Learning through language: Implications in a mathematics class

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ABSTRACT

Mathematics is a subject that can be said to be have a language all of its own. The language of mathematics, the language of teaching and the language of the students all impact on the mathematics classroom. With the ever-increasing numbers of ESOL and NESB students in our classrooms there is a need for an awareness of the benefits when we use language activities particularly in the mathematics classroom. The New Zealand Mathematics Curriculum has mathematical processes as a central focus. Communicating mathematical ideas is a sub-strand of mathematical processes. With these two thoughts as background stimulus this research examines the effect that *learning through language* activities used in a mathematics classroom have on student understanding and communication. *Learning through language* is active learning strategies for the classroom and is based on the philosophy that all teachers need an understanding of language processes. They can then build language-based interactive strategies into the teaching of their subject. *Learning through language* aims to help teachers cater for the language and learning needs of their students especially those from Non-English speaking backgrounds. The research findings indicate that the use of *learning through language* activities in the mathematics classroom has a positive effect on the willingness of students' to communicate in mathematics. There is also an indication that the quality of this communication has improved. Student understanding has not been affected by the use of these strategies, but it was difficult to draw any major conclusions based on evidence collected.

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1.1 Purpose of the Study

The purpose of this study was:

To develop teacher resources based on the material and activities presented in the *Learning through Language* course. The resources are designed for use with Form Four Mathematics Classes.

To trial the resources and to look at the effect that these resources have on student understanding of mathematics and the quality and willingness of students to communicate mathematical ideas.

1.2 Learning through Language

In 1996 I attended a *Learning through Language* course. The content of the course was covered over four days. Homework tasks were set. These tasks were to be completed in the classroom before the next workshop.

The workshop group was made up of eight members of the school's teaching staff. The teachers came from a variety of curriculum areas. Perenise Tapu was the workshop tutor and was also a teacher at the school.

> *Learning through Language is a school-based, cross-curriculum staff development programme for secondary teachers. It is designed to improve students' learning by the implementation of language-based teaching/learning strategies.*

> *A range of topics is covered in the four day workshops including: the role of language in learning; student, teacher and subject language; language-based activities to integrate into content teaching; and interactive strategies to promote learning.* (Learning through Language Course Certificate 1996)

During the four days different topics were covered. The modules for each day are outlined below:

Day One:	Module 1	Focus on the Learner
	Module 2	Preparing for Learning
	Module 3	Preparing to read Text
Day Two:	Module 4	Vocabulary
	Module 5	Talk in the Classroom
Day Three:	Module 6	Listening Skills
	Module 7	Thinking through and discussing ideas
	Module 8	Writing to Learn
Day Four:	Module 9	Extracting and Organising
	Module 10	Writing to Communicate Information

In 1988 Fran Edwards, Sylvia Hill, Ann Hunt and Ruth Penton compiled and initiated a staff development programme based on research done in 1981. This research was on Reading and Learning in the Junior Secondary School. The staff development programme also incorporated other developments from New Zealand, Australia and England.

This in-service programme, *Learning through Language: active learning strategies for the classroom,* is school-based and cross-curricular, and it provides practical strategies for teachers and students (Mabbett, 1991).

The *Learning through Language* (known as LTL) staff development programme has been successfully running in secondary schools since 1988, and was the basis for a Ministry-supported contract that started in 1991 (Penton 1995).

> *"The LTL course is based on the philosophy that all teachers need an understanding of language processes so that they can build language-based, interactive strategies into the teaching of their subject. It aims to help subject teachers cater for the language and learning needs of students, especially those from language backgrounds other than English."*
>
> (Penton 1995, Page 1)

1.3 Justification

> *"At present, children are passing through mathematics courses without needing to communicate mathematical ideas clearly and precisely. They do not develop mathematical literacy. They are unable to make use of mathematical notation as a tool for reasoning, and consequently their mathematical thinking is limited to problems they can solve mentally or by rote-learned procedures."* (MacGregor 1990a, Page 31)

While this statement holds true for many students, some progress has been made in New Zealand. The Mathematics in the New Zealand Curriculum (1992) document has addressed this issue. Through the Mathematical Processes strand all students are expected to communicate mathematical ideas. The document states:

> **Communicating Mathematical Ideas**
>
> ***Achievement Objectives***
>
> *Within a range of meaningful contexts, students should be able to:*
>
> - use their own language, and mathematical language and diagrams, to explain mathematical ideas
> - record and talk about the results of mathematical exploration
> - report the results of mathematical explorations concisely and coherently

Suggested Learning Experiences

Students should be:

- recording in words, pictures, and concrete materials
- presenting mathematical ideas and results to others
- explaining, discussing and presenting arguments
- working co-operatively as part of a group by listening attentively, generating ideas, and participating in reflective discussion

(Page 28-29)

The LTL course and philosophy supports the *Communicating Mathematical Ideas* strand of the Mathematics Curriculum. The strategies and techniques used are of benefit to all students as they encourage and support communication through the four macro skills of language: speaking, listening, reading and writing.

The LTL course notes gives the following description and value for learners.

COMMUNICATIVE ACTIVITIES

A communicative activity is one in which a student must carry out the set task by communicating with another student(s). This gives practice in speaking and listening in a pair or small group.

Value for learners

1. Learning by interacting in small groups is a natural communication setting. (family groups, friendship groups)
2. A learner is far more likely to experiment with (new) language in a small group of peers than in front of the whole class.
3. When the set task relies on talking and listening in pairs or small groups, there is significant, relevant language practice for all students.
4. Learning is much more active when each student must interact with the material and another student to complete the task.

5. A 'problem-solving' style of activity promotes co-operative learning and interpersonal skills.
6. Learners are encouraged to use their own ideas and take greater responsibility for their own ideas and others' learning.
7. Students are developing confidence in handling the language and the content, as well as social and communication skills.
8. Communicative activities give practice, reinforcement and repetition of key vocabulary and concepts.

1.4 Research Questions

What effect does the use of *Learning through Language* activities in a mathematics class have on:

- Student understanding.
- The willingness of students to communicate mathematically.
- The quality of the students' mathematical communication.

Chapter Two Literature Review

2.1 Introduction

This literature review discusses previous research and opinion related to the use of learning through language activities in Form Four Mathematics Classes. As there is little literature on learning through language, comment and research related to this field has been reviewed as well.

As an overview the following areas have been used to provide a basis for reviewing publications related to learning through language.

Literature discussing the connection between mathematics and language is an obvious starting point for the review. There is extensive research and comment on mathematics and language. (Ellerton, 1989; Halliday, 1991; Hill and Edwards, 1991; Lilburn and Rawson, 1993; MacGregor, 1993; Meiklejohn and Hedley, 1993; Miller, 1993; Munro, 1990 and Reeves, 1990).

Ballagh and Moore, 1990; Clarkson and Thomas, 1993; Lewis, 1992 and MacGregor, 1993; provide a wealth of information relating to English as a Second Language (ESL) learners, multicultural and bilingual classrooms. This area is imperative in any discussion on learning through language, especially in relation to New Zealand classrooms. ESL learners are increasing in proportion in New Zealand classrooms.

Reading and writing in the mathematics classroom are discussed. Then there is a more in-depth coverage of the specific learning through language activities (Hill and Edwards, 1991; LTL Course notes, 1996). Included in this section is research and comment on the use of concrete activities (Burnett, 1992; Colburn, 1830/1970; and Hickerson, 1952) and group work (Burnett, 1992; Cooper et al, 1993 and Miller, 1993).

The literature review finishes by looking at work done in algebra. Algebra was the topic chosen for the main research phase of the thesis. Here the emphasis has been on the use of concrete activities and approaches to teaching algebra and particularly the work done in New South Wales (MacGregor and Quinlan, 1992; Quinlan, 1991a; Quinlan and Collis, 1990a, 1990b; and Quinlan et al, 1989).

The intention of this literature review is to provide background information, research and comment related to learning through language.

2.2 Language, Communication and Mathematics

2.2.1 Introduction

What is it about communication that makes it an essential part of any learning process? Lilburn and Rawson (1993) talk about the need we have to communicate when making a decision, or when we are unsure about something. What most of us do in this situation is sound out people around us, people whose opinions we value. We discuss the ideas with others, toss the ideas around in our own minds, weigh others' opinions up against our own, and continue to look at them from a variety of viewpoints until we reach a final decision. (See Bain, 1988 and Hill and Edwards, 1991)

Lilburn and Rawson feel that

> *"we should provide children with the opportunity to go through a similar process to clarify their mathematical thinking. This means that our classrooms during maths lessons will have children working and talking together in the same way they do in other lessons."* (Page 3)

Students should be discussing problems before attempting to solve them. Discussion is a powerful tool. Students are provided with opportunities to prove facts, ask questions and debate ideas when they are discussing (Hill and Edwards 1991).

There is a need to communicate in the mathematics classroom and language stems from this need to communicate. (Cangelosi, 1988; Munro, 1990; and Reeves, 1990)

> *"Mathematical language also stems from a real desire children have to want to talk to an appreciative audience and its development on wanting to listen to a competent user. Unless a classroom climate enables talk to be with, rather than at or to children, language development, including mathematical language will not flourish.*
>
> *It is TEACHER TALK + CHILD TALK = MATHEMATICS"*
>
> (Reeves, 1990, Page 92-93)

Mathematical communication is difficult to many students, even some of those who are good mathematics students (Meiklejohn and Hedley, 1993). Students whose English language competence is low also have difficulties, and are being disadvantaged when needing to describe procedures orally (MacGregor, 1993).

2.2.2 Mathematics and Language

There is a wealth of opinion on mathematical language and mathematics and language (Cangelosi, 1988; Ellerton, 1989; Hill and Edwards, 1991; Lilburn and Rawson, 1993; MacGregor, 1993 and Meiklejohn and Hedley, 1993).

Much relates to the use of mathematical language in relationship to real life contexts. For example Meiklejohn and Hedley talk about the need for students to have the opportunity to use mathematical language and vocabulary, and how if they don't have the opportunity they don't internalise the language. They talk about students needing to be exposed to many contexts, with these contexts giving purpose for the language. Cangelosi also supports this premise. He talks about students developing a *meaningful grasp* after applying a concept to real-life problems.

Meiklejohn and Hedley also state that

> *"considerable effort needs to be made to assist these students to see examples of the concepts outside a mathematical classroom and context."* (Page 256)

This is supported by Lilburn and Rawson (1993) who talk extensively about the children's mathematical language development being based on a need to first express real life experiences in their own words and then recording that in mathematical words and symbols.

One of the main areas of concern is that students are not learning the language of mathematics, because they are not using the language of mathematics, and their teachers are not using the language of mathematics (Ellerton, 1989; Lilburn and Rawson, 1993; Meiklejohn and Hedley, 1993 and Miller, 1993). And sometimes it is more appropriate to work on the language itself, instead of assuming students have the language to learn with (Halliday, 1991).

There is a need for mathematics teachers and language teachers to work together to develop strategies for teaching language in mathematics (Halliday, 1991 and MacGregor, 1993). Halliday advocates that the language-learning process should be a part of all teacher training courses, including secondary mathematics teachers.

> *"Together they* (mathematics and language teachers) *will need to analyse the language skills required for certain mathematical topics, decide what language expectations they should have for each class or each group of students, establish how best to teach the necessary skills, and plan activities that put these skills to use."* (MacGregor, 1993, Page 58)

The above ideas are best summarised by John Munro (1990). Munro considered the notion of a language-based mathematics curriculum. He said that the assumptions for such a curriculum would include the following:

- *An idea is learned when the learner perceives a purpose or reason for learning it, when the idea may solve an existing problem.*
- *The learner needs to expect that they can learn the idea, and to see that significant other around him believe also that they can learn it.*
- *The learner learns by acting on his environment, by manipulating, experimenting, predicting, sharing their findings with others and so on.*
- *In the learning situation the learner constructs a series of hypotheses about an idea that they need to trial and may modify in the light of subsequent corrective feedback.* (Page 21)

It is clear from the literature that language has a place in the mathematics classroom; That this language is best developed in collaboration between mathematics teachers and language teachers, and that the students must see the real world link up to obtain a meaningful grasp of the language and the mathematical concepts.

2.3 English for Speakers of Other Languages (ESOL)

2.3.1 Introduction

Learning through language was developed to meet the needs of the ever-growing number of ESOL or Non-English Speaking Background (NESB) students appearing in all schools. Lewis (1992) and Penton (1995) have written about the New Zealand situation, while Ballagh and Moore (1990); Clarkson and Thomas (1993) and MacGregor (1990b, 1993) have written in relation to this issue in Australia. Diane Miller (1993) has also published in this area and she noted that there is an increasing number of students in the United States who are learning English as a second language.

The standard of some students' English language competence may interfere with their progress in mathematics. Research studies (Clarkson and Thomas, 1993 and MacGregor, 1993) show that there are two types of students whose mathematical progress may be affected by their English language competence. One type is the student who is proficient in their first language, but has insufficient English language proficiency to communicate what they are thinking or to make sense of what they read. The other type is those for whom English is their natural language, but are not aware that their formal and written English is not proficient and therefore is causing difficulties in learning mathematics. These students are also dependent on the efforts of all their teachers, including their mathematics teachers for language development.

It is essential that teachers at all levels of schooling are aware of their role in the teaching of language skills to all students, NESB students and ESB students. MacGregor (1993) states that

> ***"Many primary teachers are aware that language development is an essential component of mathematics learning and know how to promote it. However secondary mathematics teachers, who have no training in language teaching techniques, may not know what to do to help students with low levels of***

English language competence. They also are unsure whether they, as teachers of mathematics, should be concerned about students' writing in English that is poorly expressed or incoherent." (Page 51)

It is obvious then that the strategies for helping ESOL or NESB students will also be useful for students of low English proficiency who have English as their only language. Students with special needs will also benefit from these strategies.

2.3.2 Strategies for Teaching ESOL or NESB Students

The Mathematical Literacy Project in Australia identified key elements of ESOL teaching and used these to develop a framework to teaching mathematics to these types of students (Ballagh and Moore 1990). These key elements should create a classroom environment where students are provided with opportunities to use mathematical language, develop essential communication skills and be involved in group problem solving.

- *Extensive use of concrete-visual materials. These serve two interrelated purposes: as the first step in developing concepts (going from the concrete to the abstract) and (of particular relevance to NESB students) as a visual/experiential reference point for the discussion and development of language skills.*
- *Structuring of activities to encourage student discussion, hence strengthening the command of language and the understanding of concepts.*
- *Use of strategies for development of essential grammar and vocabulary that could be readily adopted for mathematical language.*
- *A balance of the four macro skills of language development in lesson design – reading, writing, speaking and listening.*

(Ballagh and Moore 1990, Page 79)

Based on the above key elements Ballagh and Moore also developed eight principles for teaching mathematics to NESB Students.

These principles listed below have assisted Ballagh and Moore to develop effective language-based teaching approaches to use in their classrooms.

(a) Lesson design should incorporate those elements necessary to ensure that students

i. Master mathematical language for example, vocabulary, grammatical structures, special meanings of otherwise common words, use of logical connectives and conditionals

ii. Are able to bridge the language gap between English language and mathematical symbolic language.

(b) Physical models, visual materials, charts and other concrete aids should be used extensively.

(c) Students should be allowed to explore concepts using their own informal English, building on what they already know before they are presented with formal language. Much of the formal operational language used in mathematics is difficult for NESB and ESB students alike. If students are allowed to develop an understanding of mathematical operations and concepts using the language that they already understand, there is a greater likelihood that they will make an effective transfer to the use of the more formal language.

(d) Students should be encouraged to talk extensively about mathematics. Opportunities for such discussion should become an indispensable part of every mathematics lesson.

(e) The environment in the mathematics classroom should engender a spirit of co-operation and actively involve students in their learning. It is important in mathematics groups that are characterised by mixed abilities, cultural and language differences, and gender differences that a spirit of trust and co-operation be developed. Students must feel secure if they are to take risks in using and developing competency in the language of mathematics. It is vital

that physical surroundings be pleasant and conducive to communication and participation.

(f) A "right answer" mentality should be discouraged. Teachers should demand from their students clear reasoning and correct language use in oral and written form. Students should develop an appreciation of mathematics as a modelling and problem-solving tool and be able to apply it to real life situations.

(g) Contexts for problem solving should be carefully chosen. Students from other cultures may not share the teacher's understanding on the context of problems until it is explained in detail. Students need to develop a clear understanding of the problem through discussion and use of physical models, diagrams, and so on, before proceeding to the solution.

(h) Good questioning technique and sensitivity to teacher language use are essential skills for an effective facilitator of learning in the mathematics classroom. In class discussions each student's answers or opinions should be valued. Each student should feel confident if they take the risk of contributing to class discussion.

i. An active association between mathematics teachers and language teachers needs of students will best serve the language.

(Page 80-83)

Miller (1993) has a variety of approaches to produce a multicultural classroom environment that will be more conducive to cross-cultural teaching. She suggests four approaches to achieve this.

<u>Be sensitive to other cultures</u>. Teachers should create opportunities to use examples from various cultures regarding the use of mathematics, the recording of mathematics, the context of mathematics, and the learning of mathematics.

> *Understand different cultural expectations from classroom discourse. These issues include asking and answering questions, verbal and non-verbal communication, male-female relationships, classroom groupings, and teachers' status.*
>
> *Encourage students to express their sociocultural understanding of mathematics. Certain teaching strategies encourage students to use mathematical language in the classroom. For example, asking students to develop their own problems to investigate allows them to cast their stories in a familiar cultural context. Students from non-English-speaking backgrounds can be allowed to write their problems in their own language first and translate them to English at a later time.*
>
> *Clearly write and carefully pronounce new mathematics vocabulary. The learning styles of students vary. Some learn best by hearing, whereas others learn best by seeing. When introducing new mathematics vocabulary, teachers should place each term within a meaningful context and give both an oral explanation and a written definition.* (Page 315-316)

The ideas that both Ballagh and Moore and Miller are suggesting are similar. The strategies listed above are all part of the LTL approach to teaching. (See also Hill and Edwards 1991). However many of the ideas suggested will require a move in teaching style for many teachers. There are teachers who already incorporate these ideas and strategies as part and parcel of their classroom approach and students in these classes are being provided with plenty of opportunities to develop language skills.

Burnett (1992) backs this up. She writes that there has been a change in classroom instruction, and the move has been away from formal procedures to more child centred approaches that encourage children to talk about their ideas and conjectures. See also Bickmore-Brand (1990), Cangelosi (1988), Penton (1995) and Tobin and Fraser (1988).

2.4 Reading and Writing in the Mathematics Classroom

2.4.1 Introduction

Ballagh and Moore (1990) talk about having a balance of the four macro skills of language development in lesson design; the four macro skills being reading, writing, speaking and listening. The previous two sections have focused more on the speaking and listening. This next section examines some of the literature surrounding reading and writing in the mathematics classroom.

2.4.2 Reading in the Mathematics Classroom

There are skills that people develop as their reading ability increases. These natural human characteristics of reading are often seen as "faults" when related to mathematical reading, that is, reading mathematics problems.

Published reports such as MacGregor (1991) and Munro (1990) talk about redundancy of words (or lack of redundant words) as one of the problems in mathematical word statements.

Often we complain that our mathematics students don't read carefully. But do we read carefully? Usually we don't read carefully because there is no need to. When we read an ordinary novel we often skip or miss out words. These words are redundant in terms of our getting the overall gist of the statement. Generally, the statement in the novel contains a degree of redundancy.

Mathematical word statements on the other hand often have a high density of concepts, and a low degree of redundancy. Several concepts have been compressed into a minimum of words. Students need to see and use every piece of information provided to extract the meaning. This is even more critical if the student is not familiar with the context of the problem.

Students' prior knowledge (or lack of) also has a major impact. If what students are reading refers to information that is not in the student's prior knowledge, then misunderstandings will occur (Hill and Edwards 1991).

Students may also add words that are not there, and change words. In ordinary language, a few altered, omitted or added words often make no real difference to the meaning. In mathematical word statements they can make a major difference.

> *"For example, a girl read 'more women than men' as 'more women to men'; a boy read 'more dogs than cats' as 'more dogs and cats'."*
>
> (MacGregor, 1991, Page 111)

Concern has also been expressed over students' lack of mathematical literacy and comprehension skills (Lilburn and Rawson, 1993 and MacGregor, 1988). Poor comprehension skills are acting as a barrier to understanding. Linguistic procedures that are useful when dealing with ordinary language are acting as a barrier to using mathematical language.

> *"Many children are handed worksheets that contain ideas well within their grasp, but they are unable to answer any of the questions because they cannot interpret what is being asked. This is not a lack of mathematical knowledge, but a problem of language ability. Unfortunately it is often interpreted as a lack of maths ability. Because children interpret written words in different ways it is important to make sure that all children know what they are doing before they start. We can do this by having questions or statements read out aloud by different children and then, after discussion and sharing ideas, have children restate the problem or task in their own words – thus using everyday language to describe mathematical situations."*
>
> (Lilburn and Rawson, 1993, Page 4)

2.4.3 Writing in the Mathematics Classroom

The 1992 Mathematics in the New Zealand Curriculum statement makes reference to students writing in mathematics. Under suggested learning experiences it states that students should be: explaining results in words and pictures; reporting in words and diagrams; making written reports; reporting in formal mathematical language (Page 29*)*.

In Australia the Mathematics Curriculum and Teaching Program (MCTP) included reference to pupils writing about mathematics. The following seven points from MCTP support the idea that writing about mathematics enhances learning:

1. *Mathematical experiences can be "captured" for later recall.*
2. *Pupils can use "natural" language arising from real contexts.*
3. *Writing, like talking, can facilitate internal organisation of mathematical relationships.*
4. *Pupils' work provides a springboard for discussion about the concepts being explored.*
5. *For many children, writing is an enjoyable, creative experience.*
6. *Writing can take place as a co-operative group task.*
7. *The written piece of work can assist teachers in formal and informal assessment.* (Lovitt & Clarke 1988, in Ellerton & Clements 1992, Page 154)

Concern has been expressed in many publications regarding the limitations, traditionally, of writing in mathematics classes (Clarke, 1986; Johnson, 1983; Marks and Mousley, 1990; Miller, 1993 and Wong and Herrington, 1992). There is concern that the writing has been limited to presenting workings in a step-by-step and logical way; that students are given little opportunity to express themselves except for formal written solutions and that students were almost never given the chance to do meaningful writing in mathematics classes.

The benefits of writing (Johnson, 1983; MacGregor, 1990b and Waters and Montgomery, 1993) include student awareness of their own thought process. Having students write about mathematical concepts and ideas requires a more active involvement in the learning process. Students are required to mentally re-order and organise their thoughts, and therefore synthesise their understanding of the concepts studied.

Waters and Montgomery go on to say that writing is also invaluable in highlighting a student's understanding of what mathematics is and guide teachers towards specific assistance that the student might need. (See also Keith, 1988)

> *"It is obvious that writing in mathematics can be a window to the child's mathematical understandings. Diaries, written explanations of 'how-to', logs, proposals, reports and resumes all allow the teacher and the child to evaluate progress."* (Waters and Montgomery, 1993, Page 206)

Various writing activities have been used in school mathematics programs (Ellerton and Clements, 1992; Havens, 1989; Keith, 1988; Swinson, 1991 and Waywood, 1992). Activities such as impromptu writing prompts, summarising activities, essays, letter writing, re-writing, journal writing, diaries, proposals, and reports. Distinguishing between recording their mathematics as opposed to writing their mathematics will encourage students to write about mathematics rather than about the lesson (Pengelly, 1990).

Ellerton and Clements (1992) report that with each of these forms of writing, some hard questions, relating to students, teachers and curriculum developers need to be addressed by mathematics education researchers. Their research issues concerning students and teachers are outlined below.

> ***Research issues concerning students.** If students participate regularly in a particular form of "writing mathematics" are they likely:*

1. *To perform as well as students who do not participate in this form of "writing mathematics" on standard tests of mathematical skills, concepts and principles?*

2. *To link more readily their mathematical understandings with their personal worlds?*

3. *To become more efficient and effective at monitoring their own mathematical thinking so that they improve their own problem-solving and problem-posing performances?*

4. *To develop more positive affective (for example, liking/disliking, confidence) responses to mathematics and mathematical situations? In particular, are students likely to develop feelings of ownership over mathematics they construct or write about?*

5. *To become more aware of their own abilities, attitudes and preference in mathematics, and to be prepared to modify these in response to their own reflections?*

Research issues concerning teachers. From a teacher's point of view, is the classroom time spent on "writing mathematics" activities time wasted or time well spent from the point of view of developing students' mathematical understandings and positive attitudes? Also, from a cost-benefit point of view, do students' mathematics scripts provide teachers with sufficiently valuable diagnostic and predictive data to justify the time taken for students to produce the scripts, and for teachers to read and comment on them, and to maintain pertinent records? Also, certain ethical questions arise: for example, are teachers entitled to assess a student's journal entries?

(Page 158)

It is important that all sides of the argument are addressed. Ellerton and Clements (1992) observed that many children do not enjoy "writing mathematics" and that negative attitudes and wrong understandings of mathematics could be nurtured if we force them to do so. Waters and Montgomery (1993) caution the use of writing in mathematics purely for assessment purposes.

> *"If children are asked to write and then hand in their writing for marking, the writing becomes akin to a test. Children will write only in areas in which they are truly 'safe'. Risk-taking dialogues won't occur in a threatening, judgemental environment."* (Page 206)

Writing is important both for the cognitive processes used during writing activities and for the diagnostic benefit afforded by the teacher. These processes include organising and structuring knowledge by defining connections and relationships between what is already known and new knowledge. There is benefit to the students and to the teachers.

2.5 Learning through Language

2.5.1 Introduction

In this section a more specific look will be taken at the types of activities used in the learning through language course. Literature is reviewed and activities defined. Research and discussion on concrete activities (Burnett, 1992; Colburn, 1830/1970; Hickerson, 1952 and Pengelly, 1990) is covered first.

The importance of group work and ideas on use and introduction (Ballagh and Moore, 1990; Bickmore-Brand, 1990; Burnett, 1992; Cooper et al, 1993; Hill and Edwards, 1991; Lilburn and Rawson, 1993 and Miller 1993), and then more specific comment on activities such as Cloze (Hill and Edwards, 1991; Hornsby, 1992), brainstorming (Dennett, 1991 and Hill and Edwards, 1991), and the different listening, vocabulary, talking and writing activities covered in the learning through language course notes (Hill and Edwards, 1991 and Language across the Curriculum (photocopied resource book), no author, no date).

2.5.2 Concrete Activities

As far back as the nineteenth century educators have been espousing the value of concrete materials and activities.

Burnett (1992), Colburn (1830/1970), Hickerson (1952) and Pengelly (1990) all advocate that students' active participation in the learning process by developing new ideas using concrete and practical examples is essential.

Concrete aids should be available for students to use though *'fingers do about as well as anything"* (Colburn 1830/1970).

Conceptual understanding is developed through a sequence of activities. The activities move gradually from concrete in nature, to more pictorial or semi symbolic, to finally abstract concepts. Burnett (1992) represents this process in the following Triad Model.

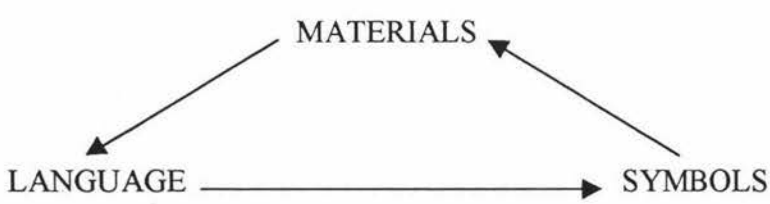


Figure 1: Triad Model (Page 223)

The role of language in this process is very important, as it possesses the ability to develop the link between the concrete referents and the symbolic notation of a particular mathematical concept (Burnett). The more opportunities students are given to experiment with communication in both oral and written forms the greater the development from the informal language they use to describe mathematical concepts into formal mathematical language.

2.5.3 Group Work

Talking and listening in particular involve working with at least one other person. That is for students to use language skills they need to work in groups. A group can be a pair of students, a small group of 3-4 students, or the whole class. Groups can be mixed ability, similar ability and can be split along gender or cultural lines. They can be self-selected, teacher-directed or created by some other basis; for example, line the class up from shortest to tallest and group according to height.

Group work and problem solving provide students with an environment that fosters the acquisition of concepts and processes in a meaningful way (Burnett, 1992; Cooper et al, 1993; Hill and Edwards, 1991 and Miller, 1993).

> *"Peer group discussions are another way in which students can exercise their mathematics vocabulary and construct meaningful understanding of the formal language of mathematics."* (Miller 1993, Page 315)

Group work encourages student talk (Bain, 1988; Bickmore-Brand, 1990 and Burnett, 1990). Student talk is likely to be: more developed; more involved and in a risk-free environment. More students are given the opportunity to talk when working in pairs or small groups.

It is important that group work is well thought out (Bain 1988) and teachers clearly understand their role (Lilburn and Rawson 1993).

> <u>*Helpful hints for teachers using group work*</u>
>
> ***"Try to remember that your major role during these activities is to encourage children to talk about and share their ideas, to use everyday language to describe mathematical situations, and to restate problems in their own words. Try to avoid only saying 'No' or 'Yes' in response to a***

child's answer. Instead, try asking how or why they reached that answer. Don't influence what they say. Body language, such as a nod of the head or raising our eyebrows can give away what we are thinking in many instances. Remember that it is important for you to find out what the children are thinking and not what they think you want to hear, so that you can use this information to diagnose problem areas and so assist the children. If you can manage to do this, you will have gone a long way towards letting children know that you value their ideas, not just the correct answer." (Lilburn and Rawson 1993, Page 5)

Group work promotes a positive climate for learning. It also increases the language practice opportunities and improves the quality of student talk. It improves social communication skills as students ask questions, listen carefully and take turns. Students receive more individual attention and there is an active involvement in processing ideas. (LTL course notes 3/6)

Most of the activities used in the LTL course support the use of group or pair work. Students cannot listen, talk or share without working with at least one other person.

2.5.4 Cloze

Cloze refers to the 'reading closure' practice where readers are required to fill in blanks left in the text. They must use whatever knowledge and experience they have. The strategies used to do this are the same as those used in reading. Cloze exercises therefore can be used to help children develop and refine reading strategies.

Cloze exercises are one of the strategies employed in *learning through language.*

Hornsby's (1992) description of a written cloze follows a cycle of:

→ prediction → justification → comparison → discussion

Hill and Edwards (1991) suggest that the use of the cloze procedure is an effective method of checking students' understanding of mathematical knowledge. This process can be self-checking, especially with Senior Students. They can use a cloze completion activity to test their own understanding of concepts. By sharing their responses with one another opportunities for peer tutoring and discussion arise.

2.5.5 Brainstorming

Brainstorms are free word associations related to the topic. Brainstorms are a useful method to find out students' prior knowledge.

Dennett (1991) talks about her work using Brainstorming with adult English language students. She says that her main objective with Brainstorming is to elicit from the students on any given topic, as much "real-life language" as she can. This environment is providing an opportunity for students to identify what they know, and shows them what they need to know.

Brainstorming provides the opportunity for students to see more clearly the purpose of their work (Hill and Edwards 1991). With the starter "What do we know about ... ?" ideas are recorded and then students can work as individuals or groups and pursue a particular idea. Goal setting, presentation, research and evaluation are all outcomes from this.

Some of the benefits of brainstorming that Dennett found are listed below. These are applicable to the classroom. The language issues are critical in classrooms with high numbers of ESOL learners.

- *Success breeds success, which also increases self-esteem and confidence. The students became involved in the sessions and were rarely absent from class.*

- *Many students improved their reading skills because what they were reading was directly related to what they had been speaking about.*
- *Many students improved their speaking and listening skills. Because they were actively participating in the Brainstorming and discussions it was necessary for them to speak clearly to make themselves understood.*
- *Students who didn't actively participate in Brainstorming sessions still developed their reading, writing and listening skills.*
- *Classroom observers such as a colleague and others who videoed the class commented that the students were very interested and involved in their lessons.* (Page 103)

Brainstorming is a useful technique. It can be used at the end of the unit as a summary of what students have learnt. It can also be used to provide a word list for structure diagrams (another LTL technique, but not used in the research).

2.5.6 Clines

A cline is a sloping line upon which words with different shades of meaning are placed. A list of words and a cline are provided. Some words are already placed on the cline. Students work in pairs to place the remaining words or provide their own words on the given cline. (LTL course notes 2/4).

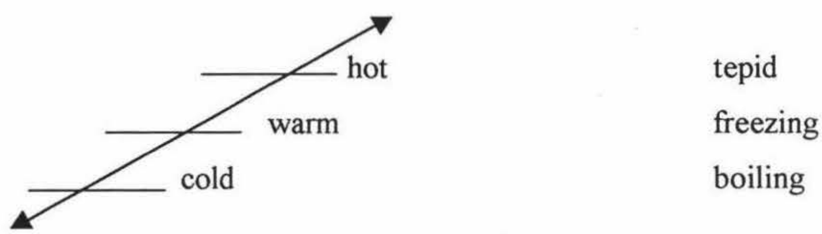


Figure 2: A water temperature cline

2.5.7 Co-operative Logic Problems

Co-operative logic problems require students to work as a group to solve the problem. Usually each member of the group has part of the information required to solve the problem. By talking they share the information they have on their clue card. The students are not allowed to show their clue to the rest of the group. They cannot look at anyone else's card. They share by reading out their information. (EQUALS 1989)

Each member of the group is involved in the solving process, even if for some the only involvement is to read out their clue. One of the stipulations is that all members of the group must understand the answer, and how they got the answer. It is then up to the rest of the group to make sure that everyone understands. The teacher (or observer) could ask any one in the group for the response. Therefore all must be prepared to answer the question, or problem.

2.5.8 Definition Activity

Prepare a list of key words and ask students to predict the definitions. Some other activity is provided to enable students to revise and confirm their definitions. (LTL Course notes 2/4)

Follow up activities could include students using the word in sentences or entering the definition in a personal dictionary or glossary.

2.5.9 Grids

Grids consist of a list of words or problems down one side of the page, with another list (e.g. the answers or features) across the top. Through discussion, students mark the features or answers that are correct for each problem. Grids give students the opportunity to show and expand their understanding of known words and concepts. It also helps them to distinguish between shades of meaning. (LTL course notes 2/4)

2.5.10 Matching Activities

Matching activities are part of a set of activities called barrier activities. With barrier activities students are placed in pairs. Each member of the pair has part of the information, or different information that must be shared to complete the task. Each student will have their own resource sheet, or own cards depending on the activity. (LTL course notes 2/5) See also Language across the Curriculum.

With the matching activities the students work with cards. In most instances there are two sets of cards. One set with the symbols, and one set with the words that describe the symbols. The cards are split into the two piles. The student with the word cards reads out what is on their card. The other student finds the symbols that match the words. The two cards are put down side by side. Once all the cards have been matched, the pairs of cards are put in order. The order may make a worked problem, or it may be ordering powers of ten for example.

2.5.11 Picture Dictation

The teacher reads out a series of instructions or a description of a picture or diagram. Students need to draw or create the picture as the instructions are read out. An outline may be provided. (LTL course notes 3/6). See also Language across the Curriculum.

2.5.12 Sequencing

A series of statements describing a process, giving instructions or worked examples are prepared. These are split up. Give each student in the group one statement. This is read aloud to other members of the group. It is not shown to anyone else. Students in the group must place themselves or their statements in the correct order. Everyone reading his or her piece in order checks this.

Variations on the theme involve getting students to memorise their statements, giving out the statements to pairs to put in order, or giving a series of diagrams or drawings instead of words. (LTL course notes 2/5) See also Language across the Curriculum.

2.5.13 Writing to Learn

Writing is an aid to learning. Students are given the opportunity to write in their own words. They are encouraged to find out what they already know and articulate their thoughts. Questions such as:

What I know about...
What three (3) questions would I like to ask about...
What three (3) important facts have I learnt about...

Other writing-to-learn strategies that were not used in the research include:

- *Predict/Observe/Explain;*
- *drafting ideas before producing a finished product;*
- *using their own first language or everyday language to think through their ideas before using 'subject' language;*
- *expressing their feelings and concerns about their learning;*
- *and writing to evaluate what they did or have learnt.*

(LTL course notes 3/8)

2.5.14 LTL Guide for preparing a unit of work

In the LTL course notes (no reference number) there is a useful guide indicating which activities were suited for which inclass task. For example to introduce a topic or to practice listening skills. Table 1 presents this information.

Table 1: LTL Guide for preparing a unit of work

Learning Outcome	Strategies
Introducing a topic and finding out prior knowledge	Structured overview
Vocabulary – new words	Cloze exercises, particularly the interactive cloze Clines, Clusters, Grids Word games Definition Activity
New information (learning by talking)	Communicative activities to get students talking e.g. Sequencing Ranking Matching Barrier activities (AB games) Information transfer
New information (learning by writing)	KWL sheet (What I **K**now, **W**hat 3 questions, 3 facts I have **L**earnt) Mapping Diagramming Graphic Outlines Mind maps
Understanding new information (comprehension)	Three level guide
Listening skills	Picture dictation Picture matching Verb story Rub out cloze Listening grids
Writing skills (communicating information)	Sequencing sentences into paragraphs Sorting and linking sentences Using models and outlines as guides

2.6 Algebra

2.6.1 Introduction

The main research unit was based in the mathematics strand of algebra. In this section comment is made on research in this area, particularly the work done in New South Wales by Brother Quinlan on the use of concrete manipulative in algebra.

2.6.2 A Concrete Approach to Algebra

The Algebra Research Group was founded in 1986. Their task was to develop an approach to the teaching of early algebra that made use of concrete manipulative as aids for clarifying basic concepts. An action research program was put in place. This involved the group writing student worksheets and teachers' notes and trialing them in various classrooms. As a result of feedback, redrafting and retrialing, the material was published in a series of four books called *A Concrete Approach to Algebra* (Quinlan, Low, Sawyer and White 1989).

Many of the activities and teaching suggestions made in the book have been incorporated into the 1989 NSW mathematics syllabus for Years 7 and 8 (the first two years of secondary schooling). Activities such as building geometric patterns out of toothpicks followed by small group discussion of the numerical relationship seen in the patterns. The relationship is first described in everyday language and later with algebraic symbols (MacGregor and Quinlan 1992).

Evaluation studies were undertaken and various reports written (Quinlan 1990a, 1990b, 1991a, 1991b and Quinlan and Collis 1990a, 1990b). For example Quinlan and Collis investigated the rationale for a concrete approach to algebra. They used theories such as Halford's (1987) Structure Mapping Theory. Indications from their investigations show that students taught by the manipulative approach were more likely to view algebraic symbols as representing numbers rather than objects than those students taught by the traditional textbook approach (MacGregor and Quinlan 1992).

Quinlan (1991a) has reported independently on this research and change to the curriculum. He commented in relation to the new mathematics syllabus approved by the Board of Secondary Education for NSW for Year 7 and 8. He also made comment about the fact that the document not only listed topics to be taught, but made recommendations that would impact on teaching practice.

> *"Perhaps the most striking departure from tradition was presented in the algebra section, where the use of concrete materials was recommended in topic after topic with the advice that algebra is best learned by progressing from experience with concrete materials through oral language to written language and then to symbolic representation."* (Page 28)

Quinlan also reported on research interviews carried out in November and December 1990. Students were interviewed six months after they had been introduced to algebra using these concrete models. Almost all of them could still use the models correctly. Students were asked to model cases they had previously answered incorrectly. All students arrived at the correct answer using the models.

MacGregor (1991) has reported on a number of algebra initiatives. She stresses that there is a need to address the low achievement and comprehension of this strand of mathematics. The comprehension of algebraic statements requires comprehension of ordinary English. Students had difficulty with the following two abstract test items:

> *A number p is eight more than another number q, and*
> *The value of y is eight times the value of z* (Page 64)

Students need to know the meaning of, for example, 'eight more' and 'eight times'. And to write an equation in the conventional notation they need to know some basic rules of algebraic notation. Figure 3 shows hypothetical relationships between a mathematical statement expressed in English, its representation in mathematical notation and its meaning.

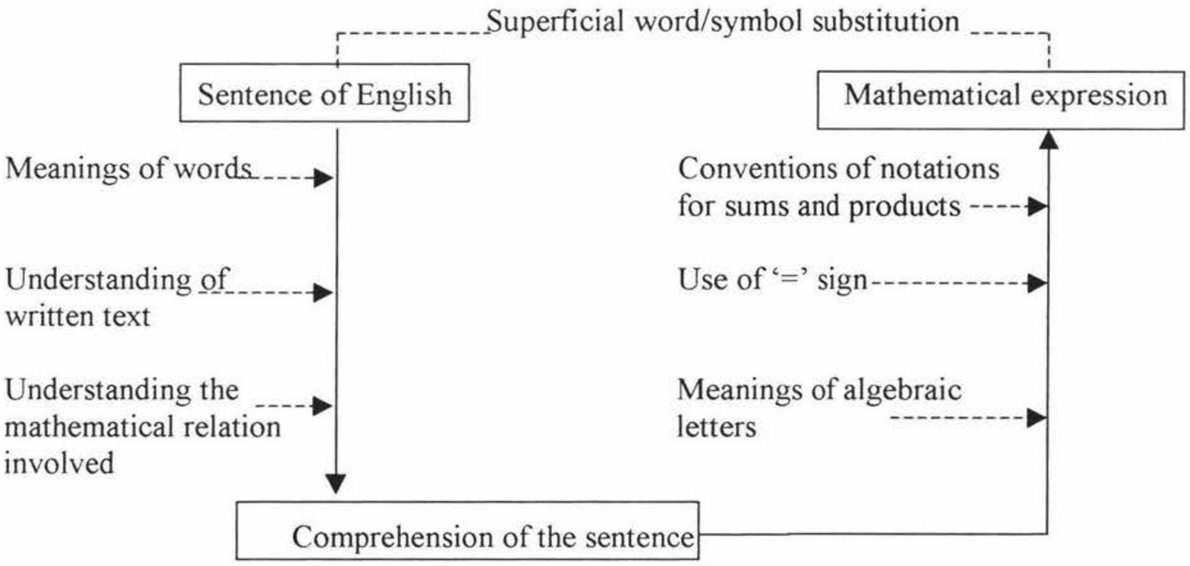


Figure 3: Model of the relationships between a sentence, its meaning, and its expression in mathematical notation. (Page 65)

Students' ability to comprehend the English language affects their ability to do mathematics. Examples from the above two questions are based on the model in Figure 3.

> *"On the basis of this model, errors in the two items have several possible causes:*
>
> - *misinterpreting the meanings of certain words (e.g. 'value');*
> - *not correctly interpreting the phrase structure of the sentence (e.g. interpreting 'p is 8 more than q' to mean that p is 8);*
> - *not understanding the mathematical concept or concepts expressed (e.g. being unsure whether 'more than' indicates to add or to multiply);*
> - *not interpreting algebraic letters appropriately;*
> - *not using the conventional meanings of mathematical signs, especially the '=' sign;*

- *not using or knowing conventions of mathematical notation (e.g. that the product of 8 and z is written as 8z);*
- *attempting superficial translation from words to mathematical symbols, without concern for meaning (e.g. writing p8>q to represent 'p is 8 more than q').* (Page 64-65)

MacGregor examined these seven components of the translation process and looked at the extent to which they influenced students' success in constructing equations. Other commentators on the level of basic algebra skills being poor include Watson (1988) and Victorian HSC examiners (1986).

MacGregor concluded that a certain proportion of students' errors in the test items were caused by factors not specifically related to algebra skills. The factors she listed are:

- *inadequate reading skills such as recognition of words, parsing ability and awareness of the syntax of meaning;*
- *poorly developed concepts of the operations of arithmetic; and*
- *careless or informal and imprecise use of mathematical notation.*

Within the domain of algebra, it was shown that errors were caused by two additional factors:

- *misconceptions about the meaning of algebraic letters; and*
- *misunderstanding of what an equation signifies.* (Page 121)

Psycholinguistic factors are a major cause of student difficulties and misunderstandings. Many teachers of mathematics may not be aware of this (MacGregor 1990a, 1991). There needs to be more interactions between researchers of mathematical education and workers in linguistics.

2.7 Summary

Language development is an important facet of mathematics learning. By providing opportunities for students to practice the four macro skills of language in the mathematics classroom, mathematics teachers are enhancing the language acquisition of students.

Students with English as a second language struggle in the mathematics classroom. The reason in many cases for this struggle is more to do with their lack of English skills than their lack of mathematics skills. Some native speaking English students also struggle with the language of mathematics. This is due to the fact that their formal English is weak, whereas their conversational English is strong. These students don't see themselves as having a problem with English. The ESOL students are more likely to reflect on the fact that they struggle due to a language problem, whereas the native speaking English students don't.

The use of language activities in mathematics classrooms benefits all students, not just those from non-English speaking backgrounds. The strategies employed by many language activities encourage students to communicate in pairs and small groups. By communicating about the mathematics students are required to organise their thoughts, clarify their thinking, and develop cognitive links.

Learning through language activities provide a springboard for developing language skills. Students are engaged in the four macro skills of language. They work in pairs or small groups at times, and discussion and interaction between students is encouraged. The environment is one of risk taking and of sharing. Students are encouraged to support one another and often act as a peer tutor. These types of interactions are powerful tools for the development of language.

Chapter Three Methodology

3.1 Research Method

Action research was the chosen research method. Action research follows a cyclic pattern whereby the researcher and others undertake a process of planning, implementing, observing and reflecting (Kemmis and McTaggart 1990). Of course we all do this to a certain degree in our classrooms, but to do action research is to plan, implement, observe and reflect more carefully and more rigorously than one would normally.

The phases of action research with their cyclic nature are shown in the action research spiral below in Figure 4 (Kemmis and McTaggart).

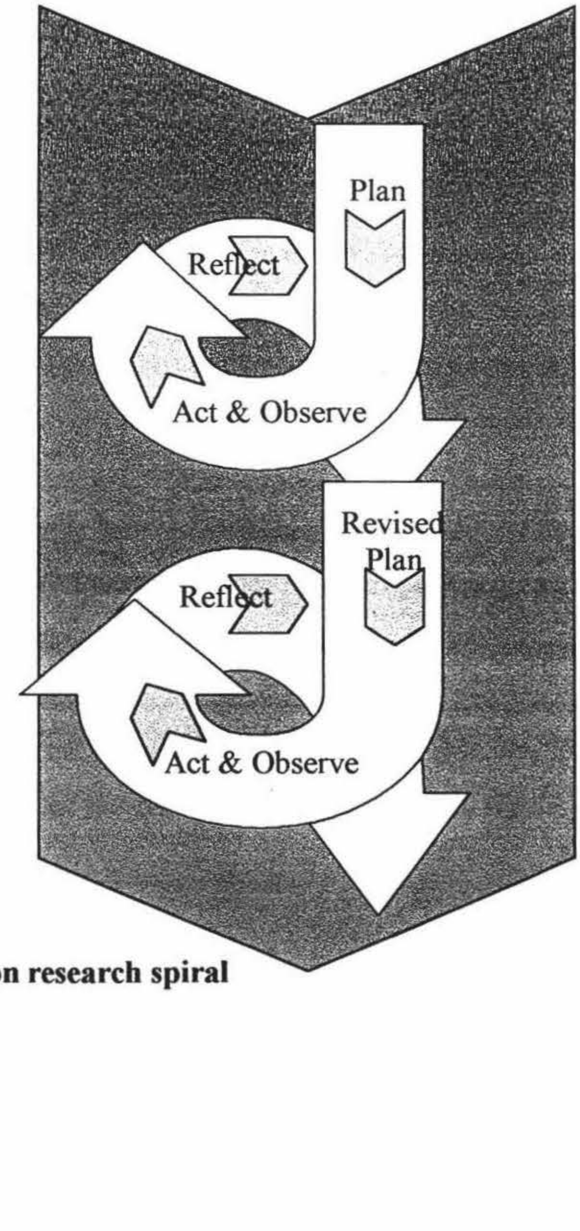


Figure 4: The action research spiral **(Page 8)**

Kemmis and McTaggart describe the plan as critically informed action. Action that must be flexible enough to change and adapt due to unforeseen effects and constraints. The risks of social change and constraints both material and political must be recognised.

The action is deliberate and controlled. It takes place in real time. Real constraints, material and political are encountered. The fact that the action is observed serves to distinguish action research from action in usual situations.

Observation has the function of documenting the action carefully. The observation must be planned to provide a broad base for reflection. The observation must be responsive, open-eyed and open-minded. Observation plans also need to be flexible.

Reflection recalls the action as it was recorded. Discussion among participants is part of the reflection process and is a useful aid. Reflection provides a basis for a revised plan. Reflection is evaluative and requires the action researchers to weigh their experience, decide whether the effects were desirable and then make recommendations for proceeding.

> *"Action research is a dynamic process in which these four moments are to be understood not as static steps, complete in themselves, but rather as moments in the action research spiral of planning, action, observing and reflecting."*
>
> (Kemmis and McTaggart 1990, Page 12)

The research phase in the current research was based on the action research principles described above. The spiral influence of the planning, acting, observing, reflecting, modifying the planning and so forth is described in Figure 5 on the next page.

The research had three cycles or phases to it. Cycle One was the initial trial period, Cycle Two was the first trial of the main unit on algebra, and Cycle Three was a re-trial of the main unit on algebra.

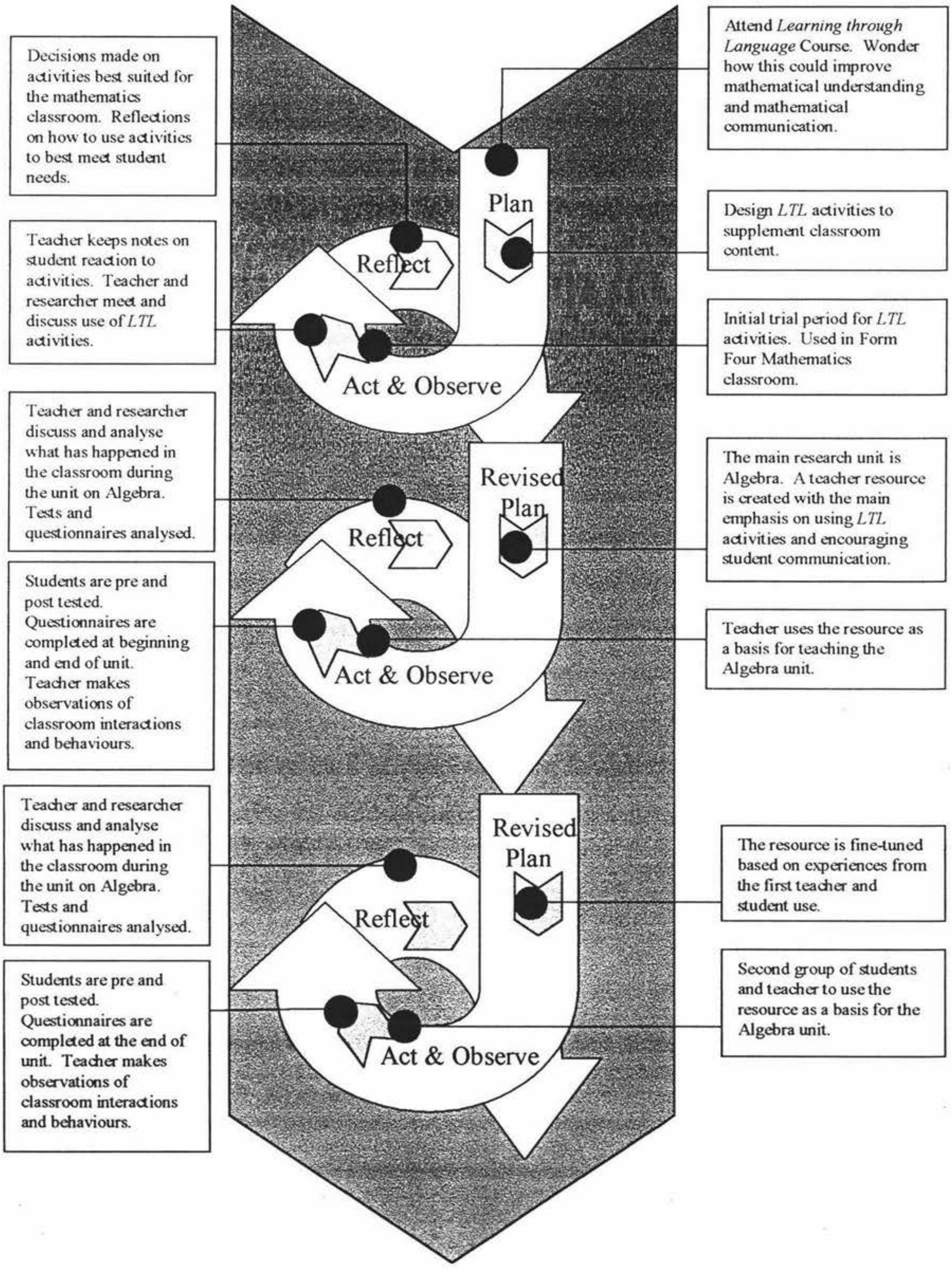


Figure 5: The action research spiral applied to current research

3.2 Internal Validity

3.2.1 Triangulation

Results are validated when using two or more methods of data collection that yield comparable data. This process is called triangulation.

Denzin (1978) broadly defines triangulation as

> *"the combination of methodologies in the study of the same phenomenon."*
>
> (Page 291)

> *Given basic principles of geometry, multiple viewpoints allow for greater accuracy. Similarly, organisational researchers can improve the accuracy of their judgements by collecting different kinds of data bearing on the same phenomenon.* (Jick 1979, Page 602)

In this research study two sources of data collection have been used for each of the two main areas. These two areas are student understanding, and student communication.

Student understanding – The use of pre and post-tests has provided one source of data. The other source of data is from teacher observations.

Student communication – The two sources of data in this area are student questionnaires and teacher observations.

3.3 Sample

The sample consists of the four teachers used in this research. The research required the co-operation of classroom teachers. The objectives were explained to the teachers, and four teachers known to the researcher agreed to participate. The teachers were selected using a convenience sample.

3.3.1 Major Sample

School A

Teacher One: Second year teacher, working in a medium-sized, multicultural, large city, and co-educational school.

School B

Teacher Two: A year eleven teacher, working in a medium sized, multicultural, large city, and single sex girls' school.

Teacher Three: A second year teacher, working in a medium sized, multicultural, large city, and single sex girls' school.

Teacher Four: A year nine teacher, working in a medium sized, multicultural, large city, and single sex girls' school.

3.3.2 *Minor Sample*

All the students in the research phase of the thesis were involved because their teacher was one of the teachers described in the major sample.

Table 2: Student Group A Co-educational Form Four Mathematics class consisting of

Beginning of the Year:	18 students
Beginning of the Algebra Unit:	18 students
End of the Algebra Unit:	21 students

Table 3: Student Group B Single sex girls' Form Four Mathematics classes consisting of

Class and Teacher	Number of students at the beginning of the Algebra Unit	Number of students at the End of the Algebra Unit
Class One: Teacher Two	22	23
Class Two: Teacher Two	27	27
Class Three: Teacher Three	18	19
Class Four: Teacher Three	18	17
Class Five: Teacher Four	27	26
Class Six: Teacher Four	23	23

3.4 *Instruments Used*

3.4.1 Questionnaires

> *"Questionnaires that ask specific questions about aspects of the classroom, curriculum or teaching method are a quick and simple way of obtaining broad and rich information from pupils."* (Hopkins 1985, Page 72).

The questionnaire used in the research thesis was an attitudinal questionnaire. It asked students whether they agreed, disagreed or neither agreed nor disagreed with the statement. The statements were focused on the mathematical communication aspect of the research.

Table 4: Advantages and disadvantages of using questionnaires
(Hopkins 1985, Page 74)

ADVANTAGES	DISADVANTAGES
• easy to administer; quick to fill in	• analysis is time consuming
• easy to follow up	• extensive preparation to get clear and relevant questions • difficult to get questions that explore in depth
• provides direct comparison of groups and individuals	• effectiveness depends very much on reading ability and comprehension of the child
• provides feedback on: attitudes adequacy of resources adequacy of teacher help preparation for next session conclusions at end of term	• children may be fearful of answering candidly • children will try to produce 'right' answers
• data is quantifiable	

3.4.2 Informal Discussions

These were between teacher and researcher. Some of the discussions were audio taped but most of them were not. One of the advantages is that "*specific in depth information*" (Hopkins, 1985) is obtained from discussions. The main disadvantage of this method is that it is time consuming.

3.4.3 Pre and Post Tests

All students doing the algebra unit were pre and post tested on key skills. The key skills tested reflected the learning outcomes expected from the unit of work. This gives a before and after record of the student's key skill ability.

Chapter Four Resource Book Development

4.1 Introduction

When the topic and ideas for this thesis were being developed one of the key goals, aside from the research aspect on understanding and communication, was to develop a resource that would be of use to teachers. One of the desired outcomes was that there would be a resource that could be used again and again (if successful) by teachers. This resource could be used to prepare of a unit of learning, and contribute towards developing mathematical communication – an essential part of the mathematical processes strand.

Instead of just creating ideas for teachers to use in the classroom, the activities were put together to make a cohesive sequence of lessons. The creation of the resource book on Algebra (see Appendix Two) was the outcome of this goal.

4.2 Initial Trial of Activities

The first stage was to find out what type of *LTL* activities best suited the mathematics classroom. Phase One of the in-class research covered this stage. During Term One various *LTL* activities were used with the class in School A.

From this initial trial period we developed a core of activities that worked and discarded ones that didn't. The activities in the first phase were based on the content the teacher was covering at the time. These were a mixture of individual activities and a sequence of activities. Only some of the classroom content objectives were covered using *LTL* activities in this first Term.

4.3 *Algebra Resource Book*

Over the course of the April School Holidays and during the first two of weeks of Term Two the Algebra Resource Book was developed. An approximate outline of the stages of developing the resource book are outlined in Table 2 below:

Table 5: Development stages of resource book

1. Preparing objectives to be taught based on Level 5 objectives from Mathematics in the New Zealand Curriculum.
2. Objectives confirmed by the school.
3. Mapped out teaching sequence including number of lessons and objective(s) to be covered in that lesson.
4. Brainstormed ideas on possible activities. Thought about what *LTL* activities I might use in each part of the unit.
5. Collected together all the resources I had at home and borrowed some textbooks from the school. (These were to use as additional reference.)
6. Prepared the pre-test based on the learning objectives.
7. Repeated a process of creating and writing out activities. Once the activities were drafted out on paper I then typed them up on the computer.
8. With each objective I developed a teaching sequence that included both *LTL* activities and references to other work on algebra. This was complemented with teacher notes on how deliver to the students.
9. Answers for all the *LTL* activities were prepared and included in the teacher notes.
10. Helpful hints, textbook references, and other content to be covered, but not included in the booklet was also included in the teacher notes.
11. Instructions for preparing resources was typed and included at the end of the teacher notes.
12. Produced a post-test with parallel questions to pre-test.
13. Met with the teacher and went through the activities in the book.

4.3.1 Objectives

The agreed objectives for the algebra unit were:

Level 5 – Mathematics in the New Zealand Curriculum (1992, Page 148)

Exploring equations and expressions

Within a range of meaningful contexts, students should be able to:

1. evaluate linear expressions by substitution
2. solve linear equations
3. combine like terms in algebraic expressions
4. simplify algebraic fractions
5. factorise and expand algebraic expressions
6. use equations to represent practical situations

As well as the above content objectives relating to the algebra strand, we were also interested in the communicating mathematical ideas from the Mathematical Processes strand. The objectives that are relevant to this level are:

Communicating Mathematical Ideas

Within a range of meaningful contexts, students should be able to:

1. use their own language, and mathematical language and diagrams, to explain mathematical ideas
2. devise and follow a set of instructions to carry out a mathematical activity
3. record information in ways that are helpful for drawing conclusions and making generalisations
4. report the results of mathematical explorations concisely and coherently (Page 28)

4.3.2 Proposed Teaching Sequence

Table 6: Proposed teaching sequence

What	Description	Lessons – proposed
Pre-test		1
Objective 1	Evaluate linear expressions by substitution	1 - 2
Objective 3	Combine like terms in algebraic expressions	2
Objective 4	Simplify algebraic fractions	2
Objective 5	Expand algebraic expressions	2
Objective 5	Factorise algebraic expressions	1 - 2
Objective 2	Solve linear equations	4
Review		1
Post-test		1

In reality both the teacher at School A and the first teacher at School B spent 6 weeks on the unit.

4.3.3 Brainstorming Session

When I first started to develop the unit I had very little to base my activities on. To increase the pool of ideas I brainstormed what activities I could create and where they might fit into the overall plan. In the following table are some examples from that brainstorming session. The last column in the table indicates whether or not the activity was used in the resource book.

Table 7: Outcomes from brainstorming session

Objective	Activity	Used/Not used
1	Cline. Which is largest/smallest? $a=2$, $b=1$, $c=4$ $a + 2b$, $3a + c$, $a + 3c$, $b + 2c$	Used
3	Cline. $a + 2a$, $3a + 4a - 2a$, $3a + 4a$, $a + a + a + a$ Make up their own to give to their friend	Not used

<table>
<tr><th>Objective</th><th>Activity</th><th>Used/Not used</th></tr>
<tr><td>2</td><td>Picture Matching.
$3y + 6 = 12$ Solve the equation $3y + 6 = 12$
$3y = 6$ Subtract 6 from each side
$y = 2$ Divide each side by 3</td><td>Not used in this form, used as a cloze exercise.</td></tr>
<tr><td>2</td><td>Sequencing.
[$3y = 6$]
[$y = 3$] Put in correct order.
[$3y + 6 = 12$]</td><td>Did not use this sequencing activity. Used other sequencing activities for algebraic fractions and backtracking.</td></tr>
<tr><td>3</td><td>Grids.
<table>
<tr><th></th><th>$2a$</th><th>$4a$</th><th>$5a$</th></tr>
<tr><td>$a + 4a$</td><td></td><td></td><td>✓</td></tr>
<tr><td>$2a + 2a$</td><td></td><td>✓</td><td></td></tr>
<tr><td>$5a + a - 2a$</td><td></td><td>✓</td><td></td></tr>
<tr><td>$5a - a + a$</td><td></td><td></td><td>✓</td></tr>
<tr><td>$3a - 2a + a$</td><td>✓</td><td></td><td></td></tr>
</table></td><td>Used.</td></tr>
<tr><td>All</td><td>Word Finds.
Algebraic, Variable, Expression, Expanding, Factorising, Equation, Simplify</td><td>Used.</td></tr>
<tr><td>2</td><td>Cloze.
3__ + 7 = 13 Three *a* plus seven equals _____
Subtract seven from both sides
3*a* = ___ Three ____ equals six
___ = 2 *a* equals ______</td><td>Used, but not in this form.</td></tr>
</table>

4.3.4 The Final Activities

The table below lists the *LTL* activities used in the final unit. The activities are listed alphabetically based on their given *LTL* name. A description of each activity is given. The algebra objective reference, and where appropriate the communicating mathematical ideas reference is given also.

Table 8: LTL activities used in the Resource Book

LTL Activity	Description	Algebra Objective	Communicating Mathematical Ideas Objective
Clines	Values or words are placed on the cline (sloping line) in order. In this case from smallest to largest.	1	1
Cloze	Students need to fill in gaps of worked problems. They use clues from the surrounding text to help them do this.	2	1
Cooperative Logic	Students share parts of a problem by reading out their clues.	2	3
Definition Activity	Provides an opportunity to explore definitions. Students initially give their own definition, then an opportunity is provided to explore the words.	1 2 3 4 5	1
Grids	Problems are listed down one side of the grid and possible answers across the top. The box that includes both the problem and the correct answer are ticked.	3	1

LTL Activity	Description	Algebra Objective	Communicating Mathematical Ideas Objective
Listening Activities	Bingo	4	
Matching Activities	Matching information from two or more sets of cards.	4	1
Picture Dictation	The teacher reads out a series of instructions or a description of a picture/diagram. Students to draw, or create as the description is read out. An outline may be provided.	3 5	3
Sequencing	Worked examples have been cut up and then mixed up. They need to be put back into order.	2 4 5	1 4 (If report back is used)
Word Hunt	An opportunity to brainstorm meanings of algebraic terms and then to use the word to find other words.		1
Writing to Learn	Students are given the opportunity to write in their own words, using everyday language and are encouraged to find out what they already know – articulate thoughts.	1 2 3 4 5	1

4.3.5 The Book

The resource book developed out of a need to provide more than just the activities for the teacher. It was fine to give the activities to the first teacher and talk through with her about the activities. To talk about things like how the activities should be introduced and what materials and photocopying was needed. But how would the other teachers get on when I was not available to meet with them and do the same?

The solution was to create not only the resources, but also a teacher's guide, or teacher notes to support the activities and the introduction of the ideas. So the resource book was created along with the *LTL* activities. The resource book contains more than just the *LTL* activities. It contains ideas on how to approach the introduction of certain objectives. It contains textbook references. It contains a collection of other activities that are not *LTL* activities, but are activities that are still valuable (researcher's opinion) in the teaching and introduction of the algebraic concepts. It contains ideas that build on the Form Three algebra units that the two schools use.

4.4 Summary

The development of the resource book was an essential part of the research phase of the thesis. It required a pulling together of all the ideas surrounding the use of learning through language activities in a mathematics class and the ideas surrounding algebra at level 5 (Mathematics in the New Zealand Curriculum).

The feedback from teachers on the book has been positive. Their comments and input have helped to create the version presented in *Appendix Two*.

Chapter Five Procedures

5.1 Introduction

The research process was based on the action research spiral. (See Figure 5: The action research spiral applied to current research – page 38.) This action research process had three phases or cycles.

5.2 Phase One
Trial of Learning through Language Activities

People involved: Researcher, Teacher One, and School A Form Four Mathematics Class.

The first phase started by looking at the *Learning through Language (LTL)* course notes and deciding which types of *LTL* activities would be best suited to trial in a mathematics class.

Cloze, Grids, Clines, Matching Activities, Brainstorming, Picture Matching and Co-operative Logic were the *LTL* activities that we decided to use. A description of these activities is in the Literature Review – 2.5 Learning through Language (See *Appendix One* for the activities used in Term One.)

Initially the main teacher and the researcher met and discussed what content objectives were to be covered in this trial period. The mathematics curriculum units covered in this phase were number and measurement. The intention was to supplement the teacher's program and to work within the bounds of the objectives covered by this part of the Form Four course.

The pattern established for this phase was: - researcher develops resource, researcher and teacher discuss the use of the resource in the class, teacher uses resource in the class, researcher and teacher discuss how the resource was received and used by the students and any problems arising.

The resources were prepared usually a week in advance of their being used in the classroom. At each meeting the content of the new resource was decided. This resource was developed over the following week. At the time of making the decision about what resource was needed, we were looking ahead two weeks. This time delay was due to the fact that we only meet weekly. For example: If standard form was to be taught in Week 3, then in Week 1 we would decide that was the topic; before the next meeting the resource was prepared. Week 2 the resource would be handed over to be prepared for teaching in Week 3.

In this phase we were trying to find out which of the *LTL* activities that we had chosen to trial, would be suitable to use in the mathematics classroom, and which of the *LTL* activities were not.

Towards the end of term one, which was the initial trial period, the students were given the questionnaire to complete for the first time. The delay in giving them the questionnaire was a hiccup in the Ethics Committee's handling of the Ethics Proposal for the research thesis. Ultimately, we would have preferred to give them the questionnaire for the first time at the beginning of the year before they had been exposed to any of the *LTL* activities.

By the end of term one we had a clear idea of the types of *LTL* activities that would be suitable for use in the mathematics classroom and the main resource was prepared by the researcher over the April school holidays.

5.3 Phase Two
Main Research Unit on Algebra

People involved: Researcher, Teacher One, and School A Form Four Mathematics Class.

Algebra was chosen as the main research unit. The main reason was that both the researcher and Teacher One felt that algebra was an area that needed development. Student motivation and interest (and often teacher motivation and interest) in this area was generally low.

Algebra is a mathematics strand that is used extensively in other mathematics strands, and extensively in senior mathematics courses. This is often one of the poorest units in terms of student understanding. MacGregor (1991) reports on a number of algebra initiatives and the need to address the issue of low achievement and comprehension of this strand of mathematics. Further comment on this is in 2.6 Algebra.

The content objectives for the unit were based on the Level 5 algebra objectives in Mathematics in the New Zealand Curriculum statement. We focused on the objectives under the sub heading of *Exploring equations and expressions*. As the research questions were also dealing with communication, the Mathematics Processes strand called *Communicating Mathematical Ideas* was also included as a main focus.

With the unit of work written (See *Appendix Two*) Teacher One undertook the second phase. There was much more direction by the researcher in terms of approach and actual classroom activity than in the previous phase. In the first phase only parts of the units of work were covered by *LTL* activities, whereas in this second phase the main emphasis was on using *LTL* activities. The timing of this second phase was the beginning of term two.

There were a number of meetings between the researcher and the teacher to discuss the implementation of the resource. These meetings also provided support and feedback.

Opportunity for clarification was also provided, predominantly through phone conversations.

Before students started the unit on algebra they were pre-tested using the algebra key skills test (See *Appendix Three*). This was a mastery test covering six objectives, with a variation in difficulty for some of the objectives. There were 12 questions in total, with three parts to each question. Students needed to get two or three parts correct to obtain mastery for that skill.

The objectives covered in the key skills test were - combine like terms in algebraic expressions (Questions-1–3); evaluate linear expressions by substitution (Questions-4–5); solve linear equations (Questions 6-9); expand algebraic expressions (Question 10); factorise algebraic expressions (Question 11) and simplify algebraic fractions (Question 12).

At the end of the unit on algebra the students were given a parallel post-test covering the same content objectives and at the same level of difficulty. The students also completed the questionnaire for the second time at the end of the unit.

Once this phase was completed, typing errors and interpretation errors were corrected and the resource was given to other teachers to use in their Form Four Mathematics Classes. These other teachers were at a different school.

Some time was also spent with other Form Four Mathematics Teachers in the first school sharing with them the activities used, and they were invited to use the activities if they wanted to. These teachers are not considered in the research thesis.

5.4 Phase Three

Main Research Unit on Algebra - Second School

People involved: Researcher, Teachers: Two, Three and Four, and School B Form Four Mathematics Classes.

The updated unit of work was given to a group of three teachers at another school. Over the end of term two and term three these teachers have undertaken to use the resource unit on algebra in their Form Four Mathematics Classes. The initial request was for only one teacher to be involved, but as all were keen there seemed to be no point in saying no.

They followed the same process as in phase two with the pre and post-testing. They also used the questionnaire, but this was only given at the end of the unit of work on algebra.

The researcher met with the first teacher at the school to use the resource and discussed the different activities and emphases with that teacher. The first teacher then shared this information with subsequent teachers as they started the unit, and modified some of the instructions and work to be covered to meet the needs of the students at this second school.

Meetings have taken place with the teachers to discuss how they found the resource and to get their reaction on student understanding and the value of the resource as a tool to improve student understanding and communication.

Chapter Six Results

6.1 Introduction

This chapter contains the findings from the pre and post-tests, questionnaires and teacher meetings. These findings represent all the data collected relating to the research.

The results are presented in three parts, which broadly cover:

Student Understanding	Results 6.2
Willingness to Communicate Mathematical Ideas	Results 6.3
Ability to Communicate Mathematical Ideas	Results 6.4

The first section looks at student understanding and is based primarily on the pre and post-tests. This is supplemented with responses from teachers in the meetings. The findings presented focus principally on the mastery attainment in the tests with reference in some cases to the total score (out of 36) achieved. The results are presented as a combination of an overall picture of the seven classes, individual class results, and individual student results where appropriate.

The second section looks at student willingness to communicate mathematical ideas. Findings in this section are based on the student questionnaires and the teacher meetings. The student questionnaires provide the main results with information from the teacher meetings providing supporting evidence.

The third section is on student ability to communicate mathematical ideas. The information presented in this section is based solely on dialogue from the teacher at school A.

Codings used in this relate to either a student or a teacher. Student codings are **S**tudent, School **A**, Class **1**, Student **4**. So SB324 refers to a student at School B in Class Three. They were the 24th student listed in the responses. Similarly for teachers, **T**eacher, School **B**, Teacher **3**. TB2 refers to Teacher Number 2 who teaches at School B.

6.2 Student Understanding

6.2.1 Introduction

All students who were in the seven classes were pre and/or post tested. The total number of students falling into each category is outlined in Table 9 below.

Table 9: Number of students sitting pre and/or post test

Description	Number
Sat both pre and post-test	147
Sat pre test only	6
Sat post test only	9

The findings reported in this section relate to students who sat both the pre and post-tests.

The number of students in each class varies. Table 10 shows the breakdown of student numbers by class.

Table 10: Breakdown of student numbers by class

<table>
<tr><td>School</td><td>A</td><td colspan="6">B</td><td rowspan="2">Total</td></tr>
<tr><td>Class</td><td>1</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>Number of Students</td><td>16</td><td>27</td><td>21</td><td>17</td><td>17</td><td>26</td><td>23</td><td>147</td></tr>
</table>

The overall percentage who mastered each question (pre and post-test) are presented as a combined result of all seven classes. The total mastery score (out of 12) is presented on a class by class basis. Variations on the general trend are presented on an individual basis.

6.2.2 Mastery – Individual Questions

The pre and post-tests had 12 questions. Each question had three parts to it. In order to obtain mastery in a particular question, students needed to get two or three parts correct. See *Appendix Four* for complete breakdown of student test results.

Individual mastery in each question, pre and post-test, has been combined to get an overall result. A comparison in the overall percentage of students mastering each question is shown in Figure 6 below.

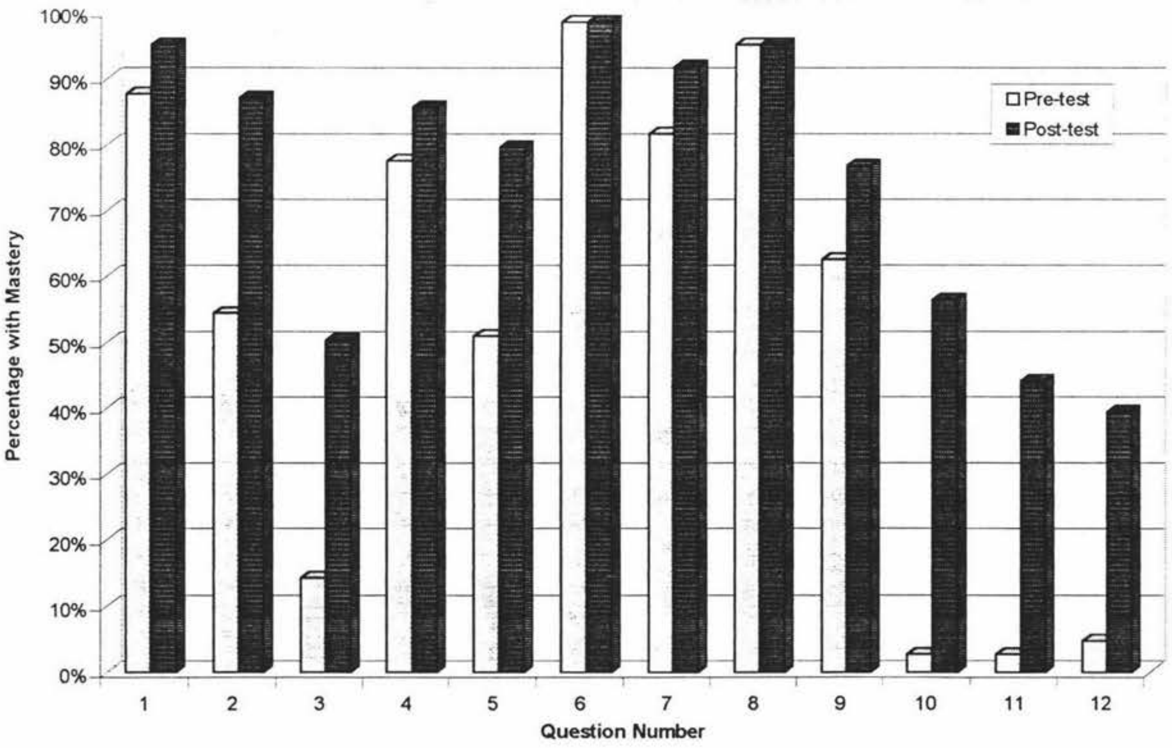


Figure 6: Percentage of students achieving mastery in each question

The percentage of students with mastery in each question improved in ten of the questions attempted. In question six and eight there was no change. The more difficult questions (Questions 3, 10, 11, 12) showed the greatest improvement. (See Table A4.8 in *Appendix Four* for the breakdown by class for each question.)

6.2.3 Total Number of Questions Mastered

The following box and whisker graphs show the movement in number of questions mastered. The results are shown for each class. In every class the overall movement was upwards. In other words students achieved mastery in more questions in the post-test than they did in the pre-test.

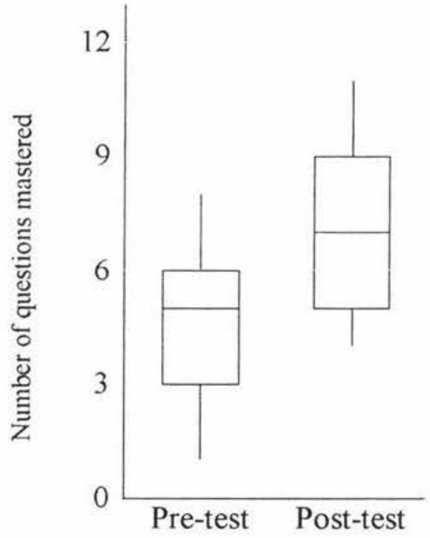


Figure 7: School A Class One

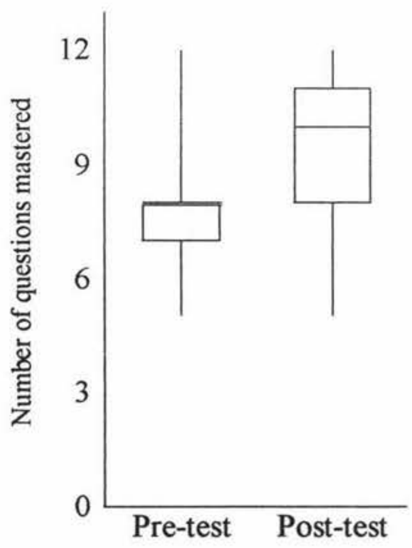


Figure 8: School B, Class One

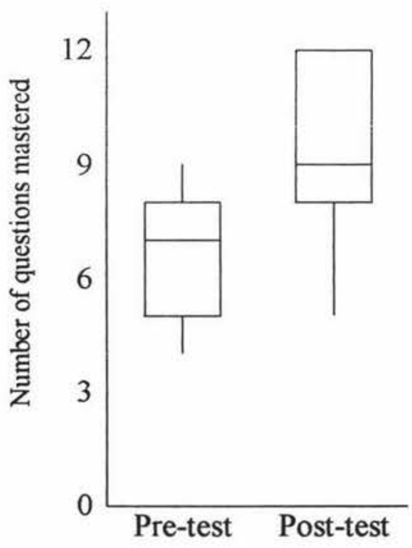
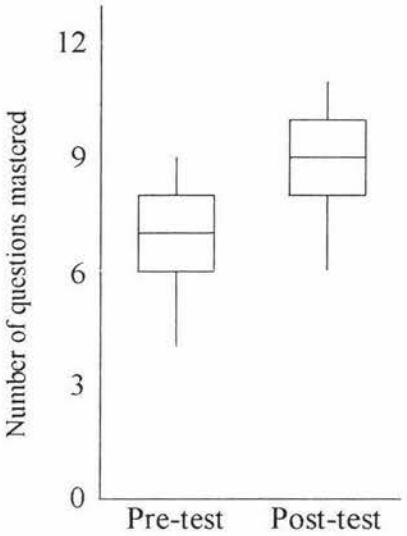


Figure 9: School B, Class Two



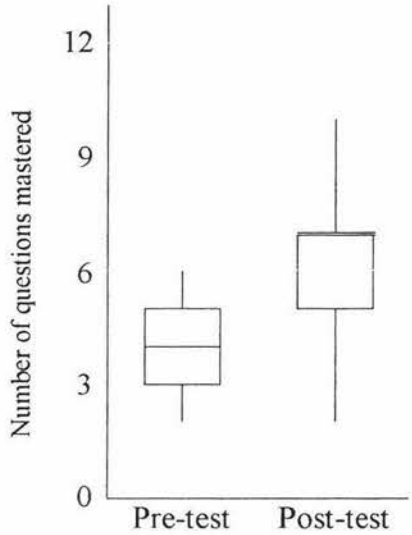
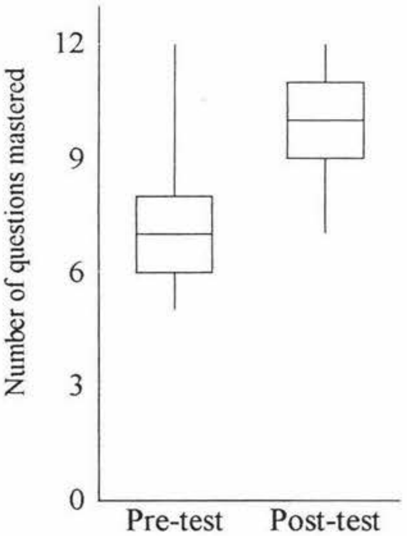


Figure 10: School B, Class Three

Figure 11: School B, Class Four



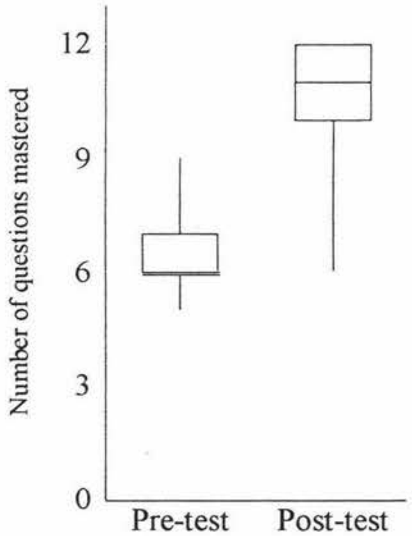


Figure 12: School B, Class Five

Figure 13: School B, Class Six

In Figure 7 (School A) 75% of the students have scored higher in the post-test than the median for the pre-test. Figures 8, 9, 10, 11, 12, and 13 show that at least 75% of the students at School B scored equal or higher in the post-test than the upper quartile of the pre-test for their class.

Figures A4.1 through A4.7 in *Appendix Four* show individual student's pre and post-test mastery.

While the general trend shows that the students increased the number of questions that they mastered from the pre-test to the post-test this was not true for all students.

For five students the number of questions they achieved mastery in dropped. That is their post-test mastery was lower than their pre-test mastery. Their results indicating the changes are outlined in Table 11 below.

Table 11: Students for whom the level of mastery dropped from pre-test to post-test

Student	Variations from Pre to Post test		Questions the same for both pre and post-test	Mastery (/12)	Mark (/36)
SA116	Pre	3	1, 2, 4, 5, 6, 7, 8	8	20
	Post			7	23
SB125	Pre	4, 9	1, 2, 6, 7, 8	7	21
	Post			5	14
SB317	Pre	3, 8, 12	1, 4, 6, 7	7	20
	Post	2, 11		6	20
SB412	Pre	1, 8	6, 7	4	12
	Post			2	12
SB510	Pre	2	1, 4, 5, 6, 7, 8, 9	8	20
	Post			7	23

Two of these five students actually improved their total mark out of 36, even though they dropped in terms of mastery. Two students stayed at the same total mark and one student (SB125) dropped considerably.

Eleven students maintained the same level of mastery. That is they mastered the same number of questions in both the pre-test and the post-test. Of these eleven students seven mastered exactly the same questions in both the pre-test and the post-test (see Table 12).

Table 12: Students who maintained the same level of mastery in the same questions

Student	Questions mastered	Mastery (/12)	Mark (/36)	
SB11	1, 2, 4, 5, 6, 7, 8, 9	8	Pre	21
			Post	24
SB14	1, 2, 4, 5, 6, 7, 8, 9	8	Pre	24
			Post	26
SB15	1, 2, 4, 5, 6, 7, 8, 9	8	Pre	22
			Post	25
SB23	1, 4, 5, 6, 7, 8, 9	7	Pre	20
			Post	21
SB33	1, 2, 4, 5, 6, 7, 8, 9	8	Pre	22
			Post	26
SB119	All questions	12	Pre	33
			Post	36
SB18	All questions	12		36

The other four students had one of the questions that they mastered in the pre test not mastered in the post-test, but mastered another question to maintain the same level of mastery (see Table 13).

Table 13: Students who maintained the same level of mastery, one question different

Student	Variations from Pre to Post test		Questions the same for both pre and post-test	Mastery (/12)	Mark (/36)
SA13	Pre	7	1, 4, 5, 6, 8	6	18
	Post	2			23
SB124	Pre	7	1, 4, 5, 6, 8, 9	7	22
	Post	2			24
SB127	Pre	4	1, 2, 5, 6, 7, 8, 9	8	22
	Post	10			25
SB36	Pre	8	1, 3, 4, 5, 6, 7, 9	8	25
	Post	2			26

All of the students except the one student (SB18) with all 36 parts correct improved their total mark (out of 36) from their pre-test to their post-test.

Thirty students improved their mastery from the pre test to the post-test, but some of the questions mastered in the pre-test were not mastered in the post-test. See Table A4.9 in *Appendix Four* for details on these 30 students. Only one student (SB114) decreased their total mark out of 36. The rest of the group of 30 students increased their total mark.

The remaining 101 students all increased their mastery. They maintained all the questions mastered in the pre-test, and added to this in the post-test. All improved their total score out of 36.

Of the 147 students that sat both the pre and post-tests, two lowered their total score out of 36, three stayed at the same score (one of these scored 36 for both tests) and the remaining 142 improved their total mark.

Student knowledge of key skills has increased from the pre-test to the post-test. One would expect this to happen as a result of their being taught the unit, regardless of the method. It is not possible to state that the improvement from pre to post-test was due solely to the use of *learning through language* activities.

What one can be sure about is that for the vast majority of the students the use of *learning through language* activities was not detrimental to their knowledge of key skills. (Two students lowered their overall score, five lowered their overall mastery, three students maintained their overall score, and ten students maintained their overall mastery.)

6.2.4 Teacher Response to Student Understanding

The general feeling from the teachers involved in teaching the algebra unit was that the students' understanding was better than it has been in previous years. The following responses from the teachers highlight this observation:

TA1 *"They (this year's class) worked out more concepts for themselves (compared to last year's class), they actually thought about it, they didn't just assume what I said was correct."*

TB2 *"The algebra unit feels good in comparison to last year."*

TB3 *(In response to a question on the understanding of the remedial group) "Yeah, although it probably won't reflect in their test marks, as they don't sit tests well."*

"I just found that they seemed to have more of an understanding and I took it really slowly."

(And what about your other group?) "I thought they had heaps of knowledge from third form and I found that they caught on really quick, most of them, others of them struggled a bit and couldn't come to grips with the different concepts of it, but on the whole they picked it up quite well. ... I don't remember my fourth form classes last year having the same sort of knowledge of it. They seemed to pick it up quite well."

6.3 *Willingness to Communicate Mathematical Ideas*

6.3.1 Introduction

How willing are students to communicate about mathematics in the classroom? Does the use of *learning through language* activities increase their willingness to communicate in the mathematics classroom?

The questionnaire (see *Appendix Five*) was used to get student responses in this area. Students at School A were given the questionnaire twice. Once at the end of Term One (before the algebra unit) and then again at the end of the algebra unit. Students at School B were given the questionnaire to complete at the end of the algebra unit.

Nine students from School A completed the questionnaire twice. An analysis of their responses and movement in perspective from the first questionnaire to the second questionnaire is given.

130 students completed the questionnaire at the end of the algebra unit. A breakdown in the number of students for each teacher is given in the table below.

Table 14: Number of students who completed the questionnaire.

	School A	School B		
	Teacher One	Teacher Two	Teacher Three	Teacher Four
Student Numbers	12	44	29	45

The results presented for the questionnaire given at the end of algebra unit focus on the combined results of all four teachers' classes. These findings focus on which statements the students agreed or disagreed with. Teacher comments are also used to provide supporting evidence.

6.3.2 Students Who Completed Two Questionnaires

Has there been a movement in the students' perception of themselves in relationship to the comments stated in the questionnaire? That is has there been a movement from the first questionnaire to the second questionnaire?

The students could respond: agree, neither agree or disagree, or disagree. Movement up was defined as: movements from disagree to neither agree or disagree, or to agree; or a movement from neither agree or disagree to agree. Movement down was defined as: movements from agree to neither agree or disagree, or to disagree; or a movement from neither agree or disagree to disagree.

Some students made no movement at all. If they agreed initially, it was hard for them to move up. Agreed in the first questionnaire, and agreed in the second questionnaire is combined with those who moved up to show the total upward movement. The same applies for disagree. If they disagreed in both questionnaires they were combined with the movement down to get the total number with a downward movement.

The following table summaries the option or category that had the majority of responses. (Table A6.2, *Appendix Six* has the total for each category) Numbers with an * (asterisk) did not have a majority, but had the most, with 4/9 students falling into that category.

Table 15: Summary of statement category

Description	Statement Numbers
Statements with an upward movement. (Student perception moved up, or remained the same at agreed.)	1, 2, 8, 10*, 12, 15, 16, 17, 18, 19, 20, 22*, 27,30
Statements with a downward movement. (Student perception moved down, or remained the same at disagreed.)	7*, 11*, 13, 14, 24, 25*, 26, 28*, 29

6.3.2.1 Statements in the upward movement category

1. In maths I like to copy notes from the blackboard
2. In maths I like to write notes in my book
8. In maths I like to draw diagrams
10. In maths I like to find the rule for a problem
12. In maths I like to solve problems in a group
15. In maths I enjoy using worksheets to do the work.
16. In maths I enjoy using the matching activities.
17. In maths I enjoy doing maths games.
18. In maths I enjoy doing work that involves working with one other person.
19. In maths I enjoy doing work that involves working with a group of people.
20. In maths I enjoy working by myself.
22. I feel confident talking about my ideas in maths to one other student.
27. I feel confident writing about my ideas in maths when I am the only person to see them.
30. I feel confident writing about my ideas in maths when my ideas are asked for in a test.

Statements 12 and 19 relate to group work specifically. Statements 15, 16, 18, and 22 also support group work ideals. Students feeling positive about group work support an increased willingness to communicate.

6.3.2.2 Statements in the downward movement category

7. In maths I like to listen to other students talking
11. In maths I like to solve problems by myself
13. In maths I enjoy doing work from the board.
14. In maths I enjoy using a textbook to do the work.
24. I feel confident talking about my ideas in maths to my maths teacher.
25. I feel confident talking about my ideas in maths to the whole class while seated at my desk.
26. I feel confident talking about my ideas in maths to the whole class in a report back situation from the front of the class.

28. I feel confident writing about my ideas in maths when I am going to show them to another student.
29. I feel confident writing about my ideas in maths when my maths teacher is going to see them.

Students are still not willing to communicate in a high-risk area. That is they are not willing to communicate to the whole class or the teacher. Students are also reluctant to write their ideas down.

Last year I spent some time looking at writing in mathematics classes as a preliminary exercise to doing the research this year. One of the interesting things I noted was that students were happy to tell me what they had done, or how they solved a problem, but they wouldn't write it down.

My conclusion then was that if it was written it was committed to and couldn't be changed. Where as if they voiced the idea, they could then reformulate it if it wasn't right. They could guess and test easier with oral communication than they could/ or would with written communication.

These students indicated that they were not willing either to talk to the teacher or write down ideas that the teacher could read.

6.3.2.3 Summary

The size of the group is really too small to draw any major conclusions from. The statements in the upward movement category reflect a positive alignment with the types of activities used in *learning through language*. This indicates that these activities should have a beneficial effect on their communication. If they enjoy doing the activities and they enjoy working in groups, then the language and conceptual understanding link is more likely to occur than in an environment where this is not happening.

6.3.3 End of Algebra Unit Questionnaire

130 students completed the questionnaire at the end of the algebra unit. This includes the nine students discussed above. The students were taught by four different teachers, and were in seven different classes.

Individual student questionnaire results and a summary of individual teachers' class results can be seen in *Appendix Six*.

Most students agreed with the statements on feeling confident talking about their ideas in maths to one other person, or to a group of students sitting in the same area. They didn't feel strongly one way or the other about talking to their teacher. However the only two statements that collectively the students disagreed with were the two statements where they were to talk about their ideas to the whole class, whether from their seat, or the front of the class.

Again there was strong agreement with the group work ethic. The students agreed with the statements referring to enjoying working with one other person, or with a group of people. They also agreed with the statement that they like to work by themselves.

Students also agreed with the statements I like to solve problems by myself, and I like to solve problems in a group.

One of the teachers also commented that with some of the activities the students wanted to work on their own.

TA1 *Really liked the sequencing, but didn't want to talk about it. Would have preferred to work on their own.*

Students seem to be more willing to communicate verbally about their ideas.

R[#] *"In summary then, your perception is they are more willing to communicate, and most of that is verbal and their verbal communication is improving..."*

TA1 *"And it is not just with each other, they are more willing to communicate with me as well. There were times when they would ask a question, I would ask what is the problem, and then they would say "never mind", and they have stopped doing that."*

The written communication doesn't take too much of a back seat. The students feel confident to write down their ideas if they are the only person to see them (with only 6 students disagreeing with this statement). However in terms of liking to write down their ideas it was mixed. 34 students agreed. 64 students neither agreed nor disagreed, and 32 disagreed.

The teacher from School A talked about the students needing *"heaps more practice on writing"* and also needing good examples of writing.

The emphasis on verbal communication versus written communication came through. The only writing statement where the students were in agreement with was the one where they were the only person to see what they had written.

[#] Researcher

Table 16: Statements that fell into the "agreed" category when most popular option was taken

Statement Number	Statement	Number out of 130 selecting agreed
1.	In maths I like to copy notes from the blackboard.	58
2.	In maths I like to write notes in my book.	79
6.	In maths I like to listen to the teacher talking.	59
7.	In maths I like to listen to other students talking.	54
8.	In maths I like to draw diagrams.	60
10.	In maths I like to find the rule for a problem.	66
11.	In maths I like to solve problems by myself.	67
12.	In maths I like to solve problems in a group.	75
13.	In maths I enjoy doing work from the board.	59
14.	In maths I enjoy using a textbook to do the work.	54
15.	In maths I enjoy using worksheets to do the work.	85
17.	In maths I enjoy doing maths games.	101
18.	In maths I enjoy work that involves working with one other person.	97
19.	In maths I enjoy work that involves working with a group of people.	72
20.	In maths I enjoy working by myself.	60
22.	I feel confident talking about my ideas in maths with one other student.	61
27.	I feel confident writing about my ideas in maths when I am the only person to see them.	79

The students are willing to communicate with one another. Was this due to the use of *learning through language* activities or not?

Teacher responses indicate that students were more willing to communicate after doing the work. The teacher from School A said that similar types of activities were used with their Sixth Form Certificate class and they were now communicating more.

Responses from the teachers are listed below.

TA1 *"They were talking about maths in their own language, at the beginning of the year they weren't talking about maths."*

"They are also confident disagreeing with the teacher and providing support for their answers."

"They are communicating about maths much more than the form four class last year. Students are talking about maths, even if it is not a specific communication activity."

In response to: Does the use of LTL activities improve students' willingness to communicate in the mathematics classroom?

TA1 *"Yes, definitely, and it has continued into the next unit."*

TB3 *"My remedial class doesn't like talking at all. They don't like it, they like to do it all by themselves, because they are quite embarrassed if they get it wrong and that showed through. My other class was quite willing to, every time you asked a question; almost every hand went up. They were quite willing to tell you what they got, and how they got it and how they did it and things like that. Most of the time they didn't use mathematical terms."*

The other two teachers encourage communication in their classes as a matter of course. They were happy that their students were willing to communicate, but this was not unusual.

TB4 *"They were communicating, they do that with all the topics, and they talk amongst themselves. I actually encourage that. They do it in any topic."*

TB2 *"Made an effort with language, but feel I always do anyway."*

6.4 Quality of Mathematical Communication

6.4.1 Introduction

Results in this section on the quality of mathematical communication are very slim. This is due mainly to the method of collecting data. Teacher responses were the only source of data. Teacher One at School A provides the only source of data for this section.

6.4.2 Responses from Teachers

The teacher at School A was very positive regarding the quality of her students mathematical communication. The statement following sums up her feelings.

TA1 *"Three students have shocked me in terms of their response, very confident, words were really mathematical, and they were telling everyone else."*

She also commented on how the students were now communicating about mathematics and how that has continued into the next unit of work.

TA1 *"They were talking about maths in their own language, at the beginning of the year they weren't talking about maths."*

TA1 *"In the new topic that they are doing (Statistics) they are still demonstrating the communication skills developed in the earlier topics."*

Students were also comfortable disagreeing with the teacher.

TA1 *"They are also confident disagreeing with me and providing support for their answers."*

6.5 Summary

The use of LTL activities in Form Four Mathematics classes has helped to increase students' willingness to communicate. It has not been detrimental to their understanding, with most students increasing their performance in the test from the beginning of the unit to the end. The quality of their mathematical communication has improved in the one class where there is evidence.

The findings were based on student pre and post-tests, student questionnaires and teacher observations. 147 students completed both the pre and post-tests. 130 students completed the questionnaire at the end of the algebra unit, and nine students completed the questionnaire before the algebra unit, and at the end of the algebra unit.

Teacher observations from four teachers are included in the findings and provide evidence for all research questions.

Chapter Seven Discussion of Findings

7.1 Student Understanding

What effect does the use of *learning through language* activities have on student understanding? This part of the discussion deals with this question, the first of the research questions. The discussion relates to the results presented in section 6.2 – Student Understanding.

7.1.1 Mastery – Individual Questions

In the pre and post-test there were 12 questions. Of these 12 questions ten showed an increased percentage of students who attained mastery, in that question (See Figure 6, Section 6.2.2, page 59), in the post-test when compared with the pre-test. There was a range of percentage change from the pre to the post-test. The pattern was that the lower the mastery level in the pre-test, the higher the percentage change and the higher the level of mastery in the pre-test, the lower the percentage change to the post-test.

In the other two questions (6 and 8) the percentage of students with mastery remained the same. Questions 6 and 8 both had a high percentage of students with mastery in the pre-test. 98.6% for question 6 and 95.2% for question 8. Given such a high percentage of students who attained mastery initially it is not surprising that this level of mastery remained constant through to the post-test.

Overall there has been an improvement in students' ability to do each question from the pre-test to the post-test. We can say that students can now do skills that they couldn't do at the beginning of the algebra unit.

7.1.2 Total Number of Questions Mastered

The results in section 6.2.3 deal with the total mastery attainment out of the 12 questions. Figures 7 – 13 (pages 60 –61) show that the overall level of mastery has improved.

In School A 75% of the students scored higher in the post-test than the median score in the pre-test. If a "pass" was considered as 6/12 then 69% of the students scored a "pass" mark. In the pre-test only 38% scored a "pass" mark. Figure 7 shows clearly this improvement in mastery for the students from School A.

This pattern of improvement follows through into School B's results. In all the classes in School B 75% or more scored higher in the post-test than the upper quartile mark for the pre-test. Figures 8 – 13 show the results for School B.

As was discussed in section 6.2.3 not all students showed an improvement in mastery. Five students dropped their overall mastery level and eleven students remained at the same mastery level. The remaining 131 students all increased their level of mastery from the pre-test to the post-test.

Student ability to do the skills required in the test has improved from pre-test to post-test. It is not possible to make a statement about their understanding, unless we assume correct response means correct understanding. It is possible to learn a process or procedure without understanding how that process or procedure works.

However there is no evidence that the contrary is true. That is, student's understanding has not improved. It would appear that the use of *learning through language* activities has not been detrimental to student outcomes and has enhanced the students' ability to do the skills required in the algebra unit based on the evidence collected in the pre and post-tests.

7.1.3 Teacher Responses

It is easier to get a picture on student understanding from the teachers' perspective. It should be appreciated that the teachers were not required to find out student understanding by interviewing students, or doing anything extra to what they would normally do in the course of teaching a topic. Comments that are reported in section 6.2.4 provide the basis for this discussion. (Comments in section 6.2.4 relate to teacher observations in a normal classroom setting, with no additional evidence collected by the teacher.)

The general feeling from the teachers was that the level of understanding was higher this year in comparison to last year. Comments such as *"they worked out more concepts for themselves (compared to last year's class) and they actually thought about it"* and *"I just found that they seemed to have more of an understanding"* support this.

While these comments don't conclusively imply that understanding has increased, they at least indicate that there has been a positive movement towards better understanding by the students. The teachers felt there was something different to last year, and it was a positive difference.

7.1.4 Summary

Student understanding has been affected by the use of *learning through language* activities in the classroom. The evidence, while not conclusive, indicates there has been a positive influence on student understanding. Further comment on the link between understanding and the use of language skills is given in the next section, 7.2 Willingness to Communicate Mathematical Ideas.

7.2 Willingness to Communicate Mathematical Ideas

What effect does the use of *learning through language* activities in a mathematics class have on the willingness of students to communicate mathematically?

7.2.1 Introduction

Communicating mathematical ideas is one of the mathematical processes skills emphasised in the mathematics curriculum. It is intended that it will be learnt and assessed within the context of the mathematics strands of algebra, geometry, measurement, number and statistics.

The importance of language and communication has been recognised in all national curriculum statements, not just mathematics.

> *"The close relationship between language and learning has always been acknowledged, albeit usually in lip service only. As the national curriculum documents are discussed and implementation planned, the language implications are beginning to be recognised. Communication is one of the essential skills and most of the documents include communication strands within their achievement objectives."* (Penton 1996, Page 4)

The importance of students' willingness to communicate is not restricted to meeting national curriculum statement requirements. Communication of ideas is linked to conceptual understanding.

> *"The very act of explaining what they do and do not understand will help pupils to consolidate their understanding."*
>
> (Bain 1988, in Hill and Edwards 1991, Page 65)

Discussion in this section relates to results presented in Chapter Six, 6.3. Communicating mathematical ideas is a requirement of the national mathematics curriculum statement and aids in understanding.

7.2.2 Group Work

Students enjoy working in the group situation.

In the questionnaire statements 12 and 19 relate directly to working in a group. Statements 16, 17, 18, 21 and 22 involve interaction with other students and imply group work. A group is defined as two or more students.

Statements 12, 16, 17, 18, 19 and 22 were in the upward movement category for the nine students who completed both questionnaires. Statement 21 was in the neutral category for these students. For these nine students group work was a rewarding experience and they were willing to be involved.

The combined results for all 130 students saw statements 12, 17, 18, 19 and 22 in the agreed category with 16 and 21 in the neutral category. This indicates clearly that overall the students enjoy working in a group.

Students who feel positive about, or agree with group work statements indicate student willingness to communicate.

Miller (1993) said that:

> *"... working in small, collaborative groups gives more students the opportunity to communicate orally and helps them to make the link between language and conceptual understanding."* (Page 315)

Burnett (1992), and Tobin and Fraser (1988) have acknowledged the importance of the small collaborative group as a positive environment for students to communicate in.

Group work also provides an environment in which students can acquire concepts and processes in a meaningful way (Burnett, 1992; Cooper et al, 1993; Hill and Edwards, 1991 and Miller, 1993).

The national mathematics curriculum statement (1992) states that:

> *"Students should be working co-operatively as part of a group by listening attentively, generating ideas, and participating in reflective discussion."*
>
> (Page 29)

The students have agreed with the statements referring to enjoying working with one other person, or with a group of people. They also agreed with the statement that they like to work by themselves.

This is not surprising, as Anne Watson (Plenary Speaker – NZAMT Conference, Palmerston North, 1997) talked about the things we think of as important, as a teacher, and as a learner. Anne said that:

> *"as a teacher I think it is important that students are working in groups to learn, but as a learner I work on my own, possibly sharing later"*

Students also agreed with the statements: "I like to solve problems by myself", and "I like to solve problems in a group".

This was also backed up by one of the teachers who commented that with some of the activities the students wanted to work on their own.

TA1 *"Really liked the sequencing, but didn't want to talk about it. Would have preferred to work on their own."*

Overall the results above indicate a willingness of students to communicate mathematical ideas in the group environment. The use of *learning through language* activities provides opportunities for group work to take place in the classroom.

Bain (1988), Bickmore-Brand (1990) and Burnett (1990) have reported on the importance of using group work to encourage student talk. Talking is considered in the next section.

7.2.3 Talking and Listening

Talking and listening require students to interact with at least one other person. Students who are willing to work in groups are putting themselves in an environment which is conducive to talking and listening. But how willing are the students to talk and listen?

Statements 6 and 7 relate to listening. Statements 9, 21, 22, 23, 24, 25 and 26 relate to talking.

Statements 6 and 7 were both in the agreed category when the results of all 130 students were combined. It is interesting to note that only 19 and 24 students respectively disagreed with these statements. Statement 6 relates to listening to the teacher talking and statement 7 to other students talking.

Generally students are willing to listen. When it comes to talking the students are not as willing to communicate their mathematical ideas.

The only talking statement that was in the agreed category for all 130 students was statement 22 – "I feel confident talking about my ideas in maths with one other student". This other student is most likely to be a friend and therefore "safe". The other talking situations covered by the questionnaire are "risky". There is the opportunity for failure in a very public arena. The situations that are risky are reporting back to the class, talking to a group of students and telling others their ideas.

The two statements with the highest risk factor must be statements 25 and 26. These were the only two statements that the collective group of 130 students placed in the disagreed category. The biggest "loser" was reporting back to the class from the front of the classroom. Of the 130 students 54% do not feel confident reporting back from the front of the classroom.

This finding supports the use of group work as an effective environment for developing mathematical communication, especially oral communication.

The teachers' perception is that the students were more willing to communicate verbally at the end of the research period than they were at the beginning. The following comment by Teacher A support this.

> *"And it is not just with each other, they are more willing to communicate with me as well. There were times when they would ask a question, I would ask what is the problem, and then they would say "never mind", and they have stopped doing that."*

Students are willing to talk about their ideas in maths, but in small secure environments. Harrison (1973) and Ballagh and Moore (1990) refer to students being given the opportunity to develop their understanding through talking with one another, and the teacher. Students who are willing to communicate are also providing themselves with the right environment to improve their understanding.

The teachers feel much more positive about students' willingness to communicate. Comments such as *"They were talking about maths in their own language, at the beginning of the year they weren't talking about maths"* from the teacher at School A. And from teacher three at School B *"They were quite willing to tell you what they got, and how they got it and how they did it and things like that."*

7.2.4 Reading and Writing

In the questionnaire there were five statements that related to writing down ideas. Statements 4, 27, 28, 29 and 30. Students are only confident writing about their ideas in maths if they are the only person to see them (Statement 27).

The students who completed the questionnaire twice are not so confident writing when it will be shown to another student, or shown to the maths teacher. But these same nine students were confident writing about their ideas when asked for in a test situation.

On the other hand these students were happy to write if it wasn't their ideas they were being asked for. This is indicated by agreement with the following statement. In maths I like to ... copy notes from the blackboard, write notes in my book, and draw diagrams. In maths I enjoy ... doing the work from the board, using a textbook to do the work, and using worksheets to do the work.

Students are used to doing certain types of writing in the mathematics class and a concerted effort is required to change both the students and the teachers' perception of what types of writing needs to be done in the maths classroom.

"In maths I like to read". This statement is the only statement relating to reading and had no showing in any of the results. The students do not feel strongly one way or the other about reading in the maths class.

Students are willing to write in the maths classroom, but need to be encouraged to do more writing about their ideas in maths, and more work on recording their results and making written and oral reports. The use of *learning through language* activities will help to develop this type of writing and other language skills. Activities such as *writing to learn* and the other LTL writing activities that are mentioned in the Literature Review, section 2.5.13, page 29.

7.2.5 Summary

The use of *learning through language* activities has had a positive effect on students' willingness to communicate mathematical ideas. Students are more willing to communicate in the mathematics class. They are more willing to participate in group work and other activities that support communication. Student responses to the questionnaire and the teachers' comments support this.

7.3 Quality of Mathematical Communication

What effect does the use of *learning through language* activities in a mathematics classroom have on the quality of the students' mathematical communication?

There has been an improvement in the quality of the mathematical communication in the classroom of the teacher at School A.

> *"Three students have shocked me in terms of their response, very confident, words were really mathematical, and they were telling everyone else."*

> *"In the new topic that they are doing (Statistics) they are still demonstrating the communication skills developed in the earlier topics."*

> *"They were talking about maths in their own language, at the beginning of the year they weren't talking about maths."*

As has been mentioned earlier there is not sufficient evidence to make any conclusive statements about the quality. The three statements above provide some evidence towards concluding that *learning through language* activities are instrumental in improving the quality of the mathematical communication in the classroom.

Chapter Eight Conclusions

8.1 Learning through Language Activities

Learning through language activities have had a positive impact on the areas looked at in this research study. The areas addressed by this study are understanding and mathematical communication.

Student understanding has benefited from the use of *learning through language* activities. The students were more independent in their learning and were prepared to think about what they were doing.

The students showed an increased willingness to communicate in the mathematics classroom. Their willingness to communicate indicates that students are now in a position to consolidate their understanding. The act of explaining what it is they do or do not understand provides a vehicle for the consolidation of concepts.

Group work is also instrumental in providing an environment for communication and understanding. Group work was well received by many of the students. Group work is an essential part of the language process. Small collaborative groups provide endless opportunities for communication and give students the chance to make the link between language and the understanding of concepts (Burnett, 1992; Miller, 1993 and Tobin and Fraser, 1988).

High-risk areas of communication such as talking to the whole class still need developing. Students prefer to communicate in the safety of the small group.

Work is still required on writing in the mathematics classroom. For example, students are willing to write in the context of doing exercises or copying notes from the board. However they are not keen to write when it is their ideas about mathematics they are being asked for. Encouragement and support are needed to make students feel comfortable and therefore they are more prepared to take the risk and write their ideas down.

Mathematics classrooms need to provide opportunities for all four macro language skills to occur: reading, writing, talking and listening.

There is an indication from the research that the quality of student communication has improved with the use of *learning through language* activities. The students are now talking about mathematics using mathematics language. They are also confident in debating an answer with one another and also with the teacher.

One of the beneficial outcomes from the research study has been the increased willingness to communicate in the mathematics classroom. Mathematical processes are an important aspect of the mathematics curriculum (Mathematics in the New Zealand Curriculum, 1992) and communication is one of the three processes covered. The intention of the curriculum is that the mathematical processes will be integrated throughout the five content strands.

Learning through language activities provides an excellent opportunity to use and develop communication skills in the mathematics classroom. The nature of these activities encourages and supports language skills.

The activities used in the research will provide a basis for further development in this area. There is a need to increase the number of secondary teachers in mathematics and other subjects with language skills. *Learning through language* training and other similar courses are invaluable in this regard. In training secondary teachers there is a need to take language issues on board (Halliday 1991 also advocates this – see page 9, Literature Review) and spend time working on language skills for use in the classroom with all trainees.

Primary teachers have the language skills and know how to promote it in the mathematics context (MacGregor, 1993). Secondary mathematics teachers usually have no training in language teaching techniques and are unsure whether it is their job to correct poorly expressed and incoherent English.

The development of resources in this area is time consuming and more than ever there is a need to combine our collective strengths and share what we create. Many initiatives stall before they even get started due to the high demand they place on teacher time. Teachers have little or no discretionary time in the current assessment and administrative paper war.

Learning through language: active learning strategies for the classroom is an initiative that has long term benefits for all our classrooms not just mathematics classrooms. The strategies are beneficial to all students, not just students from Non-English-Speaking-Backgrounds, which there is an increasing number arriving in New Zealand.

8.2 Suggestions for Future Research

8.2.1 Suggestions for Improvement of Current Research

The first area where improvement can be made is in the administration of the questionnaire. The information that I was going to focus on was how students' perspectives changed from before using the language activities to after. School B teachers only gave the questionnaire after the unit was taught. Therefore the information used to analyse all the student responses to the questionnaire was based on the one questionnaire. I was at fault in not expressing clearly to the teachers at School B that I needed students to complete two questionnaires – a before and an after.

The second area also relates to the administration of the questionnaire. Due to an accidental hold up with the Ethics Committee the questionnaire for School A could not be given at the beginning of the year. When these students completed the questionnaire the first time they had already been exposed to *learning through language* activities. Ideally this questionnaire should have been administered at the beginning of the year. School B students should have been given the questionnaire then as well.

Teacher responses need more work. Through no fault of the teachers, the information in the responses provided insufficient evidence to answer all the research questions fully. Evidence collected from the teachers needs to be a combination of informal discussions and formal interviews based on a detailed interview schedule.

The teachers should be interviewed before they start. This will establish the teacher's current perspective on their students' understanding and communication. Once the teachers have completed the unit of work they must be interviewed again to establish their current perspectives on their students' understanding and communication. This would provide a better basis to make comparisons on the movement of students' understanding, willingness to communicate and the quality of communication.

The last area where an improvement could be made is the collection of evidence on student's understanding. Ideally a group of students should be monitored and interviewed individually to ascertain their level of understanding. The interviewing would need to be done before, during and after the unit of work. This one-on-one interaction would provide much more detailed information of the student's understanding.

The improvements suggested are with the benefit of hindsight. Anyone looking to do research in this area may like to reflect on these areas when planning their project.

8.2.2 Questions Arising from Current Research

The following questions arise from this research:

1. What effect does the use of concrete approaches in the mathematics classroom have on student understanding?

When the resource book for algebra was developed it was based on two strategies. *Learning through language* strategies and the *concrete approach to algebra* strategies as outlined in Quinlan et al's (1989) *A Concrete Approach to Algebra*. This research focused primarily on the outcomes from using the *learning through language* activities. It would be worthwhile doing further investigations into the use of concrete activities and methods in other mathematics content areas.

2. What other strategies can be used to develop mathematical communication?

Learning through language activities are beneficial in improving students' willingness to communicate, especially when it comes to oral communication. What other strategies could be used to develop mathematical communication, especially writing mathematical ideas.

3. What are the advantages (and disadvantages) of using *learning through language* strategies as a method of instruction versus using a traditional method of instruction in the mathematics classroom?

In particular are there areas of the mathematics curriculum that would benefit from using *learning through language* strategies and are there areas that would not? This method seems to be effective, but is it better? What hard evidence is there to convince mathematics teachers to move from what they are currently doing?

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Appendix One

Activities used in Term One

WORD HUNT - NUMBER ONE

- *Make as many other words as possible from the letters in the following word.*
- *You cannot use letters that are not in the word.*

ROUNDING

WORD HUNT - NUMBER TWO

- *Make as many other words as possible from the letters in the following word.*
- *You cannot use letters that are not in the word.*

SIGNIFICANT

WORD HUNT - NUMBER THREE

- *Make as many other words as possible from the letters in the following word.*
- *You cannot use letters that are not in the word.*

ESTIMATION

RULES FOR ROUNDING

- *Copy the following information into your book.*
- *Fill in the blanks where you can.*
- *Use the words on the right for the blanks.*

We often use rounded _______ because they are good enough for our purposes. Rounded numbers are ________ numbers.

Rounding to ______ nearest ten

When rounding to the _______ ten, look at the digit in the **one's place**.

Numbers ending in **5 or more** are rounded to the nearest ___ above.		Numbers _______ in **4 or less** are rounded to the nearest ten below.	
number	**_______ number**	**_______**	**rounded number**
35	40	21	20
26	30	42	40
57	60	53	50
38	40	64	60
59	60	73	70
127	130	282	280

___________ to the nearest hundred

When rounding to the nearest ________, look at the digit in the **ten's place**.

Numbers having **5 or** _____ in the **ten's place** are rounded to the nearest hundred above.		Numbers _______ **4 or less** in the **ten's place** are rounded to the nearest hundred _______.	
number	**rounded number**	**number**	**rounded number**
352	400	213	200
268	300	428	400
1570	1600	2539	2500

Rounding to the nearest ___________

When rounding to the nearest thousand, look at the _______ in the _________ **place** and round up or down.

- *See if you can complete the rule for thousands by looking at the previous rules.*

Words for the blanks

ten

rounded

approximate

number

ending

nearest

below

hundred

more

having

digit

Rounding

hundred's

numbers

thousand

the

SIGNIFICANT FIGURES AND DECIMAL PLACES

- *Show how many significant figures and how many decimal places are used in each number in the following table by inserting ticks in the appropriate spaces.*
- *Paste into your maths book when finished.*

	3 significant figures (3sf)	2 significant figures (2sf)	1 significant figure (1sf)	1 decimal place (1dp)	2 decimal places (2dp)	3 decimal places (3dp)
1. 0.627						
2. 0.0815						
3. 2.125						
4. 5.05						
5. 12.3						
6. 0.94						
7. 735						
8. 7.03						
9. 12.01						
10. 0.004						
11. 1000						
12. 929						
13. 0.271						
14. 290						
15. 300						
16. 0.2						

MORE ON SIGNIFICANT FIGURES AND DECIMAL PLACES

- *Copy the following information into your book.*
- *Fill in the blanks (gaps) where* you can.

1. 2.75 has ____ significant figures and 2 ___________ ___________.
2. 13.5 has ____ decimal places and 3 ___________ ___________.
3. 12.29 has 4 ________ ________ and 2 ________ __________.
4. 0.001 has ___ significant figures and ___ decimal places.
5. 0.024 has ___ decimal places and ___ significant figures.
6. 5600 has 0 _________ _________ and 2 _________ _________.
7. 5.2 has ___ decimal places and 2 __________ __________.
8. 800 has ___ significant figures and ____ decimal places.
9. 902 has ___ decimal places and 3 __________ __________.
10. 0.15 has 2 _________ ________ and 2 ___________ __________.

ROUNDING TO ONE DECIMAL PLACE

- *Place ticks in the appropriate spaces, to round the given numbers to one decimal place.*
- *Paste into your maths book when finished.*

		12.3	12.4	12.5
1.	12.29			
2.	12.57			
3.	12.449			
4.	12.377			
5.	12.238			
6.	12.42			
7.	12.339			
8.	12.306			
9.	12.49			
10.	12.549			

ROUNDING TO TWO DECIMAL PLACES

- *Place ticks in the appropriate spaces, to round the given numbers to two decimal places.*
- *Paste into your maths book when finished.*

		0.21	0.22	0.23
1.	0.227			
2.	0.213			
3.	0.2209			
4.	0.202			
5.	0.2388			
6.	0.226			
7.	0.219			
8.	0.2206			
9.	0.2148			
10.	0.2059			

HARDER ROUNDING PRACTICE

- *Round each number to one significant figure, one decimal place and two decimal places.*
- *If the answer to one or all of these is included along the top, tick the boxes along the row to show this.*
- *Paste into your maths book when finished.*

	3	3.0	2.6	2.60	2.5	2.50
1. 2.517						
2. 2.603						
3. 3.259						
4. 3.05						
5. 2.579						
6. 2.593						
7. 2.609						
8. 3.526						
9. 2.98						
10. 2.497						
11. 3.485						
12. 2.596						
13. 2.47						
14. 2.63						
15. 2.6795						

WORD HUNT - NUMBER FOUR

- *Make as many other words as possible from the letters in the following words.*
- *You cannot use letters that are not in the word.*

STANDARD FORM

WORD HUNT - NUMBER FIVE

- *Make as many other words as possible from the letters in the following words.*
- *You cannot use letters that are not in the word.*

POWERS OF TEN

WORD HUNT - NUMBER SIX

- *Make as many other words as possible from the letters in the following word.*
- *You cannot use letters that are not in the word.*

ORDINARY

MEMORY GAME

- *Split the cards into the two different coloured piles. (One pile will be ordinary numbers, the other pile will be standard form numbers.*
- *Turn the cards in each pile upside down and mix each pile up.*

TO PLAY

- *Turn over one card from each pile.*
- *If they are equal (that is the ordinary number stands for the standard form number) take the pair and have another turn.*
- *If they are not equal. Turn them back over and the next person has their turn.*
- *Continue until all the cards have been paired.*
- *THE WINNER IS THE PERSON WITH THE MOST PAIRS.*

5.32×10^{-1}	5.32×10^{0}	5.32	0.246	7.8
7.8×10^{-1}	2.46×10^{0}	5320	0.0078	0.00246
3.79×10^{-1}	8.9×10^{0}	0.0532	0.78	89000
1.01×10^{-1}	3.79×10^{0}	53.2	0.532	0.00089
1.01×10^{-2}	7.8×10^{0}	78	0.379	89
5.32×10^{-2}	8.9×10^{1}	78000	3790	0.101
8.9×10^{-3}	1.01×10^{1}	246	0.0101	101000
3.79×10^{-3}	7.8×10^{1}	2.46	10.1	379
2.46×10^{-3}	5.32×10^{1}	2460	101	0.00379
7.8×10^{-3}	2.46×10^{-1}	8.9	0.0089	3.79
8.9×10^{-4}	2.46×10^{3}			
8.9×10^{4}	5.32×10^{3}			
1.01×10^{5}	2.46×10^{2}			
7.8×10^{4}	1.01×10^{2}			
3.79×10^{3}	3.79×10^{2}			

MATCHING ONE

- *Split the cards into two piles. One pile of ordinary numbers, one pile of standard form numbers.*
- *Match the standard form number with the correct ordinary number.*
- *Order the matched pairs from the largest to the smallest.*

5.32×10^{2}	532
5.32×10^{-4}	0.000532
5.32×10^{0}	5320
5.32×10^{4}	0.532
5.32×10^{1}	53.2
5.32×10^{-2}	0.0532
5.32×10^{3}	532000
5.32×10^{-3}	5.32
5.32×10^{-1}	53200
5.32×10^{5}	0.00532

MATCHING TWO

- *Split the cards into two different coloured piles.*
- *For each pile split the cards into "words" and "numbers".*
- *One person is to read out the "words" one at a time and the other person finds and matches the correct "number".*
- *Do this for both piles. Make sure each person has a turn at reading and choosing.*
- *Match the words and numbers from one pile with the words and numbers from the other pile.*

ten to the power of two	10^6	one hundred thousand	$\frac{1}{10}$
ten to the power of negative one	10^{-1}	ten thousand	100,000
ten to the power of one	10^3	one hundred	1
ten to the power of negative three	10^{-2}	one thousandth	1,000,000
ten to the power of four	10^2	ten	1000
ten to the power of negative two	10^0	one hundredth	10
ten to the power of three	10^4	one thousand	$\frac{1}{100}$
ten to the power of six	10^{-3}	one tenth	100
ten to the power of zero	10^5	one	$\frac{1}{1000}$
ten to the power of five	10^1	one million	10,000

MATCHING THREE

- *Split the cards into THREE different coloured piles.*
- *One pile is the ORDINARY NUMBER.*
- *The other two piles make up the STANDARD FORM NUMBER - the "number" part and the "power of ten" part.*
- *Match three cards to make a correct statement. ORDINARY NUMBER = STANDARD FORM NUMBER*

109500=	3110=	11=	9857=
10.9=	55000=	86000=	1010=
3.95=	24500=	6.35=	370=
620=	910=	700000=	745=
76.5=	585=	5.9=	42=

1.095	3.11	1.1	9.857
1.09	5.5	8.6	1.01
3.95	2.45	6.35	3.7
6.2	9.1	7	7.45
7.65	4.2	5.9	5.85

$\times 10^0$	$\times 10^1$	$\times 10^2$	$\times 10^3$	$\times 10^4$	$\times 10^5$
$\times 10^0$	$\times 10^1$	$\times 10^2$	$\times 10^3$	$\times 10^4$	$\times 10^5$
$\times 10^0$	$\times 10^1$	$\times 10^2$	$\times 10^3$	$\times 10^4$	$\times 10^5$
$\times 10^0$	$\times 10^1$	$\times 10^2$	$\times 10^3$	$\times 10^4$	$\times 10^5$
$\times 10^0$	$\times 10^1$	$\times 10^2$	$\times 10^3$	$\times 10^4$	$\times 10^5$

FILL IN THE BLANKS

- *Complete the standard form number by filling in the "number" part in the following problems.*

1.	547=	__________	x 10^2
2.	1100=	__________	x 10^3
3.	68500=	__________	x 10^4
4.	91.4=	__________	x 10^1
5.	24=	__________	x 10^1
6.	5798=	__________	x 10^3
7.	5.36=	__________	x 10^0
8.	501000=	__________	x 10^5
9.	730=	__________	x 10^2
10.	9.53=	__________	x 10^0

VOLUME - PICTURE MATCHING

- *Students will need their own set of pictures. Teacher needs the list of instructions.*
- *A student to cut out their pictures and place in order as the teacher reads the instructions. - Glue into book in correct order.*
- *Students to read back the text from the pictures. (Get different students to do different pictures)*
- *Students to write the sequence in their own words and these are checked against the original statements.*

TO FIND THE VOLUME OF A TRAPEZIUM SHAPED PRISM (Swimming Pool)

A. Jackie has to find the volume of water in the swimming pool. The swimming pool is deeper at one end than the other. The shape of the pool if you look from the side is a trapezium.

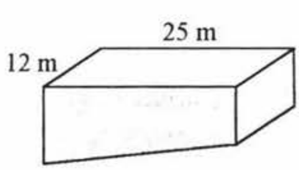
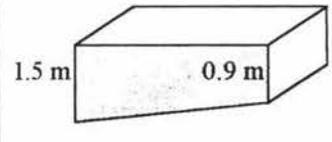
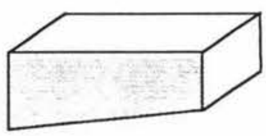
B. The depth of the pool at the deep end is 1.5 m. The depth of the pool at the shallow end is 0.9 m.

C. The pool is 25 m long. The pool is 12 m wide.

D. Jackie knows that the volume of a prism is base area x height.

E. The base shape of the pool is a trapezium. Therefore the base area is half the sum of the deep end plus the shallow end times the length of the pool.

F. The volume of water in the pool is base area times 12 (which is the height of the prism).

1. 	2. **$BA = \frac{1}{2}(1.5 + 0.9) \times 25$ $= 30\ m^2$**	3. 
4. **$V = 30 \times 12$ $= 360\ m^3$**	5. 	6. **$V = BA \times h$**

AREA - PICTURE MATCHING

- *Students will need their own set of pictures. Teacher needs the list of instructions.*
- *A student to cut out their pictures and place in order as the teacher reads the instructions. - Glue into book in correct order.*
- *Students to read back the text from the pictures. (Get different students to do different pictures)*
- *Students to write the sequence in their own words and these are checked against the original statements.*

A. Mau needs to know the area of the front of his house so he can buy the right amount of paint. The front of the house is a pentagon, but it can be broken into a triangle and a rectangle.

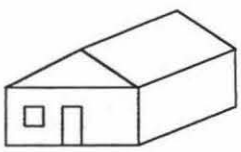
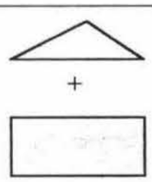
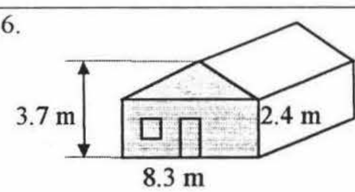
B. The width of the front of the house is 8.3 m. The height of the walls is 2.4 m. The height to the top of the roof is 3.7 m.

C. Mau knows that he needs to find the area of the triangle and the area of the rectangle and add them together to get the total area.

D. Area of the triangle is half x base x height.

E. Area of the rectangle is base x height.

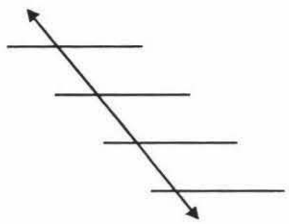
F. Total area is the sum of the triangle area and rectangle area. Total area is 25.3 metres squared to one decimal place.

1. 	2. **Total Area** **= 5.395 + 19.92** **= 25.312m²** **= 25.3 m² (1dp)**	3. **A = ½ x 8.3 x 1.3** **= 5.395m²**
4. 	5. **A = 8.3 x 2.4** **= 19.92m²**	6. 

LENGTH - CLINE

- *Place the measurements on the right on the cline in the correct position.*
- *The longest length is at the top, the shortest at the bottom.*

1.



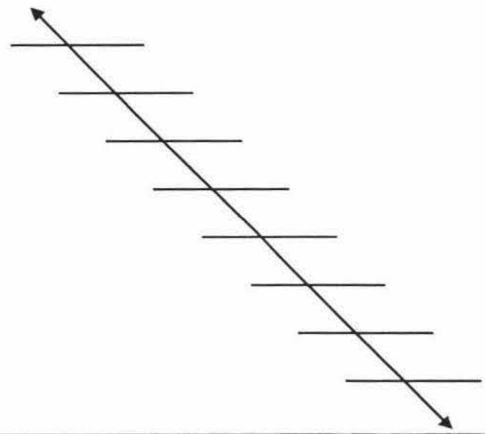
mm

m

cm

km

2.



43 cm

4 m

51 mm

32 mm

3 cm

1030 mm

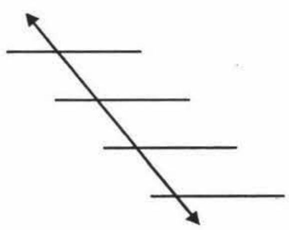
102 cm

1.3 m

AREA - CLINE

- *Place the measurements on the right on the cline in the correct position.*
- *The largest area is at the top, the smallest at the bottom.*

3.



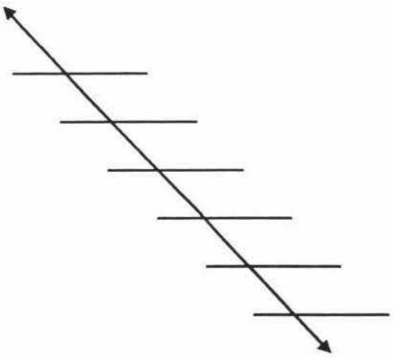
ha

m^2

mm^2

cm^2

4.



$3\ m^2$

$500\ cm^2$

$0.1\ m^2$

$125,000\ mm^2$

$235\ cm^2$

$25000\ mm^2$

CO-OPERATIVE PROBLEM SOLVING
MEASUREMENT

- *Hand out all the clue cards to people in your group face down.*
- *Each person can read what is on his or her clue card to the group, but they may not show their clue card to anyone else.*
- *You may not look at anyone else's card.*
- *Use all the clues to solve the problem.*

MEASUREMENT NO. 1 • *Use your clue to help solve the group's problem. You may read your clue to the group, but don't show anyone else your clue.* • Jackie and Eugene have to find the area of the horse paddock.	MEASUREMENT NO. 1 • *Use your clue to help solve the group's problem. You may read your clue to the group, but don't show anyone else your clue.* • Eugene measures the longest side and finds that it is 52 m long.
MEASUREMENT NO. 1 • *Use your clue to help solve the group's problem. You may read your clue to the group, but don't show anyone else your clue.* • Jackie measures the shortest side and finds that it is half the length of the longest side.	MEASUREMENT NO. 1 • *Use your clue to help solve the group's problem. You may read your clue to the group, but don't show anyone else your clue.* • Jackie and Eugene are members of the Te Atatu Pony Club.

MEASUREMENT NO. 2 • *Use your clue to help solve the group's problem. You may read your clue to the group, but don't show anyone else your clue.* • Mau and Annie are in the fourth form mixed Netball team.	MEASUREMENT NO. 2 • *Use your clue to help solve the group's problem. You may read your clue to the group, but don't show anyone else your clue.* • The length of each third on the Netball court is 10.1 m.
MEASUREMENT NO. 2 • *Use your clue to help solve the group's problem. You may read your clue to the group, but don't show anyone else your clue.* • The width of the Netball court is 15.6 m.	MEASUREMENT NO. 2 • *Use your clue to help solve the group's problem. You may read your clue to the group, but don't show anyone else your clue.* • Find the perimeter of the Netball court.
MEASUREMENT NO. 2 • *Use your clue to help solve the group's problem. You may read your clue to the group, but don't show anyone else your clue.* • The diameter of the centre circle is 1 m.	MEASUREMENT NO. 2 • *Use your clue to help solve the group's problem. You may read your clue to the group, but don't show anyone else your clue.* • Find the circumference of the centre circle. (Use $\Pi = 3.1$)

MEASUREMENT NO. 3	MEASUREMENT NO. 3
• *Use your clue to help solve the group's problem. You may read your clue to the group, but don't show anyone else your clue.* • The cylinder holds exactly 3 tennis balls.	• *Use your clue to help solve the group's problem. You may read your clue to the group, but don't show anyone else your clue.* • Find the volume of the cylinder. Use $\Pi = 3.1$.
MEASUREMENT NO. 3 • *Use your clue to help solve the group's problem. You may read your clue to the group, but don't show anyone else your clue.* • Each tennis ball is 6 cm in diameter.	MEASUREMENT NO. 3 • *Use your clue to help solve the group's problem. You may read your clue to the group, but don't show anyone else your clue.* • The tennis balls fit snugly in the cylinder.

MEASUREMENT	MEASUREMENT
• *Use your clue to help solve the group's problem. You may read your clue to the group, but don't show anyone else your clue.* •	• *Use your clue to help solve the group's problem. You may read your clue to the group, but don't show anyone else your clue.* •
MEASUREMENT • *Use your clue to help solve the group's problem. You may read your clue to the group, but don't show anyone else your clue.* •	MEASUREMENT • *Use your clue to help solve the group's problem. You may read your clue to the group, but don't show anyone else your clue.* •

Appendix Two

Algebra – A Teacher's Resource Book

CONTENTS

MATHEMATICAL CONTENT OBJECTIVES

MiNZC Level	Description	MiNZC Page Ref.
5	**Exploring equations and expressions** *Within a range of meaningful contexts, students should be able to:* • evaluate linear expressions by substitution • solve linear equations • combine like terms in algebraic expressions • simplify algebraic fractions • factorise and expand algebraic expressions • use equations to represent practical situations	**148**

MATHEMATICAL PROCESSES OBJECTIVES

Description	MiNZC Page Ref.
Communicating Mathematical Ideas *Within a range of meaningful contexts, students should be able to:* • use their own language, and mathematical language and diagrams, to explain mathematical ideas • devise and follow a set of instructions to carry out a mathematical activity • record information in ways that are helpful for drawing conclusions and making generalisations • report the results of mathematical explorations concisely and coherently	**28**

From Mathematics in the New Zealand Curriculum (MiNZC), published by Learning Media, Ministry of Education 1992.

TEACHING VOCABULARY - SUGGESTIONS FOR TEACHERS

1. Speak clearly and fairly slowly, using simple language where possible. For students' vocabulary to increase, teachers' vocabulary must decrease.

2. Use consistent language for giving instructions and classroom management. Use stress and pause to emphasise important words.

3. Identify the key vocabulary in a topic and devise appropriate language activities to teach these words. Be aware of words that may present difficulty, even if they are not the specialised words of the subject.

4. Establish what meaning students already have for words and work from the known.

5. Introduce new vocabulary by:
   - using concrete examples;
   - paraphrasing or giving a parallel;
   - using first language;
   - breaking down a word.

6. Reinforce spoken vocabulary by writing it on the blackboard and/or drawing a picture or diagram.

7. Provide many opportunities for students to practise new words. E.g. in different contexts and in all language modes.

8. Help students to develop strategies for working out new words. E.g. using context, dictionaries, glossaries etc.

9. Model a variety of strategies for communicating word meanings. E.g. gesture, mime, simple drawings, diagrams etc. This will encourage students to use similar strategies.

10. Create a supportive environment for exploring words and taking risks with language.

MAKE WORDS FUN!

LTL course notes 2/4

WHAT CAN TEACHERS DO TO IMPROVE LISTENING?

1. Use simple, clear speech and be aware of speed.

2. Plan to speak for a short time only and break up your talking into sections, checking on understanding as you go.

3. Make sure students have a purpose for listening.
   e.g. I'm going to tell you three things about ......;
   I want you to listen carefully to these instructions.

4. Prepare your students for listening by giving an overview, introducing new vocabulary, or setting the context.

5. Sometimes give students specific tasks to do as they listen. e.g. fill in a diagram or write key words.

6. Remember to write key words on the blackboard, especially for NESB students. Use visual aids to support what you say.

7. Train your students to listen to instructions, using these headings: e.g. WHAT to do (action); HOW to do it (manner); WHEN to do it (sequence). Ask students to repeat these points.

8. Give other language activities such as reading, talking or writing to vary and support the listening.

9. Plan for listening as part of your programme and devise ways of assessing it.

LTL course notes 3/6

NOTES ON USE

Materials required: ☺ indicates materials to be photocopied

All photocopiable materials referred to in this section are located at the back in the section photocopiable masters.

References for further practice: I have given references to the textbooks I had access to at the time.

Making Sense With Mathematics, **MSM**, Books **1A, 1B, 2A, 2B**
Its a Mathematical World Book 2 (The original), **MW2**
Complete Mathematics for Fourth Form Students, **Catley**
National Curriculum Mathematics, **NCM**, Level **5**/ Book **1**

The **bold** represents the abbreviations used in this section.

There is also space in the tables in this section for teachers to add any other references they might have. I am aware that these are only some of the texts used.

May be more appropriate for use with Form Three When this is noted at the beginning of the activity, this indicates that the trial teachers felt it was more appropriate to use with Form Three students.

However if you have not had the opportunity to do concrete activities with the students the year before, then it would be beneficial to use this.

If students have used these concrete activities in Form Three then a quick refresher of what was done would be useful.

These activities would also be useful for weaker students.

In the end it is your call. Assess the needs of the students within your class. Some may need the concrete activities for a long time, some will use them only very briefly.

Other comments in (italics) These are comments based on feedback from trial teachers.

WORD HUNT

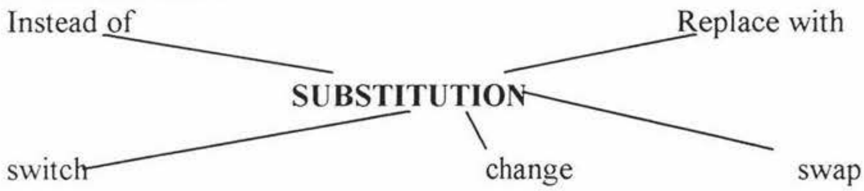
How to:

- For each of the words given students have two tasks.

1. Brainstorm what the word means. They may like to do this in their group, or with their partner.
2. Find as many other words from that word.

Example 1.

BRAINSTORM



WORD FIND

- bus
- bust
- butts
- bit
- button
- sit
- sin
- son
- suit
- stub
- tub
- ton
- tin
- tuition

When to use:

- At the beginning of each new part of the algebra section, especially when using specialist terms.
- Possible terms to use. You decide when to use them.
    - *ALGEBRAIC*
    - *EQUATION*
    - *EXPRESSION*
    - *EXPANDING*
    - *FACTORISING*
    - *SIMPLIFY*
    - *SOLVING*
    - *SUBSTITUTION*
    - *VARIABLE*
    - *and any others you can think of.*
- The word find part could be used as a reward for early finishers. The brainstorming part is important for all students.

SUBSTITUTION

Materials required:

Teacher:
- Prepared OHTs - ☺ 1, 2, 3
- Set of shapes and 1 cm^2 pieces - ☺ 4, 5, 6, 7

Students:
- 1 cm^2 sheets - ☺ 4
- Set of shapes and 1 cm^2 pieces - ☺ 4, 5, 6, 7
- Practice Sheet - ☺ 8

PART ONE: ***(May be more appropriate for use with Form Three)***

Concrete Activities

These activities using concrete activities are based on the work done in Australia by Quinlan et al (1989). For further information on this see *A Concrete Approach to Algebra, Books 1-4.*

In this activity students are counting squares to find the value of the substitution.

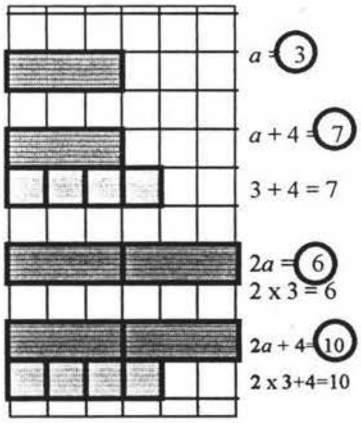
How:

- Let a represent a certain number of square centimetres of area.
- Get each student to choose their own 'a'. It will be 3, 4 or 5.
- Use 1cm^2 to represent numbers. For example 3 will be 3 - 1 cm^2 shapes.

Example 1.

(Use OHT Substitution - 1)

1. Let a cm^2 be the area of your shape.
2. Build and shade a on your grid paper.
   - Repeat for $a + 4$; $2a$; $2a + 4$.
3. Label each drawing to give its area in terms of a, count and write down the number of cm^2 in each of your shaded areas.
4. Compare answers with someone else who used a different value for a.



Example 2.

(Use OHT Substitution - 2)

Repeat procedures as above.

1. $b = 4$;
2. $3b + 2 = 3 \times 4 + 2 = 14$;
3. $3(b + 2) = 3 \times (4 + 2) = 3 \times 6 = 18$

Further

Provide examples for further practice if necessary.

Practice:

c; *c* + 5; 4*c* + 5; 4(*c* + 5); 4(5*c*)

PART TWO:

Calculator Use In this activity students are using the calculator to find the value of the substitution.

How:

1. Build the expression first.
2. Write down the calculator keystrokes.
3. Write down the value on the calculator after every keystroke.
4. Do this for 3 different values of *d*.

Example 1. Build 2(*d* + 5).

(Use OHT Calculator Use - 3)

Keystroke	*d* = 1 Display	*d* = 4 Display	*d* = *(Own choice)* Display
2	(Casio) 2	2	
x	2	2	
(	(	(	
d value	1	4	
+	1	4	
5	5	5	
)	6	9	
=	12	18	

When completing the tables with students ensure they understand that different calculators will give different results. (I often have "Casio" answers in one colour, and "Sharp" answers in another.)

Example 2. Build 2(5*d*).

(Use OHT Calculator Use - 3)

Keystroke	*d* = 2 Display	*d* = 5 Display	*d* = *(Own choice)* Display
2	(Casio) 2	2	
x	2	2	
(	(	(	
5	5	5	
x	5	5	
d value	2	5	
)	10	25	
=	20	50	

Further Practice:

Write the key strokes for the following expressions.

1. $5(c + 2)$
2. $2b + 5$
3. $3a + 2b$
4. $7ab$
5. $2c - 3b$
6. $2a + 3c - 4$
7. $5(a + 2) - 4b$
8. $2c + 3(b - 2)$

Answers:

1.	2.	3.	4.	5.	6.	7.	8.
5	2	3	7	2	2	5	2
x	x	x	x	x	x	x	x
(	*b value*	*a value*	*a value*	*c value*	*a value*	(	*c value*
c value	+	+	x	-	+	*a value*	+
+	5	2	*b value*	3	3	+	3
2	=	x	=	x	x	2	x
)		*b value*		*b value*	*c value*	)	(
=		=		=	-	-	*b value*
					4	4	-
					=	x	2
						b value	)
						=	=

Substitution Practice:

Once this is completed and checked give them values for *a, b, & c.* Encourage students to use the calculator to find the answers.

For example: $a = 2, b = 1, c = 4$.

Answers:

1. 30
2. 7
3. 8
4. 14
5. 5
6. 12
7. 16
8. 5

PART THREE:

Clines — Clines are a Learning through language activity.

How to:

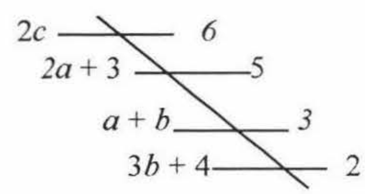
(Use practice sheet Substitution - 8)

A number of expressions are provided. A cline (sloping line) is provided with some values already placed on the axis.

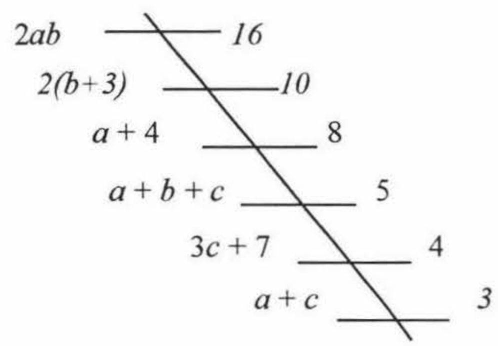
Students work in pairs. Using the given values for *a, b & c*, students need to find the value of the expression. They then place the expressions and their values on the given cline. (Note: not all values, or expressions will need to be placed as some are already on the cline) *The expressions are placed from largest value to smallest value.*

Answers:

A.



B.



References for further practice:

Text	Reference	Page
MW2	Ex 13, 14, 15	60
Catley	Ex 9:16A - 9:16B	230
NCM 5/1	Ex 10:12, 10:16	294

COLLECTING LIKE TERMS

Materials required:

Teacher:
- Prepared OHT - ☺ 9
- Set of shapes - ☺ 10
- $1cm^2$ pieces - ☺ 4, (or beans)

Students:
- Set of shapes - ☺ 10
- $1\ cm^2$ pieces - ☺ 4, (or beans)
- Practice Sheet - ☺ 11

PART ONE: *(May be more appropriate for use with Form Three)*

Concrete Activities

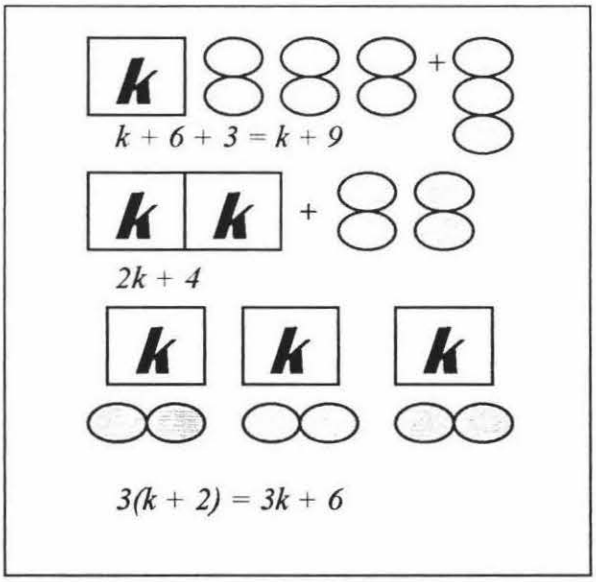
Again building on activities from the book *A Concrete Approach to Algebra.*

How:

- Students are using a shape to represent "*k*" and either $1\ cm^2$ or beans to represent numbers.
- Students build the shapes, add (or subtract) as required and then record the resulting shapes.

Example 1.

(Use OHT Collecting Like Terms - 9)



1. Build $k + 6$.
- Add 3 to this.
- Write in terms of k.

2. Build $2k$.
- Add 4 to this.
- Write in terms of k.

3. Build $k + 2$.
- Triple this.
- Write in terms of k.

The above is an illustration of what might be the outcome of the exercise. Students should be encouraged initially to draw up in their book the concrete example as well as the algebraic representation.

Further Practice:

Do the following without building.

Teacher should read out each problem slowly to allow students time to write the algebraic representation. Allow time to answer problem also.

1. $6k + 2$, add 5 to this
2. Twice $3k + 4$
3. Triple $4k$
4. $k + 2$, add $2k$ to this
5. $3(k + 2)$, add k to this
6. $4(k + 1)$, add 5 to this

Answers:

1. $6k + 2 + 5 = 6k + 7$
2. $2(3k + 4) = 6k + 8$
3. $3(4k) = 12k$
4. $k + 2 + 2k = 3k + 2$
5. $3(k + 2) + k = 3k + 6 + k = 4k + 6$
6. $4(k + 1) + 5 = 4k + 4 + 5 = 4k + 9$

Reverse the problem:

Give students the answer, they make the question.

For example: $3k + 7$ *$= 3k + 4 + 3$;*
$= 3(k + 2) + 1$;
$= 2k + 4 + k + 3$;

Do these ones.

1. $8w + 3$
2. $7g + 8$

PART TWO:

Grids

Grids are a Learning through Language activity.

How to:

Grids consist of a list of problems down one side of the page, with another list (e.g. the answers) across the top. Through discussion, students mark the answers that are correct for each of the problems.

Activity: Students to complete the grids. *(First grid quite easy – you may like to make up more)*

(Use activity - C.L.T. Grids - 11)

Answers:

Simplify the expression and tick the correct box for the answer.	$2a$	$3a$	$4a$	$5a$
$a + 4a$				✓
$2a + 2a$			✓	
$3a - 2a + a$	✓			
$5a + a - 2a$			✓	
$a + a$	✓			
$a + a + a$		✓		
$6a - 2a - a$		✓		
$5a - a + a$				✓
$7a + 2a - 5a$			✓	
$3a - 2a + a$	✓			
		✓		
				✓
			✓	
	✓			

Students to make up questions for the last four. The answer must match the tick.

Answers:

Simplify the expression and tick the correct box for the answer.	$6k + 2$	$5k + 8$	$3k + 4$	$4k + 5$
$3k + 2 + 3k$	✓			
$2k + 2 + k + 2$			✓	
$2(2k) + 5$				✓
$3(k + 1) + 1$			✓	
$2k + 2 + 3k + 6$		✓		
$3k + 3 + 1$			✓	
$2(3k) + 2$	✓			
$2(2k + 2) + 1$				✓
$4k + 2 + 3$				✓
$3k + 8 + 2k$		✓		
$2(3k + 1)$	✓			
$5(k + 1) + 3$		✓		
	✓			
				✓
			✓	
		✓		

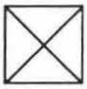
Students to make up questions for the last four. The answer must match the tick.

PART THREE: *(May be more suitable for Form 3 Students)*

Picture Dictation Picture dictation is a Learning through Language activity.

How to: The teacher reads out a series of instructions or a description of a picture/diagram. An outline may be provided.

Instructions:

- Get students to draw a 13 x 7 grid in their book.
- Label the rows 1 - 7. Row 1 is the top row. Row 7 is the bottom row.
- Explain that you are going to give them instructions to draw a Tukutuku panel. Each square in the panel is either blank, or has a cross in it. Demonstrate this.
- Blank square  Square with a cross 
- Read the following instructions out.
    - Row 1. Top row. The instructions are from left to right.
    - The first square on the left is a blank square. The next square is a cross. Four blank squares, then another cross, then 3 blank squares and finally 3 crosses follow this.
    - Row 2. This row starts with a cross, then a blank, then another cross. Two blank squares and three crosses and then three blank squares follow this. This is finished off with a cross and finally a blank square.
    - *(For the last four rows I have just given the symbols. You can make up the words.)*
    - Row 3. 3X + b + 5 X + b + 3X
    - Row 4. 3b + X + 5b + X + 3b
    - Row 5. 3X + b + 5X + 2b + X + b
    - Row 6. X + b + X + 2b + 3X + 2b + 3X
    - Row 7. 3X + b + 5X + 2b + X + b

What the picture should look like:

	X					X				X	X	X
X		X			X	X	X				X	
X	X	X		X	X	X	X	X		X	X	X
			X						X			
X	X	X		X	X	X	X	X			X	
	X				X	X	X			X	X	X
X	X	X		X	X	X	X	X			X	

Further activites:

- Get students to use algebra to describe each row.

- Find the total number of crosses and blanks in each row.

1. 8b + 5x
2. 7b + 6x
3. 2b + 11x
4. 11b + 2 x
5. 4b + 9x
6. 5b + 8x
7. 4 b + 9x

- Find the total number of crosses and blanks in the pattern.
  41b + 50x

- Get students to design own Tukutuku panel. Make it 8 x 5 in size.

- Dictate their panel to their partner using blanks and crosses.

- They then write their panel out using algebra, compare with what their partner has written.

- Swap so each person has a turn at calling out the picture.

Further practice:

Students need to be given the opportunity to practice combining terms with x^2 *&* x; a *&* ab; and other such combinations.

References for further practice:

Text	Reference	Page
MSM 1A	Ch 11, Ex 1	122
MSM 2A	Ch 11, Ex 1	115
MW2	Ex 1, 2, 3	47
Catley	Ex 9:1A - 9:4B	223
NCM 5/1	Ex 10:5, 10:7	283

FACTORS, INDICES, MULTIPLYING & DIVIDING ALGEBRAIC TERMS

Materials required:

Teacher:

- Bingo board - ☺ 12
- Bingo clue cards - ☺ 13
- Factor board - ☺ 14
- Products for factor board - ☺ 15
- Prepared OHT - ☺ 18

Students:

- Factor cards - ☺ 16
- Matching exercise, Powers of a - ☺ 17
- Concrete activity - ☺ 19
- Sequencing exercise, Multiplying - ☺ 20
- Sequencing exercise, Simplifying fractions - ☺ 21

PART ONE:

Factors

Initially reviewing, or developing, the understanding of factors and simple multiplication of numbers and letters.

How to:

- Develop ideas on factors, both numbers and letters.

  FACTOR X FACTOR = PRODUCT

  - If $2a$ is a product, then 2 and a are factors, $2 \times a = 2a$
  - If 30 is a product, then 5 and 6, 10 and 3, 15 and 2, 30 and 1 are factors, $5 \times 6 = 30$, $10 \times 3 = 30$, $15 \times 2 = 30$, $30 \times 1 = 30$
  - If ab is a product, then a and b are factors, $a \times b = ab$
- Revise listing factors of numbers
  - 12 - 12, 1, 6, 2, 4, 3
  - 70 - 70, 1, 10, 7, 14, 5, 35, 2
  - plus other numbers
- List factors of algebraic terms
  - $4a$ - 4, 1, 2, a
  - $5ab$ - 5, 1, a, b
  - $15b$ - 15, 1, 5, 3, b
  - $7a^2b$ - 7, 1, a, b etc.

Practice 1:

Bingo #1:

Listening Activity

Listening is central to learning and language acquisition. (*LTL course notes*)

How to:

(Use Bingo board - 12 and bingo clue cards - 13)

- Get students to select 4 terms from the bingo board and write them in their maths book.
- Shuffle the clue cards.
- Call out clues for the terms slowly (choose only one clue when there is more than one given).
- Place the cards over the terms on the board once they have been called out.
- Students work out the answer, and cross out the term if it is one of the ones they selected.
- First to cross out all four terms calls bingo and wins.
- Check their terms from the board.

Practice 2:

Bingo #2:

Listening Activity

How to:

(Use Factor board - 14 Products for factor board - 15 Factor cards - 16)

- Students each receive a factor card.
- Teacher shuffles product cards and then chooses a card.
- Teacher calls out the products and places them on the factor board.
- Students work out the factors of the product. If there is more than one set of factors then they can only use one set.
- Students cross off the factors on their factor card. They can only cross off one number or letter for each factor. For example: $20b^2$ gives factors of $2 \times 2 \times 5 \times b \times b$ or $4 \times 5 \times b \times b$. Students can therefore cross off two 2s, a 5 and two *b*s; or they can cross off a 4, a 5 and two *b*s.

Cover carefully with an example.

(Use $24ab$ - can be $2 \times 2 \times 2 \times 3 \times a \times b$ or $4 \times 2 \times 3 \times a \times b$, not both) It is also important to note that factors other than 2, 3, 4, 5, 7, *a*, or *b* are not required.

NOTE: They can just cross of some of the factors, they don't have to cross off all the factors for one product. In above example they could cross off 2, 2, *a*, and *b* if that is all they have left on their board.

- The first student to cross off all their factors wins.
- Check their answer using the Factor board (This should be covered with coverseal or similar).
- Get students to call out the biggest numbers first. Cross off factors as they call them out using a non-permanent pen. This allows for easy checking.

PART TWO:

Indices:

Powers of *a*:

Matching exercise:

Students have to match information. Students usually work in pairs.

For example: one student has the words, one has the symbols.
one has the pictures or diagrams, one has the explanations.

How to:

(Use Powers of a - 17)

- Students work in pairs for this activity.
- Hand each pair of students a set of prepared pieces.
- Get students to sort the pieces into those with words, and those with symbols. There will be twice as many symbols as words.
- One student to take the words, the other the symbols.
- The student with the words reads out what is on the cards one at a time.
- The other student initially finds the symbol for the words. e.g.:*a* to the power of negative two *matches* a^{-2}
- Together they can put them in order from the largest power of *a* to the smallest power of *a*.
- Once this is done they can add the last symbols, which is each power of *a* in expanded form.

Rules for Indices:

Concrete Activity:

The ideas developed here could be applied to other rules of indices.

How to:

(Use OHT - 18)

(Students Use a x a - 19)

[a x a] x [a x a x a] a^2 x a^3	• a^2 x a^3 $= a^5$
[a x a] x [a x a x a x a] a^2 x a^4	• a^2 x a^4 $= a^6$
[a] x [a x a] a x a^2	• a x a^2 $= a^3$
[a x a x a] x [a x a] x [a] a^3 x a^2 x a	• a^3 x a^2 x a $= a^6$

Suggested teaching sequence.

- Give students set of "a x a" cards between a group.
- Present the problem.
- Get them to find an a^2 and an a^3 card.
- Draw up on OHT (or build using the "a x a" shapes at the bottom of the OHT sheet).
- What is this if we were to write as a single power of a?
- Count up the a's. There are 5, so it must be a^5. Therefore a^2 x $a^3 = a^5$.
- Work through the remaining examples on the OHT.
- Can anyone see a pattern?
- Explain to class.
- Generalise rule.
- $a^n \times a^m = a^{n+m}$

Further practice:

Problems.	Answers.
1. $a^2 \times a^4 \times a =$	1. a^8
2. $a^2 \times a^2 =$	2. a^4
3. $a^3 \times a^3 =$	3. a^6
4. $a \times a^3 \times a =$	4. a^5
5. $a^4 \times a =$	5. a^5
6. $b^4 \times b^7 =$	6. b^{11}
7. $c^{11} \times c^3 =$	7. c^{14}
8. $d^2 \times d^5 =$	8. d^7
9. $e^8 \times e^2 =$	9. e^{10}
10. $f^{10} \times f^5 =$	10. f^{15}

Cover the rest of the rules for indices.

- $(a^n)^m = a^{n \times m}$
- $a^0 = 1$
- $\frac{a^n}{a^m} = a^{n-m}$
- $\frac{1}{a^m} = a^{-m}$

PART THREE: ***Multiplying number and letters:***

Sequencing: Students are given worked examples that have been split up. They need to put them back in order.

For these ones there is the problem, the working, the answer. Three parts, and there are fifteen problems grouped together.

How to:

(Use Multiplying - 20)

- Students need to group together the three parts for each problem. Once they have the three parts they should put them in the correct order to make a correctly worked problem.
- Start at problem #1 and work through to #15.
- Remind them to always multiply the numbers together and then the letters.

  e.g. $3a \times 2a = 3 \times 2 \times a \times a = 6a^2$

- Students should discuss what they have found out. Encourage them to write what happens in their own words.

PART FOUR: ***Simplifying Algebraic Fractions:***

How to:

(Use Simplifying Algebraic Fractions - 21)

- Similar processes to multiplying except the problems are broken up into four parts and there are only seven problems. (*Start at problem #1 and work through in order.*)
- The problem, expanding factors, removing common factors, answer.
- Remind them to simplify the numbers and letters separately.
- Discuss what they have found. Write what happens in their own words.

References for further practice:

Text	Reference	Page
MSM 1A	Ch 11, Ex 2	125
MW2	Ex 4	48
Catley	Ex 9:5A - 9:14C	225
NCM 5/1	Ex 10:3, 10:9	280

EXPANDING

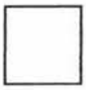
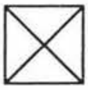
Materials required: *Students:* • Sequencing Activity, Expanding - ☺ 22

PART ONE: *(Trial teachers didn't have time to complete this activity.)*

Picture dictation: Picture dictation is a Learning through Language Activity.

How to: The teacher reads out a series of instructions or a description of a picture/diagram. An outline may be provided.

Instructions:

- Get students to draw a 27 x 4 grid in their book.
- Label the rows 1 - 4. Row 1 is the top row. Row 4 is the bottom row.
- Explain that you are going to give them instructions to draw a Tukutuku panel. Each square in the panel is either blank, or has a cross in it. Demonstrate this.
- Blank square  Square with a cross 
- Read the following instructions out.
    - Row 1. Top row. The instructions are from left to right.
    - The first four squares from the left are blank squares. The next square is a cross. This is followed by 8 blank squares. The next square is a cross. This is followed by 8 blank squares again, a cross and finally 4 blank squares.
    - *(For the next three rows I have just given the symbols. You can make up the words.)*
    - Row 2. 3b + 3X + 6b + 3X + 6b + 3X + 3b
    - Row 3. 2b + 2X + b + 2X + 4b + 2X + b + 2X + 4b + 2X + b + 2X +2b
    - Row 4. b + 2X + 3b + 2X + 2b + 2X + 3b + 2X + 2b + 2X + 3b + 2X + b
- Bring in the algebra.
- Write the total number of blanks and crosses for rows 1 to 4.
    1. 24b + 3X
    2. 18b + 9X
    3. 15b + 12X
    4. 15b + 12X

- Each row has the same pattern repeated three times.
  1. (4b + X + 4b)
  2. (3b + 3X + 3b)
  3. (2b + 2X + b + 2X + 2b)
  4. (b + 2X + 3b + 2X + b)

- Write the base pattern out.

- Total the base pattern.

1. 8b + X
2. 6b + 3X
3. 5b + 4X
4. 5b + 4X

- Each is repeated three times, or tripled.

1. 3(8b + X)
2. 3(6b + 3X)
3. 3(5b + 4X)
4. 3(5b + 4X)

- Expand, 3 lots of 5b is 15b, 3 lots of 4X is 12X, compare with total they started with. Do for all four rows.

1. 3(8b + X) = 3 x 8b + 3 x X
   = 24 b + 3X
2. 3(6b + 3X) = 3 x 6b + 3 x 3X
   = 18b + 9X

3&4. 3(5b + 4X) = 3 x 5b + 3 x 4X
   = 15b + 12X

PART TWO:

Sequencing: Students are given worked examples that have been split up. They need to put them back in order.

For these ones there is the problem, the working, the answer. Three parts, and there are fifteen problems grouped together.

How to:

(Use Expanding - 22)

- Students need to group together the three parts for each problem. Once they have the three parts they should put them in the correct order to make a correctly worked problem. (*Start with problem #1 and work through in order.*)

- Students should discuss what they have found out. Encourage them to write what happens in their own words.

References for further practice:

Text	Reference	Page
MSM 2A	Ch 11, Ex 2, 3	119
MW2	Ex 5 - 7	48

FACTORISING

How to: This section is still baffling me in terms of Learning through Language activities. I am leaving this small section up to you!

References for further practice:

Text	Reference	Page
MSM 2A	Ch 11, Ex 2, 3	119

LINEAR EQUATIONS

Materials required:

Students:
Three bean salads - ☺ 23
Three bean salads extension - ☺ 24
Recording sheets - ☺ 25, 26
Backtracking - ☺ 27, 28
Linear Equations - ☺ 29A - 29D

PART ONE:

Three bean salads

This activity is based on a similar activity from *Family Math* by Jean Stenmark, Virginia Thompson and Ruth Cossey.

How to:

1. Making the salads

(Use Three Bean Salads - 23)

- Each salad has red beans, pinto (brown) beans and haricot (white) beans.
- In groups of no more than three, students share their clues, by reading them to the group, and then make the salad.
- Students record the number of each type of bean needed for each salad. (Use recording sheet 1 - 25)

2. Extending making the salads

(Use Three Bean Salads - Extension - 24)

- Get students to make up the clues for the salads given.
- Get students to make up the salads and the clues.

PART TWO:

Converting the words into symbols.

Using the three bean salads from above the students practice writing the words using algebraic symbols.

How to:

- Let r stand for red beans, let p stand for pinto beans, and let h stand for haricot beans. (They may prefer b for brown and w for white. Ask them.)
- For each problem write three algebraic statements based on the information given.
- Students should be encouraged to work in the same groups again. Each person is responsible for writing the statement for

his or her part of the problem. They share their answers. All must agree on the answer before it is written down. They can hand in a group response, but all are responsible. (Use recording sheet 2 - 26)

Example 1. For salad 1. (Solutions given are from left to right from the master.)

1. $r + p + h = 20$ There are 20 beans in all in this salad.
2. $p = 3$ There are three pinto beans in this salad.
3. $r = 2p$ There is twice as many red beans as pinto beans.

The last one can cause difficulty. Students often want to write it as $2r = p$. It is worthwhile spending a little time this.

Example 2. For salad 10.

1. $r + p + h = 25$ There are 25 beans in all in this salad.
2. $r = p + 5$ There are five more red beans than pinto beans.
3. $h = r + 6$ There are six more haricot beans than red beans.

For all the solutions above any correct rearrangement of the same statement is correct also.

Answers:

2.	1. $r + p + h = 18$	2. $r = 10$	3. $h = r/2$
3.	1. $r = 18$	2. $p = r/6$	3. $r + p + h = 33$
4.	1. $r + p + h = 35$	2. $h = 4$	3. $p = 7h$
5.	1. $r + p + h = 12$	2. $r = 6$	3. $p = r/2$
6.	1. $h = 4$	2. $p + h = r$	3. $r = 2h$
7.	1. $r + p + h = 57$	2. $p = 57/3$	3. $r = p - 17$
8.	1. $r + p + h = 7$	2. $r = 2$	3. $r + h = 5$
9.	1. $h = 3$	2. $r = 2h$	3. $p = r/6$
11.	1. $r + p + h = 30$	2. $h = 6$	3. $r = p$
12.	1. $r + p + h = 14$	2. $r = 3$	3. $p = r + 4$
13.	1. $r + p + h = 24$	2. $p + h = r$	3. $h = r - 3$

14. 1. $r + p + h = 28$ 2. $p = 6h$ 3. $h = 3$

15. 1. $r = 4$ 2. $p = r + 2$ 3. $r + p = h$

16. 1. $r = 2$ 2. $p = 2r$ 3. $h = 2p$

17. 1. $r + p + h = 59$ 2. $h = 7p$ 3. $r = 3p/7$

PART THREE:

Backtracking: Ideas developed from the work on Backtracking from the RIME pack. Reference - A7#7

"Backtracking is an 'insert' before formal processes are developed. It functions to delay formal approaches until the links between an algebraic statement and some for of reality are firmly established, for example:

$$\frac{2(x+3)-4}{2} = 7$$

The lesson attempts to describe the left side of that equation in meaningful terms. In this case it is simply a summary of what happened to an unknown number (the x). The plot is unravelled through 'first-principles', the realm of reality. The answer is then returned to the world of reality."

See the RIME pack for detailed instructions. I am assuming some knowledge of the procedures of backtracking.

How to: THE IDEA

- The teacher invites a class member to write a number on the board. Once the class all know the number it is rubbed off the board. The teacher is not to know the number.
- The teacher (emphasising that the number is unknown to him or her) asks the pupils to perform the following operations on the number, keeping the answers to them.

Example 1.

- **Multiply the number by 3:** Teacher draws up on board:

x 3

Now add 5: The diagram becomes:

x 3 + 5

Now multiply that by 2:

x 3 + 5 x 2

Finally, take away 4:

x 3 + 5 x 2 - 4

- The teacher now asks the class for the final answer and announces that he or she will be able to 'figure out' the starting number. This is done (for those that have never backtracked - take the students answer and work backwards doing the opposite operation - i.e. + 4; divide by 2; - 5; divide by 3 - this should give the starting number). Check with the students that it is correct.

- Repeat this again for another number, and another set of instructions.

Example 2.

- **Multiply by 4; add six; divide by two; subtract one:**

  x 4 + 6 ÷ 2 - 1

- By now someone is usually saying "I know what to do", so let them (this might be one or more students) go outside the room and give the rest of the class another number to use. Get the students back in and work through another example with them.

Example 3.

- **Add 3; multiply by ten; subtract five:**

  + 3 x 10 - 5

- Let them work out what the starting number was.

- You may like to try this a couple of times, as more students click onto what is happening. Make up the operations, be careful of division especially.

- Once a number of students seem comfortable with getting the 'right' answer, talk about the different strategies they are using. Get them to explain to the others in the class how to "backtrack".

- Do a couple more examples on the board, where the teacher knows the answer, and gives the operations, the students work out the teacher's number.

LINK UP WITH ALGEBRAIC EQUATIONS

- Let's go back to the first problem.

Example 1.

- Multiply by 3, add five, multiply by 2, and subtract 4.
- Write this using algebra. Firstly we have an unknown number, let's call that number n;
- What happened first?, x 3 **$3n$**
- Then add five: **$3n + 5$**
- Multiply by two: what? $3n$ or the 5 or all of it? **$2(3n + 5)$**
- Subtract 4: **$2(3n + 5) - 4$**
- Linking up the idea of what happens first to the variable, what happens next and so on. Do with a couple of other examples.

Example 2.

- **Multiply by 4; add six; divide by two; subtract one:** $\frac{4n+6}{2} - 1$

Example 3.

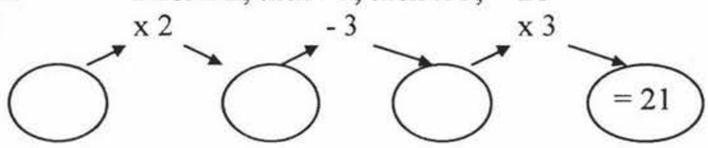
- **Add 3; multiply by ten; subtract five:** $10(n+3) - 5$

FROM THE EQUATION TELL THE STORY OF WHAT HAPPENS TO n

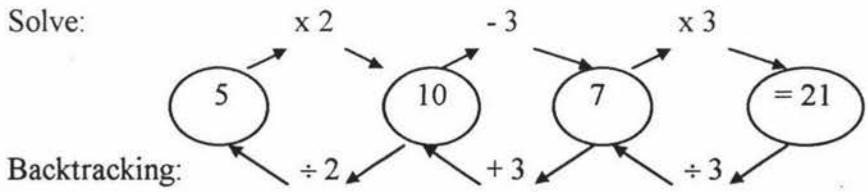
Example 1.

$3(2n - 3) = 21$ First x 2; then - 3; then x 3; = 21

Draw up as:



Solve:



This gives the solution $n = 5$

Practice:

Sequencing Activity:

Students are given worked examples that have been split up. They need to put them back in order.

In these problems there is: the equation to be solved, the variable, the operations that happen to the variable, the answer. There are thirteen problems altogether.

How to:

(Use Sequencing - Backtracking - 27 & 28)

- Students work in pairs. The cards are split up into two parts. The equation and the parts for backtracking.
- Split the equations into two. Put one pile aside for now.
- One student takes the first pile of equations. The other student takes all the parts.
- The student with the equation calls out the parts they need. First the variable: then the operations on the variable in the correct order, and finally the answer.
- The other student finds the parts and puts them down in the order called out. The equation is then placed with the parts. Both students have to agree with the order.
- Continue in this fashion until the first student has finished their pile of equations. Then swap.
- The other student now has the other pile of equations, they call out the variable, the parts and the answer and the first student finds them and places them in order.

Solve the equations:

Once all the equations are put in order with their parts, students can then work together to solve the equations using backtracking.

Answers:

1. $2(x - 3) = 12$	x	- 3	x 2	=12	Ans. 9		
2. $7x + 11 = 25$	x	x 7	+11	= 25	Ans. 2		
3. $\frac{3r+1}{5} = 2$	r	x 3	+ 1	÷ 5	= 2	Ans. 3	
4. $2(z - 3) - 5 = 3$	z	- 3	x 2	- 5	= 3	Ans. 7	
5. $\frac{n}{3} - 1 = 4$	n	÷ 3	- 1	= 4	Ans. 15		
6. $4y - 7 = 5$	y	x 4	- 7	= 5	Ans. 3		
7. $3(2p + 1) = 15$	p	x 2	+ 1	x 3	= 15	Ans. 2	
8. $\frac{r+1}{3} = 8$	r	+ 1	÷ 3	= 8	Ans. 23		
9. $\frac{w}{8} - 1 = 1$	w	÷ 8	- 1	= 1	Ans. 16		
10. $\frac{2r+10}{3} = 6$	r	x 2	+ 10	÷ 3	= 6	Ans. 4	
11. $\frac{3(w-1)}{2} = 12$	w	- 1	x 3	÷ 2	=12	Ans. 9	
12. $3(2z - 1) + 8 = 23$	z	x 2	- 1	x 3	+ 8	= 23	Ans. 3
13. $2(3(x + 1) - 8) = 20$	x	+ 1	x 3	- 8	x 2	= 20	Ans. 5

PART FOUR:

Formalising the Algebra

Time to move onto more formal solving of equations. Link the ideas from backtracking to the formal algebra.

The backtracking method will not work with equations that have variables on both sides of the equals sign.

How to:

Teachers will probably want to cover some example problems on the board to formalise the algebraic process.

Make sure the language used is consistent with the practice cloze exercises that follow.

Practice the process:

Cloze exercises: Cloze exercises have gaps in the text. Students fill in the gaps by using the clues from the surrounding text.

How to:

(Use Solving Equations - 29 A-D)

- Give each student a copy of the prepared task and explain how to fill in the gaps.
- Students should work on their own to fill in the gaps.
- Students are then placed in pairs or small mixed groups to discuss their responses and decide which alternatives are better.
- The teacher then leads a class discussion, using students' responses. Possible alternatives are discussed and useful strategies pointed out.

e.g.. using context to work out an unknown word
using headings and graphics

- Emphasis should be placed on words that make sense and are appropriate rather than the "correct" word.

REMEMBER: A cloze activity is not a test of knowledge; Discussion is an essential part.

References for further practice:

Text	Reference	Page
MSM 1B	Ch 16	14
MSM 2B	Ch 9	92
MW2	Ex 9 -15	50
Catley	Ch 10	238
NCM 5/1	Ex 11:4, 11:5, 11:7, 11:13, 11:14, 11:17, 11:20	308

REVIEW ACTIVITIES

Materials required:

Students: Writing to Learn - ☺ 30
Definition Activity - ☺ 31
Matching Activity - ☺ 32

PART ONE:

Writing to Learn: Students are given the opportunity to write in their own words, using everyday language and are encouraged to:

find out what they already know - articulate thoughts

How to:

(Use Writing to Learn - 30)

- Students write about the algebra topic using the sheet provided.
- Encourage all students to complete the sheet. Hand them in without names, use this as a basis for reviewing the topic.

PART TWO:

Definition Activity: This Learning through Language activity provides the opportunity to explore definitions. Students initially give their own definition, then opportunity is provided to explore the words - in this instance using a matching activity. Students then revise their definitions if they feel they need to.

How to:

(Use definition activity - 30 & matching activity - 31)

- Students fill in the first column headed own definition for each of the algebraic terms. Encourage them to use an example as well to support their words.
- Once they have completed this first part, get them to put their sheets away, and in pairs they work on the definition matching activity. This should provide an opportunity for them to discuss which definition is most suitable, and should help identify words they are still not familiar with.
- Once students have matched up words and definitions in their pairs, get two sets of pairs together, compare and revise their answers. If they don't agree on a definition they need to work on this definition to establish which is the correct one.

During this part of the process I would identify student pairs within the class to provide a definition for each of the words. Let them know before hand that you would like them to provide the definition, and tell them they are right. This way no one is in a vulnerable position when providing the answer.

- Collate all the definitions on the board. Get students to write in the revised definition for the words that they were unsure of.

Appendix Three

Pre and Post Tests

All students who were in the class(es) at the time (be it the beginning, or the end of the unit on Algebra) sat the pre and/or post-tests.

The twelve questions each have three parts. The students needed to get two or three parts to obtain mastery in that question.

The pre and post-tests were based on the achievement objectives for the Algebra unit.

The objective/skill tested in the different questions are:

1. Combining like terms in algebraic expressions – all variables the same.
2. Combining like terms in algebraic expressions – two different variables.
3. Combining like terms in algebraic expressions – same variable, different exponent.
4. Evaluate linear expressions by substitution – all values whole numbers.
5. Evaluate linear expressions by substitution – all values are integers.
6. Solve linear equations – one operation, box in place of number.
7. Solve linear equations – two operations, box in place of number.
8. Solve linear equations – one operation, variable in place of number.
9. Solve linear equations –two operations, variable in place of number.
10. Expand algebraic expressions.
11. Factorise algebraic expressions.
12. Simplify algebraic fractions.

Question 6 tests Level 3 of the curriculum, Question 7 is Level 4 and the remainder test Level 5.

FORM FOUR ALGEBRA PRE-TEST NAME: ________________

Key Skills Write answers in the boxes in the space provided. Use back of sheet for working.

1.	Collect together like terms, (simplify the expression)			
	a. $a + a + a =$	b. $5b + 3b + b =$	c. $5c - 3c + 2c =$	
2.	Collect together like terms, (simplify the expression)			
	a. $a + 3b + 2a =$	b. $5b + 3c + 2c - b =$	c. $6x + 2y - 3x + 2y =$	
3.	Collect together like terms, (simplify the expression)			
	a. $a^2 + 3a + 2a^2 =$	b. $5b^2 + 3b + 2b - b^2 =$	c. $x^2 + 2x + 3x^2 + 2x =$	
4.	By replacing the variable with the correct number find the value of the expression, (evaluate the expression). Use $a = 2,\ b = 7,\ c = 4$			
	a. $a + b + c =$	b. $2b - c =$	c. $5a + 3c =$	
5.	By replacing the variable with the correct number find the value of the expression, (evaluate the expression). Use $a = 5,\ b = -1,\ c = 3$			
	a. $a + b + c =$	b. $2b - c =$	c. $5a - 3c =$	
6.	Place the correct value in the box to make the statement true. (Solve the equation)			
	a. $\square + 5 = 8$	b. $15 - \square = 8$	c. $2 \times \square = 12$	
7.	Place the correct value in the box to make the statement true. (Solve the equation)			
	a. $2 \times \square + 5 = 13$	b. $35 - 3 \times \square = 20$	c. $2 \times \square - 8 = 12$	
8.	Find the value for x, which makes the statement true. (Solve the equation for x.)			
	a. $x + 6 = 13$	b. $15 - x = 3$	c. $2x = 22$	
9.	Find the value for x, which makes the statement true. (Solve the equation for x.)			
	a. $5x + 4 = 14$	b. $5 - 2x = 3$	c. $4x + 12 = 32$	
10.	Expand.			
	a. $3(a+2) =$ __xa+ __x__ = ___a + ___	b. $x(x-3) =$	c. $2d(4+d) =$	
11.	Factorise.			
	a. $3a+9 =$ ___(___+___)	b. $d^2 - 6d =$	c. $2x^2 + 4x =$	
12.	Simplify these expressions.			
	a. $\frac{6a}{a} =$	b. $\frac{x^5}{x^2} =$	c. $\frac{15y^4}{3y^6} =$	

Appendix Four

Test Results

Presented in this Appendix are the raw scores for all students who sat the pre test, and/or the post-test.

Results are presented in tabular form. The table shows for any student the number of parts correct in each question, their total score out of 36, and then their total mastery out of 12. Questions which gained mastery are shaded grey.

Student results are grouped with pre test results on the first row and post-test results on the second row. Where students have sat only one of the tests this is indicated clearly.

Following each class's test results is a graph displaying the pre and post-test mastery.

Presented in this Appendix:

SCHOOL A

Table A4.1: Algebra Test Results School A

Student Number	1	2	3	4	5	6	7	8	9	10	11	12	Total	Mastery
1	1	0	0	0	0	3	2	2	0	0	0	0	8	3
	3	3	0	2	1	2	2	3	0	0	0	0	16	6
2	0	0	0	0	0	2	0	0	0	0	0	0	2	1
	3	0	0	0	0	3	2	3	0	0	0	0	11	4
3	2	0	0	2	3	3	2	2	0	0	0	1	15	6
	3	3	0	2	2	3	1	2	0	1	1	1	19	6
4	3	2	0	3	3	3	2	2	0	0	0	1	19	7
	3	1	1	3	3	3	3	3	0	2	2	0	24	8
5	2	1	0	2	0	2	0	3	3	0	0	1	14	5
	3	2	0	2	2	3	3	3	3	2	1	1	25	9
6	3	2	0	2	0	3	1	2	2	0	0	1	16	6
	3	3	3	2	2	3	2	3	3	1	1	1	27	9
7	2	0	0	0	0	3	1	2	0	0	0	0	8	3
	2	0	0	0	0	3	2	3	0	0	0	1	11	4
8	3	0	0	3	2	3	3	2	1	0	0	0	17	6
	2	3	0	2	2	3	3	3	3	1	0	1	23	8
9	3	3	0	1	0	3	1	1	0	0	0	0	12	3
	3	1	0	1	1	3	2	2	0	1	0	0	14	4
10	3	2	0	2	1	3	2	2	0	0	0	0	15	6
	3	2	2	3	1	3	3	3	1	3	0	2	26	9
11	1	0	0	0	0	0	0	2	0	0	0	0	3	1
	3	2	0	1	1	2	1	2	0	1	0	1	14	4
12	2	2	0	2	1	3	2	2	3	0	0	2	19	8
	3	3	3	3	2	3	3	3	3	2	2	0	30	11
13	3	1	0	3	1	3	3	2	0	0	0	0	16	5
	3	3	2	3	2	3	3	3	2	0	1	0	25	9
14	2	0	0	1	0	3	2	2	0	0	0	0	10	4
	3	2	3	3	3	3	3	3	3	3	3	0	32	11
15	0	1	0	2	1	3	2	3	0	0	0	0	12	4
	1	2	2	1	1	3	2	2	0	0	0	0	14	5
16	3	3	2	2	2	3	3	2	0	0	0	0	20	8
	3	3	0	3	3	3	3	3	1	0	0	1	23	7

Student Number	1	2	3	4	5	6	7	8	9	10	11	12	Total	Mastery
Pre test only														
17	2	3	3	3	2	3	2	2	0	0	0	0	20	8
18	2	0	0	0	0	3	0	1	0	0	0	0	6	2
Post test only														
19	3	2	0	2	1	3	2	3	3	0	0	1	20	7
20	3	0	0	0	0	0	0	0	0	0	0	0	3	1
21	3	3	3	1	1	2	3	2	1	0	0	0	19	6
22	1	1	0	1	2	3	3	2	0	0	0	0	13	4
23	3	2	0	3	3	3	2	3	2	0	0	0	21	8

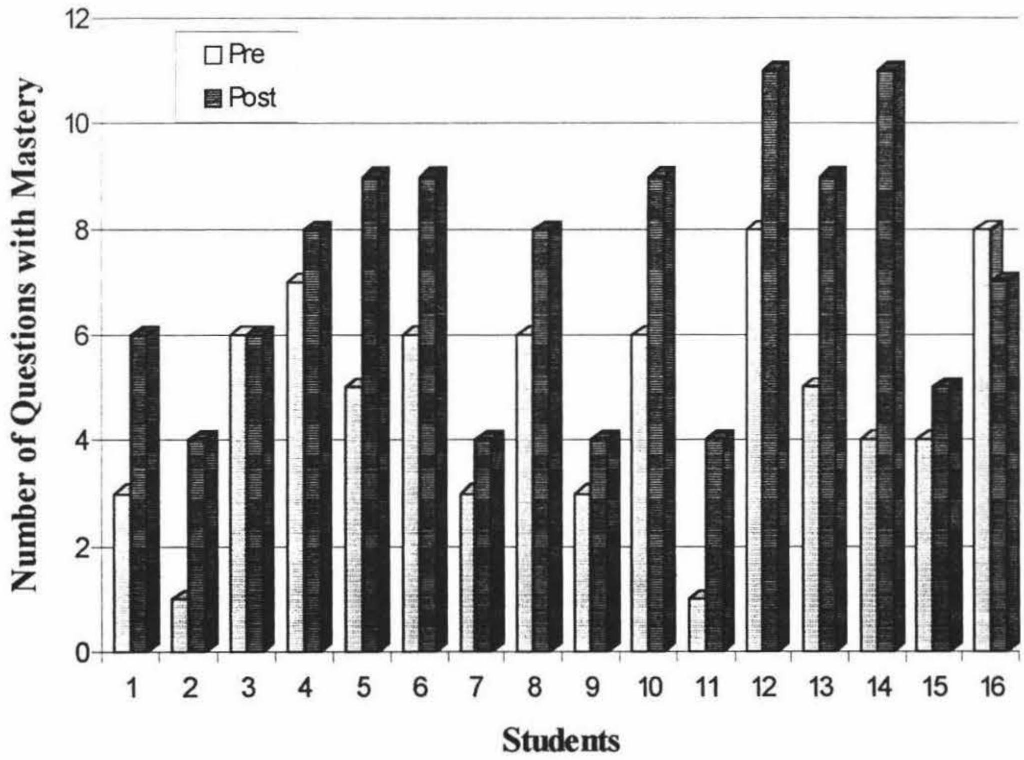


Figure A4.1: Number of questions with mastery. Pre and post test comparison.

SCHOOL B – CLASS ONE

Table A4.2: Algebra Test Results School B, Class One

([illegible])

Student Number	1	2	3	4	5	6	7	8	9	10	11	12	Total	Mastery
1	2	2	0	3	2	3	3	3	3	0	0	0	21	8
	3	2	0	3	3	3	3	3	3	1	0	0	24	8
2	3	2	3	3	3	3	3	3	3	1	0	0	27	9
	3	3	2	3	3	2	3	3	3	3	3	1	32	11
3	3	3	0	2	2	3	1	2	2	0	0	0	18	7
	3	3	0	3	3	3	3	2	1	2	0	0	23	8
4	3	3	1	3	3	3	3	2	3	0	0	0	24	8
	3	3	0	3	3	3	3	3	3	1	0	1	26	8
5	3	3	0	3	2	3	3	2	3	0	0	0	22	8
	3	3	0	3	3	3	3	3	3	1	0	0	25	8
6	3	3	3	3	3	3	3	3	3	1	0	1	29	9
	3	2	3	3	3	3	3	3	3	3	3	3	35	12
7	3	1	0	3	1	3	2	3	3	1	1	0	21	6
	3	3	2	3	3	3	2	3	1	3	2	1	29	10
8	3	3	3	3	3	3	3	3	3	3	3	3	36	12
	3	3	3	3	3	3	3	3	3	3	3	3	36	12
9	3	3	0	3	3	3	3	3	3	1	1	0	26	7
	3	3	3	3	3	3	3	3	2	3	3	2	34	12
10	3	3	1	2	3	3	2	2	3	0	0	1	23	8
	3	3	2	3	3	3	2	3	3	0	0	1	26	9
11	3	2	0	3	3	3	2	3	3	0	1	0	23	8
	3	2	2	2	1	3	2	3	3	1	2	2	26	10
12	3	3	0	3	2	3	2	3	3	0	0	0	22	8
	2	3	0	2	3	3	2	3	3	1	1	1	24	8
13	3	1	0	2	1	3	1	3	2	0	0	1	17	5
	3	3	3	3	3	3	3	3	3	2	2	0	31	11
14	2	3	0	1	3	2	3	3	1	0	0	1	19	6
	2	1	2	2	2	3	2	2	0	1	0	1	18	7
15	2	3	0	3	3	3	3	2	3	1	0	0	23	8
	3	2	3	3	3	3	3	3	3	1	1	1	29	10
16	3	2	0	3	2	3	3	3	0	1	1	0	21	7
	1	2	0	3	2	3	2	3	3	3	2	2	26	10
17	3	3	1	3	3	3	3	3	3	1	0	0	26	7
	3	3	0	3	3	3	3	3	3	2	2	3	31	11
18	1	3	0	3	3	3	3	3	2	0	0	0	21	7
	3	3	1	2	3	3	2	3	3	2	1	1	27	9
19	3	3	3	3	3	3	3	3	2	2	3	2	33	12
	3	3	3	3	3	3	3	3	3	3	3	3	36	12

Student Number	1	2	3	4	5	6	7	8	9	10	11	12	Total	Mastery
20	3	3	0	3	2	3	2	3	3	0	0	0	22	8
	3	3	3	2	2	3	2	3	3	2	2	1	29	11
21	3	3	0	3	2	3	2	3	2	0	0	0	21	8
	3	3	0	3	3	3	3	3	3	3	3	3	33	11
22	3	3	0	0	0	3	3	2	0	0	0	1	15	5
	3	3	3	3	3	3	3	3	2	2	0	2	30	11
23	3	1	0	0	0	3	3	3	3	0	0	0	16	5
	3	3	2	3	3	3	3	3	3	0	2	1	29	10
24	3	1	0	3	3	3	3	3	3	0	0	0	22	7
	3	2	1	3	3	3	1	3	3	1	1	0	24	7
25	3	3	0	2	1	3	3	3	2	1	0	0	21	7
	3	3	0	0	1	3	2	2	0	0	0	0	14	5
26	3	0	0	3	2	3	2	3	1	0	0	0	17	6
	2	3	3	3	3	3	3	3	3	1	0	0	27	9
27	2	3	0	3	3	3	3	3	2	0	0	0	22	8
	2	3	0	1	3	3	3	3	3	2	1	1	25	8

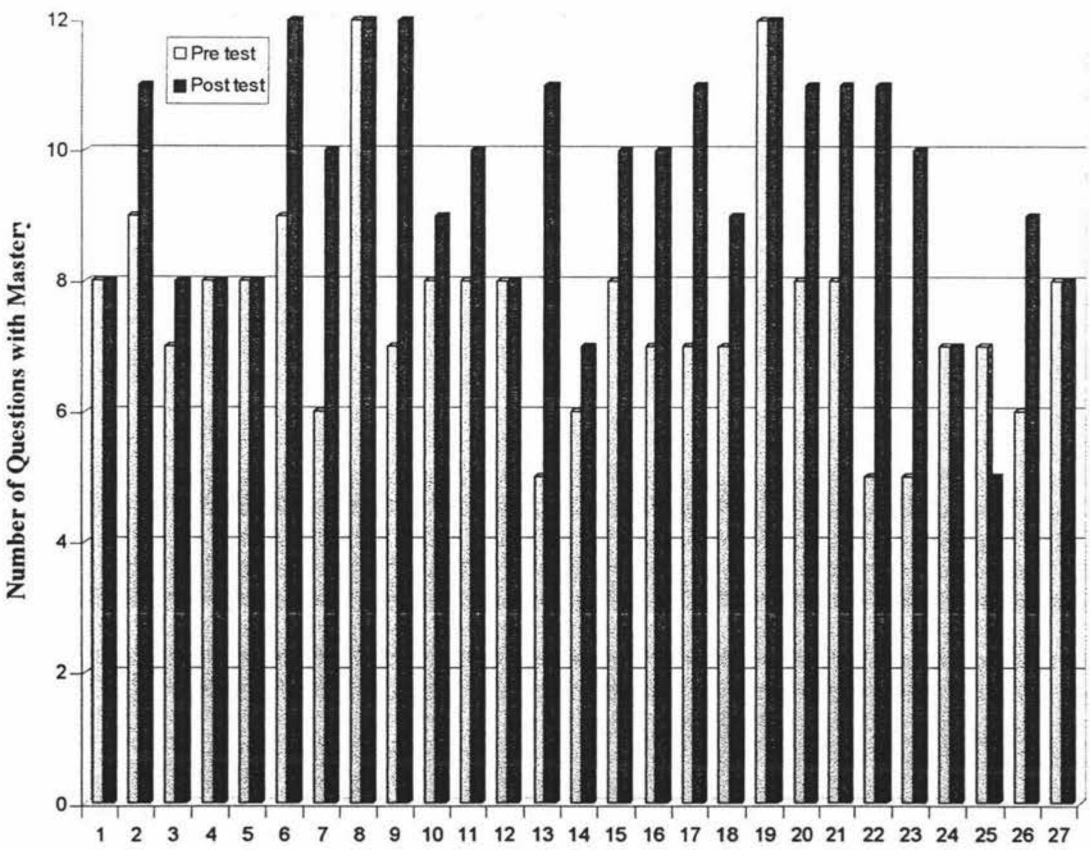


Figure A4.2: Number of questions with mastery. Pre and post test comparison.

SCHOOL B – CLASS TWO

Table A4.3: Algebra Test Results School B, Class Two

([illegible])

Student Number	1	2	3	4	5	6	7	8	9	10	11	12	Total	Mastery
1	3	0	0	3	2	3	3	3	3	0	0	0	20	7
	3	3	3	2	3	3	3	3	3	3	3	2	34	12
2	2	0	0	3	3	3	3	3	2	1	1	0	21	7
	3	2	0	3	1	3	3	3	2	1	2	3	26	9
3	3	0	0	2	3	3	2	2	3	1	0	1	20	7
	2	1	0	3	3	3	2	3	3	1	0	0	21	7
4	2	1	0	2	0	3	3	3	3	0	0	0	17	6
	3	3	0	3	1	3	3	3	3	2	0	0	24	8
5	3	3	0	3	3	3	2	2	0	0	0	0	19	7
	3	3	3	2	3	3	3	2	3	3	2	3	33	12
6	2	0	0	1	1	3	3	3	0	0	0	0	13	4
	2	1	0	0	2	3	3	3	0	0	0	0	14	5
7	3	3	0	3	2	3	3	3	3	0	0	0	23	8
	3	3	3	3	3	3	3	3	3	1	0	0	28	9
8	3	1	3	3	3	3	3	3	3	2	1	1	29	9
	3	3	3	3	3	3	3	3	3	3	2	1	33	11
9	3	0	0	1	2	3	3	3	2	0	0	0	17	6
	0	2	0	3	3	3	3	3	3	3	1	0	24	8
10	2	0	0	3	1	3	2	1	0	0	0	1	13	4
	3	2	0	3	3	3	1	3	3	1	0	1	23	7
11	1	1	0	3	1	3	3	3	3	0	0	0	18	5
	3	3	3	3	2	3	3	3	3	0	1	0	27	9
12	3	1	0	3	1	3	2	3	0	0	0	0	16	5
	3	2	3	3	3	3	2	3	3	1	1	0	27	9
13	2	0	3	3	3	2	3	2	3	1	0	0	22	8
	3	3	3	3	3	3	3	3	2	3	2	2	33	12
14	3	1	0	3	1	2	3	2	0	0	0	0	15	5
	3	3	0	3	2	3	3	2	2	2	1	1	25	9
15	0	0	0	3	3	3	3	3	3	0	0	2	20	7
	3	3	3	3	2	3	3	3	2	3	3	3	34	12
16	3	2	0	3	3	3	3	3	3	0	0	0	23	8
	3	3	2	3	3	3	3	3	3	2	2	2	32	12
17	3	3	0	3	3	3	2	2	0	0	0	0	19	7
	3	3	0	3	3	2	2	2	3	2	1	1	25	9
18	3	3	0	3	1	3	2	2	0	1	0	0	18	6
	3	3	0	3	3	3	3	3	3	3	2	1	30	10
19	3	3	0	3	2	3	2	3	3	1	1	0	24	8
	3	3	2	3	2	3	3	3	2	2	3	2	31	12

Student Number	1	2	3	4	5	6	7	8	9	10	11	12	Total	Mastery
20	3	2	0	3	2	3	1	2	0	0	0	1	17	6
	3	3	1	3	3	3	1	3	3	3	3	2	31	10
21	3	0	0	0	0	3	2	3	3	0	0	0	14	5
	2	2	0	0	0	3	3	3	3	1	0	2	19	7
Pre test only														
22	3	2	0	3	1	3	3	3	3	1	1	0	23	7
Post test only														
23	3	3	3	3	3	3	2	2	3	1	1	1	28	9
24	3	3	2	3	3	2	3	3	3	2	3	2	32	12

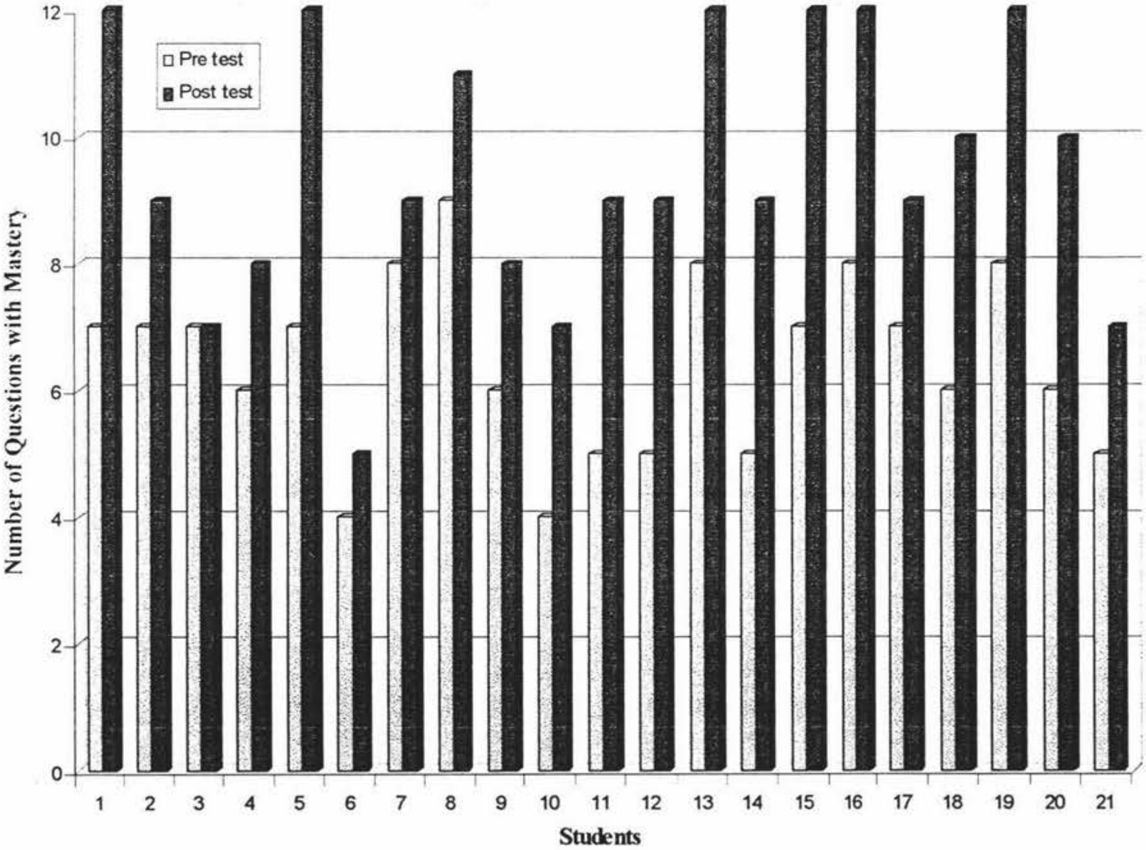


Figure A4.3: Number of questions with mastery. Pre and post test comparison.

SCHOOL B – CLASS THREE

Table A4.4: Algebra Test Results School B, Class Three

( )

Student Number	1	2	3	4	5	6	7	8	9	10	11	12	Total	Mastery
1	3	3	0	2	3	3	3	3	3	1	0	1	25	8
	3	2	0	3	3	3	2	3	3	1	2	3	28	10
2	3	2	3	3	3	3	2	3	3	1	0	0	26	8
	3	3	1	3	2	3	2	3	2	3	2	3	30	11
3	2	2	0	3	3	3	3	3	3	0	0	0	22	8
	3	2	0	3	3	3	3	3	3	1	1	1	26	8
4	3	1	2	2	1	3	2	3	3	0	0	0	20	7
	2	2	0	1	2	3	2	3	3	2	1	0	21	8
5	2	3	0	3	3	3	3	2	2	0	0	0	21	8
	3	3	0	3	3	3	3	3	3	1	1	3	29	9
6	3	1	2	3	3	3	3	3	3	1	0	0	25	8
	3	3	3	3	2	3	3	1	2	1	1	1	26	8
7	3	1	0	2	3	3	3	3	2	0	0	0	20	7
	2	1	0	3	3	3	2	2	2	2	0	2	22	9
8	3	2	0	3	2	3	2	2	3	1	0	0	21	8
	3	3	0	3	3	3	3	3	2	2	1	1	27	9
9	3	2	0	3	3	3	3	3	3	1	0	0	24	8
	3	2	0	3	3	3	3	3	2	2	0	2	26	10
10	2	0	0	3	1	3	3	3	3	0	0	0	18	6
	3	2	0	3	3	2	3	3	3	1	0	2	25	9
11	2	1	0	2	0	3	1	3	3	0	0	1	16	6
	3	2	2	3	2	3	2	3	3	2	2	1	28	11
12	3	3	0	3	0	3	3	3	3	0	0	0	21	7
	2	3	0	3	3	3	3	2	3	1	1	0	24	8
13	3	1	2	2	3	3	1	3	3	1	0	0	22	7
	2	3	0	3	3	3	2	3	0	3	3	2	27	10
14	1	2	0	1	0	2	3	2	0	0	0	0	11	4
	2	3	0	3	3	3	2	3	0	3	3	2	27	10
15	1	0	0	3	1	3	3	3	3	0	0	0	17	5
	2	0	0	2	2	3	3	3	3	0	1	0	19	7
16	3	1	0	3	3	2	1	2	3	0	0	0	18	6
	3	2	0	3	3	2	1	2	2	1	0	2	21	8
17	2	1	2	3	0	3	2	3	1	1	0	2	20	7
	3	3	0	3	1	3	2	1	0	1	2	1	20	6
Pre test only														
18	3	2	0	0	0	3	3	3	2	0	0	0	16	6
Post test only														
19	3	3	0	3	3	3	2	3	3	2	2	2	29	11
20	3	3	0	1	2	3	2	3	0	1	0	0	18	6

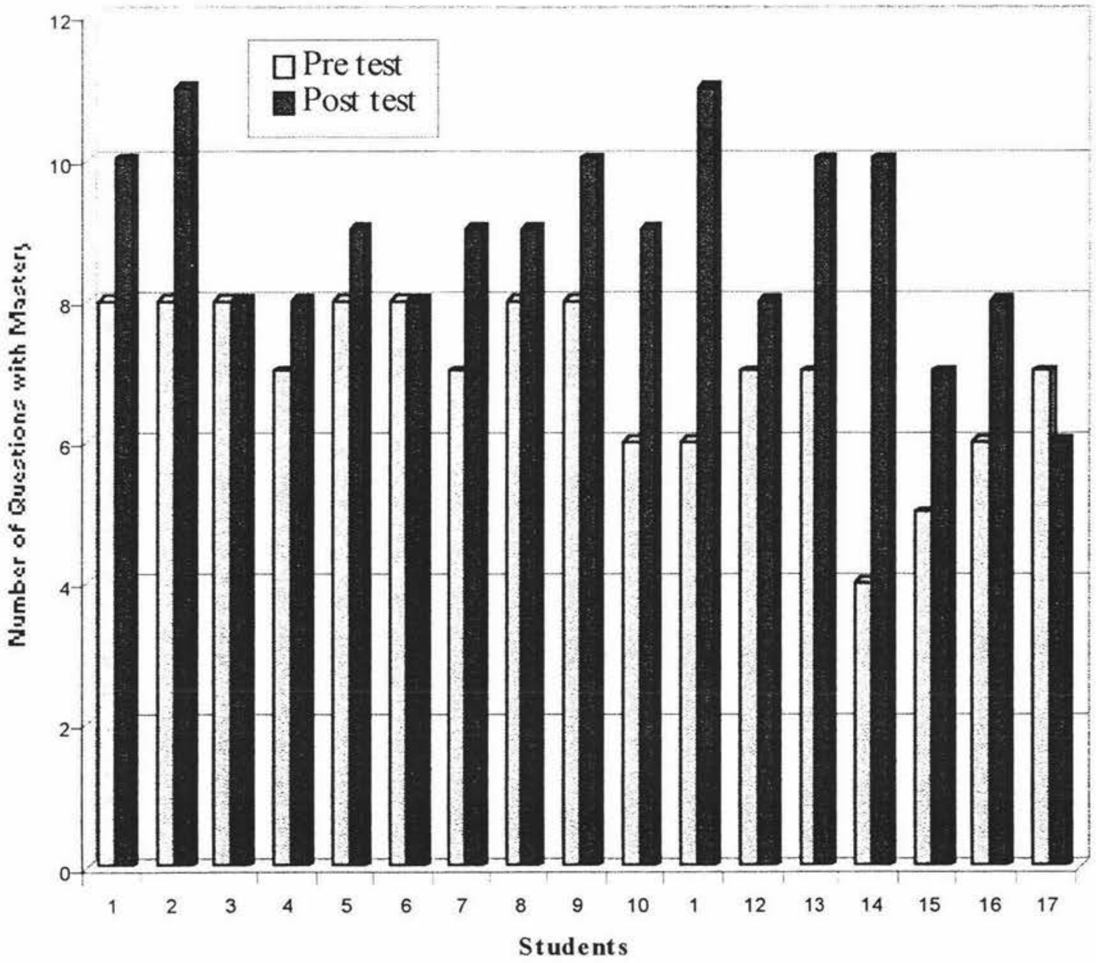


Figure A4.4: Number of questions with mastery. Pre and post test comparison.

SCHOOL B – CLASS FOUR

Table A4.5: Algebra Test Results School B, Class Four

( )

Student Number	1	2	3	4	5	6	7	8	9	10	11	12	Total	Mastery
1	2	1	0	3	1	3	3	2	1	0	0	0	16	5
	2	2	2	3	3	3	0	2	1	1	1	1	21	7
2	3	0	0	3	1	3	2	3	2	0	0	0	17	6
	3	2	0	3	3	2	3	2	1	1	0	1	21	7
3	2	0	0	1	1	3	3	2	0	0	0	0	12	4
	3	3	2	1	0	3	2	1	2	1	1	1	20	6
4	0	0	0	1	0	3	2	2	1	0	0	0	9	3
	3	2	0	1	1	3	2	2	2	1	1	1	18	6
5	2	0	0	1	1	3	1	3	3	0	0	0	14	4
	3	3	0	1	1	3	2	3	3	2	0	0	21	7
6	1	0	0	2	0	3	2	2	0	0	0	0	10	4
	3	3	1	1	1	3	2	2	1	1	1	0	19	5
7	0	0	0	1	0	3	2	3	0	0	0	0	9	3
	3	1	0	1	1	3	2	2	0	0	0	0	13	4
8	2	0	0	1	0	3	1	1	0	0	0	1	9	2
	2	3	1	3	3	3	3	2	3	0	0	0	23	8
9	0	0	0	0	0	3	1	3	0	0	0	0	7	2
	3	3	3	2	1	3	3	3	1	1	0	0	23	7
10	3	0	0	0	0	3	2	2	0	0	0	0	10	4
	2	2	0	1	0	3	3	2	3	1	1	0	18	6
11	0	0	0	0	0	2	2	2	0	0	0	0	6	3
	2	1	0	2	1	3	0	3	0	2	0	0	14	5
12	3	0	0	1	0	3	2	2	0	0	0	1	12	4
	1	1	0	0	0	3	3	1	1	1	0	1	12	2
13	3	2	0	1	1	3	1	3	2	0	0	0	16	5
	3	2	3	2	1	3	2	3	1	1	0	3	24	8
14	2	0	0	3	1	2	2	3	2	0	0	0	15	6
	2	2	0	3	3	0	2	3	3	1	0	1	20	7
15	0	0	0	0	0	3	2	0	0	0	0	0	5	2
	1	0	0	0	0	3	2	2	0	0	0	0	8	3
16	2	0	0	2	0	3	2	2	0	0	0	0	11	5
	3	1	0	2	1	3	3	2	3	0	0	2	20	7
17	2	1	0	0	0	3	2	1	0	0	0	0	9	3
	3	3	2	3	2	3	3	3	2	2	1	0	27	10
Pre test only														
18	2	0	0	0	0	3	0	2	0	0	0	0	7	3

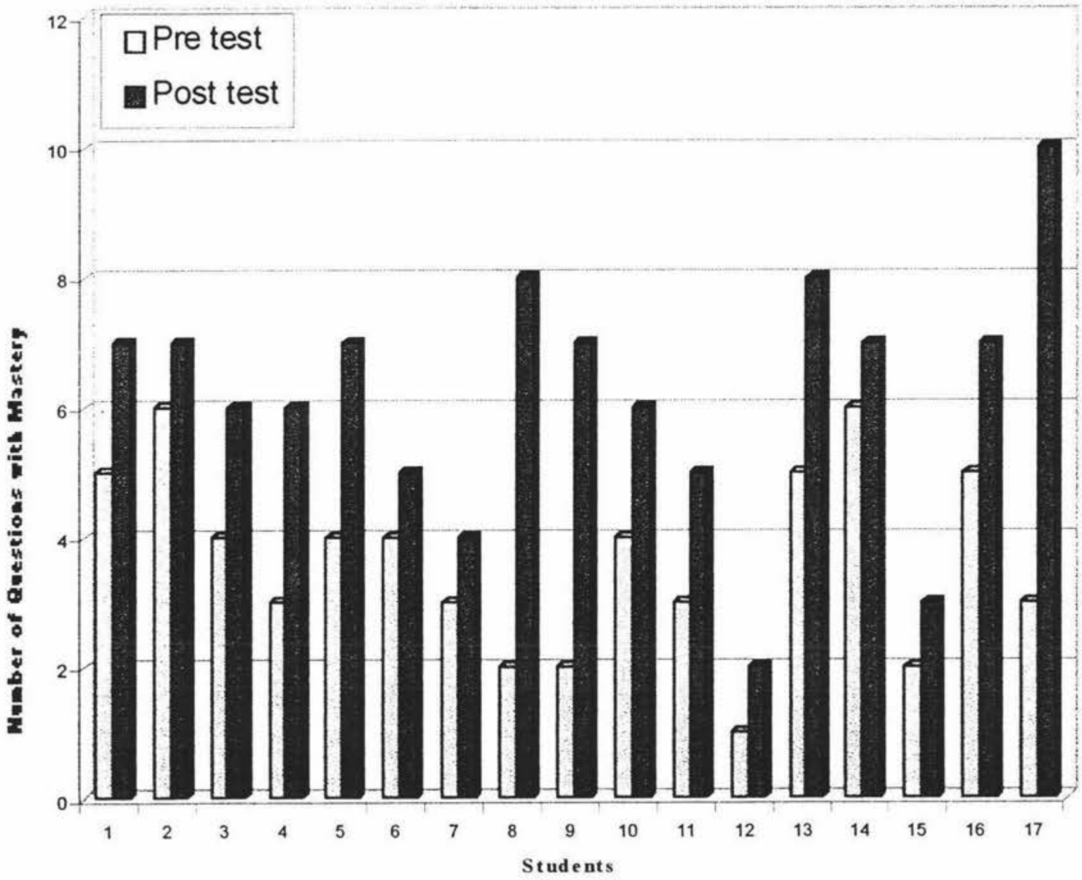


Figure A4.5: Number of questions with mastery. Pre and post test comparison.

SCHOOL B – CLASS FIVE

Table A4.6: Algebra Test Results School B, Class Five

Student Number	1	2	3	4	5	6	7	8	9	10	11	12	Total	Mastery
1	2	3	0	0	0	3	2	3	1	1	0	0	15	5
	2	3	1	3	3	3	3	3	2	1	3	1	28	9
2	3	2	0	1	3	3	3	3	3	1	0	1	23	7
	3	2	3	3	2	3	3	3	3	3	3	2	33	12
3	3	3	2	3	1	3	3	3	1	0	0	0	22	7
	2	2	0	3	3	3	3	2	2	1	0	2	23	9
4	3	3	0	1	2	3	3	3	2	1	1	0	22	7
	3	3	2	2	1	3	3	3	3	2	1	0	26	9
5	3	1	0	2	3	3	3	3	0	0	0	1	19	6
	3	3	3	2	3	3	3	2	2	2	3	1	30	11
6	3	3	0	3	2	3	3	3	3	0	0	1	24	8
	3	2	3	3	3	3	3	1	2	1	2	3	29	10
7	3	2	0	2	3	3	3	3	3	1	0	0	23	8
	3	3	3	3	3	3	3	3	2	2	3	1	32	11
8	3	2	0	3	3	3	2	3	3	0	0	0	22	8
	2	3	3	3	3	3	2	3	3	1	3	2	31	11
9	3	3	2	3	3	3	3	3	2	1	2	0	28	10
	3	3	3	3	3	3	2	3	3	3	3	3	35	12
10	2	2	0	3	2	2	3	3	3	0	0	0	20	8
	3	1	0	3	3	3	3	3	3	1	0	0	23	7
11	2	1	0	3	2	3	3	3	3	0	1	1	22	7
	3	3	3	3	3	3	3	3	3	3	3	3	36	12
12	3	3	0	2	3	3	3	3	0	1	0	1	22	7
	3	3	3	3	3	2	2	2	2	3	2	1	29	11
13	3	3	0	3	1	3	1	3	2	0	0	0	19	6
	3	3	2	3	3	3	3	3	3	3	3	1	33	11
14	3	3	0	3	2	3	3	3	3	0	0	0	23	8
	3	3	1	3	3	3	3	2	2	3	3	1	30	10
15	3	3	0	3	3	3	3	3	3	0	0	0	24	8
	3	3	3	3	3	3	3	2	2	1	1	2	29	10
16	3	2	0	2	1	3	1	2	0	1	0	0	15	6
	3	3	3	3	3	3	1	2	2	2	3	2	30	11
17	2	1	2	3	3	3	3	3	3	0	0	0	23	8
	3	3	0	3	3	3	3	3	3	3	1	0	28	9
18	3	3	3	3	2	3	3	3	2	1	0	0	26	9
	3	3	3	3	3	3	3	2	1	3	3	1	31	10
19	3	3	0	3	1	3	1	3	3	0	0	0	20	6
	3	3	3	3	3	3	2	3	1	3	1	1	29	9

Student Number	1	2	3	4	5	6	7	8	9	10	11	12	Total	Mastery
20	3	3	3	3	3	3	2	3	3	3	2	2	33	12
	3	3	3	3	3	3	3	3	3	3	3	2	35	12
21	3	3	0	2	2	3	3	3	3	0	0	0	22	8
	3	2	3	3	2	3	2	3	3	3	3	2	32	12
22	3	1	1	3	3	3	3	3	3	1	0	1	25	7
	3	2	3	3	3	3	1	2	2	1	1	3	27	9
23	3	3	0	3	0	3	3	2	0	0	0	0	17	6
	1	1	0	3	3	2	2	3	2	2	0	1	20	7
24	3	2	0	3	3	3	1	3	3	0	0	1	22	7
	2	3	1	3	3	3	3	2	3	3	2	1	29	10
25	3	3	2	2	3	3	2	3	1	0	0	0	22	8
	2	2	3	3	3	3	3	3	3	3	1	3	32	11
26	2	0	0	3	1	2	0	3	3	1	0	0	15	5
	3	3	0	3	3	3	3	2	2	2	0	1	25	9
Pre test only														
27	3	2	0	1	1	3	2	2	0	0	0	0	14	5

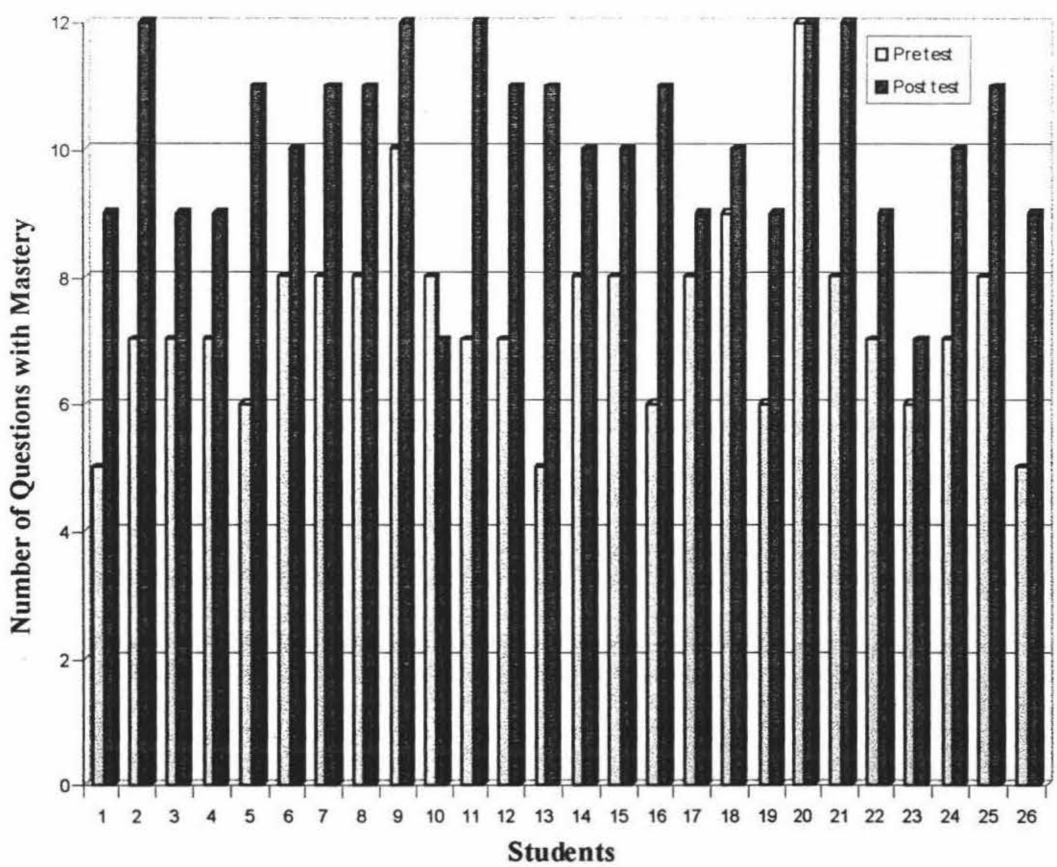


Figure A4.6: Number of questions with mastery. Pre and post test comparison.

<u>SCHOOL B – CLASS SIX</u>

Table A4.7: Algebra Test Results School B, Class Six

([illegible])

Student Number	1	2	3	4	5	6	7	8	9	10	11	12	Total	Mastery
1	3	3	0	2	0	3	3	3	3	1	0	1	22	7
	3	3	3	3	3	3	3	3	3	3	3	2	35	12
2	2	2	2	0	0	3	1	2	1	0	0	1	14	6
	2	3	3	3	3	2	2	2	2	3	1	1	27	10
3	3	2	0	2	2	3	0	3	3	0	0	0	18	7
	3	3	1	3	3	3	2	3	3	3	3	2	32	11
4	3	1	0	3	1	3	2	2	0	0	0	0	15	5
	3	3	3	3	3	3	2	3	3	3	3	2	34	12
5	2	1	0	3	1	3	2	2	3	0	0	1	18	6
	3	3	3	3	3	3	3	3	2	3	3	3	35	12
6	1	2	0	2	1	2	2	3	3	0	0	0	16	6
	3	3	3	3	3	3	2	3	2	1	0	2	28	10
7	3	2	0	2	2	3	1	3	0	0	0	0	16	6
	3	3	3	3	2	3	1	2	0	3	1	0	24	8
8	3	0	0	3	3	3	2	3	3	0	0	1	21	7
	2	3	3	3	3	3	2	3	3	3	2	3	33	12
9	3	3	0	0	0	3	3	3	0	0	0	0	15	5
	2	0	0	3	2	3	3	3	1	1	1	1	20	6
10	3	3	0	3	3	3	3	3	3	1	0	0	25	8
	2	3	2	3	3	3	2	3	3	3	3	3	33	12
11	3	2	3	3	3	3	2	3	3	0	0	0	25	9
	2	3	3	3	3	3	2	3	3	3	2	1	31	11
12	1	3	2	2	3	3	3	3	3	1	1	1	26	8
	3	3	3	3	2	3	2	3	1	3	2	2	30	11
13	3	1	0	3	1	3	3	2	1	0	0	0	17	5
	2	2	0	1	3	3	2	3	3	0	1	3	23	8
14	3	3	0	3	1	3	3	3	3	1	0	1	24	7
	2	3	3	3	3	3	3	3	3	3	3	2	34	12
15	3	2	0	0	0	3	3	3	1	1	0	0	16	5
	3	3	3	3	3	3	3	3	3	3	3	2	35	12
16	3	3	0	0	0	3	3	3	3	0	0	0	18	6
	3	3	0	3	3	3	3	3	2	2	2	2	29	11
17	2	3	0	0	0	3	2	2	3	0	0	0	15	6
	3	3	2	3	3	3	2	3	2	3	3	2	32	12
18	3	0	0	2	2	3	3	3	2	0	0	2	20	8
	3	2	3	3	2	2	3	3	1	3	3	2	30	11
19	2	2	0	3	2	3	1	3	2	1	0	1	20	7
	3	3	3	3	3	3	2	1	3	3	3	1	31	10

Student Number	1	2	3	4	5	6	7	8	9	10	11	12	Total	Mastery
20	3	3	0	2	1	3	2	3	3	0	0	0	20	7
	3	2	3	3	3	3	3	3	3	2	3	2	33	12
21	1	0	0	3	3	3	2	3	2	0	0	0	17	6
	3	3	3	3	3	2	3	3	3	3	1	2	32	11
22	2	0	0	2	1	3	2	3	0	0	0	0	13	5
	3	0	0	0	0	2	3	3	2	2	2	1	18	7
23	3	3	3	2	3	3	3	3	3	0	0	0	26	9
	3	3	3	3	3	3	3	3	3	1	3	1	32	10

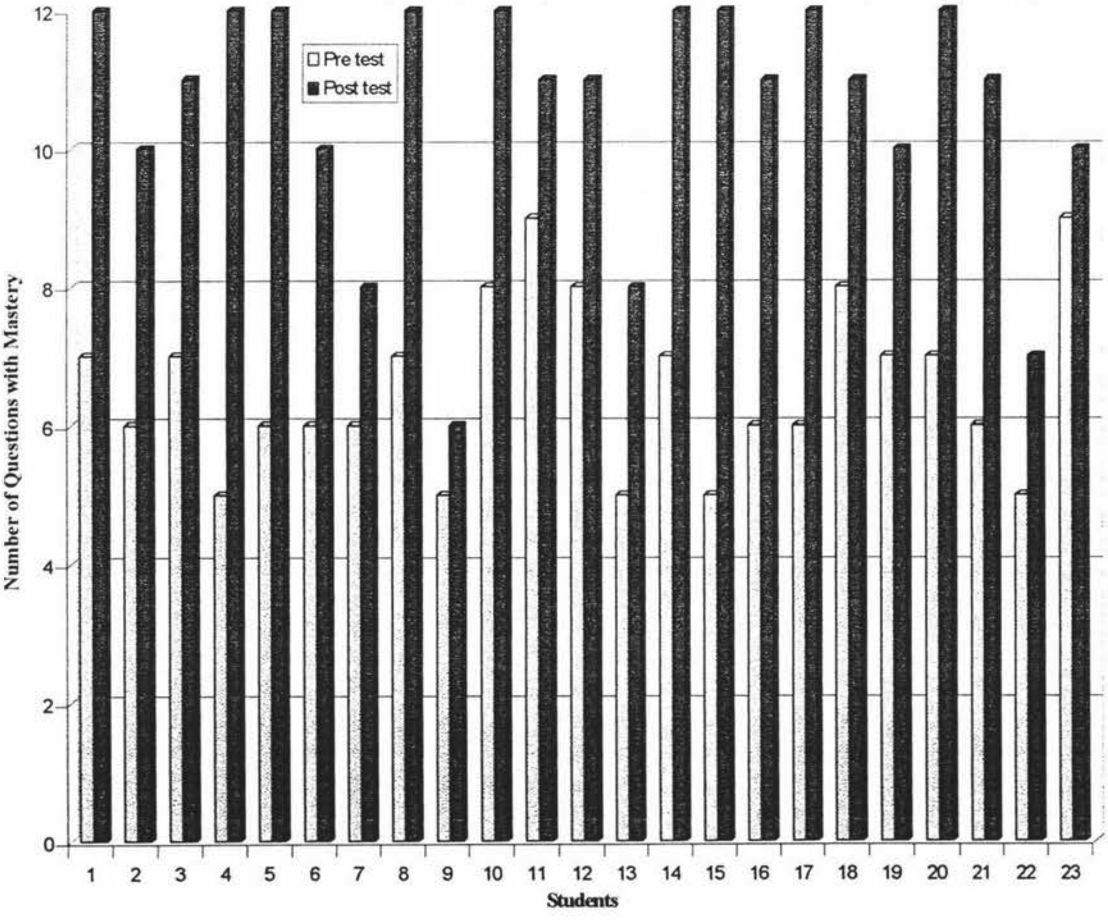


Figure A4.7: Number of questions with mastery. Pre and post test comparison.

Table A4.8: Percentage of students obtaining mastery in each question

Question	School																											
	A		B											
	1		1		2		3		4		5		6	
1	75	94	96	96	90	95	88	100	65	82	100	96	87	100
2	38	75	78	93	33	90	47	88	6	71	81	92	70	91
3	6	38	15	56	10	48	29	12	0	29	19	69	17	78
4	63	69	85	89	86	90	94	94	29	53	92	100	78	91
5	25	56	78	89	52	76	59	94	0	29	73	96	43	96
6	94	100	96	96	100	100	100	100	100	94	100	100	100	100
9	56	88	89	93	95	90	82	94	76	88	81	92	83	96
8	88	100	96	96	95	100	100	88	82	88	100	96	100	96
9	19	38	81	81	62	95	88	82	24	47	73	92	70	83
10	0	31	7	59	5	62	0	47	0	18	4	73	0	83
11	0	19	7	48	0	48	0	35	0	0	8	62	0	74
12	6	6	7	33	5	43	6	53	0	12	4	46	4	70

Pre-test: percentage of students with mastery

Post-test: percentage of students with mastery

Table A4.9: Students who improved mastery from pre to post-test, but changed some of the questions they mastered.

Student	Variations from Pre to Post test		Questions the same for both pre and post-test	Mastery (/12)	Mark (/36)
SA14	Pre	2	1, 4, 5, 6, 7, 8	7	19
	Post	10, 11		8	24
SA19	Pre	2	1, 6	3	12
	Post	7, 8		4	14
SA115	Pre	4	6, 7, 8	4	12
	Post	2, 3		5	14
SB13	Pre	9	1, 2, 4, 5, 6, 8	7	18
	Post	7, 10		8	23
SB17	Pre	9	1, 4, 6, 7, 8	6	21
	Post	2, 5, 10, 11		9	29
SB111	Pre	5	1, 2, 4, 6, 7, 8, 9	8	23
	Post	3, 11, 12		10	26
SB114	Pre	2	1, 5, 6, 7, 8	6	20
	Post	3, 4		7	17
SB116	Pre	1	2, 4, 5, 6, 7, 8	7	21
	Post	9, 10, 11, 12		10	26
SB22	Pre	5	1, 4, 6, 7, 8, 9	7	21
	Post	2, 11, 12		9	26
SB29	Pre	1	5, 6, 7, 8, 9	6	17
	Post	2, 4, 10		8	24
SB210	Pre	7	1, 4, 6	4	13
	Post	2, 5, 8, 9		7	23
SB32	Pre	3	1, 2, 4, 5, 6, 7, 8, 9	9	26
	Post	10, 11, 12		11	30
SB34	Pre	3, 4	1, 6, 7, 8, 9	7	20
	Post	2, 5, 10		8	21
SB313	Pre	3, 9	1, 4, 5, 6, 8	7	22
	Post	2,7,10,11,12		10	27
SB42	Pre	9	1, 4, 6, 7, 8	6	17
	Post	2, 5		7	21

Student	Variations from Pre to Post test		Questions the same for both pre and post-test	Mastery (/12)	Mark (/36)
SB43	Pre	8	1, 6, 7	4	12
	Post	2, 3, 9		6	20
SB53	Pre	3	1, 2, 4, 6, 7, 8	7	22
	Post	5, 9, 12		9	23
SB54	Pre	5	1, 2, 6, 7, 8, 9	7	22
	Post	3, 4, 10		9	26
SB56	Pre	8	1, 2, 4, 5, 6, 7, 9	8	24
	Post	3, 11, 12		10	29
SB517	Pre	3	1, 4, 5, 6, 7, 8, 9	8	23
	Post	2, 10		9	28
SB518	Pre	9	1, 2, 3, 4, 5, 6, 7, 8	9	26
	Post	10, 11		10	31
SB519	Pre	9	1, 2, 4, 6, 8	6	20
	Post	3, 5, 7, 10		9	29
SB522	Pre	7	1, 4, 5, 6, 8, 9	7	25
	Post	2, 3, 12		9	27
SB523	Pre	1, 2	4, 6, 7, 8	6	17
	Post	5, 9, 10		7	20
SB69	Pre	2	1, 6, 7, 8	5	15
	Post	4, 5		6	20
SB612	Pre	9	2, 3, 4, 5, 6, 7, 8	8	26
	Post	1, 10, 11, 12		11	30
SB613	Pre	4	1, 6, 7, 8	5	17
	Post	2, 5, 9, 12		8	23
SB618	Pre	9	1, 4, 5, 6, 7, 8, 12	8	20
	Post	2, 3, 10, 11		11	30
SB619	Pre	8	1, 2, 4, 5, 6, 9	7	20
	Post	3, 7, 10, 11		10	31
SB622	Pre	4	1, 6, 7, 8	5	13
	Post	9, 10, 11		7	18

Appendix Five

Questionnaire

MATHEMATICS QUESTIONNAIRE

- This questionnaire forms part of a Masters Thesis.
- By completing this questionnaire you are giving consent for the results to be used in the thesis.
- There is no way you will be able to be identified in the final write up as the questionnaire will not have your name on it.
- If you are unsure as to what this means please ask your teacher.

Please complete the questionnaire by circling the face that best describes how ***you feel*** about the statement.

	AGREE	NEITHER AGREE OR DISAGREE	DISAGREE
• In maths I like to copy notes from the blackboard	☺	😐	☹
• In maths I like to write notes in my book	☺	😐	☹
• In maths I like to think about ideas for myself	☺	😐	☹
• In maths I like to write my ideas down	☺	😐	☹
• In maths I like to read	☺	😐	☹
• In maths I like to listen to the teacher talking	☺	😐	☹
• In maths I like to listen to other students talking	☺	😐	☹
• In maths I like to draw diagrams	☺	😐	☹
• In maths I like to tell others my ideas	☺	😐	☹
• In maths I like to find the rule for a problem	☺	😐	☹
• In maths I like to solve problems by myself	☺	😐	☹
• In maths I like to solve problems in a group	☺	😐	☹
• In maths I enjoy doing work from the board.	☺	😐	☹
• In maths I enjoy using a textbook to do the work.	☺	😐	☹

	AGREE	NEITHER AGREE OR DISAGREE	DISAGREE
• In maths I enjoy using worksheets to do the work.	☺	😐	☹
• In maths I enjoy using the matching activities.	☺	😐	☹
• In maths I enjoy doing maths games.	☺	😐	☹
• In maths I enjoy doing work that involves working with one other person.	☺	😐	☹
• In maths I enjoy doing work that involves working with a group of people.	☺	😐	☹
• In maths I enjoy working by myself.	☺	😐	☹
• In maths I enjoy doing work that involves discussing my ideas.	☺	😐	☹
• I feel confident talking about my ideas in maths to one other student.	☺	😐	☹
• I feel confident talking about my ideas in maths to a group of students sitting in the same area as me.	☺	😐	☹
• I feel confident talking about my ideas in maths to my maths teacher.	☺	😐	☹
• I feel confident talking about my ideas in maths to the whole class while seated at my desk.	☺	😐	☹
• I feel confident talking about my ideas in maths to the whole class in a report back situation from the front of the class.	☺	😐	☹
• I feel confident writing about my ideas in maths when I am the only person to see them.	☺	😐	☹
• I feel confident writing about my ideas in maths when I am going to show them to another student.	☺	😐	☹
• I feel confident writing about my ideas in maths when my maths teacher is going to see them.	☺	😐	☹
• I feel confident writing about my ideas in maths when my ideas are asked for in a test.	☺	😐	☹

Appendix Six

Questionnaire Statements and Summary of Results

Statements used in the Questionnaire Results Write Up

1. In maths I like to copy notes from the blackboard
2. In maths I like to write notes in my book
3. In maths I like to think about ideas for myself
4. In maths I like to write my ideas down
5. In maths I like to read
6. In maths I like to listen to the teacher talking
7. In maths I like to listen to other students talking
8. In maths I like to draw diagrams
9. In maths I like to tell others my ideas
10. In maths I like to find the rule for a problem
11. In maths I like to solve problems by myself
12. In maths I like to solve problems in a group
13. In maths I enjoy doing work from the board.
14. In maths I enjoy using a textbook to do the work.
15. In maths I enjoy using worksheets to do the work.
16. In maths I enjoy using the matching activities.
17. In maths I enjoy doing maths games.
18. In maths I enjoy doing work that involves working with one other person.
19. In maths I enjoy doing work that involves working with a group of people.
20. In maths I enjoy working by myself.
21. In maths I enjoy doing work that involves discussing my ideas.
22. I feel confident talking about my ideas in maths to one other student.
23. I feel confident talking about my ideas in maths to a group of students sitting in the same area as me.
24. I feel confident talking about my ideas in maths to my maths teacher.

25. I feel confident talking about my ideas in maths to the whole class while seated at my desk.
26. I feel confident talking about my ideas in maths to the whole class in a report back situation from the front of the class.
27. I feel confident writing about my ideas in maths when I am the only person to see them.
28. I feel confident writing about my ideas in maths when I am going to show them to another student.
29. I feel confident writing about my ideas in maths when my maths teacher is going to see them.
30. I feel confident writing about my ideas in maths when my ideas are asked for in a test.

Table A6.1: Questionnaire Results School A - Students who completed both Questionnaires

(Letters indicate each student, 1 indicates first questionnaire, 2 indicates second questionnaire. A indicates the student agrees with the statement, N indicates they neither agree nor disagree, and D indicates they disagree with the statement.)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A1	D	A	A	A	D	D	A	A	A	A	A	A	A	D	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
A2	A	A	N	N	N	N	N	N	N	N	N	A	N	D	N	A	A	A	A	N	N	N	A	N	N	N	N	N	N	N
B1	A	A	A	A	N	A	N	A	N	A	A	A	A	N	N	D	N	A	A	N	N	N	N	A	D	N	A	N	A	N
B2	A	A	N	N	N	A	N	A	N	A	N	A	N	N	N	N	N	A	A	N	N	N	N	N	N	N	N	N	A	A
C1	A	A	A	N	D	N	N	A	N	N	N	N	A	D	N	A	A	N	A	A	N	A	A	A	N	D	D	A	A	D
C2	N	N	A	A	D	N	N	A	N	N	N	N	A	D	N	A	A	N	A	A	N	A	A	A	A	N	N	D	N	N
D1	N	D	D	D	N	A	N	A	N	N	N	N	D	D	A	N	A	A	D	N	N	A	D	N	D	D	D	N	D	D
D2	N	D	D	D	N	N	D	A	D	D	D	A	D	D	N	A	A	A	A	D	D	N	D	D	D	D	A	D	D	D
E1	A	A	N	D	N	A	A	A	A	N	D	A	A	N	A	A	A	A	A	A	A	A	N	N	D	N	N	N	N	N
E2	A	A	A	N	A	N	A	A	A	N	N	A	N	D	A	A	A	A	A	A	N	N	D	N	D	N	A	N	N	N
F1	A	A		D	A	A	N	A	N	D	N	A	A	A	A	N	N	A	N	A	N	A	N	N	N	N	N	N	N	A
F2	A	A	A	N	A	N	N	A	A	N	N	A	A	A	A	D	A	A	N	A	N	A	A	N	N	D	D	A	D	A
G1	N	N	N	D	D	N	A	N	N	D	N	N	N	D	N	N	A	N	A	N	N	N	A	A	A	D	N	N	N	D
G2	A	N	N	N	A	N	N	N	N	N	N	N	N	D	N	N	A	A	A	N	N	N	N	N	A	N	N	N	N	N
H1	A	A	N	N	N	A	A	A	A	A	A	A	A	N	A	N	A	A	A	A	A	A	N	A	N	A	A	N	A	A
H2	A	A	A	N	N	A	A	N	A	A	A	A	A	N	A	N	A	A	A	A	A	A	A	A	N	A	A	A	A	A
I1	N	D	D	D	N	N	N	A	D	D	D	D	N	D	D	D	N	N	N	N	N	D	D	D	D	D	D	D	N	D
I2	N	D	D	D	N	N	D	A	D	D	D	A	D	D	N	A	A	A	A	D	D	N	D	D	D	D	A	D	D	D

Table A6.2: Summary of student choice movement from Questionnaire One to Two

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
U*	6	5	4	4	3	3	2	6	3	4	2	7	3	1	5	6	8	8	8	5	2	4	4	2	3	2	5	2	2	5
N	1	1	1	1	4	3	3	1	3	1	3	2	1	2	3	2	1	1	1	2	4	2	1	2	2	2	1	3	2	1
D#	2	3	4	4	2	3	4	2	3	3	4	0	5	6	1	1	0	0	0	2	3	3	4	5	4	5	3	4	5	3

* U indicates either the student moved from disagree to agree or neither, neither to agree or stayed with agree

D indicates either the student moved from agree to disagree or neither, neither to disagree or stayed with disagree

Shading indicates the question was selected for comment, and which movement, up or down.

Table A6.3: First Questionnaires School A

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A1	D	A	A	A	D	D	A	A	A	A	A	A	A	D	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
B1	A	A	A	A	N	A	N	A	N	A	A	A	A	N	N	D	N	A	A	N	N	N	N	A	D	N	A	N	A	N
C1	A	A	A	N	D	N	N	A	N	N	N	N	A	D	N	A	A	N	A	A	N	A	A	A	N	D	D	A	A	D
D1	N	D	D	D	N	A	N	A	N	N	N	N	D	D	A	N	A	A	D	N	N	A	D	N	D	D	D	N	D	D
E1	A	A	N	D	N	A	A	A	A	N	D	A	A	N	A	A	A	A	A	A	A	A	N	N	D	N	N	N	N	N
F1	A	A		D	A	A	N	A	N	D	N	A	A	A	A	N	N	A	N	A	N	A	N	N	N	N	N	N	N	A
G1	N	N	N	D	D	N	A	N	N	D	N	N	N	D	N	N	A	N	A	N	N	N	A	A	A	D	N	N	N	D
H1	A	A	N	N	D	A	A	A	A	A	A	A	A	D	A	N	A	A	A	A	A	A	N	A	N	A	A	N	A	A
I1	N	D	D	D	N	N	N	A	D	D	D	D	N	D	D	D	N	N	N	N	N	D	D	D	D	D	D	D	N	D
K1	D	A	A	A	N	A	A	A	A	A	D	A	D	D	A	A	A	A	A	D	A	A	A	A	A	A	A	A	A	A
L1	A	A	N	A	A	N	N	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	N
M1	A	N	A	A	A	A	A	N	D	A	N	A	A	N	A	A	N	N	A	A	A	A	A	A	A	A	A	A	A	A
N1	A	A	N	N	D	D	D	A	D	N	N	N	N	N		N	A	N	N	A	A	N	N	N	N	N	A	A	A	N
O1	A	A	N	D	N	N	N	A	D	A	D	N	A	N	A	N	A	D	A	D	D	D	N	N	D	D	N	N	N	D
P1	A	A	A	A	A	A	N	A	N	N	N	A	A	A	A	A	N	A	A	N	N	A	A	N	A	A	A	A	A	A
Q1	A	A	A	N	N	A	A	A	N	A	A	A	A	N	A	A	A	N	A	N	A	A	A	A	A	N	N	N	A	N
R1	N	N	A	A	N	A	A	A	A	A	N	A	A	D	A	A	A	A	A	N	A	A	A	D	A	A	D	D	D	A
S1	N	A	A	D	N	A	N	A	D	A	N	N	N	A	D	A	N	N	A	N	N	A	A	A	A	A	A	A	A	A

Table A6.4: Summary of all First Questionnaires School A

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	11	14	9	7	3	11	8	16	6	10	5	11	12	4	12	10	12	10	14	8	9	13	10	10	9	8	9	8	11	8
N	5	2	6	4	10	5	9	2	7	5	9	6	4	6	3	6	6	7	3	8	8	3	6	6	4	5	5	8	5	5
D	2	2	2	7	5	2	1	0	5	3	4	1	2	8	2	2	0	1	1	2	1	2	2	2	5	5	4	2	2	5
Most	A	A	A	A D	N	A	N	A	A N	A	N	A	A	D	A	A	A	A	A	N	A	A	A	A	A	A	A	N	A	A

Table A6.5: Second Questionnaires School A

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A2	A	A	N	N	N	N	N	N	N	N	N	A	N	D	N	A	A	A	A	N	N	N	A	N	N	N	N	N	N	N
B2	A	A	N	N	N	A	N	A	N	A	N	A	N	N	N	N	N	A	A	N	N	N	N	N	N	N	N	N	A	A
C2	N	N	A	A	D	N	N	A	N	N	N	N	A	D	N	A	A	N	A	A	N	A	A	A	A	N	N	N	N	N
D2	N	D	D	D	N	N	D	A	D	D	D	A	D	D	N	A	A	A	A	D	D	N	D	D	D	D	A	D	D	D
E2	A	A	A	N	A	N	A	A	A	N	N	A	N	D	A	A	A	A	A	A	N	N	D	N	D	N	A	N	N	N
F2	A	A	A	N	A	N	N	A	A	N	N	A	A	A	A	D	A	A	N	A	N	A	A	N	N	D	D	A	D	A
G2	A	N	N	N	A	N	N	N	N	N	N	N	N	D	N	N	A	A	A	N	N	N	N	N	A	N	N	N	N	N
H2	A	A	A	N	D	A	A	N	A	A	A	A	A	N	A	N	A	A	A	A	A	A	A	A	N	A	A	A	A	A
I2	N	D	D	D	N	N	D	A	D	D	D	A	D	D	N	A	A	A	A	D	D	N	D	D	D	D	A	D	D	D
T2	A	A	N	A	A	A	A	A	A	A	A	A	N	A	A	N	A	A	A	N	N	A	A	A	N	D	A	A	A	A
U2	N	N	N	D	D	N	N	A	N	N	N	N	N	N	A	N	A	N	N	D	N	N	N	N	N	N	N	N	N	N
V2	A	D	N	N	D	A	A	N	N	A	N	A	A	D	A	A	A	A	A	A	A	D	A	A	D	D	A	A	A	D

Table A6.6: Summary of all Second Questionnaires School A

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	8	6	4	2	4	4	4	8	4	4	2	9	4	2	6	6	11	10	10	5	2	4	6	4	2	1	6	4	4	4
N	4	3	6	7	4	8	6	4	6	6	8	3	6	3	6	5	1	2	2	4	8	7	3	6	6	6	5	6	5	5
D	0	3	2	3	4	0	2	0	2	2	2	0	2	7	0	1	0	0	0	3	2	1	3	2	4	5	1	2	3	3
Most	A	A	N	N	All	N	N	A	N	N	N	A	N	D	A N	A	A	A	A	A	N	N	A	N	N	N	A	N	N	N

Table A6.7: Questionnaires School B – Class One

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	A	A	A	N	A	A	A	N	N	A	N	A	N	A	A	N	A	A	A	N	N	A	A	N	A	N	A	A	N	N
B	N	N	N	N	N	N	N	N	N	N	N	A	N	N	N	N	N	A	A	N	N	N	A	D	D	D	A	D	D	D
C	A	A	D	D	D	N	N	N	N	A	A	D	A	D	A	N	A	A	A	A	N	A	A	N	D	N	A	A	N	A
D	N	A	A	D	A	N	A	A	A	A	A	N	A	A	A	A	A	A	A	A	A	A	A	A	A	A	N	A	N	A
E	A	A	D	N	A	D	A	A	A	D	D	A	A	D	N	A	A	A	A	D	A	N	A	D	N	N	A	N	D	A
F	D	D	A	N	N	D	A	A	N	A	N	A	D	A	A	N	A	A	A	A	N	N	N	D	N	A	A	N	D	N
G	N	A	D	D	N	D	D	N	A	N	N	N	N	A	A	A	A	A	D	N	N	A	A	N	A	N	A	A	N	A
H	N	A	A	A	D	N	A	A	A	A	N	A	N	D	A	A	A	A	N	N	A	A	A	A	D	D	A	A	A	D
I	N	N	D	D	D	A	A	D	N	N	A	A	A	N	A	N	D	A	A	A	N	A	A	A	D	D	A	N	A	N
J	N	N	A	A	D	N	A	A	N	A	A	D	N	D	A	A	N	N	A	A	N	N	A	N	N	N	D	A	A	N
K	N	A	A	A	N	N	A	N	N	N	A	D	A	A	N	D	N	N	D	A	N	N	N	N	N	D	A	N	A	A
L	A	A	A	A	A	A	A	A	N	A	A	A	N	N	N	D	A	N	A	N	N	A	A	N	N	N	N	N	N	N
M	D	D	A	A	A	N	D	N	D	A	N	A	D	D	A	A	A	A	D	N	A	N	D	D	D	D	A	D	D	A
N	A	N	N	D	D	N	N	N	N	A	D	N	A	N	A	D	A	A	A	N	N	N	A	N	N	D	N	N	N	N
O	N	N	A	D	A	D	A	N	A	N	D	A	A	A	A	N	N	A	A	D	A	D	D	D	D	D	A	D	D	N
P	A	D	A	A	D	A	A	N	D	A	A	N	A	D	N	N	N	A	N	A	N	A	D	D	D	D	A	N	D	D
Q	N	N	N	N	D	N	N	N	D	N	A	D	N	N	A	N	A	A	D	A	D	N	N	A	D	D	A	N	A	N
R	A	A	N	A	N	A	A	N	N	A	A	A	A	A	A	A	A	A	A	A	A	N	N	N	N	N	A	A	A	A
S	A	A	N	A	A	N	N	N	A	N	N	A	N	D	A	A	A	A	N	A	N	A	D	D	D	D	A	N	D	N
T	A	A	N	D	N	D	D	N	N	A	A	N	D	A	A	A	D	A	A	A	N	D	N	D	D	N	A	N	D	D
U	N	A	N	N	N	D	N	N	N	A	N	N	N	N	N	A	A	N	N	D	N	A	A	D	N	D	N	N	D	N
V	A	N	D	A	A	N	A	D	N	N	A	A	N	A	A	N	A	N	A	A	A	N	N	N	D	N	A	N	N	A

Table A6.8: Summary of all Questionnaires School B – Class One

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	10	12	10	9	8	5	13	6	6	13	11	12	9	9	16	10	15	17	14	12	7	10	12	4	3	2	17	7	6	8
N	10	7	8	6	7	11	6	14	13	8	8	6	10	6	6	9	5	5	4	7	14	10	6	9	8	9	4	12	7	10
D	2	3	4	7	7	6	3	2	3	0	3	4	3	7	0	3	2	0	4	3	1	2	4	9	11	11	1	3	9	4
Most	A N	A	A	A	A	N	A	N	N	A	A	A	N	A	A	A	A	A	A	A	N	A N	A	ND	D	D	A	N	D	N

Table A6.9: Questionnaires School B – Class Two

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	N	N	N	D	N	N	A	D	D	N	N	N	A	A	A	A	A	N	N	N	N	N	A	N	N	N	A	N	N	N
B	A	A	N	N	A	A	D	A	N	A	A	A	N	N	A	A	A	N	A	A	N	A	A	A	N	N	A	A	A	A
C	A	N	N	N	A	A	A	N	D	N	D	A	A	N	N	A	A	A	N	D	D	A	N	N	D	D	A	A	N	D
D	N	N	N	N	N	N	N	N	N	N	A	A	N	N	A	A	A	A	A	A	A	A	A	A	N	N	A	N	N	N
E	A	A	A	N	N	N	D	A	N	A	A	N	D	A	A	A	A	A	N	N	A	D	D	D	A	D	A	N	N	N
F	A	A	N	N	N	N	N	N	N	A	N	N	A	A	A	A	A	A	A	A	N	N	N	N	N	N	N	N	N	N
G	N	N	A	N	N	D	N	N	N	D	A	A	N	D	D	D	A	A	A	N	A	N	D	D	D	D	A	N	N	D
H	N	A	N	D	N	A	D	A	D	N	N	A	N	N	A	A	N	A	A	D	A	N	N	D	D	D	A	N	N	A
I	N	N	N	N	A	A	N	A	N	A	A	A	A	D	A	A	A	A	A	A	N	A	N	N	D	D	A	A	A	N
J	N	A	A	A	N	D	A	A	A	N	A	A	D	D	D	A	A	A	A	N	A	N	A	D	D	D	N	N	D	N
K	A	A	D	A	D	A	N	A	N	D	N	D	A	A	A	A	A	A	D	N	A	A	N	D	D	N	N	A	D	A
L	N	A	A	A	D	D	A	D	A	N	A	N	A	A	A	A	A	A	A	D	N	A	D	A	N	N	A	A	A	A
M	A	A	N	N	A	N	A	A	A	A	N	A	A	A	A	A	A	A	A	N	A	N	A	D	A	A	A	D	D	N
N	N	A	N	A	N	N	N	A	N	A	A	N		N	A	A	A	A	N	A	N	N	N	N	N	D	A	N	A	A
O	N	A	N	D	D	N	N	N	N	N	A	A	A	N	N	N	A	N	A	N	N	D	D	D	D	D	A	N	D	N
P	N	A	N	N	N	A	N	A	A	A	A	A	N	N	A	N	A	A	A	A	A	A	A	A	A	A	N	A	A	A
Q	A	A	N	D	D	D	D	D	D	N	A	A	D	A	A	D	A	A	A	A	D	A	A	N	N	D	A	A	A	A
R	D	N	A	N	N	N	A	A	A	A	A	N	N	N	N	N	A	N	N	N	N	N	N	N	N	N	A	A	A	N
S	D	N	A	N	D	A	A	D	N	N	A	A	D	D	A	N	A	A	A	N	N	N	N	N	N	D	A	A	N	N
T	N	A	N	D	N	N	A	A	N	D	D	A	N	A	N	A	N	A	A	D	N	N	N	D	D	D	A	A	D	D
U	A	A	A	A	A	A	D	D	A	A	A	A	A	A	A	D	A	A	A	D	A	A	A	D	A	D	A	A	A	A
V	A	A	N	N	N	N	N	A	D	N	N	A	N	A	N	N	N	N	N	N	D	A	N	N	N	N	N	N	N	N

Table A6.10: Summary of all Questionnaires School B – Class Two

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	9	15	7	5	5	8	8	12	6	9	14	15	9	10	15	14	19	17	15	7	9	10	8	4	4	2	17	11	8	8
N	11	7	14	12	12	10	9	5	11	10	6	6	8	8	5	5	3	5	6	10	10	10	10	9	10	8	5	10	9	11
D	2	0	1	5	5	4	5	5	5	3	2	1	4	4	2	3	0	0	1	5	3	2	4	9	8	12	0	1	5	3
Most	N	A	N	N	N	N	N	A	N	N	A	A	A	A	A	A	A	A	A	N	N	A/N	N	ND	N	D	A	A	N	N

Table A6.11: Summary of all Questionnaires School B – Classes One and Two

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	19	27	17	14	13	13	21	18	12	22	25	27	18	19	31	24	34	34	29	19	16	20	20	8	7	4	34	18	14	16
N	21	14	22	18	19	21	15	19	24	18	14	12	18	14	11	14	8	10	10	17	24	20	16	18	18	17	9	22	16	21
D	4	3	5	12	12	10	8	7	8	3	5	5	7	11	2	6	2	0	5	8	4	4	8	18	19	23	1	4	14	7
Most	N	A	N	N	N	N	A	N	N	A	A	A	A/N	A	A	A	A	A	A	A	N	A/N	A	ND	D	D	A	A	N	N

Table A6.12: Questionnaires School B – Class Three and Four

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	N	A	D	D	D	N	A	N	N	A	D	D	A	A	A	D	A	N	N	N	D	N	N	N	D	D	N	D	N	D
B	D	N	A	N	N	A	N	D	A	A	N	N	N	N	A	N	A	N	N	N	N	N	A	N	A	A	A	A	A	A
C	D	A	N	A	N	A	N	A	A	A	N	N	A	A	A	N	N	A	N	A	A	A	N	A	A	N	N	N	A	N
D	N	A	A	A	N	A	D	A	A	N	N	A	A	N	A	N	A	A	N	N	A	N	N	A	N	N	A	N	N	A
E	A	A	A	N	D	A	N	A	N	N	A	D	A	D	N	N	A	N	N	A	N	D	N	A	N	A	N	N	A	A
F	A	D	A	N	D	D	A	D	N	D	N	A	N	D	N	N	A	A	A	N	D	D	D	N	D	D	N	N	A	N
G	N	N	N	D	A	A	N	A	N	N	N	A	N	N	N	A	A	N	A	N	N	A	N	A	D	A	N	D	N	N
H	D	N	N	N	N	A	N	N	N	N	A	N	A	D	D	N	A	N	N	N	A	A	A	A	A	A	A	A	A	A
I	N	A	D	N	N	A	A	N	N	D	D	A	A	A	A	N	A	N	A	N	D	N	N	N	N	N	D	N	N	N
J	N	A	N	A	N	A	A	D	N	N	N	A	D	A	N	N	N	N	A	A	N	N	N	N	N	D	N	N	N	N
K	D	A	D	D	D	A	D	D	D	D	D	A	N	D	N	N	A	A	A	D	N	N	N	N	N	D	N	D	D	D
L	N	N	N	N	N	N	N	N	N	A	A	N	N	N	N	N	A	N	N	N	N	N	N	N	N	N	N	N	N	N
M	D	D	A	A	D	N	N	D	N	A	A	N	D	A	A	A	A	A	A	A	D	A	A	N	D	A	A	N	N	N
N	A	A	N	N	A	A	A	N	D	A	A	A	A	D	A	A	A	A	A	A	A	A	A	A	A	N	A	A	A	N
O	A	A	N	D	N	N	D	A	D	N	A	A	A	N	N	N	A	A	N	N	D	N	D	N	D	D	A	N	A	N
P	N	N	N	A	A	A	A	N	N	N	N	A	D	D	A	A	A	A	A	N	A	A	N	N	N	D	N	A	N	N
Q	N	D	A	N	N	A	N	N	D	N	A	A	A	N	N	N	A	N	N	A	D	N	D	N	D	D	N	N	D	N
R	N	N	N	D	A	A	A	A	D	N	A	N	A	N	D	N	A	N	D	N	D	D	D	D	D	D	A	A	D	N
S	A	N	A	N	D	D	D	D	N	A	A	N	N	D	A	N	D	A	D	A	N	A	D	A	D	N	N	N	N	A
T	N	A	N	N	D	N	N	D	N	N	A	N	A	D	A	N	A	A	N	A	D	D	D	N	D	D	A	N	N	A
U	A	A	A	A	N	A	A	A	N	A	N	A	A	D	N	N	A	A	A	D	N	N	N	N	N	D	N	N	N	A
V	N	N	N	N	D	N	A	A	N	D	A	A	A	D	A	N	A	A	A	N	A	A	A	A	A	A	A	A	A	N
W	N	A	N	N	A	N	N	N	N	N	A	A	A	N	A	N	A	A	N	N	A	A	D	N	N	D	N	N	N	N
X	A	A	N	A	D	N	A	N	A	N	A	A	N	D	N	N	A	A	A	A	A	A	A	A	A	A	N	A	A	N
Y	N	N	N	A	D	N	A	A	A	A	A	A	N	A	A	A	A	A	N	A	A	N	N	A	A	N	A	N	N	N
Z	N	A	A	N	A	A	D	A	N	D	A	N	A	N	A	A	A	A	A	N	D	A	D	A	D	A	A	N	A	A
1	A	A	N	A	N	A	A	A	N	A	N	A	A	D	N	N	A	A	N	N	A	N	N	N	D	D	N	D	N	N
2	A	A	A	N	N	A	N	A	A	A	N	N	A	D	N	N	A	N	A	N	N	A	A	A	A	A	A	A	A	A
3	A	A	A	D	D	A	A	N	A	A	A	N	A	D	A	A	A	A	D	N	A	A	A	A	A	A	D	N	N	A

Table A6.13: Summary of all Questionnaires School B – Class Three and Four

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	10	17	11	9	6	18	13	12	7	12	16	16	18	6	15	7	26	18	13	10	11	13	8	13	9	10	12	8	11	10
N	14	9	15	14	12	9	11	10	17	12	10	11	8	9	12	21	2	11	13	17	9	12	13	15	9	7	15	17	15	17
D	5	3	3	6	11	2	5	7	5	5	3	2	3	14	2	1	1	0	3	2	9	4	8	1	11	12	2	4	3	2
Most	N	A	N	N	N	A	A	A	N	A N	A	A	A	D	A	N	A	A	A N	N	A	A	N	N	D	D	N	N	N	N

Table A6.14: Questionnaires School B – Class Five and Six

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	A	A	D	N	D	A	N	A	N	D	N	A	A	N	N	N	A	A	A	N	D	N	N	N	N	N	N	N	N	D
B	A	A	N	N	N	A	N	A	N	A	A	N	A	D	A	A	A	A	N	N	N	A	D	A	N	N	N	A	A	N
C	D	N	N	N	N	A	A	A	N	N	N	N	A	A	A	A	A	N	A	N	D	N	A	N	D	D	A	N	D	N
D	A	A	N	A	D	A	A	N	A	N	N	A	A	A	A	N	A	A	A	D	N	A	A	A	N	N	N	A	A	N
E	A	A	N	N	D	N	A	D	N	D	D	A	D	D	A	A	A	A	A	N	A	A	A	A	N	A	N	A	A	N
F	D	A	N	N	A	D	A	N	A	A	A	N	N	A	D	A	A	A	N	N	N	A	D	D	D	D	A	N	N	N
G	D	A	N	N	D	D	A	D	D	A	A	A	D	A	N	D	A	A	A	A	D	A	A	N	A	D	A	D	D	N
H	D	D	A	A	A	A	N	N	D	A	D	A	D	A	A	A	A	A	D	A	D	A	N	A	D	D	A	A	A	A
I	A	A	N	D	D	A	A	D	A	N	A	A	A	D	N	N	A	A	A	A	N	N	N	A	N	N	D	D	N	A
J	A	A	A	D	N	D	D	A	D	N	A	N	A	N	N	N	A	A	N	N	D	D	D	N	D	D	A	D	D	A
K	A	N	A	D	N	A	A	D	D	N	A	A	N	N	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
L	D	N	A	A	D	A	D	A	N	A	A	D	A	A	A	D	D	D	D	A	D	A	A	A	A	A	A	A	A	N
M	N	A	N	D	D	A	N	A	A	A	A	A	A	A	A	N	A	A	N	A	A	A	A	A	A	A	A	A	A	N
N	A	N	N	N	D	A	N	A	D	D	N	A	A	A	A	D	A	A	A	A	N	A	N	A	D	D	A	N	N	N
O	D	A	N	N	D	N	A	N	N	A	A	D	N	N	A	D	N	N	D	A	N	A	D	N	N	D	N	N	N	N
P	N	A	N	A	N	A	A	A	A	D	A	A	A	D	A	A	A	A	N	A	D	A	N	A	D	N	A	N	A	D
Q	A	A	D	N	N	A	A	A	N	A	N	N	A	A	A	N	N	N	N	N	N	N	D	D	D	D	N	N	N	D
R	A	A	N	N	N	N	A	A	A	A	A	N	N	D	A	N	N	A	N	A	N	D	A	N	D	D	A	N	A	A
S	N	N	N	D	D	N	N	N	D	N	N	N	N	A	A	N	N	N	N	A	N	N	N	D	D	D	A	A	A	A
T	A	A	N	N	N	N	N	A	A	A	N	A	N	D	N	N	A	A	N	N	A	N	N	A	N	D	A	N	N	A
U	A	A	N	N	N	A	A	D	D	N	A	A	A	A	A	A	A	A	N	N	N	D	D	D	D	D	A	D	A	N
V	D	A	D	N	N	N	N	N	N	A	A	D	A	N	A	N	A	N	D	A	D	A	A	A	N	D	N	N	N	N
W	A	A	A	N	N	N	D	A	N	A	A	D	N	A	N	N	D	D	D	A	N	N	N	N	N	N	A	N	N	N
X	N	N	A	A	N	A	D	N	N	A	N	N	N	A	A	N	A	A	A	N	N	A	D	N	D	D	N	N	N	A
Y	A	A	N	N	D	D	D	D	D	D	D	N	A	A	N	D	N	A	N	A	D	A	D	D	D	D	A	D	N	D
Z	D	A	N	D	N	A	A	A	N	A	N	N	N	A	A	A	A	A	A	A	N	N	N	N	N	N	A	A	A	A
1	A	A	A	N	A	A	D	A	A	A	A	N	A	A	A	N	N	A	N	A	N	A	D	A	D	D	A	A	A	A
2	A	N	D	N	N	N	N	A	N	N	N	A	N	N	A	A	A	A	A	N	N	N	A	D	N	N	D	D	D	N
3	N	N	N	N	N	A	A	A	N	A	A	D	A	A	A	N	A	D	D	A	D	N	D	A	A	D	N	A	A	A
4	A	A	N	N	D	A	N	N	A	A	A	A	A	A	A	N	N	A	A	A	A	A	A	A	D	D	N	A	A	N
5	A	A	N	N	D	A	N	N	A	A	A	A	A	A	A	N	N	A	A	A	N	A	A	A	D	D	N	N	N	N
6	A	A	N	A	A	A	D	A	N	A	D	N	A	A	A	N	N	N	N	A	D	N	A	N	D	D	A	N	A	A
7	D	D	D	D	D	N	N	D	D	D	D	A	N	N	N	D	A	A	A	D	D	D	D	N	D	D	A	D	N	D
8	D	N	D	D	D	N	D	N	D	D	D	A	N	N	N	D	N	A	A	D	D	D	D	N	D	D	A	N	A	N
9	D	D	N	N	N	N	N	N	N	N	N	N	D	N	A	N	A	N	N	N	N	N	N	N	N	N	N	N	N	D
10	D	D	A	D	A	D	N	N	N	A	N	A	N	A	A	A	A	A	A	N	A	N	N	D	D	D	A	N	D	A
11	N	D	A	A	A	A	A	A	A	A	A	A	N	A	A	A	A	A	A	A	A	A	N	D	N	N	A	A	D	A
12	N	N	N	N	D	D	D	N	D	A	A	D	N	N	N	N	D	A	D	A	D	A	D	N	D	D	A	N	N	D
13	N	A	N	N	N	N	N	A	N	A	A	N	N	N	N	A	A	A	N	N	N	N	N	N	N	D	N	N	N	N
14	A	A	A	A	N	A	N	A	A	A	A	A	N	A	A	N	A	A	N	A	N	A	N	N	N	D	A	N	N	N
15	A	A	A	A	N	A	N	A	A	A	N	N	N	A	A	N	A	A	N	A	N	A	N	N	D	D	A	N	A	A
16	N	N	N	D	D	D	A	D	A	N	A	A	N	A	A	D	N	A	A	A	A	A	A	N	A	A	N	N	N	A
17	D	A	N	N	N	N	N	A	N	A	A	A	D	A	A	A	A	A	A	N	N	N	N	N	N	D	N	N	N	A
18	N	A	N	D	D	A	N	N	N	A	N	D	N	A	A	N	N	A	N	A	N	A	A	A	A	D	N	A	A	D
19	D	A	N	N	N	N	N	N	A	A	A	A	N	N	A	A	A	A	A	N	A	N	N	N	N	D	A	D	N	N

Table A6.15: Summary of all Questionnaires School B – Classes Five and Six

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	21	29	10	9	6	24	16	22	15	28	24	23	19	27	33	15	30	35	20	26	9	24	15	17	7	5	27	14	19	17
N	10	11	29	25	21	14	20	15	20	10	15	15	21	12	11	22	12	7	18	16	22	16	17	20	17	10	16	23	20	20
D	14	5	6	11	18	7	9	8	10	7	6	7	5	6	1	8	3	3	7	3	14	5	13	8	21	30	2	8	6	8
Most	A	A	N	N	N	A	N	A	A	A	A	A	N	A	A	N	A	A	A	A	N	A	N	N	D	D	A	N	N	N

Summary of all Questionnaires *(Second Questionnaire for School A)*

Table A6.16: Agree

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	8	6	4	2	4	4	4	8	4	4	2	9	4	2	6	6	11	10	10	5	2	4	6	4	2	1	6	4	4	4
B1	10	12	10	9	8	5	13	6	6	13	11	12	9	9	16	10	15	17	14	12	7	10	12	4	3	2	17	7	6	8
B2	9	15	7	5	5	8	8	12	6	9	14	15	9	10	15	14	19	17	15	7	9	10	8	4	4	2	17	11	8	8
B3/4	10	17	11	9	6	18	13	12	7	12	16	16	18	6	15	7	26	18	13	10	11	13	8	13	9	10	12	8	11	10
B5/6	21	29	10	9	6	24	16	22	15	28	24	23	19	27	33	15	30	35	20	26	9	24	15	17	7	5	27	14	19	17
Total	58	79	42	34	29	59	54	60	38	66	67	75	59	54	85	52	101	97	72	60	38	61	49	42	25	20	79	44	48	47

Table A6.17: Neither Agree or Disagree

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	4	3	6	7	4	8	6	4	6	6	8	3	6	3	6	5	1	2	2	4	8	7	3	6	6	6	5	6	5	5
B1	10	7	8	6	7	11	6	14	13	8	8	6	10	6	6	9	5	5	4	7	14	10	6	9	8	9	4	12	7	10
B2	11	7	14	12	12	10	9	5	11	10	6	6	8	8	5	5	3	5	6	10	10	10	10	9	10	8	5	10	9	11
B3/4	14	9	15	14	12	9	11	10	17	12	10	11	8	9	12	21	2	11	13	17	9	12	13	15	9	7	15	17	15	17
B5/6	10	11	29	25	21	14	20	15	20	10	15	15	21	12	11	22	12	7	18	16	22	16	17	20	17	10	16	23	20	20
Total	49	37	72	64	56	52	52	48	67	46	47	41	53	38	40	62	23	30	43	54	63	55	49	59	50	40	45	68	56	63

Table A6.18: Disagree

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	0	3	2	3	4	0	2	0	2	2	2	0	2	7	0	1	0	0	0	3	2	1	3	2	4	5	1	2	3	3
B1	2	3	4	7	7	6	3	2	3	0	3	4	3	7	0	3	2	0	4	3	1	2	4	9	11	11	1	3	9	4
B2	2	0	1	5	5	4	5	5	5	3	2	1	4	4	2	3	0	0	1	5	3	2	4	9	8	12	0	1	5	3
B3/4	5	3	3	6	11	2	5	7	5	5	3	2	3	14	2	1	1	0	3	2	9	4	8	1	11	12	2	4	3	2
B5/6	14	5	6	11	18	7	9	8	10	7	6	7	5	6	1	8	3	3	7	3	14	5	13	8	21	30	2	8	6	8
Total	23	14	16	32	45	19	24	22	25	17	16	14	17	38	5	16	6	3	15	16	29	14	32	29	55	70	6	18	26	20

Table A6.19: Summary of totals from all Questionnaires

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	58	79	42	34	29	59	54	60	38	66	67	75	59	54	85	52	101	97	72	60	38	61	49	42	25	20	79	44	48	47
N	49	37	72	64	56	52	52	48	67	46	47	41	53	38	40	62	23	30	43	54	63	55	49	59	50	40	45	68	56	63
D	23	14	16	32	45	19	24	22	25	17	16	14	17	38	5	16	6	3	15	16	29	14	32	29	55	70	6	18	26	20

Table A6.20: Most popular choice from all Questionnaires

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	A	N	N	N	A	A	A	N	A	A	A	A	A	A	N	A	A	A	A	N	A	AN	N	D	D	A	N	N	N

Table A6.21: Individual teacher results from all their Questionnaires

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	A	A	N	N	All	N	N	A	N	N	N	A	N	D	A/N	A	A	A	A	A	N	N	A	N	N	N	A	N	N	N
B1/2	N	A	N	N	N	N	A	N	N	A	A	A	A/N	A	A	A	A	A	A	A	N	A/N	A	ND	D	D	A	A	N	N
B3/4	N	A	N	N	N	A	A	A	N	A/N	A	A	A	D	A	N	A	A	A/N	N	A	A	N	N	D	D	N	N	N	N
B5/6	A	A	N	N	N	A	N	A	A	A	A	A	N	A	A	N	A	A	A	A	N	A	N	N	D	D	A	N	N	N

Appendix Seven

Notes from Teacher Meetings

A7.1 Notes from meetings with Teacher One

A7.1.1 Jan 31 - Notes

Talked over the project using the research proposal. Covered the main objectives. Planned the order of teaching. The first topic is on Number. I was to look at work for week beginning 10 February. This is on estimation. Discussed that work would need to be prepared for the week ahead, not the current week.

A7.1.2 Feb 2 - Notes

Made a selection of activities. Cloze – rules for rounding; grid boxes – significant figures and decimal places; word hunt – maths vocabulary. Dropped off to teacher. No chance to talk through the activities.

A7.1.3 Feb 17 – based on taped conversation

Met and talked through how the activities went.

Rules for Rounding (10LTL 1): took ages, really long. They worked well in the first period, really got into it. Normally hyper kids got into it and discussed in groups. Was not completed in the first period, and in the second period seemed to take a long time. The boys moaned about copying out the words.

Pip – They don't want to write. Will this change?

The girls were really into it. Three of the girls said easy, rest of the girls said ok.

Pip – Suggested they could fill in the blanks and then glue into their books. Would this be better?

The teacher felt this could be better. Students take a real pride in their work.

*** WRITING THINGS OUT IS A PROBLEM

10LTL 2: The girls found this easy. Boys took a long time. It was writing again, but easier to do than Table #1 problem.

Table #1 caused some concern to the students because there was no column for four significant figures. Students wanted this column for things like 1000, and 0.004. There was also discussion on why 0.8 was bigger than 0.75.

The wordhunt was used. The teacher first brainstormed what the word might mean. The students then glued the brainstorming session into their books. This provided a basis for vocabulary introduction.

*** USE KEY WORDS AS A BRAINSTORMING SESSION, THEN DO WORD HUNT.

Table #2 Question 3. 12.449 gives a rounded answer of 12.5. They first rounded the nine to get 12.45 and then the five rounded the four to get 12.5. Some suggested the answer of 12.4. The students were actually arguing against one another, and supporting their side, 12.4 or 12.5.

Table #2 & 4. Enjoyed pasting into their book once completed. They could do the work.

Pip – Encourage students to write why an answer is wrong.

Table #4 The students did not like it when there was no space for their answer. For example number 5 is 0.24. Therefore they got stuck and wouldn't move on until they got it right.

General comments: The students made the teacher feel guilty when they had to write. It wasn't that they students couldn't do the work, they are just lazy.

A7.1.4 March 6 – based on taped conversation

Brainstorming was done in class, and for homework the word hunt. The brainstorming on powers brought out ideas like 1, 10, 100, 1000, 10^2, 2^{10}, multiplying by ten. Talked about why 100 and 1000 were powers of ten.

Matching Two – really keen to get into it and not listen to the instructions. They really enjoyed doing this activity. Needed time to set up this type of activity. Teacher thought they would have more problems, especially with the negatives, but they didn't. They used the order of the cards/numbers to help them work out the negatives. Matching three found the patterns better than in Matching two. Found patterns such as the number of zeros, eg 10^1 was 1 zero, and 10^2 was 2 zeroes. Students wanted to do the work by themselves, need to be encouraged to cooperate to solve the problem.

One student shared his "rule" for standard form with the class. Others had the same ideas but didn't want to share. (Matching Three). They developed the ideas based on this activity. Last year the fourth form found standard form much harder.

*** MATCHING ACTIVITIES WERE ENJOYED BY THE STUDENTS

Went through activities for next time. Using a selection from Mathematics and Learning through Language. A resource coordinated by the LTL team and the Secondary Maths Advisors in Auckland.

We also brainstormed possible questionnaire questions.

A7.1.5 March 14 - Notes

Discussed the next unit of work. We looked at the objectives for Measurement. These are: Level 4 Measurement Nos 1-3, 5 and Level 5 No. 1

A7.1.6 April 4 - Notes

Met and discussed use of activities for measurement. Volume and Area Picture Matching, Length – Cline; Cooperative Logic Problems.

A7.1.7 April 8 - Notes

Positive feedback on clines and picture matching exercise.

A7.1.8 June 12 – Main report back from Algebra Unit – based on taped conversation

General

After a bit of practice the girls spoke up more.

It was better when they had something to paste in. They didn't like having to write out things.

Needed more time. Not enough time allowed for the topic.

Really liked the sequencing, but didn't want to talk about it. Would have preferred to work on their own. Work was needed to encourage them to communicate. It only took two or three activities to encourage them to talk.

Some boys were better than the girls at doing things, other boys just wouldn't communicate. Most of the boys had improved their communication in their own way. Some were very laid back, very casual, didn't use much maths language. They were talking about maths in their own language, at the beginning of the year they weren't talking about maths.

Calculator use activity – hard to see what the point of it all was. They are still not keen on writing out work to do. If they can be provided with the resource to write on, then they are more likely to do it. Lazy. Enjoy doing the pasting into their book.

Clines were a problem as the activity was incorrect. First one went well, but the second one was a problem.

Collecting like terms, needed to do some more building. Individuals needed beans for a long time. All of the class needed beans to complete the Further Practice in Part One of Collecting Like Terms. (c.f. School Two who felt the concrete wasn't necessary as they had covered this quite fully in Form Three) Homework associated with this was well done. Only three not completed, normally only three would have done the homework.

Picture dictation went well. Easily summarised the number of blanks and crosses. They dictated to their partner. Some of the guys used it as an opportunity to talk.

Factors – took ages. Students are much more comfortable with answering questions and providing solutions. They are also confident in disagreeing with the teacher and providing support for their answers.

In the new topic that they are doing (Statistics) they are still demonstrating the communication skills developed in the earlier topics. They are communicating about maths much more than the form four class last year. Students are talking about maths, even if it is not a specific communication activity.

Bingo - #1 went well. #2 a bit harder, instructions to teacher not clear. Still some confusion over factors of 2a. Confusion between 2a and a^2 .

Matching exercise on powers went well. Generalised the rule. Mix up with $a^2 \times a^3 = a^5$ wanted to do 2 x 3, but eventually someone said it was 2 + 3. Used the manipulatives to help find the answer. Not all needed to use the manipulatives.

Sequencing - picked up the multiplying quite quickly. Decided that the reason for the numbers out the front in 2 x 3 x a x a was to make it easier. However the algebraic fractions were a bit more difficult.

Expanding – no picture dictation, did the sequencing. Pattern was developed, but needed more work on negatives.

Bean salads, went well. They were good at, arguing, used the recording sheet. All groups but one student on task. Congratulating one another on the answers, extra good lesson. The hands on were good. Break from the book work type of problem. Wanted to do for the rest of the term. The second sheet, writing the equations, was too hard for half a dozen students, needed to pick and choose the easier problems for the slower students to do. Split the salads amongst the groups and give each three or four salads to do. You can grade the problems by the level of the students.

Backtracking – They enjoyed the backtracking introduction. Sequencing activity took a handful of students longer. Also a disruptive return to class of a student. Some groups wanted to solve it with out doing the sequencing.

Questions

Pip - Do you think your group this year picked up on the ideas better than your group did last year.

They worked out more concepts for themselves, they actually thought about it, they didn't just assume what I said was correct.

They were more actively involved?

Yes

Did you feel like you were doing less blackboard teaching than in the past?

Yes – heaps less.

Test results – there was improvement in all of them.

Yes

Q2 – Does the use of LTL activities improve students' willingness to communicate in the mathematics classroom?

Yes, definitely, and it has continued into the next unit.

Q3 – Has what they are saying improved? Is it better, more coherent mathematically,

Some of the students are still the same, three students have shocked me in terms of their response, very confident, words were really mathematical, they were telling everyone else. A group of girls are talking more, not all mathematical words, but talking more.

What about writing?

Heaps more practice on writing. Need good examples of writing.

Better on verbal communication vs. written communication.

Yes

Book work generally

Better and neater than earlier in the year. Most still want to put answers only. A couple have real pride in their work, and often do more than is needed.

Summary – your perception is they are more willing to communicate, and most of that is verbal, and their verbal communication is improving.,

And it is not just with each other, they are more willing to communicate with me as well. There are times when they would ask a question, the teacher would ask what is the problem, and then they would say "never mind", and they have stopped doing that.

Any other comments

Best unit I have ever done and good response and output from the students.

A7.2 Notes from first meeting with Teacher Two

The concrete activities in Substitution and Collecting Like Terms only need to be revisited, rather than teach, as there is a good coverage of this in the Form 3 Algebra unit at this school. Appropriate for the Remedial level student.

The clines were a different activity, and would have liked more to do. More difficult ones. Links weren't necessarily established between the calculator activity and the clines.

The first grid was too easy.

Concrete materials – some students became independent quickly. This would probably be due to them using them in Form 3. If they were not so confident they continued to use the concrete materials as needed.

Picture dictation. Would be more suitable as a Form 3 activity at this school. Was too easy for Form 4. The students worked in pairs and gave one another lots of helpful instruction. Purposeful.

Factors and bingo boards. The first bingo board was easy and completed quickly. The second one was more difficult, but good for factorising. Discovered that the teacher instructions were not sufficient to do the activity in the way intended. The big problem was needing to cross off all of the factors of 8ab for example, when they could have just crossed off 2, a, b if they wanted.

Matching exercise for powers of a was no problem. Approximately 50% of the students doing the indices work used the concrete manipulatives.

In the matching activity in multiplying numbers and letters the students could match the correct parts, but didn't know what they were doing.

Three bean salads. Worked in groups. Some problems with part two finding the equations.

100% success with the backtracking.

Did the review activities also.

Students had a good idea of the meanings of expand and factorise in the review activities. They found expression, term and equation the most confusing terms to define.

Trigonometry was easier because the students had done the algebra unit. The algebra unit feels good in comparison to last year. Made an effort with language, but feels that they always did anyway.

A7.3 Notes from meeting with all Teachers at School B

A7.3.1 23 October – based on taped conversation

Pip – run down on how it went with your classes

Teacher Three – School B

I thought it went really well with one of my classes, which was the remedial class. They got the most benefit from it. With my other normal class they actually found some of it quite repetitive.

But on the whole they actually enjoyed the activities and working from the overhead instead of just the whiteboard, and they enjoyed doing the sheets, and the little game things. They enjoyed it, but sometimes found it quite repetitive, whereas my remedial class loved it and could keep on going and doing it. I think they benefited from it the most because as they were doing it they were picking up the skills that I was trying to get across to them.

Was it repetitive with your other group because they had met the skills before?

Yes I think so

That was the same discussion we had wasn't it (addresses to Teacher Two – School B) When *I wrote this I wrote it for the School A group, who had very little Form Three Algebra and I think that you were talking about next year changing it around quite a bit.*

Teacher Two - School B

The teacher of the accelerate thirds hasn't done it yet with them, she will do it with them, and I think she will get the most benefit out of all of us. Those kids will be much better when they go into the fourth form accelerate class, I think, she has only just started it.

My fourth form remedial class like all the activities and they would come in and say could we do that thing again, but they were actually learning from it so I didn't actually mind doing things over and over again with them, but the other class got a bit bored after the first go sort of thing.

I think they were meeting the skills that I had set out for them.

It could be that the activities need to be upskilled or go to a higher level, if you wanted to use that type of activity.

In terms of the instructions did they make sense, or did Teacher Two need to interpret a lot of them.

They made sense, and because she had made the activities up and just handed me a box and all the base work was done. (General discussion on making up the resources.)

I think it was quite well explained in the book. It was very easy to follow.

Both Teacher One and Teacher Two have worked with me, and they would have known a little bit more about where I was coming from than you would have, and that is why I was interested in how you found it.

I found it was quite clear.

In terms of understanding, you were saying that your remedial group was getting understanding from it.

Yeah, although it probably won't reflect in their test marks, as they don't sit test well. When we were actually doing it as a group, we did quite a bit of it as groups, rather than as pairs, I got them into fours and you could hear the better ones explaining it to the other ones, and they were explaining it so well, I was just so amazed. There is about four or five who are a little bit higher than the others and they were really good and they caught on really well.

Is that unusual for that group (the remedial group)?

I haven't seen it in any other topic this year.

Would you have done activities similar to this in other topics?

Sort of year, I did some matching stuff with the trig, and stuff like that, but not quite the same, and I've done other sort of activities, and mainly done it in pairs. But I thought I would try it in fours and put one good kid in each group, and that seemed to work really well, because they are not like good kids if you like they are quite slow. I just found that they seemed to have more of an understanding and I took it really slowly. With my other class what I did in one lesson with them, I would often spread over two lessons with my remedial and I think that worked heaps better.

They are as a remedial class, because I did a couple of critiques on that class, and they are a very focused class, they're not what I would call ordinary, you've got them working really well.

The Principal picked up on that as well when she did my critique, she couldn't believe for a remedial class how intent they were on listening and doing things like that

And actually doing, that this was a class, it wasn't a cop out, that they were actually doing valid stuff.

(The students sit the same test as the rest of the fourth form, and usually score low; this doesn't reflect how far they have come.)

(A test suited more to their ability level would) They could actually see their success, because they have come so far and it's a real shame sometimes just seeing that they get 30% in a normal test, but that to me is really good, so yeah.

So understanding with the remedial group you felt was good, what about the other group?

I thought they had heaps of knowledge from third form and I found that they caught on really quick, most of them, others of them struggled a bit and couldn't come to grips with the different concepts of it, but on the whole they picked it up quite well. They did comment to me that they enjoyed the activities, cause it was something different than just doing the normal book work sort of thing. I just felt that they had a lot of previous knowledge, far more than I would have expected. I don't remember my fourth form classes last year having the same sort of knowledge of it. They seemed to pick it up quite well.

Last year I thought the same, I thought gosh, its not that we hadn't done the work in the third form, it's just that the third form unit is much better now and everyone is onto what is happening in there. So the kids are getting more from their third form unit, and they're retaining it obviously. Some of them not immediately, but as soon as you start to do the activity its like oh I remember this, I know how to do this.

Maybe it is also because the form four unit this year is based on the third form unit, whereas last year it wasn't.

Teacher Two can we just go through the understanding bit in terms of your classes, just to reiterate how you felt about the unit. I have a note there about the trig unit you felt was handled better this year, than in previous years, was it because you had done the algebra first?

Because we had done the algebra unit first I could then say, this is where we need algebra, so we are not just guessing, how do we find out what x is, it's on the bottom now, so what do we do? We all did those types of things with them this year.

It would have been better for me to do algebra before trig?

I don't think it would have made any difference, because I did measurement before number this year, and it all ties in, so if I had done trig first, then done algebra, I would have talked retrospectively about where would we use it. I think that it was quite good, and my feeling at the time was that the kids were getting considerable success and they were doing something that they understood already. They do need lots of practice. I ran out of time.

At the time yes I thought this is great, and some of the activities I thought yes they really can do that really well, but then when we took the activity away they hadn't got what it was that we were doing. Had to go back and do that in a different way. They enjoyed the activities, but they didn't really want that many. They found by the time I was getting the last one out they would go, sigh, *and that became quite negative. I was the only person who didn't want the beans and the cardboard (the students did).*

Definitely we would take that and use as a base, our kids are coming stronger into the fourth form and we would keep some of those activities and just put different ones in. We only had four weeks for that module, and we could easily have five or six. It is a real problem only having four periods a week, and all the interruptions we have had this year.

The thing that I have noticed with both Schools is that we have tried to cram this topic into four weeks, and haven't really done it justice, because it is so critical.

I would like to move onto the communication side of it. The communication is the hard one for me because I can't get the evidence from the students. One of the big things for me was for them to be communicating mathematical ideas well, using good language, and not necessarily maths language.

Were they communicating more, where they more prepared to talk in the class than they had in the past, and what was the quality of that?

My remedial class doesn't like talking at all. They don't like, they like to do it all by themselves, because they are quite embarrassed if they get it wrong and that showed through. My other class was quite willing to, every time you asked a question; almost every hand went up. They were quite willing to tell you what they got, and how they got it and how they did it and things like that. Most of the time they didn't use mathematical terms,

But it was quite valid language that they were using. Everyone could understand.

Half the time I'm up on the board, and I'm not using proper mathematical terms and I say what are we doing here, and they say oh we've got to take x from that side and they could explain it back to you, and they knew what they were doing, because when you gave them the exercises they could do it, but my remedial class are not big on that.

There are a couple of issues with the remedial class is that often those kids actually need to work like that because if you give them too much freedom, they have not self control at all, and that's why they're like my 6 maths one class. Which is absolutely abysmal, they are either working flat out, or they are not there at all. This group that are coming through will probably be much better, they have really good base for working. I was really impressed when I saw them.

My other class likes to talk about things, they are a nice bunch of girls and they are quite competitive. In the classroom doing exercise they are quite competitive, and they, if I ask a question and I just pick somebody the others go, ooo, because I picked her, and when I reward the contribution they all say I had that as well. I found it more with the algebra and trig, they were quite willing to do it and write it on the board.

(Discussion about class structure in the school.)

That was with the Algebra unit (providing success for students) if we did a worksheet, you know the tick ones, well we were just working them out, and when we went through they were getting them all right.

A7.3.2 Notes from Teacher Four Meeting – Taped conversation.

Pip – What I want to talk about is the three questions for my research. Their understanding, their ability to communicate and their willingness to communicate.

I found at the start of the algebra unit that they remembered what we did in form three. A lot of it was revision for them. But we didn't get onto quadratics.

I think they are quite happy with algebra.

They came in with good third form knowledge, how did you feel about the stuff is newish. What was your feeling at the end of the unit? Do you feel they had a good understanding of what they had done during the time?

I think they had. They had good marks,

A progress on from third form?

Yes. They still have to try and accept that x and y; letters are standing for numbers. I think that's just practice.

In terms of when they talk to one another, were they talking more during this unit, focusing on mathematical things, more so than in the past, and was the language that they were using, even if it wasn't mathematical, was it good explanations, or good... were they communicating with one another using good language?

They were communicating, they do that with all the topics, and they talk amongst themselves. I actually encourage that. They do it in any topic. I was quite happy with the marks.

The cumulative test, they can't remember over a long time. They were adding unlike terms.

I was quite happy with the algebra unit.