Advanced Second Order Functional Differential Equations

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Abstract

Hall and Wake [1989] showed that an advanced first order equation arising in a cell growth model has a Dirichlet series solution. If the effects of dispersion are included, the cell growth model leads to a second order equation. We show that this equation also has a Dirichlet series solution, which is unique and positive and that it has one maximum. We then investigate the general second order equation with constant coefficients, and show that these equations also have Dirichlet series solutions and that certain qualitative properties such as uniqueness and positivity are preserved for a range of coefficients. Although the solution to the equation arising in a cell growth model with dispersion is a probability density function of the cell size, $y(0) \neq 0$. There are however parameter choices such that $y(0) = 0$ and this motivates our study of the eigenvalue problem. Our final chapter concerns general equations with variable coefficients. We can express a first order equation as a Fredholm integral equation of the second kind and the existence of a solution thus follows using results for Fredholm equations. In addition, we study some classes of second order equations, and show that certain equations have a series solution involving Bessel or Airy functions.

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