Enacting Challenging Tasks: Maximising Opportunities for Students' Mathematical Learning

A thesis presented in partial fulfilment of the

requirements for the degree of

Master of Education

at Massey University, Palmerston North, New Zealand

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2016

Abstract

Three teachers of year 7 and 8 learners explored pedagogical approaches that exemplified current research on maximising opportunities for students to engage with and learn from challenging mathematics tasks. This study examined the learning opportunities afforded by the task enactments in the teachers' classrooms. The study also considered teachers' perspectives on a planning and lesson structure that exemplified explored approaches, and the challenges teachers experienced in implementing the tasks and approaches.

Reforms in mathematics education that have called for change in how teachers view mathematical knowledge, the value and purpose of social interaction in the classroom, and teachers' role as participants in classroom discourse, have influenced pedagogical approaches to the enactment of classroom tasks. Relevant literature was reviewed that illustrated the importance of tasks in affording opportunities for students to engage in meaningful mathematical practices and discourse, and construct conceptual mathematical understanding. Evidence was provided that teachers' pedagogical decisions and actions play a significant role in optimising opportunities for student learning from tasks, and that teachers' task implementations are mediated by their intentions, goals, knowledge, attitudes and beliefs.

The qualitative methodology chosen for this study aligned with case study and design-based research approaches. Multiple data sources were collected, and systematic analysis and triangulation of data alongside collaboration between the researcher and participant teachers strengthened the research findings.

The study revealed the influence of task selection on the type of mathematical activity afforded value in classrooms. The planning template and lesson structure prompted purposeful decision-making that strengthened teachers' task enactments, including explicit consideration of mathematical ideas inherent in tasks, students' prior understandings, and the role of task variations in supporting students' access to tasks. The study demonstrated that different enactments from the same planning resulted in contrasting opportunities for student learning. A noteworthy difference was the extent to which the mathematical ideas inherent in the task were explicitly addressed by teachers.

The results revealed the impact of teachers' decisions when selecting and implementing classroom tasks, and offered insights into purposeful pedagogical actions that teachers could incorporate into their practice to maximise opportunities for their students to learn mathematics.

Acknowledgements

I would like to acknowledge and thank the people who made this study possible. Thank you to my principal and Board of Trustees for their encouragement and support not only this year, but over the four years leading up to this research project. They have consistently encouraged me to work towards a vision for ambitious mathematics education practices in our school, to take up opportunities to extend my own practice and understanding, and supported me to take on further study.

I wish to acknowledge and thank my main supervisor, Professor Glenda Anthony, who provided positive and generous support, as well as invaluable professional advice and input into this study. My thanks extend to Dr Jodie Hunter, my second supervisor, for her supportive professional input and feedback.

Most importantly I wish to thank the teachers who participated in the study. They bravely opened their classrooms and their teaching practice to scrutiny, and willingly committed time and energy to challenging their practice. I would also like to thank the students in the classrooms whose enthusiasm for mathematics learning sustains my passion for teaching, and who were a constant reminder of why this study was worth doing.

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Chapter 1: Introduction

1.1 Background to the Study

There is widespread agreement that teachers' selection and implementation of classroom tasks significantly influences students' development of conceptual mathematical understanding (e.g., Anthony & Walshaw, 2007; Blackwell, Trzesniewski, & Dweck, 2007; Boaler, 2016; Middleton & Jansen, 2011; Stein, Smith, Henningsen, & Silver, 2009; Sullivan, Clarke, & Clarke, 2013; Sullivan, Walker, Borcek, & Rennie, 2015). Carefully chosen tasks based on cogent and significant mathematics play an important role in motivating and activating student thinking, promoting mathematical discourse and reasoning, fostering connections between important mathematical ideas, and developing an orientation to persistence (Anthony & Walshaw, 2007; Blackwell et al., 2007; Middleton & Jansen, 2011; Sullivan & Davidson, 2014). The choice of tasks determines the mathematical content students learn, their perceptions of their potential to make sense of mathematics, and their view on the nature, worth and relevance of mathematical activity (Anthony & Walshaw, 2009; Sullivan, Clarke, & Clarke, 2013).

> Day-in and day-out, the cumulative effect of students' experiences with instructional tasks is students' implicit development of ideas about the nature of mathematics – about whether mathematics is something they can personally make sense of, and how long and how hard they should have to work to do so. (Stein et al., 2009, p. 1)

Studies have established links between gains in student achievement and the adoption of inquiry-based approaches using rich tasks (e.g., Boaler & Staples, 2008; Stein & Lane, 1996). From a five-year project analysing task implementation in hundreds of classrooms Stein et al. (2009) reported that students' learning gains were greatest in classrooms where tasks consistently encouraged high-level student thinking and reasoning, and least where tasks were consistently procedural.

When implementing rich tasks in classrooms, specific pedagogical actions of teachers optimise opportunities for student learning. The Third International Mathematics and Science Study (TIMMS) 1999 Video Study of lessons from Year 8 classrooms made comparisons of a number of teaching dimensions including "the ways classrooms are organised in different countries, the kinds of mathematics problems presented to students, and the ways problems are worked on during classroom lessons" (Hiebert & Stigler, 2004, p. 11). No significant difference was reported between higher and lower performing countries in the relative emphasis given to problems designed to teach skills and those designed to give students opportunities to develop a connected understanding of mathematical ideas. However, a compelling difference was noted in teachers' implementations of tasks. In higher performing countries teachers were more likely to implement tasks in ways that gave students opportunities to illuminate the connections and relationships embedded in tasks, whereas in less high performing countries the teachers "almost always stepped in and did the work for the students or ignored the conceptual aspect of the problem when discussing it" (Hiebert & Stigler, 2004, p. 12).

Mathematical tasks with high-level cognitive demands are difficult to implement well. Difficulties and reluctance to implement high-level tasks are related to teachers' intentions for the task, their mathematical knowledge, and their beliefs regarding learning mathematics (Sullivan, Clarke, & Clarke, 2013). During task enactment there is a tendency for tasks to be transformed by teachers' use of traditional teaching routines into less demanding tasks thus limiting their learning potential (Stein, Grover & Henningsen, 1996), or for teachers to over-explain tasks in an attempt to make them more accessible to struggling students (Hiebert & Stigler, 2004).

In the intermediate years of schooling additional factors come into play that can affect the successful adoption of challenging tasks and associated instructional practices. The mathematics concepts that students (and teachers) are expected to understand become more complex and can challenge teachers' own mathematical knowledge, fuelling a tendency to adopt traditional procedural teaching approaches (Sullivan, Walker et al., 2015; Young-Loveridge, 2007).

Reforms introduced in New Zealand through the widespread implementation of the National Numeracy Development Project (NDP) explicitly advocated the importance of multiple solution strategies for mathematical problems. However, an unintended interpretation of the NDP materials led to endorsement of essentially procedural instruction of teacher-selected calculation strategies as quasi-algorithms (Young-Loveridge, 2010). An inclination to procedural instruction is also driven by concerns that students who have not mastered the basics from the primary curriculum will not be able to access higher-level tasks and instead require a programme concentrating on simplification and repetition (Sengupta-Irving, 2016).

The use of ability groupings within or across classes is prevalent in New Zealand intermediate schools, even though there is little or no evidence to support that these practices support students' learning (Education Review Office, 2013). Students in the resulting 'low ability' groups are commonly denied access to rich learning experiences (e.g., Boaler & Sengupta-Irving, 2016; Clarke, Cheeseman, Roche, & van der Schans, 2014). This exacerbates adolescents' already heightened consciousness of social comparisons and ability self-awareness during a critical time for the formation of their perceptions of themselves as learners (Blackwell et al., 2007).

Adopting teaching approaches that use challenging tasks, that emphasise reasoning and problem solving, and that support learners to choose their own solution paths requires teachers to challenge their own mindsets. It demands that they seize opportunities to practise implementing rich tasks in ways that emphasise mathematical reasoning, problem solving, and collaboration. International studies have shown that by focusing on the details of teaching practice as an object of study, challenging mathematical tasks can be enacted in ways that promote the development of a rich connected understanding of big mathematical ideas for all students. These details include how teachers pose problems, how they interact with students about mathematical content, how they work on problems with students, and how they engage with students sharing and justifying solution strategies. This study aimed to build on this research within a New Zealand context.

1.2 Research Objectives

The purpose of this study was to explore how teachers can maximise the mathematical opportunities of cognitively demanding tasks in ways that benefit student learning. The study involved supporting three teachers of Year 7 and 8 students to enact challenging mathematical tasks in ways that promoted the development of a rich connected understanding of big mathematical ideas. As part of the support the study explored the characteristics of appropriately challenging tasks and pedagogical actions that teachers use to encourage students to persist and to engage with mathematical practices that develop conceptual understanding.

The study addressed the following research questions:

1. How do teachers' enacted pedagogies occasion opportunities for students to engage with and learn from cognitively demanding tasks?
2. What challenges do teachers experience in implementing cognitively demanding tasks?
3. Which elements of a planning approach and recommended lesson structure do teachers find helpful in implementing cognitively demanding tasks?

1.3 Overview

Chapter 2 reviews literature that provides a theoretical lens through which this study can be viewed. Perspectives on mathematics teaching and learning that inform particular approaches to classroom tasks are reviewed, followed by studies on mathematical tasks and their enactment in classrooms. Pedagogical actions that maximise students' opportunities to learn from their work on tasks are outlined.

Chapter 3 describes the research methodology used in this study. Data collection and analysis methods are discussed, the research setting and schedule are described, and ethical considerations are reviewed.

The results of the study are presented and discussed in chapters 4 to 8. Chapter 4 outlines the support for teachers' provided by the study. Chapters 5 to 7 document the task implementations in the participant teachers' classrooms. Chapter 8 reviews the teachers' perspective on the recommended approaches. Chapter 9 presents the conclusions of the study.

Chapter 2: Literature Review

2.1 Introduction

There are fundamental assumptions about the nature of mathematics and mathematics teaching and learning that underpin this research and inform particular approaches to classroom tasks. Section 2.2 of this literature review examines theoretical perspectives on mathematics, teaching and learning. It outlines the nature of mathematical activity, and current theories of learning that indicate the orientation of classrooms as communities of mathematical inquiry. Section 2.3 examines the role of students' disposition to persist with cognitively demanding tasks.

Section 2.4 examines the research on classroom mathematics tasks. It outlines the literature on task purposes, types and frameworks including aspects of tasks that are associated with maximising opportunities for students' mathematical thinking. The role of big mathematical ideas in classroom tasks is reviewed in section 2.5, and the role of teachers' knowledge is considered in section 2.6.

Section 2.7 reviews studies on the enactment of mathematical tasks in classrooms and outlines pedagogical approaches that support students' learning of mathematical ideas. Section 2.8 outlines constraints that can influence teachers' use of tasks. The literature on differentiating learning when using demanding tasks with diverse learners is reviewed in section 2.9.

2.2 Perspectives on Mathematics, Teaching and Learning

Many students in New Zealand schools are not successful in mathematics, and their lack of success is customarily attributed to their lack of inherent 'mathematical ability'. Many teachers view mathematics as an indisputable static body of knowledge and procedures to be memorised. Associated traditional teaching approaches of teacher procedural demonstration followed by multiple items for individual student practice provide succour for this perspective.

2.2.1 The Nature of Mathematics

An alternative notion of the nature of mathematics incorporates the activities and practices of working mathematicians (Boaler, 2002; Dossey, 1992; Hersh, 1997). Holton (2010), a working mathematician, supports this view:

> Mathematics is a way of looking at the world and trying to sort out its problems. It seems to have presently accumulated an enormous number of facts, ideas and theorems. But, despite the way that maths is still taught in almost every classroom in the world, it is more than a collection of algorithms that, for some unknown reason appear to have to be known. There is also a creative side to the subject – a place where new maths suddenly appears, sometimes even miraculously. And the thing we have tried to suppress from the general public is that it is created by human intervention. (p. 21)

Teachers' conceptions of mathematics have a significant influence on the way in which mathematics is characterised and communicated in their classrooms (Boaler, 2002; Dossey, 1992). Teachers' knowledge, attitudes and beliefs in turn inform decisions they make when selecting instructional material (Sullivan, Clarke, Michaels, & Mornane, 2012). The prevailing procedural view is characterised by the propensity of teachers to deliver a programme of "routine unrelated mathematical tasks which involve the application of learnt procedures and by stressing that each task has a unique, fixed and objectively right answer" (Ernest, 1991).

Proponents of reform in mathematics education advocate a different approach based on the understanding of mathematics as a "personally constructed or internal set of knowledge" (Dossey, 1992, p. 22). The emphasis in this perspective is on the doing of mathematics and the practices involved in that doing. The activity of learning mathematics is a vehicle for acquiring mathematical knowledge, but as opposed to a traditional perspective that would regard the activity as distinct from the mathematical knowledge that is developed, more recent theories consider that "the practices of learning mathematics define the knowledge that is produced" (Boaler & Greeno, 2000, p. 172). Although the goal for students in a traditional or reform classroom may be to learn similar mathematical content, students will have learned different forms of knowledge mediated by the beliefs that they develop about the nature of mathematics and learning in response to different teaching approaches.

Mathematical knowledge and understanding is also mediated by collaboration, social interaction and context (Boaler & Greeno, 2000). Social theories of learning position learners and learning within their social and cultural contexts.

2.2.2 Social Theories of Learning

Notions of constructivism began with Piaget, who viewed learners as information processors and knowledge as "that produced by the learner's use of cognitive processes" (Walshaw, 2007, p. 29). Piaget argued that learners construct knowledge as they make connections between new and existing experiences that occur when they interact with the environment. He contended that learning is the result of disequilibrium caused by exposure to cognitive conflict or ambiguity. Learners seek to maintain a state of cognitive equilibrium so must reorganise their cognitive structures to achieve a new state of equilibrium. (St George & Sewell, 2014; Walshaw, 2007).

Later constructivists have placed more emphasis on the learners' *active* construction of knowledge. Anthony (1996) argued that *active learning* refers to a learning environment in which the students are given a sense of autonomy and control over the direction of their learning activities, and also indicates "a quality of the pupils' mental experience in which there is *active* intellectual involvement in the learning experience characterised by increased insight" (p. 350). If teachers adhere to the first interpretation only, and provide their students with open-ended tasks, and hands-on activities, successful construction of meaningful knowledge will not necessarily result (Anthony, 1996). The second interpretation of *active learning* suggests that effective learning tasks need not only to be hands-on but also to be minds-on, and that the teacher plays an important role in upholding a high press for conceptual thinking (Anthony, 1996; Taber, 2011). Taber (2011) described teaching informed by constructivist learning theory as optimally guided instruction, a balance between explicit teacher strategies including providing suitable learning tasks, and scaffolding and monitoring student-centred activity.

Social constructivist or socio-cultural theories, originating in the work of Lev Vygotsky, argued that knowledge acquisition is fundamentally a social rather than cognitive process. Vygotsky maintained that while taking place in individual minds, all learning results from social interaction, meaning is socially constructed through communication, activity and interactions with others, and learning is mediated through cultural tools in particular through language (Simon, 1995; Swan, 2005). An important concept in Vygotsky's theory of learning is the zone of proximal development. The zone of proximal development (ZPD) is described as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86).

The notion of teaching as scaffolding or responsive guided assistance helps to explain how socio-cultural theory is enacted in classrooms (St. George & Sewell, 2014). If a learner is attempting a task that is beyond their current level of expertise, but within their ZPD, then scaffolding such as modelling, questioning or fostering connections can be provided so that the learner can achieve the task with support (Taber, 2011). Lerman (1998) argued that the zone of proximal development lies within learning activity and that the task prompts that activity. He proposed that the zone might be aligned with the group or classroom, rather than exclusively with the individual learner. Featherstone et al. (2011) suggested that group-worthy tasks that fall into the zone are those where no one in the group can complete the task alone but students can make progress if they collaborate. "Learning occurs as students work in the space in which they can succeed only with the aid of others and become able to perform tasks independently for which they earlier needed help" (p. 109). Tasks that enable this type of learning afford multiple entry points and pathways through the task, and offer learners with varying prior knowledge opportunities to contribute and to use each other as intellectual resources (Sullivan, Mousley, & Jorgensen, 2009).

Situated theories of learning describe social interactions as a mechanism for learning, and argue that learning is embedded in ongoing participation within a community (Walshaw, 2007). Neither individual learning nor social participation is privileged in this theory. Recent research in mathematics education provides support for socio-cultural situative theories of learning (Boaler, 2000, 2002; Boaler & Greeno, 2000). Boaler (2002) found that students from traditional mathematics classes who predominantly worked through textbooks were less able to use mathematics in new situations that required novel practices than students from classes where practices of negotiation, interpretation and creating shared mathematical understandings occurred. She argued that social practices not only provide a context for mathematics learning, but furthermore participation in social practices is what mathematics learning is.

2.2.3 Communities of Mathematical Inquiry

Opportunities for learners to construct meaningful mathematical knowledge in the classroom are strengthened by an environment that supports and values participation of all students, where social norms encourage collaboration and negotiation of meaning, and where multiple perspectives are respected and incorporated into collective knowledge creation (Swan, 2005). The New Zealand Curriculum implicitly supports this view and challenges teachers to cultivate within their classes a learning community where "everyone, including the teacher is a learner; learning conversations and learning partnerships are encouraged; and challenge, support and feedback is always available" (Ministry of Education, 2007, p. 34).

Building on the work of Cobb and McClain (1999), Sullivan, Zevenbergen, and Mousley (2002) described a socio-mathematical framework that explains two complementary norms of activity within a community of mathematical inquiry. The first dimension, mathematical norms, are described as "principles, generalisations, processes and products that form the basis of the mathematics curriculum and serve as the tools for teaching mathematics itself" (p. 650). The second dimension, socio-cultural norms, are described as the "usual practices, organisational routines and modes of communication that impact on approaches to learning, types of responses valued, views about legitimacy of knowledge produced and responsibilities of individual learners" (p. 650). Within an inquiry classroom neither mathematical nor socio-cultural norms are prioritized. Their interrelationship offers explanation of how the pedagogical actions of teachers as they enact classroom tasks, and the intellectual activity of students as they engage with the tasks, result in mathematical meaning negotiated through discursive dialogue (Hunter, 2008; Sullivan et al., 2002).

Teachers in inquiry classrooms reposition themselves as participants in the discourse, and engage in collaborative endeavours with learners resulting in co-construction of shared meaning (St George & Sewell, 2014). Although this constitutes a shift in power and authority from the teacher alone to sharing with the learner, the teachers' role is no less critical (Hunter, 2008; Sengupta-Irving, 2016). The inquiry classroom teacher fosters student responsibility for active learning, supports student authority to solve mathematical problems for themselves, ensuring learners are publicly credited as authors of their ideas, and promotes student accountability for how their mathematical thinking connects with others (Hunter, 2008; Sengupta-Irving, 2016; Stein, Engle, Smith, & Hughes, 2008). Practices in inquiry classrooms may include small group collaborative activity that maximises students' opportunities to use language and tools to make sense of mathematics, and whole class discussions where students share and justify their solution methods, and evaluate and reflect on strategies (Anthony & Hunter, 2005; Hunter, 2008).

2.2.4 Variation Theory

A further theory of learning that provides a useful framework for thinking about learning related to classroom tasks is variation theory (Marton, Runesson, & Tsui, 2004). Unlike other theories that hypothesise about learning independent of the content to be learned, variation theory focuses on the learning of specific content. The goals of teaching are described as *objects of learning*; what is planned by teachers as *intended objects*, what is implemented in classrooms as *enacted objects*, and learning as experienced by students, *lived objects* (Askew, 2012). Marton et al. (2004) argued that discernment is a core process in learning, and variation in learning experiences provides opportunity for discernment. Variation theory provides a framework for thinking about the design and sequence of tasks that maximize the likelihood of the desired mathematical objects being discerned, and the teachers' intended object of mathematical learning matching the students' lived object.

2.3 The Role of Persistence

In order to engage with cognitively demanding tasks students need to be willing to persist in the face of difficulty. Clarke et al. (2014) describe persistence as student activity that includes, "concentrating, applying themselves, believing they can succeed, and making an effort to learn" (p. 47). The association between cognitively demanding tasks and persistence is described in *PISA in Focus* (Organisation for Economic Co-operation and Development, 2014) as follows:

> Teachers' use of cognitive-activation strategies, such as giving students problems that require them to think for an extended time, presenting problems for which there is no immediately obvious way of arriving at a solution, and helping students to learn from their mistakes, is associated with student drive. (p. 1)

A barrier to implementing challenging tasks is that teachers consider that many students lack persistence when confronted with a problem they are not sure how to solve (Sullivan, Aulert et al., 2013; Sullivan, Clarke, & Clarke, 2013; Sullivan, Walker et al., 2015). Finding ways to encourage students to persist involves understanding the motivational characteristics of learners that contribute to increased persistence, enhancing the value of mathematical activity and optimising learners' orientation to engage with challenge (Middleton & Jansen, 2011; Sullivan et al., 2011).

Drawing on a meta-analysis of research studies on classroom culture, Rollard (2012) recommended that classrooms foster a learning goal orientation rather than a performance goal orientation. Learning goal orientation is characterised by the learners' desire to develop conceptual understanding and skills, and incorporates beliefs that success is the result of hard work (Dweck & Leggett, 1998). Multiple studies (e.g., Harackiewicz, Barron, Pintrich, Elliot, & Thrash, 2002; St George, Riley, & Hartnett, 2014) have found that students who develop learning goals are more likely to persist in the face of challenge, demonstrate self-regulatory adaptive behaviours, are more likely to take responsibility for success and less likely to deny responsibility for failure. Performance goal orientation is characterised by the desire to compare favourably against the performance of others, i.e. an outward appearance of competence. Students who adopt performance goals are more likely to avoid risk taking and challenging tasks due to fear of failure, to believe that ability is the cause of their successes, and that lack of ability causes their failures (Dweck & Leggett, 1998; Sullivan et al., 2011).

The work of Carol Dweck (2002, 2010) has been influential in establishing that people hold two different kinds of beliefs about intelligence. Those with a fixed mind-set believe that ability or intelligence is fixed for each individual whereas those with a growth mind-set believe that ability or intelligence is malleable and can be augmented through learning. Students whose views of intelligence have been reoriented toward a malleable view are more likely to be learning oriented, more comfortable with taking risks and more willing to persevere with challenging tasks (Blackwell et al., 2007; Dweck, 2007, 2010).

Instructional practices that engender the development of a learning goal orientation and growth mindset can have a major impact on students' motivation, persistence and self-belief (Dweck, 2007; Middleton & Jansen, 2011). Strategies that encourage students to persist include teachers' expectations of student success, valuing errors as part of learning, belief that students can learn even if they do not complete the task, empowering students to choose their own approaches to the task, and allocating enough time for lesson review (Sullivan, Aulert et al., 2013; Sullivan et al., 2011).

2.4 Classroom Mathematics Tasks

Tasks given to students serve as bridges between student learning and key mathematical concepts. Tasks are defined by Sullivan, Clarke, and Clarke (2013) as "information that serves as the prompt for student work, presented to them as questions, situations and instructions that are both the starting point and the context for their learning" (p. 13), and by Stein and Smith (1998) as a "segment of classroom activity that is devoted to the development of a particular mathematical idea" (p. 9).

2.4.1 Task Types and Purposes

The focus of the first definition above is the information teachers present. The focus of the latter is the classroom activity that surrounds the tasks. Together, tasks and activity largely determine the nature of student learning. Tasks that require students to perform a memorised procedure in a routine manner, for example, will lead to different learning than tasks that require students to think conceptually and make connections (Stein & Smith, 1998; Stein et al., 2009). Anthony and Walshaw (2007), in a synthesis of research evidence, elaborate on a range of conceptual task types and learning opportunities as follows:

> Tasks that require students to engage in complex and non-algorithmic thinking promote exploration of connections across mathematical concepts; tasks that require students to model their thinking promote reflection; tasks that require students to discern invariants and variation, and structure, promote generalisation; tasks that require students to interpret and critique data promote the disposition of ‘scepticism’; tasks that require students to ‘notice and wonder’ promote the disposition of curiosity; and tasks that provide opportunities for ‘mathematical play’ promote conjecture and exploration. (p. 95)

Although initially teachers select a task type and purpose that aligns with their goals for instruction, the nature of tasks can change as they are implemented in the classroom.

2.4.2 Mathematical Task Framework

Stein et al. (1996) developed a framework (Figure 2.1) to represent how tasks unfold as they pass through three phases. In phase 2, the task as it appears in instructional materials (i.e. phase 1) may undergo change as the teacher introduces it in the classroom. The task introduction may vary from an in-depth discussion of mathematical ideas to simply handing out problem sheets and instructing students to get started (Stein & Lane, 1996). The third phase represents the task as implemented by the students. How students approach the task and engage with the content is ultimately what influences student learning.

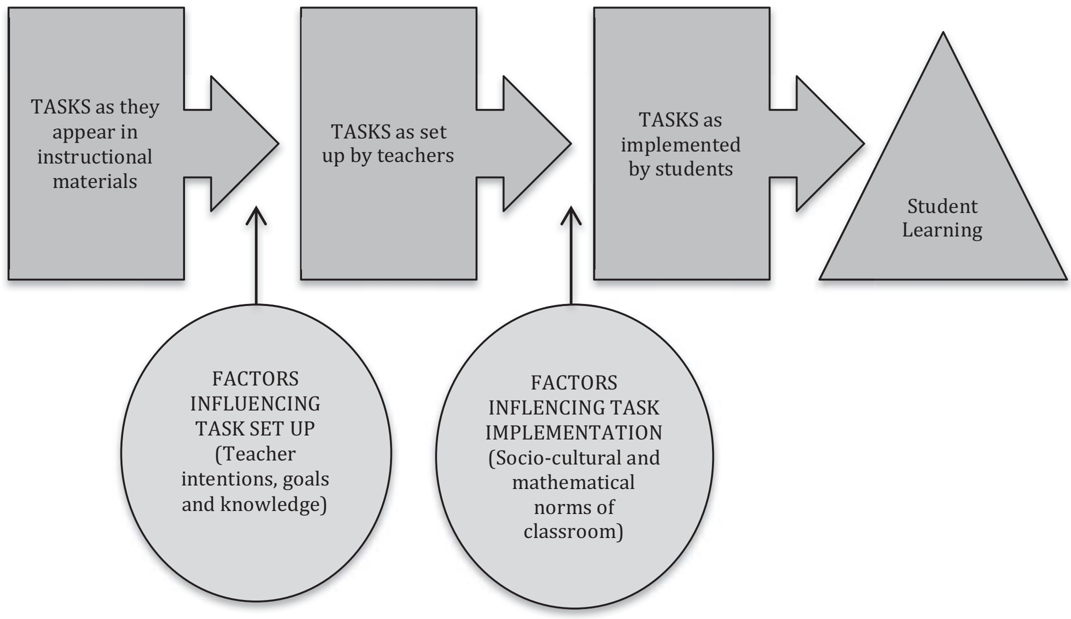

Figure 2.1 Mathematical Tasks Framework (Stein et al., 1996) showing relationship among various task-related variables and student learning.

This framework represents what many researchers maintain, (e.g., Hiebert & Stigler, 2004; Horoks & Robert, 2007; Sullivan, 2009; Sullivan, Clarke, & Clarke, 2013) namely that tasks can change their character once implemented, as they are mediated by teacher and student goals, knowledge and intentions and the socio-cultural and mathematical norms of the classroom. Marton et al. (2004) described this process of converting tasks to lessons, from the frame of reference of variation theory, as intended, enacted and lived objects, and pointed out that even when a lesson is enacted as the teacher intended, the lived objects of students are their subjective experiences and may result in different learning than either intended or enacted.

2.4.3 Worthwhile Tasks

Worthwhile tasks are those that promote reasoning, explanation, justification, thinking, creativity and reflection, and encourage students to actively construct and make connections between networks of mathematical ideas (e.g., National Council of Teachers of Mathematics, 2014; Sullivan, Clarke, & Clarke, 2013).

Scholars and researchers have largely agreed on the characteristics of tasks that promote meaningful engagement in this way. Tasks that generate multiple solution paths such as trial and error, discovering and using patterns, or applying a generalised approach promote the rich discourse of explanation and justification (Stein & Lane, 1996). Problems that afford multiple connected representations and different possible solution types encourage students to explore and look for patterns, and promote communication and discussion of alternatives (Sullivan, Mousley et al., 2009). Non-routine tasks provide optimal conditions for cognitive development, for evaluation of existing understandings and relational construction of new knowledge (Sullivan, Clarke, & Clarke, 2009). Sullivan et al. (2011) also recommended tasks with multiple entry points that afford students opportunities to plan their own approach, to choose their own strategies, goals, and level of accessing the task.

From a motivational perspective, Middleton and Jansen (2011) recommend that when selecting tasks teachers consider complementary dimensions of interest, challenge, and control. They contend that students' interest in a task can be engaged by a meaningful context or communication of the inherent value of the activity; an appropriate level of challenge requires students to expend effort to achieve success; and provision of elements of choice, be it between different activities, solution methods or alternative representations, empowers students and supports their sense of agency.

2.4.4 Cognitive Demand

Researchers have examined tasks according to the nature of thinking processes required for students to engage successfully with the task. Baumert et al. (2010) characterised tasks as "cognitively activating" and Stein et al. (1996) as "high cognitive demand". The classification system developed by Stein et al. resulted in four categories of cognitive demand, two of which describe higher levels and two of which describe lower levels of demand. Mathematical tasks with higher-level demands prompt students to actively explore and understand the nature of mathematical concepts and relationships or foster the use of procedures that are meaningfully connected with concepts or understanding. Tasks that encourage the use of procedures or algorithms in ways that are not connected to understanding or concepts, or that focus on memorisation or the replication of previously memorised facts or procedures place lower-level cognitive demands on students. Stein et al.'s (1996) taxonomy is summarised in Table 2.1.

Table 2.1 Levels of cognitive demand ordered from highest to lowest, their definitions and examples.

Level of demand	Definition	Examples (Stein et al., 2009, p. 3)
Doing mathematics (highest)	Students are required to use complex, non-algorithmic thinking to solve problems where no predictable, pre-determined approach or pathway is explicitly suggested by the task, instructions or a worked example.	"Shade 6 small squares in a 4x10 rectangle. Using the rectangle, explain how to determine the percent, decimal and fractional part that is shaded."
Procedures with connections	Students use a procedure in a manner that maintains and/or develops rich understanding of mathematical concepts or ideas. Although following a suggested pathway through the problem, they do so in a manner that maintains close connections to underlying conceptual ideas.	"Using a 10x10 grid identify the decimal and percent equivalents of $\frac{3}{5}$"
Procedures without connections	Students use a previously demonstrated procedure with no attention to why or how the algorithm works with limited, if any, connection to underlying mathematical ideas.	"Convert the fraction $\frac{3}{8}$ to a decimal and a percent."
Memorisation (lowest)	Students reproduce previously learned facts, rules or definitions or are required to commit facts, rules or definitions to memory.	"What are the decimal and percent equivalents for the fractions ½ and ¼?"

In using the framework to analyse the demand of classroom tasks, Stein et al. (2009) warned that superficial features, such as the use of manipulatives, real-world contexts or requiring multiple steps could mask a low-level task so that it appeared to be a high-level task, and asserted that it is the thinking processes that are elicited by the task that should be considered.

Multiple researchers (e.g., Boaler & Staples, 2008; Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013; Marshall & Horton, 2011; Stein & Lane, 1996; Sullivan, Mousley et al., 2009) have established links between dimensions of instructional tasks, associated thinking processes and learning gains. Stein and Lane (1996) analysed hundreds of tasks using the conceptual task framework above and established that the greatest student gains related to tasks that "engaged students in high-levels of cognitive processing, especially those that encouraged non-algorithmic forms of thinking associated with the doing of mathematics" (p. 74). However, they also found that high-demand tasks that were implemented in ways that students were not engaged in high-level thinking resulted in only moderate student performance gains. Although selection of a worthwhile task did not guarantee students' engagement at a high level, it was noted that starting with a procedural task would almost certainly guarantee its absence.

Boaler and Staples' (2008) study of two schools with very different approaches found that teachers who used mixed-ability groups using exploratory, open tasks resulted in students achieving significantly better results than those using a more traditional text-based approach. They reported that it was the lowest attaining students who benefitted most materially from these approaches.

In a study investigating the relationship between teaching and learning, Hiebert and Wearne (1993) found that students who were given fewer tasks that were longer and more focused on mathematical ideas as opposed to mechanical procedures were associated with greater gains in student performance.

2.5 The Role of Big Mathematical Ideas

The importance of presenting students with tasks that engage with worthwhile mathematics and explicitly focusing instruction on big mathematical ideas is widely recognised (e.g., Anthony & Walshaw, 2007; Askew, 2013; Siemon, Bleckly, & Neal, 2012). It is, however, difficult to find agreed upon definitions of big mathematical ideas in the literature or succinct information to guide teachers in planning instruction. The New Zealand Curriculum (Ministry of Education, 2007) categorises mathematical content into three strands, each represented by a number of achievement objectives. An example, "students will solve problems and model situations that require them to use a range of multiplicative strategies" (Ministry of Education, 2007, level 4 achievement objective) illustrates the lack of description of any big mathematical ideas that underpin this activity. Teachers, in looking to curriculum documents for guidance, organise the content of their mathematics programmes accordingly into topics (e.g. addition and subtraction, measurement-area) that do not in reality reflect the recommended focus on big mathematical ideas (Sullivan, Clarke, Clarke et al., 2012).

From scholars there is also ambiguity. Schifter and Fosnot (1993) defined big mathematical ideas as "the central, organising ideas of mathematics – principles that define mathematical order" (p. 35). Charles (2005), on the other hand, emphasised the learning of mathematics, and defined a big mathematical idea as "a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole" (p. 10). Askew's (2013) suggestion that big ideas should be both "mathematically significant as well as individually and conceptually significant" (p. 7), links the previous definitions by connecting historical paradigm shifts in mathematical reasoning and learners' shifts in reasoning as they grasp big ideas.

Charles' (2005) definition is complemented with an associated list of 21 possible big mathematical ideas with examples provided of mathematical understandings for each. This list significantly contains no information on possible learning trajectories beyond what is implied by the format of the list. Because of limited guidance for teachers from curricula or research on incorporating big mathematical ideas, a clear grasp of both subject matter knowledge and pedagogical content knowledge is required to inform teachers' decisions as they select mathematics tasks, plan lessons, make adaptations during instruction and reflect on their students' progress.

2.6 Teacher Knowledge

Teachers' subject matter knowledge encompasses knowledge of the mathematics needed to solve the task, knowledge of how to represent mathematical ideas and provide explanations, and knowledge to understand students' various solution strategies (Ball, Thames, & Phelps, 2008). Teachers' pedagogical content knowledge includes knowledge of learning progressions, knowledge of how to sequence and evaluate instruction and knowledge of students and how they learn (Ball et al., 2008). Simon (1995) argued that both perspectives are interrelated, that teachers' perceptions of students' mathematical understandings are structured by their understandings of the mathematics in question.

To frame a programme of instruction around tasks that focus on key mathematical ideas and build on students' existing understandings, and to situate the tasks in contexts that are engaging but do not become the end goal of the learning experience, nor blur the mathematical content, is complex. Sleep (2012) argued the importance of teachers articulating their mathematical purpose in the planning stages so that associated tasks and student activity can be oriented to engage students with the mathematical point of the lesson.

Research studies have demonstrated the challenge this presents. In a study that examined teacher knowledge and the use of tasks, Sullivan, Clarke et al. (2009), found that creating a lesson out of a task (which is bigger, $\frac{2}{3}$ or $\frac{201}{301}$?) was problematic. Many teachers were not aware of the range of solution strategies students might use, had difficulty describing the mathematical content of the task, and struggled to translate the task into a worthwhile lesson.

An exploratory study (Charalambous, 2008) investigating the unfolding of tasks in a series of lessons delivered by two teachers, one with high subject matter knowledge and one with low, reported that the teacher with the high knowledge consistently maintained the cognitive demand of the tasks during presentation and enactment, urged her students to use multiple representations and pressed students to articulate and justify their thinking. On the other hand, the teacher with low knowledge often proceduralised the tasks, placed more emphasis on remembering and applying formulae and rules, and maintained the level of demand in only half of the lessons under study. It cannot be assumed that a teacher who understands the content necessarily understands effective methods to teach it. However, without a clear grasp of mathematical ideas, there is a tendency for teachers to adopt surface features of curriculum reform and attempts to make a mathematics task interesting can sacrifice mathematical meaning and accuracy (Anthony & Walshaw, 2007; Sleep, 2012; Stein et al., 2009).

2.7 Enacting Challenging Tasks in the Classroom

A worthwhile task alone cannot guarantee student learning. Various authors have described explicit pedagogical actions that teachers can incorporate into their practice to maximise students' opportunities to learn from tasks. These include talk moves (Chapin & O'Connor, 2007); the 'advancing children's thinking' framework (Fraivillig, 2001); the mathematics communication and participation framework (Hunter & Anthony, 2011); steering instruction to the mathematical point (Sleep, 2012); practices for orchestrating productive mathematics discussions (Smith & Stein, 2011) and maintenance of high-level cognitive demand (Stein et al., 2009). Drawing from these complementary frameworks three inter-related and overlapping themes emerge: the maintenance of cognitive demand, maintenance of mathematical focus and fostering productive discourse.

2.7.1 Maintenance of Cognitive Demand

Mediated by teachers' and students' goals, knowledge, beliefs, and attitudes as they are implemented in the classroom, it is not unusual for the cognitive demands of a task to be unwittingly or purposefully altered during the implementation phase (Stein et al., 2009). Reasons for this may include teachers' desire to avoid anticipated negative reactions from students (Sullivan, Walker et al., 2015), or their assumption that the best way to assist students who are facing difficulty accessing the task is to specify step-by-step instructions or an explicit procedure to follow (Sullivan et al., 2011). As a consequence, teachers instead of students are doing the mathematics, the thinking and reasoning, and students are deprived of important opportunities to learn. These teacher actions may also occasion students' views of themselves as helpless learners, and affect their motivation and orientation to persist (Dweck, 2007; Middleton & Jansen, 2011). Other common practices associated with decline in cognitive demand include an emphasis on correctness, completeness, and speed, often at the expense of prioritising meaning and conceptual understanding (Stein & Smith, 1998).

Various authors have advocated the importance of teachers consciously planning to maintain the cognitive demand of tasks (e.g., Jackson et al., 2013; Stein & Smith, 1998; Sullivan, Walker et al., 2015). In their comprehensive research on the use of classroom mathematics tasks Stein and Smith (1998) identified factors associated with the maintenance and decline of high-level demands (see Table 2.2).

Table 2.2 Factors associated with maintenance and decline of high-level tasks (Stein & Smith, 1998, p. 274).

Factors Associated with the Maintenance of High-Level Cognitive Demands
1. Scaffolding of student thinking and reasoning is provided.
2. Students are given the means to monitor their own progress.
3. Teachers or capable students model high-level performance.
4. Teacher presses for justifications, explanations, and meaning through questioning, comments, and feedback.
5. Tasks build on students' prior knowledge.
6. Teacher draws frequent conceptual connections.
7. Sufficient time is allowed for exploration – not too little, not too much.

Factors Associated with the Decline of High-Level Cognitive Demands
1. Problematic aspects of the task become routinized (e.g. students press the teacher to reduce the complexity of the task by specifying explicit procedures or steps to perform; the teacher "takes over" the thinking and reasoning and tells the students how to do the problems.
2. The teacher shifts the emphasis from meaning, concepts, or understanding to the correctness or completeness of the answer.
3. Not enough time is provided to wrestle with the demanding aspects of the task, or too much time is allowed and students drift into off-task behaviour.
4. Classroom-management problems prevent sustained engagement in high-level cognitive activities.
5. Task is inappropriate for a given group of students (e.g. students do not engage in high-level cognitive activities because of lack of interest, motivation, or prior knowledge needed to perform; task expectations are not clear enough to put students in the right cognitive space).
6. Students are not held accountable for high-level products or processes (e.g. although asked to explain their thinking, unclear or incorrect student explanations are accepted; students are given the impression that their work will not "count" toward a grade).

These suggestions concur with those suggested by other authors (e.g., Chapin & O'Connor, 2007; Fraivillig, 2001). Fraivillig (2001) recommended three interrelated pedagogical actions to support students' problem solving: *eliciting* encourages multiple solution pathways and fosters discussion, justification, and elaboration; *supporting* fosters students' tenuous understanding of their own and others' solutions methods and includes reminding students of conceptually similar problems; and e*xtending* focuses on the analysis and comparison of students' solutions and pathways to encourage generalisation.

2.7.2 Maintenance of Mathematical Focus

An important role for the teacher during the implementation phase is to ensure that student activity remains aligned with the mathematical focus of the task (Anthony & Walshaw, 2007; Sleep, 2012). Sleep identified seven teaching moves to promote student engagement in the intended mathematical work, which she described as "steering instruction to the mathematical point":

1. attending to and managing multiple purposes,
2. spending instructional time on *mathematical* work,
3. spending instructional time on the *intended* mathematics,
4. making sure *students* are doing the mathematical work,
5. developing and maintaining a mathematical storyline,
6. opening up and emphasising mathematical ideas, and
7. keeping a focus on meaning. (p. 942)

Reminding us that classroom activity can encompass multiple goals, both mathematical and non-mathematical, Sleep (2012) recommended attending to non-mathematical purposes, for example the use of contexts chosen to engage students' interest, in ways that do not detract from the intended mathematical work. A pertinent illustration can be found in the work of Hill et al. (2008) who described lessons where efforts to make mathematics engaging resulted in a focus on artistic endeavours with little or no mathematical value. Sleep's third point relates to teachers' strategic selection of problems and student examples that draw attention to the mathematics that is, in the lexicon of variation theory, the intended object of learning. A mathematical storyline responds to the need for coherence and a considered progression of mathematical ideas, again indicating the importance of teachers' pedagogical content knowledge.

Sullivan, Clarke, and Clarke (2013) suggested that teachers hone the mathematical focus of the lesson by working through the task prior to instruction utilising their understanding of their students' prior knowledge. By anticipating typical student responses teachers can prepare an appropriate lesson trajectory or storyline. This does not mean that the teachers' role is to sanction particular approaches as being correct, but rather to shape the activity to build on the collective sense-making of students (Stein et al., 2008). Although elicitation and acceptance of multiple solution methods is a common feature of reform practices, the importance of pressing students' use of more efficient solution methods while remaining sensitive to their current mathematical understandings, should also be stressed (Fraivillig, 2001; Stein et al., 2008).

2.7.3 Fostering Productive Discourse

Orchestrating productive conceptual classroom discourse is arguably teachers' most sophisticated tool in advancing mathematical learning (e.g., Chapin & O'Connor, 2007; Hunter, 2008; National Council of Teachers of Mathematics, 2014; Smith & Stein, 2011). Task selection influences the type and level of discourse that is possible. Meaningful discourse is generated by students' justification of idiosyncratic solution strategies, which is unlikely to eventuate if limited thinking and reasoning is required to access the task.

Chapin and O'Connor (2007) researched types of discourse used in classrooms, particularly the use of "talk moves" that support academically productive discourse. The talk moves suggested are *revoicing* to clarify responses; *repeating* another student's contribution; *eliciting student reasoning*; *adding on* to another's strategy to promote connections; and *waiting* to give all students enough time to compose their response thereby communicating an expectation that all students participate in the discourse.

Communities of mathematical inquiry promote students' engagement in mathematical discourse and argumentation. Hunter (2008) reported how a group of teachers used a purposefully designed communication and participation framework, described as a "set of collective reasoning practices related to the communicative and performative actions that support effective mathematical inquiry practices" (Hunter & Anthony, 2011, p. 105), to scaffold the establishment of an inquiry community. Lave and Wenger (1991) argued that participation in a community of practice changes as skills, attitudes and knowledge develop and the learner moves from peripheral to full participation. This concurs with Hunter's (2008) findings that participation in the learning community changed for both teachers and students as they became proficient in the adoption of discursive practices. The challenges of implementing ambitious discourse practices are implicitly acknowledged within the framework (Hunter, 2008), which sequences gradual shifts in practice over three phases. She argued that scaffolding students' participation in mathematical reasoning at higher intellectual levels resulted in improved argumentation skills, content understanding and increased teacher expectations.

The work of Smith and Stein (2011) addressed teachers' use of student responses to challenging tasks to orchestrate whole-class discussions. Like Fraivillig (2001), Smith and Stein argued that eliciting students thinking and accepting multiple solution strategies was not enough; the teacher needed to guide and sequence classroom discussion towards important and worthwhile mathematics. Their model of five practices specifically designed to support teachers to conduct whole-class discussions following students' work on challenging tasks includes:

1. *anticipating* likely student responses to challenging mathematical tasks;
2. *monitoring* students' actual responses to the tasks;
3. *selecting* particular students to present their mathematical work during the whole-class discussion;
4. *sequencing* the student responses that will be displayed in a specific order; and
5. *connecting* different students' responses and connecting the responses to key mathematical ideas. (Smith & Stein, 2011, p. 8)

The contribution of Smith and Stein (2011) to the literature has been embedding of these practices as a coherent package, where the use of each depends on the others. Anticipation requires the teacher to solve the task they are planning for their students, to view the task through their students' eyes, and consider students' possible interpretations or misinterpretations (Stein et al, 2008). While monitoring students' work the teacher pays close attention to the mathematics within students' conversations and solution strategies. During this phase the teacher selects particular students to share their work with the class and sequences responses to maximise the likelihood of the mathematical goals of the lesson being achieved and to maintain the coherence or mathematical storyline of the lesson (Sleep, 2012; Stein et al., 2008). To conclude the lesson, the teacher supports students to make connections between mathematical ideas inherent in the strategies and representations shared. Discernment of the same mathematics represented in different strategies and judgement on the efficiency of alternative strategies enhances students' understanding of the underlying mathematics (Stein et al., 2008). In selecting strategies for presentation, teachers should not signal that strategies need to be validated by the teacher as worthy; or undermine students' authority and sense-making by seeming to value some responses over others (Anthony & Walshaw, 2007; Stein et al., 2008).

Leatham, Peterson, Stockero, and Van Zoest, (2015) added a further practice of *recognising* to the five above. They focused on teachers' recognition of critical moments in the enactment of lessons, on "expressions of student thinking that lose their instructional value if they are not acted on in the moment" (Leatham et al., 2015, p. 90). They argued that the intersection of three critical characterisitcs, student mathematical thinking, mathematical signifiance and pedagogical opportunity, distinguish moments that provide potential to productively build on student thinking from others. This suggests that when deciding which path to pursue at critical moments in the lesson, teachers should consider more than mathematical content alone, but should take into account students' thinking that may indicate paths that are more productive than others in their potential to develop mathematical sense-making (Anthony, Hunter, Hunter, & Duncan, 2015).

2.8 Potential Constraints

Constraints teachers experience in using demanding tasks are predominantly related to the crucial role that teachers' goals, knowledge, attitudes and beliefs play in informing pedagogical decisions and classroom socio-cultural and mathematical norms. For example, Sullivan (2009) reported teachers' reluctance to implement high-level tasks over concerns that students may be reluctant to take risks or expend effort in their learning and preferred closed tasks. Teachers also expressed concerns they were "not always sure what maths will come out of it" (Sullivan, 2009, p. 729) which suggests a lack of confidence in drawing on their own mathematical knowledge to respond to students' idiosyncratic strategies.

In instances where the activity of task enactment involves innovation in pedagogy, Horoks and Robert (2007) argued that the strength of teachers' professional habits could act as a constraint:

> Some teachers are reluctant to let the students work in small groups even if they are convinced of the usefulness of this type of activity. Some of them cannot easily give up their control over students while others are afraid of the possible noise in the class notably because it might disturb their colleagues. (p. 280)

Further challenges stem from meeting the needs of learners from diverse cultural and mathematical backgrounds.

2.9 Differentiating Learning

Various authors have addressed characteristics of mathematics lessons that are successful with students from diverse backgrounds or those who experience difficulty (e.g., Hunter, 2008; Hunter & Anthony, 2011; Lambert & Stylianou, 2013; Sullivan, Mousley et al., 2009; Sullivan, Mousley, & Zevenbergen, 2004). Students who lack understanding of aspects of a complex task need supports to get started (Jackson, Shahan, Gibbons, & Cobb, 2012). Likewise, supports should be considered for students who are able to solve the task easily or quickly.

Currently, many teachers address the needs of students experiencing difficulty by setting alternative goals for them, for example teaching them as a separate group, with different mathematical goals from the rest of the class, most likely at lower levels of cognitive demand and procedural in nature (Boaler & Sengupta-Irving, 2016; Stein et al., 1996). To mitigate these practices, Sullivan, Clarke and Clarke (2013) suggested that students are more likely to learn significant mathematics and participate in the classroom community if teachers offer prompts that allow those students experiencing difficulty to engage in experiences related to the core task rather than pursuing alternative goals. Other authors (e.g., Boaler, 2016; Lambert & Stylianou, 2013) have advocated that using open-ended tasks that offer multiple means of representation, engagement and strategic action provide greater access to mathematics for a wide variety of learners.

2.9.1 Enabling and Extending Prompts

Sullivan and his colleagues (e.g., Sullivan et al., 2004; Sullivan, Mousley et al., 2009) proposed that a task variation that still afforded an appropriate problem solving opportunity could be posed to students who experienced difficulty engaging with the core task. Students proceed with the support of the variation, or *enabling prompt*, and then once successful return to work on the original task (Sullivan et al., 2004). Effective variations might include "reducing the required number of steps, simplifying the modes of representing results, making the task more concrete or, reducing the size of the numbers involved" (Sullivan, Clarke, & Clarke, 2013, p. 19). Sullivan et al. (2004) argued that supports such as these should be offered discreetly. Less satisfactory approaches they observed included additional prompts offered to the class in public statements or as an "anybody who is unsure stay behind with me" approach, both of which draw unnecessary attention to those experiencing difficulty, nurture learned helplessmess and affect students' inclination to seek help.

There is a danger that students experiencing difficulty with tasks that afford multiple solution pathways do not extend their current levels of proficiency and merely reinforce current misconceptions (Sullivan, Mousley et al., 2009). Sullivan and his colleagues recommended careful monitoring of students as they work and the provision of supporting prompts as required. They noted "incidents throughout the research where relatively open-ended questions allowed teachers to see where individuals and groups of students had a misunderstanding that needed whole-class attention" (p. 34), a position that concurs with Leatham et al. (2015) on recognising critical moments in lesson enactment where action is required.

Students who finish the core task quickly can be provided with supplementary tasks, or *extending prompts*, that are related to the core task but elicit abstraction or generalisation and extend their thinking and activity (Sullivan et al., 2004). Extending prompts have proved effective in keeping higher-achieving students productively engaged and supporting higher level, generalisable understandings (Sullivan, Clarke, & Clarke, 2013). The purpose of extending prompts is to develop mathematical thinking on the same context as the core task, rather than students proceeding to the next stage of the lesson before the rest of the class (Sullivan, Mousley et al., 2009).

2.10 Summary

Pedagogical approaches to the enactment of classroom tasks have been influenced by reforms in mathematics education that have called for a change in how teachers view mathematical knowledge, the value and purpose of social interaction in the classroom and teachers' role as participants in classroom discourse. Teachers' knowledge, attitudes and beliefs about learning and mathematics inform their pedagogical decisions, and the socio-cultural and mathematical norms in the classroom.

Worthwhile mathematics tasks are those that promote reasoning, explanation, justification and collaboration. In affording students opportunities to engage in meaningful mathematical practices and discourse, worthwhile tasks encourage active construction of networks of interconnected mathematical ideas.

Multiple studies demonstrate that once implemented in classrooms, tasks are mediated by teacher and student goals, knowledge, and intentions and there is a tendency to reduce their cognitive demand and their efficacy for student learning. The literature recommends explicit pedagogical actions that optimise opportunities for student learning from challenging tasks, which include actions associated with the maintenance of cognitive demand and mathematical focus, and strategies for fostering productive discourse.

A challenge exists for teachers in implementing demanding tasks in ways that cater for the needs of learners from diverse cultural and mathematical backgrounds. There is a growing body of evidence that supports initiation of learning through the use of a challenging task and differentiation of learning through task variations.

Chapter 3: Research Design

3.1 Introduction

This chapter outlines the perspective and approaches that informed the design of the study. Section 3.2 describes how this study conforms to the principles of case study and design-based research approach. The role of the researcher is outlined in section 3.3 and data collection methods in section 3.4. The setting and participants are introduced in sections 3.5 and the research schedule in 3.6. Section 3.7 describes the data analysis used in the study and includes discussion of validity and reliability. Ethical considerations are outlined in section 3.8.

3.2 Methodology

The purpose of this study was to explore how teachers maximised the mathematical opportunities of cognitively demanding tasks in ways that benefitted student learning. Three teachers implemented three tasks in their classrooms using a range of pedagogical approaches previously explored as part of a support programme facilitated by the researcher. Several propositions informed the design of the research. The first proposition was that adoption of ambitious pedagogies where all students are supported to productively engage with cognitively demanding mathematical tasks is beneficial for student learning. The second was that teachers experience constraints that hinder their use of such tasks in their classrooms, and the third that suggestions of pedagogical approaches and a lesson structure to facilitate the enactment of demanding tasks would be useful for teachers.

The choice of methodology was influenced by several factors. These included the role of the researcher and participants in the research, the desire to embrace holistic insights into teacher activity, and the socio-constructivist paradigm fundamental to this study. The pedagogical approaches to classroom tasks proposed by the researcher likely differed from the participant teachers' usual teaching practice so elements of innovation and intervention were further considerations.

3.2.1 Case Study

The research design aligns with the characteristics of a multiple case study approach, enabling the exploration of similarities and differences within and between cases, and supporting predictions and connections based on theory (Baxter & Jack, 2008; Hancock & Algozzine, 2011). A case study approach to research "involves an empirical investigation of a particular contemporary phenomenon within its real life context using multiple sources of evidence" (Robson, 1993, p. 146). A case is defined as a "phenomenon of some sort occurring in a bounded context" (Punch, 2009, p. 119).

The case or phenomenon under investigation in this study was participant teachers' enactment of a mathematical task in a classroom of year 7 and 8 learners, a system bounded by time and activity. Each case consisted of the enactment of the same task in the three teachers' classrooms. As such, the three different tasks resulted in three cases for discussion. The research was guided by propositions and theoretical frameworks gleaned from the literature that informed the analysis of teachers' actions and the details of their practice as the objects of study (Baxter & Jack, 2008). Denscombe (2010) argued that the use of more than one research method meshes comfortably with the case study approach, which supported the assertion that a case study can be produced from design-based research.

3.2.2 Design-Based Research

Design-based research, an approach used "for the study of learning in context through the systematic design and study of instructional strategies and tools" informs the formative nature of this study (Design-Based Research Collective, 2003, p. 5). Key features of designed-based research include innovation, intervention, and iteration (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003). By designing its elements and by conjecturing how these elements function together to support learning, design-based research aims to understand what Cobb et al. (2003) describe as a *learning ecology:*

> Elements of a learning ecology typically include the tasks or problems that students are asked to solve, the kinds of discourse that are encouraged, the norms of participation that are established, the tools and related material means provided, and the practical means by which classroom teachers can orchestrate relations amongst these elements. (p. 9)

The learning ecology this study aimed to understand related to the types of tasks and teacher practices that impact on students' engagement with mathematical ideas, their participation in productive mathematical discourse, and their willingness to persist in the face of challenge. The features of design-based research incorporated in this study included an intervention comprising a programme of support including suggested tasks and lesson elements to prompt participant teachers to adopt innovative practices, and iteration in that the implementation of each task was informed by the reflection of the participant teachers and the researcher on previous lessons.

3.3 Role of the Researcher

The Design-Based Research Collective (2003) argued that this approach to research provides the means for practitioners and researchers to "work together to produce meaningful change in contexts of practice" (p. 6). They argued that the relationship between the researcher and teachers impacts on the success of the intervention as collaboration not only ensures that local goals and constraints are considered alongside the researcher's agenda, but also that shared ownership and commitment to the design enhances the likelihood of the innovation enduring in teachers' practice.

The socio-constructivist paradigm that underpins this research recognises the importance of subjective creation of truth and meaning without completely rejecting notions of objectivity. Teachers' decisions and actions are the focus of this study so collaboration between the researcher and participants enabled the researcher to better understand participants' perspectives and their actions. The challenges of the researcher's role as facilitator and objective observer of an intervention was described by the Design-Based Research Collective (2003) as "the dual intellectual roles of advocate and critic" (p. 7).

The researcher in this study is an experienced teacher of year 7 and 8 students and is a colleague of the teacher participants. Punch (2009) warns that possible disadvantages of researching your own school or context include "bias and subjectivity, a vested interest in the results, a lack of generalisability and ethical considerations" (p. 45). An awareness of these issues allows them to be managed, and highlights the important role of the research supervisor as a check for possible subjectivity or vested interest. The researcher recorded field notes and reflections to validate recollections, prevent errors and detect bias, but as Punch (2009) acknowledged, "there is no such thing as a position-free project" (p. 45).

3.4 Data Collection

Consistent with both case study and design-based research, multiple data sources and multiple collection techniques were employed in order to gather a complete picture of each case (Baxter & Jack, 2008; Cobb et al., 2003). Data generated by the study included qualitative data from video records of classroom lessons, self-reporting from teacher responses to free format questionnaires, semi-structured interviews and documentary data, as well as quantitative data from Likert-style survey items.

3.4.1 Observation

Classroom observations are a characteristic of design-based research methodology (Cobb et al., 2003). This study recorded each of the three teachers implementing each of the three tasks, nine lessons in total. The focus of the study was exploration and understanding of teacher actions that support learning during the enactment of the task, so data gleaned from the observations included the way that teachers posed the problems, how they interacted with students about the mathematical content, how they worked on problems with students, and how they prompted and engaged with students sharing and justifying solution strategies. The iterative nature of the research approach enabled a sharper focus of the observations on successive task implementations.

Observations as a record of events are open to subjective analyses. The videotaped observations in this study were available to the participant teachers as well as the researcher which served to reveal or confirm participants' perspectives, and established the data source as both trustworthy and rich. Field notes and teacher input following the first round of observations informed the development of descriptive categories that in turn informed a more structured approach to subsequent observations.

3.4.2 Questionnaire

Teachers completed a questionnaire to elicit their perspectives at the end of the project (Appendix A). The teachers rated descriptive items on a 5-point Likert-type scale where 1 measured the most negative attitude and 5 the most positive with a notionally neutral score of 3 for each description. Teachers also responded to free format questions designed to elicit data that provided illumination of the research questions.

3.4.3 Group Interview

Group interviews provide opportunities for "the explicit use of group interaction to produce data and insights that would be less accessible without the interaction found in a group" (Punch, 2009, p. 147). A semi-structured format was used for the group interview at the end of the project (Appendix B). Participants' responses and discussion centred on themes that offered insights aligned with the research questions, but the format allowed for additional questions that clarified or explored the responses. The interview was recorded to facilitate the accurate recording of the data.

3.4.4 Documentary Data

Further data collected in the study was documentary data. This played an important role in facilitating data triangulation (Punch, 2009). The documents collected revealed participants' perspectives and interpretations and included teachers' planning and evaluations, and students' written recording of their mathematical solution strategies that illustrated student understanding.

3.5 Research Setting and Participants

The research was undertaken at a New Zealand urban full primary school serving a low socio-economic community. The participant teachers involved in the project were three teachers employed in the senior syndicate of 80 year 7 and 8 students working in three multi-level classes. The teachers represent a range of age, experience and confidence with mathematics teaching. They have been assigned pseudonyms (Sally, Nanette, and Louise) and all have been allocated the female gender in order to protect their identities.

3.6 Research Schedule

The teaching study comprised three phases conducted over three months.

Phase One

In Phase One the participant teachers attended professional learning sessions facilitated by the researcher. In the first session characteristics of worthwhile tasks were explored and in the second, pedagogies associated with successful implementation of tasks. The third session incorporated a suggested lesson structure, selection of the first task and collaborative planning of the first lesson. The next chapter describes the content of the professional learning sessions.

Phase Two

Phase Two comprised iterations of teaching and reflection. The teachers' implementations of the tasks were observed and videoed. As soon as practicable after teaching the lesson the teachers reflected on the task and its implementation. The researcher and participant teachers met to reflect collectively on the first lesson and to plan the second.

The iterative approach of collaborative planning followed by classroom implementation and reflection was repeated for three tasks. Further research data collected in phase two included field notes, samples of student work, audio recordings of planning meetings and teachers' written planning.

Phase Three

Phase three occurred after the enactment of task three. The participant teachers completed the final questionnaire and took part in a semi-structured group interview.

3.7 Data Analysis

A feature of design-based research is that data collection and analysis occur concurrently (Denscombe, 2010). In this study the researcher and participant teachers' collaborative reflection on the data and ongoing analysis focused the teachers' attention on their instructional decisions and effectuated adaptations to their practice in subsequent iterations. It also served to sharpen the researcher's focus of the study. The organisation of the research findings in chapters 5 to 7 represents each case study task. Chapter 8 presents the teachers' perspectives on the usefulness of the structures suggested by the project. Chapter 9 presents the conclusion including claims and assertions generated from the research strengthened by retrospective analysis of the data, a distinctive feature of design-based research.

Case study research typically comprises data from multiple sources converged in the analysis process rather than handled individually as "each data source is one piece of the puzzle with each piece contributing to the researcher's understanding of the whole phenomenon" (Baxter & Jack, 2008, p. 554). This study used approaches to analysis of qualitative data of reduction, display and drawing and verifying conclusions from data (Miles & Huberman, 1994). Video and audio data was transcribed and used together with teachers' qualitative responses. Stated intentions of the research were to document teachers' processes of enactment and perspectives, so triangulating multiple sources and kinds of data was utilised to increase confidence in both the researcher's interpretation and in teacher self-reported data. Reduction of the data illuminated similarities, differences, themes and patterns. Punch (2009) argued "good qualitative analysis involves repeated and iterative displays of data" (p. 175). In this study data displays assisted in organising, compressing, comparing and assembling the information. A repertoire of analytic techniques applied to the same body of data served to illuminate different aspects and enabled linking of the data to propositions.

3.7.1 Validity and Reliability

Within qualitative research validity refers to the accuracy and appropriateness of the data collected (Denscombe, 2010). In this study triangulation of multiple data sources and types strengthened confidence in the data, as did collaboration between the researcher and the participant teachers enabling facts to be checked and interpretations to be corroborated or revised. This was particularly important as the dual role of researcher as facilitator and observer has implications for the credibility and trustworthiness of the research findings. Partnership and iteration, both features of design-based research, address the validity of research findings and typically result in increasing alignment of theory, design, practice and measurement over time (Design-Based Research Collective, 2003). Retrospective analyses were contrasted with the analyses conducted while the study was in progress to strengthen credibility of the research claims.

Reliability is concerned with consistency (Punch, 2009). In this study the researcher and participants' roles have been clearly described, multiple data sources and types have been used and systematic analysis carried out. A lack of formal measures of student learning resulting from the teaching experiment could constitute a challenge to the reliability of the research, but the students were unlikely to achieve measurable differences in learning within the short time frame of the project, so any claims about student learning gains result from teachers' indirect reporting. This encompassed teachers' perceptions of changes in students' persistence as well as mathematical understanding and utilised teachers' holistic knowledge of their students and their learning.

3.8 Ethical Considerations

The research project was designed and conducted in accord with the Massey University Code of Ethical Conduct for Research, Teaching and Evaluations Involving Human Participants (Massey University, 2015). The project was reviewed and approved by the Massey University Human Ethics Committee prior to data collection. Ethical considerations taken into account included respect for persons, minimisation of risk to participants, respect for privacy and confidentiality, and social and cultural sensitivity.

Benefits and risks that required consideration included the time commitment of the participant teachers and their students to the project. Lessons under study took place as part of the normal classroom programme and the research was conducted in a way that supported the education and welfare of children and teachers. Consent was sought and obtained from the Board of Trustees of the school, participant teachers, the students and their parents or guardians. All participants were provided with relevant information on which to base their decision (Appendices C, D, & E).

A potential risk related to the school setting and the position of the researcher as a colleague of the teacher participants. Ethical dilemmas anticipated with the shift in role from colleague to researcher were minimised by clear communication of plans within the research framework to value and make visible the teachers' perspective throughout the process. Participant teachers were invited to collaborate on the analysis of data, and meetings were held at times and settings of the teachers' choice. No evaluations of teaching and learning programmes were made other than those grounded in the context of the study. Because others in the school were aware of the teachers taking part in the study, guaranteeing anonymity was not possible, but the school and participants were assigned pseudonyms and no identifying information was included in any reports.

3.9 Summary

The methodology chosen for this study was a case study and design-based research approach. Multiple data sources including observations, questionnaires, interviews and documentary data were collected during three phases of research. The three phases incorporated intervention, iterations of teaching and reflection and collection of concluding thoughts. Systematic analysis and triangulation of data alongside collaboration between the researcher and participant teachers served to strengthen the validity and reliability of the research findings reported in subsequent chapters.

Chapter 4: Intervention

4.1 Introduction

This chapter outlines Phase One of the research comprising the professional learning sessions for teachers. Section 4.2 provides an overview of the sessions and session one is described in section 4.3. The types of tasks which became the focus of the project are described in section 4.4. Section 4.5 outlines the teachers' second session. A proposed lesson structure and planning template are outlined in section 4.6. The chapter concludes with section 4.7 describing the third session.

4.2 Overview of Teacher Professional Learning Sessions

The goal of the teacher professional learning sessions was to build teachers' capacity to engage their students with cognitively challenging mathematics tasks. The research of Boston and Smith (2011), Stein et al. (2009), Smith and Stein (2011), and Sullivan and his colleagues (e.g., 2009, 2011, 2103, 2015, 2016) guided the design of the professional development sessions and selection of activities. Boston and Smith (2011) outlined the success of their own research project implementing a 'task-centric' approach to professional development. They described the benefit of programmes which engaged teachers in critical analysis of tasks and argued that learning about the cognitive demands of tasks supported and shaped teachers' thinking about the relationship between tasks and learning. Other researchers have advocated the benefits of task-centric approaches to improving teacher practice (e.g., Stein et al., 2009; Watson & Sullivan, 2008). Sullivan, Mousley et al. (2009) argued:

> An important component of understanding teaching and improving learning is to identify the types of tasks that prompt engagement, thinking, and the making of cognitive connections, and the associated teacher actions that support the use of such tasks, including addressing the needs of individual learners. (p. 18)

Activities for the sessions were planned with an awareness of the challenges that often exist in translating professional development into associated improvements in practice (Boston & Smith, 2011; Timperley, Wilson, Barrar, & Fung, 2007). With this in mind, activities that represented the everyday authentic practice of teachers were at the heart of the sessions, including opportunities for teachers to solve, understand, and assess the cognitive demands of mathematics tasks, and to experience and explore pedagogies that support student learning during task implementation. The activities were framed within the larger body of ideas about the nature of mathematics teaching and learning discussed in the literature review. The intervention aimed to equip teachers with a set of tools to assist in the adoption of pedagogical approaches that support the use of cognitively demanding tasks. These included a task analysis guide (Stein et al., 2009, p. 6; Appendix F), and a lesson structure and planning template (Appendix G) designed to prompt teachers to consider their pedagogical decisions at each phase of planning and implementation.

4.3 Session 1: What are Good Tasks and Why are they Important?

The first two sessions for teachers were part of a whole-school mathematics professional development facilitated by the researcher. The focus of the first was for teachers to solve, understand, and assess the cognitive demands of tasks as they appear in instructional materials i.e. in phase one of the Mathematical Task Framework (Stein et al., 1996). Participants solved and compared (see Figure 4.1) two tasks both focusing on the same mathematical content but requiring different levels and kinds of thinking.

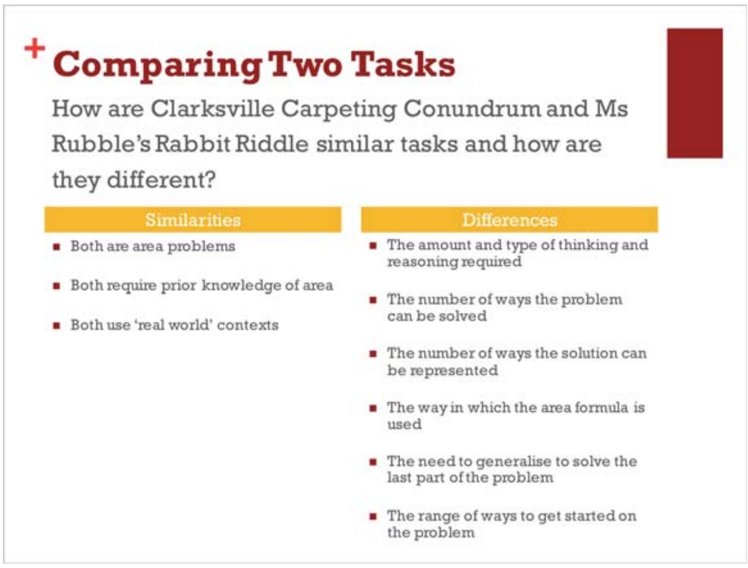

Figure 4.1 Excerpt from Teacher Professional Learning Session One (adapted from Stein et al., 2009).

Teachers used the task analysis guide (Stein et al., 2009, p. 6; Appendix F) to sort mathematics tasks based on the thinking processes they demand from students. The guide consists of lists of task characteristics at each of four levels of cognitive demand described by Stein et al. (2009) as *memorisation*, *procedures without connections*, *procedures with connections*, and *doing mathematics*. The goals of this activity were to develop shared understandings and language for discussing mathematical tasks and to raise teachers' awareness that not all tasks are equal and that task differences impact on opportunities for student thinking and learning.

Teachers were challenged to identify the level of demand of the tasks they had already planned to use the next day, and then to find or create an open-ended task that matched the criteria for *doing mathematics* and addressed the mathematical content and goals they intended to cover.

4.4 Content Specific Open-ended Tasks

There is substantial support for the argument that open-ended tasks promote meaningful engagement with mathematical ideas (e.g., Boaler & Staples, 2008; Sullivan, 2009). A task is described as open when it has more than one possible solution path or response and thus promotes openness in student activity and in interactions between the students and the mathematical content (Sullivan, Clarke, & Clarke, 2013). Sullivan, Walker et al. (2015) list four benefits of designing a task with multiple solution paths:

- It allows a low "floor" for the task in that all students can find at least one solution readily;
- There is an expectation that students will determine their own strategy for answering the questions and it is this opportunity for decision making that is engaging the students;
- There is a high "ceiling" in which students who complete the learning task can seek to propose a generalisation; and
- Having found their own solution strategy, the openness means that students can make unique contributions to class discussions. (p. 47)

The tasks used in this research project as well as incorporating these positive characteristics of open-ended tasks, were also content specific, meaning they addressed mathematical topics that form the basis of the curriculum. The tasks were also chosen to fit in with the topics that the participating teachers already planned to teach at the time the data was being collected. The content specific open-ended tasks used in this project, both in the teacher professional learning sessions and those chosen for the cases under study, were a mix of those drawn from available resources and those adapted or created by the researcher.

4.5 Session 2: How can we Implement Good Tasks to Maximise Student Engagement with Mathematical Ideas?

The set up and implementation of mathematics tasks i.e. phase two and three of the Mathematical Task Framework (Stein et al., 1996) was the focus of the teachers' second session. The session began with the researcher enacting a task explicitly modelling the use of a lesson structure and pedagogical approaches to support the maintenance of high level demand and differentiation of learning for a heterogeneous group through task variations.

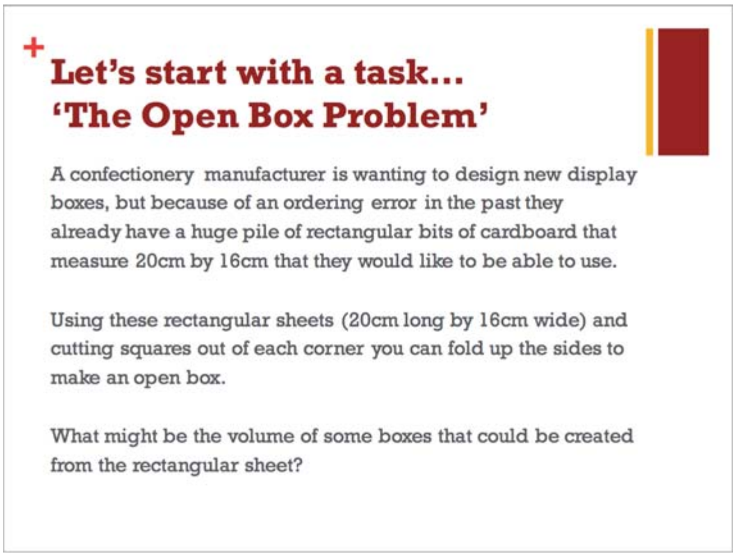

Figure 4.2 Task from Teacher Professional Learning Session Two.

The lesson structure modelled is discussed in section 4.6. The task chosen for this session (see Figure 4.2) is an adaptation of a problem widely available online or in mathematics texts.

The task affords multiple solution strategies and representations, and opportunities for visualisation and creative thinking. Various strategies to encourage generalisation and persistence were explicitly modelled (e.g., see Figure 4.3).

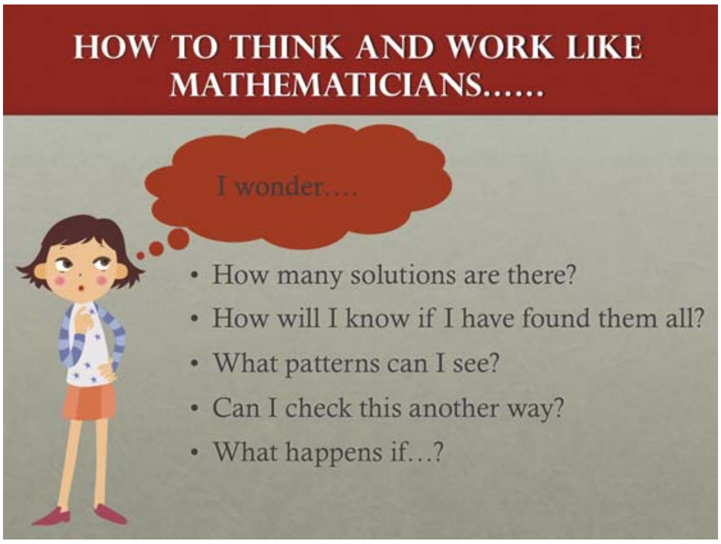

Figure 4.3 Excerpt from Teacher Professional Learning Session Two.

Pedagogies associated with the successful implementation of demanding tasks were explored. These included clear goals for the lesson based on conceptual understanding of mathematical relationships, selection of an appropriate open-ended task that has potential to help students accomplish those goals, and anticipation of student experience by exploring possible solutions, strategies, and representations (Smith & Stein, 2011). Teachers' pedagogical actions associated with the maintenance and decline of high-level cognitive demands were explored (see Figure 4.4). The session concluded with discussion of the lesson structure modelled.

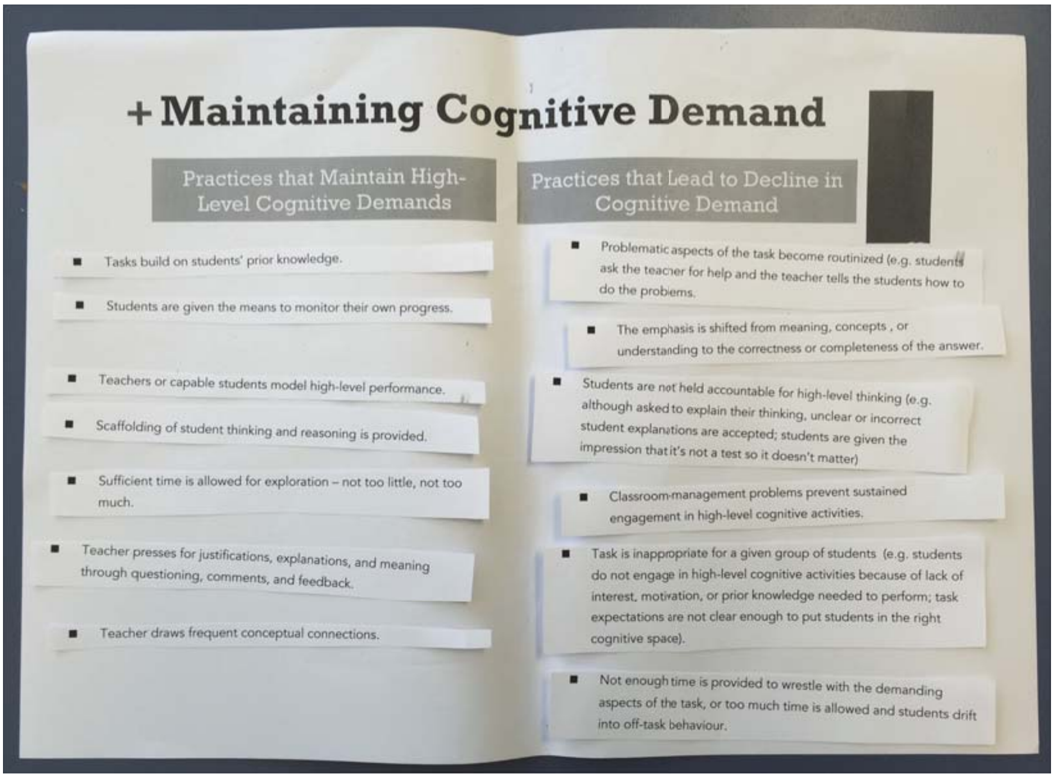

Figure 4.4 Activity from Teacher Professional Learning Session Two (adapted from Stein & Smith, 1998, p. 274).

4.6 Proposed Lesson Structure

The lesson structure advocated here is drawn from the work of Sullivan and colleagues (e.g., Sullivan, Askew et al., 2015, 2016) who explored a lesson structure that initiated learning through the use of a challenging task and differentiated learning through task variations. The work of Marshall and Horton (2011) on inquiry instruction is also influential. They analysed the structure of over 100 observed lessons particularly focusing on the order of instruction and concluded:

When teachers give students an opportunity to explore the concepts prior to an

> explanation, no matter whether the teachers or the students provide the explanation, the students think more deeply about the content. If reasoning and critical thinking are instructional goals, then these results suggest that teachers should consciously provide opportunities for students to develop their ideas for themselves. (p. 99)

A further influence is the framework for a three phase lesson structure, launch, explore, and summary, developed for use with cognitively demanding tasks by Lappan, Fey, Fitzgerald, Friel, and Phillips (2002).

4.6.1 Launch

The launch is how the task is introduced to the students. The information that teachers divulge or not at this stage is critical and requires careful planning. Jackson et al. (2012) argued that four crucial issues should be considered when planning a successful launch phase: ensuring that potentially problematic or unfamiliar contextual features of the task are discussed; key mathematical ideas inherent in the task are made explicit without hinting at particular strategies or procedures to find a solution; a common language is established to enable students to interpret the task appropriately and participate in subsequent discussion; and cognitive demand is maintained.

4.6.2 Explore

Important features of the explore phase includes students engaging with the task for themselves rather than being told what to do (Stein & Lane, 1996) and enabling prompts offered only when students have been attempting the tasks for some time (Sullivan et al., 2016). Explicit pedagogical actions that teachers can use to support learning at this stage may include *eliciting*, *supporting*, and *extending* strategies (Fraivillig, 2001), *monitoring* students' responses and *selecting* students to present their work later in discussion (Smith & Stein, 2011), *recognising* mathematically significant opportunities (Leatham et al., 2015), and actions to steer the instruction to the mathematical point (Sleep, 2012).

4.6.3 Summary

The lesson review is an orchestrated whole-class discussion where student activity on the task is explored through sharing and justifying of solution paths. The summary incorporates key pedagogical actions to maximise learning opportunities such as sequencing and connecting student responses to illuminate key mathematical ideas (Smith & Stein, 2011).

4.6.4 Planning Template

Multiple authors (e.g., Ball, 1993; Simon, 1995; Sullivan, Clarke, & Clarke, 2013) have argued the importance of teachers' purposeful planning of the content, design, and sequence of mathematics lessons. A mathematics task planning template (Appendix G) which incorporates the proposed lesson structure was developed by the researcher. It prompts teachers to consider their decisions at different phases of task planning, introduction, and implementation, and to support students to 'work like mathematicians' encouraging persistence and generalisation. Suggestions for using the practices for orchestrating mathematical discussions proposed by Smith and Stein (2011) are included, as are task variations in the form of enabling and extending prompts to support diverse learners (Sullivan, Mousley et al., 2009).

4.7 Session 3: Task Selection and Lesson Planning

The purpose of the third session was for the teachers participating in the research to select tasks for implementation and to explore the proposed lesson structure and planning template. The researcher presented examples of challenging tasks addressing an agreed upon topic. The suggested lesson structure was explored, the first task chosen from those presented and the first lesson was collaboratively planned. The task and its implementation is presented in the next chapter.

4.8 Summary

Phase One of the research, the intervention, comprised teacher professional learning sessions, the goals of which were to build teachers' understanding of cognitively demanding tasks and their capacity to implement them in ways that support student learning. The planning of the intervention sessions was informed by research. Following the third session teachers were equipped with a lesson structure, planning template, and associated pedagogies to assist in the implementation of the first task.

Chapter 5: Task One

5.1 Introduction

This chapter outlines the teachers' implementation of the first task. Section 5.2 describes the task. The remainder of the chapter describes the actions of teachers prior to, during and after the lesson as they endeavoured to maximise opportunities for student learning afforded by the task. Section 5.3 describes the teachers' collaborative preparation and planning. Section 5.4 outlines the task implementation in Nanette's class, and section 5.5 in Sally's. The two lessons described are illustrative of the themes extracted from the data from all three teachers' implementations of the first task. For this reason, description of the third lesson was redundant.

5.2 The Task

Looking for Three More

Four people in this room have an average height of 158cm.

You are one of them.

Who are the other three?

This task, adapted from Sullivan, Clarke, and Clarke (2013), is an open-ended task, in that it has a range of possible solution methods and responses. The task addresses content across all three strands of the mathematics and statistics learning area of the New Zealand Curriculum (Ministry of Education, 2007). It provides challenge in a meaningful context (height), and promotes engagement by incorporating a personal dimension for students (their own height). The task as it appears above i.e. in phase one of the Mathematical Task Framework of Stein et al. (1996) (see Figure 2.1) is a *doing mathematics task* as it requires students to explore and understand the nature of the mathematical concept of average (the arithmetic mean), no solution pathway is explicitly suggested by the task, and it requires multiple steps and complex thinking and reasoning.

5.3 Teacher Preparation and Planning

The teachers collaboratively planned the implementation of the first task using the suggested lesson structure and planning template. The planning template (Appendix H) supports teachers to consider the big mathematical ideas inherent in the task and generate a hypothetical learning trajectory that acknowledges and values both teachers' goals for instruction and students' thinking and understanding (Simon, 1995).

5.3.1 Mathematical Goals for the Lesson

The first step in the planning meeting was to identify the mathematical focus of the task which the teachers identified as 'the concept of average'. At this stage, refining this focus to specific mathematical goals for the lesson was problematic for the teachers. An illustrative response, *What is average? What is a typical height in our class?* (S-PreT1)[1] lacks specificity and only vaguely relates to the multiple concepts addressed by the task. Because teachers are more likely to use challenging tasks if they understand the mathematics involved and its potential (Sullivan et al., 2016), the researcher joined the teachers' discussion to refine appropriate goals for the lesson. It was agreed that 'average' in this problem referred to the 'arithmetic mean'. The following goals were decided:

- develop conceptual and procedural understanding of average i.e. a fair share or centre of balance conceptualisation, and procedurally know how to calculate the average from a set of scores,
- know own height in centimetres and measure height to the nearest centimetre.
- estimate sets of four scores that can average 158cm and demonstrate awareness of more than one possible solution.
- understand the influence on the average of changing one or more scores.

5.3.2 Anticipating Student Difficulties

Lesson design entails connecting teachers' mathematical understanding and their hypothesis about students' understanding (Simon, 1995). In this case the teachers thought that the task would be very challenging for many students because it requires complex reasoning involving several strands of mathematics understanding. Sally, in particular, expressed concern about *her range of students* and what this means in terms of *what I will have to set up for my students to all engage with the same task* (S-PreT1). To address anticipated difficulties, the teachers decided to focus on the measurement aspects of the task on the previous day. This included students measuring and recording their heights and displaying these publicly, thus providing visual prompts to support students to *get a sense of their own and others' heights and get buy in from the students* (N-PreT1).

The teachers had little confidence that many students had prior experience of the

[1] The following notation is used to reference the teacher planning meetings and interviews. S refers to the initial of the teacher pseudonym. PreT1 refers to the pre-task 1 planning meeting, TransT1 refers to the transcript of the lesson implementation for task 1 and PostT1 refers to the post-task 1 interview etc.

mathematical concept of average (mean). The planning indicated their decision to use a visual prompt of a line drawn at 158cm on the classroom wall to support students to understand their own height in terms of the average (158cm), and to support conceptualisation of average as a balance point. The planning also indicated that the teachers would ascertain previous experience and understanding of average during the task launch, and provide sufficient explanation to scaffold students into the task.

5.3.3 Anticipating Solutions and Strategies

The teachers worked through the task with the purpose of anticipating students' possible solutions and strategies. They anticipated that many students would use the visual prompts provided to consider their height as above or below the average and search for someone else an equal distance on the other side of average. This conceptualisation of average as a balance point, exemplifies the property that the sum of deviations from the average is zero ($\Sigma(\bar{x} - x_i) = 0$).

Some students may use the add-then-divide procedure ($\bar{x} = \frac{\Sigma x_i}{n}$), starting with a random selection of students then likely using trial and error, swapping students in and out to achieve the desired average. The teachers anticipated that this approach would support students to recognise the effect that changing scores has on the average, and consequently make informed choices of heights to include or exclude. Some students may already understand the relationship between scores and the average and intuitively manipulate the formula for average ($\bar{x} = \frac{\Sigma x_i}{n} \Rightarrow n\bar{x} = \Sigma x_i$) and look for four students with a total height of 632cm.

5.3.4 Enabling and Extending Prompts

The teachers planned enabling prompts to provide appropriate scaffolding for students struggling to get started while still supporting them to be challenged by the intended mathematics in the task. The teachers' discussion acknowledged the importance of maintaining cognitive demand and awareness of research reporting teachers' tendency to reduce the cognitive demands of tasks when assisting students facing difficulty (Stein et al., 2009; Sullivan et al., 2011). The enabling prompts planned were:

- Solve the problem with just two students who have an average height of 158cm.
- Find the average height of your group. What does this tell you?
- What if the average of the four people was 150cm?
- Find four students with a total height of 600cm. How could this help you to work on the task?

Extending prompts planned to extend the thinking and activity of students who finished the task quickly included:

- Find a second set of students whose average is 158cm.
- Use your solution to find four students whose average is 159cm.

5.3.5 Arranging for Learning

The students were typically organised into like-achievement groups for mathematics lessons, but the teachers decided in the planning meeting that for this lesson the students would choose their own groups. The benefits of mixed-ability groupings were discussed from both a mathematical and growth mindset perspective, but concerns were expressed that the students who were usually in the 'low-ability' groups would struggle with the openness of the task and the persistence required to overcome potential difficulties.

In order to support collaborative work on the task each self-selected group of three or four students could include one or more of their heights as '*You are one of them*' referred to in the task.

5.4 Nanette's Lesson: *Looking for Three More*

5.4.1 Task Launch

As indicated in the written plan Nanette had focused on measurement of height in the previous day's lesson and students' previously measured heights were displayed on the classroom wall.

It was clear that Nanette had also discussed average with the students the previous day.

> *Remember how yesterday we had 3 students up the front and we looked at average, so the average height was where I put the magnet between them. 158 is where the magnet is. 158 is the average.* (N-TransT1)

In the post lesson interview Nanette recalled:

> *We placed a magnet where we thought the average height of the students might be, focusing on the fact that it would be in the middle of the heights and then we worked out the average. I chose three people because it made it a slightly different problem, different enough to not give it all away today.* (N-PostT1).

The four-minute task launch consisted of Nanette reading the task to the students, and reminding them of the previous day's activity (as above). She concluded with:

For this task you're going to need to move around the room. You're going to need to have conversations with people. You need to find their heights. I want to see lots of movement and talking and thinking. We've got plenty of time to get on with this today, plenty of time to work it out. If you're struggling put your hand up, I'll come around. (N-TransT1)

5.4.2 Task Explore

Following the launch, Nanette was very busy helping students who had immediately accepted her offer for assistance and gathered around her. She initially responded by providing an unplanned prompt, namely the procedure for calculating average: *Get four students' heights, add them together and divide by four* (N-TransT1). Seven minutes into the explore section of the lesson Nanette approached the researcher and acknowledged the difficulty she was having responding to the students' requests for help: *I find it really hard not to tell them* (N-TransT1).

Although prompting students to use the add-then-divide rule was intended to be helpful it did not lead to a successful solution for most of these students. The prompted procedure allowed them to access the first step (calculating the average height of four chosen people), after which many struggled to make sense of the task, mostly adopting a random trial and error approach (Figure 5.1).

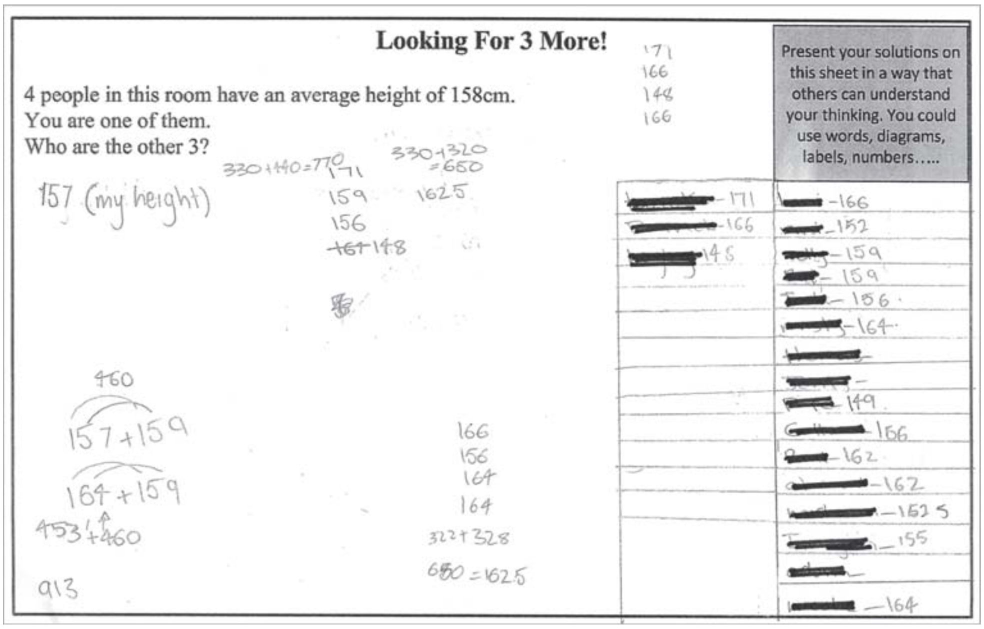

Figure 5.1 Procedure and trial and improve strategy, Example 1

Five of the 12 students who used this approach successfully found a solution. Their use of the add-then-divide rule together with a trial and improve approach promoted an emerging understanding of how the scores affected the average (Figure 5.2).

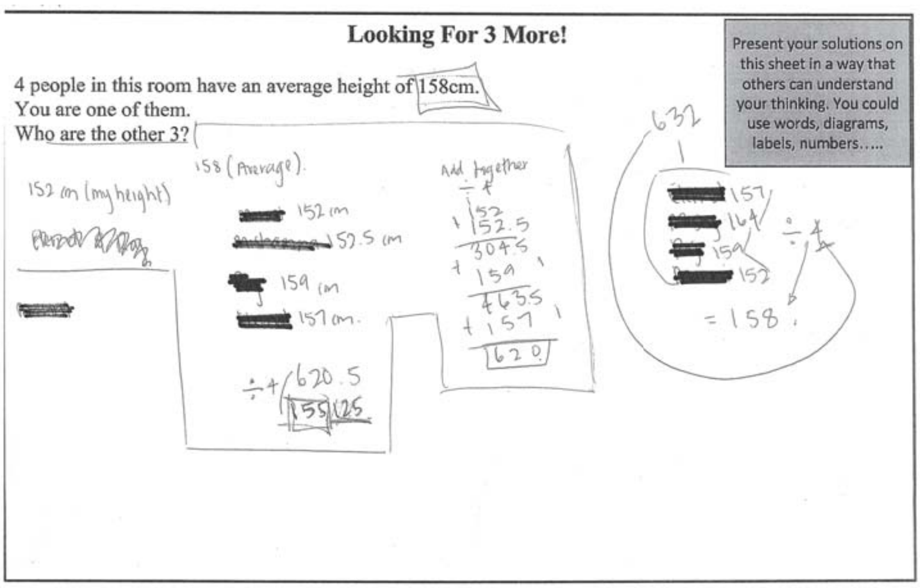

Figure 5.2 Procedure and trial and improve strategy, Example 2

A group of students had experienced difficulty getting started, but had not sought assistance from the teacher early in the lesson. When Nanette realised they were struggling she prompted: *Find the average height of your group. What does this tell you?* (N-TransT1). The students in this group were all 156cm tall and were initially confused about whether it was possible to have an average if all the scores were the same. In reaching a consensus on this issue these students conceptually explored their own understandings about average and successfully solved the task using a centre-of-balance approach.

Chris: *We are all 156cm so our average is 156*

James: *That's too short then. We need to ditch someone and get a different person. We could use (teacher).*

Anton: *She'd be nearly 2m. We need someone who's 164cm*.

Chris: *Why 164?*

Anton: *Because that is 6 over the average. We are 2 under the average each. 2,4,6; 6 for all of us so we need someone who is 6 over.*

(N-TransT1)

The total height (632cm) approach ($n\bar{x} = \Sigma x_i$) was used by four students all of whom successfully found at least one solution. This solution strategy promoted conceptual understanding that supported generalisation and several students found multiple solutions. It is possible that these students' prior understanding was more advanced, although the illustrative student work sample (Figure 5.3) suggests that this student's understanding developed as he worked through the task.

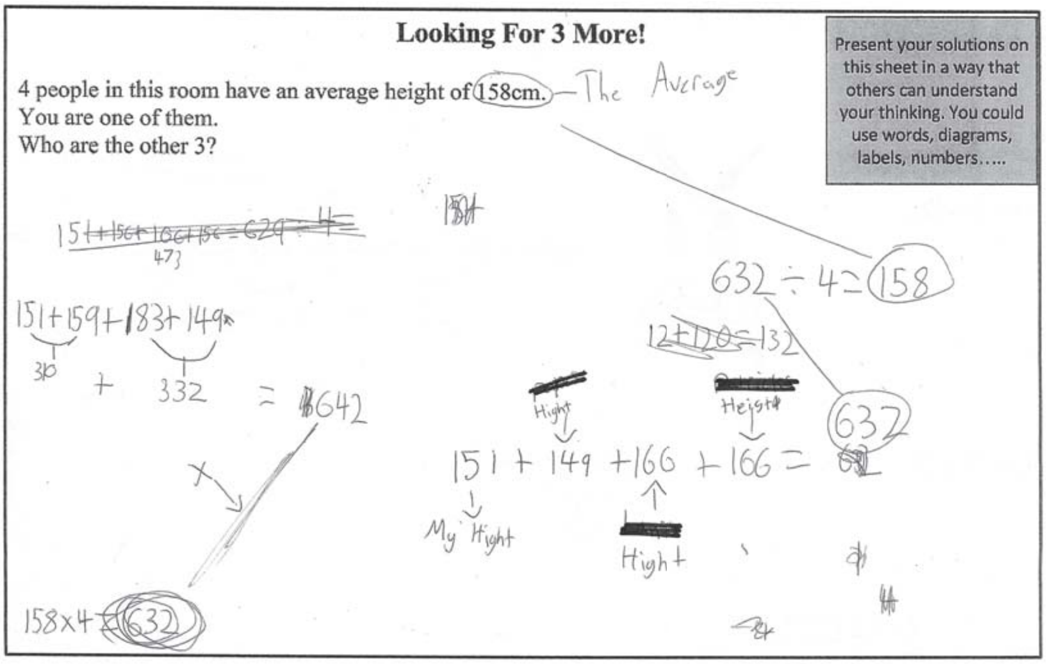

Figure 5.3 Solution using total height approach.

The students spent 30 minutes exploring the task, and of the 21 students present, 13 identified at least one correct solution.

5.4.3 Task Summary

For the lesson summary Nanette selected three groups to share their solution strategies representing the three approaches adopted. Nanette later explained her decision to start with the centre-of-balance approach. *Their strategy was different to everybody else's, it was simple and those who hadn't got it yet would be able to understand* (N-PostT1).

The group chosen to share next had successfully used the procedural approach, swapping in students' heights until the required average was reached.

Nanette: *What were their heights?*

Ray: *157, 152, 164 and 159.*

Nanette: *And what did you do with those numbers?*

Ray: *Added them together which was 632 and divided by 4, we got 158.*

Nanette: *So you added all these together and got 632. 632 was the magic number. Who else got 632?* (N-TransT1)

These students were not initially aware of the importance of 632, the total height, but Nanette's emphasis connected this strategy to the final strategy shared, the total height approach. Nanette reiterated the connection at the conclusion of the lesson.

> *So Danny started the opposite way to Ray's group. She started with finding the total height she needed altogether which is the same number 632, the average height times 4.* (N-TransT1)

5.4.4 Discussion

This section discusses Nanette's enacted pedagogies in relation to opportunities occasioned for students to engage with and learn from the task. The discussion is informed by the researcher and teacher's reflection on the data collected from the lesson observation and the subsequent teacher meeting, further strengthened by retrospective analysis.

Nanette was concerned initially that the students in her class would not make progress without assistance. They were unaccustomed to being given tasks that they did not immediately have a strategy for solving, and the classroom norms were such that the students saw it as the role of the teacher to provide one. Nanette reflected: *I had a lot of resistance for the first couple of minutes but this died down. Knowing my enabling prompts better would be something I could improve on, I just needed to be patient in the beginning to let students find their feet. They surprised me, they could do so much more than I gave them credit for* (N-PostT1). Previous research (e.g., Clarke & Peterson, 1986) has recognised that teachers' intentions to act are informed by their knowledge, beliefs, values and attitudes as well as the constraints they anticipate experiencing. Although Nanette was aware of recommended approaches to implementing challenging tasks and had anticipated and prepared for student difficulty she acknowledged how difficult this was especially as it involved altering ingrained practices.

Ultimately the students who were provided with the unplanned procedural prompt were the least likely to successfully solve the task. Nanette's adaptation lowered the cognitive demand of the task. In relation to the framework and taxonomy developed by Stein et al. (1996) the task was transformed from a *doing mathematics* task (in instructional materials) to *procedures without connections* for the students whose approach was unconnected to meaning, and *procedures with connections* for those students whose use of the procedure and trial and improve strategy resulted in an emerging understanding of how the scores affected the average.

The group of students who used the centre-of-balance approach also lacked prior knowledge, but without provision of an explicit pathway by the teacher, they used each other as intellectual resources in collective sense-making. Likewise, the students who generated multiple solutions were those that devised their own methods. This is consistent with findings in the literature (e.g., Marshall & Horton, 2011; Sullivan et al., 2016) that students who have opportunities to develop their ideas for themselves prior to an explanation think deeply about the content and are more likely to develop conceptual understanding.

Nanette made purposeful decisions selecting and sequencing strategies and responses during the lesson review. She felt confident that the sequence chosen maintained a coherent argument, but felt that there would have been value in creating a shared record of strategies so that connections could be more clearly seen.

As an early career teacher, confident in her own mathematical knowledge and ability, Nanette felt that teaching this task *challenged me to think about how students see maths. I can do the task but it's really hard for me to think how other people might do it.* (N-PostT1). Nanette's comments reflect the well documented challenges teachers face in translating their own mathematical knowledge into worthwhile lessons (Ball et al., 2008; Sullivan, Clarke et al., 2009). Ball (1993) pointed out that teachers must have a "bi-focal perspective – perceiving mathematics through the mind of the learner while perceiving the mind of the learner through mathematics" (p. 159).

5.5 Sally's Lesson: *Looking for Three More*

5.5.1 Task Launch

In Sally's class the students had also measured and recorded their heights the previous day and although there had been some discussion about average no procedure for calculating average had been provided. Sally reiterated:

> *So we're looking at Geoff's height (144), and Kyle's (163). So there's something happening between Geoff and Kyle, there's clearly a height difference isn't there? We have 144 and 163, what does that say about 158? Is it somewhere between those two?* (S-TransT1)

The remainder of the task launch addressed the establishment of classroom norms, reinforcing for students that they can learn from each other, and: *it's important to set goals and decide next steps* (S-TransT1). It was unclear whether this referred to mathematical goals.

5.5.2 Task Explore

The students worked in self-selected groups, except a 'working group' of six students the teacher deemed would need further support. She gathered them to work with her. The goals for this group were altered from those for the original task.

Sally was kept busy predominantly with the ‘working group’, leaving them on two occasions to monitor other students’ work and to offer enabling prompts. After 20 minutes Sally was aware that most students although engaged were not making progress towards a mathematical solution. Sally called the class together and reiterated her expectations.

> *I notice some people are moving forward in this now – that’s fantastic. Some people have drawn it, some people have put it in order, those kinds of things. One of things you need to be thinking about, how are we going to use that information? Leila said we could add them together and divide, and we could use google if we needed that information, ok? So we need to be thinking about some of the things we have discussed, what could we use?* (S-TransT1)

It became clear that ‘*setting goals and deciding next steps*’ Sally had emphasised earlier related to problem solving strategies displayed on the wall. These included ‘*Draw a picture.*’ and ‘*What information have you been given? How can you use this information?*’. Sally’s description of students’ strategies, namely *some people have drawn it in, some people have put it in order* (S-TransT1) was an accurate account of many student approaches, presumably prompted by the problem solving strategies recommended to them (see Figure 5.4). However, in many cases, these strategies did not result in the intended mathematics of the task being addressed.

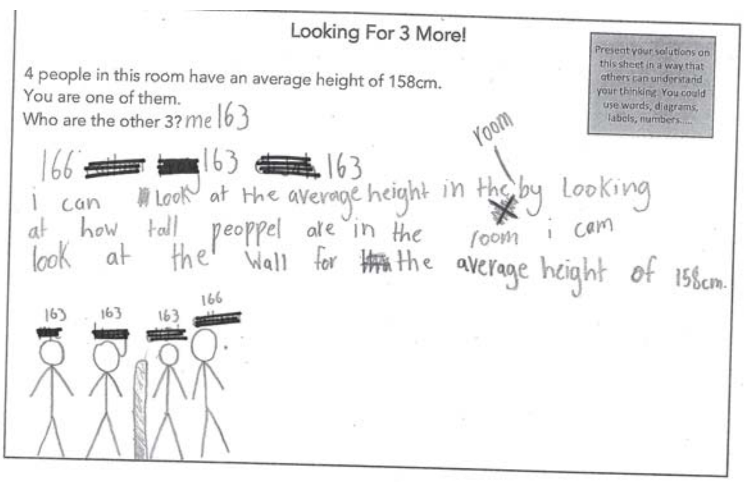

Figure 5.4 Strategy that does not address intended mathematics

Only two of the 20 students successfully solved the task.

5.5.3 Task Summary

For the lesson summary Sally selected three groups to share their solution strategies. A student from the 'working group' shared that he had chosen four people, drawn them and recorded their heights.

The second group Sally asked to share had attempted to solve the main task. Isabel's strategy (see Figure 5.5) and explanation were based on partial understanding that the average was the midpoint of two heights, and a misconception that 'averages' calculated for many combinations of two heights could then be combined to form an overall average.

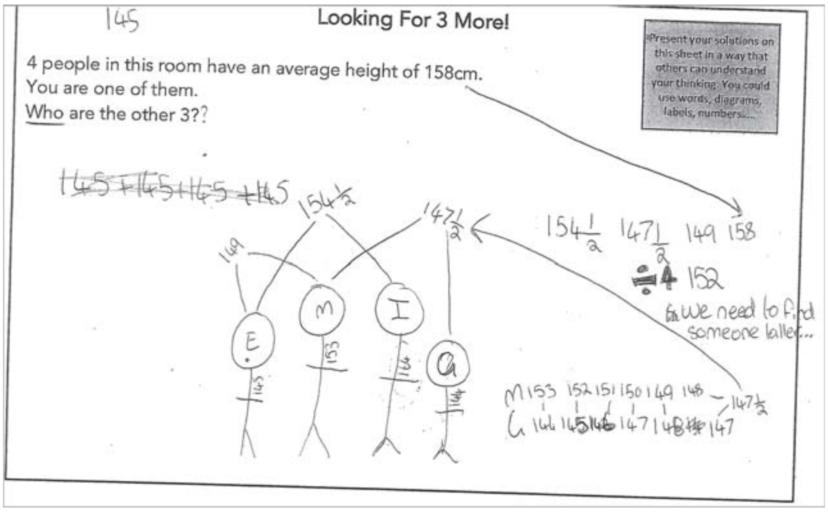

Figure 5.5 Isabel's solution demonstrating misconception

Sally questioned and prompted Isabel to clarify her reasoning, but Isabel's explanation was confusing and ultimately her misconception and incorrect solution remained unchallenged.

The summary concluded with Emma's successful solution strategy.

> *You can figure out the average by balancing it out, like 2 under average and 2 higher. I had 151, 161, 154 and 162. Take 3 off 161 and put it on 155 you get 158, and take 4 off 162 and put it on 154. Altogether the numbers would be 158, 158, 158, 158*
>
> *You can have it uneven like 1 taller and 3 smaller like 179, 152, 148, 153*
>
> *179 is 21 over 158*
>
> *152+6=158   148+10=158   153+5=158 and 6+10+5=21 so it's 21 under.*
>
> (S-TransT1)

5.5.4 Discussion

There were a group of students who Sally did not believe would be successful with the task. Unlike Nanette's approach which ultimately resulted in all students having the opportunity to engage with the main task at some level, and in some cases surprise her, Sally transformed the task demands and goals for these students right from the outset. The self-fulfilling prophecy of lack of expectation and success that entraps these students is supported by research. Students placed in low-ability groups are commonly denied access to rich learning experiences, and are therefore less likely to be exposed to higher order mathematics thinking or ideas. The resulting low achievement further exacerbates their own and others' views of themselves as helpless learners and promotes belief that they lack inherent 'mathematical ability' (Boaler & Sengupta-Irving, 2016; Clarke et al., 2014; Dweck, 2007; Middleton & Jansen, 2011).

Sally's decision to spend her time with the 'working group' affected her ability to support other students while they explored the task. Previous research (e.g., Leatham et al., 2015; Sullivan et al., 2016) has recognised the importance of monitoring and recognising students' mathematical thinking, solutions, and misconceptions during this stage of the lesson. Isabel's explanation was undoubtedly confusing, but Sally had also not adequately monitored her work during the explore phase and was subsequently ill-prepared to respond to her strategy, explanation or misconception.

The orientation of the teacher towards the development of student agency was clear, with most students allowed and encouraged to choose their own solution strategy. Sally, a confident experienced teacher conversant with the lexicon of pedagogy, prides herself on creating student-focused programmes and a classroom environment where multiple perspectives are respected and incorporated into collective knowledge creation. However, of the two complementary norms of activity that constitute the socio-mathematical framework described by Cobb and McClain (1999) and more recently by Sullivan et al. (2002), socio-cultural norms were prioritized over mathematical norms in this lesson.

The focus on non-mathematical purposes at the expense of clearly articulating the mathematical point of the lesson contributed to students' difficulty in engaging with the big mathematical ideas inherent in the task. The mathematical emphasis of the launch on the difference between heights and the average's location between those values, failed to orient the student activity towards the intended mathematics. In the lesson summary, student work was accepted without challenge, and opportunities to clarify mathematical misconceptions or to illuminate or connect mathematical ideas were not taken. This result concurs with descriptions of teachers' difficulties in steering instruction towards intended mathematical goals discussed in the literature (e.g., Sleep, 2012; Sullivan, Clarke, & Clarke, 2013).

Substantial evidence indicates teachers' knowledge of mathematics informs their intentions for lessons and influences their effectiveness in facilitating mathematical learning (e.g., Charalambous, 2008; Sullivan et al., 2016). Mathematics lessons are shaped by the planned direction of the lesson and teachers' ability to interpret and respond to students' mathematical thinking in the moment. Sally's implementation of the first task was impeded by lack of mathematical clarity. Although aware of recommended pedagogies and the desire not to routinise the activity, opportunities were missed to monitor and fully understand students' invented strategies, and she appeared to be uncertain about aspects of the mathematical content.

5.6 Summary

This chapter has described two teachers' implementations of the first task. The teachers collaboratively created a plan for instruction, although during implementation teachers adapted this to suit their teaching style and context.

Two very different implementations of the same task demonstrated that tasks change their character as they are implemented in classrooms mediated by teachers' beliefs, goals and intentions for instruction, their mathematical subject matter knowledge, and pedagogical content knowledge. The lessons confirm the applicability of the framework and taxonomy of Stein et al. (1996, 2009), and corroborate the work of other researchers regarding the impact teachers' knowledge has on their effectiveness in facilitating students' mathematics learning.

Chapter 6: Task Two

6.1 Introduction

The previous chapter demonstrated that teachers' beliefs, goals, intentions, and knowledge mediate their implementation of classroom tasks. Evidence was provided that different implementations of the same task resulted in different experiences for students and different opportunities to learn mathematics.

This chapter outlines the teachers' implementation of the second task. Similar to the previous chapter the findings are reported in four sections. Section 6.2 describes the task and section 6.3 the teachers' preparation and planning. Section 6.4 outlines the task implementation in Sally's class, and section 6.5 describes Louise's implementation.

6.2 The Task

> *Measuring Money*
>
> You have won a prize! Your prize can be one of:
>
> - two metres of $1 coins (lying flat)
> - one square metre of 10 cent pieces (edges touching, lying flat)
> - a one litre container full of 20 cent pieces
> - one kilogram of $2 coins
>
> Which prize would you choose?

This task is adapted from Beesey, Clarke, Clarke, Stevens, and Sullivan (1998). The task provides challenge in a meaningful context, namely money, although the task only nominally concerns money. Task complexity relates to students' understanding of multiple aspects of measurement, their choices of possible solution pathways, and understanding the impact of measurement precision and accuracy on their solutions.

6.3 Teacher Preparation and Planning

Topics discussed at the planning meeting included the mathematical focus of the lesson, strategies to support student persistence, accountability and collaboration, anticipated solutions and difficulties, and strategies that teachers could use in the summary section of the lesson to facilitate participation, engagement, and understanding.

6.3.1 Mathematical Goals for the Lesson

The mathematical goals for the lesson were:

- devise measurement strategies for measuring multiple attributes (length, area, capacity, weight) of various collections of money.
- generate estimations and/or apply proportional relationships to samples rather than counting all.
- understand the influence of precision and accuracy of measurements on the final solution.

6.3.2 Support for Perseverance, Accountability, and Collaboration

Uncomfortable about the ‘zone of confusion’ her class had experienced at the beginning of the previous lesson, Sally felt that scaffolding to support them to overcome initial barriers would increase the likelihood of task success. Specific strategies she suggested included:

1. explicit teaching of skills for students to work through the ‘zone of confusion’,
2. teacher rather than student selected groups, and
3. the teacher taking greater control over progress through the task by separating it into four parts and directing the students to work on one aspect of the task each day over four days.

In relation to the third point, Nanette argued *I don’t think we need to scaffold that much, not for the whole class* (N-PreT2). In directing the learning closely, the teacher risks lowering the task demands, adversely affecting opportunities for connections between the attributes measured and diminishing student authority to decide their own solution path (Stein et al., 2009). Having reflected on the tension between the shift in agency and authority towards student-centred activity advocated in inquiry classrooms and the desire for teachers to direct student learning, the teachers decided not to control students’ progress through the task so closely.

The teachers wanted to create groups where students with varying prior knowledge would support each other to access the task, a notion that is supported in the literature. Featherstone et al. (2011) described a challenging group-worthy task as one that positions the zone of proximal development within the learning activity prompted by the task and where students’ success with the task arises from and depends on collaboration. Hatano and Inagaki (1991) argued:

> A group as a whole usually has a richer data base than any of its members for problem solving. It is likely that no individual member has acquired or has ready

access to all needed pieces of information, but every piece is owned by at least one member in the group. (p. 341)

Sally reflected that *the groupings are really important as everyone in the group has got to be able to contribute at a level that will be valued and valuable to the group as a whole* (S-PostT2).

6.3.3 Anticipating Methods, Solutions, and Difficulties

There was an expectation that students without sufficient prior knowledge of measurement, in particular metric units associated with each attribute and conversions between units, would experience difficulty. The area and volume aspects of the task are more challenging. Teachers anticipated that most students would solve the task for a smaller area (or volume) and scale up their solution as required (see Figure 6.1). This could result in misconceptions with scaling up area and volume calculations, and potential for computational difficulties. A range of solutions was expected depending on the level of accuracy and precision students applied to the problem.

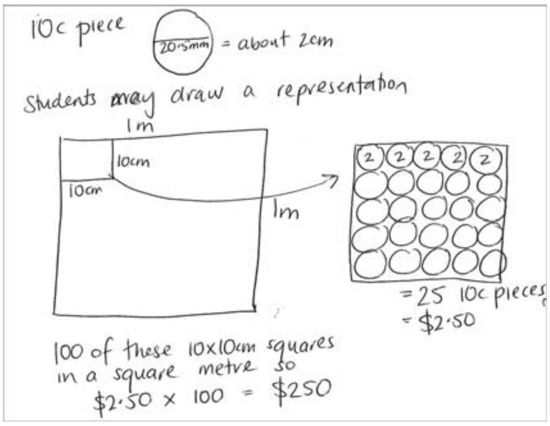

Figure 6.1 Teachers' anticipated solution strategy for area task

6.3.4 Enabling and Extending Prompts

Scaffolding in the form of enabling prompts included:

- How many $1 coins would fit on your ruler?
- Draw a 10cm by 10cm square. How many 10c pieces could you fit in this area if you put them in, or draw them in, with edges touching?
- Can you think of a way to work this out without counting each coin?
- Draw a square metre on the floor.
- What are the measurements of a one litre container?
- How many coins would fit into this side, (or area) of the container?

Likewise, extending prompts included:

- Will the same dollar value of coins fit into other containers that have a different shape but still hold one litre?

6.4 Sally's Lesson: *Measuring Money*

6.4.1 Task Launch

Sally's task launch, similar to task one, focused on students' use of learning strategies. Explicit instruction guided student activity and metacognition. *Talk with your group about the task. Make a plan how you are going to approach the task. What is the task asking of you? What do you need to do to work it out? Remember our strategies. You might draw a picture, make a table or a chart, guess and check or something else* (S-TransT2).
The students spent 12 minutes planning their approaches, during which time Sally moved amongst the groups. The class shared their plans and aspects of the task were clarified.

6.4.2 Task Explore

The students, working in groups, spent two days exploring the task. Seven out of the eight groups accurately calculated the value of two metres of $1 coins (see Figure 6.2) and of one kilogram of $2 coins.

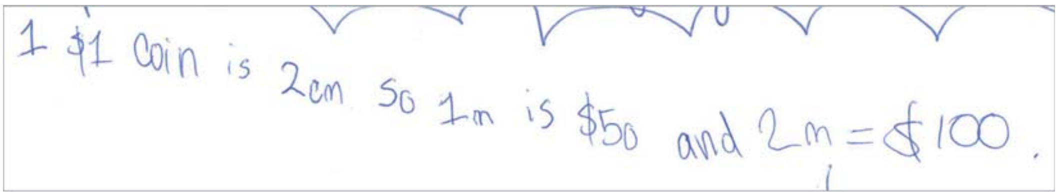

Figure 6.2 Value of 2m of $1 coins

Five groups calculated the value of one square metre of 10 cent pieces, some using the anticipated method visualising the square metre as made up of smaller more manageable areas and scaling up (see Figure 6.3), while other solution methods illustrated abstract conceptualisations not requiring the support of a visual representation (see Figure 6.4).

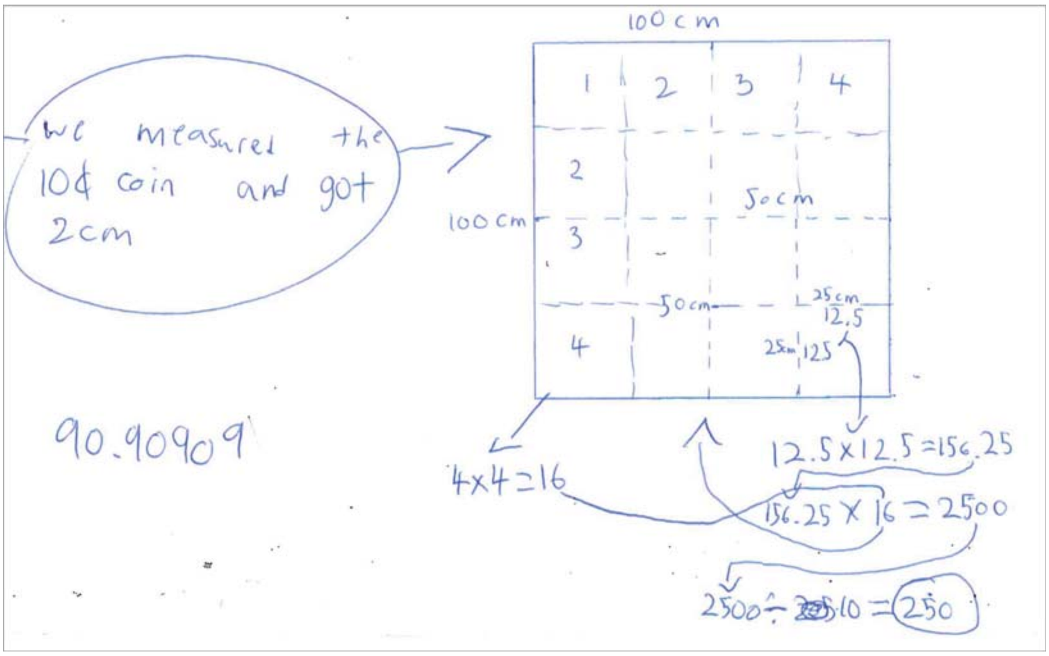

Figure 6.3 Value of square metre of 10c pieces, Example 1.

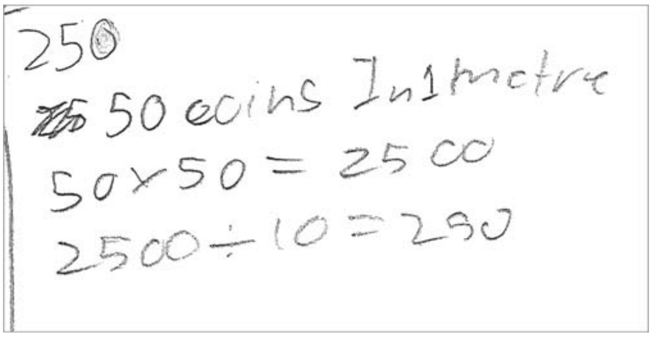

Figure 6.4 Value of square metre of 10c pieces, Example 2.

Six groups attempted to find the value of one litre of 20 cent pieces. As anticipated this task elicited varying degrees of precision resulting in divergent solutions. One group (see Figure 6.5) used the dimensions of a 20cent coin (d=2.1cm, h=1.5mm) to image a cube with sides of 2.1cm that contained a stack of 14 coins. (1.5mmx14=21mm). They calculated how many times the volume of this small cube (9.261cm^3) would fit into a litre (1000cm^3).

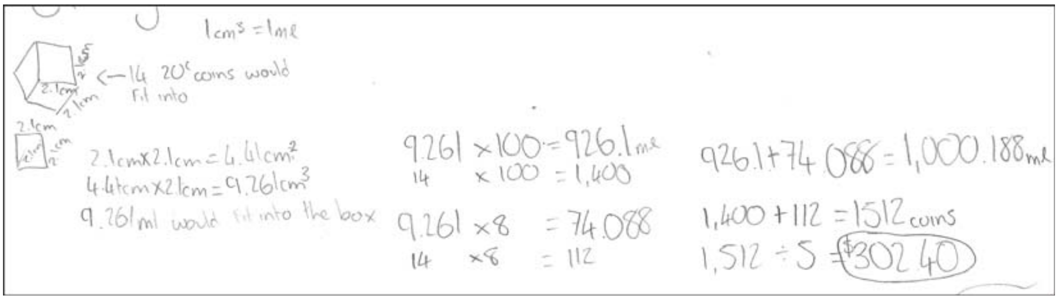

Figure 6.5 Value of one litre of 20 cent pieces, Example 1

Other groups used the containers provided to estimate how many coins would fit across the bottom and visualised stacks of coins (see Figure 6.6).

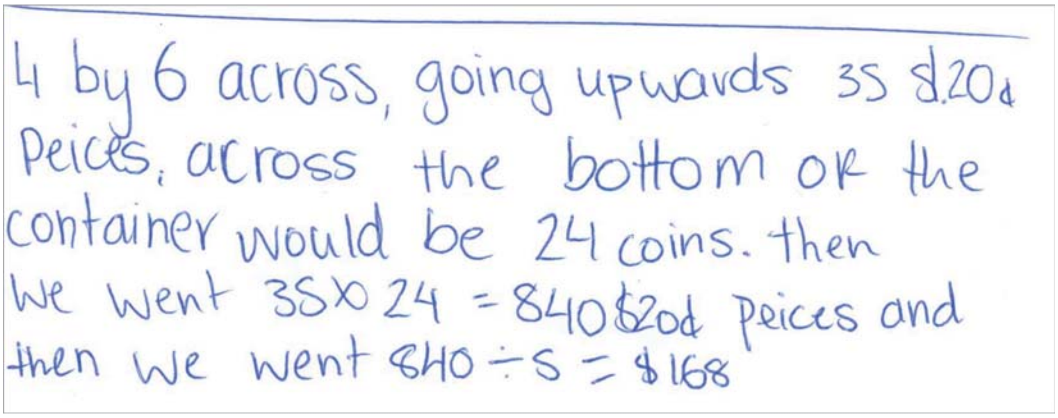

Figure 6.6 Value of one litre of 20 cent pieces, Example 2

6.4.3 Task Summary

The lesson summary consisted of mini-summary sessions where groups of students compared their solutions and methods with another group who had tackled the same task. Some students shared solutions to the length task only, thus limiting their access to the more challenging aspects of the problem. Diverse solutions were presented in the groups who shared their solutions to the area and volume tasks, resulting in examples of student-led discussions where misconceptions were challenged and issues of accuracy and precision were scrutinised. Students in one group, trying to account for the variations in their solutions, considered whether the shape of the one litre container would impact on how many coins it would hold, thereby extended their thinking about the relationship between the lengths and shapes of sides to the volume of the container.

6.4.4 Discussion

Sally reflected in the post-lesson discussion that the implementation of task two was more successful than the first and identified two reasons. Firstly, she believed the task offered greater opportunities for all students to engage, and secondly her deliberate use of specific actions facilitated greater student engagement.

The completed student work samples provided evidence that the task was more accessible by the students and suggested high levels of engagement. From a motivational perspective, the task satisfied the two complementary dimensions of challenge and control. Completion of the task was undoubtedly challenging, but some aspects were more accessible than others, exemplifying a "low floor, high ceiling" task where differentiation for diverse learners is built into the task itself. The task also afforded student decision making, offering multiple possible methods as well as which aspect of the task to work on first, thereby increasing students' sense of control. This is consistent with Middleton and Jansen's (2011) argument that opportunities for choice empower students, support their sense of agency and increase task interest.

Teachers' decisions and actions impact on engagement and learning opportunities for students. With this in mind, Sally's intentions in managing the summary section of the lesson as student-led mini-summaries, were to facilitate participation of all students, to value diverse student responses and to support students who lacked confidence. She was aware that students who engage in both the 'explore' and 'explain' components of the learning process, are more likely to gain conceptual understanding by making sense of their own and others' strategies (Marshall & Horton, 2011). In some of the mini-summary sessions, the groups of students were able to examine their work critically, formulate explanations and connections in the absence of the teacher. In others, the sharing session did not result in productive discourse that challenged students' existing mathematical thinking.

In an inquiry classroom teachers balance their role between supporting students' authority and agency for their learning, and being accountable for upholding high level of rigorous conceptual mathematical thinking (Anthony, 1996; Anthony & Walshaw, 2007). Sally's decision to hand control for orchestrating the discussion to the students potentially put at risk opportunities to move students thinking to new levels of understanding. Smith and Stein (2011) argue that at this crucial point in the lesson the teachers' role is to skillfully devise responses and questions that make the mathematics visible and understandable as "the key to connecting is to make sure that the mathematics to be learned is openly addressed" (p. 50).

6.5 Louise's Lesson: *Measuring Money*

6.5.1 Task Launch

For her task launch, Louise read the problem to the students, advised them of their working groups and gave organisational instructions for the use of available equipment.

6.5.2 Task Explore

The students spent two days exploring the task. After the two sessions, all eight groups had solutions for the length and weight aspects of the task, although not all were correct. Six groups had attempted the area task, and four had attempted the volume task. Several group solutions were based on erroneous mathematical assumptions, including that one square metre was the same as four metres, or that one litre of coins weighed one kilogram.

6.5.3 Task Summary

Louise orchestrated a lesson summary that addressed each aspect of the task separately. The length task summary is described below and is illustrative of Louise's decisions and actions for other aspects of the task.

Louise selected two groups of students to share during the lesson summary. The first group shared a counting strategy where they iterated two coins along a two metre length of string resulting in a count of 100 coins. Louise prompted the second group to connect and compare their strategy to the previous group:

> Louise: *You got the same answer, but you didn't lay the coins down. Explain how your strategy was different.*
>
> Roy: *We measured each coin. They're 2cm, so we added 50 of them to make 1m. 1m would be $50 so 2m is $100.*
>
> Ricky (interrupting): *Why didn't you just halve the 200? Each coin is 2cm, you have 2m which is 200cm, so halve the 200.*
>
> Louise: *That's an interesting way of solving it that I hadn't thought of. Very cool. So these answers are all $100, is that what everyone got?*
>
> Leslie: *I got $20.*
>
> Louise*: Did you use the same method or something different?*
>
> Leslie: *I used fractions. I started with* $\frac{1}{10}$ *of 2m would be 2 coins, so* $\frac{10}{10}$ *would be 20 coins which is $20, but I think it's wrong.* (L-TransT2)

Ricky's solution, which Louise had not noticed as she monitored student activity during the explore phase of the lesson, represented a flexible understanding of the relationship between the measurement and the number of coins. Louise was also not aware of Leslie's strategy or of her misconception that 2 coins corresponded with $\frac{1}{10}$ of 2m rather than $\frac{1}{100}$ of 2m. Although Leslie herself noted that she made an error she didn't understand what her error was, and this was not addressed.

6.5.4 Discussion

Louise reflected that her minimalist approach to the launch of this task was a reaction to her task one lesson where she had over-prepared the students for the task. She acknowledged that the first day spent on task two did not go well. *I didn't pose the task well. On the first day they didn't really know what to do and so I was working really hard moving amongst the groups to motivate them and to clarify parts of the task* (L-PostT2).

Important roles of a task launch are to clarify key mathematical ideas and relationships and to establish a common language that enables students to interpret the contextual and mathematical features of the task appropriately (Jackson et al., 2012, 2013; Stein & Lane, 1996). Louise's failure to address these at the outset appeared to affect how students participated in solving the task. The establishment of a shared understanding of key mathematical ideas may have cleared up misunderstandings and incorrect assumptions that marred some solution approaches.

Louise's lack of task introduction also affected her work in subsequent phases of the lesson. As the students were not supported to understand key aspects of the task in the launch, Louise spent the next phase of instruction explaining the task to individuals or groups of students. Even with an adequate task launch the teacher may need to provide additional or different information about the task to students, for example enabling prompts to support them to begin the task or to solve it in a productive way. However, Louise's time spent reintroducing the task during the explore phase was not productive and affected her ability to be able to *monitor* and *recognise* students' mathematical thinking, solutions, and misconceptions, and subsequently carefully *select* students' responses for the concluding whole class discussion (Leatham et al., 2015; Smith & Stein, 2011). The strategies Louise selected for sharing in the task summary lacked rigour, and neither Ricky's strategy, which provided a more efficient approach, nor Leslie's strategy, which demonstrated a misconception, were explored beyond the student presentation.

This analysis of Louise's lesson has focused on the relationship between the task launch and other phases of the lesson, and the implications this has on opportunities for student learning.

Due to the nature of this study the relationships reported here are descriptive and cannot be claimed to be causal, but are consistent with findings reported in the literature. Jackson et al. (2013) reported from a study of 165 middle-grades mathematics teachers' instruction that the task setup phase is crucial for students' opportunities to engage in rigorous mathematical activity. They suggested that teachers' decisions and actions during the task launch are "related to the extent to which students are able to participate in concluding whole-class discussions in high-quality ways" (p. 679).

6.6 Summary

This chapter has described two teachers' implementation of the second task, *Measuring Money*. The discussion has illustrated the role that the nature of the task and the task launch plays in maximizing opportunities for student learning. These lessons illustrate how opportunities for choice empower students, support their sense of agency and increase task interest. The results also concur with other studies that have identified that the setup phase of instruction influences students' participation in high-demand activity over the remainder of the lesson.

Chapter 7: Task Three

7.1 Introduction

Evidence was presented in the previous chapter that task characteristics and the task launch influence opportunities for student learning. In particular teachers' decisions and actions when launching the task impact on opportunities for student engagement during the explore and summary phases of the lesson.

This chapter describes the implementation of the final task. The task and teachers' planning for its implementation are outlined in sections 7.2 and 7.3 respectively. Section 7.4 describes the task implemented in Nanette's class.

7.2 The Task

> *Wrap the Present*
>
> I have wrapped a gift box with one metre of ribbon.
>
> The bow at the top used 30 cm of this.
>
> What might be the dimensions of the box?

This task, adapted from Sullivan, Clarke, and Clarke (2013), encourages students to connect their understanding of length to their visualisation of a 3-dimensional box. It is an open-ended task that offers students opportunities to generalise their approach to find multiple solutions.

7.3 Teacher Preparation and Planning

The teachers recorded their collaborative planning for the implementation of the task on the template provided (Appendix G). Excerpts from this planning including mathematical goals for the lesson, anticipated solutions, strategies, representations, and difficulties, and enabling and extending prompts are reproduced below in Figure 7.1.

Mathematical Ideas and Goals:

- *understanding relationship between 2-d measurements (length) and corresponding 3-dimensional box.*
- *understanding attribute of length and metric units used to measure it.*
- *Understanding attributes and language of 3-d box eg. face, corner etc.*
- *Task supports visualisation and creativity.*

Implementing the Task:
Anticipating solutions/strategies/representations/ misconceptions:
What? How?

Students may image the box (with possible misconception that 6 faces means 6 measures of ribbon instead of 8 – see enabling prompt 2)
Possible confusion re the 30cm required for the bow – see enabling prompt 1
Students may draw the box and guess and check measurements (they may find just one solution - use extending prompt 1 to encourage more solutions)

Students may create a table of values - entering multiple rows in table may lead students to see pattern → a generalised solution) e.g.

Length x 2	*Width x2*	*Height x4*	*Total of 70*
10+10	*5+5*	*10+10+10+10*	*70*
11+11	*4+4*	*10+10+10+10*	*70*

Generalised algebraic solution
e.g. 2l + 2w + 4 h = 70 using understanding that what gets taken off e.g. length can be added to width or height to keep total unchanged.

Enabling Prompts:

1. *How much ribbon is available to wrap the faces of the box?*
2. *Have a look at this box that is already tied up with ribbon. Work out how long the ribbon is without untying it. (addresses misconception that 6 faces means 6 measures of string i.e. missing 2)*
3. *If your box is a cube, how many faces does the ribbon pass over?*
4. *What if the height of the box was 5cm?*

Extending Prompts:

1. *Describe as many solutions as possible. I wonder how many there are. How will you know you have found them all?*
2. *What if the box took 2 metres of ribbon to wrap it? What might be the dimensions of that box?*
3. *What if the box was a different shape? What shape might it be? What would the possible dimensions be? How would the ribbon be tied? (provide solid shape examples)*

Figure 7.1 Excerpts from teachers' planning for task 3

7.4 Nanette's Lesson: *Wrap the Present*

7.4.1 Task Launch

Nanette showed the students some presents she had already wrapped. *As you can see I have been busy wrapping presents. I am wondering what size box we could possibly decorate with a metre of ribbon? Because I want a decent-sized bow on top, we are going to use 30cm of the ribbon to tie a nice bow* (N-TransT3). She demonstrated how to wrap the ribbon around a box, ensuring that the students understood that the ribbon covered all the faces and created a cross on both the top and bottom face. Nanette communicated enjoyment and curiosity about the task, and the students' applause after she finished tying the ribbon demonstrated that she had caught their interest. She continued the launch by eliciting input from multiple students explicitly clarifying the mathematical purpose of the task and developing shared understanding of mathematical language used in the problem (e.g. dimensions).

7.4.2 Task Explore

The students spent 40 minutes exploring the task. All groups except for one found at least one correct solution.

The group whose solution was incorrect were diverted from the intended problem by measuring the dimensions of an actual box. They calculated the ribbon required to wrap this box. Although they were not successful in solving the intended task, they did demonstrate understanding that the ribbon required was height x 4 + width x 2 + length x 2 (see Figure 7.2).

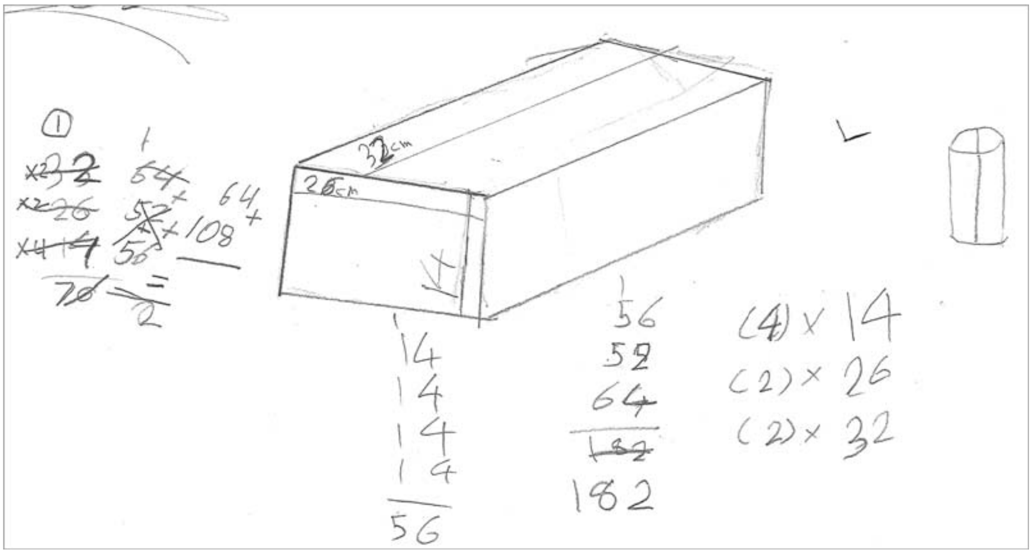

Figure 7.2 Strategy from measuring an actual box

The solution from several groups assumed the box was a cube (see Figure 7.3).

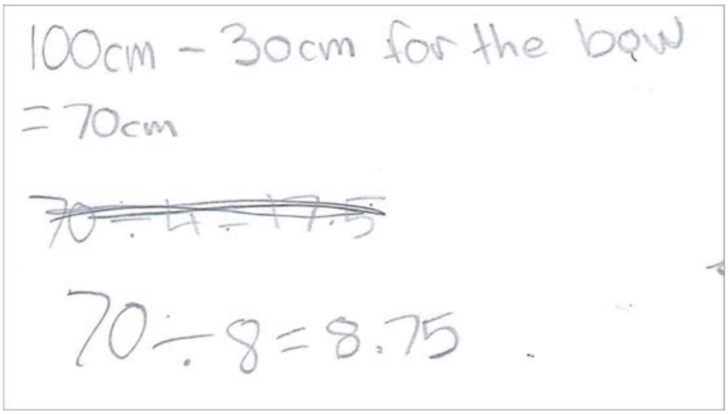

Figure 7.3 Assuming box was a cube

Common difficulties students experienced were visualising the box and representing solutions. Some students were more successful than others at creating 2-dimensional representations of the 3-dimensional shape (see Figure 7.4).

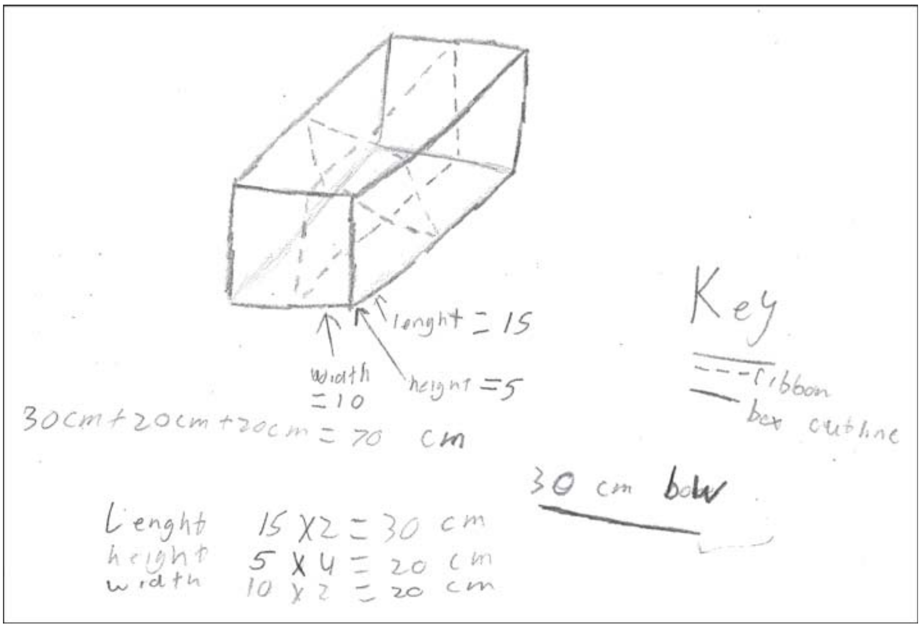

Figure 7.4 Students' solution representation

The groups who found multiple solutions presented their solutions in tables, or used symbolic notation (see Figures 7.5 and 7.6), validating these representational tools as useful for supporting the discovery of multiple solutions.

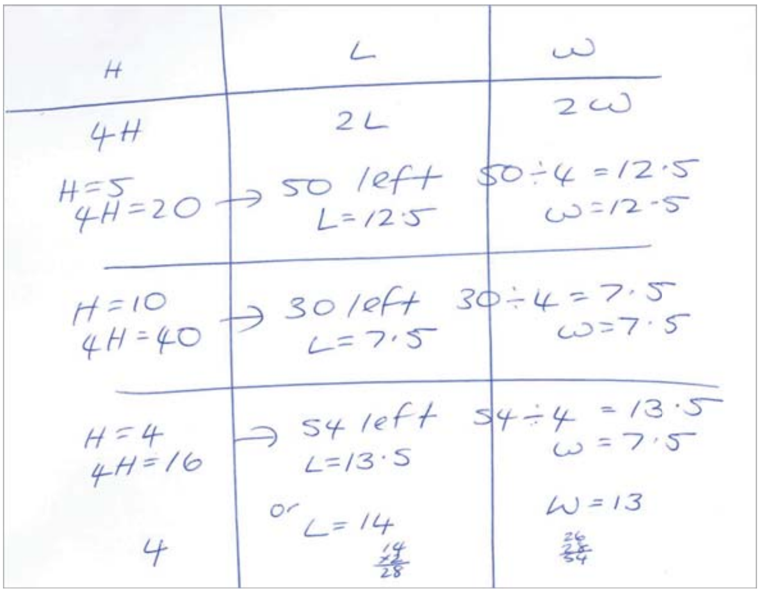

Figure 7.5 Approach resulting in multiple solutions, Example 1

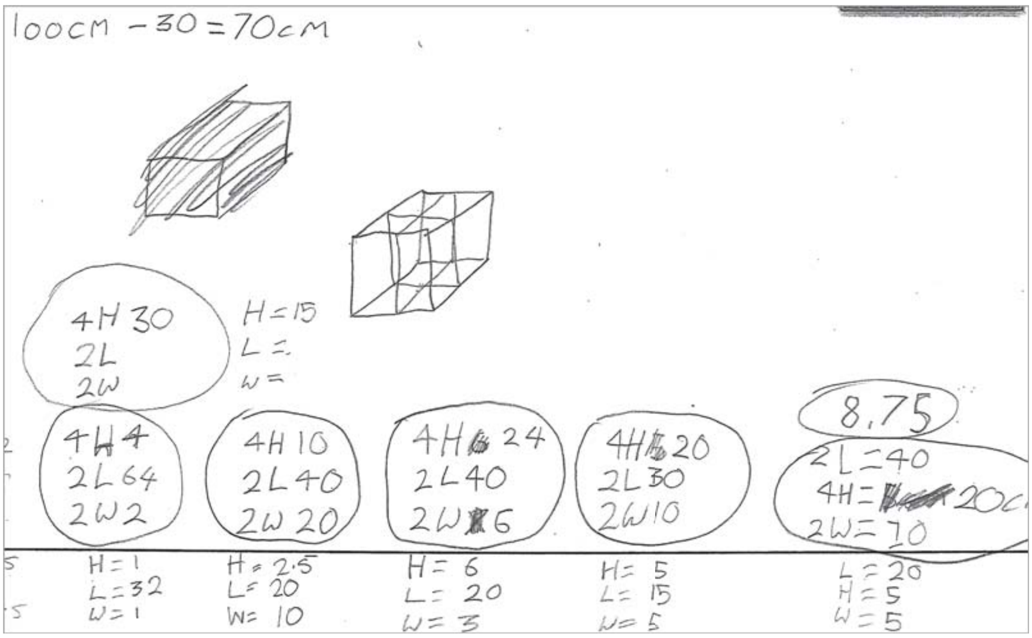

Figure 7.6 Approach resulting in multiple solutions, Example 2

7.4.3 Task Summary

Nanette asked the group of students with the incorrect solution to share their strategy first. These students demonstrated the measurements they had made on the actual box and were able to clearly articulate the connection between the box's measurements and the length of ribbon required. With the teacher's support they also articulated shared language for their measurements (height, width, length).

Nanette asked the next group to explain how they used similar thinking in their solution but used it to solve the intended problem.

> *Like Manaki, we knew we needed 4 heights but we just chose 5cm for our height (4x5=20). We chose 10cm for another side and we knew we needed two of those (2x10=20). That left 30cm for the last two sides, so it was 15cm each. 20+20+30=70 and another 30 for the bow makes 1m* (N-TransT3).

Two further groups shared their solutions. First, a group who had assumed that the box was a cube, followed by a group who had generated multiple solutions. Nanette challenged each group to compare their strategy to the one before and supported them to do this. She also pressed the students to clearly communicate and justify their solutions by eliciting input from other students. For example, she asked: *Do you believe what Danny is saying? Is she convincing you? If not, think of a question to ask her that will help her give a clearer explanation* (N-TransT3).

7.4.4 Discussion

Nanette remarked that this was her best lesson yet. *This was the most challenging task, but I really thought about and practised my introduction. It was the best I've done so far, and that made a difference to them getting started* (N-PostT3). Nanette solicited input from the students during the task launch to clarify contextual features and key mathematical ideas, and to develop a common language. The involvement of multiple students in the initial discussion served to reposition the task from a teacher problem to a problem of the community of learners, encouraging the students to take ownership. Nanette's adaptations to the task launch to suit her own teaching style resulted in a launch that clarified contextual features and key mathematical ideas and fostered students' enthusiasm. The previous chapter and the literature (e.g., Jackson et al., 2013) support the claim that characteristics of a task launch influence the quality of the concluding discussion.

Nanette's plenary session consisted of four separate presentations of different ways to solve the problem. Because she had monitored student activity during the explore phase Nanette was able to select, scaffold, and sequence the presentations in ways that focused attention on the mathematical ideas that were the purpose of the lesson and supported the development of a generalised solution. Specifically, Nanette wanted students to understand the relationship between the measurements of length and representations of a box. Her initial focus on students sharing their measurements of an actual box supported other students to connect their solutions to a concrete object. She made connections to labelled diagrams, drawn by both students and herself, and a symbolic representation ($2l+2w+4h$). Many students may have struggled to write this symbolic rule independently, but Nanette related their verbal descriptions to the rule and pressed them to use it to test their own solutions.

Although it is not possible to claim that Nanette's implementation of *Wrap the Present* directly contributed to student learning, it was apparent in her lesson that the students were engaged in the learning process. Student work samples illustrated both effort and mathematical reasoning, and 14 of the 20 students contributed to the whole-class discussion in substantive ways. Nanette's deliberate use of the five practices recommended by Smith and Stein (2011) to orchestrate a lesson summary supported students to engage in mathematical discourse and argumentation. She encouraged the students to engage in activities that support a community of mathematical inquiry, including sharing responsibility for sense-making by justifying their mathematical reasoning and connecting their ideas to those of others.

7.5 Summary

This chapter has described one teacher's implementations of the final task. Using a plan for instruction collaboratively created by all three teachers, Nanette's implementation incorporated pedagogical decisions and actions that supported her students' understanding of key mathematical ideas and their engagement in productive mathematical discourse.

Chapter 8: Teachers' Perspective

8.1 Introduction

The descriptive accounts of lessons in previous chapters offered insights into the relationship between teachers' intentions, actions, and learning opportunities for students. The data presented was derived from observations of lessons and teacher meetings and documentary data, and although analysis was corroborated by the participant teachers it was nonetheless subject to the researcher's interpretation. The data presented in this chapter is from the teachers' perspective and provides an alternative lens through which to view this study.

This chapter outlines the teachers' perspectives on the usefulness of approaches suggested for planning and implementing challenging tasks. Section 8.2 presents data on the usefulness of the planning template, and section 8.3 on opportunities afforded by the lesson structure. Challenges experienced by the teachers are discussed in section 8.4.

8.2 Planning

The data presented in this section on the usefulness of the planning template were from two sources. These were teachers' responses to a survey (Appendix A) completed by the three teachers following the implementation of the final task, and their comments made in a subsequent group interview. The survey contained both Likert scale-type items and free-response questions. Likert-type survey data by itself should be interpreted with caution, but when considered alongside teachers' qualitative responses and comments made during the interview, a credible insight into the teachers' perspective can be presented.

Teachers rated elements of planning according to their perceived importance when preparing to implement a challenging task. Table 8.1 presents the profile of responses that rated statements about planning elements from *not needed* (NN), to *very important* (VI). The two negative responses, *not needed* (NN) and *not important* (NI), and the neutral response, *not sure* (NS), were not selected by any of the teachers for any of the items so have been aggregated in the data. The teachers' exclusive selection of the positive responses, *important* (I) and *very important* (VI), indicates that they found the supports provided by the planning structure useful.

Table 8.1 Frequency of teacher responses to statements about planning

Statements About Planning	NN,NI,NS	I	VI
Teacher understanding of the big mathematical ideas inherent in the task.		1	2
Enabling prompts for students experiencing difficulty.			3
Extending prompts for students who complete the task quickly.		1	2
Anticipating possible questions.			3
Anticipating possible misconceptions.			3
Anticipating possible solution strategies.		1	2
Having completed the task yourself.			3

In an open-ended survey response Nanette stressed the *importance of doing the problem as a teacher and thinking about how the students will solve it* (N-PostT3). In the subsequent interview she elaborated on the challenges of seeing and connecting two potentially differing perspectives on the task; looking for mathematical possibilities, and also anticipating possible methods, solutions, representations, and misconceptions through the eyes of her students. She commented: *I have to really think about whether the ways students solve the problems show whether they fully understand the concepts* (N-PostT3). Her comments indicate a cognizance of the value of students' work for illuminating mathematical understandings and what Simon (1995) describes as the "ongoing and inherent challenge to integrate the teachers' goals and direction for learning with the trajectory of students' mathematical thinking and learning" (p. 121).

A further theme that emerged from teachers' responses was the value they placed on collaboration. Comments included:

> *Planning the tasks together has been really helpful. Even as teachers we see different possibilities* (L-PostT3)
>
> *Getting clear mathematical goals at the start is key and that is difficult for us so it's great to have each other's support to get those right* (S-PostT3).

The latter comment acknowledges the value of teacher collaboration in ameliorating the difficulty many teachers face in articulating the mathematical purpose of tasks, a point that aligns with discussion in this study and in the literature (e.g., Sullivan, Clarke et al., 2009; Sullivan, Clarke, Clarke, & Roche, 2013).

The teachers indicated that the planning template, although initially overwhelming, was helpful. Sally commented *there's more accountability in this planning* (S-PostT3). Nanette added *it makes you think about the task you're giving the students and why you're giving it to them. Last year I was like here is a task for you to show me that you can use my strategy. I didn't really think of the task or the students as the things that drive the learning* (N-

PostT3). This indicates a pedagogical shift in emphasis for Nanette, from the teacher as the key player in the classroom, to the students.

The teacher responses indicated both awareness of and adoption of approaches that occasion opportunities for students to engage with and learn from cognitively demanding tasks. Although no causal links can be claimed, it does appear that the template served its intended purpose in prompting teachers to consider mathematical purposes of their lesson, and relate their decisions at different phases of task planning, introduction, and implementation to evidence-based practices.

8.3 Implementation

Teachers rated statements about task implementation related to elements of the proposed lesson structure according to their perceived level of importance. Table 8.2 presents the profile of the responses. The two negative responses, NN and NI, were not selected by any of the teachers for any of the items so have been aggregated in the data presented in the table.

Table 8.2 Frequency of teacher responses to statements about task implementation

Statements About Implementation	NN,NI	NS	I	VI
Posing the Task				
Explaining the mathematical purpose of the task to the students		1	2	
Clarifying relevant mathematical language with the students			3	
Not telling the students how to solve the problem				3
Students Working on the Task				
Giving the students time to struggle with the task before intervening		1	1	1
Provision of prompts that differentiate the task (enabling and extending)			1	2
Allowing students to develop their own method of solution			1	2
Sharing of Student Activity				
Selecting particular student responses for presentation to the class and giving these students advance notice that they will be asked to explain what they have done.		1	1	1
Sequencing student responses so that the reporting is cumulative.			2	1
Making connections between student strategies.			2	1

In comparison to their perspective on the planning structure, the variance in responses on lesson implementation, including three *not sure* responses, suggests less confidence in teachers' perception of the importance of some elements. It also illuminates possible explanations for teachers' pedagogical decisions. For example, lack of surety of the importance of 'explaining the mathematical purpose of the task to the students' may account for why a teacher may explain (or not) the mathematical purpose of a task sufficiently when implementing it in her class. This indication that teachers' pedagogical decisions and actions are influenced by their intentions, goals, and knowledge concurs with both the framework used to guide this research and findings discussed in earlier chapters.

There was general agreement amongst the teachers of the importance of the task launch. Louise's comment is representative of their perspective. *I think the launch is the key, making sure the kids have the prior knowledge to be able to start the task, doing enough to trigger their interest, but not too much that you do the work for them* (L-PostT3).

In terms of advantages for student engagement and learning, comments included:

> *Students using their own strategy or adopting one from another student that makes sense to them is far more effective than us telling them what to do. Some kids and not necessarily the ones we might see as smart came up with ways that are easier than the way I solved the problem* (N-PostT3).
>
> *Some kids stepped up during the three tasks that wouldn't usually shine* (L-PostT3).

Such responses indicate teachers' awareness of the benefits afforded by multiple strategies and differing perspectives for promoting sense-making, self-efficacy, and creativity.

The teachers emphasised the important role task variations in the form of enabling and extending prompts played in the implementation of the lesson. *The enabling prompts were brilliant especially if you know them well so you don't get sucked in to telling a student how to solve the task* (N-PostT3). The prompts afforded teachers the opportunity to facilitate engagement of all students and ensure that everyone was able to engage with the same problem at a level of demand conducive to learning. Careful planning of the prompts ameliorated teachers' acknowledged difficulty that there was a tendency for 'made in-the-moment' instructional decisions to result in lowering task demands.

8.4 Challenges

Challenges teachers reported experiencing in the implementation of tasks predominantly fell into two categories: difficulties engaging 'students who are not risk-takers', and pedagogical aspects such as *when students come up with a strategy that you haven't thought of* or *stopping yourself from telling students how to solve the task* (N-PostT3).

Teachers expected that catering for the range of mathematical understanding within the class would be their greatest challenge, but reported that trying to engage students with the task who were reluctant to take risks in their learning and get started without a prescribed pathway was more challenging. The teachers identified students' fixed mindset and lack of orientation to persist as key inhibitors. They also identified improvement in students' risk-taking and persistence as the project progressed.

> *I didn't have as many issues in the last lesson as in the first. They knew they weren't expected to get the answer quickly and that was ok* (L-PostT3).
> *If we started off the year like this and ran it through the first and second terms to where we are now we would see a drastic change in kids' mindsets* (N-PostT3).

Many researchers (e.g., Blackwell et al., 2007; Dweck, 2007; Middleton & Jansen, 2011) have highlighted the influence of students' mindset on their motivation, persistence, and self-belief, but have also noted the role that instructional practices including task choice, teacher expectations, and empowerment of students to select their own pathways play in influencing students' goal orientation and mindset. This suggests that teachers' increasing expertise in posing the tasks and pedagogical choices made during their implementation, including consistent messages on the value of effort, along with the open-ended challenging nature of the tasks themselves, may have contributed to a reduction in negative responses from students. It is worth noting that comments made by teachers as the project progressed suggest a change in the teachers' mindset as well. Initial concerns about whether challenging tasks would be suitable for all of their learners (see Chapter 5) were replaced by an articulated belief in the capacity of all students to achieve success with the tasks.

8.5 Summary

This chapter has outlined the teachers' perspectives on the usefulness of approaches suggested by the study for planning and implementing challenging tasks. The teachers' responses indicated that the lesson documentation and proposed structure for implementation were helpful. Although challenges were still experienced implementing the tasks, teachers expressed the view that they were aware of improvements across the three lessons.

Alignment exists between the perspectives of the teachers articulated in this chapter and descriptions of their practice in earlier chapters. This strengthens confidence that their comments were derived from their practice rather than merely reproducing messages from the intervention, and that together the data can be used to credibly illuminate the research questions.

Chapter 9: Conclusion

9.1 Introduction

The purpose of this study was to explore how teachers can enact challenging tasks in ways that maximise opportunities for students' mathematical learning. Specifically, the research was designed to (i) examine how teachers' enacted pedagogies occasioned opportunities for students to engage with and learn from cognitively demanding tasks; (ii) explore teachers' perspective on the use of a planning approach and lesson structure that exemplified particular pedagogical approaches to the implementation of challenging tasks; and (iii) illuminate the challenges teachers experienced in implementing the open-ended tasks suggested by the study.

Through examination of three cases, this study aimed to understand a learning ecology related to the types of tasks and teacher practices that impact on students' engagement with mathematical ideas, their participation in mathematical discourse, and their willingness to participate in the face of challenge. It should however be noted that this learning ecology was situated within real classrooms, and it was beyond the scope of the study to capture all of the complexities of teaching and learning within the classroom communities. Consideration of the complexities of classroom practice, combined with the small scale of this study, means that although interpretation of the results can offer insights into the ways in which students can be supported to engage with and learn from cognitively demanding tasks, further larger scale research is required to substantiate these findings.

While mindful of the potential limitations of the study, this chapter draws together findings presented in earlier chapters which jointly serve to illustrate the important role that teachers' pedagogical decisions and actions play in mediating students' opportunities to learn from tasks. This chapter overviews how teachers in this study experienced and enacted the series of challenging tasks, and in particular how their decisions in relation to tasks, their planning and their implementation, afforded or constrained opportunities for students' mathematical learning. Implications for classroom practice and teacher professional learning, and suggestions for further research are drawn from these conclusions.

9.2 Teachers' Pedagogical Decisions in Relation to:

9.2.1 Opportunities Afforded by the Task

A central assumption of this study was that task choice is essential to effective teaching of mathematics. Although, teachers' task selection decisions were not a focus of the study, opportunities afforded by the selected tasks for promoting valued activity was a point of discussion. In particular, it was noted that teachers' and students' perceptions of what kinds of mathematical activity were valued appeared to be influenced by the nature of the tasks used. As opposed to previously valued activity, for example accurate completion of a task using a teacher prescribed strategy, activities that were prompted by engagement with the tasks in this study included exploration of mathematical ideas, student decision making and authority to choose their own approaches, justification of methods, representations and solutions, and discovery of patterns that supported generalised conceptual understanding.

As the project progressed, teachers' perceptions shifted towards tasks as vehicles for activating learning, as opposed to demonstrating performance. This was evidenced by teachers' explicit promotion of the value of effort, collaboration, multiple student perspectives, and students' selection of their own pathway through a problem. Likewise, observed improvements in students' engagement and persistence, and a reduction in their negative responses as the project progressed suggested a similar shift in students' perceptions. Sullivan, Aulert et al. (2013) described similar findings, and suggested that teachers can influence students' persistence by presenting them with challenging tasks and actively promoting a culture that values effort and challenge in their classrooms.

The presence of particular task characteristics is associated with potential for promoting valued mathematical activity that affords opportunities for students to engage with and learn from tasks (Anthony & Walshaw, 2007; Stein & Smith, 1998; Stein et al., 2009). The content specific open-ended tasks that were part of this study, for example, afforded multiple solutions, methods, and representations, and promoted generalisation and conceptual understanding of important mathematical ideas. Of particular value to the teachers participating in the study were task characteristics that enabled a wide variety of learners to access the mathematics associated with the task. Such tasks were aptly described as *tasks that are easy to start but hard to finish*. Other researchers (e.g., Boaler, 2016; Lambert & Stylianou, 2013; Sullivan, Clarke, & Clarke, 2013) have argued that open-ended tasks that offer multiple means of representation, engagement, and strategic action provide greater access to mathematics for a wide variety of learners.

9.2.2 Task Planning

In this study, teachers found the use of a suggested planning template helpful with the process of turning tasks into lessons. The template (see Appendix G) served its intended purpose of prompting teachers to make purposeful pedagogical decisions in relation to elements of planning that researchers (e.g., Smith & Stein, 2011; Sullivan, Askew et al., 2015; Sullivan et al., 2016) have argued maximise students' opportunities to learn from tasks. These include explicit consideration of the mathematical ideas inherent in the task and of students' prior knowledge they bring to the task, and what this signifies in terms of possible student approaches, representations, solutions, and potential difficulties.

The participant teachers emphasised the important role that task variations, in the form of enabling and extending prompts, played in supporting all students to access the task in meaningful ways. Thoughtful planning of these prompts provided teachers with an alternative strategy to support struggling students, and ameliorated teachers' acknowledgement that their made in-the-moment instructional decisions often resulted in lowering task demands. The tendency for task demands to be unwittingly or purposefully altered by teachers during implementation has been well documented (e.g., Stein et al., 2009). Teachers' initial concerns that the tasks presented were unsuitable for their low-attaining students, were in part mitigated by planning of and use of enabling prompts.

Teachers recognised that working through the tasks themselves was a very important element of lesson planning. Working through the task prior to instruction promoted a beneficial bi-focal perspective, namely a focus on both the mathematical potential of the task, and the opportunities and challenges it may afford for students' emerging understanding. This process, however, highlighted the different levels of knowledge that teachers brought to the discussion. Besides their knowledge of the mathematics needed to solve the task and of their students' current understandings, other areas of knowledge that influenced the planning process included teachers' theories about mathematics teaching and learning, their beliefs about valued mathematical activity, and their knowledge of mathematics in general including knowledge of appropriate representations, materials, and models. Within the workshop process, the teachers recognised the value of collaborative planning in mediating possible shortcomings in teacher knowledge. They recognised the benefit of multiple perspectives and that in working together they were able to learn from each other especially where they lacked confidence in their own level of knowledge. This was particularly evident when identifying and articulating key mathematical ideas that underpinned the tasks used.

Lessons in this study illustrated that teachers' implementation of the same task, from the same planning, did not necessarily result in the same lesson or in the same opportunities for students to learn mathematics. From the observed lessons it became clear that planned objects of learning, recorded as mathematical goals for the lesson in the teachers' plan, were transformed into different objects of learning as they were introduced and implemented in the classrooms, which in turn resulted in varying learning experiences, or lived objects, for students. The teachers' collaborative planning sessions resulted in in-depth and useful planning for each task. However, the trajectory of the lesson and subsequent opportunities for students to learn from the tasks were also influenced by the teachers' unrehearsed in-the-moment decisions made as they interpreted and responded to students' mathematical thinking as the lesson proceeded. Whether lesson adaptations were planned prior to instruction or occurred spontaneously as the lesson progressed the resulting variations reflected teachers' different intentions, understandings, teaching style, classroom norms, and expectations. These results concur with what many researchers maintain (e.g., Clarke & Peterson, 1986; Stein et al., 2009; Sullivan, Askew et al., 2015; Sullivan et al., 2016), namely that teachers' actions are informed by their intentions, and their intentions are in turn informed by their knowledge, goals, attitudes and beliefs.

9.2.3 Task Implementation

As part of the study the teachers explored the use of a three-phase lesson structure that incorporates a task launch, explore, and summary. Concurring with previous studies (e.g., Sullivan, Askew et al., 2015; Sullivan et al., 2016), teachers reported that they found this structure useful in facilitating the conversion of cognitively demanding tasks into lessons. However, this study suggests that several factors in relation to teachers use of this lesson structure impacted on students' opportunities to learn from the tasks.

Lessons observed in this study supported others' findings (e.g., Jackson et al., 2013) that the way in which the task is introduced influences how students participate in solving the task, and how teachers work during subsequent phases of the lesson. Task launches that were well rehearsed, interactive, engaged students' interest, clarified understanding of key mathematical ideas and of contextual features of the task, established expectations and support for students to engage in purposeful mathematical activity associated with the task. Moreover, an effective launch released the teacher to monitor and support learning, and to intentionally select and sequence student responses for the subsequent summary phase of the lesson. Teachers' failure to find the optimal balance between too little or too much instruction during the launch tended to result in unproductive use of teacher and student time throughout the lesson, ultimately impacting on students' opportunities to learn mathematics.

Implicit in the explore phase of the lesson structure are opportunities for students to develop their own pathway through the problem, rather than following a prescribed solution path. Consistent with findings in the literature (e.g., Marshall & Horton, 2011; Sullivan et al., 2016), students in this study who devised their own methods, particularly those who purposefully collaborated in collective sense-making, appeared to be more likely to think deeply about the content and develop conceptual understanding. Teachers recognised that not telling the students how to solve the problem was a very important element of task implementation, aligning with a constructivist perspective (e.g., Simon, 1995) that students construct their own understandings rather than absorb the understandings of their teachers. This does not however, mean that teachers' role in successful task implementations is to leave the students to their own devices, but rather to strike a balance between supporting students' agentic learning and upholding a high press for conceptual mathematical thinking.

The teachers' role in the summary phase was particularly important for promoting conceptual understanding. In the study, teachers who had noticed and monitored student activity during the explore phase were more likely to be able to sequence the students' presentations in ways that focused attention on the mathematical ideas that were the purpose of the lesson and support the development of a generalised solution. Specific teacher actions observed during successful summaries included teachers' press for justification and sense-making, eliciting multiple student perspectives on the presentations shared, and creating a shared public record of students' representations, strategies, and solutions to facilitate comparisons and connections.

The extent to which teachers are able to explicitly address the intended mathematical ideas of the lesson during the summary phase of the lesson has a significant impact on students' opportunities to learn the mathematics inherent in the task (Marshall & Horton, 2011; Smith & Stein, 2011; Sullivan, Askew et al., 2015; Sullivan et al., 2016; Sullivan, Clarke, & Clarke, 2013). Managing end of lesson summaries is challenging, and it was no surprise that this phase of the lesson afforded challenges for the teachers in this study. Missed opportunities to illuminate or connect mathematical ideas were illustrated in observed lessons when students' strategies, solutions or misconceptions were accepted without challenge, justification or clarification. In some cases, non-mathematical goals became the focus of the lesson at the expense of spending instructional time on the intended mathematics. Teachers' pedagogical actions associated with the establishment of a community of inquiry were observed, such as explicit value placed on multiple students' perspectives, learning through participation in discourse, and the establishment of classroom norms that encourage metacognition and student authority. However, in some lessons observed, these teacher actions were not accompanied by orientation of the student activity towards the rigorous mathematical content that should have been at its heart.

9.3 Concluding Thoughts, Implications, and Opportunities arising from this Research

Although it was within the scope of this study to recognise when opportunities to maximise the learning potential of tasks had been overlooked, it was beyond its scope to identify the reasons for teachers avoiding or disregarding such opportunities. However, considering the possible impact on students' opportunities to learn mathematics, further investigation is warranted as to the extent to which teachers' lessons deliver the full mathematical potential of the task, can be attributed to their own mathematical knowledge. The association between teachers' ability to maintain high press for understanding of the mathematical focus of the lesson with their own mathematical knowledge has been consistently argued in the literature (e.g., Charalambous, 2008; Sullivan, Clarke, & Clarke, 2013), and it persists as a problem for the profession.

Prior to this study, the participant teachers were conversant with inquiry approaches to teaching in other curriculum areas, but had not explored a problem-based inquiry aproach to teaching mathematics. Throughout the study they committed considerable time and energy to collaborative inquiry and the adoption of new pedagogial approaches. As the project progressed, the teachers involved incorporated the recommended pedagogies into their classroom practice and into their lexicon of lesson reflections with increasing fluency and intuition. By the end of the project, they were advocates for the benefits of using cognitively demanding tasks and were committed to continuing to develop their expertise in implementing them using pedagogies and approaches explored in this project.

This study demonstrated that building professional development efforts around cognitively demanding open-ended mathematical tasks has merit. This approach offers a vehicle for supporting teachers to challenge their pedagogical understandings and current knowledge of mathematics while meeting the demands of essential everyday activities of teacher practice. Findings from this study also suggest that there is value in further research on the role that teacher collaboration could play in a task-centric approach to teacher professional learning. The processes involved in teachers' collaborative planning include sharing their own strategies, solutions and representations as well as those they anticipate may be used by their students. These processes have potential to influence the teachers' construction of new mathematical understanding as well as the adoption of new pedagogical approaches. Other research on task-centric approaches to improving teacher practice (e.g., Boston & Smith, 2011; Johnson, Severance, Penuel, & Leary, 2016) has argued this approach increases teachers' use of cognitively demanding tasks and their understanding of how tasks influence students' learning. The proposition that this approach also has potential to address teachers' mathematical content knowledge and promote ambitious teaching practices is worthy of further investigation.

The intention of this research was to examine teachers' enactment of challenging tasks and describe and evaluate their pedagogical actions in relation to students' opportunities to learn. The study supported three teachers of year 7 and 8 learners to adopt pedagogical approaches that supported student learning from challenging tasks, and suggested the use of a flexible set of tools and approaches to support the planning and implementation of lessons. The results offer insights into issues relevant to the practice of teachers of mathematics, and confirm the applicability of findings from previous research studies (e.g., Stein et al., 2009; Sullivan, Askew et al., 2015; Sullivan, Aulert et al., 2013; Sullivan et al., 2016) to the New Zealand context, namely that teachers' decisions as they select and implement mathematics tasks for their students significantly influence opportunities for those students to engage with and learn the mathematics inherent in the tasks.

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Appendices

Appendix A: Teacher Survey

Survey To Complete At End of Project

From your experience with all 3 tasks, rate the following aspects of lesson planning and implementation according to whether you believe each aspect is important and needed in order to successfully teach with rich tasks.

Statements About Planning	Not needed	Not important	Not sure	Important	Very important
1. Teacher understanding of the big mathematical ideas inherent in the task.	❍	❍	❍	❍	❍
2. Enabling prompts for students experiencing difficulty.	❍	❍	❍	❍	❍
3. Extending prompts for students who complete the task quickly.	❍	❍	❍	❍	❍
4. Consolidating tasks.	❍	❍	❍	❍	❍
5. Anticipating possible questions.	❍	❍	❍	❍	❍
6. Anticipating possible misconceptions.	❍	❍	❍	❍	❍
7. Anticipating possible solution strategies.	❍	❍	❍	❍	❍
8. Having completed the task yourself.	❍	❍	❍	❍	❍
Statements About Implementation					
Posing the Task	Not needed	Not important	Not sure	Important	Very important
9. Explaining the mathematical purpose of the task to the students.	❍	❍	❍	❍	❍
10. Clarifying relevant mathematical language with the students.	❍	❍	❍	❍	❍
11. Not telling the students how to solve the problem	❍	❍	❍	❍	❍
Students Working on the Task	Not needed	Not important	Not sure	Important	Very important
12. Giving students time to struggle with the problem before intervening.	❍	❍	❍	❍	❍
13. Provision of prompts that differentiate the task (enabling and extending).	❍	❍	❍	❍	❍
14. Allowing students to develop their own method of solution.	❍	❍	❍	❍	❍
Sharing of Student Activity	Not needed	Not important	Not sure	Important	Very important
15. Selecting particular student responses for presentation to the class and giving these students advance notice that they will be asked to explain what they have done.	❍	❍	❍	❍	❍

Continues on the back of the sheet.

16. Sequencing student responses so that reporting is cumulative.	❍	❍	❍	❍	❍
17. Making connections between student strategies.	❍	❍	❍	❍	❍
Consolidating Learning	**Not needed**	**Not important**	**Not sure**	**Important**	**Very important**
18. Provision of consolidating task	❍	❍	❍	❍	❍

Is there anything that could be added or taken out of the lesson suggestions?

Do you have any suggestions/comments about teaching actions during the implementation of the tasks in each of the following lesson phases?

- Posing the Task:
- Students Working on the Task:
- Sharing of Student Activity:
- Consolidating Learning:

Appendix B: Semi-structured Group Interview Questions

1. Of the 3 tasks that you tried which worked the best? What made this more successful than the others?

2. What do you see as advantages of using this type of task in your teaching?

3. What makes teaching with tasks such as these difficult? What are the challenges of using this type of task?

4. What benefits (or otherwise) do you see for student learning through using rich tasks?

5. What would encourage you to use rich tasks more often in your mathematics programme?

Appendix C: Board of Trustees Information Sheet and Consent Form

Enacting Challenging Tasks: Maximising Opportunities for Students' Mathematical Learning

BOARD OF TRUSTEES INFORMATION SHEET AND CONSENT FORM

Dear

I am currently on a Teachers' Study Award carrying out a research project to complete a Masters in Mathematics Education through Massey University. My thesis is a study exploring how teachers can maximise the mathematical opportunities of cognitively demanding tasks in ways that benefit student learning. The study will focus on supporting teachers of Year 7 and 8 students to enact challenging mathematical tasks in their classrooms.

(Name of teacher/s) have informally agreed to participate in a collaborative teaching research process in which we will focus on the challenge of how we provide rich mathematical experiences for students so that all of our diverse learners can engage with the tasks and ultimately learn the maths behind them. These teachers will be formally approached following B.O.T. approval of the study. The parents and students will be informed of the nature of the study through information sheets.

The teacher involvement will entail attending an introductory after school session where we will explore some background research on the characteristics of appropriately challenging tasks and pedagogical actions that teachers use to encourage students to persist and to engage with mathematical practices that develop conceptual understanding, and teaching 3 lessons using cognitively challenging tasks with their classes. Prior to each lesson there will be an after school planning session where, with myself and the other teachers involved, we will collaboratively plan the implementation of the task. These lessons will be audio or video recorded, so that the teacher and I can reflect on them after the lesson is finished. The video will be focussed on the teacher's words and actions, not on the students, as the research study is looking at what teachers can do to support students' maths learning. If students wish, we can further ensure that they are not in the line of sight of the video at all by making certain that they are sitting in a learning space other than the room where the videoing is taking place. The teachers will fill in a survey at the end of the project I will interview them about their teaching experiences with the tasks.

The study will take place over two months incorporating the latter part of term two and the first few weeks of term three.
All project data will be stored in a secure location, with no public access, and will be used for this research only and for any publications arising from this research. All data collected for this study will be destroyed after five years. The school name and names of all participants will be changed to maintain anonymity.

Please note that you have the following rights in relation to your school participation in this study:

- withdraw from the study during the first three weeks;
- ask any questions about the study at any time during participation;
- provide information on the understanding that the participants' names will not be used unless you give permission to the researcher;
- be given access to a summary of the project findings when it is concluded.

If you have any questions about the project you are welcome to discuss them with me personally:
Kat Freeman: Phone 021 069 4750; Email katf@carisbrook.school.nz

or contact my supervisors at Massey University (Palmerston North)

Professor Glenda Anthony, Co-Director Centre for Research in Mathematics Education: Phone: 06 356 9099 Extn 84406; Email: G.J.Anthony@massey.ac.nz

This project has been reviewed and approved by the Massey University Human Ethics Committee: Northern, Application 16/19. If you have any concerns about the conduct of this research, please contact Dr Andrew Chrystall, Chair, Massey University Human Ethics Committee: Northern, telephone 09 414 0800 x 43317, email humanethicsnorth@massey.ac.nz.

Yours sincerely,
Kat Freeman

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Enacting Challenging Tasks: Maximising Opportunities for Students' Mathematical Learning

CONSENT FORM: BOARD OF TRUSTEES

This consent form will be held for a period of five years.

We have read the information sheet and have had the details of the study explained to me. Our questions have been answered to our satisfaction, and we understand that we can ask further questions at any time.

☐ We agree to ____

____

____

participating in this study under the conditions set out in the information sheet.

Signature: ____ Date: ___________________________

Name (printed): ____

Appendix D: Teacher Information Sheet and Consent Form

Enacting Challenging Tasks: Maximising Opportunities for Students' Mathematical Learning

TEACHER INFORMATION SHEET AND CONSENT FORM

Dear

I am doing a research project to complete a Masters in Mathematics Education through Massey University. My thesis is a study exploring how teachers can maximise the mathematical opportunities of cognitively demanding tasks in ways that benefit student learning. The study will focus on supporting teachers of Year 7 and 8 students to enact challenging mathematical tasks in their classrooms.

I would like to formally invite you to be part of a collaborative teaching research process in which we will focus on the challenge of how we provide rich mathematical experiences for students so that all of our diverse learners can engage with the tasks and ultimately learn the maths behind them.

Your involvement will entail:

- attending an introductory after school session where we will explore some background research on the characteristics of appropriately challenging tasks and pedagogical actions that teachers use to encourage students to persist and to engage with mathematical practices that develop conceptual understanding.
- teaching 3 lessons using cognitively challenging tasks with your class. Prior to each lesson there will be an after school planning session where, with myself and the other teachers involved, we will collaboratively plan the implementation of the task.
- filling in a survey after the third lesson
- answering some interview questions after the third lesson.
- keeping a journal recording any reflections of the process of planning and implementation of the tasks.

The lessons and final interview will be audio or video recorded. During any time when you are being recorded you may ask that the audio or video be turned off and any comments that you have made be deleted.

The study will take place over two months incorporating the latter part of term two and the first few weeks of term three.

All project data will be stored in a secure location, with no public access, and will be used for this research only and for any publications arising from this research. All data collected for this study will be destroyed after five years. Your name or anything else that might identify you will not be used in the written work, or any oral presentation or publication. The school name and names of all participants will be changed to maintain anonymity.

Please note that you are under no obligation to accept this invitation and should you decide to participate, you have the right to:

- decline to answer any particular question;
- withdraw from the study during the first three weeks;
- ask any questions about the study at any time during participation;
- provide information on the understanding that your name will not be used unless you give permission to the researcher;
- ask for the recorder to be turned off at any time during observations or interviews;
- be given access to a summary of the project findings when it is concluded.

If you have any questions about the project, you are welcome to discuss them with me personally:
Kat Freeman: Phone 021 069 4750; Email katf@carisbrook.school.nz

or contact my supervisors at Massey University (Palmerston North)
Professor Glenda Anthony, Co-Director Centre for Research in Mathematics Education: Phone: 06 356 9099 Extn 84406; Email: G.J.Anthony@massey.ac.nz

This project has been reviewed and approved by the Massey University Human Ethics Committee: Northern, Application 16/19. If you have any concerns about the conduct of this research, please contact Dr Andrew Chrystall, Chair, Massey University Human Ethics Committee: Northern, telephone 09 414 0800 x 43317, email humanethicsnorth@massey.ac.nz.

Yours sincerely,
Kat Freeman

✂---

Enacting Challenging Tasks: Maximising Opportunities for Students' Mathematical Learning

CONSENT FORM: TEACHER PARTICIPANT

This consent form will be held for a period of five years.

I have read the information sheet and have had the details of the study explained to me. Any question that I have asked has been answered to my satisfaction, and I understand that I can ask further questions at any time.

☐ I agree to participate in this study under the conditions set out in the information sheet.

☐ I agree to being audio-taped.

☐ I agree to being video-taped.

Signature: ____ Date: _____________

Name (printed): ____

Appendix E: Student and Parent Information Sheet and Consent Form

Enacting Challenging Tasks: Maximising Opportunities for Students' Mathematical Learning

STUDENT AND PARENT INFORMATION SHEET AND CONSENT FORM

Dear Students and Parents/Caregivers

My name is Kat Freeman and this year I am studying for my Masters in Mathematics Education through Massey University. My research project is a study exploring how teachers can best use challenging maths tasks in their classes in ways that support student learning.

Your teacher has agreed to participate in the study and the Board of Trustees has also given their approval. Your teacher and I are going to plan three lessons using challenging tasks that she/he will deliver to the class as part of your usual mathematics programme. These lessons may be structured a little differently than your usual maths lessons, but apart from that you might not even notice any difference.

These lessons will be audio or video recorded, so that the teacher and I can reflect on them after the lesson is finished. The video will be focussed on the teacher's words and actions, not on the students, as the research study is looking at what teachers can do to support students' maths learning. The video will not focus on your face, but as you participate in regular classroom interactions it is possible that some teacher student exchange is recorded. At no point in the reporting of the research will student names or identifying information be attached to any reported dialogue. If you wish, we can further ensure that you are not in earshot or line of sight of the video at all by making certain that you are working in a learning space other than the room where the videoing is taking place. With your permission, I might sometimes collect copies of your maths written work that illustrates the learning that you have done. You have the right to refuse to allow the copies to be taken.

All of the information gathered will be stored in a secure place and used only for this research. After the completion of the research the information will be destroyed.
Your name or anything else that might identify you will not be used in the written work, or any oral presentation or publication. The school name and names of all participants will be changed to maintain anonymity.

I would like to invite you, with your parent's permission to be part of this study. If you do not wish to take part you will be seated in a different learning space during these teaching sessions. Please discuss all of the information in this letter with your parents before you give consent to participate.

Please note that you have the following rights.

- to say that you do not want to participate in the study;
- to withdraw from the study at any time;

- to refuse to allow copies of your written work to be taken;
- to ask questions about the study at any time;
- to participate knowing that you will not be identified at any time;
- to be given a summary of what is found at the end of the study.

If you have any questions about the study you are welcome to discuss them with me personally: Kat Freeman: Phone 021 069 4750; Email katf@carisbrook.school.nz

or contact my supervisors at Massey University (Palmerston North)
Professor Glenda Anthony, Co-Director Centre for Research in Mathematics Education: Phone: 06 356 9099 Extn 84406; Email: G.J.Anthony@massey.ac.nz

This project has been reviewed and approved by the Massey University Human Ethics Committee: Northern, Application 16/19. If you have any concerns about the conduct of this research, please contact Dr Andrew Chrystall, Chair, Massey University Human Ethics Committee: Northern, telephone 09 414 0800 x 43317, email humanethicsnorth@massey.ac.nz.

Yours sincerely,
Kat Freeman

✂---

Enacting Challenging Tasks: Maximising Opportunities for Students' Mathematical Learning

CONSENT FORM: STUDENT PARTICIPANT

This consent form will be held for a period of five years.

☐ I agree to participate in this study under the conditions set out in the information sheet.

OR ☐ I do not agree to participate in this study under the conditions set out in the information sheet and have indicated my preferred arrangement below.

☐ I wish to be seated out of screenshot of the video

OR

☐ I wish to work in an alternative supervised space.

Student's Signature: ________________________________ Date: _____________

Name (printed): ____

CONSENT FORM: PARENT OF STUDENT PARTICIPANT

This consent form will be held for a period of five years.

☐ I agree to ________________ participating in this study under the conditions set out in the information sheet.

OR

☐ I do not agree to ________________ participating in this study under the conditions set out in the information sheet, and have indicated my preferred arrangement below.

- ☐ Please arrange for my child to be seated out of screenshot of the video

OR

- ☐ Please arrange for my child to work in an alternative supervised space.

Parent's Signature: _______________________________ Date: ______________

Name (printed): ____

Appendix F: Task Analysis Guide (Stein et al., 2009, p. 6)

THE TASK ANALYSIS GUIDE

Memorization Tasks

- Involve either reproducing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory.
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
- Are not ambiguous—such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.

Procedures without Connections Tasks

- Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedure being used.
- Are focused on producing correct answers rather than developing mathematical understanding.
- Require no explanations, or explanations that focus solely on describing the procedure that was used.

Procedures with Connections Tasks

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
- Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
- Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.

Doing Mathematics Tasks

- Require complex and nonalgorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example).
- Require students to explore and understand the nature of mathematical concepts, processes, or relationships.
- Demand self-monitoring or self-regulation of one's own cognitive processes.
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
- Require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Appendix G: Mathematics Task Planning Template

Maths Task Planning Sheet

<table>
<tr><td colspan="2"><u>Task:</u></td><td colspan="2">Mathematical Ideas and Goals:</td><td rowspan="5">Mathematicians' Habits:
☐ Make sense of problems and persevere in solving them.
☐ Develop, explain, justify and communicate mathematical thinking.
☐ Participate in discussions about maths.
☐ Attend to accuracy.
☐ Look for and make use of patterns and structure.
☐ Develop generalisations.

Mathematicians' Wonderings:
☐ How many solutions are there?
☐ How will I know if I have found them all?
☐ What patterns can I see?
☐ How can I check this another way?
☐ What happens if…?

Five Practices for Orchestrating Discussions:
(Smith, Hughes, Engle & Stein, 2009)
☐ Anticipating student responses.
☐ Monitoring student work.
☐ Selecting students to present.
☐ Sequencing student responses.
☐ Connecting students' responses and connecting responses to mathematical ideas.</td></tr>
<tr><td colspan="2" rowspan="2"><u>Introducing the Task:</u></td><td colspan="2">Prior Knowledge Required: include language that may need clarification)</td></tr>
<tr><td colspan="2">Resources needed:</td></tr>
<tr><td colspan="2"><u>Implementing the Task:</u>
Anticipating solutions/strategies/representations/ misconceptions:
What? How?</td><td>Monitoring and Noticing
What? Who?</td><td>Selecting and Sequencing
When? Why?</td></tr>
<tr><td colspan="4"></td></tr>
<tr><td colspan="4">Enabling Prompts:</td><td>Who?</td></tr>
<tr><td colspan="4">Extending Prompts:</td><td>Who?</td></tr>
</table>

Consolidating Task:

Reflections: (Student Learning, Teacher Practice, Explicit Teaching needed…)

Present your solutions on this sheet in a way that others can understand your thinking. You could use words, diagrams, labels, numbers…..