Awareness of Learning in the Mathematics Classroom

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ABSTRACT

The thesis reports on a teacher research project, involving a Form 3 class in a New Zealand secondary school. The study considers the importance of metacognitive behaviours in developing students' awareness of learning in mathematics. It focuses on the teacher in the classroom emphasising awareness of learning with students.

The theoretical basis of the New Zealand Mathematics curriculum, that is, constructivism and its corollary active learning, provides the impetus for the study. Classroom activities, both routine and those specifically tailored for such an investigation, are trialled. In the process, shifts and developments in the students' and teacher's knowledge and beliefs, are documented. Methods of teaching are explored and evaluated in the move towards constructivist teaching practice.

Although teacher research is a relatively new and accepted methodology, it derives from Dewey (1933) and Schon's (1983) work on reflective practice. Using the more established action research methodology as a scaffold this thesis found the open teacher research style suited the sole researcher nature of this work. Within the process of critical reflection this study of mathematics classroom practice exposes the conflicts faced when beliefs and attitudes of both students and teacher are sometimes inconsistent with those inherent in the curriculum guidelines. It also documents some of the difficulties in sustaining teacher research while coping with daily teacher class loads.

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CHAPTER ONE

Introduction

1.1 Background

Mathematics teaching involves the teacher thinking about mathematics, the mathematics of the students and the students themselves. The teacher acts as a catalyst to introduce new ideas and foster the learning of mathematics. Students build on their conceptions of mathematics by meeting new ideas which they think about, discuss and share with the teacher and their friends. The teacher models the style of learning she believes is valid and encourages students to value their learning opportunities. She also engages in an ongoing process of decision making to balance curriculum requirements with the personalities and lives of a group of individuals. Her teaching involves interactions with students within a mathematical context. Ideally the classroom is an environment which promotes thinking, discussion and enjoyment of mathematics because the learning of it is exciting and empowering.

The move in mathematical pedagogy towards developing students' power to think, reason and problem solve is a move away from traditional instruction which handed down knowledge to be learned passively as a set of academic skills. This shift is a result of increased knowledge and changing theories about how children learn. These theories have been directly influenced by a new direction in thought about the nature of mathematical knowledge. From the formalist tradition, which considered mathematics to be a discipline of truths and certainty, the conception of mathematics has changed to that of a changing and growing body of knowledge; mathematics is seen as a way of knowing and interpreting our experience. Mathematical knowledge is the result of social and cultural factors (Nickson, 1992). This recent view of mathematical knowledge is also in line with the demands of a technological society in which all forms of activity are mathematised in some way, requiring people to become more active, reflective and meaningful mathematicians (Crawford & Adler, 1996).

Mathematics educators have realised that teaching and instruction has to be appropriate to how people learn. Modern learning theory is based on constructivism. The constructivist view of learning holds that learning is an active process, building on prior knowledge, and making connections between new ideas and those already held (Begg, 1996). The New Zealand Curriculum Framework (1993) stresses that a mathematics education should provide opportunities for students to be inquiring, creative, resourceful, self-reliant and persevering. This emphasis on effective active learning is also promoted in *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992). Active learning involves the learner thinking about and monitoring the learning process as well as being involved in the learning.

The step from cognition to thinking about cognition has been defined as metacognition (Flavell, 1981) and the need to develop metacognitive strategies in the classroom is increasingly recognised as part of the active learning process (Anthony, 1996a). As such, the classroom is viewed as a place where one learns how to learn (Mitchell, 1992). Specifically in mathematics, students are expected to develop the ability to reflect critically, use mathematics to explore and conjecture, and develop the characteristics of logical and systematic thinking (Ministry of Education, 1992).

Studies on classroom based experiences promoting active learning are few and there is a need for teachers' perspectives on this issue. Teacher research can add light to how teachers and learners experience the process of shifting away from traditional classroom instruction towards active learning styles.

1.2 Changes in Teaching and Learning

The New Zealand Curriculum Framework (1993) acknowledges new emphases in learning and new challenges for teachers. Research shows that the changes proposed in the curriculum are far from being fully implemented; "the connection is tenuous at best", and "the extent to which the new approaches to teaching and assessment will change teacher behaviour have still to be demonstrated" (Garden, 1996, p. 239). To respond to the new directions teachers need to acknowledge that changes in learning will follow from changes in teaching. Mathematics achievement is related to the effectiveness of mathematics teaching. The teacher influences what is done in the classroom guided by curriculum outlines.

Reflecting on teaching and learning leads to changes in teaching practice and should be the basis for teachers' decision making (National Council of Teachers of Mathematics, 1998). The reflective teacher works towards setting up a safe stimulating environment for students which focuses on learning with understanding as the major concern. Mathematics teaching needs to move away from being *classroom-focused*, based on knowledge about effective classrooms, or *content-focused with an emphasis on performance*, based on mastery of mathematical rules and procedures, to being *learner focused*, based on the learner's personal construction of knowledge (Kuhs & Ball, 1986). The quality of the teacher's instruction depends on many factors; not least her knowledge of content, pedagogy, student learning and her beliefs about teaching and mathematics. Through reflection on teaching and learning the teacher develops these factors and improves the learning environment for her students. Good classroom practice engages students in meaningful and purposeful activity. Students perceive activity as such when they understand what they are doing and why they are doing it.

Learning mathematics is supported by encouragement to be an active mathematical thinker and being part of a class that acts as an intellectual community (Ministry of Education, 1995). An active learner will use common sense, be intuitive and imaginative, ask questions, explain solutions, and take risks (Bock, 1994). The culture of the classroom needs to support thinking, arguing and expression of ideas. By emphasising an active learning process with the use of metacognitive skills, it is believed that learning and teaching will be enhanced. This study will focus on changing teacher practice to encourage the use and development of metacognitive skills within the classroom, through teacher research based loosely on the action research model.

1.3 Research Objective

This project examines how changing teacher practice to encourage the use and development of students' metacognitive skills and knowledge in mathematics impacts on the learning that happens in the classroom. It considers the process of change from both the teacher and student perspectives. In order to change practice and increase students' and teacher's metacognitive awareness, as the teacher researcher, I will address three questions:

- What attitudes and beliefs do students hold in relation to learning mathematics?
- What metacognitive behaviours do students exhibit, in typical classroom exercises, and in specific classroom interventions designed to increase awareness of learning?
- How can everyday classroom routines, e.g., writing notes, reviewing previous work, be adapted to develop students' awareness of learning and my own awareness of their learning?

1.4 Overview

Chapter Two reviews the literature on learning and teaching mathematics in order to provide the background for this study. It summarises findings on related issues of constructivism, active learning, learning strategies and the reflective teacher.

In Chapter Three the methodology for this study is discussed with details of setting and data collection included.

Chapter Four reports the results of the study and in Chapter Five the implications of the study for teachers and reflective practice are discussed. It draws the findings from both the students' and teacher's perspectives together and makes conclusions. Directions for further research are suggested.

CHAPTER TWO

Literature Review: Theoretical background

2.0 Introduction

Interest in what distinguishes 'good' learning and teaching has motivated much research in mathematics education. Current research is largely based on the constructivist view of learning mathematics with its corollary, active student learning. The constructivist principle not only requires the learner to be active; it also demands the teacher takes a reflective stance. These responsibilities lead to a focus on strategies which facilitate learning. There has been a growing realisation that concentrating on cognitive strategies alone is no guarantee of successful teaching and learning. The metacognitive skills of reflecting, planning, monitoring and evaluating are also essential for learning. Literature supporting these issues is discussed in the following order:

1. Constructivism
2. Active Learning
3. Learning Strategies
4. Metacognitive Strategies
5. Reflective Teacher
6. Teacher as Researcher
7. Classroom Studies relevant to this project.

2.1 Constructivism in the Classroom

The constructivist stance holds that mathematics is constructed from prior knowledge and experience and negotiated with others who have similar or different levels of knowledge (Balacheff, 1991). According to von Glaserfeld (1991) constructivism is based on two principles. The first principle states that knowledge is not received passively but is built up by the learner. This means that it is not possible to simply transfer knowledge from someone who knows to someone who doesn't. The second principle states that the function of cognition is adaptive and enables the learner to construct viable explanations of experience. This means that knowledge about the "world outside" is seen as a human construction. The two principles imply learning is a human activity that occurs in a cultural context (Crawford, 1994).

While constructivism is not a teaching theory but a theory about knowledge and how people learn, it does however have implications for teachers. Noddings (1990) advises that in accepting the constructivist position teachers are consistent only in so far as they adopt teaching methods that reinforce this belief. She links the adoption of the constructivist position with the establishment of the constructivist classroom in which the teacher needs to know what and how the students think. Begg (1996) lists the implications for teachers as follows:

> •*Knowledge construction is personal and provisional, it works towards socially negotiated understandings (accepted truth) by providing a viable fit between each learner's ideas and their experiences.*
> •*The schema concept implies that making connections is important in learning, in particular it emphasises the importance of prior learning and shows how learning is both culture and language dependent.*
> •*Social interaction, communication and language are valued within the learning process.*
> •*The teacher will find it useful to know what is going on in the learner's head.*
> •*Teacher's subject knowledge is important.*
>
> (Begg, 1996, p. 8)

Mathematics in the New Zealand Curriculum states that "as new experiences cause students to refine their existing knowledge and ideas, so they construct new knowledge" (Ministry of Education, 1992, p. 12). The centrality of the constructivist position means teachers need to ensure students are provided with appropriate experiences. Experiences are appropriate in the sense that they assist students reveal and examine their prior understandings and assumptions, and consider the sense of their own and the teacher's assertions (Lampert, 1991). Lampert argues the teacher must choose and use "good problems" and institute classroom communication that enables students to understand both the problem and the solution strategies. By starting with familiar problems and listening to students' reasoning the teacher can introduce new problems, less familiar to students, that will test their assumptions and assertions. The language used in the classroom must be a deliberate pedagogical focus; it should be the teacher's intention that students learn to "talk mathematics" (Lampert, 1998).

Pirie and Kieren (1992) give four guidelines for teachers when considering students' mathematical understandings:

- *there are different levels of understanding shown by different students.*
- *there are differently pathways to similar mathematical understanding which may involve a variety of instructional acts.*
- *different people hold different mathematical understanding.*
- *students act to develop their own unique understanding.*

(Pirie and Kieren, 1992, pp. 507-508)

The most effective way for a teacher to create a constructivist environment is through showing these beliefs in action. The belief that there is no absolute mathematical understanding to be acquired by students is central to allowing for, and valuing, the students' unique understandings.

In the teaching and learning of mathematics decisions have to be made as to what "mathematics" is to be learned and how the learning, or coming to know, is to be achieved (Lampert, 1985). Cobb, Wood and Yackel (1990) noted how practising teachers do not use formal models in their interactions with students but operate pragmatically and instinctively, interacting with students and making on the spot decisions. The nature of teaching is complicated by the need for the teacher to transform the proposed curriculum into the enacted curriculum while simultaneously dealing with students' cultural differences, diversity of life experiences and the knowledge and beliefs of teacher and students (Senger, 1999). Alongside theoretical considerations the teacher must also balance practical considerations to manage a successful classroom. The better the knowledge of the subject matter and the deeper the understanding of how it is learned, the more the teacher is freed to control this balance. Experience is beneficial in the mathematics classroom.

The constructivist approach to teaching can cause something of a dilemma to the teacher in deciding how to intervene to help students learn. diSessa and Minstrell (1998) call this the "dance of ownership" and recommend that the first strategy of teacher intervention is to "let things run" so that students debate and reflect on aspects of the lesson. Student reflection enhances students' ability to construct knowledge from the activities they do. Teachers must build in time for student reflection so that they learn to use appropriate and effective strategies. Anthony (1996b) mentions the need for teachers to provide an instructional environment that allows students to use strategies to learn. Opportunities must be given to students to participate effectively in lessons. Students need time to clarify what the lesson is about; they need to develop control and autonomy over skills such as summarising, planning and revision; they need to feel full and active members of the classroom community. Knowing when and how to provide appropriate "scaffolding" or guidance is a challenge to the constructivist teacher. *Mathematics in the New Zealand Curriculum* points out "the extent to which teachers are able to facilitate this process [constructing new knowledge] significantly affects how well students learn" (Ministry of Education, 1992, p. 12).

Constructivism, like most belief systems, involves commitment and understanding by the teacher. It is implemented effectively only when "teachers' traditional beliefs about transmission approaches to learning and absorptionist views of learning are challenged" (Treagust, Duit & Fraser, 1996, p. 5). Cobb, Wood and Yackel (1991) claim it is valuable only in so far as it contributes to the learning of mathematics. If it does not then it will become irrelevant.

2.2 Active Learning

Modern theories of learning hold that students learn by acting on new information to construct meaning for themselves. Students do not learn merely by attending class; they need to think about and process the ideas they meet (Begg, 1996). Unless learners makes a deliberate attempt to incorporate new information into their memory or schema it will not be retained for later use. There are two major views on types of learning. Active learning is achieved by being involved in experimenting, formulating hypotheses and testing and validating these through learning activities and social experiences (Begg, 1996). The opposite type of learning is passive learning where it is hoped learning is passed from teacher to student through conversation and lecturing. Active learning is a direct implication of the constructivist position and is supported in *Mathematics in the New Zealand Curriculum*: "Mathematics is most effectively learned through students' active participation in mathematical situations" (Ministry of Education, 1992, p. 18). This position is similar to that of other national documents such as the American *Principles and Standards for School Mathematics: Discussion Draft* which argues that the "student understanding of mathematical ideas builds as a result of carefully designed experiences of active engagement with mathematics content" (NCTM, 1998, p. 36).

Perkins (1991) claims constructivism makes several demands on the learner which all relate to the notion of the active learner. He outlines three ways demands are made. Firstly, learners have to face the conflicts that arise when their incomplete models are challenged with new, maybe unfamiliar, models. Secondly, they have to manage their learning more effectively than in traditional situations. The props like the teacher, the directions and the overall nature of the learning tasks ask for more student involvement than conventional instruction. Finally, learners have to "buy in" to the type of learning asked of them. Some students might prefer to be told what to learn (by the 'expert', i.e. teacher) and then asked to practise it. This is much less bother to them than having to engage in the thinking required to assimilate new concepts and incorporate them in their schema. Perkins recommends teachers operate a double agenda and "engage students constructively in thinking both about X and the learning process reflectively" (p. 20).

However, it is not sufficient to expect that a student 'doing' mathematics is necessarily learning the mathematics. An active learner has to engage appropriate strategies to enhance knowledge construction, not just complete tasks. Students need to connect with the learning process by actively choosing to participate in and contribute to the lessons. They need to develop expertise in how to learn through understanding and applying cognitive and metacognitive strategies (Anthony, 1996a).

Furthermore, the teacher has to be aware of the use of these strategies by students. Teachers can help students by alerting them to helpful strategies and observing their use of them. Unless the teacher is aware of both the cognitive and metacognitive side of learning she may not help students who do not automatically adopt correct strategic learning processes. Students can learn and apply mathematical strategies, in the form of routines, without necessarily having an underlying understanding of the problem. By failing to recognise the problem they often use inappropriate and inefficient strategies. They may not incorporate the strategies properly into their schemes of learning because they have not assimilated them into their prior knowledge (Anthony, 1996a). By helping students understand the need to construct, rather than reproduce, knowledge teachers can alter and advance the learning of their students.

It is also essential to understand students' mathematics to help them reorganise their schema to accommodate new knowledge. Teaching mathematics is not merely following a set plan that can be applied to any group. In line with constructivist principles the teacher constantly evaluates the learning taking place and sets up environments to improve it.

Teachers have a responsibility to create a classroom atmosphere in which students feel they are party to the creation of interesting and challenging mathematics. To do this they need to understand the pedagogy of learning mathematics, in both the conceptual and practical senses (Lampert, 1998). The issue of students who are unwilling or unable to engage in active learning at the level the teacher feels is most appropriate to the learning needs of the class is significant. The teacher has to work with the circumstances created by the class as a group and also the class as a collection of individuals.

2.3 Learning Strategies

Mathematical knowledge learned with understanding is powerful, because this knowledge can be applied to new topics and problems (Carpenter & Lehrer, 1999). If mathematical knowledge does not have wider application than the classroom it will be of little use to the individual student or the wider society. Understanding mathematics develops through constructing personal knowledge. With suitable learning strategies students can influence the knowledge they acquire (Anthony, 1996a).

Flavell (1979) separated strategies into three groupings; cognitive, metacognitive and social. Cognitive strategies help the learners improve their understanding and retention of new knowledge. They are used to learn and apply knowledge. Cognitive strategies can be classified as rehearsal, elaboration and organisation. Cognitive strategies include rehearsal (practice and revision), elaboration (making links with prior knowledge), and organising the knowledge (Weinstein & Mayer, 1986). Pressley and Harris (1990) advocate techniques such as summary, imagery, prior knowledge activation, self-questioning, and question-answering as suitable activities.

Metacognitive strategies are used to direct and evaluate the learning process. They help the learners by making them aware of their learning and how to control it. Metacognitive strategies include identifying goals, self-monitoring, checking and evaluating work, planning and anticipating how to use learning resources, reflecting, and learning from mistakes (Anthony, 1996a). They are the deciding factors in using cognitive strategies and include reflection, planning, monitoring and evaluating. Students who work metacognitively show an awareness of the nature of mathematics and how they learn it (Bell, 1993). Specific metacognitive behaviours that are used in the mathematics classroom, include checking progress, planning and anticipating, reflecting on the work, setting goals, linking the work to beliefs and experiences, and assuming a position (Anthony, 1995; Baird & Northfield, 1992).

Social strategies are also used to control the learning process. They are based around the opportunity to learn and include interaction with peers, teacher and task. Anthony (1994) also includes resource management strategies such as seating arrangements, discussion with peers and teacher, and modifying the task demands.

2.4 Metacognition

Metacognition is the thinking behind the 'doing'. Flavell (1979) defined it as knowledge of cognitive processes as well as controlling, monitoring and evaluating the use of them. He divided metacognitive knowledge into three aspects; knowledge of personal characteristics, task knowledge and strategy knowledge. The personal characteristics are knowledge about oneself as a thinker and one's beliefs, about others as thinkers and about universally held assumptions about the nature of human thinking. Task knowledge is the understanding and appreciation of the level of complexity of the task. Strategy knowledge is the knowledge of strategies necessary to do the task. Metacognitive learners are aware of their learning, their beliefs about learning, and the strategies they use to learn. They realise they have methods at their disposal to facilitate learning and that different methods suit different situations, so they are discerning between them. In this way students purposely act to change and develop their understanding, and are consequently better learners.

Teaching students to become metacognitive learners means the teacher also acts in reflective ways to support a thinking environment (Graeber, 1991). Manning and Payne (1996) claim that teachers must operate metacognitively themselves before they can realistically expect students to develop these skills. The teacher is involved in learning as much as the students because the teacher is thinking about the nature of learning, as well as the nature of the content, in deeper ways than before (Baird & Northfield, 1992).

Teaching strategies is too simplistic a view of metacognitive teaching; strategies need to be constantly modelled, discussed and promoted. They need to be made explicit so there is a duality of awareness between knowing the problem to be solved and the methods employed to solve it. The teacher adopts a similar teaching approach to that outlined by Lester (1989) whereby there are three roles for the teacher: that of monitor, facilitator and model. The teacher-monitor directs discussion about the problem, leads students to find ways to understand the problem, and observes and guides students in their solution attempts. The teacher-facilitator asks questions and devises assignments that require students to analyse their mathematical performance. The teacher-model questions, explains, decides and acts to solve problems as a clear exemplar of problem-solving behaviour.

To improve learning and teach in a metacognitive style there are research based principles, such as those offered by Angelo (1993), which teachers can follow. These include active learning methods, focusing students' attention on awareness of their learning, making goals explicit to students, helping students make links and connections to prior knowledge, encouraging students to be organised in their learning, giving students feedback on their learning, assessing students' learning in ways that help students focus their attention on important learning, acknowledging that learning demands time and effort on the part of the student, having high expectations of students, and encouraging meaningful communication between students and students, and between teachers and students. The teacher, guided by the above principles, also must be aware of the need for students to be independent and active learners. Students learn most effectively when they take control of their own learning.

The emphasis for the metacognitive teacher is summed up well by S. Cannard in Baird and Northfield (1992):

> *It (metacognitive teaching) does not just deal with teaching methods and techniques - it is a whole new way of thinking about teaching, and classroom management. You begin to ask yourself these questions:*
>
> - *What are your **values** in a classroom? What is important to you?*
> - *How am I going to foster these values?*
> - *How much do you sacrifice from the course to teach the students techniques to not only understand, but to learn.*
>
> (S. Cannard, teacher, quoted in Baird and Northfield, 1992, p. 9)

The teacher merely facilitates the process of student learning through working with the above principles and encouraging students to use effective cognitive and metacognitive strategies. Metacognitive strategies are most effective when they are self-regulated; externally imposed activities are less likely to promote long-lasting learning (Wagner & Sternberg, 1984). Students' own strategies and questions will always be best for them and their need to develop skills in formulating these is a high priority in the classroom. In mathematics students frequently mismatch methods to problems. Their self-regulatory practices often need improving so they remember to check if they are on track, and if their methods and solutions make sense in terms of the problem. Many students either persevere too long with incorrect and/or incomplete methods, or they give up on problems too soon without reflecting on what was good and bad about their initial attempts.

Specific metacognitive strategies can also be promoted with activities. The teacher can use questions in class which remind students to self-monitor, such as, "Is this working, or should we try something else?", "What are you doing and why?" and "What are we trying to find out?". The teacher can show models of incorrect problem-solving and ask students to find the inconsistencies and correct them. The activities can be prescriptive activities, such as reflection tasks, or they can be discussion arising out of contexts. In this study both types will be trialled within the context of the classroom.

2.5 Reflective Teacher

Reforms, based on constructivist principles, acknowledge the need for changes in teaching. The New Zealand Curriculum Framework (Ministry of Education, 1993) recognises new emphases in learning bring new challenges for teachers. To respond to the directions teachers need to acknowledge that changes in learning follow from changes in teaching, because teachers' knowledge and beliefs influence classroom instruction and students' learning (Carpenter & Fennema, 1991; Steffe, 1990; Thompson, 1992). However, such changes will only come through reflection on teaching, and changes in teachers' pedagogical content knowledge and beliefs about learning and teaching (NCTM, 1991).

Despite reforms and research findings which signal changes in direction as important for mathematics teaching and learning, teachers have been slow to change (Garden, 1996; Hiebert, 1999; Steffe, 1990). There is an obvious gap between theory and classroom practice as indicated by the fact that new teaching methods are only gradually being implemented in the classroom as teachers understand and appreciate new directions in mathematical education. Even though research provides evidence of improvements in students' learning and achievement with methods that emphasise active learning; such as group work, problem solving, investigation and use of technology, teachers are still hesitant to incorporate these approaches. This could be because change in teaching usually requires taking on a new philosophy and pedagogy which may be contrary to previous educational experiences and training (Mousley, 1992). It could also be because teachers are not always aware of, or have good access to, the research available (Senger, 1999). And, when they do know about it, they can feel separated from it, in the sense that it appears theoretical, and unrelated to the practicalities of running a classroom. Change is most effective when it is motivated and initiated by the agent of the change, who in this case is the teacher.

Dewey (1933), Schon (1983) and others indicated that classroom change goes deeper than changing practice. There must be change of philosophy and intent before change in practice is effective. Dewey sees intended action following a thinking process as true reflection. 'Active, persistent, and careful consideration of any belief or supposed form of knowledge in light of the grounds that support it and the further conclusions to which it tends constitutes reflective thought' (1933, p. 9). Schon defined teaching as an activity in which 'reflection in action' can lead to experimentation which 'generates both a new understanding and a change in the situation' (1983, p. 68). The reflection motivates the action which is different to previous action because of the prior reflection. The experimental phase of teacher change is described by Senger (1999), who outlines a process of teacher change that involves moving from states of awareness (of new knowledge), to mental imaging (reflection), to experimenting, to being convinced, to finally changing. One major problem to teachers undergoing this change is that it is based on a view of mathematics as a socially constructed knowledge, with the classroom needing to reflect this as a miniature community in which students engage in collaborative mathematical practice. Teachers generally have little experience of being participants in such communities themselves, and they do not always have the confidence in their knowledge of subject content and pedagogy to react readily to students' mathematics. In addition, many teachers have a narrow, static view of mathematics, which is absolutist, in line with the formalist tradition, based on rules and procedures (Steffe, 1990; Thompson, 1992). Until teachers appreciate the nature of the subject of mathematics as open to enquiry and creation they are unlikely to make major shifts in their teaching.

There are five major shifts needed in mathematics classrooms described in the Professional Standards for Teaching Mathematics (NCTM, 1991):

- toward classrooms as mathematical communities
- toward logic and mathematical evidence as verification
- toward mathematical reasoning
- toward conjecturing, inventing and problem-solving
- toward connecting mathematics, its ideas, and its applications.

(NCTM Professional Standards for Teaching Mathematics, 1991, p. 3)

These shifts require teacher change, which involves the processes, described above by Senger (1999). By adopting a reflective teacher mode, the teacher can work towards setting up a good learning environment for students, where activity is interpreted through interactive communication rather than focusing on the result of the activity. Stein, Silver and Smith (1998) claim the best way for teachers to make these changes in their classrooms is to become part of collaborative reflective communities (as in the PEEL project (Baird & Northfield, 1992)) themselves.

2.6 The Teacher as Researcher

Research on teaching and learning is designed to understand and improve these practices. It examines concerns and problems in teaching and learning and trials methods to overcome these. Teachers who research their own practice and talk about changes and challenges within their own classrooms are not prolific in the literature. The *Handbook of Research on Teaching* (Wittrock, 1986) excludes any research done by the classroom teacher, even though there are 35 research reviews included. Most researchers talk about the classroom and their observations but they are not the classroom teacher. They do not have ultimate responsibility for the environment, nor have to commit to being there and ensuring its success. Research can alienate teachers instead of converting them by appearing too theoretical and not grounded in reality. Despite this, there is the expectation that teachers, who are the objects of this research, will become the consumers and implementors of the findings (Cochran-Smith & Lytle, 1990).

Teachers in the classroom face many constraints such as lack of time and resources as well as demands made in other areas like playground duties, sports team coaching. The context of the school environment, as opposed to the university environment, does not expect or support teacher research. Robertson and Allan (1999) list other contributing factors to the lack of teacher research as: the relative isolation people in schools work in, the busy routine, 'dailiness' nature of much of teacher's work, the need for others to help in the critical reflection process, and, the lack of research skills necessary to enable reflective practice to occur. While not conducive to research, these constraints do not prevent teachers asking questions and trying to understand and improve their classroom practice. There is a vast gap in the knowledge base for teaching about teachers' questions, the way teachers talk to and work with their students, and the way teachers interpret and assess their classroom activities (Cochran-Smith & Lytle, 1990). Teachers, through published research, can provide a rich perspective on an area that is still little understood.

Published research can also help teachers gain new understanding and knowledge. Often teachers become involved in research themselves by reflecting and trialling new ideas before they are convinced of their worth. Teacher research is an on-going accountable mechanism that supports and sustains the reflective teacher who is adapting to new directions. Increasingly as more teacher research is undertaken, the dominant theme that comes through is its relevance, practicality and potential for making change (Cochran-Smith & Lytle, 1990; D'Ambrosio, 1998; Mousley, 1992).

Teachers are led to research because they realise that new curriculums require change. By reading and reflecting they challenge assumptions and ask questions. In everyday practice they act and make judgments from their private thinking. Teachers move to a research position when they seek to clarify their practice in some organised documented way. The research position extends the everyday reflective practice to deeper thinking about beliefs and values of the nature of mathematics, the subject, and teaching and learning mathematics.

2.7 Classroom studies that are relevant to this study

Amongst others there have been two major projects which looked at active learning through developing learning awarenesses. The British Economic and Social Research Council (ESRC) Project *Awareness of Learning, Reflection and Transfer in School Mathematics* (Bell, Crust, Shannon & Swan, 1993) and the Project for Enhancing Effective Learning (PEEL project) (Baird & Mitchell, 1986) both looked at what happened when students were exposed to learning activities designed to provoke reflection and develop better understanding. These projects emphasised active learning, and students were encouraged to be more responsible for their learning. Students were taught about learning and how to learn.

PEEL was a long term Australian project, started in 1985, that supported groups of teachers in a school making consistent regular interventions to support learning in their classrooms. The aims were to:

- *Foster students' independent learning through training for enhanced metacognition.*
- *Change teachers' attitudes and behaviours to ones that promote such learning.*
- *Investigate processes of teacher and student change as participants engage in action research.*
- *Identify factors that influence successful implementation of a program to improve the quality of classroom learning.*

(Baird & Northfield, 1992, p. iii).

Working with students at high school level across disciplines, the teachers in the program incorporated deliberate activities, such as challenge of ideas, promoting and using students' questions and 'what I did wrong' checklists, to aid metacognitive learning. These activities were vast in scope and to some extent limited only by teachers' imaginations. They often arose as a possible solution to a need or concern: concerns, such as "students won't try and really understand the work - they don't know how to think". In all, seventy six different procedures or activities are described in PEEL to address eighteen common concerns or tendencies teachers saw in students who were not exhibiting good metacognitive behaviours. Obviously some strategies were more suited to certain subjects, as in role playing being adapted more easily in an English lesson, but a creative teacher could find scope for any activity within a mathematics lesson.

As teachers experimented with activities in the PEEL project they themselves changed their views of learning and classroom practice. The researchers noted that "practice precedes understanding, most of the change process occurs after teachers try new approaches" (Baird & Northfield, p. 193).

The British ESRC project (Bell et al., 1993) investigated the metacognitive skills students, in the first three years of secondary education, showed in typical mathematical learning environments and also explored the feasibility of raising the levels of student awareness by appropriate interventions.

The aims of the ESRC project were to:

- *Investigate the metacognitive skills and concepts possessed by pupils aged 10-16 in a variety of typical mathematical learning environments*
- *Study the relationship between these metacognitive abilities and the effectiveness of the learning of skills, concepts and general strategies.*
- *Develop ways of enhancing the metacognitive aspects of students' learning in different environments.*
- *Study the effect of such enhancements on mathematical learning*
- *Test the potentiality of the resulting collection of classroom material and methods for use by other teachers.*

(Bell et al., 1993, pp. 2-3)

Teachers in the project trialled a series of mathematical activities, many of them one-off interventions, in classrooms. Bell (1993) concluded that all aims were addressed satisfactorily apart from number two, which was difficult because of the timeframe of the project and the diversity of groups being studied. The focus was slightly different to PEEL in that PEEL was as much interested in teaching behaviours as student learning and ESRC concentrated more on student abilities and skills. The procedures which PEEL suggested, could be incorporated into a teaching programme and used over a series of lessons, whereas the ESRC procedures were more stand alone activities, which often incorporated student role-reversal activities. For example, student as teacher, assessor, text composer or observer (Bell et al., 1993, p. 9). PEEL focused on long-term teacher/learner change, over a period of five years during which teachers and academics met regularly. ESRC looked at the feasibility of student change through appropriate interventions, trialled over a period of one year, during which teachers and academics met monthly. ESRC was mostly evaluated in terms of student change; PEEL was mostly evaluated in terms of teacher change.

PEEL was motivated by a belief that teacher change was necessary for students to become better learners. ESRC was motivated by earlier findings that teaching from the basis of students' concepts and misconceptions and resolving the latter by exposure to cognitive conflict and discussion was more effective than teaching by the usual lecture and display methods. These new teaching methods required a shift in students' beliefs about appropriate activities in a mathematics lesson.

Each project concentrated on seven areas and gave suggestions of interventions or teaching strategies that were appropriate. The interventions were classified under headings which give general learning objectives (PEEL, pp. 216-218 and ESRC, p. 6). The areas concentrated on have been matched, as far as possible, below (Table 2.1):

Table 2.1 Areas concentrated on in PEEL and ESRC projects

PEEL	ESRC	Types of activities
Building understanding of subject knowledge	Increase awareness of mathematical content	Reflect & Review
Retrieve, clarify or restructure existing views	Increase awareness of components of mathematical activity	Discussion activities
Improve communication,collaboration and negotiation	Increase awareness of different ways of working	Collaborative activities
Improve written notes	Increase awareness of resources for learning and how to use them	Developing notes
Make numerical problem solving more reflective	Increase awareness of mathematical strategies	Strategy awareness
Monitor and control learning	Increase awareness of learning principles	Monitor activities
Make assessment more formative	Increase awareness of types and purposes of mathematical tasks	Assessment activities

At the end of the projects students were surveyed to study their shifts in views of learning. Students saw benefits to their learning from participating in the projects and could identify behaviours they had learnt from being part of the study. In the PEEL project many students were able to identify aspects and benefits of active learning, as in "students' discussions made up about fifty percent of the lesson so if you didn't listen to this or you day-dreamed you wouldn't understand anything" (Baird & Northfield, p. 46). Teachers commented on students' changes, for example, "changes include active thinking, where students challenge me, suggest things, link to their experiences; ownership and security, where they have a greater say in what's to be done; questioning, including when to ask questions and using 'self-questions'" (Baird & Northfield, pp. 168-169). Most students recognised that they had participated in activities that were different from those normally experienced in other classrooms. They commented how they "did a lot more talking; asked more questions; noticed more communication between ourselves and the teacher, and between the kids; felt the information sinks in more" (Baird & Northfield, pp. 57-60). Students seemed more aware and appreciative of the interventions used by teachers on the PEEL project because they had been ongoing and incorporated into the routine of classroom lessons. In particular, most regarded the emphasis on questioning, the knowledge of note taking skills, and diary reflections as worthwhile activities. Obviously individual students valued different interventions differently. Some interventions were perceived more positively by students than others and some were seen as particularly helpful by some students but an unnecessary chore by others.

Some of the interventions in the ESRC project were reported as 'one-offs' and students tended to react negatively to those they saw as novel or lacking purpose. Bell et al. concluded that the use of one-off interventions was mostly insufficient to enhance awareness. Additionally, it was noted that introducing new activities to a class used to routine methods of working can be difficult; "A notable obstacle was the resistance generated by the severe conflicts with students' existing concepts of learning" (1992, p. 2) which agreed with the finding by Baird and Mitchell that students hold "definite, conservative and restricted views about what constitutes learning" (1986, p. 186). Another problem was students did not always perceive the intent of the activities accurately because they were preoccupied with understanding the nature of the activity and what it demanded of them at a superficial level. The example where students interviewing students on their learning was intended to help students review by focusing on major aspects of the work but was perceived by students as not helpful in improving their work, "it helped to go over how you did it, but it didn't help you do it better" (Bell et al., p. 8). Some activities, like diary writing, were perceived as routine and boring rather than reflective exercises and were not successfully implemented by many students. Students were reluctant to complete tasks they saw as tedious.

It was shown how negative attitudes could be changed by the teacher reflecting on how to improve the activity the next time and being persistent with it. When activities, like the construction of a review booklet, which were initially seen by students as gimmicks, were worked through, most students valued the understanding gained. Students commented how "you can tell how much you understood" (Bell et al., p. 109) and when asked whether they preferred to write their own review or use one provided they voted to write their own.

Both projects illustrated that teachers have to be careful to explain the motive or purpose of an activity. Especially where the teacher's interpretations of the value and objective of the activities can also be at odds with the students'. For instance, Peter, a teacher in the ESRC project, had felt students gained good knowledge of the structure of the topic by interviewing each other, but the students did not value this aim and preferred a different aim, which was to improve their skills in the topic. The success, or otherwise, of activities can be dependent on the learning environment. For example, in an individualised learning environment discussion on specific questions can be very difficult. Running debates in the classroom relies on effective management skills on the part of the teacher; their strength can be dependent on how the teacher manages them.

Overall, the predominant theme was students' appreciating the value of discussion and communication in PEEL type classes. The ESRC project was a shorter term project. Bell cautions that while the interventions had positive effects on classroom learning it is difficult to make significant changes to "well established but restricted student views of learning" (Bell et al., 1993, p. 3).

The ESRC project had a focus on student learning so teachers' views on their own learning and practice were not given a major focus. However, teachers reported maintaining the use of interventions with their classes after the project trials had finished. The PEEL project teachers were enthusiastic about the effects of the project on their practices. They noted how they had developed their personal classroom theory and practice after being part of the project. For example, they cited specific changes like delaying judgment on student responses to questions, gaining greater sympathy to and awareness of students' learning, understanding their own learning and teaching better. In both studies the teachers were committed to the idea of active learning as being valuable. Teachers were able to identify positive changes in their appreciation of student learning and their overall understanding of the learning process.

A summary of teacher beliefs after participating in PEEL (Table 2.2) shows how shifts in thinking had taken place (adapted from Baird and Northfield, pp.192-193):

Table 2.2 Summary of shifts in teacher beliefs

Item	From	To
1	Believes learning is satisfactory and assessment is reasonably accurate	Dissatisfied with quality of learning; believes there are mismatches between what teacher thinks is happening and what is happening
2	Has a transmissive view of learning and teaching	Has a constructivist view of learning and teaching
3	Students should restate teacher's answers and should only give right answers	Students should take risks with their answers and should challenge teacher's ideas
4	Teacher decides rate of progress and students comply	Learning is a collaborative process with both teacher and students having active roles
5	Real work involves lots of copying and drill.	Copying and drill are low level tasks. Activities such as discussion make more intellectual demands on students

2.8 Summary

Curriculum reforms have been greatly influenced by constructivism. This theory has had implications for both learning and teaching. Active learning requires involved participation by the learner. The learner commits to learning and success is enhanced with effective learning strategies including metacognitive strategies that regulate and monitor learning. Earlier studies have shown that teachers can promote the development of effective use of metacognitive strategies with suitable classroom activities and social norms.

With the reform process the teacher is the key to classroom change. Classroom change calls for more thinking, reasoning, problem solving and communicating. Effective metacognitive strategies enhance students' performance in these areas. The best way to effect these changes is for the teacher to become metacognitive in the classroom because teachers teach what they know. The reflective teacher works to establish good learning environments where students become self-regulatory in their learning behaviour.

Research is a mechanism that supports the teacher making change. teacher researchers attest to the value of classroom research because of its relevance, practicality and potential for making change.

There are two major studies that have looked at teachers promoting active learning and focusing on metacognitive activities in the classroom. The PEEL project and the ESRC project both trialled different activities with students and teachers. The activities were designed to increase awareness of different aspects of mathematical learning. Both teachers and students felt their learning was enhanced after participation in the projects. Teachers, in particular, showed major shifts in beliefs about the teaching/learning process after the projects' completion.

CHAPTER THREE

Research process

3.0 Introduction

As a classroom teacher the concern to reflect the intent of the curriculum as well as its content has led me to want to study the development of students' awareness of learning in the classroom. It is felt that the variance between what is intended in the curriculum and what occurs is deeper than is often acknowledged. The aim of the study is to investigate the effects of a classroom based research program that focuses on developing students' metacognitive skills and my own developing awareness of these skills. In particular, three questions form the focus of this classroom study:

- What attitudes and beliefs do students hold in relation to learning mathematics?
- What metacognitive behaviours do students exhibit, in typical classroom exercises, and in specific classroom interventions designed to increase awareness of learning?
- How can everyday classroom routines, e.g. writing notes, reviewing previous work, be adapted to develop students' awareness of learning and my own awareness of their learning?

The study is also motivated by findings of the ESRC project, *Awareness of Learning, Reflection and Transfer in School Mathematics* (Bell et al, 1993) and findings of the PEEL project, *Learning from the PEEL Experience* (Baird & Northfield, 1992). Both of these projects looked at working in the classroom, with teachers looking at raising students' awareness of their learning. They were both honest accounts of classroom activity, with descriptions of methods and interventions that could be readily adopted into my teaching programme, and their interest lay in their potential for making positive change.

The study is being undertaken because of a belief that classroom practice, and hence student achievement, will improve with more reflective teaching and learning.

3.1 The Teacher as Researcher

This teacher as researcher project involves qualitative research methods. Qualitative research is a broad term for a wide variety of research that involves "an interpretive, naturalistic approach to its subject matter" (Denzin & Lincoln, 1994, p. 2). The stress is on the socially constructed nature of reality rather than the measuring of value-free variables, which is a characteristic of quantitative research. The main feature of this qualitative research study is the use of several methods to draw together a deeper understanding of students' and teacher's awareness of their learning. The qualitative approach suits the aim of analysing and developing students' and teacher's awareness because it helps define what works and why it works.

The method involves the teacher experimenting with an activity, collecting data and making decisions on the basis of the data and judgments as to whether the activity "works". The process is written up in the form of a narrative inquiry which analyses situations and events that take place in the classroom over a period of time. As such, it is seen as a way to support teachers who see value in gathering data and conducting research to understand their own practice (Cochran-Smith & Lytle, 1990). Following Stenhouse's (1994) definition of research as systematic enquiry made public, this method is commonly called 'teacher research'.

Essentially, the teacher as researcher is a practical approach that can be undertaken by teachers in an attempt to improve practice. It is regarded by some as more likely to lead to classroom change than formal research conducted by university researchers or practitioners. Richardson (1994) notes how teacher research can take several approaches. Firstly, there is the notion that *teaching is research*; the work of teaching is like that of a researcher, although less formal. The second approach is that of the *teacher as reflective practioner*. The third concept is that of teacher research as a form of *action research*. The fourth approach is the teacher as *formal educational researcher*, with the results contributing knowledge for others' use. This project, taking the form of a practical inquiry fits into the first three categories and its ultimate publication lends it some formality.

Cochran-Smith and Lytle (1990) gave a definition of teacher research as systematic and intentional inquiry work carried out by teachers. Systematic in the way the data gathering, recording and documentation is done, intentional as opposed to spontaneous, and inquiry because the work stems from questions the teacher asks about classroom experiences. Because of the nature of the teacher's role, the value of this work is that the view is different from that of an outside observer, even if the observer does spend a lot of time in the classroom.

Characteristically, teacher research is learner-centred, teacher-directed, collaborative, context-specific, scholarly, practical, relevant, and continual (Cross & Steadman, 1996). Teachers focus on observing and trying to improve learning so that their teaching becomes more effective. They become instigators of research rather than consumers of research with this method. As is the case in this study, the questions researched typically apply to the classroom and discipline identified, and arise from a specific concern of the teacher researcher.

The results of teacher research may be generalizable to other populations but that is not a condition of this work. teacher research builds on the knowledge base of research on teaching and learning. It requires a question, a design and consideration of the implications for practice. The questions are practical and relevant and the quality of the project is primarily measured by its contribution to the knowledge and practice of the teacher concerned. Because of the personal and practical nature of the research, teacher research is necessarily ongoing. New questions can arise leading to new projects. Changes suggested by the research require continual evaluation and modification. Thus, the research is "more a process than a product" (Cross & Steadman, 1996, p. 4).

However, it should be noted that teacher research is more than the 'good practice' of the expert teacher. Phillips (1997) defines the difference between teacher research and day-to-day teacher reflection:

Research is something I do in addition to my defined teaching job. It involves 'extra' time and energy. I could still do my job well without it. Looking at aspects of myself *that relate to effective and non-effective teaching is part of my job as a teacher. It needs to be refined and worked on to be transformed into 'research'.*

(Phillips, 1997, p. 14)

The systematic reflection on classroom interaction, learning awareness, and teacher behaviour and the nature of the data collection and analysis make this project teacher research rather than simple teacher reflection which is what many teachers already do on a day-to-day basis.

The research methodology loosely follows the action research model with its stages of planning, acting, observing, reflecting and evaluating, to form cycles which repeat, building on experiences which arise. Action research is intended to support teachers in coping with the challenges and problems of implementing change (Altrichter, Posch & Somekh, 1993). The situation or general idea that the teacher wishes to focus on is taken from some situation within their experience. A cyclical framework is planned for an aspect of that situation or idea they want to change or improve on .

The research findings are primarily written in narrative form. Connelly and Clandinin (1990) have defined narrative inquiry as the way humans make meaning of experience in a process of refiguring the past and creating purpose for the future. It is empowering for those practitioners who want to have a voice in, rather than be the objects of study. Narrative is considered to be a natural way for teachers to share and explore their practice (Johnston, 1994).

3.2 Subjects and Setting

This study was done in conjunction with my existing teaching program. It relies on my experience and insights as both the teacher, with a day-to-day knowledge of the subject matter taught and the opportunities to plan the instruction, and the researcher, who observes the learning in my classroom. The classroom is the major resource; it is the natural environment for the students and me, their teacher. The levels of interaction experienced by myself, as the teacher researcher, with the students, are the prime characteristic.

This study focuses on a form three class in a provincial New Zealand girls high school, over a period of three school terms. The roll of the school is 1150 students. The school has a high decile rating. There are nine form three classes of age range from twelve to fourteen, which represents year nine of schooling. Two classes at this level are top-streamed classes and the rest are mixed ability. Students are timetabled for five mathematics lessons per week of fifty minutes duration each, that is just over four hours of mathematics in a week, out of a total of twenty six hours instruction. The mixed ability class involved in the study has 28 students with a diverse range of abilities. The class had mathematics lessons for two morning sessions and three afternoon sessions per week.

Students in the class are experiencing their first year at secondary school and have come from a wide variety of contributing schools. Some come from the three local intermediate schools, some from three local private schools and some from outlying rural schools or other parts of New Zealand.

3.3 Ethical Considerations

The guiding principle followed was the need to be thoughtful and considerate of the needs and feelings of others. Permission to undertake the study was initially obtained from the school principal and the head of the mathematics department of the school. Approval was given by the Human Ethics Committee, Massey University.

In the first week of term the study was discussed with the class and an information sheet (Appendix A) outlining the details of the study was sent home for parents to read. Students and parents were given details of the aims of the study and invited to respond for further information, if need be. Emphasis was given to the fact that the mathematics course would not differ from the prescribed Form 3 course in content or time allocation. There was only one reply from parents. A father acknowledged the information sheet and asked if he should continue extra private tuition classes in mathematics for his daughter. Within the study no one student was singled out for direct observation or interview with the researcher.

It was important for the students that their teaching and learning fitted within the culture and atmosphere of the school. This meant that the study had to allow for all the requirements of the normal school programme. In some instances, these created obstacles to the aims of true constructivist teaching and learning. For example, common tests, sat by all students across the year level, had a strong emphasis on recall of content and skills.

Because the researcher was also the teacher the difficulty in maintaining a clear vision in terms of the project was always present. Day to day classroom matters could often obscure the big picture. School and student demands were not always predictable, and plans were sometimes obstructed by such things as school photos, sports exchange visits and trips. Sometimes specific activities had to be altered to fit changed circumstances. However, coping with these realities provided for a realistic picture of the difficulties facing teacher research and teacher change.

3.4 Overview of the Research Schedule

The study was primarily conducted in the first three terms (February to September) of a four term year. Different tasks and interventions were trialled over this time-frame. Reflections and evaluations of the interventions prompted the further development and use of trialled interventions and the choice of new ones. The activities followed main themes:

- Monitoring learning
- Strategy awareness
- Reflection and review
- Self assessment

Cycles roughly followed terms, and there were three terms. In each term there was a focus on specific areas with targeted activities.

Term 1

Baseline data in the form of questionnaires (Appendices B–D) was gathered early in the term and introductory activities included opportunities for the students to give feedback on attitudes to mathematics. Student perceptions and beliefs about mathematics learning were ascertained to guide the direction of the study.

Other feedback from the questionnaires combined with teacher intuition suggested the students perceived mathematics as an entity outside of their influence, owned and controlled by 'others'. To overcome this it was decided to encourage activities that emphasised personal ownership, and scope for control, of the subject. The main focus was on notes and analysis. The writing of notes shifted direction from direct input from the teacher to a shared input from both teacher and class. The notes were used as a mechanism to encourage students to monitor their learning.

Additional activities included this term were;

- writing similes - "mathematics is like...."
- topic test evaluations

Term 2

At the end of the first term the notes books were collected and observed for details such as completion, accuracy and individualised input. After reflection it was decided the students needed to put more value on their notes by seeing an obvious need for them. This led to the incorporation of the following interventions:

- open book tests
- students writing topic tests.

Another focus this term was the development of communication, through regular reflection and review activities. These activities involved discussion and collaboration. We also ran a series of mini-debates about working in mathematics. In particular, interventions included:

- revision questions at the start of the lesson
- discussion and remedy of common errors
- conducting mini-debates.

Term 3

Observation and reflection on the review activities and test results during term 2 led to the decision that consolidation and maintenance of these was important for continued progress. It was decided to also look at how developed their sense of awareness of the purpose of mathematical activities was. In the final phase of the study it was noted how groups had settled to become identifiable in terms of work habits, on-task performance, achievement and motivation. As well as keeping an ongoing focus with notes, reflection and review, the final concentration was on students' monitoring and controlling their own learning through self assessment. Interventions were:

- deciding the purpose of an activity
- assessing their performance after class tests.

3.5 Data Collection

The epistemological paradox (Brown & Dowling) refers to the fact that "the objectification of your activities is always a different experience from your experience of those activities themselves; the act of making your experience explicit of necessity entails its transformation"(1998, p 8). As the researcher, I was aware of the concern that the data be presented as honestly as possible.

In order to increases the trustworthiness of the data, and subsequent interpretation, a common response is to employ two or more approaches to collecting data. This is referred to as triangulation. To build a profile of students' learning awareness and metacognitive behaviours four methods of data collection were used: questionnaires, teacher's diary, informal interviews, and students' books.

Questionnaires

These were used to ascertain the students' attitudes and perceptions of mathematics before, during and after the study. They were taken from Bell's work in the ESRC study (Bell et al., 1993), and are included in Appendices B to D. The advantage in using these prepared questionnaires was they had been trialled with a sample of 350 students of similar age to those in this study. The data could be analysed and compared with the results from the larger study.

Teacher's Diary

This was used to plan the work for the class and record details of assessments. Notes of what activities were used in the lessons were recorded; the diary was used to note teacher's observations and students' comments at various points in the study. The quality of the diary entries directly relating to this study was dependent on the time of year and commitment of the researcher to other school activities. The diary nonetheless was the working document of the researcher in the position as classroom teacher and was used daily. Referring to the diary when doing routine activities like day-to-day planning, noting key events and recording students' results reminded the researcher of the need to maintain an ongoing focus with the target class.

Informal Interviews

Cobb, Wood and Yackel (1990) noted how practising teachers do not use formal models in their interactions with students but operate pragmatically and instinctively interacting with students and making on-the-spot decisions. Students often volunteered comments in class discussion, were observed in conversation with other students, or responded to questions posed by the teacher either in whole class discussion or informal conversation. These comments were sometimes recorded in note form and often added insight to the study.

Students' Books

Students kept three books; their exercise, notes, and sample/test books. Evidence in the form of samples of students' summary, organisation, problem solving and reflection was regularly taken from students' notes and sample/test books. At the end of the year five sets of students' books were retained by the researcher with the permission of the students.

3.6 Issues in Teacher Research

Traditional methodologies which incorporate quantitative data are subject to examination on grounds of reliability and validity. The debate over the reliability and validity of teacher research is well documented. Some academics (Reichardt & Rallis, 1994) approach the research with reservations because it is not easily open to critique in terms of the constructs of validity and reliability. Connelly and Clandinin (1994) remark that the method does not rely on the quantitative criteria of validity and reliability and generalisation, but on apparency, verisimilitude and transferability. Cochran-Smith and Lytle (1990, p. 4) claim that comparing teacher research with university-based research "involves a complicated set of assumptions and relationships that act as barriers to enhancing our knowledge based about teaching".

Rather, it is better to consider teacher research as a new genre of research with its own methodology, criteria of rigour, and style (D'Ambrosio, 1998). A teacher research project's chief value is for the teacher herself to better understand and transform her teaching practice; ideally her personal findings will lead to an increase in shared professional knowledge (Hatch & Shiu, 1997).

Schon describes the teacher researcher's dilemma between rigour and relevance:

> *Shall the practioner stay on the high, hard ground where he can practice rigorously, as he understands rigour, but where he is constrained to deal with problems of relatively little social importance? Or shall he descend to the swamp where he can engage the most important and challenging problems if he is willing to forsake technical rigour?*
>
> *There are those who choose the swampy lowlands. They deliberately involve themselves in messy but crucially important problems and, when asked to describe their methods of inquiry, they speak of experience, trial and error, intuition, and muddling through.*
>
> (Schon, 1983, p. 42)

Cross and Steadman (1996) see teacher research as opting for relevance over rigour for two reasons. Firstly, the problem must be relevant to the needs of the teacher researcher, who is doing the study with the main purpose of enhancing her understanding. Secondly, the main strength of the teacher researcher lies in her understanding and experience of the classroom context, not her technical research competence.

Transferability is an issue to be addressed with research. In particular, the interest is whether the results of this study would transfer (a) to other classes taught by the teacher and (b) to the practice of other teachers? As the researcher I acknowledge a deeper understanding and appreciation of the role of metacognition in learning and believe that I am better prepared to foster active learning as a result of this project. It is also hoped that teachers reading this study may recognise some of their own thoughts and practices, and may be motivated to reflect deeper on their teaching and beliefs, even if in a less formal manner.

3.7 Summary

Over three terms a third form class was monitored for their response to specific activities aimed at raising learning awareness. While reflecting on my own teaching approach, I also worked daily with the class to develop their knowledge about learning, their awareness of the nature of learning and their control of learning. The methodology was teacher research following an action research model. While the study makes no claim to generalizability it is hoped that experiences and insights it offers will resonate with teachers in similar situations. The results are presented in the next chapter.

CHAPTER FOUR

Results

4.0 Introduction

In this study activities were trialled which focused on students' use and awareness of metacognitive behaviours in mathematics learning. The study was based in the classroom and concentrated on the teacher working with the class as a focus group.

The data collected was in the form of questionnaires, researcher's field notes in the teacher's diary and students' work and observations as recorded in their books. The questionnaire data explored students' attitudes, perceptions and awareness of mathematics and ways of working in mathematics. The teacher's diary recorded the planning, details of activities and interventions and results of student assessments. It was also used to record details of informal interviews with students. The ways and times the teacher adopted the roles of monitor, facilitator and model were noted in the diary. The students' books were used to check ways students showed awareness of their learning and the strategies they used to learn.

4.1 Questionnaires

To gauge the students' perceptions, attitudes and awareness of general learning principles questionnaires were given in the early weeks of term one. The questionnaires were taken from those developed by Bell et al., (1993) in the ESRC project. Results are summarised below.

4.1.1 Questionnaire on attitudes to mathematics:

In the first questionnaire (Appendix B) the students (27) were asked how they felt about mathematics. They were given a list of twelve attitude statements, such as "I usually enjoy maths", "In maths, I feel as if I'm making progress", and the students were asked to rate them from "strongly agree" to "strongly disagree" on a 5-point scale. Their scores were recorded as 1 = strongly agree, 2 = agree, 3 = not sure, 4 = disagree and 5 = strongly disagree. The twelve statements could be paired as six themes each containing opposing attitudes: mathematics is easy/hard; mathematics is useful/not useful to me; mathematics is enjoyable/boring; in mathematics I make progress/stand still; in mathematics I work hard/am lazy; in mathematics I can use my own ideas/ use other peoples' ideas. The items were not written in a way that made the pairing obvious and they were listed in a jumbled order. The mean score of each item for the class was calculated and compared to the score of the opposite item. A high mean score indicates not sure or disagree with the item and a low mean score shows agreement. By calculating differences the strength of feeling can be shown. A positive difference shows overall class agreement with the first attitude, whereas a high negative difference shows agreement with the opposing attitude. All the differences were positive, bar one, which supported the researcher's belief that the attitude to mathematics in the class was good.

The highest difference showed up in mathematics' usefulness which suggests the students perceive mathematics as being useful to them in their lives. They also believed strongly that they make progress in mathematics lessons. Enjoyment rated positively, but not strongly so. Ability to do mathematics rated even lower; that indicated the class did not feel confident in their proficiency in mathematics and they were ambivalent about their skills in the subject. The difference for ownership of mathematics was marginally negative, meaning more students feel mathematics is others' ideas than feel mathematics is their own ideas. The results are summarised in Table 4.1:

Table 4.1 How I feel about mathematics

Attitude	Opposing view	Mean Attitude	Mean Opposing	Difference
Make progress	Stand still	1.92	3.42	1.5
Maths useful	Not useful	1.42	4.07	2.65
Enjoy maths	Maths boring	2.23	3.42	1.19
Maths easy	Maths difficult	2.5	3.4	0.9
Work hard	Lazy	2.19	3.07	0.88
Others ideas	Own ideas	2.9	2.73	-0.17

4.1.2 Questionnaire on perceptions of mathematics

The second questionnaire (Appendix C) looked at students' perceptions of mathematics under the title "Learning maths is like...". Again the students were asked to score ten items, each on a scale from "strongly agree" (1) to "strongly disagree" (5). A mean score close to one indicates strong agreement within the class. The statements were phrased as similes, such as "Maths is like a jig-saw. The ideas fit neatly together" and "Maths is like a jungle. The ideas are all jumbled up". The items represented opposite perceptions and were paired in the analysis, but this pairing was not shown in any obvious way on the questionnaire. By comparing means for the items and taking the difference between the means it was possible to gauge any strongly felt convictions in the class. A positive difference indicates a stronger leaning to the second stated perception, with a negative difference meaning more agreement with the first preference. The largest difference occurred in the practice/understand item which suggested that students rated understanding more strongly than the need for practice. Students were mostly in agreement with the need to understand mathematical ideas and were impartial to, or in disagreement with, the statement : "you don't need to understand how mathematics works: you just need to practise doing it". Students also recognised there can be several ways to solve problems. However, many students perceived mathematics as a structured rule-driven discipline. A summary is shown in Table 4.2.

Table 4.2 Learning maths is like...

Perception	Opposing view	Mean Perception	Mean Opposing	Difference
Make choices	Follow rules	2.39	1.78	0.61
Unstructured	Structured	2.96	2.13	0.83
Many ways	Only one way	1.74	2.44	-0.7
Jumble of bits	Framework	2.65	1.96	0.69
Understand	Practice	1.65	2.91	-1.26

4.1.3 Questionnaire on ways of working in mathematics

The third questionnaire (Appendix D) looked at 'ways of working in mathematics'. There was a list of seventeen different ways of working, most of which the students all claimed they did either often or sometimes. These were things like listen to the teacher, read text, work on problems and investigations. Approximately half the class claimed they discussed others' mistakes, watched others work and just over one quarter of the class claimed to copy others' answers. However, more students thought it unhelpful to watch others work than thought it helpful and most of the class felt it was unhelpful to copy others' answers. These results are shown in Table 4.3.

Table 4.3 Ways of working in mathematics

Title	often	sometimes	never	OK	help	unhelpful
copy off board	20	6	1	4	22	1
listen to teacher	21	6	0	3	22	1
read text	19	8	0	4	22	0
copy text	13	13	1	4	21	0
do short exercises	14	12	1	3	22	1
make up question	2	18	7	11	11	3
explain to partner	9	14	4	7	16	2
discuss own mistakes	8	14	5	5	20	1
discuss other mistakes	3	12	11	7	13	4
watch others work	5	13	9	8	8	9
copy others answers	0	8	19	3	5	17
discuss with partner	12	13	1	10	15	0
discuss w small group	5	16	4	9	15	1
explain to teacher	6	15	5	10	14	2
listen to partner explain	10	14	2	8	17	1
work on problems	17	9	0	3	23	0
work on investigations	11	12	3	5	20	1

4.2 Students' Attitudes and Beliefs

With the results of the questionnaires in mind the first decision was to seat the class in groups so that students could more easily and informally discuss their work and solutions with others. The students were seated in self-selected groups. Research by Cobb et al. (1991, p. 174) encourages such arrangements: "the social interactions... influence their mathematical activity and give rise to learning opportunities". There were six groups in the classroom of about four to five students in each. They were arranged around clusters of desks. Three of the groups were stable with consistent membership. The other three had fluctuating membership. These groups experienced a lot of friendship anxieties and shifting loyalties over the year. These problems were reflected back in classroom behaviour and some students were increasingly off-task as the year progressed. It was intended that students shared work and results in their groups. I repeatedly encouraged students to "use your friends" to help problem solve; and "asking someone else" gradually became a helpful strategy acknowledged by students.

One of the aims of this study was to look at what metacognitive behaviours students use regularly. It was decided to do this by observing them at work and through their written work records. The metacognitive dispositions I looked for in students were an awareness of their role in the learning process; a belief in the need for thinking about new ideas and linking them to old ones; an ability to communicate by reasoning and arguing for an idea; an appreciation in the value of arguments and questions, and other ideas, as helpful to learning. The extent to which students demonstrated these behaviours by working and acting metacognitively was monitored by activities and checklists.

4.2.1 Awareness of role in the learning process

Students were asked to compile a title page for their sample books using the idea of the simile "mathematics is like.... ". The intention was to get students to think of different aspects of learning mathematics and to express their thoughts and feelings about learning, thus showing awareness of their role in the learning process. Within the simile statements values of doing homework, practising, asking for help, and being prepared for lessons were common items. Feelings of satisfaction when understanding is reached and frustration when the work is hard were other themes. These pages were illustrated with appropriate drawings and many students produced very attractive pages.

Some similes offered were:

- reading a book - *"Don't judge a book by its cover and don't just say mathematics is boring cause it can be fun. If you read on you will know more; if you keep trying you will learn more".*
- climbing a mountain - *"Climbing a mountain is hard work, but it's satisfying when you reach the top; mathematics is hard work, but it's satisfying when you've finished and you understand the work"* (see Figure 4.1).
- flying a plane - *"You have to have the instructions before you fly off; you need instructions before starting a mathematics problem. You need lots of practice before you can fly on your own; you need practice in mathematics before you can work out the answers. When you soar across the sky you can look down at the great views; after you've achieved your best you can look back at how much and far you've come, how much you've achieved"* (see Figure 4.2).
- painting - *"Doing painting is fun; doing mathematics is fun. If you do a paint job wrong you will get in trouble; if you do your mathematics wrong you will get in trouble; if it is messy you have to do it again".*
- flying a kite - *"if you don't fly the kite enough times you won't learn the techniques like in mathematics if you don't do the homework when you're told you won't learn the basics; if the kite gets stuck in the tree ask someone to help get it down for you like in mathematics if you can't understand the question ask the teacher to help you; if you fly your kite when it's calm it won't go anywhere like if you don't like mathematics and you don't do it you will find when it gets harder you won't get anywhere".*
- talking on the phone - *"you can't have a conversation without anything to say; you can't do your work without any resources".*
- climbing a ladder - *"whenever you learn something new, you go up a step, and if you get something wrong you go down a step, but eventually you will get to the top, and when you see how many steps you've climbed, you'll know you've really achieved something".*

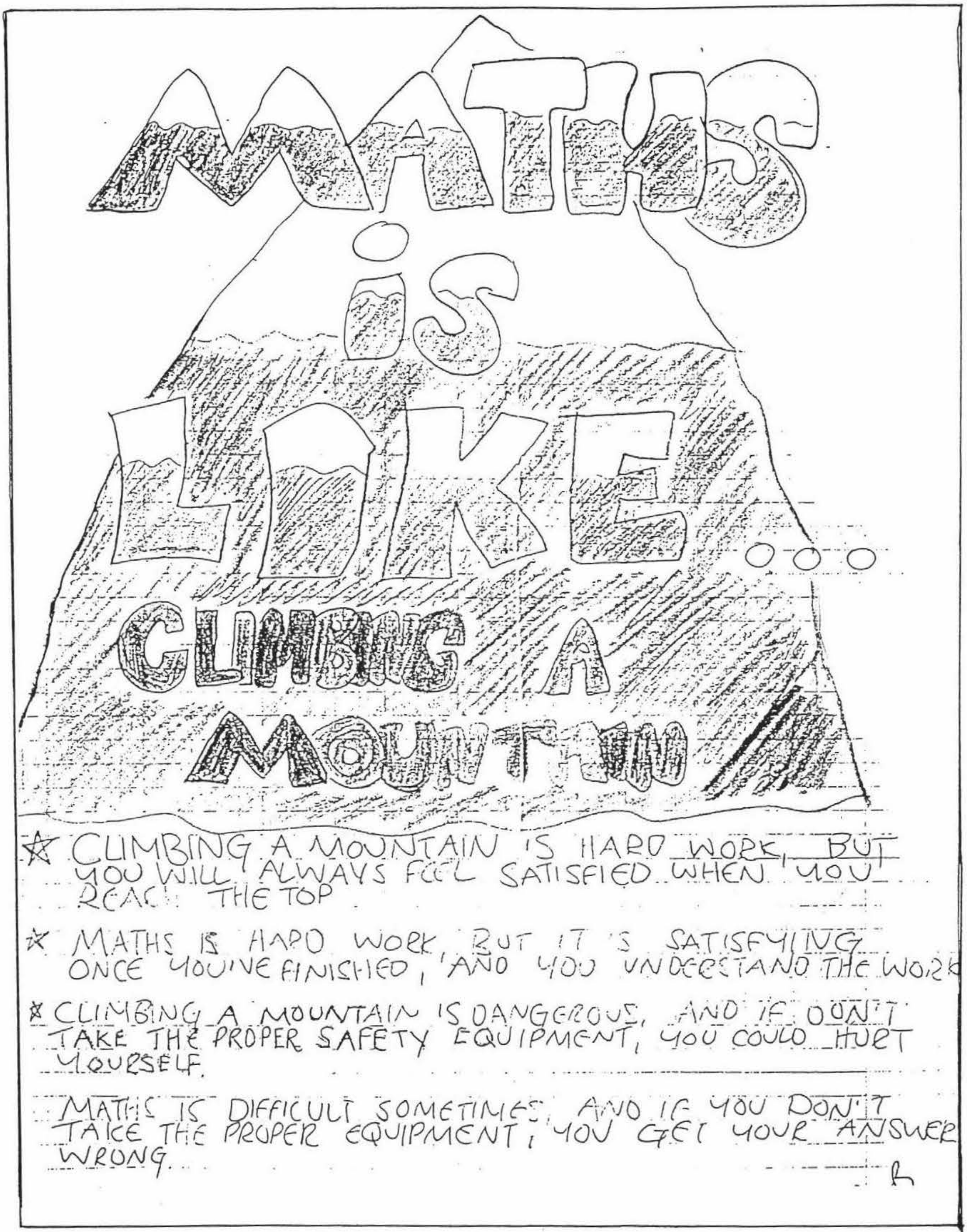


Figure: 4.1 Example of a title page, expressing the need for understanding

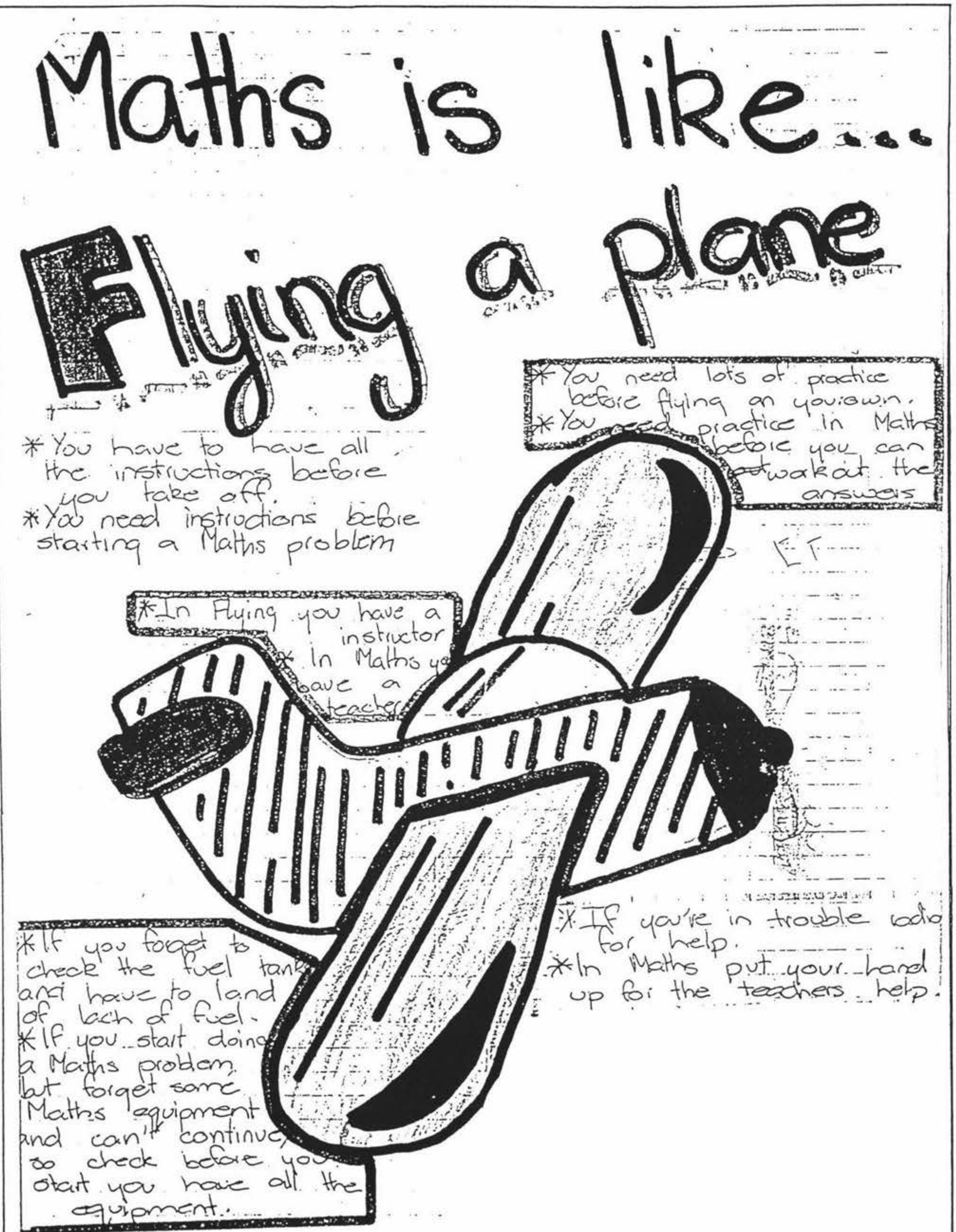


Figure: 4.2 Example of a title page, expressing the need for practice

These comments have to be appreciated in the context they were done. They were done early in the year in the front of a new book that was to be the 'sample book' of best work. The book was kept by me and shown to parents and other teachers as examples of the students' work. Thus, while these comments reflected personal values it would be reasonable to assume they were written with the recognition of a teacher and parent audience.

4.2.2 Belief in need for thinking about new ideas and making links

Note writing formed a significant part of most lessons and it was in this section of the lesson that opportunities existed for students to demonstrate their attitudes to new ideas. While new ideas were presented and explained the students were encouraged to discuss and debate these. With use of examples they could make links to old ideas they knew. During the project, the writing of notes shifted direction from direct input from the teacher to a shared input from both teacher and class. Explanations were written in the words of the students usually as a result of class discussion. Often ideas were recorded on the board to assist students write up their notes.

Students used a special book for their notes and were asked to take care with setting out and display. The organisational nature of the notes was stressed with topic numbers and sub-heading numbers. They could personalise these notes as much as possible by putting things in their own words, annotating the notes and writing down examples and hints that made the learning easier (see Figure 4.3).

Students also used their notes to write summaries and copy out examples. Despite encouragement to write their own notes many students preferred copying directly from the teacher's notes. To further develop students' confidence in note-taking, specific help was given. However when page references from the text were given to help students write summaries most tended to copy the text exactly, and only a few paraphrased. They asked, "Do we copy everything?" and were hesitant when told to copy "what you need to know, or what you feel is new learning".

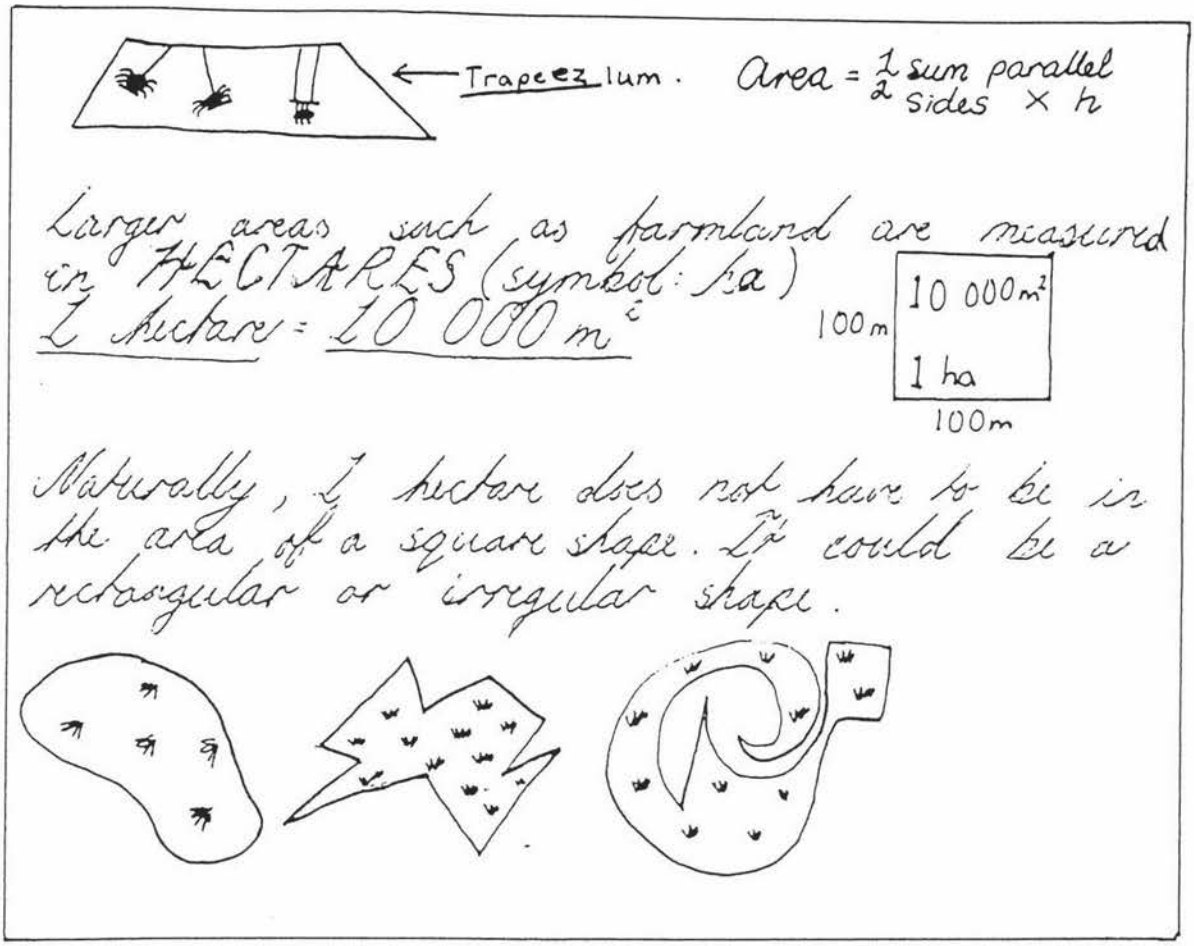


Figure: 4.3 Examples of a student's personalised notes

To help students value their notes and see an obvious need for them the notes were allowed to be used in class tests. It was felt that the value of knowing how to access, refer to and use ideas out-weighed the value of learning to replicate ideas and procedures from memory. Students who requested clarification on details that should have been in the notes were reminded to use them and reproved if they didn't have them available. In the test on angles formed by parallel lines students needed to have either learned the definitions and relationships or have notes to refer to. It was observed that for some students, even after reference to notes, found it difficult to interpret which relationship was appropriate. This emphasis on use of notes for reference did not work for those students who lacked the organisational skills and level of sophistication in interpreting material that had been assumed. Overall, students' results showed no obvious advantage or disadvantage with this method.

Mindful, that for myself as the researcher the notes were a source of observing students' attitudes to new ideas and checking their beliefs in the need for thinking about concepts, the books were collected three times and observed for evidence of these types of metacognitive activity. While written records are only one picture of any given student's learning at a particular time they gave some data to the researcher and proved the most practical solution. Teacher feedback on notes included comments which related to how comprehensive the notes were (e.g., whether all aspects of course work was included); how much the student had personalised and not directly copied the notes and how helpful they looked for learning. The process of note-taking and the potential usefulness for students of the notes was assessed as well as the standard checking for content. Many students appreciated their notes and were proud of their efforts with their books. They resented other students borrowing them and not returning them in time for the next lesson because this might make them fall behind. These students were up to date with topics, they annotated their notes with their own comments or friends' observations. Their books were well organised and provided a personal record of the curriculum content, with evidence of individual thought and effort. Two years on good students still use the notes book from this year to help them study. In the checks about one third of the students showed strong evidence of valuing and committing to learning new ideas through the use of their notes.

The pace of the work was right for most of the class, however, some students got behind because of absenteeism or because of lack of effort in class. They had clear gaps in their books, which initially, they took care to catch up with. However, for some the catch up got to be too big a task and they resolved this by leaving gaps here and there. These books indicated a decline in the initial interest to stay on top of new ideas. As the year progressed, issues arose such as peer group problems and friendship concerns, also an unpleasant bullying situation - which involved a group of students. These concerns interfered with the motivation levels of some students, and for them the goal of understanding became unrealistic. This is not to say these students' learning was impeded over the whole year, but for periods of time they were not focused on mathematics learning, and this of course had the consequence of loss of the big picture. Over the course of the year two students became chronic truants so they were always on the back foot when in class because of missed work. About one third of the class were reluctant learners to a greater or lesser degree. Three or four students showed little care to maintain order or sequence; they lost their books and wrote notes on pad paper, in the back of other exercise books, or not at all. Some of these students showed sporadic interest but they relied on me, as their teacher, to be their prime motivator. One common theme with these students was irregular attendance at school.

4.2.3 Communication

One further aspect of metacognitive behaviour the study sought to examine and develop was students' appreciation of arguments, explanations and questions as helpful to learning. One way this was realised was in regular review sessions that began each lesson. Commonly, these took the form of 5-10 quick questions. These led to discussion about ideas by acting as prompts for the students' thinking. Questions were also posed by the teacher to try to understand students' thinking of problems, and their answers were used to learn more about their thinking. Sometimes answers were given and students made up corresponding questions. For example, 'the answer is '–5', what is the question?' or 'the mean of three numbers is 10, what are the numbers?'. Further, deliberate and common mistakes were given for students to correct. For example, find the fault in this answer: 3 – -4 = -1. Other examples are shown in Table 4.4 below:

Table 4.4 Different ways of phrasing review questions

Question (old format)	Question (new format)
What is the mean of 3, 4, 5, 12?	Mean of four numbers =4, What are they?
List the multiples of 4	The multiples of 4 are 1,2,4. True/False
Expand: 9(x + y)	Correct this: 9(x + y) = 9x + y
Round 32.895 to 2 dp	Correct this: 32.895 to 2 dp = 33
How many lines of symmetry in a square?	Draw a shape with 4 lines of symmetry.

Students discussed the work amongst themselves and sometimes asked for help or further details. If possible, other students were asked to answer these inquiries or students were referred to the text or their notes. After about fifteen minutes students were encouraged to present their answers. "Explain how you got that answer" or "Show us what you did" were prompts used in class. Students enjoyed working on the board to show others their work. This focus on strategy helped students develop their explanations for working and good students, endeavouring to understand, were keen to explain their reasoning, when recognising their work hadn't achieved the 'answer in the back'. They realised there was a fault somewhere in their process and wanted to find where. Having students say "I'll tell you what I did and you tell me where I went wrong" or "this is what I did to get this and yet the book, (or the teacher) tells me it should be this" was indicative that they appreciated the reasoning process involved in mathematical solutions. Even better was when students talked through their work amongst themselves and arrived at better solutions because they saw inconsistencies and errors, or developed a better understanding of the concepts involved.

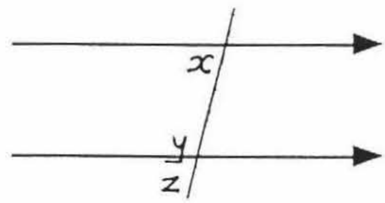
Daily review of previous work and the style of working together to solve problems helped students deepen understandings and make connections within the subject. A pattern for discussion was established that made taking risks non-threatening; no one asked how many were correct, or checked the work, other than to see if students were participating. The aim was to incorporate active and effective patterns of learning into revision and work strategies rather than the passive looking at old notes and exercises which some students believe constitutes learning. Students who took an active role gained an appreciation of not just their own role in learning but how other students' ideas contributed to their learning as well.

4.2.4 Appreciation in the value of arguments and other ideas

Working in groups contributed to an atmosphere of sharing and learning together. The discussions, either in small groups or as part of the whole class, were open and accepting. Students asked each other about the work and listened to others' explanations. This acceptance of others was a significant factor in establishing norms for classroom discussion behaviour. Students who ignored, or talked over, other students who were actively learning were censured, and exhibitions of this behaviour were minimal. Students who admitted to not understanding, by asking appropriate questions, were praised.

Their questions were used to promote discussion. An example occurred in the following conversation with L:

L:	Why do they (cointerior angles x and y) add to 180°, when the others (alternate and corresponding angles) are equal?



T:	Who knows why this happens?
M:	Those ones (x and z) are equal (corresponding angles). Those ones (y and z) add to 180° (adjacent angles on a line). So those (x and y) have to also add to 180°.
T:	That is great, M. Good reasoning. What do you think, L?
L:	I can see those (y and z) add to 180° (adjacent angles on a line). But how does x get into it?
T:	Does it have anything to do with z?
L:	If we could cut out angle z, it would fit by x.
T:	Yes. Good.
L:	So x and y also add to 180°, just like adjacent angles on a line.
M:	But they're not on a line, so we call them cointerior angles.

Specific cooperative activities were used on occasions. The EQUALS cooperative logic puzzles (Erikson, 1989) were enjoyed. These included rules, such as "make sure everyone gets to participate; listen to what other people say; ask others for their opinions", which the groups followed quite well. After the puzzles it was interesting to discuss with the class what had happened. We reflected on the process of sharing and learning, as well as the understanding that developed. When asked if they thought everyone had a chance to talk the class thought that most had talked, but some students didn't want to or couldn't be bothered. When asked if they had learned things from other group members, initially they said not, but gradually one or two admitted that some students 'knew more' and had helped them work things out. The class showed good social skills in doing cooperative activities, but the strongest group of students worked much faster than others and made slower groups lose heart. Possibly it would have been better to assign groups randomly, rather than let the students stay in their established working groups, which were self-selected.

Mostly, students worked alongside others on routine activities. Practice of new work is an essential task for ongoing learning and it occurred in most lessons. During this time the teacher circulated among the students checking homework and discussing with them on one to one or small group levels. They were encouraged to seek clarification over points and if their questions were assessed as 'easy' they would be referred to their friends or the 'back of the book' for help. Independence in learning was encouraged and fostered. Any questions which were persistent or raised wider issues were addressed to the whole class.

4.3 Student Metacognitive Behaviours

The specific learning behaviours looked for in this study were drawn from Mitchell's list of good learning behaviours (Baird & Northfield, 1992). They were separated into monitoring and constructing behaviours. Over the course of the study, different aspects of good learning behaviours were concentrated on and looked for in students.

The monitoring behaviours were:

- seeking assistance
- checking work
- planning and reflecting.

The constructing behaviours were:

- reflecting
- linking ideas
- assuming a position.

It proved challenging to monitor students' use of these behaviours because some were more transparent than others.

4.3.1 Monitoring behaviours

• Seeking assistance

The most obvious was students' seeking assistance and at times it seemed there is a fine line between students endeavouring to be independent learners, asking for guidance as a last resort, and students who are prefer to be dependent learners, relying on the teacher or their friends. Within the class several personalities stood out as being clearly in the latter category, but there were four or five students who actively learned in the former sense. It was generally acknowledged by the class that a good way to learn was to ask why something was wrong and admit if you don't understand. Asking questions was good, and there was a vast source of people to ask, ranging from the teacher, friends or family.

• **Checking work**

Students were expected to check their work. Homework and class work was to be marked, using answers supplied in the texts or from the teacher. It was stressed that this was a good learning behaviour and corrections should be written on work in ways that made sense to the student. Variations were seen here; from students who self-corrected methodically and regularly, to students who merely copied work from the answers, with little apparent reflection.

Homework gave an opportunity for students to display monitoring behaviours. Homework was rated highly by students as a desirable discipline early in the year and its value was emphasised by me throughout the year. The tasks were mostly completing textbook tasks began in class which went over the points covered in the lesson. Sometimes we used photocopied pages of resources from other books and journals but the textbook was very comprehensive and served our needs well.

Checking homework was a good opportunity to interact with students on a more personal level. Problems with the homework were talked about, as was, who had helped them at home, or what were the reasons for not doing it, and good and bad habits in homework tasks were noted. Checking homework completion was done regularly and recorded but no consequences for doing, or not doing, homework were given, apart from praise or censure. The record was useful when it came to writing the students' reports because often there was a correlation between the students overall performance and their ownership of homework tasks.

Students were encouraged to seek help at home or ring up friends if they did not 'get' the homework. This common response/excuse for not doing homework was countered by the teacher by "Did you ask someone at home?"

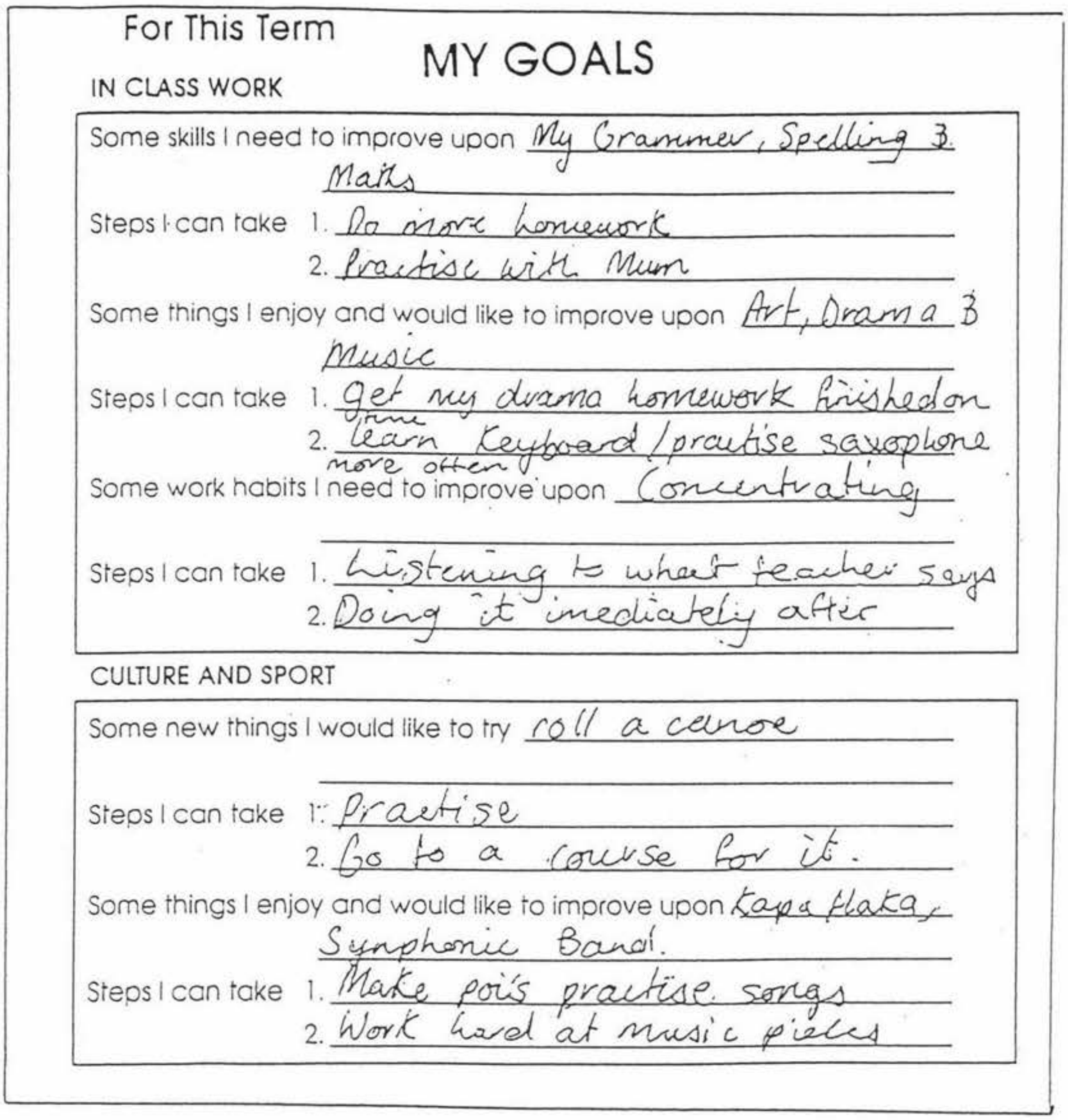
or "Could you have rung up A?". They were pleased to report if mum or dad had helped out and how they had 'learned a new way of doing it'. In this way students were encouraged to see homework as an opportunity to learn and consolidate understanding as well as practice skills.

Students were able to mark most of the homework themselves and they were encouraged to check progress periodically. Some of them obviously copied results directly from the answers page, although noted and sometimes commented on, this was not made an issue of. Reliable students used their marking to self correct and highlight points of confusion. If several students asked about some items of homework then we would go over it in class making emphases about points that obviously needed clarifying.

Homework was valued by students throughout the year; even students who did not complete homework knew that they should be doing it to improve their learning. Some students took considerable effort to catch up on missed homework and liked it recorded as done when checked the next day. "Can you change it (*the record that it was previously not done*) please?"

- **Planning**

Planning was a metacognitive behaviour that, on reflection, should have been stressed more in the study. The students used school diaries in which they recorded homework and test dates. These diaries also had term goal setting pages which gave them an opportunity to reflect on practices and skills that they felt needed more effort. Typically, students recorded they needed to keep up with homework, improve revision for tests, be more organised, and, use their diaries more. Some students filled these pages in willingly, others needed persistent checking. At the end of term these goals were self-assessed. An example of a student's diary page is shown in Figure 4.4.



For This Term

MY GOALS

IN CLASS WORK

Some skills I need to improve upon My Grammer, Spelling & Maths

Steps I can take 1. Do more homework

2. Practise with Mum

Some things I enjoy and would like to improve upon Art, Drama & Music

Steps I can take 1. Get my drama homework finished on time

2. Learn Keyboard / practise saxophone more often

Some work habits I need to improve upon Concentrating

Steps I can take 1. Listening to what teacher says

2. Doing it imediately after

CULTURE AND SPORT

Some new things I would like to try roll a canoe

Steps I can take 1. Practise

2. Go to a course for it.

Some things I enjoy and would like to improve upon Kapa Haka, Symphonic Band.

Steps I can take 1. Make poi's practise songs

2. Work hard at music pieces

Figure: 4.4 **An example of a student's diary planning page**

Apart from diary entries I gave the class little scope to demonstrate planning and other activities were not arranged for students to practise this skill. It is doubtful that without positive reinforcement or deliberate evaluation of performance in relation to planning that students fully appreciated the value of planning as an effective strategy.

4.3.2 Constructing behaviours

• Reflecting

Reflecting was purposely built into the programme. There were regular tests during the year and students were asked to reflect on their performance after these. The first was after the Skills Analysis test.

Skills Analysis is an activity done across all mathematics classes at Form 3 level at the school. It is done in the first month of the school year and repeated at the end of the year. Students are given a written test on the five areas; Number, Measurement, Geometry, Algebra and Statistics. The work tested is what a teacher would reasonably expect to have been covered in primary and intermediate years of schooling. The results provide the teacher with a profile of an individual's knowledge and the statistics create a picture of the class' knowledge. Although there are more than 25 questions the test is arranged into 25 skill areas and each area was allocated one mark; if a student got all or nearly all the work in a skill area the mark was awarded. Examples of skill areas are: Make sensible estimates and check answers; Change analogue time to digital and vice versa; Draw and interpret simple scale maps; Use a systematic approach to count "possible outcomes". Students' results showed a wide range of performances.

In order to encourage students to reflect on their overall achievement, they were asked to analyse their performance after their results were returned. They wrote comments like:

- *I'm quite good at Algebra and Numbers. Geometry and Statistics is where I could do some work.*
- *Need work on Algebra and I am alright with others, but still getting some wrong.*
- *I am quite good at Number and Geometry because I only got 2 wrong in each. Good work at Number, need work at Statistics. Overall perfect as usual.*

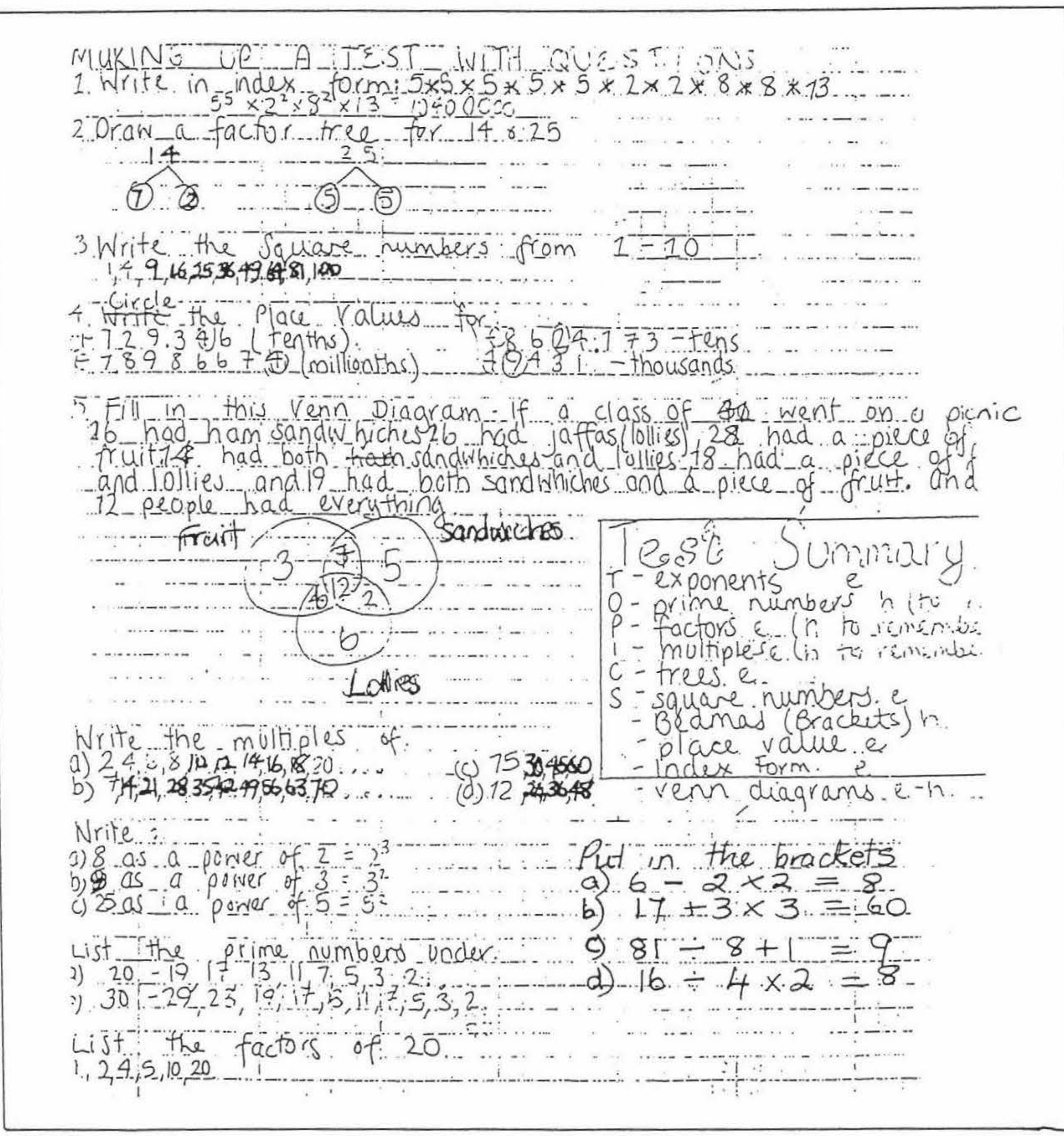
As expected, they related their comments to their scores in the different areas tested. No one highlighted specific problems within the broader areas. One encouraging sign was how positive they were about themselves, concentrating largely on what they felt their strengths were. On reflection, it would have been good to get feedback on how they felt they could have improved their knowledge and understanding of weaker areas. Informally, I did get this. Since the test was given early in the year some students felt they could blame their prior teaching, or their perceived lack of teaching, at earlier schools.

To encourage students to monitor and reflect on their learning, at the end of a topic they recorded what they had learned and what they felt they needed to learn better. Items were discussed and recorded on the board, in brainstorm format, so there was some uniformity and agreement about what a topic had entailed. Students recorded the lists, which were short, but focused them on sorting out the main features of a topic.

By looking at achievement objectives in a topic; such as in measurement 'choosing suitable metric units', 'converting metric units', 'calculating perimeter, area and volume', etc., we could discuss what had been easy to learn and why, as well as what had been hard to learn and why. When they told me they knew concepts such as 'multiples and factors' I was able to probe deeper and ask what they meant; "Give me an example". This led them to saying "I know the multiples of 3 are 3,6,9,12 etc and the factors of 3 are 1 and 3. Multiples go bigger and factors are smaller". By giving instances they showed an understanding of the content of the lessons and verbalised learning strategies which other students often picked up on.

They also wrote test questions corresponding to topics, with answers. Some wrote original and challenging questions; others copied examples from worksheets or made up trite questions. Good students answered their questions well; weaker students supplied answers that were incomplete, or wrong. The class worked willingly on these activities because it was self-directed work and gave scope for originality. They indicated that writing their own tests had helped them revise.

Writing summary lists and questions did not seem to have prompted the depth of reflection intended for the class; analysing the lists confirmed what was intuitively known. Sensible students accurately distinguished the difficult features of topics, and were able to concentrate on learning these ideas. Weaker students preferred to ignore the harder aspects of topics and were happy to study the more basic ideas, which they already knew. A's questions (Figure 4.5) shows how she used her list to think up challenging questions.



MAKING UP A TEST WITH QUESTIONS

1. Write in index form: 5×5×5×5×5×2×2×8×8×13

$5^5 \times 2^2 \times 8^2 \times 13$ = 1040000

2. Draw a factor tree for 14 & 25

14 → (7) (2)   25 → (5) (5)

3. Write the square numbers from 1 - 10

1, 4, 9, 16, 25, 36, 49, 64, 81, 100

4. ~~Write~~ Circle the place values for:

7 2 9 . 3 4 6 (tenths)   8 6 2 4 . 1 7 3 - tens

7 8 9 8 6 6 7 (5) (millionths)   7 9 4 3 1 - thousands

5. Fill in this Venn Diagram - If a class of 40 went on a picnic 16 had ham sandwiches 26 had jaffas (lollies), 28 had a piece of fruit 14 had both ~~both~~ sandwiches and lollies 18 had a piece of fruit and lollies and 19 had both sandwiches and a piece of fruit. and 12 people had everything

Fruit   Sandwiches

3   7   5

4 12 2

6

Lollies

Write the multiples of:

a) 2 4 6, 8, 10, 12, 14, 16, 18, 20 . . .   (c) 75 30, 45, 60

b) 7, 14, 21, 28, 35, 42, 49, 56, 63, 70 . . .   (d) 12 24, 36, 48

Write:

a) 8 as a power of 2 = 2^3

b) 9 as a power of 3 = 3^2

c) 25 as a power of 5 = 5^2

List the prime numbers under

1) 20 - 19 17 13 11 7 5 3 2

2) 30 - 29, 23, 19, 17, 13, 11, 7, 5, 3, 2

List the factors of 20

1, 2, 4, 5, 10, 20

Put in the brackets

a) 6 - 2 × 2 = 8

b) 17 + 3 × 3 = 60

c) 81 ÷ 8 + 1 = 9

d) 16 ÷ 4 × 2 = 8

Test Summary

T - exponents e

O - prime numbers h (to ...

P - factors e (h to remember

I - multiples e (h to remember

C - trees e

S - square numbers e

- Bedmas (Brackets) h

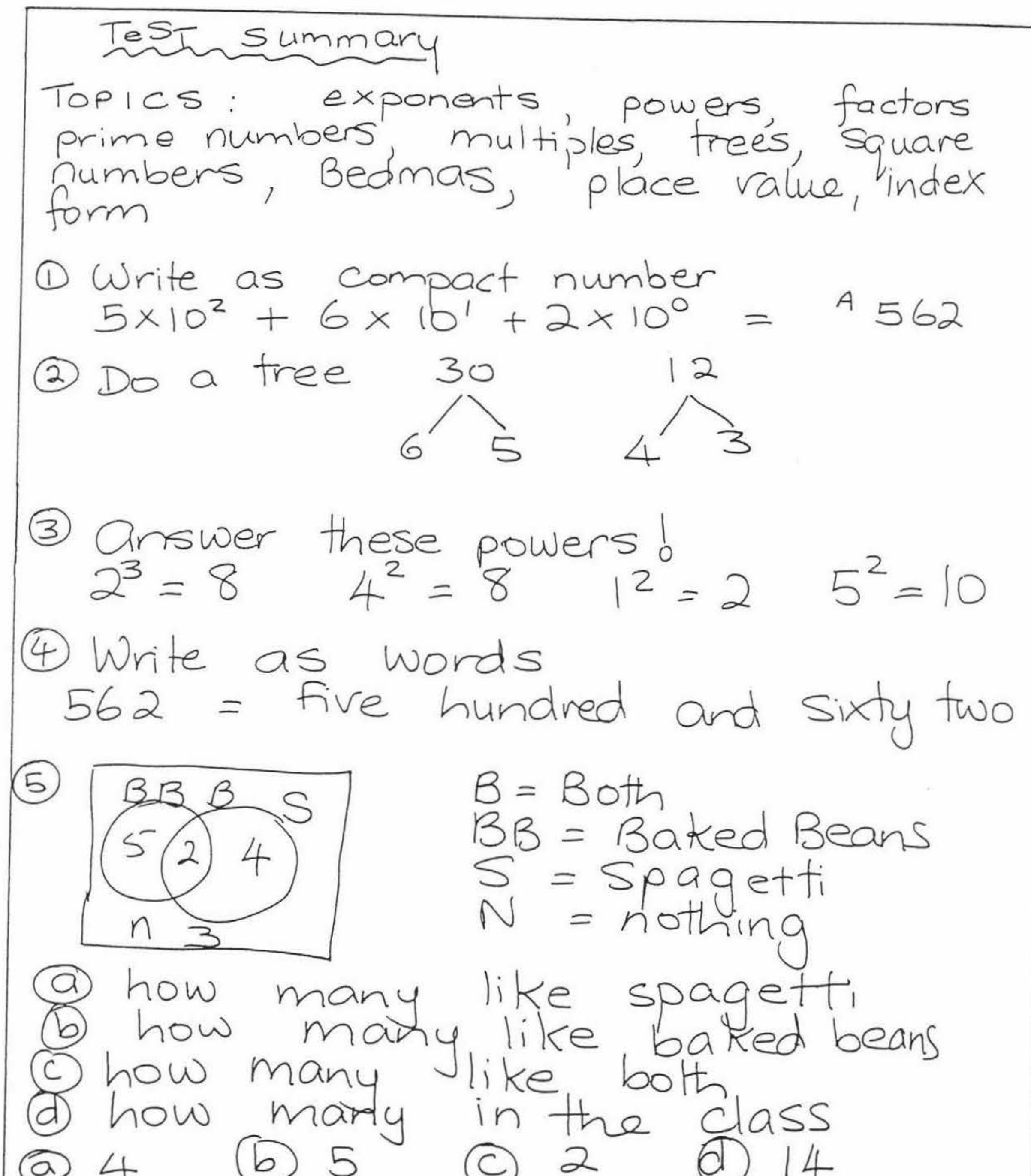
- place value e

- Index form e

- venn diagrams e-h

Figure: 4.5 An example of a summary list with challenging questions.

In contrast to the above list, B's list (Figure 4.6) was followed by easy questions, some of which she answered incorrectly.



Test summary

Topics: exponents, powers, factors prime numbers, multiples, trees, square numbers, Bedmas, place value, index form

① Write as compact number
$5 \times 10^2 + 6 \times 10^1 + 2 \times 10^0$ = A 562

② Do a tree 30 → 6, 5   12 → 4, 3

③ Answer these powers!
$2^3 = 8$ $4^2 = 8$ $1^2 = 2$ $5^2 = 10$

④ Write as words
562 = five hundred and sixty two

⑤

B = Both
BB = Baked Beans
S = Spagetti
N = nothing

ⓐ how many like spagetti
ⓑ how many like baked beans
ⓒ how many like both
ⓓ how many in the class

ⓐ 4   ⓑ 5   ⓒ 2   ⓓ 14

Figure: 4.6 An example of a student's questions and answers:

• **Linking ideas**

At the start of a new topic an overview of the content was given. This was to encourage students to work on linking ideas. Previous knowledge and concepts were recalled and supplied by students. For example, when statistics was studied several students recalled 'averages' and explained how they were obtained by adding up all the data values and dividing by the amount of data values; they explained the differences between bar, pie and line graphs.

The students were encouraged to relate ideas, not just back to other concepts already known, but also in terms of practical examples. When angles on parallel lines were studied, we visited the local playground to observe instances of parallel lines on the structures, such as the bars on the edge of the slide, the chains holding the seats of the swings, the seesaws when they all point the same direction. Back in class we discussed how parallel lines and transversals are important in constructions. The introduction of alternate, corresponding and co-interior angles followed.

Sometimes they asked "what if?" questions related to the revision. For instance when learning rotation one student asked what happens if you need to rotate a shape more that one full turn. This led to a discussion of pirouettes, dials on the oven and windmills. It was agreed that $1^1/_4$ turns = $^1/_4$ turn.

It was stressed that the ability to link new ideas to old is fundamental to learning work and that there are unifying themes, such as change, pattern and shape, that run throughout mathematics. Following Coxford's (1995) suggestion, mathematical connectors such as graphs, variables and transformations were used so that students could see the use of these ideas in many situations.

Students were also given reminders like "Remember when we did integers, what was the rule for multiplying a negative by a negative; we have to use that here as in $-2(x - 3) = -2x + 6$". They were encouraged to recall connections and comment on them. For instance, a student who likened "taking away a negative as being a good thing; hence a positive". Catch phrases like, "how will you remember this?" for recalling definitions, and "what will remind you of this?" were used in the lessons. Also, mnemonic memory strategies, such as alternate angles are in a Z shape - **A** (for alternate angles) to **Z** (for the picture) were often suggested by both teacher and students. However, other common strategies, such as "2 minuses make a plus", a popular call, which worked for 3 – -4 =7, but not for –3 –4 = 7, were analysed for their shortcomings. Words used in mathematics, like perimeter, were discussed as to how they could become part of the understanding. So, from perimeter we got 'p' for 'plus' and 'rim' for the outside; while from area we got 'are' which comes from squ'are', times square (in New York). Making informal connections was an increasingly common practice done in the classroom. Lampert (1998) comments on the inarticulate nature of much teacher-student talk and yet notes how both groups seem to understand each other.

• **Assuming a position**

Students were hesitant to challenge opinions, express disagreement or offer alternative explanations. Not many would regularly assume a position that stood out as uniquely their own. One activity done to promote this was the use of "debates". Using an idea from the ESRC project (Bell et al.,1993) cards outlying two conflicting points of view were given to two different groups of students and they were to argue for them. Some examples were:

"You learn more from working on one hard problem, than from working on ten easy problems", against "You learn more from working on ten easy problems, than from working on one hard problem". "You learn a great deal by discussing common mistakes" against "Discussing mistakes is a waste of time. The teacher should just tell us how to get things right".

This activity was sufficiently different from the normal routine lesson that it threw some students who said they thought it was silly and "Even though I don't agree with my card I'll make something up, but, it won't be true and I'll really agree with the opposite idea". To give students some guidelines it was agreed that five supporting statements would be given and whether or not they personally agreed with them was unimportant. Some interesting points of view were put forward; the main value of the activity probably was the open discussion about the different ways of learning, and the way the activity highlighted for me, how deep seated some of the student's beliefs were.

4.4 Students' Awareness of Purpose of Activities in Mathematics

Metacognitive teaching works at raising students' awareness of their learning. For students to take greater control of their learning they must become more alert to what and how they are studying. Making metacognitive aims explicit to students is important in raising their awareness. The behaviours that help learning and the reasons for doing things in class were discussed with the class throughout the study and highlighted with specific activities.

4.4.1 Deciding the purpose of an activity

A small study was done with the class to see how aware they were of purposes of mathematical activities. An intervention was taken from the ESRC project (Bell et al.,1993) whereby students were given examples of various mathematical activities and asked to rate their purpose. They were not expected to do the activities, merely to decide why they might be done the way they were presented. From a list of seven purposes they were asked to rate each from 2 = a main purpose, 1 = helped a bit, 0 = not a purpose, in terms of the activity.

Activity 1

The first activity described deciding on the correct answer for an ordering of decimal numbers (Bell et al., 1993, p. 16).

Anne's class was given this question in a test. ***Which of these three decimal numbers is the biggest? 0.4, 0.236, 0.62***

10 people said 0.4 is the biggest. 15 people said 0.236 is the biggest. 5 people said 0.62 is the biggest. The teacher asked the class to think about the question again and, in groups of three, try to agree on the correct answer and explain your choice.

The students' evaluations showed some confusion as to why the activity was done. They probably interpreted the purpose of the activity more in terms of solving the actual problem than the discussion to agree on a correct answer. Their mean results are summarised in Table 4.5, with Bell's evaluation alongside.

Table 4.5 Students' mean evaluations of purposes of activity 1

Purpose to help the group:	Class mean	Bell's evaluation
practise calculating decimals	1	0
solve in an organised way	1.17	0
learn to work neatly	0.26	0
understand decimal numbers	1.61	2
find out who got the wrong answer	0.39	0
get better at discussing and explaining	1.57	2
see how to use maths in everyday life	1.04	0

Activity 2

This activity described drawing a geometrical design from a verbal description without being able to look at the required final outcome (Bell et al., 1993, p. 17).

Christine's maths teacher gave her this task. ***Sit back to back with a partner. You hold a picture. Your partner has a blank sheet of paper and drawing instruments., Describe your picture so your partner can draw it - without looking.***

The question sheet also showed a clear diagram of two students doing this activity and was less likely to cause confusion than the previous activity. The class mean results, which agree quite well with Bell's evaluations except for item (d) "to discover relationships between angles and shapes", are summarised in Table 4.6 .

Table 4.6 Students' mean evaluations of purposes of activity 2

Purpose to help the students:	Class mean	Bell's evaluation
practise drawing & measuring	1.09	1
plan & organise a drawing	1.17	2
learn to work neatly	0.39	0
discover relationships	1.26	0
remember words like "square", "right	1.39	1
get better at describing shapes	1.87	2
see how to use maths in everyday life	0.78	0

Activity 3

This activity described a word problem that involved calculating the cost of newspapers for a year given the cost of a daily and a Sunday paper (Bell et al.,1993, p. 12).

Paul's maths teacher gave him this problem to do. ***My Sunday paper costs $0.60 and my weekday papers cost $0.35. What will I spend in one year on newspapers.*** *This is what Paul did. He used a calculator and worked with a partner.*

Paper for Sunday	*$0.60*
Papers for weekdays	*$0.35 x 6 = $2.10*
Total for week	*$2.70*
Total for year	*$2.70 x 52 = $140.40*

The class mean results are summarised in Table 4.7.

Table 4.7 Students' mean evaluations of purposes of activity 3

Purpose to help Paul:	Class mean	Bell's evaluation
practise multiplying quickly	1.77	0
know when to multiply	1.36	2
learn to work neatly	0.18	0
think what decimal numbers mean	1.18	1
remember rules for multiplying	1.14	0
get better at discussing and explaining	0.91	1
see how to use maths in everyday life	1.27	2

This was a straight forward activity, but students rated "practise multiplying quickly and accurately" and "remember the rules for multiply" as important purposes, whereas a calculator could have been used and the activity was focused more on students' knowledge of how maths is used in everyday life and that they knew when to multiply. Students, obviously see skill based work as more important than applying mathematical knowledge.

Activity 4

This activity described a number pattern problem that involved working with a series of brick pyramids of varying heights and calculating how many bricks are needed to build the pyramids (Bell et al., 1993, p. 15).

Brick Pyramids. *This is a pyramid of 9 bricks. It is 3 bricks high. Suppose we make a pyramid 10 bricks high. How many bricks will we need? Write down everything you do. Rani worked by herself.*

The mean results are summarised in Table 4.8.

Table 4.8 Students' mean evaluations of purposes of activity 4

Purpose to help Rani:	Class mean	Bell's evaluation
practise skills like making tables	1.69	1
think about organising her work	1.22	2
learn to work neatly	0.39	0
understand what a pyramid is	1.39	0
use words like "odd" & "square"	1.35	0
get better at writing explanations	1.3	2
see how to use maths in everyday life	1.09	0

The two obvious disparities are the students' rating of "understand what a pyramid is" and "learn and use words like 'square' and 'odd' numbers". This may have been because they themselves have trouble remembering concepts like these, and see all such activities as 'testing' their knowledge. Again students have rated skill based purposes higher than thinking and communicating purposes.

Activity 5

This activity described a party and the costing of refreshments (Bell et al., 1993, p. 14).

It will soon be the end of term party. Our class will look after selling the refreshments. About 200 people will be coming. Decide what food and drink we should get. In groups, work out how much we need to buy. Work out what it will cost. Decide how much we will sell each item for.

The mean results are summarised in Table 4.9.

Table 4.9 Students' mean evaluations of purposes of activity 5

Purpose to help the groups:	Class mean	Bell's evaluation
practise adding & multiplying	1.65	1
learn how to plan & organise	1.96	2
learn to work neatly	0.39	0
think what adding & multiplying mean	0.96	1
learn what food people like & costs	1	1
get better at discussing & explaining	1.57	1
see how to use maths in everyday life	1.26	2

Students accurately identified "plan and organise" as a significant purpose. They were fairly accurate in all their assessments for this activity.

Overall, where the students are most in agreement with educators' purposes of lessons can be seen by calculating the mean of the absolute differences of each of the scores from Bell's evaluations under the different headings, which in some cases were consistent for each activity, but in other cases varied. The descriptions of the purposes have been summarised under broad titles, as follows (see Table 4.10). A value close to zero indicates close agreement, and a value close to one indicates disagreement.

Table 4.10 Overall summary of students' mean evaluations of purposes

practice	strategies	work neatly	understand	learn/remember	discuss	use in everyday life
0.84	0.57	0.32	0.60	0.80	0.31	0.82

The students agreed fairly closely with the evaluations for working neatly and being able to discuss and explain the work. They were in most disagreement with the evaluations for the need to practice, to learn and remember work, and the use of mathematics in everyday life. They saw practising, learning (i.e. being able to recall quickly) and remembering work as important purposes of mathematical activity, whereas these are not necessarily the intended aims of activity. The application of mathematical knowledge was not seen as an important purpose when it was intended, and conversely, was rated quite important when it was not intended to be.

The results of this activity highlighted for myself the need for clarifying the purposes of work in mathematics with students.

4.5 Teacher awareness of metacognitive skills

Given the curriculum expectations to develop mathematical processes and appropriate metacognitive skills, as the teacher it was important to be aware of students' use of these skills and think about how they could be developed and taught in the classroom. This meant that as the teacher, I needed to act metacognitively in my role as well. Striving to be actively aware in planning, teaching and classroom organisation had the advantage of explicitly modelling self-regulatory practices within my teaching. The activities were not construed just for immediate and obvious effect, but for longer term and deeper effects. In other words, to introduce a new idea to the class, thought had to be given to the content of the idea, how it was best explained and that explanation would fit with the mood of the class, and, how it would tie in with other ideas previously taught in class. The teacher acting metacognitively is something argued strongly for by many researchers (Fravillig, Murphy & Fuson, 1999; Manning & Payne, 1996; Senger, 1999).

Probably the main value of this project was the focusing done on students' learning. While this is something in the day to day teaching of all classes I attend to, and am interested in, by trialling specific tasks and observing student response I came to clarify and realise aspects of learning mathematics that had been taken for granted. The main sources of the methods and activities used (Baird & Northfield, 1992; Bell, 1993) were anchored in the constructivist philosophy and presupposed an active learning style was being fostered by the teacher. These caused me to examine my own beliefs in teaching and learning, and although I felt I was already a constructivist teacher, I realised there was still a way to go for me on this pathway. For instance, letting go of the authoritative stand of the teacher with the absolutist cum instrumentalist position is not simple. There were many times when the quickest, most efficient way to teach seemed to be to say, "Take my word for it, I know that such and such is the case; now, if you learn that, you will pass".

Although I felt the study was undertaken from a personal stance of commitment and belief in working towards metacognitive teaching and learning, I was still looking for shifts in practice and deepened understanding of learning issues. The changes incurred by programme such as this one can be subtle and not immediately obvious. Some of the classroom interventions have not been repeated but many routines that were developed in the programme and seen to work have been developed and cultivated. Obviously other students in other classes taught by myself have gained from this programme. Their learning styles have been developed and challenged too.

There was good morale in the class over the year and students recognised the interest in them, as learners. It would be difficult to claim that students' behaviour changed significantly because of the interventions and the focus of the program. However, students became more aware of the need to talk about learning, to organise themselves for learning, to practise for learning, to think about learning, to link learning. Some found the increased control of the subject motivated them better; but some others found the demands of taking ownership of their learning too taxing.

CHAPTER FIVE

Discussion and Conclusion

5.0 Introduction

The goal of this study was to investigate students' dispositions to learning mathematics, and, through encouraging the development of metacognitive knowledge and behaviours, enable students to develop an awareness of their learning. With reflective teacher practice, I was able to focus on my own understanding of the nature of mathematics, mathematics learning and mathematics teaching. Classroom routines were modified to fit better with a developed understanding of learning and teaching. In short, this study was an exploration of the constructivist teaching approach.

As part of a teacher research project, I worked with the Form 3 mathematics class which I taught. Classroom activities were investigated to see the extent of students' metacognitive attitudes and beliefs about mathematics and mathematics learning. Over the course of three terms, my attention was focused on how students' learning behaviours responded to teaching methods. In order to raise students' awareness of learning, both cognitive and metacognitive strategies were stressed within classroom interventions intended to enhance learning. My own beliefs and attitudes, as the teacher, were considered to see if working from a specific metacognitive stance shifted these significantly.

In this chapter issues arising from this study of metacognition in the classroom are examined in terms of students' learning and in terms of my teacher development. Implications, arising from the above issues, are outlined and suggestions for further research are developed.

5.1 Students' Responses

At the beginning of the study students reported a positive attitude to mathematics and learning. They thought the subject was useful, and they believed that in mathematics they make progress, because they learn new ideas. These responses were similar to those found in Bell's (1993) study, for the most part. However, Bell's students felt strongly that they worked hard, whereas my students were more ambivalent about their ability and work habits in mathematics, although they were not negative about these attributes. An appreciation of the satisfaction and good feelings real achievement in mathematics brings was also expressed. An activity to make up a simile for mathematics showed beliefs in the value of practice, learning new ideas, and seeking help. Interestingly, students disapproved of copying or watching others work. They saw independent, self-motivated work as more important and helpful.

Students showed they perceived mathematics as a structured discipline with rules, but they agreed that there could be more than one way to solve a problem. Considering students' conceptions of mathematics are influenced by teachers' conceptions (Thompson, 1993), it is worth noting how conservative some of the students' ideas were. After 9 years of schooling, these views support the viewpoint that many teachers still hold fixed beliefs about the subject, and highlight the arguments of reform educators who stress that shifts in beliefs and practice must therefore ultimately filter down from teachers, who must be educated to change (Hiebert, 1999; Peterson, 1988).

The students indicated they liked to gain understanding of the subject and they believed the teacher and the text book were helpful ways towards this. As Anthony (1996a) noted, these resource strategies, used to gain help, can be ineffective if they are merely used to complete tasks, and not to construct student understanding. Teachers have to be careful that students see themselves as learners with appropriate learning goals; content coverage is low level compared to content mastery (Anthony, 1996c). Students also showed they found that working with others could be helpful to learning. This fits with the idea of the classroom as an environment for collaborative mathematical thinking and learning (Stein, Silver & Smith, 1998). Similar responses, to the above, were also shown in Bell et al.'s (1993) study.
In the study, students' self-evaluations of their work showed they were positive about aspects that they clearly understood; but they focused on areas that were low level and already established in their knowledge schema, and

they skimmed over areas where they had difficulties. They looked at their performance in terms of large topic areas and did not make serious attempts to highlight any misunderstandings within a topic; nor did they seem to attempt to find out details of what they had got wrong. Students who achieved well, did these activities in much greater depth than those who achieved poorly. This is similar to findings by Bell et al. (1993) and Baird and Northcross (1992).

5.2 Specific Interventions

Notes books were used as records of students' efforts. Class discussion and explanations were recorded as much as possible in students' own words to formulate the notes. Organisation of content was modelled by the teacher and looked for in the students' work. Students mostly maintained well organised notes. In spite of strongly agreeing that copying others' work was not helpful to learning their preference for the security of notes that came from some authority, such as a textbook or the teacher, was held strongly throughout the study. When asked to write their own summaries they tended to copy everything from the textbook.

Within the study opportunities were created for the notes book to be used as a personal resource for learning, by setting open book tests, and, by using their notes books to make study lists and write sample tests. Students related well to using notes in class tests, and this activity helped highlight the need for good personalised reference material. The advantages of using personally compiled reference notes was actively reflected on in classroom discussion. It was also noted that students preferred to use their own handwritten notes over the textbook, which, in some cases had the same material. Students who were disorganised or poor attenders developed poor notes and found this discouraging. They would prefer to use another student's notes to refer to, over the textbook. Their lack of organisation and/or commitment added to their disenchantment with the subject as the year progressed.

In review sessions students practised using concepts and ideas. Discussion, leading from reviews, gave opportunities for the teacher and students to learn metacognitive skills. Questions, both from the teacher to the class and individuals, and from students to the teacher and one another, became more frequent and of better quality. More open questions than closed questions became the norm, which was a noticeable and conscious shift for me as the teacher. However, the tendency for me to ask 'funnel' type questions was strong - it was not easy to break a habit of questioning to obtain the 'right answer. Time constraints, such as pressure to complete a topic, can cause one to relapse from teaching based on constructivist ideals to more traditional transmissive teaching. During the study, students were encouraged to judge and monitor their own understanding. Awareness of what was required to be understood and learned was strengthened by emphasising both verbal and written explanations in discussion, reviews and daily classroom activities. The quality of discussion about mathematics was enhanced through increased questioning and debate. Techniques and strategies for learning ideas were shared, as was the need for these to help learning in mathematics. Good learning behaviours, such as seeking assistance by asking why they went wrong, checking personal progress, planning, reflecting on the work, making links to beliefs, experiences and prior knowledge, and, offering opinions and ideas, were noticed, highlighted, and discussed.

Cooperation, getting along together, and valuing others' ideas were stressed as part of the working atmosphere. The fact that we were all learning together with a common aim of increasing understanding and knowledge underlined all the classroom decisions. Taking risks and seeking help were emphasised as being important to successful learning. Specific cooperative activities, such as EQUALS' (Erikson, 1989) cooperative logic puzzles and Bell's (1993) mini debates encouraged students to listen to different points of view and think widely about issues and problems. These activities enabled me, as the teacher, to appreciate more clearly how fixed student views were, but also to watch students articulate points of view, sometimes with great conviction, and sometimes with difficulty. The activities posed a challenge in terms of classroom management. Students, who were not used to the change in style of lesson, needed clear guidance. It was also noted how the students found it difficult to cope with justifying a statement they disagreed with in the mini-debates. Rules for these activities had to be stressed, to prevent dominance by one or two individuals, and ensure participation by all. Originally I had envisaged being able to take notes, while students conducted these activities, but that was impossible. My role was as a very busy 'chair'. While taking care to coordinate these activities and ensure they ran fairly and smoothly, I was mindful of the need for the students to use their group situations to solve the problems and come up with the ideas. It is all too easy for the teacher to intervene with a 'too helpful' hint, or answer, to move things along.

When studying students' perceptions of the purposes of activities in mathematics, the students showed near agreement with the teacher's perceptions in familiar closed tasks; but they were in less agreement with unfamiliar open tasks which incorporated a greater proportion of higher level skills. The need for the teacher to explain and discuss the purpose of activities in mathematics was highlighted through this intervention.

5.3 Metacognitive Behaviours in the Classroom

Metacognitive behaviours were also studied. Students' awareness of what constitutes learning, and their ability to control and monitor their own learning, was observed to greater or lesser degrees in all students. Monitoring behaviours, such as seeking assistance, checking work, and planning and reflecting, were all exhibited to varying degrees. There was a sense of needing to be an active learner, even if this was not manifest in all students at all times. Students, who made genuine efforts only occasionally, liked these to be noticed and would comment on their 'extra' efforts. Although some students used strategies effectively there were equally others who used them ineffectively. Their learning behaviours matched observations made by Anthony (1996b) whereby they either failed to realise the need for strategy use; or, had inadequate knowledge for the problem they identified; or, applied strategic knowledge ineffectively; or, chose not to solve the problem.

Students, given guidance and instruction about skills such as rehearsal (practice), elaboration (explaining knowledge in one's own words), organisation, self-regulation (monitoring one's use of resources for learning), time and study management, effort management and help-seeking behaviour, still had to choose to act in these ways themselves. It was found that high succeeding students remained motivated, and used and adapted metacognitive strategies more readily and effectively than those students who were inconsistent in their efforts.

With encouragement, students increasingly demonstrated metacognitive attitudes and behaviours, such as reflecting on their work and seeking help, deciding ways of learning, and linking new ideas back to previous knowledge. When these activities were attempted, it took personal effort, but committed students persisted. They discussed their work with others and thought of possible alternative solutions and scenarios, they devised memory strategies, they argued and discussed misunderstandings, they selectively chose problems to do, they read their textbooks independently of me, their teacher, and they showed interest in the overall structure of the mathematics they were learning. They learned effectively because their methods indicated a value of metacognitive practices and knowledge. They refined and developed existing metacognitive procedures, and adapted new ones with help. Students who already use procedures successfully, learn quicker and more readily than others. However, students' awareness of learning varied from being actively interested and keenly self-monitoring to being disinterested and easily distracted by other issues. Students who showed high interest and awareness were, not surprisingly, the strongest performers.

Although I was consciously trying to raise the profile of metacognitive strategies and increase the value of such behaviours through assessment, feed-back and self-reflection, peer modelling was often the most effective influence. If friends showed good learning behaviours, students usually did likewise. Highly motivated students tended to seek out others with similar aspirations and indifferent students gravitated to other indifferent students. What was obvious, was, students sat beside learners who were inclined to show similar traits to their own. However, motivation to learn can be positively affected by the task and the teacher, as well as the learner. Students can be directed in ways to develop better awareness of learning, though they have to make a personal commitment to learn before any guidance is effective. The teacher's challenge is to ensure that interaction between students, and between teacher and students, are both of a high quality. The learning environment must be supportive of, and value awareness of learning.

Students at high school level are greatly influenced by peers and will adapt procedures they see peers using successfully. They naturally listen to peer explanations or advice. The willingness to learn from peers appeared to be a significant factor in motivation to use metacognitive strategies. Teachers can endeavour to manage the conflict between monitoring learning and supervising behaviour by raising students' awareness of the control they can take in the learning process. If effective learning strategies are valued in the classroom then the frustrations of those students who normally resist classroom educational opportunities will diminish. The effect can snowball in a class; when enough students are working on target there is pressure for less motivated students to make efforts to become more involved.

Assessment and evaluation are powerful motivators in changing a student's learning focus. Students work on activities they see have some value. There is a challenge to teachers to manage effective valuable assessment that motivates students to use effective learning strategies. As the teacher, my tendency was to use assessment to be able to report on students' progress in terms of knowledge output. Given that assessment is usually part of a school-wide, or department-wide plan, there are restrictions on what the classroom teacher can reasonably manage to add to existing assessment structures. Activities need to be fair and equitable, so that students' achievement can be honestly explained. Assessment takes up time and teachers require efficient methods. Within this study I increased the amount of written feedback, both in terms of explaining specific misunderstandings and offering general support and encouragement. It would have been interesting to know how students responded to my advice and guidance. This was never followed through. However, I came to realise that students also need to learn how to give themselves feedback and act on it.

Home study and homework are areas that offer potential for student learning. This is when classroom content is consolidated by many students. In this study advice on specific strategies that students could adopt for effective use in independent study time was appreciated by students.

5.4 Teacher Development

I began this teacher research project after recognising in some larger studies, particularly the PEEL and ESRC projects (Baird & Mitchell, 1986; Bell et al., 1992) thoughts and practical suggestions that agreed with the ways I was thinking and trying to direct my teaching. Although I had encountered constructivist teaching ideas (e.g., through in-service courses, readings, and in conservation with colleagues) over a period of several years, and new approaches to motivating students had been experimented with, for some time, I still felt my teaching had not radically shifted emphasis for many years. Throughout this study there was a personal shift further away from the view that mathematics was a vast body of knowledge, some of which I could impart to students. My teaching moved further towards exhibiting a base of constructivist principles. I found this style of teaching came more naturally to me, as the study progressed, and I could more easily adapt activities and discussion to promote active learning.

An obvious shift has been noted in the parallel need for me as the teacher to act metacognitively while teaching. I now find that my thoughts are directed more to the students' learning of the content than the content itself. This change is most likely to have been facilitated by the fact that I felt confident that course-work was well known and understood by me. When shifting focus like this, it does mean content has to come as 'second nature', and, when teaching, my main concentration is on learning, with content incorporated as instances and examples in my language and thinking.

There has been an acknowledgement of the need for guiding principles for good teaching practice. This study has led me to an adaptation and adoption of some of those principles outlined earlier in the study (Angelo, 1991; Baird & Northfield, 1992; Begg, 1996; Pirie & Kieren, 1992). In my classroom I hope to be carrying out practice that reflects beliefs in:

1. The need for active learning by students, and for metacognitive teaching to optimise that learning.
2. Students need to be helped to develop an awareness of what needs to be learned and how it can be learned.
3. The need for my teaching goals to be made explicit to the students and for these goals to fit with their goals.
4. Links and connections from previous experience and knowledge are important for new learning to hook onto.
5. Learning involves organisation of some kind. It can be a personal method, but it needs to be able to be explained to another person.

6. Feedback is important to help students value their learning, but it can be self-feedback. Assessment, although important, is only one way of giving students feedback.
7. Learning takes effort, time and commitment.
8. Motivation to learn can be affected by the task, the environment, the student, the student's peers and the teacher, and it is alterable.
9. Learning is a social process. There needs to be quality interactions happening in the classroom between teacher and learners and between the learners themselves.

Although the process and the product of teacher research are rewarding, it still takes much time and effort. There is no drive within the teaching profession to do such work; it has to be self-motivated. I have continued with full time teaching and other related duties while completing this study. It is always challenging to find the 'space' to think deeply about professional matters in theory, in the midst of competing thoughts and responsibilities. Much of the real thinking and work has been completed in vacation periods.

Doing action research or teacher research in isolation is difficult, too. It would have been stimulating and encouraging to have been able to do this study with a team of other teachers who were similarly inspired and motivated. Conversation and discussion can stimulate and deepen thinking, pushing you in new directions, making you think and rethink about theories and practices. Although teachers discuss classroom matters daily informally, no one could have been asked to work through this extended research, unless they too had some goal to work towards and some incentive to inspire them. I have relied heavily on my university supervisor for extra motivation and drive. She is the one, with whom, I have discussed concerns and directions.

There is a real need for more teacher-release time to do research. It is increasingly seen as important for teacher professional development (Robertson & Allan, 1999). Professional development needs to be on-going and developmental. Teacher research, offering an established method of critical reflection is rewarding for teachers, and their students must benefit from the knowledge attained.

5.5 Implications

The issues above give rise to further implications for the learning and teaching of mathematics.

Every class offers a teacher its own dynamics and no class will be a duplicate of the one studied but methods and styles practised and analysed in this study have been adopted successfully with other classes. It is hoped that aspects of this study will be recognised by teachers, and, some observations may agree with others' experiences. However, whether other teachers will radically change their practice in light of this study, is unlikely, unless they adopt a teacher researcher stance.

Students have to be willing to learn but teachers can add structures to the learning context that help students. These can involve changes in teaching and assessment methods. Instruction about what leads to good learning, and advice to ensure that students engage in appropriate mathematical activity are directions the metacognitive teacher can take up.

Learning issues that were particularly noticeable in the course of this study were:

- the need for assessment to match learning and teaching goals.
- the importance of feedback for students; we need to know more about how students use teacher feedback and we need to develop students' ability to self-assess.
- the value of organisation for learning and how this must be emphasised to students; the methods used to organise oneself can be personal, but must be felt to be systematic in some way.
- planning and setting goals are good learning activities, but these must be monitored, and revisited, to be of worth.

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APPENDIX A

INFORMATION SHEET PROVIDED TO STUDENTS AND PARENTS

Dear Parent/Guardian and ______________

I am the mathematics teacher of 3B this year and I am completing my M. Ed. Studs. (Mathematics) course at Massey University. As part of my thesis I am interested in student awareness of their learning strategies in mathematics.

During the year we will follow the school Form 3 Mathematics scheme. Topics will be completed by doing exercises, investigations, and problem solving. Learning activities will also involve writing and using notes on topics studied, reflecting on what has been learned and summarising topics. Students will increase their awareness of how they learn best, and the different ways they can learn mathematics. All students will be expected to participate in all activities as these will be part of the usual class teaching programme.

The aid of students in 3B is requested to help me in my research. To support my research project I would like to use data from some of our class activities during terms one to three. Specifically, I would use examples of students' work and description of activities which are part of our classroom programme. Occasionally I may video or tape a lesson to assist in my recall of how particular activities helped student learning.

In my research I would like to include as wide a range of students as possible. I reiterate that regardless of participation in the study all students will be treated in the same manner - the study involves my reflection and evaluation of the classroom programme designed to increase student awareness of the learning process.

The information provided will be kept confidential and students' names and the name of the school will not be revealed. The information will only be used by me for my research and as my academic thesis publication. Students and parents have the right to ask questions about the research at any time.

Further questions about this research can be directed to me at school, phone ------- or to my research supervisor, Dr Glenda Anthony, Dept of Education, Massey University, phone ------- during working hours.

Yours sincerely

Eleanor Bourke

APPENDIX B

QUESTIONNAIRE ON ATTITUDES TO MATHEMATICS

How I feel about maths.

Tick one box for each statement.

	strongly agree	agree	not sure	strongly disagree	disagree
(a)Other people invented maths.					
(b)I usually find maths easy.					
(c)I usually enjoy maths.					
(d)I'm a bit lazy in maths lessons.					
(e)Maths is very useful to me.					
(f)In maths, I feel as if I'm making progress.					
(g)I can put my own ideas into maths. Sometimes I feel as if I'm inventing something.					
(h)I usually find maths difficult.					
(i)I find maths boring.					
(j)I work hard in maths lessons.					
(k)Maths is very useful to someone, but not me.					
(l)In maths, I feel as if I'm standing still.					

APPENDIX C

QUESTIONNAIRE ON PERCEPTIONS OF MATHEMATICS:

Learning maths is like ...

Tick one box for each statement.

		strongly agree	agree	not sure	strongly disagree	disagree
(a)	Learning maths is like learning a new cooking recipe. The teacher or book gives you step-by-step instructions. You just do what they say.					
(b)	Maths is like a jungle. The ideas are all jumbled up.					
(c)	Doing a maths problem is like crossing a river on stepping stones. There is only one way to go.					
(d)	Learning maths is like building a wall. You have to lay the bricks in order.					
(e)	You don't need to understand how maths works: You just need to practise doing it.					
(f)	Learning maths is like exploring an unknown country. You make lots of choices, where to go, what to do.					
(g)	Maths is like a jigsaw. The ideas fit neatly together.					
(h)	Doing maths is like finding your way through a maze. There are lots of possible paths to go down.					
(i)	Learning maths is like drawing a picture. It doesn't matter which bit you do first. it all fits together in the end.					
(j)	You need to understand each idea in maths before you use it.					

APPENDIX D

QUESTIONNAIRE ON WAYS OF WORKING IN MATHEMATICS:

Tick two boxes for each statement.

	How often do you work this way?			How much does this method help you to learn & understand math?				
	Often	Sometimes	Never	very helpful	helpful	OK	unhelpful	very unhelpful
(a)Copying work from the board.								
(b)Listening to the teacher explain.								
(c)Reading my textbook.								
(d)Copying out from my textbook.								
(e)Doing exercises from my textbook.								
(f)Making up my own questions.								
(g)Explaining something to my partner.								
(h)Discussing my mistakes.								
(i)Discussing other people's mistakes.								
(j)Watching other people working.								
(k)Copying someone else's answers.								
(l)Discussing ideas with a partner.								
(m)Discussing ideas with a small group.								
(n)Explaining something to my teacher.								
(o)Listening to my partner explain.								
(p)Working on problems.								
(q)Working on investigations.								

APPENDIX E

Application to Human Ethics Committee, Massey University, P.N.

TITLE: A Reflection on a Classroom Learning Environment with Emphasis on Encouraging Awareness of Learning Strategies

1. DESCRIPTION

1.1. Justification.

Much has been written about learning strategies of students in the classroom, but most research has been done by a third party observing the classroom process with the focus on the student. This study will be a first person documentation of the process of teacher and student adjustment to activities directed at enhancing awareness of learning strategies. This will lead to greater understanding of the interaction of teacher and student in a classroom based on constructivist principles.

1.2. Objectives.

This qualitative study will concentrate on recording details of student and teacher reaction to activities which require students to focus on their learning strategies.

1.3. Procedures for recruiting participants and obtaining Informed Consent.

The students in one of the researcher's junior classes in 1997 will be informed of the research by their form teacher. The form teacher will invite the class to assist me in my research project and give the students an information letter detailing the objectives and nature of the study. Students who wish to be included in the report will complete and return consent forms. Consent will also be required from students' parents. The Principal of the High School, and the Head of Mathematics have been consulted and an information sheet been presented to the Board of Trustees at the school. Their consent has been obtained.

1.4. Procedures in which research participants will be involved.

All students will be asked to complete journal entries outlining their thoughts on classroom learning activities. Tape and video recordings may be made of lessons to assist my recall of events. Lessons will include normal teacher/pupil interactions and discussions. Data for the research will include students' journal and work entries and teacher evaluation of activities. Participants will agree to having their written work photocopied if necessary and to having their actions reported in the study. There will be no extra involvement or time commitment required by participants.

1.5. Procedures for handling information and material produced in the course of the research including raw data and final research report(s).

A diary will be kept by the researcher. Students will note ideas, opinions and reflections in their journals. Recordings of some classroom sessions will be made. Photocopies of student work relevant to the research will be made. Only data from those students who have given their consent will be used in the analysis. This information will remain confidential to myself as teacher and researcher. In the report writing the anonymity of individual participants will be preserved. Security and confidentiality of records will be maintained at all stages of the study by keeping the records in a locked cupboard.

2. ETHICAL CONCERNS

2.1. Access to participants.

Participants will be students in one junior class taught by the researcher and they will have been given information about the nature of the research. Only data relating to behaviours of consenting participants will be analysed in the study.

2.2. Informed Consent.

Consent has been obtained from the Principal, the Board of Trustees and the Head of Mathematics. Students and parents will be informed of the nature of the project, the nature of their participation and their rights. They will exercise choice in their decision to participate or not. Student and parental indication of consent will be obtained on written forms.

2.3. Anonymity and Confidentiality.

The participants will have their names changed and the name of the school will be omitted. The only person to have access to the teacher's diary will be me. As would be the usual practice in class the students' reflections in their journals and their regular work will be private to themselves, their parents and me. The tapes will only be reviewed by me.

2.5. Potential Harm to Researcher.

There is no conflict of interest between my role as a teacher and as a researcher. This study, involving reflection on teaching practice, should be beneficial to both teacher and learner .

2.6. Potential Harm to the University.

None.

2.7. Participant's right to decline to take part.

This is outlined in the consent form. As the research will be carried out within usual classroom routines there will be no differential treatment given to participants and non-participants in class.

2.8. Uses of the information.

The information will be part of M.Ed.Studs (Maths) thesis and a copy of the report will be given to the B.O.T.

2.9. Conflict of Interest.

None.

2.10. Other ethical concerns.

None.

3. LEGAL CONCERNS

3.1. Legislation.

None.

3.2. Other Legal Issues.

None.

4. CULTURAL CONCERNS

None.

5. OTHER ETHICAL BODIES RELEVANT TO THIS RESEARCH

5.1. ETHICS COMMITTEES

None.

5.2. PROFESSIONAL CODES

None.

6. OTHER RELEVANT ISSUES

None.