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Student Perspectives and Roles in an Inquiry Mathematics Classroom

A thesis presented in partial fulfilment of the
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ABSTRACT

This study examines the perspectives and roles of students aged 9-10 years old learning mathematics in an inquiry classroom. It builds on previous work which has advocated students learning mathematics through collaborative interaction as opposed to passive transmission of knowledge and skills. In this study the students' beliefs about what they consider to be important in learning mathematics is compared to the ways in which they engage in mathematics activity. The varying roles students assume while learning mathematics and how this affects their agency are considered.

This investigation is situated in an inquiry classroom. A sociocultural perspective provides the framework for the classroom context. Relevant literature is examined to provide a rationale for how students engaged in mathematical reasoning within this environment. The pedagogic approach of the teacher in developing effective student participation in mathematical reasoning by facilitating the even distribution of authority in the classroom is offered as an alternate to customary practice. Active student engagement in mathematical discussion and debate are all viewed as highly important for the enhancement of mathematical understanding.

A qualitative research approach was implemented. The case study supported a classroom based investigation. Data were collected through individual interviews, participant and video-recorded observations and classroom artefacts. To develop the findings as one classroom case study, on-going and retrospective analyses of data were made.

Significant changes were revealed in the relationship between the students' espoused beliefs about learning mathematics and their enacted beliefs. The investigation illustrated that students were able to develop positive positional identities through active engagement in mathematical reasoning. The interaction patterns created in the classroom explicitly affected the construction of mathematical knowledge. From these findings insights are made into the type of environment which supports enhanced mathematics learning.

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TABLE OF CONTENTS

ABSTRACT		ii		
ACKNOWLEDGEMENTS				
TABLE OF (CONTENTS	iv		
CHAPTER 1	: INTRODUCTION			
1.1	Introduction	1		
1.2	Background to the study	1		
1.3	Research objectives	2		
1.4	Overview	3		
CHAPTER 2	: LITERATURE REVIEW			
2.1	Introduction	5		
2.2	Sociocultural theory	5		
	2.2.1 Zone of proximal development	6		
2.3	Inquiry classrooms	8		
	2.3.1 The role of the teacher in inquiry classrooms	9		
	2.3.2 Discourse in inquiry classrooms	10		
2.4	Classroom norms	11		
	2.4.1 Social norms	11		
	2.4.2 Sociomathematical norms	12		
2.5	Student perspectives	13		
2.6	Student identity	14		
2.7	Student agency	17		
2.8	Summary	19		
CHAPTER 3	: RESEARCH DESIGN			
3.1	Introduction	21		
3.2	Justification for methodology	21		
3.3	Researcher role	23		

	3.4	Data c	collection	24			
		3.4.1	The questionnaire	24			
		3.4.2	Observation	25			
		3.4.3	Interviews	26			
		3.4.4	Classroom artefacts	27			
	3.5	The re	esearch study: Setting, sample and schedule	27			
		3.5.1	The setting and the sample	28			
		3.5.2	The research study schedule	28			
	3.6	6 Data analysis					
	3.7	30					
	3.8	Ethica	l considerations	30			
	3.9	Summ	ary	33			
CHAF	LEARNING						
MATHEMATICS IN AN INQUIRY CLASSROOM							
	4.1	Introdu	uction	34			
	4.2	The qu	34				
	4.3	Summ	40				
	4.4		attitude scale	40			
	4.5	Summ	nary	42			
CHAF	PTER 5		THE ROLES OF STUDENTS LEARNING MA	THEMATICS			
011711	ieno	•	IN AN INQUIRY CLASSROOM				
	5.1	Introdu	uction	43			
	THE F	IRST P	HASE				
	5.2	The cl	assroom context	43			
		5.2.1	The structure of the learning sessions	44			
	5.3	Stude	nt roles within the classroom context	45			
		5.3.1	Assuming identity through student positioning	45			
	5.4	Stude	nt agency within the classroom context	47			
	5.5	The ro	ole of the teacher	47			
	5.6	Stude	nts use of social and sociomathematical norms	51			
	5.7	Collab	orative discourse	54			
	5.8	Summ	nary of the first phase	56			

5.9	The classroom context					
5.10	Student roles within	the classroom context	57			
	5.10.1 Assuming id	entity through student positioning	57			
5.11	Student agency with	in the classroom context	60			
5.12	The role of the teacl	ner	61			
5.13	Students use of soc	Students use of social and sociomathematical norms				
5.14	Collaborative discourse					
5.15	Summary		64			
CHAPTER	S: DISCUSSIO	ON AND CONCLUSION				
6.1	Introduction		65			
6.2	Student perspective	s	65			
6.3	Student identity		66			
6.4	Student agency		68			
6.5	6.5 Inquiry classrooms					
	6.5.1 The role of	of the teacher	70			
6.6	Communication and	participation patterns	71			
	6.6.1 Developin	g social and sociomathematical norms	71			
	6.6.2 Collabora	tive discourse	73			
6.7	The complex nature of teaching and learning					
6.8	Opportunities for fur	ther research	74			
6.9	Concluding thoughts	3	74			
REFERENC	ES		76			
APPENDIC	ES:					
	Appendix A: The o	questionnaire (both phases)	88			
	Appendix B: Liker	t attitude scale (both phases)	89			
	Appendix C: Math	ematics problems	91			
	Appendix D: Interv	view questions	92			
	Appendix E: Teac	her information sheet and consent form	93			
	Appendix F: Stude	ent and parent information sheet and				
	cons	ent form	96			

	Appendix G: Board of Trustees Information sneet and			
	consent form	99		
LIST OF TA	BLES			
4.1	Students' responses from phases one and two to the questions:			
	what is mathematics and why should we learn it?	35		
4.2	Students' responses from phases one and two to the questions:			
	how do we know if someone is good at mathematics and how			
	do you think real mathematicians do mathematics?	36		
4.3	Students' responses from phases one and two to the questions:			
	when you do mathematics in your class: who does the teaching			
	and the talking; and who asks the questions and gives the			
	answers; and what do you do if you get stuck?	37		
4.4	Students' responses from phases one and two to questions			
	about working collaboratively to solve mathematics problems	38		
4.5	Students' responses from phases one and two to questions			
	about the most difficult and the most fun about doing mathematics	40		
4.6	Percentage of students' responses to passive statements about			
	learning mathematics	40		
4.7	Percentage of students' responses to active statements about			
	learning mathematics	41		
	-			

CHAPTER ONE

INTRODUCTION

1.1 INTRODUCTION

This chapter provides the background context of the study. The background context outlines the international and national calls for changes to how mathematics is taught in classrooms. The rationale for providing young students with a voice to discover what they think about learning mathematics is presented. The primary research objectives of this study are identified and important terms used in this thesis are clarified. An overview of the thesis is presented.

1.2 BACKGROUND TO THE STUDY

Currently and in the past, there has been increased interest in both national and international research and curriculum reforms on the teaching and learning of mathematics within classrooms which emphasise problem solving and effective communication skills (e.g., Chapin & O'Connor, 2007; Goos, Galbraith, & Renshaw, 2004; Ministry of Education, 2007a; Yackel, 1995). Such emphasis has arisen from growing acknowledgements of changes in how mathematics classrooms are conceptualised. An essential element of the changes is the idea of teachers and students actively working together to enhance mathematical understanding through effective mathematical practices. Reform and inquiry type classrooms place student reasoning through explanation, justification and validation at the centre of classroom activity (Hunter, 2002). Features of a classroom environment in which this would occur would be one where:

- the students and teacher participate in mathematical discourse;
- dialogue and discussion of meanings are important aspects of the mathematical task;
- the students work together in small groups on demanding mathematical tasks and are prompted to elaborate on and discuss their own strategies;
- the students are accountable for choices about validity and justification;
- the role of the teacher is to stimulate perseverance in working on the mathematical tasks;

 the goal of the mathematics lesson is not to merely solve the problem, but is concerned with the arguments that support or reject solution strategies (Civil, 2002; Wood & Kalinec, 2012).

A need for further study into understanding learning mathematics in inquiry classrooms from the perspectives of the students has been specified in research (e.g., Hodge, 2008; Hunter & Anthony, 2011). Finding out what they know and think about while learning mathematics is important if learning opportunities are to be created for all students (Fraivillig, Murphy, & Fuson, 1999). However, what students *say* is important about learning mathematics and what they *do* while learning mathematics may not correlate. Perger (2007) reports that existing research centres on either examining what students say is best practice, or on their theory-in-use, which is identified through classroom observations of them engaged in regular practices. She asserts that to fully grasp what students believe is important in learning mathematics, it is essential to examine both their espoused theory and their theory-in-use. Furthermore, while international research reports the perspectives of younger students, research in New Zealand has mainly been centred on students aged 12-17 years old. It is against this background and for these reasons that this study, *Student perspectives and roles in an inquiry mathematics classroom*, was conducted.

1.3 RESEARCH OBJECTIVES

The primary aim of this study is to explore the perspectives and roles of students aged 9-10 years old who are learning mathematics within the context of an inquiry classroom. The study also seeks to examine the ways in which students construct mathematical understanding. A related objective is to explore the classroom environment connecting the effects of specific classroom practices on the participants as they engage in mathematical reasoning.

In particular, the following research questions have been addressed:

- How does participating in a mathematical community of inquiry shape students' perspectives?
- 2. How do classroom practices, and in particular the social and sociomathematical norms support the development of positive positional identities?
- 3. How does the development of positive positional identities influence student agency in learning mathematics?

In order for the reader to develop a shared understanding of what is described in this study, important terms are clarified to communicate explicit meaning:

- The term sociocultural in the context of this study should be understood as being the environment created where all learners, students and teacher, work together and use dialogue as a means of communicating what they know and as a way of making meaning of new ideas or concepts. It is also the relationship between thinking and learning and the environment within which thinking and learning occurs (Wertsch, del Rio, & Alvarez, 1995)
- Social norms are activities such as explaining, justifying, questioning different ideas, and making sense of others' explanations found in all areas of the curriculum
- Sociomathematical norms are "normative understandings of what counts as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant in a classroom" (Yackel & Cobb, 1996, p.461)
- Dialogue pertains to the discourse and talk taking place in the classroom during mathematics lessons

1.4 OVERVIEW

Chapter 2 reviews the literature from both a New Zealand and an international perspective providing a background with which this study can be viewed. The context and framework for the current study are provided through summarising and connecting appropriate and essential literature related to: active learning in an inquiry classroom; collaborative interaction and classroom discourse; social and sociomathematical norms; and students' perspectives, identity and agency.

In Chapter 3 the methodology for the study is described. The research setting and sample, data collection and data analysis are discussed and a timeframe for the case study is outlined.

Chapter 4 and Chapter 5 present the findings of the study. The perspectives of the students are described and analysed. The varying identities and roles which the participants assumed while learning mathematics are outlined. The classroom context and practices are examined and the important ways in which social and sociomathematical norms facilitated productive discourse and collaborative interaction are illustrated. Descriptions and analyses of the pedagogic actions of the teacher in developing a mathematics inquiry classroom are provided.

In Chapter 6, the results are discussed and conclusions are drawn. The implications for classroom practice and suggestions for further areas of research are described.

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

The previous chapter presented the background context of the current study. This chapter reviews national and international literature providing the theoretical framework through which this study can be viewed. As the current study draws on sociocultural perspectives of learning, this theoretical rationale is examined. The nature of inquiry classrooms and the ways in which students learn mathematics in such classrooms by engaging in dialogue and social interaction are considered. The current perspective of the zone of proximal development as a shared learning space into which students are pulled through their active participation in the inquiry classroom is illustrated. The ways in which classroom norms shape student participation are examined. The importance of considering students' perspectives, how students assume different roles while learning and how this affects their agency are examined.

In Section 2.2 relevant literature on sociocultural theory is examined. The zone of proximal development is also considered, with an emphasis on current perspectives. Section 2.3 examines the nature of inquiry classrooms and the role of the teacher and discourse. Section 2.4 highlights the social and sociomathematical norms which shape student participation in the mathematics classroom. Section 2.5 considers the significance of understanding students' perspectives about what they deem is important while learning mathematics. Section 2.6 illustrates different ways in which students develop identities while learning mathematics. Section 2.7 draws attention to the importance of student agency in the mathematics classroom.

2.2 SOCIOCULTURAL THEORY

Examining an environment which emphasises social interaction as a means of teaching and learning requires considering the contributions of a sociocultural theory of learning. Within the context of this project, the term sociocultural is used to describe an environment where according to Rogoff (1995), all learners, both students and the teacher, work together and use dialogue as a means of communicating what they know, and as a way of making meaning of new ideas or concepts. The learning environment can also be understood as being the relationship between thinking and learning, as well as the space within which thinking and learning occurs (Gutierrez, 2012; Lave & Wenger, 1991; Wertsch et al., 1995). According to Hunter (2010),

sociocultural theorists believe that collaboration and dialogue are essential for the transformation of external communication to internal thought. Ideally, the teacher and students jointly participate in activities and learning which allow them to develop a mastery of skills and potential for further learning (Bell & Pape, 2012; Mercer & Littleton, 2007; Moll & Whitmore, 1999; Rogoff, 1995). It is important to note that accumulating knowledge and skills alone does not ensure significant development in students understanding of mathematics (Kinard & Kozulin, 2008; Pirie & Kieren, 1994). Learning mathematics with understanding is a procedure whereby students absorb the mathematical practices of the wider world, and in doing so change their own behaviour and thinking around mathematics (Goos, 2004; Goos et al; 2004; Lave & Wenger, 1991; Yackel & Cobb, 1996). Therefore, it is necessary to ensure that the sociocultural (inquiry) classroom is organised socially and culturally in order to sustain and advance learning with understanding.

Central to the sociocultural theory of learning is the Vygotskian notion of instruction which is understood to be the importance of the interaction of a more capable adult or peer with a less capable, and their mutual use of social and cultural tools to develop understanding (Cobb, 2000; Ernest, 2011; Rogoff, 1995).

2.2.1 ZONE OF PROXIMAL DEVELOPMENT

The zone of proximal development (zpd) was first described by Vygotsky as being the difference between what a child is able to achieve individually and independently, and what a child can potentially do in collaboration with someone more capable or knowledgeable (Litowitz, 1993; McCallum, Hargreaves, & Gipps, 2000; Palinscar, 1986). In learning environments where all participants engage in discourse, and the teacher (or more knowledgeable peer) and students take turns assuming the role of the expert, the responsibility of learning is shared and there are possibilities to refine strategies to become more sophisticated. In this way the zpd is utilized (Brown & Ferrara, 1985; Bruner, 1986; Cole, 1985; Ernest, 2011; Litowitz, 1993; Palinscar, 1986; Rogoff, Mosier, Mistry, & Goncu, 1993). The less knowledgeable or capable student-the novice, is supported through the learning steps in such a way that understanding is internalised, and what was at first accomplished in collaboration is now able to be done independently.

The zpd is traditionally linked to the notion of scaffolding, apprenticeship and guided participation. Addison Stone (1993) describes scaffolding as a metaphor for the practice of a more able other helping an individual to complete tasks which are normally unachievable for them. In this process the facets of the task which are at first

too difficult, are managed, which allows the individual to focus on completing those parts of the task in which they are already proficient. Arguably, this process potentially allows an individual to accomplish more than if assisted to complete the task, and also expedites the development of task proficiency. Similarly, Rogoff (1995) refers to apprenticeship as being the process of developing sophisticated participation while purposefully interacting with more experienced others in set activity. Guided participation refers to how students are able to change and develop their understandings and obligations for particular activities through their own participation in communication and interaction. The zpd is traditionally viewed as being the space where the expert can draw the novice into their zone in order to refine understanding. However, the findings in Esmonde's (2012) study of cooperative group interactions and mathematics learning in three different high school mathematics classrooms demonstrate that sometimes when students are positioned as experts; it can be detrimental to learning. Her study (2012) illustrated that the experts did not assume the expected adult-like role and were not proactive in attempting to comprehend and develop from the viewpoint of the novice. In many instances the expert persisted in pursuing their own ideas irrespective of what the novices had contributed.

Current perspectives of the zpd allow reflection on the ways in which students are able to learn within the zone. In contrast to the view of an expert facilitating an apprentice into their individual zone, current perspectives consider the idea of interthinking, whereby participants are facilitated into a mutual unrestrained space. Within this shared learning space, participants have the potential for learning through unrestricted interactions with each other. The idea of participation within a mutual dialogic space broadens the traditional view of the zpd beyond scaffolding and guided participation to one where learning takes place through shared and active engagement in meaning making (Brown & Renshaw, 2000; Goos et al., 2004; Hunter, 2009; Lampert, 1991; Lerman, 1999; Mercer, 2000; Secada, 1999). From this perspective the zpd is seen as a symbolic space wherein participants are able to mutually take on others' actions through active engagement and understanding of one another's viewpoints (Hunter, 2009). Each participant endeavours to work with and make meaning of others' understanding and attitudes, including the teacher who stands for the practices of the broader community (Goos, 2004; Hunter, 2009; McCallum et al., 2000). This requires active participation from each member of the learning community.

In such a frame, learning is a process of interthinking, together, teacher and students engage in collective dialogue and activity to create a mutual space known as the intermental development zone (Mercer, 2000). This zone alters constantly as the

students and teacher are required to consult and discuss their way through the activity together. The success of the intermental development zone is only assured if communication guarantees that all minds are actively engaged, and the teacher facilitates the students in working slightly ahead of their recognised potential (Mercer, 2000). Within such a shared space, students are able to take ownership of, and direct their own learning as active participants (Goos, 2004; Hunter, 2009; Mercer, 2000).

2.3 INQUIRY CLASSROOMS

In reform mathematics classrooms the structures are reorganised so that students participate in learning by active engagement in doing and talking mathematics (Askew, 2012; Wagner, Herbel-Eisenmann, & Choppin, 2012; Yackel, 1995). This contrasts with traditional mathematics classrooms which are dominated by teacher directed instruction (initiate-response-feedback interaction pattern) (Baumfield & Mroz, 2002; Bell & Pape, 2012). The term inquiry classrooms is an alternate term for reform classrooms where the focus is on students participating in meaningful mathematical activity, explaining their thinking mathematically, and working collaboratively to construct mutual understanding (Bell & Pape, 2012; Goos, 2004; Kazemi & Stipek, 2001; Yackel, 1995).

Internationally, shifts towards reform have included increased focus in curriculum documents on mathematics as communication and problem solving (Chapin & O'Connor, 2007; Yackel, 1995). For example, within the U.S.A., the National Council of Teachers of Mathematics' (NCTM) Standards and Principles emphasise mathematics as communication. Similarly, the National Statement on Mathematics for Australian Schools (Australian Educational Council, 1991) emphasises need for students to learn mathematics collaboratively and to develop effective communication skills. In this document, students are also required to develop ability to solve mathematical problems by practising the processes of conjecture, generalisation, proof and refutation (Goos et al., 2004). Within the New Zealand context, the current New Zealand Mathematics Curriculum places an emphasis on problem solving, reasoning and communicating mathematics ideas (Ministry of Education, 2007a).

Within inquiry classrooms, the teaching and learning process is viewed as social interaction (Pirie & Kieren, 1994; Voigt, 1994). Students in inquiry classrooms are required to engage in interactive dialogue and critically consider what is presented through questioning and evaluative feedback (Bell & Pape, 2012; Kazemi & Stipek, 2001). In the context of learning mathematics; equal emphasis is placed on student

induction into mathematical practices and the understanding of the mathematics (Lampert, 2001).

Within the climate of an inquiry classroom, students are expected to put forward and support their mathematical ideas and speculations with justification and proof, and to carefully consider others' mathematical ideas. In so doing, they acquire the necessary skills to proficiently participate in a community of mathematical inquiry (Goos, 2004; Goos et al., 2004). Through their active participation the students then have potential to develop inherent beliefs in their own ability as mathematicians, in contrast to relying on teacher authority. Conclusions from research by Goos (2004) in an Australian school with students aged 15-17 years old, recognise the effects that participating in communities of practice, made up of experts and novices, have on learning. The results illustrated how mathematics is learned through participating in a community of inquiry. Goos (2004) illustrated characteristics of a culture of mathematical inquiry whereby students purposefully engaged in explaining and justifying ideas, and were expected to make meaning. Although the findings show many positive examples of students' learning in a community of inquiry, it was noted that not all students were willing to fully participate in learning mathematics in this way. Some students preferred to work as individuals and limited their participation in group discussions.

2.3.1 THE ROLE OF THE TEACHER IN INQUIRY CLASSROOMS

Teachers take an important role in the development of an inquiry environment. Goos (2004) illustrated the teacher actions which facilitated the creation of a mathematical inquiry classroom. In the first instance, the teacher engaged in deliberate acts of teaching, including: modelling processes, structuring social interactions, and linking concepts to mathematical language and symbols. Students were required to reflect and monitor their own actions and reasoning. The teacher advanced student thinking by scaffolding inquiry practices and asking questions such as "how is this; and, what is the reason for?" Furthermore, the teacher created expectations that they would explain solutions to others and guide each other's learning through collaborative activity. They were also expected to persevere rather than asking for help and develop clear explanations and justification of solutions in order to make personal sense of concepts. Other research (e.g., Moll & Whitmore, 1993; Yackel, Cobb, & Wood, 1991; Zevenbergen, 2000) illustrates the changed role of teachers in inquiry classrooms. In such classrooms the teacher's role is to facilitate and direct the action and expansion of knowledge, and by doing so, learning becomes a shared realisation.

2.3.2 DISCOURSE IN INQUIRY CLASSROOMS

There is an increasing emphasis on the role that language plays in teaching and learning in Western mathematics classrooms with reform (Zevenbergen, 2000). Utilising language as part of the learning process results in shared ideas and negotiated meanings (Mercer & Littleton, 2007; Moll & Whitmore, 1999; Solomon & Black, 2008; Zevenbergen, 2000). Learning mathematics should be a social activity whereby participants interact with each other to explore ideas. Many studies (e.g., Bauersfeld, 1980; Chapin & O'Connor, 2007; Hicks, 1998; Pratt, 2006; Sfard, 2000; Yackel, 1995) demonstrate that academically productive discourse sustains the development of mathematical reasoning. Being able to articulate one's ideas is a measure of understanding. Reasoning, perception and understanding are developed through social interaction; and discourse and language can be seen as a means for organising thinking (Bruner, 1986, Mercer & Littleton, 2007). A key purpose of engagement in mathematical discourse is for students to make sense of definitions and conjectures which are negotiated and developed (Askew, 2012; Bell & Pape, 2012; Carpenter & Lehrer, 1999; Khisty & Chval, 2002; Kinard & Kozulin, 2008; Lampert, 1990; Lave & Wenger, 1991; Wenger, 1998; Wood, Cobb, & Yackel, 1995).

A range of studies (e.g., Bruner, 1986; Voigt, 1995; Wood et al., 1995; Yackel, 1995) demonstrates the value of participating in collaborative dialogue which allows all participants to negotiate meaning and to extend and elaborate taken-as-shared ideas into something more refined and meaningful. However, in her review of research related to mathematics education and cooperative learning, Esmonde (2009) cautions the need to consider the potential learning strategies that silent students may utilise and bring to a group.

The environment for effective mathematics communication requires careful structuring and facilitation in order to allow access to learning for all participants. Findings from a study by Young-Loveridge, Taylor, and Hawera (2005) with students aged 9-11 years old showed that not all students appreciate collaborative participation in mathematical communication. These researchers described how some students centred on their own explanations rather than understanding the value of others' explanations to enhance their own conceptual development. In addition, Hunter and Anthony (2011) described how expected communicative practices involved in collaborative group work could be challenging for some groups of students. These researchers showed that unless the cultural norms inherent in the practices were closely facilitated, they could lead to students learning undesirable social behaviours or inaccurate mathematical strategies.

2.4 CLASSROOM NORMS

Classroom norms shape student participation. By mutually creating specific social norms teachers and students are able to rethink what their individual roles are and what it means to complete mathematics at school (Wood et al., 1995). Many researchers (e.g., Blunk, 1998; Forman, 1996; Greer, 1996; Hicks, 1998; Lampert, 1990; Weingard, 1998) contend that mathematical dialogue comprising of reasoning and argumentation should become commonplace classroom norms. Reasoning, argumentation, hypothesising and proving are all discourse practices (norms) in which students should participate in, in order to be able to make sense of one another's ideas (Esmonde, 2009; Krummheuer, 1995).

2.4.1 SOCIAL NORMS

Social norms are the common ways in which students take part in classroom activities across any curriculum area. They include practices of explaining, justifying, questioning and discussing different ideas, clarifying one's thinking, completing activities within groups, and making sense of others' explanations (Cobb, Yackel, & Wood, 1995; Yackel et al., 1991; Yackel, 1995). Yackel and her colleagues (1991) examined the mathematical learning of 20 students aged 7 years old. The research project used small group problem solving and the students solved mathematical problems collaboratively. The teacher intervened at intervals in each group and fostered cooperation and the mutual exchanging of ideas. Their results showed that through collaboration, effective classroom norms were constructed.

Through establishing acceptable social norms students develop social autonomy in mathematics and are accountable for their behaviour. They also develop intellectual autonomy through taking responsibility for individual learning (Wood et al., 1995). Wood and her colleagues (1995) showed how a teacher repositioned herself from monitoring and supervising students to being able to monitor and interact with the students as they completed mathematical tasks in small groups. By instilling a belief and value in social cooperation, students were able to make sense of each other's explanations and justifications and become mutually supportive. The development of positive attitudes and beliefs towards mathematics can be achieved by facilitating the development of sound sociomathematical norms.

2.4.2 SOCIOMATHEMATICAL NORMS

Sociomathematical norms are explicit to mathematical activities. They include evaluating mathematical concepts which underpin different strategies and utilising mathematical arguments to reach agreement (Hunter, 2010; Kazemi & Stipek, 2001; McClain & Cobb, 2001; Yackel & Cobb, 1996). McClain and Cobb (2001) contend that as students contribute to the establishment of these norms, they restructure their explicit mathematical beliefs and values. Furthermore, the development of explicit mathematical beliefs and values, such as the importance of being able to explain and justify their mathematical assertions, allows students to enhance their autonomy in the mathematics classroom (Cobb, 2000).

Important sociomathematical norms include the development of explanations and justifications. Some researchers (e.g., Cobb, Wood, Yackel & McNeal, 1992; McClain & Cobb, 2001) argue that in order for mathematical explanations and justifications to be sociomathematical and acceptable, they have to be acted out and validated on mathematical objects. Students offer explanations in order to clarify concepts that may not be clear to others (Cobb et al., 1992). However, as Yackel (1995) argues, this does not mean that other participants will always try to make sense of explanations.

Holding students accountable for their explanations and justifications plays an important role in developing effective sociomathematical norms. McClain and Cobb (2001) investigated the actions of a teacher in creating sociomathematical norms in a classroom with students aged 6 years old. The teacher proactively enhanced the development of sociomathematical norms by directing and intervening in students' interactions. The students were held accountable for making acceptable explanations and justifications. This meant that explanations and justifications had to be acted out on mathematical objects in order to be acknowledged. The results of their study demonstrated that students were able to make significant progress in learning mathematics with understanding through making mathematically sound judgements.

Developing the sociomathematical norms of argumentation and reasoning is important in order for students to enhance their mathematical proficiency. Successful mathematical argumentation occurs when students work together to fine-tune interpretations of what they perceive as mathematically correct (Cobb, Gresalfi, & Hodge, 2009; Krummheuer, 1995; Lampert, 1990). Mathematical questioning is an important way in which meaning is mediated (Baumfield & Mroz, 2002). Splitter and Sharp (1995) suggest that, among skills needed to develop and maintain communities

of inquiry, those associated with formulating, and asking and responding to questions occupy a unique place.

Through establishing specific sociomathematical norms, teachers empower their students to redefine the convictions and values that make up their mathematical identity (Cobb, 2000).

2.5 STUDENT PERSPECTIVES

In order to better understand student learning, it is important to consider what students learning mathematics have to say. Several studies have been conducted aimed at finding out what students think about learning mathematics (e.g., Franke & Carey, 1997; Hodge, 2008; Hunter, 2006; Hunter & Anthony, 2011; Perger, 2007; Young-Loveridge, 2005; Young-Loveridge et al., 2005).

Over the course of their study, Hunter and Anthony (2011) collaborated with one teacher to establish an inquiry classroom setting characterised by mathematical discourse. In order to examine the emerging shifts in mathematical disposition, participation and competencies, the researchers conducted interviews with a group of students aged 11-12 years old, over the course of one school year. By focusing on the students' voices these researchers were able to highlight ways in which these students came to comprehend what it means to learn and do mathematics. The results of their study illustrated that when students were supported to actively participate in a variety of collective mathematical practices, their roles in the classroom changed. The inquiry setting afforded these students the opportunity to take ownership of their learning as active participants engaging in mathematical collaboration and communication, such as discussion and argumentation. In a study by Hunter (2006), a group of 9-11 year old students from an inquiry classroom were interviewed individually in order to examine their perspectives on explanations, and how these impact on their ability to make mathematical meaning. For a year, these students had been learning mathematics in an inquiry setting where making and listening to explanations, and participating in mathematical argumentation were the norm. The focus of this study was to explore these elements from the students' perspectives. The students' responses demonstrated that they considered explanations to be thinking tools, and that mathematical argumentation was seen as a means to restructure mathematical thinking. Students were able to develop mathematical dispositions of understanding and sense making through consistent practice in making and listening to explanations. By using others' statements as thinking devices, students were able to enhance their understanding of mathematical ideas.

Young-Loveridge et al., (2005) conducted a study aimed at examining what students, aged 9-11 years old, thought about learning mathematics. Reporting on analyses of interviews of 183 students, these researchers concluded that most students placed higher value on explaining their own thinking to others as opposed to considering how others had solved mathematical problems. Franke and Carey (1997) conducted a similar study to determine what students from two different school systems perceived learning mathematics was all about. One group of students was learning mathematics in a traditional classroom, and the other in an inquiry classroom. The results of 36 individual interviews of children aged 6 years old concluded that students from inquiry classrooms held significantly different ideas about what it means to learn mathematics to those of students from more traditional learning environments. Similarly, Young-Loveridge (2005) reports on a study where 27 students aged 10-11 years old were individually interviewed in order to explore their perceptions and beliefs towards learning mathematics. Most of the students responded positively to the idea of understanding how others solved mathematical problems; furthermore, that there were many advantages to understanding and using multiple problem solving strategies. In addition, a study was undertaken by Hodge (2008) where individual students aged 7 years old were interviewed regarding their perceptions about learning mathematics. Results of this study support the notion that active engagement in mathematical discourse and valuing thinking plays a significant role in allowing students to develop positive beliefs towards learning mathematics. A study by Perger (2007) of students aged 11-13 years old examined their thoughts about learning mathematics. After students had listed what they believed were essential practices for learning mathematics (their espoused theory), they were observed during a routine mathematics class to isolate the practices they used (theory-in-use). Findings from this study highlight that in order to understand students as learners, it is important to consider what they say and what they do.

2.6 STUDENT IDENTITY

Students need to be provided with opportunities to build and develop meaningful mathematical ideas and forge positive mathematical identities (Esmonde, 2009). There are many definitions for identity in education. Cobb and his colleagues (2009) assert that the everyday meaning of identity is how one connects or belongs closely with another individual or group. In contrast, Wenger (1998) describes identity as a means to show how learning, emerging from active social interaction, transforms who we are. Boaler and Greeno (2000) present another view of identity to describe that which students have to connect with or associate with in a classroom setting to develop

mathematical identities, rather than how students see themselves personally. Learning is viewed as the process of adjustment in participation through which one develops one's viewpoint and role with respect to the practices of that activity setting (Gresalfi, 2009; Lave & Wenger, 1991; Wenger, 1998).

Developing a mathematically competent identity is jointly created in interaction and through actions that realise the expectations of others in a range of repeated circumstances (Boaler & Greeno, 2000; Cobb et al., 2009; Wenger, 1998). As learning is taking place as part of a social practice, it involves the whole person transforming into a full participant as part of a negotiated experience (Greeno & Gresalfi, 2008; Lave & Wenger, 1991; Wenger, 1998). Students who actively participate in taken-as-shared collaborative experiences are able to make sense of what they are learning. Boaler and Greeno, (2000) contend that students who do this are more likely to be able to extend their understanding and skills beyond the classroom.

An important element of identity within the context of mathematics education is how students gain awareness of what it means to do mathematics in the classroom and how they identify with mathematics. Young-Loveridge, Taylor, Sharma, and Hawera (2006) explored the viewpoints of 459 students aged 6-13 years old towards mathematics learning. They report that despite the emphasis the New Zealand Curriculum places on increased use of active mathematical processes, very few students could articulate what these processes were. Many students viewed themselves as passive recipients of knowledge in their mathematics classrooms and were not aware of their own potential mathematical competencies. The students' responses reflected a utilitarian view towards mathematics, with a number of students unable to discuss the nature of mathematics itself. These researchers concluded that there appear to be few opportunities for students to actually participate in such discussions in New Zealand mathematics classrooms. By reviewing cases in mathematics education literature, Cobb and his colleagues (2009) also examined how students develop awareness of what it means to do mathematics, including whether and to what degree they identified with mathematics. They found distinct ways in which students identify with classroom activity: some students clearly identify with classroom mathematical activity; some students simply cooperate with their teacher; some students refuse to connect with the classroom activities, and in doing so develop oppositional identities. Other studies (e.g., Boaler & Greeno, 2000; Cobb et al., 2009) illustrate that there are differing degrees to which students identify, cooperate or resist in different classrooms.

When examining identity in mathematics education, it is important to consider the different types of identity which can develop. Two key types of identity identified in research literature (e.g., Boaler & Greeno, 2000; Cobb et al., 2009; Gresalfi, Martin, Hand, & Greeno, 2009) include a normative identity and a personal identity. Students develop normative identities as a result of the ways in which they *do* mathematics in the classroom; in contrast, personal identities are developed through participation in classroom experiences. Greeno and Gresalfi (2008) categorise a further identity type, a participatory identity. In this definition, the identity of the individual corresponds with what others expect of them and what they expect of themselves, as well as how they participate in the activity.

Another important aspect of identity is the relationship between how students are positioned in the classroom and the development of identity. Esmonde (2009) suggests that there is a gap in literature pertaining to the development of positioning and identity in relation to cooperative learning in mathematics; particularly about ways in which different classroom settings and activities afford different forms of positioning. In her review of the literature, Esmonde (2009) examines the idea of identity as it emerges in social interaction in relation to the ways that individuals position themselves and are positioned by others.

Opportunities for students to learn are affected both by how students are positioned in and position themselves in groups (Esmonde, 2012; Greeno & Gresalfi, 2008). Links can be made between positioning and levels of authority in the group. Individuals may be positioned as experts or novices; or as facilitators who control the group participation structures. Students' views of their own mathematical proficiency also affect how they position themselves in the group.

Key differences may be discerned between those who take the identity of expert or novice in positioning themselves. Esmonde (2012) describes an expert as individuals given the power to choose whose ideas or contributions are valid or correct. In order for there to be an expert in the group, one or more other individuals in the group must be positioned as the novice. Without the novice in a group, there is no one to concede to the expert. Novices are individuals who concede to experts. They have positioned themselves as less capable than other members of the group. Novices are usually told what to do and they accept others' instructions, although they will, at times question the expert's contribution. Discrepancies between experts and novices are usually determined by means of a straightforward declaration by the expert. The remaining members of the group may assume the roles of facilitators. Students in this role

delegate different jobs to individuals, encourage others to ask questions, or persuade individuals to collaborate in problem solving (Esmonde, 2012).

The role of the facilitator in group collaboration affects the balance of authority. Esmonde's (2012) study showed that if no facilitators emerged in the group, the experts were more prone to contributing most of the work, putting forward their own ideas while ignoring much of what the other group members had to offer. Cobb (1995) also emphasises how the facilitator's role can result in power imbalance as one student assumes the authority of regulating ways in which students interact as they do mathematics and discuss it. Cobb (1995) identified two elements which are required for productive relationships to occur. These are the development of taken-as-shared mathematical communication, and that within these expected taken-as-shared exchanges, no individual assumes authority.

It is important to note that identity is fluid and shifts within the framework of practice as one become proficient at that practice, or as others become adept (Esmonde, 2009). Furthermore, when students are granted opportunities to develop constructive positional identities they are ranked as commanding and proficient members of the classroom community. Therefore, these processes need to be viewed as being as important to mathematical development as learning content material (Esmonde, 2009; Greeno & Gresalfi, 2008).

Several studies (e.g., Franke & Carey, 1997; Hunter, 2006; Hunter & Anthony, 2011; Young-Loveridge, 2005) concur that in order for students to develop positive mathematical beliefs and identities through effective mathematical communication, a secure, collaborative environment is essential.

2.7 STUDENT AGENCY

Developing constructive identities in the mathematics class affords students the opportunity to develop agency. There are multiple definitions of student agency which relate to how students work within the mathematics classroom. Solomon and Black (2008) consider agency to be the way in which students have control over what they are doing. A differing explanation by Greeno and Gresalfi (2008) refers to agency as being the type of actions students are able to utilise and that are necessary in order to complete a specific mathematical activity. Student agency is also considered to be a component of collaborative meaning making (Wertsch, Tulviste, & Hagstrom, 1993).

Considering particular types of agency is important. For example, Greeno and Gresalfi (2008) specifically identify two kinds of agency. The first is: disciplinary agency,

whereby the intended outcome is predetermined by the use of recalling facts or definitions for a particular procedure or method. The second type of agency identified by these researchers is: conceptual agency, which refers to the way in which an individual purposefully determines how to solve a problem by selecting from a range of possible methods. These researchers contend that in an ideal setting students should be able to complete mathematical tasks by alternating between conceptualising a solution using abstract tools (conceptual agency) and using those tools to determine whether their hypothesis is correct (disciplinary agency). However, it is important to note that many activities within the mathematics classroom limit students to exercising disciplinary agency, as they are given tasks that involve utilising a known and expected method (Cobb et al., 2009; Greeno & Gresalfi, 2008). A key finding from a study by Boaler and Greeno (2000) was that students are more effortlessly able to develop identities as effective learners when they are placed in positions with more conceptual agency.

For students to exercise conceptual agency they need to be positioned to deliberately select from a range of methods with connection and understanding (Cobb et al., 2009; Greeno & Gresalfi, 2008). Furthermore, Gresalfi and her colleagues (2009) argue that assessments of how competence is being constructed are possible when the classroom setting runs smoothly and all aspects are aligned and supported by accountability.

The concept of agency is closely aligned with student autonomy. Cobb (2000) describes intellectual autonomy as pertaining to how students become aware of how to utilise their individual intellectual abilities when engaged in mathematics. Through his research, Cobb (2000) has seen teachers directing and facilitating communities of students who no longer have to rely on or appeal to the authority of the teacher or textbook, but are able to validate mathematical truths through collaborative discourse. He suggests that it is not enough for students to simply offer a wide range of mathematical contributions; they must be able to determine what can be seen to be mathematically different solutions, insightful solutions or proficient solutions, as well as satisfactory mathematical explanations.

The development of student agency can be viewed as continuing progress from comparatively peripheral participation in classroom activities to more significant participation where students are able to depend on their own findings rather than the textbook or teacher (Cobb, 2000). Similarly, Yackel and Cobb's (1996) study examined how students build up mathematically sound values and beliefs and linked this to

becoming academically independent in mathematics. In classrooms where students are afforded the chance to be actively involved in mathematical discussions and where teachers share processes of effective mathematical problem solving, rather than transmitting information, increased student agency results (Boaler & Greeno, 2000).

2.8 SUMMARY

Advocates of reform in mathematics education have called for a move away from student passivity to active engagement in the mathematical processes of problem solving and communicating mathematical ideas. Inquiry mathematics classrooms reflect the aims of reform education in that they are places where students are able to become active participants of an effective learning community, and where mutual understanding is able to be constructed collaboratively. Inquiry classes reflect sociocultural theory with the centrality of learning occurring as social interaction. Emerging from the idea of learning through social interaction is the Vygotskian concept of the zpd. Traditionally, the zpd has been described as being the difference between a student's individual potential and that which a student can achieve in collaboration with those more capable. In contrast, more recent views of the zpd consider the idea of interthinking, whereby students are pulled into a free space, enabling them to learn through their shared and active interactions with one another. For this symbolic space to be effective, students and teacher are required to critically consider and make meaning of others' mathematical contributions.

In order for students to learn how to participate and contribute effectively in the mathematical community of inquiry, they have to learn and enact specific social and sociomathematical norms. The development of these norms requires the teacher to reposition himself/herself as part of the learning community, as opposed to being the sole authority figure in the classroom. A range of studies has concluded that when students are able to build a repertoire of mathematical values and beliefs through their enactment of sociomathematical norms, they are empowered to become academically independent in mathematics.

In order to understand what affects students' learning, it is important to grant them affordances to share their perspectives on what they consider to be important when learning mathematics. When students are given opportunities to offer insights into their perspectives, understanding develops as to how students enhance their ability to reason mathematically.

Students are able to develop mathematically competent identities through their active engagement in negotiating meaning within a community of learning. Differing types of identities are identified in the research literature. Importance is also placed on how identity is developed by the ways in which students position themselves and are positioned by others during social interaction. This affects opportunities to learn and is connected to the concept of authority in the group.

As inquiry classroom environments enhance opportunities for students to actively participate in mathematical discussion and problem solving, increased student agency results. In traditional mathematics classrooms students tend to exercise disciplinary agency, recalling facts and predetermined methods to solve problems. In contrast, inquiry classrooms value both the use of conceptual and disciplinary agency, allowing students to conceptualise possible outcomes using abstract tools to determine their mathematical worth.

CHAPTER THREE

RESEARCH DESIGN

3.1 INTRODUCTION

The preceding chapter provided the theoretical framework for the current investigation. This chapter outlines the design and methods used in the study. Section 3.2 provides justification for the selection of a qualitative approach for this project and describes the use of case study design. Section 3.3 outlines the role of the researcher. Section 3.4 discusses the data collection methods used in this investigation. Section 3.5 describes the setting, the participants and the research schedule. Section 3.6 details the data analysis. Section 3.7 considers the steps taken to guarantee the validity and reliability of the findings of the study. Section 3.8 highlights the ethical considerations for this study.

3.2 JUSTIFICATION FOR METHODOLOGY

The aim of this project is to investigate the perspectives and roles of 9 and 10 year old students participating in a mathematical community of inquiry, in one primary school classroom. Therefore, a qualitative approach has been selected as appropriate for this project. Qualitative research accentuates gathering rich, descriptive data in natural settings, uses inductive thinking, and focuses on understanding and making meaning of the social occurrences in the setting (Bogdan & Knopp Biklen, 2007; Merriam, 1998; Yin, 2011). Qualitative research aims to capture how different facets work together to shape the whole, with the emphasis being on making meaning from the point of view of the participants in the setting (Merriam, 1998).

This research investigation utilises a case study approach as the aim is to extract meaning and learning through fieldwork, in a bounded system, and provide thick, rich and detailed descriptions to illustrate findings (Berg, 2009; Check & Schutt, 2012; Cohen, Manion & Morrison, 2000; Lodico, Spaulding & Voegtle, 2010; Merriam, 1998). Qualitative case studies are commonly used in the area of education (Merriam, 1998), and therefore in the current study a single case study, situated in the bounded system of a particular setting-one school classroom, was undertaken. Furthermore, this is an exploratory case study, characterised by a range of data being collected in order to allow for thick description aimed at illustrating and supporting theoretical assumptions, as well as attempting expanding on knowledge that would supplement further research

(Merriam, 1998; Willis, 2008). The intent is that this case study investigation will yield results that are generalisable to theory (Berg, 2009; Bloor & Woods, 2006).

The rationale behind employing a case study design in this project is that the defining characteristic that sets case study apart from other types of qualitative research is that it is bounded. Limits and boundaries surround the phenomenon to be explored and in this case, the classroom and the students form the bounded system (Berg, 2009; Bloor & Woods, 2008; Bogdan & Knopp Biklen, 2003; Cohen et al., 2000; Lodico et al., 2010; Merriam, 1998). The purpose of this case study is to find meaning, explore processes and gain an understanding of how participating in a mathematical community of inquiry influences student perspectives and roles (Johnson & Christensen, 2000; Lodico et al., 2010; Merriam, 1998). Case study design seeks to provide an exclusive example of real people in authentic environments (historical, social and cultural contexts), allowing readers to understand concepts more thoroughly, as opposed to merely providing hypothetical theories or principles (Berg, 2009; Bloor & Woods, 2006; Bogdan & Knopp Biklen, 2003; Cohen et al., 2000; Johnson & Christensen, 2000; Lodico et al., 2010). As Berg (2009) writes, case study design aims to capture a holistic depiction and elucidation of a phenomenon.

There are numerous types of case study designs, each having their own specific purpose, length, procedure and complexity (Bogdan & Knopp Biklen, 2003; Lodico et al., 2010; Merriam, 1998). Case studies may also be single case studies or multiple cases or sites (Lodico et al., 2010; Merriam, 1998). Merriam (1998) states that various types of case study research are specifically employed in studies around education. For example, ethnographic case studies, whereby the study pivots around the culture of a school, a group of students or classroom behaviour (Bogdan & Knopp Biklen, 2003; Merriam, 1998). According to Merriam (1998), case studies are also classified according to their disciplinary tendencies, their purpose, or by a mixture of the two. They may be solely descriptive in nature, but more often tend to be a synthesis of either description or interpretation, or descriptive and evaluation (Merriam, 1998). Case studies may also be categorised as being intrinsic, instrumental or collective: Intrinsic case studies aim to understand a particular case, the case being a person, program, institution or activity that is significant in its own right; instrumental case studies study distinct cases in order to better understand broader topics; collective or comparative case studies study and compare multiple cases in order to allow for deeper understanding into a specific issue-the aim of the comparative case study, in particular, is to obtain insight into what is unique about each case, as well as what they have in common (Berg, 2009; Lodico et al., 2010).

Two concerns raised about utilising a case study as a form of inquiry in this investigation are: that the researcher may not adhere to disciplined procedures and would let preconceived notions affect findings; and that case studies do not offer a firm foundation for scientific generalisation (Berg, 2009; Yin, 2009). However, Yin (2009, p. 15) proposes that "case studies...are generalisable to theoretical propositions and not to populations or universes...and your goal will be to expand and generalise theories." So, in order to maintain objectivity, extensive, on-going reflexive examination of the researcher's own assumptions, beliefs and biases took place (Bogdan & Knopp Biklen, 2003; Johnson & Christensen, 2000; Lodico et al., 2010; Merriam, 1998).

3.3 RESEARCHER ROLE

As is characteristic of qualitative research, the researcher is the main instrument in qualitative studies (Merriam, 1998). As such, the role of the researcher in this study was as the sole collector of data. This role was undertaken as observer as participant with more of a leaning towards researcher participant (Merriam, 1998). This meant that the actions of the researcher as observer and collector of data, were made clear to the participants and the researcher was an accepted part of the setting, but the researcher did not participate in the lessons as teacher or student. Prior to any formal observations commencing, the researcher spent considerable time in the classroom getting to know the students and observing classroom practice. A professional and collaborative relationship with the teacher developed. These actions meant that the researcher became an accepted and familiar member of the setting prior to any formal data collection commencing. As the researcher's role was overt, all data were collected openly. The researcher's interpretation of events was shared with the teacher as the study progressed, with the understanding that the teacher was welcome to disagree with any of the analyses or offer alternate or contradicting viewpoints (Cohen et al., 2000). The researcher is an experienced primary school classroom teacher with an interest in how students learn mathematics with understanding. The researcher's own experience at utilising an inquiry approach to teaching mathematics with students of a similar age to the study meant that she was familiar with expected classroom practices and potential outcomes. On the other hand, this also meant that the researcher entered the study with certain assumptions and biases which needed to be carefully monitored over the duration of the project.

3.4 DATA COLLECTION

Qualitative data collection is drawn from the following actions: observing, interviewing, gathering and studying classroom artefacts, and using intuition to examine and interpret the social relationships amongst participants (Yin, 2011). Data refer to the material collected from the project setting which form the basis of the analysis-"data are both the evidence and the clues" (Bogdan and Knopp Biklen, 2007, p. 117). Case study design does not employ one sole data collection method (Bloor & Woods, 2006; Merriam, 1998). By making use of numerous methods of collecting data, rich and distinct forms of information are produced (Johnson & Christensen, 2000; Lodico et al., 2010; Merriam, 1998; Walford, 2009). Yin (2009) also asserts that no one type of data collection method has complete advantage over another. Within this case study design, the researcher collected data by conducting the study in the environment under scrutiny.

Data collection tools utilised in this project were a questionnaire, a Likert attitude scale, video observations of the participants, interviews, classroom artefacts, and detailed field notes (commentaries of the lessons observed and reflections on the interviews). All data collected were triangulated in order to verify findings and to ensure the validity of the project.

3.4.1 THE QUESTIONNAIRE

As part of the fieldwork, and in order to be able to report on what the participants thought about the topic or believed to be true, the researcher needed to be aware of what they were thinking, and so a questionnaire (Appendix A), as a tool, was utilised. The advantages of using a questionnaire are that they are comparatively easy to manage and are standardised (Bicknell, 1998). Using questionnaires also has the advantage of possibly eliciting more honest responses than from an interview (Bicknell, 1998). The participants were all given the same sets of questions to respond to without any intervention from the teacher or researcher. Confidentiality was guaranteed as participants were explicitly told not to write their names on their questionnaires. The teacher and researcher made no attempt to identify any of the questionnaires with any of the participants. The same questionnaire was used at the beginning and end of the data collection period to investigate students' views about learning mathematics. The questionnaire was designed to capture whether or not students held a passive or active belief about themselves during problem solving activities in mathematics lessons. The analysis of the questionnaire consisted of comparing and contrasting the responses from the first and second data collection. Furthermore, the students' responses were also compared to their behaviour and responses during the observations and the interviews to determine whether their initial views were espoused or enacted. Emerging themes and patterns were determined.

A Likert attitude scale (Appendix B) was also used to measure the students' attitudes towards how they viewed themselves while learning mathematics. The Likert scale, developed by Rensis Likert in 1932, is the most widely used form of rating scale developed to determine attitudes directly (McLeod, 2008). These scales are intended to measure attitudes or opinions and are ordinal scales measuring levels of agreement/disagreement. By completing a five point Likert attitude scale, participants were able to communicate how much they rejected or accepted a specific statement. To ensure the validity of the Likert scale, students were instructed not to write their names on them. The participants in the current study were given the options of choosing from five points (highly disagree, disagree, neutral, agree and highly agree) on a linear scale to indicate how much they disagreed (rejected) or agreed (accepted) with a specific assertion about their roles and perspectives while tackling mathematical activities.

3.4.2 OBSERVATION

Qualitative researchers' observations are considered a form of primary data (Yin, 2011). During this investigation, observations were made through video-recording, with the aim of capturing students' conversations and behaviour. Running commentaries in the form of field notes were also taken. Videoing allows for the expansive detail of the classroom setting to be captured. Viewing the footage offers time for reflection on what has been observed (Gamorin Sherin, Linsenmeier, & van Es, 2009). As the sole collector of data, the researcher was the only one video recording in the setting; it was, therefore, impossible to capture all that was happening during the lessons observed. It was also necessary to be mindful that what the researcher was recording may not have even been relevant or important to the study. Furthermore, the researcher was also aware of the observer effect of how her presence may modify the environment. Observations were made at the start of the project, and at the end of the project, thus reducing bias and "lack of representativeness" (Yin, 2011, p.145). By later transcribing the video-recorded lessons, the researcher was also able to reflect retrospectively on what had occurred in the setting. Relevant episodes of interest were analysed directly from the footage, through repeated viewings, with a focus on detailing the communication patterns of the participants and the enactment of social and

sociomathematical norms. The emerging themes and patterns were matched against the theoretical framework.

Comprehensive field notes are utilised in qualitative research as a means of capturing as much detail of the action in the setting as possible (Bloor & Woods, 2006; Cohen et al., 2000; Glesne, 2006; Goos, 2004; Hall, 2000; Johnson & Christensen, 2000; Lodico et al., 2010). Running commentaries on teacher action, student action, board work and materials used, were recorded as quickly as possible during and after each observation by the researcher. The expectation was that, in reading the notes later, the researcher would be able to instantly recall an accurate account of events that took place.

3.4.3 INTERVIEWS

In qualitative research, interviews are used to collect descriptive information in the participants' own words (Berg, 2009; Bogdan & Knopp Biklen, 2007). The aim is for the researcher to be able to understand the world of the participant, and to test, expand on or confirm intuitions (Walford, 2009; Willis, 2008; Yin, 2011).

In the current study, stimulated recall interviews were conducted with individual students to ascertain their interpretations of filmed excerpts. Video-stimulated recall is a popular tool in capturing participants' immediate and specific reflections and perspectives (Cheeseman, 2008; Muir, 2010; Pirie, 1996; Stough, 2001; Theobald, 2008). Most of the literature reports on the use of video-stimulated recall in stimulating teacher reflections regarding classroom interactions and there is little research on describing the use of video-stimulated recall with children or the effects thereof (Cheeseman, 2008; Stough, 2001).

All of the interviews in the current study were conducted in a vacant office adjacent to the classroom setting. During the video-stimulated interviews, individual students were asked to watch a video-record of the observed lesson which acted as a stimulus for reflection. Although the researcher had a script, the interviews were conversational in style and questions were adapted during the course of the interview as required. A small number of segments were chosen for replay and students were invited to interpret their own and their partners' talk. These interviews generally lasted 10-15 minutes and were audio taped for later transcription to supplement the lesson field notes. Comments were requested through questions such as "what was happening here?" See Appendix D for details of the interview scripts.

There are researchers who have highlighted several methodological issues with the use of video-stimulated recall interviews (Pirie, 1996; Stough, 2001; Theobald, 2008).

One of these concerns is the validity of the students' responses to occurrences, which may be distorted when viewed with the researcher (Pirie, 1996; Stough, 2001; Theobald, 2008). Researchers also emphasise that the time lapse between observing the lessons and conducting the stimulated recall interview may influence whether a child is able to offer a true reflection and perspective on events as they occurred, as opposed to that child reconstructing a version of events (Pirie, 1996; Stough, 2001; Theobald, 2008). Furthermore, adults do not often consider children to be reliable or competent in reporting effectively on occurrences concerning themselves (Pirie, 1996; Theobald, 2008). However, the significance of giving children a voice is being increasingly emphasised, as the understanding that children have the power to expose critical issues which are important to them is identified and welcomed (Duffield, Allan, Turner, & Morris, 2000; McCallum et al., 2000; Theobald, 2008; United Nations, 1989). The Child Rights movements that have originated from the signing of the United Nations Convention on the Rights of the Child strive to allow children to have a voice and the right to be part of any decisions made concerning them (Perger, 2008; Theobald, 2008; United Nations, 1989). "If one is to consider children as competent beings with rights to participate in and have a say over their lives, then the use of video-stimulated accounts is a valid one" (Theobald, 2008, p.4).

3.4.4 CLASSROOM ARTEFACTS

Analysing documents within the field involved examining written examples of how the participants collaboratively solved mathematical problems during the observed lessons. During this study, the researcher was constantly spiralling back and forth into literature so as to confirm, reflect on or refine findings. A research journal with the purpose of reflecting on the research process, including the choice of research design and entering any potential emerging biases, assumptions and interpretations of events was also kept.

3.5 THE RESEARCH STUDY: SETTING, SAMPLE AND SCHEDULE

This section describes the setting for this investigation, the details of the participants and the phases of the study.

3.5.1 THE SETTING AND THE SAMPLE

This project took place at an urban primary school during term one and term two of the 2013 school year. This school has a Decile¹ rating of 6. The students at this school mainly come from middle to low socio-economic home environments and represent a range of ethnicities.

The investigation took place in one classroom in this school. The teacher invited by the researcher to participate in this project was identified as teaching mathematics in an inquiry setting. This is the teacher's second year of teaching. During preparations for this project, this teacher and the researcher developed a collaborative working relationship characterised by trust.

Initially, 24 out of a class of 28 Year Five and Six students aged 9-10 years old agreed to participate in this study. During the course of the study two participants left this class. Data were collected from all of the participants. The students in this class mostly solved mathematic problems in heterogeneous groups consisting of boys and girls working across a range of Mathematical Curriculum Levels from Level Two to Level Four of the New Zealand Curriculum Document; equating to Numeracy Level Stages Five, Six, and Seven of the Numeracy Professional Development Projects (Ministry of Education 2007a; Ministry of Education, 2007b).

3.5.2 THE RESEARCH STUDY SCHEDULE

This investigation was conducted over approximately 4 months (March-June, 2013) and consisted of two phases of data collection.

It is emphasised that the first collection of data took place as the teacher was *beginning* to establish a mathematical community of inquiry within this class. The second collection of data took place phase after this process had been evolving and developing for approximately sixteen weeks.

Preliminary phase

Prior to any data collection several meetings between the researcher and the teacher took place. The purpose of the meetings was to outline and discuss the research plan, the objective of the study, the sampling, the tools, and the timeframe for data collection. The researcher was also introduced to the participants and the aims of the project were

¹ Each state and integrated school in New Zealand is ranked into deciles, low to high, on the basis of an indicator. The decile indicator measures the extent to which schools draw from low or high socio-economic communities. Decile 1 is the lowest and decile 10 is the highest.

shared with the students. In order to familiarise the students and teacher with her presence, the researcher spent considerable time in the class before commencing with any formal data collection.

First phase

This phase comprised of the first data collection in the setting. This involved the 24 student participants completing the first questionnaire (Appendix A) and the Likert attitude scale (Appendix B) during one of their scheduled mathematics lessons. Later that week, the teacher divided the total number of participants into two heterogeneous groups. Four consecutive mathematics lessons were then filmed-involving each group alternating over the four days. The research data collected during the observed lessons included video footage of the problem solving activities, written samples of group work, teacher lesson objectives, and researcher field notes. Individual video-stimulated recall interviews, with each participating student, were conducted immediately after each filmed lesson. Additional data included reflective discussions between the researcher and the teacher following each lesson.

The format of each of the four lessons was standard, allowing for natural variations due to the unique nature of each heterogeneous group; and the fact that the teacher was also in the process of establishing the norms for participation in a mathematical community of inquiry. The lessons were 50-60 minutes in length, beginning with the teacher presenting a contextual mathematical problem to the group. Details of each problem presented are provided in Appendix C. On each day, one group of 12 students was then divided further into smaller peer groups, ranging in size, but with an average of three-four participants in each group (The remaining students not involved in the day's observation, completed independent mathematical problems). These small groups then attempted to solve the problem collaboratively for approximately 15-20 minutes. During this time the teacher roved the room, interjecting in group discussions necessary; furthering the dialogue and development of social and sociomathematical norms. After this time, the small groups were called together to form one large group of 12. The teacher then facilitated the students' discussion of how they had solved the problem.

Second phase

The aim of this phase was to carry out a second collection of data following the students' on-going participation (16 weeks) in a mathematical community of inquiry. The data collection followed the same format as phase one: participants completed the

same questionnaires and Likert attitude scale; were observed and filmed over four consecutive mathematics lessons, followed by individual video-stimulated recall interviews. The researcher and teacher also reflectively discussed each lesson.

Third phase

Analysis of data was on-going throughout this investigation. Phase three of this study comprised of retrospective analyses of the collected data.

3.6 DATA ANALYSIS

The aim of analysing data is to make sense of it (Merriam, 1998). Analysis of data, in this case study, meant making sense of the mathematical practices of the teacher and students in the mathematical community of inquiry. Data collection and analysis were carried out concurrently in order to generate categories and develop theoretical insights, as well as to manage the project effectively. This involved analysing the questionnaires and Likert attitude scales in order to develop an understanding of the participants' perceived beliefs and attitudes towards learning mathematics. Comparisons were made between the espoused beliefs that emerged from analysis of the questionnaires and Likert scales, with the enacted beliefs which evidenced in the video footage and the stimulated recall interviews. To complete this process comprehensively, all video and audio recordings were completely transcribed and revisited many times in order to identify themes. The video-stimulated recall interviews supplemented the video footage, so that social interactions could be clarified. Patterns were captured and categories generated to allow for coding of the collected data and to lead to a comprehensive and detailed description of this case. Themes were examined against the assortment of collected data; including the classroom artefacts, field notes and teacher reflections.

3.7 VALIDITY AND RELIABILITY

Reliability is the degree to which, if a study was repeated, would the same results be found (Merriam, 1998; Yin, 2009). Reliability is grounded on the supposition that there is one sole reality and that studying it again and again will produce the same results (Merriam, 1998). However, there is an inherent problem with achieving this in qualitative research design in that it deals with human behaviour, which is fluid and so there are no yardsticks to take duplicate measures and ascertain reliability (Bogdan & Knopp Biklen, 2003; Merriam, 1998). Therefore, the qualitative researcher must ensure that the results are consistent with the data; in other words, the results need to made sense (Bogdan & Knopp Biklen, 2003; Merriam, 1998).

Validity deals with two main concerns. Firstly, internal validity deals with the extent to which the researcher examines or measures what they believe they are examining or measuring rather than what might be inferred (Bicknell, 1998; Yin, 2009). Secondly, external validity refers to what extent the findings of the case study may be generalisable to other situations and settings (Bogdan & Knopp Biklen, 2003; Merriam, 1998; Yin, 2009).

Ensuring validity and reliability in this case study required that this investigation was carried out in an ethical manner; that careful regard was given to the design of the study; that the data collection method and analysis were carefully considered and documented; and that attention to how the results were revealed in the final report were given. Research validity was further enhanced through multiple sources of data collection. Furthermore, triangulation of the collected data was vital to ensure the validity of this study-a variety of data (running commentaries, transcribed and analysed footage, stimulated recall interviews, and analysis of documents) were collected, analysed and compared in order to understand or confirm any findings. The researcher's supervisors were also asked to review interpretations and findings as a further strategy to enhance validity.

3.8 ETHICAL CONSIDERATIONS

Research adequacy was maintained by ensuring the research objective was clear and that the design of the investigation led to meeting the objective (Massey University, 2010). This was an overt study and therefore, the purpose of the study was made clear to all of the participants and their guardians, and informed consent sought. Voluntary and written consent was obtained by adhering to Massey University's Human Ethics Committee's 'Code of Ethical Conduct for Research, Teaching and Evaluation involving Human Participants'. Informed consent was gained in writing from the Board of Trustees of the school, the teacher, the students and their guardians-as the students were under the age of 15 years (Appendices E, F, & G). To ensure that ongoing trust was sustained, it was made clear to the guardians what and how data would be gathered throughout the duration of the study (Lodico et al., 2010). The students and their guardians retained the right to withdraw themselves/their child from participation at any point. The teacher also retained the right to withdraw from this investigation at any time.

Throughout this study the researcher adhered to the Massey University's Human Ethics Committee's 'Code of Ethical Conduct for Research, Teaching and Evaluation involving Human Participants' (Massey University, 2010). The tenets of informed

consent, confidentiality, doing no harm, sensitivity, and honesty were maintained at all times. First and foremost, the researcher was compelled to uphold what was best for the education and welfare of the students in this class, to compromise this would be in conflict with the fundamental principle of doing no harm. All participants in the study, including the principal, the Board of Trustees, the teacher, students and their guardians were fully informed of the intentions and length of the study, including how the presence of the researcher could impact on the daily routines of the specific setting (the classroom), and/or the wider school. Because the study was undertaken within the participants own classroom during routine mathematics lessons, harm to the students was reduced.

Within a case study, the researcher is required to initiate and foster close and trusting relationships with key participants within the environment. The researcher spent considerable time in the setting prior to any formal data collection commencing. Harm to the teacher was reduced by open and honest communication throughout the study, as well as allowing the teacher to set appropriate times for interviews, discussions and analysis. There remained the ethical dilemma pertaining to the potential change in the professional and collaborative relationship between the teacher and the researcher. It was not anticipated that judgement on the teaching practice of the teacher would be made as this was not relevant to the objective of the study; rather the study aimed to focus on the perspectives of the students. Respect for all participants as people and not mere objects of research was essential to ensure that relationships were not compromised and that trust was secure and sustained over the duration of the study (Berg, 2008; Bogdan & Knopp Biklen, 2003; Cohen et al., 2000; Lodico et al., 2010). This, therefore, involved allowing the teacher to view parts of the analysis and for the researcher to consider suggestions; and should disagreement in interpretation arise, to assure the teacher that his remarks or evaluations were written into the final report.

It is imperative that anonymity and confidentiality are guaranteed and that the issue of privacy is neither invaded during a study or refuted once the research is complete (Berg, 2009; Bloor & Woods, 2006; Cohen et al., 2000). The potential vulnerability of participants in the current study, particularly if confidentiality or anonymity were to be breached was considered and respected by the researcher. Regarding anonymity, the researcher took extreme caution during filming to ensure that unintentional filming of students who had not consented to participate in the study did not occur; if it occurred, however, the researcher was ethically bound to ensure that none of the footage was used as part of the study and was destroyed. The impact of changes in the natural interaction and behaviour of the participants when being filmed was also considered.

Prior to any filming taking place, the teacher or researcher explained and discussed the purpose of the filming, asking the students to ignore the equipment as much as possible. Practice sessions occurred to allow the students to get used to filming taking place. Pseudonyms were used throughout the writing of the current study.

3.9 SUMMARY

A qualitative research design was chosen as the most appropriate method for this investigation. In particular a single case study design was implemented. Multiple forms of data were collected by the researcher, including a questionnaire and a Likert attitude scale, on-site observations which were filmed and fully transcribed, video-stimulated recall interviews which were audio-recorded and wholly transcribed, comprehensive running commentaries, reflective discussions with the teacher, and classroom artefacts. To ensure the reliability and validity of this study, the researcher maintained a high level of ethical consideration, and the data collection and analysis were carefully documented. Ethical principles were maintained at all times throughout this investigation to ensure that no harm would come to any of the participants. The findings and discussion of this investigation are reported in the ensuing chapters.

CHAPTER FOUR

THE PERSPECTIVES OF STUDENTS LEARNING MATHEMATICS IN AN INQUIRY CLASSROOM

4.1 INTRODUCTION

The literature chapter drew attention to how learning mathematics in an inquiry classroom provides opportunities for students to become active participants in their learning. Understanding what students think about learning mathematics in an inquiry classroom is important to consider. In this chapter, the perspectives of the students are illustrated. At the beginning and at the end of the study, students completed a written questionnaire and a Likert attitude scale. Student voice, as expressed through their responses is described. Importantly, it must be emphasised that the first phase of the current study was completed at the start of the school year, just as the teacher was beginning to reorganise the classroom structures to reflect an inquiry classroom environment. By the second phase, the learning structures had been reorganised and the classroom environment reflected the collaborative learning features of an inquiry classroom.

Section 4.2 illustrates the responses and analyses of the questionnaires. Questions from the questionnaire which were similar in nature were grouped together. Section 4.3 details the responses and analyses of the Likert attitude scales. The passive and active statements from the Likert attitude scales were grouped together.

4.2 THE QUESTIONNAIRE

During both phases of data collection the students completed the written questionnaire (Appendix A). The written questionnaire sought to find out what they thought about learning mathematics. Responses from phase one and phase two of the current study are presented in the Tables below.

Table 4.1 shows students' responses to the first two questions: what is mathematics and why should we learn mathematics?

TABLE 4.1 Students' responses from phases one and two to the questions: what is mathematics and why should we learn it?

FIRST PHASE (n=24)	SECOND PHASE (n=22)
a) 24 stated that mathematics was a subject involving numbers and methods and had to be learned and/or memorised	 a) 19 stated that mathematics was about solving problems and asking questions in order to understand. One student stated it was a subject learned at school, and two stated that it was about memorising times tables and rules
b) 24 described the utility of mathematics and stated learning mathematics was about giving the teacher an <i>answer</i> to a problem and was useful in helping secure a job involving money or building	 b) 19 stated that learning mathematics would help them with everyday life, e.g., in a job, with money; for their future. One student stated it would help them become a mathematician; and two stated it would help with learning in general

The responses from the first phase illustrated that all of the students thought about mathematics as a school subject. This involved learning about numbers and methods by rote in order to present answers for tests or the teacher. They took a utilitarian view of mathematics and believed that it was useful to learn in order to secure a good job in the future. The teacher and/or procedural methods were believed to hold the authority in mathematics.

In contrast, a shift in perspectives can be seen in the second phase where most responses emphasised that learning mathematics meant active engagement in reasoning. In this later phase the students believed that mathematics was about reasoning and solving problems as opposed to finding an answer for an authority figure (the teacher) or a test.

Table 4.2 shows the responses of the students to questions which aimed to examine how students characterised competency in mathematics.

Table 4.2 Students' responses from phases one and two to the questions: how do we know if someone is good at mathematics and how do you think real mathematicians do mathematics?

FIRST PHASE (n=24)	SECOND PHASE (n=22)
a) 16 provided the following responses as the <i>only way</i> to know if someone was good at mathematics: They give the answers straight away; they answer the teacher's questions quickly and correctly. The remaining responses stated that ability should be evaluated through answers and tests, and obtaining 100% would indicate that someone was good at mathematics	a) 16 stated that the behaviour of students during mathematics lessons would determine whether or not they were good at mathematics; e.g., working excellently with others, sharing ideas; clear explanations; collaborative discourse, taking risks and speaking up even if unsure and asking questions. Six students stated that testing would indicate whether someone was good at mathematics
b) Responses described mathematicians as older people and that they solved long and complicated problems, way beyond the capabilities of the students	b) 13 stated that they were real mathematicians, as mathematicians did mathematics just as they did in class: e.g., by co-operating when proving solution strategies; by learning from others; by asking questions and helping each other understand. Nine students stated that mathematicians just solved very difficult problems and were really good at mathematics

The responses from the first phase indicated that most students believed that mathematical competency was knowledge driven rather than something which was constructed. None of the students believed that any of them had the ability to be real mathematicians. These perspectives would have been formed based on their prior experiences in mathematics classrooms in previous years at school.

Changes in classroom structures which had occurred by the second phase of the current study resulted in a shift in perspectives. Most students now believed that mathematical competency was about being actively engaged in mathematical activity and constructing understandings through collective sense-making. Most of the students perceived that they were real mathematicians who collaboratively engaged in problem solving.

Table 4.3 illustrates students' perspectives on how mathematics was learned in their classroom.

Table 4.3 Students' responses from phases one and two to the questions: when you do mathematics in your class: who does the teaching and the talking; and who asks the questions and gives the answers; and what do you do if you get stuck?

	FIRST PHASE (n=24)	=24) SECOND PH				
a)	Who does the teaching? 18 stated that the <i>teacher</i> did the teaching. Three stated that <i>everyone</i> did the teaching. Three stated that a <i>good maths solver</i> did the teaching	a)	Who does the teaching? 15 stated that everyone did the teaching. Six stated that the teacher did the teaching			
b)	Who does the talking? 10 stated that everyone did the talking; and 10 stated that the teacher did the talking. Two stated that the students did the talking; and the remaining two students stated that only people who knew the answer talked.	b)	Who does the talking? 18 stated that everyone did the talking. Four stated that the teacher did the talking.			
c)	Who asks the questions? 10 stated that everyone asked questions. Seven stated that the teacher asked the questions. Five stated that the students asked questions; and two stated that anyone who didn't understand asked the questions	c)	Who asks the questions? 14 stated that <i>everyone</i> asked the questions. Seven stated that <i>students</i> asked the questions. One student stated that the <i>teacher</i> asked the questions			
d)	Who gives the answers? 18 stated that the <i>students</i> gave the answers. Three stated that the <i>teacher</i> gave the answers; and three stated that <i>everyone</i> gave the answers	d)	Who gives the answers? 19 stated that everyone gave the answers. Three stated that the teacher gave the answers			
e)	What do you do if you get stuck? 17 stated that they would ask someone who knew the answer, or the teacher questions. The remaining seven students reported that they would try to work it out themselves	e)	What do you do if you get stuck? 20 stated that they would ask questions if they got stuck. Two stated that they would keep trying different ways until they solved the problem			

Although some of the responses in the first phase illustrated beliefs in active student participation, most students indicated that the mathematical authority lay with the teacher or with students who were perceived as having mathematics ability. Many students believed that only students should answer questions. Other responses illustrated a passive approach to constructing reasoning and a dependence on more knowledgeable others when stuck.

Clearly, in the second phase, a pronounced shift in perspectives was noted. Authority in the mathematics classroom was now perceived as evenly distributed and there was

a sense of everyone being legitimate members of the learning community. Classroom observations supported these perspectives.

The responses illustrated in Table 4.4 are to questions concerning the actions students should take while problem solving in mathematics lessons.

Table 4.4 Students' responses from phases one and two to questions about working collaboratively to solve mathematics problems

FIRST PHASE (n=24)	SECOND PHASE (n=22)
a) Do you like to work with others during mathematics lessons? Why? 19 stated that they liked to work with others during mathematics because it made it faster and easier when working together. Five indicated that they did not like to work with others as they would get distracted	a) Do you like to work with others during mathematics lessons? Why? 20 said yes, they liked to work with others during mathematics lessons as it helped them understand; they could share ideas; and they would learn more strategies. Two said no, it was distracting to work with others
b) Is it important to be able to explain to other children how you solved a problem? Why? 23 agreed it was important to be able to explain your solution strategy to others as it would prove your answer was true and help others learn	b) Is it important to be able to explain to other children how you solved a problem? Why? 22 stated that it was important to be able to explain their solution strategies to others in order to ensure they understood it themselves, and to share their learning and ideas with other students. Many stated that it helped them learn and proved their thinking to others. One student added that it was an enjoyable challenge to prove their answers to others
c) Is it important to understand how someone else solved a problem? Why? 23 agreed it was important to understand someone else's explanation. Some gave reasons that it would show you were listening if the teacher asked you to repeat it; others stated it would provide a quicker way to solve the problem and help people understand; and it was a way of sharing strategies. One student stated that if you knew the answer there was no reason to listen to how someone else solved it	c) Is it important to understand how someone else solved a problem? Why? 22 stated it was important to understand someone else's solution because they would then learn it and could use it. It was a way of sharing learning

Responses from the first phase indicated positive beliefs in active participation in problem solving through collaborative reasoning. However, what they perceived as

important was them explaining to others rather than collaborative construction of reasoning or them listening and learning from others.

The responses in the second phase illustrated beliefs in active participating in problem solving through collaborative reasoning. Moreover, classroom observations showed that these beliefs were enacted.

Table 4.5 illustrates the students' perspectives regarding what they consider to be difficult and fun about learning mathematics.

Table 4.5 Students' responses from phases one and two to questions about the most difficult and the most fun about doing mathematics

FIRST PHASE (n=24)	SECOND PHASE (n=22)				
a) What is the most difficult thing about doing mathematics? There was a range of responses from everything to specific mathematical concepts or operations and having to answer questions quickly	a) What is the most difficult thing about doing mathematics? 12 stated that ensuring they understood and could explain their thinking to others in a group were the most difficult things about doing mathematics. The remaining 10 stated that ratios and proportioning were the most difficult things				
b) What is the most fun about doing mathematics? Most of the students were able to state an aspect of mathematics that was fun, e.g., problem solving; working collaboratively; easy things; games. Four stated that there was nothing fun about learning mathematics, as it was hard	b) What is the most fun about doing mathematics? 20 stated that working with others, listening to others, arguing about different ideas, discussion, and asking questions were the most fun things about learning mathematics. Two mentioned operating on numbers and getting the right answer as being the fun part of learning mathematics				

Responses in the first phase showed that most students believed that difficulties in mathematics were linked to specific concepts or operations. Most students indicated that actively engaging in mathematics activity was fun. However, classroom observations did not show that all students were actively participating in mathematics tasks.

In the second phase, students described parts of active participation as being difficult as well as some conceptual aspects of mathematics. Students believed that it was fun to actively participate in mathematical reasoning. Classroom observations illustrated that most students actively participated in problem solving.

4.3 SUMMARY STATEMENT

In both phases of the current study it was evident that students held positive beliefs about engaging in learning mathematics. However, classroom observations in the first phase generally did not support the students' perspectives. The students stated that they believed in active participation and collaboration to solve mathematics problems, yet were not observed to do so. A significant shift was noted in the second phase of the current study. Students had again stated positive perspectives towards learning mathematics and were now observed to enact their beliefs.

4.4 LIKERT ATTITUDE SCALE

During both phases of data collection the students completed a Likert attitude scale (Appendix B). Students were given the opportunity to disagree, agree or remain neutral about statements regarding learning mathematics. The passive statements have been grouped together, followed by the active statements. In each category column (e.g. disagree, neutral, agree) the first percentages represent the first phase of data collection and the second percentages represent the second phase. In all instances, the analyses of the statements disregard the neutral responses.

Table 4.6 illustrates the percentage of students' responses to the passive statements.

Table 4.6 Percentage of students' responses to passive statements about learning mathematics

Passive Statements	Disagree		Neutral		Agree	
I prefer working on my own to solve	29%	27%	50%	50%	21%	23%
problems in maths						
Our teacher is the only person I	54%	82%	25%	9.0%	21%	9.09%
should ask for a correct answer						
Maths is all about remembering facts	21%	41%	42%	27%	37%	32%
I am good at maths if I can get the	17%	27%	25%	32%	58%	41%
correct answer by remembering rules						
It is more important to get the right	58%	77%	29%	9.0%	13%	14%
answer than to explain how to solve						
the problem						
My teacher thinks I am good at	0%	9.09%	46%	23%	54%	68%
maths						

In the first phase, based on responses to four out of six of these statements, the majority (n=24) of students rejected passively learning mathematics. On the other hand, many students agreed that mathematics was about remembering facts and

believed that getting the correct answer by remembering rules equated with being good at mathematics.

In the second phase, although the majority (n=22) of students indicated that they were good at mathematics if they got the right answer by remembering rules, the responses to the remaining statements illustrated that, generally, most students rejected the passive statements.

Table 4.7 illustrates the percentage of students' responses to the active statements.

Table 4.7 Percentage of students' responses to active statements about learning mathematics

Active Statements	Disagree		Neutral		Agree	
I enjoy learning maths in this class	4%	0%	17%	18%	79%	82%
Talking to other people is an important part of learning maths	4%	0%	29%	14%	67%	96%
It is important to listen when others are explaining their thinking in maths	0%	0%	4%	4%	96%	96%
If I disagree with someone's explanation I speak up and say so	4%	4.5%	13%	18%	83%	77%
I think of myself as a real mathematician when I am doing maths	21%	9%	25%	36%	54%	54%
I am not afraid to ask someone to prove their answers	8%	0%	33%	18%	58%	82%
I can justify my solutions to others in maths lessons	21%	9%	33%	36%	46%	54%
I can generalise strategies with other numbers during maths lessons	33%	18%	17%	23%	50%	59%
Making mistakes is part of learning in maths lessons	0%	0%	13%	9%	87%	91%
It is ok to disagree with the teacher about the answers during maths lessons	0%	0%	17%	9%	87%	91%
Having an argument about maths means I am learning	17%	4%	17%	18%	66%	78%
Working with other students makes maths easier for me	8%	0%	25%	50%	67%	50%
I think I am good at maths	8%	14%	29%	9%	63%	77%
Maths is more about problem solving than remembering facts and rules	17%	14%	58%	36%	25%	50%

The responses in the first phase illustrated that most (n=24) of the students held active beliefs about learning mathematics. Many students indicated how listening to each other, taking risks to ask questions, making mistakes; and disagreeing with the teacher were all legitimate ways of learning mathematics. However, classroom observations did not support these perspectives.

In the second phase, the responses indicated that most (n=22) of the students continued to hold active beliefs about learning mathematics. They accepted that making mistakes was part of learning; talking and listening to others was important;

and that having arguments about mathematics meant they were learning. Classroom observations fully supported these perspectives.

4.5 SUMMARY

Gaining insight into the perspectives of the students was a central goal of the case study. This was accomplished by considering responses from student questionnaires. While the students' responses in the first phase of the study illustrated positive perspectives about learning mathematics, classroom observations did not support these beliefs. Significant changes can be seen in the second phase of the study when the students' stated perspectives were matched against what was observed in the classroom. The findings presented in this chapter are discussed in detail in Chapter Six.

CHAPTER FIVE

THE ROLES OF STUDENTS LEARNING MATHEMATICS IN AN INQUIRY CLASSROOM

5.1 INTRODUCTION

The literature chapter drew attention to how learning mathematics in an inquiry classroom provides opportunities for students to become active participants in their learning. Evidence was provided that when students build constructive mathematical values and beliefs through their enactment of sociomathematical norms, they are able to become more autonomous mathematics learners. By actively engaging in negotiating meaning within a community of learning, students are empowered to develop mathematically competent identities. In this chapter, the case study is organised around distinct themes. Findings are presented sequentially to promote a full description of the case. The classroom environment, the ways in which activities were structured for learning mathematics, and how those practices promoted student identity and agency are described.

Sections 5.2 to 5.7 describe the findings of the first data collection phase: Section 5.2 outlines the classroom context. The structure of the learning sessions is described. Section 5.3 outlines the varying roles students assumed through positioning within the classroom context. Section 5.4 describes student agency. In Section 5.5, the teacher's pedagogical actions are described in order to explain how the learning environment was created. Section 5.6 describes the students' use of social and sociomathematical norms. The ways in which the students' engaged in collaborative discourse is described in Section 5.7. This is followed by a summary of the first phase. Sections 5.9 to 5.14 outline the findings of the second data collection phase, followed by a summary.

THE FIRST PHASE

The first phase took place at the start of the school year as the teacher was beginning to establish an inquiry learning environment.

5.2 THE CLASSROOM CONTEXT

At the beginning of the study, the teacher explained, in interview, that he was in the process of reorganising classroom structures to facilitate the development of a learning environment reflecting an inquiry classroom. He expressed his realisation that many

students would face changes to their prior experiences in mathematics classrooms. He stated that expectations were being created for active student engagement in learning mathematics through the development of effective participation and communication skills.

Observations of mathematics lessons illustrated that without teacher prompting; few students asked questions or offered explanations. Teacher interventions were regularly enacted while effective classroom structures were being established.

5.2.1 THE STRUCTURE OF THE LEARNING SESSIONS

In all mathematics learning sessions, the class was divided into two heterogeneous groups. One group worked independently on mathematical tasks. The second group worked with the teacher for the hour of mathematics instruction. Their lesson began as a group. After the teacher talked to them briefly they were divided into small collaborative problem solving groups consisting of 3-4 students. One mathematics problem, situated in a real-life context was presented. The small groups were required to collaboratively clarify the context of the problem, and work at solving and recording solution strategies for the problem for 15-20 minutes. Then they returned to the larger group setting for the remainder of the mathematics lesson, approximately 20-25 minutes. In this concluding session, solution strategies were shared and discussed. Any questions or problems encountered were also voiced.

As the students worked in their small groups, the teacher roved purposefully amongst them, listening to explanations and only intervening to progress thinking or position children to participate in the discourse. He listened carefully to understand the students' reasoning, and intervened when necessary to strengthen classroom social and sociomathematical norms.

In the concluding session, the teacher carefully selected which groups were to explain their solution strategies. He described the factors that affected his selection of the first group to provide the first explanation and subsequent explanations. He explained how the sequencing of the group explanations provided other groups with opportunities to reflect on their own ways of reasoning. He also considered explanations which offered a more effective, refined or varied approach to the problem. Or he would select a group which had struggled to find an effective means of solving the problem. Within the discussion, all groups were given opportunities to clarify their strategy solutions.

5.3 STUDENT ROLES WITHIN THE CLASSROOM CONTEXT

On-going and retrospective examination of data provided insight to how students reasoned and achieved mathematical competency in the classroom. Observed classroom episodes and responses from student interviews illustrated the varying roles students assumed while participating in mathematics lessons.

5.3.1 ASSUMING IDENTITY THROUGH STUDENT POSITIONING

In every small group situation during this phase of the study, it was noted that students positioned themselves in distinct ways. Responses from individual interviews supported classroom observations that students assumed varying roles through positioning themselves at levels of student competency. Students explained why they listened more to particular students, or why they felt they had to explain the solution to the others, or why they facilitated the interactions in the group. The following extract from an individual interview illustrates the viewpoint of a student who had positioned another student as being *better* at mathematics that her. The student, when shown the video-record and questioned about it stated:

Researcher: Kim, I notice here that you don't say much, why is that?

Kim: Well, Robert knew the answer and he was explaining

Researcher: Do you think you could have explained how to solve the problem?

Kim: Umm, no, I don't think so. I didn't really get it, I didn't know the answer

Researcher: Did you think Robert's answer was correct?

Kim passed the authority of determining what was correct to the teacher.

Kim: Well, the teacher said it was correct, so I think Robert was right. He could do it, I couldn't

This illustrated that because Kim was unable to offer a solution strategy, and Robert could, she had deferred to him as an authority and therefore did not even question his explanation.

In interview, some students stated that those students who could *do* mathematics easily or who were students known to do well in tests were *better* at mathematics. These students conceded that they were *not* as *good* at *mathematics* and were therefore passive in their acceptance of any of this group of students' explanations and justifications. In turn, when interviewed another group of students outlined how they

considered themselves to be *better at mathematics* than some in the group. In all instances, the way in which individual students positioned themselves aligned with how others positioned them. In the lesson observations, the students who had been positioned by other students as being *better at mathematics* took control of the problem solving sessions in the small groups and predominantly controlled the talking. Some students chose to passively engage in the small and large group interactions by facilitating the smooth running of the group or by directing others to explain or listen. Others resisted active participation in problem solving. For example, one particular observation illustrated how a student dominated a concluding group discussion session. Callum assumed an authoritative role and led the discussion by directing which students were to explain their ideas. Interested in seeing the outcome, the teacher stepped back and observed the interaction without intervention. None of the other students questioned Callum's control of the discussion and did as they were directed. During an interview, one student was asked why he had accepted Callum's authority so readily.

Levi: Well, we all know that Callum is really good at maths

Researcher: What do you mean?

Levi: He is clever; he knows the answers straight away and gets the best marks for

tests

Researcher: What about you? Do you think you are good at maths?

Levi: Sort of, sometimes I know stuff, but I am not as clever as Callum

Through Levi's responses he illustrated that Callum was good at mathematics because of his ability to respond quickly to questions and achieve well in tests. Therefore he positioned himself in a passive role in response to the authoritative role he had given Callum. Callum, at interview stated that he thought he was the best at mathematics in the class, as he was quick to respond with the correct answers to questions. When prompted further, Callum revealed that he liked to show other students how to "do it" as he considered that he was an authority in mathematics. Analyses of lesson observations highlighted that Callum was keen to share his ideas with others, but was reluctant to listen to anyone else except the teacher. He had positioned himself as a classroom authority, only deferring to the higher knowledge of the teacher and considered that other students had little knowledge to offer.

5.4 STUDENT AGENCY WITHIN THE CLASSROOM CONTEXT

Students were given mathematical problems that could be solved in a range of ways. Some students recalled facts or used set methods they had been taught in previous classrooms to solve problems. Several students solved mathematical problems by employing written algorithms. In interview, one student explained her reliance on a written method to solve the problem².

Researcher: So what was happening here?

Pamela looks at the video record

Pamela: I got the answer?

Researcher: How did you get it?

Pamela: I used algorithm, like I know. I can easily do it with algorithm

To ascertain whether Pamela could use multiple strategies to solve the problem she is asked if she could have solved it differently.

Researcher: Could you have solved it in a different way?

Pamela: No, if I use algorithm then I know I will get the right answer

Pamela's stance showed that rather than validating her own reasoning using the mathematics, she had positioned rules and procedures as the mathematical authority. Similar situations and responses were observed in this phase of the study. When asked by the teacher or others to explain algorithms, *none* of those who used them could clearly explain how these written methods worked, although they remained convinced that if they had followed the procedures they had previously been taught then they must be correct.

5.5 THE ROLE OF THE TEACHER

In interview, the teacher described how he aimed to establish an inquiry classroom. He explained how the structure of the learning sessions and development of norms and expectations occurred. He asserted that he aimed to create a learning space which supported the students' collaboration and the development of positive mathematical identity. He stated:

It is vital that students are given clear expectations on how they are to act individually and collaboratively within the classroom. Together, the students and I promote our learning space as a safe environment where everyone is encouraged to take risks in

² Taine has some cows. Taine buys 29 more cows. Now he has 81 cows. How many cows did Taine have before?

their thinking and verbalise their thoughts. We establish the expectation that we must

support each other and work together to solve problems. By supporting them to ask

questions and seek clarification, their confidence and understanding will increase. The

students must believe that they can do it

In the first instance in the lesson observations, when the large group discussions

commenced he established clear social norms which required the students to be active

members of the learning community. To reinforce this expectation, before a group

began to explain a solution strategy for a problem³ he stated:

So we are going to start here; John, you are going to start explaining, but before you

start explaining, what is your responsibility (gesturing to the larger group) as citizens of

this community? What do you have to do now?

Students: Pay attention

The teacher prompts further to ascertain whether students know what this looks like.

Teacher: How is John going to know that you are paying attention? What are you going

to be doing?

Chris: Looking at him

The teacher emphasises the importance of Chris' statement by revoicing what he has

said.

Teacher: Looking at him. Right, all of you move in a little closer

Through the teacher's actions it is emphasised that paying attention and looking at

someone does not necessarily mean active engagement. When John begins to explain

and it is evident his voice is too soft the teacher stops him and asks:

Teacher: Can you hear what he is saying?

Students: No

Teacher: So what do you need to do?

Students: Make him speak louder

David: Can you speak louder, John?

³ Taine has some cows. Taine buys 29 more cows. Now he has 81 cows. How many cows did

Taine have before?

48

Through this action the teacher indicated that the explaining student had a responsibility to speak loud enough to be heard. At the same time he emphasised that

it was the responsibility of the listening community to listen and if they could not hear

then they needed to tell the explainer to raise his voice. He also focused on the need

for all listeners to make sense of the reasoning being explained. This was illustrated

later in the explanation when the teacher observed a student who looked puzzled:

Teacher: Pamela, do you understand what John is talking about?

Pamela: No

Teacher: So what should you do if you don't understand?

Through this action he puts the responsibility back with the student but uses the

opportunity to reinforce the whole group's need to actively engage in the reasoning

being explained. When Pamela looks blankly at him he asks the whole group:

Teacher: Who knows what we should do?

Lavinia: Like, tell them in a different way

Teacher: You mean, ask them to explain it in a different way?

Lavinia: Yes

Throughout this session, the teacher constantly scanned the listening students. When

he noticed a lack of attention or confusion, he provided students with opportunities to

alter their behaviour by asking open-ended questions, rather than telling them what to

do or how to do it. As a result, although the students still looked to the teacher for

support and guidance in this community under construction, he consistently positioned

them as being responsible for their sense-making of the proffered explanations.

Through these actions he was indicating their need to become more agentic in their

meaning making.

In this initial stage of the study, in interview, the teacher explained that in order to

develop student agency, he considered that all students needed to actively engage in

mathematical activity. He described how he regularly intervened in student interactions

and modelled examples of how to reason mathematically by making effective

explanations and justifications.

In lesson observations, social norms for how the groups were to work together were

repeatedly and explicitly addressed as illustrated in the following lesson excerpt:

49

Teacher: So what is your responsibility when you work together to solve this problem?

Xavier: We need to work out the answer to the question

Noting Xavier's focus on getting the answer rather than constructing a reasoned explanation the teacher observes that the expectations for how the students are to work together are not clear to all students. In response, he probes all the students' thinking with an open-ended question.

Teacher: How are you going to do that?

Sue: I think we have to write it down so everyone in our group can see our answer

Building on Sue's understanding of the expectations, the teacher probes to embed deeper understanding of this.

Teacher: Do you think you have to do something before that?

Luke: We have to talk about the question

Luke's reply adds more detail. In order to extract deeper understanding of the expectation, the teacher prompts further.

Teacher: What do you mean by that?

The teacher pushes Luke to clarify what he means and then revoices to emphasise that they must all make sense of the problem before constructing an explanation. He asks an open-ended question to gauge whether the students know what to do next.

Luke: Well, like make sure we understand the question

Teacher: Yes, you have to think together about what the problem is asking you to do; you have to understand what it is asking before you try to solve it. What happens when you have all understood what the problem means?

The teacher prompts the students with open-ended questions to extend their statements in order to find out whether they understand how to develop a group explanation.

Robert: We have to try and figure out the answer

Teacher: How will you do that?

Robert: Well, we used to have to figure it out on our own before I was in this class, but well, now I know that is not how we do it in maths anymore

Teacher: Can you tell us more about what you mean?

Robert: I know we have to work with our buddies in our groups, so not on our own

May-Lin: Yes, we have to talk to each other to find out what everyone is thinking about

the answer

Teacher: Just about the answer?

Janine: I think you mean how to work it out, how we get the answer

Noting that the students keep referring to the answer, the teacher reminds them of the expectations and responsibilities each group member has when finding a solution strategy. He reminds them of the importance of providing a group explanation. To probe whether students have detailed understanding of what a group explanation is, he asks another open-ended question.

Teacher: Can anyone add more about how this works? No? Ok, let me remind you that everyone in the group needs to work through a solution step-by-step making sure you understand every part of it. It is important to find a group explanation. What do I mean by that?

David: Everyone in the group must understand every step to get to the answer. Anyone in our group must be able to explain properly so that everyone can understand

David has provided proof that students are beginning to understand the importance of working together to find a group explanation to a problem. Students are expected to participate in discussions aimed at developing shared understandings of the expectations of collaborative sense-making in mathematics. Through active engagement in such discussions, students are granted affordances to increase their agency.

Research (Yackel, 1995) has illustrated that the development of student authority as mathematicians in the classroom are constructed through the ways in which they engage in the sociomathematical norms enacted within the classroom community.

5.6 STUDENTS USE OF SOCIAL AND SOCIOMATHEMATICAL NORMS

The ways in which the development of classroom norms was fostered is illustrated in this section. The following two episodes occurred during the large group sharing

51

session and illustrated how students' realisation of the importance of providing acceptable explanations for their solution strategies⁴ was advanced.

Teacher: Now, Anthony, I would like your group to share how you solved the problem. I would like you to explain carefully how you got the answer. You need to show us your thinking step-by-step as you explain to us all how you got the answer

The teacher encourages Anthony to share the group's strategy. In response, Anthony hesitates indicating his role may not be as an authority in mathematics. When Anthony hesitates, the teacher explains why he believes it is important that Anthony share the strategy with the larger group. He restates the importance of listening and making sense of the reasoning. He shows them how to make a conceptual explanation and how to make sense of it step-by-step. These actions press the students to shift beyond the social norm of making explanations to more closely draw on what makes a conceptual explanation as a sociomathematical norm.

Teacher: Anthony, don't worry, we are all taking a risk, What is important is that you try and explain your answer so we can hear your thinking and all of us can understand how your group was thinking when you worked the answer out. Just try your best

Anthony: First, we had to work out how many lollies Sarah had at the beginning...

With teacher prompts and guidance, Anthony explained his group's solution strategy.

Later in this session, the teacher directed a different group to explain their solution strategy.

Angela: We just did 177 and 25 and that equals 202

The teacher notes that Angela is focused solely on explaining to him. He directs her to look at the other students while explaining and share her explanation with everyone. Through his actions the teacher is indicating that all members of the group need to be responsible.

Teacher: Angela, when you are explaining, I would like you to look around at everyone's face and see if everyone is listening, or if someone has a question. What we are doing is bringing the control of the lesson to the person who is explaining. See if you have everyone's attention, look for people with their hands up, or even better listen for when someone says "excuse me Angela". Everyone in this larger group has a responsibility to listen and speak up so that we know that everyone understands each explanation

⁴ At her party Sarah provides some lollies. She gives away 177 of these lollies and is left with 25. How many lollies did she start with?

Angela: We started with 177 and then we added 25 and our answer is 202

Noting that the other students are sitting passively and looking either at him, Angela or around the room, the teacher interrupts Angela's explanation. His action recognises that he has observed the way in which the students are placing responsibility on him to sense-make rather than assuming a more active role themselves. In response he insists that Angela provides the mathematical reasons for her actions and models some examples of the sorts of questions the others should ask.

Ok, so now we need to start getting some real mathematical talk going on. Angela, where did you get 177 from? Where did you get 25 from? Why have you added them together? These are examples of the sorts of questions you all need to ask Angela so that she can explain mathematically how her group solved the problem. Angela, please continue

His expectations cause Angela to present the first part of her explanation in detail. She then assumes further responsibility for her fellow students to make sense of the reasoning she is using by asking if anyone has any questions.

Angela: Well, we know that Sarah has some lollies, but we don't know how many. We do know she gave 177 lollies away and she has 25 left over. Alright, so she started with some lollies. Any questions?

When no one responds she continues

Angela: So we knew that we couldn't just take 25 away from 177 because she didn't have 177 lollies she must have had more

The teacher's initial press on his students to actively engage in listening and sensemaking and subsequent actions provides space for other voices to contribute. Cam asks her to justify what she has stated causing a deeper conceptual explanation.

Cam: How do you know she had more?

Angela: Because after she gave 177 away she still had 25 lollies left over, so she must have had more to begin with

Cam nods in agreement

Angela: So it was something take away 177 is 25. We knew the best way to work it out was to add the 25 Iollies back on to the 177 Iollies and then we would know how many she had at the beginning. So, first we took 77 away from 177 and that gave us 100

Another student autonomously requests that Angela explain where the 77 came from.

Tony: Where did you get 77 from? Why did you do that?

Again this deepens the conceptual reasoning as Angela adds more detail to her

explanation.

Angela: From 177 Iollies, because we knew if we took 77 away we would get 100 which

makes it is easy to add 25

In turn, the teacher recognised the opportunity this exchange has afforded to increase

student agency in providing and assessing clear mathematical explanations. He

presses Angela to further explain the mathematics involved in her solution strategy,

again reinforcing the sociomathematical norm of what makes a clear mathematical

explanation.

Teacher: Angela, why did you think it was easier to add 25 to 100, rather than adding

25 to 177? How do you make this explanation mathematical, what is the maths

involved?

Angela: Well, because the number 100 has zeros in it, where the tens and ones are, so

we can add any number easily because they will be in place of the zeros

5.7 COLLABORATIVE DISCOURSE

Initially, at the beginning of the study, there were no set patterns of how students talked

together while solving mathematics problems. Many students talked past each other,

each intent on being the mathematical authority and having their own explanations or

answers heard. Some students resisted active participation altogether and unless the

teacher was present to draw them into the discussion, they positioned themselves as

passive onlookers. Other students worked individually and few attempts were made to

jointly clarify any aspect of the task, as the following excerpt shows⁵.

Group (Manu, Pamela, Max)

Max: Here, I will read the question to the group

Max reads the question, Pamela appears to be listening, and Manu stares into space

Max: So who would you rather be and why?

⁵ Sally and David have agreed to work for their mum over the holidays. The pay they get will vary though. Sally will get \$10 for the first day she works and \$2 more for every day she works after that. David, on the other hand, will get \$1 for the first day he works, but for each he works

from then on his pay will be doubled. Who would you rather be and why?

54

Pamela hears the problem and solves it in her head. She announces the solution and

starts writing it on the page as an algorithm.

Pamela: David got \$28 and Sally got \$22 (writing this down) so David got more money

Manu continues to stare into the distance.

Max: Yeah but who would you rather be? Who would you rather be and why?

Pamela: David

Max: Why?

Pamela states the answer and asserts that she would rather be David because he gets

more money. She does not offer any further justification for her reasoning and Max

does not press her to explain further.

Pamela: David and this is why, because David has more money that Sally (writing while

she is speaking)

Max: more money, more money

Pamela notices she has made an error in her algorithm and begins writing it again,

without further explanation.

Pamela: and that is why I want to be David...did David get paid doubles? (Pamela looks

at her algorithm)

Pamela: I think I did it wrong. I am doing it again.

Neither Max nor Manu ask her to explain her algorithm and spend the rest of this

episode in silence while Pamela reworks her algorithm

Although Max and Pamela verbalised their thoughts throughout this episode, they did

not work together to find a solution. Throughout the above interaction, Manu sat quietly.

At the end of this episode, Max too fell silent and left the working out solely to Pamela.

The students in this group have not worked collaboratively. There was little evidence of

them listening carefully to each other as they pursued their individual ideas. Each child

positioned themselves as individuals. These actions illustrated that they considered

that mathematics was not a constructive process in which collaborative discussion and

sense-making had a place.

Following this lesson, using a video-record, Manu was interviewed:

Researcher: What was happening here?

55

Manu: Nothing

Researcher: What do you mean?

Manu: I was trying to work it out?

Researcher: Did you know how to do it?

Manu: No

Researcher: Ok, who knew how to do it?

Manu: Pamela

Researcher: Did you have any questions for Pamela; did you ask her any questions?

Manu: No

Researcher: Why not?

Manu: I was confused. I didn't know anything

These responses illustrated that although Manu was confused, he did not see his role as needing to ask any questions or seek clarification. In response to further questions Manu stated that Pamela knew the answer and that would satisfy the teacher when they returned to the larger group discussion. He expressed that he was too shy to ask any questions. Manu had positioned himself in a passive role and placed Pamela and the mathematical procedure she was using as the authority which he considered the teacher wanted.

5.8 SUMMARY OF THE FIRST PHASE

In this first phase, the classroom structures were beginning to be reorganised to allow the development of effective mathematics learning. When asked in interview, many students stated that working with others was important in learning mathematics. However, it was evident from classroom observations that many students believed the teacher and written methods held the authority in the classroom. On-going teacher interventions were required in order to facilitate the development of taken-as-shared expectations in how groups were to work together. In order to distribute authority evenly and to increase student agency, the students were regularly positioned to actively participate in collaborative discourse and sense-making. It was evident that some students willingly engaged in the challenges of learning mathematics in a different environment to what they had previously experienced. Other students resisted change.

THE SECOND PHASE

The second phase took place approximately sixteen weeks later (almost two school terms). An inquiry learning environment had been under construction for almost two terms.

5.8 THE CLASSROOM CONTEXT

In developing an inquiry classroom, the teacher facilitated students' participation in learning by active engagement in doing and talking mathematics. The aim was for students to be able to explain their thinking mathematically and work collaboratively to make sense of mathematics. For this to happen authority needed to be evenly distributed and students had to develop agency. By the second phase there had clearly been significant shifts in the sharing of mathematical authority in the classroom as in interview; the teacher explained he intervened less frequently in student interactions than at the beginning of the study. He explained that effective participation and communication skills were becoming embedded as the school year progressed. Students recognised and understood the importance of taking responsibility for their own learning and sense-making by collaborating with others. The importance of making clear explanations and understanding others' contributions was becoming common practice. A learning environment which recognised the relationship between thinking and learning had been established through the reorganisation of the classroom structures. As a result of the establishment of taken-as-shared expectations, the students no longer required recurring teacher intervention to actively engage in problem solving.

5.9 STUDENT ROLES WITHIN THE CLASSROOM CONTEXT

By the second phase, observed classroom episodes and responses from student interviews illustrated that students had built and developed meaningful mathematical ideas and had created positive mathematical identities. Students had developed awareness of what it means to do mathematics through social interaction. The ways in which students were positioned while problem solving provided them with further opportunities for learning.

5.9.1 ASSUMING IDENTITY THROUGH STUDENT POSITIONING

Assuming identity through positioning based on perceived mathematical competency no longer occurred. Students did not categorise themselves as being better or worse than anyone else at mathematics. The following is from an episode of one small group solving a problem⁶ which illustrates that now the students positioned them self and each other as having mathematical contributions which they knew would add productively to their mathematical activity.

Group (May-Lin, Robert, Alice)

Robert has conjectured that the way to solve the problem is to skip count up to the total length of rope required.

Robert: It is 1.5m and if you added another one...tied another one it is 3m. So, you keep adding 1.5m until you get 13m. What do you think?

Alice has listened carefully making sense of what he has said and in response she states that their group should use the method.

Alice: Let's try doing it that way, ok, May-Lin?

The three students together add the 1.5m sections of rope by skip counting, but realise that this takes their total over 13m. Alice assumes responsibility and states:

Alice: We need 13m. We need to think about this

May-Lin states that they need to contextualise this situation as real-life. She draws on the context of the problem to develop a realistic reasoning. The three students talk and listen using productive discourse to finally agree that they have a solution.

May-Lin: I wonder, I think they need the extra 0.5m to tie around the poles

Robert: Can you say that again?

May-Lin: 0.5m, you know to actually tie around the poles. That is real life isn't it?

Alice: Yes, Wendy wants to tie across these two poles so she will need some extra to make the tie around the poles. That makes sense

May-Lin: Wendy's goal is 13m to stretch between the poles and then she can use the extra to tie around the pole. So how many pieces of rope will she need?

Alice/Robert: 9

May-Lin: So we have solved it

⁶ Wendy wants to have a rope long enough to stretch between two poles 13m apart, but she only has pieces of rope 1.5m long. How many of these pieces would she need to tie together to stretch between the two poles?

58

A zpd was created and authority shared. Each student respected what was conjectured responding to the mathematics rather than the person.

Individual interviews with other students supported the observation that students no longer positioned themselves as *better* or *weaker* at mathematics than anyone else in their group or the class. The following excerpt from an individual interview presents a view which was similarly expressed by others. It shows how students believed they had progressed to more evenly distributing the mathematical authority as they engaged in mathematical activity. It also shows how they knew now how they could learn from each other.

Researcher: How do you think it has happened that everyone is taking part and talking more when you are solving problems in maths?

Cam: We have been doing a lot of learning to take control without the teacher having to interfere and stuff, so we are like learning from each other more

Researcher: Could you talk a little more about what that is?

Cam: Well, it's like us learning from each other and for us to have to learn the skills ourselves by asking for help

Researcher: Do you think that will make you better at maths?

Cam: Kinda, yeah because it helps. Like before a lot of people didn't talk much, but after we learned how to talk to each other and ask questions, people were talking a lot more...so yeah, it did help us get better

Cam illustrated the shared understanding of the importance of everyone working together so that they could all learn more. Individual students were no longer identified as having more or less authority in the mathematics classroom than others.

The following excerpt from an individual interview illustrated another student's point of view on why he believed he was now as good as anyone else at mathematics:

Researcher: John, do you think you are better at maths now than you were before?

John: Yes because as you saw last time, I was crying because I wasn't really sure. But now I manage to speak up and ask questions of what they mean and that's how I have been able to explain properly because I have been asking questions. I just never used to ask questions and I never used to understand. Last time you were here, if we didn't agree we wouldn't say, but now we do

John has recognised his obligation to actively interact with others in order to make sense of mathematics. His statement emphasised that students believed that authority was evenly distributed across all the members of the classroom community and nobody was positioned by their fellow students as being more or less mathematically competent.

5.10 STUDENT AGENCY WITHIN THE CLASSROOM CONTEXT

At this stage, the students positioned themselves and each other as having responsibility to contribute reasoned mathematical contributions. Their accountability to contribute constructively is illustrated in the following excerpt as the students worked collaboratively on constructing a mathematical explanation to a problem⁷ which could be solved through various means. Leo begins by conjecturing a possible solution strategy.

Leo: I thought it could work if we did 300 take away 100 which is 200. Then if we do 70 take away 40 is 30, and then we do 6 take away 8

Kim listens and then challenges the final section. Her action illustrates the way in which an intermental zone (Mercer, 2000) has been constructed and all the children are engaged in meaning-making.

Kim: I can see how the first two bits work but I can't see how you do the end bit, you can't do it like that. How do you take 8 away from 6?

Leo rethinks what he has conjectured and, seeking further sense-making asks Kim to help him clarify his reasoning. Kim provides an alternate method of solving the problem while at the same time holds herself accountable to Leo for suggesting a solution which made sense.

Leo: Oh, yeah, let me look at that again. It works easily for the hundreds and the tens because I am taking a smaller number away from a bigger number. I need to think about this. Mmm, I am stuck. Kim, what you were thinking about doing at that bit?

Kim: Yeah, I realised that the 8 was bigger than 6, so I thought, what if I kept the 76 and took 8 away from 76. So that makes it 68. Then I can do 68 take away 40 is 28. What do you think?

Leo: That makes sense

-

⁷ For her birthday, Mary got an iPod with some songs on it. She downloaded another 148 songs and she now has 376 songs in total. How many songs were on her iPod when she first got it?

Clearly, these students had constructed a zpd in which clear conceptual mathematical explanations were explored, validated and justified through using their mathematical reasoning as the authority. The sociomathematical norm of constructing mathematical explanations and justifications were taken-as-shared. At the same time they held each other accountable to communicate their ideas and negotiate meaning.

5.11 THE ROLE OF THE TEACHER

In interview, the teacher described how he facilitated the smooth running of mathematics lessons by intervening in student interaction only when necessary. He explained the effect of the consequences of his actions in creating an effective learning environment.

I believe that collaborative learning can only work in an environment where the children feel safe and confident to express themselves fully. Only after this safe environment has been established will you witness the whole class expressing themselves. It is also paramount that the teacher learns to step back and allows the children to work out a problem or discuss something without interruption. Over time these students' confidence has grown and when they work collaboratively they are able to recognise lost focus and actively engage others through questioning

The teacher has emphasised that a collaborative learning environment needs to be nurtured. His belief in the importance of evenly distributing mathematical authority in the classroom is evident in the way he repositioned himself. These actions created affordances for student collaboration and careful deliberation resulting in increased student agency.

5.12 STUDENTS' USE OF SOCIAL AND SOCIOMATHEMATICAL NORMS

Now most students questioned and probed the explanations until justification and proof were provided. This occurred both in the small and large group sessions. The teacher encouraged student persistence and perseverance in pursuing mathematical arguments both in interaction and in a way that promoted metacognitive reasoning.

Teacher: If you think you have an answer, prove it. Ask yourself questions

Individual interview responses supported observations that students acknowledged the importance of explanation and justification. For example, when a student was shown a video-record and asked what was happening he stated:

Luke: Well, Pamela said that she had worked the answer out in her head, but I was confused because I didn't know the answer

Researcher: Go on

Luke emphasises that Pamela is obligated to provide proof for her solution strategy.

Luke: So we told Pamela to tell us how she got the answer. She started telling us, but I didn't understand, so I knew I had to ask her to explain it differently, otherwise I wouldn't get it. I asked her to prove how she got the answer

Researcher: And did she prove it?

Luke: Yes, because after she proved it, I got it, I could see how she got there

This extract illustrated that the students knew that they were expected to provide acceptable explanations and that this was the individual responsibility of each student. At the same time they knew it was also their responsibility to question until they had full understanding. Students had become more agentic as they recognised their obligations to contribute to making sense of the mathematics.

Many students realised that sense-making encompassed not only understanding others' explanations but also understanding the meaning of mathematical difference. The following episode described how students furthered their mathematical understanding through negotiating the meaning of mathematical difference as they regrouped to share solution strategies8.

Alice: We added \$100 and \$100 together and that gave us \$200. Then we added \$50 and \$40 and that gave us \$90. Then we added \$3 and \$7 so that gave us \$10. Then we added all of these amounts together and got \$300

Teacher: Did anyone solve this problem in a different way?

Tony: Our group said split \$153 into \$100, \$50, and \$3. Then we split \$147 into \$100, \$40, and \$3. Then we added the \$100's together and then we went \$40 + \$50 is \$90

Sonja challenges Tony's explanation and describes how it is identical to Alice's strategy. She asserts that her group's strategy is different and explains why.

Sonja (interjecting): But that is the same as Alice, you have just swapped the groups around. I think our one is actually different

Sonja then explains how her group recognises the relationship of \$157 and \$143 to doubles and solves it by doubling \$150 to get \$300

⁸ Tana had \$153 saved to buy a new skateboard. For his birthday he got given \$147. How much money has Tana got altogether?

These actions show how the students knew they had the authority to compare the similarities and differences between their group's solutions strategy and others. Generalisations had begun to be constructed within the sociomathematical norms enacted in the classroom.

5.13 COLLABORATIVE DISCOURSE

In order for students to make sense of the mathematics they were expected to effectively communicate their ideas and reasoning through active participation in all mathematical activity. When conjectures were made by others', all students were consistently obligated to make sense of these. During this phase of the study, discourse patterns were purposeful and collaborative. Students worked together to clarify the mathematics problems before discussing possible solution strategies. The following episode illustrated how one group of students found an effective solution strategy⁹ together.

Group (May-Lin, Chris, Julia)

The students systematically deconstruct the problem by clarifying what the problem is and what they have to solve.

May-Lin reads the question to the small group

Chris: So Nic has got four pieces of wood, they are all 2.5m long and he wants to saw them

Julia: Into 1m long pieces

Chris: Yeah. So what do we have to try and do?

May-Lin: Figure out how many pieces of 1m he can get out of all of the planks

Chris makes a conjecture which is accepted by the other students. May-Lin demonstrates her understanding of the conjecture by visually representing it. Julia understands the idea but recognises a problem with the next step to the solution.

Chris: Yes, we could go 2.5m x 4. Do you think?

The two girls nod in agreement. May-Lin then draws four planks and writes 2.5m beneath each one. She then writes 2.5×4

Julia: but how do we get them into 1m pieces?

-

⁹ Nic has bought four planks of 2.5m each. How many 1m planks can he saw out of these planks?

May-Lin shares her idea of how the planks can be divided into the required lengths. Chris demonstrates his understanding of this idea by continuing the explanation. Julia has followed their conjectures and is able to add meaningfully to finding an effective solution strategy.

May-Lin: This already has 2m, so he could already get one...I mean, two out of these (She points to each drawn plank)

Chris: So, you just saw it in half and then you got two and then 0.5m is left

Julia: Yes

May-Lin: So each plank would be two pieces and then 0.5m is left

Julia: Now we just have to do this with each plank and add up the pieces, agreed?

May-Lin/Chris: Yes

Step-by-step within on-going discussion, the students were able to create and record the notational solution to the problem. It was evident that it was now a shared expectation that students consistently utilise collaborative discourse to jointly negotiate effective solution strategies in mathematics lessons.

5.14 SUMMARY

This chapter has outlined the journey the students took as a community of learning was established. In interview, the teacher provided explanation of how he established an inquiry classroom over the course of the study. The ways in which classroom structures were reorganised to facilitate the development of an effective learning environment reflecting inquiry classrooms were described. How students were positioned while working to solve mathematics problem were illustrated. Authority in groups which had been assigned based on levels of assumed mathematics competency during the first phase was now evenly distributed. How students reasoned and achieved mathematical competency was illustrated by the ways in which they engaged in collaborative discourse to solve mathematics problems. Descriptions were provided for how students explained and justified solution strategies and defined mathematical difference. The significance and implications of the findings presented in this chapter are discussed in the following chapter.

CHAPTER SIX

DISCUSSION AND CONCLUSION

6.1 INTRODUCTION

The previous two chapters presented an analysis of the findings of the current study. Student perspectives, as outlined in responses to questionnaires were described and the participation and communication patterns were illustrated. In this chapter, the findings are discussed and situated in the theoretical framework of the current study. The transformation of the interaction patterns and the alignment of the students' initial ideas about learning mathematics with their subsequent ideas are presented.

Section 6.2 discusses student perspectives and how these transformed over the course of the study. Section 6.3 discusses the ways in which student identity developed. Section 6.4 discusses the development of student agency. Section 6.5 links the classroom context of the current study to sociocultural theory and reform mathematics education. The role of the teacher is also discussed. In Section 6.6 the communication and participation patterns, the development of social and sociomathematical norms, and collaborative discourse are discussed. Section 6.7 highlights the effects that the complex nature of teaching and learning had on the current study. Section 6.8 outlines opportunities for further research. Section 6.9 presents the conclusion.

6.2 STUDENT PERSPECTIVES

As paralleled in other studies (e.g., Franke & Carey, 1997; Hodge, 2008; Hunter, 2006; Hunter & Anthony, 2011; Perger, 2007; Young-Loveridge, 2005; Young-Loveridge et al., 2005) in order to understand what the students in the current study thought about learning mathematics, it was important to afford them the opportunity to share their perspectives. Students were given a voice through their written responses to statements and open-ended questions about learning mathematics, and by careful consideration of their responses in individual interviews.

Many of the written responses from the first phase of the study illustrated that most students held positive attitudes towards learning mathematics. Many also stated strong beliefs in their own ability to be effective learners of mathematics. Many stated that they understood mathematics to be a collaborative problem solving activity in which

communication was essential. However, further analyses of their statements illustrate their utilitarian attitudes to mathematics. They believed mathematics to be external knowledge which they either had or did not. They also saw mathematics not as something they constructed but rather as something transmitted to them by the teacher. Similar to what Young-Loveridge et al. (2006) found, many of these students viewed themselves as passive recipients of knowledge in the mathematics classroom, despite the New Zealand Curriculum emphasising increased student engagement in problem solving and communicating mathematical ideas. It also became apparent from the first classroom observations that many student statements were not consistent with student behaviour. Initially, what they said they believed and what they did while engaged in mathematical activity was not aligned. This links to Perger's (2007) observations that, at times students' espoused theory may not relate to their theory-inuse. While many students claimed it was important to listen to others and ask questions, classroom observations showed them to be passive onlookers who gave those who they deemed held the mathematical knowledge the authority in the problem solving process.

Analyses of data during the second phase illustrated that most students had a positive sense of self as a learner and reasoned in mathematics. They maintained their belief that active participation in collaborative dialogue while problem solving was an important part of learning and through listening to others they learned. In contrast to the first phase of the current study, the responses to the written questionnaires from the second phase closely matched what was observed during mathematics lessons in the classroom. It was evident through comparison of written statements, classroom observations and interviews that students had advanced to solving mathematics problems in ways they believed and stated were important. It was clear, from their perspectives that effective mathematics learning involved knowing how to explain or question ideas and mathematical concepts. Furthermore, it was evident that student perspectives and the development of student identity and agency were closely related.

6.3 STUDENT IDENTITY

How students were positioned while engaged in mathematical activity affected the development of student identity. Similar to the findings of Greeno and Gresalfi (2008), in the first phase, distinct participatory identities were identified during problem solving sessions. During group interactions in mathematics lessons, at least one student would be positioned as an expert. The experts took control of the authority in the group and were quick to verbalise their ideas and solution strategies. Their authority was not

questioned. Greeno and Gresalfi (2008) described how other students were either positioned as facilitators or novices and this was replicated in this study. The facilitators would take control of recording the strategies or would direct the actions of the other students. They themselves usually contributed little to group discussions. The facilitators never disputed the authority of the experts, nor would they challenge the solution strategy offered by the expert. Facilitators gave the impression that they understood everything the expert put forward. Novices were identified by their lack of participation. They did not contribute towards possible solution pathways and accepted experts' contributions without question. At times, they would voice confusion but not ask for clarification.

Analyses of interview data showed that many of the identified facilitators or novices believed that the expert in their group was an authority in mathematics and therefore, their solution strategy had to be correct. As other researchers (e.g., Esmonde, 2012; Greeno & Gresalfi, 2008) have found, it was apparent that most of the students assumed varying identities based on their own or others' perceived views of their mathematical competence. In this first phase, as a result of how students assumed identity through positioning, group authority was unevenly distributed. There was little evidence of balance and cohesion in how students worked together.

In the second phase, it was evident that a significant shift had occurred in how students were positioned. The distribution of authority had become more evenly balanced. Two studies by Hunter (2006) and Hunter and Anthony (2011) previously illustrated that the actions of the teacher in creating explicit expectations for mathematical learning are important. Similarly, in this study the students were positioned as being responsible for understanding others' explanations and for communicating their thinking clearly so that others could learn from them. Occasionally, some students positioned themselves as experts, facilitators or novices, but generally the heterogeneous groups consistently interacted cohesively. When students were confused they spoke up, questions were asked and clarity was sought. A noticeable development was that students consistently spent time systematically deconstructing the various facets of each mathematical problem before finding effective solution strategies. Many students, who had previously resisted active participation were confidently asking questions or presenting possible solutions. Other studies (e.g., Cobb, 1995; Esmonde, 2009; Franke & Carey, 1997; Greeno & Gresalfi, 2008; Young-Loveridge, 2005) have paralleled this finding. As these other studies previously also showed there was evidence in this study that through the development of a secure, collaborative environment, students had been granted opportunities to develop constructive positional identities. In the second phase, the

students stated that it didn't matter who was better at mathematics, as long as they all understood the mathematics and could learn from each other. Over the course of the current study, the development of constructive positional identities in the mathematics class allowed students to develop agency over their mathematics learning.

6.4 STUDENT AGENCY

The actions that students utilised while attempting to solve each mathematical problem, determined how much control they had in sense-making of the reasoning. At no time during the study did the teacher state that any of the problems were to be solved in a particular way. Each problem presented opportunities for students to exercise conceptual agency and select from a range of solution methods. In order to make connections between solution strategies, students were required to convince others of the reasonableness of their solution method. However, during the first phase, there was little evidence of conceptual agency being utilised. Many students relied on an authority figure to provide the solution. Some students relied on utilising disciplinary agency (Greeno & Gresalfi, 2008), by recalling facts or using set methods to solve problems. As other studies have shown (e.g., Cobb et al., 2009; Solomon & Black, 2008) these rules and procedures were given the authority in the mathematics classroom. Students who solved problems using written algorithms were also unable to provide clear explanations of how these worked, despite being challenged to do so.

In the second phase, most students were utilising conceptual agency (Cobb et al., 2009; Greeno & Gresalfi, 2008) to solve problems. As Boaler and Greeno (2000) showed in their research, now the students in this classroom were able to judge the similarities and differences between each others' solutions strategies. Clearly, now through the comparison of solution strategies, the students were able to further their understanding of mathematical difference. Reasoning and negotiating meaning through active participation in learning activities meant that students were afforded opportunities to become effective learners. Regarding agency, this interaction involved the cognitive demand of *doing* mathematics rather than just applying procedures.

Typically, in traditional mathematics classrooms, it is expected that the teacher makes implicit judgements about what counts as legitimate mathematical explanations. As paralleled in a study by Cobb and his colleagues (2009) analyses of data from the first phase indicated that some students believed the teacher to be the most important authority in the mathematics classroom. Students believed they were obligated to meet only the teacher's expectations. In contrast, in the second phase students believed authority to be more evenly distributed. They assumed responsibility to contribute to

decisions about the reasonableness of a solution method and the legitimacy of solutions. As Cobb (2000) and Yackel and Cobb (1996) explain, as students negotiated meaning and became accustomed to validating mathematical truths through collaborative discourse, they developed opportunities to become academically independent in mathematics. Most students based assessment of their own competence on whether the teacher and other students jointly determined that their contributions to class discussions gave rise to insights into the mathematics problem. These actions suggest that students had come to view themselves as capable of fulfilling their specifically mathematical obligations in the class. They believed they possessed the ability, facility and legitimacy to contribute to, take responsibility for and shape the meanings that mattered. These students had become agentic.

6.5 INQUIRY CLASSROOMS

The students' mathematical skills and understanding enhanced as the learning environment developed. As other studies (e.g., Askew, 2012; Wagner et al., 2012; Yackel, 1995) found, when classroom structures are reorganised so that students learn mathematics through active engagement in problem solving, the aims of reform mathematics education which emphasises reasoning and communication skills, are reflected. The grouping of the students into heterogeneous groups provided opportunities for learning on multiple levels. By actively participating in collaborative problem solving, students were able to learn through limitless interactions with each other. Many other researchers (e.g., Goos, 2004; Hunter, 2009; Mercer, 2000) illustrated that when the teacher and students worked and communicated with each other, an intermental development zone was created which afforded students opportunities to take ownership of their own learning. The same findings emerged in other research (e.g., Brown & Renshaw, 2000; Goos et al., 2004; Lampert, 1991; Lerman, 1999; Secada, 1999). As these researchers explained, with a more current perspective of the zone of proximal development the nature of shared and active participation in reasoning was also reflected in the classroom environment of the current study. Furthermore, the findings illustrate as other researchers (e.g., Bell & Pape, 2012; Hunter, 2010; Mercer & Littleton, 2007; Moll & Whitmore, 1999; Rogoff, 1995) have explained that the active participation of the students in learning mathematics through effective collaboration, with the emphasis on social interaction developed into a shared expectation.

6.5.1 THE ROLE OF THE TEACHER

The role that the teacher played in creating the learning environment was important. The teacher held firm beliefs about the value of the pedagogical approach he adopted in his classroom. He believed that his students could make sense of mathematics through the creation of a mathematical inquiry classroom. He facilitated students' development of cognitive and metacognitive strategies by asking significant mathematical questions, enabling collaboration, and holding students accountable for each others' learning. He explicitly enforced an approach to learning mathematics that encompassed responsibility to others.

As a study by Goos (2004) found, the explicit actions taken by the teacher to create a supportive, effective learning environment resulted in students taking ownership of their learning. In the first phase, as the teacher set about establishing an inquiry classroom, he spent time reinforcing the structure of the learning sessions and the development of classroom norms and expectations. He modelled processes and carefully structured social interactions, intervening when necessary. The expectation was created that students had to collaborate in order to successfully negotiate meaning. This expectation was regularly reinforced throughout the current study. These explicit actions have been shown to be important in many studies (e.g., Boaler & Greeno, 2000; Goos, 2004; Moll & Whitmore, 1993; Yackel et al., 1991; Zevenbergen, 2000).

In the second phase, it was evident that expectations were embedded and consequently, teacher intervention was required less frequently. In granting students' affordances to take responsibility for their own actions during mathematics lessons, the teacher effectively repositioned the authority in the classroom. His actions were similar to those described in previous studies (e.g., Moll & Whitmore, 1993; Wood et al., 1995; Yackel et al., 1991; Zevenbergen, 2000). He had aligned himself as part of the learning community and facilitated active student participation. The teacher had established the practice that all students were accountable for ensuring that everyone understood the mathematics well enough to explain it. A shared expectation had developed that group tasks remained unfinished until each member could explain and justify their answer. The teacher regularly reinforced the value of collaboration by insisting that everyone work together and understands what others contribute. His intervention and prompting throughout the study regularly consisted of asking questions, and in doing so a thinking space was created which supported and strengthened expectations.

6.6 COMMUNICATION AND PARTICIPATION PATTERNS

At the beginning of the study, the teacher stated that developing effective ways in which students interacted with each other was of primary importance. Through regular modelling, prompting and intervention, the teacher aimed to develop active student engagement in mathematical activity, collaborative discourse and effective social and sociomathematical norms. Active engagement in mathematical activity meant that students were expected to collaboratively make sense of the mathematics while finding solution strategies for mathematics problems. Learning how to do this successfully meant that effective social and sociomathematical norms had to be developed. This finding is similar to those described by (Blunk, 1998; Esmonde, 2009; Forman, 1996; Greer, 1996; Hicks, 1998; Krummheuer, 1995; Lampert, 1990; Weingard, 1998).

6.6.1 DEVELOPING SOCIAL AND SOCIOMATHEMATICAL NORMS

The classroom norms consisted of reasoning and argumentation. Social norms included explaining, justifying, questioning and discussing different ideas, completing activities within groups and making sense of others' explanations. As other researchers (e.g., Cobb et al., 1992; Hunter, 2010; Kazemi & Stipek, 2001; McClain & Cobb, 2001; Yackel & Cobb, 1996) explained previously, the sociomathematical norms were explicit to mathematical activity. As these researchers previously showed in their studies, in this study it was important that students were able to explain and justify their solution strategies.

During the first phase, it was evident that these norms had not yet been firmly established. Analyses of data illustrated that students believed it was more important to record the correct answer to the mathematics problem than to be able to offer clear explanations. After being presented with mathematics problems, little discussion took place prior to students recording a range of answers on the big pieces of paper and students were observed to be working individually to solve the problems. In the large group discussions, all of the students directed their explanations to the teacher and accepted his rejection or acceptance of their solution strategies without question. Students waited for the teacher to ask questions or to comment on what was offered and always listened intently to the teacher but not to each other. This finding is consistent with the findings of other studies (e.g., Baumfield & Mroz, 2002; Bell & Pape, 2012).

On most occasions in the large group sharing sessions, one student would offer their group's explanation and the remaining students would sit quietly, without asking

questions unless directed to do so by the teacher or were inattentive, fiddling, drawing or staring into space. When the teacher asked students to revoice what had been stated previously, they were unable to do so. This resistance to actively participating in the mathematics learning was paralleled in a study by Yackel (1995) which argued that even though explanations are offered, others' may not feel compelled to make sense of these.

Throughout the current study, the students were purposefully and regularly scaffolded into explaining their ideas and questioning others' contributions during small group and large group sessions, and in this way these norms developed into shared expectations. As other researchers (e.g., McClain & Cobb, 2001; Wood et al., 1995; Yackel et al., 1991) explain, these actions are important to the construction of a learning community.

In the second phase, significant changes were observed. Students had developed social and intellectual autonomy in mathematics by developing acceptable social and sociomathematical norms. Students did not require on-going prompting from the teacher to state and enact expectations. As paralleled in research (e.g., Cobb, Yackel, & Wood, 1995; Yackel et al., 1991; Yackel, 1995) the social norms of explaining, justifying, questioning and discussing different ideas, clarifying one's thinking, completing activities within groups, and making sense of others' explanations took time to become taken-as-shared sociomathematical practices. Students expected to evaluate mathematical concepts that underpinned different strategies and use mathematical arguments to reach agreement. A consistently high level of engagement was maintained as students individually and collectively recognised the shared expectation of making mathematical sense of explanations. All students made concerted efforts to ensure that everyone in the group understood the mathematics by questioning each other until explanations were clear. As found in other research (e.g., Wood et al., 1995) students were able to develop firm beliefs and values by working together and mutually supporting each other to make sense of the mathematics. The social norms of explanations and justifications had further advanced to become sociomathematical norms as these were expected to be acted out on mathematical objects. These findings are consistent with many other researchers (e.g., Hunter, 2010; Kazemi & Stipek, 2001; McClain & Cobb, 2001; Yackel & Cobb, 1996). Students were able to seek, recognise and evaluate mathematical difference which gave them autonomy to reject or legitimise a range of possible solution strategies.

It was evident that social and sociomathematical norms were embedded and students could make sense of explanations through collaborative discourse.

6.6.2 COLLABORATIVE DISCOURSE

Analyses of data revealed regular ways in which students collaborated to solve problems. During the first phase, similar to what has been reported in other studies (e.g., Baumfield & Mroz, 2002; Bell & Pape, 2012) students expected their interactions to follow the traditional patterns of teacher directed initiate-response-feedback. In order to develop collaborative discourse, in instances where students only addressed the teacher, he directed them to share their explanations with everyone. The teacher intervened when other students were confused and they were encouraged to ask specific questions or to ask for solution strategies to be explained in different ways. At times, in order to develop their understanding of how to provide clear explanations, the teacher guided the students step-by-step through an acceptable explanation. Students were encouraged to persevere in asking questions until they were satisfied with the response. At first, some students found this difficult and resisted. However, by the second phase, it was evident that by encouraging students to persevere in asking questions and expecting clear expectations, collaborative discourse became an accepted means of engaging in and making sense of mathematics. Working together had provided opportunities to voice thoughts, explain or defend solutions, and to invite clarification. As students resolved conflicts they were granted opportunities to rethink how they had solved a problem and were able to build structures for alternate solution methods. Through the creation of taken-as-shared expectations, students had evolved into active meaning makers in the mathematics classroom. These findings are similar to those reported in other studies (e.g., Bauersfeld, 1980; Bruner, 1986; Chapin & O'Connor, 2007; Hicks, 1998; Pratt, 2006; Sfard, 2000; Voigt, 1995; Wood et al., 1995; Yackel, 1995).

6.7 THE COMPLEX NATURE OF TEACHING AND LEARNING

This investigation took place within a real classroom. Classrooms, by nature are complex places consisting of multifaceted layers of teaching and learning. When interpreting the results of this study, it is important that the small sample size, the explicit pedagogical approach taken by the teacher, and the ways in which these particular students were supported to make sense of mathematics are considered. The results can only suggest emerging perceptions into how students can be supported to make mathematical meaning and develop a more agentic identification within an inquiry mathematics classroom.

6.8 OPPORTUNITIES FOR FURTHER RESEARCH

A number of opportunities for further research have been identified from the results of this investigation. This study took place in one primary school classroom with students aged 9-10 years old. Examining and comparing the perspectives and roles of younger, older, or different gender students in a mathematics inquiry classroom warrants further research. Further research would be to explore how teachers in other types of classrooms scaffold students into sense-making. How other teachers may develop a pedagogic approach to establishing an inquiry classroom is worthy of exploration. Additionally, further research would be to investigate how students in different decile level schools construct mathematical understanding in an inquiry classroom.

6.9 CONCLUDING THOUGHTS

This study examined the perspectives and roles of students in an inquiry mathematics classroom. Their perceptions offered insight into what they believed was important while engaging in mathematical activity. Results illustrated that students valued active participation in the problem solving process and being given open-ended contextual problems that called for them to explain their thinking. Initially, it was evident that many of these beliefs were espoused, however, the results illustrated that the interaction patterns created in the classroom explicitly influenced how students built mathematical knowledge. Through the development of an effective learning environment reflecting sociocultural theory and the aims of reform mathematics education, significant shifts in student behaviour were evident. In the second phase, in alignment with the goals of reform mathematics education the idea that mathematics is about communicating, explaining and justifying ideas was reflected in the students' actions. Students believed that mathematics was about problem solving and were able to utilise conceptual agency to complete mathematics tasks with growing ease. The students described in the study were supported to actively engage in conceptualising multiple solutions paths which led to more advanced levels of mathematical thinking.

In the second phase, discourse had become collaborative. Through enactment of sociomathematical norms, explanations were mathematical in nature. Clear explanations and negotiated mathematical difference had become taken-as-shared expectations. Students took guidance from each other during on-going interactions and through active participation rather than requiring teacher affirmation. They trusted their own statements and were able to justify their selected solution pathways. In seeking individual clarification and deeper understanding of the mathematics, students

challenged others' conjectures and statements until satisfied. As a result of being encouraged to persist when faced with challenging problems, positive positional mathematics identities were created and mathematical authority became evenly distributed. Collaboration and interactions which had initially been fragmented became cohesive. Collaboration with others was seen as pivotal to learning mathematics and students held them self accountable for helping each other learn. Competency and agency in mathematics progressed from solely focusing on finding correct solutions to contributing clear explanations, asking questions and learning from others. By participating in the social practices that embody the wider mathematical community, students in this classroom developed intellectual autonomy. Students from this inquiry based classroom understood their obligations to actively engage in mathematical communication and learning.

The evidence from this research would indicate that in a classroom where the focus is on collaborative problem solving and value is placed on the intellectual contributions of all students, mathematical reasoning becomes an integral part of classroom activity. By actively engaging in mathematics activity by working collaboratively and being obligated to explain and justify, students' learning was sustained as a sense-making activity causing deeper conceptual understanding of mathematics.

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APPENDIX A: Student Questionnaire (Both phases)

- 1. What is mathematics?
- 2. Why should we learn mathematics?
- 3. How do we know if someone is good at mathematics?
- 4. How do you think real mathematicians do mathematics?
- 5. When you do mathematics in your classroom:
 - a) Who does the teaching?
 - b) Who does the talking?
 - c) Who asks the questions?
 - d) Who gives the answers?
 - e) What do you do if you get stuck?
- 6. Do you like to work with others during mathematics lessons? Why?
- 7. Is it important to be able to explain to other children how you solved a problem? Why?
- 8. Is it important to understand how someone else solved a problem? Why?
- 9. What is the most difficult thing about doing mathematics?
- 10. What is the most fun about doing mathematics?

APPENDIX B: Likert attitude scale (Both phases)

For the following statements please tick the box that you most agree with

1. I enjoy learning	g maths in this o	class		
Strongly disagree	Disagree	Neutral	Agree	Strongly agree
2. I prefer working	g on my own to	solve problems i	n maths	
Strongly disagree	Disagree	Neutral	Agree	Strongly agree
3. Talking to other	r people is an ir	mportant part of	earning maths	
Strongly disagree	Disagree	Neutral	Agree	Strongly agree
4. It is important t	to listen when o	thers are explain	ing their thinking	in maths
Strongly disagree	Disagree	Neutral	Agree	Strongly agree
5. Our teacher is	the only person	I should ask for	a correct answer	
Strongly disagree	Disagree	Neutral	Agree	Strongly agree
6. If I disagree wi	th someone's ex	xplanation I spea	ak up and say so	
Strongly disagree	Disagree	Neutral	Agree	Strongly agree
7. I think of myse	If as a real math	nematician when	I am doing math	S
Strongly disagree	Disagree	Neutral	Agree	Strongly agree
8. I am not afraid	to ask someone	e to prove their a	inswers	
Strongly disagree	Disagree	Neutral	Agree	Strongly agree
9. I can justify my	solutions to oth	ners in maths les	sons	
Strongly disagree	Disagree	Neutral	Agree	Strongly agree
10. I can generalise strategies with other numbers during maths lessons				
Strongly disagree	Disagree	Neutral	Agree	Strongly agree

11. Maths is all about remembering facts				
Strongly disagree	Disagree	Neutral	Agree	Strongly agree
12. I am good at i	maths if I can g	get the correct a	nswer by remen	nbering rules
Strongly disagree	Disagree	Neutral	Agree	Strongly agree
13. It is more imp	ortant to get th	e right answer t	han to explain h	ow to solve the problem
Strongly disagree	Disagree	Neutral	Agree	Strongly agree
14. Making mistal	kes is part of le	earning in maths	lessons	
Strongly disagree	Disagree	Neutral	Agree	Strongly agree
15. It is ok to disa	gree with the t	eacher about th	e answers durin	g maths lessons
Strongly disagree	Disagree	Neutral	Agree	Strongly agree
16. Having an arg	gument about r	naths means I a	am learning	
Strongly disagree	Disagree	Neutral	Agree	Strongly agree
17. Working with other students makes maths easier for me				
Strongly disagree	Disagree	Neutral	Agree	Strongly agree
18. I think I am go	ood at maths			
Strongly disagree	Disagree	Neutral	Agree	Strongly agree
19. My teacher thinks I am good at maths				
Strongly disagree	Disagree	Neutral	Agree	Strongly agree
20. Maths is more about problem solving than remembering facts and rules				
Strongly disagree	Disagree	Neutral	Agree	Strongly agree

APPENDIX C: Mathematics problems

- 1. Taine has some cows. Taine buys 29 more cows. Now he has 81 cows. How many cows did Taine have before?
- 2. At her party Sarah provides some lollies. She gives away 177 of these lollies and is left with 25. How many lollies did she start with?
- 3. Sally and David have agreed to work for their mum over the holidays. The pay they get will vary though. Sally will get \$10 for the first day she works and \$2 more for every day she works after that. David, on the other hand, will get \$1 for the first day he works, but for each he works from then on his pay will be doubled. Who would you rather be and why?
- 4. Wendy wants to have a rope long enough to stretch between two poles 13m apart, but she only has pieces of rope 1.5m long. How many of these pieces would she need to tie together to stretch between the two poles?
- 5. For her birthday, Mary got an iPod with some songs on it. She downloaded another 148 songs and she now has 376 songs in total. How many songs were on her iPod when she first got it?
- 6. Tana had \$153 saved to buy a new skateboard. For his birthday he got given \$147. How much money has Tana got altogether?
- 7. Nic has bought four planks of 2.5m each. How many 1m planks can he saw out of these planks?

APPENDIX D: Interview questions: (Both phases)

Pick and choose from these:

- What is happening here?
- I found that really interesting can you tell me more?
- I was really confused at this bit; can you tell me what was happening?
- What were you thinking here?
- What were you trying to accomplish here?
- What were you feeling here?
- Why did you say that?
- So it was your idea...then what happened?
- How did you decide?

APPENDIX E: Teacher information sheet and consent form

Dear

As you know I am to be on study leave from my school from 4 March to 24 November to complete a thesis for a Master of Education at Massey University. My thesis is a qualitative study examining the perspectives and roles of students learning mathematics in an inquiry classroom.

Together we have discussed ways in which students in primary school learn mathematics. Now I am formally inviting you to be a part of this research as I examine the students' perspectives and roles while learning mathematics in an inquiry classroom. Your role in this project will be as the mathematics teacher of the student participants.

Permission to participate in the study will be sought from both the parents/caregivers of the students in your class and the students themselves. The students and their parents/caregivers will be given full information and consent will be requested in due course. Consent will be twofold: one for individual interviews, and consent for participating in a case study which tracks how students learn mathematics.

I will interview you and the students. These interviews will take place at the start of the investigation and towards the end of the investigation. The time involved for your interview will be no more than 20 minutes. The interviews for each student will also be no more than 20 minutes. The interviews with you and students will be audio-recorded.

During this project, four consecutive mathematics lessons will also be videotaped at the beginning of the study, and four consecutive mathematics lessons will be videotaped at the end of the study. You and the students will be interviewed following these lessons. Work samples from each lesson will also be collected and photo-copied. The interviews and observations will take place in the classroom and be part of the normal mathematics programme.

The time involved in the complete study for you will be no more than 15 hours over a period of two school terms. The students and their parents/caregivers will be given full information and consent will be requested in due course. Specifically, permission to allow the students to be filmed and to participate in individual interviews will be sought from both the parents of the students and the students within the class.

All project data collected during individual interviews and filming will be stored in a secure location, with no public access and used only for this research and any publication arising from this research. After completion of five years, all data pertaining to this study will be destroyed in a secure manner. All efforts will be taken to maximise confidentiality and anonymity for participants. Names of all participants and the school will not be used once information has been gathered and only pseudonyms and non-identifying information will be used in reporting.

Please note that you are under no obligation to accept this invitation. If you decide to participate you have the right to:

- Decline to answer any particular question;
- Withdraw from the study after four weeks;
- Ask any questions about the study at any time during participation;
- Provide any information on the understanding that your name will not be used unless you give permission to the researcher;
- To ask for the audio or video-recorder to be turned off at any time during the interviews and any comments you have made be deleted;
- Be given access to a summary of the project findings when it is concluded.

If you have any further questions about this project you are welcome to discuss them with me personally:

Generosa Leach. Phone: 0221 764362. Email: gleach@vodafone.co.nz

Or contact either of my supervisors at Massey University

- Dr Roberta Hunter (09) 414 0800 ext. 41480. Email. <u>R.Hunter@massey.ac.nz</u>
 Institute of Education, Private Bag 102 904, North Shore, Auckland 0745
- Jodie Hunter (06) 356 9099 ext. 84405. Email. <u>J.Hunter1@massey.ac.nz</u>
 Institute of Education, Massey University Manawatu, Private Bag 11 222,
 Palmerston North, 4442

This project has been evaluated by peer review and judged to be low risk. Consequently, it has not been reviewed by one of the University's Human Ethics Committees. The researcher(s) named above are responsible for the ethical conduct of this research.

If you have any concerns about the conduct of this research that you wish to raise with someone other than the researcher(s), please contact Professor John O'Neill, Director, Research Ethics, telephone (06) 350 5249, email humanethics @massey.ac.nz

CONSENT FORM: TEACHER PARTICIPANT THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

I have read the Information Sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I agree/do not agree to the interview being sound recorded.

I agree/do not agree to the interview being image recorded.

I agree to participate in this study under the conditions set out in the Information Sheet.

Signature:	Date:
Full Name - printed	

APPENDIX F: Student and parent information sheet and consent form

Dear

I am doing a research project for a Master of Education at Massey University. I am going to examine the perspectives and roles of students learning mathematics in an inquiry classroom.

I would like to invite you with your parent's permission to be involved in this study. (Teacher's name), your teacher has also agreed to participate in this study. The Board of Trustees has also given their approval for me to invite you to participate, and for me to do this research.

If you agree to be involved, I will interview you about what you think about learning mathematics. There will be several interviews; some will be at the beginning of my project and some will take place at the end of my project, which will be towards the end of term two. The interviews will take about 20 minutes each. The interview will be audio-recorded and you may ask that the recorder be turned off and that any comments you have made be deleted if you change your mind or are not happy about what you said.

I will also be observing you participating in some mathematics lessons at the beginning of my project and some mathematics lessons towards the end of my project. (Teacher's name) will be teaching you at this time and these lessons will be part of your normal mathematics programme, whether you agree to be in the study or not. These lessons will also be video-recorded and you may at any time ask that the video recorder be turned off and any comments you have made deleted. With your permission I might sometimes collect copies of written work or charts you make to support your mathematical thinking. You have the right to refuse to allow the copies to be taken.

Taking part in this research will not in any ways affect your learning, but rather may help you clarify what you know about being a mathematician. The interview and observations will take place in the classroom and be part of the normal mathematics programme.

All the information gathered will be stored in a secure location and used only for this research. After completion of the research the information will be destroyed. All efforts will be taken to maximise your confidentiality and anonymity which means that your

name will not be used in this study and only non-identifying information will be used in reporting.

I ask that you discuss all the information in this letter fully with your parents before you give your consent to participate.

Please note that you have the following rights:

- To say that you do not want to participate in the study;
- To withdraw from the study at any time;
- To ask for the audio or video recorder to be turned off at any time during the lessons or interviews and any comments you have made be deleted;
- To refuse to allow copies of your written work to be taken;
- To ask questions about the study at any time;
- To participate knowing that you will not be identified at any time;
- To be given a summary of what is found at the end of the study.

If you have any further questions about this project you are welcome to discuss them with me personally:

Generosa Leach. Phone: 0221 764362. Email: gleach@vodafone.co.nz

Or contact either of my supervisors at Massey University

- Dr Roberta Hunter (09) 414 0800 ext. 41480. Email. <u>R.Hunter@massey.ac.nz</u>
 Institute of Education, Private Bag 102 904, North Shore, Auckland 0745
- Jodie Hunter (06) 356 9099 ext. 84405. Email. <u>J.Hunter1@massey.ac.nz</u>
 Institute of Education, Massey University Manawatu, Private Bag 11 222,
 Palmerston North, 4442

This project has been evaluated by peer review and judged to be low risk. Consequently, it has not been reviewed by one of the University's Human Ethics Committees. The researcher(s) named above are responsible for the ethical conduct of this research.

If you have any concerns about the conduct of this research that you wish to raise with someone other than the researcher(s), please contact Professor John O'Neill, Director, Research Ethics, telephone (06) 350 5249, email humanethics@massey.ac.nz

CONSENT FORM: STUDENT PARTICIPANTS THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

I have read the Information Sheet and have had the details of the study explained to me. My questions have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I agree/do not agree to the interview being sound recorded.

I agree/do not agree to the interview being image recorded.

I agree to participate in this study under the conditions set out in the Information Sheet

Child's Signature:	Date:	
Full Name - printed		
	ARENTS/CAREGIVERS OF STUDENT PARTICIPANTS M WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS	
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APPENDIX G: Board of Trustees information sheet and consent form

Dear Sir/Madam

My name is Generosa Leach. I have been a teacher at (School name) for 6 years and am to be on study leave from 4 March to 24 November to complete a thesis for a Master of Education at Massey University. My thesis is a qualitative study examining students' perspectives and roles in an inquiry mathematics classroom.

(Teacher's name) has tentatively agreed to participate in this study as the mathematics teacher of the students involved in this project. The teacher will be formally approached following B.O.T. approval of this study. The parents and students will be informed of the nature of the study through information sheets and a discussion in class, and permission to participate in the study will be sought. Consent will consist of two parts: one for individual interviews, and consent for participating in a case study which tracks how students learn mathematics in an inquiry classroom. Interviews involving the teacher and students will take place at the start of the investigation and towards the end of the investigation. The time involved for the teacher and students for each interview will be no more than 20 minutes. The interviews with the teacher and students will be audio-recorded.

During this project, four consecutive mathematics lessons will be video-recorded at the beginning of the study, and four consecutive mathematics lessons will be video-recorded towards the end of the study. (Teacher's name) and the students will also be interviewed following these lessons. Work samples from each lesson will also be collected and photo-copied.

The time involved in the complete study for the teacher will be no more than 15 hours over a period of two school terms. The teacher, the students and their parents/caregivers will be given full information and consent will be requested in due course. Specifically, permission to allow the students to be filmed and to participate in individual interviews will be sought from both the parents of the students and the students within the class.

All project data collected during individual interviews and filming will be stored in a secure location, with no public access and used only for this research and any publication arising from this research. After completion of five years, all data pertaining to this study will be destroyed in a secure manner. All efforts will be taken to maximise confidentiality and anonymity for participants. Names of all participants and the school

will not be used once information has been gathered and only pseudonyms and nonidentifying information will be used in reporting.

Please note that the Board of Trustees is under no obligation to accept this invitation. If you decide to participate you have the right to:

- Decline to answer any particular question;
- Withdraw from the study after four weeks;
- Ask any questions about the study at any time during participation;
- Provide any information on the understanding that your name will not be used unless you give permission to the researcher;
- Be given access to a summary of the project findings when it is concluded.

If you have any further questions about this project you are welcome to discuss them with me personally:

Generosa Leach. Phone: 0221 764362. Email: gleach@vodafone.co.nz

Or contact either of my supervisors at Massey University

- Dr Roberta Hunter (09) 414 0800 ext. 41480. Email. <u>R.Hunter@massey.ac.nz</u>
 Institute of Education, Private Bag 102 904, North Shore, Auckland 0745
- Jodie Hunter (06) 356 9099 ext. 84405. Email. <u>J.Hunter1@massey.ac.nz</u>
 Institute of Education, Massey University Manawatu, Private Bag 11 222,
 Palmerston North, 4442

This project has been evaluated by peer review and judged to be low risk. Consequently, it has not been reviewed by one of the University's Human Ethics Committees. The researcher(s) named above are responsible for the ethical conduct of this research.

If you have any concerns about the conduct of this research that you wish to raise with someone other than the researcher(s), please contact Professor John O'Neill, Director, Research Ethics, telephone (06) 350 5249, email humanethics@massey.ac.nz

CONSENT FORM: BOARD OF TRUSTEES THIS CONSENT FORM WILL BE HELD FOR A PERIOD OF FIVE (5) YEARS

We have read the Information Sheet and have had details of the study explained. Our questions have been answered to our satisfaction, and we understand that we may ask further questions at any time.

We agree to	
	ditions set out in the Information Sheet.
Signature:	Date:
Full Name – printed	