1} a_{k,\ell'}^2 b_{k,\ell'} b_{k,\ell'}^* b_{k,\ell'} b_{k,\ell'}^* \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} e^{j \frac{2\pi}{N} [(k-i)n + k\xi(n,m)]} e^{-j \frac{2\pi}{N} [(k'-i)n' + k'\xi(n',m')]} \right\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{k=0}^{N-1} \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} E \left\{ e^{j \frac{2\pi}{N} [(k-i)(n-n') + k[\xi(n,m) - \xi(n',m')]]} b_{k,\ell'}^2 b_{k,\ell'}^{*2} \right\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{k=0}^{N-1} \sum_{m=1}^L \sum_{n=0}^{N-1} E \left\{ e^{j \frac{2\pi}{N} [(k-i)(n-n') + k[\xi(n,m) - \xi(n',m')]]} \right\} E(b_{k,\ell'}^2) E(b_{k,\ell'}^{*2}) \\
&= \frac{\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N^2} E \left\{ \sum_{k=0}^{N-1} L * N \right\} = \frac{\sigma_s^2 * L * N * N}{N^2} \\
&= L \sigma_s^2 \sigma_k^2 \sigma_k^{*2}
\end{aligned}$$

(4.19)

When $k = k', m = m', n \neq n', \ell = \ell''$

$$\begin{aligned}
\text{Var}(\alpha_{i,\ell'}) &= \frac{1}{N^2} E \left\{ \sum_{k=0}^{N-1} a_{k,\ell'}^2 b_{k,\ell'} b_{k,\ell'}^* b_{k,\ell'} b_{k,\ell'}^* \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} e^{j \frac{2\pi}{N} [(k-i)n + k\xi(n,m)]} e^{-j \frac{2\pi}{N} [(k'-i)n' + k'\xi(n',m')]} \right\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{k=0}^{N-1} \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} E \left\{ e^{j \frac{2\pi}{N} [(k-i)(n-n') + k[\xi(n,m) - \xi(n',m')]]} \right\} E(b_{k,\ell'}^2) E(b_{k,\ell'}^{*2}) \\
&= \frac{\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N^2} \sum_{k=0}^{N-1} \sum_{m=1}^L \sum_{n=0}^{N-1} \sum_{n'=0, n \neq n'}^{N-1} e^{j \frac{2\pi}{N} [(k-i)(n-n')]} e^{-\frac{4\pi^2}{N^2} k^2 \sigma_j^2 [1 - \rho(n-n')]} \\
&= \frac{2L \sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N^2} \sum_{k=0}^{N-1} \sum_{\Delta n=1}^{N-1} (N - \Delta n) \cos \left[\frac{2\pi}{N} (k-i) \Delta n \right] e^{-\frac{4\pi^2}{N^2} k^2 \sigma_j^2}
\end{aligned}$$

(4.20)

When $k = k', m \neq m', n = n', \ell = \ell''$

$$\begin{aligned}
\text{Var}(\alpha_{i,\ell'}) &= \frac{1}{N^2} E \left\{ \sum_{k=0}^{N-1} a_{k,\ell'}^2 b_{k,\ell} b_{k,\ell}^* b_{k',\ell} b_{k',\ell}^* \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} e^{j\frac{2\pi}{N}[(k-i)n+k\xi(n,m)]} e^{-j\frac{2\pi}{N}[(k'-i)n'+k'\xi(n',m')]} \right\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{k=0}^{N-1} \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} E \left\{ e^{j\frac{2\pi}{N}[(k-i)(n-n') + k[\xi(n,m) - \xi(n',m')]} \right\} E(b_{k,\ell}^2) E(b_{k',\ell'}^{*2}) \\
&= \frac{\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N^2} \sum_{k=0}^{N-1} \sum_{m=1}^L \sum_{m'=1, m \neq m'}^L \sum_{n=0}^{N-1} e^{-\frac{4\pi^2}{N^2} k^2 \sigma_j^2 [1 - \rho(N(m-m'))]} \\
&= \frac{2\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N} \sum_{k=0}^{N-1} \sum_{\Delta m=1}^{L-1} (L - \Delta m) e^{-\frac{4\pi^2}{N^2} k^2 \sigma_j^2}
\end{aligned} \tag{4.21}$$

When $k = k', m \neq m', n \neq n', \ell = \ell''$

$$\begin{aligned}
\text{Var}(\alpha_{i,\ell'}) &= \frac{1}{N^2} E \left\{ \sum_{k=0}^{N-1} a_{k,\ell'}^2 b_{k,\ell} b_{k,\ell}^* b_{k',\ell} b_{k',\ell}^* \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} e^{j\frac{2\pi}{N}[(k-i)n+k\xi(n,m)]} e^{-j\frac{2\pi}{N}[(k'-i)n'+k'\xi(n',m')]} \right\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{k=0}^{N-1} \sum_{m=1}^L \sum_{m'=1, m \neq m'}^L \sum_{n=0}^{N-1} \sum_{n'=0, n \neq n'}^{N-1} E \left\{ e^{j\frac{2\pi}{N}[(k-i)(n-n') + k[\xi(n,m) - \xi(n',m')]} \right\} E(b_{k,\ell}^2) E(b_{k',\ell'}^{*2}) \\
&= \frac{\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N^2} \sum_{k=0}^{N-1} \sum_{m=1}^L \sum_{m'=1, m \neq m'}^L \sum_{n=0}^{N-1} \sum_{n'=0, n \neq n'}^{N-1} e^{j\frac{2\pi}{N}[(k-i)(n-n')]} e^{-\frac{4\pi^2}{N^2} k^2 \sigma_j^2 [1 - \rho(N(m-m') + (n-n'))]} \\
&= \frac{4\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N^2} \sum_{k=0}^{N-1} \sum_{\Delta m=1}^{L-1} \sum_{\Delta n=1}^{N-1} (N - \Delta n)(L - \Delta m) \cos \left[\frac{2\pi}{N} (k-i)\Delta n \right] e^{-\frac{4\pi^2}{N^2} k^2 \sigma_j^2}
\end{aligned} \tag{4.22}$$

When $k \neq k'$ or $\ell \neq \ell'$

$$\text{Var}(\alpha_{i,\ell'}) = 0$$

From Equation 4.13, the variance of $\beta_{i,\ell'}$ can be expressed as:

$$\begin{aligned}
\text{Var}(\beta_{i,\ell'}) &= E(\beta_{i,\ell'}^2) - [E(\beta_{i,\ell'})]^2 = E(\beta_{i,\ell'}^2) \\
&= E \left\{ \left[\frac{1}{N} \sum_{\ell=1, \ell \neq \ell'}^{N_{\text{user}}} \sum_{k=0}^{N-1} a_{k,\ell} \sum_{m=1}^L C_{m,\ell} C_{m,\ell}^* b_{k,\ell} b_{k,\ell}^* \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}[(k-i)n+k\xi(n,m)]} \right. \right. \\
&\quad \left. \left. * \frac{1}{N} \sum_{\ell=1, \ell \neq \ell', \ell'' \neq \ell'}^{N_{\text{user}}} \sum_{k'=0, k \neq i}^{N-1} a_{k',\ell''} \sum_{m'=1}^L C_{m',\ell''} C_{m',\ell''}^* b_{k',\ell''} b_{k',\ell''}^* \sum_{n'=0}^{N-1} e^{-j\frac{2\pi}{N}[(k'-i)n'+k'\xi(n',m')]} \right] \right\}
\end{aligned}$$

When $k = k', m = m', n = n', \ell = \ell''$

$$\begin{aligned}
\text{Var}(\beta_{i,\ell'}) &= \frac{1}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^{L-1} \sum_{m=1}^{L-1} \sum_{n=0}^{N-1} \sum_{n=0}^{N-1} E\{a_{k,\ell}^2 C_{m,\ell} C_{m,\ell}^* C_{m,\ell} C_{m,\ell}^* b_{k,\ell} b_{k,\ell}^* b_{k,\ell} b_{k,\ell}^* e^{j\frac{2\pi}{N}[(k-i)n+k\xi(n,m)]} e^{-j\frac{2\pi}{N}[(k-i)n'+k\xi(n',m')]} \} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^{L-1} \sum_{m=1}^{L-1} \sum_{n=0}^{N-1} \sum_{n=0}^{N-1} C_{m,\ell} C_{m,\ell}^* C_{m,\ell} C_{m,\ell}^* E\{e^{j\frac{2\pi}{N}[(k-i)(n-n')+\xi(n,m)-\xi(n',m')]} * b_{k,\ell} b_{k,\ell}^* b_{k,\ell} b_{k,\ell}^* \} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^{L-1} \sum_{m=1}^{L-1} C_{m,\ell}^2 C_{m,\ell}^{*2} E\{e^{j\frac{2\pi}{N}[(k-i)(n-n')+\xi(n,m)-\xi(n',m')]} \} E(b_{k,\ell}^2) E(b_{k,\ell'}^{*2}) \\
&= \frac{\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N^2} * N * L * N * Nuser \\
&= \sigma_s^2 \sigma_k^2 \sigma_k^{*2} L Nuser
\end{aligned} \tag{4.23}$$

When $k = k', m = m', n \neq n', \ell = \ell''$

$$\begin{aligned}
\text{Var}(\beta_{i,\ell'}) &= \frac{1}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^{L-1} \sum_{m=1}^{L-1} \sum_{n=0}^{N-1} \sum_{n=0, n \neq n'}^{N-1} E\{a_{k,\ell}^2 C_{m,\ell} C_{m,\ell}^* C_{m,\ell} C_{m,\ell}^* b_{k,\ell} b_{k,\ell}^* b_{k,\ell} b_{k,\ell}^* e^{j\frac{2\pi}{N}[(k-i)n+k\xi(n,m)]} e^{-j\frac{2\pi}{N}[(k-i)n'+k\xi(n',m')]} \} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^{L-1} \sum_{m=1}^{L-1} \sum_{n=0}^{N-1} \sum_{n=0, n \neq n'}^{N-1} C_{m,\ell} C_{m,\ell}^* C_{m,\ell} C_{m,\ell}^* E\{e^{j\frac{2\pi}{N}[(k-i)(n-n')+\xi(n,m)-\xi(n',m')]} b_{k,\ell} b_{k,\ell}^* b_{k,\ell} b_{k,\ell}^* \} \\
&= \frac{\sigma_s^2 L}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^{L-1} \sum_{m=1}^{L-1} \sum_{n=0, n \neq n'}^{N-1} E\{e^{j\frac{2\pi}{N}[(k-i)(n-n')]} e^{\frac{4\pi^2}{N^2} k^2 \sigma_j^2 [1-\rho(n-n')]} \} E(b_{k,\ell}^2) E(b_{k,\ell'}^{*2}) \\
&= \frac{2\sigma_s^2 L \sigma_k^2 \sigma_k^{*2}}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{\Delta n=1}^{N-1} (N-\Delta n) \cos\left[\frac{2\pi}{N}(k-i)\Delta n\right] e^{\frac{4\pi^2}{N^2} k^2 \sigma_j^2} \}
\end{aligned} \tag{4.24}$$

When $k = k', m \neq m', n = n', \ell = \ell''$

$$\begin{aligned}
\text{Var}(\beta_{i,\ell'}) &= \frac{1}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^{L-1} \sum_{m=1}^{L-1} \sum_{n=0}^{N-1} \sum_{n=0}^{N-1} E\{a_{k,\ell}^2 C_{m,\ell} C_{m,\ell}^* C_{m,\ell} C_{m,\ell}^* b_{k,\ell} b_{k,\ell}^* b_{k,\ell} b_{k,\ell}^* e^{j\frac{2\pi}{N}[(k-i)n+k\xi(n,m)]} e^{-j\frac{2\pi}{N}[(k-i)n'+k\xi(n',m')]} \} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^{L-1} \sum_{m=1}^{L-1} \sum_{n=0}^{N-1} \sum_{n=0}^{N-1} C_{m,\ell} C_{m,\ell}^* C_{m,\ell} C_{m,\ell}^* b_{k,\ell}^2 b_{k,\ell'}^{*2} E\{e^{j\frac{2\pi}{N}[(k-i)(n-n')+\xi(n,m)-\xi(n',m')]} \} \\
&= \frac{\sigma_s^2 N}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^{L-1} \sum_{m=1, m \neq m'}^{L-1} \sum_{n=0}^{N-1} C_{m,\ell} C_{m,\ell}^* C_{m,\ell} C_{m,\ell}^* e^{\frac{4\pi^2}{N^2} k^2 \sigma_j^2 [1-\rho(N(m-m'))]} E(b_{k,\ell}^2) E(b_{k,\ell'}^{*2}) \\
&= \frac{2\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{\Delta m=1}^{L-1} C_{m,\ell} C_{m,\ell}^* C_{m,\ell} C_{m,\ell}^* (L-\Delta m) e^{\frac{4\pi^2}{N^2} k^2 \sigma_j^2}
\end{aligned} \tag{4.25}$$

When $k = k', m \neq m', n \neq n', \ell = \ell''$

$$\begin{aligned}
\text{Var}(\beta_{i,\ell'}) &= \frac{1}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} E\{a_{k,\ell}^2 C_{m,\ell} C_{m',\ell}^* C_{m,\ell} C_{m',\ell}^* b_{k,\ell} b_{k,\ell}^* b_{k,\ell} b_{k,\ell}^* e^{j\frac{2\pi}{N}[(k-i)n+k\xi(n,m)]} e^{-j\frac{2\pi}{N}[(k-i)n'+k\xi(n',m')]} \} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^L \sum_{m'=1, m' \neq m}^{N-1} \sum_{n=0}^{N-1} \sum_{n'=0, n' \neq n}^{N-1} C_{m,\ell} C_{m',\ell}^* C_{m,\ell} C_{m',\ell}^* E\{e^{j\frac{2\pi}{N}[(k-i)(n-n') + k[\xi(n,m) - \xi(n',m')]} b_{k,\ell} b_{k,\ell}^* b_{k,\ell} b_{k,\ell}^* \} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^L \sum_{m'=1, m' \neq m}^{N-1} \sum_{n=0}^{N-1} \sum_{n'=0, n' \neq n}^{N-1} C_{m,\ell} C_{m',\ell}^* C_{m,\ell} C_{m',\ell}^* e^{j\frac{2\pi}{N}[(k-i)(n-n')]} e^{-\frac{4\pi^2}{N^2} k^2 \sigma_j^2 [1 - \rho(LN(m-m') + (n-n'))]} E(b_{k,\ell}^2) E(b_{k',\ell''}^{*2}) \\
&= \frac{4\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{\Delta m=1}^{L-1} \sum_{\Delta n=1}^{N-1} C_{m,\ell} C_{m',\ell}^* C_{m,\ell} C_{m',\ell}^* (N - \Delta n)(L - \Delta m) \cos[\frac{2\pi}{N}(k-i)\Delta n] e^{-\frac{4\pi^2}{N^2} k^2 \sigma_j^2}
\end{aligned} \tag{4.26}$$

When $k \neq k'$ or $\ell \neq \ell'$

$$\text{Var}(\beta_{i,\ell'}) = 0$$

From Equation 4.14, the variance of $\mu_{i,\ell'}$ can be expressed as:

$$\begin{aligned}
\text{Var}(\mu_{i,\ell'}) &= E(\mu_{i,\ell'}^2) - [E(\mu_{i,\ell'})]^2 = E(\mu_{i,\ell'}^2) \\
&= E\left\{ \frac{1}{N} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} a_{k,\ell} b_{k,\ell} b_{k,\ell}^* \sum_{m=1}^L C_{m,\ell} C_{m',\ell}^* \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N} k \xi(n,m)} * \frac{1}{N} \sum_{\ell=1, \ell \neq \ell', \ell' \neq \ell}^{Nuser} a_{k',\ell'} b_{k',\ell'} b_{k',\ell'}^* \sum_{m'=1}^L C_{m',\ell'} C_{m',\ell'}^* \sum_{n'=0}^{N-1} e^{-j\frac{2\pi}{N} k' \xi(n',m')} \right\}
\end{aligned}$$

when $k = k', m = m', n = n', \ell = \ell''$

$$\begin{aligned}
\text{Var}(\mu_{i,\ell'}) &= \frac{1}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} E\{a_{k,\ell}^2 C_{m,\ell} C_{m',\ell}^* C_{m,\ell} C_{m',\ell}^* b_{k,\ell} b_{k,\ell}^* b_{k,\ell} b_{k,\ell}^* e^{j\frac{2\pi}{N} k \xi(n,m)} e^{-j\frac{2\pi}{N} k \xi(n',m')} \} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} C_{m,\ell} C_{m',\ell}^* C_{m,\ell} C_{m',\ell}^* b_{k,\ell}^2 b_{k,\ell}^{*2} E\{e^{j\frac{2\pi}{N} k [\xi(n,m) - \xi(n',m')]} \} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{m=1}^L \sum_{n=0}^{N-1} E\{C_{m,\ell}^2 C_{m',\ell}^{*2}\} E(b_{k,\ell}^2) E(b_{k',\ell''}^{*2}) \\
&= \frac{\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N} LN_{user}
\end{aligned} \tag{4.27}$$

When $k = k', m = m', n \neq n', \ell = \ell''$

$$\begin{aligned}
\text{Var}(\mu_{i,\ell'}) &= \frac{1}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{\text{Nuser}} \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} E\{a_{k,\ell}^2 C_{m,\ell} C_{m',\ell}^* C_{m,\ell} C_{m',\ell}^* b_{k,\ell} b_{k,\ell}^* b_{k',\ell} b_{k',\ell}^* e^{j\frac{2\pi}{N}k\xi(n,m)} e^{-j\frac{2\pi}{N}k\xi(n',m')}\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{\text{Nuser}} \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} C_{m,\ell} C_{m',\ell}^* C_{m,\ell} C_{m',\ell}^* E\{e^{j\frac{2\pi}{N}k[\xi(n,m)-\xi(n',m')]} b_{k,\ell} b_{k,\ell}^* b_{k',\ell} b_{k',\ell}^*\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{\text{Nuser}} \sum_{m=1}^L \sum_{n=0}^{N-1} \sum_{n'=0, n' \neq n}^{N-1} E\{e^{j\frac{2\pi}{N}k[\xi(n,m)-\xi(n',m)]}\} E(b_{k,\ell}^2) E(b_{k',\ell'}^{*2}) \\
&= \frac{\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{\text{Nuser}} \sum_{m=1}^L \sum_{n=0}^{N-1} \sum_{n'=0, n' \neq n}^{N-1} E\{e^{-\frac{4\pi^2}{N^2}k^2\sigma_j^2}\} \\
&= \frac{2\sigma_s^2 L \sigma_k^2 \sigma_k^{*2}}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{\text{Nuser}} \sum_{\Delta n=1}^{N-1} (N - \Delta n) e^{-\frac{4\pi^2}{N^2}k^2\sigma_j^2} \}
\end{aligned} \tag{4.28}$$

When $k = k', m \neq m', n = n', \ell = \ell''$

$$\begin{aligned}
\text{Var}(\mu_{i,\ell'}) &= \frac{1}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{\text{Nuser}} \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} E\{a_{k,\ell}^2 C_{m,\ell} C_{m',\ell}^* C_{m,\ell} C_{m',\ell}^* b_{k,\ell} b_{k,\ell}^* b_{k',\ell} b_{k',\ell}^* e^{j\frac{2\pi}{N}k\xi(n,m)} e^{-j\frac{2\pi}{N}k\xi(n',m')}\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{\text{Nuser}} \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} C_{m,\ell} C_{m',\ell}^* C_{m,\ell} C_{m',\ell}^* E\{e^{j\frac{2\pi}{N}k[\xi(n,m)-\xi(n',m')]} b_{k,\ell} b_{k,\ell}^* b_{k',\ell} b_{k',\ell}^*\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{\text{Nuser}} \sum_{m=1}^L \sum_{m'=1, m' \neq m}^L \sum_{n=0}^{N-1} C_{m,\ell} C_{m',\ell}^* C_{m,\ell} C_{m',\ell}^* e^{-\frac{4\pi^2}{N^2}k^2\sigma_j^2[1-\rho(N(m-m'))]} E(b_{k,\ell}^2) E(b_{k',\ell'}^{*2}) \\
&= \frac{2\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N} \sum_{\ell=1, \ell \neq \ell'}^{\text{Nuser}} \sum_{\Delta m=1}^{L-1} C_{m,\ell} C_{m',\ell}^* C_{m,\ell} C_{m',\ell}^* (L - \Delta m) e^{-\frac{4\pi^2}{N^2}k^2\sigma_j^2}
\end{aligned} \tag{4.29}$$

When $k = k', m \neq m', n \neq n', \ell = \ell''$

$$\begin{aligned}
\text{Var}(\mu_{i,\ell'}) &= \frac{1}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{\text{Nuser}} \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} E\{a_{k,\ell}^2 C_{m,\ell} C_{m',\ell}^* C_{m,\ell} C_{m',\ell}^* b_{k,\ell} b_{k,\ell}^* b_{k',\ell} b_{k',\ell}^* e^{j\frac{2\pi}{N}k\xi(n,m)} e^{-j\frac{2\pi}{N}k\xi(n',m')}\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{\text{Nuser}} \sum_{m=1}^L \sum_{m'=1, m' \neq m}^L \sum_{n=0}^{N-1} \sum_{n'=0, n' \neq n}^{N-1} C_{m,\ell} C_{m',\ell}^* C_{m,\ell} C_{m',\ell}^* E\{e^{j\frac{2\pi}{N}k[\xi(n,m)-\xi(n',m')]} b_{k,\ell} b_{k,\ell}^* b_{k',\ell} b_{k',\ell}^*\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{\text{Nuser}} \sum_{m=1}^L \sum_{m'=1, m' \neq m}^L \sum_{n=0}^{N-1} \sum_{n'=0, n' \neq n}^{N-1} C_{m,\ell} C_{m',\ell}^* C_{m,\ell} C_{m',\ell}^* e^{-\frac{4\pi^2}{N^2}k^2\sigma_j^2[1-\rho\{N(m-m')+(n-n')\}]} E(b_{k,\ell}^2) E(b_{k',\ell'}^{*2}) \\
&= \frac{4\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{\text{Nuser}} \sum_{\Delta m=1}^{L-1} \sum_{\Delta n=1}^{N-1} C_{m,\ell} C_{m',\ell}^* C_{m,\ell} C_{m',\ell}^* (N - \Delta n)(L - \Delta m) e^{-\frac{4\pi^2}{N^2}k^2\sigma_j^2}
\end{aligned} \tag{4.30}$$

When $k \neq k'$ or $\ell \neq \ell'$

$$\text{Var}(\mu_{i,\ell}) = 0$$

4.2.2 ICI due to colored timing jitter

When the sampling interval is small, the timing jitters could be correlated at adjacent symbols, which are called colored timing jitters. We will derive the variance of $\alpha_{i,\ell}$,

$\beta_{i,\ell}$, and $\mu_{i,\ell}$ due to the colored timing jitter in this section.

The variance of $\alpha_{i,\ell}$ can be written as following from Equation 4.12,

$$\begin{aligned} \text{Var}(\alpha_{i,\ell}) &= E(\alpha_{i,\ell}^2) - [E(\alpha_{i,\ell})]^2 = E(\alpha_{i,\ell}^2) \\ &= E \left\{ \frac{1}{N} \sum_{k=0, k \neq i}^{N-1} a_{k,\ell} b_{k,\ell} b_{k,\ell}^* \sum_{m=1}^L \sum_{n=0}^{N-1} e^{j \frac{2\pi}{N} [(k-i)n + k\xi(n,m)]} * \frac{1}{N} \sum_{k'=0, k' \neq i}^{N-1} a_{k',\ell} b_{k',\ell} b_{k',\ell}^* \sum_{m'=1}^L \sum_{n'=0}^{N-1} e^{-j \frac{2\pi}{N} [(k'-i)n' + k'\xi(n',m')]} \right\} \end{aligned}$$

When $k = k', m = m', n = n', \ell = \ell''$

$$\begin{aligned} \text{Var}(\alpha_{i,\ell}) &= \frac{1}{N^2} E \left\{ \sum_{k=0}^{N-1} a_{k,\ell}^2 b_{k,\ell} b_{k,\ell}^* b_{k',\ell} b_{k',\ell}^* \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} e^{j \frac{2\pi}{N} [(k-i)n + k\xi(n,m)]} e^{-j \frac{2\pi}{N} [(k'-i)n' + k'\xi(n',m')]} \right\} \\ &= \frac{\sigma_s^2}{N^2} \sum_{k=0}^{N-1} \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} E \left\{ e^{j \frac{2\pi}{N} [(k-i)(n-n') + k[\xi(n,m) - \xi(n',m')]]} b_{k,\ell}^2 b_{k',\ell}^{*2} \right\} \\ &= \frac{\sigma_s^2}{N^2} \sum_{k=0}^{N-1} \sum_{m=1}^L \sum_{n=0}^{N-1} E \left\{ e^{j \frac{2\pi}{N} [(k-i)(n-n') + k[\xi(n,m) - \xi(n',m')]]} \right\} E(b_{k,\ell}^2) E(b_{k',\ell}^{*2}) \\ &= \frac{\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N^2} E \left\{ \sum_{k=0}^{N-1} L * N \right\} = \frac{\sigma_s^2 * L * N * N}{N^2} \\ &= L \sigma_s^2 \sigma_k^2 \sigma_k^{*2} \end{aligned}$$

(4.31)

When $k = k', m = m', n \neq n', \ell = \ell''$

$$\begin{aligned}
\text{Var}(\alpha_{i,\ell'}) &= \frac{1}{N^2} E \left\{ \sum_{k=0}^{N-1} a_{k,\ell'}^2 b_{k,\ell} b_{k,\ell}^* b_{k',\ell'} b_{k',\ell'}^* \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} e^{j\frac{2\pi}{N}[(k-i)n+k\xi(n,m)]} e^{-j\frac{2\pi}{N}[(k'-i)n'+k\xi(n',m')]} \right\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{k=0}^{N-1} \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} E \left\{ e^{j\frac{2\pi}{N}[(k-i)(n-n')+k[\xi(n,m)-\xi(n',m')]} \right\} E(b_{k,\ell}^2) E(b_{k',\ell'}^{*2}) \\
&= \frac{\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N^2} \sum_{k=0}^{N-1} \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0, n \neq n'}^{N-1} e^{j\frac{2\pi}{N}[(k-i)(n-n')]} e^{-\frac{4\pi^2}{N^2} k^2 \sigma_j^2 [1-\rho(n-n')]} \\
&= \frac{2L\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N^2} \sum_{k=0}^{N-1} \sum_{\Delta n=1}^{N-1} (N-\Delta n) \cos \left[\frac{2\pi}{N} (k-i)\Delta n \right] e^{-\frac{4\pi^2}{N^2} k^2 \sigma_j^2 [1-\rho(\Delta n)]}
\end{aligned} \tag{4.32}$$

When $k = k', m \neq m', n = n', \ell = \ell''$

$$\begin{aligned}
\text{Var}(\alpha_{i,\ell'}) &= \frac{1}{N^2} E \left\{ \sum_{k=0}^{N-1} a_{k,\ell'}^2 b_{k,\ell} b_{k,\ell}^* b_{k',\ell'} b_{k',\ell'}^* \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} e^{j\frac{2\pi}{N}[(k-i)n+k\xi(n,m)]} e^{-j\frac{2\pi}{N}[(k'-i)n'+k\xi(n',m')]} \right\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{k=0}^{N-1} \sum_{m=1}^L \sum_{m'=1, m \neq m'}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} E \left\{ e^{j\frac{2\pi}{N}[(k-i)(n-n')+k[\xi(n,m)-\xi(n',m')]} \right\} E(b_{k,\ell}^2) E(b_{k',\ell'}^{*2}) \\
&= \frac{\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N^2} \sum_{k=0}^{N-1} \sum_{m=1}^L \sum_{m'=1, m \neq m'}^L \sum_{n=0}^{N-1} e^{-\frac{4\pi^2}{N^2} k^2 \sigma_j^2 [1-\rho(N(m-m'))]} \\
&= \frac{2\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N} \sum_{k=0}^{N-1} \sum_{\Delta m=1}^{L-1} (L-\Delta m) e^{-\frac{4\pi^2}{N^2} k^2 \sigma_j^2 [1-\rho(N\Delta m)]}
\end{aligned} \tag{4.33}$$

When $k = k', m \neq m', n \neq n', \ell = \ell''$

$$\begin{aligned}
\text{Var}(\alpha_{i,\ell'}) &= \frac{1}{N^2} E \left\{ \sum_{k=0}^{N-1} a_{k,\ell'}^2 b_{k,\ell} b_{k,\ell}^* b_{k',\ell'} b_{k',\ell'}^* \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} e^{j\frac{2\pi}{N}[(k-i)n+k\xi(n,m)]} e^{-j\frac{2\pi}{N}[(k'-i)n'+k\xi(n',m')]} \right\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{k=0}^{N-1} \sum_{m=1}^L \sum_{m'=1, m \neq m'}^L \sum_{n=0}^{N-1} \sum_{n'=0, n \neq n'}^{N-1} E \left\{ e^{j\frac{2\pi}{N}[(k-i)(n-n')+k[\xi(n,m)-\xi(n',m')]} \right\} E(b_{k,\ell}^2) E(b_{k',\ell'}^{*2}) \\
&= \frac{\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N^2} \sum_{k=0}^{N-1} \sum_{m=1}^L \sum_{m'=1, m \neq m'}^L \sum_{n=0}^{N-1} \sum_{n'=0, n \neq n'}^{N-1} e^{j\frac{2\pi}{N}[(k-i)(n-n')]} e^{-\frac{4\pi^2}{N^2} k^2 \sigma_j^2 [1-\rho[N(m-m')+(n-n')]]} \\
&= \frac{4\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N^2} \sum_{k=0}^{N-1} \sum_{\Delta m=1}^{L-1} \sum_{\Delta n=1}^{N-1} (N-\Delta n)(L-\Delta m) \cos \left[\frac{2\pi}{N} (k-i)\Delta n \right] e^{-\frac{4\pi^2}{N^2} k^2 \sigma_j^2 [1-\rho(N\Delta m+\Delta n)]}
\end{aligned} \tag{4.34}$$

When $k \neq k'$ or $\ell \neq \ell'$

$$\text{Var}(\alpha_{i,\ell}) = 0$$

From Equation 4.13, the variance of $\beta_{i,\ell}$ can be expressed as:

$$\begin{aligned} \text{Var}(\beta_{i,\ell}) &= E(\beta_{i,\ell}^2) - [E(\beta_{i,\ell})]^2 = E(\beta_{i,\ell}^2) \\ &= E \left\{ \frac{1}{N} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} a_{k,\ell} \sum_{m=1}^L C_{m,\ell} C_{m,\ell}^* b_{k,\ell} b_{k,\ell}^* \sum_{n=0}^{N-1} e^{j \frac{2\pi}{N} [(k-i)n + k\xi(n,m)]} \right. \\ &\quad \left. * \frac{1}{N} \sum_{\ell=1, \ell \neq \ell', \ell'' \neq \ell', k'=0, k' \neq i}^{Nuser} \sum_{m=1}^{N-1} a_{k',\ell''} \sum_{m=1}^L C_{m,\ell''} C_{m,\ell''}^* b_{k',\ell''} b_{k',\ell''}^* \sum_{n=0}^{N-1} e^{-j \frac{2\pi}{N} [(k'-i)n' + k'\xi(n',m')] } \right\} \end{aligned}$$

When $k = k', m = m', n = n', \ell = \ell''$

$$\begin{aligned} \text{Var}(\beta_{i,\ell}) &= \frac{1}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^L \sum_{m=1}^L \sum_{n=1}^{N-1} \sum_{n=0}^{N-1} E \{ a_{k,\ell}^2 C_{m,\ell} C_{m,\ell}^* C_{m,\ell} C_{m,\ell}^* b_{k,\ell} b_{k,\ell}^* b_{k,\ell} b_{k,\ell}^* e^{j \frac{2\pi}{N} [(k-i)n + k\xi(n,m)]} e^{-j \frac{2\pi}{N} [(k-i)n' + k\xi(n',m')] } \} \\ &= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^L \sum_{m=1}^L \sum_{n=1}^{N-1} \sum_{n=0}^{N-1} C_{m,\ell} C_{m,\ell}^* C_{m,\ell} C_{m,\ell}^* E \{ e^{j \frac{2\pi}{N} [(k-i)(n-n') + k(\xi(n,m) - \xi(n',m'))]} * b_{k,\ell} b_{k,\ell}^* b_{k,\ell} b_{k,\ell}^* \} \\ &= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^L \sum_{m=1}^L C_{m,\ell}^2 C_{m,\ell}^{*2} E \{ e^{j \frac{2\pi}{N} [(k-i)(n-n') + k(\xi(n,m) - \xi(n',m'))]} \} E(b_{k,\ell}^2) E(b_{k,\ell}^{*2}) \\ &= \frac{\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N^2} * N * L * N * Nuser \\ &= \sigma_s^2 \sigma_k^2 \sigma_k^{*2} L Nuser \end{aligned} \tag{4.35}$$

When $k = k', m = m', n \neq n', \ell = \ell''$

$$\begin{aligned} \text{Var}(\beta_{i,\ell}) &= \frac{1}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^L \sum_{m=1}^L \sum_{n=1}^{N-1} \sum_{n=0}^{N-1} E \{ a_{k,\ell}^2 C_{m,\ell} C_{m,\ell}^* C_{m,\ell} C_{m,\ell}^* b_{k,\ell} b_{k,\ell}^* b_{k,\ell} b_{k,\ell}^* e^{j \frac{2\pi}{N} [(k-i)n + k\xi(n,m)]} e^{-j \frac{2\pi}{N} [(k-i)n' + k\xi(n',m')] } \} \\ &= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^L \sum_{m=1}^L \sum_{n=1}^{N-1} \sum_{n=0}^{N-1} C_{m,\ell} C_{m,\ell}^* C_{m,\ell} C_{m,\ell}^* E \{ e^{j \frac{2\pi}{N} [(k-i)(n-n') + k(\xi(n,m) - \xi(n',m'))]} b_{k,\ell} b_{k,\ell}^* b_{k,\ell} b_{k,\ell}^* \} \\ &= \frac{\sigma_s^2 L}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^L \sum_{m=1}^L \sum_{n=0, n \neq n'}^{N-1} E \{ e^{j \frac{2\pi}{N} [(k-i)(n-n')] } e^{\frac{4\pi^2}{N^2} k^2 \sigma_j^2 [1 - \rho(n-n')]} \} E(b_{k,\ell}^2) E(b_{k,\ell}^{*2}) \\ &= \frac{2\sigma_s^2 L \sigma_k^2 \sigma_k^{*2}}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{\Delta n=1}^{N-1} (N - \Delta n) \cos \left[\frac{2\pi}{N} (k-i) \Delta n \right] e^{\frac{4\pi^2}{N^2} k^2 \sigma_j^2 [1 - \rho(\Delta n)]} \} \end{aligned} \tag{4.36}$$

When $k = k', m \neq m', n = n', \ell = \ell'$

$$\begin{aligned}
\text{Var}(\beta_{i,\ell'}) &= \frac{1}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} E \left\{ a_{k,\ell}^2 C_{m,\ell} C_{m,\ell}^* C_{m',\ell} C_{m',\ell}^* b_{k,\ell} b_{k,\ell}^* b_{k',\ell} b_{k',\ell}^* e^{j \frac{2\pi}{N} [(k-i)n + k\xi(n,m)]} e^{-j \frac{2\pi}{N} [(k-i)n' + k\xi(n',m')]} \right\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} C_{m,\ell} C_{m,\ell}^* C_{m',\ell} C_{m',\ell}^* b_{k,\ell}^2 b_{k',\ell}^{*2} E \left\{ e^{j \frac{2\pi}{N} [(k-i)(n-n') + k(\xi(n,m) - \xi(n',m'))]} \right\} \\
&= \frac{\sigma_s^2 N}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^L \sum_{m'=1, m' \neq m}^{L-1} \sum_{n=0}^{N-1} C_{m,\ell} C_{m,\ell}^* C_{m',\ell} C_{m',\ell}^* e^{-\frac{4\pi^2}{N^2} k^2 \sigma_j^2 [1 - \rho(N(m-m'))]} E(b_{k,\ell}^2) E(b_{k',\ell}^{*2}) \\
&= \frac{2\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{\Delta m=1}^{L-1} \sum_{n=0}^{N-1} C_{m,\ell} C_{m,\ell}^* C_{m',\ell} C_{m',\ell}^* (L - \Delta m) e^{-\frac{4\pi^2}{N^2} k^2 \sigma_j^2 [1 - \rho(N\Delta m)]} \\
&= \frac{2\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{\Delta m=1}^{L-1} C_{m,\ell} C_{m,\ell}^* C_{m',\ell} C_{m',\ell}^* (L - \Delta m) e^{-\frac{4\pi^2}{N^2} k^2 \sigma_j^2 [1 - \rho(N\Delta m)]}
\end{aligned} \tag{4.37}$$

When $k = k', m \neq m', n \neq n', \ell = \ell'$

$$\begin{aligned}
\text{Var}(\beta_{i,\ell'}) &= \frac{1}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0, n' \neq n}^{N-1} E \left\{ a_{k,\ell}^2 C_{m,\ell} C_{m,\ell}^* C_{m',\ell} C_{m',\ell}^* b_{k,\ell} b_{k,\ell}^* b_{k',\ell} b_{k',\ell}^* e^{j \frac{2\pi}{N} [(k-i)n + k\xi(n,m)]} e^{-j \frac{2\pi}{N} [(k-i)n' + k\xi(n',m')]} \right\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^L \sum_{m'=1, m' \neq m}^{L-1} \sum_{n=0}^{N-1} \sum_{n'=0, n' \neq n}^{N-1} C_{m,\ell} C_{m,\ell}^* C_{m',\ell} C_{m',\ell}^* E \left\{ e^{j \frac{2\pi}{N} [(k-i)(n-n') + k(\xi(n,m) - \xi(n',m'))]} b_{k,\ell} b_{k,\ell}^* b_{k',\ell} b_{k',\ell}^* \right\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{m=1}^L \sum_{m'=1, m' \neq m}^{L-1} \sum_{n=0}^{N-1} \sum_{n'=0, n' \neq n}^{N-1} C_{m,\ell} C_{m,\ell}^* C_{m',\ell} C_{m',\ell}^* e^{j \frac{2\pi}{N} [(k-i)(n-n')]} e^{-\frac{4\pi^2}{N^2} k^2 \sigma_j^2 [1 - \rho(N(m-m') + (n-n'))]} E(b_{k,\ell}^2) E(b_{k',\ell}^{*2}) \\
&= \frac{4\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{k=0, k \neq i}^{N-1} \sum_{\Delta m=1}^{L-1} \sum_{\Delta n=1}^{N-1} C_{m,\ell} C_{m,\ell}^* C_{m',\ell} C_{m',\ell}^* (N - \Delta n)(L - \Delta m) \cos \left[\frac{2\pi}{N} (k-i)\Delta n \right] e^{-\frac{4\pi^2}{N^2} k^2 \sigma_j^2 [1 - \rho(N\Delta m + \Delta n)]}
\end{aligned} \tag{4.38}$$

When $k \neq k'$ or $\ell \neq \ell'$

$$\text{Var}(\beta_{i,\ell'}) = 0$$

From Equation 4.14, the variance of $\mu_{i,\ell'}$ can be simplified as:

$$\begin{aligned}
\text{Var}(\mu_{i,\ell'}) &= E(\mu_{i,\ell'}^2) - [E(\mu_{i,\ell'})]^2 = E(\mu_{i,\ell'}^2) \\
&= E \left\{ \frac{1}{N} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} a_{k,\ell} b_{k,\ell} b_{k',\ell}^* \sum_{m=1}^L C_{m,\ell} C_{m,\ell}^* \sum_{n=0}^{N-1} e^{j \frac{2\pi}{N} k \xi(n,m)} * \frac{1}{N} \sum_{\ell=1, \ell \neq \ell', \ell' \neq \ell}^{Nuser} a_{k,\ell} b_{k',\ell} b_{k,\ell}^* \sum_{m=1}^L C_{m,\ell} C_{m,\ell}^* \sum_{n=0}^{N-1} e^{-j \frac{2\pi}{N} k \xi(n,m)} \right\}
\end{aligned}$$

when $k = k', m = m', n = n', \ell = \ell''$

$$\begin{aligned}
\text{Var}(\mu_{i,\ell'}) &= \frac{1}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{N_{\text{user}}} \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} E\{a_{k,\ell}^2 C_{m,\ell} C_{m,\ell}^* C_{m',\ell} C_{m',\ell}^* b_{k,\ell} b_{k,\ell}^* b_{k',\ell'} b_{k',\ell'}^* e^{j\frac{2\pi}{N}k\xi(n,m)} e^{-j\frac{2\pi}{N}k\xi(n',m')}\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{N_{\text{user}}} \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} C_{m,\ell} C_{m,\ell}^* C_{m',\ell} C_{m',\ell}^* b_{k,\ell}^2 b_{k',\ell'}^{*2} E\{e^{j\frac{2\pi}{N}k[\xi(n,m)-\xi(n',m')]} \} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{N_{\text{user}}} \sum_{m=1}^L \sum_{n=0}^{N-1} E\{C_{m,\ell}^2 C_{m,\ell}^{*2}\} E(b_{k,\ell}^2) E(b_{k',\ell'}^{*2}) \\
&= \frac{\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N} L N_{\text{user}}
\end{aligned} \tag{4.39}$$

When $k = k', m = m', n \neq n', \ell = \ell''$

$$\begin{aligned}
\text{Var}(\mu_{i,\ell'}) &= \frac{1}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{N_{\text{user}}} \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0, n' \neq n}^{N-1} E\{a_{k,\ell}^2 C_{m,\ell} C_{m,\ell}^* C_{m',\ell} C_{m',\ell}^* b_{k,\ell} b_{k,\ell}^* b_{k',\ell'} b_{k',\ell'}^* e^{j\frac{2\pi}{N}k\xi(n,m)} e^{-j\frac{2\pi}{N}k\xi(n',m')}\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{N_{\text{user}}} \sum_{m=1}^L \sum_{m'=1}^L \sum_{n=0}^{N-1} \sum_{n'=0, n' \neq n}^{N-1} C_{m,\ell} C_{m,\ell}^* C_{m',\ell} C_{m',\ell}^* E\{e^{j\frac{2\pi}{N}k[\xi(n,m)-\xi(n',m')]} b_{k,\ell} b_{k,\ell}^* b_{k',\ell'} b_{k',\ell'}^*\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{N_{\text{user}}} \sum_{m=1}^L \sum_{n=0}^{N-1} \sum_{n'=0, n' \neq n}^{N-1} E\{e^{j\frac{2\pi}{N}k[\xi(n,m)-\xi(n',m)]}\} E(b_{k,\ell}^2) E(b_{k',\ell'}^{*2}) \\
&= \frac{\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{N_{\text{user}}} \sum_{m=1}^L \sum_{n=0}^{N-1} \sum_{n'=0, n' \neq n}^{N-1} E\{e^{-\frac{4\pi^2}{N^2}k^2\sigma_j^2[1-\rho(n-n')]}\} \\
&= \frac{2\sigma_s^2 L \sigma_k^2 \sigma_k^{*2}}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{N_{\text{user}}} \sum_{\Delta n=1}^{N-1} (N - \Delta n) e^{-\frac{4\pi^2}{N^2}k^2\sigma_j^2[1-\rho(\Delta n)]}
\end{aligned} \tag{4.40}$$

When $k = k', m \neq m', n = n', \ell = \ell''$

$$\begin{aligned}
\text{Var}(\mu_{i,\ell'}) &= \frac{1}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{m=1}^L \sum_{m'=1}^{L-1} \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} E\{a_{k,\ell}^2 C_{m,\ell} C_{m,\ell}^* C_{m',\ell} C_{m',\ell}^* b_{k,\ell} b_{k,\ell}^* b_{k',\ell} b_{k',\ell}^* e^{j\frac{2\pi}{N}k\xi(n,m)} e^{-j\frac{2\pi}{N}k\xi(n',m')}\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{m=1}^L \sum_{m'=1}^{L-1} \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} C_{m,\ell} C_{m,\ell}^* C_{m',\ell} C_{m',\ell}^* E\{e^{j\frac{2\pi}{N}k[\xi(n,m)-\xi(n',m')]} b_{k,\ell} b_{k,\ell}^* b_{k',\ell} b_{k',\ell}^*\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{m=1}^L \sum_{m'=1, m' \neq m}^{L-1} \sum_{n=0}^{N-1} C_{m,\ell} C_{m,\ell}^* C_{m',\ell} C_{m',\ell}^* e^{-\frac{4\pi^2}{N^2}k^2\sigma_j^2[1-\rho(N(m-m'))]} E(b_{k,\ell}^2) E(b_{k',\ell'}^{*2}) \\
&= \frac{2\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{\Delta m=1}^{L-1} \sum_{n=0}^{N-1} C_{m,\ell} C_{m,\ell}^* C_{m',\ell} C_{m',\ell}^* (L-\Delta m) e^{-\frac{4\pi^2}{N^2}k^2\sigma_j^2[1-\rho(N\Delta m)]} \\
&= \frac{2\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{\Delta m=1}^{L-1} C_{m,\ell} C_{m,\ell}^* C_{m',\ell} C_{m',\ell}^* (L-\Delta m) e^{-\frac{4\pi^2}{N^2}k^2\sigma_j^2[1-\rho(N\Delta m)]}
\end{aligned} \tag{4.41}$$

When $k = k', m \neq m', n \neq n', \ell = \ell''$

$$\begin{aligned}
\text{Var}(\mu_{i,\ell'}) &= \frac{1}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{m=1}^L \sum_{m'=1}^{L-1} \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} E\{a_{k,\ell}^2 C_{m,\ell} C_{m,\ell}^* C_{m',\ell} C_{m',\ell}^* b_{k,\ell} b_{k,\ell}^* b_{k',\ell} b_{k',\ell}^* e^{j\frac{2\pi}{N}k\xi(n,m)} e^{-j\frac{2\pi}{N}k\xi(n',m')}\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{m=1}^L \sum_{m'=1, m' \neq m}^{L-1} \sum_{n=0}^{N-1} \sum_{n'=0, n' \neq n}^{N-1} C_{m,\ell} C_{m,\ell}^* C_{m',\ell} C_{m',\ell}^* E\{e^{j\frac{2\pi}{N}k[\xi(n,m)-\xi(n',m')]} b_{k,\ell} b_{k,\ell}^* b_{k',\ell} b_{k',\ell}^*\} \\
&= \frac{\sigma_s^2}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{m=1}^L \sum_{m'=1, m' \neq m}^{L-1} \sum_{n=0}^{N-1} \sum_{n'=0, n' \neq n}^{N-1} C_{m,\ell} C_{m,\ell}^* C_{m',\ell} C_{m',\ell}^* e^{-\frac{4\pi^2}{N^2}k^2\sigma_j^2[1-\rho\{N(m-m')+(n-n')\}]} E(b_{k,\ell}^2) E(b_{k',\ell'}^{*2}) \\
&= \frac{4\sigma_s^2 \sigma_k^2 \sigma_k^{*2}}{N^2} \sum_{\ell=1, \ell \neq \ell'}^{Nuser} \sum_{\Delta m=1}^{L-1} \sum_{\Delta n=1}^{N-1} C_{m,\ell} C_{m,\ell}^* C_{m',\ell} C_{m',\ell}^* (N-\Delta n)(L-\Delta m) e^{-\frac{4\pi^2}{N^2}k^2\sigma_j^2[1-\rho(N\Delta m+\Delta n)]}
\end{aligned} \tag{4.42}$$

When $k \neq k'$ or $\ell \neq \ell'$

$$\text{Var}(\mu_{i,\ell'}) = 0$$

From the above formula, we can calculate BER for the MC-DS-CDMA system due to the timing jitters both in AWGN channel and multi-path Rayleigh fading channel and compare the BER performances when the timing jitters are white, colored and zero, respectively.

5. BER performance test methods

5.0 Introduction

In this chapter the BER performance test methods are introduced based on the analysis in previous chapters. The test methods show different BER performance affected by white, colored and zero timing jitters in AWGN channel and multi-path Rayleigh fading channel.

The software used in the testing is Matlab. Matlab (Matrix laboratory) is an interactive software system for numerical computations and graphics. It was originally designed to simplify the implementation of numerical linear algebra routines and then used to implement numerical algorithms for a wide range of applications [44]. In our research, algorithms is designed to generate the MC-DS-CDMA transmission, simulate the AWGN channel and multi-path Rayleigh fading channel, recover the original input data, and calculate the transmission bit error rate (BER) according to the transmitted and received data in Matlab.

5.1 Testing method in AWGN channel

The received data of the MC-DS-CDMA system on the effect of white, correlated and zero timing jitters in AWGN channel with different parameters are generated respectively and compared with original input data. Then the BER can be calculated under different signal-to-noise ratio according to the transmitted and received data.

5.2 Testing methods in multi-path Rayleigh fading channel

Method one:

True simulation is applied in this method. The MC-DS-CDMA transmission, channels, timing jitters are simulated as the model described in Chapter Three and recovered data is collected. Then the BER can be calculated over different level of signal-to-noise ratio

(SNR) from the comparison of recovered data and input data.

Method two:

In this method Monte Carlo simulation with AWGN variance calculated from simulation is used. Monte Carlo simulation relies on repeated random samples to calculate their results and is widely used in systems with a large number of uncertain inputs. The input data is the same as method one. The mean and variance of recovered data is calculated according to Equation 4.17 and the AWGN variance is computed from the true simulated channels.

Method three:

Method three is similar to Method two. The mean and variance of recovered data is calculated from Equation 4.17 but the AWGN variance is calculated from SNR setting in the algorithm.

Method four:

Monte Carlo simulation is also used in this method. All interference variances of recovered data are calculated from formulas derived in Chapter Three, which are Equation 4.7-4.11.

Method five:

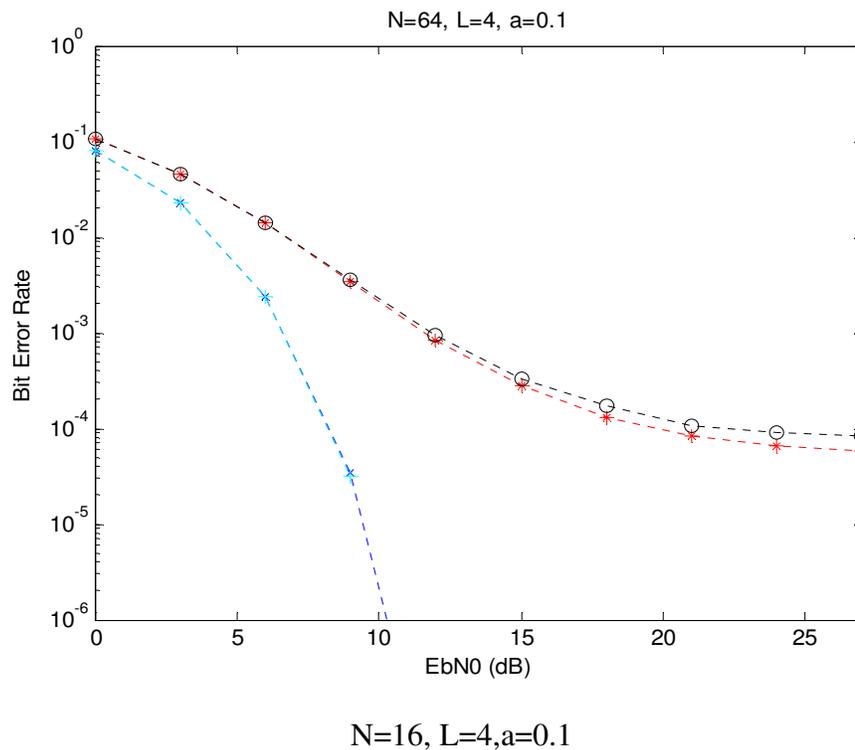
In this method all interference variances of recovered data are calculated from simplified formulas in derived Chapter Three, which are Equation 4.31-4.42.

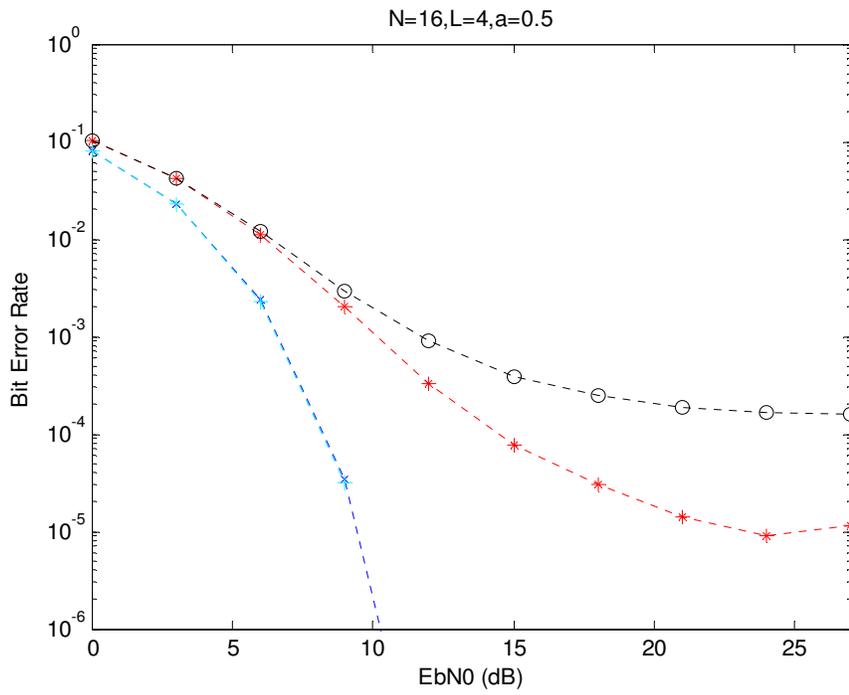
6. Results and discussion

In this section results of the BER performance of MC-DS-CDMA system on the effect of timing jitter are outlined. 'N' denotes the number of sub-carriers; 'L' is the length of spreading code; 'a' corresponds to the level of correlation of the timing jitters, for instance, 'a=0' corresponds uncorrelated timing jitter and 'a=1' means fully correlated timing jitter.

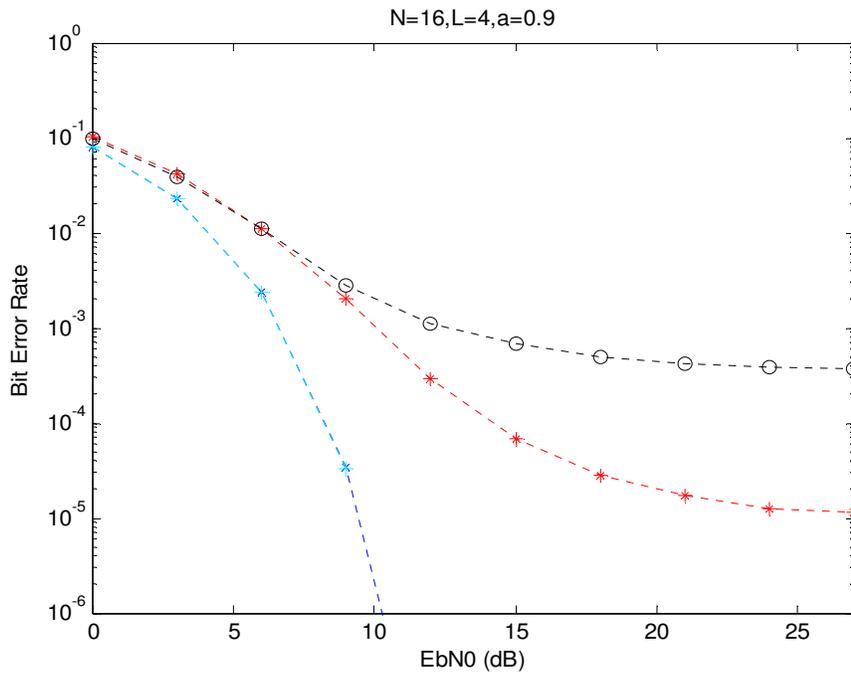
6.1 BER performance on the effect of timing jitters in AWGN channel

The results of the BER performance on the effect of white, correlated and zero timing jitters are presented. The dashed curve with circle denotes the BER performance affected by white timing jitter. The dashed curve with star presents the BER performance due to correlated timing jitter. The dashed curve with plus shows the BER performance on the effect of zero timing jitter and the dashed curve with x-mark is the ideal BER performance without interference.

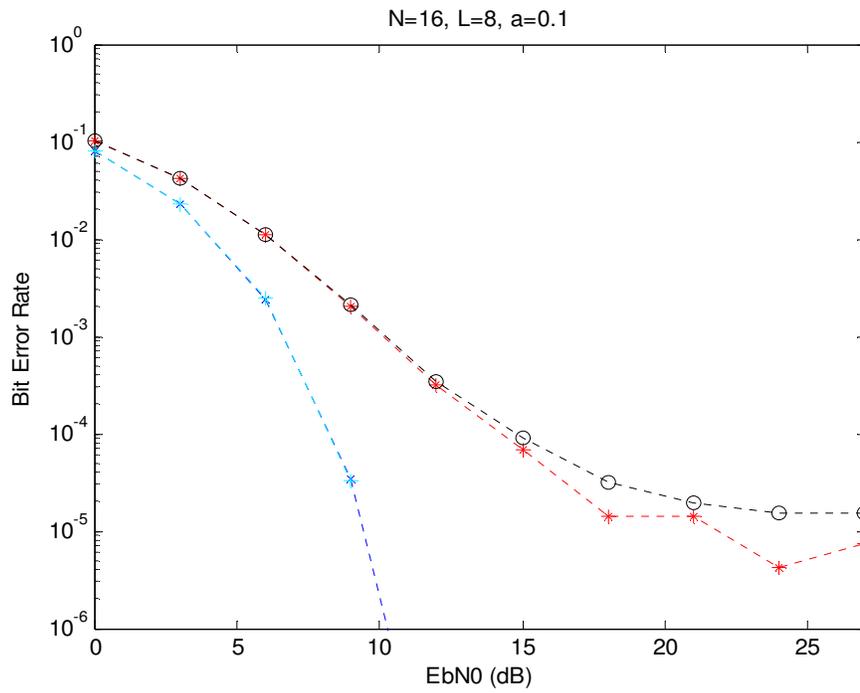




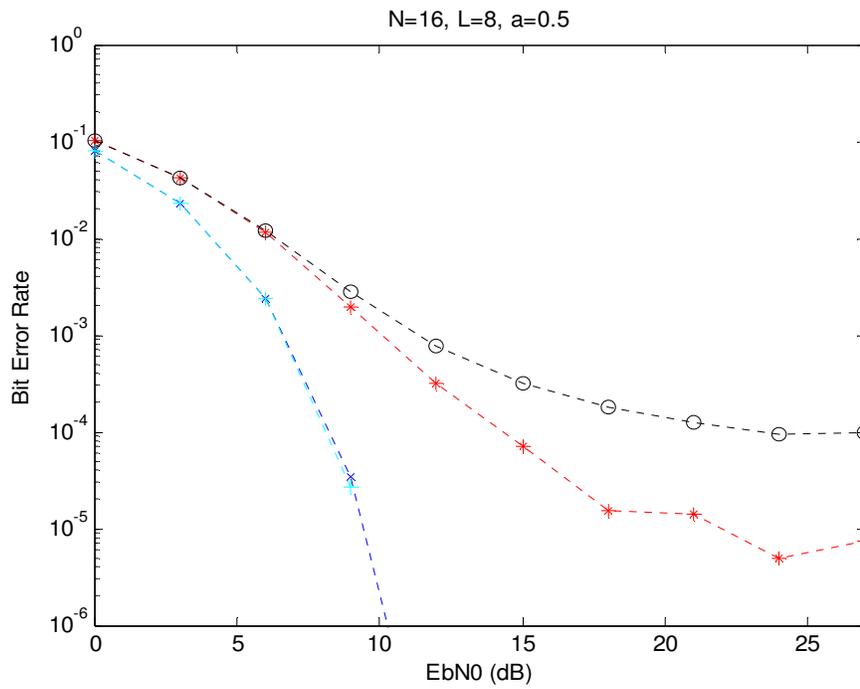
N=16, L=4, a=0.5



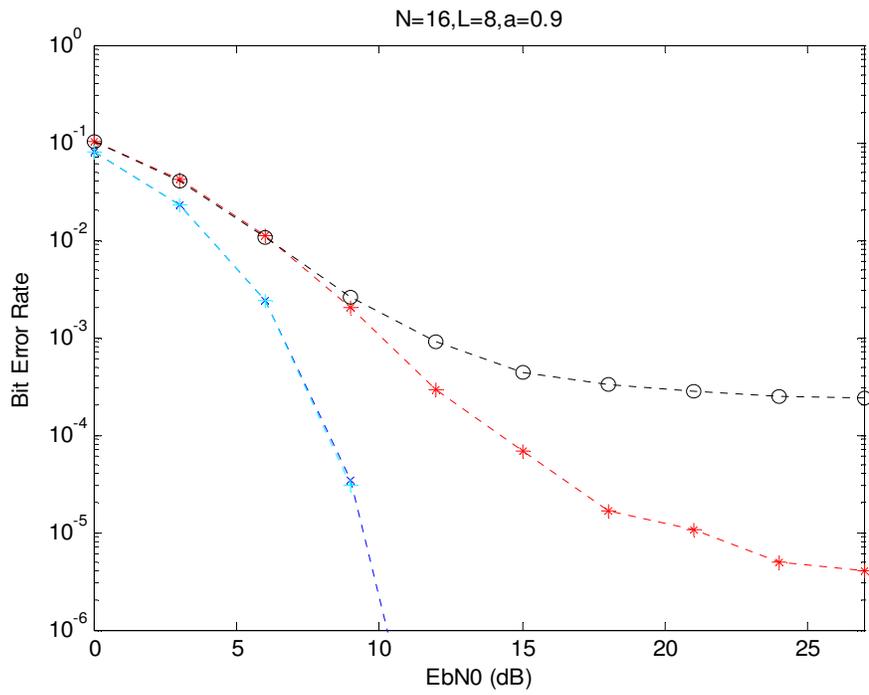
N=16, L=4, a=0.9



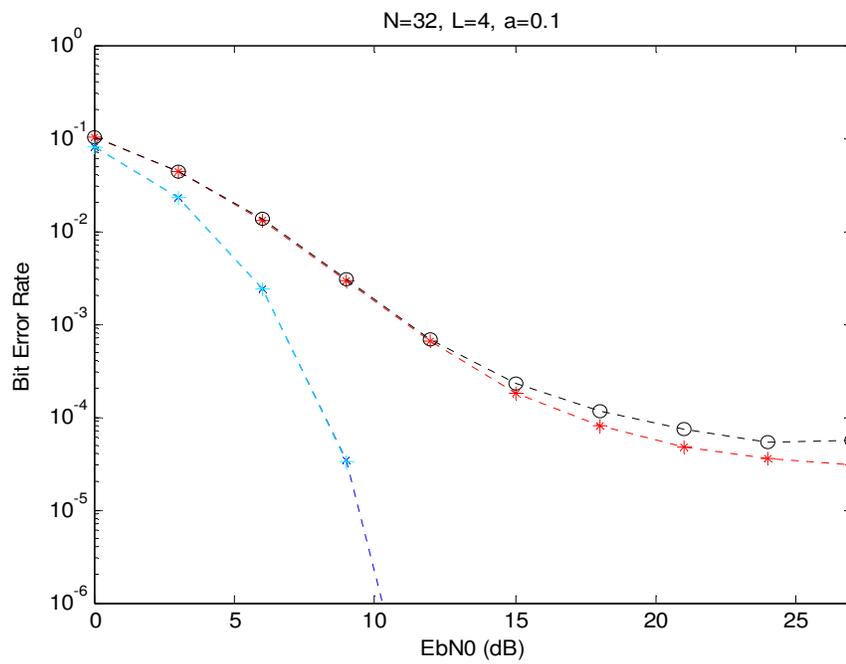
N=16, L=8, a=0.1



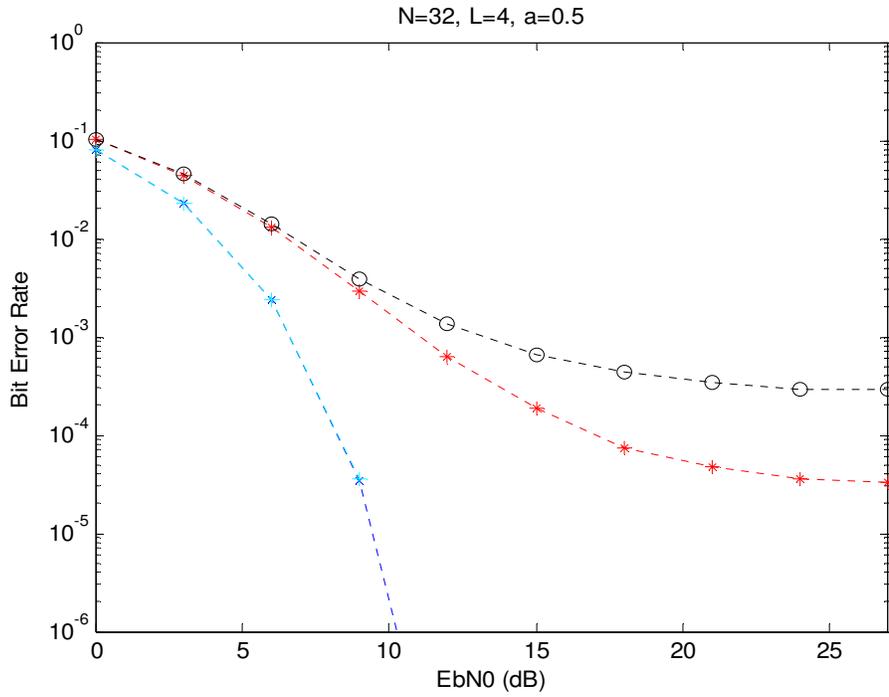
N=16, L=8, a=0.5



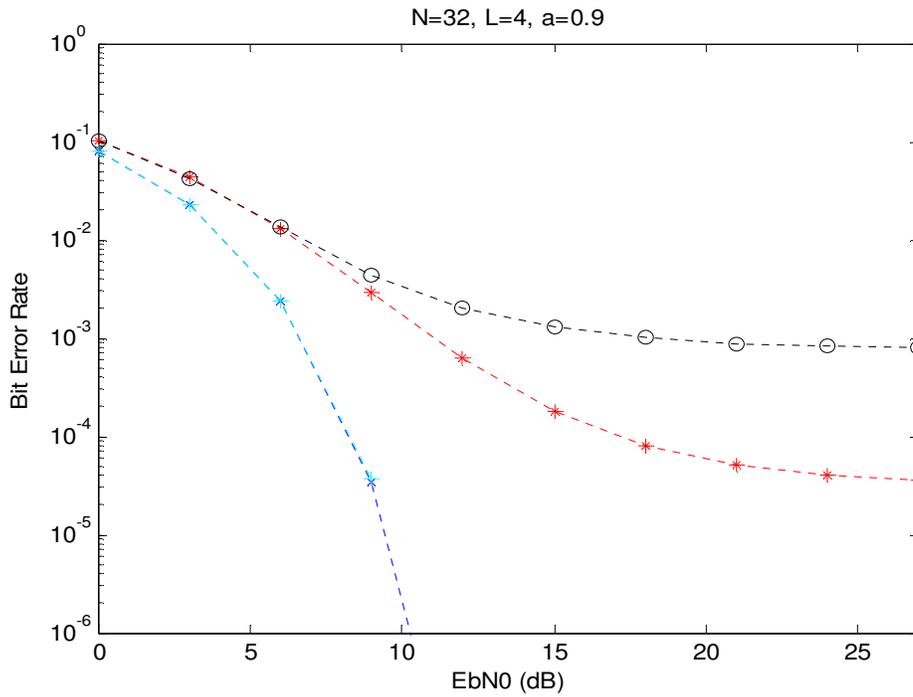
N=16, L=8, a=0.9



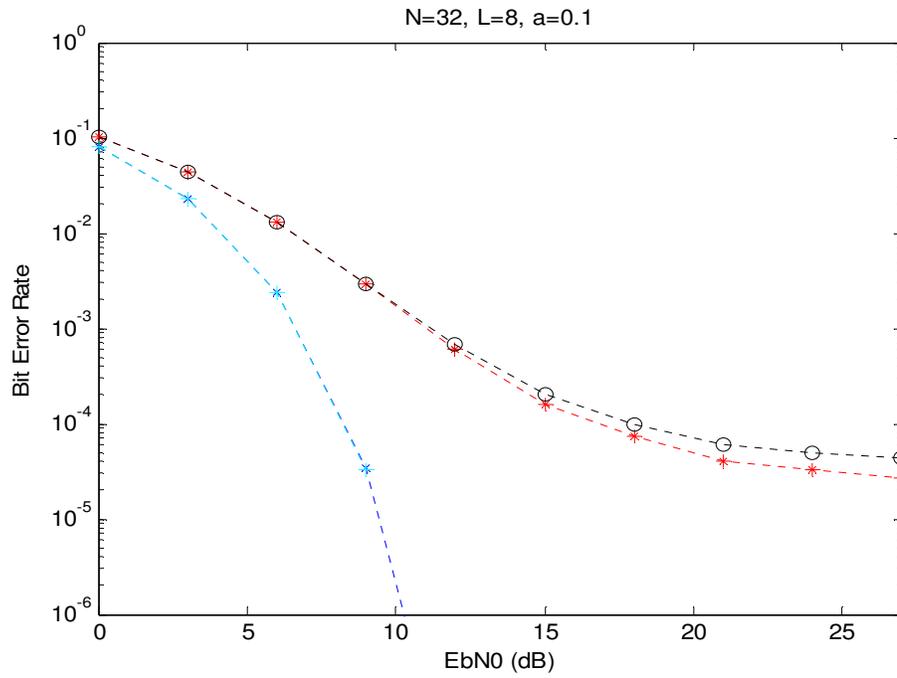
N=32, L=4, a=0.1



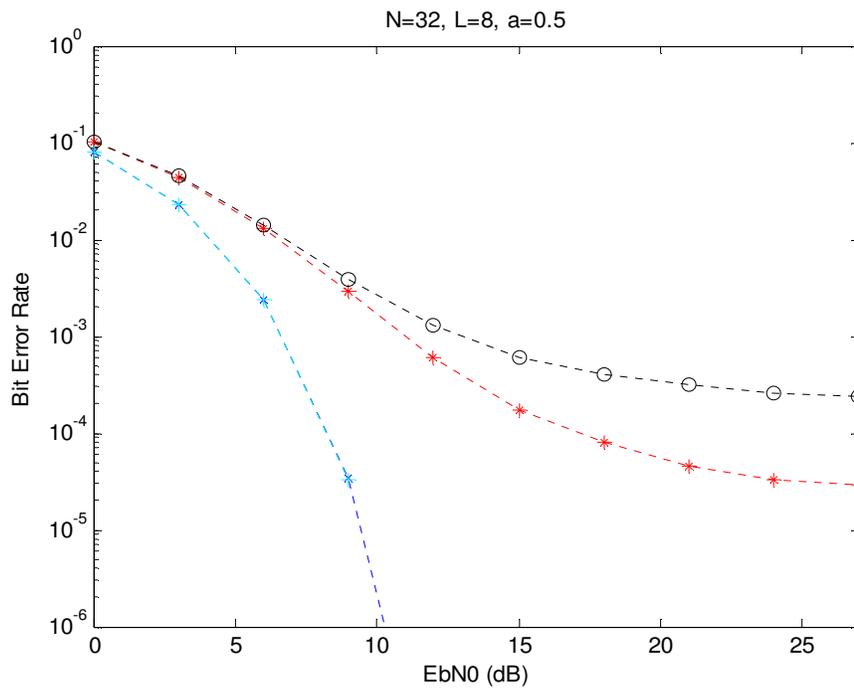
N=32, L=4, a=0.5



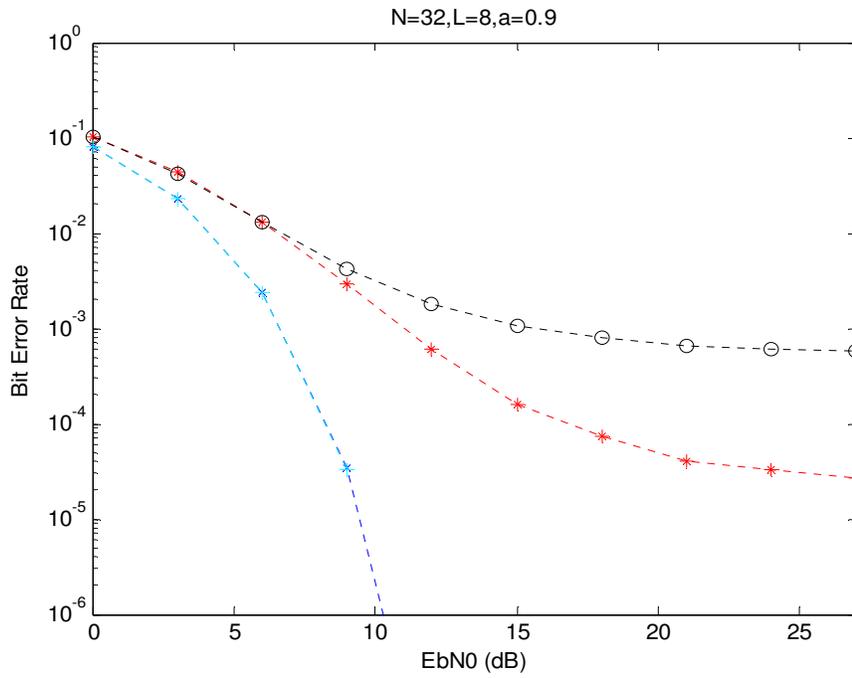
N=32, L=4, a=0.9



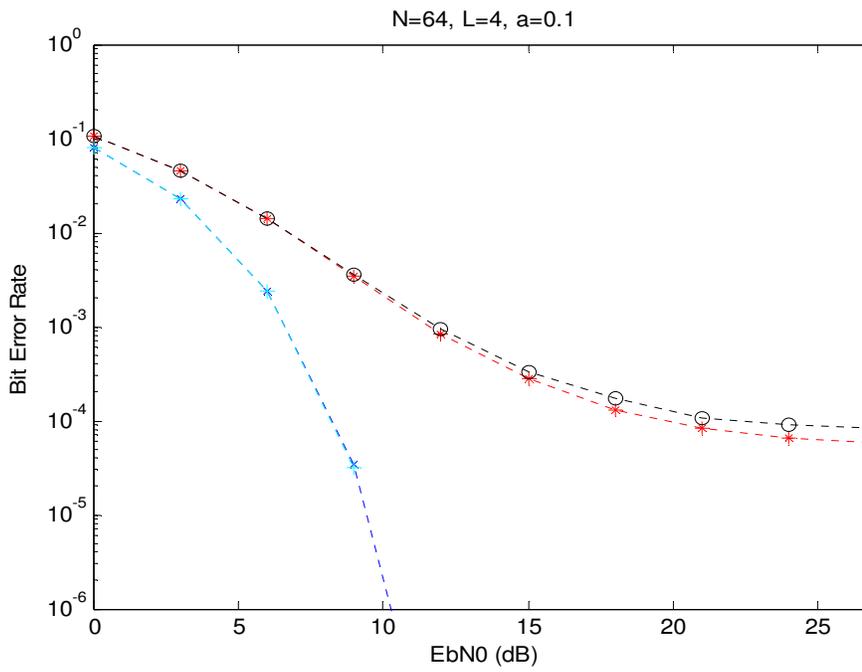
N=32, L=8, a=0.1



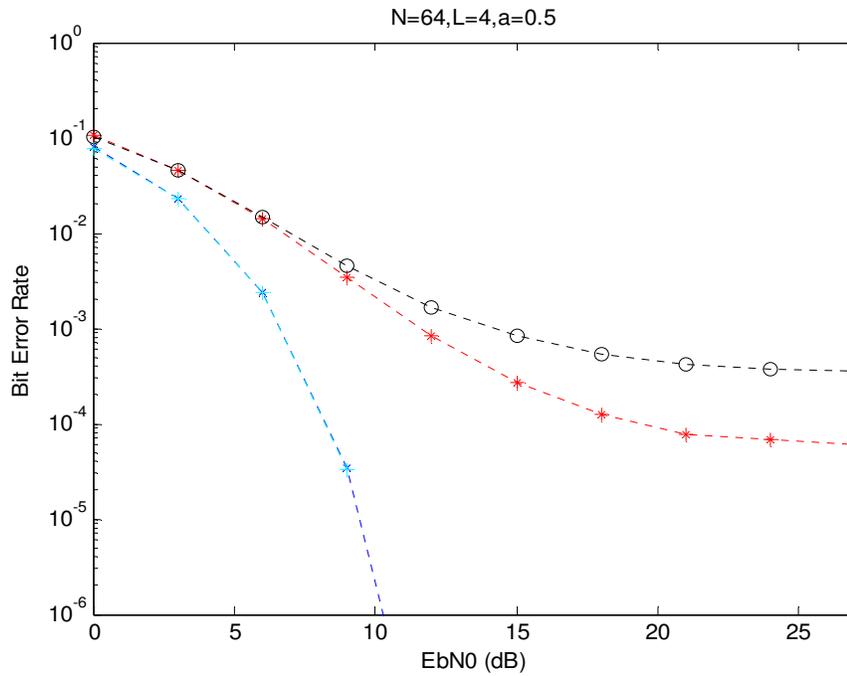
L=32, L=8, a=0.5



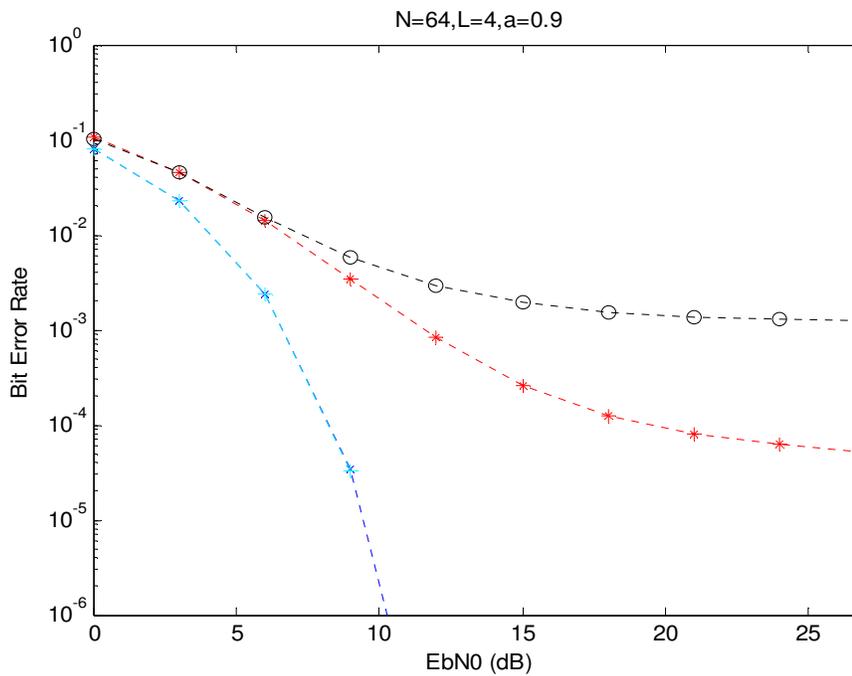
N=32, L=8, a=0.9



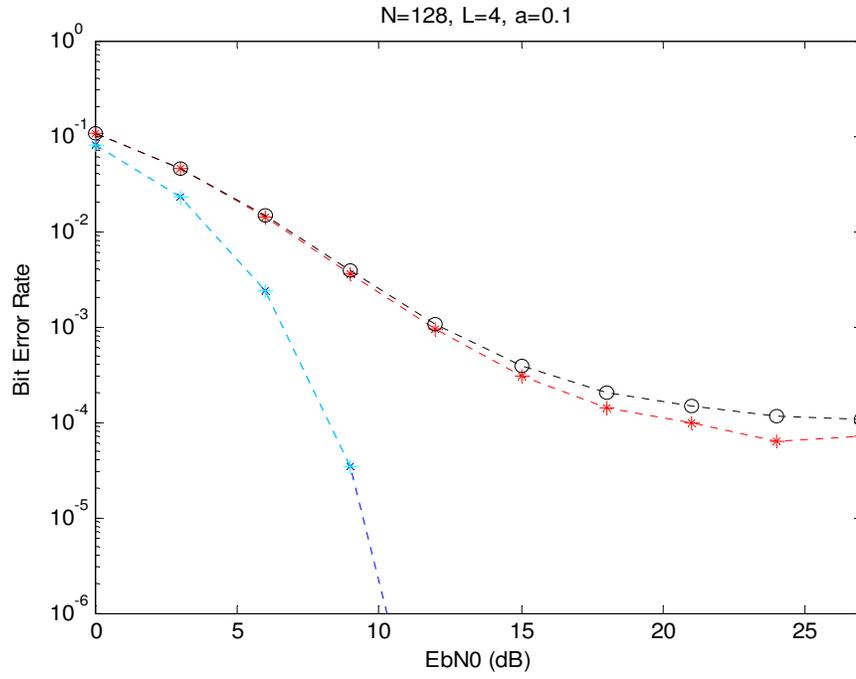
N=64, L=4, a=0.1



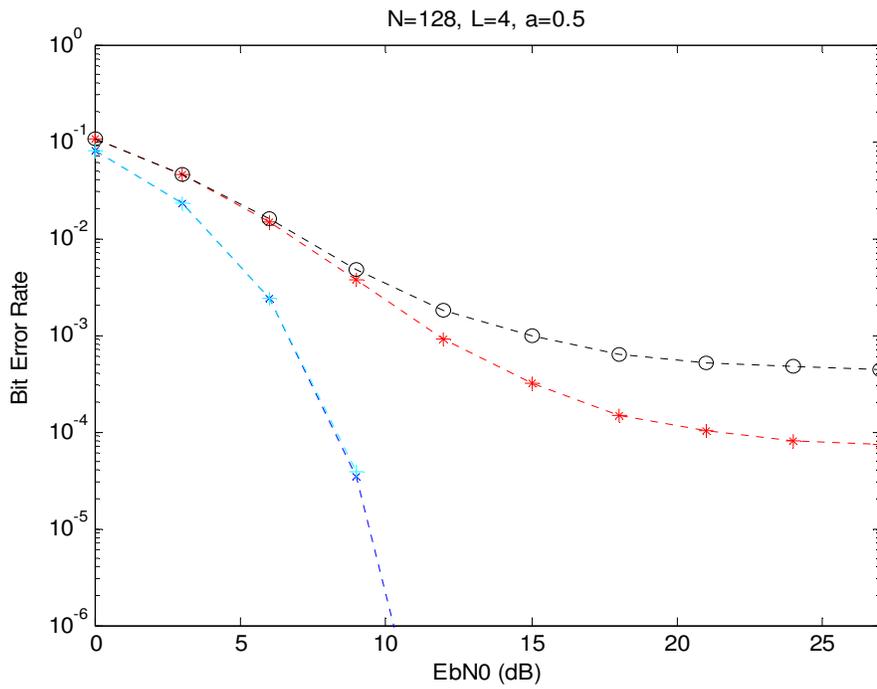
N=64, L=4, a=0.5



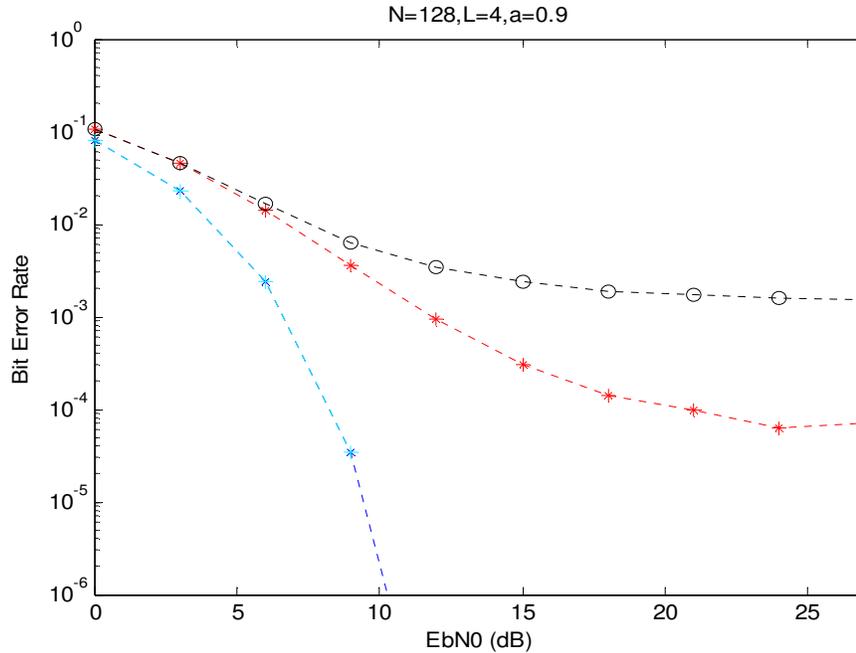
N=64, L=4, a=0.9



L=128, L=4, a=0.1



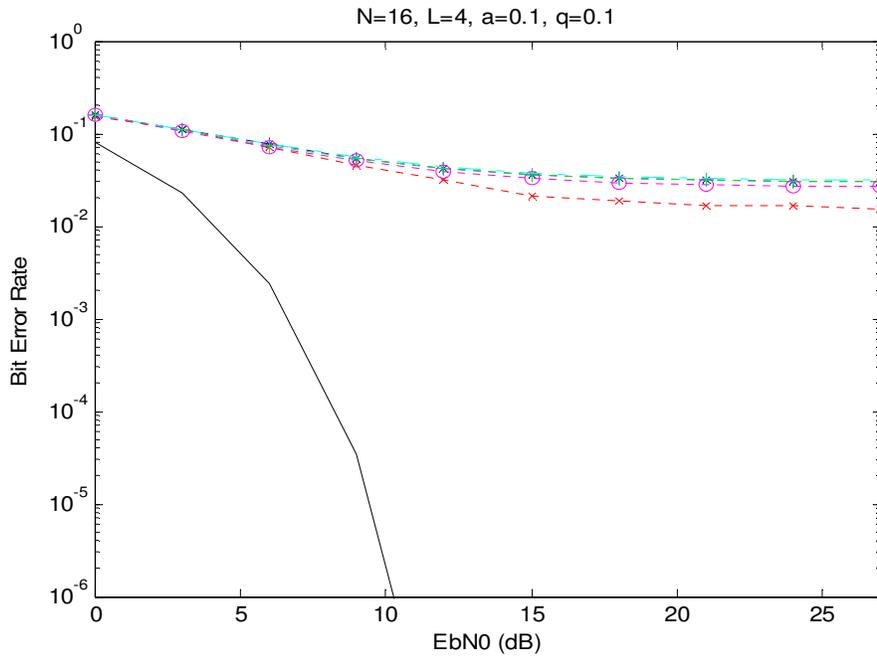
L=128, L=4, a=0.5



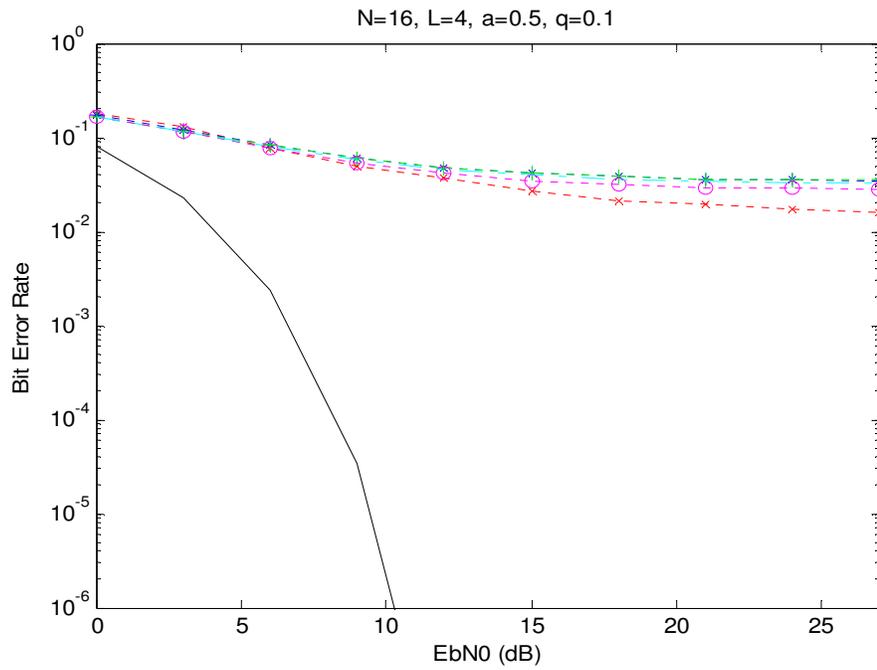
N=128, L=4, a=0.9

6.2 BER performance on the effect of timing jitters in multi-path Rayleigh fading channel

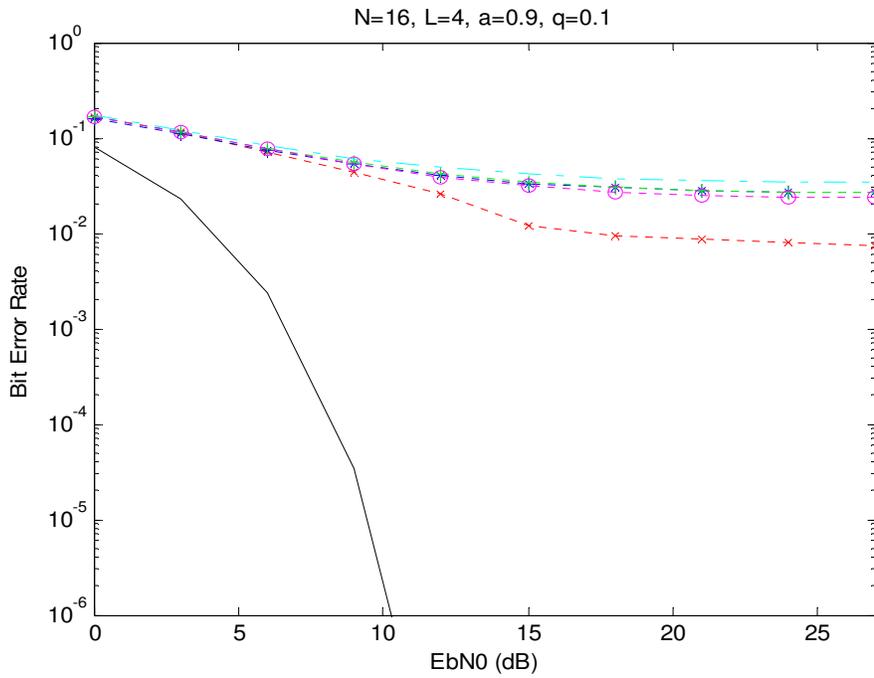
The results of the BER performance on the effect of timing jitter in multi-path Rayleigh fading channel are shown in this section. The dashed curve with x-mark is the results of the BER performance from testing method one. The dashed curve with star presents the results of the BER performance from testing method two. The dashed curve with plus denotes the results of the BER performance from testing method three. The dashed curve with circle shows the results of the BER performance from testing method four. The dashdot curve gives the results of the BER performance from testing method five and the solid curve is the ideal BER performance without interference.



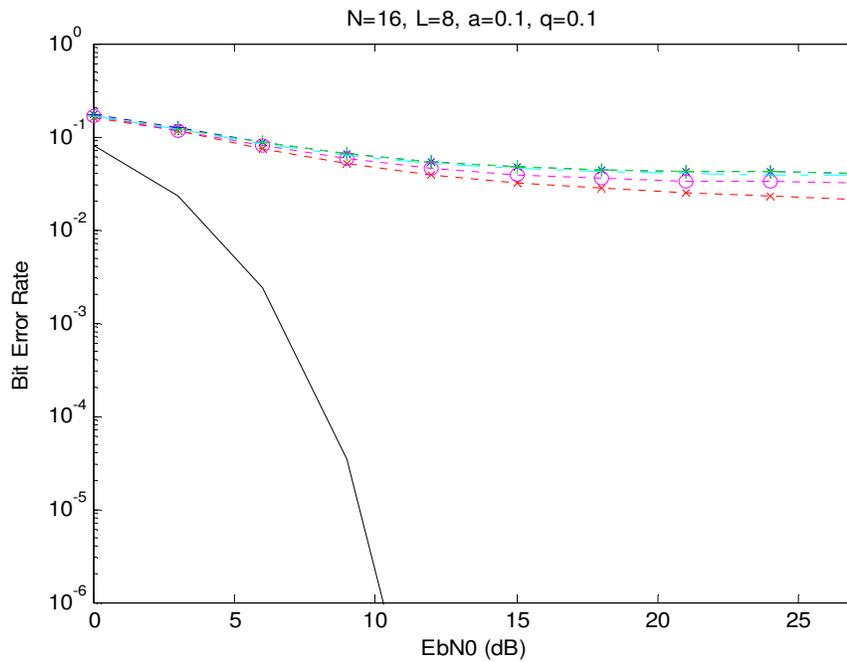
N=16, L=4, a=0.1 q=0.1



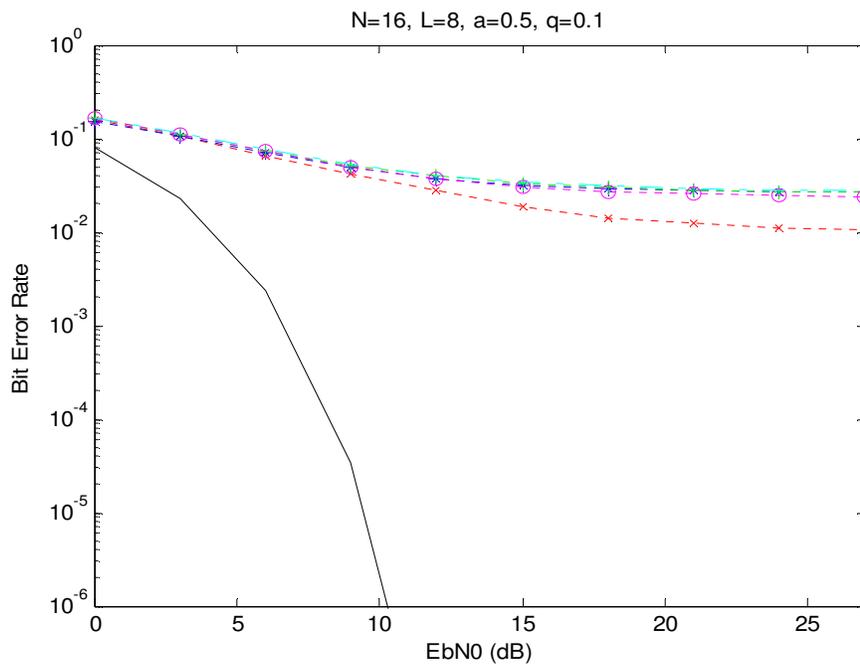
N=16, L=4, a=0.5 q=0.1



N=16, L=4, a=0.9 q=0.1



N=16, L=8, a=0.1 q=0.1



N=16, L=8, a=0.5 q=0.1

7. Conclusion

In this thesis, the BER performance of MC-DS-CDMA system under the effects of timing jitters has been analyzed in the presence of AWGN channel and multi-path Rayleigh fading channel. It can be observed from the results in Sections 6.1 & 6.2 that in general Gaussian approximations give pessimistic BER estimates. Furthermore, the higher the correlation between timing jitters, the less the impact on BER. The number of sub-carriers does not appear to affect the BER significantly. The role of spreading factor in BER performance with timing jitters needs further study.

Reference

1. Je-hong Jong, *Performance Analysis of Coded Multicarrier Spread-Spectrum Systems in the Presence of Multipath Fading and Nonlinearities*, IEEE Communication Magazine, Volume 49, pp. 168-179, January 2001.
2. Li-Chun Wang, and Chih-Wen Chang, *On the performance of multicarrier DS-CDMA with imperfect power control and variable spreading factors*, IEEE Communication Magazine, Volume 24, pp. 1154–1166, June 2006.
3. T. J. Lim, S. Edwards, and Sr. Rogers, *An introduction to multicarrier modulation: notes for ECE1520 Data Communications*, Department of electrical and computer engineering University of Toronto, pp.1-16.
4. — —, *OFDM Tutorial*, WAVE Report, 4th Wave Inc., 2007.
5. D. Matiaë, *OFDM as a possible modulation technique for multimedia applications in the range of mm waves*, *Introduction to OFDM*, second edition, TUD-TVS, pp.1-18, October 1998.
6. R. W. Chang, *Synthesis of band-limited orthogonal signals for multichannel data transmission*, The Bell Systems Technical Journal, Volume 45, pp. 1775-1796, December 1966.
7. B. R. Saltzberg, *Performance of an efficient parallel data transmission system*, IEEE Transactions on Communication Technology, Volume COM-15, December 1967.
8. R. R. Mosier, and R. G. Clabaugh, *Kineplex, a bandwidth efficient binary transmission system*, AIEE Transactions, Volume 76, pp. 723- 728, January 1958.
9. — —, *Orthogonal frequency division multiplexing*, U.S. Patent 3,488,445, filed November 14, 1966, issued January 6, 1970.

-
10. W. Y. Zou, and Y. Wu, *COFDM: an overview*, IEEE Transactions on Broadcasting, Volume 41, No. 1, March 1995.
 11. S. B. Weinstein, and P. M. Ebert, *Data transmission by frequency division multiplexing using the discrete Fourier transform*, IEEE Transactions on Communication Technology, Volume COM-19, No. 15, October 1971.
 12. E. Bidet, C. Joanblanq, and P. Senn, *A fast 8K FFT VLSI chip for large OFDM single frequency networks*, Proceedings of the International Conference on HDTV 94, Torino, Italy, October 1994.
 13. R. V. Nee, and R. Prasad, *OFDM for Wireless Multimedia Communications*, Artech House, Boston, London, 2000.
 14. G. C. Porter, *Error distribution and diversity performance of a frequency-differential PSK HF modem*, IEEE Transactions on Communication Technology, Volume COM-16, pp. 567-575, August 1968.
 15. M. S. Zimmerman, and A. L. Kirsch, *The AN/GSC-10 (KATHRYN) variable rate data modem for HF radio*, IEEE Transactions on Communication Technology, Volume COM-15, pp. 197-205, April 1967.
 16. P. A. Bello, *Selective fading limitations of the Kathryn modem and some system design considerations*, IEEE Transactions on Communication Technology, Volume COM-13, pp. 320-333, September 1965.
 17. B. Hirosaki, *An orthogonally multiplexed QAM system using the discrete Fourier transform*, IEEE Transactions on Communications, Volume COM-29, pp. 982-989, July 1981.

-
18. B. Hirosaki, *A 19.2 kbits voiceband data modem based on orthogonally multiplexed QAM techniques*, Proceedings of IEEE International Conference on Communications, pp.21.1.1-5, 1985.
 19. W. E. Keasler, and D. L. Bitzer, *High speed modem suitable for operating with a switched network*, U.S. patent No. 4,206,320, June 1980.
 20. E. F. Casas, and C. Leung, *OFDM for data communication over mobile radio FM channels-part 1: analysis and experimental results*, IEEE Transactions on Communications, Volume. COM-39, No.5, May 1991.
 21. P. S. Chow, J. C. Tu, and J. M. Cioffi, *Performance evaluation of a multichannel transceiver system for ADSL and VHDSL services*, IEEE Journal on Selected Area in Communications, Volume SAC-9, No.6, pp.900-919, August 1991.
 22. P. S. Chow, J. C. Tu, and J. M. Cioffi, *A discrete Multitone transceiver system for HDSL applications*, IEEE Journal on Selected Area in Communications, Volume SAC-9, No. 6, pp.909-919, August 1991.
 23. B. Sueur, D. Castelain, G. Degoulet, M. Riviere, and B. LeFloch, *Digital Terrestrial Broadcasting of Audiovisual Signals*, Spectrum 20120 conference, Toronto, September 1992.
 24. B. LeFloch, M. Alard, and C. Berrou, *Coded Orthogonal Frequency Division Multiplex*, Proceedings of the IEEE, Volume 83, No. 6, June 1995.
 25. Q. Wang, and L. Y. Onotera, *Coded QAM Using a Binary Convolutional Code*, IEEE Transactions on Communications, Volume 43, No.6, June 1995.
 26. — —, *an introduction to direct-sequence spread-spectrum*, application note 1890, Maxim Integrated Products, Feb 2003.

-
27. — —, *Spread Spectrum Systems, chapter 8 module 7: Wireless Communications*, ICT Technologies, pp.74-87.
 28. K. Feher, *Wireless Digital Communications: Modulation and Spread Spectrum Applications*, Prentice Hall PTR, Prentice-Hall, Inc., 1995.
 29. L.Hanzo, L-L. Yang, E-L.Kuan, and K. Yen, *single-and multi-carrier CDMA: multi-user detection, space-time spreading, synchronization, networking, and standards*, IEEE press, IEEE communications society, 2003.
 30. S. Hara, and R. Prasad, *Overview of Multicarrier CDMA*, IEEE Communication Magazine, Volume 35, pp. 126–133, December 1997.
 31. E. Sourour, and M. Nakagawa, *Performance of Orthogonal Multicarrier CDMA in a Multipath Fading Channel*, IEEE Transactions on Communications, Volume 44, No. 3, pp. 356-67, March 1996.
 32. M. K. Simon, and M. S. Alouini, *BER Performance of multicarrier DS-CDMA systems over generalized fading channels*, IEEE communication theory mini-conference, pp.72-77, June1999.
 33. J. Hou, and M. H. Lee, *Dual OVVSF Codes with Lower Correlations*, IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences, Volume E89-A, Issue 11, pp. 3363-3367, November 2006.
 34. M. H. Lee, *A new reverse jacket transform and its fast algorithm*, IEEE Transactions on Circuits and Systems II, Volume 47, No. 1, pp. 39-47, January 2000.
 35. W. U. Bajwa, A. Sayeed, and R. Nowak, *Sparse Multipath Channels: Modeling and Estimation*, 13th IEEE Digital Signal Processing Workshop and 5th IEEE Signal

Processing Education Workshop, pp. 320-325, January 2009.

36. A. C. Brooks, and S. J. Hoelzer, *Design and Simulation of Orthogonal Frequency Division Multiplexing (OFDM) Signaling: Final Report*, Electrical and Computer Engineering Department, Bradley University, pp. 1-50, May 2001.
37. K. V. S. Hari, et. al., *Channel Models for Fix Wireless Applications*, IEEE 802.16 Broadband Wireless Access Working Group, pp. 1-36, July 2001.
38. H. Steendam and M. Moeneclaey, *Sensitivity of orthogonal frequency division multiplexed systems to carrier and clock synchronization errors*, Signal Processing, Volume 80, pp. 1217-1229, July 2000.
39. H. Steendam, *The Effect of Timing Jitter on MC-DS-CDMA*, IEEE Transactions Communications, Volume 52, No. 3, pp. 467-472, March 2004.
40. U. Onunkwo, Y. Li, and A. Swami, *Effect of Timing Jitter on OFDM-Based UWB Systems*, IEEE Journal on Selected Areas in Communications, Volume 24, No. 4, pp. 787-793, April 2006.
41. L. Tomba, and W. A. Krzymien, *A model for the analysis of timing jitter In OFDM systems*, Proceedings of IEEE International Conference on Communications, Volume 3, pp. 1227-1231, June 1998.
42. L. Tomba, and W. A. Krzymien, *An analysis of the effect of chip timing jitter in MC-CDMA systems*, IEEE Transactions on Vehicular Technology, Volume 48, No. 6, pp. 1889-1898, November 1999.
43. T. L. Carroll, and L. M. Pecora, *synchronization in chaotic systems*, Physical Review letters, 64(8), pp. 821-825, February 1990.

-
44. K. Black, *Matlab Tutorial*, Department of Mathematics, Clarkson University.
 45. T. Pollet, P. Spruyt, and M. Moeneclaey, *the BER Performance of OFDM systems using non-synchronized sampling*, Globecom 1994, San Fransisco CA, Paper S7.6, pp. 253-257, November 1994.
 46. M. Moeneclaey, *the effect of synchronization errors on the performance of orthogonal frequency-division multiplexed (OFDM) systems*, Proc. COST 254, Emerging techniques for communication terminals, Toulouse, France, pp.41-45, July 1997.
 47. H. Steendam, and M. Moeneclaey, *the effect of carrier frequency offsets on downlink and uplink MC-DS-CDMA*, IEEE Journal on Selected Area in Communications, Volume 19, No. 12, pp. 2528-2536, December 2001.

Appendix: Complete Matlab codes

A1 OVSF code:

```
function [L,Code,R,C] = ovsf(L,index)
clc
if index==1
    j4=[1,1,1,1;1,-i,i,-1;1,i,-i,-1;1,-1,-1,1];
    I=eye(4);
    pr=I([1,4,3,2],:);
    pc=I([2,3,4,1],:);
    mj4=pr*j4*pc;
    cj4=mj4;
    cj2=cj4;

    if L>4
        temp=cj2;
        for k=1:L/8

            clear cj1;
            for l=1:2^(k+1);
                cj1=temp(l,:);
                cjt(2*l-1:2*l,:)=[cj1 cj1;cj1 -cj1];
                clear cj1
            end
            temp=cjt;
            clear cjt;
        end
        clear cj2;
        cj2=temp;
    end
    Code=cj2;

    for m=1:L;
        cj=cj2(m,:);
        for t=1:L;
            a=circshift(cj,[0 1-t]);
            r=cj.*conj(a);
            x=sum(r);
            y(t)=x;
        end
        R(m,:)=y;
    end
end
```

```

for m=1:L-1;
cj2a=cj2(m,:);
for n=(m+1):L;
cj2b=cj2(n,:);
    for t=1:L;
        b=circshift(cj2b,[0 1-t]);
        r=cj2a.*conj(b);
        x=sum(r);
        c(t)=x;
    end
    T(n,:)=c;
end
C(:,:,m)=T;
end
end

```

```

if index==2
    H=hadamard(L);
    for l=1:L;
        W(l,:)=H(l,:);
    end
    Code=W;

```

```

for m=1:L;
    w=W(m,:);
    for t=1:8;
        a=circshift(w,[0 t-1]);
        r=w.*conj(a);
        x=sum(r);
        y(t)=x;
    end
    R(m,:)=y;
end

```

```

for m=1:L-1
    wa=W(m,:);
    for n=(m+1):L
        wb=W(n,:);
        for t=1:8;
            b=circshift(wb,[0 1-t]);
            r=wa.*conj(b);
            x=sum(r);
            c(t)=x;
        end
        T(n,:)=c;
    end
end

```

```
end
C(:,:,m)=T;
end
end
```

A2 Simulation code of the MC-DS-CDMA system in AWGN channel:

```
close all;
clear
clc

usernumber=1;
[L,code,R,C] = ovsf(4,2);
W=[];
Ts=(1/528)*10^(-6);
N=16;;
Nsample=N*1;
a=0.5;
q=0.1*Ts;

nn=0:(L*Nsample-1);
auto=a.^abs(nn);
cv=toeplitz(auto);
ch=chol(cv);

ref_W=[];
ref_w=code(usernumber,:);
for l=1:L;
    ref_signal_w=repmat(ref_w(l), [1,(Nsample+Nsample/4)]);
    ref_W=[ref_W,ref_signal_w];
end
```

```

Num_total=1*10^2;
for Num=1:Num_total;
display (Num);

ran=randn(1,Nsample*L);

erro_m=ran*ch;
erro_r=randn(1,Nsample*L)*q/Ts;%white sampling jitter
erro_c=erro_m*q/Ts;%correlated sampling jitter
erro_i=zeros(1,Nsample*L);%no sampling jitter
erro=[erro_r;erro_c;erro_i];

for u=1:L

input=rand(1,N-4)>=0.5;
inputcode(u,:)=input;
mo = input*(-2)+1;

mo_f=zeros(1,N);
%mo_4=circshift(mo_3,[0 3]);
%mo_f=[mo_2,mo_4];
mo_f(:,2:(N-4)/2+1)=mo(:,1:(N-4)/2);
mo_f(:,(N-(N-4)/2+1):N)=mo(:,((N-4)/2+1):(N-4));

for tt=1:3
    for n_symbol=1:L
        index=(n_symbol-1)*Nsample;
    if tt~=3,
        for n=1:Nsample

```

```

    for k=1:N
        d(k)=mo_f(k)*exp(j*2*pi*(k-1)*((n-1)+erro(tt,index+n))/Nsample);
    end
    iff_sig(n)=sum(d);
end

else
    iff_sig= ifft(mo_f)*Nsample;%Inverse discrete Fourier transform%
end

ad=iff_sig(:,(Nsample-Nsample/4+1):Nsample);%get guard interval%
sign=[ad,iff_sig];%insert the guard interval%

index_lg=(n_symbol-1)*(Nsample+Nsample/4)+1;
index_rg=n_symbol*(Nsample+Nsample/4);

signal(1,index_lg:index_rg)=sign;
    end

w=code(u,:);
for l=1:L;
    signal_w= repmat(w(l), [1,(Nsample+Nsample/4)]);
    W=[W,signal_w];
end

out2=signal.*W;
user(u,:,tt)=out2;
W=[];
end
end

```

```

user_m=sum(user,1);
[m1,m2,m3]=size(user_m);

Ksnr=0;
DIFF=10*log10(N*L/(N-4));
for SNR=0-DIFF:3:27-DIFF
Ksnr=Ksnr+1;
EbN0(Ksnr)=SNR+DIFF;

Pawgn=(N-4)/10^((SNR)/10);
awgnx=randn(m2,1).*sqrt(Pawgn/2);
awgny=randn(m2,1).*sqrt(Pawgn/2);
Noise=complex(awgnx,awgny);

for tt=1:3;
user_mm=user_m(:,tt);
user_f=user_mm.';
awgn_sig=user_f+Noise;

s=awgn_sig.*conj(ref_W. ');

y0=reshape(s,(Nsample+Nsample/4),L);
y1=permute(y0,[2 1]);
y2=sum(y1); %returns sums along the same columns of an array%
y3=y2/L;
y4=y3(:,(Nsample/4+1):(Nsample+Nsample/4));
fft_sig=fft(y4);%Discrete Fourier transform%

del_1=fft_sig(:,2:((N-4)/2+1));
del_2=fft_sig(:,(N-(N-4)/2+1):N);

```

```

del_sig=[del_1,del_2];%remove the inefficient bits from data stream%
out=1-(real(del_sig)>=0);%pskdemod(del_sig,2);%bpsk demodulation%

if Num==1
    n_error(tt,Ksnr)=0;
end
n_error(tt,Ksnr)=n_error(tt,Ksnr)+sum(xor(inputcode(usernumber,:),out));
end
end
end

BER_r=n_error(1,:)/((N-4)*Num_total);
BER_c=n_error(2,:)/((N-4)*Num_total);
BER_i=n_error(3,:)/((N-4)*Num_total);
BER=0.5*erfc(sqrt(10.^(EbN0/10)));

semilogy(EbN0,BER,'bx:',EbN0,BER_r,'r*:',EbN0,BER_c,'ko:',EbN0,BER_i,'c+:');
axis([0 27 10^(-6) 10^(0)]);
xlabel('EbN0 (dB)');
ylabel('Bit Error Rate');
title(['N=',num2str(N),' L=',num2str(L),' a=',num2str(a), ]);

```

A3 Simulation code of the MC-DS-CDMA system in multi-path Rayleigh fading channel:

```

close all;
clear
clc

[L,code,R,C] = ovsf(4,1);

```

Nuser=L;%total number of users

usernumber=1;

Ts=(1/528)*10⁽⁻⁶⁾;

N=16;%16;

Nsample=N*1;

a=0.9;

% q=0.1*Ts;

q=0.1;

Qs=1;

nnn=0:(L*Nsample-1);

auto=a.^abs(nnn);

cv=toeplitz(auto);

ch=chol(cv);

Num_total=100;

data=zeros(Num_total,N);

datacoded=zeros(Num_total,N);

Gi=zeros(Num_total,N);

Ai=zeros(Num_total,N);

Bi=zeros(Num_total,N);

Ui=zeros(Num_total,N);

M=4;

G=sum(exp(-0.69*(0:M-1)));

KK=Num_total;

h1=complex(randn(M,KK),randn(M,KK)).*repmat(sqrt(exp(-0.69*(0:M-1)')),1,KK)/sqrt(2)/sqrt(G);

tmp=fft(h1,Nsample);

```

tmpp=permute(tmp,[2 1]);

Noise=complex(randn(Num_total,Nsample*L), randn(Num_total,Nsample*L));
Ni=zeros(Num_total,N);

for Num=1:Num_total;
display (Num);

ran=randn(1,Nsample*L);
erro_m=ran*ch;
erro_c=erro_m*q;

erro=reshape(erro_c,N,L);
erro=permute(erro,[2 1]);

Noise_t1=Noise(Num,:);
Noise_t2=reshape(Noise_t1,N,L);
Noise_t3=permute(Noise_t2,[2 1]);

input=zeros(L,N);
inputcode=zeros(L,N);

for u=1:L;
in=rand(1,N)>=0.5;
mo = in*(-2)+1;%pskmod(input,2);
input(u,:)=in;
inputcode(u,:)=mo;
end
data(Num,:)=input(usernumber,:);
datacoded(Num,:)=inputcode(usernumber,:);

```

```

for i=0:N-1;
    temp=0;
    for m=1:L;
        for n=0:N-1;
            temp=temp+exp(j*2*pi*i*erro(m,n+1)/N)*tmpp(Num,i+1);
        end
    end
    Gii(Num,i+1)=temp/N;

end

for i=0:N-1;
    temp=0;
    for k=0:N-1;
        if k==i;
            continue;
        else
            for m=1:L;
                for n=0:N-1;

temp=temp+inputcode(usernumber,k+1)*exp(j*2*pi*((k-i)*n+k*erro(m,n+1))/N)*tmpp(
Num,k+1);

                end
            end
        end
    end
    Aii(Num,i+1)=temp/N;
end

for i=0:N-1;
    temp=0;

```

```

for k=0:N-1;
    if k==i;
        continue;
    else
        for l=1:Nuser;
            if l==usernumber;
                continue;
            else
                for m=1:L;
                    for n=0:N-1;

temp=temp+inputcode(l,k+1)*code(l,m)*conj(code(usernumber,m))*exp(j*2*pi*((k-i)*n
+k*erro(m,n+1))/N)*tmpp(Num,k+1);

                        end
                    end
                end
            end
        end
    end
    Bii(Num,i+1)=temp/N;
end

%=====

%=====
=====
for i=0:N-1;
    temp=0;
    for l=1:Nuser;
        if l==usernumber;

```

```

        continue;
    else
        for m=1:L;
            for n=0:N-1;

temp=temp+inputcode(l,i+1)*code(l,m)*conj(code(usernumber,m))*exp(j*2*pi*i*erro(m
,n+1)/N)*tmpp(Num,i+1);
                end
            end
        end
    end
    Uii(Num,i+1)=temp/N;
end

for i=0:N-1;
    temp=0;
    for m=1:L;
        for n=0:N-1;

temp=temp+conj(code(usernumber,m))*Noise_t3(m,n+1)*exp(j*2*pi*i*n/N)*conj(tmpp
(Num,i+1));
            end
        end
        Nii(Num,i+1)=temp/N;
    end
end
Gi=Gii;
Ai=Aii;
Bi=Bii;
Ui=Uii;
Ni=Nii;

```

```

%=====calculate variance of Ai, Bi, Ui for Method 4=====
Var_Ai=zeros(1,N);
Var_Bi=zeros(1,N);
Var_Ui=zeros(1,N);
for i=0:N-1,
    temp=0;
    for k=0:N-1
        if k==i
            continue
        else
            for m=1:L
                for mm=1:L
                    for n=0:N-1
                        for nn=0:N-1
                            d=abs((m-mm)*N+n-nn);
                            temp=temp+exp(-4*pi^2/N^2*k^2*q^2*(1-a^d))*exp(j*2*pi/N*(k-i)*(n-nn));
                        end
                    end
                end
            end
        end
    end
    Var_Ai(i+1)=Var_Ai(i+1)+real(temp)/N^2;

    temp=0;
    for l=1:Nuser
        if l==usernumber
            continue
        else

```

```

        for mm=1:L
            temp1=0;
            for n=0:N-1
                for nn=0:N-1
                    d=abs((m-mm)*N+(n-nn));
                    temp1=temp1+exp(-4*pi^2/N^2*i^2*q^2*(1-a^d));
                end
            end
        end

temp=temp+temp1*code(l,m)*code(l,mm)*conj(code(usernumber,m))*conj(code(usernumber,mm));

        end
    end
end
Var_Ui(i+1)=Var_Ui(i+1)+temp/N^2;

end

%=====

%==calculate variance of Ai, Bi, Ui for Method 5=====
%=====

for i=0:N-1;
    %DD2=0;
    for k=0:N-1;

        d21=0;
        d22=0;
        d23=0;
        d24=0;

```

```

if k==i;
    continue;
else
    for m=1:L
        for mm=1:L
            for n=0:N-1;
                for nn=0:N-1
                    d1=(m-mm);
                    d2=(n-nn);
                    if m==mm && n==nn;
                        d21=d21+L*Qs^2;
                        % D21=[D21,d21];
                    elseif m==mm && n~=nn
                        %nnn=n-nn;

d22=d22+L*(Qs^2/N^2)*(N-abs(d2))*cos(2*pi*(k-i)*abs(d2)/N)*exp(-4*pi^2*k^2*q^2
*(1-a^abs(d2))/N^2);

                    elseif m~=mm && n==nn;
                        %mmm=m-mm;

d23=d23+(Qs^2/N)*(L-abs(d1))*exp(-4*pi^2*k^2*q^2*(1-a^abs(d1*N))/N^2);

                    elseif m~=mm && n~=nn;

d24=d24+(Qs^2/N^2)*(N-abs(d2))*(L-abs(d1))*cos(2*pi*(k-i)*abs(d2)/N)*exp(-4*pi^2
*k^2*q^2*(1-a^abs(N*d1+d2))/N^2);

                    end
                    DD2=(d21+d22+d23+d24);
                end
            end
        end
    end
end

```

```

                                %DD2=d21+d22+d23+d24;
                                end
                                end
                                end
                                end

Di2(i+1)=DD2/(N^2);

for l=1:Nuser
    if l==usernumber
        continue
    else
        for k=0:N-1;
            if k==i;
                continue;
            else
                for m=1:L;
                    for mm=1:L
                        d31=0;
                        d32=0;
                        d33=0;
                        d34=0;
                        for n=0:N-1;
                            for nn=0:N-1
                                d1=(m-mm);
                                d2=(n-nn);
                                if m==mm && n==nn;
                                    d31=d31+L*Qs^2*Nuser;
                                elseif m==mm && n~=nn
                                    d32=d32+L*(Qs^2/N^2)*(N-abs(d2))*cos(2*pi*(k-i)*abs(d2)/N)*exp(-4*pi^2*k^2*q^2

```

```

*(1-a^abs(d2))/N^2);
                                elseif m~=mm && n~=nn;

cd=code(l,m)*code(l,mm)*conj(code(usernumber,m))*conj(code(usernumber,mm));

d33=d33+(Qs^2/N)*(L-abs(d1))*cd*exp(-4*pi^2*k^2*q^2*(1-a^abs(N*d1))/N^2);
                                elseif m~=mm && n~=nn;

cd=code(l,m)*code(l,mm)*conj(code(usernumber,m))*conj(code(usernumber,mm));

d34=d34+(Qs^2/N^2)*(N-abs(d2))*(L-abs(d1))*cos(2*pi*(k-i)*abs(d2)/N)*cd*exp(-4*pi
i^2*k^2*q^2*(1-a^abs(N*d1+d2))/N^2);

                                end
                                end
                                end
                                DD3=(d31+d32+d33+d34);
                                end
                                end
                                end
                                end
                                end
                                end
                                end

Di3(i+1)=DD3/N^2;
%=====
%=====

for l=1:Nuser
    if l==usernumber
        continue
    else

```

```

for m=1:L;
    for mm=1:L
        d41=0;
        d42=0;
        d43=0;
        d44=0;
        for n=0:N-1;
            for nn=0:N-1
                d1=(m-mm);
                d2=(n-nn);
                if m==mm && n==nn;
                    d41=d41+L*Qs^2*Nuser/N;
                elseif m==mm && n~=nn
                    %nnn=n-nn;

d42=d42+L*(Qs^2/N^2)*(N-abs(d2))*exp(-4*pi^2*i^2*q^2*(1-a^abs(d2))/N^2);
                    elseif m~=mm && n==nn;

cd=code(l,m)*code(l,mm)*conj(code(usernumber,m))*conj(code(usernumber,mm));

d43=d43+(Qs^2/N)*(L-abs(d1))*cd*exp(-4*pi^2*i^2*q^2*(1-a^abs(N*d1))/N^2);
                    elseif m~=mm && n~=nn;
                        %mmm=m-mm;
                        %nnn=n-nn;

cd=code(l,m)*code(l,mm)*conj(code(usernumber,m))*conj(code(usernumber,mm));

d44=d44+(Qs^2/N^2)*(N-abs(d2))*(L-abs(d1))*cd*exp(-4*pi^2*i^2*q^2*(1-a^abs(N*d
1+d2))/N^2);
            end
        end
    end
end

```

```

                                end
                                DD4=d41+d42+d43+d44;
                            end
                        end
                    end
                end

            end

            Di4(i+1)=DD4/(N^2);
        end

%===calculate variance of Ai, Bi, Ui for Method 5=====
Ksnr=0;
Z=zeros(Num_total,N);

DIFF=10*log10(L);
for SNR=0-DIFF:3:27-DIFF
    Ksnr=Ksnr+1;
    EbN0(Ksnr)=SNR+DIFF;
    Pawgn=N/10^((SNR)/10);
    Z=real((Gi.*datacoded+Ai+Bi+Ui).*conj(tmpp)+Ni*sqrt(Pawgn/2));
    out=1-(Z>=0);%bpsk demodulation%
    error_bits=xor(data,out);
    BERs_M1(Ksnr,:)=sum(error_bits)/Num_total;%BER of each subcarrier
    BER_M1(Ksnr)=sum(BERs_M1(Ksnr,:))/N;%overall BER

%Method 2: Monte Carlo simulation with AWGN variance calculated from
simulation=====
Z_var2=mean(real((Ai+Bi+Ui).*conj(tmpp)+Ni*sqrt(Pawgn/2)).^2);
ber_conditional2=erfc(real(Gi.*conj(tmpp))./(repmat(real(sqrt(Z_var2*2)),Num_total,1).
*(abs(tmpp)))))*0.5;

```

```

BERs_M2(Ksnr,:)=sum(ber_conditional2)/Num_total;%BER of each subcarrier
BER_M2(Ksnr)=sum(BERs_M2(Ksnr,:))/N;%overall BER

%====Method 3: Monte Carlo simulation with AWGN variance calculated from
SNR setting====%
Z_var3=mean(real((Ai+Bi+Ui).*conj(tmpp)).^2)+Pawgn/2*(L/N);
ber_conditional3=erfc(real(Gi.*conj(tmpp))./(repmat(real(sqrt(Z_var3*2)),Num_total,1).
*(abs(tmpp))))*0.5;
BERs_M3(Ksnr,:)=sum(ber_conditional3)/Num_total;
BER_M3(Ksnr)=sum(BERs_M3(Ksnr,:))/N;

%====Method 4: Monte Carlo simulation with all interference variance calculated
from formula====%
Z_var4=(Var_Ai+Var_Bi+Var_Ui)/2.*var(tmpp).*var(conj(tmpp))+Pawgn/2*(L/N);
ber_conditional4=erfc(real(Gi.*conj(tmpp))./(repmat(real(sqrt(Z_var4*2)),Num_total,1).
*(abs(tmpp))))*0.5;
BERs_M4(Ksnr,:)=sum(ber_conditional4)/Num_total;
BER_M4(Ksnr)=sum(BERs_M4(Ksnr,:))/N;

%====Method 5: ====%
Z_var5=(Di2+Di3+Di4)/2.*var(tmpp).*var(conj(tmpp))+Pawgn/2*(L/N);
ber_conditional5=erfc(real(Gi.*conj(tmpp))./(repmat(real(sqrt(Z_var5*2)),Num_total,1).
*(abs(tmpp))))*0.5;
BERs_M5(Ksnr,:)=sum(ber_conditional5)/Num_total;
BER_M5(Ksnr)=sum(BERs_M5(Ksnr,:))/N; end
BER=0.5*erfc(sqrt(10.^(EbN0/10)));

semilogy(EbN0,BER,'k',EbN0,BER_M1,'rx:',EbN0,BER_M2,'b*:',EbN0,BER_M3,'g+:',
EbN0,BER_M4,'mo:',EbN0,BER_M5,'c-');
axis([0 27 10^(-6) 10^(0)]);
xlabel('EbN0 (dB)');

```

```
ylabel('Bit Error Rate');  
title(['N=',num2str(N),' L=',num2str(L),' a=',num2str(a),' q=',num2str(q)]);
```