PORTFOLIO RISK DIVERSIFICATION, COHERENT RISK MEASURES AND RISK MAPPING, RISK CONTRIBUTION ANALYSIS AND THE SETTING OF RISK LIMITS

A thesis presented in partial fulfillment of the requirements for the degree of Master of Business Studies in Finance at Massey University

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2003

Abstract

This study aims to investigate the nature and sources of portfolio risks during normal as well as abnormal market conditions. The benefits of portfolio diversification will be studied first. Portfolio risk as measured by the volatility and beta will be calculated as the number of the positions is increased until the marginal diversification benefits obtained are at its optimal. Other measures based on statistical measures such as quantiles, quantile differences and quantile ratios for central tendency and asymmetry presence and significance of extreme events of skewness and kurtosis will also be used. This study is conducted on the daily data for the period August 9, 1998 to June 30, 2003, for 25 stock markets worldwide: Australia, Brazil, Chile, France, Germany, Hong Kong, Japan, India, Indonesia, Ireland, Israel, Italy, Mexico, New Zealand, Singapore, South Africa, South Korea, Spain, Sweden, Switzerland, Taiwan, Thailand, Turkey, United Kingdom and United States. Based on the theory of central limit theorem (CLT) and hence jointly normal distributions, the relationship between portfolio diversification and value at risk (VaR) as a coherent risk measure is examined. Diversification benefits based on two simulation models namely: the geometric Brownian motion (GBM) and Fréchet random walk (FRW) which serve as the ideal models are also investigated.

The second part of the study focuses on the main sources of risk or risk hot spots in a portfolio using component VaR $\left(VaR_C\right)$, incremental VaR $(IVaR)$, and delta or marginal $(DVaR)$. Finally, the portfolio risk will be monitored using a risk mapping or risk decomposition method. The risk of a given position is mapped onto a much smaller number of primary risk factors. In this study, individual country's stock index will be used as proxy for equities, government bond index and risk free rate for fixed interest, spot foreign exchange rate and forward one month, three month and one year exchange rate and gold and crude oil for commodities.

In general, the results for the tail-risk measures are similar to what has been found for the center of the portfolio risk measures and covariance plays a significant role in the assessment of the risk inherent to real portfolios based on the greater diversification benefits gained from the two simulated models, whose log-returns were generated independently. Diversification "works" well under normal market conditions.

Acknowledgements

I would like to express my deepest gratitude to my supervisor Dr. John W. Dalle Molle, first for his work in making this thesis possible, second for his guidance and continued support even when he is in Singapore. It was an honor and privilege to have John as my supervisor. Not only did John teach me how to conduct a good piece of research, he also taught me other important lessons in life.

Thanks also to the research officer, Fong Mee Chin for her assistance in obtaining data from DataStream.

A special thanks to my parents and siblings for all their support and encouragement through my time at Massey University. Mum and Dad, you guys are the greatest!

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Introduction

Most studies of the benefits of diversification focus on portfolio risk measures that are functions of the properties of the center region of a log-return distribution. The distribution of the log-returns of an equity portfolio should be at least approximately Gaussian if the portfolio is considered diversified. This is because the consequences of the benefits of the diversification effects of a well-diversified portfolio will have "averaged" away the significance of the extreme values and extreme dependencies. Statistically, this is just another way of stating the central limit theorem for a sum of centered and normalized sums. If the portfolio is diversified and the log-returns are approximately jointly Gaussian, the variance/covariance VaR forecast is a coherent risk measure. In general, even if the returns are fat-tailed, there will be diversification benefits, justifying the use of the simple model to implement VaR and expected shortfall estimators to be implemented using the historical risk measurement method. The expected shortfall is, in general a coherent risk measure, but the historical VaR method may not be coherent. This study concerns an empirical analysis of the statements and concepts just mentioned. Specifically, the analysis concerns the quantification of the extent and nature of the risk reduction arising as a consequence of the effects of diversification on a number of different categories of equity portfolios. The risk reduction will be measured using both an unexpected and an exceptional (or extreme) risk metric, which were the value at risk (VaR) and the expected shortfall, respectively. Two simulation models will be used as the reference models namely: Fréchet random walk (FRW) and geometric Brownian motion model (GBM).

As an extension to the results presented, the empirical risk mapping experiment will be investigated, and risk mapping is the next step for portfolio with extremely large numbers of positions from numerous financial markets in numerous different country. The idea is risk mapping may be able to simplify the risk measurement process for portfolios with extreme large numbers of position, where the position may be changing often. The analysis is static, implying that the effects of liquidity changes and portfolio composition changes will not be considered.

PART ONE
LITERATURE REVIEW

A review of the literature relating to portfolio diversification, coherent risk measures, risk mapping and the setting of risk limits is provided in Part one. Examining the nature and the sources of portfolio risks during normal and abnormal market conditions are the focuses of this thesis. The literature review creates the context within which these areas of the finance literature have developed. Although the principles behind portfolio diversification were firmly established in the finance literature in the 1950s, it was not until Grubel (1968) that diversification in an international context was formally considered. Chapter 1 reviews the early literature relating to portfolio diversification in an international setting and the effects of the increasing integration of world financial markets. The October 1987 global stock market crash highlighted for many, the extent to which world equity markets had become increasingly integrated and the merits of international diversification given this comovement and the relative transaction costs. Chapter 2 considers the assumption of normal distributions of returns underlying stock prices and the use of linear VaR models as opposed to other VaR models such as historical VaR or Monte Carlo simulations. Chapter 3 considers the various approaches to selecting risk factors and risk mapping and the importance of setting risk limits as a supplement to the one number VaR. This is useful as then institutional investors and international equity investors can actively use it for portfolio and risk management purposes. Besides, VaR itself has serious fundamental flaws. It is based on volatilities and correlations that can work in normal market conditions but break down in times of market crises.

Chapter 1: Merits of International Diversification

The issue of portfolio diversification in world equity markets became prominent with the article of Grubel (1968). Chapter 1 reviews some of the important articles to appear in the early literature; particularly the benefits of domestic diversification using naïve diversification, the establishment of the necessary and sufficient conditions for ascertaining gains from portfolio diversification in an international setting and the effects of an increasingly globalized world. The relationship between diversification and coherent risk measures is also briefly discussed.

1.1 Benefits of Domestic Diversification Using Naïve Diversification

Naïve or simple diversification implies that an equal proportion of wealth is allocated to each security in the portfolio. Naïve diversification can be defined as "*not putting all your eggs in one basket*." Naïve diversification ignores the covariance between securities. In general, diversification works best when the asset exhibit zero or low correlations among each other. Also diversification works well when there are a number of short and long positions.

For a portfolio composed of n securities, $w_i = 1/n$ for $i = 1, 2, \ldots, n$, which is due to the equal weight assumption inherent to naïve diversification (Elton, Gruber, Brown and Goetzmann, 2003, p. 51-61; Francis, 1991, sections 9-1 through 9-3; and Francis and Archer, 1979, section 9.1). This implies that the variance of the log-returns of the portfolio can be rewritten as follows:

$$VAR_{t_0}[R(t)] = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j COV_{t_0}[r_i(t), r_j(t)]$$

$$= \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} COV_{t_0}[r_i(t), r_j(t)]$$

$$= \frac{1}{n^2} \left(\sum_{i=1}^{n} VAR_{t_0}[r_i(t)] + \sum_{i=1}^{n} \sum_{j=i+1}^{n} COV_{t_0}[r_i(t), r_j(t)] \right)$$

noting that $VAR_{t_0}[r_i(t)] = COV_{t_0}[r_i(t), r_i(t)]$.

Now let the following relationships hold for the variances and covariances:

$$VAR_{t_0}[r_i(t)] < VAR$$

for $i=1,2\ldots,n$ and

$$COV_{t_0}[r_i(t), r_j(t)] < COV$$

for $i=1,2,\ldots,n$ and $j=i+1,2,\ldots,n$; where VAR and COV are bounded positive constants.

$$VAR < \infty \text{ and } COV < \infty,$$

respectively. VAR and COV may be viewed as the average variance and average covariance. First, the case where $COV = 0$, which implies the log-returns of the n securities are uncorrelated:

$$VAR_{t_0}[R(t)] = \frac{1}{n^2}\sum_{i=1}^{n} VAR = \frac{1}{n} VAR$$

As the number of securities n increases, i.e. $n \longrightarrow \infty$;

$$VAR/n \to 0, \text{ implying } VAR_{t_0}[R(t)] \to 0$$

This implies that the portfolio variance approaches zero is there are enough uncorrelated assets. If $COV \neq 0$, which is the case for correlated assets and using these relationships:

$$VAR_{t_0}[r_P(t)] < \frac{1}{n^2}\left(nVAR + n(n-1)COV\right)$$

or

$$VAR_{t_0}[r_P(t)] < \frac{1}{n}\left(VAR + (n-1)COV\right).$$

As the number of securities n increases, i.e. $n \longrightarrow \infty$;

$$VAR/n \to 0,\ (n-1)/n \to 1 \text{ and } (n-1)COV/n \to COV.$$

One aspect of the interpretation of this comment is that the cumulative contribution of the variance terms becomes negligible. The sum of the variance terms is referred to as the firm-specific risk or diversifiable risk. Another aspect of this comment is that the cumulative of the covariance terms does not become negligible. The cumulative effect of the covariance terms is referred to as the market-specific risk or nondiversifiable risk.

Rearranging the above expression for the value of the portfolio illustrates the effects of diversification:

$$VAR_{t_0}[r_P(t)] < \frac{1}{n}(VAR - COV) + COV$$

The first term represents $1/n$ times the difference between the average variance of the log-returns of the individual securities VAR and average covariance of the log-returns of the individual securities COV. The first term is reduced as securities are added to the portfolio, which illustrates the effects of diversification. The second term represents average covariance of the log-returns of the individual securities COV. The minimum portfolio variance for correlated assets may be obtained for portfolios with very large numbers of positions, and is equal to the average covariance between all the individual securities (Dalle Molle, 2003).

Naïve diversification using 15 to 20 randomly selected stocks can reduce the risk of a portfolio by approximately 50 percent (on the average). The diversification benefits with respect to risk reduction of adding more stocks is insignificant after 15 to 20 stocks have been randomly included in a portfolio. This observation is referred to as superfluous diversification. The following table illustrates the effects of diversification for United States (U.S.) equities listed on the New York Stock Exchange (reproduced from Elton, Gruber, Brown and Goetzmann, 2003, p. 59).

Table 1.1 – Portfolio Diversification Effects	
Number of Securities	**Expected Portfolio Variance**
1	46.619
2	26.839
4	16.948
6	13.651
8	12.003
10	11.014
12	10.354
14	9.883
16	9.530
18	9.256
20	9.036
25	8.640
30	8.376
35	8.188
40	8.047
50	7.849
75	7.585
100	7.453
150	7.321
200	7.255
250	7.216
500	7.137
1000	7.097
Infinity	7.058

Note that the average equity variance was 46.619 and the average covariance between the equity was 7.058.

1.2 Comovements between World Equity Markets

In this section the necessary condition for ascertaining gains from portfolio diversification in an international setting is considered. The necessary condition is that the correlation coefficients between world equity markets must be less than the correlation coefficients between domestic assets – otherwise one can obtain the risk-reduction benefits of portfolio diversification by investing in the domestic market alone. Grubel (1968) laid the theoretical foundations for portfolio diversification in an international setting. Although Markowitz (1952) and Sharpe (1964) had clearly established the tenets of portfolio theory and developed selection techniques for portfolio optimization, it was not until Grubel (1968) that international diversification were formally considered in the literature. Grubel (1968) highlighted that those investors who did not consider foreign assets were ignoring a potentially important source of low correlation.

Grubel (1968) used the special case of a two-country two-asset investment model, where as long as less than perfect correlation exists between the two countries, diversification of investment will benefit investors. Grubel's finding offered nothing particularly new relative to portfolio optimization except that it highlighted the importance of considering international assets. Further, the outcome of the special case is unlikely to hold when there is more than one domestic asset and diversification internally is an option. Grubel's empirical study examined the potential gains to U.S. investors from international diversification which he found to be potentially large. Grubel's investigation of portfolio diversification provided a descriptive explanation for the need for continued international capital flows.

Subsequent authors starting with Levy and Sarnat (1970) looked at international portfolio diversification normatively. Levy and Sarnat (1970) used efficient frontiers and the market equilibrium model developed by Litner (1965) and Sharpe (1964) to determine the proportion of investment in each of the various markets in their sample, at different interest rates. Levy and Sarnat (1970) were able to clearly illustrate the benefits accrue to the U.S. investor for diversifying outside the domestic market. However, they added that there were only marginal benefits for moving from a U.S. stock only portfolio to one containing stocks from high income, common market or Western European countries. Only when the U.S. investor diversified to include countries such as Japan and South Africa and the developing countries of South America and Asia that a significant improvement in the efficient frontier resulted. These countries tend to have lower correlations with the U.S. market, and thus are able to provide the benefits of international diversification to the investor.

Grubel and Fadner (1971) pointed out two important influences not encountered by U.S. assets which might lead to lower inter-country correlation coefficients and make foreign assets attractive to U.S. portfolio holders. First, returns on foreign assets are influenced by business cycles, natural and man-made catastrophes and government policies whose effects are limited to or strongly felt in the economies of the affected countries. Second, the capital value changes of assets caused by exchange rate variation influence the variance of returns on foreign assets. Using the *ex post* variance-covariance matrices of returns for industry sub-indices from U.S., United Kingdom (U.K.), and West German stock indices, Grubel and Fadner (1971) attempted to measure the significance of those two factors. They discovered that the pairwise inter-country correlation coefficients were positively low and lower than intra-country coefficients. Solnik (1974) also revealed that inter-country differences in business cycles enhance international portfolio diversification. Solnik (1974) examined the level of risk reduction as more stocks are included in a portfolio. For portfolios that contain assets from outside the domestic market, the risk reduction was greater than for portfolios that contain only domestic assets. Even after accounting for exchange rate fluctuations, the risk was still less for portfolios that contain foreign assets.

Implicit in these early studies, is the hypothesis that international diversification will lead to larger gains than ordinary 'pure diversification' gains as a result of increasing the universe of available securities within a single country. The early studies can also be summarized by their focus on the gains for foreign market diversification from a U.S. investor perspective. Lessard (1974) summarized the early tests as:

> "The early tests of Grubel (1968) and Levy and Sarnat (1970) relied on low correlations between the national markets and the performances of *ex post* efficient internationally diversified portfolios to establish these greater gains. However, the low correlations among markets may or may not indicate large potential gains relative to domestic diversification depending on the correlations among groups of stocks in each market and the *ex post* efficient portfolios are, at best, indicative of gains." (p. 379).

Research by Agmon (1972) and Lessard (1976) examined the concept of segmented versus integrated capital markets. Lessard (1973, 1974) highlighted the difference between gains from domestic versus inter-country portfolio diversification alluded to above. These two areas contributed a new angle to portfolio diversification in an international setting and were likewise important in developing literature in this area. They are considered below.

Agmon (1972) highlighted that the literature up until that point had considered national capital markets on a segmented markets approach, where national capital markets are treated as separate entities, almost independent to each other like Grubel's two-country, two-asset model. As such, one might naturally assume low correlation coefficients between markets. Agmon (1972) noted that "different currency areas separated political organizations and trade barriers have been given as *a priori* evidence for the segmentation of the international capital market." (p. 839). Agmon (1972) argued that the alternative hypothesis, that prices of capital assets in the global capital market behave as if there is one multinational perfect capital market should also be considered and that an examination of the behavior of capital asset prices reveal that the price behavior is consistent with the one market hypothesis.

In terms of Grubel's two-country, two-asset framework, Agmon (1972) argued that, while still important, the interesting question relates to how free trade, deregulation between two countries impact on the composition of investment portfolios. This is an issue that could not be examined by Grubel (1968) as the full risk-return profile for each of the markets is not captured by the market index used in his study. Agmon (1972) emphasized that while it is correct in terms of Grubel's theoretical two-country, two-asset world to look at portfolio diversification in terms of correlation, the empirical analysis conducted by him was based on market indices which comprised of many shares and thus are only close approximations to diversified portfolios. In comparing two portfolios, domestic and foreign, the relevant measure is a function of the covariance between the return of any given asset and the return on the investor's portfolio. It may be that, even in situations like the one presented by Grubel (1968), investors cannot benefit from diversification between countries.

Agmon (1972) disagreed with Grubel's revelation that, given equity markets are segmented, correlation coefficients substantially less than one between any index of non-U.S. equity markets and the U.S. market index would give U.S. investors potential welfare gains from international diversification, the benefits accruing once the barriers among equity markets were removed. Because composite market indices do not capture all the possibilities for diversification within a local market, the fact that two indices are weakly correlated does not necessarily imply the superiority of international diversification over internal diversification. Consequently, one cannot be sure that internal diversification would not give the same (or better) efficient sets. The one market hypothesis may have an advantage here since it implies that all the potential gains from inter-country and internal diversification are already reflected in the current prices of capital assets traded on the world market. Using individual stock returns from four countries, Agmon (1972) outlined a framework of a single equity market and demonstrated that price behavior in these countries was consistent with the single market hypothesis. In spite of the barriers that exist in multinational equity markets, there was a strong relationship among the four equity markets. In particular, Agmon (1972) noted that the price movement in the majority of German shares resembled U.S. shares. Agmon (1972) highlighted the importance of not simply accepting the segmented markets hypothesis.

McDonald (1973) also cautioned the need in interpreting the studies of Grubel (1968) and Levy and Sarnat (1970).

> "In the context of portfolio choice these results must be interpreted with caution, as one cannot demonstrate with *ex post* returns on market indices alone the extent to which international diversification is desirable. The important question is whether efficient (*ex ante*) multinational portfolios of individual securities dominate efficient portfolios constructed from stocks in a single country, and on this issue the evidence is limited." (pp. 1161-1162).

McDonald (1973) argued that the issue also depends on the effective degree of integration of the world's equity markets. McDonald (1973) suggested reality falls somewhere between the two hypotheses discussed by Agmon (1972): the fully-integrated one-market hypothesis; and the fully-segmented market hypothesis, because of impediments in multinational investment. In the former, the capital asset pricing model $(CAPM)$ of capital market equilibrium would include portfolios of domestic common stocks from a number of nations lying along a common capital market line; for the latter, one would expect a unique capital market line in each national market.

> "In fully-integrated markets, a portfolio which purchased common stocks in a second country would gain only the 'pure diversification' advantage of access to a larger part of the total universe of securities. The segmented –market hypothesis implies potential advantages from international investment beyond those associated with pure diversification, as more favorable ratios of expected return to non-diversifiable risk may be available in foreign markets." (McDonald, 1973, p.1162).

McDonald (1973) used the investment performance of French mutual funds as examples of portfolios diversified outside the domestic market and discovered that the funds generally produced superior risk-adjusted returns than funds invested only in domestic assets.

Lessard (1976) also examined the issue relating to segmented versus integrated markets. The following passage captures the importance of segmented versus integrated markets in establishing the theory of portfolio diversification in an international context.

> "The low correlations between the country factors represent the key to gains from international diversification. The magnitude of these gains will depend, however, on whether markets are segmented or integrated internationally. In the former case, assuming the validity of the capital asset pricing model, prices and expected returns are determined by the undiversifiable risk of each security in the context of the appropriate national portfolio. In the latter, prices and expected returns are determined by the undiversifiable risk of each security in the context of the world portfolio. With fully integrated markets, the advantage to international diversification is a pure diversification effect, a reduction in the non-systematic risk of the portfolio. With fully segmented markets gains might be even greater, since prices would adjust to reflect the fact that some previously undiversifiable risk was becoming diversifiable." (Lessard, 1976, p.34).

Unfortunately, it is very difficult to actually determine which of the two conflicting theories most accurately describes portfolio diversification in an international setting. The reality lies somewhere between the two extremes.

Lessard (1973) studied the communality among returns within individual countries relative to the communality across countries. Using an Investment Union (IU) approach for four Latin American countries, multivariate analysis of the return structure for individual stocks was used to investigate the likelihood of greater gains for inter-country diversification over domestic diversification. For gains from inter-country diversification to be greater, two important conditions must be satisfied. Firstly, returns within each country must share a common element of variance, and secondly, the common elements for each country must be largely independent of those from the other countries. Using principal component analysis, Lessard (1973) concluded:

> "...that even though the principal components for each country are not absolutely independent, it is possible to explain an average of 93 per cent as much variance for each country as is explained by the principal components with four completely *independent* factors." (p. 625).

This is an important finding as it again highlights the benefit of inter-country diversification over domestic diversification. It shows that while the benefits of domestic diversification are limited due to the common trend in stock returns, potential benefits are much greater by diversifying outside the domestic market because common trends are much harder to find between countries. Lessard (1973) attributes this to different levels of economic activity at different times and the different monetary and fiscal policies of different governments. Though different methodologies were applied, Lessard's findings are consistent with what McDonald (1973) found for French mutual funds.

Lessard (1974) studied the benefits of diversifying outside the domestic market in contrast to domestic diversification by considering the stochastic process generating returns. Lessard (1974) highlighted that in previous studies which have used a $CAPM$ market model, national markets have been found to be characterized by a strong market factor consistent with a single-factor stochastic process. Because of the linkages between national markets, one would expect some relationship between market factors in different countries. However, Lessard (1974) found that only a small proportion of the variance of national portfolios is similar in an international context. Lessard (1974) demonstrated the considerable risk reduction available through portfolio diversification in an international setting.

The literature reviewed in this section was important in establishing portfolio diversification in an international setting. Grubel (1968), Levy and Sarnat (1970), Grubel and Fadner (1971) and Solnik (1974) all highlighted that lower correlation coefficients between national equity markets meant that additional benefits were available to investors who diversified outside the domestic market. Agmon (1972), McDonald (1973) and Lessard (1976) looked at the segmented versus integrated market hypothesis, providing a context in which the early studies of portfolio diversification in an international context should be examined. They showed that it is essential to consider the assumptions being made in testing the benefits of portfolio diversification in an international setting. Lessard (1973, 1974) provided further evidence to support the notion of additional benefits offered by portfolios diversified outside the domestic market, over portfolios with domestic assets only.

1.3 Integration of World Equity Markets

The fundamental rationale for international portfolio diversification is that it expands the opportunities for gains from portfolio diversification beyond those that are available through domestic investment. However, if international stock market correlations are higher than normal as found in empirical literature (refer to section 1.2), then international diversification will fail to yield the promised gains and correlation breakdowns will occur. This is especially true during times of crises when the market is in stress with low liquidity. In this section the worldwide impact of the 1987 stock market crash, the 1997 Asian crises and the long-term linkages between equity markets using the Vector Autoregression (VAR) technique are examined. It will be interesting to see to what extent globalization has impacted on the linkages between equity markets. If strong linkages between markets are found then the rationale for diversifying outside the domestic market to benefit from risk reduction through portfolio diversification becomes questionable.

The October 1987 stock market crash attracted reasonable interest not only in the academic literature, but also from regulatory authorities due to its worldwide scope. The findings in the early literature revealed low comovement between world equity markets. The 1987 crash raised the obvious question of whether world equity markets had become more integrated? Table 1.2 shows that prices in October 1987 dropped all around the world.

Table 1.2

STOCK PRICE INDEX PERCENTAGE CHANGES IN MAJOR MARKETS: CALENDER YEAR 1987 AND OCTOBER 1987

	Local Currency Units		U.S. Dollars	
Countries	**1987**	**October**	**1987**	**October**
Australia	-3.6	-41.8	4.70	-44.9
Austria	-17.6	-11.4	0.70	-5.8
Belgium	-15.5	-23.2	3.1	-18.9
Canada	4.0	-22.5	10.4	-22.9
Denmark	-4.5	-12.5	15.5	-7.3
France	-27.8	-22.9	-13.9	-19.5
Germany	-36.8	-22.3	-22.7	-17.1
Hong Kong	-11.3	-45.8	-11.0	-45.8
Ireland	-12.3	-29.1	4.7	-25.4
Italy	-32.4	-16.3	-22.3	-12.9
Japan	8.5	-12.8	41.4	-7.7
Malaysia	6.9	-39.8	11.7	-39.3
Mexico	158.9	-35.0	5.5	-37.6
Netherlands	-18.9	-23.3	0.3	-18.1
New Zealand	-38.7	-29.3	-23.8	-36.0
Norway	-14.0	-30.5	1.7	-28.8
Singapore	-10.6	-42.2	-2.7	-41.6
South Africa	-8.8	-23.9	33.5	-29.0
Spain	8.2	-27.7	32.6	-23.1
Sweden	-15.1	-21.8	-0.9	-18.6
Switzerland	-34.0	-26.1	-16.5	-20.8
United Kingdom	4.6	-26.4	32.5	-22.1
United States	0.5	-21.6	0.5	-21.6

Reproduced from Kamphuis, Kormendi and Watson (1989, p. 37).

The Asian turmoil of 1997 erased almost three-fourths of the dollar capitalization of the equities markets in Indonesia, Korea, Malaysia, the Philippines and Thailand. By the end of 1997, many loans became non-performing and the crisis spread through Southeast Asia after the Thai government abandoned its support of the baht as exhibited by these percentage declines in the foreign exchange rates of a number of Southeast Asia countries: the Korean won fell 47.44 percent, the Indonesian rupiah fell 55.9 percent, the Malaysian ringgit fell 34.8 percent and the Philippine peso fell 28.3 percent. The Korean companies: Hanbo Steel Group and the Kia Car Company both went bust. Many of the top ten banks in the region became technically insolvent. The Hong Kong investment bank Peregrine Investments filed for bankruptcy with debts of US$400 million (Dalle Molle, 2003). The presence of contagion or inter-dependence among economies of a certain region becomes important with the diminishment of the advantages to investors of international diversification.

Jeon and Von Furstenberg (1990) argued that the increasing comovement between equity markets might be the result of two things. It may be caused by the decreasing regulation of world financial markets and the resultant increase in integration which has led to an increase in the efficiency with which capital is allocated and news is processed worldwide. Alternatively, world equity markets may just be reacting increasingly to each other even if there are no news developments of global economic significance that would account for such comovement. Regardless of the explanation, if markets are indeed subject to greater comovement, then the benefit of diversifying outside the domestic market will be reduced. Jeon and Von Furstenberg (1990) studied the changes in price relations among the world's major stock markets that might have been precipitated by the crash of October 1987.

Using the Vector Autoregression (VAR) technique, Jeon and Von Furstenberg (1990) focused on the correlation of daily price movements from 1986 to 1988 for the New York, Tokyo, London and Frankfurt stock markets. The 35-month period is divided into two sub periods (January 6/7, 1986 to October 13/14, 1987, and October 21/22, 1987, to November 24/25, 1988) to study whether there have been changes in interrelationships among stock prices in the major world equity markets since the stock market crash of October 1987. Their findings are worth noting despite the short time study period and the small sample. They discovered that the extent of international comovement in stock indices has increased significantly since the 1987 October stock market crash.

Espitia and Santamaria (1994) also used the VAR technique to examine the linkages between world equity markets. Using daily data during the period of 1987 to 1992, their sample consisted of indices for the stock exchanges of Japan, Spain, Italy, Germany, France, U.K. and the U.S.. The early studies of Grubel (1968) and others argued that diversification reduces risk without sacrificing expected return but Espitia and Santamaria (1994) emphasized that to justify this argument, a prerequisite is needed: that is, capital markets must be independent in the process of price formation. If the markets move in parallel, then the opportunities for diversification are eliminated. The transmission of shocks in one market to other markets implies that linkages between markets exist which might reduce the benefits of international portfolio diversification. They found that diversification outside the domestic market does not appear to have an excessive economic rationale. Only if diversification is implemented by choosing stocks whose differential characteristics give them a specific behavior relative to the local stock market on which they are quoted will there be some use in such diversification. They also found that the effects of a shock to the New York market last longer in the period from 1987 to 1992 (up to four days) than in the 1980 to 1985 period (two days). Like Jeon and Von Furstenberg (1990), this finding illustrates that the comovements of the markets have increased.

1.4 Coherent Risk Measures and Value At Risk (VaR)

Coherent risk measures refer to risk measures such as the expected shortfall, which is the expected loss given a loss greater than VaR occurs. Artzner, Delbaen, Eber and Heath (1999) define a risk measure as been coherent if it satisfies the following conditions i.e. letting a set V be real-valued random variables and a function

$$\rho: V \rightarrow \Re:$$

1) Translation invariance: adding cash to the portfolio decreases its risk by the same amount. This property is intuitive, only risk is measured in terms of the final net worth rather than changes in value i.e. $X \in V, \Rightarrow \rho(X+a) = \rho(X) - a$.

2) Sub-additivity: the risk of the sum of sub-portfolios is smaller or equal than the sum of their individual risks, in other words merging of portfolios should not create additional risk. Been sub-additive ensures that a risk measure behaves reasonable when aggregating positions. Sub-additivity could also be a matter of concern for regulators, where firms might be motivated to break up into affiliates to satisfy capital requirements. i.e. $X,Y,X+Y\in V\Rightarrow\rho(X+Y)\leq\rho(X)+\rho(Y)$.

3) Positive homogeneity of degree 1: if the size of every position in a portfolio is doubled, the risk of the portfolio should be twice as large i.e. $X\in V,h>0,hX\in V\Rightarrow\rho(hX)=h\rho(X)$.

   Note that this rules out liquidity effects associated with portfolio that have large amounts in any given individual position.

4) Monotonocity: if losses in portfolio A are larger than losses in portfolio B for all possible risk factor return scenarios, then the risk of portfolio A is higher then the risk of portfolio B i.e. $X\in V,X\geq 0\Rightarrow\rho(X)\leq 0$.

Properties (2) and (3), which refer to sub-additivity and homogeneity, respectively, imply that the convexity of the risk measure ρ and this corresponds to risk aversion on the part of regulators/supervisors.

Generally, VaR is not regarded as a coherent risk measure even though it satisfies the properties of translation invariance, positive homogeneity and monotonocity. According to Artzner, Delbaen, Eber and Heath (1999), this is because: "

a) VaR does not behave nicely with respect to the addition of risks, even independent ones, thereby creating severe aggregation problems.

b) The use of VaR does not encourage and, indeed, sometimes prohibits diversification because VaR does not take into account the economic consequences of the events, the probabilities of which it controls." (p. 218).

In short, VaR does not satisfy the sub-additvity property. There are situations where a portfolio can be split into sub-portfolios such that the sum of the $VaRs$ of the sub-portfolios is smaller than the VaR of the total portfolio. This may cause problems if the risk-management of a financial institution is based on VaR limits for its individual trading books. On this point, Artnzer, Delbaen, Eber and Heath (1999) argued that if quantiles were computed under a distribution for which all prices are jointly normally distributed which is discussed in more detail in section 2.1, then VaR is a coherent risk measure since the quantiles do satisfy the sub-additvity condition requiring probabilities of exceedence to be smaller than 0.5 i.e. $\sigma_{X+Y} \leq \sigma_X + \sigma_Y$ for each pair of random variables. Since for a normal random variable X, the VaR is:

$$VaR_\alpha(X) = -(E_p[X] + \Phi^{-1}(\alpha) \cdot \sigma_p(X))$$

with Φ the cumulative standard normal distribution and since $\Phi^{-1}(0.5) = 0$, the sub-addivity condition is met.

In fact, if one does not intend to aggregate risks computed by independent units but rather to allocate risk, then VaR being coherent is unnecessary because one can use the incremental VaR or $IVaR$ (Mina and Xiao, 2001).

1.5 Conclusions

If world equity markets are at least partially integrated, there will be some incentive for investors to diversify outside the domestic market, depending on the comovement between assets in the domestic market. The increasing integration of world equity markets however tends to offset these incentives to diversify outside the domestic market. In the first section, the finding in the early literature that markets are partially integrated was confirmed.

The Vector Autoregression (VAR) methodology is suitable for testing the transmission mechanism between equity markets and capturing the linkages and efficiency between markets. Studies that have employed the VAR technique were reviewed in the second section. The results of these studies have generally found significant linkages between equity markets which mean comovement between these markets is greater than the early literature suggests. The increasing integration of equity markets might explain this. The evidence from testing in this section questions the rationale for investing outside the domestic market to reduce risk in the portfolio.

The properties of coherent risk measures as defined by Artzner, Delbaen, Eber and Heath (1999) is also reviewed namely: translation invariance, sub-additivity, positive homogeneity and monotonocity. Based on these properties, it was demonstrated that VaR is not a coherent risk measure as it violates the condition of sub-additvity. Except if diversification under the assumption of jointly normal distribution of returns works, then only then VaR is a coherent risk measure suggesting the property of sub-additivity is met. VaR been sub-additive is equivalent of saying diversification never increases the level of risk.