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DYNAMIC ROUTING WITH COMPETITION:
FOUNDATIONS AND STRATEGIES

A THESIS PRESENTED IN PARTIAL FULFILMENT
OF THE REQUIREMENTS FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY IN
OPERATIONS RESEARCH AT
MASSEY UNIVERSITY

Mark Richard Johnston

1999

Errata and Addenda

P 7 line 2	Replace “evaluate these difficulty” by “evaluate the difficulty”.
P 17 line 11	Replace ‘[35]’ by ‘[35, page 475]’.
P 19 line –12	Replace “Brideau and Cavalier” by “Brideau and Cavalier [28, page 1]”.
P 21 line –17	Replace “They adopt” by “Desrochers, Lenstra and Savelsbergh [54, page 323] adopt”.
P 21 line –1	Replace “which will be useful to” by “that will be useful in”.
P 23 line 18	Replace ‘[60]’ by ‘[60, page 12]’.
P 26 line 13	Replace ‘interesting’ by ‘interestingly’.
P 26 line 14	Replace ‘[146]’ by ‘[146, page 1105]’.
P 28 line 10	Replace ‘[7]’ by ‘[7, page 206]’.
P 31 line 14	Replace “Is it possible communicate between decision makers?” by “Is communication possible between decision makers?”.
P 34 line –16	Delete ‘of these’.
P 39 line –6	Replace “move continuously at the same constant speed in the Euclidean plane” by “move continuously at unit speed in the Euclidean plane and may instantaneously change direction”.
P 40 line 3	Replace ‘[7]’ by ‘[7, pages 11–18]’.
P 43 line –7	Replace “principles make” by “principles that make”.
P 46 line 3	Insert after the first sentence: “Let d_{Xi} be the distance from player X ’s current location to the location of prize i (at unit speed this is equivalent to the minimum possible time for player X to reach prize i).”
P 46 line 23	Replace “players prize i ” by “players share prize i ”.
P 47 line –17	Delete ‘a’.
P 47 line –15	Delete ‘it’.
P 47 line –11	Replace “median prize of move” by “median prize or move”.

P 49 line –18	Replace “at least on prize” by “at least one prize”.
P 52 line 4	Replace “subpath, but not necessarily committing to entire, but” by “subpath, not necessarily committing to an entire subpath, but”.
P 53 line –13	The definition given for <i>Farthest Insertion</i> is that of <i>Most Expensive Insertion</i> . For <i>Farthest Insertion</i> , select vertex k not on the subpath such that d_{ik} (for $k \in \text{subpath}$) is maximized. Insert vertex k between subpath edge (i, j) such that $d_{ik} + d_{kj}$ is minimized.
P 54 line –1	Replace “guaranteed to player $\mathcal{A}$ ” by “guaranteed to player $\mathcal{A}$ if targeted first”.
P 55 line –1	Replace “higher likelihood” by “higher likelihood of capture”.
P 58 line 8	Delete ‘we’.
P 59 line –14	Insert new paragraph “Let p denote a problem instance defined by the number of prizes, the location of each prize, the value of each prize, the initial location of each player, the overall deadline, and the step size. The guaranteed value of a maximal guaranteed subpath for player $\mathcal{A}$ is denoted $\Gamma_{\mathcal{A}}(p)$ and the guaranteed value of a maximal guaranteed subpath for player $\mathcal{B}$ is denoted $\Gamma_{\mathcal{B}}(p)$.”
P 61 line –1	Replace ‘paranoid’ by ‘GUARANTEE-SUBPATH’.
P 63 line –10	Replace “We can calculate the cooperative value, Ω , which is” by “For a problem instance p , we can calculate the cooperative value, $\Omega(p)$, which is”.
P 63 line –5, –1	Replace ‘ Ω ’ by ‘ $\Omega(p)$ ’.
P 64 line 8	Replace “select between them.” by “select between them?”
P 68 line –15	Replace “is the defined” by “is then defined”
P 68 line –13	Replace ‘apply’ by ‘applying’.
P 93 line 6	Delete comma.
P 93 line 13	Replace “ $d_{A1} < d_{B1}$ and $d_{A2} = d_{B2}$ ” by “ $d_{Ai_1} < d_{Bi_1}$ and $d_{Ai_2} = d_{Bi_2}$ ”.
P 93 line 19	Replace “ $\mathcal{A} \rightarrow i_1$ and $\mathcal{B} \rightarrow i_1$ is a Nash equilibrium in which the players share the sequence $i_1 \rightarrow i_2$ ” by “ $\mathcal{A} \rightarrow i_2$ and $\mathcal{B} \rightarrow i_2$ is a Nash equilibrium in which the players share the sequence $i_2 \rightarrow i_1$ ”.

P 93 line 22 Replace " $\mathcal{A} \rightarrow \arg\max\{v_1, v_2\}$ and $\mathcal{B} \rightarrow i_2$, for a reward of $\max\{\frac{1}{2}(v_1 + v_2), v_1\}$ to player $\mathcal{A}$ and $\min\{\frac{1}{2}(v_1 + v_2), v_2\}$ to player $\mathcal{B}$ " by " $\mathcal{A} \rightarrow \arg\max\{v_{i_1}, v_{i_2}\}$ and $\mathcal{B} \rightarrow i_2$, for a reward of $\max\{\frac{1}{2}(v_{i_1} + v_{i_2}), v_{i_1}\}$ to player $\mathcal{A}$ and $\min\{\frac{1}{2}(v_{i_1} + v_{i_2}), v_{i_2}\}$ to player $\mathcal{B}$ ".

P 94 line -12 Replace 'max' by 'min'.

P 94 line -11 Replace 'of' by 'or'.

P 94 line -10 Replace 'max' by 'min'.

P 103 line 18 Replace ' $\mathcal{B}$ ' by ' $\mathcal{A}$ '.

P 109 line -13 Replace ' $\mathcal{B}$ safety-window' by ' $\mathcal{B}$ -safety-window'.

P 117 line -12 Replace last '+' by ' $\geq$ '.

P 118 Replace ' $\Omega(\mathcal{A}, \mathcal{B})$ ' by ' $\Omega(p)$ ', replace ' $\Gamma(\mathcal{A})$ ' by ' $\Gamma_{\mathcal{A}}(p)$ ', and replace ' $\Gamma(\mathcal{B})$ ' by ' $\Gamma_{\mathcal{B}}(p)$ '.

P 118 line 15 Replace "every prize of sequence of prizes" by "every prize or sequence of prizes".

P 118 line 17 Replace 'paranoid' by 'guaranteed'.

P 118 line -1 Delete the redundant condition "and $d_{A i_2} + d_{i_1 i_2} < d_{B i_1}$ ".

P 119 line 1 Replace " $\mathcal{A} \triangleright (i_2 \rightarrow i_1)$ is accessible but $\mathcal{A} \triangleright (i_1 \rightarrow i_2)$ is not accessible" by " $\mathcal{A} \triangleright (i_1 \rightarrow i_2)$ is accessible but $\mathcal{A} \triangleright (i_2 \rightarrow i_1)$ is not accessible".

P 119 line 6 Replace ' $\Omega(\mathcal{A}, \mathcal{B})$ ' by ' $\Omega(p)$ '.

P 133 line -4 Replace ' $\Gamma(\mathcal{A}|\mathcal{B})$ ' by ' $\Gamma_{\mathcal{A}}(p)$ ' and replace ' $\Omega(\mathcal{A}, \mathcal{B})$ ' by ' $\Omega(p)$ '.

P 136 Replace ' $\Gamma(\mathcal{A}|\mathcal{B})$ ' by ' $\Gamma_{\mathcal{A}}(p)$ ', replace ' $\Gamma(\mathcal{B}|\mathcal{A})$ ' by ' $\Gamma_{\mathcal{B}}(p)$ ', and replace ' $\Omega(\mathcal{A}, \mathcal{B})$ ' by ' $\Omega(p)$ '.

P 139 line -1 Replace ' $\Omega(\mathcal{A}, \mathcal{B})$ ' by ' $\Omega(p)$ '.

P 146 line 2 Replace 'engine' by 'engines'.

P 170 line -13 Replace 'define' by 'determine'.

P 187 line -12 Replace 'prosed' by 'proposed'.

P 192 line -8 Delete 'the'.

P 196 line 15 Replace ' $\Gamma_{\mathcal{A}}(\mathcal{A} \triangleright x \mathcal{B} \triangleright y)$ ' by ' $\Gamma_{\mathcal{A}}(\mathcal{A} \triangleright x, \mathcal{B} \triangleright y)$ '.

P 198 line 3 Replace "which make" by "that would make".

P 202 In the first table replace ' $v_{i_1} < i_2 v_{i_3}$ ' by ' $v_{i_1} < 2v_{i_3}$ ' and in the second table replace ' $v_{i_2} < i_2 v_{i_3}$ ' by ' $v_{i_2} < 2v_{i_3}$ '.

P 206 line -20, -18 Replace ' $\Gamma(j, \mathcal{X})$ ' by ' $\Gamma_{\mathcal{X}}(j)$ '.

P 206 line -19 Replace ' $\Gamma(j)$ ' by ' $\Omega(j)$ '.

P 210 line 19 Replace 'that' by 'there'.

P 210 line -15 Replace 'provide' by 'provides'.

P 213 line 10 Replace 'defined' by "defined by".

P 213 line -17 Replace 'local-optima' by "local optimum".

P 217 line 4 Replace "swap to clusters" by "swap two clusters".

P 220 line 22 Delete "in an $\mathcal{X}$ -future-family".

P 225 line -10 Before "Table 7.2(d) illustrates" insert the sentence "This exhibits a potential drawback of enforcing the *cluster-no-return* rule."

P 233 line -13 Replace '[29]' by '[29, page 111]'.

P 241 line -10 Replace 'lemma' by 'theorem'.

P 245 line 10 Replace ' $\mathcal{A} \rightarrow 14 \rightarrow 12 \rightarrow 12$ ' by ' $\mathcal{A} \rightarrow 14 \rightarrow 12 \rightarrow 13$ '.

P 245 line 11 Replace ' $\mathcal{A} \rightarrow 11 \rightarrow 9 \rightarrow 6 \rightarrow 7 \rightarrow 8$ ' by ' $\mathcal{B} \rightarrow 11 \rightarrow 9 \rightarrow 6 \rightarrow 7 \rightarrow 8$ '.

P 245 line -11 Replace 'playerA $\rightarrow 14 \rightarrow 12 \rightarrow 13$ ' by ' $\mathcal{A} \rightarrow 14 \rightarrow 12 \rightarrow 13$ '.

P 245 line -11 Replace '199' by '179'.

P 247 The caption for Table 7.4(d) should read "Player $\mathcal{B}$ FAMILY-PRIZE-GUARANTEE Subpath".

P 251 line -15 Replace 'possible' by 'possible'.

P 254 line 30 Add to the end of the paragraph: "In each case the two player game table indicates the *structure* of the root game table and ' $\square$ ' indicates an entry remaining to be evaluated."

P 261 line -6 Replace "clusters [2] and 35]" by "clusters [2] and [3]".

P 281 line 6 Replace "which is even more confusing" by "which is interesting since both players were previously trying to avoid a tactical conflict".

P 281 line -10 Add to the end of the paragraph: "Cycling occurs when players perpetually retargeting in response to the opponent retargeting. It would eventually be resolved as the players move closer to their respective candidate target clusters but may be detected and resolved earlier by switching to a MAXIMIN evaluator."

P 283 line 10 Replace 'governed' by "governed by".

P 284 line -11 Replace "be following in" by 'follow'.

P 286 line 1 Replace "to straightforward" "to be straightforward".

P 287 line 16 Move $\square$ to the right hand side.

P 302 line -15 Delete "in a player $\mathcal{X}$ future-family".

P 305 line -20 Delete "in a player $\mathcal{A}$ future-family".

P 305 line -11 Replace ' $\mathcal{G}_h + 1$ ' by ' $\mathcal{G}_{h+1}$ '.

P 337 line –7 Replace ‘between’ by ‘from’.

P 340 line –9 Insert: “In the latter two grid-structures, players are located near to the centre of a grid-cell to minimize the edge effects in the initial planning paths.”

P 366 line 7 Replace “measure a strategies” by “measure a strategy’s”.

P 387 line –10 Replace “but a similarly” by “but are similarly”.

P 389 Delete prize 1 from Table 9.7(b) and Table 9.7(d).

P 392 line –10 Replace “is one which is hard to predict” by “is one for which it is hard to predict”.

P 393 Delete prize 1 from Table 9.9(b) and Table 9.9(d).

P 400 In Table 10.1 replace ‘//PRIZE-PARANOID’ by ‘//PRIZE-GUARANTEE’.

P 404 line –14 Replace “A ORIGINAL-DT” by “An ORIGINAL-DT”.

P 407 The second part of Figure 10.1 should have label ‘(b)’.

P 422 line –8 Replace first ‘cluster’ by ‘clusters’.

P 433 line –1 Delete one ‘the’.

P 438 line –6 Replace “grid-size. which” by “grid-size, which”.

P 462 Rename Section 11.2.1.1 as “Characterisation of Strategies and Problem Instances”.

P 462 Add the following paragraph to Section 11.2.1.1: “Problem design (via templates, or more refined bad-case examples) needs to be structurally *catalogued*, if possible. Also, the concept of fairness, methods for ‘quickly’ estimating the value of a game, and playing against a ‘capricious’ game designer need to be investigated. It is also necessary to further distinguish between inherently difficult problems, and those which are difficult for the solution strategies.”

- [202] VOLGENANT, T., AND JONKER, R. On some generalizations of the travelling salesman problem. *Journal of the Operational Research Society* 38, 11 (1987), 1073–1079.
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Glossary of Symbols

The Players

$\mathcal{A}, \mathcal{B}$	The two specific players.	39
$\mathcal{C}$	A third specific player.	467
$\mathcal{X}, \mathcal{Y}$	Generic players, not specifically stating which is player $\mathcal{A}$ or which is player $\mathcal{B}$.	45
$(x_A, y_A), (x_B, y_B)$	The initial location of the players.	39

Prizes, Clusters and Families

$V = \{1, 2, \dots, n\}$	The set of all prizes.	39
(x_i, y_i)	The location of prize i .	39
v_i	The value of prize i .	39
$[\alpha]$	A cluster, elements are prizes $j \in [\alpha]$.	211
$\eta_{[\alpha]}$	Required PCTSP value for cluster $[\alpha]$.	218
$\mathcal{C}$	A clustering, elements are clusters $[\alpha] \in \mathcal{C}$.	211
$\mathcal{F}$	A family (also a clustering).	216
$\mathbb{F}$	Set of families, elements are families $\mathcal{F} \in \mathbb{F}$.	216

The hierarchy of membership is $j \in [\alpha] = Q \in \mathcal{Q} \in \mathbb{Q}$.

Distances and Times

λ	Overall deadline.	39
Δ	Step size.	45
d_{ij}	Distance between prize i_1 and prize i_2 .	46
d_{Xj}	Distance from player $\mathcal{X}$'s current location to prize j .	46
$D_{[\alpha][\alpha']}$	Distance between clusters.	212
$d_{jC_{[\alpha]}}$	Distance between prize j and the centroid of cluster $[\alpha]$.	212
τ_{Aj}	Earliest arrival time of player $\mathcal{A}$ at prize j .	149
τ_{Bj}	Earliest arrival time of player $\mathcal{B}$ at prize j .	149

Problem Instances

p	CPCP problem instance.	59
$P : \text{nl}lvab\lambda$	Prize class (P-class) of problem instances.	372
$C : \text{ml}lv\kappa ab\lambda$	Cluster class (C-class) of problem instances.	375
$D : \text{hcbssa}$	Density class (D-class) of problem instances.	381

Player Strategies

$\mathcal{S}$	Set of player strategies.	381
$\mathcal{S}_{\infty}$	Infinite set of all possible player strategies.	381
$\mathcal{S}_{A\infty}$	Infinite set of all possible strategies for player $\mathcal{A}$ for the two prize CPCP.	122
$\mathcal{S}_{B\infty}$	Infinite set of all possible strategies for player $\mathcal{B}$ for the two prize CPCP.	122
a, b	Individual player strategies.	122

Game Values

$\Gamma_A(p)$	Maximum value guaranteed to player $\mathcal{A}$.	59
$\Gamma_B(p)$	Maximum value guaranteed to player $\mathcal{B}$.	59
$\Omega(p)$	Cooperative value.	118
$\kappa_A(p; \mathcal{A} \sim a, \mathcal{B} \sim b)$	Score for player $\mathcal{A}$.	401
$\kappa_B(p; \mathcal{A} \sim a, \mathcal{B} \sim b)$	Score for player $\mathcal{B}$.	401
$\kappa_A(\mathbb{P}; \mathcal{A} \sim a, \mathcal{B} \sim \mathcal{S})$	Effectiveness: either MIN-MIN, MIN-MEAN or MEAN-MEAN.	401
$\kappa^{\diamond}(p)$	Overall problem MAXIMIN score.	402
$\Xi(p; \mathcal{A} \sim a, \mathcal{B} \sim b)$	Number of steps.	402
$\tau_A(p; \mathcal{A} \sim a, \mathcal{B} \sim b)$	Processor time.	402
$\tau_B(p; \mathcal{A} \sim a, \mathcal{B} \sim b)$	Processor time.	402
$\xi(p)$	Difficulty of problem instance p .	402
$\bar{\xi}(\mathbb{P})$	Average difficulty of a problem instance class $\mathbb{P}$.	402
$v_A(p; \mathcal{A} \sim a, \mathcal{B} \sim b)$	Dynamic simulation battle value to player $\mathcal{A}$.	381
$v_B(p; \mathcal{A} \sim a, \mathcal{B} \sim b)$	Dynamic simulation battle value to player $\mathcal{B}$.	381
$v_A^*(p), v_B^*(p)$	Nash equilibrium value of the problem instance.	381
$v_A^{\downarrow}(p), v_B^{\downarrow}(p)$	Infinite maximin value of the problem instance.	381
$v_A^{\diamond}(p), v_B^{\diamond}(p)$	Computational maximin value of the problem instance.	385

$v^*(p), v^H(p)$	Optimal and heuristic value of a solution to a combinatorial optimization problem p .	390
ρ_H	Worst case performance of a heuristic H on a combinatorial optimization problem.	390
$\xi(p)$	Difficulty of problem instance p .	392

Paths, Game Trees and Branch and Bound Trees

j	A game tree node.	155
i	A branch and bound tree node.	205
$\Gamma_A(j)$	The guarantee value for player A corresponding to game tree node j .	160
$\Gamma_B(j)$	The guarantee value for player B corresponding to game tree node j .	160
$\Omega(j)$	The cooperative value corresponding to game tree node j .	160
P	Planning subpath.	72
P_A	Planning path for player A .	62
P_B	Planning path for player B .	61
$v_A(P_A, P_B)$	The value of prize on subpath P_A claimed outright by player A plus half the value of those prizes on P_A shared with player B on subpath P_B .	155

Step Planning

$\mathcal{X} \rightarrow i$	Player $\mathcal{X}$ moves one step towards prize i .	92
$\mathcal{X} \rightarrow X$	Player $\mathcal{X}$ moves directly to location X .	92
$\mathcal{X} \triangleright i$	The 'look through' scenario that player $\mathcal{X}$ is committed to travelling all the way to prize i .	92
$\mathcal{X} \triangleright (i_1 \rightarrow i_2)$	The 'look through' scenario that player $\mathcal{X}$ is committed to travelling all the way to prize i_1 then all the way to prize i_2 .	97
$\mathcal{X} \triangleright (i_1 \rightarrow i_2 \rightarrow i_3)$	The 'look through' scenario that player $\mathcal{X}$ is committed to travelling all the way to prize i_1 then all the way to prize i_2 then all the way to prize i_3 .	103

$\mathcal{X} \triangleright \{i_1, i_2\}$	The 'look through' scenario that player $\mathcal{X}$ is committed to travelling all the way to at least one of prizes i_1 and i_2 .	98
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Tactical Planning

Q	Prizes remaining unclaimed.	49
Q_A, Q_B	Prizes remaining and accessible to each player.	147
P_A, P_B	Planning paths for each player.	148
t	Projected time stamp.	148
t_A, t_B	Projected time stamps.	168
$\mathcal{L}_A, \mathcal{L}_B$	Lead prizes for each player.	169
$\mathcal{D}_A, \mathcal{D}_B$	Direct lead prizes for each player.	169
$\mathcal{R}_A, \mathcal{R}_B$	Follow prizes for each player.	169
$\mathcal{F}_A, \mathcal{F}_B$	Follow pairs for each player.	170
$\odot$	Probe.	148
$\mathcal{X} \rightarrow P_X \odot x$	Probe.	148
$\oplus$	Commitment.	169
$\mathcal{X} \rightarrow P_X \oplus x$	Commitment.	169
W	Window scenario and corresponding feasibility window.	170
$\mathcal{X} \triangleright (W \rightarrow \{i_1, i_2\})$	The 'look through' scenario that player $\mathcal{X}$ is committed to travelling all the way to at least one of prizes i_1 and i_2 via feasibility window W .	171

Strategic Planning

FF	Final family requirement enforced.	219
Q_A, Q_B	Families remaining and accessible to each player.	220
$\mathcal{Q}_A, \mathcal{Q}_B$	Clusters remaining and accessible to each player.	220
S_X	Set of target-clusters of player $\mathcal{X}$.	220
$\mathbb{P}_A, \mathbb{P}_B$	Cluster planning paths.	230
W	Cluster window scenario and corresponding cluster window feasibility constraints.	257

$\mathcal{X} \triangleright [ci]$	The ‘look through’ scenario that player $\mathcal{X}$ is committed to travelling all the way to some prize in cluster $[ci]$.	231
$\mathcal{B} \triangleright \{[cy_1], [cy_2]\}$	The ‘look through’ scenario that player $\mathcal{B}$ is committed to travelling all the way to at least one prize from cluster $[cy_1]$ or cluster $[cy_2]$.	256
$\mathcal{B} \triangleright ([cy_0] \rightarrow \{[cy_1], [cy_2]\})$	The ‘look through’ scenario that player $\mathcal{B}$ is committed to travelling all the way to at least one of prize from cluster $[cy_0]$ and then travelling all the way to at least one prize from cluster $[cy_1]$ or cluster $[cy_2]$.	257
$\mathcal{A} \rightarrow \mathbb{P}_A \oplus [cx_0] \triangleright [cx_1]$	Player $\mathcal{A}$ moves through cluster $[cx_0]$ while committed to cluster $[cx_1]$ as the next look-through cluster.	238
$\mathcal{B} \rightarrow \mathbb{P}_B \triangleright \emptyset$	Player $\mathcal{B}$ has no look-through commitment.	243
$\mathcal{B} \rightarrow \mathbb{P}_B \oplus [cx_0] \triangleright \emptyset$	Player $\mathcal{B}$ moves through cluster $[cx_0]$ and has no further look-through commitment.	243
$\mathcal{B} \rightarrow \mathbb{P}_B \triangleright [cx_0]$	Player $\mathcal{B}$ is committed to cluster $[cx_0]$ as the next look-through cluster.	253

Grid Planning

$\mathbb{G}_A, \mathbb{G}_B$	Grid planning paths.	344
$\mathcal{A} \rightarrow \mathbb{G}_A \oplus_D (i, j)$	Commitment to grid cell (i, j) via the intra-cell-direct-path through the current $\mathcal{A}$ -grid-cell.	344
$\mathcal{A} \rightarrow \mathbb{G}_A \oplus_H (i, j)$	Commitment to grid cell (i, j) via the intra-cell-harvest-path through the current $\mathcal{A}$ -grid-cell.	345
$\mathcal{A} \rightarrow \mathbb{G}_A \oplus_P (x, y)$	Commitment to grid cell (i, j) carrying last-direct, last-harvest and all-direct paths via the current $\mathcal{A}$ -grid-cell.	348

Abstract

Operations Research studies a wide range of problems, including long-term, strategic, business planning and short-term, operational, logistical planning. Long-term business decisions revolve around the market demand for goods or services, whereas logistics focuses on efficient scheduling of production and distribution. However, vehicle routing and scheduling problems in a dynamic environment require short-term, operational planning in conjunction with computationally expensive, short-term tactical considerations.

This thesis investigates a model of competition in the distribution of goods to customers, in which a number of independent carriers compete to deliver goods to a fixed set of customers. Assuming that the price and quality of the goods are consistent, each customer is indifferent towards which carrier actually delivers the required goods, but will only accept delivery from the first to arrive at their location. The main source of uncertainty is planning for competition against other independent carriers.

Firstly, we consider the basic elements of competition vehicle routing and scheduling problems, and propose a Reference Model for Competition Routing Problems, synthesising the literature from vehicle routing and game theory. The general problem involves a number of independent decision makers, each representing a carrier company with a private fleet of vehicles, and a fixed set of customers to be serviced. We also formulate the Competitive Prize Collection Problem (CPCP), involving two independent decision makers with one vehicle each. The CPCP encapsulates the core elements of competition within a two player version of the Prize Collecting Travelling Salesman Problem.

Secondly, we consider which strategic, tactical, and operational planning elements are important in the design of strategies for effective performance on the CPCP. We propose a Strategic Planning Architecture (SPA), i.e., a strategy framework based on hierarchical planning at nested planning horizons. This incorporates strategic and tactical planning engines based on modelling the decision problem at each planning horizon as a multiple

stage game. Dynamic monitoring processes match these strategic plans to the predicted and observed movements of the opponent. Strategies which implement the SPA are designed to cover a range of planning horizons and problem sizes.

A series of computational tournaments on problems of different sizes and characteristics suggests that strategies which address contingent planning, cognizance of opponent, and planning based on existing natural structure, are the most effective of those considered. In the process, benchmark sets of robust strategies, and challenging problem instances, are established against which the effectiveness of strategies may be evaluated.

The significant conclusion is that for small problems, strategic considerations are more effective than routing, but for large problems, routing considerations are more effective than strategic. Problems in between require a balance between strategy, response, and routing considerations. Routing only is not sufficient; response requires good strategic information. The CPCP remains a deceptively simple problem which is computationally demanding at all scales of planning, from small problems to large problems. There is considerable scope for the study of further strategies, especially those able to classify, learn from and adapt to, the observed behaviour of the opponent, and for extrapolating these results to a richer set of competition routing problems.

Acknowledgements

When you are a bear of very little brain, and you think of things, you find sometimes that a thing which seemed very thingish inside you is quite different when it gets out into the open and has other people looking at it.

— WINNIE THE POOH (A.A. MILNE)

This thesis is dedicated to the many people who have been supportive, and encouraging, in numerous ways: tiny, small, medium, large and *jumbo*. I am grateful to IBM New Zealand, Landcare Research, and Massey University for their generous financial and technical support.

Thanks to my supervisor, Dr. John Giffin, for his exhortation to “Ph.D. thinking” and his attempts to eschew obfuscation (a.k.a. the red pen). Thanks to Professor Michael Hendy for many (eventually) useful discussions about theorems, the fruit of which is on pages 94 and 241. Thanks also to John Kay for his patience in making sense of this gallimaufry, and to Andy Smith for his unexplainable belief in me.

Thanks to my Mum and Dad, family, and all my friends, for their encouragement and patience, for putting up with my “top secret project”, and for friendly “research” into competition (through cricket and rugby seasons). Thanks also to Radio Sport and CricInfo for keeping me in touch with the *real* world.

Finally, thanks must go to my speedy typist, and my dedicated C, UNIX, and MATLAB guru and L^AT_EXnician, for prompt and excellent technical assistance (that’s me).

Table of Contents

Acknowledgements	v
Table of Contents	vii
List of Figures	xi
List of Tables	xv
List of Algorithms	xix
1 Introduction	1
1.1 The Field: Paths, Routes, Schedules and Games	1
1.2 The Proposition: Motivation for Competition Routing Problems	5
1.3 The Challenge: Research and Thesis Overview	6
I Foundations	9
2 Problem Modelling	13
2.0 Introduction	13
2.1 Combinatorial Subset Selection Routing Problems	14
2.2 VRSP Classification	20
2.3 Dynamic Vehicle Routing and Scheduling Problems	23
2.4 Decision Makers and Game Theory	25
2.5 Reference Model for Competition Routing Problems	28
2.6 Core Problem: Competitive Prize Collection Problem	38
2.7 Research Questions	41

3	Elements of Strategy	43
3.0	Introduction	43
3.1	Algorithms, Heuristics, Strategies and Simulations	44
3.2	Prize Ranking and Selection Strategies	49
3.3	Combinatorial Subproblem Based Strategies	58
3.4	Necessary Principles of Strategy	64
3.A	Appendices to Chapter 3	67
4	Strategic Planning Architecture	77
4.0	Introduction	77
4.1	Aggregation and Contingency Planning	78
4.2	Cognizance of Opponent and Response Monitoring	81
II	Strategies	87
5	Optimal and Heuristic Analysis of Tiny Problems	91
5.0	Introduction	91
5.1	Two Prize Problem	92
5.2	Window Feasibility Subproblem	103
5.3	Strategies for Two Prize Problem	104
5.4	Two Prize Problem with Finite Deadline	118
5.5	Tiny Tournament	122
5.6	Tactical Approach to Three Prize Problem	131
6	Tactical Planning for Small Problems	145
6.0	Introduction	145
6.1	Tactical Engine: PRIZE-PARANOID	148
6.2	Tactical Engine: ORIGINAL-DT	153
6.3	Tactical Engine: PRIZE-DT	168
6.4	Dynamic Monitoring: Small-DMS	187
6.A	Appendices to Chapter 6	201
7	Strategic Planning for Medium Problems	209
7.0	Introduction	209
7.1	Cluster and Family Structures	210
7.2	Family-Cluster Precedence Constrained Tactical Subproblems	217
7.3	Strategic Cluster Building Blocks	227
7.4	Strategic Engine: CLUSTER-PARANOID	260
7.5	Strategic Engine: CLUSTER-DT	263
7.6	Dynamic Monitoring: Medium-DMS	283
7.A	Appendices to Chapter 7	297

8 Strategic Planning for Large Problems	335
8.0 Introduction	335
8.1 Strategic Approach to Large Problems	337
8.2 Grid Structures	340
8.3 Strategic Grid Planning	343
8.4 Tactical Grid Planning	349
8.5 Dynamic Monitoring: Grid-DMS	351
8.A Appendix to Chapter 8	357
 III Computational Evaluation	 363
9 Problem Design	367
9.0 Introduction	367
9.1 Classes of Problem Instances	368
9.2 Prediction of the Expected Value of the Game	381
9.3 Sensitivity Analysis	386
9.4 Bad-Case Problem Instance Design	390
10 Computational Tournaments	397
10.0 Introduction	397
10.1 Preliminary Computational Tournaments	403
10.2 Final Computational Tournaments	443
11 Conclusions and Recommendations for Future Research	457
11.1 Conclusions: Foundations and Strategies	457
11.2 Recommendations for Future Research	462
A Competition Routing Kernel Environment (CR₄KE_T)	473
A.1 CR ₄ KE _T Tournament	473
A.2 CR ₄ KE _T Simulator	473
A.3 CR ₄ KE _T Viewer	474
A.4 CR ₄ KE _T Generator	478
A.5 CR ₄ KE _T Sensitivity	478
Bibliography	479
Index of Problems	492

List of Figures

2.1	Entity-Relationship Model for VRSP	22
2.2	Entity-Relationship Model for CRP	30
3.1	Guarantee and Avoidance	55
3.2	A Guarantee Partition.	59
3.3	Structure of ABA -path Components	63
3.4	Lin and Kernighan Two-Exchange Definition	69
3.5	Basic Local Subpath Operators	70
3.6	Generalized Subpath Insertion and Deletion operators	71
3.7	Additional Subpath Two-Exchanges	72
3.8	Structure of ABA -PATH heuristic	75
4.1	Generic Dynamic Monitoring System	84
5.1	Static Cases of the Two Prize Problem	92
5.2	Dynamic Case of the Two Prize Problem	94
5.3	Angles θ_A and θ_B	95
5.4	Two Prize Median Location	98
5.5	Player Location Sensitivity of Two Prize Problem	101
5.6	Prize Location Sensitivity of Two Prize Problem	102
5.7	Three Prize Problem	106
5.8	Initial target prizes of A and B	137
5.9	Nine cases for THREE-ORIGINAL-DT	138
5.10	Nine cases for THREE-PRIZE-DT	141
6.1	Example Tactical Problem	150
6.2	Evaluators of an ORIGINAL-DT Game Tree Node	156
6.3	Direct-lead, lead and follow subregions for player A	169
6.4	Alternative Evaluators for PRIZE-DT	176

6.5	Monitors and Frames for Small DMS	189
6.6	PRIZE-DT evaluators incorporating a <i>prize-ts</i>	199
6.7	Subpath appendage two-exchange	204
7.1	A Family-Cluster Structure	216
7.2	Example Tactical Problem with Clusters	224
7.3	Cases of one cluster look-through dominance	232
7.4	Possible deadlock scenario: $\mathcal{A} \rightarrow [1] \rightarrow [2]$ and $\mathcal{B} \rightarrow [2] \rightarrow [1]$	235
7.5	Possible deadlock scenario: $\mathcal{A} \rightarrow [1] \rightarrow [3]$ and $\mathcal{B} \rightarrow [2] \rightarrow [3]$	236
7.6	Possible deadlock scenario: $\mathcal{A} \rightarrow [1] \rightarrow [3]$ and $\mathcal{B} \rightarrow [2] \rightarrow [4]$	237
7.7	Direct-cluster-lead move-through cases.	239
7.8	Cluster-lead move-through cases from player $\mathcal{A}$'s perspective.	240
7.9	Specialist Case (I) Move-through for CLUSTER-GUARANTEE	244
7.10	Example Strategic Problem	246
7.11	State transitions between states (F1)–(F8).	270
7.12	State transitions between states (L1)–(L3) and (F6)–(F8).	271
7.13	Nested Frames for Medium DMS	284
7.14	Monitors and Frames for Medium DMS	285
7.15	Subpath appendage two-exchange	303
7.16	Inter-cluster distance calculation	307
7.17	Cluster Feasible Window Cases (i)–(iv).	319
7.18	Cluster Feasible Window Cases (v)–(viii).	323
7.19	Cluster Feasible Window Cases (ix)–(xi).	328
8.1	Examples of 4×4 grid-structures	341
8.2	Grid-cell entry/exit locations	342
8.3	Generic Strategic Plan	344
8.4	Example scenario of a GRID-DT search.	347
8.5	Example scenario of a GRID-PATH search.	349
8.6	Manhattan grid-cell distance: $m = 4$	350
8.7	Four types of adjacency considerations	351
8.8	Monitors and Frames for a Grid-DMS	352
8.9	Construction of Static Grid	358
8.10	Construction of Single Player Dynamic Grid	358
8.11	Construction of Two Player Dynamic Grid	360
8.12	Positioning Players for Two Player Dynamic Grid	360
9.1	Hotspot Prize Density Function $f(x, y)$	378
9.2	Results from Evaluation of Static Predictors	386
10.1	Small Preliminary Tournament: Results by Strategy	407
10.2	Small-Medium Preliminary Tournament: Results by Strategy	417
10.3	Medium Preliminary Tournament: Results by Strategy	425

10.4	Medium-Large Preliminary Tournament: Results by Strategy	434
10.5	Large Preliminary Tournament: Results by Strategy	440
10.6	Small Final Tournament: Results by Strategy	445
10.7	Small-Medium Final Tournament: Results by Strategy	447
10.8	Medium Final Tournament: Results by Strategy	450
10.9	Medium-Large Final Tournament: Results by Strategy	453
10.10	Large Final Tournament: Results by Strategy	454
11.1	A Grid Network CPCP	465
11.2	Dependencies between Team and Cooperative CPCPs	469
A.1	CRiKET Viewer	477
A.2	CRiKET Viewer Controls	477

List of Tables

2.1	List of Attributes of RTDPs (reproduced from Séguin et al [192])	25
5.1	Tiny Tournament: Participating Strategies	125
5.2	Possible Outcomes of Two Prize Problem	126
5.3	Tiny Tournament: Results by Tiny Subclass	128
5.4	Tiny Tournament: Results by Player $\mathcal{A}$ Strategy	129
5.5	Tiny Tournament: Results by Player $\mathcal{B}$ Strategy	130
5.6	Cases of Three Prize Strategic Analysis for $\lambda = \infty$	136
6.1	Tactical Example of PRIZE-GUARANTEES	152
6.2	Tactical Example of PRIZE-PARANOID	154
6.3	Tactical Example of ORIGINAL-DT MINIMAXIMIN Game Tables	165
6.4	Tactical Example of ORIGINAL-DT	166
6.5	Tactical Example of Player $\mathcal{A}$ PRIZE-DT Game Tables	186
6.6	Tactical Example of Player $\mathcal{B}$ PRIZE-DT Game Tables	188
7.1	Tactical Example of FAMILY-PRIZE-GUARANTEES	225
7.2	Tactical Example of Player $\mathcal{A}$ FAMILY-PRIZE-DT Game Tables	226
7.3	Tactical Example of Player $\mathcal{B}$ FAMILY-PRIZE-DT Game Tables	228
7.4	Strategic Example of Player $\mathcal{A}$ CLUSTER-GUARANTEES	247
7.5	Strategic Example of Player $\mathcal{A}$ CLUSTER-GUARANTEES	248
7.6	Strategic Example of Player $\mathcal{B}$ CLUSTER-GUARANTEES	249
7.7	Strategic Example of Player $\mathcal{B}$ CLUSTER-GUARANTEES	250
7.8	Strategic Example of CLUSTER-GUARANTEES	252
7.9	Strategic Example of CLUSTER-PARANOID Game Tables	261
7.10	Strategic Example of CLUSTER-PARANOID	262
7.11	Strategic Example of CLUSTER-DT Root Game Tables	272
7.12	Strategic Example of CLUSTER-DT Scenario $\mathcal{A} \triangleright [5]$ and $\mathcal{B} \triangleright [2]$	273
7.13	Strategic Example of CLUSTER-DT Scenario $\mathcal{A} \triangleright [4]$ and $\mathcal{B} \triangleright [2]$	274

7.14	Strategic Example of CLUSTER-DT Scenario $\mathcal{A} \triangleright [5]$ and $\mathcal{B} \triangleright [4]$	276
7.15	Strategic Example of CLUSTER-DT Scenario $\mathcal{A} \triangleright [1]$ and $\mathcal{B} \triangleright [1]$	277
7.16	Strategic Example of CLUSTER-DT Scenario $\mathcal{A} \triangleright [1]$ and $\mathcal{B} \triangleright [2]$	278
7.17	Strategic Example of CLUSTER-DT Scenario $\mathcal{A} \triangleright [5]$ and $\mathcal{B} \triangleright [3]$	279
7.19	Examples of Strategic Concepts	282
7.20	Counting the number of distributions of six distinguishable objects into three distinguishable containers for all possible sizes of the containers.	299
7.21	Counting the number of distributions of six distinguishable objects into three distinguishable containers for all possible sizes of the first container.	299
9.1	P-Class: Prize Location Component Subclasses	370
9.2	Examples of P-Class Problem Instances	373
9.3	Examples of C-Class Problem Instances	376
9.4	Examples of D-Class Problem Instances	382
9.5	Static Predictors of the Expected Value of the Game	385
9.6	Static Sensitivity to Player Location	388
9.7	Static Sensitivity to Prize 1 Location	389
9.8	Dynamic Sensitivity to Player Location	391
9.9	Dynamic Sensitivity to Prize 1 Location	393
9.10	Example of a Bad-Case Problem Instance	395
10.1	Participating ‘-DT’ Strategies in the Tournaments	400
10.2	Small Preliminary Tournament: Participating Strategies	405
10.3	Small Preliminary Tournament: Results by Strategy	409
10.4	Small Preliminary Tournament: Results by Prize Subclass	412
10.5	Small-Medium Preliminary Tournament: Participating Strategies	415
10.6	Small-Medium Preliminary Tournament: Results by Strategy	418
10.7	Small-Medium Preliminary Tournament: Results by Cluster Subclass	420
10.8	Medium Preliminary Tournament: Participating Strategies	423
10.9	Medium Preliminary Tournament: Results by Strategy	426
10.10	Medium Preliminary Tournament: Results by Cluster Subclass	429
10.11	Medium-Large Tournament: Participating Strategies	431
10.12	Medium-Large Preliminary Tournament: Results by Strategy	435
10.13	Medium-Large Preliminary Tournament: Results by Cluster Subclass	437
10.14	Large Preliminary Tournament: Participating Strategies	438
10.15	Large Preliminary Tournament: Results by Strategy	441
10.16	Large Preliminary Tournament: Results by Density Subclass	442
10.17	Small Final Tournament: Results by Strategy	446
10.18	Small-Medium Final Tournament: Results by Strategy	448
10.19	Medium Final Tournament: Results by Strategy	451
10.20	Medium-Large Final Tournament: Results by Strategy	452
10.21	Large Final Tournament: Results by Strategy	455

A.1 Specification of Single Format Problem Instance File 475

A.2 Specification of Bulk Format Problem Instance File 476

List of Algorithms

3.1	model SIMULATION ENVIRONMENT	47
3.2	strategy EQUIDISTANT	47
3.3	strategy CONSISTENT RANDOM PRIZE	49
3.4	strategy NEAREST NEIGHBOUR	50
3.5	strategy FARTHEST NEIGHBOUR	50
3.6	strategy MAX VALUE	50
3.7	strategy MIN VALUE	51
3.8	strategy GREEDY	51
3.9	strategy SUBGRAVITY GREEDY	51
3.10	strategy GREEDY EN ROUTE INSERTION	53
3.11	strategy GREEDY PAIR	54
3.12	strategy GUARANTEED NEAREST NEIGHBOUR	55
3.13	strategy GUARANTEED GREEDY	55
3.14	strategy NEAREST NEIGHBOUR AVOIDANCE	56
3.15	strategy GREEDY AVOIDANCE	56
3.16	strategy TARGET SET NEAREST NEIGHBOUR AVOIDANCE	57
3.17	strategy TARGET SET GREEDY AVOIDANCE	57
3.18	strategy GUARANTEE-SUBPATH	60
3.19	strategy TARGET-SET GUARANTEE-SUBPATH	60
3.20	strategy PRIZE-GUARANTEE	60
3.21	strategy HARVEST PATH	61
3.22	strategy <i>ABA</i> -PATH	62
3.23	strategy TARGET-SET <i>ABA</i> -PATH	63
3.24	heuristic FAST PRIZE GUARANTEE	73
3.25	heuristic FAST HORIZON-OP	73
3.26	heuristic FAST HARVEST PATH	74
5.1	procedure WINDOW FEASIBLE	105

5.2	strategy TWO-PRIZE AHEAD PURE-ATTACK	110
5.3	strategy TWO-PRIZE AHEAD PURE-DEFEND	110
5.4	strategy TWO-PRIZE BEHIND PURE-ATTACK	110
5.5	strategy TWO-PRIZE BEHIND PURE-DEFEND	111
5.6	strategy TWO-PRIZE AHEAD CEILING-ATTACK	111
5.7	strategy TWO-PRIZE BEHIND CEILING-ATTACK	111
5.8	strategy TWO-PRIZE AHEAD THRESH-DEFEND	112
5.9	strategy TWO-PRIZE BEHIND THRESH-DEFEND	112
5.10	strategy TIT FOR TAT BEHIND (BEARING)	113
5.11	strategy TIT FOR TAT BEHIND (LOCATION)	114
5.12	strategy TIT FOR TAT AHEAD (BEARING)	114
5.13	strategy TIT FOR TAT AHEAD (LOCATION)	115
5.14	strategy TIT FOR TWO TATS AHEAD	115
5.15	strategy RANDOM PRIZE	115
5.16	strategy PREVIOUS MEDIAN	116
5.17	strategy TWO-PRIZE BEHIND LAST-RESORT MEDIAN	117
5.18	strategy TWO PRIZE MEDIAN BIAS	132
5.19	procedure WINDOW DISTANCE	134
6.1	function EXHAUSTIVE-MAXIMIN ORIGINAL-DT	158
6.2	function α - β -MAXIMIN ORIGINAL-DT	159
6.3	function NODE-BOUNDED α - β -MAXIMIN ORIGINAL-DT	161
6.4	function NODE-BOUNDED α - β -MINIMAX ORIGINAL-DT	162
6.5	function NODE-BOUNDED α - β -MINIMAXIMIN ORIGINAL-DT	163
6.6	definition $\bar{v}_A(j; \mathcal{A} \triangleright x, \mathcal{B} \triangleright \{y_1, y_2\})$	179
6.7	function α - β -GENERALIZED-MINIMAX PRIZE-DT	181
6.8	function α - β -GENERALIZED-MAXIMIN PRIZE-DT	183
6.9	function α - β -GENERALIZED-MAX PRIZE-DT	185
6.10	function α - β -GENERALIZED-MIN PRIZE-DT	185
6.11	monitor SMALLDMS-PRIZE-MONITOR	192
6.12	monitor SMALLDMS-PRIZE-REFINE	192
6.13	monitor SMALLDMS-STEP-MONITOR	197
6.14	procedure COOP_SLAVE(i)	206
6.15	procedure GUARANTEE_SLAVE(i)	207
7.1	heuristic AGGLOMERATIVE CLUSTERING	212
7.2	definition TARGET-CLUSTERS	221
7.3	procedure RESOLVE DEADLOCK TWO CLUSTER	235
7.4	procedure RESOLVE DEADLOCK THREE CLUSTER	236
7.5	procedure RESOLVE DEADLOCK FOUR CLUSTER	237
7.6	monitor MEDIUMDMS-GLOBAL-MONITOR	288
7.7	monitor MEDIUMDMS-CLUSTER-MONITOR	289

7.8	monitor MEDIUMDMS-CLUSTER-REFINE	289
7.9	monitor MEDIUMDMS-PRIZE-MONITOR	290
7.10	procedure RANDOM PARTITION	298
7.11	procedure RANDOM k -PARTITION	300
7.12	procedure RANDOM CLUSTER SIZES	301
7.13	procedure FAMILY_COOP_SLAVE(i)	305
7.14	procedure DISTANCE_DP(j)	306
7.15	procedure ALL_DISTANCE_SLAVE(i)	309
7.16	procedure PCTSP_DISTANCE_SLAVE(i)	310
7.17	function POSSIBLE_DP(i)	311
7.18	function POSSIBLE_HEURISTIC(k, τ)	312
7.19	procedure FAMILY_GUARANTEE_SLAVE(i)	314
7.20	function FAMILY_FEASIBLE_SLAVE(i)	315
7.21	strategy TWO PRIZE MEDIAN BIAS	317
8.1	monitor GRIDDMS-GRID-MONITOR	353
8.2	monitor GRIDDMS-PRIZE-REFINE	353
8.3	monitor GRIDDMS-PRIZE-MONITOR	354
8.4	monitor GRIDDMS-STEP-MONITOR	355