IMAGE REGISTRATION USING FINITE DIMENSIONAL LIE GROUPS

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Abstract

D'Arcy Thompson was a biologist and mathematician who, in his 1917 book 'On Growth and Form', posited a 'Theory of Transformations', which is based on the observation that a smooth, global transformation of space may be applied to the shape of an organism so that its transformed shape corresponds closely to that of a related organism. Image registration is the computational task of finding such transformations between pairs of images.

In modern applications in areas such as medical imaging, the transformations are often chosen from the infinite-dimensional diffeomorphism group. However, this differs from Thompson's approach where the groups are chosen to be as simple as possible, and are generally finite-dimensional. The main exception to this is the similarity group of translation, rotation, and scaling, which is used to pre-align images. In this thesis the set of planar Lie groups are investigated and applied to image registration of the types of images that Thompson considered. As these groups are smaller, successful registration in these groups provides more specific information about the relationship between the images than diffeomorphic registration does, as well as providing faster implementations. We build a lattice of the Lie groups showing which are subgroups of each other, and the groups are used to perform image registration by minimizing the L^2-norm of the difference between the group-transformed source image and the target image. A robust, practical, and efficient algorithm for registration in Lie groups is developed and tested on a variety of image types.

Each successful registration returns a point in a Lie group. Given several related images (such as the hooves of several animals) it is possible to find smooth curves that pass through the Lie group elements used to relate the various images. These curves can then be employed to interpolate points between the set of images or to extrapolate to new images that have not been seen before. We discuss the mathematics behind this and demonstrate it on the images that Thompson used, as well as other datasets of interest.

Finally, we consider using a sequence of the planar Lie groups to perform registration, with the output from one group being used as the input to the next. We call this multi-registration, and have identified two types: where the smallest group is a subgroup of the next smallest, and so on up a chain, and where the groups are not directly related, i.e., separated on the lattice. We demonstrate experimentally that multi-registration can provide more information about the relationship between images than simple registration. In addition, we show that transformations that cannot be obtained by a single registration in any of the groups considered can be successfully reached.

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