“It Makes Me Feel Proud Of Who I Am”: Developing Functional Thinking Through Culturally Located Tasks

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Abstract

Success in algebra plays a major role in equity and lifelong opportunities well beyond the mathematics classroom. Nationally, and internationally, high failure rates in algebra see many non-dominant students excluded from equitable higher education, career, and economic opportunities. There appears to be limited research focused on non-dominant students and the development of algebraic thinking through culturally located tasks. This study examines the representations Māori and Pāsifika students use when engaging with contextual functional tasks, and the ways Māori and Pāsifika students generalise culturally located tasks involving functions.

A design based research intervention and qualitative research methods, drawing on a Pāsifika research methodology, were selected as most appropriate for the study. Twelve 10-12 year old Māori and Pāsifika students from a low socio-economic, urban school in New Zealand participated. Students engaged in an intervention of eight lessons focused on developing functional thinking with growing patterns drawn from Māori and Pāsifika cultures. A range of data were collected and analysed, including interviews, field notes, video recorded classroom observations, and photographs of student work.

Findings revealed that when Māori and Pāsifika students were given opportunities to draw on their cultures to make sense of functional relationships, they constructed increasingly sophisticated and abstract representations to identify, communicate, and justify generalisations. There was significant growth in their conceptual understanding of both contextualised and decontextualised growing patterns. Additionally, aligning tasks with non-dominant students' traditions and experiences strengthened students' mathematical and cultural identities.

This study offers a contribution to the literature regarding how culturally contextualised tasks support non-dominant students to engage in early algebra, in particular, the representation and generalisation of functions. To address disparities and structural inequities in mathematics education, educators must acknowledge that students bring their own cultural knowledge and strengths to the classroom, and provide opportunities for all students to learn mathematics in ways they see as relevant to their cultural identities and communities. Recognising that mathematics is inherently cultural is a key lever for equity.

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E kore e mutu ngā mihi aroha ki a koutou.

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Chapter One: Introduction

1.1 Introduction

This chapter provides the background to the study and establishes the context of the research. Section 1.2 highlights mathematical achievement challenges for Māori and Pāsifika students in New Zealand schools, and the need for educators to recognise mathematics as a cultural endeavour if equitable outcomes are to be achieved. Section 1.3 explains the rationale of the study, and section 1.4 presents the specific research questions this study investigates. Finally, an overview of the chapters is provided in section 1.5.

1.2 Background to the study

New Zealand communities, like those around the world, are becoming increasingly culturally diverse. Within New Zealand, Māori and Pāsifika peoples are two of the fastest growing groups compared to the overall population (Statistics New Zealand, 2018). Current projections indicate that Māori and Pāsifika children will make up the majority of New Zealand primary school students by 2040. The implications for the education system are far-reaching, as these changing demographics require a greater capacity to respond to diverse cultures, and optimise mathematical learning opportunities for all students (Alton-Lee, 2003; Hunter & Hunter, 2018; Seah & Andersson, 2015).

In New Zealand, national and international data (e.g. Educational Assessment Research Unit and New Zealand Council for Educational Research, 2018; Ministry of Education, n.d.) show that Māori and Pāsifika students are underachieving in mathematics compared to all other cultural groups. For a number of years, New Zealand governments have sought to reform and improve educational outcomes for Māori and Pāsifika students. Education policies and Ministry of Education initiatives, such as the Ka Hikitea Māori education strategy (Ministry of Education, 2013a) and Pāsifika Education Plan (Ministry of Education, 2013b), promote an equitable education system, focused on raising academic success and reducing disparities in outcomes. However, this vision is yet to become a reality.

Evidence from the research (e.g. Hunter & Hunter, 2018; Louie & Adiredja, 2020; Rubel, 2017) suggests that the persistent underachievement of non-dominant students within mathematics education can be attributed to three interrelated factors: deficit theorising;

prevailing beliefs that mathematics is culture free; and structural inequities within the education system. A common thread between the three components, is that they all problematise non-dominant student's cultures. Deficit theories assign blame for disparities in educational outcomes to students themselves, and perceived deficiencies in their family, community, and cultural backgrounds (Hunter & Hunter, 2018; Louie & Adiredja, 2020; Rubel, 2017). Related to deficit theories are pervasive Eurocentric beliefs that mathematics is culture and value free, and involves universally accepted facts, rules and procedures (Hunter & Hunter, 2018; Louie, 2017; Nasir, 2016). Rather than questioning assumptions about the education system non-dominant students are positioned within – for example, which students and their language, values, knowledge and ways of being are privileged while other groups of students are disadvantaged - the structures of the education system are framed as neutral and normal, and students and their cultures as the problem that needs to be fixed (Bills & Hunter, 2015; Davis, 2018; Louie & Adiredja, 2020).

In contrast to focusing on gaps in achievement, a significant body of research (e.g. Bills & Hunter, 2015; Hunter & Hunter, 2018; Louie, 2017; Wager, 2012) questions the gaps in opportunities for all students to learn mathematics in ways they see as relevant to their cultural practices and cultural identities. These research studies show that privileging non-dominant students' cultural capital in the mathematics classroom has a powerful influence on equitable educational outcomes. When learning is viewed as inherently cultural, non-dominant students' values, languages, traditions, and worldviews, are not deficits to be overcome but assets which can propel students to academic success (Si'ilata et al., 2018).

1.3 Rationale

Over the past two decades the teaching and learning of algebra has been a prominent research area in mathematics education (e.g. Blanton et al, 2015; Kaput, 2017; Rivera & Becker, 2011). Algebra is considered critical to students' mathematical development, and has been labelled as a gateway to academic and professional success (e.g. Blanton et al., 2018; Knuth et al., 2016; Morton & Riegle-Crumb, 2019). However, for many non-dominant students algebra is not a gateway to opportunity, it is a gatekeeper. Nationally, and internationally, high failure rates in algebra exclude non-dominant students from equitable economic, citizenship, higher education and career opportunities - particularly those related to the twenty first century STEM subjects of science, technology and engineering (Hunter & Miller, 2018; Knuth et al., 2016; Morton & Riegle-Crumb, 2019). Research into the teaching and learning of algebraic concepts is important, because success in algebra plays a major role in equity and lifelong opportunities well beyond the mathematics classroom for non-dominant students.

A key aspect of learning to think algebraically is exploring functional thinking, including generalising the relationship between two variables, and representing, justifying, and reasoning with these generalisations (e.g. Blanton et al., 2018; Wilkie, 2016). Teachers commonly use visual growth patterns to develop functional thinking, because growing patterns provide a means for students to focus on the underlying mathematical structure of a pattern and explore generalisation (Beatty, 2014; Markworth, 2012; Miller & Warren, 2012; Wilkie, 2016). However, the types of growing patterns presented and discussed in the mathematics classroom make a significant difference to the ways in which all students are provided with opportunities to develop functional understandings (Beatty, 2014; Markworth, 2012).

The majority of studies into the development of students' functional thinking through growing patterns are with students from majority backgrounds, or, if they feature non-dominant students, patterns are generally decontextualized rather than culturally located (Hunter & Miller, 2018). There appear to be few international studies which have considered how young non-dominant students' engage in functional reasoning, and the role of culture in this process.

Within the New Zealand context both Māori and Pāsifika cultures have a long and rich mathematical heritage with a strong emphasis on geometric patterning used, for example, within craft design and architecture (Finau & Stillman, 1995; Meaney et al., 2013). There is some evidence that drawing on the mathematics embedded within Pāsifika patterning provides a powerful means of supporting young culturally diverse students to develop understanding of growing patterns and engage in functional thinking (Hunter & Miller, 2018). However little is known about Māori and Pāsifika students' representations of growing patterns, and how they move through the stages of mathematical generalisation. The current study aims to extend upon the limited research available regarding how culturally contextualised patterning tasks support non-dominant students to develop their understanding of growing patterns, represent their functional thinking, and engage in the generalisation process.

1.4 Objectives

The purpose of the study is to explore how 10-12 year old Māori and Pāsifika students' draw upon culturally embedded mathematics to develop algebraic understandings and make sense of functional relationships.

In particular, the study addresses the following research questions:

1) What representations do Māori and Pāsifika students use when engaging with contextual functional tasks?

2) How do Māori and Pāsifika students generalise culturally located tasks involving functions?

1.5 Overview

Chapter Two summarises national and international literature that is focussed on the role of culture in the teaching and learning of mathematics. It also defines the role of functional thinking in constructing early algebraic understanding, looking particularly at the research on representing and generalising functions. The literature review describes how culturally contextualised tasks provide opportunities to improve non-dominant students outcomes in mathematics education, as well as strengthening cultural knowledge and understanding.

Chapter Three presents the research design and methods used in the study, and explains the data collection and analysis. The participants and research setting are introduced, and the timeframe for the study is outlined. Considerations regarding ethics, the role of the researcher, and the validity and reliability of the research are also addressed.

Chapter Four integrates the findings, and discussion of the findings, to present the results of the study. The chapter examines the shifts that occurred in how non-dominant students were able to articulate and symbolise generalisations, and use multiple representational forms to support generalisation. The key developments in student thinking are identified and analysed, supported by evidence from the data and the theory discussed in the literature review.

Finally, Chapter Five concludes the research by addressing the research questions, presenting a summary of the key themes, and providing recommendations for educators. It addresses the limitations of the current study, and provides suggestions for areas of future research.

Chapter Two: Literature Review

2.1 Introduction

The previous chapter introduced the background to the study and established the context of the research. This chapter summarises relevant literature that highlights the links between mathematics and culture. This is followed by a review of research studies investigating the development of functional thinking in early algebra. Section 2.2 introduces the sociocultural research framework and discusses mathematics as a cultural endeavour. It draws attention to the disconnect between mathematics education and the cultural worlds of Māori and Pāsifika learners. Section 2.3 examines relevant literature focused on the use of contextualised tasks aligned with non-dominant students' cultural practices. It considers how culturally located tasks offer opportunities to both improve educational outcomes in mathematics, and build cultural knowledge and understanding. Section 2.4 defines the role of functional thinking in developing early algebraic understanding, looking particularly at the literature on generalising and representing functions. This section also outlines difficulties with functions, and the limitations on students' awareness and ability to successfully generalise the functional relationship between two varying quantities, depending on the task. Finally, section 2.5 highlights research investigating growing patterns and the development of algebraic thinking in indigenous cultures.

2.2 Sociocultural framework

The theoretical framework underpinning this research is a sociocultural view of learning mathematics grounded in the work of Vygotsky (1986), who saw the social environment as instrumental to children's learning. A sociocultural framework emphasises relationships and culture as essential facets of learning and development (Nasir & de Royston, 2013; Planas & Valero, 2016). From this perspective, understanding mathematical learning requires a focus on how students participate in mathematical activities with other students, and how they draw on cultural artefacts and practices, to solve problems in a local context (Nasir & Hand, 2006). A central concept of sociocultural theory is that children construct and modify their mathematical capabilities and competencies through the cultural practices they engage in their everyday lives, because they are culturally and socially situated learners (Planas & Valero, 2016). Therefore a sociocultural framework draws our attention to the consideration of relationships between cultural and content knowledge in mathematics and provides a lens to understand and support students using knowledge from their cultural backgrounds to construct new mathematical knowledge and understanding in the classroom (Nasir et al., 2008).

Sociocultural theory highlights that learning is inherently cultural, and mathematics classrooms are cultural and social spaces. Different forms of knowing and being are validated, based on historical, economic, political, and social power structures that serve to perpetuate inequities in both schools and society (Davis, 2018; Nasir et al., 2008; Nieto, 2010). Rather than focusing on gaps in achievement researchers (e.g. Louie, 2017; Wager, 2012) question the gaps in opportunities for all students to learn mathematics in ways they see as relevant to their cultural identities and communities.

2.2.1 Concept of culture

Nieto (2010) defines culture as:

> The ever-changing values, traditions, social and political relationships, and worldview created, shared, and transformed by a group of people bound together by a combination of factors that can include a common history, geographic location, language, social class, and religion (p. 136).

Researchers (e.g. Nasir et al., 2008; Nieto, 2010) regard culture as a social construct because culture cannot exist outside of social interaction and culture is produced, as well as reproduced, between people in local contexts. Closely related to the concept that culture is socially constructed is the concept that culture is learned (Fecho, 2016; Nasir et al., 2014). This differentiates culture from ethnicity, as culture is not a passive legacy passed down through our genes or inherited, but is dynamic and learned through interactions with families, communities and other social groups (Nieto, 2010).

2.2.2 Māori and Pāsifika cultures

Māori are indigenous to New Zealand, and are a dynamic, heterogenous population made up of divergent groups and cultural identities (Greaves et al., 2015). Pāsifika is an umbrella term used to describe a diverse, heterogeneous group of people who originate, or identify in terms of ancestry or heritage, from the Pacific Islands of Samoa, Tonga, Niue, Cook Islands, Tokelau, Tuvalu and Fiji (Coxon et al., 2002; Samu, 2015). Each group is unique in terms of how they identify with particular ways of knowing, being, and viewing the world (Mahuika et al., 2011). There are, however, a set of common values which Māori and Pāsifika peoples share. For example, family and collective responsibility are an integral part of life for both Māori and Pāsifika peoples (Hunter et al., 2019). Other core cultural values include reciprocity, respect, service, inclusion, relationships, spirituality, leadership, love, and belonging (Ministry of Education, 2013b).

2.2.3 Culture and mathematics

Mathematics is traditionally thought of as a discipline that is objective and independent of values and culture (Davis, 2018; d'Entremont, 2015). The perception that mathematics is culture-free, and consists of neutral, universal truths, is perpetuated by assumptions such as mathematical properties remain the same regardless of who is doing the mathematics, for example, five plus five is always 10 (Nasir, 2016; Parker et al., 2017). Research studies (e.g. Hunter & Hunter, 2018; Louie, 2017; Nasir, 2016; Parker et al., 2017) show both students and teachers frequently consider mathematics learning and their cultures as distinct entities. For example, Parker et al. (2017) describe the challenges involved in developing mathematics teachers' cultural responsiveness when the belief that mathematics is culture-free has been normalised within mathematics education. Louie (2017), and Nasir (2016), show that mathematics educators often frame mathematics as a culturally neutral, fixed body of knowledge to be received and focus on developing universal approaches to instruction, rather than culturally specific ones. In the New Zealand context, Hunter and Hunter (2018) report statements from middle years Pāsifika students that indicate a strong belief their cultural background is non-mathematical.

Every culture has mathematics and the mathematics students know, understand, and come to school with is linked to the particular cultural practices and cultural identity of the learner (Nasir et al. 2008; Wager, 2012). Students from non-dominant cultures enter school with rich mathematical experiences, backgrounds, and knowledge (Nasir et al. 2008; Wager, 2012). For example, Barton and Fairhall (1995) identify mathematical aspects of Māori culture implicit in geometric patterns in raranga (weaving), kowhaiwhai (painting) and whakairo (carving). Hunter et al. (2019) provide examples of the mathematics embedded within

Pāsifika homes and communities such as food preparation, making traditional clothing, and patterns in drumming.

However, educators frequently under-utilise or dismiss the cultural assets of diverse students (Aguirre et al., 2017; Wager, 2012). When all children are expected to conform to the dominant groups' practices emphasised in school mathematics, there is a disconnect between diverse students and what they perceive to be another group's mathematics which is alien to their reality (Aguirre et al., 2017; Wager, 2012). Many non-dominant students believe that mathematics has been developed and is owned by a community they are not part of (Barta et al., 2012). Louie (2017) argues that a key product of the dominant culture characterising mathematics education, (i.e. the model of everyday instruction and classroom practices), is exclusion. Students who do not conform to the dominant stereotypes about what people who are good at math are like, typically White or Asian males from economically privileged backgrounds, are excluded from developing positive mathematical identities through narrow definitions of what counts as mathematical activity and mathematical ability. Indigenous students in Matthews et al.'s (2005) study on indigenous perspectives of mathematics education in Australia, perceived mathematics as an assimilation process and a subject where they had to become white in order to succeed. Similarly, students interviewed by Hunter and Hunter (2018), examining practices which have marginalised Pāsifika students in mathematics classrooms, expressed "a belief that to be successful in mathematics you must enter a "white-space" (p. 5). A "white-space" is a pervasive space in the education system representing dominant white values, knowledge, culture and language as the culturally neutral norm, thereby marginalising or alienating non-dominant learners from their own cultural and mathematical identities (Battey & Leyva, 2016; Davis & Martin, 2018; Hunter & Hunter, 2018).

2.3 Contextualisation of tasks

A mathematical task is defined as a set of problems or a single complex problem, that focuses students' attention on a particular mathematical idea (Stein et al., 1996). Mathematical tasks directly determine what learning opportunities are made available to students and are significant in establishing how students come to "view, develop, use, and make sense of mathematics" (Anthony & Walshaw, 2009, p. 13).

Tasks embedded within cultural contexts provide opportunities for non-dominant students to see their experiences reflected in school mathematics, and to recognise that the activities they engage in at home and in their communities involve mathematics which is meaningful and valued (Lipka et al., 2005; Nasir et al., 2008; Wager, 2012). Meaney and Lange (2013) report on students transitioning between home and school contexts. They highlight the potential limitations on learning caused by discontinuities when the two contexts have significant differences in what counts as valued mathematical content and knowledge. These researchers contend that having to mediate between in and out of school knowledge systems results in students more likely to be unable to perform well at school in mathematics topics they master easily at home. On the other hand, drawing on connections between the mathematics culturally diverse students learn in the classroom and the mathematics embedded in their everyday lives, repositions students as having valid cultural capital in their own mathematics classrooms (Hunter & Hunter, 2018; Wager, 2012). Matthews et al. (2005) emphasise that contextualising mathematics has the potential to change the educational environment so that indigenous students believe they can perform well in the education system. Both Leonard et al. (2010), and Martin (2009) undertook research on opportunity and access in mathematics for underrepresented minority students in the USA. They report that when non-dominant students mathematics knowledge is valued alongside school mathematics knowledge students develop their mathematical identity and disposition, and feel empowered as learners and doers of mathematics.

Children create meanings for mathematical concepts when they work within contexts that already have meaning for them. There are rich opportunities for students to develop new mathematical knowledge, and construct progressively more abstract understandings, as they generate formal mathematics from cultural ideas (Lipka et al., 2005; Nasir et al., 2008; Wager, 2012). As part of a long term project on mathematics in cultural contexts, Lipka et al. (2005) undertook a case study in rural, predominantly Yup'ik villages where students simulated building a fish rack structure used to dry salmon. The cultural context enabled rich explorations into how area can change while perimeter stays the same, and students explored properties of a rectangle, developed geometric proof that their bases were in fact rectangles, and investigated how quadrilaterals are related. The students participated in challenging mathematics based on a practical problem that community members of their Yup’ik villages solve every time they build a fish rack.

Researchers (e.g. Beatty & Blair, 2015; Davis & Martin, 2018) report that aligning tasks with non-dominant students' cultural practices not only improves educational outcomes in mathematics, but strengthens diverse students' cultural identities. Davis and Martin's (2018) research focused on teaching practices and assessment that stigmatises African American students. They argue that for mathematics education to be liberatory for African American students, they must receive an education that produces growth not only in their mathematical skills, but builds on their cultural knowledge base. Beatty & Blair (2015) explored mathematical content knowledge based on the Ontario curriculum expectations and the mathematics inherent in indigenous cultural practices through Algonquin loom beading. The research focus was both the students' mathematical thinking and cultural connections. Lessons in the study emphasised the cultural importance of looming, providing an opportunity for students to connect to their own cultural heritage. Community members, educators and students expressed pride in the mathematical thinking the children were demonstrating, such as patterning, multiplicative thinking and spatial reasoning, alongside the development of deeper cultural knowledge and understanding.

2.4 Functional thinking

Functional thinking entails generalising relationships between two or more varying quantities, representing and justifying these relationships in multiple ways (such as words, symbols, tables, or graphs), and reasoning with these generalised representations to understand and predict function behaviour (Blanton et al., 2015; Stephens et al., 2017; Wilkie & Clarke, 2015). Functional thinking provides an important entry point for developing algebraic understanding, and is regarded by many mathematicians as a powerful, unifying strand, because functional thinking is threaded through all of mathematics, is a crucial part of mathematical development, and leads to a deeper understanding of the structural form and generality of mathematics (Blanton et al., 2018; Kaput, 2017; Kieran et al., 2016).

Researchers have found that elementary school children are capable of deeper functional analysis than previously thought, and that functional thinking begins at grades earlier than typically expected (e.g. Blanton & Kaput, 2011; Brizuela et al., 2015; Kieran et al., 2016). Blanton & Kaput's (2011) examination of children's capacity for functional thinking found, for example, the types of representations students use, the ways students organise and track data, and how they express functional relationships, can be scaffolded in instruction beginning with the very youngest students, at the start of formal schooling.

2.4.1 Pathways of student thinking related to functions

Learning trajectories are used in mathematics education to provide a research based representation of the ways student thinking in a particular domain develops over time (Fonger et al., 2018; Sarama & Clements, 2019). The levels of thinking in the progressions of a learning trajectory are not intended to be interpreted as stages students progress through in a linear sequence, but as a depiction of the growing sophistication shown in their reasoning (Stephens et al., 2017).

Several researchers (e.g. Blanton et al., 2015; Markworth, 2010; Stephens et al., 2017; Wilkie, 2014) have developed learning trajectories specifically for functional thinking, describing the typical development of progressions of student thinking in generalising functional relationships. Generalisation is at the core of functions, and involves noticing a commonality in terms of a sequence, and deliberately extending the range of reasoning from specific situations, to more general ideas and conclusions identifying patterns, structures and relationships (Blanton et al., 2017; Kaput, 2017; Wilkie, 2016). Three types of functional thinking are evident from the learning trajectories, and serve as a framework to plan for, interpret, analyse and assess the kinds of functional reasoning found in mathematics classrooms: recursive, covariational, and correspondence thinking.

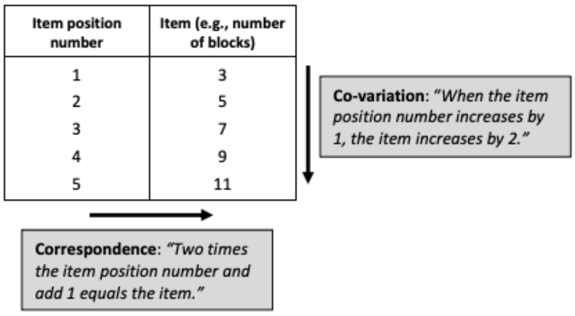
Recursive thinking involves looking for a relationship in a single sequence of values, and indicating how to obtain a number in a sequence given the previous number or numbers (Stephens et al., 2017). When students display recursive thinking, they consider the relationship between successive terms in a pattern by referring to a sequence of distinct, particular instances only, and add the constant from term to term to extend the pattern (Blanton et al., 2015; Miller, 2016; Stephens et al., 2017). For example, Stephens et al. (2017, p.154) asked students to describe patterns they noticed in a task involving a growing pattern of seats and tables being joined for a birthday party. “The number of people is going up by 2s” was evidence of recursive thinking, because the response of adding two each time suggests the student was attending to only one variable, the people, and therefore only the recursive structure of the pattern.

Students engaging in covariational thinking analyse how two quantities are coordinated and vary in relation to each other (Blanton & Kaput, 2011; Blanton et al., 2015; Stephens et al., 2017). For instance, Blanton and Kaput (2011) report on a cutting string task, where children could describe the covarying relationship between the number of cuts on a piece of string and the resulting number of pieces of string when the string is folded in a single loop. Students were not looking for a recursive pattern such as adding two every time, but a relationship between the two variables: "Every time you make one more snip it's two more" (p. 51). A critical marker of this level is that, similar to the recursive level, children could describe a relationship within specific cases but not as a generalised functional relationship over a series of instances (Blanton et al., 2015; Carraher & Scheilman, 2016).

With a correspondence approach the focus is on identifying the correlation between the variables in one set and the related variables in another set (Wilkie & Clarke, 2015). Students must pay attention to the correspondence between the two variables and identify an explicit rule so that they can calculate a variable no matter which term they are looking at (Wilkie, 2014). For example, Wilkie (2014) reports on students giving a rule for an upside-down T plant growing pattern in words: "if you multiply the day number by 3 and add 1 more, you will be able to find the total number of leaves for the plant on any day" and symbols: "'t' is the total number of leaves on the nth day. The rule for the 'upside-down T' plant is t = 3n + 1." (p. 25). A distinction of children's thinking at this level is their focus on the structure of the pattern, their awareness of what constitutes a functional relationship, that is, a correspondence between two variables, and explicitly stating a function rule which describes a generalised relationship (Rivera & Becker, 2011; Stephens et al., 2017; Wilkie, 2014). Figure 1 provides an example of a covariation and correspondence approach to understanding functional relationships in a sequence.

Figure 1

A Covariation and Correspondence Approach for a Growing Pattern (as cited in Wilkie, 2012, p. 4)



2.4.2 Layers of generalisation

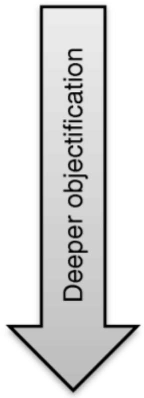
As students' functional reasoning develops, so does their capability to articulate the underlying structure of growing patterns algebraically (Cooper & Warren, 2008; Radford, 2010). Radford (2010) undertook a six-year long program of research with students as they moved from Grade 2 to Grade 6, to determine their transition from non-symbolic to symbolic expressions of algebraic thinking in pattern generalising activities. From his research, Radford (2010) theorised that the ways students express relationships and convey generalisations develops through three levels of algebraic generality—factual, contextual, and symbolic generalisation. As students notice pattern structures as recursive, covariational or correspondence relationships, they express these structures in factual, contextual, or symbolic forms (Warren et al., 2016).

Radford (2010) views the process of generalisation as 'objectification', where students progress from arithmetic thinking to algebraic thinking, and recognise functional relationships in more abstract and analytic ways (see Figure 2). At an early level, factual generalisation is in the form of a concrete rule which allows children to calculate a numerical value for particular instances. For example, "there will be 10 triangles, and since there are three matches for each triangle, there will be 30 matches altogether" (Kanbir et al., 2018, p.100). Contextual generalisation focuses on more descriptive terms such as "you add the next figure from the top row", and is language driven to explain the generalisation (Radford, 2018). The generalisation still refers to material objects in the sequence, but has moved to a new layer of generality where students' attention has shifted from specific numbers to the variables and their relationship (Radford, 2010, 2018; Twohill, 2018). Symbolic generalisation requires a different perspective on the mathematical objects involved, and students use algebraic notation, including letters, symbols or signs, to express the generalisation (Cooper & Warren, 2008; Kieran et al., 2016; Radford, 2010). The focus is now relational, and children represent their thinking symbolically, without reference to specific or situated instances (Twohill, 2018; Wilkie, 2019).

Figure 2

Pattern Generalisation as Objectification, Showing Layers of Generality and Different Types of Thinking (as cited in Wilkie, 2016, p. 338)

Factual generalization *(emerging algebraic thinking – the "raw material")*
– Formation of a way to calculate particular instances of a variable; – Indeterminacy is not enunciated or named but is implicit; it might be expressed in actions (gestures, words, rhythm).
Contextual generalization *(algebraic thinking)*
– Contextual references to variables in the pattern (e.g. mixture of mathematical symbols and terms in natural language); – Indeterminacy is made linguistically explicit.
Symbolic generalization *(algebraic thinking)*
– Generality is expressed in the alphanumeric semiotic system of algebra.



2.4.3 Representations to support reasoning and generalising

Researchers (e.g. Blanton, 2008; Blanton et al., 2018; Cañadas et al., 2016), provide evidence that elementary school children can develop and use a variety of representational tools to help them reason with functions, describe recursive, covarying, and correspondence relationships, symbolise relationships, and express generalisations.

Individually, students use representations as a tool to make sense of ideas and explore the problem in their own way. Socially, representations are a way for children to communicate their reasoning with others as they represent, explain and justify generalised relationships in diverse ways (Blanton, 2008; Stephens et al., 2017; Tripathi, 2008). Investigating student-created representations also gives educators and researchers insights into students' functional thinking and emerging conceptual understandings (Blanton, 2008; Stephens et al., 2017; Tripathi, 2008).

Representation is a dynamic process, and students move through different phases in their choice of representation, showing various levels of complexity in the ways they represent the relationship between two terms in a growing pattern (Blanton et al., 2015; Bobis & Way, 2018; Stephens et al., 2017). Blanton (2008) discusses representation as referring to both the process and the product of expressing an idea. For instance, children may represent their thinking with pictures or tables showing the process of their developing awareness of the functional relationship. As students develop their understanding of the relationship between the variables they represent a generalisation of the relationship - the product of their thinking - using representations such as words or symbols. Blanton (2008) describes children's representations transitioning from direct modelling, to mathematising, to mathematical understanding, as children's representations develop from the more concrete to more abstract.

2.4.4 Forms of representation

Different forms of representation can be helpful to support the development of algebraic reasoning. These representations include concrete, verbal, numerical, graphical, contextual, pictorial, or symbolic components (Tripathi, 2008).

Through drawing, students may identify the underlying functional relationship between two variables, and express a generalisation. Cañadas et al. (2016) report on children using drawing to make the specifics of a problem explicit, for example, students drew desks and people to represent the context of a growing pattern. They used both drawings and natural language to express and justify the relationships between the two varying quantities: "I draw desks first and then I draw the people and then I counted by 2's." (p. 95). Conversely, Moss and McNab (2011) designed a teaching intervention to support second grade students understanding of linear functions, through geometric and numeric representations of growing patterns. All of the students had worked with repeating patterns, but none with growing patterns. Results showed that when students used visual representations of geometric and numeric patterns they were able to notice the constant in visual arrays, represent generalisations in their natural language, and identify and express two part rules for growing patterns (e.g. $y=ax+b$). Moss and McNab (2011) concluded that when visual representations were prioritised students were better able to find, express, and justify functional rules.

Using concrete materials to manipulate both variables in a growing pattern gives students the opportunity to model the growing pattern, examine the pattern structure and construct generalisations (Cooper & Warren, 2008). Twohill (2018) investigated the strategies that nine and ten year old students used when constructing general terms for shape patterns. Prior to constructing the pattern terms of tiles, two students tended strongly towards recursive thinking with comments such as “each time you’re adding two”. After using materials, one student identified the pattern as a top and bottom row containing $n + 1$, and n, tiles respectively. The other student identified the terms as containing “*t* diagonal pairs, with an additional tile on the top right corner” (p. 224).

A common representation for solving functions problems is creating a t-chart, or function table, where students make two columns of data and record corresponding entries for the independent and dependent variables (Blanton & Kaput, 2011). Blanton et al. (2015) report on children in the early grades initially creating function tables as a means to organise covarying data, but by Grade 2, they were beginning to use tables as a tool for thinking about the data and reflecting on the relationships and patterns they could see in the table. Similarly Blanton and Kaput (2011), discuss the shifts from students in the early grades using t-charts as a place to record numbers, to students in the middle grades using t-charts as a tool that can be used to determine relationships in data and an important structure in mathematical reasoning. For instance, a student in a task where students were to find the number of body parts a growing snake would have on day ten and day n stated: “I know that on day 10 the snake will have 101 body parts and I know that on day n the snake will have n x n +1. I know this because I used my t-chart and I looked for the relationship between n and body parts” (p. 11). This suggests that the t-chart’s structure helped the student use it as a tool to compare data and find relationships.

As students are plotting a graph they can directly see how relations change, rather than, for example, reading a table showing a limited number of discrete input and output values. Caddle and Brizuela’s (2011) exploration of fifth grade students discussing linear graphs found that using a graph prompted different reasoning than other representations, and gave a more complete view of the function. Similarly, Brizuela and Earnest (2017) examined how young, multi-ethnic, students worked with multiple representations to choose the best deal between a grandmother either doubling a child’s money, or tripling his money and then taking away $7. Brizuela and Earnest (2017) found students questioned their understanding about the nature of the relationship between the variables as they constructed a graph and interpreted the best deal, and the functions became more explicit in a graph than with other representations.

Students' natural language descriptions are another form of generalising and representing mathematical concepts. Cañadas et al. (2016) investigated second grade students articulating their ideas about functional relationships. They found the use of natural language is helpful for students in the early grades because of their familiarity with it, and students used natural language to express relationships as a recursive pattern (counting by twos) or a functional relationship (doubling). Interestingly, Stephens et al. (2017) and Blanton et al. (2015) both unexpectedly found that students were generally more successful representing generalisations using variable notation than using words. For example, in Stephens et al. (2017) 66% of students provided a correct function rule for a growing stars pattern in variables: $x \cdot x = y$, and only 32% in words: "The picture number times itself equals the number of stars" (p. 157). This clearly challenges the notion that variables as a varying quantity should not be introduced until secondary school.

Researchers (e.g. Blanton et al., 2015; Brizuela et al., 2015; Stephens et al., 2017; Wilkie, 2014) underline the importance of students using letters or symbols to represent variables and generalise functional relationships through explicit rules. Brizuela et al. (2015) provides evidence that children as young as six can use variable notation in meaningful ways to express relationships between quantities and represent generalisations. They argue that allowing variable notation to become part of mathematical language as children represent their algebraic ideas gives them opportunities to develop a deep and powerful means of representing generalised relationships. Wilkie (2014) reports on upper primary students being keen to explore and experiment with letters to represent variables in algebraic equations. This tended to flow naturally from students' own attempts to create a number sentence for their rule which often contained a mixture of numbers and words, for example: "The answer is the day × 3 + 1" (p. 26). Both Wilkie (2014) and Brizuela et al. (2015), consider students should be given opportunities to use variables and variable notation early in their formal schooling, in order to give children multiple opportunities to fully explore and develop their understandings over time.

Researchers (e.g. Cañadas et al., 2016; Neilsen & Bostic, 2018; Stephens et al., 2017; Tripathi, 2008) provide evidence of the benefits of using multiple representations in order to develop a deeper, richer and more flexible understanding of functional relationships. When children have multiple ways to represent an idea they can choose representations that are intrinsically meaningful to them (Blanton, 2008; Daryaee et al., 2018). Using multiple representations enables students to develop a deeper understanding of underlying mathematical structures, because different mathematical representations highlight different aspects of mathematical relationships (Neilsen & Bostic, 2018; Stephens et al., 2017; Tripathi, 2008). Each representation provides a different way for students to examine and compare the relationships, and students learn about and deepen their understanding of functions when they are explored while making connections across diverse representations (Carraher & Scheilman, 2016; Daryaee et al., 2018).

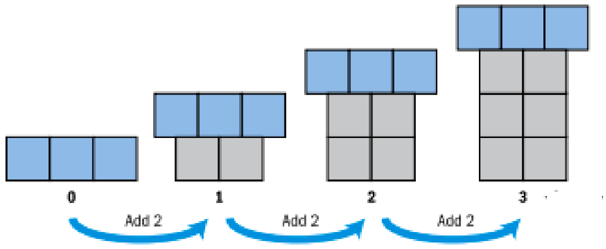
2.4.5 Difficulties with functions: from patterns to generalisation

Although young students are capable of sophisticated functional thinking, there are a number of common difficulties related to the development of functional thinking reported in the research literature.

Students often have difficulty transitioning from looking at the relationship between successive terms in a pattern as recursive, to focusing on the relationship between variables and viewing patterns as functions (e.g. Blanton et al., 2015; Isler et al., 2015; Stephens et al., 2017; Wilkie, 2014). Research identifies numerical and visual geometric growing patterns as a common vehicle for supporting the development of functional thinking (Blanton & Kaput, 2011; Cooper & Warren, 2011). However, the growing patterns typically used, and the ways in which the pattern structures are presented, can limit students' awareness and ability to successfully generalise the functional relationship between variables and across instances (Beatty, 2014; Miller, 2016). Beatty (2014) shows when students work with growing patterns (such as Figure 3), they are able to describe the pattern and extend it to the next position based on additive reasoning, but have difficulty predicting values for terms further down the sequence (i.e., the 10th position, 25th position, 100th position, *n*th position).

Figure 3

A Typical Growing Pattern and Recursive Approach to Finding the Next Position (as cited in Beatty, 2014, p. 1)



Many students over-generalise the applicability of proportional methods to growing pattern contexts, believing that proportional reasoning can be applied to all linear relations because they increase (or decrease) at a constant rate (e.g. Ayalon & Wilkie, 2019; Lannin, 2003). Lannin (2003) provides an example of joined cubes in a row and “smiley” stickers on rods of ten cubes. A student claimed that because a rod of ten cubes has 42 stickers, you could multiply 42 by two for a rod length of 20. The student was unaware that when she used this method she was overcounting the extra stickers where the two rods joined. In this case the student was focused on the numeric relationships (examining the increase in the number of stickers in a table), without connecting her reasoning to the context and considering the functional relationship between the variables.

Moving beyond particular cases and expressing generality can create a challenge for students. Lannin’s (2005) research shows students relying on empirical evidence to support general statements. This difficulty appears to be due to the traditional focus in mathematics on calculating only a particular instance of a situation, rather than determining a general relation. Lannin (2005) describes students using guess-and-check strategies, and experimenting with various operations and numbers provided in the problem situation, to construct a generalisation. This trial and error approach led to students guessing a rule without considering why the rule might work, and attempting to find a formula to fit a particular instance of the pattern. Lannin (2005) emphasises the importance of students constructing and justifying a general relation in the problem context, and developing arguments which are independent of particular instances.

The role of variable as a varying quantity is an essential tool to develop and express functional relationships, but is one with which students may struggle (e.g. Blanton et al., 2017; Isler et al., 2015). Students' typical difficulties with variables include believing that variables stand for a fixed unknown quantity, label, or attribute (e.g., l stands for leaves, d stands for day, or *x* is always three), rather than as a symbol that can stand for any real number in a functional relationship (Blanton et al., 2015; Blanton et al., 2017; Isler et al., 2015; Wilkie, 2014). Other conceptions regarding variables include thinking that two different variables (e.g. x, y) in the same equation cannot represent the same value, and believing that the value of a variable has something to do with its position in the alphabet (Brizuela et al., 2015; Wilkie, 2014).

2.5 Growing patterns and indigenous cultures

The majority of studies into the development of students' functional thinking are with students from majority backgrounds, or if they feature indigenous students then tasks are generally decontextualized or shared context rather than drawing on culturally located tasks (Hunter & Miller, 2018). It appears that there is limited research that focuses on indigenous students and the development of algebraic thinking through engaging with their culture within the mathematics curriculum.

Miller's (2014) study with Year 2 and 3 students in an urban indigenous school in North Queensland investigated the role of culture in young indigenous students mathematical generalisation of growing patterns. Results indicated that the type and context of the pattern impacts on indigenous students' abilities to access the structure and relationship between the variables. Young Australian indigenous students were more successful extending and generalising growing patterns that came from the natural environment (e.g. identifying the relationship between possum tails and eyes) than they were extending and generalising growing patterns represented by decontextualized geometric shapes. Miller selected growing patterns for tasks deliberately to ensure that the functional relationship was transparent and explicitly represented. This was achieved by using materials where the variables were explicit (i.e. pattern term cards and coloured tiles) or could not be physically separated (i.e. plastic toy kangaroos and crocodiles). Miller (2016) conjectured that representing growing patterns in this manner assisted students to attend to both variables in the pattern, and potentially pushed them towards functional thinking rather than recursive thinking.

Further evidence demonstrates the importance of educators and researchers understanding students' cultural representations, such as gesture in indigenous contexts, and providing opportunities for students to engage in and express their own mathematical understandings in culturally appropriate ways. Miller's (2014) research explored how young indigenous students use gesture to generalise growing patterns, and brings attention to the importance of cultural interpretations of gesture and actions within the classroom. Miller provides two case studies of students using gesture to support their explanations, articulate their understanding of the structure of growing patterns, and generalise the rule of growing patterns. Miller argues that these students are demonstrating contextual and symbolic generalisation within Radford's (2010) three layers of generality, and these young indigenous students are using gesture to represent generalisations of geometric growing patterns, rather than alphanumeric symbolism. If these cultural signs are missed or misinterpreted the true understanding of students' knowledge will potentially be unseen.

In the New Zealand context there is some evidence that drawing upon the mathematics implicit in Pāsifika and Māori patterning provides a powerful means of developing culturally diverse students' early algebraic reasoning and understanding of functional patterns (Hunter & Miller, 2018). Hunter and Miller's (2018) research concentrates on the use of culturally located patterns from Pāsifika and Māori cultures to develop young culturally diverse students' understanding of functional patterns and support generalisation. Hunter and Miller focused on a pattern from a Cook Island tivaevae (a communally sewn traditional quilt) which grew in multiple directions. Their evidence showed that when mathematics is embedded in a cultural context, in this case the structure of a tivaevae pattern, young culturally diverse students are able to make a meaningful connection to the mathematics, begin to see covariation, develop their understanding of growing patterns, and articulate generalisations as they see the structure of the pattern growing in multiple ways.

2.6 Summary

Despite mathematics being positioned as a value and culture free subject area, researchers (e.g. Hunter & Hunter, 2018; Nasir et al. 2008; Wager, 2012) have shown that mathematics is a cultural product, and is closely tied to the cultural identity of the learner. As this literature review depicts, setting mathematical tasks in contexts centred on non-dominant students traditions, experiences, and cultures, gives more equitable opportunities to participate in, and develop, higher level mathematical thinking (Anthony & Walshaw, 2009; Hunter & Miller, 2018). All children can be successful in mathematics when their understanding is linked to meaningful cultural referents (Ladson-Billings, 1997).

Evidence from research studies highlights that young children are capable of sophisticated sense making of functional relationships, and generalising and representing these relationships in diverse ways (Blanton et al., 2015). However, in contrast to the research, Māori and Pāsifika students are well behind their peers in national and international measures of mathematics achievement (e.g. Educational Assessment Research Unit and New Zealand Council for Educational Research, 2018; Ministry of Education, n.d.). There appear to have been limited studies which have investigated young non-dominant students' understanding of growing patterns, and little is known about Māori and Pāsifika students' representations of growing patterns, and how they move through the stages of mathematical generalisation.

The objective of this study is to provide insight into the representations Māori and Pāsifika students use when engaging with contextual functional tasks, and how Māori and Pāsifika students generalise culturally located tasks involving functions.

Chapter Three: Methodology

3.1 Introduction

The previous chapter discussed the literature related to the current study. This chapter outlines the research design and methods used in the study. Section 3.2 provides a justification for the selection of design based research and qualitative methods used. This section also details the Ula model, and how the current study aligns with this Pāsifika theoretical framework. Section 3.3 describes the role of the researcher, and data collection methods are explained in section 3.4. The participants and research setting are introduced in section 3.5. Section 3.6 outlines the research project and the instructional sequence that forms the basis of the study. Section 3.7 describes the data analysis, and discusses the validity and reliability of the findings of the research. Finally, section 3.8 elaborates ethical considerations.

3.2 Justification for methodology

The choice of methodology was influenced by the aim of this study. This was to provide insight into how Māori and Pāsifika students generalise culturally located tasks involving functions, and the representations Māori and Pāsifika students use when engaging with contextual functional tasks.

Design based research is a prominent methodology in mathematics education research, and is appropriate for developing research based solutions to complex problems in educational practice (Plomp, 2013; Prediger et al., 2015). Several key features define design based research, namely "it is interventionist, theory driven, context-specific, collaborative and contains a … focus on local impact and theory generation" (Crippen & Brown, 2018, p. 490). Design based research was selected as the most appropriate methodology for the current study for the reasons outlined in the following paragraph.

Firstly, design based research aligns with the sociocultural perspective that underpins the current study. Both design based research and sociocultural theories emphasise the social aspects of learning, such as collaboration, the active construction of knowledge, and the integration of cultural experiences into the learning process (Prediger et al., 2015; Steffe & Thompson, 2000). Secondly, design based research is well suited to studying real-life issues in naturalistic environments (Anderson & Shattuck, 2012). The current study is classroom based because there are complexities and social interactions in natural settings that would not be present if the study took place out of context (Anderson & Shattuck, 2012). Thirdly, a goal of design based research is to investigate the possibilities for educational improvement by supporting and studying the development of new or different forms of practice (Cobb et al., 2015). The goal of the current study is to understand how students develop the generalisation and representation of functional thinking through culturally located tasks. An interventionist methodology such as design research that aims to bring about the intended developments in order to study them is therefore appropriate.

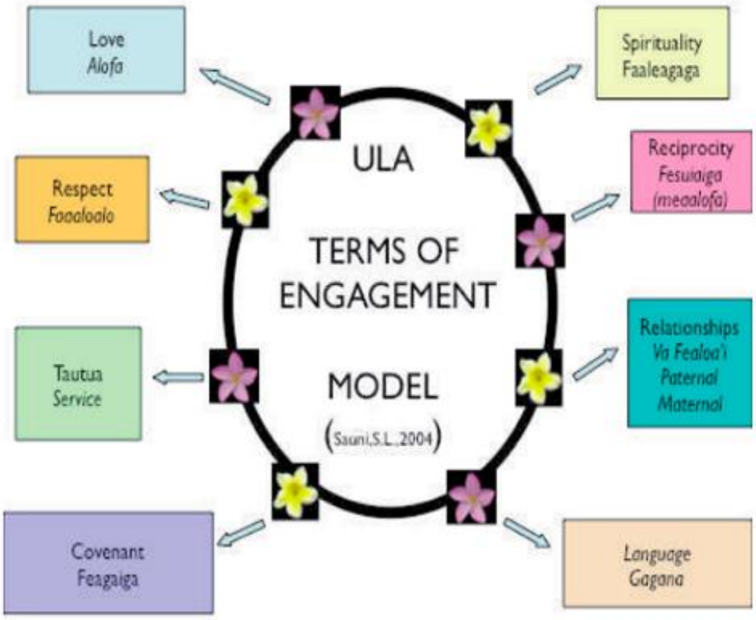
Design based research is descriptive and explanatory by nature, so design based researchers often make use of qualitative methods to study learning in the design intervention (Reimann, 2016). The essence of nearly all qualitative studies is a focus on people, and narrative information about their perceptions, actions, beliefs or behaviours (Merriam & Tisdall, 2016; Yin, 2016). A qualitative approach to data collection and analysis was appropriate for this study, in order to provide an insight into Māori and Pāsifika students lived experiences in an upper primary school mathematics classroom. A qualitative approach allowed the researcher to draw on student voices which not only provided insights into participants points of view, but also privileged the voices and experiences of Māori and Pāsifika students.

3.2.1 Pāsifika framework

In order to reflect a culturally appropriate theoretical framework for the research a Pāsifika research methodology, the Ula model, was drawn upon (Sauni, 2011). The Ula model (see Figure 4) is a metaphor for collaborative engagement based on fa'asamoa: Samoan cultural principles, values and beliefs (Sauni, 2011).

Figure 4

The Ula Model for Collaborative Engagement (as cited in Sauni, 2011, p. 57)



The circle of flowers represents the use of cultural values throughout the research study (Sauni, 2011). The space inside the ula represents the va (the space between), which is not an empty space but holds the ula together through relationships (Anae, 2010; Sauni, 2011).

The Ula model also supports the current study to be responsive to Māori participants through integrating core cultural values such as whanaungatanga (whanau or family type relationships), manākitanga (caring) and kotahitanga (unity). To teu le va - cherish, nurture and care for the relational spaces - is a way of affirming the importance of relationships, and engaging with both Māori and Pāsifika participants within the same research project (Naepi, 2015).

In the current study building trusting relationships, and creating a comfortable atmosphere for the participants and the researcher to engage in meaningful dialogue, were prioritised. Reciprocity was encouraged by the researcher sharing their personal identity with participants, listening carefully and respectfully, and utilising a strength based approach – participants were viewed as experts when sharing their experiences, views and understandings. Being open and approachable, and sharing a sense of humour when appropriate, also helped students feel at ease during interviews and classroom observations (Sauni, 2011; Vaioleti, 2006).

3.3 Researcher Role

In qualitative research studies the researcher is "the primary instrument for data collection and analysis" (Merriam & Tisdell, 2015, p. 15). Accordingly, the researcher in the current study was the sole collector of data. This enabled the researcher to maximise the efficiency of data collection and the quality of the data collected (Merriam & Tisdell, 2015). For example, immediately responding to and adapting questions in an interview, or checking with participants to validate the researcher's interpretation of events.

During classroom observations the researcher's role was observer as participant, primarily observing and gathering information, but having some level of interaction with participants (Merriam & Tisdell, 2015; Mertler, 2019). This meant the researcher sat in with the class and participated in small ways, as a means for generating a more complete understanding of the groups' activities, but was not integrally involved in the lesson.

The relationship between the researcher and the teacher was professional and collaborative. Unlike traditional studies in education that pose the researcher as expert, design based research and the Ula model allow for a reciprocal relationship as the teacher and researcher share knowledge and resources (Jung & Brady, 2015; Sauni, 2011). During the research lessons the teacher had primary responsibility for teaching the lesson, but the researcher was available to assume a tuakana (older sibling) role if required, and provide support, advice, or further information in the moment. For example, helping the teacher decide how she might select and sequence students solution strategies in order to highlight important mathematical ideas.

Participants understood the researcher's role as an observer and collector of data, and data was openly collected (Merriam & Tisdell, 2015). The students knew the researcher as a mentor in mathematics, and were used to the researcher working in classrooms supporting teachers for pedagogical change in mathematics. This meant that participants felt comfortable when observations took place. However, participants who know they are being observed can behave in ways differently than they normally would (Merriam & Tisdell, 2015; Yin, 2016).

For example, the participants in the current study may have tried to present themselves in a favourable manner or regulate their behaviour in reaction to the presence of the researcher (Merriam & Tisdell, 2015).

The researcher's experiences in using an inquiry approach to teach mathematics with Māori and Pāsifika students meant familiarity with expected classroom practices and potential outcomes of the lessons. However, this strength can also be seen as a weakness, due to bias or over-familiarity with the research context leading to critical data being missed (Merriam & Tisdell, 2015). When the researcher is the primary instrument for data collection and analysis, it is important to be aware of biases and assumptions that might impact the study, and monitor how they may be shaping the collection and interpretation of data (Merriam & Tisdell, 2015).

3.4 Data Collection

Collecting data from different sources in the learning environment is consistent with both design based and qualitative research methodologies (Merriam & Tisdell, 2015; Prediger et al., 2015). Using multiple methods for data collection produces rich and detailed information, and enhances the validity of findings (Merriam & Tisdell, 2015).

Data collection tools utilised in the current study were interviews, field notes, video recorded classroom observations, and photographs of student work.

3.4.1 Interviews

Interviewing is a valuable way of gaining in-depth information related to participants' experiences, perspectives, and constructions of reality (Flick, 2018; Seldman, 2019). Interviews are used to find out from people things which cannot be directly observed, such as thoughts, feelings and intentions (Merriam & Tisdell, 2015).

Conducting interviews in culturally appropriate ways is essential within Māori and Pāsifika contexts. Prior to interviews in the current project participants were given a brief explanation about the purpose of the study, the conduct of interviews, confidentiality, and consent. During interviews a talanoa (conversation) format was employed (Vaioleti, 2006). The interview questions were asked in a conversational rather than inquisitorial manner, allowing respectful, reciprocal interactions. Each interview lasted approximately 15 minutes and took place in a quiet breakout space within the participants' classroom. All interviews were video recorded and wholly transcribed for coding and analysis.

Two types of interviews were used in the current study: semi-structured individual interviews and task-based interviews.

3.4.1.1 Semi-structured interviews. Semi-structured interviews were undertaken before and after the intervention (see Appendix A1). Semi-structured interviews sit in the middle of a continuum between structured and unstructured interviews (Merriam & Tisdell, 2015). The questions in semi-structured interviews are open-ended and flexibly worded (Merriam & Tisdell, 2015). In the current study, interview questions exploring participant's perceptions of connections between their cultural identities and mathematics were developed from the literature review. The interview protocol provided a framework of ideas to investigate with participants, while also leaving room for participants to share their experiences beyond the expectations of the researcher. Semi-structured interviews provided space for reciprocity, as the researcher could respond to participant's ideas in the moment, and participants could elaborate on the points of each question that were meaningful to them.

3.4.1.2 Task based interviews. Task-based interviews have been used by researchers in qualitative research in mathematics education to gain insights into participants developing knowledge, and ways of explaining, reasoning, justifying and representing mathematical ideas (Assad, 2015; Pepper et al., 2018). Task based interviews were utilised for the assessment tasks, and additionally, subsequent to lessons in the teaching sequence (see Appendix A2). During task-based interviews participants were asked to think aloud while reflecting on a previously completed task, in order to gain an in-depth perspective into shifts in student thinking which might not have been obtained from observations in the classroom setting. A semi-structured interview protocol allowed for open-ended prompting if required, depending on the judgement of the researcher in response to participants descriptions.

3.4.2 Observations

Observations are important in qualitative research because they provide researchers with a first-hand account of the activity being studied while it is occurring (Cohen et al., 2018;

Merriam & Tisdell, 2015). Researchers have the opportunity to notice behaviours that might otherwise be taken for granted or go unnoticed by participants, which leads to richer, and more valid and authentic data (Cohen et al., 2018; Merriam & Tisdell, 2015).

As with other forms of data collection, observational data must enable the research questions to be answered (Cohen et al., 2018). In the current study eight semi-structured observations were made. Semi-structured observations are conducted when the researcher has considered what will be observed and recorded beforehand, and developed loose categories of data to gather (Cohen et al., 2018). Key themes that emerged from the literature review served as prompts to guide the observations (see Appendix B). This meant the researcher had a lens for the observations, but could also adapt the focus in the moment based on what emerged during the course of the lesson (Punch & Omacea, 2014).

Observations were recorded in the form of field notes and video recordings. Field notes were written up as soon after the observation as possible in order to add details and provide a summary of what had been observed. Alongside factual descriptions of what happened, the field notes had a reflective component. Reflective comments were based on, for example, initial interpretations, and tentative themes and ideas that emerged during the lesson. In this respect the researcher was engaging in preliminary data analysis alongside data collection, and could note things to ask, observe, or look for in the next round of data collection (Merriam & Tisdell, 2015).

3.4.3 Video recording of lessons

Video recording is a qualitative research method that captures complexities of social activity, which is not possible through observation alone (Cohen et al., 2018; Wang & Lien, 2013). Three features of video-recording underpin it’s strengths in qualitative research: video recording provides a real-time sequential record; captures authentic behaviours in their situational context; and is a permanent record that can be viewed multiple times (Cohen et al., 2018; Wang & Lien, 2013).

In order to gather a rich picture of classroom learning, and to provide a broad scope for data collection, all lessons in the current study were video recorded. Two video cameras were used during observations. While participants worked on the problem in small groups, a video camera was focused on two groups. The students included in these groups varied in different lessons. The purpose was to investigate the ways participants were engaging with the task, building conceptual understanding, and reasoning about the structure of the patterns and relationships. When the teacher was facilitating a larger group discussion there was one camera recording the teacher and participants, in order to document the collaborative discourse and shifts in thinking that were occurring.

Video recording the lessons allowed many details to be captured which would otherwise have been missed (Basil, 2011; Wang & Lien, 2013). For example, video recording captured non-verbal cues, reactions and gestures when participants were working in their small groups. The video records allowed the data to be analysed more than once, which helped to overcome the "fleeting nature" of observation (Basil, 2011, p.251). This allowed for reflection on what had been observed, and the possibility of reinterpretation, because the researcher wasn't restricted by memory of the lesson (Wang & Lien, 2013).

All video recordings were downloaded at the conclusion of the lesson, and wholly transcribed.

3.4.4 Student work

Digital photographs of student's responses to tasks were taken during lessons, as a means of capturing participants varied forms of representation and development of generalisations. This provided concrete evidence of how students were engaging with the pattern structures and the types of representations students were using. Names and faces were not photographed to assure anonymity.

3.5 Participants and research setting

The research was conducted with a group of twelve students from one Year Six to Eight class (10-12 year olds), in a low socio-economic, high poverty, urban school in New Zealand. Ethnicities of the twelve students were Māori (40%), and Pāsifika (60%). The study participants were selected from a larger group of possible participants by purposive sampling. Qualitative research methods are likely to produce large amounts of data, and therefore the number of participants was limited to keep the study manageable (Merriam & Tisdell, 2015; Punch & Omacea, 2014). The teacher and the researcher selected the group of twelve based on who was likely to be available and willing to participate, and able to clearly communicate their experiences and opinions (Merriam & Tisdell, 2015; Punch & Omacea, 2014).

The school was selected because they are part of a longitudinal mathematical research project called Developing Mathematical Inquiry Communities (DMIC), designed to address the persistent underachievement of Māori and Pāsifika students in mathematics (Hunter & Hunter, 2018). The teacher was an experienced educator who has long-term involvement with DMIC professional development, and core Māori and Pāsifika values underpin her pedagogy and classroom practices. She regarded her active participation in this research as professional development and a means to reflectively inform her instructional practice.

3.6 The research study schedule

Phase one

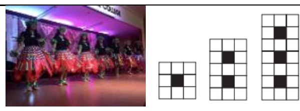
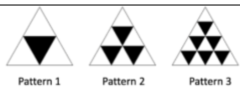
The first phase of the study began with individual semi-structured interviews with participants and the teacher. Additionally, a pre-intervention assessment was administered to ascertain what students already knew about representing and generalising functional relationships. Participants showed their algebraic thinking through parallel assessment tasks, one contextualised and one decontextualised, representing the same function. Questions were intended to ascertain students' understanding of growing patterns, including their ability to predict further positions in the pattern and describe, in general terms, the relationship between the pattern and its position. The assessment tasks mirrored the types of tasks and discussions that occurred during the teaching phase, and aligned with Level 4 learning outcomes of the New Zealand Curriculum (see Appendix C).

The teacher and the researcher analysed participant responses to the assessment tasks, and collaboratively built an understanding of the progressions of thinking that could be seen, and the shifts in thinking to be developed through the intervention. Data from the pre-intervention assessment and interviews, evidence from the academic research discussed in the literature review, and the teacher and researcher's personal and professional experience, were used to construct a series of contextual tasks (see Appendix D1-8). The tasks were designed to build on current student understandings, and increase in complexity over a sequence of lessons.

The mathematics in all the tasks was embedded in a cultural context relevant to the students' backgrounds. Patterns were drawn from Māori and Pāsifika cultures and chosen both for their context, and the ways they lent themselves to multiple ways of seeing and representing the pattern's growth. For example, it was likely that some students would see the pattern growing recursively, while others would view the pattern in a way that lent itself to finding an explicit rule (Markworth, 2010). Table 1 provides an overview of the cultural context, pattern and function tasks used in the teaching intervention.

Table 1

Cultural Context, Pattern and Function Type of Tasks Used in the Lesson Sequence

	Context	Pattern (visual)	Possible function type
Task 1	Sāsā		$y = 3x + 1$
Task 2	Ngatu		$y = 8x + 4$
Task 3	Vaka		$y = 6x - 1$
Task 4	Titi		$y = 6x + 3$
Task 5	Tukutuku panel		$y = 4x - 6$
Task 6	Tivaevae		$y = 24x + 4$
Task 7	Fala		$y = x^2 + x + 4$
Task 8	Kapa haka		$y = \frac{x(x+1)}{2}$

After the tasks were drafted, a subset of the tasks were piloted in a class with similar aged students at the same school. The teacher and students participating in the pilot work were not participants in the study. The pilot work provided the researcher with an understanding of how students made sense of the task components and helped with testing the appropriateness of the tasks, for example, the language used, the mathematical content, and the cultural context. The results of the pilot work were discussed with the teacher.

Phase two

The second phase of the project involved teaching the series of problems. The instructional sequence that formed the basis of the research was comprised of eight tasks taught over eight one-hour lessons. The teaching intervention took place over a five-week period in Term 4 (October / November). The number and length of lessons was intended to give participants enough opportunities to engage with the concepts, and to gain traction toward algebraic understanding.

Each lesson followed the same structure. First, the teacher launched the problem in a whole group setting, then participants worked in small groups to discuss the problem, look for relationships, and represent the relationships in multiple ways including words, symbols, or variable notation. After participants had enough time to explore the problem in small groups, the teacher would facilitate a larger group discussion of students' approaches, connecting students thinking to the big mathematical idea which underpinned the lesson.

The collaborative and iterative nature of design-based research allowed the teacher and the researcher repeated opportunities to discuss and reflect on the students' responses to each task in the sequence. After each lesson the teacher and the researcher reviewed the lesson, analysed classroom events and student responses, and made any modifications required for the next lesson based on, for example, what was noteworthy about how children were talking about or representing the functional relationship.

Additional data collected at this stage included audio recordings of the collaborative planning meetings and field notes regarding reflective discussions between the teacher and researcher.

Phase three

The final phase of the data collection consisted of participants completing a post-intervention assessment, and a concluding semi-structured interview.

Table 2 summarises the research activities and data gathering strategies implemented during each phase of the study.

Table 2

Summary of Research Activities and Data Gathering Strategies Implemented During Each Phase of the Current Study

	Research activity	Data gathering strategy
Phase One	Individual semi-structured interviews Two pre-intervention assessment tasks Collaborative planning of teaching unit and series of tasks with teacher Pilot tasks	Semi-structured interviews video recorded and transcribed Think-aloud video recorded and transcribed Photographs of student work
Phase Two	Algebra intervention (8 lessons) Task based interviews with participants Reflective discussion and collaborative planning after each lesson	Semi-structured observations Obervational field notes: descriptive notes and reflective comments Group work video recorded and transcribed Large group discussion video recorded and transcribed Photographs of student work Task based interviews video recorded and transcribed Audio recordings Researcher notes
Phase Three	Post-assessment tasks Final student semi-structured interviews	Think-aloud video recorded and transcribed Photographs of student work Semi-structured interviews video recorded and transcribed

3.7 Data Analysis

Data analysis is the process of making meaning out of the data (Merriam & Tisdell, 2015). Analysing the data in this study meant making sense of the nature of students' generalisations and representations, when patterns and relationships they know from their cultural experiences were the basis for functional thinking.

In both qualitative and design based research, two phases of data analysis are carried out: ongoing and retrospective analysis (Merrian & Tisdell, 2015).

In the current study ongoing analysis was an interactive and iterative process throughout the data collection phase. For instance, analysis of the pre-intervention assessment informed design of the tasks. After lessons, the researcher and teacher's collaborative analysis informed the teaching of the following task, and the next stage of data collection. This analysis continued in a cycle throughout the series of tasks. Additionally, data was collected and analysed concurrently. Transcribing interviews and video-footage throughout the lesson sequence allowed the researcher to highlight items of potential interest and begin to informally identify themes from the data.

Once all the data from assessments, transcripts from interviews and video observations, field notes, and student work were collected, retrospective and formal analysis began.

3.7.1 Coding and developing themes

Thematic analysis was the method used for "systematically identifying, organizing, and offering insight into patterns of meaning" across the data set (Braun & Clarke, 2012, p. 57). Thematic analysis is a flexible method of qualitative data analysis and allows the researcher to derive meaning and understanding from the data, in order to answer the research questions being addressed (Braun & Clarke, 2012; Merriam & Tisdell, 2015).

The systematic analysis of the data began with coding. Initially the data were coded with respect to functional thinking, Māori and Pāsifika cultures, and culturally located tasks. Codes were a combination of descriptive and interpretative. In the next phase analysis shifted from codes to themes in a manner consistent with the thematic analysis proposed by Braun and Clarke (2012). The basic process of generating themes involved gathering the codes into possible themes, then "collapsing or clustering codes that shared some unifying feature together, so that they reflected and described a coherent and meaningful pattern in the data" (Braun & Clarke, 2012, p. 63). The subsequent phase involved reviewing the developing themes in relation to the coded data set to determine that they did in fact address the research questions. Eventually the following themes were identified: generalisations to support functional thinking; representations to support functional thinking; contextual tasks; and culturally located Māori and Pāsifika learners. See Appendix E1 for the thematic analysis table used to group codes into themes, and Appendix E2 for illustrative excerpts of coded transcripts removed and grouped on to a table.

3.7.2 Validity and reliability

All research involves analysing data in a reliable way, to produce conclusions that are valid (Merriam & Tisdell, 2015). Reliability, in terms of replication, is challenging in qualitative research because human behaviour is dynamic and the social environment being studied relies on context (Merriam & Tisdell, 2015). Replication of a study will not produce the same results. However, qualitative studies can convey consistency, transferability, and trustworthiness (Flick, 2018; Merriam & Tisdell, 2015).

Transferability can be achieved through thick descriptions, detailing the setting, participants, and themes. This technique allows readers to experience the events being described, and judge for themselves how the findings could be transferred to other contexts. Additionally, thick descriptions provide enough evidence to prove that the results are consistent with the data collected, and the conclusions are trustworthy and make sense (Merriam & Tisdell, 2015).

Using multiple sources of data helps to eliminate bias that can result from relying on one research method. In the current study many sources of data collection were used and the data collection method and analysis was systematic. An audit trail detailing how the study was carried out, and how the findings were drawn from the data, strengthens reliability (Merriam & Tisdell, 2015). Triangulation of multiple data sources also supports confidence in the data (Merriam & Tisdell, 2015). In the current study, data has been triangulated wherever possible. For example, data from interviews and classroom observations were cross checked and compared, in order to verify that both sets of data were telling the same story (Atkins & Wallace, 2012).

Additionally, the iterative nature of design based research strengthened the credibility of the research findings. In the current study, the ongoing discussion, reflection, and modification of lessons, made it possible to test developing theories from earlier lessons in later lessons. Retrospective analyses were contrasted with the informal analyses conducted while the study was in progress to strengthen the credibility of the research claims.

Diversity amongst participants allows for a greater range of application of the findings (Merriam & Tisdell, 2015). In the current study participants were of Samoan, Tokelauan, Cook Islands and Māori descent. Some were New Zealand born, and others Pacific born. They varied in terms of home language, gender, age, and time at school in New Zealand. Identifying patterns that exist across a range of people increases the chance that patterns can be applied to another example, compared to collecting a narrower range of data (Holley & Harris, 2019).

Member checks are the procedure where a study's findings are shared with participants, who can give feedback to correct or otherwise improve the accuracy of the study (Merriam & Tisdell, 2015; Yin, 2016). In the current study tentative interpretations of interviews and observations were checked with participants, who were asked if they accurately represented their realities. Collaboration between the researcher and the teacher enabled facts to be checked and interpretations to be corroborated or revised. Similarly, the researcher discussed themes and emerging findings with colleagues and supervisors for peer review. Asking them to comment on whether the findings were plausible based on the data adds another layer to the study's validity.

3.8 Ethical considerations

The research was designed and conducted in a responsible manner, in accordance with the Massey University Code of Ethical Conduct for Research, Teaching and Evaluations Involving Human Participants (Massey University, 2015). The project was reviewed and approved by the Massey University Human Ethics Committee prior to data collection. Ethical considerations taken into account included respect for participants with informed consent, respect for privacy and confidentiality, and social and cultural sensitivity.

Written consent was obtained from all participants, including the school Principal, Board of Trustees, students, parents or guardians, and the teacher (see Appendices F1-F5). This research involved children under the age of fifteen years old, therefore consent from their parents or guardians was sought and obtained. A detailed information sheet was provided alongside the consent form to ensure that participants had clear details of why the research was being conducted, what was involved throughout the research, and what they were consenting to (see Appendices G1-3). All participants in this study were allocated pseudonyms, and care was taken to exclude any identifying information about the teacher, students or school within any written reports. Participants also had the right to withdraw from the research at any time.

Sensitivity to social and cultural issues was observed at all times by the researcher, for example, maintaining the daily opening and closing karakia, the classroom routines, the social groupings selected by the teacher, and respecting any silences during interviews.

Risks that required consideration included the time commitment of the teacher and her students to the project. Disruptions were avoided as lessons under study took place as part of the normal classroom programme. Meetings were held at times and locations that suited the teacher so as not to burden her with an increased workload.

3.9 Summary

This chapter has outlined the research design and methods used in the study, including the rationale for selecting design based research and qualitative methods of data collection and analysis. The Ula model for Pāsifika engagement provided a culturally appropriate framework to inform the research. A variety of methods to collect data were used, including interviews and classroom observations. Data was analysed using thematic analysis, identifying codes and generating themes. Triangulating data supported the credibility of the interpretations, and conducting the research in a thoroughly documented and ethical manner ensured the reliability and validity of results. The findings and discussion of the study are presented in Chapter Four.

Chapter Four: Findings and Discussion

4.1 Introduction

The previous chapter provided an overview of the research design and methods used in the study. This chapter presents the findings and discussion in relation to how Māori and Pāsifika students generalise culturally located tasks involving functions, and the representations Māori and Pāsifika students use when engaging with contextual functional tasks. Section 4.2 outlines the initial functional understandings of the twelve Māori and Pāsifika students involved in the study. Section 4.3 draws on the data analysis from the series of eight contextual tasks to describe student learning and provide insight into the development of students' capabilities in representing and generalising functional relationships. Section 4.4 highlights connections between the cultural contexts of the tasks, and the cultural worlds of Māori and Pāsifika learners. Finally, section 4.5 presents student understandings of functional relationships at the conclusion of the teaching intervention.

4.2 Students' initial understanding of functional relationships

Prior to commencing the teaching intervention participants completed two parallel assessment tasks, one contextualised[1] and one decontextualised[2]. The purpose of the assessment tasks was to measure the growth of students' functional thinking over the course of the teaching intervention. Student responses were categorised according to the type of relationship described: recursive, covariational, or correspondence; the type of generalisation

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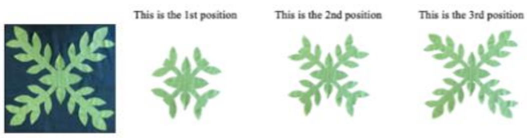
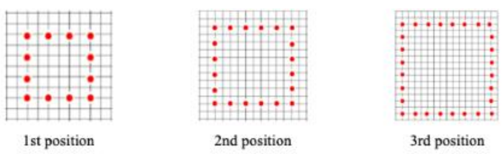
1 A group of Mamas are working on a tivaevae design.



Look at this pattern and think about how it is growing.
How many leaves would it have for the 7th position? What about the 17th position? What about the 76th position?
Show how the pattern grows using a table, ordered pairs and / or a graph.
Write the rule for the pattern in words, numbers, or symbols.

2

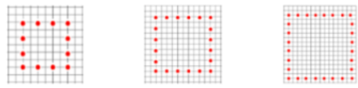


Look at this pattern and think about how it is growing.
How many dots would it have for the 7th position? What about the 17th position? What about the 76th position?
Show how the pattern grows using a table, ordered pairs and / or a graph.
Write the rule for the pattern in words, numbers, or symbols.

expressed: factual, contextual, or symbolic; and the type of representation used. Table 3 shows student responses to the tasks.

Table 3

Participants' Functional Thinking and Representations Pre-Teaching Intervention

		Contextualised task Percentage of participants (n = 12) 	**Decontextualised task** Percentage of participants (n = 12) 
Functional thinking	Not evident		42%
	Recursive thinking	83%	58%
	Covariational thinking	17%	
	Correspondence thinking		
Generalisation	Not evident		42%
	Factual generalisation	92%	58%
	Contextual generalisation	8%	
	Symbolic generalisation		
Representation	No representation		42%
	Drawing	25%	50%
	Using visual image to count	75%	8%
	T-chart		
	Multiple representations		

The data shows that at this stage the majority of participants were using concrete, numerical thinking, and not yet reasoning about or expressing functional relationships. None of the participants identified the correspondence relationship between variables, or used symbolic representations to express generalisations. Describing how the pattern was growing recursively, by a single sequence of values, was the most common way of thinking about pattern structures in both the contextual (n = 10) and decontextualised tasks (n = 7). These results are consistent with the findings of a number of researchers (e.g. Blanton et al., 2015; Radford, 2010; Stephens et al., 2017; Wilkie, 2014), showing students have a tendency to initially notice recursive relationships when learning to generalise growing patterns. Learners commonly focus on the concrete nature of geometric patterns and count the value of the dependent variable, rather than look for a more general relationship between an element of the pattern and it's position.

Almost half of the participants (n = 5) did not represent the decontextualised pattern in any way, while all participants used either the visual representation of the pattern to count (n = 9), or drew the next position (n = 3), to make sense of how the contextual pattern was growing. These results align with those described by Hunter and Miller (2018) and Miller (2014), showing students are more able to readily generalise culturally located tasks than growing patterns represented by decontextualised geometric shapes.

4.3 Representing and generalising functional relationships

Analysis of the data collected from lesson observations and interviews showed that these Māori and Pāsifika students used a variety of representational tools to reason about functions, and generalised recursive, covarying and correspondence relationships in increasingly sophisticated ways. This mirrors what has been reported in research literature (e.g. Blanton et al., 2015; Markworth, 2010; Stephens et al., 2017; Wilkie, 2014) in relation to the typical trajectory of dominant groups of students' functional thinking. The results resonated with previous findings that students may use a mixture of strategies simultaneously, or move bidirectionally through levels of understanding, in learning to represent and generalise growing patterns. The findings and discussion therefore present key shifts in student thinking. Shifts particularly occurred in how students were able to (1) express generalisations, from describing additive relationships using natural language to representing multiplicative relationships symbolically, and (2) use multiple representational forms to identify, communicate and justify generalisations.

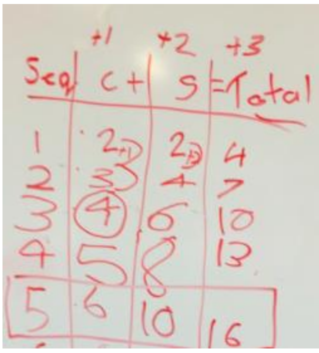
4.3.1 Using T-charts to support functional thinking

The t-chart, or function table, is identified in the literature as an important structure in student's mathematical reasoning (Blanton & Kaput, 2011; Blanton et al, 2015; Stephens et al., 2015). In the current study, the t-chart was the most commonly used tool to make sense of, explore, and represent functional relationships across the series of tasks. Over the eight lessons students used t-charts in gradually more complex ways. Students initially created t-charts to organise data and more easily notice patterns, and later used the t-chart as a tool to compare data, find relationships, and derive a function rule. These findings align with international research (e.g. Blanton & Kaput, 2011; Stephens et al., 2015), showing that as students' functional thinking developed, they transitioned from using the t-chart as an opaque object, a place to record numbers, to a transparent object, used to determine relationships in data and make explicit connections between the variables.

In the first sāsā task[3], all groups of students independently chose to construct a t-chart as an organisational tool, and many of the groups developed factual generalisations where they counted the number of claps and slaps. In the follow-up interview a student said they created a t-chart: "to show all our information and to have our information in order. It's easier to understand because you know where to put the numbers and what they represent". Following small group activity, a group of students were selected to share their explanation and justification with the larger group. The group showed their t-chart and identified recursive patterns in the data by looking down the t-chart to find the additive difference between terms: "We decided to see how the pattern would grow. In the claps column you're adding one. The rule for the slaps column is adding two every time. And the rule for the total column is adding three each time" (see Figure 5).

Figure 5

T-chart Showing Recursive Relationship in the Dependent Variable



Mathematically exploring the problem context was a first step towards functional thinking, and the students developed a systematic way to collect and represent their data which was not

[3] Tevita's group are practicing the sequences for their siva at Polyfest.
The first sequence is: clap, slap, slap, clap
The second sequence is: clap, slap, slap, clap, slap, slap, clap
The third sequence is: clap, slap, slap, clap, slap, slap, clap, slap, slap, clap
If the sequence keeps on going how many claps and slaps will there be in the 11th sequence? What about the 29th? What about the 83rd?
What's a rule that tells us now many claps and slaps there are no matter how long the siva sequence is? Can you show this rule in words and numbers?
How can you show the ways the pattern grows? Can you use pictures? A table? A graph? A diagram?

evident in the pretest. However, previous research studies (e.g. Blanton & Kaput, 2011; Markworth, 2010) argue that although t-charts are an important representation for children's mathematical reasoning, they can position students to focus on recursive patterns in the dependent variable. While a recursive approach allowed students to predict the next position of the pattern, it did not support covariational thinking about a functional relationship between variables.

By the third task[4] the teacher, Sarah, knew the students were confident using t-charts to identify recursive patterns in the data going down a column, but she wanted them to think about the horizontal relationship between the dependent and independent variables across the columns. During the large group discussion the teacher stepped in to press the students to look at the t-chart in another way, as shown in the following vignette:

Large group discussion
Sarah: Ana had another clever way of thinking about how that pattern was growing that was going to be really helpful to find the tenth pattern. Ana what did you guys think might be part of your guys rule? Ana: Ten times six. Sarah: Where did that times six come from? Ana: We're timesing the pattern number by how many fauato [coconut fibre twine]. Sarah: So for the fourth pattern what would we do? Ana: For the fourth one, four times six. Sarah: Ana can you write that beside the 23 on the t-chart? Write 24. Mia: So that hasn't worked. Sarah: So if we tried that rule for the third pattern? Tai: Three times six, 18. Sarah: Write that next to the 17.

[4] The vaka at Matauala Hall has a pattern where the fauato (coconut fibre twine) joins the planks. What would happen if the vaka kept getting longer and longer. How many pieces of fauato would there be if the pattern went up to iva (nine)? What about hefuluiva (nineteen)? What about ivahefulu (ninety)?

Tahi  Lua 

What's a rule you could use to find the number of pieces of fauato for any number of the pattern?
Use as many different representations as you can to show your thinking.

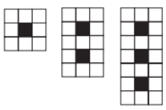
Ihaia: Wait, you’re always adding one. Mia: Unless you want to times the pattern and take away one? Sarah: Stop the bus! Mia can you explain that and point to the pattern? Mia: For the first pattern it’s five. Times the pattern by six then take away one. The minus one is coming from these lines and that’s the extra one in the pattern. Sarah: So what was the difference between them? Mia: Instead of plus six, plus six, plus six you can times six and take away one. Sarah: Exactly. So instead of going down the t-chart you’ve gone across. Solomon: Downwards takes ages. Mia’s way works way easier. Observation three: vaka task

Evident in the vignette is a shift in focus from thinking additively about what came next in the column to looking across the t-chart to find an explicit rule. For the first time students had used the t-chart to think about the relationship between the variables, and the t-chart became a tool to identify then generalise the pattern into a functional relationship.

When students began to work on the next task[5] in their small groups, it was apparent that they were making connections to the previous lesson and independently using the t-chart as a tool to determine a functional relationship between quantities in the task. The following vignette illustrates how one group focused on using the data in the t-chart to generate a rule for the function:

Small group discussion
Anaru: We found a rule, it’s adding by six Mia: We’ve got plus six, so let’s try going across.

5



Titi (dance skirts) from Tuvalu have bright colours and eye-catching designs. If this was going to be the design on the wide strip of a titi, how many white and how many black squares would be in the fifth position? What about the 85th? What about the 99th?
How could you work out how many white and black squares for any position in the pattern?
Show how the pattern grows in as many different ways as you can.

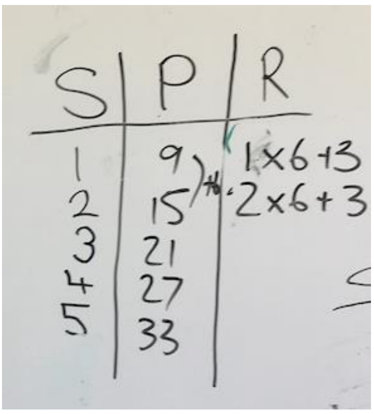
Solomon: Plus six. What do you mean by going across? Oh times. So one times six.
Mia: Plus three. It's one times six plus three makes nine so it's making the pattern.
Solomon: One times six plus three. See if it works for two.
Mia: It does work. Two times six plus three equals fifteen. Try it with three times six.
Solomon: Look we got it. It's times six plus three. The three comes because you need to make the whole number of the pattern.
Mia: We found out that we had to add three to make .... for this one, it's one times six plus three makes nine so it's making the pattern. The total number of squares.
Anaru: Is that our rule?
Solomon: Yep, that's our rule. So strip times six, that's the pattern. Then add three.

Observation four: titi task

In this case the group structured the t-chart with three columns: S (representing the strip or position number); P (representing the pattern or number of squares); and R (representing the rule). They used the t-chart to work out what they needed to do to the independent variable (*S*) to get to the dependent variable (*P*). The group used the third column to represent the rule: multiplying the pattern number by the common difference of six, and adding three (see Figure 6).

Figure 6

The Three Columned T-chart



There are similarities with Markworth (2010), who found a three-column table advanced students' functional thinking by providing a location to explicitly pay attention to the horizontal relationship between the independent and dependent variables, and identify a functional rule. The difference between the current study and Markworth (2010), is that the Māori and Pāsifika students in the current study spontaneously used a three-column table, rather than the teacher providing a direct instruction to do so.

The tukutuku task[6] was next in the series of problems. Unlike the previous patterns presenting the position number and the growing pattern as variables, the tukutuku task involved looking for a relationship between the number of kaho (rods) and the number of tuinga (crosses) in the pattern. The kaho started from three, rather than one as in the previous tasks, which made the function more difficult to derive: "It took us heaps of tries". However, all groups of students used a t-chart as a tool to find the common difference and "go sideways not go down. It won't take as long going sideways as going down". One student explained how they were going to use the recursive relationship to find the explicit rule, before they even began solving the task:

> We're going to use a t-chart on our way to find a rule. We're going to find out what we're adding each time then we're going straight to multiplication, we'll multiply by that, then we might subtract something or add something.

Rivera and Becker (2011) found middle school students in the third year of their study used the same process for generalising numerically using differencing. The constant difference property enabled students to find the *n*th element of a pattern. For example, in the tukutuku task the students used a t-chart to find the common difference between successive values in the second column was $4n$, then each term was always six less. In both the current study and Rivera and Becker's (2011) research, this conceptually developed constant difference strategy was eventually taken as shared and became a common classroom practice.

6

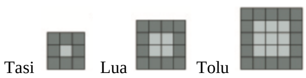
Tukutuku panels can be made from kaho (wooden rods) with tuinga (cross stitches). This kaokao is a traditional design that symbolises the strength of a warrior.
Imagine you continued this pattern until there were 9 kaho. How many tuinga would there be? What if there were 18 kaho? What if there were 38 kaho?
What's a rule you could use to find the number of tuinga needed for any number of kaho?
Represent your group's thinking in as many different ways as you can.

The final two tasks required students to use t-charts with an even greater degree of flexibility, because the patterns were more complex quadratic relationships. Quadratic functions involve powers of two, for example, square numbers. The sequences were non-linear, so the strategy of looking for a common difference in order to formulate the multiplicative relationship between a number and its position could not be applied. On interview students explained the patterns they had observed in the t-chart for the fala task[7]:

> The white squares was so much harder because we noticed that they were going up by different numbers. It took us a while but we figured out that the boxes were adding three then five then seven then nine. We found out that it was also adding two, so three plus two is five then five plus two is seven then seven plus two is nine.

Although the group were looking at the pattern recursively, the t-chart helped them identify an important property of quadratic functions: the second differences are the differences that have the common value (see Figure 7). For example, this pattern has a second difference of 2, so it will be connected to the sequence of square numbers.

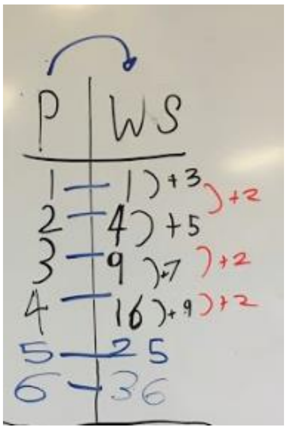
7



Talia was looking at the border of her mat. She thought she could find a way to figure out how many squares there would be in any part of the pattern.
If Talia wanted to figure out a rule for how many border squares there would be at any number of the pattern what could she do?
What about a rule for the squares in the middle?
Show your groups thinking in as many ways as you can.

Figure 7

T-chart Representing the Quadratic Pattern



Eventually the group saw that the x values were all square roots of the y values, suggesting that the t-chart helped structure these Māori and Pāsifika students thinking about relationships between quantities:

> Isaiah: Wait look. In these times tables these numbers are in it. You get it? Look five times something equals twenty five. Four times something equals sixteen. Three times something equals nine.
> Junior: So we times the pattern by the same number!

Overall, these examples highlight the transition of the t-chart from a representation for organising data, to a tool for reasoning about functional relationships between variables. Matching previous research (e.g. Blanton & Kaput, 2011; Blanton et al, 2015; Stephens et al., 2015), these students used the t-chart as an important structure in their mathematical reasoning, and it became a representation that they could look through to see new relationships between the variables in increasingly challenging tasks.

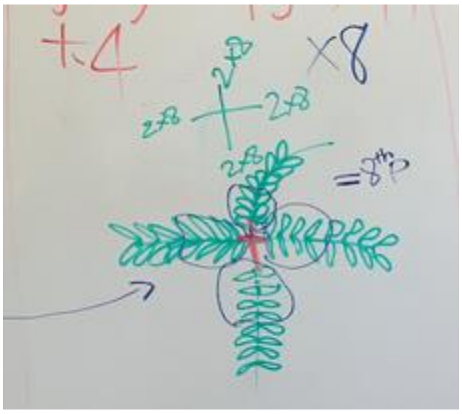
4.3.2 Visual representations to support generalisation

Throughout lessons in the current study the visual interpretation and analysis of familiar cultural patterns was another effective means of non-dominant students representing and generalising functional relationships. Research (e.g. Moss & McNab, 2011; Rivera and Becker, 2011; Wilkie & Clarke, 2015) shows that visual strategies are more powerful for generalising functional relationships than numerical strategies, such as finding the constant difference, alone.

Some students in the current study used drawing to explicitly identify the structure of the contextual patterns and make sense of the relationship between the variables. For example, in the ngatu problem[8] a group drew the eighth position, the near generalisation in the task, to show the multiplicative structure of the growing pattern (see Figure 8):

Figure 8

Visual Representation of the Multiplicative Structure of the Growing Pattern



In a task based interview one of these students explained how they had seen the pattern growing, as the pattern number two times up each of the four stalks:

Task based interview
Aroha: We drew it. We drew the pattern. We were trying to find out how much stems and leaves you would have on the eighth pattern. First, we decided to draw the stem then we

8

Ngatu is Tongan tapa cloth. In Samoa the same cloth is called siapo and in Niue it is hiapo. Ngatu can sometimes tell a story, using symbols from nature and geometric patterns. Numbers written on the tapa indicate its langanga, or length.
How many stems and leaves would there be if this ngatu pattern was 8 langanga long? What about 50 langanga?
Show how the pattern grows in multiple ways.
What is a rule that tells us now many stems and leaves there are no matter how many langanga long the ngatu is?

drew eight leaves on each side of the stem. Then we wrote our drawing in an equation, two times eight four times.
Interviewer: So where did the two times eight come from?
Aroha: Two times eight because it adds on in two leaves. The eight comes from this langanga.
Interviewer: So what if it was the fiftieth langanga ?
Aroha: Two times fifty, two times fifty, two times fifty, two times fifty. It's always timesed by the langanga, the pattern number.
Interviewer: And what about the four stems in the middle?
Aroha: Plus four.

Task based interview: ngatu task

Previous research (e.g. Miller, 2016; Cooper & Warren, 2008) reports challenges for students in identifying the multiplicative structure of growing patterns. These difficulties include not visualising patterns growing spatially, and the tendency to focus on a single data set using additive strategies for describing generalisations. In the current study engaging visually with the ngatu pattern gave students the opportunity to see it's underpinning structure and connect the visual representation to multiplicative changes. As a result, this group were able to express a correspondence description of the relationship between the ngatu leaves and the pattern: "multiply the pattern by two, then multiply by four", or as a student pointed out: "you can just multiply the pattern number by eight".

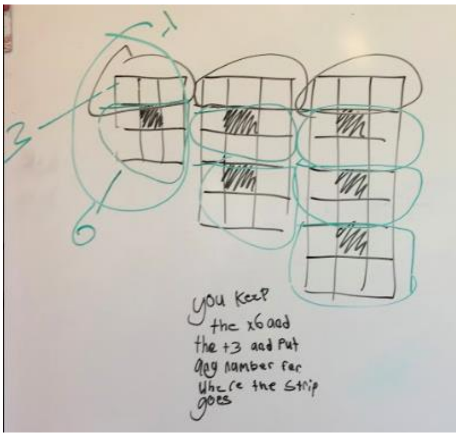
Students also used the visual representation of patterns to explain and justify their generalisations. For example, in the larger group discussion of the titi pattern, one group justified the rule they constructed using constant differencing from the t-chart, with the way they saw the structure of the pattern. When students were probed for an explanation of the rule they could show how the pattern grew in relation to the position number, and how parts of the pattern changed and parts stayed the same, based on the visual configuration (see Figure 9):

> The one is the pattern number. The three was from the top, those three. This is the six we're timesing, these six square boxes. There's one six in the first pattern, and two sixes in the second pattern so it's times two. Three sixes and so on. See how every

pattern has an odd line? Like they all just have one row of three you have to add on up the top.

Figure 9

Using the Visual Representation to Justify the Function $\gamma = 6x + 3$



During a reflective interview after the lesson, another group showed how they could see the structure of the pattern growing differently, starting with eight white squares and counting the black squares separately. Their rule was:

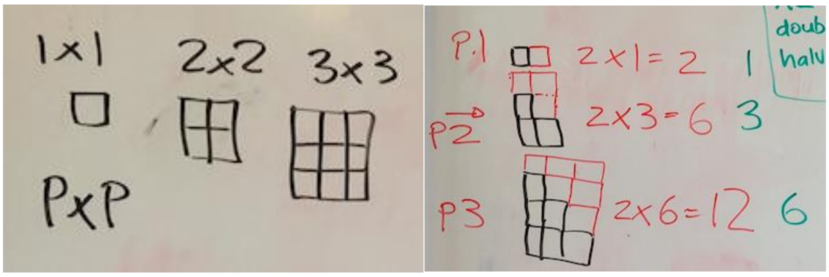
> It's always one less than the pattern number that you're up to. So for the 85th pattern we're doing 84 times five plus eight because the eight is like the extra part of the pattern that makes however many white squares. So it's one less than the pattern times five plus eight. It's a bit confusing but it works too.

Although their explicit rule 'worked', they realised that compared to the first expression of the rule, it involved a lot of calculation and was less efficient. These Māori and Pāsifika students were developing deeper algebraic thinking as they realised that there is more than one way a functional relationship can be represented, and there are multiple ways of seeing how a pattern grows (see Section 4.3.5.). This contrasts with the findings of Wilkie's (2014) research where few upper primary students were able to find more than one way of visualising and generalising the structure of a growing pattern themselves.

Visualising quadratic relationships provided further evidence that these Māori and Pāsifika students were developing flexibility with pattern recognition. In the kapa haka problem[9], engaging with the pattern visually supported students to make connections between the triangular numbers in the kapa haka task and square numbers in the fala task, in order to analyse and trial a variety of solutions. The visual representation of the triangular numbers supported the students to identify both the geometric and numerical structure of the kapa haka pattern (see Figure 10).

Figure 10

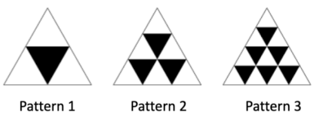
Visual Representation of Triangular Numbers to Explore the Function $y = \frac{x(x+1)}{2}$



In a reflective interview following the lesson, a student explained:

> Me and my group we just tried to think of other ways and we went back to the other problem. Then we saw, we tried using the same strategy. We made the triangles into a square, so it was like the fala one. We found out the rule was pattern times pattern plus one, then we had to divide it by two because it was half, we were halving. We made the triangles into a square but half of it was like fake. So we had to take that fake half away again.

9 Mātua and Whaea are coming up with designs for Te Haeata Awatea kapa haka uniform. They like this triangular pattern.



If the pattern kept growing, how many black triangles would pattern 7 have? What about pattern 17? What about pattern 27? How could Mātua and Whaea work out how many black triangles any pattern number would have? Can you show this rule in words and numbers?
How can you show the ways the pattern grows? Can you use pictures? A table? A graph? A diagram?

The ways these Māori and Pāsifika students used visual representations to develop pattern generalisation aligns with prior research (e.g. Moss & McNab, 2011; Rivera and Becker, 2011; Wilkie & Clarke, 2015) showing visual representations are a powerful means to support students to bridge the gap between the concrete situations represented in tasks, and the abstract mathematical structures underlying the growing patterns.

4.3.3 Natural language to express generalisations

Tasks in the current study provided students with opportunities to use natural language around familiar cultural contexts to express generalisations. The use of natural language is considered by researchers (e.g., Cañadas et al., 2016; Moss & McNab, 2011; Radford, 2018) to have an important role in developing algebraic thinking, because it allows learners to make sense of, and describe algebraic concepts, using language they know. Tasks in the current study integrated Māori and Pāsifika languages. For example, students used langanga interchangeably with pattern in the ngatu task, and kaho and tuinga for the variables in the tukutuku task. Students were encouraged to draw on their home languages as part of the mathematical discussions. On interview, a student explained: "Sometimes if my Samoan friends are struggling to speak English we speak Samoan to help them so we all understand stuff".

Prior research (e.g. Blanton & Kaput, 2011; Moss & McNab, 2011; Radford, 2018) shows that as students' functional reasoning develops, so does their capacity to generalise growing patterns in natural language. Similarly, in the current study when non-dominant students were provided with opportunities to explore functional relationships from cultural contexts, they transitioned from expressing concrete rules describing numerical values to representing the functional relationship between variables through natural language.

Natural language was used in the first task to express factual reasoning as students attended to particular instances of the sāsā pattern. For example: "For the slaps it adds on two. It goes plus two, plus two, plus two. It's two, four, six, eight, ten". Through natural language these students indicated how to obtain the next number in the sequence given the previous number, and expressed the recursive rule as an action within the pattern. The relationship between the variables remained implicit.

A contextual, natural language generalisation in the second task illustrated a shift in student reasoning from factual to covariational thinking. Students coordinated the relationship between the leaves and the pattern position. For example: “every time you move to the next pattern the leaves grow by eight and the stem stays at four”. Contextual generalisations are descriptive and language driven, and the language used revealed that the students algebraic thinking was becoming more general (Radford, 2018). For example, performing an action “every time you move” on objects “the leaves grow”, rather than on concrete numbers.

In the following tasks students conveyed correspondence thinking using more precise natural language. For example, in the tukutuku task a group wrote: “if you times the rod with the crosses then take away six that tells you how many crosses there are”. This natural language generalisation provided evidence that these students were aware of the correlation between the two variables, and could explicitly state a rule which described a generalised relationship.

Wilkie (2014) found that students’ attempts to represent a rule in words often led naturally to their interest in symbols to replace the variables. In the current study students spontaneously began to express function rules using a mixture of natural language and symbols as they started to look for more productive ways to represent generalisations. That is not to say that students needed to be able to describe a functional relationship in words before they could use variable notation. As reported in the literature (e.g. Blanton et al, 2015; Brizuela et al., 2015; Stephens et al., 2017), the use of natural language serves as a mediator between a real world context, and symbolic expression. The data in the current study showed that students learnt to express functional relationships in their own language, in symbolic language, or a combination of the two, as they made sense of functions in meaningful ways.

4.3.4 Symbolic representation

Early on in the current study, a group proposed using symbols to represent the variables in the ngatu task:

> We were trying to find out how many stems and leaves would there be if the pattern was 8 langanga long. First we did eight langanga times eight patterns then we added the leaves because we needed to know how much there were altogether. After that we started to shorten it by just writing the first letter to L, which is the L for leaves, and the first letter for pattern which is P. So $8L \times P + 4$

Research (e.g. Wilkie, 2014) shows a common misconception is to use symbolic letters as abbreviated words, and care must be taken to ensure that students do not confuse the variable as shorthand for a word or phrase. The teacher was aware of this, and sought to draw out more of the students' thinking about the symbolic representation of variables, illustrated in the following vignette:

Large group sharing
Sarah: Actually who asked was it algebra? TJ: Me. I don't know what algebra is. Solomon: Algebra is patterns. Ana: It's when letters stand for something. You write letters that stand for other things. Junior: So pattern number times eight leaves plus four for the stem equals *T* for total. Aroha: Or it could be *x*. Sarah: What does *x* stand for? Mia: Anything. Sarah: So Tane what did you think that *x* represents? Tane: It represents anything. Sarah: You're absolutely right it represents anything. For this pattern the *x* might represent the leaves and stems. But when you're doing algebra that *P* could be anything. It could be a square. Aroha: So a triangle could represent any number and the eight is the pattern. Ana: What about circle? Aroha: Yes it isn't only triangle that represents something. Sarah: This is such clever, clever thinking. Observation two: ngatu task

It was apparent that while their understanding of variables was in early stages, these Māori and Pāsifika students did have some awareness that symbols could represent "quantities that have variability" (Wilkie, 2016, p.335). Aligning with previous research (e.g. Blanton & Kaput, 2011; Brizuela et al., 2015; Wilkie, 2014) suggesting that young children can begin to use variable notation to represent relationships between quantities, the spontaneous use of symbols conveyed the generality with which these students were beginning to think about the functional relationship.

In the following task, students continued to experiment with expressing the rules using variable notation. In the vignette below, these Māori and Pāsifika students were further developing their symbol sense as they collaboratively discussed the use of letters to represent variable quantities:

Large group discussion
Ihaia: Can you use like a letter? It's always something times six fauato. Can you use any letter? Lolo: Any letter Mia: So *B*, *B* times six and then minus one equals … Ihaia: Any number. What about another letter? The letter's going to be for the fauato. Tai: We can just do anything it's still a rule Junior: A for answer? Tai: No, no matter what letter you put in it's still always going to come up with the fauato Junior: So it can be any letter. No matter what the letter is it's still going to be the pattern times six and minus one fauato Observation three: vaka task

In this vignette, recognising what changed and what stayed the same allowed students to generalise the pattern, and to correctly use variables to represent the explicit functional relationship. For example, Ihaia knew the six fauato were going to remain constant but the pattern number would change, stating: "it's always something times six fauato". He understood that the total number of fauato would be "any number", dependent on the pattern number. It was also evident that students realised that the structure of the pattern wasn't going to change ("pattern times six and minus one" or (B x 6)-1=A), no matter what symbol was used for the variables (see Figure 11). Rather than focusing on numbers, they were focusing on the structure of the functional relationship between the pattern number (in this case B) and the total number of fauato (A).

Figure 11

Variable Notation Characterising the Rule as an Equation

Discussions about the use of brackets and order of operations emerged naturally within this contextual, socially constructed learning. For example:

> Mia: Sarah can we do brackets?
> Tai: Are you doing brackets?
> Sarah: Why are you going to do that Mia?
> Mia: So you know what to do first. Because after we did take away one. But we needed to times by six first so we do brackets.

Additionally, in constructing a rule for the fala pattern, some students were able to show a symbolic representation for the total number of squares. After collaboratively constructing the rule for the internal squares: p^2, and the border squares: (p x 4) + 4= y: they combined the two functions to form a single equation:

> Altogether it's p squared, then the brackets over there, plus p x 4 plus 4 because you need to add the black squares to the white squares. So it's $p^2 + (p \text{ x } 4) + 4 = y$ to find how many squares altogether in the fala.

This group of students became openly excited when they discovered an expression that worked. Intertwining the linear and quadratic patterns symbolically was unexpected, and showed the importance of providing students with opportunities to reason and create equations using the context of the pattern.

By the sixth task[10], students were looking for more efficient ways to represent the generalisation, and for the first time a group used symbolic representation before a natural language generalisation. Stephens et al. (2015) described this “symbolic advantage” emerging in more complex problems (p.158). When problems increased in complexity, students in Stephens et al.’s research were more successful representing functions in symbols than in words. This was evident with students in the current study finding it easier to write $(x \text{ x } 24) + 4 = y$ than state, “you always times the position by 24 then you add 4 to get the number of leaves on the tivaevae”.

In the final tasks there was evidence that symbolic thinking was beginning to lead student thinking when expressing the rule. For example, in the kapa haka problem a group shared: “The first rule we came up with was P x 2 – 1”. The evidence here shows that the students did not have to translate a natural language generalisation into symbols, or refer to the contextual generalisation using natural language (e.g. “every time the number of triangles on the bottom row gets bigger by one triangle”). The students had moved to a conceptual level of understanding, where natural language was receding into the background to make space for symbolic thinking, and the more abstract signs of symbolic generalisations (Radford, 2018).

4.3.5 Multiple representations

Research (e.g., Hunter & Miller, 2018; Miller, 2016) shows that the type and context of growing patterns impacts on indigenous students’ abilities to access the relationship between variables. In the current study the contextual patterns for each task were specifically selected to allow the students multiple ways to see, and represent, the underlying structure of the growing patterns. In the ngatu problem, for example, the motif used in the task grew in four directions and each group of students represented the pattern growing in a different way: a t-

10

The Porirua Mama’s tivaevae group meet at Te Akapuanga Hall to sew tivaevae every week. Every Mama has their own special design, and every tivaevae tells a story. Mama donut wants to sew a tivaevae for her grandson. She has decided to make a fern pattern.


First position


Second position

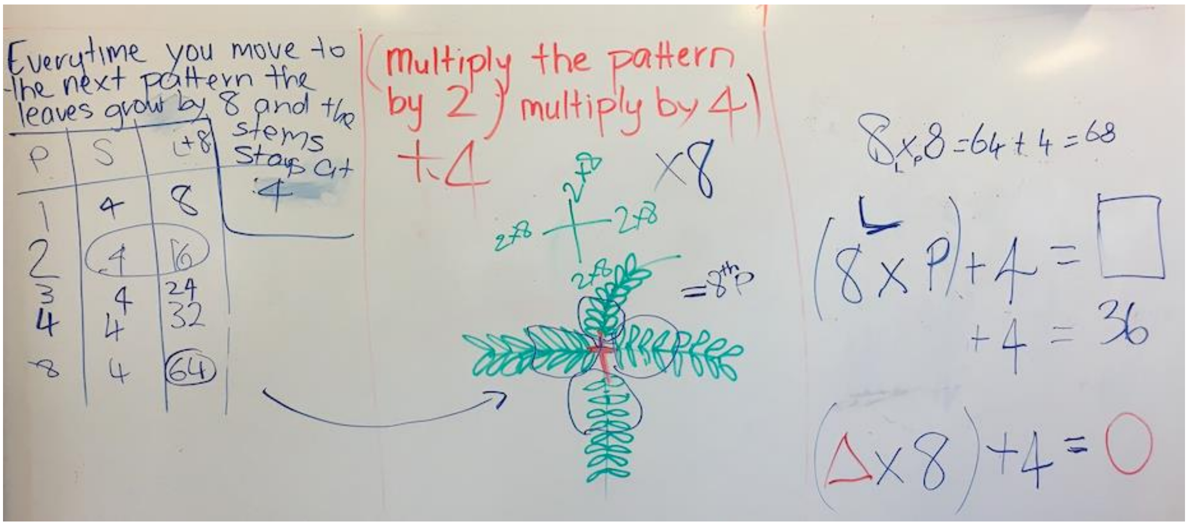

Third position

If Mama Donut’s pattern keeps growing how many leaves will there be in the 15th position? What about the 23rd position? What’s a rule that tells us now many leaves there are no matter how what position the tivaevae pattern is?
Show how the pattern grows in as many ways as you can.

chart, visual, natural language description and symbolic representation. Each representation provided an alternative way for students to examine the structure of the pattern and the relationships between variables, and highlighted different aspects of functional thinking (see Figure 12).

Figure 12

Multiple Representations and Generalisation of the Ngatu Pattern



At the end of every lesson the teacher engaged the whole group of students in discussion to ensure that all students were able to make sense of each other's explanations, make connections between the representations, and access the progressively more sophisticated levels of representing and generalising the functional relationships that students had developed. In the vignette below the teacher was scaffolding students to consider how the representations were connected:

Large group discussion
Sarah: So just jumping back to that really clever thinking you've done all the way through around looking at how the ngatu pattern was growing and then these guys were saying using those letters to represent how the pattern's growing. So what was the first groups rule? TJ: Every time you move to the next pattern the leaves grow by eight and the stems stay at four. Sarah: And they showed us that ...?

Ana: With a t-chart.

Sarah: So what's this group done here that's a little bit different?

Mia: They multiplied the langanga by two four times because there's four of those stems.

Solomon: You could multiply by eight to times the whole thing together.

Sarah: This was helpful for us because we could actually see where that eight was coming from. So you're thinking about how this plus eight from the t-chart fitted in with this times eight over here in the drawing. What about here?

Ihaia: Eight leaves times the pattern number, plus four.

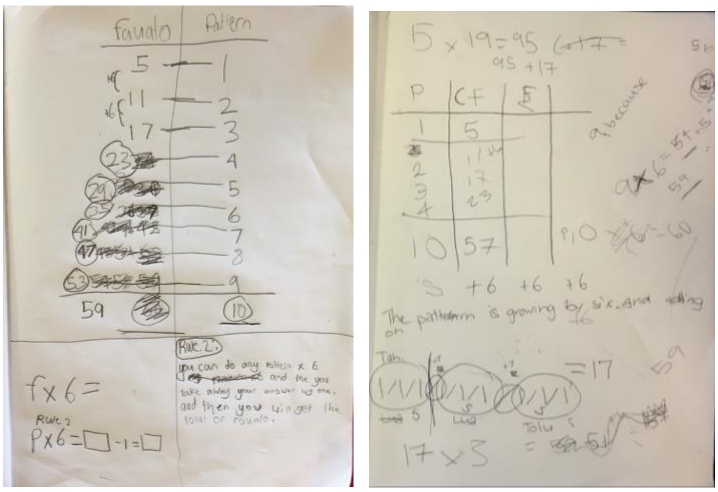
Sarah: So one thing the middle group did really well was represent the pattern by drawing it. And this group showed us how we can use different symbols to represent different numbers in algebra.

Observation two: ngatu task

In the following lesson, students built on what they had learnt from the ngatu problem and, without teacher direction, constructed multiple representations of the patterns in their collaborative groups. Groups used several different representational tools, such as t-charts, drawing, and natural language generalisations, to develop their reasoning about the relationships between variables. This was significant because in the first two problems each group used only one representation independently to try to understand the nature of the pattern (see Figure 13).

Figure 13

Multiple Representations in Group Workbooks



The use of multiple representations became common practice in both small group work and the larger group discussion at the end of each lesson. Research shows that students' flexibility with multiple representations promotes deeper mathematical insights, and students gain a more thorough understanding of functional relationships when they represent them in more than one way (e.g. Blanton et al., 2015; Cañadas et al., 2016; Stephens et al., 2017). When pressed for why they thought multiple representations were important a student stated that:

> A t-chart helps you see the numbers because you're just looking sideways and it's just easy to look for the pattern. And when we draw the pattern we can check our numbers and it's an easier way to show our thinking. When we write our drawing in an equation when we share back our thinking everyone can understand what our drawing meant.

Another student shared:

> I like that I can include t-charts and drawing. Somehow it's easy for me and I can understand it and you can see what the t-chart represents and you don't have to go back looking in your mind for what they represent. The t-chart shows the numbers, and drawing shows where the numbers came from. It all links up and I get it.

These students recognised that when they used multiple representations to solve a task, they could check their reasoning, because they arrived at the same conclusion in different ways. Student responses also confirmed that they understood the connections between different types of representations, and were able to triangulate natural language, numerical and visual strategies to justify their reasoning to peers. This aligns with prior research (e.g. Moss & McNab, 2011; Rivera & Becker, 2011) showing that making connections across representations gives students diverse opportunities to identify, communicate and justify functional rules.

4.3.6 Challenges with functions

Prior to the teaching intervention the researcher considered that students might have trouble with common misconceptions related to the development of functional thinking reported in the research literature. For example, the difficulty in shifting students' perspectives away from recursive thinking to functional thinking or believing that variables stand for a fixed unknown quantity rather than as a symbol that can stand for any real number in a functional relationship (Blanton et al., 2015; Stephens et al., 2017; Wilkie, 2014). However, the Māori and Pāsifika students in the current study did not struggle with these challenges in ways that were anticipated.

Interestingly, none of the participants in the current study represented the functional relationship between variables with a graph. Wilkie (2016) found the same with the 12-13 year old students in her research. In a semi-structured interview the teacher commented that:

> We didn't go into graphs. They were excited about being algebraic thinkers so I think if we'd gone longer we would have. We really wanted them to see the generalisation, and looking at the variables and the rules, and the graphs would've just added another element. We wanted to embed what we were working on.

In order to continue building a robust understanding of functions an important next step would be to connect students' functional understanding to graphically representing the relationship. The need for the provision of diverse opportunities to explore representations is reinforced in the research literature, and graphs in particular are recognised as prompting different reasoning, and giving a more complete view of the function, than other representations (e.g. Brizuela & Earnest, 2017; Caddle & Brizuela, 2011).

4.4 Culturally located learners engaging with contextual tasks

Data from the study revealed that the use of contextual tasks affirmed Māori and Pāsifika students' cultural identities. When students were asked how it felt to work on tasks related to their cultures, students expressed the idea that contextual tasks normalised their cultures in the mathematics classroom.: "It feels the same, I just feel like I'm Māori". Another student shared the feeling of belonging the contextual tasks engendered, stating: "It makes me feel welcomed. It makes me feel like I belong here". Students communicated feelings of pride in their culture being represented by the tasks. For example:

> I felt proud. I was proud because Tokelau’s not a big culture and not a big island. But being mentioned in this problem it’s like there’s a big amount of people that know about it. They care about our culture, and know things about our culture.

Rather than perceiving mathematics at school as separate from their cultures, these students experienced strong cultural alignment with their Māori and Pāsifika identities and their mathematics class. These findings are consistent with previous national and international research (e.g. Beatty & Blair, 2015; Davis & Martin, 2018; Hunter & Hunter, 2018; Hunter & Miller, 2018), showing that contextual tasks promote positive cultural identities through valuing the cultural capital that non-dominant students bring to the mathematics classroom.

Students shared their cultural expertise, and learnt about the cultures and traditions of each other, through the tasks. In the following vignette a group were discussing the titi (Tuvalu dance skirt) task after the lesson:

Small group discussion
Solomon: I never knew that they call it titi too because we call it titi in Tokelau. Mia: Same in the Cook Islands. The things you wear on your waist. Solomon: What about Samoa? Kupa: We have our laei, or our lavalava Solomon: Tongans they use mats instead of titi. Kupa: We use mats too. My Nana I think she has material that makes mats. She’s made a mat before in Samoa. Solomon: My Nana makes those fau’s, you know these fau’s we wear in Polyfest? They’re cool as. She makes those hair things for the girls, like the thing in their hair. She makes these small as bands and she puts in on there with some flowers and some things for their hair. Kupa: What’s it called? Like the Tongans have? Solomon: Nah, like a pin. She uses a pin. Mia: My nana makes ei’s. Do you know what those are? With the flowers that go around your head? Solomon: Yeah. Mia: And fans

Solomon: All the Islander nanas make fans.

Observation four: titi task

The discussion of mathematics through the lens of students' own cultural perspectives allows them to reflect and appreciate not only their own culture, but the culture and traditions of others (d'Entremont, 2015). In an interview following the lesson, a student signalled that contextual tasks drew Māori and Pāsifika students together as a collective: "It makes me happy. Because everyone's connected to each other and to other cultures. If they learn some problems about their cultures everyone's connected". The importance to the students of shared cultural values, such as relationships, reciprocity and respect, were evident in the data. During a reflective interview another student explained: "I learnt about someone else's culture and they feel valued… We're learning what other people know and what you know. So, basically, you're sharing what you know". These results correspond with those described in d'Entremont's (2015) research. According to d'Entremont (2015), exposing culturally diverse students to the contributions of members of their own and other cultures helps them gain a sense of belonging, as well as respect for the mathematical thinking of other cultures.

The teacher drew on the mathematical expertise of students' tupuna (ancestors, particularly grandparents), to support students to recognise and appreciate the mathematical nature of cultural craft work, and take pride in the mathematical richness of their cultural activities. During the follow-up interviews students were able to recognise algebra in their cultural heritage, for example: "I learnt that Maori and Tokelauan use algebra. And I didn't even know. It made me feel cool because it's not usual to use algebra while making something". Students' pride in their cultural heritage led them to consider their families were good mathematicians: "My grandma's things she makes. Like how the pattern grows, she has to remember it, how it's going up, what the pattern is, how long it needs to be. My ancestors used algebra way before me".

Providing Māori and Pāsifika students with tasks that are built around their cultural knowledge and experiences allows them to be positioned as experts, resulting in increased confidence and feelings of empowerment as learners and doers of mathematics (Hunter & Hunter, 2018). During an interview a student explained:

> I never knew how to do algebra. I thought it was tricky as college problems but the difficulty is alright and it's fun. When it involves our culture once you hear it's something about your culture you're like the expert because you know about a lot of things and you're like "oh yep, this is me, I know it" and then you just like relax and have fun while you solve the problem. It gives you confidence. And it gets your brain working.

In the current study, culturally located tasks gave Māori and Pāsifika students opportunities to draw upon the mathematics embedded in their cultures to make sense of functions and develop algebraic reasoning. For example, as soon as one student saw a photo of a tukutuku panel during the launch of a task he commented: "they probably started from three triangles and kept adding", immediately identifying mathematics embedded in the cultural pattern. During an interview the teacher stated that part of the reason she believed the learning from these tasks was "important and meaningful to the kids" was because the mathematics was "so embedded in the culture and the authentic contexts. We often try to find a context to fit the maths whereas this was the maths very much fitting the context". This reflects prior research, (e.g. Wager, 2012), who found that focusing on exploring the mathematics embedded in cultural activities provided teachers with a roadmap to get beyond a superficial incorporation of culture. In other words, the context was not a setting for the task, but students were accessing the mathematics inherent within their everyday cultural and social lives.

In line with previous studies (e.g., Hunter and Miller, 2018; Miller, 2016), the data in the current study showed that non-dominant students were successful in accessing the mathematical structure of growing patterns that came from cultural contexts. The teacher launched each problem in contextual and cognitive ways, so that students could make a meaningful connection to the cultural context and the mathematics in the task. Below is an example of the teacher launching the sāsā problem:

Launching the task
Sarah: OK, break it down. Kupa what's going on with Tevita's group? Kupa: Tevita's group are practicing the sequence for the siva at Polyfest. Sarah: OK. Who can share something about siva?

Ana: It’s like a dance .
Tai: A siva, there’s not only one, and there’s always a story behind it. You know how Toa Samoa have their dance? That has a story behind it. It’s like using your strength, like this man he’s strong as and he’s confident to do anything.
Sarah: Thank you for sharing Tai. So what are you using in this dance? What’s the story with the sequence?
Mia: It’s a pattern.
Kupa: It equals the movements.
Junior: A sequence is like a trail of dance moves.
Sarah: Very clever sharing. Aroha what’s the story with the first sequence?
Aroha: You clap once, then slap two times, then you clap again.
Sarah: So what’s the story with the second one Ana?
Ana: They’re doing it two times. Clap, slap, slap, clap, slap, slap, clap.

Observation one: sāsā task

In the vignette the teacher was making the context of the problem explicit, and orienting students to the structure of the pattern. The familiarity of the sāsā movements provided students with the opportunity to access the underlying mathematical structure of the pattern, and identify the relationship between the sequence and the number of slaps and claps. During an interview a Samoan student stated:

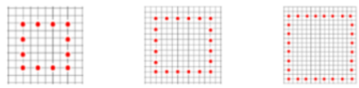
> The problem ... included our culture and what we already knew. You could do it. So people already know what to do when you’re slapping or clapping and they did it, they tried it, to figure out what the answer was. And like how many slaps or claps there were in each sequence.

When contexts are unfamiliar and irrelevant to students, learners are faced with the challenge of making sense of the context before they can access the mathematics within the task. National and international research (e.g. Hunter et al., 2019; Nasir et al., 2008; Wager, 2012) shows that drawing on student’s cultural ways of being provides a more cognitively challenging connection to mathematical concepts, and allows students to focus on deeper problem solving. The use of culturally relevant tasks therefore acted as a lever for equity by offering opportunities for non-dominant students to engage in high level algebraic reasoning and functional thinking.

4.5 Phase three: students' final understanding of functional relationships

Table 4

Participants' Functional Thinking and Representations Post-Teaching Intervention

		Contextualised task Percentage of participants (n = 12) 	**Decontextualised task** Percentage of participants (n = 12) 
Functional thinking	Recursive thinking	25%	25%
	Covariational thinking		
	Correspondence thinking	75%	75%
Generalisation	Factual generalisation	8%	17%
	Contextual generalisation	34%	25%
	Symbolic generalisation	58%	58%
Representation	Drawing		
	Using visual image to count		8%
	T-chart	20%	17%
	Multiple representations	80%	75%

The results of the post-test provides evidence that there was significant growth in students' understanding of growing patterns, and in their ability to represent and generalise functional relationships. At the pre-test all students (n=12) experienced difficulties in identifying the correspondence relationship between variables, expressing generalisations symbolically, and using multiple representations. By the post-test three quarters of the participants (n = 9) could describe how both the contextual and decontextualised patterns were growing using correspondence thinking, and over half (n = 7) were generalising both patterns symbolically. Almost all participants represented both the contextualised (n = 10) and decontextualised generalisation (n = 9) in more than one way, including t-charts, visual, natural language and symbolic representations. These results are consistent with the findings of a number of researchers (e.g. Blanton et al., 2015; Radford, 2010; Stephens et al., 2017; Wilkie, 2014), providing evidence that over a sequence of lessons young students are increasingly able to make sense of functional relationships, and represent and generalise these relationships with growing complexity.

Changes between the pre-test and post-test data shows that these Māori and Pāsifika students became more proficient and sophisticated in how they represented and generalised functional relationships, not just in the contextual tasks, but the decontextualised tasks as well. The post-test data therefore suggests that culturally contextual tasks helped non-dominant students learn both contextual and decontextualised mathematics. These results concur with other research studies (e.g. Lipka et al, 2005; Nasir et al., 2008; Wager, 2012) showing that contexts that are meaningful for students provide opportunities for them to develop new mathematical knowledge, and construct progressively more abstract understandings, as they generate formal mathematics from cultural ideas.

4.6 Summary

This chapter has presented the findings in relation to how twelve Māori and Pāsifika students generalised culturally located tasks involving functions, and the representations Māori and Pāsifika students used when engaging with contextual functional tasks. It has offered evidence that Māori and Pāsifika students are capable of sophisticated sense making of functional relationships, and representing and generalising these relationships in diverse ways. This chapter has described how shifts particularly occurred in how students were able to articulate and symbolise generalisations, from natural language descriptions of additive relationships to symbolic representations of multiplicative relationships, and use multiple representational forms to support generalisation. This chapter has offered evidence of contextual tasks providing opportunities for non-dominant students to construct abstract algebraic understandings from cultural contexts. It has described how aligning tasks with non-dominant students cultures provided opportunities for culturally diverse students to make meaningful connections to the mathematics presented in the lessons, and strengthened both cultural and mathematical identities. Finally, the post-test assessment was analysed and the shifts in functional thinking in both the contextual and decontextualised assessment tasks were presented.

In this study, four overarching themes were evident throughout the analysis: generalisations to support functional thinking; representations to support functional thinking; contextual tasks; and culturally located Māori and Pāsifika learners. The next chapter will present a summary of the themes identified, address the research questions, and provide key findings for educators to create opportunities for non-dominant students to draw upon the mathematics embedded in their cultures to make sense of functions and develop high level algebraic thinking.

Chapter Five: Conclusion

5.1 Introduction

The previous chapter presented the findings and discussion in relation to how young Māori and Pāsifika students draw upon mathematics embedded in culturally located tasks to develop algebraic understandings, and make sense of functional relationships. This chapter concludes by reviewing the main findings in relation to the research questions. Section 5.2 summarises the research questions. The key themes and recommendations are presented in section 5.3. Limitations of the study are addressed in section 5.4. Finally, section 5.5 outlines suggested areas for future research.

5.2 Summary of research questions

In order to focus on non-dominant students and the role of cultural contexts in developing functional thinking, two research questions were generated from the literature.

5.2.1 What representations do Māori and Pāsifika students use when engaging with contextual functional tasks?

A review of the research literature (e.g. Blanton et al., 2018; Cañadas et al., 2016; Cooper & Warren 2011) showed that young children are able to develop and use a variety of representational tools to help them reason with functions, and express generalisations. However, there appear to have been limited studies which have investigated the representations young non-dominant students' use when engaging with contextual functional tasks.

In the initial phase of the current study, many of the Māori and Pāsifika students had difficulty representing a decontextualised growing pattern. By contrast, all the Māori and Pāsifika students were able to construct a visual representation of the culturally located pattern, and draw or count how the pattern was growing. During the second phase of the study, contextual Māori and Pāsifika patterns were used for a series of algebra tasks focused on functional thinking. The contextual patterns for each task were specifically selected to provide the students with diverse ways to access the relationship between variables, and represent the underlying structure of the growing patterns. Through collective problem solving with the culturally embedded tasks, students constructed increasingly sophisticated ways of representing functions such as: t-charts, natural language, visual, and symbolic representations. As students' functional thinking developed they transitioned from using representations as a place to record numbers and look for patterns, to using representations as a tool to find relationships and derive a function rule. Students' representations developed from the concrete, such as a pictorial representation of the pattern, to the abstract, such as using symbolic notation for variables. Students used multiple representations and made connections across different representations, to identify, communicate and justify the functional relationship between variables. By the final phase of the study the majority of students were representing both contextualised and decontextualised patterns in multiple ways.

These Māori and Pāsifika students' increasingly sophisticated representations of functions were consistent with the learning trajectory of dominant groups of students reported in the literature (e.g. Blanton et al., 2015; Markworth, 2010; Stephens et al., 2017; Wilkie, 2014). When non-dominant students were given opportunities to draw on their cultures to make sense of functional relationships, they demonstrated significant growth in their ability to develop a deeper, richer and more flexible understanding of representing functions.

5.2.2 How do Māori and Pāsifika students generalise culturally located tasks involving functions?

The importance of generalisation to algebraic thinking is highlighted in the literature (e.g. Blanton et al., 2017; Stephens, 2017; Wilkie, 2016). Researchers (e.g. Blanton et al., 2017; Radford, 2010; Stephens, 2017; Wilkie, 2016) have identified several layers of algebraic thinking that children engage in, as they develop more abstract ways to generalise functional relationships: recursive, covariational, and correspondence thinking. Like the representation of functions, little appears to have been recorded about how non-dominant students move through these stages of mathematical generalisation.

In the current study, the majority of students initially focused on the concrete nature of growing patterns. They described how patterns were growing recursively, indicating how to obtain the next number in a sequence from the previous number or numbers. In the second phase of the study culturally located tasks provided a meaningful context for students to engage in generalisation by collaboratively exploring the underlying mathematical structure of the growing patterns. Students began to look for the multiplicative relationship between two variables, and analyse how quantities varied in relation to each other in a more generalised way. Over the series of lessons there were significant shifts in the ways these Māori and Pāsifika students identified growing patterns, mathematical structures and relationships, and expressed generalisations. The transition, from describing additive relationships using natural language to representing multiplicative relationships symbolically, conveyed the generality with which these students were thinking about functional relationships in familiar growing patterns. By the final phase of the study many of the students could express how both the contextual and decontextualised patterns were growing using a correspondence description of the relationship, and over half were generalising both the contextual and decontextualised patterns symbolically.

It was evident that there was significant growth in these Māori and Pāsifika students' conceptual understanding of growing patterns, and in their ability to generalise functional relationships with growing complexity. Contexts that were meaningful for non-dominant students provided opportunities for them to construct progressively more abstract understandings, as they generated formal mathematics from cultural ideas. This trajectory mirrored the typical development of dominant groups of students generalisations, which has been reported in the research literature (e.g. Blanton et al., 2015; Markworth, 2010; Stephens et al., 2017; Wilkie, 2014).

5.3 Key findings, implications and recommendations

This section presents a summary of the themes which were evident throughout the analysis, and provides key recommendations that have been drawn from the study. The recommendations are directed towards educators providing opportunities for non-dominant students to draw on cultural assets in order to engage in high level functional thinking.

<u>Representations to support functional thinking</u>

Contextual patterns provide non-dominant students with meaningful ways to access the functional relationship between variables, and represent the underlying structure of growing patterns in diverse ways. It is important for educators to provide opportunities for non-dominant students to draw on their cultural funds of knowledge and explore multiple ways of representing functions. As a result, non-dominant students can construct increasingly sophisticated representations, and make connections across multiple representations, to identify, communicate, and justify generalisations.

Generalisations to support functional thinking

Educators should provide culturally contextual tasks that are familiar to non-dominant students, in order to support them to make sense of the underlying mathematical structure of growing patterns, and explore the functional relationship between variables. Contexts that are meaningful for non-dominant students provide a solid foundation for constructing progressively more sophisticated generalisations, as students generate abstract mathematics from familiar cultural contexts.

Contextual tasks

The type and context of growing patterns presented and discussed in the mathematics classroom have a significant impact on the ways in which all students are provided with opportunities to develop functional understandings. Educators should specifically select culturally located patterns for tasks, to acknowledge non-dominant students cultural worlds, and support them to develop their understanding of growing patterns and functional relationships in meaningful ways. Contexts serve as an anchor for understanding, and culturally located tasks support non-dominant students to develop increasingly sophisticated understandings of the mathematical structures of both contextual and decontextualised patterns.

Culturally located Māori and Pāsifika learners

Educators must provide opportunities for all students to learn mathematics in ways they see as relevant to their cultural identities and communities. Recognising that mathematics is inherently cultural is a key equity issue for non-dominant students, who have traditionally been marginalised in mathematics. Aligning tasks with non-dominant students traditions, experiences, and worldviews provides opportunities for non-dominant students to be positioned as experts, make meaningful connections to mathematics, and strengthen both their cultural and mathematical identities.

It is important that educators consider implementing these key recommendations in their classrooms. These recommendations contribute to non-dominant students successfully engaging with algebra, and developing rich understandings of the structural form and generality of functional relationships.

5.4 Limitations of the study

As with any research study, there are limitations in the current study which must be taken into consideration when interpreting the results. The current study took place within one inquiry classroom, where collaborative problem solving and active participation in mathematical discourse were expected ways of engaging in, and making sense of, mathematics. As a result, generalisation of the findings for teachers and students of non-inquiry classrooms is limited. Given the complex nature of teaching and learning within a real classroom, the short time-frame, and the relatively small number of participants, the interpretation of the results can only provide an emerging understanding of the ways Māori and Pāsifika students draw upon culturally embedded mathematics to develop algebraic understandings and make sense of functional relationships.

5.5 Suggested areas for further research

The current study investigated the development of a group of Māori and Pāsifika student's functional thinking over a period of one school term. Given that algebraic thinking is complex and unfolds across a curriculum, a longitudinal study investigating non-dominant student's algebraic understanding over a longer period of time would be warranted. Additionally, a larger scale study with a greater number of participants would determine if the findings are applicable in other contexts and locations. The intervention in the current study was very successful in the context of an inquiry classroom. Further research might determine the impact of the teacher and the classroom culture on the ways non-dominant students engage with contextual tasks and collaboratively construct high level algebraic thinking.

None of the participants in the current study represented the functional relationship between variables graphically, indicating the potential for further development in exploring this type of representation. This would be worthwhile as graphical representations of functions are important in mathematics and in the application of algebra across curriculum areas, for example, physics and biology. A more comprehensive, sustained study could address the ways contextualised tasks support non-dominant students' to develop understanding across a range of algebraic concepts, for example, equations, equivalence, and properties of arithmetic. Furthermore, future work could explore the impact of culturally located tasks when students transition to more traditional secondary school settings. That is, does basing algebra instruction on culturally embedded tasks in the primary context increase the likelihood of non-dominant students' success in the study of more advanced mathematics, particularly algebra, in secondary school and beyond.

5.6 Final thoughts

Findings presented from this study offer a contribution to the literature regarding how culturally contextualised tasks support non-dominant students to develop their understanding of growing patterns, represent their functional thinking, and engage in the generalisation process. Prior to the study, little was known about how Māori and Pāsifika students might engage in formally (within a school setting) representing and generalising functional relationships. In the study, Māori and Pāsifika students demonstrated an aptitude for representing and generalising both linear and non-linear functions, and made equivalent progress to dominant groups of students reported in the literature. This contradicts non-dominant students' performance on national and international measures of mathematics achievement. The use of culturally relevant tasks acted as a lever for equity by blurring the line between cultural knowledge and content knowledge, and provided opportunities for non-dominant students to engage in high level algebraic reasoning and functional thinking. To address disparities and structural inequities in mathematics education, it is crucial for educators to challenge deficit theories problematising student's cultures, acknowledge that students bring their own cultural knowledge and strengths to the classroom, and empower all students to learn mathematics in ways they see as meaningful to their cultural identities, their families, and communities.

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Appendices

Appendix A1: Semi-structured initial and final interview

Hello, I am Bronwyn. Thank you for coming to talk with me. Today I'll be asking you some questions about your culture and your experiences with mathematics. Is it ok with you if I record our conversation? All of our conversation is confidential so I won't report anything from this interview to your teacher or other people you know. I'm doing this as part of a research project that looks at how students solve algebra problems. I'm hoping that we can find ways to make learning mathematics better for lots of different students, so I'm glad you are able to participate. Remember though if you change your mind we can stop the interview at any time. Do you have any questions before we begin?

1. Do you think mathematics is important to know? What are some things mathematics is important for? Why?
2. Can you tell me about your culture?
3. How do you feel being your culture in your mathematics classroom?
4. Where do you see mathematics in your culture? Where do you use mathematics in your culture?
5. Are these things used in your mathematics classroom?
6. If yes, how?
7. When your teacher uses problems in mathematics that relate to your culture how does it make you feel?
8. Do you think problems related to your culture help you understand mathematics better? Why?

Thank you for helping me today. Is there anything else you want to say or any questions you want to ask?

Appendix A2: Task based interview

Thank you for coming to talk with me. Today I'll be asking you some questions about the problem that you worked on in your mathematics lesson. Is it ok with you if I record our conversation? All of our conversation is confidential so I won't report anything from this interview to your teacher or other people you know. I'm doing this as part of a research project that looks at how students solve algebra problems. I'm hoping that we can find ways to make learning mathematics better for lots of different students, so I'm glad you are able to participate. Remember though if you change your mind we can stop the interview at any time. Do you have any questions before we begin?

1. Can you explain how you solved this problem in maths today?
2. Can you describe how the pattern is growing? Did you find a rule for how the pattern is growing? How do you know?
3. What representations did you use? Why? How did that help you?
4. How did you feel about the task? Did the context of the task help you solve the problem? How?

Thank you for helping me today. Is there anything else you want to say or any questions you want to ask?

Appendix B: Record sheet used for classroom observations

Date:	Observational notes	Reflective comments
Engagement with context		
Using context to access mathematical structure		
Functional thinking		
Recursive (single sequence of values e.g. + 2)		
Covariational (vary in relation to each other: specific cases: “every time you add a table you add 2 more people”)		
Correspondence(identifies generalised relationship between the two variables)		
Generalisation		
Factual (concrete to calculate a variable for a particular instance)		
Contextual (language driven, descriptive)		
Symbolic (relational, algebraic notation, generality expressed)		
Representations		
Representations as process (supporting thinking / reasoning) E.g.: materials, gesture, pictures, numerical, words, symbols, tables, graphs		
Representations as product (proving / justification) E.g.: materials, gesture, pictures, numerical, words, symbols, tables, graphs		
Connections		

Appendix C: Assessment tasks

A group of Mamas are working on a tivaevae design.

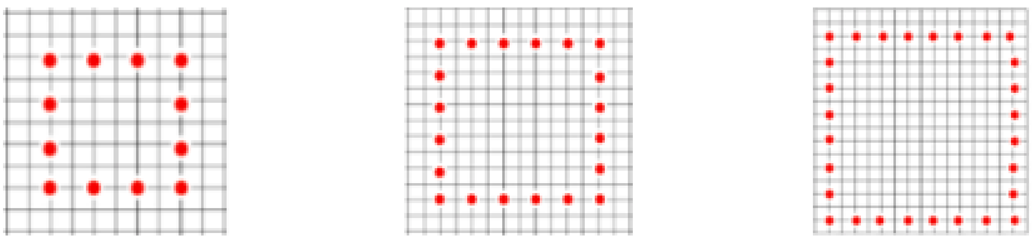
This is the 1st position This is the 2nd position This is the 3rd position

Look at this pattern and think about how it is growing.

How many leaves would it have for the 7th position? What about the 17th position? What about the 76th position?

Show how the pattern grows using a table, ordered pairs and / or a graph.

Write the rule for the pattern in words, numbers, or symbols.



1st position 2nd position 3rd position

Look at this pattern and think about how it is growing.

How many dots would it have for the 7th position? What about the 17th position? What about the 76th position?

Show how the pattern grows using a table, ordered pairs and / or a graph.

Write the rule for the pattern in words, numbers, or symbols.

Appendix D1: Task one: sāsā

Tevita's group are practicing the sequences for their siva at Polyfest.

The first sequence is: clap, slap, slap, clap
The second sequence is: clap, slap, slap, clap, slap, slap, clap
The third sequence is: clap, slap, slap, clap, slap, slap, clap, slap, slap, clap

If the sequence keeps on going how many claps and slaps will there be in the 11^{th} sequence? What about the 29^{th}? What about the 83^{rd}?

What's a rule that tells us now many claps and slaps there are no matter how long the siva sequence is? Can you show this rule in words and numbers?

How can you show the ways the pattern grows? Can you use pictures? A table? A graph? A diagram?

Appendix D2: Task two: ngatu

Ngatu is Tongan tapa cloth. In Samoa the same cloth is called siapo and in Niue it is hiapo. Ngatu can sometimes tell a story, using symbols from nature and geometric patterns. Numbers written on the tapa indicate its langanga, or length.

How many stems and leaves would there be if this ngatu pattern was 8 langanga long? What about 50 langanga?

Show how the pattern grows in multiple ways.

What is a rule that tells us now many stems and leaves there are no matter how many langanga long the ngatu is?

Appendix D3: Task three: vaka

The vaka at Matauala Hall has a pattern where the fauato (coconut fibre twine) joins the planks. What would happen if the vaka kept getting longer and longer. How many pieces of fauato would there be if the pattern went up to iva (nine)? What about hefuluiva (nineteen)? What about ivahefulu (ninety)?

Tahi 

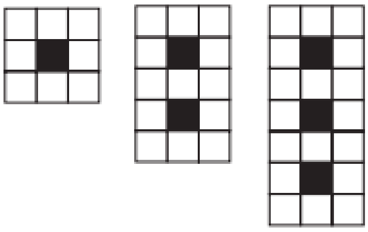
Lua 

What's a rule you could use to find the number of pieces of fauato for any number of the pattern?

Use as many different representations as you can to show your thinking.

Appendix D4: Task four: titi

Titi (dance skirts) from Tuvalu have bright colours and eye-catching designs.



If this was going to be the design on the wide strip of a titi, how many white and how many black squares would be in the fifth position? What about the 85th? What about the 99th?

How could you work out how many white and black squares for any position in the pattern?

Show how the pattern grows in as many different ways as you can.

Appendix D5: Task five: tukutuku panel

Tukutuku panels can be made from kaho (wooden rods) with tuinga (cross stitches). This kaokao is a traditional design that symbolises the strength of a warrior.

Imagine you continued this pattern until there were 9 kaho. How many tuinga would there be? What if there were 18 kaho? What if there were 38 kaho?

What's a rule you could use to find the number of tuinga needed for any number of kaho?

Represent your group's thinking in as many different ways as you can.

Appendix D6: Task six: tivaevae

The Porirua Mama's tivaevae group meet at Te Akapuanga Hall to sew tivaevae every week. Every Mama has their own special design, and every tivaevae tells a story. Mama Donut wants to sew a tivaevae for her grandson. She has decided to make a fern pattern.

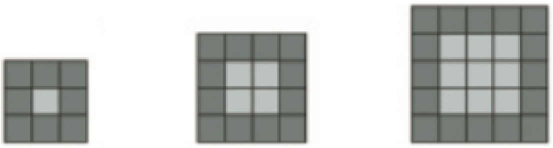
First position | Second position | Third position

If Mama Donut's pattern keeps growing how many leaves will there be in the 15th position? What about the 23rd position?

What's a rule that tells us now many leaves there are no matter how what position the tivaevae pattern is?

Show how the pattern grows in as many ways as you can.

Appendix D7: Task seven: fala



Tasi Lua Tolu

Talia was looking at the border of her mat. She thought she could find a way to figure out how many squares there would be in any part of the pattern.

If Talia wanted to figure out a rule for how many border squares there would be at any number of the pattern what could she do?

What about a rule for the squares in the middle?

Show your groups thinking in as many ways as you can.

Appendix D8: Task eight: kapa haka

Mātua and Whaea are coming up with designs for Te Haeata Awatea kapa haka uniform. They like this triangular pattern.

If the pattern kept growing, how many black triangles would pattern 7 have? What about pattern 17? What about pattern 27?

How could Mātua and Whaea work out how many black triangles any pattern number would have? Can you show this rule in words and numbers?

How can you show the ways the pattern grows? Can you use pictures? A table? A graph? A diagram?

Appendix E1: Thematic analysis table used to group codes into themes

Theme	**Generalisations to support functional thinking**	**Representations to support functional thinking**	**Contextual tasks**	**Culturally located Māori and Pāsifika learners**
Codes	recursive thinking covariational thinking correspondence thinking factual generalisation contextual generalisation symbolic generalisation role of variable mathematical connections	visual representation T-chart rule in natural language rule in symbols multiple representations	mathematics embedded in context seeing the pattern in multiple ways structure of the pattern identifying and articulating relationships challenging tasks constructing new mathematical understandings	launching the task Māori / Pāsifika values cultural assets cultural connections to mathematics building cultural knowledge feelings about culture in mathematics class cultural connections with others

Appendix E2: Thematic analysis

Illustrative excerpts grouped in table of themes (interview transcripts)

Theme	Initial interviews	Task based interviews	Final interviews
Generalisations to support functional thinking	This one's 12. This one's 2, 4, 6, 8, 10, 12, 14, 16, 18, 20. This one's 28. The next one would have 36. I used my fingers and counted by 8. I used 12 then counted up to 20. Then plus 8 then plus 8 then plus 8 again. I keep on adding 8. [PW]	Every time you move to the next pattern the leaves grow by 8 and the stems stay at 4.[RC]	You do position number, so for example one, times it by how much it's adding – going up – which is eight. And once you've done that you add four. The rule is position times eight plus four [JS]
Representations to support functional thinking	 When I draw it I can count how it's adding up. It adds on one leaf on every corner every position.[RP]	I: So X did you use a t-chart and the drawing? X: I looked at the drawing first so I knew what it was adding and what it was about. And I used a t-chart to get the answer to the problem. [XD]	I saw it was adding eight and that it was position times 8. So position times 8 plus 4 equals y. *Y* is the answer, but it represents anything. [AF]
Contextual tasks	I: Which pattern do you think was easiest to figure out how it was growing? R: Tivaevae. The numbers are more obvious. You could just look at it and see - three, three, three, three … The pattern was a lot clearer for me to see. [RS]	The six came from how the pattern was growing. And the three was from the first one (pointing). Like what Z said there was always an odd one out in the titi pattern. [XD]	I think I've improved a lot at maths. In everything. Before once I was in the pit I just gave up. But now it's like trying to figure out a rule and using other rules. The problem helps me want to keep going. It's interesting and it makes you want to know the answer because the problem makes you, there's always one thing you want to know. It's not telling the whole thing and you want to know the whole story. And once you know it you feel like you've finished something. [ZT]
Culturally located Māori and Pāsifika learners	I: So when your teacher uses a problem like the Matariki one do you think that helps you understand the maths better? X: Not really because it's just telling me knowledge that I might already know. They're kind of separate. The equation is like maths [XD]	I: So do you think when the problem's in a cultural context like that it helps you figure out the algebra? J: Yes it helps us because that might be your culture and you know a lot about it. Like when you know about it you'll feel confident and then you're loaded with ideas. [JS]	When Kate pulled out the dress yes, because I was like "I've worn that to do kapa haka". And it was just cool! It is cool. Whenever I do maths about my culture it feels like my culture means something to this school. And I'm happy because this is New Zealand, it's Aotearoa.[RR]

Appendix F1: Principal consent form

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MĀTAURANGA

CONSENT FORM: PRINCIPAL

THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

Developing functional thinking through culturally located tasks

I have read the information sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

PRINCIPAL CONSENT

We agree / do not agree (circle one) for Kate Collins and Room 11 students to participate in this study under the conditions set out in the information sheet.

Date:

Principal Signature:

Full Name - printed:

Appendix F2: Board of Trustees consent form

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MĀTAURANGA

CONSENT FORM: BOARD OF TRUSTEES

THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

Developing functional thinking through culturally located tasks

We have read the information sheet and have had the details of the study explained to us. Our questions have been answered to our satisfaction, and we understand that we may ask further questions at any time.

BOARD OF TRUSTEES CONSENT

We agree / do not agree (circle one) for Kate Collins and Room 11 students to participate in this study under the conditions set out in the information sheet.

Date:

Board Chairperson Signature:

Full Name - printed:

Principal signature:

Full Name - printed:

Appendix F3: Student participant consent form

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MĀTAURANGA

CONSENT FORM: STUDENT PARTICIPANT

THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

Developing functional thinking through culturally located tasks

I have read the information sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

STUDENT CONSENT

I agree / do not agree (circle one) to being video-recorded during mathematics lessons

I agree / do not agree (circle one) to completing video-recorded interviews about my mathematics lessons

I agree / do not agree to participating in this study under the conditions set out in the information sheet

Date:

Student name:

Student Signature:

Appendix F4: Parents of student participants consent form

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MĀTAURANGA

CONSENT FORM: PARENTS OF STUDENT PARTICIPANTS

THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

Developing functional thinking through culturally located tasks

I have read the information sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

PARENT CONSENT

I agree / do not agree (circle one) to ___________________________________ completing video-recorded interviews about their mathematics lessons

I agree / do not agree (circle one) to ___________________________________ being video-recorded during mathematics lessons

I agree / do not agree (circle one) to ___________________________________ participating in this study under the conditions set out in the information sheet

Date:

Parent / Caregiver name:

Parent signature:

Appendix F5: Teacher participant consent form

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MĀTAURANGA

CONSENT FORM: TEACHER PARTICIPANT

THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

Developing functional thinking through culturally located tasks

I have read the information sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

TEACHER CONSENT

I agree / do not agree (circle one) to be video-recorded

I agree / do not agree to participate in this study under the conditions set out in the information sheet

Date:

Teacher name:

Teacher Signature:

Appendix G1: Student and parent research information sheet

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MĀTAURANGA

Developing functional thinking through culturally located tasks

STUDENT AND PARENT INFORMATION SHEET

Dear ____

I am doing a research project for a Master of Education at Massey University. I am going to investigate how Māori and Pāsifika students learn algebra through mathematics problems that are set in a cultural context.

I would like to invite you with your parent's permission to be part of this research project. The Board of Trustees has also given me their approval to do this research, and to invite you to participate.

If you agree to be involved I will do two assessment tasks, one at the beginning of Term 4 and one at the end of Term 4. The assessment tasks are to understand what you already know about growing patterns, and then what you have learnt. I will also interview you about your experiences with mathematics at the beginning of Term 4 and at the end of Term 4. The time involved will be no more than 30 minutes. The interviews will be video recorded and you may ask that the camera be turned off and that any comments you have made be deleted if you change your mind or are not happy about what you said.

The algebra lessons will be taught in your classroom and will be video recorded. During classroom mathematics activities you may at any time ask that the video recorder be turned off and any comments you have made deleted. With your permission I might sometimes interview you to find out how you have thought about the problem, collect copies of your written work and charts you make to support your explanations to the group during the unit. You have the right to refuse to allow the video or copies of your work to be taken.

The mathematics activities you do in class will be the same whether you agree to be in the study or not. The interview and observations will take place in the classroom and be part of the normal mathematics programme. It is possible that talking about your learning may help you clarify what you know about algebra and what you need to know next.

All data will be stored in a secure location, with no public access and used only for this research. In order to maintain anonymity the school name and names of all children will be assigned pseudonyms in any publications arising from this research. At the end of 2020, a summary of the study will be provided to the school and made available for you to read.

Please note that you have the following rights in response to the request to participate in this study:

- To say you do not want to participate in the study
- Decline to answer any particular question in interviews
- In any interview or video observation have the right to ask for the video tape to be turned off at any time
- Withdraw from the study at any point
- Ask any questions about the study at any time during participation
- Provide information on the understanding that your name will not be used unless you give permission to the researcher
- Be given access to a summary of the project findings when it is concluded

If you have further questions about this project you are welcome to discuss them with me personally:

Bronwyn Gibbs
Phone: 02102733472
Email: b.e.gibbs@massey.ac.nz

Or contact my supervisors at Massey University (Albany)

Doctor Jodie Hunter: Institute of Education
Phone: (09) 414 0800 ext. 43518
E-mail: J.Hunter1@massey.ac.nz

Professor Roberta Hunter: Institute of Education
Phone: (09) 414 0800 ext. 43530
E-mail: R.Hunter@massey.ac.nz

This project has been evaluated by peer review and judged to be low risk. Consequently, it has not been reviewed by one of the University's Human Ethics Committees. The researcher named in this document is responsible for the ethical conduct of this research.
If you have any concerns about the conduct of this research that you want to raise with someone other than the researcher, please contact Professor Craig Johnson, Director – Ethics, telephone 06 3569099 ext 85271, email humanethics@massey.ac.nz.

Appendix G2: Board of Trustees research information sheet

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MĀTAURANGA

Developing functional thinking through culturally located tasks

BOARD OF TRUSTEES RESEARCH INFORMATION SHEET

I am doing a research project for a Master of Education at Massey University. I am investigating the representations Māori and Pāsifika students use when engaging with contextual tasks, and how Māori and Pāsifika students generalise culturally located tasks involving functions in algebra.

The research will involve students doing two short assessment tasks and interviews at the beginning and end of Term 4. Kate will teach an algebra unit across 8 lessons. The mathematics lessons will take place in the classroom and be part of the normal mathematics programme.

I am writing to formally request your permission to:

- Allow students to participate in video recorded interviews about both their experiences of mathematics and how they have worked on these problems
- Allow students to complete an assessment task at the start and end of the project
- Allow students to have some of their mathematics lessons video-recorded in the classroom

All data will be stored in a secure location, with no public access and used only for this research. In order to maintain anonymity the school name and names of all children will be assigned pseudonyms in any publications arising from this research. At the end of 2020, a summary of the study will be provided to the school and made available for you to read.

Please note that you have the following rights in response to the request to participate in this study:

- Decline to participate
- Withdraw from the study at any point
- Ask any questions about the study at any time during participation
- Provide information on the understanding that your name will not be used unless you give permission to the researcher
- Be given access to a summary of the project findings when it is concluded

If you have further questions about this project you are welcome to discuss them with me personally:

Bronwyn Gibbs
Phone: 02102733472
Email: b.e.gibbs@massey.ac.nz

Or contact my supervisors at Massey University (Albany)

Doctor Jodie Hunter: Institute of Education
Phone: (09) 414 0800 ext. 43518
E-mail: J.Hunter1@massey.ac.nz

Professor Roberta Hunter: Institute of Education
Phone: (09) 414 0800 ext. 43530
E-mail: R.Hunter@massey.ac.nz

This project has been evaluated by peer review and judged to be low risk. Consequently, it has not been reviewed by one of the University's Human Ethics Committees. The researcher named in this document is responsible for the ethical conduct of this research.
If you have any concerns about the conduct of this research that you want to raise with someone other than the researcher, please contact Professor Craig Johnson, Director – Ethics, telephone 06 3569099 ext 85271, email humanethics@massey.ac.nz.

Appendix G3: Teacher research information sheet

MASSEY UNIVERSITY
INSTITUTE OF EDUCATION
TE KURA O TE MĀTAURANGA

Developing functional thinking through culturally located tasks

TEACHER RESEARCH INFORMATION SHEET

Dear Kate,

As you know I am doing a research project for a Master of Education at Massey University. My thesis is investigating the representations Māori and Pāsifika students use when engaging with contextual tasks, and how Māori and Pāsifika students generalise culturally located tasks involving functions in algebra.

I am formally inviting you to be a part of a collaborative teaching design based research process in which we look at the ways children represent and generalise their functional thinking using tasks set in a cultural context. I would like to work with you to plan and teach a series of lessons designed to develop students functional thinking.

The children and their parents / caregivers will be given full information, and permission to participate in the study will be sought from both the parents of the children in your class and the children themselves.

If you agree to be involved I will do two assessment tasks with your students, one at the beginning of Term 4 and one at the end of Term 4. The assessment tasks are to understand what students already know about growing patterns, and then what they have learnt. I will also interview students about their experiences with mathematics at the beginning of Term 4 and at the end of Term 4. The time involved for each child for each interview will be no more than 30 minutes. The interviews will be video recorded.

We will plan a unit of mathematics which you will teach. The lessons will be taught in your classroom and will be video recorded. During classroom mathematics activities you may at any time ask that the video recorder be turned off and any comments you have made deleted. The time involved in the complete study for you will be no more than thirty-five hours, over the period of one school term.

All data (electronic audio files and surveys) will be stored in a secure location, with no public access and used only for this research. In order to maintain anonymity the school name and names of all children and teachers will be assigned pseudonyms in any publications arising from this research. Near the end of the study a summary will be presented to you to verify accuracy, and following any necessary adjustments, a final summary will be provided to the school and yourself.

Please note that you have the following rights in response to the request to participate in this study:

• Decline to participate
• Decline to answer any particular question
• In any interview or video observation have the right to ask for the audio/video tape to be turned off at any time
• Withdraw from the study at any point
• Ask any questions about the study at any time during participation
• Provide information on the understanding that your name will not be used unless you give permission to the researcher
• Be given access to a summary of the project findings when it is concluded

If you have further questions about this project you are welcome to discuss them with me personally:

Bronwyn Gibbs
Phone: 02102733472
Email: b.e.gibbs@massey.ac.nz

Or contact my supervisors at Massey University (Albany)

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