Copyright is owned by the Author of the thesis. Permission is given for a copy to be downloaded by an individual for the purpose of research and private study only. The thesis may not be reproduced elsewhere without the permission of the Author.

"Mary, we will count it with you"
Inclusion of all Students in the Large Group Mathematical Discussion

A thesis presented in partial fulfilment
of the requirements for the degree of

Master of Education
in
Mathematics Education

at Massey University, Manawatū, New Zealand

Jennifer Ann James

2023

Abstract

Achievement in mathematics contributes to lifelong opportunities well beyond the mathematics classroom. Nationally, and internationally, high failure rates in mathematics see many marginalised students excluded from equitable higher education, career, and economic opportunities. Current studies in mathematics education emphasise the power of mathematical discourse to recalibrate equity. This study is an exploration of teacher actions that promote inclusion of marginalised students during the mathematical discussion in an inquiry model called Developing Mathematical Inquiry Communities (DMIC). While many studies focus on inclusion of marginalised students in mathematical discourse there have been no studies located specifically in the large group discussion under the DMIC model.

A case study, using qualitative research methods, was selected as most appropriate for the study. Two primary teachers, with experience teaching under the DMIC pedagogical model, participated in the current study. Both teachers engaged, respectively, in four mathematics lessons focused on their facilitation of the large group discussion. A range of data were collected and analysed, including interviews, classroom observations, photographs of student work, and teacher planning.

Findings revealed that when marginalised students were given opportunities to contribute, they exceeded their teacher's expectations. Specific teacher actions across seven pedagogical tools effectively promoted inclusion of marginalised students in the mathematical discussion. Teachers were enabled to enact these specific acts of inclusion through a shift in beliefs. Of significance was the belief shift where teachers transitioned from a fixed intelligence belief system to a belief in the fluidity of intelligence. Employing a growth mindset supported this belief shift.

This study adds to the literature in how teachers can include marginalised students in mathematical discourse. Evidence is provided which suggests that when educators explicitly address structural inequities in mathematics education, opportunities are provided for not only marginalised students but all students to bring their own strengths to discussion in relevant and meaningful ways.

Acknowledgements

I owe deep gratitude to my supervisors, Professor Roberta Hunter, and Professor Jodie Hunter. Thank you for believing in me and encouraging me to undertake a master's degree. I have been privileged to benefit from your extensive expertise. It has been an honour to work through this journey with both of you.

I wish to acknowledge the teacher participants. Thank you for being so open in sharing your practice, perspectives, and enthusiasm for developing inclusive pedagogy throughout the study. I acknowledge your flexibility around our scheduling during the pandemic, which was a testament to your unwavering professionalism. I feel privileged to have shared your journeys towards creating more spaces in the mathematical discussion for your marginalised learners. Thank you to all the students in both classrooms for allowing me to observe your wonderful mahi and, especially, to Mary and Duncan for showing us all what you can do when given the opportunity. Thank you to the principal and the board of trustees for allowing me to undertake the research within your school setting.

Thank you to all my colleagues in the DMIC (Developing Mathematical Inquiry Communities) team: the researchers and the mentors (who work tirelessly every day with teachers and children). I also acknowledge all the teachers, management, and educationalists I work with who share my vision for a brighter future in mathematics. It is my mahi, alongside all of you, and our reflections that has led me towards identifying my passion for inclusion of all students in mathematical discussions.

Finally, I would like to thank my family. Thank you, Acacia, Sophie, and Poppy, for the sacrifices you made so your mother could study. I am especially grateful to my husband, Matt, who assumed the lion's share of parenting and still agreed to read my work and give feedback. I am grateful to be married to someone who, like myself, has been involved in education for over twenty-five years, and is as passionate as I am about providing children with equitable opportunities to learn.

Table of Contents

List of Tables

List of Figures

Chapter 1: Introduction

1.1 Introduction

The current study examines the actions two teachers took, over the course of four lessons, to promote the inclusion of marginalised students in the large group mathematics discussion. The discussion is one phase of an inquiry model mathematics lesson. The teachers are in their fourth year of an equity reform professional learning and development course. This course is called Developing Mathematical Inquiry Communities (DMIC).

This chapter outlines the background to this study and introduces the context of this research. Section 1.2 highlights the global macrocosm of societal inequity that dictates the microcosm of institutional inequity at an educational level. It discusses how issues of inequity adversely impacts on specific groups of society and, in the classroom, specific members of the class. Section 1.3 explains the rationale of the study, and section 1.4 outlines the specific research questions and aim of the study. Section 1.5 provides an overview of the chapters in this thesis.

1.2 Background to the Study

The quest for equitable inclusion for all students in a mathematics classroom is wider reaching than simply an educational aim. It is a global political issue with social justice at its core. A just society is one where every member receives equitable opportunities to participate in the distribution of wealth and privileges available within that society (Archer, 2018). Apple (1992) states that mathematical literacy is awarded high status in educational reforms because it has socio-economic utility for those who already possess economic capital. Therefore, students who exit education with mathematical literacy have a greater advantage of accessing the wealth and privileges available in society than their mathematically illiterate contemporaries.

However, Basile and Lopez (2015) claim, those who hold economic and governmental power in the United States of America are not interested in social justice. They are interested in creating an elite workforce to advance their agenda, which is protecting its security and superpower status in an increasingly global market, with an astonishingly fast rate of advancing technology. This agenda promotes creaming off the most mathematically literate graduates to enter their workforce (Berry, 2018). The mathematically illiterate are dispensable. Similarly, Oakes and Lipton (1991) claim British society has a system favouring those with economic and cultural capital, which creates a self-fulfilling prophecy of inequity. According to Francis et al.

(2017), this inequity is reflected in today's British schooling system by the demographical distribution represented in mathematics ability groupings. Archer (2018) elaborates on Francis et al.'s claim, stating that white middle class students are most likely to be in the top sets, or ability groups, while working class and black students are most likely to be in the bottom sets.

Our system in Aotearoa reflects the same inequities as the United States of America and Britain. The current New Zealand Ministry of Education's Statement of Intent (2021) declares a priority focus on improving the outcomes for "all ākonga | learners, particularly underserved groups" (Ministry of Education, 2021, p. 14). This includes Māori, Pāsifika, students with special education needs, disabled students, and students from low socio-economic areas. Despite this well-intentioned statement, marginalised students remain overrepresented in the tail of underachievement in mathematics (Hunter & Civil, 2021). Averill (2018) attributes the lack of impact of these initiatives to the deep entrenchment of the dominant framework in New Zealand policy, history, and societal norms. The dominant framework Averill (2018) refers to is New Zealand's homogenous governance power base, which has its roots firmly planted in colonialism. While policy, even well-intentioned policy such as the Ministry's statement of intent, remains rooted in traditional western ideology it will be inherently exclusive.

This entrenchment of exclusion raises important questions around the ethics of mathematics education, especially when based on the premise that this entrenchment is deliberate (Apple, 1992; Berry, 2018). A government agenda to protect economic interests and generate the right type of workers for the workforce creates a utilitarian perspective of mathematical literacy, which, in turn, encourages a bias towards promotion of the social and economic interests of those with power (Berry, 2018). The marginalisation of race, sex, class, disability, and other identities in classrooms reflect in mirror image to how they exist in the macrocosm societal hierarchies (Louie, 2017). Apple (1992) urges us to consider these two questions: does mathematics education perpetrate, or at least contribute to, social inequity? Is inclusion of all in mathematics education an economic necessity or a moral obligation for society?

1.3 Rationale

There are many studies which outline deliberate teacher actions to effectively shift pedagogy towards a more equitable and inclusive framework (Boaler, 2006; Hunter & Civil, 2021; Langer-Osuna, 2017; Louie, 2017). One commonality between these studies is the emphasis on discourse and the power of mathematical talk to recalibrate equity in the mathematics classroom. All these studies provide recommendations of specific teacher actions to elicit equitable discourse in an inquiry mathematics lesson. The current study seeks to investigate the impact of the specific teacher actions on inclusion of marginalised students in the large group mathematical discussion.

The current study is situated in the large group discussion phase of a mathematics lesson because this phase provides powerful opportunities to address the exclusion of marginalised students. The opportunities are powerful because this is the most communally participatory phase of the lesson. Lotan (2022) explains that participation is important for marginalised students because there is a direct link between increased participation and increased achievement. Foote (2018) noted that collaborative mathematical discourse was one of three key common findings across a range of studies in the United States of America for engaging marginalised students. Since the large group discussion is a co-construction of shared understanding, deeply enriched by all students' thinking, this is an ideal situation for teachers to engage marginalised students. Teachers have the power here to be the gateway for their marginalised students to access deeper understanding of key mathematical concepts.

Unfortunately, teachers are sometimes inadvertently the gatekeeper preventing marginalised students from entering and contributing to the discussion (Hunter & Hunter, 2018). Averill (2018) explains this is because teachers are embedded in the exclusive societal framework described previously. Usually, teachers learnt as students in the same framework, so they are prone to repeating the same acts of exclusion their own teachers enacted in their childhood classrooms. Exclusion is the only schema many teachers have (Hunter & Hunter, 2018). Often, teachers are members of the hegemonic ethnicity, class, and culture. This means the exclusive framework they learnt in as children favoured them. Hunter and Hunter (2018) state that this creates a pedagogical challenge for teachers who are attempting to address inequity.

1.4 Objectives

The purpose of the current study is to explore the actions teachers take to include marginalised students in the large group mathematical discussion. The study aims to address the following questions:

1. What teacher actions effectively include marginalised students in the large group mathematical discussion?

2. What enables teachers to take actions which effectively include marginalised students in the large group mathematical discussion?

1.5 Overview

Chapter two reviews national and international literature focused on equitable pedagogy in mathematics under the inquiry model of teaching and learning. The review focuses on literature that identifies specific pedagogical tools designed to promote inclusion of marginalised students in collaborative mathematical discourse. The review identifies teacher actions, within these tools, which promoted inclusion of marginalised students in these classroom-based studies. Chapter three outlines the methodology and research design used in this study. Data collection and analysis methods are described here. The participants and research setting are introduced in this chapter. The timeframe of the study is outlined. Ethical considerations and the role of the researcher are discussed. The validity and reliability of the research is addressed here.

Chapter four presents the findings and analysis of this study. Teacher actions, which promoted inclusion, are described, and analysed through the evidence of the data. The beliefs and perspectives of each participant are reported here, with description and analysis of the respective journeys both teachers took towards a more inclusive framework. Chapter five discusses the results of the study supported by the theory discussed in the literature review. Conclusions are drawn and implications for classroom practice are described here. Limitations of the current study are addressed, as well as suggestions for areas of future research.

Chapter 2: Literature Review

2.1 Introduction

This review examines effective teacher actions that promote inclusion of marginalised students in mathematical discourse drawn from national and international literature. The focus is on how these identified actions could impact specifically on marginalised students' engagement in the context of the large group mathematical discussion Section 2.2 outlines the theoretical framework of this study. Section 2.2.1 gives an overview of the inquiry model of teaching. Section 2.3 reviews seven tools identified through relevant classroom-based studies that support inclusion of marginalised students in mathematical discourse. The seven tools are: the five practices, teacher talk moves, mathematical practices, fostering social and socio-mathematical norms, utilising cultural funds of knowledge, addressing status issues, and holding high expectations of all students.

Section 2.3.1 reviews relevant literature related to the five key practices to elicit deep understanding. Section 2.3.2 examines literature in relation to teacher talk moves in relation to increasing participation of marginalised students. Section 2.3.3 outlines, through relevant literature, the importance of making mathematical practices explicit. Sections 2.3.4 and 2.3.5 explore, respectively, the impact of establishing norms and cultural funds of knowledge through relevant literature. Section 2.3.6 investigates literature focused on the effect of status on student participation. Section 2.3.7 explores literature related to the impact of high teacher expectations on marginalised students. 2.4 summarises the chapter.

2.2 A Socio-Cultural Perspective

This thesis takes a socio-cultural perspective. To understand the development of student engagement in mathematical discussions, and teacher facilitation of this engagement, a learning theory that accounts for the complexity of mathematics education is required. Takeuchi (2018) explains how social constructivism theory developed from acknowledgement of both individual sense making and social processes being essential to learning mathematics. This is rooted in Vygotsky (1978) challenging the traditional view of learning as a closed phenomenon. Vygotsky asserted that, instead of a solo pursuit, learning blooms through collaboration with others.

Additionally, Cobb (1995) states that, through the lens of social constructivism learning theory, students already possess valuable knowledge and ideas to bring to the discussion. Mathematical dialogue and learning require interplay between two or more parties and therefore cannot be described as completely individual, nor completely social; instead, it occurs in the interplay between the two (Amineh & Asl, 2015). Constructivist theories, such as Piagetian theory, view knowledge construction as filtering through an individual's prior experience and knowledge (Cobb, 1995). This interplay creates a socio-constructivist learning system whereby learners actively construct new knowledge and understanding based on the interaction between their own and others' ideas (Calor et al., 2019).

Takeuchi (2018) adds that socio-cultural theory builds on socio-constructive theory to promote the capabilities and understandings people already have established in their everyday lives. The social process of building a shared understanding during a mathematical discussion lifts the discipline out of the abstract and places it firmly within a cultural context. Rogoff and Mejía-Arauz (2022) claim this is where mathematics comes alive and touches the lives of students. Similarly, McDermott (1993) emphasises that learning should be situated within the understanding of what is around the learner, to be learned. The inquiry-based teaching approach for mathematics incorporates socio-cultural theory by providing students opportunities to explore their thinking through co-constructed inquiry (Goos, 2004; Henningsen & Stein, 1997; Jorgenson, 2014).

2.2.1 Inquiry Classrooms

Over the past two decades, increasing attention has been given to the promotion of an inquiry-based approach in mathematics. Researchers (Jackson et al., 2012; Stein et al., 2008) claim inquiry-based approaches have the potential to improve student achievement, develop students' conceptual understanding, and promote equitable opportunities for success. The definition of inquiry-based mathematics, within the current study, builds on that suggested by Lampert (2001). Lampert (2001) promotes a pedagogy of student-led learning through complex tasks with multiple solution pathways aimed at increased conceptual understanding of mathematical principles and ideas. This pedagogy aims to connect different aspects of mathematics and students' representations, or methods, with students' prior knowledge.

Many variants of inquiry lessons include student-led exploration through group work, discussions, a whole class activity, or some sort of shared evaluation of thinking (Leach, 2014; Lawler 2018; Stein et al. 2022). According to Leach et al. (2014), an inquiry mathematics lesson provides multiple opportunities for students to discuss mathematical ideas and build understanding. Furthermore, Hackenberg (2010) states that these opportunities empower students because teachers value their knowledge and reasoning and view this as worthy of bringing to the table. While there is autonomy for students in terms of their thought processes Lawler (2018) reminds us that the teacher role remains crucial. Lawler refers to the teacher's role in an inquiry classroom as that of a facilitator, rather than a downloader of information. Teachers and students share responsibility to develop communities of learners who share, reflect, and build on their own and others' thinking (Lawler, 2018; Leach et al., 2014). The teachers in the current study follow a particular inquiry model developed in Aotearoa, known as Developing Mathematical Inquiry Communities (DMIC).

Developing Mathematical Inquiry Communities (DMIC) is an equity reform professional learning and development (PLD) programme designed specifically for the Aotearoa and South Pacific context (Hunter et al., 2018). DMIC pedagogy incorporates the established international research in inquiry teaching and learning of Cohen and Lotan (1997), known as complex instruction, and Boaler's (2006) research, which locates this inquiry model specifically in mathematics. Hunter, who developed the DMIC model, brings a uniquely socio-cultural focus to the inquiry pedagogy (Hunter et al., 2018). DMIC PLD incorporates this socio-cultural perspective to enable a strength-based approach to addressing the persistent underachievement of marginalised students, especially Māori and Pāsifika students, caused by structural inequities (Hunter et al., 2018). Alton-Lee et al. (2011) claim that this socio-cultural element of the DMIC pedagogy supports teachers to take a strength-based approach towards marginalised students.

2.3 Seven Teacher Tools for Inclusion

This section summarises seven pedagogical tools, identified by relevant literature, to support teachers to equitably facilitate mathematical discourse. The focus is on teacher actions, using the seven tools, which promote the inclusion of marginalised students in the large group discussion phase of an inquiry mathematics lesson.

2.3.1 The Five Practices

Stein et al.'s (2008) "five practices" framework is a practical guide for teachers enacting the inquiry model pedagogy (p. 322). Stein et al. outline five key practices for teachers to engage in sequentially through five phases of a lesson: anticipate, monitor, select, sequence, and connect. Firstly, plan and anticipate the lesson. Secondly, launch the task. This is followed by monitoring the group work, then selecting and sequencing to share back. Finally, connect students' ideas, generalise, and reflect. Stein et al. advise teachers, when setting up the discussion, to focus on two of these practices: selecting and sequencing. Furthermore, Smith & Stein (1998) state that teachers who select and sequence with an ethic of care maximise the inclusion of marginalised students. However, the practices employed prior to this, and how effectively these were employed, directly impact the success of the selection and sequencing process. Stein et al. (2008) assert that, of all the instructional practices a teacher can employ, it could be argued that none are as significant as anticipating the task that the students will be engaging with when studying mathematics.

Anticipating means predicting student responses to the task including possible misconceptions. It requires teachers to solve the problem in as many ways as possible and decide which strategies will be most useful to address the mathematical goal intended for the lesson. The findings of Moscardini's (2010) study of twenty-four Scottish primary students, aged six-eleven years, with moderate learning difficulties show students thriving on the same problems as their peers. Moscardini's study highlights the importance of students developing their own strategies and teachers capitalising on students' intuitive understandings of mathematics. Additionally, Louie (2017), recommends teachers anticipate possible divergent solution pathways students with learning difficulties might take as this supports teachers to understand and hold high expectations of all their students.

Monitoring student understandings involves close observation of how students explore the task (Smith & Stein, 1998). The challenge with this monitoring, Stein et al. (2008) explain, is to simultaneously aim to comprehend how students are making sense of the task and align the, often disparate, strategies and ideas to the key mathematical concepts of the lesson. Teachers need to utilise this stage to determine the engagement, validity, and the ways in which students are making sense of the big mathematical ideas. Some students might be using strategies that are amiss, but teachers must not dismiss these as they provide valuable clues into students' mathematical thinking (Erath, 2018; Stein et al., 2008). Erath (2018) adds that the effectiveness of the monitoring impacts how successful the selection process will be for the subsequent discussion. Horn (2007) explains that noticing what struggling students can do or represent, rather than what they cannot do, provides teachers with a ripe opportunity to validate these students.

Selecting refers to teachers selecting which small groups will share their work with the large group. Stein et al. (2008) state that sequencing refers to the order of these contributions. According to Stein et al., the overall aim of the discussion is to co-construct a shared understanding of the mathematical goal for the lesson. Selecting the groups can prove challenging. Stein et al. found that selecting all, or many groups, can turn a focused discussion into a show-and-tell activity. However, Stein et al. caution against selecting only one group because teachers tend to select the group with the most sophisticated strategy. This is a problem, Louie (2017) explains, because selecting only exemplary work reinforces a result-driven ethic that creates exclusion, encourages competition, and produces a unidimensional trajectory for students.

Wood et al.'s (2006) study corroborates Louie's (2017) findings, stating that uni-dimensionality reduces divergent thinking and excludes marginalised students. Stein et al. (2008) found that teachers often resisted selecting groups that struggled with the task, or made mistakes, because they wanted to save students from embarrassment and prevent other groups from exposure to incorrect, or confusing, solutions. Warshauer (2015) states that, in fact, selecting solutions containing mistakes is mathematically powerful and re-calibrates beliefs around errors. Additionally, Erath (2018) explains that teachers can re-calibrate these entrenched beliefs by regularly using common misconceptions as teaching points. Teachers must consider equitable inclusion when choosing how to sequence the selected explanations.

Sequencing is most effective when the planning of the task itself was effective. Therefore, the success of the anticipation and monitoring phases impacts upon the success of sequencing (Stein et al., 2008). According to Stein et al. (2008), the creation of open task design that allows for multiple solution pathways and, often, multiple correct answers. Louie (2017) explains that sequencing multiple representations of solutions, and possibly multiple answers, enables teachers to draw on a variety of solutions to collectively explore and validate. Lawler (2018) states that the greater the breadth of diversity, the stronger the teacher's message is to students that there is not only one correct way, and their heuristic thinking is valued.

During the facilitation of the large group discussion, teachers should sequence various strategies or ideas to build upon each other, thus ending the lesson by connecting all the ideas into a cohesive mathematical sequence (Batista & Chapin, 2019). Sullivan (2008) states that this requires teachers developing strong mathematical content knowledge and deep understanding of the key mathematical concepts. Improving teacher content knowledge, Stein et al. (2008)

claim, is achieved through detailed anticipations of the task. Successful sequencing supports teacher facilitation of the discussion as teachers are focused on unpacking the trajectory of ideas in the lesson.

2.3.2 Talk Moves

Teacher talk moves are a tool to support facilitation of discourse that generates a shared mathematical understanding. Chapin and O'Connor's 1998–2002 longitudinal intervention study in one urban school in the United States of America, with four hundred students and eighteen teachers, aimed at improving mathematics outcomes for at-risk students (Chapin & O'Connor, 2007). The students were aged between ten and thirteen years old. A large emphasis was placed on eliciting academically productive student talk (Chapin & O'Connor, 2007). Five teacher talk moves were identified as supportive of teachers' facilitation of academically productive student discourse.

The five talk moves include re-voicing, repeating, adding on, reasoning, and wait time (Chapin et al., 2009; O'Connor et al, 2016). Re-voicing is where a teacher or student re states another student's utterance in their own words to check if he or she has understood the conjecture, idea, question, or argument correctly. This move is powerful because it clarifies the thinking for everyone in the discussion (Chapin et al., 2009). The second talk move, repeating, is a valuable tool where teachers ask a student to repeat, verbatim, another student's utterance (O'Connor et al., 2016). Students do not have to understand it to be able to repeat it. Chapin and O'Connor (2007) found that teachers who encouraged students to repeat other students' ideas tuned students in to listening to each other. Additionally, this move gave a voice to students who would otherwise remain silent.

When teachers invite students to 'add on' to another student's idea, conjecture, or argument the mathematical voice is spread across the class (Chapin et al., 2009). Chapin and O'Connor (2007) describe how sometimes 'add-ons' become repetitive, but they still have the advantage of including more voices in the discussion. Other times adding on serves to create more nuances in the discussion. Takeuchi (2018) claims these nuances enrichen and build on the shared understanding of the large group. Reasoning is where the teacher invites students to engage further with an idea, argument, or conjecture by asking why they think that or if they agree or disagree. Chapin et al. (2009) found that this talk move supported robust discussion because it encouraged students to actively make sense of others' ideas and check these against their own reasoning.

Wait time requires teachers to allow students some thought processing time before answering a question or sharing their idea. Chapin and O'Connor (2007) noticed that teachers who found techniques to support wait time, such as counting eight seconds in their head, were rewarded with thoughtful contributions from their students. These researchers noted that often students assumed the teacher would move on to another student if they hesitated over their response and this was usually the case. However, when specific attention was given to the use of wait time and teachers employed wait time regularly, students came to realise they were obligated to contribute (Chapin et al., 2009).

Additionally, students came to expect a reasonable wait time in which to gather and articulate their thoughts. Louie (2017) claims wait time diminishes the valorisation of speed in the mathematics classroom by slowing down the pace of the discussion. This sends a powerful message to students that the teacher values a process rich discussion, as opposed to a mere transportation towards the answer (Louie, 2017). Takeuchi (2018) asserts that in terms of talk moves that re-calibrate the norm of exclusion to inclusion in the large group discussion, wait time is possibly the most powerful teacher move.

Kazemi and Hintz (2014) add two more talk moves to the repertoire: revise and turn and talk. Revising requires teachers to invite students to revise their thinking as new insights come to light. Kazemi & Hintz note that, in their work with teachers, when this move is employed, students come to view understanding as an ever-evolving process because constructing new knowledge requires refining along the way as new pieces of information are discovered. Turn and talk is a move employed by teachers to circulate the discussion by getting students to turn and talk with their partner or small group. Teachers gain useful information as they listen to this talk. Kazemi and Hintz found this move supported teachers around their choices for who to call on next to contribute to the discussion. Furthermore, Langer-Osuna (2017) claims this move is beneficial for orienting peers to view each other as rich resources. Louie (2017) found that re-framing mathematical authority away from the teacher, or textbook, and towards peers increases inclusion because students view each other as allies instead of competitors in the discussion.

2.3.3 Mathematical Practices

Students need a set of practices to support their engagement in mathematical dialogue and communication (Engle & Conant, 2002). These practices include using representations, explaining, justifying, generalising, questioning, and mathematical argumentation (Ball &

Bass, 2003; Kazemi & Hintz, 2014; Maher & Martino, 1996; Wood, 1999; Yackel & Cobb, 1996). The draft New Zealand mathematics curriculum refresh (Ministry of Education, 2022) highlights the importance of these mathematical practices as central to students successfully meeting learning outcomes in mathematics. This updated focus is significant because all teachers in Aotearoa, are now expected to incorporate these mathematical practices into their mathematics teaching. Boaler (2019) states that engagement with mathematical practices deepens students' conceptual understanding of mathematics. This produces mathematical identities as knowers and doers of mathematics (Stein et al., 2008; Warshauer, 2015).

The role of the teacher, explains Lawler (2018), is to facilitate the flow of the discussion while keeping the authority with the students, who are the owners of their intellectual property. This requires careful consideration of the amount and quality of the teacher's contribution to the discussion. Students need scaffolding, for example, around how to justify robustly. However, directing students by asking questions on their behalf, or becoming overly prescriptive, robs students of their opportunity to improve (Selling, 2016). The balance between the two is where teachers enable mathematical practices to evolve through the right prompt or question, at the right time. Selling (2016) claims that this serves to deepen the discussion or direct the flow towards the mathematical goal of the lesson; but what are the right questions and prompts? And when should they be used? Selling proposes teachers make mathematical practices explicit primarily after students have participated in them, so students are cognisant that they have engaged in a mathematical practice.

Selling's (2016) study of twenty-six students from three middle and high school mathematics classrooms in the United States of America was part of a larger study. The students in Selling's study were aged between eleven and eighteen years old. Selling's study examined how teachers make mathematical practices explicit in mathematical discourse. The student demographic was racially and ethnically diverse: 74% Latino or Hispanic, 11% African American, 11% Asian, 2% Filipino, 1% Native American, and 1% White. Selling analysed twenty-six discussions from three mathematics classrooms and developed the findings into a framework of eight types of teacher moves that made mathematical practices explicit.

Selling's (2016) framework outlines three over-arching moves: initiating, sustaining, and reprising. An initiating talk turn facilitates the beginning of student participation in a dialogue. A sustaining talk turn presses on, or further elicits, student participation in mathematical practices. A reprising talk turn happens when the teacher explicitly reflects on student participation in mathematical practices. The third talk move, reprisal, is further broken into sub-parts: naming, highlighting, evaluating, explaining the rationale, connecting, and framing.

Naming refers to explicitly naming the mathematical practice the student engaged in. Highlighting involves highlighting aspects of student engagement in mathematical practices. Evaluating requires a verbal evaluation of student engagement in mathematical practices, such as specific praise. Explaining requires the teacher to reprise the mathematical practice by explaining the goal or rationale for engaging in it to the student. Connecting refers to connecting different students' engagement in mathematical practices. The last reprisal move, framing, supports students to see the mathematical practice they engaged in as having future utility by framing it expansively over time and activities (Selling, 2016). Selling found that, initially, the most used talk move was the initiating move.

The earlier discussions revealed that teachers frequently initiated discourse with a student. For example, asking why they chose a particular method. Selling noticed that this usually occurred before a mathematical practice had been engaged with. Often the student's response was simply accepted, and the teacher moved on. For students, as well as teachers, expecting more requires a deliberate re-calibration. Louie's (2017) study returned similar findings to Selling's. Louie states that making students' internal thought processes external requires explicit unpacking because, for many students, mathematics has been an internal and independent discipline for years prior to learning in this model. Selling explained how, in later discussions, teachers sustained conversations by adding another comment or asking a further question. Selling found that teachers who sustained the conversation were more likely to elicit the student to subsequently engage in a mathematical practice.

Subsequent discussions revealed the addition of the reprisal move. Selling (2016) described how the teachers in her study reprised the mathematical practice by either naming, highlighting, evaluating, explaining, connecting, or framing the practice (Selling, 2016). For example, one teacher used an initiate talk move by asking his student 'where did you count nine?' The student responded by showing his teacher where the nine squares were. The teacher then used a sustaining talk move and told him to write a nine on the side to show this. He consequently engaged in the mathematical practice, of representing through labelling, by writing the numeral 9 on the left side of his page. The teacher subsequently engaged the reprisal move of naming: "exactly, so what you just did, Leo, is label the dimensions" (Selling, 2016, p. 519). Selling asserts that the specification makes the implicit explicit for this student who may have been unaware that labelling dimensions was a mathematical practice before this conversation.

Through this process students begin appropriation of mathematical practices in deliberate and meaningful ways. Selling (2016) found that the reprisal talk-move, in concert with initiating and sustaining, proved most effective for making aspects of mathematical practices explicit. These moves provide students with the opportunity to develop a shared understanding (Cobb et al., 1993). Over time, students develop their own meanings for these mathematical practices relative to their prior experiences and the different roles they adopt during the discussion (Selling, 2016). Moschkovich (2004) noted how learners appropriate and transform their usage of mathematical practices through watching how their peers use them in their different roles. The discussion phase is, therefore, a key opportunity for students to engage in, and be exposed to, a range of mathematical practices (Louie, 2017). Van Es et al. (2017) state that marginalised students benefit from this opportunity to explore mathematical practices with their peers under the skilful guidance of their teacher.

2.3.4 Norms

Establishing social and socio-mathematical norms in an inquiry mathematics lesson supports the inclusion of marginalised students by addressing status issues and developing stronger mathematical identities through new patterns of behaviour (Louie, 2017). Social norms refer to protocol around organisational routines, practices, and behaviour as a member of an inquiry community. Socio-mathematical norms refer directly to the mathematical practices discussed earlier in this review that support student engagement in high level cognition (Louie, 2017; Selling, 2016). Many studies illustrate the importance of intwining both types of norms (e.g., Hufferd-Ackles et al.; 2004; Lester, 2007; Louie, 2018; McCrone, 2005; Yackel, 1996). According to these studies, the combination of both is needed to push collective discourse to its most cognitively potent formula: argumentation, challenge, and debate.

The establishment of a collaborative protocol supports relational equity (Boaler, 2019). Relation equity, during the large group discussion, requires students to recognise and accept that other groups used different strategies or pathways to solve the same problem. Furthermore, students who are relationally equitable realise these different ideas and, sometimes, different points of view, enrichen their own understanding rather than contradict it. Hunter and Hunter (2018) state that, once embedded, social and socio-mathematical norms ensure pro-social and inclusive interaction.

Social norms include active listening, sharing ideas, asking questions, respecting differing points of view, and taking turns (Yackel & Cobb, 1996). They require explicit and consistent teaching. Socio-mathematical norms are developed once social norms are embedded. One way to establish these is for teachers to reposition themselves as participants in the discourse and emphasise student responsibility for active listening and sense-making (Shah & Crespo, 2018). The establishment of a safe space is vital. Teachers who create a supportive learning environment promote social and intellectual risk-taking (Averill, 2018).

Teachers should employ a range of strategies to foster social norms, including explicit instruction of the need for collegiality (Boaler et al., 2022; Louie 2017). Over time students grow comfortable with making conjectures, agreeing, and disagreeing, and verbally revising thinking (Shah & Crespo, 2018). Inter-thinking becomes the new norm (Leach, 2014). Social norms, such as active listening, develop into socio-mathematic norms because they are experientially real to students (Civil & Hunter, 2015).

Hunter's (2008) study examining how socio-cultural norms supported marginalised students to engage in mathematical practices developed from a larger classroom-based design. The study was conducted at a New Zealand urban primary school and involved four teachers and one hundred and twenty students aged between seven and thirteen. The majority of students came from low socio-economic home environments and were of Pāsifika or New Zealand Māori ethnic groupings. The findings of this study showed how attention placed on sociocultural and mathematical norms was significant in developing communal dialogue and individual and collective responsibility to sense-make. Students who hail from collectivist traditions, such as the Pāsifika and Māori students in Hunter's (2008) study, find the interdependence required for a co-construction of understanding comes naturally to them as this is their cultural norm.

Hunter (2008) found that teachers who incorporated collectivist values in the group norms positioned indigenous and Pāsifika students as rich resources for their peers. For example, one teacher prefaced the sharing back discussion with this comment: "remember you are a member of our whanau, so you need to be loud and proud and confident … we are all ready to think and listen" (Hunter, 2008, p. 35). Metaphors, such as *whānau* reminds everyone that the expectation is to work together as a family. Other metaphors, such as *waka*, where all students row together, or *Siapo,* which is made collectively but with individual skills, emphasise individual strengths interconnecting to form one unit. Pāsifika values include reciprocity, family, relationships, spirituality, leadership, collectivism, love, and belonging (Hunter, 2008). Triandis et al. (1988)

assert that collectivist values, such as these, are the antithesis to western values, which include competition, formal relationships, secularity, individualism, and a uniform adherence to the status quo. Teachers who make explicit use of analogies that promote collectivist values, during the discussion, recalibrate individualistic thinking towards the collective (Civil & Hunter, 2015).

2.3.5 Cultural Funds of Knowledge

Creating contexts for mathematics tasks that are relevant, meaningful, and interesting to students helps them hook into the task and see mathematics outside the classroom (Alcalá et al., 2018). Beatty (2018) emphasises the importance of teachers defining cultural knowledge in a broader sense than just ethnicity. Cultural knowledge encompasses ethnicity, religion, beliefs, values, activities, and traditions relating to a community or a sub-set of a community, including popular culture. Saxe (2012) claims that developing culturally sustaining contexts requires deep knowledge about students' lives.

During a mathematical discussion, Warshauer (2019) cautions, there are often moments in the centre of an explanation where the context gets lost while the numbers are explored. At this point students are in danger of losing the context, so the solution becomes implausible. For example, students with a solution of dividing the remaining quotient in half is only valid if the context allows. So, cutting a cake in half may be contextually valid, for instance, whereas cutting a person in half is not (Warshauer, 2019). Batista and Chapin (2019) advise teachers to bring students back to the context through careful questioning and ground the problem back in what is experientially real.

Hunter and Civil (2021) encourage teachers to tap into the cultural funds of knowledge in the communities of their students. Community funds of knowledge, in this review, is defined as "historically accumulated and culturally developed bodies of knowledge and skills essential for household or individual functioning and well-being" (González et al., 2006, p. 133). Teachers who use community funds of knowledge raise the cultural capital of their students (Bills & Hunter, 2015). Bennet et al. (2009) believe that Bourdieu and Coleman's (1991) description of cultural capital as familiarity with the legitimate culture within a society refers, in an educational setting, to the dominant culture within the classroom. Students who belong to this culture hold cultural capital, or status, which privileges their knowledge compared with their peers from non-dominant cultural backgrounds. Evidence suggests that cultural capital, passed on through families, helps children do better in school (Saxe, 2012). Bills and Hunter (2015) claim that teachers who raise the cultural capital of students from non-dominant cultures disrupt the status-quo of an inequitable cultural knowledge base in the classroom.

Since this knowledge is passed on through families, accessing community funds of knowledge supports raising the cultural capital of marginalised students (Civil, 2007). Civil discusses the ways in which Latino/a parents' and children's mathematical knowledge is used in their everyday lives, such as ancient wisdoms around gardening. Civil describes using a community fund of knowledge to develop a culturally sustaining task incorporating graphing the growth of amaryllis, which included developing the concept of scale. Allowing marginalised students' cultural funds of knowledge to form part of the shared understanding enriched everyone's mathematical understanding and empowered these Latino/a students.

Smith (2012) positions the sacredness of cultural wisdom, grounded as it is in an exclusive knowledge system, as a taonga, or treasure, to be gifted to others. Gibbs' (2020) study on twelve Māori and Pāsifika students, aged between ten and twelve, explored students making sense of functional relationships in algebra. Gibbs found that, when students were given opportunities to draw on their cultures, there was a significant shift in their conceptual understanding. Since these tasks drew on cultural funds of knowledge related to Māori and Pāsifika traditions the mathematical and cultural identities of these students increased. Unfortunately, Milne (2020) asserts that, in most classrooms, mathematical tasks lack this contextual wisdom.

2.3.6 Status

Students hold beliefs about their own attributes and capabilities, and those of their peers, which affects the way they interact with each other in collaborative situations (Horn, 2007). Additionally, state Featherstone et al. (2011), students' expectations of their own and each other's intellectual and social capabilities feed into group dynamics, creating another brand of self-fulfilling prophecy. Langer-Osuna (2017) describes two types of status: intellectual and social. Having intellectual authority means a student is positioned as a credible source of mathematical information. This includes peers seeking help from this student or positioning this student as the main driver of the intellectual work in the group. Langer-Osuna (2017) frames social authority as students who have the right to issue directives to their peers, or in some way have management over their group. Often students with social authority are described by their peers as having many friends and being popular or cool (Horn, 2007).

Intellectual and social authority can be self-appointed, peer-appointed, or teacher-appointed. While both intellectual and social authority can co-exist the distinction between the two is important since they can, also, exist in isolation (Langer-Osuna, 2017). For example, students may assign intellectual authority to a socially popular student, which gifts this student power to direct the flow of mathematical discussion and hold sway over ideas despite knowing less about the task than their peers (Langer-Osuna, 2017). Status is fluid so it is important to note that peers and teachers can re-assign status to students. For example, teachers and peers can attribute intellectual authority to a student who previously held low intellectual status (Webster & Foschi, 1988).

Teachers also hold beliefs about their students' attributes and capabilities, appoint high and low status to their students even when they are not aware of this (Webster & Foschi, 1988). These beliefs affect the way teachers interact with different students in collaborative situations (Horn, 2007). Teachers hold immense power in this instance as students look to their teacher for cues regarding the social and intellectual structure of the classroom (Boaler, 2006). Webster and Foschi (1988) explain how teachers can re-calibrate status by placing value on students with low status or on skills which they usually do not emphasise as valuable. Re-calibrating status, while possible, is challenging for teachers due to human nature's inherent predilection to generalise attributes and characteristics belonging to groups of people. Webster and Foschi (1988) coined the term "status generalisation" to describe this phenomenon (p. 3).

Webster and Foschi (1988) explain how, for example, society has generalised women as being more emotional and irrational than men; therefore, men hold a higher intellectual status than women. Webster and Foschi (1988) elaborate that one status generalisation can extend to other groups of people, for example, the belief that people with white skin are more intelligent than those with black skin. Although beliefs, such as these, evolve over time and grow to be regarded as false and harmful stereotypes, the power they hold must not be underestimated. Status generalisations remain deep in our human psyche, so we are not always aware we have them (Webster & Foschi, 1988).

Some researchers refer to this as unconscious, or implicit, bias and emphasise how, because people are unaware of it, it is outside of their control (Equality Challenge Unit, 2013). It is triggered by the human brain having to make quick judgments or assessments of people and situations (Equality Challenge Unit, 2013). Teachers, in their busy, human populated environment, must make quick decisions and assessments multiple times per day (Lotan, 2022).

However, if teachers examine the values and judgements that underpin their actions this can lead to awareness around any biases they may hold. This new self-awareness supports teachers to actively re-calibrate the status of their students.

Therefore, the teacher, who facilitates the flow of the discussion is well placed to address status issues (Hufferd-Ackles et al., 2004). According to Leach et al. (2014), teachers hold a powerful position to disrupt the status quo by addressing and re-calibrating status rankings during this phase. Teachers who know their students well are at an advantage when facilitating inclusion in the discussion (Goos, 2004). For example, a marginalised student with low status in the mathematics classroom might feel that she and mathematics have nothing in common. However, on weekends she helps her father dismantle and reassemble lawnmower engines not realising this is mathematics. Boaler (2019) states that teachers have an opportunity to use this knowledge to position students as experts, so in this case the girl is an expert in lawnmower assembly.

This action is powerful, Boaler (2019) explains, because it positions the student as a competent mathematician in the eyes of her peers. Langer-Osuna (2017) describes how teachers initially avoided singling these students out. Black (2004) corroborates Langer-Osuna, and, further, suggests this often stems from an ethic of care around not wanting to embarrass students perceived to be struggling, by asking them questions or encouraging them to contribute when they do not understand. Wagner and Herbel-Eisenmann (2014) claim that this ethic of care derives from teachers' low expectations of the intelligence of their students whom they perceived to be struggling.

Louie's (2018) findings demonstrate that when emphasis shifts from accuracy, speed, and results to effort, exploration, and process students see that their thinking, right or wrong, is valued. This aligns with Boaler's (2006) findings from her longitudinal research at three high schools in California, United States of America, where she re-framed student ability. Boaler's study spanned four years and three high schools, including over six hundred hours' worth of classroom observations of mathematics lessons. One of the three schools, Railside High, was more urban and had a more diverse ethnic and socio-economic demographic than the other two. Assigning competence is one re-framing strategy that proved highly effective for raising status (Boaler, 2006). The teachers publicly positioned students, perceived by their peers as incompetent, as intellectually worthy by using specific praise. Boaler (2006) showed how this was most effective when the feedback was genuine, public, based on intellectual accomplishments, and related to the task. Boaler's study found that Railside High's achievement results exceeded those of the more affluent and white high schools' due to teachers employing actions to promote equity through raising status (Boaler, 2006). The teachers at Railside High had to examine their implicit biases and belief systems before they could effectively employ teacher actions that effectively re-calibrated status in their mathematics classrooms (Boaler, 2006).

2.3.7 High Expectations

For equitable facilitation of the large group discussion teachers must hold high expectations of all their students. When teachers believe that all children can learn, they are more likely to produce greater learning gains in their students (Bishop & Berryman, 2006; Timperley & Robinson, 2001). Wilkinson and Townsend (2000) reported that the best-practice teachers in their study held a developmental notion of ability. This means that, rather than believing intelligence to be fixed and finite, these teachers believed intelligence was fluid and incremental.

Rubie-Davies' (2015) study investigated how these beliefs impacted on teacher expectations of students. Rubie-Davies (2015) reported how many teachers in her study differentiated their instruction for students depending on whether they had low or high expectations of their students. High expectation students were given challenging, fun activities where they had a degree of autonomy. Low expectation students were given low-level repetitive tasks and were more closely directed and monitored by the teacher (Rubie-Davies, 2015). The large group discussion, therefore, may cause concern for teachers who believe in fixed intelligence because they will worry how their low expectation students can contribute to the discussion without explicit teaching (Wilkinson & Townsend, 2000).

Rubie-Davies (2015) states that students learn what they have been given the opportunity to learn. Rubie-Davies' (2015) study drew on Merton's original (1948) self-fulfilling prophecy theory. Merton outlined how, if we believe something to be true, we act in particular ways that can cause our beliefs to become true. In the classroom this prophecy plays out through students' internalisation of their teacher's low expectations (Rubie-Davies, 2015). These students subsequently lack motivation because, in their minds, they have already failed so why bother trying? This lack of motivation serves to re-confirm the teacher's low expectations of these students and so develops a vicious cycle that continually perpetuates this self-fulfilling prophecy (Rubie-Davies, 2015). Teachers who expect all their students to participate in the same engaging and challenging learning experiences create a new self-fulfilling prophecy (Rubie-Davies, 2015). Over time, this creates a new ethic of care based on high expectations.

Merton's self-fulfilling prophecy theory is congruent with the work of Diener and Dweck (1978), which examined how children respond to failure. Diener and Dweck's (1978) study, based in a school in the United States of America, examined ninety-four students aged between ten and eleven years. Diener and Dweck (1978) identified two main types of responses: one termed 'helpless' and the other 'mastery-oriented' (p. 451). Diener and Dweck's (1978) findings showed that children with a helpless response perceived their failure as fixed and irrevocable, but those with a mastery orientation persevered despite the failure and concentrated on mastering the task. Dweck's (2006) growth mindset theory, further developed from her earlier work, relates to implicit theories of intelligence. Dweck (2006) expanded from her previous classroom-based studies, with students, to other groups of people including parents, teachers, managers, and athletes. Dweck (2006) explained that people who hold a growth mindset believe intelligence can be developed and capabilities are enhanced through the learning process. People with a growth mindset welcome challenge and persevere through adversity. The focus is on the process more than the result.

Intelligence beliefs are powerful in shaping teacher actions. Dweck (2006) describes how a German researcher illustrated that, when teachers believed intelligence was fixed, students who began their academic year as low achievers were still achieving below-average levels at the year's end. However, teachers who believed that intelligence could be increased, saw their students who began the year as low achievers transition to average or high achievers at the year's end. Dweck (2015) states that a powerful way to shift from a fixed mindset to a growth mindset is to add the word *yet* to a finite statement. While teachers can employ this technique with their students to encourage self-belief this can equally apply to teachers themselves when they need to shift their beliefs of their students. For example, adding *yet* to the statement 'student B has not achieved the learning outcome' allows an opportunity for growth.

Holding a growth mindset supports teachers to foster this same mindset in their students. Louie (2017) recommends that teachers aiming to shift from the exclusive belief framework of fixed intelligence towards an inclusive framework, where intelligence is viewed as fluid, promote productive struggle in their mathematics classrooms. Productive struggle, here, means students grappling with a challenging task in a mathematically productive way. To achieve this, state

Stein et al. (2022), teachers must design challenging tasks with multiple entry and exit points so students can enter and exit a common task within their own learning trajectory.

Warshauer (2015) emphasises the importance of teachers creating rich tasks that are designed to elicit productive struggle. Warshauer (2015) states that teachers will know they have successfully elicited productive struggle if they notice all students grappling with important mathematical ideas while they struggle to come to grips with the task. Moscardini (2010) advises that a challenging task should provide a plethora of opportunities for all students to experience productive struggle and achieve deep understanding. Furthermore, Henningsen and Stein (1997) state that the perseverance and resilience gained through struggle leads to improved mathematical performance over time. Hackenberg (2010) believes that every student has the capability to contribute something important to the discussion.

2.4 Summary

This chapter reviewed relevant literature related to promoting inclusion of marginalised students in mathematical discourse (e.g., Chapin & O'Connor, 2009; Civil 2007; Hunter 2008; Kazemi & Hintz, 2014; Langer-Osuna, 2017; Rubie-Davies, 2015, Selling, 2016; Stein et al., 2008). The review highlighted specific teacher actions across seven pedagogical tools that teachers can enact to promote the inclusion of their marginalised students in the large group mathematical discussion. The importance of utilising the five practices was outlined in relation to setting marginalised students up for success in the discussion. Teacher talk moves were reviewed with a focus on how these can be employed to promote the inclusion of marginalised students in the discussion.

Explicit scaffolding of mathematical practices, using teacher talk moves, was reviewed in relation to increasing marginalised students' engagement in mathematical practices. Fostering the norms and using marginalised students' cultural funds of knowledge was shown, through the literature, to be integral to disrupting the status-quo of inequitable participation in mathematical discussions. Finally, addressing status issues and developing high expectations of all students was examined through the literature with a focus on how teacher belief systems impact on students' perceptions of themselves. It highlighted the importance of growth mindsets, for both students and teachers, and a belief in the fluidity of intelligence.

Chapter 3: Methodology

3.1 Introduction

The previous chapter discussed the literature related to the current study. This chapter outlines the research design and methods used in the current study. Section 3.2 provides a justification for the selection of a case-study design using qualitative methodology. Section 3.3 presents the study sample, setting, and schedule. Section 3.4 describes the role of the researcher. Section 3.5 elaborates ethical considerations of the current study. Section 3.6 presents the data collection instruments and methods and describes the data analysis. Section 3.7 discusses the validity and reliability of the study. Section 3.8 provides a summary of the chapter.

The researcher in the current study has a background in primary school teaching. Currently, the researcher is involved in delivering an equity reform professional development programme for teachers of new entrant to year ten students in the field of mathematics education. The teacher participants in the current study have been involved in this professional development programme and have a pre-existing professional relationship with the researcher. Consequently, the research design is unapologetically qualitative. It draws from the Māori ethical model Te Ara Tika to navigate the inherent ethical considerations of the current study and views the whakapapa between the researcher and the participant as a positive element of the study.

3.2 Justification of Methodology

The current study is a qualitative case study grounded in critical theory. Qualitative research is an umbrella term that incorporates a range of different inquiry methods including field study, case study, ethnography, narrative, naturalistic, inductive, and participant observation (Barth & Thomas, 2012). All qualitative methods have the same primary motive, which is to understand their participants and their social world (Nerland, 2022). Since participants, in the current study, were integrally linked to their social world, analysis of their realities necessarily considered the society they existed in. Consequently, a qualitative methodology was appropriate for the current study.

The current study took a case study approach because this approach enabled social phenomenon to be explored through a critical lens (Punch & Oancea, 2014). The aim of the current study was to gain insight into the social phenomenon of teacher actions, and the beliefs that enabled these actions. A case-study design was selected because it uncovered thoughts, perceptions, values, beliefs, and feelings experienced by the participants (King & Horrocks, 2010). Emphasis was placed on really getting inside the world and delving deeply into the perspective of the participants to make sense of the drivers behind their human behaviour (Lindqvist & Forsberg, 2022). The approach was empathetic (Barth & Thomas, 2012). The main premise of a case study design is that human reality is constructed by individuals' interaction in and with society (Saldaña, 2014).

The aim of the current study was to understand the teachers' viewpoints, perceptions, and beliefs in depth, in relation to the society they existed in (Merriam & Tisdell, 2015). Case studies are bound in time, setting, personnel, and context that provides a snapshot of a unique situation in a specific setting within a defined timeframe. This aligns to the current study as the case was set in one school with the same two teachers over one timeline. The focus was holistic because it sought to understand the wholeness and unity of the case (Saldaña, 2014). Since the current study was a co-constructed sense making exercise between teachers and the researcher the case study approach was appropriate.

The current study used an explorative approach to case study. There are different types of case studies. Explorative case studies aim to answer questions such as what and how. The current study examined what actions promoted inclusion and how teachers were enabled to take these actions. An explorative case study approach provided the researcher an opportunity to explore understanding around factors that enabled teachers to effectively include marginalised students in mathematical discourse and expanded the range of current interpretations in this area (Nerland, 2022). Teacher actions, and the resulting student responses, were interpreted by the researcher and the teachers based on a range of evidence of student thinking, communication, and representations. Explorative case study aims to encapsulate multiple individual viewpoints and ways of understanding a situation (Battey & Leyva, 2018). There was a range of viewpoints and interpretations expressed by teachers in the current case study. Since there were multiple layers of complexity inherent in the interpretation of teachers' actions and beliefs an explorative case study design was applicable.

The current study was grounded in critical theory. Critical theory, rooted in sociology, is the philosophy that culture is a social construct and social problems stem from social structures and cultural assumptions rather than from individuals (McArthur, 2022). Therefore, power structures, such as the government, are socially constructed and shaped by culture. To disrupt inequitable power balances the culture must change rather than the individuals who populate the current institution, company, or department. Individual teachers are part of a much wider social construct than that of their classroom or school. A much larger culture of exclusion exists in society as a whole, influencing the beliefs and behaviour of individuals in almost imperceptible ways (Louie, 2017). A methodology enabling critical theory was vital since teachers needed to understand their unconscious bias before a pedagogical shift could occur (Freire, 1970). These biases are often reflective of belief systems at play in society at large.

A qualitative study using critical theory enabled the researcher to interpret phenomenon within the classroom as a microcosm of what was occurring outside of the classroom (Saldaña, 2014). The current study had an emancipatory ideology because the findings led to shifts in teacher belief systems. An approach grounded in critical theory has the potential to contribute to the emancipation of society through deep insights into the conditions people find themselves in and the subsequent development of new ideologies that liberate people (McArthur, 2022). In the current study new belief systems influenced teacher actions that promoted equitable inclusion during the large group discussion.

3.3 The Study: Setting, Sample, and Schedule

The research was conducted in two classrooms, at a large urban-rural primary school in Aotearoa, across three school terms during the 2022 academic year. The two participants were both experienced practitioners. A pre-study interview was conducted, with each teacher, followed by four observations of a mathematics lesson over the course of the study and a post-study interview at the conclusion of the study (see Appendices A1-A2).

3.3.1 The Setting and the Sample

The school has an EQI rating of 474[1]. This rating indicates a mixed demographic of low and mid socio-economic families attending this school. 53 % of the student population are New Zealand European. The current study sample included two teachers and fifty-two students. Most of the fifty-two students were New Zealand European. Twenty-four of the students, aged between seven and nine years, were in a year four and five classroom. The remaining twenty-

[1] The Equity Index is a statistical model that indicates the extent to which a school's students face socio-economic barriers that could get in the way of them achieving at school. Student numbers are averaged at an individual school level to produce an EQI number for each school between 344-569. A higher EQI number indicates that a school has students facing greater socio-economic barriers (Ministry of Education, 2022).

eight, aged between ten and thirteen years, were in a year seven and eight (intermediate) classroom.

The two teacher participants in this study, Mrs Ulster, and Mrs Walter (pseudonyms), were experienced practitioners with more than five years teaching experience. Mrs Ulster taught the year four and five classroom. Mrs Walter taught the year seven and eight classroom. Both teachers had completed a three-year equity reform professional development programme in mathematics. They were in their fourth year of teaching mathematics within this pedagogy and were passionate about teaching for equity through inquiry mathematics teaching. Both teachers expressed an interest in being involved in this study because they felt the large group discussion was the most challenging phase of a mathematics lesson to facilitate. They were both concerned about specific students in their classrooms, who they felt were not adequately included during this phase of their mathematics lessons. They hoped this study would provide them with support to address these concerns.

3.3.2 The Schedule

The current study was conducted over three school terms (thirty weeks) in 2022. Data were collected via interview transcripts, video transcripts, student work, and teacher planning over three phases.

3.3.2.1 Phase One

The first phase of the study began with individual semi-structured interviews with participants during term one. (The original schedule for phase one scheduled pre-study interviews and initial observations but this was reduced to only the pre-study interviews due to a Covid-19 related disruption). Questions followed a set framework designed around the seven tools for inclusion identified in the literature. The intention of the pre-study interviews was to explore teacher beliefs, perceptions, and values around inclusion.

3.3.2.2 Phase Two

The second phase of the study was conducted over ten weeks during term two and included four observations (two per teacher) and the collection of student and teacher artefacts. The observations were spaced out so that the initial observations occurred at the beginning of the term and the second observations at the end. The observations were video recorded. The researcher took informal field notes during the observations alongside a structured framework based on the seven tools for inclusion, identified in the literature. The intention of the observations was to record teacher actions that promoted the inclusion of marginalised students in the large group discussion. The intention of photographing teacher planning was to evidence the precipitation of effective teacher actions, or a shift in this regard. The intention of photographing student work was to evidence student responses to effective teacher actions, or a shift in this regard.

3.3.2.3 Phase Three

Research was conducted over ten weeks during term three and included a further four observations (two per teacher) and the post-study interviews. The third and fourth observations, like the first and second observations, were spaced out across the term to provide teachers with time to synthesise learning from previous observations. The intention of these observations was the same as in phase two except the focus leaned more towards the shift in actions between the earlier observations and the later ones. Copies of student work and teacher planning were gathered during these observations, with the same intention as phase two. At the conclusion of term three a post-study semi-structured interview was conducted between the researcher and participants. The intention of the post-study interviews was to investigate shifts in beliefs, values, and perceptions between the pre-study interview and this one. The schedule for the current study is presented in Table 1.

Table 1

Summary of Research Activities and Data Gathering Strategies Implemented During Each Phase of the Current Study

Phase	Research Activity	Data
1. Term One	• Individual semi-structured (pre-study) interviews	• Semi-structured interviews audio recorded and transcribed
2. Term Two	• Observations of mathematics lessons: – Mrs Ulster: two lessons observed – Mrs Walter: two lessons observed • Study of student work and teacher planning for these lessons	• Four large group discussions video recorded and transcribed (two per teacher) • Observational field notes: descriptive notes and reflective comments • Photographs of student work and teacher planning
3. Term Three	• Observations of mathematics lessons: – Mrs Ulster: two lessons observed – Mrs Walter: two lessons observed • Final (post-study) semi-structured interviews • Study of student work and teacher planning for these lessons	• Four large group discussions video recorded and transcribed (two per teacher) • Observational field notes: descriptive note and reflective comments • Semi-structured interviews audio recorded and transcribed • Photographs of student work and teacher planning

3.4 Researcher Role

The current study aimed to employ a transparent approach to the role of the researcher, taking the view that such openness was positive and useful. In qualitative research studies the researcher is "the primary instrument for data collection and analysis" (Merriam & Tisdell, 2015, p. 15). The researcher in the current study was the sole collector and analyser of the data. This enabled the researcher to maximise efficiency of data collection and ensure the quality of the data collected. The qualitative paradigm is premised on reducing the distance between the researcher and the participants (Pihama, 2015). The researcher in the current study had a dual role as a researcher and a mentor of mathematics. While the researcher did not mentor the participants, both participants knew the researcher in her capacity as a mathematics mentor.

This pre-established relationship with both participants reduced the distance between both parties and was viewed as a positive element within the study. There was no attempt made, within this paradigm, to hide the values, experiences, perceptions, skills, and expectations the researcher necessarily brought with her. The current study was unapologetically qualitative as the quest for objectivity would have been redundant and insidiously reductionist (Smith, 2012). It was, therefore, appropriate to acknowledge how influential the role of the researcher was in the current study. Clearly stating researcher biases, values, and judgements from the outset of the research design supported the transparency the researcher in the current study sought (Noble & Smith, 2015). This stance recognised the inter-relationship between the researcher and participants. Additionally, it promoted the researcher to reflect on her unconscious bias and preconceived expectations or perceptions (McArthur, 2022).

To mitigate potential bias the researcher deliberately sought teacher participants whom she was not mentoring. This protected the purity of the data and supported all parties to have clarity of purpose. Even with this clarity and definition between roles, the researcher was aware that she was perceived by the teacher participants as the expert in the room during the observations. Participants who know they are being observed can alter and regulate their behaviour in a way that is different to their normal behaviour pattern (Merriam & Tisdell, 2015). There was a danger that the teachers would not feel relaxed enough to teach naturally. To mitigate this, the researcher placed an emphasis on the co-construction of understanding between the researcher and participants. Since understanding human behaviour and interpreting the belief structures that underpin it is a complex undertaking this co-construction of knowledge was necessary. The teachers always remained the experts on their own, and their students' motivations for behaviour and the researcher acknowledged this.

3.5 Ethical Considerations

The current study used Massey University's (2017) Code of Ethical Conduct for Research, Teaching and Evaluations Involving Human Participants and the Te Ara Tika Māori ethical model (Hudson et al., 2010), to guide ethical decisions and considerations. The research proposal was submitted to Massey University Human Ethics Committee in 2021. It was reviewed and approved prior to data collection. The ethical considerations in the current study are outlined under four categories:

1. Tika: Intent and conflict of interest.
2. Mana: Consent, confidentiality, and privacy
3. Whakapapa: Relationships, reciprocity, and respect
4. Manaakitanga: Socio-cultural considerations.

3.5.1 Tika: Intent and Conflict of Interest

The intent and aims of the current study were clearly stated in the information sheet, which was provided to the principal, the Board of Trustees, alongside the invitation to participate (see Appendix E4). The intent of the current study was to explore the ways teachers support inclusion of marginalised students in the large group mathematical discussion. The aim was emancipatory because the researcher aimed to identify deliberate teacher actions that supported inclusion. The participants in the current study understood the intention of the study and shared its aim. This mutual purpose mitigated conflict of interest issues through a shared goal to research openly for the benefit of marginalised students (Hudson et al., 2010).

As aforementioned, the researcher had a dual role as a mathematics mentor and researcher. There was the potential for conflict of interest. The researcher provided the participants with an explicit explanation of the different intentions between participating in a case study and being mentored. The condition that the researcher would not also be the participants' mentor was always part of the criteria for participation, to avoid a conflict of interest.

3.5.2 Mana: Consent, Confidentiality, and Privacy

Coercion to participate was a possible ethical dilemma in the current study. Teachers could have potentially perceived the invitation to participate as a management recommendation since their principal had signed them up to the associated professional learning and development (PLD). The Massey University Code of Ethics states that the researcher is ethically obliged to make all invitees explicitly aware that declining the invitation, or withdrawing at any point, does not affect their participation or experience in the PLD in any way (Massey University, 2017). Written consent was obtained from all participants, including the school Principal and Board of Trustees (see Appendices E1-E3). The researcher in the current study made the voluntary nature of the invitation explicit to the principal when seeking his consent to use his school as a research site.

The researcher initially issued an invitation for expressions of interest generically to the whole staff via an open invitation email with a strong emphasis on the voluntary nature of expressing interest. Mrs Walter and Mrs Ulster expressed an interest and sought further explanation around the expectations of their commitment. The researcher provided them both with a detailed, hard copy, information sheet and consent form (see Appendices E1-E4). Once a schedule was proposed, with various options for timetabling the interviews, both Mrs Walter and Mrs Ulster felt satisfied that the study would not infringe on their own time commitments adversely and enthusiastically consented to participate in the current study.

To protect the privacy of all participants the researcher recorded all personal details, such as names, anonymously using codes and pseudonyms. Assurance was given that it should not be possible to identify any participants or the school from the research report. Interviews were conducted between the researcher and each participant offsite to protect the privacy of participants (Forsey, 2012). To ensure confidentiality all information was handled sensitively and protected the confidentiality of the participants. Safe custody of the data was maintained. The recorded interviews were transcribed by the researcher only.

3.5.3 Whakapapa: Relationships, Respect, and Reciprocity

Relationship dynamics between the participant and researcher are of utmost importance. Smith (2012) refers to the importance of the three r's: relationships, reciprocity, and respect, when researching with people. Establishing trust is the first step to developing productive relationships. Within the Māori ethical model, whakapapa, the pre-established relationship between the researcher and participants, is viewed as an advantage because trust already exists (Smith, 2012). The researcher in the current study had positive, pre-established relationships with both participants. This supported participants to place their trust in the unfamiliar processes of research such as the somewhat uncomfortable and formal situation of filming lessons and audio-recording interviews.

Respecting participants includes understanding that time is precious. Teachers are time poor and have many demands to meet within this limited timeframe. To mitigate this intrusion the researcher designed the schedule with a focus on minimum encroachment on participants' time and energy (Miles et al., 2018). Observations occurred during these teachers' normal mathematics lessons, with no change to their ordinary timetable. The interview times were mutually negotiated. An interview environment conducive to creating a rapport is important (Lee & Goodman, 2009). Interviews in the current study were held at each participant's location of choice where privacy was protected. Reciprocity requires the researcher to reciprocate time, energy, and gratitude in payment for the gift of knowledge imparted by the participants (Pihama, 2015). The researcher in the current study presented both participants with a koha upon completion of the research.

3.5.4 Manaakitanga: Socio-cultural Considerations

It is important for the researcher to establish socio-cultural bonds with participants (Pihama, 2015). The researcher and participants in the current study are from the same ethnic group, gender, and generation, which enabled the researcher to draw on many commonalities to establish a socio-cultural bond. Additionally, the Māori ethical model, Te Ara Tika (Hudson et al., 2010), enabled the researcher to consider her spirit, or wairua, when researching. It is important to have a clear wairua through each stage of the research process to enact manaakitanga (Smith, 2012). The researcher in the current study utilised this ethical tenet during the pandemic related disruptions to the schedule. While the disruptions were beyond the researcher's control she felt ethically compromised having to cancel and re-schedule observations. This disruption subsequently caused the observations, when they did occur, to be closer together than was planned. With a clear wairua the researcher apologised to the participants and communicated honestly about the situation not being ideal.

3.6 Data Collection

The mode of data collection in the current study was qualitative. Instruments of collection included structured observations, semi-structured interviews, and copies of artefacts relevant to the study, such as student work and teacher planning. Qualitative methods enabled the researcher to access the understandings, responses, and perceptions of the participants in the real-world context of a busy classroom (Punch & Oancea, 2014). The data collection instruments, selected as appropriate for the current study, were based upon their effectiveness to analyse teacher actions and beliefs that promoted inclusion of students in the mathematics discussion (Merriam & Tisdell, 2015).

Data were collected from eight separate observations of mathematics lessons over two terms (four observations per teacher). Observational field notes were semi-structured because these were recorded using a framework based on the seven tools identified in the literature (see Appendix B). Data was mined specifically from the large group discussion via video recording, which was wholly transcribed. Data were collected from four separate semi-structured interviews (see Appendices A1-A2). Each teacher was individually interviewed once prior to the first observation and once post the final observation. The interviews were semi-structured because they followed a set question format but were designed to be open (see Appendices A1–A2). All interviews were audio recorded and transcribed by the researcher at a later date.

Data were collected from photographic copies of relevant artefacts, including student work and teacher planning, collected at each observation. Photographs were taken, during the large group discussion, of collective work on the whiteboard. Photographs were taken of small group work, where relevant. Photographs of teacher anticipations of tasks, planning, and notes were taken, where relevant. Document analysis of the collected video and audio transcripts, field notes, and artefacts was subsequently undertaken following the process outlined in the following data analysis section.

3.6.1 Data Analysis

Data analysis is the exercise of constructing meaning out of the data (Noble & Smith, 2015). Analysing the data in the current study meant making sense of the teachers' actions and the beliefs that enabled these actions to occur. This section outlines the process of analysing the three sources of data collected: observations, semi-structured interviews, and artefacts.

3.6.1.1 Observations

Teacher actions were analysed through the framework of seven pedagogical tools, identified through the literature, to promote the inclusion of marginalised students in mathematical discourse. Each teacher action, which promoted inclusion, was placed in one of the frames under these codes: the five practices, talk moves, mathematical practices, norms, culture, status, and high expectations (see Appendix D1).

3.6.1.2 Semi-structured Interviews

The semi-structured interview data was analysed for shifts in teacher beliefs using an inclusionary and exclusionary pedagogical framework. Initial codes were developed based on themes identified in the pre-study interviews: the pedagogy, belief changes, mathematical practices, talk moves, status, norms, culture, students with specific needs, the share back, a common task, anticipations and misconceptions, and high expectations. Data, from the post-study interviews, were recorded onto the same framework and shifts in perceptions or beliefs were identified in a different font colour. This framework clearly identified the shift of beliefs moving out of the exclusionary table and into the inclusionary (see Appendix D2).

The completed framework tables subsequently underwent a secondary analysis where teacher actions and beliefs were compared by combining the two frameworks together. Results were analysed for alignment between shifts in beliefs and actions. For example, one belief shift, where Mrs Ulster raised her expectations of Mary's capability to be involved in the discussion, was cross referenced with her teacher actions related to high expectations. In this example, the shift in teacher actions aligned with the shift in beliefs (see Appendix D3).

3.6.1.3 Artefacts

Student work and teacher planning were analysed under the same codes as the actions and beliefs. These results were subsequently cross-referenced with the combined teacher beliefs and actions framework to scrutinise for alignment between beliefs, actions, and artefactual evidence. For example, Mary's participation where she capably records two thirds in fractional notation with peer support is corroborated by clear artefactual evidence. The artefact (see Figure 2, section 4.5) clearly displays the dots drawn by Mary's peer and Mary's own tracing over the dots. This aligned with the teacher actions which enabled her to stay in the discussion and the teacher belief that Mary must be included (see Appendices D1-D3).

3.7 Quality Criteria

All quality research involves analysing data in a reliable manner that produces valid conclusions (McArthur, 2022). Reliability is challenging in qualitative research because human behaviour is complex, and social research relies on context (Merriam & Tisdell, 2015). Replication of a qualitative study may not produce the same results as contexts are ever evolving, as is human behaviour. However, validity can be enhanced through consistency, transferability, and trustworthiness (Noble & Smith, 2015).

3.7.1 Validity and Reliability

The current study tested the consistency of the results obtained from the data using a within methods triangulation data analysis. Transferability was generated through substantial descriptions of the setting, participants, and themes. Trustworthiness was enhanced through clearly outlining the qualitative elements of the study. The findings were shared with the participants prior to completing final analysis to check that interpretations aligned, corroborate the findings, and re-adjust where there were discrepancies.

Consistency was achieved in the current study through using multiple data sources and applying a within methods approach to triangulation. There are three types of triangulation: between methods, between researchers, and within methods (McArthur, 2022). The data in the current study was collected from three sources: interviews, observations, and artefacts. Each analysed data set was subsequently triangulated through merging all three into one framework to scrutinise for cohesion under the same codes (see Appendix D3). Similar patterns and trends emerged across all three data sources. Identified themes which over-lapped between each type of data were cross-checked for relatability to code frequency, increasing the inter-data reliability and making use of the triangulation within methods data analysis.

Transferability was attained in the current study through substantial descriptions, which thoroughly detailed the setting, participants, and themes. This enabled readers to experience the chronicled events and determine the transferability of the findings to other contexts for themselves (Noble & Smith, 2015). Furthermore, substantial descriptions generate substantial evidence to prove that the findings are consistent with the data collected, and the conclusions are trustworthy (McArthur, 2022). In the current study, the plethora of descriptions from field notes, transcripts, artefacts, and notations on framework tables was overwhelming at times for the researcher. However, after multiple passes through the raw data undeniable themes and patterns began to emerge. The data sources were initially kept separate, and themes were organised into singular frameworks with their own codes. It then became clear that all three frameworks aligned and were able to be combined into one framework with a shared set of codes and themes. This supported the researcher to confidently draw conclusions from the data.

Trustworthiness was enhanced in the current study in the three ways recommended for qualitative research (Nerland, 2022). Firstly, the researcher clearly stated the research rationale, aims, and the question which needed to be addressed. Secondly, the researcher’s role was explained, especially in relation to possible biases, assumptions, and conflict of interest. Thirdly, the researcher thoroughly outlined the research schedule, timeline, data-gathering procedures, arrangements of interviews and observations, relationships with participants, and the categories developed for analysis (Nerland, 2022).

3.8 Summary

This chapter has outlined the research design and methods used in the study, including the rationale for selecting a case-study approach and qualitative methods of data collection and analysis. A variety of methods to collect data were used, including interviews, classroom observations, and student and teacher artefacts. Data was analysed using thematic analysis, identifying codes, and developing themes. Triangulating data supported the credibility of the interpretations. The Te Ara Tika Māori ethical model (Hudson et al., 2010) provided a culturally appropriate ethical framework to conduct research with the participants. The findings of the current study are presented in Chapter Four.

Chapter 4: Findings and Analysis

4.1 Introduction

The previous chapter outlined the design and methods used in the current study, including discussing the data collection methods used in this investigation. This chapter will analyse the data collected over the course of the study. The data will be analysed in terms of which teacher actions were employed and their impact on inclusion of marginalised students in the mathematical discussion. The analysis is situated within the framework of the seven teacher tools required for an effective mathematics lesson under the inquiry model, as identified in the literature review.

Section 4.2 analyses teacher actions that promoted inclusion through use of the five practices. Section 4.3 examines teacher actions that promoted inclusion through use of teacher talk moves. Section 4.4 reviews teacher actions that promoted inclusion through eliciting student mathematical practices. Section 4.5 investigates teacher actions that promoted inclusion through fostering social norms. Section 4.6 explores teacher actions that promoted inclusion through accessing cultural funds of knowledge. Section 4.7 examines teacher actions that promoted inclusion through attending to status issues. Section 4.8 analyses teacher actions that promoted inclusion through holding high expectations of all students. Section 4.9 summarises the chapter.

4.2 The Five Practices – Teacher Actions that Promoted Inclusion

Both teachers already incorporated the five practices in their mathematics lessons. However, they held reservations about how they were meeting the needs of specific students during the large group discussion, known colloquially as the *share back*. They found it difficult to "include everyone's voice and ideas" and were concerned that a few students were not able to access the mathematical thinking because key concepts went "over their heads" (initial interviews). Particularly, one student in each class, caused major concern for both teachers because they had specific needs which differed to their peers. "Mary," Mrs Ulster said, "is an outlier as she has learning difficulties" (initial interview). "Duncan," Mrs Walter said, "is an outlier because he is significantly behind his peers" in his achievement of mathematics (initial interview). A major motivator for both teachers' participation in this study was to find ways to include Mary and Duncan in the discussion.

Both teachers reflected on how their shift within the first two practices, anticipating and monitoring, was refined by having a specific lens on inclusion. “When I am doing the anticipations now, I think what Duncan might get stuck on” (Mrs Walter, final interview). This teacher action moved the teacher’s focus beyond the generic anticipation of how her whole class would tackle the task and towards how specific students would engage with the task.

Mrs Ulster reflected that this refined anticipation supported her to know what prior knowledge she should tune Mary into during the launch. “We tend to reflect now as a group, like ‘remember last time when we counted all the dots?’ We think about what we could have done differently based on what we learnt from the previous share back and connect” (Mrs Ulster, final interview). By considering prior knowledge from the previous lesson, Mary got additional support to recall important information from her last lesson which she needed in that day’s lesson. Furthermore, Mrs Walter’s specific anticipations about Duncan informed her planning of the launch. “I think… what Duncan might need to know – like a little bit of understanding such as ten tenths make a whole and I make a note of this for my launch” (Mrs Walter, final interview). These specific anticipations led to specific teacher actions during the launch, with the aim of supporting marginalised students to engage in the same task as their peers.

During small group time Mrs Ulster and Mrs Walter monitored Mary and Duncan closely. Their focus was on the impact of their actions during the launch and how this affected Mary and Duncan’s level of engagement. Mrs Ulster noticed that, while Mary began counting each individual dot, she quickly altered to skip counting the array in fives with her buddy. Mrs Walter noticed that Duncan confidently divided his group’s picture of a loaf of bread into ten even pieces. Additionally, he was validated by his peers for starting their solution trajectory. Mary and Duncan used the prior knowledge their teachers had cued them into to successfully engage in the same task as their peers.

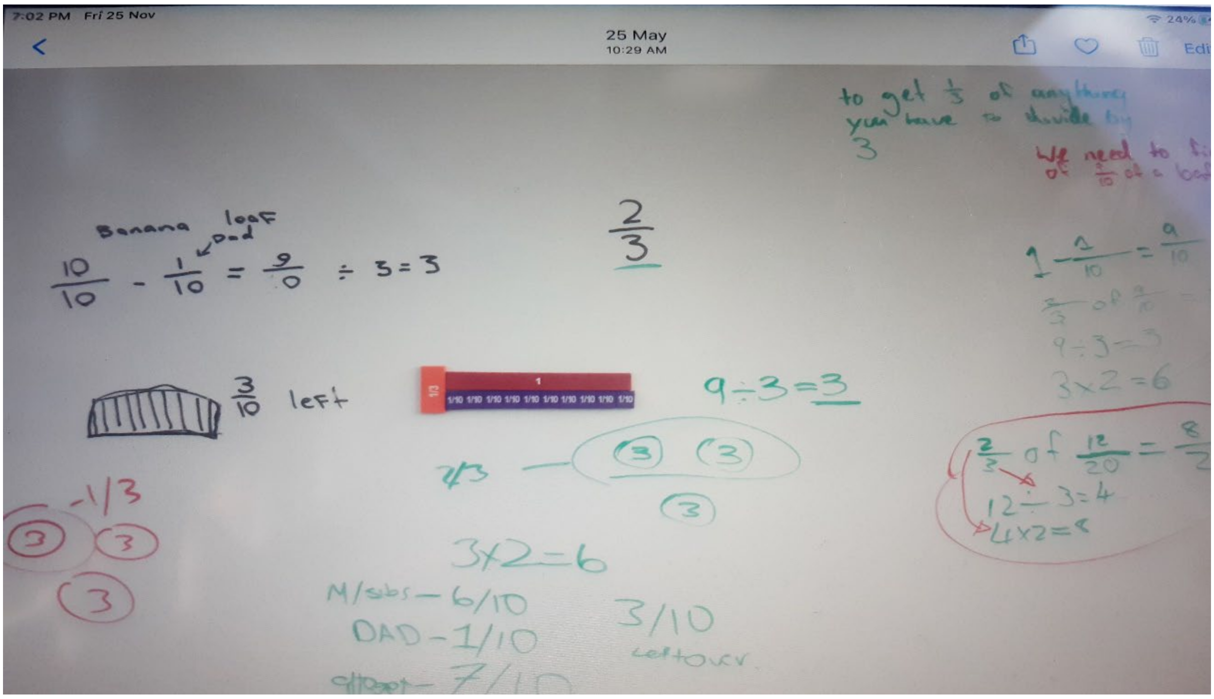
The engagement in the task led to successful engagement in the large group discussion. The third and fourth practices, sequencing and selecting, were critically considered, and then refined by both teachers drawing a lens focused on inclusion. Mrs Ulster described how she shifted to selecting and sequencing a range of different solutions, including partial solutions. This allowed her to select groups whose solutions were partially complete or correct. This provided a greater diversity of students and representations during the discussion: “Mary might only be accessing a tiny bit of the lesson, but she can share back what she can with the support of her group*”* (Mrs Ulster, final interview). This was a significant belief shift from Mrs Ulster’s initial interview, where she explained how Mary was excluded from mathematics lessons because she could not access “any of it” (initial interview). Mrs Ulster supported Mary and her buddy to share with everyone how they had skip counted using the array model. Mrs Ulster’s actions sent a message to Mary that she valued her input in the discussion. Simultaneously Mrs Ulster’s actions told the whole class that everyone’s contribution was valuable.

Analogous with Mrs Ulster, Mrs Walter refined her teacher actions in terms of selecting and sequencing, by actively seeking and selecting a wider variety of mathematical representations: “The visual representations are really coming along. My kids value drawing pictures because it is actually hard to draw a picture of level four thinking. It is not seen as babyish anymore. It is valuable and clever” (Mrs Walter, final interview). Through the teacher action, of drawing on a greater breadth of diversity in representations, the students came to realise that there was not only one ‘correct way,’ and their heuristic thinking was valued. Mrs Walter selected Duncan’s group to launch off the discussion by sharing their starting point. This was a deliberate teacher action aimed at validating Duncan’s ideas and contribution.

Mrs Walter prepped Duncan with the bit she wanted him to unpack so he was able to do this with the support of his peers. This was a deliberate teacher action aimed at setting Duncan up for success. Duncan was hesitant at first. He hid behind the large sheet of paper he and his group had used to work the problem out on. After a short pause Duncan explained: “one whole is the same as ten tenths, so Michaels’s Dad took one tenth then that left nine tenths …of the loaf of bread” (Duncan, observation one). While Duncan explained, another group member drew a picture of the loaf of bread on the whiteboard, as shown in Figure 1.

Figure 1

Mrs Walter's Whiteboard – Michael's Loaf of Bread Task – The Share Back



Mrs Walter subsequently validated the group for doing the hardest bit and "launching us all off" on the solution pathway. That was a pivotal moment for Duncan as he was not used to sharing. Subsequent groups built on Duncan's group's ideas until the whiteboard had a cohesive sequence of ideas building from early fraction concepts to more sophisticated. Mrs Walter continued to refer to "Duncan's important idea" that students were building on. Duncan remained engaged throughout the rest of the discussion, even asking a question at one point, which was uncommon.

Mrs Walter's careful sequencing allowed her to select sophisticated and less sophisticated solution strategies that promoted the inclusion of Duncan in the discussion. Mrs Walter reflected how this set the tone for Duncan's subsequent pattern of engagement during discussions: "he often contributes now. Everyone is used to Duncan sharing now" (Mrs Walter, final interview). Enacting the five practices, with a lens on inclusion, enabled Mrs Walter and Mrs Ulster to engage their marginalised students. The success of each practice impacted upon the success of the subsequent practice. Every refined teacher action, in each practice, precipitated and orchestrated the inclusion of marginalised students in the large group discussion.

4.3 The Talk Moves – Teacher Actions that Promoted Inclusion

Both teachers already incorporated talk moves in their lessons but admitted it was "sporadic" and "not often enough" (initial interviews). Mrs Walter reflected on how she sometimes used talk moves because "you are supposed to" (initial interview). As the study progressed both teachers became intentional about why and when to use the talk moves. Specifically, they focused on how the talk moves could support them to engage their marginalised students in the large group discussion. For example, Mrs Ulster found power in using the revoice move to re-engage Mary when she had tuned out during the discussion. Mrs Ulster asked the student sitting right next to Mary to revoice an important mathematical idea or conjecture. The outcome was that Mary had an additional opportunity to hear a crucial piece of information.

Mrs Ulster started "doing a little more focus on using repeat" (final interview*).* She used this move with Mary quite often. Mary was not always able to revoice an idea in her own words but was able to repeat verbatim what another student had said. Mrs Ulster deliberately used this move to give Mary a voice in the discussion, even if Mary did not understand exactly what she was repeating, her voice was included. The eventual outcome of this was that Mary contributed voluntarily to the discussion. Furthermore, Mrs Walter employed the add on talk move with a specific purpose. When a marginalised student contributed a conjecture, idea, question, or explanation to the discussion Mrs Walter asked if anybody could add on to this important idea or question. By positioning the contribution as worthy of being added to, this teacher action validated the marginalised student in front of their peers. Both teachers used the revoice, repeat, and add on talk moves to spread the mathematical ideas across the whole group which brought marginalised voices into the discussion.

Both teachers refined their use of the reasoning talk move: "I am really pushing them for the 'why" (Mrs Ulster, final interview). Mrs Walter talked about digging deeper: "I am going deeper with them [talk moves]. I use them to get reasoning now" (final interview). Mrs Walter often pushed for reasoning by prompting her students with "do you agree or disagree?" (Initial interview). Mrs Walter refined this prompt by reminding her students that they were disagreeing or agreeing with the mathematics and not the person. "I like the way the talk move agree/disagree is saying that it is not so and so you disagree with but the math" (Mrs Walter, final interview). Furthermore, Mrs Walter specified to her students that it was okay to be unsure if you did not know if you agreed or disagreed. These refinements promoted inclusion in the large group discussion because her marginalised students did not feel attacked if their idea was disagreed with, or shame if they said they did not know.

To remind herself to use the reasoning talk move Mrs Ulster wrote *because* in big letters on her whiteboard. Every time a student gave an unsophisticated explanation during the discussion Mrs Ulster prompted with *because* to push her students into justifying, as shown in the following vignette:

Large Group Discussion: The Array Task

Teacher: Cooper – how many in this group?

Cooper: ten

Teacher: in the whole group?

Cooper: oh, forty

Teacher: forty and how many in the bottom one?

(Cooper looks uncertain)

Richard: it is twenty

Teacher: because?

Richard: because I just counted them

Teacher: how?

Richard: I counted by two's

Observation four: the array task

Evident in the vignette is how Mrs Ulster employed the revising talk move and how she pushed for justification using "because." Mrs Ulster invited Cooper to revise his thinking by prompting him with "the *whole* group," which led Cooper's buddy, Richard, to offer a different answer ("twenty"). Mrs Ulster pushed Richard to expand his (correct) answer using her new "because" prompt. This resulted in the whole group realising *how* Richard got twenty (counting "by two's"). During the connect phase of the lesson, after the discussion, Mrs Ulster facilitated the whole group to count the array in twos, like Richard had, which spread the understanding across the group. For students who were struggling with thinking multiplicatively this unpacking of Richard 's thinking was vital.

Both teachers used wait time frequently and deliberately to allow their students space to think at their organic pace. Mrs Ulster not only employed this move but named it and explicitly explained to her students why taking your time was important: “I am giving you wait time… I am going to put a question to your group, and I want to give you a few moments to think…use your really good diagrams” (Mrs Ulster, observation four). The move was dove-tailed to match the mathematical practice of explaining a mathematical representation. Through this action Mrs Ulster explicitly showed that she valued her students’ individual thought processing times and gave them the message that the discussion was not a race but a process. The first time Mrs Ulster used the wait time move on Mary it was because Mary could not quite get the words out for her response. Mrs Ulster said she would give her time to think and would come back to her. Mary looked surprised when Mrs Ulster remembered to check back in with her during the discussion. While Mary transitioned to this new expectation of her participation, she increased the frequency of her requests to get a drink, use the bathroom, or leave the lesson. Mary slowly grew accustomed to being given wait time, which led to her being a regular contributor: “Mary often joins in [the discussion] now. It is incredible!” (Mrs Ulster, final interview).

In congruence with Mrs Ulster, Mrs Walter practised wait time to increase the variety of contributions in the discussion. When Mrs Walter asked Duncan to contribute, she always gave him some wait time. Mrs Walter found she did not have to go to another student and come back to Duncan, he just required a little bit of thinking time before he responded. Over the course of the study, students started giving their peer’s wait time during the discussion when it was needed. Creating space to think had become the new norm. Mrs Walter often used wait time in conjunction with the revising talk move. Sometimes all it took was for the teacher to pause once an explanation was given, or a conjecture offered, before the student revised their thinking. Both teachers reflected on how this talk move allowed for ambiguity to become a normal part of the mathematical discussion. “I tell them I do not mind if they do not know, as long as they say they do not know” (Mrs Ulster, final interview). This created a safer space for all students to voice their thinking because being wrong, partially wrong, or unsure was normalised as part of what it means to do mathematics.

Both teachers already regularly employed the turn and talk move during the launch of their lessons but not often in the discussion. With their new lens on inclusion both teachers refined this move by using it during the discussion to increase the amount and variety of voices contributing. They employed this move when there was a lull in conversation, when there were too many students talking at once, and when they wanted their marginalised students to contribute. Both teachers reflected on how this talk move helped them remain mindful of the amount of teacher voice in the discussion. Both teachers reflected on how this move provided their marginalised students with the opportunity to share their ideas with just one buddy. “They all talk with everyone and anyone now” (Mrs Ulster, final interview). This talk-move oriented peers to view each other as credible resources. Additionally, it gave the teachers an opportunity to listen to the student dialogue and notice where their students were at in terms of understanding. The turn and talk move precipitated a new way to bring the voice of marginalised students into the discussion.

Both teachers refined their use of talk moves through an intentional focus on using them as a tool to promote inclusion of marginalised students. Each talk move provided a unique way to access these student’s thinking and prompt their participation. Over the course of the study, both teachers used all the talk moves frequently with the purpose of including their marginalised students in the discussion. Who, how, when, and why each move was used was always intentional. The outcome was a greater diversity of voices contributing to the discussion and a continual flow of conversation, enriched by more divergent ideas and questions.

4.4 Mathematical Practices – Teacher Actions that Promoted Inclusion

Both teachers already provided opportunities and encouraged their students to use mathematical practices in the discussion. However, at the beginning of the study, they both felt there was a lot of room for improvement. “I am not doing the mathematical practices well enough; I can tell you this!” (Mrs Walter, initial interview). Both teachers reflected that student responses and explanations during the discussion were very much a work in progress. Mrs Walter reflected that the practices “come from me – not the kids” (initial interview). Mrs Ulster admitted that sometimes eliciting mathematical practices felt like “pulling teeth” (initial interview). Both teachers were especially frustrated at their marginalised students’ lack engagement with mathematical practices. These students did not ask questions, explain their thinking, or contribute to the discussion. “They just switch off” (Mrs Walter, initial interview). Mrs Ulster was so frustrated by Mary’s lack of engagement that she no longer included her in the lessons, let alone the discussion. “Mary can’t join in – she is too low and does not have the concentration span” (Mrs Ulster, initial interview).

As the study progressed both teachers made a significant shift in eliciting mathematical practices through their heightened focus on using talk moves. “It’s the talk-moves, isn’t it? I am using them to get the math’s practices now, not just to get kids talking!” (Mrs Ulster, final interview). Employment of the reasoning talk move, for example, elicited student engagement in sense making and justifying. “Justification is the big one. I really push back on that one now. That word ‘because’ is swimming around in my head all the time now!” (Mrs Ulster, final interview). Justification became the new norm during discussion time in Mrs Ulster’s class. Mrs Walter, too, wanted justification to be a discussion norm. At the beginning of the study Mrs Walter already elicited justification: “pushing for justification comes naturally because you want to see how much they are really understanding” (initial interview). Mrs Walter’s frustration was around having to be the one to elicit the justification, she wanted students to start to do this of their own volition. While at first Mrs Ulster used her talk moves to elicit justification, and other mathematical practices, over time her students expected it of each other and themselves. Mrs Walter experienced the same shift in her classroom: “they are doing these now! [mathematical practices] Not me!” (Final interview).

The improvement in justification led to an overall better flow of discussion. “There are not as many (unproductive) pauses as there used to be during the discussion. Students are filling those gaps in conversation…I have noticed a real rise in student engagement – all students” (Mrs Ulster, final interview). Students supported their peers to justify by asking questions to clarify, sense make, and clear up misconceptions. In turn, their peers explained with justification and through multiple representations. Mary, like her peers, asked questions. Although these were often superficial or irrelevant, she was participating and knew asking questions was valued. Mary explained her thinking, with support, and even justified her idea when pushed to. All students were seeking a common understanding and viewed each other as pivotal in this pursuit.

The shift from implicit mathematical practices, where students did not always know they were engaging in them, towards explicit mathematical practices required a refinement of teacher actions. Using talk moves to elicit the practices was effective but the biggest shift occurred when the teachers made mathematical practices explicit through naming the practice and explicitly stating its value. For example, the students knew their teacher valued multiple representations because this had been explicitly stated. Both Mrs Walter and Mrs Ulster named the practices and explained why they valued them. They initiated dialogue, sustained it, then reprised it so all students were made explicitly aware of its presence and its value in the discussion.

For example, Mrs Ulster highlighted the representation Greg used to explain his thinking because she wanted everyone to see the value in using arrays to think multiplicatively. Mrs

Ulster initiated the discussion with Greg, but it was her sustain move which was the catalyst for his specific mention of the diagram, as shown in the following vignette:

Making Mathematical Practices Explicit: *Representations*   Array Task

(*initiate)* **Teacher:** Can you just remind me what I asked you to look at now Greg?

Greg: how we explained it differently – fifteen rows of four.

(*sustain)* **Teacher:** What did you explain it with?

Greg: a diagram!

Teacher: yes, does anyone know what this kind of diagram is called?

Ollie: an allay!

Teacher: yeah – but is an a*rr*ay not an a*ll*ay. (Teacher writes array on the whiteboard and circles it). Do you remember the game we play where you throw the dice and draw rectangles and squares?

Students: oh yeah!

Teacher: so, an array is where you are drawing it in lines and columns

(*reprisal*) so, using your arrays {*naming}* you all drew your diagrams, {*highlighting}* they were amazing! {*evaluating}* – so this group here – your diagrams are outstanding because they are so easy to read {*Explaining}*. So, if you use your diagrams, called arrays, it is going to make it easier for you {*Framing}*.

Observation four: array task

Evident in the vignette is Mrs Ulster using all three talk moves, initiate, sustain, and reprise as well as the five sub-sets of the reprisal move: naming, highlighting, evaluating, explaining, and framing. Mrs Ulster named the practice of using a representation, an array in this example, before highlighting it as a clever tool that many students used. She continued to reprise the practice by evaluating it, specifying that using this model to count is clever, and explaining what makes a good array. Finally, Mrs Ulster framed the practice across time so her students were explicitly aware that arrays would help them in future lessons, and it was not restricted to an isolated task on one day.

The refinement of teacher actions to elicit mathematical practices resulted in an increase of student engagement in all mathematical practices, including the marginalised students. Both teachers normalised confusion and named this as an important part of sense making "Gina: say you do not understand what they are doing" (Mrs Walter, observation three). Duncan and Mary noticed their peers getting confused and heard their teacher validating this as an important mathematical practice. “I don’t mind if you do not understand but you must ask questions” (Mrs Ulster, observation three). Clearly, the teacher was promoting practices such as articulating partial ideas, seeking clarification, or saying you do not understand as important. Duncan began asking questions during the discussion because his teacher made the mathematical practices of sense making normal. Mary volunteered her ideas in the discussion because this was the norm, and her contributions were always valued.

The increased frequency and quality of mathematical practices created a better flow of discussion with an organic pace, a greater variety of voices and representations, a greater breadth of ideas, and a higher quality of discourse supported by robust justification and mathematical argumentation. Those outcomes promoted the inclusion of marginalised students in the large group discussion because the net spread wider to allow for all ideas, including divergent ones, the focus shifted to the process, not the result, and concepts were unpacked through representations such as pictures and diagrams, which made them more accessible.

4.5 Norms – Teacher Actions that Promoted Inclusion

Both teachers already had well embedded social norms around the organisational routines and procedures related to the discussion. There were specific expectations displayed around behavioural protocol during the discussion time, such as waiting until afterwards to use the bathroom or get a drink. For certain marginalised students, however, expectations were lowered. Mary was permitted to leave the discussion to get a drink, go to the toilet, or to do something with the teacher aide because she was bored. Mrs Walter kept Duncan present on the mat throughout the duration of the discussion but felt sorry for him: “I just can’t see what he’s getting out of it” (Mrs Walter, initial interview). While most behaviour was pro-social during the discussion both teachers reported certain students “who are really frustrated that they can’t just get on with it because they have to explain it to somebody else…they do not want to see other people’s point of view, and this bothers me” (Mrs Ulster, initial interview). Both teachers believed that the norms needed improving in terms of inclusivity during the discussion time.

One of the significant shifts, for both teachers, occurred around teacher actions related to the collaborative norms in the groups. Both teachers shifted from expectations which focused on the individual towards placing the responsibility on the whole group. “Ruth, you are part of this group, and you need to join in… stand with your group, remember you are part of this group”

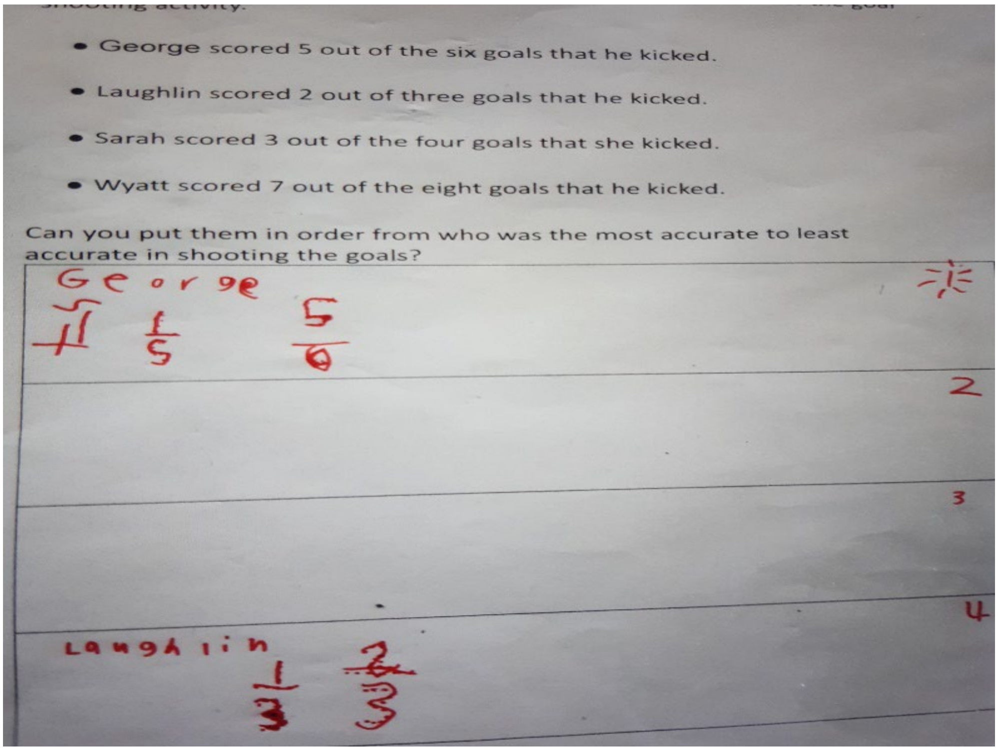
(Mrs Walter, observation one). While individual accountability was important, and the teacher action to remind Ruth that she was part of a group was valid, there also needed to be responsibility placed upon the group to include Ruth. Both teachers reflected on the shift in collaborative norms and how this had a positive impact on their marginalised students. They made their expectations around collaborative participation explicit. "I actually need everyone to take part and I am not going to let anyone just sit there! This has spilled over into my independent half too…they now work together when they need to. They are all doing really well!" (Mrs Ulster, final interview).

The explicit expectation manifested in student internalisation of collaboration. Evidence that students shifted from being a collection of individuals to a collaborative was demonstrated during the time where they worked independently of the teacher. During that time students naturally co-constructed knowledge and helped each other without their teacher being present. Mrs Walter, too, noticed this shift at independent time in her classroom "for independent time I have a selection of level three and four tasks available for students to self-select. They often choose to work together on these…the children are trying their hardest to make sure everyone understands. They are doing really, really well" (Mrs Walter, final interview). The shift towards the pro-social was described by both teachers in terms of a whole culture shift in the classroom. "There is a good environment in here now, where kids will talk but sometimes, they're in a bad mood or acting the clown, but we all accept each other" (Mrs Walter, final interview).

Over the course of the study, both teachers refined their teacher actions by positioning themselves as part of the learning community. One subtle, yet powerful, shift occurred through their language, transitioning from *I* and *me* towards *we*, *us,* and *team,* or phrases such as "come on guys, let's work this out" (Mrs Walter, observation four) and similar language which implied togetherness. In turn, that language shift prompted a change in the language students used with each other. During the final observation Mary hesitated when it was her turn to share. One of her group members said, "come on, Mary, we will count it with you." During observation two, students had to order a set of fractions, as shown in Figure 2.

Figure 2

Mary's Supported Fraction Recording: Fraction Ordering Task



Mary's buddy supported her to record the fractions she wanted to write but was not sure how to. Mary's buddy drew the vincula and numerals in dots so Mary could trace around the dots. Rather than viewing Mary as incapable and doing all the work for her while she sat watching, or staring off into space, Mary's buddy found a way to include her. Mary's contribution to the work was valid and valued by her buddy and, consequently, by all her peers at share back time when they saw her fraction notation of one third and two thirds. The outcome, during share back, was a very proud Mary who puffed her chest out and clearly felt capable because *she* had written the fractions on the page. Mary had a right to stand up and be counted.

Mary's participation in discussions increased after that share back. A new self-fulfilling prophecy emerged. The cycle of teacher validation caused peer validation, which caused self-validation, which caused more teacher validation and so the cycle continued. Whenever Mary fell back into self-doubt, as she did in the final observation where she hesitated, her peers did not accept her defeat. Instead, they rallied her to participate: "Mary, we will count with you." We have got this, they told her, we are in this together. That was evidence of inclusive collaboration.

Teacher actions focused on making collaborative norms normal caused the culture shift in both classrooms. Students became comfortable explaining, justifying, making conjectures, agreeing, and disagreeing, and verbally revising thinking. The shift was a direct result of teacher actions that promoted social and collaborative norms. Two of the most notable outcomes from the shift was the diversity in voices during the share back and the evidence of inter-thinking as the new norm, as shown in the following vignette:

Large Group Discussion: All-Blacks and Springboks Height Task
Eric, Lee, and Gina explaining how they calculated the average height to their peers

Teacher: just talk now, to each other, discuss this.

Eric: the All Blacks average height is a lot taller than the Springboks

Lee: yes

Teacher: a *lot* taller?

Eric: yes

Lee: all their heights, they're kind of all about the same height but the Springboks are like there are some real short ones and some real tall ones so lots of different heights…

Gina (adding on) staggered

Eric: the All Blacks are closer together in height

Gina: most of them are in the 190's (cm)

Teacher: most of them?

Gina: *most* All Blacks

Lee: ah – not most of them

Gina: most All Blacks are 190–200 cm tall

Teacher: so how many is that? (T brings in the students on the mat) what did you guys' down here notice?

Annie: (on mat) there are forty-four All Blacks and fifty Springboks

Teacher: so, what do you think that means?

Sam: (on mat) there is clearly more Springboks than All Blacks

Moses: (on mat) maybe it is their strategy to keep their players fresh (the Springboks) so they can keep rolling new players off the field, so they have lots of energy. Maybe that is how they won the world cup!

Teacher: (deliberately bringing in new voices to the discussion) who was it who had a group statement wondering about their being less All Blacks than Springboks?

Taylor: (on mat) us and we wondered if it was because they are fitter

Teacher: you think having less players might mean that the All Blacks are fitter?

Ash: (on mat) or there could be more injuries in the Springboks

Teacher: yes, because on average they were taller than the Springboks. What would height and injury have to do with each other?

Genesis: (on mat) when you are taller you are bigger so this might make you a bit stronger.

Moses: if you are taller, you do not get as many head tackles.

Observation four: All-Blacks and Springboks height task

Evident in the vignette is how Mrs Walter promoted inter-thinking amongst her students by orchestrating the flow of the dialogue while keeping the intellectual property with her students. Mrs Walter began the discussion by asking the students who were sharing back to discuss with each other first. That action set the tone for student-led dialogue. When the dialogue became stilted Mrs Walter prompted for more justification. Additionally evident in the vignette is Mrs Walter's deliberate use of teacher interjection with the rationale of bringing more voices into the discussion. Mrs Walter had a direct lens on inclusion, bringing in a different wondering from a different group and checking in with students on the mat to see what they noticed so they connected their work with what their peers were saying. Furthermore, Mrs Walter prompted mathematical argumentation by asking "most of them?" That action precipitated disagreement between Gina and Liam, which was deepened by the teacher bringing more students into the discussion.

There was still some ambiguity surrounding the average height of the All Blacks compared to the Springboks at the end of the discussion, which was cleared up by the teacher during the connect. This vignette demonstrates how comfortable these students felt with ambiguity and how they jumped into the discussion anyway. Many different voices joined the discussion, including all the students sharing back and some students on the mat. The students were clearly bouncing off each other as one idea flowed into another, evidencing inter-thinking. Additionally, the vignette highlights how comfortable students were with partial ideas, such as noticing that the sample set size and the numbers within the sample set were interrelated in a way that affected the average, yet they were not exactly sure how.

Over the course of the current study, both teachers refined their teacher actions related to fostering social and socio-mathematical norms. Student engagement in these norms led to greater ambiguity, different ideas, and different viewpoints in the discussion. The new norm of discussing as a collective sense-making exercise, enriched by variety of voices and ideas, caused a greater acceptance of each other. Students viewed *all* their peers as credible resources. The new peer orientation towards marginalised students supported students like Mary and Duncan to gain a true sense of belonging in the discussion.

4.6 Cultural Funds of Knowledge – Teacher Actions that Promoted Inclusion

Both teachers were used to planning contexts to suit their students as this was an integral part of the professional development which they had engaged in for three years. During the initial interview both teachers referred to culture and its importance in terms of promoting inclusion. Mrs Ulster spoke about the possibility of trying out different ideas for culturally sustaining task contexts as a tool to include Mary in the lesson. She thought jigsaw puzzles might be a good one as Mary loved puzzles and could easily complete a 300-piece puzzle. Mrs Walter spoke about Duncan and trying to find out what his life outside of school consisted of.

Over the course of the study, both teachers' focus on culture fluctuated. During observation two Mrs Walter altered the context in a task to suit her students. Mrs Walter changed the schoolbag outlet suggested in the original task to a company which was more patronized by her students (see Appendix C6). Mrs Walter unpacked the context with students during the launch, as was her well-practised norm since beginning the DMIC PLD. However, Mrs Walter did not revisit the context at the start of the share back discussion. At one point in the discussion Mrs Walter asked her students "why do rebel sports need to know this information?" Here she was pushing for the task to become experientially real for her students.

During observation one, Mrs Walter used a context of cutting a loaf of bread for breakfast to teach fractional concepts. The problem was about how much of the loaf was left after Michael and his dad used some of the loaf for breakfast (see Appendix C5). Mrs Walter engaged all her students in a discussion about bread during the launch. While this problem was about a fresh homemade loaf of bread, waiting to be sliced, she referred to other types of bread her students might be familiar with. Mrs Walter covered different brand names, such as Tip-Top, Ploughman's, Quality Bakers, and different types of bread such as pre-sliced, gluten-free, white, whole meal, grain bread, fired bread, fruit bread, and so on. Every student had made at least one connection to their prior knowledge of bread by the end of the discussion.

Mrs Walter drew on Duncan's choice of bread, which was freshly baked white bread, and made links between finding tenths and Duncan cutting a loaf of bread for breakfast. Duncan's peers drew a loaf of bread on a piece of paper and encouraged Duncan to divide it into ten even strips. Duncan cut the first strip off and pretended to be Michael's dad eating his one tenth of the loaf. This supported Duncan to clearly see the remaining nine tenths of the loaf. During the final observation Mrs Walter tapped into cultural funds of knowledge and used this cultural capital to raise the intellectual status of marginalised students in the discussion, as shown in the following vignette:

Large Group Discussion: All-Blacks and Springboks Height Task Moses is the cultural fund of knowledge

(*T Move Initiate*)

Teacher: was it Moses who made the comment that what we are missing here is weights that go with it?

Moses: …if comparing the weight of say the All Blacks playing the Springboks then you compare their pack weight to the Springbok's pack weight. So, from when other people get subbed on it shows their pack weight in the scrum and it keeps changing when people get subbed.

(T Move Sustain)

Teacher: So, do you think the weights have more effect on the game than the height?

Moses: yeah, because Cheslin Colby is a South African winger, and he is one of the best wingers in the world and I think he might be the shortest in international rugby. He is 167 cm tall.

(T Move Sustain & push for justification)

Teacher: but did his height make any difference to how good he is at his profession?

Moses: no – he made it into the world ruby team – the fifteen and he was number fourteen, the winger.

Teacher: that is right so what math's did you work out?

Observation four: All-Blacks and Springboks height task

Evident in the vignette is how Mrs Walter positioned Moses, a marginalised student in her class with low intellectual status, as an expert during a task about calculating averages of rugby players' heights (see Appendix C4). She tapped into his expertise on the All-Blacks rugby team. Interesting to notice, here, how Mrs Walter used teacher talk-moves, initiate, and sustain, to draw out Moses' expertise. Mrs Walter deliberately brought Moses into the discussion using the initiate move to position him as the cultural fund of knowledge. Moses saw what he loved and had experienced in life reflected in the task and how his knowledge fund was useful to his peers' making connections to the big idea of the lesson. Mrs Walter sustained this by asking him to use his cultural expertise to draw a mathematical conclusion about weight versus height. That action sent the message to Moses and his peers that she valued Moses as a mathematician who used his rugby expertise to make sense of the mathematics.

During observation three Mrs Ulster added to the context of healthy eating by using the real context of the school breakfast club which many of her students regularly patronised (see

Appendix C2). The context was enthusiastically discussed during the launch and small group inquiry time. The context was meaningful for Mrs Ulster's students. During the discussion one student, Richard, made three unsuccessful interjection attempts to point out what he had noticed about the data in relation to the context.

Initially Richard interjected "but there are twenty-two people in this class" (Richard, observation three). Nobody heard or responded to him. Later, he tried to point out that another group's data set has a population of "two more than us" (Richard, observation three). When Mrs Ulster asked Richard if he also included the teacher and teacher-aide in his survey results he replied "yes, but we did not put it on our table because we did not write you or Mrs Filcher down" (Richard, observation three). Richard's noticing was contextually and mathematically important. He had noticed that the data were skewed because students had included the teacher and teacher-aide in the results, yet they were not eligible to attend breakfast club. This was an ideal opportunity to make the task experientially real through discussion of how important it is to keep the purity of the data sample set.

Over the course of the study, Mrs Ulster focused more deeply on cultural funds of knowledge with a lens on inclusion. Mrs Ulster reflected on using real life contexts to support her students with new terminology as well as mathematical ideas: "they did not know what an *array* was, so I connected them to a game we have in class with rectangles with rows of squares inside and they all went 'oh!" (Mrs Ulster, final interview). During the final observation, Mrs Ulster began the large group discussion by revisiting the context of the task. The task was a multiplication problem which required the use of arrays to solve it. The task context was a bossy ant ordering her ant family into even rows (see Appendix C3).

The context was kept alive throughout the discussion and was used to make the mathematics experientially real for students. Mrs Ulster continuously referred to the context even when re stating an equation. For example, she said "eight rows of five ants" instead of simply eight rows of five. This prevented the context getting lost in the numbers. Mary was intrigued by the ants and the stories that Mrs Ulster told them about the ants. When the mathematics became tricky Mrs Ulster brought Mary back into the fold of the discussion through the context: "you count the ants, Mary, and he will track your counting as you go" (Mrs Ulster, observation four). The outcome of that teacher action was a newly alert Mary who keenly counted the little ants and referred to the ants when she found the answer. Counting the ants held meaning.

Both teachers promoted inclusion in the large group discussion through designing culturally sustaining contexts for their tasks, positioning students as cultural experts or funds of knowledge, and making the mathematics experientially real.

4.7 Status – Teacher Actions that Promoted Inclusion

Both teachers were already aware of the impact status had on classroom dynamics and individual students' self-efficacy. Studying and reflecting on the status issues at play in their classrooms was an integral part of the DMIC PLD. During the initial interview both teachers shared how status issues in the large group discussion affected the participation of their marginalised students.

Mrs Ulster reflected on the inequitable distribution of intellectual authority: "there are students, perceived by their peers to have high status… and group members look to them and expect them to have all the answers, but they can be on the wrong track!" (Mrs Ulster, initial interview). This indicates that the students in Mrs Ulster's class positioned each other within a hierarchy of credibility as a source of mathematical information. Mrs Ulster reflected on how this hierarchy affected the students who were positioned, by their peers, as an intellectual authority: "they get really frustrated [having to be the main driver] because they just want to get on with it and not have to explain it to someone who is not listening or does not understand" (Mrs Ulster, initial interview). These high-status students saw no value in the mathematical practice of explaining their thinking.

Of most concern to Mrs Ulster, however, was Mary's low intellectual status. Mary's status was so low her peers did not even perceive her a member of the learning community because she was excluded from mathematics lessons. Mary had an individual programme of learning which she completed with her teacher aide during mathematics lessons. "This is the first year I haven't included one student in my groups because she [Mary] is so low" (Mrs Ulster, initial interview). There was no possibility of raising Mary's status while she was not even physically present at the discussion. During initial observations Mrs Ulster physically re-positioned students on the mat to increase engagement during the mathematical discussion. Apart from Mary's exclusion, the ethic of care in Mrs Ulster's classroom had already shifted from keeping students safe, by not expecting them to contribute, to valuing the process towards sense making. Mrs Ulster explicitly assigned intellectual competence to students with low status during the discussion in a deliberate teacher move to raise status.

During the final interview Mrs Ulster reported feeling "a lot better" about the status issues in her classroom. The biggest shift, she felt, was Mary's raised status. "I used to think Mary cannot take part with us but now I think Mary can do math's with us. She joins in with us every day now. It is incredible!" (Mrs Ulster, final interview). Mrs Ulster included Mary in every lesson and discussion over the course of the study. The outcome of that teacher action, Mrs Ulster believed, was Mary's peers now viewed her as a member of their learning community: "everyone is happy to have Mary in their group" (Mrs Ulster, final interview). Mrs Ulster re-calibrated the status-quo in her learning community by raising the status of her marginalised students. Mrs Ulster's most marginalised student, Mary, now possessed a strong sense of belonging in the mathematical discussion with her peers. Mary's peers came to view her participation as normal and welcome.

In fact, most students came to view all their peers as resources worthy of consulting: "nobody complains anymore 'oh, we've got X in our group!" (Mrs Ulster, final interview). Mrs Ulster reflected how her teacher actions around eliciting mathematical practices helped students with high status see the value in explaining, justifying, and questioning. She noted that the students were now aware that the discussion was "a journey, and not a race to the answer" (Mrs Ulster, final interview). However, Mrs Ulster reported, there was one exception: "I still have one high status girl. I make sure I spread her around the groups, so nobody gets her all the time. She is a work in progress" (Mrs Ulster, final interview). Unlike in the initial interview, Mrs Ulster approached this concern with confidence in her teacher actions to re-calibrate this status issue: "I always get someone else in her group to share back first so she has to wait her turn" (Mrs Ulster, final interview). Mrs Ulster's teacher action, making the girl with high status wait, sent a strong message to the girl and her peers that everyone's ideas and thoughts are valued. Mrs Ulster became aware of her power as the key influencer of what is valued in the learning community.

Mrs Ulster used this to not only re-calibrate high status but to raise low status: "sometimes students say stuff quietly at share back and others talk over them. I remind them to trust themselves…this is when we have some really good argumentation" (Mrs Ulster, final interview). Mrs Ulster demonstrated a shift in terms of how valuable she views all her students to be when she reflected how, sometimes, her students with low status "are actually correct and the students with high status are mistaken" (Mrs Ulster, final interview).

During the final observations Mrs Ulster assigned competence more often, on more varied occasions, and to a greater variety of students. No longer was it just used to praise mathematical thinking but, also, to praise connection with peer's thinking and mathematical practices. This was an important shift because it signalled that Mrs Ulster had broadened her values around what being good at mathematics meant to her. Additionally, a new teacher action Mrs Ulster employed was the re-assignation of competence. Like the talk moves initiate, sustain, and reprise, Mrs Ulster used the re-assignation move to explicitly raise status, as shown in the following vignette:

Large Group Discussion	**Final Observation**	**Re-Assigning Competence**
Teacher: You are amazing!		
Horton *shakes his head in denial*		
Teacher: don't shake your head at me, Horton! You are amazing!		
Horton *shy smile*		
Teacher: you are making such worthwhile contributions. You are amazing.		
Horton *smiles again*		

Evident in the vignette is how Mrs Ulster initially assigned competence to Horton, but Horton rejected the assignation. Instead of conceding defeat, Mrs Ulster reassigned competence to Horton. This time it was not rejected. Mrs Ulster then specified why Horton was competent. Analogous with making mathematical practices explicit by naming them, when competence is named, students understand what constitutes as competency in mathematics. The repetition of praise was vital because it was this pendulum swing to the positive extreme that re-calibrated Horton's deficit mindset.

Initially Mrs Walter, too, expressed a sense of frustration about how status was distributed amongst her class. She, particularly, reflected on the inequitable participation during her large group discussion: "A lot of them are switched off…these are the ones who do not see themselves as effective mathematicians…they think share back is the time for the smart kids to shine and they have to sit quietly" (Mrs Walter, initial interview). Although Mrs Walter gave all her students the explicit message that the large group discussion was a chance for everyone to share their thinking and ask questions to deepen understanding, this was not how it was interpreted by all her students.

Most of all, Mrs Walter was concerned with the status and engagement of her most marginalised student, Duncan. Duncan's status was "low across the board" (Mrs Walter, initial interview). Mrs Walter felt there was not "any value whatsoever" in the share back for him as he had "no idea" what was being discussed (Mrs Walter, initial interview). When asked how she was addressing her concern around students with low status, Mrs Walter replied "I just try to get then all involved in some way" (initial interview). During early observations Mrs Walter got all students involved through capitalizing on her role as the facilitator of the discussion to call specific students into the conversation. For example, Mrs Walter deliberately invited a student with low status to participate by asking them to unpack a representation she knew they felt confident explaining. The teacher action was effective because it orchestrated an opening for a voice which would otherwise not been heard.

Over the course of the study, Mrs Walter developed a greater repertoire of teacher actions that served to raise the status of her marginalised students, especially Duncan: "it [the study] has really worked to raise Duncan's status amongst his peers" (Mrs Walter, final interview). Mrs Walter reflected on the power of assigning competence to position Duncan as a worthy source of information in front of his peers: "for example, saying Duncan can you share that idea with everybody because it is an important idea that everyone needs to hear" (Mrs Walter, final interview). Mrs Walter realised her own power to influence what was valued in the large group discussion. When her students heard her placing value on someone's ideas, they placed value upon it. One outcome of this, Mrs Walter said, was "everyone loves being in Duncan's group!" (Mrs Walter, final interview). The raising of Duncan's status was re-calibrated through a deliberate series of teacher actions that included re-positioning Duncan at the epicentre of discussions and deliberately valuing his contributions. Mrs Walter, however, felt less of a shift had occurred in her classroom regarding high status:

> The kids who were identified at the beginning of this study [with high status] probably still hold high status. I tried putting all the high-status students together in groups and let them fight it out …but sometimes they are so busy arguing about whose strategy works best they do not come to a solution! (S. Walter, personal communication, August 8, 2022).

Mrs Walter did not view this as an opportunity to shine a light on how students with low status were operating more effectively as mathematicians. Mrs Walter perceived it as a problem for the high-status students as they had not reached a solution pathway and usually these students are the ones who "find the tasks easier and finish their independent work" (Mrs Walter, final interview). Mrs Walter viewed this status hierarchy as intrinsically natural:

> I do not necessarily think it is troublesome in the classroom [having high status] because in life everyone has different, well not strengths, but like, for different jobs, you have to have different attitudes. These high-status kids will probably go into jobs where they will need to be bossy and show leadership whereas the other ones might take jobs where they need to be team players. (S. Walter, personal communications, August 8, 2022)

Both teachers raised the status of their marginalised students through refined teacher actions which re-calibrated their status in the large group discussion. The extent to which status issues overall were re-calibrated in the discourse community depended on teacher beliefs around the status hierarchy and how intrinsic or fluid it was.

4.8 High Expectations – Teacher Actions that Promoted Inclusion

One of the key pedagogical shifts both teachers made was an increase in teacher expectations of their students' efforts, work ethic, and capabilities. During the initial interview both teachers were asked what the following statement meant to them: "what does the phrase '*being a high expectation teacher'* mean to you?"

Mrs Ulster responded by sharing the teacher action she employed to get reluctant contributors to participate "I specifically say, 'you need to ask a question" (Mrs Ulster, initial interview). Mrs Ulster prepared her reluctant students by rehearsing their role prior to share back: 'I am going to ask you a question and this is what it is going to be, and you will answer it" (Mrs Ulster, initial interview). This indicated a high expectation ethic of care because Mrs Ulster set up the expectation that participation was going to occur and was not negotiable. Mrs Ulster set up supports around this so her marginalised students were scaffolded to experience success when contributing to the discussion. Mrs Ulster stated that the phrase 'a high expectation teacher' did not equate to expecting mastery from all students during the share back but "that they know how to be a group member" (Mrs Ulster, initial interview). Although Mrs Ulster did not mention mathematical practices, talk moves, or norms specifically in relation to high expectations here, her comments indicated that the seed was planted to make these connections over the course of the study.

In terms of beliefs around student capabilities Mrs Ulster's comments indicated she had already begun the shift from a traditional pedagogy towards an inquiry model: "I used to want them to learn the way I wanted them to learn but now it's about not jumping in there too quick…letting them get in there and trusting them" (Mrs Ulster, initial interview). These teacher actions and beliefs described a high expectation ethic of care where the teacher allowed students to grapple with confusion without being rescued by the teacher.

However, with her most marginalised student, Mary, expectations were lowered. "Mary likes the thought of sharing but when it comes to it, she just can't" (Mrs Ulster, initial interview). Mrs Ulster described how Mary was not involved in mathematics lessons. This was a teacher-imposed exclusion based on not wanting to expose Mary's inadequacies and cause humiliation. Mrs Ulster stated that she "might try putting her [Mary] in a group – for the study, but the teacher aide will have to be with her" (Mrs Ulster, initial interview). Mrs Ulster demonstrated an openness to including Mary, but parameters had to be set around Mary's participation, in this case it was having the support of the teacher aide. Initial lesson observations evidenced Mrs Ulster's low expectations. Early in the discussion Mary requested to leave. Mrs Ulster accepted this request without question. Interesting to note that directly before Mary requested to leave, she contributed to the discussion but received no response.

There was a shift in Mrs Ulster's expectations of her most marginalised student, Mary, during the final interview. Mrs Ulster stated that she "always includes Mary now" (Mrs Ulster, final interview). Furthermore, she had lifted the parameters of Mary's participation: "I used to think Mary can do math's with the teacher aide… now I do not even want the teacher aide in the room!" (Mrs Ulster, final interview). Mrs Ulster's ethic of care shifted from keeping Mary safe to making Mary struggle productively. Mrs Ulster reflected that Mary still had "global learning delays" and sometimes "she dips out, other times she stays for the whole lesson" (Mrs Ulster, final interview). Mrs Ulster was culturally responsive to Mary's needs by allowing her to dip in and out, but her teacher actions are inclusive. Mary was invited to every lesson and was viewed as a valuable member of the learning community. "I now think Mary can do math's with us or she is not part of the class" (Mrs Ulster, final interview). This showed a shift in thinking from an exclusive framework, where some students can belong to the learning community and some cannot, to an inclusive framework where everyone is included and expected to participate.

It was a shift from low expectations to high expectations, but the expectations were equitable. Not everyone participates in the same way. This demonstrated a culturally sustaining inclusive pedagogy. Mrs Ulster reflected on how she felt her class was more of a community now Mary was part of it. She never felt comfortable excluding Mary. Mrs Ulster found a way to include Mary in the share back. “For Mary I always tell, well not always, that this is the bit that you are going to share back” (Mrs Ulster, final interview). This was a deliberate scaffolding, within Mary’s zone of proximal development, which enabled Mary to deliver a worthwhile contribution to her group’s share back. Mrs Ulster now believed that everyone had the capability to contribute something important to the discussion.

The final lesson observation re-iterated Mrs Ulster’s reflections. Mary was an active participant in the discussion. Approximately halfway through the discussion Mary requested to leave. Mrs Ulster denied the request: “Mary, your help is needed here.” Mrs Ulster’s action positioned Mary as an important member of the discussion. Additionally, Mrs Ulster *physically* repositioned Mary by moving her. However, she did so in a way which promoted her agency by offering her a choice: “would you like to move next to me or next to Richard?” Mary moved to sit next to the teacher. Mrs Ulster’s high expectations enabled Mary to keep participating.

Subsequently, Mrs Ulster took a further teacher action to keep Mary engaged by cueing Mary into what she was about to request her to do to support Richard. This was a very deliberate move which raised Mary’s status amongst her peers because Richard held high status. Here the teacher sent her whole class the message that *Mary* can help *Richard,* rather than always the other way around. A little while later, Mary requested to leave again. This time it was because she wanted “morning tea.” Mrs Ulster responded that she could “have morning tea soon but first let’s check our counting.” Mary became involved with the mathematical practice of double-checking the array she and Richard counted together. Towards the end of the lesson Mrs Ulster noticed Mary skip counting and, in genuine celebration, publicly praised Mary’s successful counting. Mary grinned ear to ear. Another student said they did not hear Mary’s counting. Mrs Ulster explained this was because it was quiet. Another student said, “Mary skip counted like the rest of us” (student, observation four). Mary’s peers positioned her as *one of them*. No longer was Mary *other*.

Mrs Walter reflected that, for her, the phrase being a high expectation teacher meant “everyone will be able to access something out of the task and get something out of it” (Mrs Walter, initial interview). This showed a high expectation belief system that all children can learn. However, expectations were lowered when it came to her most marginalised student, Duncan: “I just can’t see what he is getting out of it” (Mrs Walter, initial interview). Analogous with Mrs Ulster, Mrs Walter had a fixed mindset around the capabilities of her most marginalised student. However, Mrs Walter, like Mrs Ulster, was open minded to including Duncan and promoting his opportunities to contribute.

Initial observations showed that Duncan was used to being present at the discussion phase but was not accustomed to being an active participant. Mrs Walter held high expectations for many of her students. This was evidenced by her teacher actions where she deliberately and frequently checked on certain students who she knew might have trouble sense making and held them individually accountable. Mrs Walter showed a high ethic of care for her students by pushing them out of their comfort zone and requiring them to ask questions, seek clarification, and share their thinking: “Tim, do you want to explain the tally chart?” (Mrs Walter, observation two). In doing so Mrs Walter created a new prophecy in her learning community. The belief that productive struggling was natural, and effort was valued. Duncan was excluded from this new message, however, because he was never called upon. Through not calling on Duncan, Mrs Walter showed a low expectation ethic of care for him.

There was a significant shift, indicated in the final interview, in Mrs Walter’s thinking around teacher expectations. Mrs Walter reflected on her feelings of increased confidence and capability now to “take a lesson under the DMIC model that all kids can access and be successful at their level of being successful” (Mrs Walter, final interview). Mrs Walter attributed this increased confidence to her shift in focus. Mrs Walter reflected how she no longer focuses on everyone reaching the same level of mastery in a lesson: “I just feel like you can get something out of the lesson for everyone, but it might not be the same for everyone” (Mrs Walter, final interview). This was evidence of a shift from the homogenous belief that students should achieve the same learning outcome to a heterogenous belief that, within a common task, students would achieve different outcomes.

Mrs Walter specifically reflected on the result of this shift in mindset in relation to her most marginalised student: “Duncan’s low status has been removed…they [his peers] know he says valuable things and I get him to share back. Now I think it is almost like kids think ‘yay we have Duncan in our group – he will definitely have something to say” (Mrs Walter, final interview). Duncan, like Mary, now belonged firmly in his learning community. Mrs Walter reflected on Duncan’s improved identity as a capable mathematician: “he really believes that his contribution is as valuable as anyone else's. He even comes up now and asks for more math's work – he would never have done that before. He sees himself as a mathematician" (Mrs Walter, final interview).

The rise of Mrs Walter's expectations resulted in the rise of Duncan's participation. Duncan's increased participation led to his revised identity as a capable mathematician in his own and his peers' eyes. Mrs Walter's final lesson observations reiterate her reflections. Discussions were sequenced in way which enabled students, like Duncan, to launch the solution pathway yet allowed for a high ceiling to the task. As the facilitator of the discussion Mrs Walter expertly ensured every student contributed to the discussion and these contributions were validated. The students did not patronise Duncan by clapping and saying, "well done," as they had done in earlier observations. They just accepted this was the bit he was sharing and that his idea or explanation would be built upon by others.

Mrs Walter's teacher actions directly prior to the discussion phase were a vital element of this success. During the small group inquiry phase of the lesson Mrs Walter monitored Duncan's group carefully. Mrs Walter monitored Duncan's thought processes so that she could track his trajectory within the task and scaffold his share back role to maximise what he had been able to do. Additionally, this provided Mrs Walter an opportunity to capture any reasoning Duncan came up with. His reasoning was then highlighted by Mrs Walter as she selected his group for sharing back and Duncan was provided the chance to practise how he would explain his thinking at the share back.

The more Duncan shared the more he surprised Mrs Walter with his level of thinking. During the initial interview Mrs Walter stated that Duncan was working "at level one "of the New Zealand mathematics curricula, which aligns to ages five and six. Over the course of the study, Mrs Walter revised her thinking and placed him at level two, which aligns to ages seven and eight. This was a much more accurate placing. However, during the final lesson observations Duncan displayed some level three thinking, which aligns to ages nine and ten. This is only one curriculum level below the expected achievement level for his age. This highlights the importance of not underestimating students as well as the shift which can occur when teacher expectations are raised.

Both teachers raised their expectations of their most marginalised students as their belief systems shifted from a low expectation ethic of care to a high expectation ethic of care. The outcome of their raised expectations was raised participation of their marginalised students. The result of this increased participation led to greater engagement in the key concepts of the task. As student engagement deepened new mathematical identities were formed and a new pattern of participation emerged. The marginalised students became key contributors to the mathematical discussion and everyone's perception of these students shifted from an exclusive framework to an inclusive framework.

4.9 Summary

This chapter began by analysing the refinement of teacher actions to promote inclusion in the large group mathematical discussion using specific pedagogical tools: the five practices, talk moves, eliciting mathematical practices, fostering norms, and cultural funds of knowledge. Teacher reflections, in the interviews, were analysed alongside observations of teacher actions. Analysis of both actions and beliefs were supported by analysis of student artefacts. Student representations of thinking included: diagrams, pictures, symbols, writing, graphs, and equations. These representations were shared during the discussion phase and were studied to evidence conclusions drawn from teacher reflection and action.

The shift in teacher actions aligns with the shift in beliefs. Additionally, students' work reflects this shift. The analysis around status and teacher expectations indicates that the greater the shift in expectations and belief systems around inclusion, the greater the shift in teacher actions. As both teachers' understanding of the pedagogical tools deepened their use of these tools became more frequent, more varied, and more intentional (see Appendices D1-D3). Both teachers refined their actions by employing them with specific timing and rationale. While every teacher action was unique, and used for a unique purpose, the aim remained the same. Every action was used to promote the inclusion of marginalised students in the mathematical discussion.

The result of these refined actions was an increase in their marginalised students' participation in the mathematical discussion. With this new norm of participation teachers re-evaluated their belief systems around the capabilities, effort, and work ethic of their marginalised students. Consequently, both teachers lifted their expectations of their marginalised students. This proved challenging for the teachers in terms of their shift from a fixed intelligence belief system to a belief in the fluidity of intelligence. Where higher expectations extended not only to students with low status, but to those who held high status there was a re-calibrated status within the learning community where all students viewed their peers as viable resources for learning mathematics. Everyone's point of view was accepted, even if it differed to your own, and consequently everyone was accepted; no matter how divergent the thinking was. The result was an enriched discussion, relational equity, and greater opportunities for all students to successfully access the key mathematical concepts discussed.

Chapter 5: Discussion and Conclusion

5.1 Introduction

The preceding chapter analysed the findings in relation to teacher actions that promoted the inclusion of marginalised students in the large group mathematical discussion. This chapter will discuss the findings in relation to themes which emerged and make links to the literature. Additionally, this chapter concludes the study through a review of the key findings related to the research questions, a summary of key implications arising from the current study with suggestions for educators, consideration of potential future research opportunities, and an overall summary of the thesis.

Section 5.2 will discuss the importance of planning for inclusion and how the five practices supported teacher actions to achieve this. Section 5.2.1 will discuss teacher actions that facilitated the emersion of real mathematical identities in marginalised students through teacher talk moves and eliciting mathematical practices. Section 5.2.2 will discuss teacher actions that promoted marginalised students to develop a real identity in the learning community through fostering the norms, utilising cultural funds of knowledge, and raising status. Section 5.2.3 will discuss the shifts in teacher belief systems as a new ethic of care emerged through holding high expectations of all students. Section 5.2.4 will summarise these four themes.

Section 5.3 summarises the research questions. Section 5.4 presents implications for educators. Section 5.5 outlines limitations of the study. Section 5.6 considers potential future research opportunities. Finally, section 5.7 provides an overall summation with concluding thoughts.

5.2 Planning for Inclusion

Placing a lens on inclusion began with both teachers planning for the inclusion of their marginalised students. One of the most impactful shifts both teachers made, using the five practices model, was refining how they anticipated the task. This aligns with Stein et al.'s (2008) research, which highlights how anticipation is a significant instructional practice because the success of the subsequent four practices hinges upon this. Furthermore, both teachers added to their generic anticipation of how students would tackle the task by focusing on how their marginalised students would approach it. They looked at possible misconceptions these students might encounter, as well as divergent pathways they might take solving the task. The more these teachers planned this way the more proficient they became at predicting how their marginalised students would fare with any given task. This supported the teachers with their ongoing cycle of planning and monitoring their marginalised students during lessons. This is similar to Hunter and Hunter's (2018) research indicating that anticipating in this way supports teacher understanding of marginalised students' learning.

This increased understanding was further utilised by the teachers in their specific plans for the launch. Both teachers used their anticipations to ensure that their marginalised students were prompted in the launch to remember necessary prior knowledge to tackle the task. This resulted in increased engagement of their marginalised students during the small group inquiry and corroborates Hunter and Hunter (2018) claim that a thoroughly planned launch supports marginalised student's engagement in the task. Both teachers, based on their refined anticipations, began to monitor their marginalised students with specific focus. Stein et al. (2008) assert that this focus leads to a more effective selection and sequencing process, and therefore a more effective discussion. The findings of the current study support this assertion.

Stein et al.'s (2008) findings demonstrate how teachers struggle with the complex task of selecting and sequencing a clear trajectory of ideas. The teachers in the current study did struggle, initially, but, as their content knowledge improved, they were able to see the trajectory more clearly. In alignment with Sullivan's (2008) findings, their content knowledge improved as their planning became more thorough. Both teachers, initially, struggled with how and when to include their marginalised students in the selection and sequencing. Their shift in this area corroborated Erath's (2018) assertion that divergent, erroneous, or partial thinking enrichens the discussion rather than de railing or diminishing it. As the teachers selected and validated the first step of a solution or an unusual pathway, they saw how this deepened everyone's understanding and, subsequently their belief system shifted. This backs up Hunter and Hunter's (2018) claim that teacher beliefs impact on inclusion in the selection process for the discussion.

Through selecting a more diverse set of ideas, and therefore a greater variety of students, to share back during the discussion students began to see that their teacher valued the process rather than just the result. Unpacking errors became a valuable exploration. Partial ideas could hold gold, just as much as fully formed ones. Making mistakes, getting confused, and taking a risk to make a conjecture were normalised as part of how the discussion process went. This is the powerful re-calibration Warshauer (2015), and Erath (2018) refer to, where entrenched negative beliefs around mistakes and misconceptions transform into valuable teaching points. The normalisation of this process sends the message to students that their heuristic thinking is valued, as previously described by Lawler (2018). In the current study, planning with a focused lens on inclusion set marginalised students up for success in the task. This subsequently enabled them to take part and feel a sense of belonging in the discussion as Hunter and Hunter (2018) found in their study.

5.2.1 Creating Mathematical Identities as Knowers and Doers of Mathematics

As the teachers became more cognisant of the purpose of talk moves, they began using them to elicit mathematical practices. With a lens on inclusion, both teachers became skilled at eliciting their marginalised students to engage in mathematical practices. While the five practices supported teachers to select and sequence their marginalised students' participation in the discussion, the mathematical practices gave students the tools they needed to behave like mathematicians. This promoted new mathematical identities within these students as knowers and doers of mathematics.

The biggest shift both teachers made in their usage of talk moves was refining their rationale for when, why, and how often to employ these moves. Aligned with Chapin and O'Connor's (2007) findings, the revoice move proved to be a powerful teacher action when these teachers used it to clarify the thinking for everyone in the discussion. Both teachers realised how revoicing related to the mathematical practice of sense-making. They began to employ this talk move to support the sense-making of their marginalised students. Similar to Chapin and O'Connor's (2007) findings, this provided the teachers an opportunity to check in on the understanding of the re-voicer. Additionally, this move was used when marginalised students needed another opportunity to hear the same information in a different way.

The repeating and 'add-on' talk moves enabled the teachers to provide an opportunity for their marginalised students to engage in the mathematical practice of explaining. Like Chapin and O'Connor (2007) stated, it did not matter if students did not understand what they were explaining when repeating someone else's idea because the move served to tune these students back into the conversation and this increased engagement. When teachers invite students to 'add on' to another student's idea, conjecture, or argument the mathematical voice is spread across the class and the discussion deepens (Chapin & O'Connor, 2007). As well as including more voices in the discussion this served, as Takeuchi (2018) argued, to add nuance to the discussion. An unexpected addition to this talk move was its dual use with the assigning competence move. This served a dual purpose of adding nuance to the discussion to deepen it and raising the status of a marginalised student.

The reasoning and revising talk moves supported both teachers to elicit the mathematical practice of mathematical argumentation. Chapin and O'Connor's (2007) findings show how the reasoning talk move supports robust discussion because it encourages students to actively make sense of others' ideas and check these against their own reasoning. While robust discussion did occur in both classrooms it built up slowly over the course of the study. Using the reasoning talk move on its own was not enough to elicit robust debate during the discussion. When teachers invited students to revise their thinking, they sent their students the message that being incorrect or confused was a normal part of the sense making process. This aligns with Kazemi and Hintz's (2014) findings that teachers who regularly use the revise talk move support students to view understanding as an ever-evolving process because constructing new knowledge requires refining along the way.

Both teachers employed the turn and talk move to enable their marginalised students to engage in the mathematical practices of explaining and questioning. The teachers gained valuable information about their understanding as they listened to this talk. Kazemi and Hintz (2014) found that this supported teachers around their choices for who to call on next to contribute to the discussion. The teachers in the current study, with their lens on inclusion, listened to their marginalised students to understand their reasoning to contribute back to the whole group. Langer-Osuna's (2017) claim that this move has the benefit of orienting peers to view each other as rich resources proved true. The marginalised students in the current study became valuable assets in the discussion because they asked pertinent questions which required their peers to further justify their conjecture or argument. This enabled the teacher to select marginalised students, and their turn-and-talk buddy, to share this robust justification. The teacher action of asking them to contribute their important question or share their thinking with the wider group further solidified these students as knowers and doers of mathematics in the eyes of their peers.

Wait time supported marginalised students' engagement with all the mathematical practices. This aligns with Takeuchi's (2018) assertion that, in terms of talk moves which re-calibrate the norm of exclusion to inclusion in the large group discussion, wait time is possibly the most powerful talk move. Chapin and O'Connor (2007) noticed that teachers in their study who employed wait time were rewarded with thoughtful contributions from their students. This occurred in the current study. Additionally, as Chapin and O'Connor (2007) found in their study, students in the current study were reluctant to fill the silent void at first. They assumed, like Chapin and O'Connor (2007) noted, that the teacher would move on to another student if they hesitated.

As the teachers in the current study employed wait time regularly, their marginalised students realised they were obligated to contribute and began contributing. Like Chapin and O'Connor (2007) found, all students came to expect a reasonable wait time in which to gather and articulate their thoughts. The teacher action of providing wait time, slowed down the pace of the discussion and created more breathing space for everyone. This aligns with Louie's (2017) claim that wait time diminishes the valorisation of speed in the mathematics classroom and sends a powerful message to students that the teacher values a process rich discussion. Additionally, students in the current study began giving each other wait time as this new pace became everyone's default setting. The marginalised students thrived within this slower pace.

As these students in the current study engaged more effectively in mathematical practices their understanding of the key concepts of the task strengthened. This is in concert with Boaler's (2016) assertion that engagement with mathematical practices deepens students' conceptual understanding of mathematics. Stein et al. (2008) and Warshauer (2015) claim that this produces mathematical identities as knowers and doers of mathematics. This occurred through both teachers' use of talk moves as described above but alongside this, as Lawler (2018) advised, they shifted their role from the authority to the facilitator of the discussion. This required both teachers to consider the amount and quality of their contribution to the discussion. Selling (2016) advises teachers to scaffold students around what an effective mathematical question might be or how to justify robustly without being drawn into asking questions on their behalf. To achieve this delicate balancing act both teachers followed this pattern, enabling mathematical practices to evolve through the right talk move prompt at the right time.

The teachers in the current study achieved this by using Selling's (2016) teacher talk moves of initiating, sustaining, and reprising. Selling's (2016) framework supported the teachers in the current study to elicit student metacognition around mathematical practices. Teachers initiated students to engage in a mathematical practice, sustained this engagement, and then reprised it by naming it and highlighting why it is important (Selling, 2016). Through this, students came to see the value in engaging in mathematical practices and understood when and why each practice might be needed. For example, Mrs Walter's students came to view visual representations as clever and valuable because they demonstrated and deepened conceptual understanding. Previously, visual representations such as drawing a picture, or a diagram, were viewed as unsophisticated and of lower cognitive order. In alignment with Cobb's (1995) findings, students in the current study began to use visual representations to support their explanation during the discussion. Representations formed an integral part of reaching a shared understanding.

Over time the students in the current study developed their own meanings for these mathematical practices relative to their prior experiences and the different roles they adopted during the discussion (Selling, 2016). In alignment with Moschkovich's (2004) findings, the students in the current study appropriated and transformed their use of mathematical practices through observing their peers use them. Marginalised students benefited from the opportunity to explore mathematical practices with their peers under the skilful guidance of their teacher as the students in Hunter's (2008) study did. As the marginalised students' confidence in engaging in mathematical practices grew, they formed strong identities as knowers and doers of mathematics.

5.2.2 Creating Community of Learning Identities: A Sense of Belonging

Over the course of the study, both teachers supported their marginalised students to develop a true sense of belonging in the learning community through teacher actions related to fostering norms, accessing cultural funds of knowledge, and re-calibrating status. Hunter (2008) states that establishing social and socio-mathematical norms in an inquiry lesson helps address status issues and develop stronger mathematical identities through new patterns of behaviour. Social norms were strengthened by teacher actions that focused on promoting the inclusion of marginalised students.

As social norms and mathematical practices became intrinsic behaviour during the discussion, socio-mathematical norms naturally established. This aligns with Hunter's (2021) claim that once embedded, social and socio-mathematical norms ensure pro-social and inclusive interaction. Both teachers created supportive learning environments that promoted social and intellectual risk-taking. Teachers in the current study, like those in Hunter's (2008) study, gave explicit instruction of the need for collegiality. This instruction led to a stronger collaborative protocol which fostered relational equity (Boaler, 2019). Both teachers explicitly taught relational equity through norms that required students to recognise and accept different ideas and points of view.

Collaborative, or group, norms were strengthened by the incorporation of inclusive language such as *we* and *us* instead of *I* and *me*. Both teachers positioned themselves as members of the learning community. Teachers used terms such as ‘team’ and ‘everyone’ to tune students into the collective. This aligns with Hunter’s (2008) findings. As students started to see themselves as part of a collective, instead of a collection of individuals, they began to view each other as allies in the discussion and this supported relational equity. These collectivist values were a catalyst for the shift away from individualism because they were in direct opposition to the colonial values as described by Triandis et al. (1988) As a sense of collectivism settled into the learning community the discussion became a truly shared exploration which led to shared success.

This collectivism supported these teachers to develop a more culturally sustaining pedagogy which enabled them to dig deep into the design of contexts which were relevant, meaningful, and interesting to their students (Hunter, 2008). Additionally, they made the contexts experientially real for their students (Warshauer, 2019). With a lens on inclusion, they thought about which contexts would reach their marginalised students. Like Civil (2007) described, this connection proved especially powerful for increasing the engagement of their marginalised students who did not usually see their lives reflected in their mathematics tasks. When the teachers in the current study tapped into the cultural expertise of their marginalised students, they noticed a rise in engagement.

The rise in engagement supported marginalised students’ conceptual understanding of the task (Gibbs, 2020). Another outcome of the teacher actions in the current study was the raising of their marginalised students’ cultural capital, which repositioned them from the margins of the discussion to the centre (Bills & Hunter, 2015; Civil, 2018, Gutiérrez & Rogoff, 2003; Jorgenson, 2014). A new identity emerged from this repositioning as students began to feel a true sense of belonging, within the learning community (Civil & Hunter, 2015). They became integral contributors to the discussion.

Both teachers focused on raising the intellectual status of their marginalised students. They went through a journey of examining their judgments, values, and beliefs about all their students and how this affected status in the classroom. This was congruent with Webster & Foschi’s (1988) assertion that examining entrenched beliefs takes time and deliberation. Aligned with the recommendations of Webster and Foschi (1988), both teachers reflected on their implicit biases around their most marginalised students and, as a result, decided to push past unhelpful judgements about their students' attributes and capabilities.

This led to a decision to actively assign competence to Mary and Duncan, as well as other marginalised students. This action, assigning competence, was the most impactful teacher action they employed to raise status. This corroborates Boaler's (2006) findings where assigning competence proved highly effective for raising status. Additionally, Mrs Ulster re-assigned status when the first assignation was rejected. This new finding has implications for teachers who are attempting to raise status using the assigning competence technique. Perseverance is key since a low self-efficacy can be very ingrained and it may take assigning and re-assigning multiple times to effect change.

After a sustained focus on raising status these marginalised students' beliefs about their capabilities improved and, subsequently, so did the way they interacted with peers during the discussion (Langer-Osuna, 2017). Similar to Featherstone et al.'s (2011) claim, when the students in the current study shifted their expectations of their own and each other's intellectual and social capabilities, this fed into group dynamics and created a new norm of participation. Marginalised students came to be viewed by their peers as a credible source of mathematical information with something of value to contribute to the discussion (Langer-Osuna, 2017). Fostering the norms, accessing cultural funds of knowledge and re-positioning status supported marginalised students to gain a true sense of belonging within their learning community. Furthermore, peers reinforced this new identity by accepting and actively encouraging these students to take an integral role in the large group discussion.

5.2.3 Ethic of Care – Shifting Beliefs

Every single one of the previously mentioned shifts in practice occurred because of an internal shift. This shift was the changing belief systems both teachers underwent as they re-calibrated their ethic of care from low expectations of marginalised students to high expectations. When teachers believe that all children can learn, they are more likely to produce greater learning gains in their students (Bishop & Berryman, 2006; Timperley & Robinson, 2001). Both teachers faced barriers on their journey here. Originally, they viewed their marginalised students as intrinsically 'low' and, therefore, this fixed state of being had to be catered to rather than changed.

Mrs Ulster catered to it by providing Mary with a separate learning programme in mathematics which occurred with the teacher aide and outside of the learning community. This is a low ethic of care, based on keeping Mary safe. Mrs Walter included Duncan in the learning community but worried she was not catering to his especially "low" learning "ability" (Mrs Walter, initial interview). Mrs Walter did not believe that Duncan was safe. Wilkinson and Townsend (2000) report that the best-practice teachers in their study hold a developmental notion of ability. This means that, rather than viewing intelligence as fixed and finite, these teachers viewed intelligence as fluid and incremental. As both teachers in the current study opened their minds to a different perspective and focused on including their marginalised students their beliefs around ability shifted.

Dweck's (2006) work around growth mind sets showed how powerful intelligence beliefs are in shaping teacher actions. This proved true with the teachers in the current study. Following advice on how to promote inclusion both teachers began employing actions aimed at increasing the inclusion of their marginalised students in the discussion. Their marginalised students exceeded their expectations and began participating in ways they never had before (Warshauer, 2015). This served to develop a growth mindset in both teachers and further teacher actions were employed with greater confidence in their students. As the teachers' expectations of their students rose these students' growth mind sets increased. This developed into a growth cycle where teacher and student expectations fed into each other and precipitated continual new growth. A new ethic of care emerged.

Rubie-Davies (2015) states that students learn what they have been given the opportunity to learn. When the teachers in the current study provided equitable opportunities for their marginalised students, these students were able to learn and discuss the same task as their peers. This creates a new self-fulfilling prophecy where the marginalised students internalise their teachers' high expectations, instead of low expectations, and find the motivation to engage with the task (Rubie-Davies, 2015). This new high expectation ethic of care sends a new message to the marginalised students. Both teachers are now, effectively, telling these students that they care and respect them enough to expect them to productively struggle with a challenging task. Like Louie's (2018) assertion, this new prophecy became the belief that productive struggling was natural, and effort was valued. However, both teachers realised over the course of the study, this does not mean their marginalised students are left to drown in the deep water of concepts beyond their current reach. As Stein et al. (2022) state, the task design, while challenging, has multiple entry and exit points so students can enter and exit a common task within their own learning trajectory.

Kazemi and Hintz (2014) state that their struggle is rewarded in the discussion as any remaining mysteries of the problem are solved, and misconceptions cleared up. This occurred in the current study as students, including marginalised students, had *lightbulb* or *a-ha!* moments as they listened to the explanations of their peers. Hunter and Hunter (2018) believe that everyone in the learning community has the capability to contribute something important to this discussion. Over the course of the current study, both teachers came to believe this and found ways for their marginalised students to contribute genuinely valuable questions, ideas, representations, and explanations to the discussion.

Moscardini (2010) states that this includes promoting and explicitly naming skills not traditionally viewed as mathematical. Both teachers in the current study now promote skills not traditionally associated with mathematics. For example, Mrs Walter positioned a marginalised student as an expert on the All Blacks. While this is not a traditional 'mathematics skill' it served to position him as a credible source worthy of consulting in the discussion since the context was rugby. This approach increases the catchment surface of the net to 'catch' many forms of success (Boaler, 2016). This new belief system in the fluidity of intelligence created a new ethic of care. Both teachers used this high expectation ethic of care with increasing confidence as they refined their teacher actions. They were rewarded with all the aforementioned results.

5.2.4 Summary

Over the course of the study, both teachers effectively included marginalised students in the large group discussion through a combination of refined teacher actions. These actions were supported by a changing belief system which developed over time as the teachers' perspectives and understanding around inclusion shifted. This outcome is supported by the literature, reviewed in this study, around the seven tools for promoting inclusion (e.g., Chapin & O'Connor 2009; Hunter & Civil, 2021; Kazemi & Hintz, 2014; Langer-Osuna, 2017; Rubie-Davies, 2015; Selling, 2016; Stein et al., 2008).

The findings of the current study identified four themes as instrumental to affecting a pedagogical shift in relation to inclusion of marginalised students in the large group discussion: (i) planning for inclusion; (ii) creating knowers and doers of mathematics; (iii) creating identities within the learning community, and (iv) establishing an ethic of care based on high expectations of all students. The refinement of teacher actions and beliefs evolved in a growth cycle where actions and beliefs were necessarily intwined. It was the combination of actions across all four themes, underpinned by an inclusive pedagogical belief system, which enabled the inclusion of marginalised students in the large group discussion.

5.3 Summary of Research Questions

The objective of this study was to explore the actions teachers take to effectively include marginalised students in the large group mathematical discussion. The study aimed to address two key questions:

1. What teacher actions effectively include marginalised students in the large group mathematical discussion?

2. What enables teachers to take actions which effectively include marginalised students in the large group mathematical discussion?

5.3.1 What teacher actions effectively include marginalised students in the large group mathematical discussion?

A review of the research literature (e.g., Chapin et al., 2009; Hunter & Civil, 2021; Kazemi & Hintz, 2014; Langer-Osuna, 2017; Rubie-Davies, 2015; Selling, 2016; Stein et al., 2008) identified and examined seven tools for promoting an effective discourse community. The current study identified specific teacher actions which effectively promoted the inclusion of marginalised students in the large group discussion (see Appendix D1).

Many of the teacher actions identified in the current study as effective for promoting inclusion in the large group discussion matched those which were identified as effective promoting a discourse community in the research literature (e.g., Chapin et al., 2009; Hunter & Civil, 2021; Kazemi & Hintz, 2014; Langer-Osuna, 2017; Rubie-Davies, 2015; Selling, 2016; Stein et al., 2008). The findings of the current study, in terms of teacher actions, corroborate the findings in the reviewed literature. For example, assigning competence was outlined as an effective teacher action to ensure equitable inclusion of all students in the discourse community (Boaler, 2016). Teachers in the current study employed this teacher action to promote the inclusion of their marginalised students in the large group discussion. Employing this action regularly and intentionally, alongside other teacher actions, served to raise the status of their marginalised students which led to greater participation and engagement in the discussion.

Some of the teacher actions identified in the current study, while related, extended the actions suggested by previous researchers and provide new and different ways in which teachers can enact them. For example, the re-assigning of competence employed by Mrs Ulster adds to our understanding related to the assigning competence move. In the current study this move originated from an unsuccessful attempt to assign competence to a student. Rather than conceding defeat, Mrs Ulster re-assigned competence to this student and explicitly named what she was doing and why. This perseverance proved effective.

5.3.2 What enables teachers to take actions which effectively include marginalised students in the large group mathematical discussion?

The findings of the current study suggest that belief systems powerfully influence the actions which teachers take to include marginalised students in the large group discussion. The importance of believing in the fluidity of intelligence, and the elastic potential to grow intelligence, is highlighted in some of the research literature (e.g., Hunter & Civil, 2021; Langer-Osuna, 2017; Rubie-Davies, 2015). As both teachers came to believe that every student in their class could access meaningful learning from the discussion, their marginalised students produced greater learning gains. These findings align with Bishop and Berryman's (2006) study, where teachers who believed their marginalised students would and could learn achieved higher achievement results than students with teachers who did not.

Both teachers' participation in the DMIC professional learning and development programme launched the shift from an exclusive framework to an inclusive framework. They had already culled practices such as competition, timed testing, and ability grouping from their mathematics programmes. However, they were still concerned about their most marginalised students. Mrs Walter wondered if perhaps the DMIC inquiry model did not work for these particular students and maybe they "needed something else" (Mrs Walter, initial interview). Over the course of the study, both teachers came to realise that the "something else" resided inside them. This was a shift from the belief system of fixed intelligence to the belief in the fluidity of intelligence. These findings support Wilkinson and Townsend's (2000) report that the best-practice teachers in their study held a developmental notion of ability. Initially both teachers in the current study viewed intelligence as finite, but over the course of the study they shifted towards viewing intelligence as incremental.

The inception of this shift was both teachers' willingness to explore their current belief systems and perceptions. This led to an openness to employing recommended teacher actions, purported to support inclusion. They enacted these recommendations, despite not being convinced, because they held growth mind-sets. Growth mind-sets had already enabled both teachers to achieve many successes over the course of their DMIC professional development and was a significant factor in their openness to participating in the current study. Dweck and Yeager (2019) state that a powerful way to shift from a fixed mindset to a growth mindset is to add the word *yet* to a finite statement. For example, adding *yet* to the statement 'I do not [yet] believe that Mary can learn with her peers in the discussion' opens a small window of possibility. Both teachers approached their first endeavour at promoting inclusion of their marginalised students in the large group discussion with this growth mindset.

Beliefs about intelligence are powerful in shaping teacher actions (Dweck, 2012). This proved true in the current study as both teachers' shifting beliefs began to inform their next actions. However, initially, it was teacher actions which shaped the first shift in beliefs. Both teachers employed a set of recommended actions without a belief that they would work. Their expectations remained low since they had been enacting DMIC pedagogy in their classrooms for three years without much change for their most marginalised students. However, this lens on inclusion was new and it made a difference, as it did in Louie's (2017) study. Both teachers' growth mindsets precipitated the openness required to attempt the recommended teacher actions. The result of the teacher actions was their marginalised students exceeding their expectations and participating in the discussion. The first seed of a new belief system was planted.

Both teachers re-evaluated their expectations and renewed their efforts to continue employing these actions. Once again, their marginalised students rose to the occasion. This created a snow-ball cycle of actions and beliefs where, each time teacher actions were successful, teacher expectations would rise and spur on new teacher actions and so on. This cycle was the catalyst for the belief system shift from an ethic of care based on low expectations of marginalised students to an ethic of care based on high expectations. Each time their marginalised students exceeded their expectations they witnessed the fluidity of growth in these students and came to believe that their capabilities as mathematicians were not fixed. During the post-study interviews both teachers described a shift in their belief system around intelligence. Both teachers shifted further toward an inclusive framework, but their journeys were slightly different. This was reflected in the extent to which they re-calibrated their belief systems around intelligence.

5.4 Implications and Recommendations for Educators

This section provides key recommendations extracted from the current study. The recommendations are directed towards educators aiming to promote the inclusion of their marginalised students in the large group discussion.

5.4.1 Actions

A combination of multiple actions, across four themes, must be consistently and intentionally employed to promote inclusion of marginalised students in the large group discussion. Orchestrating a large group discussion to effectively include all members of the discourse community, including marginalised students, requires careful teacher facilitation. Multiple teacher actions, described in the current study, support this facilitation:

1. Planning for inclusion by utilising the five practices
2. Creating knowers and doers of mathematics by utilising teacher talk moves to elicit mathematical practices
3. Developing real identities in the learning community through fostering the norms, utilising cultural funds of knowledge, and raising status.
4. Showing an ethic of care for all students, including marginalised students, through teacher actions which show high expectations of all.

It was this combination of actions, employed with a lens on inclusion, which proved successful in the current study. This holds significant implications for educators to develop their understanding and repertoire of actions under a lens for inclusion. If establishing this lens is difficult, educators may need to examine their belief system and perceptions around intelligence.

5.4.2 Beliefs

Educators who hold the belief in the fluidity of intelligence and a developmental notion of capability are advantaged when it comes to promoting inclusion of marginalised students in the large group discussion. This belief enables educators to hold high expectations of all their students, including marginalised students. Additionally, this belief system creates an ethic of care based on high expectations. All teacher actions described in this study are enacted within this ethic of care. Inclusive practice is underpinned by inclusive pedagogy. This holds implications for the significance of educators examining their belief systems and, if this proves difficult, developing a growth mindset.

5.4.3 Growth Mindset

Educators with a growth mindset are at an advantage when aiming to include their marginalised students in the large group discussion. The teachers in the current study were enabled to make effective changes in their practice through a shift in their belief system around intelligence. This shift took hold after the teachers observed the outcome of their actions. Once both teachers witnessed their most marginalised students participating in the large group discussion, as a direct result of their teacher actions, they began to believe that their actions were impactful. However, their initial set of teacher actions were employed without this belief. Instead, both teachers relied on their growth mindset to allow an openness that enabled them to enact the first set of actions required to promote inclusion for their marginalised students in the large group discussion. This holds implications for educators around the significance of developing a growth mindset and applying this to examining and reflecting upon deeply ingrained beliefs and perceptions.

5.5 Limitations of the Study

Although qualitative in nature, sample size is a limitation of this study. Having only two participants compromises generalisability. Both participants were a homogenous group for gender, age, and ethnicity. It would be advisable to ensure the results of the current study were comparable across a more diverse population. The short time frame is another limitation. It must be acknowledged that the findings may not be representative of all teachers who have undertaken the DMIC professional learning and development project. The current study was undertaken in two inquiry classrooms where daily communal mathematical discourse was the norm. Consequently, generalisation of the findings for teachers of non-inquiry classrooms is limited.

Subjectivity is another limitation of this study. Interpreting people's belief systems is intrinsically subjective. Despite lengthy discussions and open-ended interview questions the participants' responses were interpreted and analysed by one researcher. To mitigate this subjectivity the researcher analysed these beliefs in relation to teacher actions, which were easier to objectively observe. These were then triangulated with analysis of the artefacts evidencing student work. It must be acknowledged that, even considering these mitigating efforts, there remains the possibility of differing interpretations of the data.

5.6 Suggested Areas for Further Research

The current study examined how teachers promoted the inclusion of marginalised students in the large group discussion over three short school terms disrupted by a brief hiatus due to the covid 19 pandemic. A shift of belief systems was a significant finding in this study. Since shifting belief systems is an organic process which can take an indeterminate amount of time a longitudinal study could be warranted. A larger scale study with a greater and more diverse sample set would establish if the findings in this current study can be generalised. Additionally, it would be advisable to have multiple researchers collect and analyse the data sets to increase inter-data reliability and limit subjectivity around interpreting participants' beliefs and perceptions.

None of the participants in the current study utilised cultural funds of knowledge which drew on the cultural heritage of their marginalised students. While Mary is from the dominant Pakeha ethnicity, Duncan is Māori. Mrs Walter explained that Duncan does not identify with his Māori roots hence why she sought other contexts, which she knew would interest him, to reach into his world. Mrs Walter did not want to embarrass Duncan by highlighting a cultural heritage that he does not see himself in. However, Hunter and Hunter (2018) state that it is important to use cultural heritage contexts, such as a Māori tuku tuku panel, but without reference to specific students. Hunter and Civil (2021) advocate utilising contexts which hold mathematical wisdom of indigenous and marginalised cultures with explicit explanation of their inherent mathematical value. This would show Duncan that he comes from a rich mathematical heritage without embarrassing him. It would be advisable to ensure participants include specific cultural heritage contexts in some of their tasks to investigate the impact this has, over time, on the inclusion of marginalised students in the discussion.

5.7 Final Thoughts

The findings of this study contribute to the literature around inclusion of marginalised students in co-constructed mathematical discourse under the inquiry model. Prior to the study no research had been conducted specifically on the inclusion of marginalised students in a DMIC pedagogy-based mathematics classroom during the large group discussion. Both teachers in the study shifted their marginalised students from the margins of the discussion and established them as important contributors to the progression of a shared understanding. Through deliberate teacher actions, underpinned by the belief that all students can learn, both teachers re-positioned and re-framed their marginalised students as confident knowers and doers of mathematics with a strong identity within their learning community.

As teacher expectations of these students lifted, their participation increased, and teachers developed a new ethic of care based on high expectations. This led to an improvement in the marginalised students' engagement in the key concepts being discussed which, in turn, led to deeper understanding. Over the course of the study, the outcomes of this sustained lens on inclusion, for marginalised students, was a positive shift in mathematical achievement, identity, and status amongst peers. This contradicts deficit theories in education based on fixed intelligence and the assertion that ability is natural or inherent. To challenge and disestablish this exclusive, colonial, and inequitable framework, educators must actively promote an inclusive, collectivist, and equitable framework where students, like Mary and Duncan, belong and thrive.

References

Alcalá, L., Rogoff, B., & López Fraire, A. (2018). Sophisticated collaboration is common among Mexican-heritage US children. *Proceedings of the National Academy of Sciences*, *115*(45), 11377–11384. https://doi.org/10.1073/pnas.1805707115

Alton-Lee, A., Hunter, R., Sinnema, C., & Pulegatoa-Diggins, C. (2011). *BES Exemplar 1 Developing communities of mathematical inquiry*. https://thehub.swa.govt.nz/assets/documents/42483_BES-Exemplar1_0.pdf

Amineh, R. J., & Asl, H. D. (2015). Review of constructivism and social constructivism. *Journal of Social Sciences, Literature and Languages*, *1*(1), 9 -16. https://www.semanticscholar.org/paper/Review-of-Constructivism-and-Social-Constructivism-Amineh-Asl/38903f4a7255496f75124d639e14e9b810c17370

Apple, M. W. (1992). Do the standards go far enough? Power, policy, and practice in mathematics education. *Journal for Research in Mathematics Education*, *23*(5), 412–431. https://doi.org/10.5951/jresematheduc.23.5.0412

Archer, L., Francis, B., Miller, S., Taylor, B., Tereshchenko, A., Mazenod, A., Pepper, D., & Travers, M. C. (2018). The symbolic violence of setting: A Bourdieusian analysis of mixed methods data on secondary students' views about setting. *British Educational Research Journal*, *44*(1), 119–140. https://doi.org/10.1002/berj.3321

Averill, R. (2018). Using rich investigative mathematics activities towards embedding culturally responsive and culturally sustaining mathematics teaching: Helpful, but insufficient. In R. Hunter, M. Civil, B. Herbel-Eisenmann, N. Planas, & D. Wagner (Eds.), *Mathematical discourse that breaks barriers and creates space for marginalized learners* (pp. 195–212). Sense. https://doi.org/10.1163/9789004378735_010

Ball, D. L., & Bass, H. (2003). Making mathematics reasonable in school. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 27–44). National Council of Teachers of Mathematics.

Barth, M., & Thomas, I. (2012). Synthesising case-study research: Ready for the next step? *Australia Environmental Education Research*, *18*(6), 751–764. https://doi.org/10.1080/13504622.2012.665849

Battey, D., & Leyva, L. A. (2018). Making the implicit explicit: Building a case for implicit racial attitudes to inform mathematics education research. In T. G. Bartell (Ed.), *Toward equity and social justice in mathematics education* (pp. 21–41). Springer. https://link.springer.com/book/10.1007/978-3-319-92907-1

Batista, L. N., & Chapin, S. H. (2019). Creating a discourse community. *Teaching Children Mathematics, 25*(5), 298–304. https://doi.org/10.5951/teacchilmath.25.5.0298

Basile, V., & Lopez, E. (2015). And still I see no changes: Enduring views of students of color in science and mathematics education policy reports. *Science Education*, *99*(3), 519–548. https://doi.org/10.1002/sce.21156

Beatty, R. (2018). Connecting Algonquin loomwork and western mathematics in a grade 6 math class. In T. G. Bartell (Ed.), *Toward equity and social justice in mathematics education.* (pp. 105–127). Springer. https://link.springer.com/book/10.1007/978-3-319-92907-1

Bennett, T., Savage, M., Silva, E. B., Warde, A., Gayo-Cal, M., & Wright, D. (2009). *Culture, class, distinction*. Routledge. https://www.routledge.com/Culture-Class-Distinction/Bennett-Savage-Silva-Warde-Gayo-Cal-Wright/p/book/9780415560771

Berry, R. (2018). Disrupting policies and reforms in mathematics education to address the needs of marginalized learners. In T. G. Bartell (Ed.), *Toward equity and social justice in mathematics education* (pp. 3-21). Springer.

Bills, T., & Hunter, R. (2015). The role of cultural capital in creating equity for Pasifika learners in mathematics. In M. Marshman, V. Geiger, & A. Bennison (Eds.), *Mathematics education in the margins: Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 109-116). MERGA. Education Sciences (IES) of the U.S. Department of Education. https://files.eric.ed.gov/fulltext/ED572423.pdf

Bishop, R., & Berryman, M. (2006). *Culture speaks: Cultural relationships and classroom learning*. Huia. https://huia.co.nz/products/culture-speaks

Black, L. (2004). Differential participation in the whole-class discussions and the construction of marginalised identities. *Journal of Educational Enquiry*, *5*(1), 34-54. http://www.literacy.unisa.edu.au/jee/Papers/JEEVol5No1/Paper%203.pdf

Boaler, J. (2006). How a de-tracked mathematics approach promoted respect, responsibility, and high achievement. *Theory into Practice*, *45*(1), 40-46. https://doi.org/10.1207/s15430421tip4501_6

Boaler, J. (2019). Developing mathematical mindsets: The need to interact with numbers flexibly and conceptually. *American Educator*, *42*(4), 28-40. https://files.eric.ed.gov/fulltext/EJ1200568.pdf

Boaler, J., Brown, K., Lamar, T., Leshin, M., & Selbach-Allen, M. (2022). Infusing mindset through mathematical problem solving and collaboration: Studying the impact of a short college intervention. *Education Sciences*, *12*(10), 694. https://doi.org/10.3390/educsci12100694

Bourdieu. P., & Coleman, J. S. (Eds.). (1991). *Social theory for a changing society.* Westview Press. https://doi.org/10.4324/9780429306440

Calor, S., Dekker, R., van Drie, J. P., Zijlstra, B., & Volmann, M. (2019). "Let us discuss math." Effects of shift-problem lessons on mathematical discussions and level raising in early algebra. *Mathematics Education Research Journal, 32,* 743-763. https://doi.org/10.1007/s13394-019-00278-x

Chapin, S. H., & O'Connor, C. (2007). Academically productive talk: Supporting students' learning in mathematics. *The Learning of Mathematics*, *69*, 113–128. https://scholar.google.com/citations?view_op=view_citation&hl=en&user=nED5cHgAAAAJ&citation_for_view=nED5cHgAAAAJ:d1gkVwhDpl0C

Chapin, S. H., O'Connor, M. C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn, Grades K-6.* Math Solutions.

Civil, M. (2007). Building on community knowledge: An avenue to equity in mathematics education. In N. Nasir & P. Cobb (Eds.), *Improving access to mathematics: Diversity and equity in the classroom* (pp. 105-117). New York Teacher's College Press.

Civil, M. (2018). A commentary on identifying and connecting to family and community funds of knowledge. In T. G. Bartell (Ed.), *Toward equity and social justice in mathematics education* (pp. 145-153). Springer.

Civil, M., & Hunter, R. (2015). Participation of non-dominant students in argumentation in the mathematics classroom. *Intercultural Education, 26*(4), 296-312. https://doi.org/10.1080/14675986.2015.1071755

Cobb, P., Wood, T., & Yackel, E. (1993). Discourse, mathematical thinking, and classroom practice. In E. A. Foreman, N. Minick, & C. A. Stone (Eds.), *Contexts for learning: Sociocultural dynamics in children's development* (pp. 91-119). Oxford University Press.

Cobb, P. (1995). Cultural tools and mathematical learning: A case study. *Journal for Research in Mathematics Education*, *26*(4), 362-385. https://doi.org/10.5951/jresematheduc.26.4.0362

Cohen, E. G., & Lotan, R. A. (Eds.). (1997). *Working for equity in heterogeneous classrooms: Sociological theory in practice*. Teachers College Press. https://eric.ed.gov/?id=ED412328

Diener, C. I., & Dweck, C. S. (1978). An analysis of learned helplessness: Continuous changes in performance, strategy, and achievement cognitions following failure. *Journal of Personality and Social Psychology*, *36*(5), 451. https://doi.org/10.1037/0022-3514.36.5.451

Dweck, C. S. (2006). *Mindset: The new psychology of success*. Random House.

Dweck, C. S. (2015). Carol Dweck revisits the growth mindset. *Education Week*, *35*(5), 20-24. https://www.edweek.org/leadership/opinion-carol-dweck-revisits-the-growth-mindset/2015/09

Dweck, C. S., & Yeager, D. S. (2019). Mindsets: A view from two eras. *Perspectives on Psychological Science*, *14*(3), 481–496. https://doi.org/10.1177/1745691618804166

Engle, R. A., & Conant, F. R. (2002). Guiding principles for fostering productive disciplinary engagement: Explaining an emergent argument in a community of learners' classroom. *Cognition and Instruction, 20*(4), 399-483. https://doi.org/10.1207/S1532690XCI2004_1

Equality Challenge Unit. (2013). *Unconscious bias and higher education*. https://diversity.caltech.edu/documents/19785/unconscious-bias-and-higher-education-compressed.pdf

Erath, K. (2018). Creating space and supporting vulnerable learners: Teachers' options for facilitating participation in oral explanations and the corresponding epistemic processes. In R. Hunter, M. Civil, B. Herbel-Eisenmann, N. Planas & D. Wagner (Eds.), *Mathematical discourse that breaks down barriers and creates space for marginalized learners* (pp. 39-59). Sense. https://doi.org/10.1163/9789463512121_003

Featherstone, H., Crespo, S., Jilk, L., Oslund, J., Parks, A., & Wood, M. (2011). *Smarter together: Collaboration and equity in the elementary math classroom.* National Council of Teachers of Mathematics. https://www.google.com/url?sa=i&rct=j&q=&esrc=s&source=web&cd=&ved=0CAQQw7AJahcKEwjQ5siak7z8AhUAAAAAHQAAAAAQAw&url=https%3A%2F%2Fwww.nctm.org%2Fstore%2FProducts%2FSmarter-Together!-Collaboration-and-Equity-in-the-Elementary-Math-Classroom%2F&psig=AOvVaw1hoVtlPzJVY7Vgno-1lDrs&ust=1673410781305502

Foote, M. (2018). A commentary on supporting teachers in addressing the needs of marginalized students. In T. G. Bartell (Ed.), *Toward equity and social justice in mathematics education* (pp. 327 -331). Springer.

Forsey, M. (2012). Interviewing individuals. In S. Delamont (Ed.), *Handbook of qualitative research in education* (pp. 364–376). Edward Elgar. https://doi.org/10.4337/9781849807296.00035

Francis, B., Archer, L., Hodgen, J., Pepper, D., Taylor, B., & Travers, M. C. (2017). Exploring the relative lack of impact of research on 'ability grouping' in England: A discourse analytic account. *Cambridge Journal of Education, 47*(1), 1-17. https://doi.org/10.1080/0305764X.2015.1093095

Freire, P. (1970). *Pedagogy of the oppressed*. Herder & Herder.

Gibbs, B. E. (2020). "*It makes me feel proud of who I am": Developing functional thinking through culturally located tasks* [Master's Thesis, Massey University]. http://hdl.handle.net/10179/16237

González, N., Moll, L. C., & Amanti, C. (Eds.). (2006). *Funds of knowledge: Theorizing practices in households, communities, and classrooms*. Routledge. https://www.routledge.com/Funds-of-Knowledge-Theorizing-Practices-in-Households-Communities-and/Gonzalez-Moll-Amanti/p/book/9780805849189

Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education*, *35*(4), 258-291. https://doi.org/10.2307/30034810

Gutiérrez, K. D., & Rogoff, B. (2003). Cultural ways of learning: Individual traits or repertoires of practice. *Educational Researcher, 32*(5), 19-25. https://doi.org/10.3102/0013189X032005019

Hackenberg, A. J. (2010). Mathematical caring relations in action. *Journal for Research in Mathematics Education*, *41*(3), 236-273. https://doi.org/10.5951/jresematheduc.41.3.0236

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, *28*(5), 524-549. https://doi.org/10.5951/jresematheduc.28.5.0524

Horn, I. S. (2007). Fast kids, slow kids, lazy kids: Framing the mismatch problem in mathematics teachers' conversations. *Journal of the Learning Sciences, 16*(1), 37-79. https://doi.org/10.1080/10508400709336942

Hudson, M., Milne, M., Reynolds, P., Russell, K., & Smith, B. (2010). *Te Ara Tika. Guidelines for Māori research ethics: A framework for researchers and ethics committee members*. Health Research Council of New Zealand. https://www.hrc.govt.nz/resources/te-ara-tika-guidelines-maori-research-ethics-0

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education*, *35*(2), 81-116. https://doi.org/10.2307/30034933

Hunter, R. (2008). Facilitating communities of mathematical inquiry. In M. Goos, R. Brown, & K. Makar (Eds.). *Navigating currents and charting directions: Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia* (pp. 31–39).MERGA https://files.eric.ed.gov/fulltext/ED503747.pdf

Hunter, R., & Civil, M. (2021). Collaboration in mathematics: Taking a sociocultural perspective*. Research Advances in Mathematics Education*, *19,* 7-20. http://www.aiem.es

Hunter, R., & Hunter, J. (2018). Opening the space for all students to engage in mathematical practices within collaborative inquiry and argumentation. In R. Hunter, M. Civil, B. Herbel-Eisenmann, N. Planas, & D. Wagner (Eds.). *Mathematical discourse that breaks barriers and creates space for marginalised learners* (pp. 1-22). Sense. https://cerme.nz/dmic/dmic-publications/

Hunter, R., Hunter, J., Anthony, G., & McChesney, K. (2018). Developing mathematical inquiry communities: Enacting culturally responsive, culturally sustaining, ambitious mathematics teaching. *Set: Research Information for Teachers*, *2*, 25. https://doi.org/https://doi.org/10.18296/set.0106

Jackson, K. J., Shahan, C., Gibbons, L. K. and Cobb, R. A. (2012). Launching complex tasks. *Mathematics Teaching in the Middle School, 18*(1), 24-29. https://doi.org/10.5951/mathteacmiddscho.18.1.0024

Jorgenson, R. (2014). Social theories of learning: A need for a new paradigm in mathematics education. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Curriculum in focus: Research guided practice – Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia* (pp. 311-318). MERGA. http://hdl.handle.net/10072/66450

Kazemi, E., & Hintz, A. (2014). *Intentional talk*: *How to structure and lead productive mathematical discussions.* Stenhouse.

King, N., & Horrocks, C. (2010). *Interviews in qualitative research.* Sage. https://www.academia.edu/27948728/Interviews_in_Qualitative_Research_by_KING_NIGEL_and_CHRISTINE_HORROCKS

Lampert, M. (2001). *Teaching problems and the problems of teaching*. Yale University Press. http://www.jstor.org/stable/j.ctt32bpsx

Langer-Osuna, J. (2017). Authority, identity, and collaborative mathematics. *Journal for Research in Mathematics Education, 48*(3), 237-247. https://doi.org/10.5951/jresematheduc.48.3.0237

Lawler, B. R. (2018). Learning to support student discourse in an urban high school district. In R. Hunter, M. Civil, B. Herbel-Eisenmann, N. Planas & D. Wagner (Eds.), *Mathematical discourse that breaks barriers and creates space for marginalized learners* (pp. 121-147). Sense. https://doi.org/10.1163/9789463512541_007

Leach, G., Hunter, R., & Hunter, J. (2014). Teachers repositioning culturally diverse learners as doers and users of mathematics. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.). *Curriculum in focus: Research guided practice – Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia, 2* (pp. 351–361). MERGA https://www.semanticscholar.org/paper/Teachers-Repositioning-Culturally-Diverse-Students-Leach-Hunter/106d539f011b556b2d46655376d67902f6578076

Lee, L., & Goodman, J. (2009). Traversing the challenges of conducting research with young, immigrant children: The case of Korean children. *Interchange, 40*(2), 225-244. https://doi.org/10.1007/s10780-009-9091-6

Lester, F. K. (Ed.). (2007). *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics*. IAP. https://www.amazon.com/Handbook-Research-Mathematics-Teaching-Learning/dp/1593115865

Lindqvist, H., & Forsberg, C. (2022). Constructivist grounded theory and educational research: Constructing theories about teachers' work when analysing relationships between codes. *International Journal of Research & Method in Education*, *6*, 1-11. https://doi.org/10.1080/1743727X.2022.2095998

Lotan, R. A. (1997). Complex instruction: An introduction. In E. G. Cohen & R. A. Lotan (Eds.), *Working for equity in heterogeneous classrooms: Sociological theory in practice* (pp. 15-30). Teachers College Press. https://web.stanford.edu/~hakuta/www/archives/syllabi/E_CLAD/Course3/lotan.htm

Lotan, R. A. (2022). Equitable classrooms. In M. Webster & L.S., Walker (Eds.), *Unequals: The power of status and expectations in our social lives* (pp. 178-200). Oxford University Press. https://www.amazon.com/Unequals-Power-Status-Expectations-Social/dp/019760000X

Louie, N. (2017). The culture of exclusion in mathematics education and its persistence in equity-oriented teaching. *Journal for Research in Mathematics Education*: *The National Council of Teachers of Mathematics, 48*(5), 488-519. https://doi.org/10.5951/jresematheduc.48.5.0488

Maher, C. A., & Martino, A. M. (1996). The development of the idea of mathematical proof: A 5-year case study. *Journal for Research in Mathematics Education*, *27*(2), 194-214. https://doi.org/10.2307/749600

Massey University. (2017). *Code of ethical conduct for research teaching and evaluations involving human participants*. Massey University Palmerston North. https://www.massey.ac.nz/massey/fms/PolicyGuide/Documents/c/code-of-ethical-conduct-for-research,-teaching-and-evaluations-involving-human-participants.pdf

McArthur, J. (2022). Critical theory in a decolonial age. *Educational Philosophy and Theory*, *54*(10), 1681-1692. https://doi.org/10.1080/00131857.2021.1934670

McCrone, S. S. (2005). The development of mathematical discussions: An investigation in a fifth-grade classroom. *Mathematical Thinking and Learning*, *7*(2), 111-133. https://doi.org/10.1207/s15327833mtl0702_2

McDermott, R. P. (1993). The acquisition of a child by a learning disability. In S. Chaiklin & J. Lave (Eds.), *Understanding practice: Perspectives on activity and context* (pp. 269-305). Cambridge University Press.

Merriam, S. B., & Tisdell, E. J. (2015). *Qualitative research: A guide to design and implementation* (4th ed.). John Wiley & Sons. https://www.wiley.com/en-us/Qualitative+Research:+A+Guide+to+Design+and+Implementation,+4th+Edition-p-9781119003618

Merton, R. K. (1948). The self-fulfilling prophecy. *The Antioch Review*, *8*(2), 193-210. https://doi.org/10.2307/4609267

Miles, M. B., Huberman, A.M., & Saldaña, J. (2018). *Qualitative data analysis: A methods sourcebook*. Sage. https://us.sagepub.com/en-us/nam/qualitative-data-analysis/book246128

Milne, A. (2020). Colouring in the white spaces: The warrior-researchers of Kia Aroha College. *Curriculum Perspectives*, *40*(1), 87-91. https://doi.org/10.1007/s41297-019-00092-2

Ministry of Education. (2021). *Statement of intent 2021–2026*. https://assets.education.govt.nz/public/Documents/Ministry/Publications/Statements-of-intent/SPG-0775-Statement-of-Intent-2021-2026_AW-Web-1.pdf

Ministry of Education. (2022). *Te Mātaiaho: A draft Te Tiriti-honouring and inclusive curriculum framework*. https://curriculumrefresh.education.govt.nz/get-involved?fbclid=IwAR0SmKLeVhScHXJh0V4yw3NsJk9aqVa3lOIIApv0Id52rMjmx846NthK9p0

Moscardini, L. (2010). 'I like it instead of maths:' How pupils with moderate learning difficulties in Scottish primary special schools intuitively solved mathematical word problems. *British Journal of Special Education, 37*(3), 130-138. https://doi.org/10.1111/j.1467-8578.2010.00461.x

Moschkovich, J. N. (2004). Appropriating mathematical practices: A case study of learning to use and explore functions through interaction with a tutor. *Educational Studies in Mathematics, 55*(1-3), 49-80. https://doi.org/10.1023/B:EDUC.0000017691.13428.b9

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel.* US Department of Education. https://files.eric.ed.gov/fulltext/ED500486.pdf

National Science Foundation. (2015). *Women, minorities, and persons with disabilities in science and engineering* (Special report NSF 15-311). https://www.nsf.gov/statistics/wmpd

Nerland, M. (2022). Researching professional learning in complex environments: Opportunities and challenges from a qualitative research perspective. *Methods for Researching Professional Learning and Development*, *33*, 603-618. https://doi.org/10.1007/978-3-031-08518-5_27

Noble, H., & Smith, J. (2015). Issues of validity and reliability in qualitative research. *Evidence-based Nursing, 18*(2), 34-35. http://dx.doi.org/10.1136/eb-2015-102054

Oakes, J., & Lipton, M. (1991). *Making the best of schools: A handbook for parents, teachers, and policymakers*. Yale University Press. https://www.amazon.com/Making-Best-Schools-Handbook-Policymakers/dp/0300051239

O'Connor, C., & Michaels, S. (1996). Shifting participants' frameworks: Orchestrating thinking practices in group discussion. In D. Hicks (Ed.), *Discourse, learning and schooling* (pp. 63-103). Cambridge University Press. https://www.researchgate.net/publication/313151810_Shifting_participant_frameworks_Orchestrating_thinking_practices_in_group_discussions

O'Connor, C., Michaels, S., Chapin, S., & Harbaugh, A. G. (2016). The silent and the vocal: Participation and learning in whole-class discussion. *Learning and Instruction*, *48,* 5-13. https://doi.org/10.1016/j.learninstruc.2016.11.003

Pihama, L. (2015). Kaupapa Māori theory: Transforming theory in Aotearoa Kaupapa Rangahau. In L. Pihama, S. J. Tiakiwai, & K. A. Southey (Eds.), *A collection of readings from the Kaupapa Rangahau Workshop Series* (pp. 6-15). Te Kotahi Research Institute. https://hdl.handle.net/10289/11738

Punch, K., & Oancea, A. (2014). *Introduction to research methods in education*. Sage. https://www.scirp.org/(S(351jmbntvnsjt1aadkozje))/reference/referencespapers.aspx?referenceid=2041174

Rogoff, B., & Mejía-Arauz, R. (2022). The key role of community in learning by observing and pitching in to family and community endeavours. *Journal for the Study of Education and Development*, *45*(3), 494 - 548. https://doi.org/10.1080/02103702.2022.2086770

Rubie-Davies, C. (2015). *Becoming a high expectation teacher: Raising the bar.* Routledge Taylor and Francis Group. https://www.routledge.com/Becoming-a-High-Expectation-Teacher-Raising-the-bar/Rubie-Davies/p/book/9780415713375

Saldaña, J. (2014). *Thinking qualitatively: Methods of mind*. Sage. https://us.sagepub.com/en-us/nam/thinking-qualitatively/book242817

Saxe, G. B. (2012). *Cultural development of mathematical ideas: Papua New Guinea studies*. Cambridge University Press. https://doi.org/10.1017/CBO9781139045360

Selling, S. K. (2016). Making mathematical practices explicit in urban middle and high school mathematics classrooms. *Journal for Research in Mathematics Education*, *47*(5), 505-551. https://doi.org/10.5951/jresematheduc.47.5.0505

Shah, N., & Crespo, S. (2018). Cultural narratives and status hierarchies: Tools for identifying and disrupting inequity in mathematics classroom interaction. In R. Hunter, M. Civil, B. Herbel-Eisenmann, N. Planas, & D. Wagner (Eds.), *Mathematical discourse that breaks barriers and creates space for marginalised learners* (pp. 23-37). Brill. https://doi.org/10.1163/9789463512121_002

Smith, L. T., Tuck, E., & Yang, K. W. (Eds.). (2018). *Indigenous and decolonizing studies in education*. Routledge. https://www.routledge.com/Indigenous-and-Decolonizing-Studies-in-Education/book-series/IDSE

Smith, M. S., & Stein, M. K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School*, *3,* 344-350. https://doi.org/10.5951/MTMS.3.5.0344

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, *10*(4), 313–340. https://doi.org/10.1080/10986060802229675

Stein, M. K., Russell, J. L., Bill, V., Correnti, R., & Speranzo, L. (2022). Coach learning to help teachers learn to enact conceptually rich, student-focused mathematics lessons. *Journal of Mathematics Teacher Education, 25*(3), 321-346. https://doi.org/10.1007/s10857-021-09492-6

Sullivan, P. A., & Wood, T. (2008). *Knowledge and beliefs in mathematics teaching and teaching development*. Sense. https://research.monash.edu/en/publications/knowledge-and-beliefs-in-mathematics-teaching-and-teaching-develo

Takeuchi, M. A. (2018). Conversions for life: Transnational families' mathematical funds of knowledge. In T. G. Bartell (Ed.), *Toward equity and social justice in mathematics education* (pp. 127-145). Springer. https://eric.ed.gov/?id=ED621902

Timperley, H. S., & Robinson, V. M. (2001). Achieving school improvement through challenging and changing teachers' schema. *Journal of Educational Change*, *2*(4), 281–300. https://doi.org/10.1023/A:1014646624263

Triandis, H. C., Bontempo, R. N., Villareal, M. J., Asai, M., & Lucca, N. (1988). Individualism and collectivism: Cross-cultural perspectives on self-ingroup relationships. *Journal of Personality and Social Psychology, 54*, 323-338. https://doi.org/10.1037/0022-3514.54.2.323

Van Es, E. A., Hand, V., & Mercado, J. (2017). Making visible the relationship between teachers' noticing for equity and equitable teaching practice. In E. Schack, M. Fisher, & J. Wilhelm (Eds.), *Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks, research in mathematics education.* https://doi.org/10.1007/978-3-319-46753-5_15

Vygotsky, L. S., & Cole, M. (1978). *Mind in society: Development of higher psychological processes*. Harvard University Press. https://www.hup.harvard.edu/catalog.php?isbn=9780674576292

Wagner, D., & Herbel-Eisenmann, B. (2014). Identifying authority structures in mathematics classroom discourse: A case of a teacher's early experience in a new context. *ZDM*, *46*(6), 871-882. https://doi.org/10.1007/s11858-014-0587-x

Warshauer, H. K. (2015). Strategies to support productive struggle. *Mathematics Teaching in the Middle School, 20*(7), 390-393. https://doi.org/10.5951/mathteacmiddscho.20.7.0390

Webster, M., & Foschi, M. (1988). *Status generalization: New theory and research*. Stanford University Press. https://psycnet.apa.org/record/1988-98267-000

Wilkinson. I. A.G., & Townsend M. A. R. (2000). From Rata to Rimu: Grouping for instruction in best practice New Zealand Classrooms. *The Reading Teacher, 53*(6), 460-471. https://www.jstor.org/stable/20204823

Wood, T. (1999). Creating a context for argument in mathematics class. *Journal for Research in Mathematics Education*, *30*(2), 171-191. https://doi.org/10.2307/749609

Wood, T., Williams, G., & McNeal, B. (2006). Children's mathematical thinking in different classroom cultures. *Journal for Research in Mathematics Education*, *37*(3), 222-255. http://www.jstor.org/stable/30035059

Yackel, E., & Cobb, P. (1996). Socio-mathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, *27*(4), 458-477. https://doi.org/10.2307/749877

Appendices

Appendix A1: Semi-structured Initial Interview

1. You have been teaching mathematics under the DMIC pedagogical model for at least three years now. Can you talk a bit about how you are finding this? (Highlights/ lowlights, barriers/challenges, successes etc.)
2. Have any of your beliefs and/ or values changed over these three years? Which ones? How?
3. As you know, the mathematics lesson in the DMIC pedagogy follows distinct phases – anticipate, launch, S.G.I, share back, connect, reflect – can you talk about how you find the share back phase? (E.g., Are there any barriers, challenges, highlights, or observations about the share back you would like to share?)
4. In what ways do you currently support different groups of students with specific needs during share back? (E.g., what are your thoughts about student capabilities during share back?)
5. What are your thoughts around everyone engaging in, and sharing back, one common and challenging task?
6. How does anticipating the possible student solutions and misconceptions in the task support/guide your focus during the share back and the connect?
7. What does the phrase 'being a high expectation teacher' mean to you?
8. Do you/how do you currently use the talk moves during the share back?
9. Do you/ how do you use mathematical practices during the share back?
10. What status issues (high status and low status) do you see, currently, occurring amongst your students so far this year, generally? And in mathematics lessons? Does this effect the share back? How?
11. Is there anything else you would like to comment on, reflect, or ask regarding the share back, marginalised students, or inclusion?

Appendix A2: Semi-structured Final Interview

1. How are you feeling about the DMIC pedagogy and teaching using this approach now?
2. Do you feel any of your beliefs or values have changed at all, or deepened, or become more prominent since we last had this interview?
3. How are you finding the share back phase now?
4. In what ways are you currently supporting students with different needs during the share back?
5. How do you feel about everyone engaging in one common task now?
6. How do you find anticipating the tasks before the lesson supports your share back?
7. What does the phrase 'being a high expectation teacher' mean to you now? Can you reflect on how this compares to what you said in the initial interview?
8. How are the talk moves being used during the share back now?
9. How are you finding your focus on mathematical practices now in the share back?
10. Status Issues – has there been any shifts in status for any students since our last interview?
11. Is there anything else you would like to comment on, reflect, or ask regarding the share back, marginalised students, or inclusion?
12. How was it for you, participating in this study?

Appendix B: Record Sheet Used for Classroom Observations

Observation #				
Teacher:				
Date:				
Task:	**INCLUSIVE**	**EXCLUSIVE**	**Missed Opportunities**	**Reflective Comments**
Five Practices	Actions: Outcomes:	Actions: Outcomes:		
Talk Moves	Actions: Outcomes:	Actions: Outcomes:		
Math Practices	Actions: Outcomes:	Actions: Outcomes:		
Norms	Actions: Outcomes:	Actions: Outcomes:		
Culture	Actions: Outcomes:	Actions: Outcomes:		
Status	Actions: Outcomes:	Actions: Outcomes:		
Expectations	Actions: Outcomes:	Actions: Outcomes:		

Appendix C1: Fraction Ordering Task

Fractions

Level 2 (Mrs Ulster's class)

Task 7: Super Striker Soccer

At the Super Striker Soccer competition, these were the results of the goal shooting activity.

- Ruby scored 5 out of the six goals that she kicked.
- Daniel scored 2 out of three goals that he kicked.
- Tasa scored 3 out of the four goals that he kicked.
- Sesimani scored 7 out of the eight goals that she kicked.

Can you put them in order from who was the most accurate to least accurate in shooting the goals?

Appendix C2: Breakfast Club Task

Statistics

Level 2 (Mrs Ulster's class)

Task 2: Breakfast Club

Breakfast Club is putting in an order to the supermarket.

What things might they want to find out?

What questions could you ask to gather data?

How can you collect data to answer this question?

Record your results to present to the class.

Can you represent this in different ways?

Appendix C3: The Array Task

Number: Multiplication

Level 2 (Mrs Ulster's class)

Task 1: The Bossy Little Ant

What a bossy ant the littlest ant is. She likes the ants to be organised in rows no matter how many ants there are.

If there 5 rows of ants and 10 ants in each row, how many ants are there altogether?

If there 8 rows of ants and 4 ants in each row, how many ants are there altogether?

If there 15 rows of ants and 4 ants in each row, how many ants are there altogether?

Make sure that you can explain and justify your reasoning with both a picture and numbers.

Appendix C4: Springboks and All-Blacks Height Task

Statistics

Level 4 (Mrs Walter's class)

Task 10: Heights of the All-Blacks and Springbok rugby players

Level 4/Year 7-8: Statistics 21

Task 10 (optional task)

Springboks	Stem	All Blacks
7 7	**16**	
9 8 7 6 5 4 4 3 3 2 2 0	**17**	1 2 6 7 7 7 9
9 9 9 9 9 7 6 6 6 5 5 4 4 4 4 4 3 3 2 1 0 0	**18**	2 3 3 3 4 5 6 6 7 7 8 9 9
8 8 8 6 4 3 1 1 0	**19**	0 0 0 0 1 2 2 2 3 4 5 5 6 7 7 7 7 8 8 9 9 9
6 5 3 0 0	**20**	2 4

This stem and leaf graph shows the heights of the players in the Springboks and All Blacks squads. The Springboks heights range from 167 cm to 206 cm and the All-Blacks heights range from 171 cm to 204 cm.

What statements can you make to compare players' heights shown on this stem and leaf graph? Use mode, median, mean, range and distribution to describe some statements.

Use "I notice" and "I wonder" statements and include evidence and justification from the graph.

Appendix C5: Michael’s Loaf of Bread Task

Fractions

Level 4 (Mrs Walter’s class)

Task 8: Michael’s Loaf of Bread

Michael’s father ate $\frac{1}{10}$ of a loaf of bread before Michael made lunch for his brothers and sisters.

Michael used $\frac{2}{3}$ of the loaf of bread that was left.

How much of the loaf did Michael use and how much was left?

Appendix C6: Rebel Sports Task

Statistics

Level 4 (Mrs Walter’s class)

Task 3: School Bags

Rebel Sports is looking at stocking a new brand of school bags for children.

They would like to ensure that the bags will be suitable and durable for students across a range of ages.

Make “I wonder” statements related to this topic.

Use the data card sets to help you give advice to the Warehouse.

Represent your findings in a table of data and as graphs.

Make statements about your findings using the data and draw conclusions that will provide advice to the Warehouse and the characteristics of the bags that they should stock.

Appendix D1: Thematic Analysis Framework Teacher Actions

Five Practices Teacher Actions	**Talk Moves** Teacher Actions	**Mathematical Practices** Teacher Actions	**Social Norms** Teacher Actions	**Cultural Funds** Teacher Actions	**Status** Teacher Actions	**High Expectations** Teacher Actions
T Refined anticipations -specifically anticipated how their marginalised students will tackle the task and what they might get stuck on	T use revoice move on a student seated next to a marginalised student, so the marginalised student has another opportunity to hear important information.	T pushed students' explanations into justifications using the reasoning talk move and prompts such as 'because'	T placed the responsibility for inclusion of group members on all group members as a collective.	T designed a mathematical context which was meaningful and relevant to the marginalised student and reaches into their life outside of school.	T included all students in mathematics lessons and the discussion (Mary had to dip in and out at first while she got used to it).	T held individual students accountable for their sense making during the discussion. For example, prompt them to ask a question/seek clarification.
During the launch -T reminded students of prior knowledge from pervious lesson	T used repeat talk move on a marginalised student to bring their voice into the discussion.	T scaffolded and modelled effective questioning and made explicit the reason for asking questions.	T explicitly stated what is expected for participation norms and why this was important.	T re-visited the context immediately before the large group discussion.	T explicitly stated to students that the process is valued (not just the answer).	T held high expectations for effort and work ethic. Value these explicitly and promote these instead of *ability*.
T monitored marginalised students' thinking and use of prior knowledge during the small group inquiry time	T reminded students when using the reasoning talk move that it was the mathematics not the students being reasoned.	T made mathematical practices explicit by naming and highlighting these with all students.	T set up high expectations for independent time students to help each other and work together when needed	T made the task experientially real by ensuring the context was realistic and plausible.	T explicitly stated that mistakes are a normal and valuable part of sense making	T allowed and promoted productive struggle. The discussion was not a race to the correct answer/solution. The process was important. Ambiguity or confusion was normal while sense making.

Five Practices Teacher Actions	**Talk Moves** Teacher Actions	**Mathematical Practices** Teacher Actions	**Social Norms** Teacher Actions	**Cultural Funds** Teacher Actions	**Status** Teacher Actions	**High Expectations** Teacher Actions
T selected marginalised students to share back their work or part of their work	T used the prompt 'because' to push students beyond explanation and into justification.	T used the initiate, sustain, and reprise talk move to make mathematical practices explicit	T positioned herself as the facilitator, as a member of the learning community.	T made the task experientially real by providing rationale for solving it. E.g., we need this graph to prove to Rebel Sports why they need to stock more bags with phone pockets.	T valued every contribution whether partially, completely, or not at all correct or valid. Explain to students how every conjecture or idea deepens the discussion.	T trusted students by allowing them to struggle – giving them wait time,
T sequenced the series of ideas for discussion by specifically explaining which bit marginalised students were to share and supported them to practice their explanation in their group	T used the revise talk move to normalise ambiguity as a necessary part of doing mathematics	T validated all students when they engage in mathematical practices	T used inclusive language such as *we* and *us* instead of *I* and *me*. T used a metaphor for their community	T made links to real life situations which you know your marginalised students are familiar with.	T positioned all students as peers worthy of consultation	T accepted and celebrated a diversity of ideas, including divergent thinking.
T publicly validated marginalised student's contribution	T used wait time to support students to value their organic thought processing times. T ensured she gave wait time to marginalised students,	T validated marginalised students when they engaged in mathematical practices even when the question was superficial or the explanation basic.	T explained to students how their mathematical practices were going to become the social norm / just how mathematics is done in a learning community	T used cultural capital/ expertise of marginalised students to raise their intellectual status in the discussion: their expertise on the All-Blacks rugby team.	T elicited mathematical practices and stated how engaging with this was *doing mathematics* and this process was she valued as a teacher.	T expected every student to participate but allowed dipping in and out for most marginalised students, Mary, who had a very small concentration span.

Five Practices Teacher Actions	**Talk Moves** Teacher Actions	**Mathematical Practices** Teacher Actions	**Social Norms** Teacher Actions	**Cultural Funds** Teacher Actions	**Status** Teacher Actions	**High Expectations** Teacher Actions
T selected a range of representations to be shared – pictures, diagrams, different strategies, models, etc.	T used the add-on talk move to add on to marginalised students' ideas, so they were positioned as important contributors to the discussion.	T made it explicit that confusion, mistakes, and partial ideas are all a normal part of sense making.	T used talk moves to bring marginalised students into the discussion	T validated students when they made connections between the context and the mathematical content	T fostered social and collaborative norms to value listening to a differing point of view to your own.	T expected all students to contribute to the discussion but allowed a choice to promote student agency when needed: *you can sit there or here while you tell us your idea.*
T explored, validated, and shared divergent thinking pathways as well as more conventional or homogenous ways	T used the turn and talk move to allow marginalised students opportunities to talk with just one buddy when the large group felt too threatening.	T explicitly validated students when they said *I don't know* or *I don't understand* or *I am not sure* to show students that articulating understanding, including uncertainty, is what mathematicians do	T validated students when they engaged in social and socio-mathematical norms	T attached familiar contexts or terms to unfamiliar contexts or terms to help students make the connections	T assigned competence to marginalised students by specifically naming what was competent and why. T praised the competency.	T gave *take up* time so students were aware they must share and what they were expected to share. T supported them to practice their explanation with peer support.
T looked for the *gold* in marginalised students' work. T highlighted their strengths by getting them to share this in the discussion.	T used turn and talk move to orient students to view each other as credible resources	T validated multiple representations, so students valued diversity in thinking and saw more than one way to show math's smarts.	T validated students using each other as resources or offering their support to others	T continuously referred to the contextual features of a task even when reading out the equation: *four rows of eight ants* instead of *four times eight.*	T re-assigned competence when rejected by marginalised student the first time.	T gradually increased concentration span by giving a timeframe. For example, *you can have morning tea after you have explained your array.*

Five Practices Teacher Actions	**Talk Moves** Teacher Actions	**Mathematical Practices** Teacher Actions	**Social Norms** Teacher Actions	**Cultural Funds** Teacher Actions	**Status** Teacher Actions	**High Expectations** Teacher Actions
T ditched the *mastery* lens on the learning outcomes for the lesson. Instead, T expected that each student would move from one point of understanding to a deeper level of understanding within their own learning trajectory for that lesson.	T used turn and talk move to formatively assess marginalised students' understanding		T specifically praised groups who collaborated and co-constructed their solution and selected them to share their solution in the discussion. T told everyone why they were chosen to share.	T tapped into students' cultural funds of knowledge (rugby)	T called specifically on marginalised students to contribute, share, or explain something she knew they could.	T encouraged students to accept marginalised students as 'one of them' and not as in need of special help or attention or treatment.
	T used turn and talk move to notice what marginalised students said and mined for gold which was then shared with whole group				T deliberately positioned marginalised students as important contributors to the discussion and made this validation public and explicit.	T positioned marginalised students as capable in front of their peers by sometimes pairing them with a student who held high intellectual status. T selected this group to share but got the marginalised student to take a leading role and the student with highs status to take a supporting role.

Appendix D2: Thematic Analysis Framework Teacher Beliefs

Framework of Inclusionary and Exclusionary Beliefs: Sample

INCLUSIONARY BELIEFS	EXCLUSIONARY BELIEFS
High Expectations	***High Expectations***
"I include Mary now. She sometimes stays for the whole lesson and sometimes she does not.	"This is the first year where I haven't included one student in my groups because she is so low. I just feel she is never going to get to that…she is definitely one of the lowest children I have seen."
"I have changed my whole lesson time now [to before morning tea] so the teacher aide is not in the room. Mary does not work with the teacher aide at mathematics time now. I do not even want the teacher aide in the room because Mary will go "oh, I want to do something with Mrs Filcher!"	Mary has a teacher aide. She would not be able to cope without a teacher aide.
"Mary can sit for a bit longer now!"	"Mary cannot sit on the floor for longer than two minutes. Two minutes is always her max."
"I used to think Mary can do math's with the teacher aide and she cannot take part with us but now I think Mary can do math's with us or she is not part of the class."	"Mary likes to sit with the teacher aide at the table. I give her a job like cutting up the fraction pieces…she has no idea what she is doing but she can match the colours."

Appendix D3: Thematic Analysis Used to Group all Codes to into Themes

Inclusionary/Exclusionary Framework of Teacher Beliefs and Actions

INCLUSIONARY Teacher Beliefs	**EXCLUSIONARY** Teacher Beliefs	**INCLUSIONARY** Teacher Actions	**EXCLUSIONARY** Teacher Actions
High Expectations	***High Expectations***	***High Expectations***	***High Expectations***
"Mary can sit for a bit longer now!" **"Mary sometimes stays for the whole lesson and sometimes she does not."**	"Mary cannot sit on the floor for longer than two minutes. Two minutes is always her max."	**Declines Mary's request to leave and tells Mary that her "help is needed" in the discussion.**	Accepts Mary's first request to leave the discussion

Appendix E1: Principal Consent Form

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MĀTAURANGA

PRINCIPAL CONSENT FORM – SCHOOL

THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

INCLUSION OF MARGINALISED STUDENTS
IN THE LARGE GROUP DISCUSSION

I have read the Information Sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I agree/do not agree to the study taking place at [name of school to be inserted]

Signature: .... **Date:** ....

Full Name – printed ....

Appendix E2: Board of Trustees Consent Form

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MĀTAURANGA

CONSENT FORM: BOARD OF TRUSTEES

THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

INCLUSION OF MARGINALISED STUDENTS
IN THE LARGE GROUP DISCUSSION

We have read the information sheet and have h We have read the information sheet and have had the details of the study explained to us. Our questions have been answered to our satisfaction, and we understand that we may ask further questions at any time.

We agree / do not agree (circle one) for Mrs Ulster in Room ( ) and Mrs Walter in Room ( ) to participate in this study under the conditions set out in the information sheet.

Date: ____________________

Board Chairperson Signature: ____

Full Name – printed:____

Principal signature: ____

Full Name – printed:____

Appendix E3: Teacher Participant Consent Form

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MĀTAURANGA

Inclusion of marginalised students
in the large group discussion

PARTICIPANT CONSENT FORM – INDIVIDUAL
THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

I have read the Information Sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I agree/do not agree to being filmed during mathematics lessons.

I agree/do not agree to being audio-recorded during interviews.

I agree to participate in this study under the conditions set out in the Information Sheet.

Signature: .... **Date:** ....

Full Name – printed ....

School Name ....

Appendix E4: Information Sheet

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MĀTAURANGA

Inclusion of all students in the large group discussion

INFORMATION SHEET

I am currently doing research that will focus on how teachers create inclusion for students in mathematical discussions. I am interested in how teachers make space for marginalised learners in the large group discussion phase of the mathematics lesson under the inquiry model. As you are an experienced teacher in the DMIC pedagogical approach I would like to investigate the pedagogical moves you make during the large group discussion to ensure all students have an equitable opportunity to participate and learn. I warmly invite you to participate in this study. I will be collecting data from you and your classroom for approximately three terms (with some flexibility /room for pandemic related delays).

Your participation in the project will involve **two interviews,** one at the beginning and one at the end of the project, of approximately one hour's duration each. Additionally, participation involves **four mathematics lesson observations.** The lesson observations focus on the **share back phase** of the lesson. I would like to audio-record the interviews, and these will be transcribed. I would like to film the four share backs (not the whole lesson) and these video recordings will be transcribed.

All data (electronic audio, video files, and transcribed recordings) will be stored in a secure location, with no public access and used only for this research. The school's name and names of teachers and children will be assigned pseudonyms in any research presentations or publications arising from this project. At the end of the study, a summary of the research will be provided to the school and made available for you to read.

Please note that you have the following rights in response to the request to participate in this study:

- decline to participate.
- decline to answer any question (in interviews).
- in any interview have the right to ask for the audio tape to be turned off at any time.
- withdraw from the study during the data collection phase.
- ask any questions about the study at any time during participation.
- provide information on the understanding that your name will not be used unless you give permission to the researcher.
- be given access to a summary of the project findings when it is concluded.

If you have further questions about this project, you are welcome to discuss them with me personally:

Jennifer James
Massey University
Institute of Education ([redacted])
Email: J.James@massey.ac.nz

Or email my chief supervisor:

Professor Roberta Hunter
Massey University, School of Education.
Phone: (09) 4140800 Extension 43530.
Email. R.Hunter@massey.ac.nz

This project has been evaluated by peer review and judged to be low risk. Consequently, it has not been reviewed by one of the University's Human Ethics Committees. Jennifer James is responsible for the ethical conduct of this research. If you have any concerns about the conduct of this research that you want to raise with someone other than the researcher, please contact:

Professor Craig Johnson, Director (Research Ethics), humanethics@massey.ac.nz