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EVAPORATIVE CRYSTALLIZATION OF ALPHA-LACTOSE
MONOHYDRATE

A thesis presented in partial fulfilment of the requirements
for the degree of

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Dedicated to

Rasikbihari....

Abstract

Evaporative crystallization has been used by Fonterra Cooperative Group (New Zealand) for producing lactose. It represents an important step during lactose manufacturing where control over crystal size can be obtained, a critical parameter governing the yield and end use. The art of operating these crystallizers has been developed by observation and not from scientific principles. This project was undertaken to understand the mechanisms controlling the crystal size in evaporative crystallizers.

A review of the existing literature showed that secondary nucleation is the major source of nuclei in industrial crystallizers. Based on the review, attrition, contact and fluid shear induced nucleation were identified as the probable secondary nucleation mechanisms in the studied system. Experimental investigation on each of the three mechanisms was carried out separately on a laboratory scale.

It was found that the crystal size had the most significant effect on attrition, followed by impeller speed, which together implies that the crystal collision energy intensity is the dominant factor producing new fragments. Contact nucleation was also found to be controlled by crystal-impeller collisions. It was found that at the studied supersaturation there exists a minimum kinetic energy of contact below which secondary nucleation would not occur. This threshold value was used as the basis to assess the contribution of various mechanisms at the industrial scale. Shear nucleation was found to be independent of shear above 5000 s^{-1} .

A mathematical model describing the operation of the industrial crystallizer was formulated. Sensitivity analysis was conducted by simulating the model for a range of operational and kinetic parameter values. It was found that the crystal size is affected most by secondary nucleation. The volume weighted mean size approximately halved with a 5.5 times increase in the secondary nucleation rate.

The model was refined to accommodate size dependent growth rate and growth rate dispersion. The kinetic parameters were fitted to match the measured size distribution from the industrial crystallizer. A range of simulations were conducted for various theoretical and empirical models and compared to that of plant measurements. Based on the results it was proposed that the majority of secondary nucleation is expected to occur in the pump and the boiling zone.

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