TRANSPORTATION MODELS
OF TIME ALLOCATION

A Contribution to Objective Consumption Theory

A thesis in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Economics at Massey University.

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1990

MASSEY UNIVERSITY

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To my Parents
of happy memory

ABSTRACT

This thesis investigates the optimal allocation of time by a rational agent in terms of his behaviour settings and social requirements. Time is considered as a scarce resource and as an objective measure of activities. Conceptually the models of time allocation are transportation models and share the same mathematical structure.

The findings of eco-behavioural science suggest that the behaviour of an agent, as an individual decision maker, will be shaped by environments. Behaviour settings, corresponding to sources in the transportation models, are used to define environments. As a member of society the agent is required to meet parameters of social position, a set of requirements corresponding to sinks in the transportation models. Time use studies provide quantitative measures of the agent's activities. Hence the model is able to specify constraints on the agent's time use in terms of behaviour settings and social relations.

The core model shows the relationship between groups, or classes, of agents and their lifestyles. The agent as rational decision maker is faced with the choice of meeting the demands of social position by activities in selected environments, while minimizing the total cost of the lifestyle. Each activity uses up time and incurs a money cost. The optimal solutions specify both the type and level of the activities which the agent undertakes in order to meet the parameters of social position. An equivalent program (the dual) exists. The agent is faced with the choice of maximising the net imputed value of time

use, so long as the net value of a unit of time is less than or equal to the per unit cost.

Conceptually there are two transportatition models. Both are concerned with the particular case of a student as a rational decision maker. In the slack model the focus is on the activities of a particular student. By way of contrast the focus in the tight model is on the activities of the average student, and there is a time distribution not only at sources but also at sinks. This model is useful to social accounts. Three equivalent formulations of the transportation model are outlined.

A technology matrix, defined as the agent's socio-economic production function, denotes the set of production processes available to the agent, given behaviour settings (environments) and parameters of social position. An element of the socio-economic production function is termed an activity. The choice of certain activities by the agent represents a particular lifestyle described by a specific time distribution. Social income, defined as the value of social position plus net earned income is a scalar measure (in dollars) of the agent's lifestyle.

To show that the models are operational, simple 2 x 2 and 3 x 3 models are introduced and extended in the final three chapters. A methodology is developed for obtaining per unit costs. A step-by-step approach is used to derive a 5 x 5 cost matrix from two sets of actual data, obtained independently. The effects of changes in the parameters of the time allocation models are analyzed.

PREFACE

Economic investigations of time allocation can be regarded as a venture into relatively unexplored territory. When Soule (1955) stated that time was the scarcest resource, and proposed that time should be regarded as coordinate with land, labour and capital, he was breaking fresh ground.

While the transportation models of time allocation developed in this thesis represent a completely different approach from that of Soule, his questions raise some fundamental issues. From a wider perspective, so too do the questions suggested by Braudel in his masterly survey of the rise of capitalism. He pointed to the social dimension in economics and revealed the impact of capitalism on patterns of human activity.

The first part of the thesis introduces the research program and outlines the social and historical factors that shaped the environment in which workers carried out their activities. The section concludes with an outline of two significant, but different, models of time allocation. Each extends the boundaries of economics.

The transportation models of time allocation are developed in the second part of the thesis. The models owe much to the insights of the pathfinders, and are the outcome of wrestling with unanswered questions and answers unquestioned.

The starred sections (**) which begin in Chapter 4 provide a formulation of the models within the framework of activity analysis. This mode sheds light on the agent's production function. These more technical sections can be omitted in a first reading without loss of continuity.

ACKNOWLEDGEMENTS

I wish to thank my supervisors for stimulating discussions and for their encouragement. The chief supervisor Professor Paul van Moeseke provided a rigorous critique of each draft. Mrs. Jean Goldsmith made practical suggestions about presentation. In their different ways both emphasized the need to complement theory with numerical examples.

Of those kind enough to read or to check sections of the thesis I would like to thank the following: Susan Guthrie for commenting on early versions of the theoretical model; Lorraine Stachurski for verifying most of the solutions to the numerical examples, and for reading the final draft; Soh Yong-Chuan for independently verifying most of the solutions, and also for checking the calculations in Chapter 7; Hugo Valera-Alvarez for verifying solutions of several numerical examples.

Thanks also go to Bruce Baker, Stuart Birks, Maurice Birchler and John Moriarty for providing access to, or advice on, the use of appropriate software. I would also like to thank Toni Snowball and Justine Douthett for undertaking most of the typing, and Gwen Flavell and Wendy King for kindly providing secretarial assistance.

To the Head, and to the Acting Head, of the Economics Department my thanks for practical support. Last, but by no means least, I wish to express gratitude to my family, and thanks to the friends and colleagues, too many to mention by name, who have given encouragement along the way. Their important contribution is gratefully acknowledged.

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CHAPTER 1

THE RESEARCH PROGRAM

1.1 Outline of the Problem

This investigation is concerned with the optimal allocation of time. Linear economic models are formulated to quantify optimal time use by a rational agent given the cost of activities and the social constraints on activity choice.

The problem of how to determine the optimal allocation of scarce resources is basic to economics. For the economist resources are scarce if their availability in relation to human needs is limited. Scarcity in an economic system can be linked primarily to either constraints on demand or to constraints on supply. Only scarce resources have an economic value. The scarcer the resource, the higher the value placed on its use. The relative scarcity of resources is not constant. Evidence suggests that changes in environment, technology, ecology, demography and social conditions, and the list is not exhaustive, can bring about significant shifts in relative scarcity and therefore in prices.

How are the basic categories of scarce resources determined? The term "factors of production" is used to specify the basic categories of scarce resources. In this tradition all the inputs of production can be classified as belonging to one of the classical factors of production, namely land, labour and capital, with entrepreneurship sometimes included. Time is not explicitly considered as an input in to the production process, except as a labour input, and is not assigned an economic value per se.

In received static consumption theory, income is considered as the only limiting factor on the agent's choice. In the basic model goods are consumed per unit of time. Effectively, time is considered as a free good. This is in contrast to the real world, where ordinary language usage suggests that time is a scarce resource. People have experienced the scarcity of time in their activities and decisions. The worker speaks of "overtime". The manager asks whether a new computer system is "worthwhile". Both say that they are "short of time". People wish to know how to "save time". The rise of fast food outlets is not only about food and packaging, but also about the increasing scarcity of consumers' time. The opportunity cost of time is expressed succinctly as "time is money". Explicit statements are to be found in the literature of contemporary management studies.[1] As well, the expression also reflects conscious choices facing long-distance traders in pre-modern economies.[2] From the pragmatic perspective of guidelines for survival in business, support can be found for the insight of Soule (1955) that time is the scarcest of all the economic categories of basic resources.

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1 "In fact, as a strategic weapon, time is the equivalent of money, productivity, quality, even innovation." Stalk (1988), p. 41. Also pp. 41-51.
For similar comments see "Time is money", The Economist (1988), p. 66 and p. 71.

2 This is a counter example to Linder's suggestion that "When the first economists defined their sphere of interest, the scarcity of time was hardly noticeable" Linder (1970) p. 9. He is proposing a tentative hypothesis to explain why economists traditionally have not provided any theory of time allocation. The counter example is from Gottfried (1983), p. 81.

There is a problem. There are sufficient grounds for regarding time as a scarce resource. Yet time has not been formally treated as a scarce resource in received market theory. This thesis is intended to contribute to a new approach, termed the transportation models of time allocation. The models are fully operational. Since an activity analysis model of production is used, the term "commodity" is used to specify the basic inputs and outputs. In an activity analysis model of production all inputs such as consumption goods or services are designated by the generic term "commodity". Time use is the one commodity common to the diverse range of human activities. In the production process all inputs are transformed into outputs, which can also be regarded as commodities.[3] For this reason it is not technically correct to say that the time allocation models include time as a fourth factor of production[4], in addition to land, labour and capital. Conceptually, however, the time allocation models represent an approach that is, perhaps, more far reaching. Not only do the models regard time as a primary input, along with, for example, capital, but the focus is on time use. Time is a necessary input for all activities. Other inputs are used up, to be sure, and therefore contribute to the cost of the activity, but the activity levels and the constraints on activity choice are all measured in time units. Because the models are allocation models they are completely different from "time-specific" models in which production and consumption are considered as activities which take place over time.[5] That is, activities are "processes in time" and time is a context, not a scarce resource which is used up.

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3 It is in this sense that we can speak of the "production of commodities by commodities". cf. Sraffa (1960).
4 As suggested by Soule (1955).
5 The term "time specific" is used in Winston (1982).

In contrast to economics other disciplines have made significant advances by providing an analysis of time. For the physical sciences, Einstein's revolutionary paradigm incorporated time as a fourth dimension. In the social sciences the pioneering work of Strumlin[6] and the comparative multinational time budget studies associated with Szalai (1972) have extended the scope of sociological research. Because time is a "hard" variable, able to be measured by observers, and those measurements then checked for consistency, time budgets can be used to quantify the lifestyles of individuals and of occupational groups.[7] Time use research has been utilized by Ås (1982) to derive certain social indicators of well being. Extensive research by Roger Barker (1968) has established a definite relationship between behaviour and environment. Because behaviour can be measured by time budgets there is a link between time use and the environment in which behaviour takes place. This suggests that activities can be described by time use together with an appropriate measure of spatial location.

In economics, Karl Fox (1987) has developed an integrated framework for meeting problems which go beyond the present frontiers. In this model Fox makes use of the concepts of behaviour setting - the basic unit of observation provided by Barker's studies - time allocation matrix, and time based social system accounts. The models of Becker (1965) and Moeseke (1985), (1989)[8] representing the paradigms for time allocation in a classical utility model and in an objective consumption model, respectively, break fresh ground. In general, however, economists are reluctant both to make use of ideas from other social sciences, and to develop models where time is a scarce resource.

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6 For some interesting background see Zuzanek (1979), pp. 188-213.
7 The real subject of time budgets is time use - the use agents make of their time - (and not the time endowment itself) cf. Szalai (1972), p. 2.
8 It generalizes the previous (1985) model to an ethonomic (sic) model, and also shows how the model fits in with the Marxian paradigm.

The transportation models of time allocation follow the approach of the Becker and Moeseke models. The common starting point is that time is a scarce resource, and that all activities require time as an input. Insights from other disciplines are integrated into the time allocation models. In particular, the models make use of the concept of a behaviour setting, and the link between behaviour and environment, developed by Barker; the comparative time budget research of Szalai; the interaction between environment and society surveyed in Braudel (1981), (1982), (1984); and, to a lesser extent, the concept of generalized media of social interchange found in Parsons (1967). The conceptual approach is holistic and the framework of the models appropriate to systems economics, as in Fox and Miles (1987).

1.2 Aims of the Research

1. To provide a precise and rigorous formulation of linear models of time allocation. Conceptually the models are transportation models. The first model is termed the "slack model". There is a complete time distribution at sinks but not at sources. The second model is termed the "distribution model". There is a distribution at sources in addition to the distribution at sinks.

2. To show that the models are operational. The models are intended as a contribution to socio-economic production theory and to objective consumption theory. They are relevant for social accounting.[9]

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9 Fox (1985), (1987). Juster and Land (1981).

3. To present a multidisciplinary perspective for economic analysis. The models are intended as a contribution to systems economics.[10]

1.3 Methodology

Mathematical models are chosen so as to reflect the underlying logic of the problem. The characteristics of the problem are formulated within a transportation model. Three equivalent formulations are provided, namely the standard transportation model, the standard programming model expressed in vector-matrix notation, and the activity analysis model of Koopmans (1951).[11] There is notational consistency. An activity matrix specifies the technology whereby inputs are transformed into outputs. The transportation problem can be regarded as a special form of linear programming. Linear programming is itself a very special case of activity analysis. To justify the theory in an empirical context numerical examples are provided. The primal solves an allocation problem in terms of the levels of optimal activities, measured in minutes. For the dual, the shadow-prices solve a valuation problem measured in cents/minute. Quantitative values are derived for the agent's social income and saving over a set of changing conditions. Effects of changes in the parameters, including the technology matrix, are examined.

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10 Fox and Miles (1987).

11 Use of activity analysis makes it possible to extend the perspective of the models. In particular it provides a concise statement of changes in technology. Activity analysis, which represents a more technical formulation of the transportation models, is set out in the starred sections at the end of each chapter, beginning with Chapter 5. These can be omitted in a first reading without any loss of continuity.

1.4 Formulation of the problem

An agent must carry out different activities to meet the requirements of his social position. Each activity generates income expenditure and uses up time. Certain environments enable the agent to use time more efficiently. The time required for activities is to be allocated to environments in such a way that all required activities are completed at minimum overall cost, using only the available time endowment.

1.5 Significance of the Study

The models are original because:

1. transportation problems have been used to solve the problem of the optimal allocation of time, considered as a scarce economic resource.

2. the value of time use within a set of environments has been quantified.

3. the lifestyle of an agent has been quantified, given systematic changes in the technology of the socio-economic production function.

4. a methodology for deriving per unit costs, using time budget and money expenditure data, has been developed.

The study represents a multidisciplinary approach to the theory of time allocation. The models are fully operational and make a contribution to production theory and to objective consumption theory. Economic interpretations of certain variables are new. Parameters at sources and sinks are expressed in time units, using an approach analogous to that of revealed preference.

1.6 Assumptions and Limitations

Some assumptions:

Non additivity of time.[12]

The observed time endowment is always strictly limited for any given day. In particular the upper limit is denoted by the 16 hour waking day.

The agent, as rational optimizer, meets the requirements of social position at minimum cost, given the constraints on activities.

The agent's socio-economic production function can be approximated by a linear technology.

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12 cf Szalai (1972), p.2.

Some limitations:

Because many activities of an agent are processes of uncertain duration, and many choices are made in the face of uncertain outcomes, a stochastic model would better reflect the complexity of every day activity.

A static model means that each decision by an agent does not change the current situation into a new situation. This dynamic aspect of time use is not captured by our time allocation models.

A linear model is only an approximation to more general functional relationships.

For time budget studies, considerable resources are required to build up existing data bases in order to provide adequate quantitative information necessary to make the model operational at regional or national level. For New Zealand no such data bases exist.

The categories for behaviour settings and for social requirements are new and may include problems of classification not yet apparent.

The cost matrix relates to market costs. By omitting the opportunity cost of foregone earnings[13], the models underestimate the economic cost of an agent's activities.

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13 Because of practical considerations.

CHAPTER 2

MEASUREMENT AND VALUATION OF TIME USE

2.1 Outline

This chapter is a prelude to the formulation of the transportation models of time allocation. Social and historical perspectives are provided. Examples are given from 19th century England.[1] The basic idea of an activity is introduced and used to describe a production technology. The discussion provides an intuitive approach to the allocation of time, and shows how the production technology shaped the environment in which workers carried out activities. The specific time distribution imposed on workers represents their lifestyle.

2.2 Time use in production

The focus is on the mode of the production process in which time is regarded as a necessary input. Adam Smith's classical example of the pin factory describes how an input-output transformation can be analysed into a number of separate processes. Some processes require different amounts of time for completion, while the completion of several basic processes can be seen as a

1 Morishima considers that for an understanding of economic theory it is not enough to know the mathematical framework. "There must also be some considerable knowledge of the social, institutional and historical foundations of that theory" Morishima, (1984), p. 9.

necessary condition for the start of a further stage in the transformation process.[2] A lack of synchronization will produce bottlenecks. In modern production, for example, there may be a delay between the completion of an intermediate product and its use as an input in the next stage of production.[3] Each separate delay means an increase in the time to supply the consumer. An interchange of different production processes involves managing time use. "Just in time" (JIT) production strategies involve time money trade-offs, so that, conceptually, time use becomes equivalent to money cost. JIT represents a new production environment.[4] For any given set of processes there is an upper limit on the time available for any one day period. For example quality control, information processing, the transforming of materials all use up time. Also, for more complex technologies, the need to synchronize processes becomes more urgent. Clearly time is a scarce resource which gives rise to opportunity costs.

A modern production system can be broken down into a number of separate processes. These will be termed "activities". Each of the finite set of activities involves the transformation of inputs, such as capital equipment, raw materials and time, into outputs. Time use is part of all the activities and is the link between each separate activity. This approach to production processes is applicable to both industrial and household sectors.

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2 Refer to critical path.

3 "Time in business is cost. The elimination of time in development, avoids many time-based costs that would otherwise have arisen during the time eliminated" Examples are a decrease in damage, pilferage and decay. Also less investment in working capital is needed ... Simmonds (1989), p. 15.

4 "JIT is the creation of a flexible environment in which everyone seeks to eliminate waste and to keep things simple so as to effect a continuous improvement in overall business performance" Lea and Parker (1989), pp. 10-13.
Motivations for cost accounting changes based on JIT are discussed in Foster and Horngren (1987), pp. 19-25.
A survey of changes in manufacturing software for implementation of a JIT environment is given in Rao (1989), pp. 18-20.

In 19th century England, industrial development required a substantial level of investment. A French observer in 1812 estimated the London brewery of Barclay and Company to represent an investment of £500,000, and this at a time when per capita income in England was £14.2. This example from Braudel[5] suggests that, given the massive scale of investment required, relative to the average yearly wage of a worker, ownership not only conferred power to set up systems that placed control of the means of production effectively beyond the reach of workers, but also imposed a regimented lifestyle. The effects of the high level of investment required were far reaching. Using the textile industry in England circa 1840 as an example, two effects are considered. The workers now operated in a different environment. One feature was a lifestyle regulated at the work place by the movement of steam powered machinery. Another feature was the journey to work. As well, a web of social relations was created between owners, workers and capital.

Inventions such as the mule and other technical innovations resulted in the demise of the self employed spinner and then, as a result of the invention of the power loom, together with changing demand for textiles, the self employed hand-loom weaver. Given massive increases in demand for cotton garments, the home based weaver could not match the productivity and output price of the factory. He was forced to become a factory worker.

In return for the use of time for labour the factory worker received a wage. Labour time became a commodity,[6] bought and sold on the market like any other commodity. As a market transaction this represented a voluntary contract

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5 Braudel (1984), p. 597.

6 There are differences. (Labour) time, unlike a normal commodity, cannot be stored.

between rational agents. However the location of work was no longer the home where, together, the wife and children of a weaver took part in his work. Instead of a family orientated lifestyle, social life was regimented by the mode of factory production. Work was now fixed by clocktime, not the seasons of the year. Work was contracted within a block of time exclusive of non-work related activities. The journey to work clearly involved time use, implicit to the contract for services of labour time for wages, but without monetary return. In the larger manufacturing cities of the north of England such as Manchester,[7] Birmingham and Leeds, the journey to work could entail a considerable amount of time [Engels (1958)]. At the factory, control was effectively exercised over the quality and intensity of time use. Monetary penalties were exacted where a worker's output was considered to be inferior. Late arrival of a worker resulted in a fine. If the practice was harsh, the reality is clear. Time is a scarce resource; time is money.

The term "environment" is introduced and used in an intuitive way to describe both the agent's behaviour and the location. Previously our weaver controlled the means of production. He owned both his place of work and his loom. Control of plant and equipment gave factory owners power to set up systems that "institutionalized measures of control" [Parsons (1968)]. For example, factory owners determined that machines should be operated at maximum capacity, so that labour had to be synchronized with time use for machines. Hence the introduction by factory owners of 24 hour production and the night shift. To protect their interests, workers formed their own associations, which required time use outside work hours for social, cultural and political activities. Despite vigorous opposition by owners, a reduced working day of

7 For a comment of conditions in Manchester, Braudel 1984, pp. 564-565.

12 hours became law in England in 1844. The decision to work defined a lifestyle which can be defined as a specific time distribution imposed on the worker. The decision entailed a set of institutional controls on his time use. It was as if a net had been cast over his behaviour.

Since 1974 many structural changes in modern economies have reflected the need to shift away from mass production of goods based on a source of cheap energy.[8] Accordingly we could expect to find evidence that the value placed on the worker's time has increased, with emphasis on retraining.[9] It is significant that Japan, a country completely dependent on external sources of oil supply, is the leader in time based production systems.[10] The worker's time is less controlled by the tempo of machinery in such systems. Increasingly, a premium is placed on innovation, on ability to design better information networks and on theoretical research. Higher educational qualifications are required, so that there is a quantitative improvement in time use - the same tasks are now carried out in less time - and as well, less energy intensive technologies are being developed.

Managing time use is critical to the success of time based production strategies.[11] Increasingly it is important to look at time as a variable.

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8 The secular trend for decreasing oil prices was reversed in late 1973. From 1973, before the OPEC price rise, to 1980, oil prices rose by 554 per cent. Hawken (1983), pp. 27,29.

9 a) For some comments, Peters (1989).

b) JIT is much more than a production control method. It represents a "philosophy of manufacturing", with, for example, team work and intergroup activities replacing individualism. Hopkins (1989).

10 "In autos, Japanese companies can develop new products in half the time - and with half as many people - as the US and German competition". Stalk (1988), p. 49.

11 Chew has proposed that measures of productivity should assign some value to the amount of time consumed. He provides an example where the introduction of a productivity index that focussed on "turnaround time" helped to cut plant prototype production time from 20 weeks to three days. Chew (1988).

Time is a "hard" variable, an objective quantifiable measure of activities. Time based measures can be compared over different economic systems. Hence, unlike money, time is an invariant measure with respect to space. Time is also an invariant measure with respect to periods, as in quantifying historical data for the same country. For time-based micro level accounts, data of many different types can be incorporated without the need for a common numeraire [Juster (1981)]. For time, as a measure of activities, there are no problems of comparison requiring adjustments to arbitrarily chosen bases due to inflation, exchange rates, or different systems of pricing. Data banks for prices are far more extensive than those for use of time, to be sure, but the methodology and practical experience needed for producing time based data cannot be regarded as insignificant. For example, Szalai (1972) contains an extensive bibliography of selected time budget literature for 16 countries.

2.3 Summary

In developing foundations for models of time allocation examples have been drawn from social and institutional processes relating to industrialization in 19th century England, and from contemporary time based management strategies. Constraints on an agent's time have been described in terms of environment and social relations. The first has an effect on behaviour and therefore on activities, while the second relates to expectations imposed on an agent. Time can be seen as a key variable, an objective, invariant measure of activities.

CHAPTER 3

SOME MODELS OF TIME ALLOCATION

3.1 Paradigms

Two models which represent a fundamental departure from the received theory of consumer behaviour are outlined.[1] The models of Becker (1965) and Moeseke (1985) represent the paradigms for use of time, as a scarce resource, in a classical utility model, and in an objective consumption model, respectively. Both models extend the scope of consumer theory to non-market behaviour. Conceptually the models are quite different. To enable comparisons to be made more readily, each model is outlined separately and then formulated. The Moeseke model develops and extends the range of applications of the earlier Fox-Moeseke model (1973) to include practical measurement of social parameters. Quantification was introduced in the Fox-Moeseke model, to be sure, but actual measurement of social income in this model while possible in principle would present serious difficulties in practice.[2] However the Fox-Moeseke model is important in its own right and requires comment because - to give only one reason - it lucidly outlines a justification for "crossing the boundary between economics and other social sciences".[3] In fact, by bringing concepts of an eco-behavioural system to bear on economic problems the Fox-Moeseke model breaks new ground. The models of Becker and Moeseke are then

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1 For other seminal articles which represent a departure from received theory, but which do not consider time as a scarce resource, see:
(1) Lancaster (1966), (2); (1971)
(2) Muth (1966)

2 for a comment, Moeseke (1985), p. 264

3 Fox-Moeseke (1973) see also Fox 1984(1), Fox (1987)

compared and contrasted. Different linear models are then suggested. They are the transportation models of time allocation, to be formulated in Chapter 5.

3.2 The Becker model

Becker's theory of the allocation of time is developed within a framework characterized by maximizing behaviour, market equilibrium and stable preferences. For Becker it is this framework which constitutes the economic approach, and it is the approach "used relentlessly and unflinchingly"[4] rather than the content which, he considers, sets economics apart from other disciplines. Within this framework he moves far beyond what, for many economists, had previously come to be recognized as the conventional boundaries of economic investigation.[5] In fact he holds that applications of the economic approach include all human behaviour. "Rather, all human behaviour can be viewed as involving participants who maximize their utility from a stable set of preferences and accumulate an optimal amount of information and other inputs in a variety of markets."[6] Becker's aim is to predict human behaviour, to provide a set of testable hypotheses based on a theory of actual behaviour, while avoiding explanations in terms of "non-rational" behaviour. The applications include many activities of agents which are cases of non-market behaviour, while prices may be either money prices or imputed non-market shadow prices.

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4 Becker (1976), p. 5.
5 For a discussion on Becker's extension of the boundaries of 'final' production, see Boss (1990) pps. 248-250.
6 ibid, p. 14.

Becker formulates a non-linear optimisation model for the allocation of time in non-work activities, excluding human capital which has already been analysed separately. He introduces non-working time into a utility function and postulates a household production function with goods and time as the arguments. For Becker the systematic incorporation of non-working time into a traditional utility function is the point of departure. As well, he introduces the concept of full income, the maximum money income that would be earned if all time available were used for work. The concept of cost is applied to time in the same way as it is applied to goods. Foregone use of non-working time for one activity, a visit to the theatre rather than attending a seminar, will involve an opportunity cost, so that the full cost of an activity is the sum of the market prices and the value of the foregone time used up. The model integrates production and consumption. Households combine goods and time to produce basic commodities and choose the optimal combination by maximizing a utility function, subject to a total resource constraint.[7] The demand for time and goods, the inputs of the household production function, is a derived demand. Time and goods are required because they are necessary for the production of basic commodities from which utility is obtained.[8] The concept of full income makes it possible to solve the problem of maximizing utility subject to the time and goods constraints in two stages. An expression for full income is formulated applying goods and time constraints. Then utility is maximized subject only to the full income constraint.

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7 For some alternative utility type models of the allocation of time, see: Linder (1970); De Serpa (1971), (1973); Evans (1972); Bruzelis (1979).

8 Becker's commodities can be compared with the "characteristics" possessed by goods in the models of Lancaster (1966),(2) (1971).

There is a more concise formulation of the model in Becker (1971). By comparison with the original paper (1965), there are some variations in the use of symbols. The following outline is intended to give something of the flavour of the model without being too technical.

Formulation

Each household is considered to maximise a utility function

$$U = U(Z_1, Z_2, \ldots, Z_m) \qquad (3.1)$$

where Z_i is the basic consumption good produced by the household by combining ordinary goods X_i i=1, ..., n and inputs of household time T_i i=1, ..., m.

The production function is given by

$$Z_i = f_i(X_i, T_i; R) \qquad (3.2)$$

where f_i is the production function for Z_i and R denotes other variables, including the effects of education, climate and other "environmental" variables. There are two constraints:

(1) The total expenditure on market goods is limited by the money income available, as in

$$\sum_{i=1}^{m} p_i X_i = I \qquad (3.3)$$

where I denotes income

(2) The time constraint is given by

$$\sum_{i=1}^{m} T_i = T_c = T - T_w \qquad (3.4)$$

where T is total time, T_w is work time and T_c is non-work time. As in (3.2), T_i denotes inputs of non-work time.

Income not only equals the total expenditure on goods as in (3.3) but is also equal to the sum of all factor payments, and can be written as

$$wT_w + V \qquad (3.5)$$

where w denotes the average wage rate and V denotes other income.

The separate goods and time constraints can be represented by a single total resource constraint by substituting for T_w in (3.4):

$$\sum_{i=1}^{m} p_i X_i + \sum_{i=1}^{m} wT_i = wT + V = S \qquad (3.6)$$

If w were constant, and if all time were used for market work S would denote the "full" income consisting of market work together with other income.

Shadow prices

The maximization problem is solved by forming a Lagrangian function as follows:

$$L = U(Z_1, Z_2, \ldots, Z_m) - [(\sum_{i=1}^{m} wT_i + \sum_{i=1}^{m} p_i X_i) - S] \quad (3.7)$$

The constraint can be simplified by expressing the general production functions f_i in (3.2) as

$$X_i = a_i Z_i \quad ; \quad T_i = b_i Z_i \quad (3.8)$$

where a_i and b_i are fixed input-output coefficients

The Lagrangian function now becomes

$$L = U(Z_1, Z_2, \ldots, Z_m) - (\sum_{i=1}^{m} \pi_i Z_i - S) \quad (3.9)$$

where $\pi_i = a_i p_i + b_i w$

From first order conditions,

$$L_{Z_i} = U_{Z_i} - \pi_i = 0$$

and

$$L_{Z_j} = U_{Z_j} - \pi_j = 0 \quad (3.10)$$

Hence $$\frac{MU_i}{MU_j} = \frac{\pi_i}{\pi_j} = \frac{wb_i + p_i a_i}{wb_j + p_j a_j} \quad (3.11)$$

The ratio of the marginal utilities of any two commodities Z_i and Z_j will equal the ratio of their marginal costs, π_i/π_j. Each marginal cost is the sum of two products, namely the wage rate and the marginal input-output coefficient with respect to time, together with the price of market goods and the marginal input-output ratio with respect to goods. The shadow price of time is given by $b_i w_i$, and the sum of $a_i p_i$ and $b_i w$ is the shadow price of a unit of Z_i. The shadow prices represent opportunity costs.

Becker develops a number of "empirical implications". These suggested applications include hours of work, the productivity of time, income elasticities, and transportation. Becker provides a theoretical model for the allocation of time in activities which require the consumption of time for travel. To take just one empirical implication, his formulation provides important conceptual tools in quantitative applications concerned with measuring travel time values. In transport economics considerable empirical work has been carried out in estimating the value of time saving, in making forecasts of estimated demands, and in other cost benefit studies. By comparison there has been notably less research on the economic theory side, as noted in Bruzelius (1979). Given the lack of symmetry between theory and empirical studies the contribution of Becker to this field takes on a special significance. His formulation of the problem allows for the opportunity cost of time to be incorporated into activities that require travel time. Time is a scarce resource, used as an input into the 'production' of travel activity. The full cost of the trip includes not only the money expenditures for the trip, but also the foregone use of time as an input into some other activity.[9]

3.3 The Fox-Moeseke model

Conceptually, the Fox-Moeseke model can be regarded as a link between the utility consumption model of Becker and the objective consumption model of

9 For a critique of the Becker model: Pollack and Wachter (1975).

Moeseke. In this sense it is a "transitional" model. A utility function, u(x) is used in the formulation of the programming model (P) in the Fox-Moeseke model, which suggests that this model is, as it were, a half-way house along the road to objective consumption theory. In fact utility theory is not an essential feature of the Fox-Moeseke model. The final section removes the utility function which served as a scaffolding.

The Fox-Moeseke model consists of three parts: the first explores the development of national accounts by Kuznets and others, and makes a contribution to social accounts which extends, and crosses, the boundaries of economics. The model makes use of Barker's concept of "behaviour settings" and Parson's concept of "generalized media of social discourse". Social income (SI), a scalar measure, is defined as the sum of the equivalent dollar values of all rewards during the current accounting period that are derived from the endowments.

Total income = $SI+p_1x_1$,	where x_1 is the fraction of total time spent in paid employment and p_1 is the wage or salary received per time unit.
Personal income = p_1x_1+y	where y denotes the sum of income from property and transfer payments.
Total income - personal income = $SI+p_1x_1 - (p_1x_1+y)$ = $SI-y$	where $SI-y$ denotes the equivalent dollar value attributed to rewards such as status and prestige.

The second part of the paper outlines the mathematical formulation of the model and presents a rigorous derivation of the mathematical properties of the model. The agent faces the programming model

(P)

maximize $u(x)$

Subject to

$$\mathbf{Ax} \leqq \mathbf{b} \qquad (3.12)$$

$$\mathbf{x} \geqq \mathbf{0} \qquad (3.13)$$

where the set $X \equiv \{\mathbf{x} \geqq 0 | \mathbf{Ax} \leqq \mathbf{b}\}$ is termed the feasible set of possible activity levels (time allocations to alternative settings).

If $\mathbf{v}^*$ is given the standard interpretation as a price system for endowments $\mathbf{b}$ (in terms of maximand u) then the solution of (P), implies

$$\mathbf{v}^*\mathbf{b} \equiv \Sigma \mathbf{v}_i{}^*\mathbf{b}_i = v_1{}^*y + \ldots + v_m{}^*b_m \qquad (3.14)$$

where y, denoting income not dependent on personal effort during the current accounting period, is one of the endowments.

SI is defined as:

$$SI \equiv \mathbf{v}^*\mathbf{b}/\mathbf{v}^*{}_1 = \Sigma(\mathbf{v}_i{}^*/v_1{}^*)\mathbf{b}_i$$

$$= (v_1{}^*/v_1{}^*)y + (v_2{}^*/v_1{}^*)b_2 + \ldots + (v^*{}_m/v_1{}^*)b_m \ ; \ (v_1{}^*>0) \qquad (3.15)$$

$$= y + (v_2{}^*/v_1{}^*)b_2 + \ldots + (v_m{}^*/v_{1*})b_m$$

Since the first term, namely the unearned income, is in dollars (because $y+p_1x$ represents money income), then the remaining terms, which measure the value of lifestyle, are also in dollars. The right hand side is the money value of endowments. Clearly Social Income (SI) has the same dimension as y, namely dollars. This is the key. Social Income is defined and then shown to be measured in dollars.

Total income is then $p_1 x^*_1$+SI.

What is the relationship between Social Income and utility theory? It is proved that utility theory is not essential to the scalar measure of social income. In the Fox-Moeseke model utility can be regarded as a heuristic device, to be discarded in the final part of the paper. In particular it is shown that:

1. u can, and need, only be specified up to a monotonic transformation and that SI is *invariant* (emphasis in the Fox-Moeseke paper) on the class of such transformations.

2. given that the unit of measurement for any one medium in $\mathbf{Ax} \leq \mathbf{b}$ is arbitrary, the choice of $\mathbf{x}^*$ is invariant under replacement of units.

The Le Chatelier Principle is used to show that if but one of the b_i changes, then the marginal value v_i^* of the ith endowment changes in the opposite direction. Also, an activity level is increased if its cost decreases, and vice versa.

If u is known, the value of SI is given by (3.15).

u can be computed by quadratic approximation, as follows:

$$u(x) = a+qx+\tfrac{1}{2}xQx \qquad (3.16)$$

The final section introduces empirical approaches to a study of income distributions. This represents an application of the mathematical model. Interpersonal comparisons of well being are possible, without assigning numerical values to an agent's 'utility'. The aim is develop a classification of media of social interchange as a necessary condition for a fully developed system of social accounts.[10]

3.4 The Moeseke Model

In order to meet the requirements of his social position, an agent has to carry out a number of activities. These require time and entail money expenditures on goods and services. The activities represent a distribution of time use over a given endowment, for example the 16 hour "working day", and therefore quantify the agent's lifestyle or behaviour pattern. Since costs of goods and services are known it is also possible to describe the agent's economic position. Every activity has an economic cost which can be expressed as an average in dollar terms per unit of time.

Activities provide the means of linking, on the one hand, the inputs of time and money, and on the other, the agent's social requirements, given by a range of commitments, such as professional, social, civic, religious. The social parameters, "the number of social commitments inherent in his position",[11] can be regarded as the output of a socio-economic production function, itself a function of activities. The pivotal role of activities makes it possible to assign objective values, measured in dollars, to an agent's social position.

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10 For a time based system of social accounts Fox and Ghosh (1981). Also Fox (1983, 1984, 1985)

11 Moeseke (1985) p. 264.

Social income is by definition equal to the dollar value of the agent's time plus money income from work. The first component is a measure of the status attached to the agent's social position. As well, the model specifies a relationship between social income, savings and dissaving.

By solving a linear program we can predict the agent's behaviour pattern from his economic position and his social position. The agent's environment influences behaviour, so that a change in environment can result in a different behaviour pattern, given constant economic and social positions.

The Moeseke model extends the range of economic investigation. This is done by making use of concepts from other social sciences. The agent's different activities can be related to the concept of "behaviour settings", as developed by Barker (1968) in the framework of eco-behavioural science. The significance of the eco-behavioural perspective for time use models is outlined in Chapter 4. The parameters of social position can be compared with the concept of "generalized media of social interchange" introduced by the sociologist Talcott Parsons. These concepts from other disciplines are integrated within the framework of linear economic theory. The model therefore represents a socio-economic approach to human behaviour. "Measurement of social parameters is now not only objective but Practicable."[12]

The method is an optimal linear program where an objective function - net cost of lifestyle - is minimized subject to both time and social constraints. The agent has a time endowment, termed the "waking day" which is exogenous. The

12 - ibid, p. 264.

total time use for activities cannot exceed the given endowment. As well, each of the agent's requirements of social position must be met.

Formulation:

The objective consumption model is formulated as a linear program, subject to constraints. The agent minimizes net cost subject to a time constraint and his social constraints.

The time constraint:

The total time endowment consists of the 16 hour waking day. This is taken as the time unit. Fractions of time are used up either in activities or in work. The agent's lifestyle or behaviour pattern is denoted by the (n+1)-tuple

$$(\mathbf{x}) = (x_0, x_1, \ldots, x_{n+1}) \quad , x_j \geqq 0 \ , \ j = 1, \ \ldots, \ n$$

where x_j denotes the average fraction of the total time endowment spent on activity j. x_0 denotes the average time fraction spent at work. The remaining n-tuple $(x_1, \ldots, x_n)$, written $\mathbf{x}$, denotes the agent's activities.

The time constraint is therefore

$$x_0 + x_1 + \ldots + x_{n+1} \leqq 1 \qquad (3.17)$$

which can be expressed concisely as

$$-x_0 - \mathbf{ux} \geqq -1 \qquad (3.18)$$

where $\mathbf{u}$ is the n-tuple of units (1, ..., 1)

The direction of the inequality has been changed to correspond with the direction of the inequalities for all other constraints.

The social constraints:

The agent undertakes activity j, j=1, ..., n in order to meet the requirements of social position. The parameter of social position is denoted b_i, i=1,...,m. The units for the b_i vary. In the numerical example provided, units of measurement include: publications, family outings, votes, attendances at meetings. The relationship between the x_j and the b_i can be denoted as

$$b_i = b_i(x_0, \ldots, x_j, \ldots, x_n) \quad i=1, \ldots, m \quad j=1, \ldots, n. \tag{3.19}$$

where the b_i is the output requirement of the ith socio-economic production function. For a requirement $b_i>0$. A negative requirement is possible, such that $b_i<0$ denotes an endowment. For example, complimentary books, free travel. To what extent does a particular activity contribute towards meeting a social requirement? Activity j satisfies requirement i to the extent of a_{ij} units, per unit of activity. For the ith social requirement

$$a_{i0}x_0 + a_{i1} + \ldots + a_{in}x_n \geqq b_i \tag{3.20}$$

Each activity of the agent must contribute towards meeting a social requirement, and the sum of the j activities must at least meet the ith social requirement. Each activity results in a column vector of outputs, each of which is capable of contributing to the requirements of social position. Where some elements of the activity vector are zero, no contribution is made towards meeting the corresponding element in the column vector of social requirements. Hence

$$A(\mathbf{x}) \leqq \mathbf{b} \qquad (3.21)$$

where A is the m x(n+1) matrix of coefficients a_{ij}.

The matrix A is a linear approximation to the socio economic production function in (3.19).

The objective function:

For each element of the (n+1)-tuple(x) there is an associated cost. Activity j costs $\$c_j$ per day or $\$c_j/16$ per waking hour. Net income from work is a negative cost, denoted $-c_0$, taking $c_0 > 0$. The objective function to be minimized is therefore

$$-c_0x_0 + \mathbf{cx} \qquad (3.22)$$

The program is

$$\text{(M)} \qquad \min \quad -c_0x_0 + \mathbf{cx} \qquad (3.23)$$

$$\text{subject to} \quad \begin{array}{rcl|l} -x_0 - \mathbf{ux} & \geqq & -1 & v_0 \\ & & & \\ A(\mathbf{x}) & \geqq & \mathbf{b} & \mathbf{v} \end{array} \qquad (3.24)$$

$$(\mathbf{x}) \geqq 0 \qquad (3.25)$$

The dual variables v_0 and $\mathbf{v} = (v_1, \ldots, v_m)$ appear to the right of the corresponding constraints. The predicative properties of the solutions to (M) can be summarized as:

$$[(\mathbf{c}) \text{ and } (\mathbf{b})] \Rightarrow (\mathbf{x})$$

with **A** the link between (**x**) and (**b**) where,

(**x**) is the (n+1)x1 column vector	$(\mathbf{x}) = x_0, x_1, \ldots, x_n$	denotes the agent's behaviour pattern.
(**c**) is the 1x(n+1) row vector	$(\mathbf{c}) = -c_0, c_1, \ldots, c_n$	denotes the agent's economic position.
(**b**) is the mx1 column vector	$(\mathbf{b}) = b_1, b_2, \ldots, b_m$	denotes the agent's requirements of social position.
A is the m x (n+1) matrix	$\mathbf{A} = [a_{ij}]$	denotes the matrix of coefficients quantifying the agent's environment.

Note: **A** is a linear approximation to the socio-economic production function

$$b_i = b_i(x_0, x_1, \ldots, x_n) \quad i = 1, \ldots, m$$

In particular $\quad A_0 = b_i(x_0)$

First, a scalar measure of the value of the agent's position is derived.

The dual of the program M is:

(M II) $\max \quad -v_0 + \mathbf{vb}$

subject to

$$-v_0 + \mathbf{vA}_0 \leqq c_0 \tag{3.26}$$

$$-v_0 + \mathbf{vA}_j \leqq c_j, \quad j=1, \ldots, n \tag{3.27}$$

$$v_0, \mathbf{v} \geqq 0 \tag{3.28}$$

For optimal solutions x_0^*, $\mathbf{x}^*$

$$-c_0x_0^* + \mathbf{cx}^* = -v_0^* + \mathbf{v}^*\mathbf{b} \tag{3.29}$$

Hence $$c_0x_0^* + \mathbf{v}^*\mathbf{b} = \mathbf{cx}^* + v_0^* \tag{3.30}$$

where v_0^* is the dollar value of the agent's time and $\mathbf{v}^*\mathbf{b} = v_1^*b_1 + v_2^*b_2 + \ldots + v_m^*b_m$ is the value of the agent's social position.

Economic interpretation:

$c_0x_0^*$ is the net income from work. Since the left-hand side of (3.29) is in dollars the right-hand side of the equality will also be in dollars. $\mathbf{v}^*\mathbf{b}$ is the value of social requirements. Since the agent gains a salary and, as well, social status from time use, this scalar measure of the agent's total gains is $c_0x_0^* + \mathbf{v}^*\mathbf{b}$.

Definition

Social Income is the amount corresponding to the value of an agent's social position plus money income.

$$SI = c_0 x_0^* + \mathbf{v}^*\mathbf{b} = \mathbf{c}\mathbf{x}^* + v_0^*$$

The agent is paid and therefore gains money from work. But as well the agent gains status from work. From x_0, the time fraction spent at work, the agent gains in two different and complementary ways. This is not double counting. To be sure, both paid income and status are generated from the same time flow but they represent different types of gain. In the numerical example provided, an author's output from work is measured by the number of pages written. For this output money income is paid. The author gains status from his output. This is a different type of gain from the payment for pages. Such statements as "I have always been overpaid to do that which I would pay to do"[13] clearly suggest a difference between gains from money and gains from status, job-satisfaction ... The concept of SI makes it possible to assign dollar values to both.

Saving and Impatience

The Moeseke model also offers a social theory of saving. From (3.29)

$$c_0 x^* - \mathbf{c}\mathbf{x}^* = v_0^* - \mathbf{v}^*\mathbf{b} \qquad (3.31)$$

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13 Samuelson (1983)

That is, net income from work minus consumption expenditure = value of time minus value of social requirements.

The left-hand side measures savings. If the right-hand side is negative the agent dissaves or is a net borrower. Clearly social requirements are more valuable to the agent than time. In the model this difference is termed "impatience". Both and savings impatience can be measured in dollars.

The analysis has determined optimal solutions for an agent wishing to minimize cost for a given time budget. The model can also accommodate a time minimizing program under a given budget constraint.

The program is

$$(T) \qquad \min \; v_0 + \mathbf{vx}$$

$$\text{subject to} \quad \begin{array}{rcl|l} c_0x_0 - \mathbf{cx} & \geqq & 0 & w_0 \\ \mathbf{A(x)} & \geqq & \mathbf{b} & \mathrm{w} \\ \mathbf{(x)} & \geqq & \mathbf{0} & \end{array}$$

If v_0^*, $\mathbf{v}^*$ is a dual solution to (M) then, given certain conditions, it can be shown that

$$w_0^* = 1/v_{0*} \;, \; \mathbf{w}^* = \mathbf{v}^*/v_0$$

is a dual solution to **T**, assuming $v_0^*>0$. If v_0^*, $\mathbf{v}^*$ are interpreted economically as the marginal value of a time unit in dollars, then conversely, w_0^*, $\mathbf{w}^*$ are interpreted as the marginal value of a dollar per unit of time.

Example.

If $v_0^* = \$100$, $v_1^* = \$50$, $v_2^* = \$25$, say, then $w_0^* = \frac{1}{100}$, $w_1^* = \frac{50}{100}$,

$w_2^* = \frac{25}{100}$. That is, for w_0^*, one dollar taken as the money unit is worth $\frac{1}{100}$ of a time unit.

A further paper, Moeseke (1986), specifies the relevant equivalence propositions between money programs M and time programs T, given constraints and related dual variables. Of course trade-offs between cost, as one objective, and time as another, or again between time and cost, represent just one type of general bi-objective (or vector-max) program. The generalized problem has been formulated, and the equivalence propositions defined, in terms of both primal and dual variables, Moeseke (1987).

3.5 Comparisons and Contrasts

Both models are fundamental reformulations of the received theory of consumer behaviour. They extend the scope of the theory to non-market behaviour. Both incorporate time as scarce resource. The following table sets out some comparisons and contrasts.

Table 3.1

	Becker model	Moeseke model
Method	non-linear constrained optimisation.	linear constrained optimisation.
Type of consumption model	utility.	objective.
	House holds choose best combinations of commodities	The agent carries out activities, measured as

	by maximizing a utility function.	fractions of a time unit, to meet observable social requirements.
	Comment: The model is non-objective. Goods and time can be measured. However there are fundamental problems in measuring utility which has subjective elements.[14] There is no unique utility function fully consistent with observed behaviour. There is no objective unit for measuring utility, and this is one reason why aggregation is not possible in the model.	Comment: The model is able to make use of the techniques developed by Barker (1968) and associates for measuring data. Aggregation is possible and practicable. In practice sampling would be used for making estimates.
Relationship to production theory	Households are both consumers and producers. Production model: $Z_i = f_i(X_i, T_i)$ (1)	The matrix A quantifies the agent's environment and represents the interface between two different parts - the social and the economic. A is a linear approximation to the socio-economic production function $b_i = b_i(x_0, \ldots, x_n)$ $i=1, \ldots, m$ (1)
Extended concept of income	$S = \Sigma p_i b_i Z_i + L(Z_1, \ldots, Z_m)$[15] (2) Full income is equal to direct spending on market goods plus the total earnings foregone to gain utility. $S = \Sigma p_i b_i Z_i + t_i w Z_i$ (3) $\Sigma p_i b_i Z_i$: value of consumption at market prices; $t_i w Z_i$: value of time used for consumption	$SI = v^* b + c_0 x_0^*$ (2) Social income is equal to the value of social position plus net income from work. $S = cx^* + v_0^*$ (3) cx^*: value of consumption at market prices; v_0^*: value of time

14 For a comment on classical utility theory see Moeseke (1985), p. 265. For a more detailed analysis: Moeseke (1969).

15 The total earnings foregone by the decision to gain utility is denoted by L. For the derivation of (2) see Becker (1965), p94.

$= V + Tw$ (4)

other (non-work) income — income from work, using the total time endowment (taking average earnings, w, as constant)

Comment: The right hand component of (3) in the Becker model excludes time spent at work.
By way of contrast the corresponding term v_0^*, in (3) of the Moeseke model measures the value of time which includes both time spent on meeting social requirements and time spent on work. Since the left hand components are the same, the difference in the extended concept of income for the two models is clearly to be found in the imputed value of different time uses.

Applications

Consumption theory
Michael (1973) Gronau (1977)

Life cycle decisions, including investment in human capital. Michael (1972); Ghez and Becker (1975). Becker (1976) contains a wide range of applications. For an important model of human capital which influenced Becker's model see Ben-Porath (1967).

Social accounting
Time based accounts combining expenditure and time use data. Earlier developed models of time based social accounts can be found in Fox (1981, 1983, 1984, 1985), and in Fox and Ghosh (1981)

Value of travel time.
As Becker comments, "transportation is one of the few activities where the cost of time has been explicitly incorporated into economic discussions". Gronau (1976).

Objective consumption Theory
Moeseke and Goldsmith (1987), using a game model. The consumption matrix was introduced as a matrix game by Moeseke (1981). The first empirical application was developed by Goldsmith (1983).

Comment: This outline has been kept very brief. In fact the range of applications is extensive. In some cases it is difficult to attribute the application to any one source. For example see Gronau (1973).

For a more recent model see Moeseke and Goldsmith (1990).

3.6 Transportation models of time allocation

In the remaining chapters of the thesis the transportation models of time allocation are introduced and developed. These linear models are different from the Becker and Moeseke models. There are two main points of departure. All parameters of social position, will be measured in a standard time based unit. This can be applied to the parameters of agents whose social requirements, while objective, would otherwise be measured in a variety of units, making aggregation difficult in practice. The model will introduce a further resource set, the set of locations, defined by behaviour settings, which can be used to quantify the behaviour of an agent in different environments. This is equivalent to adding a further set of constraints - at sources - to the model. The distribution of the time endowment at sources can be expected to generate a set of differential rents measured by imputed values. The production function, defined by the socio-technological matrix, is quite general and in principle can be used for firms as well as for households and individuals. The transportation models of time allocation integrate and extend the boundaries of what was regarded as "economic territory".

CHAPTER 4

BEHAVIOURAL BACKGROUND TO THE MODELS

4.1 Introduction

This chapter builds on the intuitive approach of Chapter 2, and begins the formal development of the transportation models of time allocation. These models are intended not only to provide quantitative measures of an agent's activities but also to raise questions that change our way of looking at the world.[1] As a first step, this chapter develops some of the necessary conceptual tools and suggests links between them. The economic approach is integrated with insights from other disciplines. In particular the following questions are considered: How can we measure the activities which an agent undertakes? How do environments and social relations represent constraints on the behaviour of an agent?[2] How can they be measured? Concepts from eco-behavioural science, from time-budget studies and to a lesser extent from sociological studies of the relations between economy and society are used. The next step is to show that, by integrating these concepts within the framework of a central economic problem, namely the efficient use of resources, and using transportation models, it becomes possible to shed fresh light on the behaviour of an agent.

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1 See, for example, Kac (1969) "The main role of models is not so much to explain and predict - though ultimately these are the main functions of science - as to polarize thinking and to pose sharp questions". Also Gauss (1989), p. 58.

2 We extend the intuitive approach of Chapter 2.

4.2 Integration of Economics with other Disciplines

Making use of concepts from other disciplines not only makes it possible to extend the breadth and scope of economics, but also ensures that answers which an economist offers are more likely to contribute to our understanding. For example, Fox and Miles (1987)[3] suggest a systems approach to economic analysis[4]. Society is simply too complex to be characterized by the insights of a single discipline. This is not to suggest that the economist learns a different method. The approach used in developing the time allocation models is that of linear economic theory, but the perspective is extended. What does it mean to integrate concepts from other disciplines within the framework of an economic problem? Just this. The approach is still the approach of the economist, but because the perspective is extended the models take on further dimensions. In this way the transportation models of time allocation can be said to represent the integration of economics with findings from other disciplines.

4.3 Behaviour and Environment

Do environments represent constraints on the behaviour of an agent and if so how can these constraints be measured? This section shows how the eco-behavioural view of human behaviour can provide insights into the way environment shapes behaviour. The term "eco-behavioural science" was introduced by Roger Barker in 1969. We now present an outline of Barker's

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3 "However, we believe systems economics will be particularly important in conceptualizing problems which include social and/or environmental as well as economic components" p.xvii.

4 Systems economics can be seen in the light of general systems theory, originated by Ludwig von Bertalanffy, a movement to foster the development of basic concepts which extend across different disciplines.

concept of a behaviour setting and then show the relevance for the time allocation models. Over many years, Barker carried out a meticulous exploration into the relationship between environment and behaviour. He considered that the behaviour of an individual was never isolated. Rather, his approach required a careful observation of human behaviour within a total environmental system. One significant contribution Barker made was to reduce complex patterns of human behaviour to fundamental elements, analogous to atoms within a complex molecular structure. The basic unit of observation is the behaviour setting, a key concept of eco-behavioural science. To pursue the analogy, just as the atom has a certain structure, expressed, for example, in terms of electrons and protons, so too, conceptually, the behaviour setting has a certain structure.

The behaviour setting is specified by a space-time locus and represents a "characteristic standing pattern of behaviour and milieu". The boundaries of the behaviour setting are shaped independently of the observer. This is not to deny however that, in practice, technical skills are required to identify a behaviour setting. For Barker, a behaviour setting involves both human and non-human components, for example a ballet class, an auction sale, a doctor's waiting room. Each behaviour setting incorporates a program of activities. In every case, people must be present. Certain programs are similar. Thus several ballet classes held in different sites could well constitute a "family" of behaviour settings.

Barker observed an empirical relationship between behaviour settings with similar programs, and the kinds of behaviour observed within such groupings. While the individual remains within the boundaries of the behaviour setting he is constrained to act by a vector of forces belonging to the setting. Deviant behaviour results in loss of contact with the setting. For example the same people behave in a certain way in a church, but in a quite different way at an auction sale. In both cases the environment shapes behaviour, but the particular way in which it is shaped depends on the nature of the environment. Barker's studies are based on observation and on the measurement of objectively determined data. From the perspective of eco-behavioural science he provided an operational model of human activities.

These findings are now related to the transportation models of time allocation. The assertion that environment shapes behaviour, based on empirical evidence from the research of Barker (1968) and Barker and Associates (1978) becomes an assumption of the models. In the models, each activity undertaken by an agent takes place in a specific environment and uses up time. The focus is on the aggregation of many separate activities into five well defined categories, and the concept of behaviour setting is used to refer to these categories. This is a departure from Barker, who used the behaviour setting as the basic unit of observation, and used other terms for aggregations of the basic units. Hence the concept of behaviour setting in the time allocation model represents an aggregation of the basic units of observation which are behaviour settings in Barker's studies. The use of the term "behaviour setting" in the models is at once an acknowledgement of Barker's pioneering research and a reminder that environment shapes the behaviour of an agent.

In the transportation models of time allocation all the different environments in which the agent undertakes activities are classified into five behaviour settings. The categories are provided in Table 4-1.

TABLE 4-1 ENVIRONMENTS CLASSIFIED BY BEHAVIOUR SETTINGS

Behaviour Setting	Example
1. State (S)	a class studying Computer Science in a state university
2. Household (H)	preparation of a family meal
3. Private Enterprise (P)	a housewife buying groceries at a supermarket
4. Voluntary Associations (V)	a meeting of a support group for mothers of pre-school children
5. Workplace (W)	a carpenter working on a building site

Notes: 1. Workplace represents a behaviour setting distinct from all other behaviour settings because the workplace environment is defined as one in which income is earned. The environment therefore shapes behaviour in a way that the other environments do not. In the models the concept of environment has been extended to include money costs.

2. The term "voluntary associations" is a catchall to include any environment not otherwise specified, such as places of assembly for prayer or worship and places used for non commercial hobbies. This category is used to close the environments/endowments system.

3. Because the activities which form the program of a behaviour setting have both a spatial and a time dimension we can associate the spatial dimension with an objectively determined time use. Intuitively it would seem that time use measures can be associated with behaviour settings. For example, the meeting of a support group for mothers of pre-school children takes place in a specific location and the duration of the meeting can be measured in time units. In the following section measures of time use are examined. In the final section these concepts are used to describe an activity within a three-dimensional space, time and cost framework.

There is a transition from the classification of behaviour settings as developed by Barker, and the way in which environments are classified by behaviour settings in the time allocation models. The choice of behaviour settings in these models extends the categories used by Barker to incorporate households. The next step is to relate the five categories of behaviour settings in Table 4-1 to the OECD categories of time use, as described by Ås (1982).

4.4 Time Use Studies

How can we measure the activities which an agent undertakes? As a preliminary step some requirements related to the agent's use of time are outlined:

1. A precise methodology is required. This is a necessary step in designing and formulating the transportation models of time allocation. On the one hand the methodology must rest on sound conceptual foundations, and on the other hand, it must also provide a systematic, objective means of collecting and classifying data. The methodology must be flexible enough to be adapted to the behavioural patterns of special subgroups, for example university students.

2. The particular aim is to measure activities of an agent by sequence and duration of time use.

3. As well, it is necessary that the agent's activities can be assigned a per unit cost so that it becomes possible to quantify optimal trade-offs between time and money.

The first two requirements are now considered in some detail. The third requirement is investigated in section 4.5.

Studies designed to measure the use of time are usually referred to in the literature as "time use", "use of time", or "time budget" studies. A time budget is a log or diary of the sequence and duration of activities engaged in by an individual over a specified period, most typically the 24 hour day.[5] It is worth noting that the terminology is not fixed and the term "time diary" is also used Ås (1978). Most time activities cover the 24 hours of a single day. In the time allocation models the agent's activities are analysed over a 16 hour period, referred to as the 16 hour waking day.

A necessary limitation of this whole study is the non-simultaneity of time. We assume time used up for one activity will use up time otherwise available for another activity. This is not an empirical observation. However this assumption makes it possible to measure trade-offs in the models. Individuals trade off time spent on one particular activity by spending less, or more, time on another activity, thereby incurring opportunity costs. One advantage of time budget studies is that no individual is able to call on more time use than any other over the specified period, for example the 16 hour waking day. Although the total time endowment is unknown for each individual over a life cycle, all individuals have, and can only have, at their disposal the same time endowment over the fixed period of the time budget. The amount of time spent on different activities by any one person can be expressed as a distribution, taking the fixed period of time, such as the 16 hour waking day, as a unit. Time budget studies make possible objective description of how a particular individual uses time, and provide a criterion that allows for direct comparison between individuals and groups of people both in the same locality, and in different countries. While time is a "hard" variable, able to be measured accurately by observers, it should be noted that the complex nature of human activities presents some difficulties.

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5 - Converse, (1968).

The pioneering time use studies were carried out by S. G. Strumlin in the USSR during 1922.[6] Time use budgets have been used extensively by researchers in- the social sciences, notably in sociology, human geography and urban planning.[7] The perspective is different from that of economics. Time is not considered from the economist's concept of a scarce resource, nor are valuations in money terms generally used. However these observations reflect the fact that, with few exceptions, economists have not yet made contributions in a field where, increasingly, interdisciplinary work is being carried out by scholars in other social sciences.[8]

There is a standard approach to time use studies. The basic model classifies time use under four categories. More detailed schemes are developed from this foundation. The four categories are set out in Table 4.2 below, following Ås (1982):

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6 Zuzanek, (1979).
7 (a) Carlstein and Thrift (1978).
(b) Gutenschwager (1973).
(c) Chapin (1968).
8 Ferber (1973) comments: "Most consumption economists seem to have reacted to the growing popularity and usefulness of interdisciplinary approaches by, if anything, drawing blinders about their eyes even more tightly lest they be contaminated by other disciplines" p. 1332.
However, for more recent evidence, clearly reflecting changes in outlook among some economists, see the summary of an international conference on socio-economics in Jackson (1989).

TABLE 4-2 BASIC CATEGORIES OF TIME USE

1.	necessary time	-	basic physiological needs, including sleep, meals, health and hygiene.
2.	contracted time	-	paid work or full time study, for example at university.
3.	committed time	-	involves certain behaviour which must be done because of an earlier decision. For example, getting married and having children involves housework, child care, maintenance of dwelling... A feature of committed time is that it is both productive and non-market.
4.	free time	-	this is a residual time use. It refers to whatever time remains after time has been allocated to the first three categories. It is usual to distinguish between free time and leisure.[9]

Starting from these four basic categories of time use, the next step is to list human activities and assign them to one of the four basic categories. The preparation of the Multinational Comparative Time Budget Research Project included a detailed activity code of 96 activities, with significant entries in the four basic categories subdivided. [Szalai (1972) pp. 561-564]. Thus, for the second category, contracted time, the entry "paid work" is subdivided into a further ten lesser categories including second job and work breaks. Using aggregation, Szalai has derived a less detailed 37 activity code from the 96 activity code. For certain lifestyles, specific activity codes have been developed from the four basic categories. For example the model designed for the time use survey of students attending Reading University, Tomlinson et. al. (1973), used a time budget for 12 basic activities.

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9 (a) Ås (1982), p. 95.
(b) de Grazia (1962) analyzes the nature of leisure and distinguishes between free time and leisure. Leisure is an activity which represents a productive use of time desired for its own sake.
(c) "Leisure involves sacrifice and effort. The sacrifice is in the opportunity cost of giving up free time". Gramm (1987), p. 173.

Table 4-1 represents a "pragmatic way" of classifying the activities of an agent into behaviour settings. This table can be compared with Table 4-2 by analogy. For example, in Table 4-2 such activities as study and income generating work involve contracted time. Hence contracted time can be associated with the state and workplace behaviour settings. Committed time, which is both productive and non-market is clearly associated with the household behaviour setting. Since the codes developed by Szalai are based on the OECD categories of time use set out in Table 4-2, by analogy Table 4-1 can be compared with these more detailed codes. In practice an observer would use a detailed time activity code, say the Szalai 96 activity code, and then using a standardized set of guidelines, assign each of the separate time uses to one of the five behaviour setting categories of Table 4-1. Conceptually, Table 4-1 represents a link between the behaviour setting approach of Barker and the detailed activity code of Szalai for measuring use of time. Table 4-1 combines both environments and time use.

4.5 Requirements of Social Position

Do social relations represent constraints on the behaviour of an agent and if so, how can they be measured? The sociologist Talcott Parsons (1967), (1968) contributed to our understanding of the way in which social control is exercised. When agents take part in social interaction they are influenced by a set of expectations which tend to shape or control their use of resources. Parsons considers that there are "generalized media of interchange" which are essential features of social interaction processes. For example money is a generalized medium of inducement, and power a generalized medium of coercion. These concepts can be related to the models of time allocation. An agent's lifestyle, entails the imposition of patterns of social relations all of which generate expectations. Peer groups, institutions and market forces, as well as customs, conventions and guidelines act as forms of social control, and effectively are constraints on the agent's use of time. These forms of social control will be termed "requirements of social position". They represent parameters which can be expressed as measures of time use. In this sense time use could perhaps be seen as analogous to Parson's concept of generalized media of social interaction.

By way of comparison with the transportation problem, time use activity codes represent the agent's requirements of social position, equivalent to demands at sinks for a scarce homogeneous resource. In the transportation models of time allocation there are five requirements of social position. These requirements are denoted by a five activity code which represents a set of aggregations of more detailed behaviour as provided in the 96 activity code of Szalai (1972). There is no theoretical reason why the number of social requirements has to be the same as the number of behaviour settings. While the five activity code is related to the lifestyle of university students it could be generalized for all groups. The only change needed would be to give category 3, termed "Academic" a more extensive designation such as "Education", where human capital formation is seen as a lifelong process.

TABLE 4-3 CLASSIFICATION OF SOCIAL REQUIREMENTS

Social Requirement	Description
1. Health (H)	Eating and drinking at home or canteen, personal hygiene, physical training, yoga, incidental daytime sleep.
2. Job (J)	Income generating activities.
3. Academic (A)	Full time attendance at lectures, seminars, tutorials ... Private study including preparation of assignments and reading.
4. Socio-cultural (SC)	Domestic work (non market), including preparation of meals at home, laundry, indoor cleaning, child care. Civic activities. Religious activities and charitable works. Entertainment, active leisure including hobbies, playing a musical instrument. Passive leisure, including watching television.

5. Commercial (C) — Transactions in the market place including purchasing of goods and services. Repair services, including vehicle maintenance.

Notes: 1. travel time is treated as a complement of other activities. Travel to and from the workplace, including waiting for means of transport, belongs to category 2, Job. Travel to a shopping centre, including waiting for means of transport, belongs to category 5, Commercial. Time use for travel is therefore treated as a derived demand.

2. For a full time student, category 2, Job, includes work during vacation and part time work outside regular attendance at university.

3. Category 4, Socio-cultural is a catch-all to include all activities not otherwise specified, such as reading books, reading newspapers, dancing and concerts. This category is used to close the requirements system.

4. The categories are related to New Zealand conditions where all the universities are state universities. While fees are subsidised, it is necessary for full time students to meet costs of living through an income stream. For most this entails part time and/or seasonal work.

The next step is to find a means of determining requirements levels quantitatively. A wide range of empirical studies, taken across different countries, has shown that a person's daily routine follows a structured, regular pattern of time use. In the context of time budgets, for relatively industrialized regions from 12 countries, researchers have noted an "array of similarities" in the average time spent in 37 primary activities and have provided a detailed analysis to explain what is, given the extent of cultural, political and economic diversity among the countries, an interesting result.[10] A study by Szalai suggested that time allocations could be used to infer the values placed on certain activities, and that certain groupings of activities were characteristic of particular social groups.[11] It seems not unreasonable to suppose that, by aggregating from the information obtained by measuring the activities of a representative sample of full time university students, it is both possible and practicable to obtain quantitative estimates of requirements of social position. These estimates are expressed in terms of time use. It has also been shown, as in Table 4.2, that it is possible to classify the requirements of social position by means of a five activity code. The research of Szalai suggests that there are parameters of social position which are characteristic of particular groups.[12] Specific measurements for parameters relating to full time students are provided in Chapter 8. By way of comment, the method of obtaining quantitative estimates of the requirements of social position could perhaps be regarded as analogous to a revealed preference approach.

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10 for some supporting evidence, Szalai (1966), Ch.6, pp.113-144.
11 ibid.
12 ibid.

Activities undertaken to meet the same social requirement can take place in different behaviour settings. Is this significant? Barker's research suggests that it is. As has been stated, it is an assumption of the time allocation models, supported by empirical results, that environments shape behaviour. Hence the choice of an environment, in which social requirements are to be met, entails an allocation of time use which may, or may not, be efficient, depending on the type of behaviour shaped by the particular environment. For example, child care could take place in either the household, state or private enterprise behaviour settings. An example of child care taking place in the state behaviour setting would be a creche at the university. However where payment is made to a day care centre in return for looking after a child, a market transaction is involved and therefore the appropriate behaviour setting is private enterprise. What is common to each of these cases is the use of time, which can therefore be seen as a link between social requirements and behaviour settings, the environments chosen to meet a demand.

Time budget studies measure the use which agents make of a time endowment in terms of duration and sequence. This approach can provide continuity between Chapter 2, where the rise of a modern industrial society was shown to have shaped our concept of time as a commodity, and Chapter 6 where a commodity is first defined, in the framework of activity analysis, and then given an economic interpretation in terms of time use.

4.6 The Cost of an Activity

Each activity of an agent is considered to have a per unit cost, measureable in money terms. For example, the activities that an individual takes part in to meet the requirements of social position will require inputs of a wide range of goods and services such as food, fuel and health services. In the models of time allocation the per unit cost of an activity is the sum of all per unit money costs incurred as necessary inputs in order to meet an output requirement, namely a parameter of social position. This is analogous to the money cost of transforming one unit of input into an output, given appropriate technology assumptions. Since all activities require time the cost of an activity is the cost of the time used up by the activity.[13] We have therefore specified a direct or observed cost, which is a per unit market cost, expressed in cents per minute. Formal definitions for direct and indirect costs are provided in Chapter 6. Time and money are therefore both used up by an agent in order to meet the requirements of social position. Not only do environments shape behaviour, expressed in time use, but they also generate different per unit market costs.

To determine the per unit cost of an activity, we divide all costs of an activity into two mutually exclusive components. They are:

1. the per unit cost of travel. This consists of all per unit transport costs, such as travel to work and travel to a shopping centre, associated with a particular activity. Such costs are classified according to environment and social requirement. Travel costs could include expenditure on a motor vehicle, including for example, fuel, depreciation and insurance, together with expenditure on public inner-city transport, airfares and travel insurance.

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13 - compare Moeseke (1985) p. 264.

2. the per unit cost of all remaining inputs.

Every activity can therefore be quantified in terms of space, time and cost. The derivation of the per unit cost is introduced in Chapter 7. A more detailed analysis is provided in Chapter 8.

A realistic model should allow for time or money constraints to be imposed. We assume that the agent's income will be derived from work only. For the time constraint, all the agent's activities must take place within the 16 hour working day. Time for sleep is fixed at 8 hours. We may also impose a money constraint so that the agent's expenditure must be met completely from income. The money constraint may be relaxed so that dissaving is allowed.

**4.7 Activity Analysis

Time budget studies measure the use which agents make of a time endowment in terms of duration and sequence. In activity analysis the time endowment is considered as a negative requirement, measured by the negative elements of the requirements vector. The real subject of time budget studies is the use which agents make of their time and this quantitative approach reflects a congruence with the way in which the rise of a modern industrial society shaped the way in which agents used their time, as outlined in Chapter 2. Time budget studies provide a link between the behaviour of an agent, as developed in this chapter, and the socio-economic matrix[14], expressed in the framework of activity analysis which is formulated in Chapter 5, and given an economic interpretation in Chapter 6.

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14 - compare Moeseke (1985).

In an activity analysis model all inputs such as consumption goods or services are designated by the generic term "commodity". In the production process all such inputs are transformed into outputs, which can also be regarded as commodities. It is in this sense that we can speak of the "production of commodities by commodities".[15] In the context of human activities the particular production function we are concerned with is defined by "the socio-technological matrix". Is there any one commodity common to such diverse functions as preparing a meal, going to a concert, wall papering a room? All these actions require the use of time. For each activity, considered as a quantitative pattern of behaviour, an agent uses up a specific amount of time, and this can be measured using a time budget.

A distinction was made between the different ways in which time could be used to meet the same social requirement. The models can differentiate between the ways in which this homogeneous resource is available as an input, the supply of an endowment, at a source, and as an output, the demand for time use, at a sink. This distinction can be maintained in activity analysis, where inputs of technically the same commodity at different locations are taken to represent different commodities. The same type of distinction is relevant in the case of outputs. Activity analysis provides an effective means of describing a production technology, and in particular the socio-technological matrix. We can therefore begin with time budgets as a means of recording and analyzing streams of behaviour, and finally make use of a socio-economic production function to describe the way an agent makes use of time so as to meet the requirements of social position.

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15 . ref Sraffa, P., Production of Commodities by Means of Commodities. (CUP, Cambridge, 1960).

CHAPTER 5

FORMULATION OF THE MODELS

5.1 Outline

Conceptually the linear models of time allocation are transportation problems. This chapter provides the first technical formulation of the transportation models of time allocation. As a means of linking the concepts of behaviour setting and time use, developed in Chapters 3 and 4 respectively, with the mathematical formulation set out in this chapter, a simple numerical example is introduced and solved. An explanation of the parameters and constraints is provided. The core model is then stated. Conceptually there are two transportation models. In the slack model there is a distribution at sinks but not at sources. In the tight model there is a time distribution not only at sources but also at sinks. The starred sections (**) provide a formulation of the transportation problem within the framework of activity analysis. This mode sheds light on the agent's production function and extends the range of applications of the time allocation models. The starred sections can be omitted in a first reading without loss of continuity.

5.2 Numerical Example

The basic transportation model of time allocation is termed the core model. To show that this model is operational a simple 2 x 2 numerical example is introduced and solved.

An agent is faced with the choice of meeting the demands of social position by carrying out activities in selected environments, while minimizing the total cost. Following Barker we assume that the agent's behaviour is shaped by his environments. As an individual decision maker the agent chooses the lifestyle. As a member of society, the agent has to meet certain parameters, the requirements of social position, which can be regarded as imposed. They can represent either unwritten or formally determined minimum requirements for a particular lifestyle. The requirements of social position are effectively criteria for "continuing acceptability" - to peers, institutions and agencies. All social requirements are observable. The optimal solutions specify both the type and level of the activities which the agent undertakes in order to meet the parameters of his social position. Hence the optimal solutions quantify the agent's consumption patterns.

What has been done in the simple numerical example is to use quantitative measures which are plausible. The data given in Tables 5-1 and 5-2 must be regarded as hypothetical.[1]

In this numerical example there are two sources (campus, household) and two sinks (study, relaxation).

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1 By contrast use is made of actual time budgets for students' daily activity patterns and estimates of costs of living (for a different group of students) in Chapters 7 through 9.

Program is:

$$\min \quad c_{11}x_{11} + c_{12}x_{12} + c_{21}x_{21} + c_{22}x_{22}$$

subject to the following constraints

$$\begin{array}{lcl} x_{11} + x_{12} & \leq & a_1 \\ x_{21} + x_{22} & \leq & a_2 \\ x_{11} + x_{21} & \geq & b_1 \\ x_{12} + x_{22} & \geq & b_2 \end{array}$$

Environments	Social Requirements
1. Campus (C)	1. Study (S)
2. Households (H)	2. Relaxation (R)

Time used up in activities

x_{11} denotes the number of minutes the campus environment is to provide for meeting the study social requirement.

x_{12} denotes the number of minutes the campus environment is to provide for meeting the relaxation social requirement.

x_{21} denotes the number of minutes the household environment is to provide for meeting the study social requirement.

x_{22} denotes the number of minutes the household environment is to provide for meeting the relaxation social requirement.

Costs of activities

c_{11} denotes the per unit cost (in cents/min) of studying on campus

c_{12} denotes the per unit cost (in cents/min) of relaxation on campus

c_{21} denotes the per unit cost (in cents/min) of studying in the household environment

c_{22} denotes the per unit cost (in cents/min) of relaxation in the household environment

Constraints

The amount of time spent on campus ($x_{11} + x_{12}$) cannot exceed a_1 mins.

The amount of time spent in the household environment ($x_{21} + x_{22}$) cannot exceed a_2 mins.

The amount of time spent on study ($x_{11} + x_{21}$) must be at least b_1 mins.

The amount of time spent on relaxation ($x_{12} + x_{22}$) must be at least b_2 mins.

Data

$a_1 = 500$	$a_2 = 700$	$b_1 = 580$	$b_2 = 380$
$c_{11} = 3.4$	$c_{12} = 3.6$	$c_{21} = 3.2$	$c_{22} = 3.1$

The solution is:

$x_{11}^* = 260$; $x_{12}^* = 0$; $x_{21}^* = 320$; $x_{22}^* = 380$

The market cost of this lifestyle is: (3.4 x 260) + (3.2 x 320) + (3.1 x 380) which is equal to 3086 cents or $30.86 per day.

Explanation:

Every activity uses up time and takes place in a certain environment. As well, every activity entails a money cost measured in cents per minute. Any activity can therefore be specified by a space-time locus together with a per unit cost. The term "relaxation" is a catchall to close the environments system. Each activity takes place in either the campus or the household environment. The term "household" is a catchall to close the social requirement system. Every activity is carried out to meet either the study or the relaxation social requirement. In this numerical example, the number of environments is equal to the number of social requirements so that the activity matrix is square in terms of environments and social requirements. In the transportation models of time allocation the activity matrix does not have to be square. Some 2 x 3 and 5 x 6 numerical examples are provided in Chapter 7.

All constraints are measured in minutes taken over the 16 hour waking day. Constraints at sinks represent minimal requirements. For example to gain a pass the student must spend at least 580 minutes on study. The total time requirement of 960 minutes is equivalent to the 16 hour waking day. To gain an A pass more than 580 minutes on study would be required.

Why are there constraints at sources? Lack of parking space, restrictive public transport schedules, shortage of study places in the library, security provisions on campus, represent just some of the practical constraints on the use of time on campus. We suppose that the maximum amount of time the student can spend on campus over the 16 hour waking day is 500 minutes. Likewise there are practical and logistical constraints on the use of the household behaviour setting. We suppose that the maximum amount of time the student can spend in the household is 700 minutes. Because the endowments at behaviour settings exceed 960 minutes, we note that these parameters do not define a distribution. The household and campus environments give rise to different costs for the same type of activity, such as meals, social activities. There is a further reason for identifying constraints at sources. As already noted, empirical studies by Roger Barker showed how environment influences behaviour. For some activities the campus environment, rather than that of the household, makes it possible to use time more effectively. The reverse may also be the case. For the numerical example, information and requirements can be summarized by means of an activity matrix and a cost matrix, provided in Tables 5-1 and 5-2. This format will be used for futher numerical examples.

TABLE 5-1 ACTIVITY MATRIX

from: \ to:	study (S)	relaxation (R)			
campus (C)	x_{11}	x_{12}	$\leqq$	500	a_1
household (H)	x_{21}	x_{22}	$\leqq$	700	a_2
	$\geqq$ 580	$\geqq$ 380			
	b_1	b_2			

Now it might seem from the constraints of the model that the student is compelled to spend 500 minutes studying on campus. This is not so. To make the point, we relax any institutional requirements, for example mandatory attendance at lectures, that would restrict the student's freedom to study away from campus. The inequality signs are a reminder of the constraints which the agent faces. The use of inequality signs in Table 5-1 is a heuristic device.

Economic interpretation

$a_1 \leq 500$ 500 minutes is the maximum amount of time an agent could spend on campus.

$a_2 \leq 700$ 700 minutes is the maximum amount of time an agent could spend in the household.

The a_i do not define a distribution, and $\Sigma a_i > \Sigma b_j = 960$.

TABLE 5-2 COST MATRIX

$$c = \begin{bmatrix} 3.4 & 3.6 \\ 3.2 & 3.1 \end{bmatrix}$$

Note: $c_{ij} > 0$, all i, j

Each activity involves an economic cost. With each of x_{11}, x_{12}, x_{21}, x_{22} there is an associated cost. Since every $c_{ij} > 0$, the agent will not use more than 960 minutes.

In Table 5-2, c_{11} denotes a per unit cost of 3.4 cents/min which is the market price of meeting the study social requirement in the campus environment. The per unit costs are obtained by first obtaining estimated dollar costs for the academic year and then classifying these by behaviour settings (environments). The next step is find the estimated time spent on student activities in minutes, using the same type of matrix as set out in Table 5-1. Table 5-2 is obtained by dividing each of the four dollar cost components for the academic year by the appropriate estimated time used up in any environment to meet a social requirement.

<u>Interpretation of solutions</u>

Optimally the student should spend 260 minutes studying on campus and 320 minutes studying at home. The relaxation requirement is met completely within the household environment. In this example there is unused capacity of 240 minutes at the campus behaviour setting. The market cost of this lifestyle is 3086 cents, or $30.86 per day. An optimal solution can provide further information. Measures of the value of using time in one environment, relative to another, and of the marginal value of using time to meet a social requirement can be found. Further it is possible to determine the range of values of behaviour setting and social requirements parameters, and also of per unit costs, for which the student's pattern of using time in environments, is unchanged. For example, if the academic social requirement lies between 320 and 820 minutes the basic variables remain the same. However the numerical example is not intended to duplicate the more detailed presentation, starting in Chapter 7, which explicates the more theoretical approach of Chapters 5 and 6.

5.3 Core Model

As mentioned in section 5.2, the basic transportation model of time allocation is termed the <u>core model</u>. The core model shows in symbols the relationship between groups, or classes, of agents on the one hand, and their lifestyles on the other. More formally, there is a mapping in choice space between the classes of agents, such as student, engineer, politician, and their lifestyles.

Each agent starts with a given time endowment (the "supply") of 960 minutes, equivalent to the 16 hour waking day, and carries out activities in a set of well-defined environments a_i i=1, ..., m (the behaviour settings) in order to meet a set of social requirements b_j j=1, ..., n (the "demands").[2] Both the a_i and the b_j are measured in minutes. The core model does not indicate whether or not the a_i and the b_j represent distributions.

Each activity uses up time and incurs a money cost. x_{ij} measures the amount of time (in mins) used up in the ith behaviour setting in order to meet the jth social requirement. The x_{ij} are the basic building blocks of the model. The per unit cost c_{ij} (in cents/min) measures the cost of a unit x_{ij}.[3] The total cost of the agent's activities is given by $\sum_i \sum_j c_{ij} x_{ij}$. For each unit of time use there are per unit values, v_j, u_i, associated with social requirements j and behaviour settings i.

The agent as rational decision maker is faced with the choice of meeting the demands of social requirements, making use of a given endowment, while minimizing the costs of activities.

An equivalent program (the dual) exists. The agent as a rational decision maker is faced with the choice of maximizing the net (imputed) value of time use, so long as the net value of a per unit of time use is less than, or equal to, the per unit cost.

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2 Behaviour settings and social requirements are defined in Chapter 4.

3 A simple derivation of per unit costs, c_{ij}, is given in section 7.2. A step-by-step derivation is provided in section 7.4. In Chapter 8 a step-by-step approach is used to derive a 5 x 5 cost matrix.

Formally, the core model is:

primal

$$\min \sum_i \sum_j c_{ij}\, x_{ij}$$

subject to

$$\sum_{i=1}^{m} x_{ij} \geqq b_j \quad \Big| \quad v_j \quad j=1, \ldots, n$$

$$-\sum_{j=1}^{n} x_{ij} \geqq -a_i \quad \Big| \quad u_i \quad i=1, \ldots, m \qquad (5.1)$$

$$x_{ij} \geqq 0 \;, \quad \text{all } i, j$$

$$c_{ij} \geqq 0 \;, \quad \text{all } i, j$$

dual

$$\max \sum_j v_j\, b_j - \sum u_i\, a_i \qquad (5.2)$$

subject to

$$v_j - u_i \leqq c_{ij} \quad \Big| \quad x_{ij} \quad i=1, \ldots, m \; ; \; j=1, \ldots, n$$

$$v_j,\ u_i \geqq 0 \quad \text{for all } i, j$$

5.4 Slack and Tight Models

Conceptually there are two transportation models. However neither represents an ordinary transportation problem because the "js" denoting sinks are not locations. The first model is termed the "slack model". There is a distribution at sinks but not at sources. The second model provides a conceptual approach to having a complete time distribution, that is, a distribution at sources in addition to the distribution at sinks. This is termed the "tight model". Costs are such that $c_{ij} \lesseqgtr 0$. For expenditure by the agent, $c_{ij} > 0$. However the agent may obtain income and then $c_{ij} < 0$. Income is treated as a negative expenditure.

The slack model

$$\Sigma b_j = 1 \qquad \Sigma a_i > 1$$

An aggregate endowment constraint is introduced to the slack model. The 960, measured in minutes, is equivalent to 16 hours of 60 minutes, and denoted by unity in (5.4). As indicated in Chapter 4, the total time endowment is given by the 16 hour waking day. This constraint is necessarily redundant if the b_j define a distribution ($\Sigma b_j=1$) and all $c_{ij} > 0$, because, if not, the objective function will not be a minimum, a contradiction of the optimal program. However if some $c_{ij} < 0$, the aggregate endowment constraint is no longer necessarily redundant. Suppose the aggregate constraint is relaxed. On an intuitive basis it can be seen that the objective function will be minimized by associating the $c_{ij} < 0$ with the greatest possible x_{ij}. As shown in example 4, section 7.2 for any such x_{ij}, whenever an a_i is greater than a b_j, the time endowment will exceed 960 minutes, a contradiction, given the assumption of a 16 hour waking day. The aggregate endowment constraint is also required for realism. The associated shadow price is denoted by w.

<u>primal</u>

$$\min \sum_i \sum_j c_{ij}x_{ij}$$

$$\text{subject to} \quad \sum_{i=1}^{m} x_{ij} \geqq b_j \quad \Big| \quad v_j \qquad j = 1,\ldots, n$$

$$- \sum_{j=1}^{n} x_{ij} \geqq -a_i \quad \Big| \quad u_i \qquad i = 1,\ldots, m \tag{5.3}$$

$$- \Sigma x_{ij} \geqq -1 \quad \Big| \quad w \tag{5.4}$$

$$x_{ij} \geqq 0, \quad \text{all } i, j$$

$$c_{ij} \lesseqgtr 0, \quad \text{all } i, j$$

In the primal the dual variables u_i, v_j, w appear to the right of the related constraints. The u_i, v_j, w denote shadow prices, representing an implicit price system which comes out of the model.[4]

<u>dual</u>

For the slack model the dual is given by (5.5). The imputed value at a source includes the shadow price of the endowments.

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4 One may note that the multipliers in the simplex tableau of the ith row equation and the jth column equation are equivalent to respectively, u_i, v_j.

dual - slack model

$$\max \quad \sum_j v_j b_j \; - \; \sum_i u_i a_i \; - \; w$$

$$\text{subject to} \quad v_j - u_i - w \leq c_{ij} \quad \Big| \quad x_{ij} \quad i=1,\dots,m;\; j=1,\dots,n \qquad (5.5)$$

$$v_j,\; u_i,\; w \geq 0 \quad \text{all } i,\, j$$

For the coefficient of w, the $\sum_{ij} x_{ij}$ (= 960 minutes) is denoted by 1 as indicated.[5]

The tight model

$$\sum b_j = 1 \qquad \sum a_i = 1$$

primal

For the tight model the primal is given by (5.3) only.

dual

For the tight model, the dual variable w is not required, since there is a distribution at sources, as explained earlier.[6] This means that the aggregate endowment constraint, with which the dual variable w is associated, is redundant. The dual of the tight model is given by (5.6). In the duals it can be seen that the objective function maximises the net revenue earned subject to the constraint that the imputed value at a sink less the imputed rent at a source may not exceed the transport cost.

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5 - section 5.4, pps. 66, 67.
6 - section 5.4, p.66.

$$\max \quad \sum_j v_j b_j - \sum_i u_i a_i$$

$$\text{subject to} \quad v_j - u_i \leqq c_{ij} \quad \Big| \quad x_{ij} \qquad i=1,\ldots,m,\ j=1,\ldots,n \tag{5.6}$$

$$v_j,\ u_i \geqq 0 \quad \text{all } i,\ j$$

A brief outline of the focus of each of the two models is given in Chapter 6.

5.5 The Transportation Problem

Introduction

The aim is to provide a framework of reference for the core, slack and tight models. Only if these models have the same mathematical structure as (5.7) and (5.8) can they be justified as transportation models. As well the formulations provide a link with the economic interpretations provided in Chapter 6.

Two equivalent formulations of the transportation problem are now given. There is notational consistency. The models are formulated so as to be precisely the same in structure as the activity analysis model.[7] In particular, this means that the primal and dual orders for the transportation and the standard linear programming models correspond with the way in which the activity analysis model is formulated, so that components of the vector of shadow prices correspond with, first, outputs, which are taken as positive, and then inputs, which are taken as negative. In the formulations there is no attempt to interpret the symbols in economic terms. The concern is the

7 see the starred section 5.7.

mathematical symbols. Each symbol stands by itself. The interpretation of the symbols is quite a separate issue and follows on logically from the mathematical formulation of the problems.

Mathematical notation and formulation of the transport problem

The classical transportation problem requires an optimal schedule of shipments of a scarce homogenous resource to be determined subject to constraints of demand, supply and cost. Suppose a scarce homogenous resource, located at m origins has to be allocated to n destinations. There is a per unit cost of shipping a unit quantity of the resource from origin to destination. The problem is to determine, at minimum cost, the number of units to be shipped from origin to destination, so that all demands are met exactly and therefore remaining supplies at origins are zero. So in this particular model supply is equal to demand, but in general there are inequalities.

This problem is a particular type of linear programming problem where a scarce homogenous resource is to be shipped from each of m origins, where fixed supplies are available in the quantities a_1, a_2, ..., a_m, in order to meet the separate requirements b_1, b_2, ..., b_n at the n shipping destinations. As in any transportation problem there are sources and sinks. A shipping origin will be termed a source, and a shipping destination will be termed a sink. It is well known in linear programming problems that a problem may be stated in the original (or primal) form and in a closely related (or dual) alternative form.

Assumptions

1. The total requirements of all sinks is not greater than the total quantity available at the sources.
2. All costs are linear.

Mathematical Notation

x_{ij} = the amount of the scarce homogenous resource to be shipped from source i to sink j to meet the jth requirement. x_{ij} denotes a flow.

a_i = the total amount of the scarce homogenous resource available for shipment from the ith source i=1,...,m. a_i denotes an endowment.

b_j = the total amount of the scarce homogenous resource required at sink j, j=1,..., n. b_j denotes a requirement.

m = number of sources.

n = number of sinks.

c_{ij} = per unit cost of shipping a unit quantity from source i. c_{ij} denotes transport costs.

u_i . = the shadow price of an endowment at source i. u_i denotes a rent.

v_j = the shadow price of a requirement at sink j. v_j denotes a rent, including value added by delivery.

The following information is considered to be given:

a_i, b_j, c_{ij}, m, n.

The problem is to determine a shipping schedule such that:

1. the supply a_i at source i will not be exceeded. This means that a source cannot be required to ship more than its capacity. Total quantity shipped from source i is

$$\sum_{j=1}^{n} x_{ij} \leqq a_i \qquad i=1, \ldots, m$$

2. the demand b_j at sink j will be satisfied. This means that the jth requirement is fully met. The total quantity shipped to sink j is

$$\sum_{i=1}^{m} x_{ij} \geqq b_j \qquad j=1, \ldots, n$$

3. the total shipping cost will be a minimum

$$\text{minimize} \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

A shipping schedule is denoted by a set of non negativity variables $x_{ij} \geqq 0$ which satisfy the m+n constraints.

primal

$$\min \sum_i \sum_j c_{ij} x_{ij}$$

$$\begin{array}{llll} \text{subject to} & \sum_{i=1}^{m} x_{ij} \geq b_j & v_j & j = 1, \ldots, n \\ & -\sum_{j=1}^{n} x_{ij} \geq -a_i & u_i & i = 1, \ldots, m \end{array} \tag{5.7}$$

$$x_{ij} \geq 0 \text{ , all } i, j$$

$$c_{ij} \lesseqgtr 0 \text{ , all } i, j$$

dual

$$\max \sum_j v_j b_j - \sum_i u_i a_i$$

$$\text{subject to} \quad v_j - u_i \leq c_{ij} \quad \Big| \quad x_{ij} \quad i=1,\ldots,m,\ j=1,\ldots,n \tag{5.8}$$

$$v_j,\ u_i \geq 0 \quad \text{all } i, j$$

It is noteworthy that the transportation problem can be interpreted as a two-person zero-sum game, where the starting point is von Neumann's treatment of the m x n assignment problem as a 2m x m^2 game, as noted in Karlin (1959).[8] It can also be shown that the transportation problem can be considered as a special case of the assignment problem. Hence the two problems are equivalent. For the transportation models of time allocation this means that the tight model can be interpreted as a two-person, zero-sum game.

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8 An assignment problem is concerned with finding the best way to assign m workers to m jobs. The suitability of a worker for a job varies. The problem is to optimally assign workers to jobs. The assignment problem can be considered as a special case of the transportation problem in which $a_i = b_j = 1$ and m=n. p. 132.

There are many variants of the transportation problem.[9] The time allocation models belong to the static deterministic model first formulated by Kantorovich (1939), Hitchcock (1941) and Koopmans (1947).[10] A variant of the transportation problem, the generalized transportation problem, indicates the possibility of including changes in technology in the time allocation models.

The minimum-time transportation problem represents a further variant. The name may suggest some affinity with the time allocation models. In fact the transportation models of time allocation are completely different, both conceptually and mathematically, from the minimum-time problem. The time allocation models treat time itself as the scarce resource to be shipped from sources to sinks, while the minimum-time model deals with shipping a resource, given time delays. In the minimum time model, time is distinct from the resource to be shipped, for example perishable goods, military equipment to a war zone. The models are therefore conceptually different, because the minimum-time model is not a model of time use but rather a model of moving a resource over time. Mathematically the models are distinct. The time allocation models are static while the minimum-time model is dynamic.[11] Optimal control theory is considered to be an "appropriate framework" for the dynamic transportation problem over time.

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9 Appa (1973) provides a summary. For some interesting comments on the analogy with a network problem, Samuelson (1952).

10 For some comments on Kantorovich and the Transportation Problem by Koopmans: Koopmans (1960), Koopmans (1962).

11 Bookbinder and Sethi (1980).

Generalized Transportation Problem

By relaxing the assumption of a unimodal matrix, the transportation problem can be extended to allow for changes in technology. By specifying two further symbols, a weighted distribution problem is formulated. This can be compared with the more generalized problem in Dantzig (1963).[12] A common application of the model in Dantzig is referred to in the literature as the "machine loading problem".[13]

The condition that the values of each element a_{ij} of the constraint matrix A equal 1, 0, -1 is relaxed.

The mathematical formulation of the generalized transportation problem is as follows:

Find non negative x_{ij} which minimize $\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij}$ such that:

row equation
$i=1, 2, \ldots, m$ $\quad \sum_{j=1}^{n} e_i x_{ij} \leqq a_i$

column equation
$j=1, 2, \ldots, n$ $\quad \sum_{i=1} r_j x_{ij} \geqq b_j$

where $e_i \geqq 0$, $r_j \geqq 0$

The mathematical notation is the same as that for the transportation problem except that two further symbols are specified.

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12 Ch. 21, pp. 413-420.
13 Eisemann (1964), also Lourie (1964).

primal

$$\min \quad \sum_i \sum_j c_{ij} x_{ij}$$

$$\text{s.t.} \quad \begin{array}{lll} \sum_i r_j x_{ij} \geq b_j & v & j=1, \ldots, n \\ -\sum_j e_i x_{ij} \geq -a_i & u & i=1, \ldots, m \end{array}$$

$$x_{ij} \geq 0, \text{ all } i, j \tag{5.9}$$

dual

$$\max \quad \sum_j v_j b_j - \sum_i u_i a_i$$

$$\text{s.t.} \quad r_j v_j - e_i u_i \leq c_{ij} \tag{5.10}$$

$$v_j, u_i \geq 0, \text{ all } i, j$$

By putting $r_j = 1$, so that the weight of the columns is unity, and then taking the special case where $e_i=1$ in (5.9), we obtain the transportation problem. The generalized transportation model of this section is less general than the weighted distribution problem formulated in Dantzig (1963). As noted, the machine loading problem represents an application of the weighted distribution problem. It has been shown that for generalized transportation problems, the effect of the shift from a unimodal model results in a radically altered topology.[14]

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14 Lourie op. cit.

5.6 Standard linear programming model - vector matrix notation

primal $\min \mathbf{cx}$

subject to

$$\begin{array}{rcl|c} \mathbf{Ax} & \leqq & \mathbf{b} & \mathbf{v} \\ \mathbf{x} & \geqq & \mathbf{0} & \end{array} \qquad (5.11)$$

dual $\max \mathbf{vb}$

subject to

$$\begin{array}{rcl|c} \mathbf{vA} & \leqq & \mathbf{c} & \mathbf{x} \\ \mathbf{v} & \geqq & \mathbf{0} & \end{array} \qquad (5.12)$$

We will adopt this canonical notation, to be referred to as "the standard linear program"

The c's of the standard canonical form correspond to the c_{ij} of the algebraic model

The standard linear program with 3 sources and 4 four sinks can be tabulated as a (3+4) x (3x4), that is, a 7 x 12 matrix.

TABLE 5.3 ROW AND COLUMN ARRAYS

$$\mathbf{A} = \begin{bmatrix} 1 & & & & 1 & & & & 1 & & & \\ & 1 & & & & 1 & & & & 1 & & \\ & & 1 & & & & 1 & & & & 1 & \\ & & & 1 & & & & 1 & & & & 1 \\ -1 & -1 & -1 & -1 & & & & & & & & \\ & & & & -1 & -1 & -1 & -1 & & & & \\ & & & & & & & & -1 & -1 & -1 & -1 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} x_{11} \\ x_{12} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ x_{34} \end{bmatrix} \quad \mathrm{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ -a_1 \\ -a_2 \\ -a_3 \end{bmatrix} \quad \mathrm{v} = [v_1 \; v_2 \; v_3 \; v_4 \; u_1 \; u_2 \; u_3]$$

$$\mathbf{c} = [\, c_{11} \;\; c_{12} \;\; c_{13} \;\; c_{14} \;\; c_{21} \;\; c_{22} \;\; c_{23} \;\; c_{24} \;\; c_{31} \;\; c_{32} \;\; c_{33} \;\; c_{34} \,]$$

Table 5-4 below can serve as a link with Table 5-5 which sets out the technology matrix **A** of the activity analysis model. suggesting intuitively that activity analysis preserves the structure of the transportation model.

TABLE 5.4 STANDARD LINEAR PROGRAMMING MODEL

primal variables / dual variables	x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	x_{22}	x_{23}	x_{24}	x_{31}	x_{32}	x_{33}	x_{34}	**parameter** $\geq$ 0
v_1	1				1				1				$\geq b_1$
v_2		1				1				1			$\geq b_2$
v_3			1				1				1		$\geq b_3$
v_4				1				1				1	$\geq b_4$
u_1	-1	-1	-1	-1									$\geq -a_1$
u_2					-1	-1	-1	-1					$\geq -a_2$
u_4 u_3									-1	-1	-1	-1	$\geq -a_3$
c_{ij}	c_{11}	c_{12}	c_{13}	c_{14}	c_{21}	c_{22}	c_{23}	c_{24}	c_{31}	c_{32}	c_{33}	c_{34}	

5.7 Activity Analysis

To motivate this section two concrete cases of technical change are given. The individual agent is not only a consumer but also a producer. Changes in technology, interpreted as changes of the agent's production function, result in changes in the value of time use, so that the opportunity cost of time use changes. Hence changes in technology impact on the full cost of goods and services. An agent attends a course on how to study. The result is a 50 percent increase of efficiency in time use for meeting the study social requirement, irrespective of the environment selected. Suppose instead that the campus environment was enhanced, resulting in a 25 percent increase in efficiency in meeting both the study and relaxation social requirements. In both cases the agent appears to "save" time. How can the concept of the socio-economic production function[15] be used to specify changes in the agent's lifestyle? 3 x 3 and 5 x 5 numerical examples involving changes in the agent's efficiency in using time are provided in Chapters, 7, 8 and 9. This represents a new approach to changes in technology.

How can we hold on to the concept of transferring a resource from an origin to a destination while at the same time providing a specific analysis of the production function? We do so by formulating the transportation problem within the framework of activity analysis. While the mathematical structure remains the same the introduction of the two basic concepts of commodity and activity makes it possible to define a technology matrix in terms of an optimal combination of finite activities, or processes. This extends the range of applications of the models of time allocation. Moreover, it

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15 As introduced in Chapter 3 ref. Moeseke (1985).

represents a more rigorous approach to the specification of a production function. Reasons for formulating a transportation problem within the framework of activity analysis are outlined.

We are concerned with an optimization problem, given constraints. Since there are exogenous constraints the model is open. The mathematical structure remains the same. On the one hand there are clear links with the "from sources to sinks" approach of the transportation problem. On the other hand, an activity analysis formulation introduces the two basic concepts of commodity and of activity so that it becomes possible not only to specify a technology matrix $\boldsymbol{A}$, in terms of an optimal combination of finite activities, or processes, but also to provide a more holistic setting for an economic interpretation. The technology matrix describes a production process in which commodities, as inputs, are transformed into other commodities, as outputs. In activity analysis the problem is to select an optimal combination, or "mix" of activities from a finite number. As in section 5.5 the concern is the mathematical symbols.

The classic statement of activity analysis was presented by Koopmans (1951).[16] The theoretical foundations were extended in Koopmans (1957). Koopmans considers that the first explicit statement of the activity analysis model can be found in a paper by von Neumann (1945).

Definitions: Activity analysis, commodity, activity.

Activity analysis is a method for determining the efficient allocation of commodities in productive systems by linear models. It postulates a set of production activities, or processes, available for a given agent or system.

A commodity is a homogenous, perfectly divisible scarce resource.

There are three types of commodities:

1. primary factors. These are resource endowments determined outside the system.
2. intermediate commodities, produced and used within the production process.
3. final commodities. These are produced as outputs to meet requirements.

In the time allocation model, the endowment of time is considered as a primary factor which is used in all productive activities. Final commodities represent the requirements which as agent has to meet, and involve the use of an endowment of time. As in Koopman's analysis of production, if the set of primary factors and final commodities do not together cover all commodities, the remaining commodities will be regarded as intermediate commodities.

Note: In activity analysis the term "commodity" includes services, raw materials, goods and time.

An activity, or process, is a particular combination of commodities, as inputs and outputs, representing an element of a production set.

An activity transforms a set of commodities, as inputs, into another set of commodities as outputs. Inputs are considered to be in fixed ratios, and outputs in fixed ratios to inputs. So an activity, or process, is a fixed quantitative relationship between inputs and outputs, variously described as a "recipe", giving the proportions of the different inputs required for a specific mode of production, Gale (1960)[17], a "blueprint", Takayama (1985)[18], or as a "black box" into which go certain items as inputs and out of which go other items as outputs, Dantzig (1963).[19] Activity analysis is not concerned with how the transformation from input into output takes place.

For the transportation problem, considered as an application of activity analysis, the transfer of a scarce homogenous resource from a source to a sink will be regarded as an activity, where technically the same commodity (the scarce homogenous resource) at two different locations is considered to represent two different commodities. The transfer of technically the same commodity from m different sources, as inputs, represents m different commodities. Likewise the transfer of technically the same commodity to n different sinks, as outputs, irrespective of the particular source i, represents n different commodities. By distinguishing between inputs of technically the same commodity and, as well, between outputs, the models incorporates m+n different commodities. Since an activity consists of transferring a commodity from source i to sink j, and since there are m sources and n sinks, there are mn activities, or processes. We distinguish between inputs and outputs by assigning a positive value to the coefficient of an output, and by assigning a negative value to commodities which are inputs.

By way of summary, for the terms *activity*, *input*, *output* one may substitute, respectively *transfer*, *supply*, *delivery*. An activity is a "blueprint" which describes a particular technique of production. Some commodities are inputs.

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16 see Koopmans (1951), pp. 33-97.
17 p. 5.
18 p.50.
19 p. 32.

These are used in fixed proportions to produce other commodities as outputs. The analog of an input is a supply at an origin, while the analog of an output at a sink is the delivery of a requirement. So an activity can be considered as a transfer of a commodity from supply to delivery. Some further definitions are given:

An activity consists of a technological relationship between inputs and outputs together with a level of operation. A_k, k=1, ..., N where N = mn denotes the kth activity (or process).

The level of an activity A_k measures the intensity of the activity and is denoted by $x_k \geqq 0$, k=1, ..., N. x_k is a scalar variable which measures a quantity.

Note: In moving from the standard linear program to the activity analysis model there is a transition in notation. Every original x_{ij} of the original model becomes an x_k in the activity analysis model. $x_{ij} \rightarrow x_k$ We use lexicographical ordering to specify particular cases. Thus $x_{11} \rightarrow x_1$; $x_{12} \rightarrow x_2$ where x_1 and x_2 denote the levels of activity of the basic activities A_1, A_2.

A transfer from an available input i, i=1, ..., m to a required output, j, j=1, ..., n now becomes equivalent to an activity level x_k, k=1, ..., N where N = mn.

In its unit level of operation the kth activity A_k is denoted by the unit vector a^k, k=1, ..., N where N = mn.

a^k is the 1 x M column vector of the kth per unit activity

$$a^k = \begin{bmatrix} a_{1k} \\ a_{2k} \\ \cdot \\ \cdot \\ \cdot \\ a_{\ell k} \\ \cdot \\ \cdot \\ \cdot \\ a_{Mk} \end{bmatrix}$$

The component of the unit vector is denoted by $a_{\ell k}$, where $a_{\ell k}$ is the amount of the ℓth commodity transformed from a unit input into an output by the operation of the kth activity.

Note: The focus is on the input, because the agent has a given endowment (In the tight model the endowment is 960 minutes). By way of contrast, in the Leontief input-output model the focus is on the output (product) and the a_{ij} is defined using a unit output.

The ℓth commodity is termed an output of the kth activity if $a_{\ell k} > 0$ and an input if $a_{\ell k} < 0$. $a_{\ell k} = 0$ means that the ℓth commodity is not used in the kth activity.

$A_k = x_k a^k$, the product of the scalar variable x_k and the unit vector a^k, associated with the kth per unit activity

Example We have defined activities as column vectors. Each row of a column vector corresponds to a commodity. Input coefficients are negative, output coefficients are positive. Inputs are, by definition, unit entries. For symmetry, net outputs precede inputs as ordered components of the column vectors. In Table 5-5, the technology matrix ***A*** defines the technology of the system. The matrix ***A*** is specified by the N = mn columns. ***A*** has the same structural form as the standard linear program with 3 sources and 4 sinks, set out in Tables 5-2 and 5-3.

Example It is possible to incorporate changes in technology into the model. Suppose that for activities A_5 through A_8, for the same input there was a 25 percent increase in the output. Then for the column vectors denoted by the unit vectors a^5 through a^8, each unit input is -1 and each corresponding output is 1.25.

If the level of activity A_k is given by x_k, then the production set Y is defined, using algebraic notation, by:

$$Y = \{y : y = \Sigma \mathbf{a}^k x_k , \mathbf{x} \geqq \mathbf{0}\}, \qquad \text{where } \boldsymbol{A} = [a_{\ell k}],$$

and, using vector-matrix notation, by:

$$Y = \{y : y = \boldsymbol{A}\mathbf{x}, \mathbf{x} \geqq 0\}$$

The vectors $\boldsymbol{A}$, x, y are shown in Table 5-6.

The matrix $\boldsymbol{A}$ can be considered as the rule for mapping from the vector x in N dimensional commodity space to the vector $\boldsymbol{A}\mathbf{x}$ in M dimensional commodity space.

TABLE 5-5 TECHNOLOGY MATRIX, $\boldsymbol{A}$

Activity A_k, k=1, ..., 12											
A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}
Activity level x_k, k=1, ..., 12											
x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
1				1				1			
	1				1				1		
		1				1				1	
			1				1				1
-1	-1	-1	-1								
				-1	-1	-1	-1				
								-1	-1	-1	-1

TABLE 5-6 THE ACTIVITY ANALYSIS MODEL

$$y = \boldsymbol{A}x$$

$\boldsymbol{A}$	x	y
7 x 12	12 x 1	7 x 1
technology matrix	intensity vector	vector of net outputs

$$\begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & \cdots & A_{12} \end{array} \\ \begin{bmatrix} 1 & 0 & & 0 \\ 0 & 1 & & 0 \\ 0 & 0 & & 0 \\ 0 & 0 & \cdots & 1 \\ -1 & -1 & & 0 \\ 0 & 0 & & 0 \\ 0 & 0 & \cdots & -1 \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{12} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \vdots \\ y_7 \end{bmatrix} \end{bmatrix} \begin{array}{l} \text{required commodities (+)} \\ \\ \text{commodities used up (-)} \end{array}$$

$p \geqq 0$ is a price vector which reflects scarcity.

p can be interpreted as a vector of implicit or shadow prices. With reference to Table 5-6 **p** would denote the M = m+n row vector, in the vector product **pAx**, congruent with the vector of shadow prices **v** = [v_1, v_2, ..., u_4] associated with equations (5.11) and (5.12).

**5.8 Changes in Technology

The aim of the generalized transportation problem is to allow for changes in technology. In this way we can allow for increases in the agent's capacity to meet commodity requirements. That is, for the same inputs, increased commodities are produced. We now describe changes in technology in the activity analysis mode. (5.13) and (5.14) can be compared with the earlier formulation given by (5.9). Changes to technology will be developed in two ways. We can introduce efficiences in transforming all inputs in order to meet a particular commodity requirement, denoted by a specified positive element of **y**, the vector of net outputs. In activity analysis this is equivalent to an increase in efficiency row wise, that is there are changes in a row of the output matrix, obtained by pre-multiplying the technology matrix *A* by a suitable matrix. The other way is to change the efficiency of the production process for specified selection of inputs, in order to meet at least one, and possibly all, commodity requirements denoted by the positive elements of **y**. In activity analysis this corresponds to an increase in efficency column wise that is there is post-multiplication of the original matrix.

Suppose the present technology matrix is denoted by A, where

$$\boldsymbol{A} \equiv \begin{bmatrix} A \\ \text{---------} \\ -B \end{bmatrix} \begin{matrix} \text{output matrix} \\ \\ \text{input matrix} \end{matrix} \quad \text{and,}$$

$$a_{ij},\ -b_{ij} \quad \epsilon\ (-1,\ 0,\ 1)$$

We now relax the assumption of unimodular $\boldsymbol{A}$. Let there be an increase of efficency row wise. The pre multiplication matrix is denoted by $\boldsymbol{R}$.

Then the new technology matrix is denoted by $\boldsymbol{A}^*$, where

$$\boldsymbol{A}^* = \boldsymbol{RA} \tag{5.13}$$

For changes in a row there is pre multiplication of the original matrix.

We might also consider an increase in efficiency column wise.

As before we relax the assumption of unimodular $\boldsymbol{A}$. Let there be an increase in efficiency column wise. As earlier, we use a mnemonic approach. $\boldsymbol{R}$ denotes a diagonal matrix associated with a change in efficiency <u>row wise</u>. $\boldsymbol{A}^*$ denotes a changed technology, with the single star denoting <u>pre</u> multiplication of the original matrix. The post multiplication matrix $\boldsymbol{E}$ refers to an increase of efficiency in the use of an endowment.

Then the changed technology is denoted by $\boldsymbol{A}^{**}$, where

$$\boldsymbol{A}^{**} = \boldsymbol{AE} \tag{5.14}$$

For increases in efficency column-wise there is post-multiplication of the original matrix.

Reasons for formulating a transportation model within the framework of activity analysis are now given:

1. It provides an alternative concept of household production to that of the Becker model. An activity analysis approach provides a basis for objective consumption theory. Production consists of a number of activities, or processes. For each activity certain inputs are transformed into certain outputs. The socio-economic production function is specified in the activity analysis mode in Chapter 7.

2. It is based on an elegant and rigorous mathematical analysis[20] which offers a theoretical foundation - convex sets in topological space - on which to build models of time allocation. As well the mathematical structure has links with graph theory.[21]

3. Through activity analysis the time allocation models can be extended to include welfare concepts. The concept of Pareto Optimality can be shown to be equivalent to a statement in activity analysis, namely the efficient combination of activities. Pricing theorems which make use of a vector of prices of commodities have been developed in activity analysis. These theorems suggest economic interpretations of non market price vectors.

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20 Takayama (1985), p. 51.
21 . Samuelson, op. cit. p. 285.
also Karlin, op. cit. Ch. 5, pp. 138, 139.

4. The activity analysis model shares structural features with other important linear models such as the Leontief input-output model.[22] There are links with other models. Koopmans has commented that the activity analysis model is "closely related to the von Neumann models".[23] The models discussed in this section can be derived from a generalized von Neumann model.[24] Linear models related to activity analysis have been used to represent quite different, even contrasting, approaches to economics. The fact that the models share a common mathematical structure suggests that given suitable interpretations, time allocation analysis can describe economic activity for different economic systems.

5. The activity analysis model provides a link between the behaviour of an agent, which is observable, and the non-observable value of the time used up. More technically there is a mapping between the agent's observable activities in N dimensional space and the shadow prices of time use associated with the M dimensional vector of requirements.

6. Changes in technology can be specified appropriately using the activity analysis model.[25]

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22 Koopmans (1949) comments: "This model differs from similar models discussed by Leontief in that the number of possible activities exceeds the number of desired end-products, thus permitting choice and substitution between production methods", p. 74.

23 Koopmans (1951). See footnote p. 458. also Koopmans and Bausch (1959). Topic 5, p. 111.

24 Karlin, op. cit. Chapter 5; also Brody, (1970), pp. 50-61.

25 Numerical examples are provided in Chapter 9.

CHAPTER 6

ECONOMIC INTERPRETATION

6.1 Introduction

The numerical examples introduced in an intuitive and non-formal way are now interpreted formally. Building on the mathematical notation and structure formulated in Chapter 5, this chapter offers an economic interpretation for the symbols already provided. The economic interpretation breaks new ground. Particular attention is given to the interpretation of the duals as shadow prices, and to expressions for social income.

In the starred section, economic interpretations are given for the two basic concepts of commodity and activity. Each activity uses up time and incurs a cost. Interpretations are provided for changes in the technology of a socio-economic production function.

6.2 Outline

The initial section deals with the economic interpretation of variables and parameters. The concept of social income, and the theory of saving, Moeseke (1985), are shown to be applicable to the time allocation models. Initially the models assume a fixed technology. This assumption is then relaxed so that, conceptually, the time allocation models become generalized transportation problems. Weights are attached to the endowments and to the requirements. The economic interpretation has particular significance.

Changes in technology are developed in two ways. There is an increase in efficiency column-wise, and there is an increase in efficiency row-wise.

Formulation

An agent must carry out a range of different activities to meet the requirements of his social position. These are the agent's social parameters. Activities can be undertaken within several types of given environments. Certain environments enable the agent to carry out an activity more efficiently and can therefore be considered to generate economic rents. Each activity can be quantified in terms of time and money cost. The agent's time is to be allocated in such away that all required activities are to be met using the available time endowment, and at minimum cost. Of course the problem of trade-offs is unavoidable. For the primal the agent as rational decision maker is faced with the choice between on the one hand, meeting the demands of social requirements and, on the other, minimizing the costs of the activities they generate. The problem is to specify the optimal combination of activities given resources and budget constraints.

Parameters and Variables

As in any transportation problem there are sources and sinks. Behaviour settings are at sources and requirements of social position at sinks. In chapter 7 simple 2 x 2, 2 x 3 and 3 x 3 models are formulated and solved. For the more developed models introduced in Chapter 8 there are five behaviour settings. They do not overlap. Different environments correspond to the supply of a scarce homogeneous resource at different sources.

The behaviour settings are:

state, household, private enterprise, voluntary associations, workplace. Workplace represents a behaviour setting distinct from all other behaviour settings, because the environment is defined as one in which income is earned. This environment therefore shapes behaviour in a way that other environments do not. Conceptually, behaviour settings are characterized by an endowment specified in terms of the separate environments. A behaviour setting (environment) is not simply equivalent to a physical location. For example the introduction of a JIT system results in a different environment, even though the location is itself unchanged.[1] The requirements of social position are: health, job, academic, socio-cultural, commercial.

6.3 Transportation models - economic interpretations

We now provide an economic interpretation of the mathematical symbols of the transportation models of time allocation.

Primal and dual of transportation problem (5.7) and (5.8)

a_i = the total number of minutes available within the ith behaviour setting $i=1, \ldots, m$.

b_j = the total number of minutes demanded by the jth social requirement $j=1, \ldots, n$.

c_{ij} = cost for behaviour setting i to provide one minute for the jth social requirement.

x_{ij} = number of minutes behaviour setting i is to provide for meeting the jth social requirement.

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1 O'Grady (1988) p.33; also p.77.

In the slack model the a_i is a distribution. In the tight model both the a_i and the b_j are distributions. A distribution is interpreted as a non-negative vector adding up to 1, where the unit denotes the 16 hour waking day.

The sources i, i=1, ..., m denote behaviour settings, interpreted as environments. The a_i denote the supply of time at source i. In the model the terms "endowment", "capacity", as used in the literature, are synonymous with a supply at source i.

The sinks j, j=1, ..., n denote social requirements. They represent demands on the agent's time endowment. The terms "social requirements", "requirement of social position", are interchangeable. In the numerical examples the term "agent" refers to a student.

Types of costs

1. Direct or observed costs

The per unit cost c_{ij} denotes the market cost of meeting social requirement j within behaviour setting (environment) i. The c_{ij} denote observable costs measured in cents per minute. As described in Chapter 4, the c_{ij} are divided into two mutually exclusive components, namely the per unit cost of travel and the per unit cost of all remaining inputs (goods and services). The c_{ij} are therefore interpreted differently from the per unit costs of the ordinary transportation problem which are for shipping only. In the core model $c_{ij} > 0$. In the slack and the tight models $c_{ij} \lesseqgtr 0$. $c_{ij} > 0$ is interpreted as an expenditure. $c_{ij} < 0$ is interpreted as net income.

2. Indirect costs

The shadow prices come out of the model, for example as in (5.1), (5.3) and (5.4). The shadow price system for valuing time use is generated by the dual problem. There is an explicit relationship between imputed prices and market prices.

The imputed, or shadow, prices[2] at sources u_i, and at sinks v_j, are interpreted respectively as differential rents and as efficiency prices. In the transportation problem the shadow prices at sources, namely the u_i, are often referred to as location rents. However the preferred term in the time allocation models is differential rents because, unlike the standard transportation model, the sources are not locations. Some environments confer advantages on an agent. For example an environment that generates a lower per unit market cost, c_{ij}, is at a premium relative to other environments. Comparison of the differential rents provides a guide to the relative advantage of using one environment rather than another.

In the slack model the shadow price w, of the time endowment, is introduced. The v_j are made up of three components, namely the per unit market cost c_{ij}, the differential rent u_i, and the value, w, of the total time endowment, cf(5.5).

In the tight model, the v_j represents an efficiency price made up of two components, the per unit market cost and the differential rent. For efficient

2 Dorfman, Samuelson and Solow (1964); Lancaster (1987); Takayama (1985)

allocation of resources the agent should be prepared to pay a premium price for time use to meet a social requirement, up to the value of the dual price at a sink. The v_j interpreted as an efficiency price can be compared with the delivered value of a resource in the transportation problem.

u_i = the imputed value, or shadow price, at the ith behaviour setting (environment)

v_j = the imputed value, or shadow price, for the jth social requirement

u_i is measured in cents per minute at sink i, i=1, ..., m

v_j is measured in cents per minute at source j, j=1, ..., n

In the slack model, a shadow price w is associated with the total time endowment. w refers to an overall consistency constraint. Unlike the u_i, w does not generate a shadow price for a particular behaviour setting i, i=1, ..., m.

w = the imputed value, or shadow price, of the total time endowment

w measures the value of one extra unit of the total time endowment at source, when used optimally. That is, w provides a money measure of the value which an additional unit contributes to the value of the total time endowment (resource capacity), where time is considered as a scarce resource.

w is measured in cents per minute for the total time endowment $\sum_{ij} x_{ij} \leq 1$.
(ref. 5.4)

The economic cost is defined as the market cost together with the opportunity cost of the activity.

In the numerical examples, as in all activity models, the opportunity cost of the activities not explicitly included in the program is ignored. The omission may be significant as the following example shows.

Example 1

An agent chooses to spend two hours at a daytime music festival in preference to his work as a consultant for which he is paid by the hour. The economic cost of the activity is the market cost of goods and services, including travel and the price of entry, together with the foregone earnings for the two hours. In the example we have assumed a unimodal transportation model.

Primal - transportation problem (5.7)

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij}$$

where $c_{ij} > 0$ denotes money expenditure in cents/min

$c_{ij} < 0$ denotes net money income from employment in cents/min, where net money income = earnings net of tax less cost of transportation to work.

This means that the cost matrix consists of both positive and negative entries, and that for each of the variables $x_{ij} \geqq 0$, covering all possible time use in environments to meet social requirements, there is a corresponding c_{ij}.

$$\sum_{i,j} c_{ij}x_{ij} = \sum_{c_{ij}>0} c_{ij}x_{ij} + \sum_{c_{ij}<0} c_{ij}x_{ij}$$

net cost of the agent's activities = expenditure + net income

rimal - core and tight models (5.1); (5.3)

he primal minimizes the total market cost of the agent's time use provided hat both a behaviour setting cannot be required to supply more than its time apacity and each social requirement is fully met.

rimal - slack model (5.3) and (5.4)

As above, but with the added constraint that the agent's total time use cannot exceed the endowment of 960 minutes, denoted by unity.[3]

Dual - transportation problem (5.8); core and tight models (5.2), (5.6)

$$\max \sum_{j=1}^{n} v_j b_j - \sum_{i=1}^{m} u_i a_i$$

The maximum net imputed value of time use is equal to the total (shadow) value of time use for social requirements less the total imputed rent over all behaviour settings.

The dual maximizes the net imputed value of time use, provided that this imputed value should not exceed the per unit cost of using a minute of time.

3 - ref. section 5.4.

Dual - slack model (5.5)

$$\max \sum_{j=1}^{n} v_j b_j - \sum_{i=1}^{m} u_i a_i - \sum_{i=1}^{m} w$$

The maximum net imputed value of time use is equal to the total shadow value of time use for social requirements, less the sum of the total imputed rent over all behaviour settings and the value of the total time endowment at source.

The dual maximizes the net imputed value of time use, provided that this imputed value should not exceed the per unit cost of using a minute of time. For the slack model, the net imputed value of time use includes the dual variable w, the imputed price associated with total time endowment.

Note: w is redundant for the tight model.

Slack and Tight Models

In the slack model the focus is on the behaviour of a particular agent. In the tight model the focus is on the behaviour of the average agent. In the tight model nothing precludes for an individual but the focus is on the behaviour of the average agent, considered to be representative of a group of individuals. Groups could be determined by occupational classifications, and would include certain unpaid categories such as housewife. Statistical data relating to the behaviour of individual agents can be used to obtain objective parameters, measured in time units, for the behaviour of the average agent.

How precisely is the model related to extended national income accounts to cover the socio-economic distinction?[4] The model provides an "integrating framework" for a range of data. All the agent's activities take place within five behaviour settings, following Barker, which can be linked with time use (Szalai) together with a set of per unit costs. As well the model imputes a dollar value to the vector of time use. An objective technology[5] connects time use in behaviour settings with a set of observable social parameters.[6] Because the parameters are measured in time units, the model is objective and aggregation is practicable. It therefore represents a contribution to a system of social income accounts. With the inclusion of an aggregate time constraint, the slack model is a transport model with side constraints.

Standard linear programming model

For the standard linear programming model the same definitions apply. An example using vector notation is provided in section 7.2.

Summary

For the transportation problem we can now bring together some of the economic interpretations with particular attention to shadow prices. For convenience the tight model is considered. Starred variables indicate optimum values. By way of summary, from the duality theorem, for an optimum:

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4 Fox and Miles (1987) consider the "two major problems" of social system accounting to be:
1. how to measure and account for non-market activities
2. how to combine social and economic indicators. p. 122.

5 cf. the starred section 6.5 for an economic interpretation.

6 This approach can be compared with Fox (1974) - social welfare indicators and Fox and Ghosh (1981); Moeseke (1989) - social accounts. Ås (1982) proposes several time based social welfare indicators.

At market prices	At imputed, or shadow, prices	
Total cost	Total value of the resource at sinks	Total value of the resource at sources
$\sum_{i,j} c_{ij}x_{ij}^{*}$ =	$\sum_{j} v_j^{*}b_j$ -	$\sum_{i} u_i^{*}a_i$
Total cost of time use, for an agent	Total value of the demand for time use over all social requirements j. The agent puts a non-market value on time use to meet social requirements which is interpreted as a set of efficiency prices	Total value of the agent's supply of time over all behaviour setting i. The agent puts a non-market money value on time use associated with behaviour settings

By way of comparison, from the duality theorem for the transportation problem

Cost of transportation = value and destinations - value at origins

For the transportation model, the cost of transportation is equal to the value created by the shift because every sink receives and every source gives, until the total endowment at sources has been completely transferred to sinks.

In the transportation models of time allocation, for an optimum, total cost of time use for an economic agent is equal to the net imputed value of time use, subject to the constraints of the model.

Generalized transportation problem

row equation $e_i \sum x_{ij} \leq a_i$ $i=1, \ldots, m; j=1, \ldots, n.$ (6.1)

column equation $r_j \sum x_{ij} \geq b_j$ $i=1, \ldots, m; j=1, \ldots, n.$ (6.2) where

e_i = weight of the <u>e</u>ndowment

r_j = weight of the <u>r</u>equirement

$e_i \geq 0, \quad r_j \geq 0$

$e_i = 0$ can be interpreted as a bottleneck at behaviour setting i.

$r_j = 0$ can be interpreted as a bottleneck associated with meeting requirement j.

An increase in e_i, equivalent to an increase in efficiency column-wise, means that there is an increase in the efficiency of time use in the ith behaviour setting. The endowment of time in the ith behaviour setting is enhanced. It is as if there were more time available for use within the ith behaviour setting.

An increase in r_j, equivalent to an increase in efficiency row-wise, means that with the same amount of time an increased jth social requirement can be met.

<u>Example 2</u>

Suppose improved information systems result in less time being needed at the ith behaviour setting to meet a social requirement. If, relative to the unimodal case representing the ordinary transportation model, there has been a 25 percent increase in efficiency, then $e_i = 1.25$.

6.4 Social Income and Savings

Social income is the value of an agent's social position plus money income. This definition is the same as that provided by Moeseke (1985). In that paper both the concept of social income and the theory of savings were first proposed and formulated as quantitative and practical measurements in objective consumption theory. Both concepts are used in the time allocation models. The formulation of social income suggests a different interpretation from that given in the Moeseke (1985) model. For comparison with the Moeseke model a formulation is now provided for the tight model. Alternative ways to derive Social Income are provided in Chapters 7 and 8, using vector matrix and activity analysis notation respectively.

In the numerical examples quantitative values are derived for the agent's social income and savings over a set of changing conditions. While the results of such applications are not without interest, a further aim is to show that the concepts of social income, and the social theory of saving, are operational in models that are quite different from the model in which they were first formulated, so that in fact these concepts can be shown to be generalized.

We know that

$$\sum_{i,j} c_{ij}x^*_{ij} = \sum_{c_{ij}>0} c_{ij}x^*_{ij} + \sum_{c_{ij}<0} c_{ij}x^*_{ij}$$

Applying the duality theorem,

$$\sum_{i,j} c_{ij}x_{ij}^* = \sum_j v^*_j b_j - \sum_i u^*_i a_i \qquad (6.3)$$

Hence,

$$\sum_{i,j} c_{ij}x_{ij}^{*} = \sum_{c_{ij}>0} c_{ij}x_{ij}^{*} + \sum_{c_{ij}<0} c_{ij}x_{ij}^{*} = \quad E \quad - \quad I \qquad (6.4)$$

E	I
total consumption for all non-work related activities <the positive component of $\Sigma c_{ij}x_{ij}^{*}$> = value of consumption.	net money income from work <the negative component of $\Sigma c_{ij}x_{ij}^{*}$> = value of time spent on work.

$\sum_{i,j} c_{ij}x_{ij}^{*}$ E > I hence dissaving

$\sum_{i,j} c_{ij}x_{ij}^{*}$ E < I hence saving

Imposing the <u>money budget</u> condition, $\Sigma c_{ij}x_{ij}^{*} \leqq 0$. (6.5)

In the Moeseke model (1985) Social Income (SI) is defined as the value of social position plus net income from work. This definition is based on the duality theorem. It should be noted that the definitions in Moeseke (1985) and in the Fox-Moeseke model (1973) do not necessarily coincide. By applying the duality theorem to the time use model we can show that:

$$\Sigma c_{ij}x_{ij}^{*} = \sum_{c_{ij}>0} c_{ij}x_{ij}^{*} + \sum_{c_{ij}<0} c_{ij}x_{ij}^{*} = \sum_{j} v_j^{*}b_j - \sum_{i} u_i^{*}a_i$$

That is $\quad E - I = \sum_{j} v_j^{*}b_j - \sum_{i} u_i^{*}a_i \qquad (6.6)$

By definition, $\quad SI = \Sigma v_j^{*}b_j \quad + \quad I$

$\Sigma v_j^{*}b_j$	I
value of social position	value of time spent on work. (= net income)

$$\text{Hence} \quad SI = \underset{\text{value of consumption}}{E} + \underset{\text{value of time endowments}}{\sum_i u_i a_i} \qquad (6.7)$$

In words

The total cost of social income is more than than the market expenditure.

Since $SI = \sum_j v^*{}_j b_j + I = E + \sum u_i a_i$

Then $I - E = \sum u^*{}_i a_i - \sum v^*{}_j b_j$

that is, savings = value of time endowments - value of social requirements

As in the Moeseke (1985) model, the agent saves or dissaves according as I-E > 0 or < 0 respectively.

In the following chapter numerical examples will be used to verify quantitatively the equivalence of determining saving (or dissaving) from (6.4) and from (6.6).

as follows:

(1) the primal

$\sum_{ij} c_{ij} x_{ij}^* > 0$ dissaving

$\sum_{ij} c_{ij} x^* < 0$ saving

(2) the dual

$\sum_j v^* b_j - \sum u_i a_i > 0$ dissaving

$\sum_j v^* b_j - \sum v_i a_i < 0$ saving

6.5 Activity Analysis - Economic interpretations

The formulation of the transportation problem as an activity analysis model introduced the two basic concepts of commodity and activity. An economic interpretation of these concepts is now given.

A commodity may be a resource, a good or a service which is tranformed by an activity. In the activity analysis mode, time is an endowment, the supply of a scarce resource, available for use within well defined environments termed "behaviour settings". In the tight model, the total time endowment of 960 minutes is distributed over environments. Time use in a specific environment represents a primary commodity. Social requirements can only be met by the agent's time use. Activities are carried out by the agent to produce final commodities in order to meet social requirements. Hence primary and final commodities represent time use. Any commodities which are neither primary nor final are classified as intermediate commodities, a catch-all to close the system. Since the focus of the model is time use, commodities such as consumer goods and services, are considered to be intermediate commodities. The model does not overlook such commodities, since the cost of an activity includes the cost of commodities transformed.

In the transportation problem the transfer of technically the same commodity, namely time use, from two different environments is considered to represent two different primary commodities. Likewise the transfer of technically the same commodity to meet different social requirements, irrespective of the particular behaviour setting (environment), represents two different final commodities.

We can classify commodities relative to the technology matrix $\boldsymbol{A} = [a_{\ell k}]$.

For the kth activity:

If $a_{\ell k} > 0$ the ℓth commodity is a final commodity

If $a_{\ell k} = 0$ the ℓth commodity is an intermediate commodity

If $a_{\ell k} < 0$ the ℓth commodity is a primary commodity

An activity or process, transforms primary commodities, as inputs, into final commodities as outputs. As mentioned, (**5.7) there is a fixed quantitative relationship between inputs and outputs. An activity if carried out at unit level, is the amount of the final commodity transformed from one unit (usually one minute) of the time endowment. Negative flows represents inputs, positive flows represent outputs. The time endowment is therefore treated as negative. This is why it has been described as a negative requirement.

The socio-economic production function denoted by the matrix $\boldsymbol{A}$ can be considered as the rule for mapping from the vector **x** in N dimensional commodity space to the vector $\boldsymbol{Ax}$ in M dimensional commodity space. This represents a transformation from activities to requirements. The vector $\boldsymbol{Ax}$ is congruent with the m+n requirements vector y. Endowments are denoted by the negative elements of **y**. Each element of $\boldsymbol{Ax}$ can be associated with a price vector p, congruent with the vector of shadow prices $\mathbf{v} = [u_1, u_2, \ldots, v_1, v_2, \ldots]$. The price vector p expressed in cents/min reflects scarcity of a requirement. It denotes the value of an additional unit of time to meet a requirement. For p, a negative value is associated with a negative requirement which represents an endowment. The value of a production set is given by $\mathbf{p}\boldsymbol{Ax}$.

The technology matrix can be considered as a link between the activities of an agent and the value of the time requirements, including endowments. Hence ctivity analysis is interpreted as a mapping from observable M-dimensional ctivity "space" to non-observable M-dimensional value "space".

hanges in Technology

nterpretations are now provided for the two ways in which changes in echnology have already been formulated. The approach is analogous to ncreases in efficiency for the generalized transportation problem (6.1) and (6.2). By way of example, we could consider that efficiencies in transforming nputs into outputs arise because a particular environment has been enhanced. Since environment shapes behaviour, a change in a particular environment will result in changes in behaviour by an agent. This means that for all activities involving time use in the enhanced environment the same input can be transformed into an increased output, or equivalently, the same output can be obtained using a decreased input. In activity analysis there is an increase in efficiency column wise.

Example 3

There are two inputs and two outputs. There are four activities, or processes.

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \qquad -B = \begin{bmatrix} -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

The technology matrix is

$$A = \begin{array}{c} \begin{matrix} A_1 & A_2 & A_3 & A_4 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix} \end{array}$$

The activities represent the possible technical processes, and are denoted A_1, ..., A_4. For a transportation problem the A_1, A_2, A_3, A_4 denote transfers from source 1 to destination 1, from source 1 to destination 2 and so on. Suppose that there is a 50 per cent increase in the efficiency of time use in the first behaviour setting for meeting both social requirement parameters. The new input and output matrices are denoted by A^{**} and B^{**}. The changed technology matrix is $\boldsymbol{A}^{**}$. The double star denotes <u>post</u>-multiplication of the original socio-economic matrix $\boldsymbol{A}$.

$$A^{**} = \begin{bmatrix} 1.5 & 0 & 1 & 0 \\ 0 & 1.5 & 0 & 1 \end{bmatrix} \qquad -B^{**} = \begin{bmatrix} -1.5 & -1.5 & 0 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

$$\boldsymbol{A}^{**} = \begin{bmatrix} 1.5 & 0 & 1 & 0 \\ 0 & 1.5 & 0 & 1 \\ -1.5 & -1.5 & 0 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1.5 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \boldsymbol{AE}$$

Hence $\qquad \boldsymbol{A}^{**} = \boldsymbol{AE}$

where E is the diagonal matrix specified by[7]

$$E = \begin{bmatrix} 1.5 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Suppose the 50 percent increase in the efficiency of time use in the first behaviour setting could be used to meet the first social requirement only.

Then $$E = \begin{bmatrix} 1.5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example 4

There is no relative increase in the efficiency of time use in any particular behaviour setting. Suppose that there is a 50 per cent increase in the efficiency of meeting the second social requirement. There is an increase in efficiency row-wise. The changed technology matrix is denoted by A^*. The single star denotes pre-multiplication of the original socio-economic matrix A

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7 E is a 4 x 4 diagonal matrix. The successive diagonal entries are eigenvalues.

$$A^{*} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1.5 & 0 & 1.5 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix} = \boldsymbol{RA}$$

Hence $\boldsymbol{A}^{*} = \boldsymbol{RA}$

where $\boldsymbol{R}$ is the diagonal matrix specified by

$$\boldsymbol{R} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For both an increase in efficiency column-wise and an increase in efficiency row-wise, the new technology is obtained by respectively post-multiplication and pre-multiplication by a diagonal matrix. The diagonal matrices are denoted $\boldsymbol{E}$ and $\boldsymbol{R}$ respectively and the new technologies by $\boldsymbol{A}^{**}$ and $\boldsymbol{A}^{*}$ respectively.

Note on dimensionality

In both numerical examples the technology matrix A was 4 x 4. As well, the matrices $\boldsymbol{E}$ and $\boldsymbol{R}$, related to changes in a row and in a column respectively were not only square but diagonal matrices. There is no a priori reason to have a 4 x 4 matrix. In the numerical examples of Chapter 8, there are five behaviour settings and five parameters of social position. There is no a priori reason why the number of behaviour settings is equal to the number of social requirements. A model with two inputs and two outputs generates a 4 x 4 matrix and all activities are required. However the numerical examples with five behaviour settings and five requirements of social position generates a 10 x 25 technology matrix. Since the number of activities exceeds the number elements in the constraint vector of inputs and outputs, at least 15 activities must be relinquished by the optimizing agent to obtain a square matrix. The dimensionality requirement entails that invariably we will end up by selecting a square matrix. The associated matrices ***E*** and ***R*** are diagonal matrices only.

In the numerical examples of Chapter 8, the transformation is specified by the 10 x 25 matrix ***A*** which can be considered as a rule for mapping from the vector x in 25-dimensional commodity space to the vector y in 10-dimensional commodity space. The associated diagonal matrices ***E*** and ***R*** are 25 x 25 and 10 x 10 respectively.

<u>Example 4</u>

For the 8 x 16 socio-economic production function denoted by ***A*** there is a 100 percent increase in efficiency in meeting the third social requirement. Then the new technology, described by a new socio-economic productions function A^* can be obtained by pre-multiplying the original matrix ***A*** by ***R*** where ***R*** is a 8 x 8 diagonal matrix, and $r_{33} = 2.0$, otherwise all diagonal elements =1. All other elements = 0.

Given that there is an upper limit on inputs and a lower limit on outputs, the reduction in dimensions reflects the fact that some combinations of inputs are non-optimal. Inefficient processes are not utilized by a rational agent. For example, certain combinations of commodities are inefficient because the relatively high costs per unit of time outweigh any corresponding gain obtained through a reduction in time use and therefore do not contribute to minimizing costs.

<u>Summary</u>

For the case of the agent as an individual consumer, the technology matrix ***A*** can be regarded as the socio-economic production function. In the activity analysis formulation of the transportation problem the matrix ***A*** can be regarded as a link between, on the one hand, certain observable behaviour measured in terms of time use and characterized by the concepts of commodity and activity, and, on the other, a set of values placed on time use, in a dual space. The technology matrix ***A***, is interpreted as a relationship between inputs and outputs, and as a mapping from observable activity "space" to a non-observable value "space".

CHAPTER 7

NUMERICAL EXAMPLES - (1) 2 X 2, 2 X 3 AND 3 X 3 MODELS

7.1 Introduction

To show that the two transportation models of time allocation are operational some simple 2 x 2, 2 x 3 and 3 x 3 numerical examples are introduced and solved. For the 3 x 3 examples the agent obtains income from work. As in the numerical example introduced in Chapter 5, an agent carries out economic activities in order to meet a set of social requirements. The optimal solutions specify both the type and level of the activities which the student undertakes in order to meet the parameters of his social position. We are solving for the agent's optimal lifestyle, in which he meets the parameters of his social position at minimum cost. Hence the optimal solutions quantify the agent's consumption patterns. Quantitative measures of social income are also obtained. In section 7.4 the problem of dimensionality, inherent in any transportation problem, is raised. To motivate the discussion, optimal solutions to a 2 x 3 numerical example are used to complement the theory.

What has been done in the numerical examples is to take reasonable cases, with quantitative measures not unrelated to the behaviour of a rational agent. On this basis the optimal solutions give plausible results both in terms of what economic theory would suggest and as an alternative means of interpreting patterns of human behaviour. While the examples are numerical they are not empirical. That is, neither the magnitudes accorded to constraints at sources and at sinks, using the concepts of behaviour setting and of social requirement respectively, nor the related per unit costs have been obtained directly from actual observations, or from an existing data base. This is hardly surprising. Data in the precise form are not available, given that the choice of parameters, using the concepts of behaviour setting and social requirement in the framework of a transportation problem, breaks fresh ground. Fox (1985) has made use of certain types of authority systems, to be sure, but the categories of the time allocation models use behaviour settings, rather than authority systems, and introduce for the first time the categories of household and workplace, so therefore represent an extension of the classifications originally proposed by Barker (1968). Further, all categories in the model are specified not only in terms of time but also of cost.

To provide magnitudes, use is made of both time budgets for students' daily activity patterns as in Tomlinson and others (1973) and estimates of the cost of living, for a different student group.[1] The way in which both sets of information are used suggests a methodology for deriving empirical values. A step by step derivation of a 2 x 2 cost matrix is provided in section 7.4. This serves as an introduction to the methodology of obtaining per unit costs for the 5 x 5 model, developed fully in the following chapter.

7.2 The Slack Model

Example 1 $c_{ij} > 0$, all i, j

A simple numerical example of two sources and two sinks is provided as an application of the concepts already developed. This example is similar to the

1 Living in Wellington Ko to noho i Whanganui-a-Tara (1989)

introductory numerical example provided in Chapter 5. The explanation provided there for constraints at sources and sinks, for parameters and for activities is relevant for the slack model now provided, and repetition is not desirable.

TABLE 7-1 **2 x 2 ACTIVITY MATRIX - SLACK MODEL**

	to: study (S)	relaxation (R)			
from: campus (C)	x_{11}	x_{12}	$\leqq$	490	a_1
from: household (H)	x_{21}	x_{22}	$\leqq$	670	a_2
	$\geqq$ 500	$\geqq$ 460			
	b_1	b_2			

In Table 7-1, as in Table 5-1, the term "relaxation" is a catch-all to include meals, laundry, exercise and all forms of creative activity. Likewise the term "household" is a catch-all and includes the student's flat and his family home. The terms are used to close the requirements and environments systems respectively. x_{21} denotes the time spent studying in the household, and x_{12} the time spent in relaxation on campus. All constraints are measured in minutes taken over the 16 hour waking day. Constraints at sinks represent minimal requirements. As in the numerical example in Chapter 5, we relax any institutional requirements that would restrict the student's freedom to study away from campus.

TABLE 7-2 2 x 2 COST MATRIX - SLACK MODEL

$$c = \begin{bmatrix} 3.2 & 2.8 \\ 2.7 & 3.9 \end{bmatrix}$$

With each of x_{11}, x_{12}, x_{21}, x_{22} there is an associated cost. Since every $c_{ij} > 0$, the agent will not use more than 960 minutes. The per unit costs in Table 7-2 are derived costs. An intuitive approach is now provided. This anticipates the more precise analysis first developed in section 7.4 for a 2x2 cost matrix and later extended in section 8.4 for a 5x5 cost matrix. It is usual to find data for aggregate yearly expenditure for an agent using a simple classification scheme. In the case of a student a standard classification would include such items as accomodation (flatting), food, and personal spending. Because the transportation models of time allocation assume that the environments shape behaviour, it is important to relate each expenditure classification to each of the two behaviour settings, namely campus and household.

Expenditure on food, for example, would include not only meals on campus, but also meals in the household behaviour setting. Likewise personal spending would involve expenditure in both behaviour settings. We can apportion total expenditure across behaviour settings to obtain separate aggregates of expenditure for the campus behaviour setting, and for the household behaviour setting. Because there are only two social requirements, and therefore one degree of freedom involved, we need only obtain the expenditure for study within the campus behaviour setting to find how the total expenditure of $3300 for the campus behaviour setting is split between the study and the relaxation social requirements, namely $1800 and $1500 respectively.[2]

In this way a 2 x 2 expenditure matrix can be obtained for the academic year. Each of the four elements of the matrix can be expressed as a daily cost. For example to obtain the c_{11} of Table 7-2, the daily cost of meeting the study social requirement on campus, expressed in dollars per day ($6.4) is divided by the time spent on study at campus over the 960 minutes of the 16 hour waking day (200 mins.), and then converted from dollars/min to cents/min ($\frac{\$6.4}{200}/100 = 3.2$ cent/min).

Note: Costs are measured in cents per minute because the units for the time use diaries are minutes. Because a common way of referring to rate of cost per time unit is dollars per hour some equivalent costs are provided. As before, the 16 hour waking day is used in computing $/day costs.

cents/min	1.0	1.5	2.0	2.5	3.0	3.5
$/hr	0.6	0.9	1.2	1.5	1.8	2.1
$/day	9.6	14.4	19.2	24.0	28.8	33.6

The first column indicates conversion factors from cents/min to $1/hr and $i/waking day. A wage of $264.00 for a 40 hour working week is equivalent to 11.0 cents/min.

For the quantitative derivation of the cost matrix see section 7.4.

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2 See Table 7-20, section 7.4.

Economic interpretation, Example 1

$a_1 \leqq 490$ This is the maximum amount of time an agent could spend on campus.

$a_2 \leqq 670$ This is the maximum amount of time an agent could spend in the household.

The a_i do not define a distribution, and $\Sigma a_i > \Sigma b_j = 960$

Solutions $x^*_{11} = 0$; $x_{12}^* = 460$; $x_{21}^* = 500$; $x_{22}^* = 0$.

$u_1^* = 0$; $u_2^* = 0$; $v_1^* = 2.7$; $v_2^* = 2.8$

The u_i denote the imputed value, or shadow price, of the endowment at the ith behaviour setting. The v_j denote the imputed, or shadow price, of time use for the jth social requirement.

In Example 1 there is unused capacity at both the campus and household behaviour settings. Hence the related shadow prices are zero. The slack variables are equal to 30 and 170 minutes respectively.

The value of the objective function, in cents, is

primal: $(0 \times 3.2) + 460(2.8) + 500(2.7) + 0(3.9)$
$= 2638$
$= \$26.38$ per diem

dual: $500(2.7) + 460(2.8) - 490(0) - 670(0)$
$= 2638$
$= \$26.38$ per diem

The value of social income, in cents, is

$$\begin{aligned} & 500(2.7) + 460(2.8) \\ &= 1350 + 1288 \\ &= 2638 \\ &= \$26.38 \text{ per diem} \end{aligned}$$

Example 2 $c_{ij} > 0$

There is a change in the cost matrix. Otherwise all parameters are the same as in Example 1. The new cost matrix is given by:

TABLE 7-3 2 x 2 NEW COST MATRIX (1)

$$c = \begin{bmatrix} 3.2 & 3.9 \\ 2.7 & 2.8 \end{bmatrix}$$

Solutions $x^*_{11} = 290$; $x^*_{12} = 0$; $x^*_{21} = 210$; $x^*_{22} = 460$

$u^*_1 = 0$; $u^*_2 = 0.5$; $v^*_1 = 3.2$; $v^*_2 = 3.3$

The agent's objective function is \$27.83 per diem. This represents his actual expenditure.

By comparison with Example 1, study is spread over both the campus, 290 minutes, and household, 210 minutes, behaviour settings respectively. The value of the objective function has increased by \$1.45 per diem. The per unit market cost of relaxation in the campus environment has changed from 2.8 to 3.9 cents/minute, an increase of 39.3 per cent. The increase in the net value of the student's relaxation time has changed from 2.8 to 3.3 cents/minute, an increase of 17.9 per cent. Previously the student met the full relaxation requirement on campus, but now there is no time used for relaxation in this environment. This result is in accord with economic theory. We would expect that if the marginal cost of an activity increases, the level of the activity is reduced.[3] In section 7.3 we examine changes in the cost matrix for the tight model, using the Le Chatelier Principle[4], and comment on the results.

Sensitivity analysis

How much can a per unit cost vary without changing the solution?

Each of the basic variables x_{11}, x_{21}, x_{22} is associated with a per unit cost namely c_{11}, c_{21}, c_{22}. If any of the per unit costs, which are the coefficients of the basic variables in the objective function, is changed to a number which lies outside an allowable range, there will be a change in the optimal solution. For example either the levels of the basic variable will change, or a basic variable will be displaced by the non-basic variable, x_{12}, or both.

For the c_{11} the allowable range is given by $2.7 \leqq c_{11} \leqq 3.8$. The remaining per unit costs, denoted by c_{21} and c_{22} are unchanged. If c_{11} is changed to a value which lies within the allowable range, there will be no change in the basic variables nor in their levels. However if c_{11} is changed to become either lower than the low range or higher than the high range, the variables in the optimal solution will change.

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3 Leblanc and Moeseke (1976).
4 ibid

Suppose:

(1) $c_{11} = 2.6$

Solutions

$x^*_{11} = 490$; $x_{21} = 10$; $x_{22} = 460$;

$u_1^* = 0.1$; $u_2^* = 0$; $v_1^* = 2.7$; $v_2^* = 2.8$

agent's objective function is \$25.89 per diem.

(2) $c_{11} = 3.9$

Solutions

$x_{12}^* = 290$; $x_{21}^* = 500$; $x_{22} = 170$

$u_1^* = 0$; $u_2^* = 1.1$; $v_1^* = 3.8$; $v_2^* = 3.9$

agent's objective function is \$29.57 per diem

In (1) the decrease in the per unit cost of study on campus results in a transfer of 200 minutes so that 490 minutes is spent on study at campus. The remaining 10 minutes needed to meet the study social requirement is spent in the household behaviour setting, and all relaxation, as in Example 2, takes place in the household. The basic variables remain the same but the levels have changed.

By way of contrast in (2) the basic variables have changed. Although the per unit cost of study on campus is the same as the per unit cost of relaxation on campus, the time used on campus, 290 minutes, is entirely for meeting the relaxation social requirement. The balance of relaxation, 170 minutes, is spent in the household behaviour setting.

Example 3

In examples 1 and 2 the activity matrices have been square in terms of environments and social requirements. As mentioned[5], the matrices do not have to be square. Example 3 represents a simple 2 x 3 model in which a further requirement is introduced. The agent contributes voluntarily a regular review article on recently published fiction. We term this the "civic activities" requirement, denoted by CA. The agent can meet this requirement in either, or both, of the campus and household environments. Suppose the activity and cost matrices are as follows:

TABLE 7-4 2 x 3 ACTIVITY MATRIX - SLACK MODEL

	to:	S	CA	R			
from:	C	x_{11}	x_{12}	x_{13}	$\leqq$	490	a_1
	H	x_{21}	x_{22}	x_{23}	$\leqq$	670	a_2
		$\geqq$ 500	$\geqq$ 100	$\geqq$ 360			
		b_1	b_2	b_3			

TABLE 7-5 2 x 3 COST MATRIX - SLACK MODEL

$$c = \begin{bmatrix} 3.2 & 3.1 & 3.9 \\ 2.7 & 2.5 & 2.8 \end{bmatrix}$$

Solutions $x^*_{11} = 290$; $x^*_{21} = 210$; $\boxed{x^*_{22} = 100}$; $x^*_{23} = 360$

$u^*_1 = 0$; $u^*_2 = 0.5$; $v^*_1 = 3.2$; $v^*_2 = 3.0$; $v^*_3 = 3.3$

The agents spends $27.53 per day.

5 - cf section 5.2.

The agent meets the civic activities requirement in the household environment. Now it might appear that the agent will simply meet the civic activities requirement in the environment with the lower per unit cost. Not so. Suppose that the per unit cost of meeting the civic activities requirement in the household environment, denoted by c_{22}, increases from 2.5 to 2.7 cents/min, ceteris paribus (cet par). The per unit cost for the household (2.7) is still lower than the 3.1 for campus. However the agent, acting optimally, will now meet the civic activities requirement on campus. As before study is shared between campus and household, but the relative amounts have changed.

Solutions $x^*_{11} = 190$; $\boxed{x^*_{12} = 100}$; $x_{21} = 310$; $x_{23} = 360$

$u^*_1 = 0$; $u^*_2 = 0.5$; $v^*_1 = 3.2$; $v^*_2 = 3.1$; $v^*_3 = 3.3$

The agent spends $27.63 per day.

In both solutions, the number of decision variables is four. This means that of the six possible combinations of environment and social requirement, namely x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, two are redundant, namely x_{12}, x_{13} in the first case, and x_{13}, x_{22} in the second. For a further social requirement, the maximum number of decision variables would remain at four, with four of the x_{ij} becoming redundant. Realism suggests that the number of social requirements, should be approximately the same as, if not equal to, the number of environments.[6]

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6 for a general statement, Lancaster (1987) pps. 248-249.

Example 4 $c_{ij} \lesseqgtr 0$

This example can be related to the program for the slack model, where (5.3) and (5.4) together denote the primal, and (5.5) denotes the dual.

We now introduce a third social requirement and term this "Job", denoted by J. A third behaviour setting, termed "Workplace", and denoted by W, is also introduced. The work place behaviour setting does not overlap with other behaviour settings. Workplace represents a behaviour setting distinct from all others because the environment is defined as one in which income is generated. This environment therefore shapes behaviour in a way that other environments do not. In the cost matrix income is denoted by a $c_{ij} < 0$. As noted in section 5.4, negative cost requires a further constraint, namely the aggregate time constraint.

TABLE 7-6 **3 x 3 ACTIVITY MATRIX - SLACK MODEL**

	to:	Study (S)	Relaxation (R)	Job (J)			
from:	Campus (C)	x_{11}	x_{12}	x_{13}	$\leq$	490	a_1
	Household (H)	x_{21}	x_{22}	x_{23}	$\leq$	670	a_2
	Workplace (W)	x_{31}	x_{32}	x_{33}	$\leq$	300	a_3
		$\geq$ 400	$\geq$ 340	$\geq$ 220			
		b_1	b_2	b_3			

As mentioned in section 5.2, the use of inequality signs is a heuristic device.

An aggregate time constraint $\Sigma x_{ij} \leq 960$ is now introduced. As previously noted this additional constraint is necessarily redundant if the b_j define a distribution and all $c_{ij} \leqq 0$, because then there is no reason to use more than 960 minutes. However if some $c_{ij} < 0$, as when the agent obtains income from work, the additional constraint is no longer necessarily redundant. It is also required for realism. The associated shadow price is denoted by w.

$$\Sigma x_{ij} \leqq 960 \quad \vdots \quad w \tag{7.1}$$

v refers to an overall consistency constraint. The shadow price w has been given an economic interpretation. (6.3).

Suppose that there is a new cost matrix, as given in Table 7.7.

TABLE 7-7 3 x 3 NEW COST MATRIX (1)

$$c = \begin{bmatrix} 3.2 & 2.8 & 1000 \\ 2.7 & 3.9 & 1000 \\ 1000 & 1000 & -9.5 \end{bmatrix}$$

The insertion of the large numbers 1000 is merely a computational device to exclude work activity from non-appropriate environments.

Solutions $x^*_{12} = 340$; $x^*_{21} = 400$; $x^*_{33} = 220$.

$w^* = 9.5$; $u^*_1 = 0$; $u^*_2 = 0$; $u^*_3 = 0$; $v^*_1 = 12.2$;

$v^*_2 = 12.3$; $v^*_3 = 0$.

The objective function, in cents, is

primal (0 x 3.2) + (340 x 2.8) + (0 x 1000) + (400 x 2.7) + (0 x 3.9) + (0 x 1000) + (220 x -9.5)

= 952 + 1080 - 2090

= -58

dual -(960 x 9.5) - (490 x 0) - (670 x 0) - (300 x 0) + (400 x 12.2) + (340 x 12.3) + (220 x 0)

= -9120 + 4880 + 4182

= -58

The agent's objective function is - $0.58 per diem. This represents a saving.

What happens in the slack model if the aggregate time constraint (7.1) is relaxed?

Example 5

In this example there is no aggregate time constraint. The parameters are unchanged from those given in Tables 7-6 and 7-7. An optimizing agent will maximize time use in the workplace behaviour setting as this generates income and therefore reduces the objective function.

Solutions $x^*_{12} = 340$; $x^*_{21} = 400$; $x^*_{33} = 300$

$u^*_1 = 0$; $u^*_2 = 0$; $u^*_3 = 9.5$; $v^*_1 = 2.7$; $v^*_2 = 2.8$;

$v^*_3 = 0$

The objective function is:

$$340(2.8) + 400(2.7) + 300(-9.5)$$
$$= 952 + 1080 - 2850$$
$$= -818$$

This represents a saving of $8.18 per diem.

In the previous example, the aggregate time constraint restricted time use in the workplace behaviour setting to 220 minutes, just sufficient to meet the job social requirement. With the removal of this constraint, as in example 4, the agent will use the maximum time available in the workplace behaviour setting, 300 minutes, an increase of 80 minutes. The agent now uses up altogether 1040 minutes, the 960 minutes of the waking day plus the additional 80 minutes in the workplace behaviour setting. The solution is consistent with the constraints at sources and sinks, but is not consistent with the assumption of a 16 hour waking day. The economic interpretation is straightforward. By restricting time for sleep the agent can work longer hours and increase earnings so that all social requirements are met, and savings increase by $7.60 per diem.

Social Income

An expression is derived for Social Income using vector matrix notation.[7] From the duality theorem, using the standard linear program, as in (5.11) and (5.12)

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7 ref. section 5.6.

$$\mathbf{cx} = \mathbf{vb} \tag{7.2}$$

with reference to Example 3.

$\mathbf{cx} = -\mathbf{c}_w\mathbf{x}_w + \mathbf{c}_n\mathbf{x}_n$, with the subscripts denoting work and non-work respectively

and $\mathbf{vb} = -960w + \mathbf{zb}$

where $\mathbf{z} = [\mathbf{u}\ \mathbf{v}]$

$= [u_1\ u_2\ u_3\ v_1\ v_2\ v_3]$

and $\mathbf{b} = \begin{bmatrix} \mathbf{b}_m, \mathbf{b}_n \end{bmatrix}$

where $\mathbf{b}_m = [-490\ -670\ -300]$
and $\mathbf{b}_n = [\ 400\ \ 340\ \ 220]$

so that, from the duality theorem

$$c_n\mathbf{x}_n - c_w\mathbf{x}_w = \mathbf{vb}_n - 960w - \mathbf{ub}_m \tag{7.3}$$

LHS: $c_n\mathbf{x}_n - c_w\mathbf{x}_w = \mathbf{E} - \mathbf{I}$

E	I
total consumption for all non-work related activities	net income from work

$c_n\mathbf{x}_n - c_w\mathbf{x}_w > 0$ **E** > **I** hence saving

$c_n\mathbf{x}_n - c_w\mathbf{x}_w < 0$ **E** < **I** hence saving

RHS: vb_n - $960w$ - ub_m

Value of Social Position	Value of the aggregate time endowment	Value of the separate time endowments at sources

From (7.3),

$$vb_n + c_w x_w = c_n x_n + 960w + ub_m$$

By definition, Social Income = Value of social position plus money income (1)

$= vb_n + c_w x_w$

$= c_n x_n + 960w + ub_m$

= Expenditure + Total value of time endowment (2)

Social Income (1), in cents = 400(12.2) + 340(12.3) + 220(9.5)

= 4880 + 4182 + 2090

= 11152

Social Income (2), in cents = 340(2.8) + 400(2.7) + 960(9.5)

= 952 + 1080 + 9120

= 11152

The agent's per diem social income is $111.52.

Money Income represents 18.74 percent of social income.[8]

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8 . "As suggested by Becker (1965) 'full income' normally exceeds money income substantially". Moeseke (1985), p. 269.

Summary

In each of the four examples the a_i do not define a distribution. It so happens in Example 1 that there is slack capacity at both sources, necessarily entailing zero shadow prices for time use endowments at each of the campus and household behaviour settings. It so happens in Example 3 that there is slack capacity at each of the three sources, necessarily entailing zero shadow prices for time use endowments at each of the campus, household and workplace behaviour settings. In general, where the a_i do not define a distribution there will be at least one slack variable, necessarily entailing at least one zero shadow price. In Example 3 the cost matrix included a $c_{ij} < 0$. An additional constraint, the aggregate time constraint, was introduced.

7.3 The Tight Model

Introduction

The preceding examples showed how it was possible to quantify the behaviour of an agent. The focus was on the activities of a particular student. There is no theoretical reason why the a_i define a distribution and in the three examples of the slack model, $\Sigma a_i > 960$ minutes. This approach has some disadvantages. For at least one source, and possibly all, unused capacity exists so that there is at least one primal slack variable. In an optimal solution only endowments which are used to capacity can generate non-zero shadow prices. It should be noted that the existence of a zero shadow price need not entail unused capacity at a source. More formally, unused capacity at a source is a sufficient condition for a zero shadow price. A zero shadow price is a necessary, but not a sufficient condition, for unused capacity. Because unused capacity necessarily entails a zero shadow price at a source, relative differences between opportunity costs of time use in different behaviour settings may not emerge in the slack model. As well, it just so happens in the four examples that where there is slack capacity at a source we can find an opportunity cost at a sink which is simply equal to the related per unit cost c_{ij}.

We now focus on the activities of the average student. The perspective could be, for example, that of an economic planner, interested in obtaining optimal measures of time use which would be representative of classified groups. This approach could provide a method for obtaining sectoral data for a time-based system of social accounts. The introduction of an additional constraint $\Sigma x_{ij} \leqq 1$, interpreted as the value of the total time endowment, makes it possible to obtain opportunity costs at sinks which exceed the magnitude of the $c_{ij} \geqq 0$.

Distribution of an endowment at sources

For every activity we can measure the amount of time that is used in a particular environment by a representative agent in order to meet a social requirement. We specify these amounts in the following way: for each agent we find the effective constraint, defined, for a given behavioural setting, as the difference between the total available capacity and the slack capacity. In Example 1, the total available time use endowment for the household behaviour setting was 670 minutes. The slack (unused) capacity was 170 minutes. Hence the effective constraint would be 500 minutes. Each capacity constraint now represents the maximal actual time use. Empirically, the distribution of such capacity constraints over the set of behaviour settings is given by $\Sigma a_i = 960$. By means of sampling it becomes possible to obtain data for each agent belonging to a representative group. By finding the mean maximal actual time use for the group we obtain the constraints, termed "effective constraints", for the average student. Since there is no slack capacity at sources, there are no shadow prices of endowments which are necessarily zero. For the slack model a justification was provided for introducing an additional constraint. However the justification no longer holds for the tight model so this constraint is relaxed. There are several advantages of using effective constraints for which the a_1 define a distribution. The tight model makes it possible to link data on time use at the microeconomic level with a system of social accounts at the national level. Time budgets of each individual agent could be used to obtain actual time use. This information, when combined with a per unit cost matrix, could then be used to specify the lifestyle of each agent.

Individuals would be classified into appropriate groups (perhaps by supplementing standard occupational categories) and then, using aggregation, averages for parameters of endowments and social requirements, and for lifestyles, (all measured in time based units) could be obtained for each group and in this way provide a <u>social accounting matrix</u>. Then, using the tight model the optimal allocations of time could be obtained for each classified group and compared with the matrix of actual time use. From the model the value of time use for the optimizing average member of each group could be obtained. This information would complement the optimal time based social accounting matrix for activities.

In the following examples, we wish to find optimal solutions for the agent's choice of environments, and also the amount of time used in these environments, in order to meet his social requirements while minimising total cost.

Example 6

This example can be related to the program described by (5.3) and (5.6).

TABLE 7-8 2 x 2 ACTIVITY MATRIX - TIGHT MODEL

		to: S	R		
from:	C	x_{11}	x_{12}	$\leqq$	360
	H	x_{21}	x_{22}	$\leqq$	600
		$\geqq$ 500	$\geqq$ 460		

TABLE 7-9 2 x 2 COST MATRIX - TIGHT MODEL

$$c = \begin{bmatrix} 3.2 & 2.8 \\ 2.7 & 3.9 \end{bmatrix}$$

Solutions $x^*_{11} = 0;\quad x^*_{12} = 360;\quad x^*_{21} = 500;\quad x^*_{22} = 100$

$u^*_1 = 1.1;\quad u^*_2 = 0;\quad v^*_1 = 2.7;\quad v^*_2 = 3.9$

The value of the objective function is, for the primal, in cents per diem

$$(0 \times 3.2) + (360 \times 2.8) + (500 \times 2.7) + (100 \times 3.9)$$

$$= 2748$$

$$= \$27.48 \text{ per diem}$$

The value of the objective function is, for the dual, in cents per diem

$$(500 \times 2.7) + (460 \times 3.9) - (360 \times 1.1)$$

$$= 2748$$

$$= \$27.48 \text{ per diem}$$

Comment: As noted for the numerical example of Chapter 5, any requirement that study take place on campus is relaxed. In practice such constraints may well apply, and the appropriate model is a transportaton problem with a side constraint. Examples are provided in Chapter 9.

Example 7 case (1) $c_{ij} > 0$. There is a change in the cost matrix. The new cost matrix is given by:

TABLE 7-10 **2 x 2 NEW COST MATRIX (1) - TIGHT MODEL**

$$c = \begin{bmatrix} 2.3 & 4.5 \\ 2.7 & 3.9 \end{bmatrix}$$

?

c_{21} has increased; otherwise all parameters are as in example 6.

Solutions $x^*_{11} = 360$; $x^*_{12} = 0$; $x^*_{21} = 140$; $x^*_{22} = 460$

$u^*_1 = 0$; $u^*_2 = 0.5$; $v^*_1 = 3.2$; $v^*_2 = 4.4$

The value of the objective function, in cents, is 3324. The agent dissaves $33.24 per diem.

Compared with example 6, there is a significant shift in the allocation of time use. The recreation social requirement is fully met by time use in the household environment. No recreation now takes place on campus. We now show that this relative change in the level of the student's activities is in accordance with economic theory.

The Le Chatelier Principle

We can make use of the Le Chatelier Principle,[9] first applied in economics by Samuelson (1951).[10] The principle can be stated as follows: "... if one alters one of the parameters (pressure, temperature of any one compound, etc) of a system of physical or chemical equilibrium, the remaining parameters will adjust so as to counteract the disturbance".[11]

As a corollary to the Le Chatelier principle, termed "the Le Chatelier Principle II", Leblanc and Moeseke (1976) proved that if the marginal cost (in terms of resource inputs) of an activity decreases the level of an activity is increased, and vice versa.[12] In example 7 the per unit cost c_{21} increased by

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9 The Le Chatelier Principle was originally formulated by the French chemist Le Chatelier in 1884.

10 As suggested by Becker (1965). Moeseke (1985) has provided a numerical illustration.

11 quoted in Leblanc and Moeseke (1976) p. 143.

12 ibid p. 145.

1.7 cents/min (from 2.8 to 4.5) and the corresponding change in the net imputed value of time use to meet the relaxation social requirement on campus, cf section 6.3, is [(4.4 - 0) - (3.9 - 1.1)] = 1.6 cents/min. This is equivalent to the price change of an available resource specified, for classical optimization, in terms of the Lagrange multipliers at their optimal values.[13]

Change in the net imputed value of time use = 1.6 cents/min - an increase.
Change in the level of the activity = -360 mins - a decrease.

An increase of 57.1 per cent in the net imputed value of time use to meet the relaxation social requirement on campus results in a decrease of 100 percent in the level of the activity, in accordance with the Le Chatelier Principle II. Hence on campus there is a shift of 360 mins from relaxation to study.

This transfer of time use on campus could be regarded as an induced transfer of a scarce resource. An increase in the net imputed value of time use for meeting the relaxation social requirement on campus brought about a decrease in the activity. Because the total relaxation requirement is now met in the household environment this means that the maximum available amount of time use for meeting the study requirement in the household environment is 140 minutes. Hence the remaining study requirement can only be met on campus. This explains why 72 percent of the study requirement is met on the environment with the higher relative cost for this activity.

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13 ibid p. 143. The authors comment that "the interpretation of the Lagrange multipliers (or dual variables) as a price system for available resources is known."

Case (2)

Suppose that, relative to the cost matrix of example 6, there is a decrease in the per unit cost of meeting the study social requirement on campus. $c_{11} = 1.4$. We can summarize the result briefly:

Change in the net imputed value of time use = -0.2 cents/min - a decrease.
Change in the level of the activity = 360 mins - an increase.

A 12.5 per cent decrease in the net imputed value of time use to meet the study social requirement on campus results in an increase of 100 per cent in the level of the activity, in accordance with the Le Chatelier Principle II.

Example 8

By way of comparison with example 6, the per unit costs of meeting the relaxation social requirement in the campus and household environments are interchanged.

The new cost matrix is given by:

TABLE 7-11 2 x 2 NEW COST MATRIX (2) - TIGHT MODEL

$$c = \begin{bmatrix} 3.2 & 3.9 \\ 2.7 & 2.8 \end{bmatrix}$$

Otherwise all parameters are as in Example 6

<u>Solutions</u> $x^*_{11} = 360$; $x^*_{12} = 0$; $x^*_{21} = 140$; $x^*_{22} = 460$;

$u^*_1 = 0$; $u^*_2 = 0.5$; $v^*_1 = 3.2$; $v^*_2 = 3.3$;

The value of the objective function, in cents, is

primal: (360 x 3.2) + (0 x 3.9) + (140 x 2.7) + (460 x 2.8)

= 2818

dual: (500 x 3.2) + (460 x 3.3) - 600(0.5)

1600 + 1518 - 300

= 2818

The agent's objective function is $ 28.18 per diem. This represents dissaving.

The value of social income, in cents, is

(500 x 3.2) + (460 x 3.3)

= 3118

= $31.18 per diem

Study now takes place in both the campus and household behaviour settings, with a shift of 360 minutes of time use to campus and away from household. Interestingly the relative per unit cost ratio between study in the two environments has not changed. Is this shift of time use in accordance with economic theory? The new cost matrix involves a switch in the relative cost ratios for relaxation in the campus and household behaviour settings. The corresponding changes in the net imputed values of time use to meet the relaxation social requirement on campus and in the household respectively are (in cents/min):

(1) (3.3 - 0) - (3.9 - 1.1) = 0.5 an increase

(2) (3.3 - 0.5) - (3.9 - 0) = -1.1 a decrease

The increase of 0.5 cents/min results in a decrease of 360 mins.
The decrease of 1.1 cents/min results in an increase of 360 mins.

An increase of 17.9 percent in the net imputed value of time use to meet the relaxation social requirement on campus results in a decrease of 100 percent in the level of the activity. A decrease of 28.2 percent in the net imputed value of time use to meet the relaxation social requirement in the household environment results in an increase of 360 per cent in the level of the activity. Because there is an increase in the net imputed value of time use to meet the relaxation social requirement on campus, and a decrease in the net imputed value of time use to meet the relaxation social requirement in the household environment, the Le Chatelier Principle II predicts that the agent will meet this requirement by increased time use in the household environment. In fact the entire relaxation social requirement of 460 minutes is now met in the household environment. Because there is no slack capacity in this environment, 360 minutes of the study requirement is now met on campus. This could be regarded as an induced transfer of a scarce resource.

The results are set out in Table 7-12.

TABLE 7-12 **LE CHATELIER PRINCIPLE - SUMMARY OF RESULTS**

Example	Cost matrix				Decision variables				Shadow prices				Objective function
	c_{11}	c_{12}	c_{21}	c_{22}	x^*_{11}	x^*_{12}	x^*_{21}	x^*_{22}	u^*_1	u^*_2	v^*_1	v^*_2	(in cents per diem)
6	3.2	2.8	2.7	3.9	0	360	500	100	1.1	0	2.7	3.9	2748
7(1)	3.2	4.5↑	2.7	3.9	360	0↓	140	460	0	0.5	3.2	4.4	3324
7(2)	1.4↓	2.8	2.7	3.9	360↑	0	140	460	1.3	0	2.7	3.9	2676
8	3.2	3.9↑	2.7	2.8↓	360	0↓	140	460↑	0	0.5	3.2	3.3	2812

Example 9 $c_{ij} \lesseqgtr 0$

By way of contrast with example 4, the overall consistency constraint is not required for the tight model.

TABLE 7-13 **3 x 3 ACTIVITY MATRIX - TIGHT MODEL**

	to:	S	R	J		
	C	x_{11}	x_{12}	x_{13}	$\leqq$	360
from:	H	x_{21}	x_{22}	x_{23}	$\leqq$	600
	W	x_{31}	x_{32}	x_{33}	$\leqq$	220
		$\geqq$ 400	$\geqq$ 340	$\geqq$ 220		

TABLE 7-14 **3 x 3 COST MATRIX - TIGHT MODEL**

$$c = \begin{bmatrix} 3.2 & 2.8 & 1000 \\ 2.7 & 3.9 & 1000 \\ 1000 & 1000 & -9.5 \end{bmatrix}$$

Solutions $x^*_{11} = 0$; $x^*_{12} = 340$; $x^*_{13} = 0$; $x^*_{21} = 400$;

$x^*_{22} = x^*_{23} = x^*_{31} = x^*_{32} = 0$; $x^*_{33} = 220$

$u^*_1 = 0$; $u^*_2 = 0$; $u^*_3 = 9.5$; $v^*_1 = 2.7$; $v^*_2 = 2.8$

$v^*_3 = 0$

Note: Hereafter only positive values of the x^*_{ij} will be given.

The dollar value of the objective function is, for the primal:

(0 x 3.2) + (340 x 2.8) + (400 x 2.7) + (0 x 3.9) +

(220 x -9.5)

= -$0.58

The dollar value of the objective function is, for the dual:

(400 x 2.7) + (340 x 2.8) + (200 x 0) - (360 x 0) -

(600 x 0) - (220 x 9.5)

= 1080 + 952 - 2090

= -$0.58

Social Income = Value of social position + income from work

= (400 x 2.7) + (340 x 2.8) + (220 x 9.5)

= 1080 + 952 + 2090

= 4122 cents or $ 41.22 per diem.

Example 10

There is a change in the cost matrix, as shown in Table 7-15 below. Otherwise the parameters are unchanged.

TABLE 7-15 3 x 3 NEW COST MATRIX (1) - TIGHT MODEL

$$c = \begin{bmatrix} 3.2 & 3.9 & 1000 \\ 2.7 & 2.8 & 1000 \\ 1000 & 1000 & -9.5 \end{bmatrix}$$

Solutions $x^*_{11} = 140$; $x^*_{21} = 260$; $x^*_{22} = 340$; $x^*_{33} = 220$

$u^*_1 = 0$; $u^*_2 = 0.5$; $u^*_3 = 9.5$; $v^*_1 = 3.2$; $v^*_2 = 3.3$;

$v^*_3 = 0$

The dollar value of the objective function is $0.12, a dissaving.

Social Income = Value of social position + income from work

= (400 x 3.2) + (340 x 3.3) + (220 x 9.5)

= 1280 + 1122 + 2090

= $44.92 per diem

By way of comparison with Example 7, study is now distributed over both the campus, 140 minutes, and household, 260 minutes, environments. All relaxation takes place in the household environment, a shift away from campus. The lifestyles generates a higher social income $44.92, compared with the previous $41.22, per diem. Instead of a daily saving of $0.58 the student now incurs a daily expenditure of $0.12.

Example 11 The cost of a fixed life style

The economic model suggests that an optimizing agent should adapt his lifestyle to changes, for example changes in the cost structure. If the agent fails to respond flexibly to changes, then it is likely that the lifestyle will be suboptimal.

If the student is a person of habit, and unlikely to react to changes in cost structure, then living costs over the academic year are likely to increase even when the per unit costs decrease. Suppose the student's constant space-time behaviour pattern is described by the optimal solution to Example 10, given in Table 7-16 below.

TABLE 7-16 SPACE-TIME BEHAVIOUR PATTERN

	to:	S	R	J
from:	C	140	0	-
	H	260	340	-
	W	-	-	220

Suppose also that there are changes in the cost matrix as shown in Table 7-17 below. Because the student's behaviour pattern is not responsive to price changes the space time behaviour pattern of Table 7-16 is no longer optimal.

TABLE 7-17 3 x 3 NEW COST MATRIX (2) - TIGHT MODEL

$$c = \begin{bmatrix} 3.3 & 3.2 & 1000 \\ 2.7 & 2.8 & 1000 \\ 1000 & 1000 & -9.5 \end{bmatrix}$$

The dollar value of the objective function is, using the primal,

$$(140 \times 3.3) + (0 \times 3.2) + (260 \times 2.7) + (340 \times 2.8) + (220 \times -9.5)$$
$$= 462 + 0 + 702 + 952 - 2090$$
$$= \$0.26 \text{ per diem.}$$

Hence, although the average element ($c_{ij} > 0$) of the per unit cost matrix has decreased, there has been an increase in the student's daily expenditure. For the cost matrix of Table 7-17 the optimal behaviour pattern, is

$$x^*_{12} = 140; \quad x^*_{21} = 400; \quad x^*_{22} = 200; \quad x^*_{33} = 220$$

For this optimal lifestyle, the daily expenditure is, using the primal,

$$(0 \times 3.3) + (140 \times 3.2) + (400 \times 2.7) + (200 \times 2.8) + (220 \times -9.5)$$
$$= 0 + 448 + 1080 + 560 - 2090$$
$$= \$0.02\text{, a saving.}$$

Hence by keeping to a fixed behaviour pattern in the face of changes in the per unit activity costs, the student loses a potential net gain of \$0.28 per diem. Over the academic year of 40 weeks this is equivalent to an additional expenditure of \$(28 x 7 x 40)/100 = \$78.40.

7.4 Dimensionality Problem

There is a dimensionality problem inherent in any transportation model. Two aspects are considered.

1. In example 3, to take a particular case, of the six decision variables only four are included in the optimal decision. Two decision variables are excluded. This is an important issue.

TABLE 7-18 EXAMPLE 3 AS A STANDARD LINEAR PROBLEM

$$\overset{A}{\begin{array}{cccccc} x_{11} & x_{12} & x_{13} & x_{21} & x_{22} & x_{23} \end{array}} \qquad \qquad \mathbf{x} \qquad \qquad \mathbf{b}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 \end{bmatrix} \quad \begin{bmatrix} 290 \\ 0 \\ 0 \\ 210 \\ 100 \\ 360 \end{bmatrix} \quad \begin{bmatrix} 500 \\ 100 \\ 360 \\ -490 \\ -670 \end{bmatrix}$$

non-basic variables (x_{12}, x_{13})

An intuitive explanation is now provided. In Table 7-18 the 2 x 3 example, including solutions, is expressed as a standard linear problem, cf (5.11). There are eleven inequality contraints (two behaviour setting constraints, three social requirements constraints and six non-negativity constraints). We can move from vectors to a more geometric perspective. Each inequality constraint determines a certain closed hyperplane in six dimensional hyperspace. The region of six dimensional hyperspace defined by the $x_{ij} \geqq 0$ $i=1,2$ $j=1,2,3$ is the non-negative orthant of R^6. The region of feasibility is the convex polytope[14] determined by the intersection of the hyperplanes. The set of vertices which corresponds with the 4 x 4 submatrix of ***A*** is the basic feasible set. The optimal point will belong to this set, and, like all other points in the set, will include two zero coordinates corresponding to the two non-basic variables. For this set we calculate the value of the objective function at every vertex and find the one with the smallest value, namely (290, 0, 0, 210, 100, 360). x_{12} and x_{13} are the decision variables excluded from the optimal solution. The problem was set in 6-dimensional hyperspace, R^6, but the solution is given in 4-dimensional hyperspace, R^4. What is the social meaning of the inherent preclusion? The agent has a choice of activities. On the one hand there is a loss of dimensionality, while on the other the rational agent is able to select those environments, and only those, which allow social requirements to be met optimally. Four per unit costs are sufficient to determine the agent's expenditure pattern.

The result can be generalized. Suppose there are m environments and n social requirements, and therefore m + n constraints and mn activities. Then in the standard linear problem, **A** is the MxN matrix, consisting of M rows and N columns, where M = m+n and N = mn. **A** denotes a transformation from R^N space to R^M space. From a geometrical perspective, making use of Caratheodory's theorem, if S is a non-empty subset of M-dimensional space, then each point of the minimal convex set that contains S is a convex combination of at most M+1(= m+n+1) points in S.[15] That is, the set of vertices which corresponds

14 A polytope is a closed and convex set, since it is defined by the intersection of convex closed sets (the closed hyperplanes, or half sets).

15 Caratheodory's theorem is the fundamental dimensionality result in convexity theory. For a proof of the theorem and proofs of the corollaries

with the (M+1) x (M+1) submatrix of **A** is, at most, the basic feasible set. The transportation problem is a special case of the standard linear problem. In this type of time allocation model, for m environments and n social requirements, it can be shown that there will be at most m+n-1 decision variables in an optimal solution.

2. Furthermore, in this type of time allocation model where there are m environments and n social requirements, there is some heuristic advantage in putting m ≈ n. The relative loss of information about environments, and therefore the agent's behaviour, is least in models where m ≈ n. The conclusion is now justified intuitively. For both a 5 x 6 and a 2 x 15 model there are 30 decision variables. For the 5 x 6 model there are, at most, 10 optimal decision variables, compared with, at most, 16 for the 2 x 15 model. Suppose that for each model all social requirements are met in such a way that no environment is used to meet more than two social requirements. To be sure, the 2 x 15 model generates more optimal decision variables (16, compared with 10 for the 5 x 6 model). However for the 5 x 6 model, the optimizing agent has the choice of using two environment to meet each of four social requirements, while for the 2 x 12 model, two environments can be used to meet only one of the 15 social requirements. Clearly, by comparison with the 5 x 5 model, there is less information provided about environments in the 2 x 12 model. Given the assumption that environments shape behaviour this represents a significant limitation.

(continued)

see Rockafellar (1970) pp. 155-161.

7.5 The Per Unit Cost Matrix

How is the per unit cost matrix derived? The minimum dollar costs, classified by requirements, are obtained for a representative group of students. This could be in the previous year. Estimates allowing for inflation are then obtained for the current year, as shown in Table 7-19.

TABLE 7-19 ESTIMATED DOLLAR COSTS FOR THE ACADEMIC YEAR

		$
1.	Flatting	3000
2.	Food	2200
3.	Student Fees	800
4.	Course Requirements	600
5.	Personal Spending	2100
6.	Other	300
		9000

The costs have been aggregated into six classifications, based on the data from Living in Wellington Ko to noho i Whanganui-a-Tara (1989).

In Table 7-20 the six expenditure classifications of Table 7-19 have been apportioned over two behaviour settings, campus and household. The aggregate expenditure for each behaviour setting is obtained by summing along rows, using the same six cost classifications as in Table 7-19.

In practice the detailed costs provided by invoices, receipts and other records would be assigned to either the campus or the household environments.

Some costs belong exclusively either to one or to the other. For example student fees are an expenditure associated with the campus environment, while flatting expenses are proper to the household environment. Other costs, such as food, belong to both and can be apportioned by observed budget data, including itemized receipts as supplied by, say, supermarkets. The data supplied can be assigned to either the campus or household environment. The $1090 represents food consumed on campus. This together with the $1110 for food consumed in the household forms the aggregate expenditure on food of $2200.

TABLE 7-20 ESTIMATED DOLLAR COSTS FOR THE ACADEMIC YEAR BY BEHAVIOUR SETTINGS

Classification / Behaviour setting	(1) Flatting	(2) Food	(3) Student Fees	(4) Requirements	(5) Personal Spending	(6) Other	Total
Campus		1090	800	600	750	60	3300
Household	3000	1110			1350	240	5700
Total	3000	2200	800	600	2100	300	9000

Table 7-21 is derived from Table 7-20. Table 7-21 introduces a further set of aggregates. For each of the two behaviour settings the aggregate cost is subdivided across rows into two components denoted by the study and relaxation social requirements. In practice, data would be obtained through the use of invoices, receipts and other records. The aggregate for each of the two social requirements can be found by summing dowm the appropriate column. For example, in Table 7-21, for the campus behaviour setting, the items Personal Spending and Other from Table 7-20, are apportioned over the study and relaxation requirements, using expenditure data. Of the remaining items of Table 7-20 associated with the campus behaviour setting, Student Fees and Course Requirements belong entirely to the study requirement, and Food belongs entirely to the relaxation requirement. The $1800 represents the cost of study on campus and is composed of Student Fees ($800), Course Requirements ($600), a Personal Spending amount ($380) and Other ($20). The $1800 is obtained by aggregating appropriate columns from Table 7-20, including components of Personal Spending and Other but excluding Food which is part of the cost of relaxation on campus.

TABLE 7-21 MINIMUM ESTIMATED COSTS FOR ACADEMIC YEAR

to: / from:	Study (S)	Relaxation (R)	Total
Campus (C)	$1800	$1500	$3300
Household (H)	$1200	$4500	$5700
Total	$3000	$6000	

From Table 7-21 we obtain daily estimated costs by dividing each amount of Table 7-21 by 280, the product of the number of weeks in the academic year, 40, and the number of days in the week, 7, and obtain average daily estimated costs. Results are provided in Table 7-22.

TABLE 7-22 2 x 2 MATRIX OF DAILY ESTIMATED DOLLAR COSTS

from: \ to:	S	R
C	6.4	5.4
H	4.3	16.0

To obtain the quantitative measures given by Table 7-23, time budgets are used using the methods of Szalai (1972) to find time actually spent in environments by a representative group of students, and then an average is obtained, as in other economic models. Of course there is no reason to suppose that the actual time use represents an optimal pattern. It should be noted that Table 7-23 is totally independent from previous cost data.

TABLE 7-23 2 x 2 MATRIX OF ESTIMATED TIME SPENT ON STUDENT ACTIVITIES IN MINUTES

from: \ to:	S	R	Total
C	200	190	390
H	160	410	570
Total	360	600	

Table 7-24 is obtained by dividing each element in Table 7-22 by the corresponding elements in table 7-23 and dividing by 100 in order to express the per unit cost in cents/min. Thus $c_{11} = (\frac{\$6.4}{200})/100 = 3.2$ cents/min; $c_{12} = (\frac{\$5.4}{190})/100 = 2.8$ cents/min.

TABLE 7-24 2 x 2 COST MATRIX IN CENTS PER MINUTE

	S	R
C	3.2	2.8
H	2.7	3.9

Table 7-2 used in the mathematical program for Example 1, section 7.2, is the same as Table 7-24.

**7.6 Activity Analysis

The activity analysis mode suggests intuitively how a change in the agent's production technology will affect the level of social income.

Example 12

The tight model is now formulated within the framework of activity analysis. The solution of example 10 is expressed in this mode. The basic variables can be expressed as activities, using the following transitions:

$$x^*_{11} \rightarrow x^*_1; \quad x^*_{21} \rightarrow x^*_4; \quad x^*_{22} \rightarrow x^*_5; \quad x^*_{33} \rightarrow x^*_9$$

The levels of the activities are, respectively:

$$x^*_1 = 140; \quad x^*_4 = 260; \quad x^*_5 = 340; \quad x^*_9 = 220$$

The intensity vector **x** is:

$\mathbf{x} = [\ 140 \quad 0 \quad 0 \quad 260 \quad 340 \quad 0 \quad 0 \quad 0 \quad 220\ ]^T$

The agent has the choice of nine activities. The optimal solution requires the agent to carry out the first, fourth, fifth and ninth activities.

The price vector **p** is:

$\mathbf{p} = [\ 3.2 \quad 3.3 \quad 0 \quad 0 \quad 0.5 \quad 9.5\ ]$

<u>Social Income</u>

The socio-economic production function is specified by the 6 x 9 matrix ***A***.

p*A*x is the value of the producton set.[16] In the time allocation models, this scalar can be interpreted economically as:

(1) saving, or dissaving, by the agent

(2) value of social position less the total value of the time endowments.

For (1) **p*A*** is the 1 x 9 vector

$[\ 3.2 \quad 3.3 \quad 0 \quad 2.7 \quad 2.8 \quad -0.5 \quad -6.3 \quad -6.2 \quad -9.5\ ]$

p*A*x is the scalar

$3.2(140) + 2.7(260) + 2.8(340) - 9.5(220)$

$= 448 + 702 + 952 - 2090$

16 Market costs are not explicitly stated.

$$= \boxed{\begin{array}{c}2102\\ \text{Expenditure}\end{array}} - \boxed{\begin{array}{c}2090\\ \text{Income from}\\ \text{Work}\end{array}}$$

$$= 12$$

The agent dissaves $0.12 per diem.

This is equal to the value of the objective function obtained for example 8.

Hence $p\boldsymbol{A}x = \sum_k c_k x_k$, where $c_k x_k > 0$ k=1, ..., N-1 denotes expenditure

and $c_k x_k < 0$ k=N denotes income

In example 12, N=9

For (2)

$$p \equiv [\; v_1 \;\; v_2 \;\; v_3 \;\; u_1 \;\; u_2 \;\; u_3 \;]$$

$$\boldsymbol{A} \equiv \begin{bmatrix} A \\ \hline -B \end{bmatrix}$$

$$\mathbf{vBx} = 3.2(140) + 3.2(260) + 3.3(340)$$

$$= \boxed{\begin{array}{l}\;\;2402\\ \text{value of}\\ \text{social}\\ \text{position}\end{array}}$$

$$\mathbf{uAx} = + (-0.5)(260) + (-0.5)(340) + (-9.5)(220)$$

$$= \boxed{\begin{array}{c} -2390 \\ \text{total value} \\ \text{of the time} \\ \text{endowment} \end{array}}$$

Hence $\mathbf{vBx} - \mathbf{uAx} = \mathbf{pAx} = \mathbf{pAx}^{+} - \mathbf{pAx}^{-}$

where $\sum_{k=1}^{N-1} c_k x_k$ is denoted by $\mathbf{pAx}^{+}$,

$c_N x_N$ is denoted by $\mathbf{pAx}^{-}$,

and the + and - signs specify expenditure and income respectively.

Social Income. = value of social position + net income

= $\mathbf{vBx} + \mathbf{pAx}^{-}$

= 2402 + 2090

= 4492

= \$44.92 per diem

Expenditure + total value of the time endowment = $\mathbf{pAx}^{+} + \mathbf{uAx}$

= 2102 + 2390

= 4492

= value of social income

2.7 Summary

1. The Slack Model

In the slack model time endowments do not define an exact distribution. This entails some under-utilization of capacity for at least one source. Hence in the slack model at least one shadow price associated with a constraint at a source is necessarily zero, and possibly all are zero, as in the case of the first example. In the particular case of the first example the quantitative measure of Social Income was equal to the money expenditure. In the slack model, for cases where at least one $c_{ij} < 0$, the introduction of an additional constraint with the associated shadow price w, makes it possible to provide a non-zero value of the time endowment, even for those instances where slack capacity exists over all behaviour settings. Hence, for the slack model, supposing unused capacity at all sources (the extreme case where all associated shadow prices are necessarily zero) it becomes possible to obtain a quantitative measure of social income in terms of both primal and dual which necessarily exceeds money expenditure. In the slack model the b_j define a distribution but the a_i do not. The focus is on a particular agent.

2. The Tight Model

In the tight model both the b_j and the a_i define distributions. In this model no shadow price is necessarily zero since there is no unused capacity at any source. The focus is on an average agent.

CHAPTER 8

NUMERICAL EXAMPLES - (2) 5 X 5 AND 5 X 6 MODELS

8.1 INTRODUCTION

The models developed in the preceding chapter are now extended. They incorporate the five behaviour settings (environments) and five requirements of social position as described in Chapter 4. As in Chapter 7, there is a slack model and also a tight model. For the slack model, the effects of changing one endowment are examined in Section 8.2. A 5 x 5 tight model is introduced in Section 8.3. The determination of selected parameters is provided, with the following chosen as representative: $b_1 = 100$; $b_3 = 500$; $a_1 = 390$. In section 8.4 a step-by-step approach is used to derive a 5 x 5 cost matrix from two sets of data, obtained independently. It so happens in the larger models that the activity matrix is 5 x 5. However the classifications for behaviour settings and environments are flexible enough to allow for extensions. There is no theoretical reason why the activity matrix has to be square in terms of behaviour setttings and social requirements. To make the point, 5 x 6 examples for both slack and tight models are introduced and solved. To motivate the starred section 8.5 a case study, "the resourceful student", is provided. The models are extended to include changes in technology. Themes introduced in this chapter, namely changes in a parameter and changes in the technology matrix, are developed in Chapter 9.

8.2 A 5 x 5 AND 5 x 6 SLACK MODELS

As in the numerical examples of Chapter 7, endowments and requirements are measured in minutes, and costs are expressed in cents/minute, both taken over the 16 hour waking day. Income is net of tax, and is regarded as a negative cost. A wage rate of 11.0 cents/min is equivalent to $264.00 for a 40 hour working week. The optimal solutions quantify the agent's consumption patterns.

Example 1

TABLE 8-1 STATEMENT OF PROBLEM - SLACK MODEL

Behaviour settings (environments) a_i	min (1)	hrs (2)	min (2)	Social requirements b_j	min (1)	hrs (2)	min (2)
State (S)	550	9	10	Health (HL)	100	1	40
Household (H)	470	7	50	Job (J)	220	3	40
Private Enterprise (P)	250	4	10	Academic (A)	500	8	20
Voluntary Associations (V)	100	1	40	Socio-cultural (SC)	100	1	40
Workplace (W)	300	5	0	Commercial (C)	40	0	40
Total	1670	27	50		960	16	0

TABLE 8-2 **5 x 5 ACTIVITY MATRIX**

		to:						
		HL	J	A	SC	C		
	S	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	$\leqq$	550
	H	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	$\leqq$	470
from:	P	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	$\leqq$	250
	V	x_{41}	x_{42}	x_{43}	x_{44}	x_{45}	$\leqq$	100
	W	x_{51}	x_{52}	x_{53}	x_{54}	x_{55}	$\leqq$	300
		$\geqq$	$\geqq$	$\geqq$	$\geqq$	$\geqq$		
		100	220	500	100	40		

The aggregate time constraint is:

$$\Sigma x_{ij} \leqq 960 \quad \Big| \quad w \tag{8.1}$$

TABLE 8-3 **5 x 5 COST MATRIX**

$$c = \begin{bmatrix} 5.2 & 1000 & 2.0 & 3.2 & 10.0 \\ 6.7 & 1000 & 3.5 & 3.9 & 17.3 \\ 2.2 & 1000 & 18.1 & 3.9 & 14.9 \\ 2.3 & 1000 & 3.7 & 3.9 & 10.0 \\ 1000 & -11.0 & 1000 & 1000 & 1000 \end{bmatrix}$$

TABLE 8-4 SOLUTIONS TO EXAMPLE 1

to: Social requirement / from: behaviour setting		Health HL	Job J	Academic A	Socio-cultural SC	Commercial C	u^*_i
State	S			500	50		0.7
Household	H						0
Private Enterprise	PE	100					0
Voluntary Associations	VA				50	40	0
Workplace	W		220				0
		13.2	0	13.7	14.9	21.0	$w^* = 11.0$

For subsequent examples data and solutions will be presented more concisely.

As a guide, the solutions of Example 1 are presented in compact form.

(M) denotes multiple solutions.

$x^*_{13} = 500$; $x^*_{14} = 50$; $x^*_{31} = 100$; $x^*_{44} = 50$; $x^*_{45} = 40$; $x^*_{52} = 220$

$u^*_1 = 0.7$; $u^*_2 = 0$; $u^*_3 = 0$; $u^*_4 = 0$; $u^*_5 = 0$; $v^*_1 = 0$; $v^*_2 = 0$;

$v^*_3 = 13.7$; $v^*_4 = 14.9$; $v^*_5 = 21.0$; $w^* = 11.0$ (M)

The objective function, in cents, is

primal $(500 \times 2.0) + (50 \times 3.2) + (100 \times 2.2) + (50 \times 3.9) + (40 \times 10.) - (220 \times -11.0)$

$= 1000 + 160 + 220 + 195 + 400 - 2420$

$= -445$

dual $(13.2 \times 100) + (0 \times 220) + (13.7 \times 500) + (14.9 \times 100) + (21.0 \times 40) - (0.7 \times 550) - (0 \times 470) - (0 \times 250) - (0 \times 100) - (0 \times 300) - (11.0 \times 960)$

$= 1320 + 0 + 6850 + 1490 + 840 - 385 - 0 - 0 - 0 - 0 - 10560$

$= -445$

The student's objective function is -$4.45 per diem. This represents a saving.

By definition, Social Income = Value of social position plus money income (1)

$= vb_n + c_w x_w$

$= c_n x_n + 960w + ub_m$

= Expenditure + Total value of time endowment (2)

Social Income (1), in cents = $(13.2 \times 100) + (0 \times 220) + (13.7 \times 500) + (14.9 \times 100) + (220 \times 11.0)$

$= 1320 + 0 + 6850 + 1490 + 840 + 2420$

$= 12920$

= $129.20 per diem

Social Income (2), in cents = (500 x 2.0) + (50 x 3.2) + (100 x 2.2) + (50 x 3.9) + (40 x 10.0) + (960 x 11.0) + (0.7 x 550) + (0 x 470) + (0 x 250) + (0 x 100) + (0 x 300)

= 1000 + 160 + 220 + 195 + 400 + 10560 + 385 + 0 + 0 + 0 + 0

= 12920

= $129.20 per diem

Money Income represents 18.73 per cent of Social Income.

What happens if the aggregate time constraint of example 1 is relaxed?

The agent will work for 300 minutes, using up all the workplace time endowment. To meet the remaining social requirements the agent uses 740 minutes. The total amount of time the agent uses is 1040 minutes, which violates the assumption of the 16 hour waking day.

$x^*_{13} = 500$; $x^*_{14} = 50$; $x^*_{24} = 50$; $x^*_{31} = 100$; $x^*_{45} = 40$; $x^*_{52} = 300$;
$u^*_1 = 0.7$; $u^*_2 = 0$; $u^*_3 = 0$; $u^*_4 = 0$; $u^*_5 = 11.0$; $v^*_1 = 2.2$; $v^*_2 = 0$;
$v^*_3 = 2.7$; $v^*_4 = 3.9$; $v^*_5 = 10.0$ (M)

Value of the objective function = -$13.25, a saving.

We now examine the results of two successive decreases in the time endowment associated with the state behaviour setting. Each decrease is of 50 minutes, so the maximum endowments, a_1, are now 500 and 450 respectively. There are no further changes from the data of Example 1.

<u>Example 2</u> $a_1 = 500$

<u>Primal and Dual Solutions</u>

Solutions $x^*_{13} = 500$; $x^*_{14} = 0$; $x^*_{24} = 40$; $x^*_{31} = 100$; $x^*_{44} = 60$; $x^*_{45} = 40$; $x^*_{52} = 220$

$u^*_1 = 0.7$; $u^*_2 = 0$; $u^*_3 = 0$; $u^*_4 = 0$; $u^*_5 = 0$; $v^*_1 = 13.2$;

$v^*_2 = 0$; $v^*_3 = 13.7$; $v^*_4 = 14.9$; $v^*_5 = 21.0$; $w^* = 11.0$; (M)

As previously the student meets the academic social requirement completely from the state (university) behaviour setting. However it is no longer possible to use this behaviour setting to meet part of the socio-cultural requirement as well. Hence x^*_{14}, previously 50, now becomes zero. A new decision variable, x^*_{24} enters the optimal solution, so that the socio-cultural requirement is met partly from the household environment, together with an increased amount of time use in the voluntary associations behaviour setting. There are no changes in the shadow prices. The objective function is - 410.00 cents, or -$4.10 per diem, which represents a 7.9 percent decrease in savings. Social income is unchanged. The increase in cost of (40 x 3.9) + (10 x 39) - (50 x 3.2) = $0.35 is balanced by the decrease (550 - 500) x 0.7 = $0.35 in the value of the time endowment associated with the state behaviour setting.

<u>Example 3</u> $a_1 = 450$

<u>Primal and Dual Solutions</u>

$x^*_{13} = 450$; $x^*_{23} = 50$; $x^*_{31} = 100$; $x^*_{44} = 60$; $x^*_{45} = 40$; $x^*_{52} = 220$

$u^*_1 = 1.5$; $u^*_2 = 0$; $u^*_3 = 0$; $u^*_4 = 0$; $u^*_5 = 0$; $v^*_1 = 13.2$; $v^*_2 = 0$;

$v^*_3 = 14.5$; $v^*_4 = 14.9$; $v^*_5 = 21.0$; $w^* = 11.0$ (M)

An additional decision variable, x^*_{23} is added to the optimal solution. The values of the other decisions variables remain unchanged. It is no longer possible for the student to meet the academic social requirement by making use of the university environment alone. Meeting the new academic requirement means that the value of time use in the state environment changes. The objective function is -335 cents or - $3.35 per diem, which represents a decrease of 18.3 percent in savings compared with Example 2. Social income is $133.20. The increase can be explained by the need to use the relatively more costly household environment for study, as a result of the decrease in the maximum available time endowment in the state (university) environment. Money income represents 18.16 percent of Social Income, a decrease. The household environment is now used for study, as well as for meeting part of the socio-cultural requirement.

Example 4

It was mentioned in section 7.2 that the activity matrix does not have to be square in terms of environments and social requirements, and a 2 x 3 example was solved. A further example, using a 5 x 6 matrix is provided:

(1) As before, the additional social requirement is civic activity, denoted by CA, where the agent voluntarily contributes a regular review article on recently published fiction. The agent can meet this non-income generating requirement in any environment except workplace.

TABLE 8-5 5 x 6 ACTIVITY MATRIX

	to:	HL	J	A	CA	SC	C		
from:	S	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	$\leqq$	550
	H	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	$\leqq$	470
	P	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	x_{36}	$\leqq$	250
	V	x_{41}	x_{42}	x_{43}	x_{44}	x_{45}	x_{46}	$\leqq$	100
	W	x_{51}	x_{52}	x_{53}	x_{54}	x_{55}	x_{56}	$\leqq$	300
		$\geqq$	$\geqq$	$\geqq$	$\geqq$	$\geqq$	$\geqq$		
		100	220	500	50	50	40		

The aggregate time constraint of example 1 is relaxed.

TABLE 8-6 5 x 6 COST MATRIX

$$\begin{bmatrix} 5.2 & 1000 & 2.0 & 3.0 & 3.2 & 10.0 \\ 6.7 & 1000 & 3.5 & 3.4 & 3.9 & 17.3 \\ 2.2 & 1000 & 18.1 & 4.0 & 3.9 & 14.9 \\ 2.3 & 1000 & 3.7 & 3.5 & 3.9 & 10.0 \\ 1000 & -11.0 & 1000 & 1000 & 1000 & 1000 \end{bmatrix}$$

Solutions (M) denotes multiple solutions.

$x^*_{13} = 500$; $x^*_{15} = 50$; $\boxed{x^*_{24} = 50}$; $x^*_{31} = 100$; $x^*_{46} = 40$; $x^*_{52} = 300$

$u^*_1 = 0.7$; $u^*_2 = 0$; $u^*_3 = 0$; $u^*_4 = 0$; $u^*_5 = 11.0$; $v^*_1 = 2.2$; $v_2 = 0$;

$v^*_3 = 2.7$; $v^*_4 = 3.4$; $v^*_5 = 3.9$; $v^*_6 = 10.0$ (M)

The civic activity social requirement (CA) is met in the household environment. The agent saves $13.50 per day.

(2) Although the per unit cost of meeting the civic activity social requirement is lowest in the state behaviour setting, the optimizing agent will not use this environment unless the per unit cost falls below 2.7 cents/min. Suppose c_{14} is 2.6 cents/min, cet par.

Solutions

$x^*_{13} = 500$; $\boxed{x^*_{14} = 50}$; $x^*_{25} = 50$; $x^*_{31} = 100$; $x^*_{46} = 40$; $x^*_{52} = 300$

$u^*_1 = 0.8$; $u^*_2 = 0$; $u^*_3 = 0$; $u^*_4 = 0$; $u^*_5 = 11.0$; $v^*_1 = 2.2$; $v^*_2 = 0$;
$v^*_3 = 2.8$; $v^*_4 = 3.4$; $v^*_5 = 3.9$; $v^*_6 = 10.0$ (M)

The civic activity requirement (CA) is met in the state environment, as indicated by the boxed x^*_{14}. The agent saves $13.55 per day.

(3) Suppose c_{44} falls from 3.5 to 3.3 cents/min. cet par. While the per unit cost of meeting the civic activity requirement in the state environment is 21.2 per cent less than the new cost of meeting this requirement in the voluntary associations environment, the optimizing agent will choose the latter environment.

Solutions

$x^*_{13} = 500$; $x^*_{15} = 50$; $x^*_{31} = 100$; $\boxed{x^*_{44} = 50}$; $x^*_{46} = 40$; $x^*_{52} = 300$

$u^*_1 = 0.7$; $u^*_2 = 0$; $u^*_3 = 0$; $u^*_4 = 0$; $u^*_5 = 11.0$; $v^*_1 = 2.2$; $v^*_2 = 0$;
$v^*_3 = 2.7$; $v^*_4 = 3.3$; $v^*_5 = 3.9$; $v^*_6 = 10.0$ (M)

The civic activity requirement (CA) is met in the voluntary associations environment. The agent saves $13.55 per day.

For each of cases (1) through (3) above, the aggregate constraint (8.1) is now introduced, and the corresponding solutions are denoted (4) through (6). The maximum time the agent can use to generate income is 220 minutes, so that less is saved per day. With the exception of $x^*_{52} = 220$, all corresponding decision variables are unchanged. Shadow prices change.

<u>Solutions</u>

(4) $x^*_{13} = 500$; $x^*_{15} = 50$; $\boxed{x^*_{24} = 50}$; $x^*_{31} = 100$; $x^*_{46} = 40$; $x^*_{52} = 300$

$u^*_1 = 0.7$; $u^*_1 = 0$; $u^*_3 = 0$; $u^*_4 = 0$; $u^*_5 = 0$; $v^*_1 = 13.2$; $v^*_2 = 0$;
$v^*_3 = 13.7$; $v^*_4 = 14.4$; $v^*_5 = 14.9$; $v^*_6 = 21.0$; $w^* = 11.0$ (M)

(5) $x^*_{13} = 500$; $\boxed{x^*_{14} = 50}$; $x^*_{25} = 50$; $x^*_{31} = 100$; $x^*_{46} = 40$; $x^*_{52} = 220$

$u^*_1 = 0.8$; $u^*_2 = 0$; $u^*_3 = 0$; $u^*_4 = 0$; $u^*_5 = 0$; $v^*_1 = 13.2$; $v^*_2 = 0$;
$v^*_3 = 13.8$; $v^*_4 = 14.4$; $v^*_5 = 14.9$; $v^*_6 = 21.0$; $w^* = 11.0$ (M)

(6) $x^*_{13} = 500$; $x^*_{15} = 50$; $x^*_{31} = 100$; $\boxed{x^*_{44} = 50}$; $x^*_{46} = 40$; $x^*_{52} = 220$

$u^*_1 = 0.7$; $u^*_2 = 0$; $u^*_3 = 0$; $u^*_4 = 0$; $u^*_5 = 0$; $v^*_1 = 13.2$; $v^*_2 = 0$;
$v^*_3 = 13.7$; $v^*_4 = 14.3$; $v^*_5 = 14.9$; $v^*_6 = 21.0$; $w^* = 11.0$

The solutions of the 5 x 6 models show that an optimizing agent will not always choose the lowest per unit cost environment to meet a social requirement. In particular the household, cases (1) and (4), and voluntary associations, cases (3) and (6), environments were chosen to meet the civic activities social requirement when, in fact, these were not the lowest per unit cost environments.

Example 5 - Changes in the cost matrix

We begin with the data for Example 1. In particular c_{13} = 2.0 cents/min where c_{13} measures the per unit cost of meeting the academic requirement by time use in the state (university) behaviour setting, equivalent to the cost of study on campus. There are now successive increases in the cost of study on campus so that, c_{13} = 3.2, 3.5 and 4.0 cents/min. The results are set out in tabular form.

TABLE 8-7 **CHANGES IN THE COST OF STUDY ON CAMPUS - SOLUTIONS**

(1) per unit cost c_{13}	(2) value of decision variables x^*_{13}	x^*_{14}	x^*_{23}	x^*_{31}	x^*_{43}	x^*_{44}	x^*_{45}	x^*_{52}	(3) value of objective function in cents
2.0	500	50	0	100	0	50	40	220	-445 (M)
3.2	450	100	50	100	0	0	40	220	135
3.5	450	100	50	100	0	0	40	220	270 (M)
4.0	0	100	470	100	30	0	40	220	279

A doubling of the cost of study on campus means that, for an optimal solution, the academic social requirement is met by using the household behaviour setting for 470 minutes and the voluntary associations behaviour setting for 30 mins. It is suboptimal to undertake any study on campus. In practice some minimum time for attendance at university (practical work in a laboratory, mandatory attendance at class tests...) will be required, and this can be allowed for in the model by including a further constraint so that, for example $x_{13} \geqq 100$.[1] While an increase in the per unit cost of studying on campus from 3.2 to 3.5 cents/min, equivalent to a 9.4 per cent increase, does not entail any change in the choice or level of decision variables, it does result in a 100 percent increase in the value of the objective function.

An alternative solution is given for a per unit cost of 3.5 cents/min:
$x^*_{13} = 30$; $x^*_{14} = 100$; $x^*_{15} = 40$; $x^*_{23} = 470$; $x^*_{31} = 100$; $x^*_{52} = 220$.
As previously, the value of the objective function is 270 cents.

8.3 5 x 5 AND 5 x 6 TIGHT MODELS

As in the numerical examples of section 7.3 the focus is now on the activities of the average student. For every activity we can measure the amount of time that is used in a particular environment by a representative student in order to meet social requirements. In the slack model the starting point was the time available at sources. By way of contrast the source constraints for the tight model measure the actual time use, and are termed "effective constraints". The method by which the mean maximal actual time use of a group was used to obtain the effective constraint was outlined in section 7.3 where the effective constraint was defined, for a given behaviour setting, as the difference between the total available capacity and the slack capacity. In the tight model each endowment constraint represents the maximal actual time use by the agent (the average student) and $\Sigma a_i = 960$. A case study, termed "the resourceful student" is now introduced. The relevant worked examples are 6, 7 and 9.

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1 Chapter 9 includes the addition of a further constraint.

Example 6

Determination of selected parameters

$b_1 = 100$

As mentioned in Chapter 4, the health category includes time use for personal hygiene, eating and drinking, physical training, yoga and incidental daytime sleep. In Table 8-6 the health social requirement is a minimum of 100 minutes, or 1 hr 40 mins. This represents 10.42 percent of total social requirements. This requirement is determined partly by peer group pressure, and partly from specifications and guidelines provided by influential persons who set conventional standards. For full time students there is some peer pressure to take part in competitive or social sports activities. Qualified medical and psychological professionals can, and do, act to set independent minimum health requirements for students, such as certain levels of nutrition and mental health. In so doing these professionals are applying skill and experience to determine objective standards which will help students to cope effectively with stress. Realism suggests that there will be convergence between these parameters and what is found by averaging across the time diaries of a representative group of full time internal students. What effectively restricts the magnitude of this constraint is the student's need to meet the academic social requirement.

$b_3 = 500$

The academic social requirement is determined by the university. To meet this requirement a student has to pass a certain number of papers over a minimum period of three, or four, years to graduate. How can this requirement be measured in minutes over the 16 hour waking day? Comparisons between papers could present some difficulty. There will be significant differences in workloads for individual papers. Then the components and style of a paper - lectures and tutorials, practical requirements such as laboratory work and field trips, computer time, hours of personal study - as well as duration (a semester, a year), will vary. Some universities use a standard unit of time, termed the credit point. This could be considered as equivalent to one hour per week, say, taken over the academic year. Papers are then assigned credit points. Course and degree requirements can then be expressed as credit points, aggregated over the necessary papers, for example minimum of 150 credit points over a three year degree, so that the "average" minimum student workload - 50 credit points per year - is equivalent to 50 hours per week. If the average student wants A passes, the social requirement becomes substantially higher. Increased time for this requirement incurs opportunity costs, such as less time for other social requirements. An academic requirement of 500 minutes, or 8 hr 20 mins, representing 52.08 percent of total social requirements, is equivalent to slightly under 42 hours taken over a 5 day week or slightly over 58 hours taken over a 7 day week.

$$a_1 = 390$$

The agent can select environments in which to meet social requirements. Choice will be determined by the suitability of the environment, and by the cost. In all cases there are effective restrictions on how much time can be spent in any environment. In the tight model the inequalities express the maximal actual time use and represent effective constraints. The restrictions are institutional (hours open for business) and practical (other social requirements may well demand a change of environment). We take each of these in turn. On campus there are usually up to date facilities with well developed health and fitness programs, as well as a canteen, all at subsidised cost, so the student, as rational agent, will make use of the state behaviour setting to meet some of the health social requirement. By comparison other behaviour settings could be less costly, but may provide poorer facilities. The actual distribution of the minimum requirement of 100 minutes can be measured by time use diaries. Suppose the student spends 50 minutes on health in the state behaviour setting (university). Then he cannot, even if he wishes, meet all the academic social requirements on campus. In any case he may not wish to. Study at home has advantages. But even so this is limited by the need to be physically present at lectures, tutorials, laboratory work, computer workshops. A further consideration is the distance of the household behaviour setting from campus. Given the need for some study on campus, the further the household from university, the relatively more time spent in the state environment compared with the household. There are constraints on the use of university facilities - 10 hours per day, say, for each of 5 days, equivalent to slightly over 7 hours taken over a 7 day week. The library may be open for longer. The constraint of 390 minutes, or 6 hrs 30 mins, for the state behaviour setting represents the maximal actual time use by the student. The method of quantifying the constraint was outlined in section 7.3. The data compiled from time budgets at Reading and Leicester Universities by Tomlinson and others (1973) provides a useful reference for the determination of selected parameters. On the other hand the Reading data cannot be regarded as representative for a different group of students in a different set of environments. Further, the concept of a social requirement was not a feature of the Reading research.

In the numerical examples of this chapter, the magnitude of the academic social requirement b_3 (= 500 minutes), which represents 52 percent of the total requirement, contributes to the way in which time is used in the state, relative to the household behaviour setting.

TABLE 8-8 **STATEMENT OF PROBLEM - 5 x 5 TIGHT MODEL**

At origins (sources) a_i	min (1)	hrs	min (2)	At destinations (sinks) b_j	min (1)	hrs	min (2)
State (S)	390	6	30	Health (H)	100	1	40
Household (H)	250	4	10	Job (J)	220	3	40
Private Enterprise (P)	75	1	15	Academic (A)	500	8	20
Voluntary Association (V)	25	0	25	Socio-cultural (SC)	100	1	40
Workplace (W)	220	3	40	Commercial (C)	40	0	40
Total	960	16	00		960	16	00

The cost matrix is the same as for Table 8-3, the aggregate time constraint (8.1) is relaxed, and the activity matrix is similar to that of Table 8-2 except that the constraints at sources are a distribution, as given in Table 8-6. The student has to choose environments in order to meet all requirements at minimum cost.

Solution to Case 1.

$x^*_{13} = 350$; $x^*_{15} = 40$; $x^*_{23} = 150$; $x^*_{24} = 100$; $x^*_{31} = 75$; $x^*_{41} = 25$;
$x_{*52} = 220$

$u^*_1 = 1.5$; $u^*_2 = 0$; $u^*_3 = 1.6$; $u^*_4 = 1.5$; $u^*_5 = 11.0$; $v^*_1 = 3.8$; $v^*_2 = 0$; $v^*_3 = 3.5$; $v^*_4 = 3.9$; $v^*_5 = 11.5$ (M)

The objective function, in cents, is

primal (350 x 2.0) + (40 x 10.0) + (150 x 3.5) + (100 x 3.9) + (75 x 2.2) + (25 x 2.3) + (220 x -11.0)

= 700 + 400 + 525 + 390 + 165 + 57.5 - 2420

= -182.5

dual (3.8 x 100) + (0 x 220) + (3.5 x 500) + (3.9 x 100) + (40 x 11.5) - (1.5 x 390) - (0 x 250) - (1.6 x 75) - (1.5 x 25) - (200 x 11.0)

= 380 + 0 + 1750 + 390 + 460 - 585 - 0 - 120 - 37.5 - 2420

= -182.5

The student's objective function is -$1.825 per diem. This represents a saving.

Social Income = Value of Social position + Income from employment

= (100 x 3.8) + (220 x 0) + (500 x 3.5) + (220 x 11.0) + (100 x 3.9) + (40 x 11.5)

= 380 + 0 + 1750 + 39 + 585 + 2420

= $51.74 per diem

Savings = Value of time - Value of Social Requirements

= (390 x 1.5) + (250 x 0) + (75 x 1.6) + (25 x 1.5) +

(220 x 11.0) - [(100 x 3.8) + (220 x 0) + (500 x 3.5) +

(100 x 3.9) + (40 x 11.5)]

= 31625 - 2980

= 182.5 cents, or $1.825 per diem

As expected, this corresponds to the optimal value of the objective function.

Example 7 - Changes in the Cost Matrix

Now the difficulties come. First, fewer jobs at overtime rates are available. There is a decrease in the wage rate. The new wage rate is equivalent to 10 cents/min. Since the value of the objective function is 37.5 cents per diem, there is dissaving. Then, following upon the decrease in income, a "user pays" system is introduced. The removal of subsidies effectively doubles all costs within the environment of the state behaviour setting. Endowments and social requirements are unchanged.

Solutions

$x^*_{13} = 390$; $x^*_{21} = 25$; $x^*_{23} = 110$; $x^*_{24} = 100$; $x^*_{25} = 15$; $x^*_{31} = 75$;
$x^*_{45} = 25$; $x^*_{52} = 220$
$u^*_1 = 0$; $u^*_2 = 0.5$; $u^*_3 = 5.0$; $u^*_4 = 7.8$; $u^*_5 = 10$; $v^*_1 = 7.2$; $v^*_2 = 0$;
$v^*_3 = 4$; $v^*_4 = 4.4$; $v^*_5 = 17.8$

The new decision variables x^*_{21}, x^*_{25}, x^*_{45} displace x^*_{15}, x^*_{41} previously obtained in the solutions to case (1).

The objective function is given by:

$$390(4) + 25(6.7) + 110(3.5) + 100(3.9) + 15(17.3) + 75(2.2) + 25(10) + 220(-10)$$

$$= 977.0$$

The student is not operating within the money budget. Dissaving is now $9.70 per diem.

Social Income = Value of social position + income (6.7)

Social Income in cents

$$100(7.2) + 220(0) + 500(4) + 100(4.4) + 40(17.8) + 220(10)$$

$$= 3872 + 2200$$

$$= 6072$$

= $60.72 per diem

Savings = Value of time endowments - value of social requirements (6.8)

Savings, in cents

$$= 390(0) + 250(0.5) + 75(5) + 25(7.8) + 220(10) - [100(7.2) + 220(0) + 500(4) + 100(4.4) + 40(17.8)$$

$$= 2895 - 3872$$

$$= 977.0$$

A dissaving of $9.70 per diem. As expected this corresponds to the optimal value of the objective function.

Interpretation of Shadow Prices

The shadow price for v^*_1 equals 7.2 cents/min. If the health social requirement were to be increased by one minute, the value of the objective function would increase by 7.2 cents. For case 2 dissaving would rise to $9.842 per diem. The dual variable v^*_1 is a measure of how much our student, as rational agent, would be prepared to pay for an additional unit of time to meet the health social requirement. Time is a scarce resource and the shadow price of 7.2 cents/min can be regarded as an efficiency price. The 7.2 cents/min is not a market price, but an implicit price made up of two components, the per unit market cost and the resource value of time use in a particular environment, interpreted as a differential rent. The student, as rational agent, would therefore be prepared to pay a premium price for time use to meet the health social requirement up to the value of the dual.

In the solution for case 2, the health requirement is met by using 25 minutes in the household environment and 75 minutes in the private enterprise environment. The respective market prices are 6.7 and 2.2 cents/min and the associated differential rents are 0.5 cents/min and 5.0 cents/min. The differential rents are shadow prices related to the use of a time endowment in a particular environment. Because the per unit cost of meeting the health social requirement is relatively lower in the private enterprise environment this environment confers an advantage to a cost minimizing agent. Hence it is assigned a relatively higher differential rent. In case 2, by comparison with case 1, the non-market price for meeting the health social requirement has increased from 3.8 cents/min to 7.2 cents/min. To meet the health social requirement in case 1, it was possible to use 25 minutes of the time endowment in the voluntary associations behaviour setting at a per unit market cost of 2.3 cents/min. In case 2 this is not possible. The 25 minutes is now used in the household behaviour setting at the higher per unit market cost of 6.7 cents/min. As expected, the increase in per unit market costs is associated with a decrease in differential rent from 1.5 to 0.5 for the repective environments, since the household environment confers a relative loss on time use of 4.4 cents/min.

$$v^*_1 = u_2 + c_{21} = 0.5 + 6.7 = 7.2 \text{ cents/min}$$
$$v^*_1 = u_3 + c_{31} = 5.0 + 2.2 = 7.2 \text{ cents/min}$$

The value of an additional minute to meet the health requirement is 7.2 cents/min, which is greater than the per unit market cost for either the private enterprise or the household environments.

Example 8 - 5 x 6 Tight Model

Suppose that the student has to meet an additional social requirement. As for example 4 this requirement is civic activity (CA), where the student voluntarily contributes a regular review article on recently published fiction. In Table 8-9 the magnitudes of social requirements are the same as for Table 8-5. However for the tight model, the total time endowment is 960 minutes, so that the endowment constraints are different from those of Table 8-5. To make the point numerical values are starred.

TABLE 8-9 5 x 6 ACTIVITY MATRIX

	to:	HL	J	A	CA	SC	C		
	S	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	$\leq$	390 *
	H	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	$\leq$	250 *
from:	P	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	x_{36}	$\leq$	75 *
	V	x_{41}	x_{42}	x_{43}	x_{44}	x_{45}	x_{46}	$\leq$	25 *
	W	x_{51}	x_{52}	x_{53}	x_{54}	x_{55}	x_{56}	$\leq$	220 *
		$\geq$	$\geq$	$\geq$	$\geq$	$\geq$	$\geq$		
		100	220	500	50	50	40		

The cost matrix is the same as for Table 8-6.

Solutions

$x^*_{13} = 350$; $x^*_{16} = 40$; $x^*_{23} = 150$; $\boxed{x^*_{24} = 50}$; $x^*_{25} = 50$; $x^*_{31} = 75$

$u^*_1 = 1.5$; $u^*_2 = 0$; $u^*_3 = 1.6$; $u^*_4 = 1.5$; $u^*_5 = 11.0$; $v^*_1 = 3.8$;
$v^*_2 = 0$; $v^*_3 = 3.5$; $v^*_4 = 3.4$; $v^*_5 = 3.9$; $v^*_6 = 11.5$

The student saves $2.075 cents per day. ?

By comparison with example 4 (1), the civic activity social requirement continues to be met in the household environment. As noted previously, this is not the lowest per unit cost environment for meeting the requirement. The student's lifestyle is more differentiated, with more activities taking place in the household environment. There are eight decision variables, an increase of two.

8.4 DERIVATION OF THE COST MATRIX

A step-by-step approach is used to derive the 5 x 5 cost matrix first given in Table 8-3. While this is clearly more detailed than the outline provided in Section 7.4, the difference is one of degree rather than of kind.

Table 8-10 is based on data compiled from time budgets at Reading and Leicester Universities for use in a model of student's daily activity patterns by Tomlinson and others (1973). Mean activity times were given in hours per working week of five days and four nights; the sum of the means was 110 hours. In the Reading data the mean for sleep, 33.3 hours, has been subtracted from the sums of means so that the data obtained by Tomlinson and others (1973) can be adapted to the 16 hour waking day of the time use model. The 29.1 hours classified as work by Tomlinson and others refers to study. As might be expected for the Reading data the mean number of hours for sleep each night, 8.325 hours, is close to the 8 hours allowed for sleep in the time use model, based on Szalai (1972).

TABLE 8-10 STUDENT'S DAILY ACTIVITY PATTERNS

	travel	work	eating	leisure	domestic activities	sport	shopping	miscellaneous
Time in hours	9.0	29.1	7.4	24.6	6.5	0.6	1.2	0.9

In Table 8-11 the classifications of Student's Daily Activity Patterns provided in Table 8-10 have been adapted to include paid work. The 18 hours allocated to earning income are over 5 days, equivalent to 3.6 hours per day. The opportunity cost of this change over 5 days is 9 hours each of study and leisure. Hence study and leisure are now reduced from 29.1 to 20.1 hours and from 24.6 to 15.6 hours, respectively. The classification Other is a catch-all for activities not previously classified.

TABLE 8-11 CLASSIFICATION OF DAILY ACTIVITIES

Activity	Time use - in hours over 5 days
1. Travel	9.0
2. Study	20.1
3. Paid Work	18.0
4. Eating	7.4
5. Leisure	15.6
6. Household Activities	6.5
7. Sport	0.6
8. Shopping	1.2
9. Other	0.9
Total (in hours)	79.3

In Table 8-12 the nine daily activities are classified by five social requirement categories. In some cases there is a single change of name only. The 20.1 hours previously classified as study now belong to the academic social requirement. In other cases the total time previously classified under a single heading is now divided across several categories. For example the 15.6 hours previously classified as leisure now belong to the health social requirement (7.8 hours) and the socio-cultural social requirement (7.8 hours). Estimates across two or more of the social requirement categories are arbitrary, as for example the breakdown of the 6.5 hours classified in Table 8-11 as household activities.

TABLE 8-12 DAILY ACTIVITIES CLASSIFIED BY SOCIAL REQUIREMENT CATEGORIES

Social requirements / Classification	health	academic	socio-cultural	commercial	work	Total
Travel	3.6	2.7	1.8	0.9		9.0
Study		20.1				20.1
Paid work					18.0	18.0
Eating	7.4					7.4
Leisure	7.8		7.8			15.6
Household activities	(0.4x6.5) = 2.6	(0.4x6.5) = 2.6	(0.1x6.5) = 0.65	(0.1x6.5) = 0.65		6.5
Sport			0.6			0.6
Shopping				1.2		1.2
Other			0.9			0.9
Total in hours	21.4	25.4	11.75	2.75	18	79.3
Equivalent time in mins over 16 hour waking day	259	308	142	33	218	960
Percent	27	32	15	3	23	100

Notes: 1. For each social requirement, the equivalent time in minutes, taken over the 16 hour waking day is obtained by multiplying the respective column total, given in hours, by 960/79.3. For example, in column one the 21.4 hours of the health social requirement, expressed in hours over 5 days, converts to 21.4 x 960/79.3 = 259 minutes taken over the 16 hour waking day.

2. Each equivalent time is then expressed as a percentage of the 16 hour waking day. The results are rounded to the nearest whole number.

In practice Table 8-13 could be obtained from time budgets by asking a further question, namely "What social requirement does this particular activity meet?". As well as a period of time use, the agent is also required to specify an environment. For each of the five social requirement classifications, the agent is asked to specify an environment for the separate activities that, when aggregated, make up total time used to meet that social requirement. For example, the student is required to specify what leisure activities take place in the state behaviour setting, in the household behaviour setting, and so on. As in any time budget the focus is on time use, so that it is sufficient to obtain the proportion of social requirement time use allocated to behaviour settings, rather than the precise amount. The proportional estimates in Table 8-13 are arbitrary.

TABLE 8-13 ESTIMATES OF PROPORTION OF SOCIAL REQUIREMENT ALLOCATED TO BEHAVIOUR SETTINGS

Social requirement / Behaviour setting	health	work	academic	socio-cultural	commercial
state	0.35		0.60	0.1	0.07
household	0.55		0.33	0.6	0.20
private enterprise	0.08		0.05	0.2	0.70
voluntary associations	0.02		0.02	0.1	0.03
workplace		1.0			
Total	1.00	1.0	1.00	1.00	1.00

Table 8-14 is obtained by using estimates to apportion each total social requirement of Table 8-12 across the state, household, private enterprise and voluntary associations respectively. In this way the separate components of each social requirement are allocated to endowments at sources. Example: How much time does an agent use in the state behaviour setting to meet the socio-cultural requirement? From Table 8-12 we can see that over the 16 hour waking day 142 minutes are used to meet the socio-cultural requirement. Table 8-13 indicates that 0.1 of the total socio-cultural requirement is estimated to be used up within the state behaviour setting. Hence, in order to meet the socio-cultural requirement the agent uses up 0.1 x 142 = 14.2 minutes in the state behaviour setting.

For most practical requirements, the daily estimated average times wold be expressed in whole numbers.

TABLE 8-14 MATRIX OF DAILY ESTIMATED AVERAGE TIME IN MINUTES FOR ACTIVITIES

Social requirement / Behaviour setting	health	work	academic	socio-cultural	commercial	Totals
state	90.65		184.80	14.20	2.31	291.96
household	142.45		101.64	85.20	6.60	335.89
private enterprise	20.72		15.40	28.40	23.10	87.62
voluntary associations	5.18		6.16	14.20	0.99	26.53
workplace		218				218
Totals	259	218	308	142	33	960

We now introduce Expenditure Tables. Table 8-15 is based on the cost of living for students. The estimates used represent minimum costs for tertiary students living in Wellington.[2] Table 8-15 incorporates three changes. All costs, with the exception of tuition fees, are increased by 20 percent, since the original estimates are minimum costs based on costs for the previous year. The item tuition fees (with Fees Grant) is increased to $600.00 to allow for increases in university tuition fees. The item flatting is subdivided into rent and non-rent components. As in the original estimates the academic year is taken as 37 weeks, including May and August vacations and public and university holidays.

2 Living in Wellington Ko to noho i Whanganui-a-Tara (1989)

TABLE 8-15 MINIMUM ESTIMATED DOLLAR COSTS FOR THE ACADEMIC YEAR

	non-rent expenses (1)	rent (2)	$
1. Flatting which includes: rent, heating and lighting	440.80	2220	2660.80
2. Food includes: preparation of meals, eating out			2300.00
3. Tuitions Fees including: Fees Grant			600.00
4. Student Fees consisting of: Association Fee $111.60; Services Levy $ 20.40; $132.00			132.00
5. Course requirements Consisting of: textbooks, stationery, course activities and equipment			480.00
6. Clothing which includes: shoes, repairs			420.00
7. Travel within city and travel home			600.00
8. Personal Spending which includes: dentist, vacation, toiletries, entertainment, incidentals			1998.00
Total Expenditure			$9190.80

Notes: 1. Costs have been aggregated into eight classifications.

2. For the classifications Flatting and Student Fees the component costs are indicated. For all other classifications only the totals are shown.

In Table 8-16 the expenditure items of Table 8-15 have been apportioned over four behaviour settings. In this way the aggregate expenditure for each behaviour setting is obtained by summing along rows.

TABLE 8-16 MINIMUM ESTIMATED COSTS FOR THE ACADEMIC YEAR BY BEHAVIOUR SETTINGS

Classification / Behaviour setting	(1) Flatting	(2) Food	(3) Tuition Fees	(4) Student Fees	(5) Course Requirements	(6) Clothing	(7) Travel	(8) Personal Spending	Total
State (S)		1000	600	132			(0.4x600) = 240	399.60	2371.60
household (H)	2660.80	1300					(0.3x600) = 180	399.60	4540.40
private (P) enterprise					480	420	(0.2x600) = 120	999	2019.00
voluntary (V) associations							(0.1x600) = 60	199.80	259.80
Total	2660.80	2300	600	132	480	420	600	1998	9190.80

Notes: 1. The behaviour setting categories correspond with those in Table 8-14. The classification of expenditure follows that given in Table 8-15. The expenditure total for each classification will of course be the same as for Table 8-15.

2. Some costs can be assigned directly to a behaviour setting. For example, all course requirements (5), are purchased at a bookshop which specializes in student needs.

3. Other costs are assigned over two, or more, behaviour settings. Meals are prepared and eaten in the household but the agent also eats meals on campus. Personal spending takes place in all behaviour settings. From receipts and other data we can relate each item of personal spending to a specific environment. For example, the $199.80 for personal spending in the voluntary associations' behaviour setting would include subscriptions and also expenses related to social activities.

4. Travel costs are determined differently. We assume the agent uses a car. Then the total cost is apportioned over behaviour settings using data obtained from time budgets. We measure, say, the time taken to travel to campus, and the time taken to travel from campus back to household. We do the same for any other travel related to state behaviour settings. We then express these totals as a proportion of the aggregate time spent travelling in the car and in this way a weighting is obtained. The weightings in Table 8-13 are arbitrary, given the lack of actual time budget data. The weightings for travel time are: state 0.4; household 0.3; private enterprise 0.2; voluntary associations 0.1. In principle the weightings are objective, and in practice can be obtained from time budgets.

5. An example of travel cost associated with household would be the expense involved in going to a picnic spot and returning.

6. For air travel we would assign the cost directly to the behaviour setting. Air travel is necessary when there is urgency, and the destination is relatively far. An example of air travel cost assigned to the voluntary association behaviour setting would be the expense involved in attendance at a funeral given that there is urgency, and the location is distant.

Table 8-17 makes use of the eight expenditure categories of Tables 8-15 and 8-16 to show how expenditure can be apportioned over social requirements. Table 8-16 showed how aggregates for each behaviour setting were derived using the seven classifications of costs for the academic year. Table 8-17 introduces a further set of aggregates. For each behaviour setting, the aggregate cost is subdivided across rows into four components denoted by the health, academic, socio-cultural and commercial social requirements. In practice, data would be obtained using invoices, receipts and other records. The aggregates for each of the four social requirements is then obtained by summing down the respective column. By way of example we consider expenditure on food. In Table 8-15, the total expenditure on food was given as $2300.00. In Table 8-16 this expenditure was shown to take place in the state ($1000) and household ($1300) behaviour settings. In Table 8-17 for the classification Food row (1) designates dollar costs in the state behaviour setting and row (2) designates dollar costs in the household behaviour setting. Totals are shown in the following row. The expenditure of $1000.00 which takes in the household behaviour setting is apportioned over the health ($928), socio-cultural ($52) and commercial ($20) social requirements. The expenditure of $1300.00 for the state behaviour setting is apportioned as follows: health ($1105) and socio-cultural ($195). In the first column, by summing vertically the components $928.00 and $1105.00, the total expenditure on food to meet the health social requirement, namely $2033.00, is obtained.

For the classification Personal Spending, rows (1) through (4) designate dollar costs in the state, household, private enterprise and voluntary associations behaviour settings respectively. Totals are shown in the following row. The sum of each of the row totals is provided in the right hand column. The total for personal spending in the state behaviour setting across all social requirements is $399.60. Total personal spending of $1998.00 can be obtained by summing row-wise ($399.60 + $399.60 + $999.00 + $199.80) or by summing column-wise ($473.00 + $479.60 + $269.40 + 776.00).

TABLE 8-17 MATRIX OF MINIMUM ESTIMATED DOLLAR COSTS FOR THE ACADEMIC YEAR BY SOCIAL REQUIREMENT AND EXPENDITURE CLASSIFICATION

Social requirement Classification	Health	Work	Academic	Socio-cultural	Commercial	Total
(1) Flatting	1064.32		798.24	532.16	266.08	2660.80
(2) Food ($2300)	(1) 928 1105 2023			(1) 52 (2) 195 247	(1) 20 20	1000.00 1300.00 2300.00
(3) Tuitions Fees			600			600.00
(4) Student Fees			132			132.00
(5) Course Requirements			480			480.00
(6) Clothing	34			237	149	420.00
(7) Travel	240		180	120	60	600.00
(8) Personal Spending ($1998)	(1) 206 (2) 223 (3) 38 (4) 6 473		(1) 157.60 (2) 76 (3) 204 (4) 42 479.60	(1) 20 (2) 89.60 (3) 27 (4) 132.80 269.40	(1) 16 (2) 11 (3) 730 (4) 19 776	399.00 399.00 999.00 199.00 1998.00
	3844.32		2669.84	1405.56	1271.08	9190.80

Notes: 1. For all cost classifications other than travel, receipts, invoices and other records are used.

2. From time budgets, respective time use for transport to meet the health, academic, socio-cultural and commercial social requirements are given by the ratios 0.4; 0.3; 0.2; 0.1 respectively. These weightings are used to apportion travel costs.

The eight expenditure classifications of Table 8-15 were apportioned over behaviour settings and over social requirements in Tables 8-16 and 8-17 respectively. Table 8-18 shows costs by behaviour settings and social requirements. Table 8-18 is significant because the eight expenditure classifications do not appear explicitly. That is, for each behaviour setting expenditure is apportioned over the health, academic, socio-cultural and commercial social requirements. For example $1230.00 is spent in the state behaviour setting in order to meet the health social requirement.

TABLE 8-18 **MATRIX OF MINIMUM ESTIMATED DOLLAR COSTS FOR THE ACADEMIC YEAR BY BEHAVIOUR SETTING AND SOCIAL REQUIREMENT**

Social requirement / Behaviour setting	Health	Work	Academic	Socio-cultural	Commercial	Cost of Behaviour Setting
State	1230.00		961.60	120	60	2371.60
Household	(1) 1400.00		130.00	320.60	29	
	(2) 1064.32		798.24	532.16	266.08	
	2464.32		928.24	852.76	295.08	4540.40
Private Enterprise	120.00		720.00	288.00	891.00	2019.00
Voluntary associations	30.00		60.00	144.80	25.00	259.80
Workplace						
Cost of social requirement	3844.32		2669.84	1405.56	1271.08	9190.8

Notes: 1. Time budgets are used to apportion travel and rent costs. For all other cost classifications receipts, invoices and other records are used.

2. From time budgets, respective time use for transport to meet the health, academic, socio-cultural and commercial social requirements are given by the ratios 0.4; 0.3; 0.2; 0.1 respectively. These weightings are used to apportion travel costs. For example, Table 8-16 shows that $240.00 was spent on

travel in the state behaviour setting. Hence, in Table 8-18, 0.4 x $240 = $96 is spent in the state behaviour setting to meet the health social requirement.

3. The $2660.80 for flatting, as given in Table 8-15, is apportioned across social requirements by the same method. In this case the respective ratios are 0.4; 0.3; 0.2; 0.1. Thus the component of flatting for meeting the health requirement in the household behaviour setting is 0.4 x $2660.80 = $1064.32. Similarly the component of flatting for meeting the socio-cultural requirement in the household behaviour setting is 0.2 x $2660.80 = $532.16.

4. Costs incurred in the household behaviour setting are of two types, (1) non-rent and (2) rent. The non-rent costs, are made up of Food $1300, Travel $180 and Personal Spending $399.6, from Table 8-16. The total cost of $1879.6 is apportioned across the row (1). The rent costs are apportioned across the row (2).

5. Totals are shown in the following row. Thus the cost of the meeting the health requirement in the household behaviour setting is, reading down the first column, $1400 + $1064.32 = $2464.32.

6. The weightings for travel and rent are arbitrary. The cost allocations which do not depend on time budgets are also arbitrary.

Table 8-19 shows the detailed breakdown of dollar costs for social requirements over behaviour settings. In the left hand column, the numbers (1) through (8) designate the eight expenditure classifications of Table 8-15. The breakdown of these expenditures over each behaviour setting was initially shown in Table 8-16.

TABLE 8-19 COMPONENTS OF DOLLAR COSTS FOR THE ACADEMIC YEAR

Dollar costs of Social requirements / Dollar Costs over Behaviour settings	Health	Work	Academic	Socio-Cultural	Commercial	
State						
(2) 1000.00	928.00			52.00	20.00	
(3) 600.00			600.00			
(4) 132.00			132.00			
(7) 240.00	96.00		72.00	48.00	24.00	
(8) 399.60	206.00		157.60	20.00	16.00	
	1230.00		961.60	120.00	60.00	2371.60
Household						
(1) 2660.80	1064.32		798.24	532.16	266.08	
(2) 1300.00	1105.00			195.00		
(7) 180.00	72.00		54.00	36.00	18.00	
(8) 399.60	223.00		76.00	89.60	11.00	
	2464.32		928.24	852.76	295.08	4540.40
Private Enterprise						
(5) 480.00			480.00			
(6) 420.00	34.00			237.00	149.00	
(7) 120.00	48.00		36.00	24.00	12.00	
(8) 999.00	38.00		204.00	27.00	730.00	
	120.00		720.00	288.00	891.00	2019.00
Voluntary Associations						
(7) 60.00	24.00		18.00	12.00	6.00	
(8) 199.00	6.00		42.00	132.80	19.00	
	30.00		60.00	144.80	25.00	259.80
Totals	3844.32		2669.84	1405.56	1271.08	9180.80

Table 8-20 is obtained from Table 8-18 by dividing each element of the matrix first by 37 to convert costs over the academic year of 37 weeks to weekly costs, and then by 7 to obtain daily costs for the 16 hour working day. Table 8-20 includes daily earned income. This item denotes an income flow and is therefore negative. Table 8-20 is provided to show the transition from the original expenditure data of Table 8-15 expressed in dollars taken over the 37 week academic year.

TABLE 8-20 **MATRIX OF DAILY ESTIMATED DOLLAR COSTS**

Social requirement / Behaviour setting	Health (HL)	Job (J)	Academic (A)	Socio-cultural (SC)	Commerical (C)
State (S)	4.75		3.71	0.46	0.23
Household (H)	9.51		3.58	3.29	1.14
Private (P) enterprise	0.46		2.78	1.11	3.44
Voluntary (V) associations	0.12		0.23	0.56	0.10
Workplace (W)		-23.98			

Results rounded to the second decimal place.

Derivation of the Cost Matrix **c**

The Cost Matrix **c** has been derived from Tables 8-20 and 8-14, using rounding where appropriate. Each daily estimated dollar cost in Table 8-20 is divided by the corresponding daily estimated average time for activities in minutes given in Table 8-14. Each element c_{ij} of the Cost Matrix is measured in cents/min. Example: $c_{11} = 5.2$. The daily dollar costs of the health social requirement in the State behaviour setting are \$4.75. Since 90.7 minutes is given to this activity, $x_{11} = 90.7$ and $c_{11} =$ \$4.75/91.7 = 5.2 cents/min (1 decimal place). Because c_{52} denotes an income flow it is therefore negative.

therefore negative. It should be pointed out that Table 8-20 has been introduced as part of a step by step approach. However it is not necessary to work directly from Table 8-20. Costs can be obtained directly from Tables 8-18 and 8-14.[3] In principle, an Archimedian number can be assigned to a c_{ij} wherever the corresponding x_{ij} is non-available. The insertion of the large numbers 1000 is an appropriate computational device. For convenience the Cost Matrix **c** is given in Table 8-21. As expected this is the same as Table 8-3.

TABLE 8-21 5 x 5 COST MATRIX

$$\mathbf{c} = \begin{bmatrix} 5.2 & 1000 & 2.0 & 3.2 & 10.0 \\ 6.7 & 1000 & 3.5 & 3.9 & 17.3 \\ 2.2 & 1000 & 18.1 & 3.9 & 14.9 \\ 2.3 & 1000 & 3.7 & 3.9 & 10.0 \\ 1000 & -11.0 & 1000 & 1000 & 1000 \end{bmatrix}$$

**8.5 CHANGES IN TECHNOLOGY

Example 9 - The case of "the resourceful student"

The following example follows on from examples 6 and 7. Faced with changes in the cost matrix, the student is in difficulties. Yet time and money budgets have to be met.

What is to be done? The student must work within a money budget. The constraint of a 16 hour waking day is enforced, so that burning the midnight

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3 In practice there are only minor computational discrepancies (due to rounding errors) between the two approaches.

oil is not an allowable option. His first step is to find part time employment that yields a higher income. But work is not easy to find at all, let alone at a better wage rate, and the most our student can obtain is equivalent to a net rate of 10.0 cents/min. The student decides that the solution to the problem is to use less of the scarce resource, time, to meet the same social requirements. He attends a course on how to study more effectively. The aim of the course is to optimize information processing while minimizing time use. As a result the student's same academic social requirement now requires less time. The same output measured quantitatively in terms of assignments, computer programs, essays ... requires less input of time. Significantly the student is now able to "save" time, while keeping within a money budget. A change in technology reflects differences in the efficiency of the student's use of endowments to meet the academic social requirement. As in section 6.6, this change in the new technology is interpreted as equivalent to an increase in efficiency row-wise, so that the new technology matrix is denoted by the 10 x 25 matrix $\boldsymbol{A}^*$, where $\boldsymbol{A}^* = \boldsymbol{RA}$.

We suppose that as a result of the course on how to study more effectively there is a 100 percent increase in efficiency of time use for meeting the academic social requirement. This means that the student is able to meet this requirement using only half the time previously required. The academic social requirement of 500 minutes can now be met in 500/2.0 = 250 minutes, resulting in slack capacity of (500 - 250) = 250 minutes for the state behaviour setting. The shadow price at the source is therefore zero. We make the simplifying assumption that the time "saved" remains slack. This results in a saving of 23.00 cents per day.

Case 3

Primal and Dual Solutions

$x^*_{13} = 140$; $x^*_{21} = 25$; $x^*_{23} = 110$; $x^*_{24} = 100$; $x^*_{25} = 15$; $x^*_{31} = 75$;

$x^*_{45} = 25$; $x^*_{52} = 220$

$u^*_1 = 0$; $u^*_2 = 0.5$; $u^*_3 = 5.0$; $u^*_4 = 7.8$; $u^*_5 = 10.0$; $v^*_1 = 7.2$; $v^*_2 = 0$;

$v^*_3 = 2.0$; $v^*_4 = 4.4$; $v^*_5 = 17.8$

The objective function in cents is

$140(4) + 25(6.7) + 110(3.50) + 100(3.9) + 15(17.3) + 75(2.2) + 25(10) + 220(-10)$

$= -23.00$

The student is operating within the money budget. Saving is $0.23 per day. Social Income is $50.72 per day, a decrease, because v^*_3 has decreased from 4.0 to 2.0, while shadow prices of other social requirements are unchanged. For the state behaviour setting there is slack capacity of 250 minutes.

CHAPTER 9

EFFECTS OF CHANGES IN THE PARAMETERS

9.1 Introduction

The purpose of this chapter is to examine the effects of changes in the coefficients of the objective function, in the parameters, and the technology matrix. The effects of the addition of a new inequality constraint are also considered. The effects of these changes are interpreted mathematically, and economic interpretations are provided. In this chapter we focus on the activities of the average student. As indicated in section 7.3 the tight model is appropriate. By using numerical examples it becomes possible to shed new light on time allocation and social behaviour.

9.2 Effects of Changes in the Vector c

New program - vector matrix notation

Primal $\min \mathbf{cx}$

subject to

$$\mathbf{Ax} \geqq \mathbf{b} \quad \Big| \quad \mathbf{v}$$

$$\mathbf{x} \geqq \mathbf{0} \tag{9.1}$$

Dual max **vb**

subject to

$$\mathbf{vA} \leqq c \quad | \quad \mathbf{x}$$

$$\mathbf{v} \geqq 0$$

(9.2)

Geometrically, the effect of a change in the vector c is to change the direction of the supporting hyperplane. If the boundary point of the new optimal hyperplane **cx** is now different, then the original $\mathbf{x}^*$ is no longer optimal.

A basic 5 x 5 activity matrix is set out in Table 9-1. The related 5 x 5 cost matrix is specified in Table 9-2, where c denotes the matrix of coefficients. Table 9-3 shows the effects of increases in the per unit cost of time use for the state behaviour setting. For Tables 9-3, 9-4, 9-5 net income from work is -8.5 cents/min. In Table 9-3, each of the rows (1) through (9) indicates the effect of a percent increase in state behaviour setting costs. For row (1), the base, the costs are unchanged. For row (2) the costs have increased by 25 percent. Hence the new state behaviour setting per unit costs are:

$c_{11} = 6.5$; $c_{12} = 1000$; $c_{13} = 2.5$; $c_{14} = 4.0$; $c_{15} = 12.5$

For row (3). the original state behaviour setting, per unit costs have increased by 50 percent. For example the new c_{11} has increased from 5.2 to 7.8 cents/min. As mentioned in section 7.2, the insertion of the large numbers 1000 is merely a computational device. Hence there is no need to increase this per unit cost.

TABLE 9-1 5 x 5 ACTIVITY MATRIX

			b_1 HL	b_2 J	b_3 A	b_4 SC	b_5 C	
		to:						
	a_1	S	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	$\leqq$ 390
from:	a_2	H	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	$\leqq$ 250
	a_3	P	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	$\leqq$ 75
	a_4	V	x_{41}	x_{42}	x_{43}	x_{44}	x_{45}	$\leqq$ 25
	a_5	W	x_{51}	x_{52}	x_{53}	x_{54}	x_{55}	$\leqq$ 220
			$\geqq$ 100	$\geqq$ 220	$\geqq$ 500	$\geqq$ 100	$\geqq$ 40	

TABLE 9-2 5 x 5 COST MATRIX

$$\mathbf{c} = \begin{bmatrix} 5.2 & 1000.0 & 2.0 & 3.2 & 10.0 \\ 6.7 & 1000.0 & 3.5 & 3.9 & 17.3 \\ 2.2 & 1000.0 & 18.1 & 3.9 & 14.9 \\ 2.3 & 1000.0 & 3.7 & 3.9 & 10.0 \\ 1000.0 & -8.5 & 1000.0 & 1000.0 & 1000.0 \end{bmatrix}$$

Note: Net income from work is now 8.5 cents/min, a different rate from that initially used in Chapter 8.

Increases in costs of the state behaviour setting

Table 9-4 shows the effects of increases in costs of the state behaviour setting on shadow prices at both sources and sinks. In practice such increases could occur through the withdrawal of subsidies and the imposition of a "user pays" regime. One effect is that the imputed value of time use in the state behaviour setting decreases. This is in accord with economic theory. The shadow price measures the differential rent at a source. As the relative cost of using a particular environment increases, the relative advantage of using the associated behaviour setting decreases. State behaviour setting costs have increased therefore the shadow price associated with this behaviour setting will decrease. On the other hand the advantage of using some environments increases, and so the shadow prices associated with these environments, for example, household and voluntary associations show a relative increase. Table 9-4 shows that, in contrast with the shadow price of the state behaviour setting, the shadow prices of time endowments for the household, private enterprise and voluntary associations behaviour settings progressively increase, though the relative values of the shadow prices do not change at the same rate.

TABLE 9-3 EFFECTS OF INCREASES IN STATE BEHAVIOUR SETTING COSTS - OPTIMAL SOLUTIONS

percent increase in state behaviour setting costs		Optimal Decision Variables x^*_{13}	x^*_{15}	x^*_{21}	x^*_{23}	x^*_{24}	x^*_{25}	x^*_{31}	x^*_{41}	x^*_{45}	x^*_{52}
(1)	0	350	40	0	150	100	0	75	25	0	220
(2)	25	350	40	0	150	100	0	75	25	0	220
(3)	50	375	15	25	125	100	0	75	0	25	220
(4)	100	390	0	25	110	100	15	75	0	25	220
(5)	125	390	0	25	110	100	15	75	0	25	220
(6)	150	390	0	25	110	100	15	75	0	25	220

TABLE 9-4 EFFECTS OF INCREASES IN STATE BEHAVIOUR SETTING COSTS ON SHADOW PRICES

percent increase in state behaviour setting costs / shadow prices	0	25	50	100	125	150
u_1^*	1.5	1.0	0.5	0	0	0
u_2^*	0	0	0	0.5	1.0	1.5
u_3^*	4.5	4.5	4.5	5.0	5.5	6.0
u_4^*	4.4	4.4	5.5	7.8	8.3	8.8
u_5^*	8.5	8.5	8.5	8.5	8.5	8.5
v_1^*	6.7	6.7	6.7	7.2	7.7	8.2
v_2^*	0	0	0	0	0	0
v_3^*	3.5	3.5	3.5	4.0	4.5	5.0
v_4^*	3.9	3.9	3.9	4.4	4.9	5.4
v_5^*	11.5	13.5	15.5	17.8	18.3	18.8

TABLE 9-5 EFFECTS OF INCREASES IN STATE BEHAVIOUR SETTING COSTS ON SOCIAL INCOME

percent increase in state behaviour setting costs		Per diem values in cents			
		Dissaving	Net Income	Value of Social position	Social Income
(1)	0	367.5	1870	3270	5140
(2)	25	642.5	1870	3350	5220
(3)	50	890	1870	3430	5300
(4)	100	1307	1870	3870.8	5740.8
(5)	125	1502	1870	4240.8	6110.8
(6)	150	1697	1870	4610.8	6480.8

Table 9-5 compares the effects of percent increases in state behaviour setting costs on dissaving and social income. The increase in social income takes place because the increasing shadow prices at sinks give rise to increasing values of social position. Net money income remains constant and decreases as a percentage of social income from 36.4 to 28.9 percent.

9.3 Effects of Changes in the Vector b

In vector matrix-notation the problem now becomes

primal $\min \ \mathbf{cx}$

subject to

$$\left.\begin{array}{l}\mathbf{Ax} \geqq \bar{\mathbf{b}} \\ \mathbf{x} \geqq 0\end{array}\right| \ \mathbf{v} \tag{9.3}$$

dual $\max \ \mathbf{v}\bar{\mathbf{b}}$

subject to

$$\left.\begin{array}{l}\mathbf{v\,A} \leqq \mathbf{c} \\ \mathbf{v} \geqq 0\end{array}\right| \ \mathbf{x} \tag{9.4}$$

The vector **b** consists of inputs, the endowments at sources, and outputs, the requirements at sinks. Geometrically, the effect of changes in the vector **b** is to change the configuration of the convex polyhedron, that is, the simplex. This may result in a change in the optimal basic feasible solutions, x^*. Changes in net income can also result in a change in the optimal basic feasible solution, x^*.

The purpose of this section is to examine the effects of changes in the endowments. The tight model is used to examine the effects of changing the maximum constraints for the endowments associated with the private enterprise and voluntary associations behaviour settings. The combined endowment of these two behaviour settings is kept constant at 100 minutes. In Tables 9-6 and 9-7 the net per unit income from work is -11.0 cents/min. By way of comparison the net per unit income from work becomes -8.5 cents/min in Tables 9-8 and 9-9. As previously, a negative sign denotes income. While the time budget is enforced the money budget is not.

The optimal solutions for changes in selected time endowments are shown in Table 9-7. Net income from work is -11.0 cents/min. In Table 9-6, while dissaving increases there is no change in either the value of social position or social income. It so happens that net income equals the value of social position. In Tables 9-8 and 9-9, net income from work decreases to -8.5 cents/min. In Table 9-8, unlike Table 9-6, there is no saving, and dissaving increases. Table 9-9 shows a similar pattern to that of Table 9-8. The only variation is found in the levels of x^*_{31} and x^*_{41} which correspond to the constraints associated with the private enterprise and voluntary associations respectively, as relatively more of the time use endowment is transferred from the private enterprise to the voluntary associations behaviour setting. In

Table 9-8, for net income from work of -11.0 cents/min, the values of social position and social income, while less than in Table (1), are, as in Table 9-6, invariant for changes in the distribution of time use over the private enterprise and voluntary associations behaviour settings.

The numerical results of section 9.2 provide a justification for making the assertion that if social income increases, then savings would decrease (dissaving would increase), given the assumption of fixed money income. Does the converse follow? That is, does an increase in dissaving imply an increase in social income, given the assumption of fixed money income. In section 9.2 the cause of a decrease in saving was the increase in at least one per unit cost for the use of a particular environment. In this section the cause of a

TABLE 9-6 EFFECTS OF CHANGES IN THE DISTRIBUTION OF ENDOWMENTS c_{52} = -11.0 cents/min

	Behaviour Setting: private enterprise	Behaviour Setting: voluntary associations	Saving	Net Income	Value of Social position	Social Income
(1)	100	0	-185	2420	2420	4840
(2)	75	25	-182.5	2420	2420	4840
(3)	60	40	-181	2420	2420	4840
(4)	50	50	-180	2420	2420	4840
(5)	40	60	-179	2420	2420	4840
(6)	25	75	-177.5	2420	2420	4840
(7)	0	100	-175	2420	2420	4840

TABLE 9-7 EFFECTS OF CHANGES IN THE DISTRIBUTION OF ENDOWMENTS - OPTIMAL SOLUTIONS c_{52} = -11.0 cents/min.

	Behaviour Setting: private enterprise	Behaviour Setting: voluntary associations	x^*_{13}	x^*_{15}	x^*_{23}	x^*_{24}	x^*_{31}	x^*_{41}	x^*_{52}
(1)	100	0	350	40	150	100	100	0	220
(2)	75	25	350	40	150	100	75	25	220
(3)	60	40	350	40	150	100	60	40	220
(4)	50	50	350	40	150	100	50	50	220
(5)	40	60	350	40	150	100	40	60	220
(6)	25	75	350	40	150	100	25	75	220
(7)	0	100	350	40	150	100	0	100	220

TABLE 9-8 EFFECTS OF CHANGES IN THE DISTRIBUTION OF ENDOWMENTS c_{52} = -8.5 cents/min

	Behaviour Setting: private enterprise	Behaviour Setting: voluntary associations	Saving	Dissaving	Net Income	Value of Social position	Social Income
(1)	100	0		365	1870	2420	4290
(2)	75	25		367.5	1870	2420	4290
(3)	60	40		369	1870	2420	4290
(4)	50	50		370	1870	2420	4290
(5)	40	60		371	1870	2420	4290
(6)	25	75		372.5	1870	2420	4290
(7)	0	100		375	1870	2420	4290

TABLE 9-9 EFFECTS OF CHANGES IN THE DISTRIBUTION OF ENDOWMENTS - OPTIMAL SOLUTIONS c_{52} = -8.5 cents/min

	Behaviour Setting private enterprise	voluntary associations	x^*_{13}	x^*_{15}	x^*_{23}	x^*_{24}	x^*_{31}	x^*_{41}	x^*_{52}
(1)	100	0	350	40	150	100	100	0	220
(2)	75	25	350	40	150	100	75	25	220
(3)	60	40	350	40	150	100	60	40	220
(4)	50	50	350	40	150	100	50	50	220
(5)	40	60	350	40	150	100	40	60	220
(6)	25	75	350	40	150	100	25	75	220
(7)	0	100	350	40	150	100	0	100	220

decrease in saving is a change in the distribution of endowments. Clearly the numerical results of section 9.3 provide a counter example to the assertion that an increase in dissaving implies an increase in social income.

The optimal solutions for the tight model illustrate the following possibilities:

(1) A change in the relative distribution of time endowments results in a change in the objective function.

(2) A change in the relative distribution of time endowments need not be accompanied by a change in the value of social position.

From (1) and (2) it follows that a decrease in saving (or an increase in dissaving) need not be accompanied by a change in the value of social position.

9.4 THE ADDITION OF NEW CONSTRAINT

When an additional constraint is added to a linear program a new optimal solution cannot improve on the original one. Hence if the optimal program of the original linear program is still feasible for the new program, it remains optimal. It so happens that this is not the case for the numerical examples in this section. The economic interpretation is introduced by an example. A person is recovering from a car accident. She is required by her doctor to spend at least 30 minutes each day on exercises at the physiotherapy department at a hospital, a state institution. We can express this condition by the additional constraint $x_{11} \geqq 30$.

In section 9.4 the constraint is considered both as a minimum and as a maximum. In both situations, the agent makes use of time in the household and state behaviour settings respectively to meet at least part of the health requirement, as designated. The economic significance of a new constraint is that the agent, faced with a trade-off between time and money, is less likely to find an optimal pattern of behaviour, because the problem is more complex.

Table 9-10 contains optimal solutions for both situations. As previously, per unit net income from work is held constant at 8.5 cents/min. Optimal solutions are given for the addition of constraint x_{11}, cases (1) through (4), and for the addition of constraint x_{31}, cases (5) through (7). By way of comparison, the optimal solution x^* of the original (without constraint) problem is included. In Table 9-10, the addition of the new constraint increases the number of decision variables. For example, case (1) has two more decision variables than the optimal solution x^* of the original problem. Case (6) also has two more decision variables, but the decision variables of these two cases are not identical. Clearly the addition of a new constraint generates a more difficult problem for the agent. The pattern of optimal solutions is not at all obvious. For more extreme cases, where the value of the additional constraint approaches, or becomes equal to, the endowment constraint this need not be so. For case (4) the number of decision variables is not increased but remains the same as for x^*, though the decision variables are not identical. By way of comment, an increase in net salary to -11.0 cents/min will result in the same set of optimal solutions as given in Table 9-10.

Table 9-11 shows the effects the new constraint has on social income. We examine two hypotheses relating to the addition of a new constraint.

(1) For successive increases in the numerical value of the new constraint, the value of social position and social income will decrease, cet par.

(2) For successive increases in the numerical value of the new constraint, saving will decrease/dissaving will increase.

The results provide evidence that the first hypothesis is too strong. For the new constraint x_{11} the value of social position decreases as the value of the new constraint increases, from 20 to 50, to be sure, but remains constant for further increases. In Table 9-11 the evidence suggests the weaker hypothesis, namely, that for successive increases in the additional constraint the value of social position and social income need not increase. Hypothesis (2) holds for all the increases in x_{11}, cases (1) through (4) but is falsified for the increase in x_{31} where, in case (6), the level of dissaving actually decreases. Clearly this is a counter example. Hence the second hypothesis must be rejected. Why does the level of dissaving decrease? An increase in the maximum size of x_{31} from 20 to 70 makes it possible for the agent to meet the health social requirement by using an additional 50 minutes of the endowment from the private enterprise behaviour setting and 50 minutes less of the endowment from the state behaviour setting at a saving of 3.0 cents/min. This more than compensates for a slight per unit loss for a compensating use of endowments to make up the slack. An increase in net salary to -11.0 cents/min will result in the same set of optimal solutions as given in Table 9-10.

TABLE 9-10 THE ADDITION OF A NEW CONSTRAINT - OPTIMAL SOLUTIONS

New constraint	x_{11}	x_{31}	x^*_{11}	x^*_{13}	x^*_{15}	x^*_{23}	x^*_{24}	x^*_{31}	x^*_{34}	x^*_{41}	x^*_{45}	x^*_{52}	
(1)	20		20	350	20	150	100	75	0	5	20	220	
(2)	50		50	325	15	175	75	50	25	0	25	220	
(3)	70		70	305	15	195	55	30	45	0	25	220	
(4)	100		100	275	15	225	25	0	75	0	25	220	
(5)		20	55	295	40	205	45	20	55	25	0	220	(M)
(6)		50	25	325	40	175	75	50	25	25	0	220	(M)
(7)		70	5	345	40	155	95	70	5	25	0	220	(M)
Solution without constraint x^*				350	40	150	100	75		25		220	

TABLE 9-11 THE ADDITION OF A NEW CONSTRAINT - EFFECTS ON SOCIAL INCOME

New Constraint	x_{11}	x_{31}	Dissaving	Net Income	Value of Social position	Social Income
(1)	20		425.5	1870	2980	4850
(2)	50		552.5	1870	2820	4690
(3)	70		642.5	1870	2820	4690
(4)	100		777.5	1870	2820	4690
(5)		20	615	1870	3270	5140
(6)		50	480	1870	3270	5140
(7)		70	390	1870	3270	5140

Note: For $x_{31} > 75$ the problem is infeasible, since the private enterprise behaviour setting time endowment $\geqq$ 75.

9.5 Effects of Changes in the Technology Matrix *A*

Let there be an increase in efficiency row-wise. With the same time endowment the agent can now meet an increased academic social requirement. On the other hand, if the academic parameter remains unchanged, the agent now requires less of the time endowment. Geometrically the effect of a change in technology is to change the shape of the constraint boundary.[1] The new technology is denoted by $\boldsymbol{A}^*$ where $\boldsymbol{A}^* = \boldsymbol{RA}$, as in (5.13).

1 For comments on the radically altered topology see Eisemann (1964) and Lourie (1964).

Table 9-12 shows selected percent increases in the agent's efficiency in meeting an academic social requirement of 500 minutes.[2] Parameters are unchanged from Tables 9-1 and 9-2.

Example 1. The solution for a 25 percent increase is expressed in the activity analysis mode. The basic variables are expressed as activities, using the following transitions:

$x^*_{13} \rightarrow x^*_3$; $x^*_{15} \rightarrow x^*_5$; $x^*_{23} \rightarrow x^*_8$; $x^*_{24} \rightarrow x^*_9$; $x^*_{31} \rightarrow x^*_{11}$; $x^*_{41} \rightarrow x^*_{16}$; $x^*_{52} \rightarrow x^*_{22}$

The levels of the activities are:

$x^*_3 = 350$; $x^*_5 = 40$; $x^*_8 = 50$; $x^*_9 = 100$; $x^*_{11} = 75$; $x^*_{16} = 25$; $x^*_{22} = 220$. (M)

The intensity vector **x** is:

$$\mathbf{x} = [\ 0\ \ 0\ \ 350\ \ 0\ \ 40\ \ 0\ \ 0\ \ 50\ \ 100\ \ 0\ \ 75\ \ 0\ \ 0\ \ 0\ \ 0\ \ 25\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 220\ \ 0\ \ 0\ \ 0\ ]^T$$

The price vector **p** is:

$$\mathbf{p} = [\ 3.8\ \ 0\ \ 2.8\ \ 3.9\ \ 11.5\ \ 1.5\ \ 0\ \ 1.6\ \ 1.5\ \ 8.5\ ]$$

The changed technology matrix $\boldsymbol{A}^*$ is a 10 x 25 matrix. For the activities A^*_3, A^*_8, A^*_{13}, A^*_{18}, A^*_{23} (where the star denotes an activity of the changed technology), the unit vectors a^3, a^8, a^{13}, a^{18}, a^{23} have outputs of 1.25 for unit inputs, otherwise unit inputs generate unit outputs. A^*_3 and A^*_8 belong to the optimal solution. Since $A^*_3 = x^*_3 a^3$, $x^*_3 = 350$ is sufficient to meet an academic social requirement = 350(1.25) = 437.5 mins.

---

2 For a changed technology, the decision variables need not be integers.

Since $A^*_8 = x_8\mathbf{a}^8$

x^*_8 = 50 is sufficient to meet an academic social requirement

= 50(1.25)

= 62.5 mins

Hence there is a slack (for the household behaviour setting) of

500 - (350 + 50)

= 100 mins

R is a 10 x 10 diagonal matrix where $r_{33} = 1.25$, otherwise all diagonal elements = 1. All other elements = 0.

In Table 9-12, as the percent increase in efficiency rises, the same social requirement can be met, while savings increase. There are two reasons for the increase in savings. Less time is required to meet social requirements, and so for the same per unit costs an increase in savings would be expected. As well, the gains in efficiency allow the agent to allocate time use away from relatively higher cost activities. For example, time use in the household environment decreases from 150 minutes to zero. Slack time is not used to meet further social requirements. Social income decreases.

Table 9-13 shows the distribution of slack time over sources. The increased efficiency of an agent in meeting the academic social requirement generates excess capacity. Table 9-14 shows that where there is excess capacity of time use at a source i, the shadow price of the ith endowment will be zero, as for u_1^* (increases in efficiency of 125 and 150 percent) and u_2^* (all cases). However the converse does not hold. For example $u_4^* = 0$ does not indicate excess capacity at the voluntary associations behaviour setting. Improving the technology makes it possible for the agent to increase saving (decrease dissaving) while meeting all social requirements. As the efficiency of meeting the academic social requirement increases, the scarcity of the endowment decreases and hence the value of time use to meet social requirements, other than job, decreases. This entails a decrease in social income.

TABLE 9-12 CHANGES IN TECHNOLOGY - SOLUTIONS

percent increase in efficiency	Optimal Decision Variable								Value of Objective Function	Social Income	
	x^*_{13}	x^*_{14}	x^*_{15}	x^*_{23}	x^*_{24}	x^*_{31}	x^*_{41}	x^*_{52}			
25	350	0	40	50	100	75	25	220	17.5	4500	(M)
50	333.3	16.67	40	0	83.3	75	25	220	-202.5	3968	
100	250	100	40	0	0	75	25	220	-427.5	3663	
125	222.2	100	40	0	0	75	25	220	-483.05	3265	(M)
150	200	100	40	0	0	75	25	220	-527.5	3220	(M)

Note: Solutions corrected to second decimal place.

TABLE 9-13 EFFECTS OF AN INCREASE IN THE EFFICIENCY OF MEETING THE ACADEMIC SOCIAL REQUIREMENT

percent increases in efficiency / slack at sources	25	50	100	125	150
a_1				27.78	50
a_2	100	166.67	250	250	250
a_3					
a_4					
a_5					
Total slack in minutes	100	166.67	250	277.78	300

Note: Solutions rounded to second decimal place.

TABLE 9-14 EFFECTS OF AN INCREASE IN THE EFFICIENCY OF MEETING THE ACADEMIC SOCIAL REQUIREMENT ON SHADOW PRICES

shadow prices \ percent increases in efficiency	25	50	100	125	150
u^*_1	1.5	0.7	0.7	0	0
u^*_2	0	0	0	0	0
u^*_3	1.6	0.8	0.8	0.1	0.1
u^*_4	1.5	0.7	0.7	0	0
u^*_5	8.5	8.5	8.5	8.5	8.5
v^*_1	3.8	3.0	3.0	2.3	2.3
v_{*2}	0	0	0	0	0
v^*_3	2.8	1.8	1.35	0.9	0.8
v^*_4	3.9	3.9	3.9	3.2	3.2
v^*_5	11.5	10.7	10.7	10.0	10.0

An improved technology means that the efficiency of the agent's use of time increases, and there is slack time. In practice the agent will be faced with new social requirements that place demands on all surplus endowments of time use at sources so that for the tight model total supply at sources is equal to total demand at sinks and there is no excess capacity.

Example 2. Let there be an increase in efficiency column-wise. The result is an enhanced environment. For an increase of 25 percent in the efficiency of time use in the state behaviour setting, it is as if the agent's time endowment in that environment had been increased by 25 percent to 390(1.25) = 487.5 mins. The 960 mins of the 16 hour waking day remains unchanged. However the agent can meet increased social requirements using the same time endowment. An analogy would be the appreciation of a nation's currency. For the same amount of the domestic currency more foreign goods can be imported. The new technology is denoted by $\boldsymbol{A}^{**} = \boldsymbol{AE}$, as in (5.14).

The solution for a 25 percent increase is expressed in the activity analysis mode. Parameters are unchanged from Tables 9-1 and 9-2 except that a_1 becomes 487.5 mins. The following transitions are used:
$x^{**}_{13} \rightarrow x^{**}_{3}$; $x^{**}_{15} \rightarrow x^{**}_{5}$; $x^{**}_{23} \rightarrow x^{**}_{8}$; $x^{**}_{24} \rightarrow x^{**}_{9}$; $x^{**}_{31} \rightarrow x^{**}_{11}$; $x^{**}_{41} \rightarrow x^{**}_{16}$; $x^{**}_{52} \rightarrow x^{**}_{22}$, where the double star denotes an activity of the changed technology.

The intensity vector **x** is:

$$\mathbf{x} = [\ 0\ \ 0\ \ 358\ \ 0\ \ 32\ \ 0\ \ 0\ \ 52.5\ \ 100\ \ 0\ \ 75\ \ 0\ \ 0\ \ 0\ \ 0\ \ 25\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 220\ \ 0\ \ 0\ \ 0\ ]^T$$

The price vector **p** is:

$$\mathbf{p} = [\ 6.06\ \ 0\ \ 3.5\ \ 3.9\ \ 9.9\ \ 1.9\ \ 0\ \ 3.86\ \ 3.76\ \ 8.5\ ]$$

The changed technology matrix $\boldsymbol{A}^{**}$ is a 10 x 25 matrix. For the activities A^{**}_1, A^{**}_2, A^{**}_3, A^{**}_4, A^{**}_5 inputs of 1.00 generate outputs of 1.25 otherwise unit inputs generate unit outputs. Since $A^{**}_3 = x^{**}_3 a^3$, $x^{**}_3 = 358$ represents enhanced time use and can meet a social requirement of 358(1.25) = 447.5 mins. Similarly, $x^{**}_5 = 32$ can meet the entire commercial requirement of 32(1.25) = 40 mins. Hence there is a slack (for the state behaviour setting) of 487.5 - (358 + 32) = 97.5 mins. The related shadow price is therefore zero. $\boldsymbol{E}$ is a 25 x 25 diagonal matrix where, for $\ell = k$, $e_{\ell k} = 1.25$, $\ell=1,\ldots,5$ $k=1,\ldots,5$; otherwise e=1. For $\ell \neq k$, $e_{\ell k} = 0$

Comment. Suppose the new technology was described as follows:

$\boldsymbol{A}^{**}$ is a 10 x 25 matrix. For the activities A^{**}_1, A^{**}_2, A^{**}_3, A^{**}_4, A^{**}_5 inputs of 1.0 generate outputs of 1.25. Otherwise units generate unit outputs. Suppose the state behaviour endowment is 390 mins. This combination of changed technology and unchanged parameter generates the same intensity vector as in the previous solution. v^*_1 increases to 2.375, otherwise **p** is unchanged. The objective function and social income are the same as for the previous solution.

This way of describing the changed technology is unsuitable because the technological relations between the input-output relations of activities A^{**}_1 through A^{**}_5 specify a socio-economic production function which is different from the changed technology matrix given by $\boldsymbol{A}^{**} = \boldsymbol{AE}$ as in (5.14).

Example 3

There is a 25 percent increase in the efficiency of time use in the household behaviour setting. It is as if the time endowment of 250 mins in this environment had been increased to 312.5 mins. The new technology is denoted by $\boldsymbol{A}^{**}$ where $\boldsymbol{A}^{**} = \boldsymbol{AE}$, as in (5.14), and $\boldsymbol{E}$ is a 25 x 25 diagonal matrix, where, for $\ell=k$, $e_{\ell k} = 1.25$ $\ell=6,\ldots,10$ $k=6,\ldots,10$; otherwise $e_{\ell k}=1$. For $\ell\neq k$, $e_{\ell k}=0$.

This means for activities in the household behaviour setting, the same social requirements can now be met using less time. Hence to meet the academic social requirement optimally, the agent uses 350 mins in the state behaviour setting, and 120 minutes in the household behaviour setting. Since 120(1.25) = 150 mins, the academic social requirement of 500 mins is met by using 470 minutes. Hence time "saved" in meeting the academic requirement is 30 mins. The socio-cultural requirement of 100 mins is met by using 80 mins in the household behaviour setting, for a "saving" of 20 mins. Total time use in the household behaviour setting is 120 + 80 = 200 mins, for a total "saving" of (312.5-250) = 62.5 mins. This is comprised of three components, namely (30 + 20 + 12.5). The first two have been described as time "saved". The 12.5 mins is idle capacity, that is, part of the enhanced endowment which is not utilized.

Optimally, the intensity vector is:

$$\mathbf{x} = [\ 0\ \ 0\ \ 350\ \ 0\ \ 40\ \ 0\ \ 0\ \ 120\ \ 80\ \ 0\ \ 75\ \ 0\ \ 0\ \ 0\ \ 0\ \ 25\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 220\ \ 0\ \ 0\ \ 0\ ]^T$$

The price vector is:

$$\mathbf{p} = [\ 5.36\ \ 0\ \ 2.8\ \ 3.12\ \ 10.8\ \ 0.8\ \ 0\ \ 3.16\ \ 3.06\ \ 8.5\ ]$$

The expenditure is $1.845. The change in technology generates a daily money saving of $0.02 and a time "saving" of 62.5 mins, given the assumptions of the model.

The cost of social income

The conclusions of the previous example, namely that an improved technology results in a saving of money together with a "saving" of time, seem at variance with experience. For example, labour saving devices in the home do not necessarily generate spare time.[3] In fact the opposite may be the case.[4]

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3 For US data which indicate that time spent at household tasks was almost unchanged over the period (1920)-(1952)-(1968) see Hall and Schroeder (1970). However, the distribution (how time is spent) has changed. Time spent on food preparation and dishwashing increased from 13.3 hours in 1920 to 18.5 hours per week in 1968 p.28.

4 Linder (1970). Ch. VII.

The model can provide an economic interpretation of the paradox that more labour saving consumer durables and increased personal productivity can result in stress - "more things to do and less time to do them."

The models have shown how to measure the effects of improved technologies. In particular, the increase in time, considered as excess capacity of an endowment, was quantified. However this is only the first stage of a process.[5] In the models we assume that environments shape behaviour, and now describe the changes in the agent's lifestyle.

What happens is that the agent will be faced with new social requirements that place demands on all surplus endowments of time use, so that for the tight model, total effective supply (the enhanced endowment) is not greater than the new demand. There are two related effects. New social requirements may entail a need for an increase in social position. For example, new social requirements may entail increased peer pressure (for the student, the new requirement is for top grades), and a need for status (for the executive, the new requirement is a more affluent life style - perhaps a house in a more exclusive suburb).

New social requirements may entail a need for increased income. More time may well be required for the workplace environment either directly, by overtime, or indirectly, through job commitments outside office hours involving for example business related travel meetings and home entertaining. For the agent there is a money/non-work related time trade-off.

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5 . Soule (1955) suggests that the structure of an economic system governed by the "invisible hand" can give rise to competitive striving for material goods and status. p. 74.

Example 4

We now postulate that slack time is allocated in equal proportions across the four non-work related social requirements. We consider successive increases in an agent's efficiency in meeting the academic social requirement. For example a 25 percent increase in efficiency generates slack capacity of 100 mins. This is now allocated in the following way:

No increase in the job social requirement.

The health, socio-cultural and commercial requirements become 125, 125 and 65 mins. The academic social requirement becomes 531.25 mins. Here the common 25 percent increase is itself increased by the change in technology. The new demands are equal to the enhanced supply. There is no slack. To focus on the effect of no excess capacity, a cet par approach is used. Net income, time allocated to work, and cost of living are held constant. The model is therefore conservative, and considerably understates the personal cost of the agent's new lifestyle.

Solutions:

The price vector **p** for a 25 percent increase in efficiency (Table 9-15) is:

$\mathbf{p} = [\ 6.7\ \ 0\ \ 2.8\ \ 3.9\ \ 11.5\ \ 1.5\ \ 0\ \ 4.5\ \ 4.4\ \ 8.5\ \ 6.7\ ]$

As efficiency increases, $v^*_3 = 2.8$ cents/min decreases. Otherwise **p** remains unchanged.

In Table 9-15 the agent's lifestyle is reflected by increasing social income. Because the value of time endowments, **uAx**, remains constant at $29.025 per day, social income increases because expenditure increases.[6]

There are two reasons for the increase in dissaving. While costs remain constant more time is used up. Then, as the agent becomes more efficient in

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6 For **uAx**, see the starred section 7.5.

meeting academic social requirement, less time is required for activities $\boldsymbol{A}^*_3$ and $\boldsymbol{A}^*_8$. To meet the increasing total social requirement more time is now required for activities A^*_5, A^*_6, A^*_9 where the per unit costs are relatively higher. For the percent increase in the agent's efficiency in meeting the academic requirement, from 2.5 to 150 percent, the corresponding increases in the cost of lifestyle are 44 and 263.7 percent.

In Table 9-15 the time for work has been held constant. Suppose this assumption is relaxed. Extra income from longer time spent working will reduce dissaving, to be sure, but involves a trade-off. Because the agent now has less time to meet non-work requirements, this remaining time is increasingly scarce and causes stress. We relax the assumption of the 16 hour waking day. The agent can meet all social requirements, including extra working time. The trade-off is less time given to sleep. This is also likely to increase stress, and could bring about a decrease in the efficiency of the agent's socio-economic production function. The health social requirement may increase (medical treatment), while the other social requirements remain unchanged. A different trade-off is for the agent to adopt a lifestyle where social requirements are less demanding. Social requirements are held constant. Excess capacity, as in Table 9-12, can be used for leisure[7].

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7 As defined by de Grazia (1962), Chapter 1, where leisure is distinguished from free time.

TABLE 9-15 CHANGES IN TECHNOLOGY - NO EXCESS CAPACITY

percent increase in efficiency	Optimal Decision Variables								Dissaving (in cents)	Social Income (in cents)
	x^*_3	x^*_5	x^*_6	x^*_8	x^*_9	x^*_{11}	x^*_{16}	x^*_{22}		
25	325	65	25	100	125	75	25	220	657.5	5430 (M)
50	308.33	81.67	41.67	66.67	141.67	75	25	220	850.83	5622.5 (M)
100	287.5	102.5	62.5	25	162.5	75	25	220	1092.5	5865 (M)
125	280.56	109.44	69.44	11.1	169.44	75	25	220	1173.06	5945.11 (M)
150	275	115	75	0	175	75	25	220	1237.5	6010.3 (M)

9.6 SUMMARY

The effects of changes in costs, in endowments, in social requirements and in the agent's socio-economic production function have been related to changes in lifestyle. As well, the effects of imposing an additional inequality constraint have been examined. In order to focus on the particular effects of each change a cet par approach has been used. The effects of these changes have been given economic interpretations. Quantitative results have been provided for such measures of lifestyle as savings and social income.

For an agent to adapt optimally to even one change may prove to be unexpectedly difficult. However an agent could well be faced with the problem of optimizing a lifestyle where several changes in parameters occur together, and where change is the rule rather than the exception. Some of the stress which people experience in their lifestyle may well result from the difficulty of making optimal trade-offs between time and money when faced with changes in cost of living, environments, social requirements, and ability to use time more efficiently. In turn, stress may bring about a decrease in the efficiency of the socio-economic production function so that more time, and not less, is required to meet the same social requirements. However if the agent now faces increased social requirements even more time is needed. Problems are compounded if income decreases. It is not only Linder's "leisure class" that are harried.[8]

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8 Linder (1970).

SUMMARY AND CONCLUSIONS

Summary

10.1 Examples from 19th century England and from contemporary time based management strategies were used to introduce the linear models of time allocation. Constraints on an agent's time use were described in terms of environments and social relations. Time was considered as a scarce resource and as a objective invariant measure of activities.

In chapter 5 a core model, was used to introduce the transportation models of time allocation. The core model linked together the concepts of behaviour setting and social requirement, developed in Chapter 4, and the economic interpretations given in Chapter 6. There is a mapping in choice space between the classes, or groups of agents, and their lifestyles. The slack model, for an individual agent, and the tight model, for the average agent, representative of a class, or group, were then developed. The models examine the lifestyle of one particular class of agent. Conceptually the linear models of time allocation are transportation models and share the same mathematical structure. Three equivalent formulations of the transportation problem were outlined. Activity analysis was the preferred form.

10.2 To show that the transportation models of time use are operational, numerical examples were provided in Chapters 7 through 9. A derivation of the per unit cost matrix from time use and money expenditure data was given in Chapter 8. The solutions to the numerical examples appear related to the real world and are in accord with economic theory, as in sensitivity analysis and in the application of the Le Chatelier Principle. Because the solutions quantify the optimizing behaviour of a rational agent, given constraints, it is possible to link the class, or group, to which an agent belongs with a particular lifestyle specified by a time distribution and a money expenditure. Using activity analysis, expressions for savings, value of time endowments and social income were formulated and then quantified.

The technology matrix ***A***, defined as the agent's socio-economic production function, was interpreted as a mapping from observable N dimensional activity "space" to non-observable M-dimensional value "space". Since the parameters and solutions are both measured in observable units, namely (hrs/mins), ($/hr, or cents/min) the models represent a contribution to objective consumption theory.

10.3 The agent as rational decision maker uses a given endowment to meet the demands of social requirements - the norms, customs and expectations of society - while minimizing the costs of activities. Activities are measured by time use (Szalai) and shaped by environments (Barker). Perspectives of sociology and eco-behavioural science have been integrated into an economic model to provide realistic constraints and to propose that the agent is both an optimizing individual who makes choices, and a member of society. The intention is to break new ground. Since the model transforms inputs into outputs and also incorporates concepts from several disciplines, it can be considered to make a contribution to systems economics.

Conclusions

The following formulations have been derived:

10.4 The conditions for which the aggregate endowment constraint, required for realism in the slack model, is not necessarily redundant.

10.5 Measures of savings and social income in terms of an agent's socio-economic production function, using activity analysis.

10.6 A matrix denoting an agent's per unit costs of time use in a set of environments in order to meet social requirements.

10.7 A changed matrix, equivalent to post-multiplication and pre-multiplication of the existing technology matrix, denoting increases in an agent's efficiency in time use, because of an enhanced environment, or because of greater ability to meet a social requirement, respectively.

The following conclusions have been obtained from numerical examples:

10.8 A fixed behaviour pattern is not necessarily efficient. Unless the optimizing agent adapts his lifestyle to changes, a decrease in per unit costs can result in an increase in living costs.

10.9 The optimizing agent will not always undertake activities in the lowest per unit cost environment in order to meet a social requirement.

10.10 Successive increases in behaviour setting costs, with income held constant, lead to increases in the values of dissaving, social position and social income.

10.11 Changes in the distribution of endowments, with income held constant, can result in changes in savings while the values of social position and social income remain constant.

10.12 Successive increases in an agent's efficiency of time use, with income held constant, will generate increasing slack time, together with increases in savings and decreases in social income. Decreases in social income take place because, as a result of an increase in an agent's efficiency of time use, most shadow prices of both endowments and social requirements decrease. Otherwise they are unchanged.

However, if social requirements increase, so that there is no excess capacity, dissaving and social income both increase.

10.13 The addition of a new constraint, with income held constant can, but need not, result in an increase of saving, while the values of social position and social income remain unchanged. If the increase in the magnitude of the new constraint brings about a shift in time use from higher to lower per unit cost environments, then savings will increase.

LIST OF SYMBOLS AND NOTATION

$\boldsymbol{A}$	fundamental M x N technology matrix.
$\boldsymbol{A}^*$	a changed technology matrix for an increase of efficiency row-wise.
$\boldsymbol{A}^{**}$	a changed technology matrix for an increse of efficiency column-wise.
A	output matrix.
$\boldsymbol{A}$	a changed output matrix.
A_k	the kth activity, k=1, ..., N.
a_i	the total amount of the scarce homogenous resources available for shipment from the ith source.
a_{ij}	element of the output matrix $\boldsymbol{A}$.
$\mathbf{a}^k$	the 1 x M column vector of the kth per unit activity.
$a_{\ell k}$	element of the technology matrix $\boldsymbol{A}$.
B	input matrix.
$\mathbf{b}$	column vector of requirements with m+n components.
$\mathbf{b}^T$	transpose of vector $\mathbf{b}$.
$\mathbf{b}_m$	vector of parameters for resources.
$\mathbf{b}_n$	vector of parameters for final requirements.
$\mathbf{b}$	a changed vector.
b_{ij}	element of the input matrix $\boldsymbol{B}$.
b_j	the total amount of scarce homogenous resource required to be shipped to sink j.
$\mathbf{c}$	cost vector.
$\mathbf{c}$	a changed cost vector.
c_{ij}	per unit cost of shipping a unit of the scarce homogenous resource from source i to sink j.

$\boldsymbol{E}$	diagonal matrix, post-multiplication.
e_i	weight of the endowment, generalized transportation problem.
$e_{\ell k}$	element of matrix $\boldsymbol{E}$
i	subscript to designate a particular source.
j	subscript to designate a particular sink.
k	number of activities k=1, ..., N where N = mn.
ℓ	number of commodities ℓ=1, ..., M where M = m+n
M	the number of rows of matrix $\boldsymbol{A}$, where M = m+n
m	number of sources i=1, ..., m.
N	the number of columns of matrix $\boldsymbol{A}$, where N = mn.
n	number of sinks j=1, ..., n
p	price vector.
$\mathbf{R}^n$	the n-dimensional real space.
$\boldsymbol{R}$	diagonal matrix, pre-multiplication.
r_j	weight of the requirement, generalized transportation problem.
$r_{\ell k}$	element of matrix $\boldsymbol{R}$
u	dual variable vector, shadow price vector for supply of resource.
u_i	ith component of vector u
u^*_i	optimal solution, shadow price of resource at source i
v	dual variable vector, shadow price vector for demand for resource.
v_j	jth component of vector **v**
v^*_j	optimal solution, shadow price of resource at sink j.
w	shadow price, total constraint, slack model.
x	M x 1 intensity vector.
$\mathbf{x}^*$	optimal vector
x_i	the ith component of vector **x**.
x_k	the scalar variable which measures the level of an activity.

x_{ij}	the amount of the scarce homogenous resource to be shipped from source i to sink j to meet the jth requirement.
x^*_{ij}	optimal solution, amount of resource shipped from source i to sink j.
$\mathbf{x} \geqq 0$	vector **x** is non-negative.
$\mathbf{x} > 0$	vector **x** is positive.
Y	the production set.
y	N x 1 column vector of net outputs.
z	vector of shadow prices. z = [u, v] for the slack model.
$\rightarrow$	a transformation
$p \Rightarrow q$	p implies q. p is sufficient for q.
$p \Leftrightarrow q$	both p implies q and q implies p. p iff q. p is a necessary and sufficient condition for q and vice versa.

NOTATION

An upper-case italic letter designates a set.

Upper-case bold face letters designate matrices.

Upper-case boldface sloping letters designate either technology matrices, or diagonal matrices used to multiply a technology matrix.

Lower-case bold face letters denote vectors.

Lower-case italic letters denote scalars.

For convenience, vectors are written in row form. The context usually makes it clear whether a particular vector is a row vector or a column vector.

Ax	implies x is an n x 1 column vector
p*A*	implies p is a 1 x m row vector.

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