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LONG MEMORY PROPERTIES OF INTERNATIONAL EX POST AND EX ANTE REAL INTEREST RATES: ASIAN, PACIFIC AND EUROPEAN COUNTRIES

A Research Thesis submitted in fulfilment of the requirements for the degree of Master of Applied Economics at Massey University

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Abstract

According to the Fisher equation efficient capital markets should compensate for changes in the purchasing power of money. This implies that in the long-run, the nominal interest rate and expected inflation should move together one-for-one. However, because expected inflation is unobservable, testing the Fisher relationship is problematic and an appropriate proxy for expected inflation must be employed.

Empirical results in the literature of the Fisher relationship have produced mixed findings concerning the validity of this relationship. Many recent studies have focused on the stationarity of the *ex ante* real rate in determining the acceptability of the long-run Fisher relationship. For the long-run Fisher effect to hold the *ex ante* real interest rate should display mean-reversion. Mean-reversion is characterised by the tendency of a time series to return to its mean after a shock. Most studies that have examined the stationarity of the *ex ante* real rate have concentrated on testing for restrictive integer orders of integration. This is restrictive because mean-reversion is confined to the covariance stationary I(0) process. However, an I(0) process is not the only process that displays mean-reversion. Fractional orders of integration can characterise a wider form of mean-reversion.

Many studies that observe the order of integration of the real interest rate use actual or realised inflation for expected inflation in order to generate the *ex post* real rate, which differs from the *ex ante* real rate only by a stationary forecast error. These studies have then used the *ex post* real rate to infer the dynamic behaviour of the *ex ante* real rate. However, because the difference between the *ex post* and *ex ante* real rates is unexpected, the large volatility of the forecast error can mask the more persistent behaviour of the *ex ante* real rate. The additional volatility is inherited by the *ex post* real rate and therefore estimates of the order of integration are biased downwards.

In this research the order of integration is estimated for real interest rates of nineteen European, Asian and Pacific countries. Two different econometric techniques are used in order to generate proxies for expected inflation, and it is found that these proxies exhibit a more persistent dynamic when compared to actual inflation. Employing an autoregressive fractionally integrated moving average (ARFIMA) model, the order of integration is estimated by using a maximum likelihood (ML) estimation technique. This estimation technique is applied to the two estimated ex ante real rates as well as the ex post real rate for each country studied. The empirical results show that estimated orders of integration display a distinct pattern. That is, the ex post real rate is found to be significantly less persistent when compared to either of the ex ante real rates estimated in this study. This is due to the additional volatility that is inherited within the ex post real rate of interest.

The Fisher relationship has also been extended to international capital and goods markets. Real interest rate parity (RIP) theory postulates that if international capital (through uncovered interest rate parity) and goods (through relative purchasing power parity) markets are efficient then the real interest rate on two perfectly comparable assets between countries and across time should equalise. Similar to the Fisher relationship, RIP has also had mixed empirical results. Early studies found limited support for RIP, on the other hand more recent studies have found evidence of real interest rate integration. In this research, a preliminary study was conducted of RIP between New Zealand and Australia. Using the same methodology mentioned above, RIP was examined for three real interest rate differentials of New Zealand and Australia. Again, these differentials differ in the method used to model expected inflation. The empirical results of RIP between New Zealand and Australia are not overwhelmingly conclusive. The order of integration of the real interest rate differentials do not differ from the order of integration of the real interest rates of New Zealand and Australia, which does not support RIP. This analysis however, does generate many possibilities for further research including data and methodological extensions.

Table of Contents

ACKNOWLEDGEMENTS	II
ABSTRACT	111
TABLE OF CONTENTS	v
LIST OF TABLES	VII
LIST OF FIGURES	
LIST OF ABBREVIATIONS	
CHAPTER ONE INTRODUCTION	
1.1 AIMS AND OBJECTIVES	
1.2 Data and Methodology	
1.3 CHAPTER OUTLINE	6
CHAPTER TWO LITERATURE REVIEW: FISHER EFFECT AND LONG	
MEMORY MODELS	7
2.1 Introduction.	7
2.2 The Fisher effect	8
2.2.1 Fisher Effect: Early Studies	
2.2.2 Rational Expectations Hypothesis	
2.2.3 Fisher Equation: Time Series Properties	
2.2.4 Empirical Studies	
2.2.5 Fisher's Paradox	
2.3 THE LONG MEMORY MODEL	
2.4 Conclusion	
CHAPTER THREE INFLATIONARY EXPECTATIONS: RELEVANT ISSU	ES 31
3.1 Introduction	31
3.2 Inflationary Expectations	32
3.3 EXPECTATIONS METHODOLOGY	39
3.4 Conclusion	49
CHAPTER FOUR DATA, UNIT ROOT TESTS, MODELS AND METHODO	LOGY
4.1 Introduction.	50
4.2 Data	
4.3 Unit Root Test Procedures	52
4.4 THE LONG MEMORY ARFIMA AND ARFIMA-GARCH MODEL	
4.5 MODEL SELECTION METHODOLOGY	
4.6 CONCLUSION	63

APPENDIX 4.1	64
CHAPTER FIVE EMPIRICAL RESULTS FOR ASIA, PACIFIC	AND EUROPE 65
5.1 Introduction	65
5.2 Unit root test results	65
5.3 Long memory model results	
5.3.1 Autocorrelation Functions	
5.3.2 Empirical Results of the ARFIMA Model	
5.3.3 Estimated ARFIMA-GARCH Models	99
5.4 SUMMARY AND CONCLUSION	104
APPENDIX 5.1	105
CHAPTER SIX THE REAL INTEREST RATES AND REAL IN	
DIFFERENTIAL OF AUSTRALIA AND NEW ZEALAND	106
6.1 Introduction	106
6.2 REAL INTEREST RATE PARITY THEORY	109
6.3 Data and Data Issues	112
6.4 Empirical Results	114
6.5 CONCLUSION	125
APPENDIX 6.1	126
CHAPTER SEVEN SUMMARY, CONCLUSION AND FURTHE	CR RESEARCH 127
7.1 Introduction.	127
7.2 CHAPTER SUMMARY AND CONCLUSION	
7.3 FURTHER RESEARCH	
REFERENCE	134

List of Tables

Table 5.1: The PP and KPSS Unit Root Test Results for European, Asian and Pacific	
Countries	67
Table 5.2: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest	
Rate of Belgium	78
Table 5.3: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest	
Rate of Canada	80
Table 5.4: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest	
Rate of France	81
Table 5.5: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest	
Rate of Germany	82
Table 5.6: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest	
Rate of Italy	84
Table 5.7: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest	
Rate of Japan	85
Table 5.8: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest	
Rate of Korea	86
Table 5.9: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest	
Rate of Malaysia	87
Table 5.10: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest	
Rate of the Netherlands	88
Table 5.11: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest	
Rate of Pakistan	89
Table 5.12: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest	
Rate of the Philippines	91
Table 5.13: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest	
Rate of the Singapore	92

Table 5.14: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest
Rate of South Africa
Table 5.15: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest
Rate of Spain
Table 5.16: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest
Rate of Sweden
Table 5.17: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest
Rate of the United Kingdom
Table 5.18: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest
Rate of the United States
Table 5.19: Estimated ARFIMA(p,d,q)-GARCH (1,1) Model for Selected ex post Real
Rates of European Asian and Pacific Countries
Table 5.20: Estimated ARFIMA(p,d,q)-GARCH (1,1) Model for Selected HP-ex ante Real
Rates of European Asian and Pacific Countries
Table 5.21: Estimated ARFIMA(p,d,q)-GARCH (1,1) Model for Selected ES-ex ante Real
Rates of European Asian and Pacific Countries
Table 6.1: The PP and KPSS Unit Root Test Results for Australia, New Zealand and the
Real Interest Rate Differentials
Table 6.2: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest
Rate of Australia for the Sample Period 1969:3-2002:2
Table 6.3: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest
Rate of Australia for the Sample Period 1978:1-2002:2
Table 6.4: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest
Rate of New Zealand 121
Table 6.5: Three Chosen Competing Estimated ARFIMA Models for the Real Interest Rate
Differentials of New Zealand and Australia $(r_{t,NZ} - r_{t,Aus})$
Table 6.6: Estimated ARFIMA (p, d, q) -GARCH $(1, 1)$ Model for Selected Real Rates of
Australia and New Zealand
Table A4.1: List of the Chosen Interest Rate Used, Sample Size of the Data and the IMF's
Series Code of Each Time Series Studied
Table A5.1: Automatic Bandwidth selections for PP and KPSS tests

Table A6.1: Automatic Bandwidth selections for PP and KPSS tests of Australia	and New
Zealand	126

List of Figures

Figure 3.1: Quarterly Inflation Expectations Time Series for the United States Over the
Period of 1981:3-2002:3
Figure 3.2: Inflation Expectations and Equivalent Real Interest Rate Time Series for Japan
Over the Period of 1957:1-2002:3
Figure 3.3: Inflation Expectations and Equivalent Real Interest Rate Time Series for Korea
Over the Period of 1976:4-2002:3
Figure 3.4: Inflation Expectations and Equivalent Real Interest Rate Time Series for the
United Kingdom Over the Period of 1957:1-2002:3
Figure 3.5: Inflation Expectations and Equivalent Real Interest Rate Time Series for the
United States Over the Period of 1957:1-2002:3
Figure 5.1: Autocorrelation Functions of Real Interest Rates for Japan at the Level and
First-Difference 72
Figure 5.2: Autocorrelation Functions of Real Interest Rates for Korea at the Level and
First-Difference 73
Figure 5.3: Autocorrelation Functions of Real Interest Rates for the United Kingdom at the
Level and First-Difference
Figure 5.4: Autocorrelation Functions of Real Interest Rates for the United States at the
Level and First-Difference
Figure 6.1 Real Interest Rate and Real Interest Rate Differential for Australia and New
Zealand Over the Period of 1978:1-2002:2

List of Abbreviations

ACF Auto-Correlation Function

ADF Augmented Dickey-Fuller

AIC Akaike Information Criterion

AR Auto-Regressive

ARFIMA Auto-Regressive Fractionally Integrated Moving Average

ARIMA Auto-Regressive Integrated Moving Average

ARMA Auto-Regressive Moving Average

BEW Bivariate Exact Whittle

CCAPM Consumption-based Capital Asset Pricing Model

CER Closer Economic Relations

CPI Consumer Price Index

ES Exponential Smoothing

FIGARCH Fractionally Integrated Generalised Auto-Regressive Conditional

Heteroskedastic

GARCH Generalised Auto-Regressive Conditional Heteroskedastic

GARMA Gegenbauer Auto-Regressive Moving Average

GNP Gross National Product

GPH Geweke and Porter-Hudak

HP Hodrick-Prescott

IFS International Financial Statistics

IMF International Monetary Fund

KPSS Kwiatkowski, Phillips, Schmidt and Shin

LMM Long Memory Modelling

MA Moving Average

ML Maximum Likelihood

MMR Money Market Rate

OECD Organization for Economic Co-operation and Development

PP Phillips and Perron

REH Rational Expectations Hypothesis

RIP Real Interest Rate Parity

RPPP Relative Purchasing Power Parity

TBR Treasury Bill Rate

TVP Time Varying Parameter

UIP Uncovered Interest Parity

VAR Vector Auto-Regressive

WPI Wholesale Price Index

Chapter One Introduction

1.1 Aims and Objectives

The ex ante real interest rate is of great importance in macroeconomics as it is used in determining all intertemporal decisions regarding savings and investment, thus influencing macroeconomic dynamics. The long-run time series properties of the ex ante real interest rate is linked to the Fisher (1930) equation. According to the Fisher equation, the real interest rate is the calculated difference between the nominal interest rate and expected inflation. Neutrality or superneutrality is at the core of classical economic theory; this stipulates that once-and-for-all movements in nominal variables do not have permanent effects on real variables. Therefore, in relation to the Fisher equation changes in expected inflation should not have a permanent impact on the ex ante real interest rate. Many economists believe that the neutrality proposition fails over business cycle horizons, although there is much debate concerning the validity of neutrality over the long-run (Atkins and Coe, 2002).

Because the *ex ante* real interest rate is the difference between nominal interest rate and expected inflation, the long-run behaviour of the *ex ante* real interest rate is linked to the behaviour of these two variables. However, since expected inflation is an unobservable variable, the *ex ante* real interest rate is also unobservable and, as a result, studying the Fisher equation is not a clear-cut case. The majority of empirical studies circumvent this problem by employing realised or actual inflation under the assumption of a stationary forecast error in order to calculate the *ex post* real interest rate and use the *ex post* real rate to infer the long-run behaviour of the *ex ante* real interest rate. Yet, as explained below, this can lead to controversial results.

Fisher (1930) originally proposed that in a world in which economic agents have perfect foresight, expected changes in the inflation rate should be immediately followed by

changes in the nominal interest rate, as borrowers and lenders adjust their behaviour keeping the real rate of interest unchanged. However, subsequent to Fisher's work a number of empirical studies, such as Nelson and Schwert (1977), Fama and Gibson (1982) and Huizinga and Mishkin (1984) argue that the *ex ante* real interest rate is in fact not constant but instead found evidence to suggest that it is time varying. Thus this evidence led studies to focus on the time series properties of the *ex ante* real interest rate in order to verify the long-run Fisher effect.

Since the work of Rose (1988) the time series properties of the ex ante real interest rate have been called into question. Rose found that unit root test results for the United States and seventeen other Organization for Economic Co-operation and Development (OECD) member countries suggested that the nominal interest rate contained a unit root, whereas the inflation rate did not. This result implied that a stable long-run Fisher effect could not exist because the degree of persistence of the ex ante real interest rate is necessarily the same as that of the dominant component, which in this case is the nominal interest rate. Rose concluded that if the ex post real interest rate is nonstationary (which differs from the ex ante real rate by a stationary forecasting error), the ex ante real rate must be nonstationary as well. In contrast, Mishkin (1992) found that both the nominal interest rate and inflation contained a unit root, which led to the finding that both the two variables are cointegrated. Therefore, if forecast errors are stationary this result implied that the ex ante real interest rate is a stationary process and is mean-reverting. Mean-reversion describes the tendency of a time series to return to a constant and finite mean after a shock. Similarly, other studies such as Evans and Lewis (1995) and Crowder and Hoffman (1996) found a cointegrating relationship between inflation and the nominal interest rate in the case of the United States.

These mixed findings mentioned above used conventional unit root test procedures, such as the augmented Dickey-Fuller (ADF) and Phillips and Perron (PP) tests. These tests assume that a particular series is integrated of the order of either I(0) or I(1). In terms of the Fisher equation for a stable long-run relationship to hold, the *ex ante* real interest rate is restricted to an I(0) stationary process. However, Lai (1997) argues that for a stable long-run Fisher

equation to exist the minimum requirement is that the *ex ante* real rate is mean-reverting; thus in the long-run the nominal interest rate and rationally expected inflation respond one-for-one to permanent shocks. Using an alternative approach Lai (1997) and Tsay (2000) found that the United States *ex ante* and *ex post* real interest rates can be properly described by a fractionally integrated I(d) process, where the order of integration d lies between zero and one. The fractional process can characterise a wide range of mean-reverting behaviour which is not accommodated by the conventional restrictive integer orders of integration. And therefore, fractional integration can verify the existence of a long-run Fisher effect.

In a recent study Sun and Phillips (2003) suggest that estimating or inferring the order of integration of the *ex ante* real rate using the *ex post* real rate can create misleading results. Sun and Phillips argue that because realised inflation contains an unexpected component (the forecast error) it tends to be more volatile than expected inflation. As a result, the extra volatility of realised inflation masks the actual slow moving persistent component buried within the volatile fluctuations. The additional short-run volatility of realised inflation is inherited within the calculated *ex post* real interest rate, which masks the more persistent *ex ante* real interest rate. Consequently, univariate long memory estimates of *d* based on the *ex post* real interest rate are underestimated because the additional short-run volatility create a seemingly less persistent time series. Using a new approach Sun and Phillips estimate the order of integration of the *ex post* real interest rate for the United States using a bivariate exact Whittle (BEW) estimator. This estimator controls for the additional short-run volatility and the empirical results estimated by Sun and Phillips show orders of integration that were larger compared to the estimates of the orders of integration using other estimation methods.

The main objective of this study is to determine whether the long memory behaviour found in the United States is exhibited within other countries. The long-run dynamics of the *ex post* and *ex ante* real interest rates for nineteen Asian, Pacific and European countries are studied here. However, instead of employing a particular estimation method that considers the additional short-run dynamic of realised inflation as carried out in Sun and Phillips, two econometric techniques are employed to generate a historical series of inflation forecasts in

addition to the use of realised inflation as a proxy for expected inflation. Each of the proxies for expected inflation used in this study differs in the degree of short-run volatility. This study aims to find that for each country the estimated order of integration of the real interest rate depends on the level of short-run volatility present in expected inflation. It is hypothesised that the way economic agents incorporate short-run dynamics in forming inflationary expectations may play a crucial role in determining the level of persistence of the *ex ante* real interest rate. This will have important implications for the mean-reverting dynamics of the *ex ante* real interest rate and therefore the validity of the Fisher effect. Overall, the empirical results suggest that for the majority of the nineteen countries studied here, the estimated long memory parameters are significantly smaller for the *ex post* real interest rate compared to both of the *ex ante* real interest rates.

In addition to this analysis, this study considers preliminary work of real interest rate parity (RIP) between New Zealand and Australia by examining the long-run time series properties of the real interest rate differentials. The RIP hypothesis describes the equilibrium condition between domestic and foreign real interest rates. If there is integration of world capital and goods markets, then real interest rates of identical financial assets tend to be equal across countries over time (Moosa and Bhatti, 1997). Because Australia and New Zealand have a close economic relationship, both the commodity and capital markets should have a certain degree of integration. This study therefore aims to examine the long-run properties of the real rate differentials to determine if a form of RIP exists between the two countries. Using the same methodology to examine the long memory properties of the real interest rates for various countries, the order of integration of the real rate differentials are examined to find evidence of a cointegrating relationship between the real interest rates of Australia and New Zealand.

1.2 Data and Methodology

The nineteen Asian, Pacific and European countries studied include: Australia, Belgium, Canada, France, Germany, Italy, Japan, Korea, Malaysia, the Netherlands, New Zealand, Pakistan, the Philippines, Singapore, South Africa, Spain, Sweden, the United Kingdom

and the United States. The analysis presented in this research requires a certain degree of consistency of data between each country. Thus, all the data utilised in this research is obtained from the International Monetary Fund's (IMF) *International Financial Statistics* (IFS) database, IMF (2002). The inflation data utilised for each country is calculated from quarterly consumer price index (CPI) data and the interest rate data is for quarterly three-month interest rates.

Lai (1997) explains that there are generally two approaches typically employed when studying the mean-reversion of the real interest rate. The first approach examines a cointegrating relationship between the nominal interest rate and inflation under the assumption of a stationary forecast error (for example, Mishkin (1992)). The second approach involves directly testing for a unit root in the real interest rate. In this study the latter approach is adopted by directly estimating the order of integration (d) of the real interest rate for the above-mentioned countries. Using the three proxies for expected inflation indicated above the real interest rate is calculated using the Fisher equation.

The first step in determining the order of integration of the real interest rates is to conduct unit root tests. Conventionally, the order of integration of a particular series is established using conventional unit root tests, such as the ADF and PP tests, which assume that a particular time series is integrated of the order of either I(1) or I(0). These conventional unit root tests fall into two categories; the first category tests the null hypothesis of a unit root (PP test) whereas the second type tests the null of stationarity (Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test). However, if the null hypothesis is rejected for both tests then this suggests the possibility that the time series may be fractionally integrated. Thus, in this study the PP and KPSS unit root tests are used to test for the stationarity of each real interest rate.

Following Tsay (2000), the order of integration d is estimated using a parametric approach, specifically, an autoregressive fractionally integrated moving average (ARFIMA) model is estimated for each real interest rate using a conditional time-domain maximum likelihood (ML) estimation procedure. The parametric approach allows for the estimation of the long-

run dynamic component (d), while controlling for any short-run dynamic behaviour (autoregressive (AR) and moving average (MA) components). Where the estimated ARFIMA models contain serial correlation in the variance of the residuals, a generalised autoregressive conditional hetroskedastic (GARCH) innovation has been estimated. The order of integration is then compared for the three real interest rates within each country in order to determine if the volatility of expected inflation has an impact on the order of integration of the real interest rate.

1.3 Chapter Outline

The rest of this study is organised as follows: Chapter two presents a review of the literature pertaining to the Fisher equation. As previously explained the Fisher equation is important as it describes the relationship between the nominal interest rate and expected inflation, thus the behaviour of the ex ante real interest rate is derived from this relationship. Chapter two also discusses the main concepts of fractional integration, in which basic properties, tests, estimation methodologies and macroeconomic application of the long memory models are described. Chapter three deals with the main issue faced in various studies of the Fisher equation, that is, the unobservable variable expected inflation. The methodologies utilised in this study to generate expected inflation are explained in detail. How inflationary expectations fit into the Fisher equation and the assumptions behind them are also examined. In addition, some of the methods employed in other studies to cope with this issue are discussed. Chapter four explains the data and methodologies employed in order to estimate and examine the order of integration for each real interest rate. Issues related to the unit root tests are discussed as well as the econometric methodology and the model selection strategy. Chapter five presents the empirical results for seventeen Asian, Pacific and European countries. Chapter six provides a preliminary analysis of real interest rate parity between Australia and New Zealand using the methodology outlined in chapter four. Finally, chapter seven concludes the study and provides suggested directions for future research.