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**THE DISSOLUTION OF
ALPHA LACTOSE MONOHYDRATE.**

**A MATHEMATICAL MODEL FOR PREDICTING
DISSOLUTION TIMES.**

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of Masters in
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Abstract

Commercially available lactose is generally available in the form of alpha lactose monohydrate. The dissolution of this is slow in comparison to other sugars due to lactose existing in solution as alpha and beta lactose. When alpha lactose monohydrate has dissolved up to the alpha lactose solubility limit, no more can dissolve until some of the alpha lactose in solution has mutarotated to beta lactose. This makes mutarotation and the solubility of alpha lactose the two main limiting factors in lactose dissolution.

A variety of factors can affect the mutarotation rate and solubility of lactose. Temperature and pH affect both significantly. The effect of carbohydrates in solution on the solubility of alpha lactose is particularly important as it has been found that beta lactose inhibits the solubility of alpha lactose.

Dissolution of alpha lactose may be considered to consist of three steps; surface disintegration from the crystal, dissolution into the bulk of solution, and mutarotation of alpha to beta in solution. Attempts to model this process have previously been unsuccessful.

For dissolution to total lactose concentrations above the alpha lactose solubility limit, it was found that the effect of beta lactose on the solubility of alpha lactose had a significant effect on the dissolution of lactose. For dissolution to total lactose concentrations below the alpha lactose solubility limit, it was found that the surface disintegration reaction was significant, particularly at low temperatures.

A model was developed for predicting the dissolution of alpha lactose. This was found to compare well to experimental results for both single size and mixed crystal size lactose. The model worked well in dissolutions with excess lactose, dissolution to total lactose concentrations above and dissolutions below the alpha lactose solubility limit.

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