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Graphical calculators in the classroom.

**“The Effects that Graphical Calculators have in
the Enhancement of Mathematical
Learning in the Classroom.”**

**A THESIS PRESENTED IN PARTIAL
FULFILMENT OF THE REQUIREMENTS FOR
THE DEGREE OF MEdSt
(Mathematics)
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Abstract

The graphical calculator is a phenomenon that is beginning to become a favourable mathematical learning and teaching tool in New Zealand secondary schools. Discussions about the use, role and the impact that this technology in secondary schools have, and will continue to be a topical issue for some time to come.

Technology use in today's mathematics classrooms is of particular interest due to current curriculum reforms. The graphical calculator is a fairly recent technological tool that can be added to the array of mathematical tools already at the disposal of both the teacher and the student.

This study examines the use of the graphical calculator in classrooms to gauge whether it has the potential to assist in the learning of mathematical concepts and to see if students become more active learners when using graphical calculators.

The graphical calculator can be used in the classroom, in as much the same way as an overhead projector or even a piece of chalk. It is a tool for illustrating a concept, opening up avenues for investigation and can enhance the learning that is going on. Secondary school mathematics today is much more than the acquisition of arithmetic and algebraic skills it encourages understanding. The graphical calculator can be considered a powerful, pocket computer and successful users require a higher level of understanding than that which is required for rote or skills based learning.

The introduction of the graphical calculator into classrooms confronts existing mathematics' content and methodologies in the imparting of mathematical knowledge from the teacher to the students. With this in mind, teachers in New Zealand need to react and prepare for this impending change.

This study looks at the impact the graphical calculator had on eight secondary schools, the mathematics teachers inside these schools, and the way in which the graphical calculator was used in their classrooms. The study also examined the implementation of the calculators and the effects that they had on teaching and the mathematical programme delivered to students in the classroom.

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Chapter 1

Introduction

“Computers and graphic calculators should be to mathematics teachers as what laboratory equipment is to science teachers.”

(Demana & Waits, 1990, pp 29.)

Technology in schools.

Discussions about the use, role and impact of technology in secondary schools have, and will continue to be a topical issue. Technology, and in particular, a student's own personal technology, needs to be accessible and relevant to the needs of the student and should support and encourage their mathematical learning. In particular, the graphical calculator represents 'hand-held' personal technology that is accessible to most students today. It is becoming a specialist tool for secondary school mathematics and science students and very soon developments in graphical calculator functions will see them looking and behaving like a computer (Jones,1995).

As mathematics curriculum developments and statements are made by the appropriate government agencies, the delivery of this mathematics curriculum will be constrained by the 'tools' that are available to the classroom teacher. Until recently, the principal tools in mathematics have been the scientific calculator (calculation tool), a ruler, protractor and

compass (measurement tools) and a vast collection of 'pencil and paper' procedures (algorithmic tools).

Computer software can also be added to these tools for the teaching and learning of mathematics. It is noted that at the time of writing this thesis both Massey and Victoria Universities were using computers in some of their undergraduate mathematics courses, promoting mathematics laboratories and tutorials, rather than lectures, and supplementing these with voluntary tutorials. Secondary schools generally have classroom suites of computers, which must be shared by all departments and hence, restricted access to computers is found by teachers in all subject areas.

The predominance of manipulative procedures has been considered the backbone of the secondary mathematics curriculum and is currently under a challenge from:

- (i) spreadsheets,
- (ii) computer algebraic and symbolic manipulator software, and
- (iii) the graphical calculator.

If these types of technology can produce the accuracy, reliability and quick results that are required, then the 'hand-procedures' cannot possibly compete with these technologies competitively. So, why does the classroom teacher spend so much class time insisting that students master the procedural mechanics of these manipulative algorithms? (Jones, 1995)

Unfortunately, many software applications on school computers are 'fitted' into the mathematics curriculum and the software may not generally have been designed for classroom use, an example of this being the spreadsheet.

To some extent the role of this software in a school is the preparation of the student for what some would call 'worldware' - the education of the student in readiness for their likely encounter with the software once they leave school. Hence, the spreadsheet was not designed specifically with the adolescent learner in mind (Jones, 1995).

In the secondary mathematics curriculum, computer software is readily available but at a cost, namely the required packages and site licences and there is also the issue of access to the school's computer suite. At times, teachers experience frustration in attempting to fit this technology into their existing mathematics courses and for some mathematics teachers the computer room still remains a 'caged and dangerous' place. It is usually a resource that is available only in small and / or poorly - timed doses, with very little support, due to the lack of computer personnel present on the school site. So the notion of having a class set of twenty or thirty graphical calculators rather than one computer, with comparable recent pricing and cost effectiveness, becomes a choice that possibly could be considered quite straight forward.

Hand held technology.

The graphical calculator has revolutionised 'hand-held' technology since the mid 1980s and is beginning to make its impact felt on the New Zealand secondary schools mathematics curriculum and its potential to change classroom dynamics when it is introduced, needs to be examined carefully. The introduction of the graphical calculator into classrooms confronts existing mathematics' content and methodologies in the imparting of

mathematical knowledge to the students, and with this in mind, teachers in New Zealand need to react and prepare for this possible change. The New Zealand Qualifications Authority in their external examinations and Unit Standard Assessments say that graphical calculators are able to be used by the students to gain credits.

The name 'graphical calculator' is somewhat of a misnomer. Today, this name only reflects one of the major features of its earlier development as its functionality has been refined and modified. Today's various models on the market shelves offer numerous extra integrated capabilities that more or less resemble features found in some computer software:

- * the ability to draw graphs, both algebraic and statistical.
- * a programming language.
- * comprehensive statistical tools.
- * the ability to store and retrieve text and graphical information.
- * linking facilities to computers and printers and other calculators.
- * word processing features.
- * numeric solving and symbolic manipulation features.

The introduction of this technology into the classroom cannot be considered a trivial one and many teachers are already very enthusiastic about its possibilities, while others remain sceptical about the graphical calculator's introduction into their own classroom teaching practices. This could be because of similar fears that existed in the 1970s when the scientific calculator disintegrated the common use of both the slide rule, and to some extent, the statistical and logarithm tables. The use of these historic mathematical tools that contained so much history, covering the last three or

four hundred years was now under threat of being lost to new technology. Were teachers to lose this history, or was it that the scientific calculator began a new phase in mathematical education and a historical event ?

The introduction of a tool such as graphical calculators into New Zealand classrooms necessitates the participation of both mathematics teachers and mathematics students, in readiness for the technological revolution that is happening worldwide, as well as in New Zealand. One must recognise that over the last fifty years or so, the development of the silicon chip and micro processors has influenced dramatic changes in global technology, industry, health and science. In consequence, our lives within society have altered radically too.

Since the 1970s electronic calculators have made an impact on the mathematical world. The computer also has allowed the mathematician to exploit the power of the spreadsheet, drawing sophisticated 3 - D algebraic and statistical graphs. Computer applications such as *The Maple*, *The Theorist*, *Mathematica* to name a few, now promise to do for algebra and calculus what the 4 - function calculator did for addition, subtraction, multiplication and division. The graphical calculator too, is beginning to free the student from the drudgery of calculations and graph plotting, long statistical calculations, rectangular - polar conversions, and finding matrix inverses by Gaussian reduction methods. Time consuming trigonometrical computations are now an easy sequence of 'key strokes'.

As stated in the Mathematics in the New Zealand Curriculum (MinNZC) document, it is assumed that technology will be available and used in the teaching and learning of mathematics.

"In an increasingly technological world the students should be encouraged to become innovative and flexible problem - solvers."

(MinNZC,1992, pp 7)

and

"This curriculum statement assumes that both calculators and computers will be available and used in the teaching and learning of mathematics at all levels."

(MinNZC,1992, pp 14)

The graphical calculator and its' role.

Why should students be encouraged to use graphical calculators?

1. The graphical calculators can be an essential part in the relating of real world problems to mathematical modelling.
2. They make school mathematics more interesting.
3. They save time on routine calculations. This time can be better spent focusing on the mathematical ideas.
4. They help to reinforce the basic number facts.
5. Linking the 'rule of three' - (i) the numerical
(ii) the graphical
and (iii) the algebraic or generalising.

Graphical calculators - what are their role in mathematics? The use of graphical calculators is fast becoming the 'pencil and paper' methods of the 1990s and the use of this technology has an appeal to students in today's mathematics classes too. Most of the students today have been educated with a computer readily available in their classroom at their primary,

intermediate and secondary school, or in their own home. However, the availability of 'hand-held' calculators with graphing capabilities has only happened in the last 10 years or so. Up until then, graphing had been restricted to either expensive computer packages or the continued use of hand methods and algorithms that the students learnt for the calculating and plotting of co-ordinate points.

Today's graphical calculators are not just restricted to graph plotting and curve sketching. With their multi - functional icon displays, they open up access to statistical data collection and their related displays, regression modelling, solving equations by matrices or by linear systems, a programming language and base operations, along with the capacity to share information with other calculators, printers and personal computers.

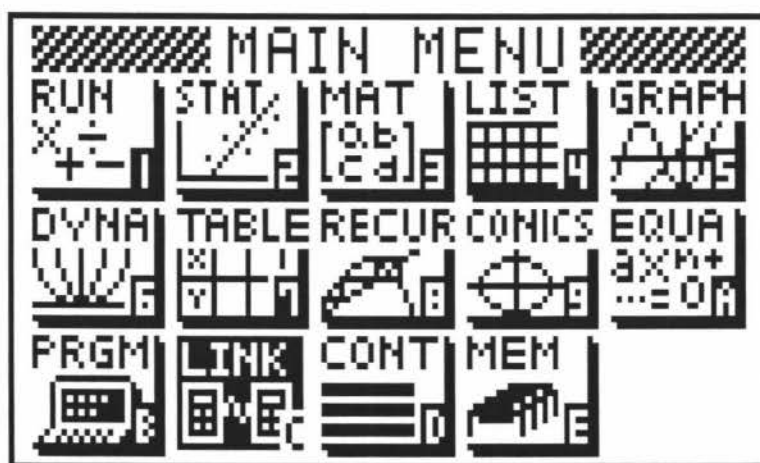


Figure 1: Main Menu for Casio cfx 9850 Graphical Calculator.

Graphical calculators have fallen steadily in price and are now at a price accessible to most who wish to take advantage of the technology. Some modern graphical calculators examples are the SHARP EL-9300, TEXAS TI 82 and the CASIO 7700 G, which all offer many computer like features.

Included in their functions are:

- * Easily written and stored programmes, used for example in evaluating programmes for evaluating limits, areas under curves, probability simulations and the solutions to equations and mathematical games.
- * The ability to graph curves such as cartesian, polar or parametric equations; graphs can be overlaid for comparisons or traced for finding 'zeros', and the interesting areas can be 'zoomed' for finer detail.
- * The ability to show powerful statistical functionality, including graphical display and several regression models.
- * Able to add, subtract, multiply, transpose and invert matrices of a reasonable size.
- * Manipulation of complex numbers.
- * A Performance that is better than most scientific calculators, showing both the input and output of calculations simultaneously - the input can also be edited.
- * Data that can be printed, stored and transferred to other calculators and to computers.

With these calculators the students can:

- * Solve equations by graphical methods including simultaneous equations and those not solvable using algebraic methods.
- * View the relationships between 'families' of functions.
- * See algebraic and graphical relationships.
- * Focus on the graph and the information that it contains rather than the manipulations needed to draw the graph.
- * Solve realistic problems using matrices.
- * Perform statistical investigations rather than statistical calculations.

Even as this research takes place, new graphical calculators are becoming available with Casio introducing a cfx 9850 model, Texas Instruments promoting their TI 92 and Sharp launching a 9600 model which are all superseding earlier models available in New Zealand in the past three years or so.

Implications of graphical calculator use.

Although the phenomenon of the graphical calculator is still relatively new, there are possibly four different view points for examining their use in the modern classroom.

- (1) Graphical calculators and student achievement.
- (2) Graphical calculators and student conceptual understanding.
- (3) Graphical calculators and problem-solving.
- (4) Graphical calculators and classroom dynamics.

Graphical calculators have the potential to dramatically affect the teaching and learning of mathematics in New Zealand schools and also to assist the learner into becoming a better problem - solver. They also facilitate the changes in roles of the student and the teacher in the classroom reflected in MinNZC. It can act as a catalyst for, and not as an obstacle to, mathematics learning, Dunham & Dick (1994). Questions that arise because of graphical calculator use can be:

- * "What aspect of the graphical calculator brings about improved understanding?",

- * “What paper and pen skills retain or lose their importance?”,
- * “Can technology use impede understanding?”,
- * “What accounts for the success or failure in implementing the use of graphical calculators?”,
- * “Will a particular gender favour the use of the graphical calculator?”

These questions are some of the starting points of consideration for this study.

Summary.

To enable students to keep abreast of developments in technology and its association with mathematical learning today, the graphical calculator is perhaps a vehicle to utilise. We, as educators, must consider the demands of technological developments, as we all move into the twenty first century, and consider their implications in the learning and teaching of mathematics today.

Are graphical calculators going to be used more and more ? Will they be considered as basic equipment for the mathematics learner, no matter at what level of mathematics the students are studying?

“A graphics calculator is best regarded as part of the standard working equipment of anybody doing high school mathematics into the third millennium, rather than a special item, reserved for ‘calculator activities’. In particular, even brief contact with a modern graphics calculator will make it clear that many

aspects of mathematics other than graphing functions are likely to be affected by this change to the standard mathematical tool kit"

(Kissane, 1997, pp (iv))

If this is the case, the graphical calculator should be used and integrated more and more into mathematics classrooms. Effective use of the graphical calculator can permit the kinds of teaching / learning environments that most teachers long for and also encourages the aims and objectives of MinNZC and keeps within the spirit in which it was written. Technology is becoming a standard tool for problem solving and decision making in science, business, government and industry. It causes significant change to what we ask and empowers students to learn, and it can assist their understanding of mathematics.

Therefore, the graphical calculator is cost effective, portable, and multifunctional, and can improve student contact time with technology, compared to the limitations to the computer. It is a valuable adjunct to reinforcing the mathematical concepts being learnt and should have a positive effect on classroom procedures and dynamics if its introduction is handled in a positive manner.

Chapter 2

History of Calculating Aids.

“It is not fit that worthy men should spend their time in calculation”

G. Liebnitz.

The calculator evolution.

A brief look at mathematics and technology through the ages shows that throughout the centuries it has been a human endeavour to search for tools that will help perform the arithmetic side of mathematics more easily and quickly. The history of mathematics and its applications to trade, surveying, navigation and astronomy, is riddled with links to increasingly sophisticated measuring and calculating devices. Mankind has progressed from tools such as the abacus, logarithm tables, and the slide rule to analytical computers and sophisticated calculators. All of these devices, modern and useful in their era, served the purpose of freeing the student, the scientist and the mathematician from the drudgery of long tiresome calculations.

The first steps in numeration.

The earliest of civilisations did not have the same needs for a number system as we do today, as the recording of numbers was more necessary as a cultural development, utilising the ability to count or as a way for

individuals to enumerate. For example, a shepherd could put one pebble in a bag for every sheep that he released from a pen to graze in the morning, and then he would remove one pebble from the bag as each sheep returned at night. If a pebble or two remained in the bag, he knew that he had to go and look for stray sheep.

First attempts of some numerical notations were generally simple pictorial systems. For example, five animals drawn on a slate or cave wall could represent five cattle beasts or seven huts representing seven families or groups. Classic examples of numerical systems were the ancient Egyptians and Romans, who realised there was a need for a numerical system for recording daily activity (trade) that utilised writing materials (clay slate) that were available.

The development of mathematical symbols and numerical systems slowly became integrated as more and more cultures interacted with each other. This interaction brought a number of different cultural numerical systems together, and were modified over a long period of time, to what we now use and call the 'Hindu - Arabic' numerical system.

Early calculations.

Demosthenes (384 - 322 BC) wrote about a need for pebbles for performing calculations that were too difficult to do in your head. In 1590, Joseph de Acosta, a Jesuit, recorded some facts about the Inca culture in North America:

“ . . these Indians make use of their kernels of grain. They place one here, three somewhere else and eight I know not where. . .As a matter of fact, they are better at calculating what one is due to pay or give than we should be with pen and ink.”

(Newman, 1956, 463 - 464)

Archeological excavations have revealed that the Aztec abacus, circa 900 - 1000 AD, was made up with counters using kernels of maize threaded through a string, and was attached to a wooden frame.

The Abacus.

The abacus, or counting frame, is an excellent example of a digital calculating aid that was first mentioned by Herodotus (c. 450 BC) as an aid used in Egypt, and is considered to be the first real calculating device. Addition, subtraction, multiplication and division can be performed on the abacus. Europeans also used the abacus and a table abacus was discovered in Greece, on the Island of Salamis, while in Britain, the table abacus appeared to be used extensively in schools. Robert Recorde (c.1510 - 1558) was particularly influential, and his book *The Ground of Artes* was published around 1540 and set the style for books on arithmetic. It was in print to the early 1700s and showed that abacus arithmetic was being taught in schools throughout this period (Buxton, 1984). As Hindu - Arabic numerals became firmly established in Europe, the use of the abacus slowly diminished.

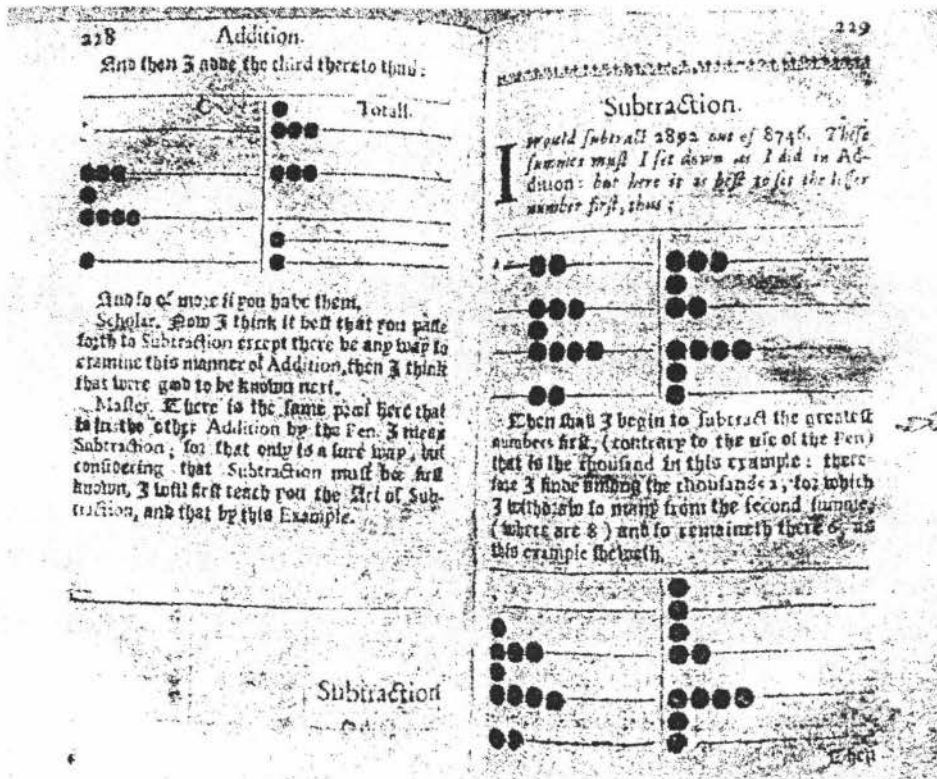


Figure 2: A page from Robert Recorde's book on arithmetic. (Aspray et al, 1990, pp 13.)

The Oriental abacus appears to have been firmly entrenched in their society dating back to as early as 500 BC, with Japanese use appearing to have begun around 1600 AD. In China, it was called a *swan pan*, and in Japan, a *soroban*. The Chinese abacus consisted of five beads in the lower bar and two beads on the top bar of each rod, while the Japanese abacus consisted of four beads on the lower bar and only one on the upper bar of each rod. It is interesting to note that the abacus is still widely used in the orient today, in spite of technological developments (Aspray, Bromley, Campbell-Kelly, Ceruzzi, & Williams, 1990).

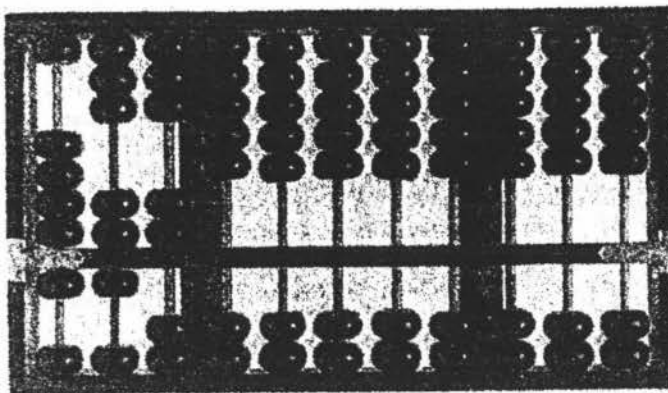
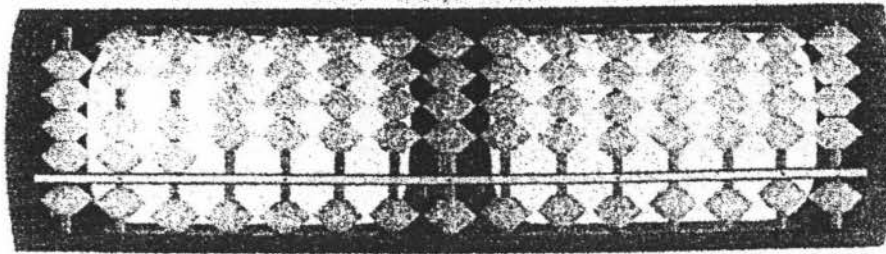


Figure 3: The Chinese abacus the *swan pan*, the Japanese abacus the *soroban*. (Aspray et al, 1990, pp 14 & 15.)

Bones and Tables.

John Napier and his 'bones' (Ivory rods), *Rabdologia*, appeared in 1617, three years after his description of logarithms. This very old method of multiplication is based on the *Gelosia* method, believed to have originated from India, and consisted of a matrix-like grid, with the multiples placed in triangular spaces. Addition is done diagonally, with the result being the answer to a multiplication problem.

For example,

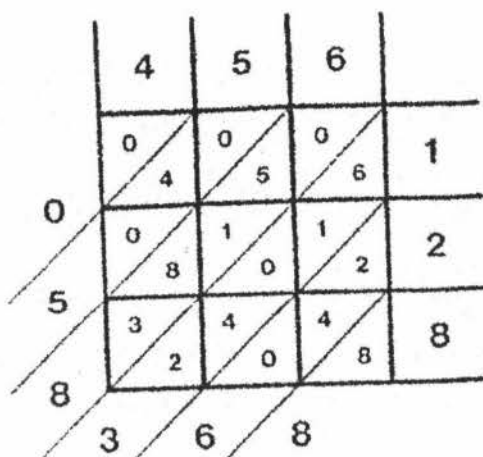


Figure 4: The Gelosia method of multiplication.

(Aspray et al, 1990, pp 18.)

Napier's bones consisted of a number of rotating rods that utilised the Gelosia principles and large multiplication problems could be performed using the 'bones'. By examining the rods closely we can see they are simply the multiplication tables themselves (Buxton, 1984).

0	1	2	3	4	5	6	7	8	9	√	√
0 0	0 1	0 2	0 3	0 4	0 5	0 6	0 7	0 8	0 9	0 1	0 3
0 0	0 2	0 4	0 6	0 8	1 0	1 2	1 4	1 6	1 8	0 4	0 6
0 0	0 3	0 6	0 9	1 2	1 5	1 8	2 1	2 4	2 7	0 9	0 2
0 0	0 4	0 8	1 2	1 6	2 0	2 4	2 8	3 2	3 6	1 6	0 8
0 0	0 5	1 0	1 5	2 0	2 5	3 0	3 5	4 0	4 5	2 5	1 2
0 0	0 6	1 2	1 8	2 4	3 0	3 6	4 2	4 8	5 4	3 6	2 1
0 0	0 7	1 4	2 1	2 8	3 6	4 2	4 9	5 6	6 3	4 9	3 4
0 0	0 8	1 6	2 4	3 2	4 0	4 8	5 6	6 4	7 2	6 4	5 1
0 0	0 9	1 8	2 7	3 6	4 5	5 4	6 3	7 2	8 1	8 1	7 2

Figure 5: A set of Napier's bones. (Aspray et al, 1990, pp 18.)

The Genaille - Lucas rulers were developed by a French engineer and this device differed from the format used in Napier's bones. The rulers consisted of a 'set of strips' which were headed with the numbers 0 through to 9, with each of these rulers divided into 9 parts, with several digits inscribed in them.

	0	1	2	3	4	5	6	7	8	9
1	0	1	2	3	4	5	6	7	8	9
2	0 1	2 3	4 5	6 7	8 9	0 1	2 3	4 5	6 7	8 9
3	0 1 2	3 4 5	6 7 8	9 0 1	2 3 4	5 6 7	8 9 0	1 2 3	4 5 6	7 8 9
4	0 1 2 3	4 5 6 7	8 9 0 1	2 3 4 5	6 7 8 9	0 1 2 3	4 5 6 7	8 9 0 1	2 3 4 5	6 7 8 9
5	0 1 2 3 4	5 6 7 8 9	0 1 2 3 4	5 6 7 8 9	0 1 2 3 4	5 6 7 8 9	0 1 2 3 4	5 6 7 8 9	0 1 2 3 4	5 6 7 8 9
6	0 1 2 3 4 5	6 7 8 9 0 1	2 3 4 5 6 7	8 9 0 1 2 3	4 5 6 7 8 9	0 1 2 3 4 5	6 7 8 9 0 1	2 3 4 5 6 7	8 9 0 1 2 3	4 5 6 7 8 9
7	0 1 2 3 4 5 6	7 8 9 0 1 2 3	4 5 6 7 8 9 0	1 2 3 4 5 6 7	8 9 0 1 2 3 4	5 6 7 8 9 0 1	2 3 4 5 6 7 8	9 0 1 2 3 4 5	6 7 8 9 0 1 2	3 4 5 6 7 8 9
8	0 1 2 3 4 5 6 7	8 9 0 1 2 3 4 5	6 7 8 9 0 1 2 3	4 5 6 7 8 9 0 1	2 3 4 5 6 7 8 9	0 1 2 3 4 5 6 7	8 9 0 1 2 3 4 5	6 7 8 9 0 1 2 3	4 5 6 7 8 9 0 1	2 3 4 5 6 7 8 9
9	0 1 2 3 4 5 6 7 8	9 0 1 2 3 4 5 6	8 9 0 1 2 3 4 5	7 8 9 0 1 2 3 4	6 7 8 9 0 1 2 3	5 6 7 8 9 0 1 2	4 5 6 7 8 9 0 1	3 4 5 6 7 8 9 0	2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8

Figure 6: A set of Genaille - Lucas rulers.
(Aspray et al, 1990, pp 21.)

The major computational problems of Napier's era were mostly related to astronomy, navigation and horoscopes, and led scientists and mathematicians to devote their time to the development of trigonometry. Logarithms, from the Greek phrase meaning 'ratio number', were the brain child of Napier and he developed a series of tables published in 1614, which had remarkably very few arithmetical errors contained in it. The small text called "Logarithmorum Canonis Descriptio", meaning 'Description of the Admirable Cannon (Tables) of Logarithms', gained popularity by the mathematicians of that time. However, it was Henry Briggs (1561 - 1631) who really popularised the concept of logarithms and he made a significant contribution in clarifying the logarithm tables in reading and usage. Johann Kepler too, saw it's usefulness and he also published a set of logarithms and within a very short period of time, the use of logarithms was worldwide (Aspray et al, 1990).

The Slide Rule.

The slide rule was developed by Edmund Gunther, who had been intrigued by the work of Briggs and the logarithm, and saw a use for logarithms in his primary interest, navigation. Initially, it involved trigonometric calculations using Gunther's 'Line of Numbers' and calculated tangents for each minute of a quadrant. Published in 1620, 'Line of Numbers' relieved the burdensome calculations of finding one's position at sea.

William Oughtred (1574-1660) later noted that Gunther's 'Line of Numbers' required the use of a 'pair of dividers' and came up with the idea of two scales marked on the side of two pieces of wood. The pieces of wood could

be slid relative to each other and thus do away with the dividers. Therefore the basic principles of the slide rule utilised the properties of the logarithm. The slide rule continued to be refined until in 1850, when Amedee Mannheim (1831-1906) designed a slide rule with a movable double sided cursor, which is standard in the 'modern slide rule' that was in use in schools until the 1970s.

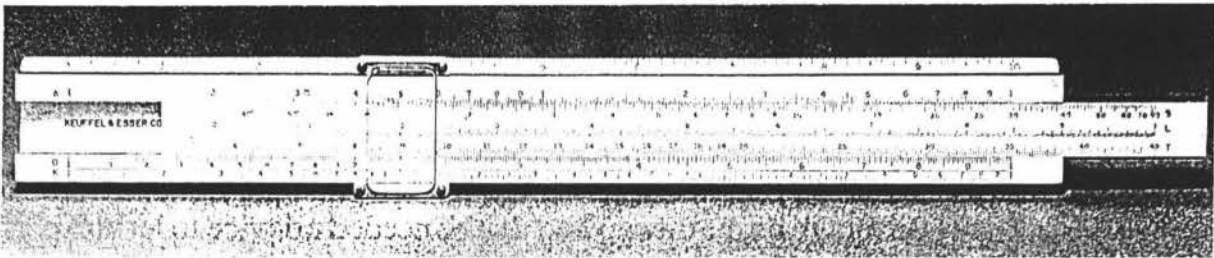


Figure 7: A modern version of the Mannheim slide rule. (Aspray et al, 1990, pp32.)

With the development of the hand held calculator in the 1970s (which offered better accuracy and convenience) the slide rule and logarithm tables lost their importance in the instruction of mathematics, and their use subsequently declined.



Figure 8: A variety of basic and scientific calculators.

Calculating machines.

Wilholm Schickland (1592-1635), Blaise Pascal (1623-1662) and Gottfried Wilhelm Leibniz (1646-1716) were also interested in easing the burden of laborious calculations. Their inventions relied on gears, movable parts, springs, ratchets, cranks and levers, all working in unison to each other. In 1642 Pascal invented the first simple mechanical calculating device and later another example, Leibniz's machine developed in 1673, the 'Step Reckoner' could add, multiply, divide and extract square roots.

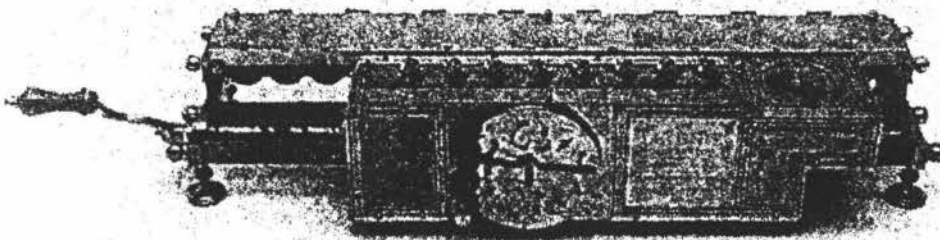


Figure 9: The Leibniz calculator. (Aspray et al, 1990, pp 49.)

A Swedish engineer, Odhner, invented a 'pin - wheel' method for mechanical calculations in 1878. From his work stemmed the decimal hand and electric calculating machines that were used worldwide until the advent of the electronic pocket calculator in the 1970s. These mechanical devices represented the magnitude of numbers by a set of physical quantities, for example the length of a shaft or the amount of a shaft's rotation. All of these forerunners to the modern calculating device focussed only on the four basic arithmetic operations.

Charles Babbage, in 1822, demonstrated his 'Difference and Analytical' machine, which also relied on gears and movable parts (Hyman, 1982). It was a mechanical contrivance that used interconnecting toothed wheels and employed similar principles to that of an odometer of a car. It provided support for his paper, 'Observations on the Application of Machinery to the Computation of Mechanical Tables'. Gaining some funding from the British Government, Babbage began to work on a larger machine and soon began to realise that there was a need for a program of instructions. Joseph Marie Jacquard! Jacquard, in 1780 invented the 'punch card', where by a pattern of holes controlled the operation of automatic weaving looms. This punch card approach was picked up some 100 years later by Herman Hollerith in 1887, who designed, compiled and tabulated data using storage in the form of patterns of holes in the punched cards.

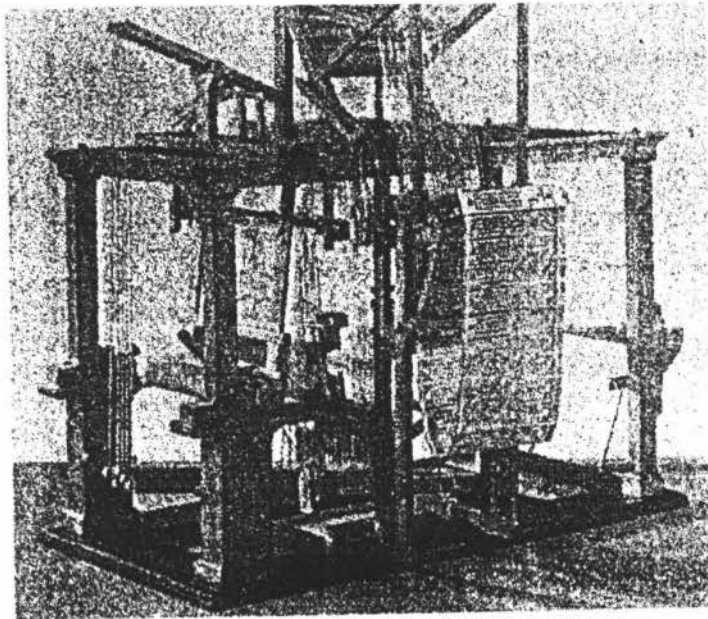


Figure 10: The Jacquard 'punch card' weaving loom. (Aspray et al, 1990, pp 87.)

Other calculating devices continued to appear like 'Logic' machines, an

example being made by Charles Stanhope (1753-1816) and his Logical Demonstration machine. Other inventions, such as Lord Kelvin's Analog - Tide predicting machine, harmonic analyser and differential analyser contributed further to the development of calculating devices.

A breakthrough in logic theory came with Claud Shannon in 1938 who discovered that there was an isomorphism between 'switching circuits' and 'propositional calculus'. Alan Turning, in 1937 gave the first description of a stored program computer and utilising Shannon's theory, revolutionised the punch card industry (Aspray et al, 1990). For example,

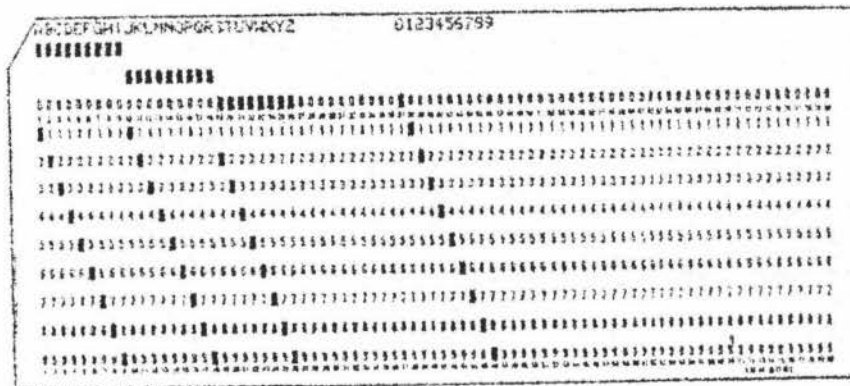


Figure 11: The 80 column IBM card.

Up until then, the analogue machine remained shaft rotation-orientated in design, but this important discovery by Shannon brought about the use of valves (vacuum tubes) and the invention of the electronic integrator, whereby voltage was used as an analogue quantity.

In the late 1940s and early 1950s, after World War Two, people like J. Von Neumann worked on logic design, and outlined proposals for computer design. Among these proposals were ideas about the storage of data in binary and the project began in 1946, which produced the Von Neumann

machine, named the Electronic Discrete Variable Automatic Computer (E.D.V.A.C.) (Tee, 1983).

Most of these computers consisted of thermionic valves and other components such as resistors and capacitors. They were big and required tender loving care, and had to be housed in rooms where temperature was rigidly controlled to minimise effects on the components.

In the late 1950s quality production of reliable transistors and other componentry meant that computers could use reduced voltage and also permitted miniaturisation of the computer. Today's sophisticated integrated circuits (I.C.) can perform calculations that required a room - sized computer some 30 years ago.

Modern types of calculators.

Calculators can be grouped into three categories nowadays:

1. The four function calculator. The cheapest, performing the four arithmetic operations - addition, subtraction, multiplication and division.
2. Advanced - function calculators, like the scientific and business calculators, where more 'features' like \log , \sqrt{x} and e^x appear. It contains specially designed circuitry for computing each such function and
3. The graphic, programmable calculators, containing more scientific features that can be used in combination or permutation and also has

the flexibility of being able to create your own programs.

In the mid-1960s, the development of the integrated circuit (I.C.) continued to be refined and semiconductor technology was used to fabricate complete logical circuits onto a single chip of semiconductable material also. Costs for components were tumbling in price and their reliability continued to be enhanced, meaning that substantial computing power could be achieved in a very small volume, with low power consumption and at a realistic cost. The classroom calculator of today continues to undergo some changes as logic boards and new techniques in their design and purpose is developed. For example, take a simple trigonometric calculation: Find the sine of 45 degrees.

Keying in this would require the calculator logic to accept:

$$45 \sin =$$

Now, with 'True Algebraic Logic' or 'Direct Algebraic Logic' the mathematical symbolism matches the inputted key stroking into the calculator, namely:

$$\sin 45 =$$

The graphical calculator is a further link in the development of technology for human use in the evolving development for better, faster and more accurate computational tools.

Summary.

As the twentieth century draws to a close and the new millennium approaches, mankind looks forward to an exciting new dawn, bringing with it new knowledge, new technologies and new discoveries. As we look back over the developments of the past, in science, industry, communications and transport, we can readily trace the importance of technology in these areas. As our knowledge increases, then so must technology develop in order to keep abreast of these new discoveries and changes. Technology has become the driving force in today's society and has transformed our social lives. No longer does a corporate need a secretarial pool or a multitude of bookkeepers to record daily activities, as personal computers and software simplify information processing and make the recording of data accurate and easy.

In our homes, technology has made our lives easier, but it has also had a dramatic impact on our lifestyles - electronic wizardry has changed the way we do our domestic chores, use our leisure time and enjoy our lives.

Developments in communications and travel now mean countries, governments and individuals are able to make instant contact, breaking down barriers of time and distance. Therefore it may be said that technology has transformed our lives in many ways but as educators, we need to consider how today's students are being prepared for the future - to teach them to use technology rather than letting the technology control them.

In the classroom setting, technology has had an increasing influence in the learning and teaching process, giving new ways of reaching solutions that

have not been available to us in the past. Students have now been exposed to computers in the classroom for some twenty years and the graphical calculator for ten years. These technological revolutions in schools are becoming an integral part of the teaching and learning programmes in classrooms. Growing confidence and competence in using this kind of technology is gaining momentum as schools realise their responsibility to use technology to acquire and build on students' existing knowledge.

Therefore as our knowledge increases in terms of global developments in technology, we as educators, are faced with the challenge of keeping pace with these developments and of course mastering to some degree the technological skills that are required. Awareness of the capabilities and uses of the new technology will encourage and help the students that sit before us, as well as keeping abreast of new learning.

Chapter 3

Comparisons between Australian and New Zealand Schools.

The New Zealand Scene.

The national curriculum for mathematics, *Mathematics in the New Zealand Curriculum*, (MinNZC) published in 1992, was phased in with year 1 to year 11 students being taught under the new curriculum by 1994, year 12 in 1995 and year 13 in 1996. The new structure in mathematics education in New Zealand schools was to reflect a fresh look at the teaching and learning of mathematics. Its aim was to provide a 'seamless' mathematics education, as the student moved from level to level, classroom to classroom, and school to school. This curriculum replaced the syllabuses *Mathematics: Junior Classes to Standard 4* and *Mathematics: Form 1 to 4* to provide a curriculum for mathematics in New Zealand schools from year 1 to year 13.

The structure of the mathematics curriculum may be described as having a series of five strands of knowledge and skills, namely number, measurement, geometry, algebra and statistics. One other strand is '**mathematical processes**' which can be separated into three processes:

- (1) Problem solving,
- (2) Logic and reasoning and
- (3) Communicating mathematical ideas.

It is these mathematical processes that are particularly new to the curriculum. Under the new guidelines, the student is given more ownership to their learning and approaches to mathematical problems and they are encouraged to use their cognitive processes and skills in deciding how to work their way through tasks. Students are also encouraged to decide for themselves what knowledge and skills they need in order to find a solution. The mathematical process strand encourages the student to think about the problem, work out a solution and to check and see if they were correct.

The communication aspect encourages telling others about their methods and solutions that they come up with. Through writing, explaining, collaborating, and illustrating their solutions are achieved in recognisable, appropriate forms of mathematics.

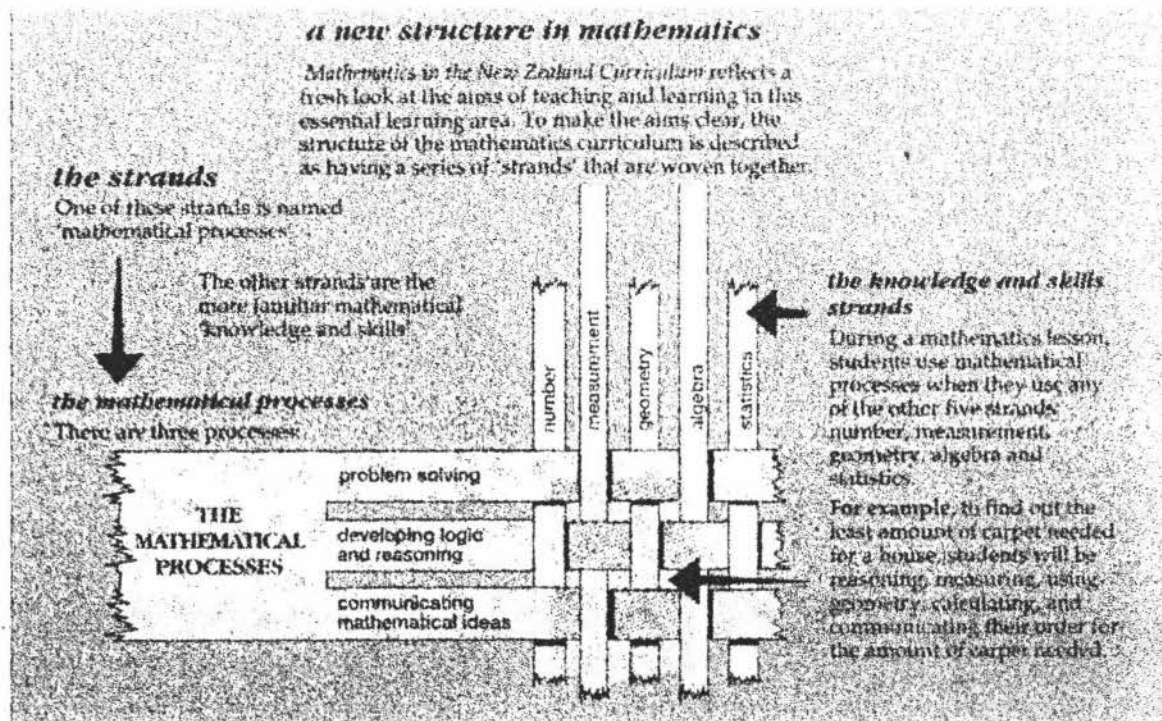


Figure 12: Mathematics in the New Zealand Curriculum: a new structure in mathematics.

(M.O.E pamphlet, 1995, pp 2)

The changing nature of the teaching of this curriculum has resulted in the move away from the more traditional areas of mathematics teaching. Now, artificial problems are replaced with a more problem solving approach and the use of examples of mathematics that have meaning and relevance to the students. Calculators and computers are encouraged, as using them can encourage the student to learn and understand the mathematical ideas, rather than providing only mechanical algorithms and routines. Mathematics should be seen as a collection of isolated skills.

"Mathematics should not be seen as a set of separate skills..."

(MinNZC, 1992, pp 15)

As the students advance in both years and their schooling, they will progress, at different speeds, through eight levels of 'achievement objectives' in each of the strands. These levels state clearly the standard of student achievement expected at each level and in each strand. Exemplars are given to show what types of indicators can measure this achievement. Catering for individual needs is addressed in the documents by 'overlapping' of the levels and the years of schooling for the students as they progress.

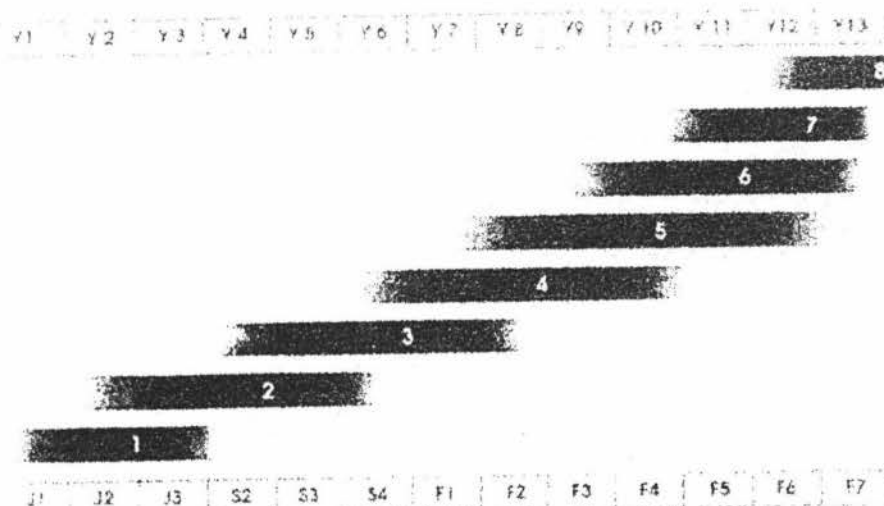


Figure 13: Achievement levels in MinNZC. (MinNZC,1992, pp 17.)

Mathematics seen in the MinNZC documents reflects a more constructivist approach to teaching and learning, shown by statements like:

“...Critical reflection may be developed by encouraging students to share ideas, ...” (MinNZC, 1992, pp 11)

and

“A chance to look for problems as well as to solve them, to create and to produce rather than reproduce what already exists...Creativity i in problem solving is recognised as one of the best traits that must be developed if outstanding achievement is to result, and it plays a major role in innovation, invention, and scientific discovery.” (MinNZC, 1992, pp 11)

These opinions reflect a growing awareness of the importance of encouraging students to be metacognitive. They suggest that application, analysis and synthesis play a major role in the acquisition of mathematical knowledge rather than simply a focus on ‘basic knowledge’ of mathematical principles and ideas.

“Closed problems, which follow a well-known pattern of solution, develop only a limited range of skills” (MinNZC, 1992, pp 11)

This quote, applied to Bloom’s taxonomy, refers to the pyramidal development of the higher-order questioning and illustrate the notion of the mathematics curriculum of the past, with emphasises on ‘closed questioning’. It is now felt that more time should be invested in higher order thinking and objectives, in contrast to Bloom’s original model, his pyramid

model should be inverted.

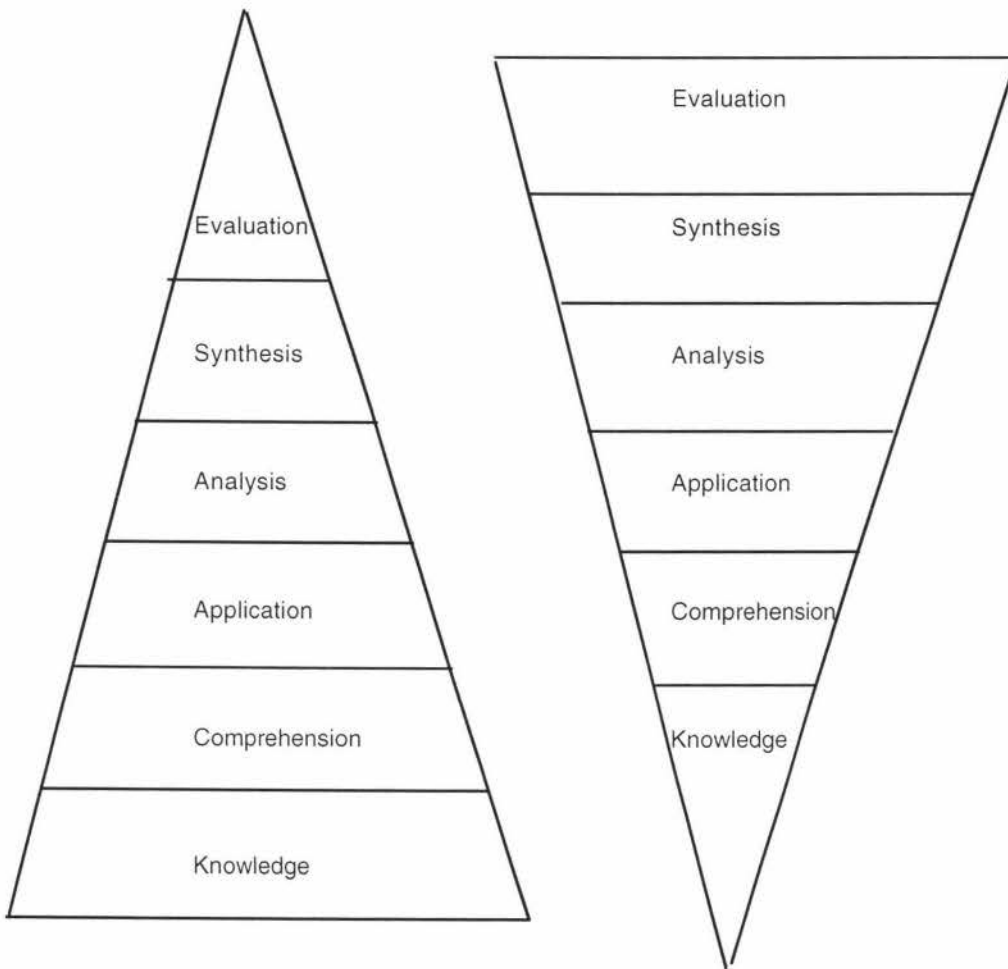


Figure 14: Bloom's taxonomy, the pyramids illustrate the notion of investing more time on 'higher level' activities.

Educational objectives can generally be divided into two categories, 'knowledge' and 'skills and abilities'. Bloom refers to knowledge as basically remembering and recalling, comprehending, applying and analysing are skills or abilities. A more interesting statement from MinNZC is that there are that there are different ways of doing mathematics, therefore no one method is correct.

“. . .creating opportunities for students to develop. . .strategies as guessing and checking, drawing a diagram, making lists, looking

for patterns, classifying, substituting, rearranging, putting observations into words, making predictions, and developing proofs."

(MinNZC, 1992, pp 11)

For example, the links that can be made between graphical, algebraic, and numerical methods and the different approaches, achieve the same 'ends' and the emphasis is placed on comprehending, applying and analysing. (See Appendix 7, 'Interest rate worksheet'.)

The phrase, 'meaningful contexts' may be seen in each achievement objective, denoting that the student should learn mathematics through real life problems that have meaning to them, rather than by the artificial problems that has held mathematics 'together' in the past.

There are, in essence, three directional focusses in the MinNZC statement. An emphasis on:

- (a) continuity and progression in the students learning,
- (b) a focus on the importance of diagnostic testing and formative assessment to enhance the teaching as well as the learning, and thirdly,
- (c) the need for mathematics to be taught and learnt within the context of problems that are meaningful to the student and to encourage an understanding to the way mathematics is applied to the world.

Assessment in mathematics in New Zealand has in the past focussed on a narrow range of skills and procedures. These 'pen and paper tests' of algorithmic skills do not always reveal student difficulties or indeed their

understanding and abilities and for that reason MinNZC encourages other avenues of alternative assessment techniques, such as written and oral appraisals, demonstrations and group or team assessments.

The School Certificate (S.C.) examination is based on the objective outlined in MinNZC up to level 6 and New Zealand Qualifications Authority (N.Z.Q.A.) Unit Standards are based on levels 6, 7, and 8 objectives. The University Bursaries (U.B.) examinations examine subsets of objectives up to level 8 although it is not expected that all year 12 and 13 students will do all of the mathematics objectives in levels 7 and 8. The prescription of the U.B. examination are also published by N.Z.Q.A..

Presently, there is 'dual' assessment in the senior mathematics courses, namely, SC and unit standards, Sixth Form Certificate (S.F.C.) and unit standards and U.B. and unit standards. It is up to each individual school to 'balance' their mathematics programmes and these two assessment systems, so as not to over-assess the students, and to provide both short term and long term goals for the students who study mathematics.

The Australian connection.

Having examined the current objectives, principles and practices currently in place in New Zealand, the researcher also had the opportunity to see what is happening currently in some Australian states.

Australia, one of our closest neighbouring countries, is also moving through a similar transition with their own mathematics curriculum. In a tour of some

Queensland secondary schools, the researcher was fortunate to have been provided with some interesting insights into the use of technology in mathematics classrooms. There is evidence that the Queensland mathematics curriculum and its subsequent classroom implementation, reinforced the notion of student cognition, metacognition learning and teaching practices in the use of the graphical calculator in classroom practices.

Each Australian State has essentially its own educational goals covering the directives made in federal policies, planning and statements in education, such as the 'National Statement on Mathematics for Australian Schools (1991)'.

In particular, Queensland offers a year 1 - 10 syllabi and then a senior, year 11 - 12 syllabi, the latter being a two year course divided into 3 levels, namely Mathematics A, B and C. There are state examinations at the end of year 10, and the results of this generally divide the student population into the appropriate mathematics course that they will follow in year 11 and 12. Assessment in Queensland secondary schools at the junior level (year 8 - 10) is by external examination, compared with School Certificate in New Zealand. At the senior level, internal assessment of the two year programme is in place at all schools and has been since 1972.

It seems that this internal assessment procedure in place in Queensland secondary schools has had a strong influence on graphical calculator use in the mathematics curriculum. Exit 'Proficiency Tests' are sat by all year 12 students and include a written set of examinations covering all areas of the curriculum. Mathematics is considered a compulsory subject to all senior

students and it is a rare case for a student to opt out of a senior mathematics programme. If they are allowed to opt out, then a 'catch up' course of mathematics is provided to assist the student's preparation for the 'Proficiency Test' as these results can influence their exiting grades from secondary education and hence their entry into tertiary courses.

Within each senior mathematics course there are three assessment levels:

- (a) Communication - skills that enable the student in order to understand, assess and convey ideas and arguments.
- (b) Mathematical techniques - the developing of familiarity with the techniques of mathematics, using mathematical instruments.
- (c) Mathematical Applications - the applying of mathematics in real life situations and extracting the relationships within the context represented mathematically, deriving a solution and justifying the solution in the context of the arguments and conclusions.

These ideas run parallel to the strand of 'mathematical processes' in MinNZC which is entwined with the other five strands. Queensland mathematics focusses on three criteria:

Criteria 1 - Communication must be clear and concise in written mathematical language. Good reasoning must be shown to support claims and arguments based on the completed problem.

Criteria 2 - Mathematical skills and practice techniques which the students are directed to use in answering the questions cover the need for the student to be able to carry out learned procedures in a mathematical sense and with the appropriate instruments.

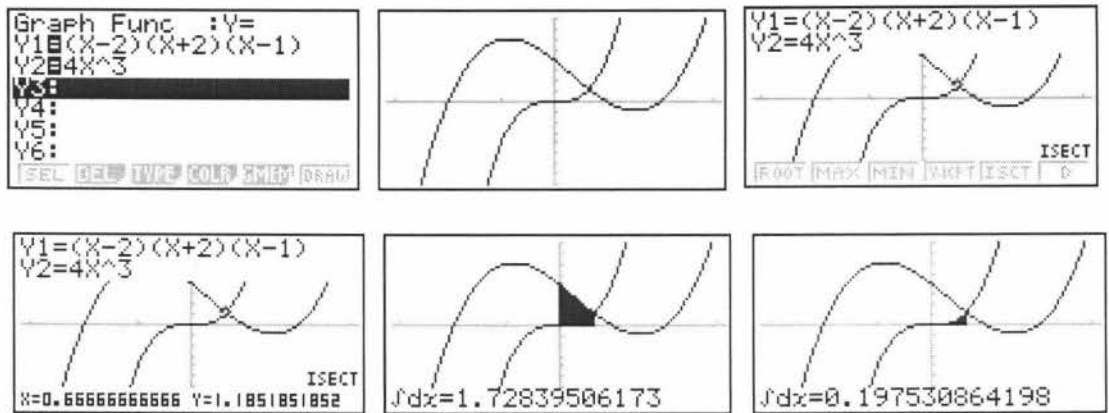
Criteria 3 - In using mathematical processes, the student is required to select the appropriate method, with questions generally open ended.

Assessment is of the logical processes that the student utilises (compare this with N.Z.Q.A. Unit Standards, Mathematical Processes) and the focus is on the student being confronted with unfamiliar problems that they are to solve. (Board of Senior Secondary School Studies, Mathematics A, 1992. pp 5 - 7)

These three criteria run parallel to MinNZC.

- (1) Communication.
- (2) Learned results and procedures and using mathematical instruments.
- (3) Life related with mathematical applications and justifications.

In the schools visited, there was an important emphasis placed on mathematical applications. Most of the class work reinforced mathematical applications and in order to do this, graphical calculators played an integral part in the mathematics being discussed and developed in the lessons. For instance, in Mathematics C, the calculus that is covered in Mathematics B is assumed, and thus students in Mathematics C usually were enrolled in the Mathematics B course. This meant that the students could progress in their development of calculus, and its applications to real world problems. For example, a problem of finding the area between the two curves $y = (x - 2)(x + 2)(x - 1)$ and $y = 4x^2$ in the diagrams on the next page.



[The emphasis is on finding the area and not finding the points of intersection.]

Figure 15: Finding the area between the two curves.

The graphical calculators were used extensively in this area to find the graphical intersection points, so that the focus remained on finding the area bounded by the two curves. The graphical calculator made this possible. Another example in Mathematics B, the technique of long division of polynomials, is not in the prescription, so a dilemma exists for providing ways for teachers to teach concepts such as solving for the x - intercepts of a particular polynomial higher than a trinomial or even a rational function.

In order to simulate real world problems, real world answers cannot usually include whole number answers. The teachers and students respond to this by using the graphical calculator's ability to find 'roots', drawing and tracing the function in question on the graphical calculator, or zooming in or setting up an iterative method.

Statements in the general objective of Mathematics A, B, C include:

"In order that the emphasis of the mathematics learned is on concepts and techniques rather than tedious and / or involved

calculations, it is appropriate in using a calculator and a computer.”

(Senior Mathematics A, B, C, 1992, pp 1)

We can compare this with the MinNZC statement:

“Graphic calculators, and computer software such as graphing packages and spreadsheets, are tools which enable students to concentrate on the mathematical ideas rather than on the routing mechanical manipulation, which often intrudes on the real point of particular learning situations.”

(MinNZC, 1992, pp 14)

The Open Access Coordinator of Mathematics, in Queensland, Australia, is similar to New Zealand's Correspondence School (without the teaching). This department of the Queensland educational sector develops courses that are to be used in Distance Education (Correspondence School with the teaching) and are incorporating the use of graphical calculators into their teaching modules. They are also encouraging the students who enrol to purchase a graphical calculator as well as, presently (results unpublished) surveying all schools in Queensland for the type and usage of graphical calculator in schools. This information will be used to assist in their preparation of course materials covering the spectrum of graphical calculators models that are available on the Australian market. The survey results will aid in providing teaching resources that would be accessible to all, rather than limited to a particular graphical calculator model type.

The New Zealand Correspondence School as at August, 1997 were not considering any of the above initiatives, in contrast to the actions of their Queensland counterparts.

The Queensland Association of Mathematics Teachers (Q.A.M.T.) also is pursuing the use of the graphical calculator in schools, and had recently held a 'resource weekend' entitled A.T.O.M.I.C. Focus was primarily on the development of resources aimed for use in Mathematics A, B and C. Worksheets utilising the graphical calculator as an investigative tool to solve real world problems and simulations were developed. Some twenty six teachers primarily from the Brisbane area attended, but others were also flown in from other parts of the State, to be on the writing team.

Ownership issues.

As mathematics is 'compulsory' the graphical calculator is already showing it has a role in influencing student learning and promoting understanding of mathematics. Generally, it is the schools in Queensland who are the purchasers of class sets of graphical calculators, however the number of graphical calculators in schools vary from 90 in a school of size 1700 students, to 30 graphical calculators for a school of size 400. Only a few students are purchasing graphical calculators for themselves.

Other Australian States, such as New South Wales, have different educational philosophies from that of Queensland, and have an education system with external examinations, similar to School Certificate and Bursary examinations in New Zealand. The students appear to be the main purchasers of the graphical calculator rather than schools and consider that since their use is permitted in the examinations, the graphical calculator is considered advantageous in achieving personal success.

In the state of Victoria, graphical calculators are permitted in the Victorian

Certificate of Education (V.C.E.) examinations from 1997. This decision by the Board of Studies (B.O.S.) was the result of a widely extended use of graphical calculators in the teaching of mathematics and a change from school sets to students individual ownership.

There are two differing stand points about a notion of 'ownership', by the teacher and the student. If the student regards the graphical calculator as a useful tool in assisting acquisition of knowledge in mathematics, then they will endeavour to utilise the graphical calculators' functions to their fullest, in order to maximise their chances in examinations. However, it is the schools who envisage the role of the graphical calculator as a useful tool for mathematical practices in the classroom, rather than an instrument that will provide easy answers.

There are, of course, down sides of graphical calculator usage. These have been seen in recent press articles, for example, the 'Sunday Mail, 20th July, 1997, "... a feeling of negativity among society as they are a vehicle for cheating." Students were found to have been using the memory capabilities to store their notes, for use in the examinations.

Now it's Cybercheat

QUEENSLAND education authorities say they will be unchasing this year for powerful new calculators that help students cheat in exams. The warning follows the revelation that Year 12 chemistry students in Victoria cheated by downloading 16-page chemistry summaries into high-powered graphical calculators during a mid-year exam. The calculators were not banned so the cheating during the June 16 exam was legal. Some Victorian schools are understood to have modified it. Graphical calculators

By CHRIS GRIFFITH

have been available since the mid 1980s and are used mainly by engineers and surveyors to plot graphs in the field.

But recent models with up to 2 kilobytes of memory store huge amounts of text which can be loaded from a home computer into the calculator through a connecting cable.

Yesterday the director of Queensland's Board of Secondary School Studies John Pittman, said calculators that stored programs or "verbal or numerical data"

would be banned this year from Core Skills tests.

"At the present time they would not be kosher but we would review that year by year and it would depend on the papers we set," he said.

He said exam supervisors had to vet and exclude new calculators they might not have seen before.

Indooroopilly High School principal Rod Bailey said that when used in memory recall testing, super-memory calculators were an electronic form of writing on your hand.

He said basic or scientific calculators, programmable calculators, and electronic foreign language translators were allowed at the school but programmable calculators including those with large text memories were banned from exams.

Stephen Parry from The Calculator Shop in Brisbane said graphical calculators started at \$150 but most cost more than \$200. He said the most common brands were Texas Instruments, Casio and Hewlett Packard.

Mr Pittman said technology "appropriate to the times" was increasingly adopted in classrooms and exams.

He said there would be no point banning super-memory calculators in writing and short response tests where the recall of information was not the issue.

"There probably were times when people thought that a fountain pen should be banned because it didn't allow students the skill of taking the quill and dipping it into the ink," he said.

The use of graphics and powerful memory calculators in Queensland schools is small compared with that in the US. This is expected to change.



CHIP CHEAT: A graphing calculator

Three killed on roads

THREE men were killed on Queensland roads on Monday. A man died when

Figure 16: Press article - "Now it's Cybercheat."

The researcher feels this is a challenge for examiners involved in preparation of examination questions to re-evaluate what they want to examine. The graphical calculator provides them with such a challenge. For example, rather than

Find the inverse of the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ that can

easily be done on the graphical calculator, a simple change can be made to ask:

Find the inverse of the matrix $\begin{pmatrix} 1 & 2 \\ a & 4 \end{pmatrix}$

This subtle change challenges the student to apply their mathematical knowledge to more higher order problems, thus introducing more metacognitive learning processes to be used in the examination setting.

There is no point in banning the graphical calculator from examinations as this would defeat the purpose of the use of technology in mathematics. It is an important mathematical tool that is as much relevant today as the slide rule or the fountain pen was in the past. When operated correctly and with proper guidelines in place to prevent cheating and misuse, it is a valuable tool in classroom and examination settings. Remember, the curriculum in both countries reflects metacognition within the learning and teaching of mathematics.

The graphical calculator in Queensland was seen to be useful primarily for matrix operation, graphing curves, statistical calculations and statistical graphing, but limited use was seen in developing and storing programmes

and calculus. The use of the graphical calculator by the teachers was seen as a mathematical tool which could reinforce learnt skills and key mathematical skills that were developed in classroom learning programmes. It is the teacher or schools' mathematics departments who see the benefit of the graphical calculator in their delivery programmes in mathematics.

Summary.

Schools, departments and classroom teachers need to reflect on the aims set out in the MinNZC criteria as it has now been implemented in secondary schools for some two years and covers all student levels. Although both New Zealand and Australia have similar mathematics programmes in place, our Australian counterparts are achieving better results with the graphical calculator in the classroom setting, while New Zealand's progress in this area is somewhat slower.

So, how will the graphical calculator compliment the instruction in New Zealand mathematics classrooms? Before we can attempt to answer this, we need to look at a number of questions that need to be addressed before we can look at alternative approaches to mathematics education with the graphical calculator.

1. Is it a useful mathematical and scientific tool for learning?
2. Does it make mathematical learning fun and interesting for the student?
3. Does it provide the students the opportunity to explore mathematics?
4. Does it help to reflect the spirit in which MinNZC was written?

5. Does it provide the students an opportunity to do "What if" scenarios?
6. Does it help the student to be reflective in their mathematics?
7. Does it provide the student the opportunity to check outcomes?
8. Does it integrate a number of mathematical packages successfully into a hand-held portable unit?
9. Does it encourage experimentation amongst students and teachers alike?
10. Is keystroking another form of mathematics for the students to learn?
11. How do teachers assess students with the graphical calculator in mind?

With these questions in mind, classroom practitioners need to see how students learn mathematics, construct their own mathematical knowledge and feel about mathematics.

Chapter 4

Related Readings.

Learning Mathematics.

One approach to mathematics learning, according to an ancient Chinese proverb is:

I hear and I forget,

I see and I remember,

I do and I understand.

How we learn mathematics.

The mathematical understanding reflected in the above quotation, coupled with the ability to apply it to real problems, and the consideration of the aims and objectives of MinNZC, is the goal of today's mathematics education programmes.

Before students come to school, children learn many things, including a great deal of mathematics, and generally do this without the help of a teacher. Most of this learning is done informally through play, such as involving scoring (numbers), playing boards and playing fields (geometry) and strategy games (logic) (Blair, 1981).

The primary school teacher often introduces such games into their lessons, so that the children are learning without realising it, and the student generally finds learning enjoyable. This approach to mathematics learning needs to be addressed by secondary school mathematics teachers too, as that learning needs to be relevant and purposeful in its application, for the students to see the relevance of mathematics in society.

“Mathematics is a coherent, consistent, and growing body of concepts which makes use of specific language and skills to model, analyse, and interpret the world. Mathematics provides a means of communication which is powerful, concise, and unambiguous.”

(MinNZC, 1992, pp 7)

Therefore, children spend a great deal of their school time unaware that they are actively ‘doing’ mathematics. Mathematics lessons have changed substantially over the years, influenced by our greater knowledge to the learning needs of children and the ways in which they learn. It is very important that our children understand the mathematics that they are learning and to then apply this knowledge to the new and varied situations which they will meet as they grow up. It is not sufficient now, if it ever was, simply to be able to do something by rote. Learners must be able to see why it works and know why certain things are as they are.

The development of mathematical ideas in children is a very gradual process. For example, a child initially may consider ‘five’ to be simply a word in a nonsense jingle, “one, two, three, four, five, six, seven, eight, nine, ten”. Later, it will become a name for one of their fingers, then the name for the number of fingers on their hand. After much experience with other collections

containing five objects, it will become the name of a common property of all these collections, that is, the number of objects in each collection. Later still, it will become just one way to represent this number of objects, others being $2 + 3$, $8 - 3$, $2 \times 2 + 1$. Gradually, refinements are made to these basic ideas. The use of activities, materials and language are all important aspects in the learning process, but in order to facilitate successful learning we must be able to understand the learning processes involved.

Cognition and metacognition - why indeed is there now all this fuss about these two educational concepts? Increasingly, educators are realising the significance of metacognition in successful classroom practices, and teaching outcomes. It is important, therefore, that as teachers, we recognise and understand how and when it is operating and working, and for that reason, it is necessary to take a brief look at previous research on metacognition and earlier definitions of learning strategies.

From a behaviouralist perspective to . . .

"We know less about the way learners approach their individual acts of learning than we do about how we, as teachers, would like them to approach learning."

(Galloway & Labarca, 1990, pp 127)

Previously, learning had been viewed as something which happened to an individual. This followed moves by psychologists and educationalists in support of a behaviouralist approach to learning, with teachers 'covering' well defined topics. The students 'learnt' the material that was presented to

them. Focus was on knowledge acquisition, rather than processes and assimilation.

Classroom researchers now have begun to focus on the role of the learner, and how the student plays a critical role in determining how much and what is learnt. Learning strategies has been shown to be a major variable in the learning process as these are behaviours that the student can have at their disposal in their learning, and which affect learner motivation and / or the way that new knowledge is selected, acquired, organised and utilised (Anthony, 1994). In order for a learner to implement effective learning strategies then one must consider their learning environment. To develop an ideal learning situation Collins, Brown & Newman (1989) suggested that attention needs to be paid to student development of:

- (a) domain knowledge,
- (b) learning strategies,
- (c) control strategies, such as monitoring, diagnostic and remedial and
- (d) a belief system appropriate to learning.

If the student possesses these attributes then they will be more successful in their learning. Poor learning outcomes are usually a result of (Alexander & Judy, 1988, Anthony, 1994):

- (a) lack of prior knowledge,
- (b) confusion about the tasks goals,
- (c) lack of mastery of domain skills,
- (d) inappropriate use of learning strategies in monitoring their understanding,
- (e) an effective use of seeking of teacher or peer assistance, and
- (f) their ineffective use of resources.

Constructivism is based on two principles (Von Glaserfeld, 1989):

1. Knowledge is not passively received, but actively built up, by the learner,
2. Cognition is an adaptive process that helps the learner to organise their world, rather than to know absolute reality (Cited in Begg, 1995).

Thus, from a constructivist perspective, the learner is seen as being responsible for being attentive to their learning and to be active by engaging in strategic learning strategies. What learners do to select, organise and relate the information to what they already know, is an important factor if that knowledge is to be learnt and remembered. If a learner knows about how they learn, then they will make the most of the learning situation (Weinstein & Mayer, 1986).

Learning strategy research, in such learning areas as reading (Palincsar & Brown, 1994), mathematics (Schoenfeld, 1985) and languages (White, 1993) has shown that the characteristics of successful learners are demonstrated by those who have the ability to select and use appropriate learning strategies and who are also able to monitor and control their learning. In contrast, less successful learners have been characterised as either not having effective learning strategies in which to utilise in their repertoire, or are not able to employ them at the appropriate time (Anthony, 1994).

Learning Strategies.

The term strategy refers to procedures for implementing a plan of action,

“implementation to a set of procedures for accomplishing something”
(Schmeck, 1988, pp 5). Hence, a logical follow on is that learning strategies are a sequence of procedures to accomplish learning (Anthony, 1994) and can be categorised into particular goal orientated areas:
Cognitive strategies can be described as tasks that enhance:

- (a) elaboration or rehearsal, which relates individual learning tasks and operates directly on incoming information, so that the student makes cognitive progress,
- (b) metacognitive strategies, such as planning and evaluation. They are used to control and monitor the learning process,
- (c) affective strategies, such as self talking to enhances concentration, and
- (d) resource management strategies, such as help seeking or modifying the task in order to utilise their learning environment.

New Zealand has seen curriculum documents produced that reflect the importance that is now given to the development and understanding of effective learning strategies for students:

“Learning how to learn is an essential outcome of school programmes”

(The Curriculum Review, Department of Education, 1987, pp 10)

and

“The curriculum should enable students to take an increasing responsibility for their learning. With their teachers they should be involved in setting goals, planning their activities, organising their studies to gain skills and understanding, and evaluation their

progress.”

(draft National Curriculum Statement, Department of Education, 1988, pp7)
and

“We need a learning environment which enables students to attain high standards and develop appropriate personal qualities. as we move towards the twenty-first century, with all the rapid technological change which is taking place, we need a work-force which is increasingly highly skilled and adaptable.”

(New Zealand Curriculum Framework, Ministry of Education, 1993, pp 1)

The use of metacognitive strategies, enables the learner to control and monitor their learning process and can be seen as an important indicator to learning success (Anthony, 1991).

Learning strategies and technology.

“In an increasing technological age, the need for innovation, and problem-solving and decision-making skills, as been stressed in many reports on the necessary outcomes for education in New Zealand. Mathematics education provides the opportunity for students to develop these skills, and encourages them to become innovative and flexible problem-solvers. ”

(MinNZC, 1992, pp 7)

One of the main aims of mathematics learning is problem solving. When a student leaves school, they will enter a world dominated by electronic gadgetry. The problems that occur in industry, business, science,

engineering and architecture involve a lot of advanced mathematics. Computers and calculators are now widely used in these areas because they can perform the mathematics quickly and accurately, however we still need to understand the mathematics in order to tell the computer or calculator what to do. With regard to the classroom setting, the graphical calculator can motivate mathematical learning and can provide exciting challenges for the students in their studies of mathematics.

Learning should be active, and students should be involved in their own construction of the knowledge as it is presented to them. The use of the graphical calculator can be an instrument to facilitate this active role and to provide assistance in the inter-connections between concepts taught.

Learning outcomes, to be successful, need to be goal orientated, thus a combination of experimentation, review, elaboration, organisation, planning, monitoring self-evaluation, and modifying of tasks asked and seeking clarification by the students will enhance the students' understanding of subject content.

The use of technology within the classroom setting will change the climate not only in curriculum delivery, but in how mathematical ideas and concepts are taught and learnt. Such activities in the classroom will move from being teacher - centred to learner - centred. The teacher will become a facilitator, rather than an expert fact - teller, who imparts the mathematical knowledge to the students that are seated before them. The emphasis from facts, skills and memorisation will move to the reasoning of mathematical relationships and investigations, from accumulation of facts to a transformation of facts, and more importantly, from drill and practice to investigation, expression,

collaboration and communication.

“The suggested learning experiences are carefully worded in active terms. This is to emphasise that mathematics is most effectively learned through students’ active participation in mathematical situations, rather than through passive acceptance and repetition of knowledge.” (MinNZC, 1992, pp 18)

and

“ Computer programs, such as LOGO, provide excellent environments for mathematical experimentation and open-ended problem solving.” (MinNZC, 1992, pp 14)

and

“ The characteristics of good problem solving techniques include both convergent and divergent approaches. This includes systematic collection of data or evidence , experimentation (trial and error followed by improvement), flexibility and creativity, and reflection - that is, thinking about the process that has followed and evaluating it critically.” (MinNZC, 1992, pp 11)

These statements also reflect a constructivist and metacognitive approach to the learning and teaching of mathematics.

As noted in the MinNZC (1992, pp9 - 10), a wide range of learning strategies are being promoted. These aim to:

- (a) *develop the ability to reflect critically on methods they have chosen,*
- (b) *develop the skills of presentation and critical appraisal of the mathematical argument or calculation,*

- (c) *use mathematics to explore and conjecture,*
- (d) *learn from mistakes as well as successes,*
- (e) *develop the ability to estimate and to make approximations, and be alert to the reasonableness of results and measurements,*
- (e) *develop accuracy, efficiency, and confidence in calculating - mentally, on paper and with a calculator,*
- (f) *recognise patterns and relationships in mathematics and the real world, and be able to generalise from these, and*
- (g) *develop the ability to think abstractly and to use symbols, notation, and graphs and diagrams to represent and communicate mathematical relationships.*

In all these learning strategies, the graphical calculator can be a vehicle that used to reinforce mathematical concepts.

The graphical calculator can be used in a wide range of mathematical activities that can also extend into other curriculum areas. For example, in statistical analysis, the graphical calculator can be used in the collection of data from electronic experiments, and the readings gathered can then be processed as a regression model and relationships investigated in a science context.

What teaching aids are presently in place.

The delivery of the curriculum is generally constrained by the 'tools' that are available to the classroom teacher. The principal tools used in mathematics classes as already discussed, are the scientific calculator, a ruler, protractor

and compass and a collection of 'pencil and paper' procedures and manipulations.

"The invention and the widespread accessibility of the graphics calculator seem likely to have a major impact on the mathematics education of most students during their high school years." (Kissane, 1997, pp v.)

Kissane's statement reflects the move in mathematics education to bring the advances in technology and its relevance into the learning environment of the mathematics learner in the 1990s.

Qualding (1982) sees society as a world that is becoming both more mathematical and a less mathematical place in which to live. That is to say, the personal skill in mathematics is lessening, due to the increased reliance on technology. Changes in the curriculum have included:

- (a) teaching / learning strategies in group work,
- (b) exploring mathematical ideas co-operatively,
- (c) developing individual strategies for computations,

The use of calculators provides ways for children to use their own methods for recording instead of conventional algorithms and mathematical processes being integrated into other strands and other subject areas. Calculators, graphical calculators and computers create possibilities for change in curriculum delivery, opening new ways of thinking about mathematics, new ways of presenting material and new ways of learning (Fey, 1989).

1. Children need to be actively involved in their mathematical learning,

- both physically and mentally.
2. Children pass through a series of stages in their learning of each mathematical concept.
 3. Children pass through these stages at different rates. This explains the different levels of development among children.
 4. Practice is important and helps with the development of fast recall and the application of learnt skills.

Implementing a Strategy.

It has been suggested that one needs to recognise the strategy that the student employs in the classroom before implementing strategy instruction programmes. There are differing views about this method, as the teaching of strategies can improve a student's on-task behaviours, only if it is known what students do, or fail to do. Kardash and Amlund (1991) support this theory, suggesting that spontaneous strategy use is important because research evidence points to the notion that students adopt preferred strategies (which sometimes are ineffective). However, these lessen the likelihood of students being amenable to strategy training and it is now considered better to determine what strategies the learner uses on their own, how these strategies relate to one another, and which strategies enhance their learning outcomes.

Metacognition.

Loosely speaking, metacognition refers to one's knowledge and control of

one's cognitive practices. ". . . *being aware of our thinking as we perform specific tasks and then using this awareness to control what we are doing.*" (Marzano, Brant, Hughes, Jones, Presseisen, Rankin & Suhor, 1988, pp 9).

Introduced by Flavell, metacognition was described as the understanding that individuals have of their thinking and learning and meant 'transcending knowledge' He concluded:

". . . knowledge concerning own's own cognitive processes and products or anything related to them, e.g., the learning -relevant properties of information or data . . . metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects on which they bear, usually in the service of some concrete goal or objective." (Flavell, 1976, pp 232).

Later, literature by Flavell (1987, pp 21) suggested that the concept of metacognition be

". . . broadened to include anything psychological, rather than just anything cognitive. . . any kind of monitoring might also be considered a form of metacognition."

This interpretation of metacognition suggests:

1. the regulation and control of cognition,
2. knowledge and beliefs about cognition, and
3. metacognitive experiences and beliefs on one's performance.

Executive control (Brown, 1985) of the learning by the learner is achieved through their learning strategies and being able to orchestrate the process

of learning (Biggs, 1984). Early metacognitive research concentrated primarily on the regulatory control aspects of metacognition in problem solving to:

- (a) monitoring and controlling one's learning,
- (b) adopting strategies of planning, self-questioning, assessing their progress and evaluating their learning.

Metacognitive knowledge is concerned with the learner knowing about their cognitive abilities, and using resources in relation to their own personal performance. Brown (1987, pp 67 - 68), states, "*Knowledge about cognition refers to the stable, storable, often fallible, and often late developing information that human thinkers have about their own cognitive processes.*" Thus, metacognitive knowledge can be fallible, so that what one believes may not always be accurate.

Cullen (1991) emphasises the need for learning situations to provide practice of reflective skills as students who engage in reflective strategies are able to address such questions as, "I need to write notes about this if I am to remember.", "Do I know how to do this type of example?", or "Do I need my calculator to do these calculations?" In Flavells' terms, these questions are judgments about one's cognitive abilities or strategies and these can either impede or facilitate the student's performance, demonstrating how effectively the learner can learn.

The student's tolerance to feelings of failure in order to understand or to remember, can be related to their expectations or goals in their learning. How often have teachers heard a student comment, "Sometimes I know I

am not getting it, but I don't care to put in the extra effort to fix it."

There is an interaction between cognitive and metacognitive experiences, goals, actions and knowledge. Flavell stated, "*Cognitive strategies are invoked to make cognitive progress, metacognitive strategies monitor it*" (1979, pp 909). Flavell's (1981) model of cognitive monitoring, figure 17 shows the interactive nature of these two concepts.

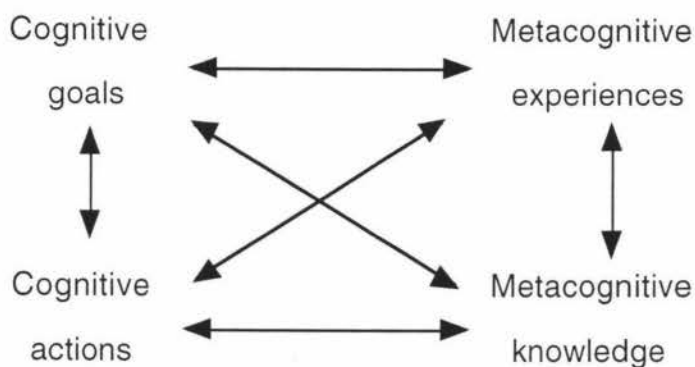


Figure 17: A model of Cognitive Monitoring. (Anthony, 1994, pp 29.)

Anthony sums this up succinctly with the following quote:

"The metacognitive knowledge that a student uses interacts with metacognitive experiences, in order to achieve the cognitive goal of learning. In the constructivist frame, metacognition promotes positive self perception, affect and motivation among students. When they re - read difficult material, or select learning activities that are appropriate to a given task, they are becoming active participants in their own performance, and are actively learning rather than being passive recipients of instruction and teacher imposed experiences."

(Anthony, 1994, pp29)

Teacher and student ability to select and employ learning strategies are considered as being central to the learning process and metacognition is a key variable in the monitoring and regulation of that learning process.

“The self regulated learner must appropriately control his or her learning processes by selecting and organising relevant information and building connections from relevant existing knowledge.” (Mayer, 1992, pp 409)

The student who is self regulating is capable of learning more independently, however it is important to remember that one of the primary goals of education is to help students develop expertise in how to learn, and to use that expertise in order to construct useful knowledge. Zimmerman and Martinez - Pons (1992) suggested that student efforts to initiate and self regulate their learning may depend on their perceptions of personal competence in the use of learning strategies, particularly when difficulties are encountered. Bloom’s taxonomy in the pyramidal form (see figure 14), places cognition and metacognition at the pinnacle of the pyramid while the constructivist and learning strategy framework places a higher importance on these concepts too.

The use of key words in learning and instruction, such as read, understand, analyse, explore, plan, implement and verify can induce the cognitive and metacognitive strategies that could be utilised by students in the classroom setting.

Cognitive and metacognitive strategies.

Cognitive strategies can be characterised by the engagement of the learner in purposeful learning and gaining new knowledge. They are able to form links to this newly acquired knowledge and also to their prior knowledge. Weinstein and Mayer (1986) suggested that cognitive strategies can be placed under three groupings: rehearsal, elaboration and organisation.

Rehearsal helps the students to store and retrieve information and includes basic learning tasks such as repetition and practice. Weinstein & Mayer (1988) noted that rehearsal practices were beneficial in student understanding as they provided opportunities for more meaningful process, such as elaboration, and organisation to take place.

Elaboration enables the student to -*“Transfer knowledge from long term memory to working memory, processing the new information coming in. Paraphrasing, summarising, linking with prior knowledge, and uses of a metaphor are examples of elaboration.”* (Weinstein et al, 1986).

Elaboration in the constructivist paradigm is related to the active generation of meaning that is personally relevant to the individual learner's prior knowledge and experiences (Weinstein et al, 1988).

Organisational strategies such as elaboration, require the student to be active by taking responsibility for organising the information for their unique requirements, and transforming that information into language that is easier to understand. Summarising and highlighting important cues and examples assist the student's organisation of new information.

"Items associated with effective strategies are typically remembered much better than items associated with ineffective strategies, and students come to realise this." (Levin, 1988, pp 197.)

Affective and management strategies.

Affective strategies can also have an impact on learning and on the student's use of metacognitive strategies. Students can experience both positive and negative emotions as an inevitable part of their learning and examples of this are more noticeable in the classroom, such as dislike of mathematics in general or delight in getting a correct answer. Affective strategies can provide motivation, reward and encouragement, reduce anxiety and maintain the student's interest as well as encouraging effective time management. Ineffective use of these strategies however can produce negativity, frustration, inattention and poor time management. Thus, affective strategies can play a critical role for some students, by changing and / or controlling their attitudes towards learning.

In the researcher's own classroom, situations can be identified where the students were using metacognitive strategies. The first example is Neil, an average scoring student in mathematics. He purchased a graphical calculator at the beginning of the year to help him with aspects of mathematics with which he has difficulties. He had hoped that the graphical calculator would solve a lot of the problems for him, which it could, however he soon realised that he also needed to put in some time to become familiar with it's operation. Now, he is always 'keystroking' the calculator when mathematics are being done in class and checking solutions on the

graphical calculator to confirm what is happening. He's always looking at how the graphical calculator can be used to help him with his mathematics.

Mary is a year 13 student in Mathematics with Calculus and she too is of average ability in the class. She recently purchased some 'Write on notes' for calculus in order to supplement the text and notes that are generally provided in class as topics develop. At home, Mary relates what we covered in class that day to what is in the 'Write on notes' and highlights these. If she has difficulty relating the examples in the 'Write on notes' to what was done in class, then at the next lesson Mary will always seek clarification so that links can be made, particularly in making connections between numerical, graphical and algebraic representations of problems. Help seeking and modifying the task enhance the desired learning outcomes (Thomas & Rohwer, 1986).

These strategies employed by the students promote learning, such as task management and also to control and modify their learning environment (Pokay & Blumenfeld, 1990). Other examples of self regulated learning are co-operative learning, asking questions, seeking clarification either from the teacher or from the students peers and looking up another text or following through examples step by step.

Implications for the classroom.

Few students learn to become active learners on their own. Collins et al (1989) suggested that the model of cognitive development is crucial. Some students receive this modelling from their home, social settings or receive

and develop these skills throughout their schooling years. They need to become more aware of their own learning strengths and weaknesses and need to be provided with experiences that would allow access to strategy use and to compare the effectiveness of different learning strategies for different tasks. As educators we need to teach not just how to do, but also, how to learn. In the constructivist framework there is an increased need for peer interaction and communication, which would encourage metacognitive behaviours in the students.

Teachers too, need to be more aware of their instructional and assessment methods, such as the particular tasks set and also provision of learner support (Costa 1992). The classroom environment needs to minimise 'end product' mentality whereby answers are given without any engagement in learning by the student. Teacher supplied summaries, too little 'wait time' after a question is asked and the acceptance of answers from other students other than from whom the question was directed at, can contribute to students not being actively involved in learning, developing and using cognitive and metacognitive learning strategies (Anthony, 1994).

Assessment should be more collaborative, and utilise higher order skills where possible, so that analysing, verifying and evaluating skills can be employed while students search for possible solutions or engage in alternative approaches to solving the posed problem. It has an effect on student learning and collaboration along with other help-seeking strategies and will encourage students to develop cognitive and metacognitive strategies, such as using resources and seeking help from their peers and adults alike.

Teaching for thinking in the classroom requires the students to be actively engaged on the task in hand and to employ higher order thinking (Swartz & Perkins, 1990). It encourages discussion, provides thought provoking questions, hands-on activities and exploratory tasks and aids progress in this direction. Formerly, reliance was placed on the teacher giving the class information or questions that focussed solely on the recall of basic facts that reinforced pencil and paper algorithms only.

Using Technology.

“Education should at least be concerned with the core goals of trying to ensure that students are able to retain, understand and actively use in a meaningful way the knowledge they have acquired.”

(Perkins, 1992, pp 17)

The interaction between knowledge and strategies is important and Skemp (1976) makes a distinction between instrumental and relational understanding. The behaviourist view results in focussing on the learning of finished products of mathematical activity with emphasis placed on procedures and closed manipulative techniques, that limit students to the solving of routine problems. This is characterised mainly by the teacher showing and telling, with students following and repeating. The constructivist view results in the learning of mathematics as a process (irrespective of the content material), emphasising meaningful developments of the concepts and generalisations and increasing the prospects of real problem - solving, by encouraging open inquiry and investigations. This method is characterised mainly by the teacher challenging, questioning and guiding,

with students doing, discovering and applying.

Students need to accommodate existing knowledge and adapt this to extend into other number systems. For example, multiplying to begin with is viewed as making things bigger, but when applied to decimals this view is lost, $0.3 \times 0.6 = 0.18$ and a similar process can be seen for division. *“Students learn facts best much better if they organise them, actively relate them to prior knowledge, use visual associations, quiz themselves, elaborate and extrapolate what they are reading or hearing.”* (Perkins, 1992, 29 - 30).

Competence in mathematics by the student can be categorised in the three different areas:

1. The ability to understand the concept.
2. The ability to learn, retain and reproduce the concept.
3. The ability to combine the above two aspects and to use effectively in a problem - solving situation.

All these aspects are interdependent on each other.

Even today, mathematicians and mathematics teachers tend to have a Platonic (formalist) view with a rigid set of rules and this still has an influence in the development of mathematics education. The teaching of mathematics today needs to be developed from the student's own personal experience and the MinNZC has begun to address this by allowing for the modelling of ideas and events and with it, discussion.

Textbook learning has generally been the fundamental tool of the teacher. The MinNZC has in part engineered the demise of the 'transmission'

teaching model and while some teachers are relishing their release from the bind of textbook instruction, others exhibit withdrawal and are seeking an alternative replacement, but to date, nothing has really fulfilled that role.

Mathematics employs the use of cognitive processes and the past starkness needs to be replaced by the student's formation of mathematics knowledge which should result in social interaction and events with which the student can identify. Learning should be more active in today's classroom, rather than passive.

“ . . . mathematics is most effectively learned through students' active participation in mathematical situations, rather than through passive acceptance and repetition of knowledge.”

(MinNZC, 1992, pp 18)

According to Thom (1973), (cited Pimm, 1987, pp 7), the construction of meaning rather than the question of rigour is the central problem facing mathematics education. Mathematics is not simply a series of techniques used only by scientists, accountants or engineers who may strip it of its beauty, reasoning and significance. If people such as Descartes, Newton, Leibniz and Kepler did not have an imagination, then developments in mathematics could not have broken away from tradition to unleash new and revolutionary concepts and mathematics would not be the beacon of light to both science and society that it is today.

Finding solutions using graphical means, rather than algebraic means can give rise to shifts in mathematical instructional importance. The learners can gain a greater control of their own learning by experimenting, observing and checking their own algorithms, bringing Aristotle's ideology into the twenty

first century.

With a decreasing emphasis on the teaching of mathematical ideas in isolation and the use of routine book work, the teacher and the textbook no longer are the exclusive sources of mathematical knowledge, with rote learning procedures becoming rare practices today.

“Teachers should avoid carrying out only tests which focus on a narrow range of skills such as the correct application of standard algorithms” (MinNZC, 1992, pp 15.)

Research findings by Anthony (1994), for example, reflect and support this, giving today’s teacher the challenge to create a learning environment that supports and encourages students to be active, constructive, and goal orientated in the knowledge acquisition process. Anthony (1994, 1995, 1996) found that presently, aspects of classroom instructional methods can unwittingly contribute to passive learning behaviours.

Active constructive knowledge - building is characterised by a balance between discovery learning and personal exploration on one hand and systematic instruction and guidance on the other (De Corte, 1995, Leder 1993). This type of teaching strategy (scaffolding) should be gradually removed, in order that students become agents of their own learning, as they progress through the school system. Thus it is logical to ask, “Could the graphical calculator help to achieve this goal ?”

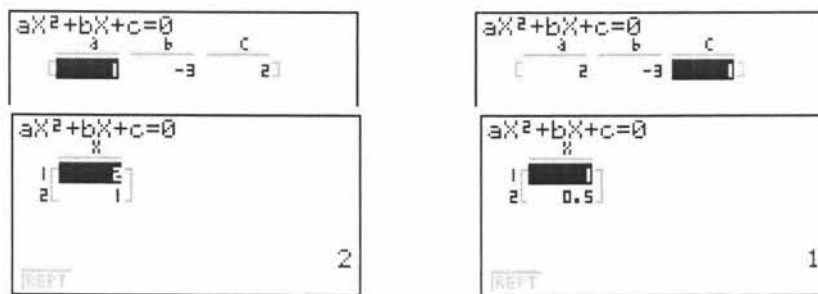
In reply to that, if the answer is in the affirmative, we then need to ask, “What can cause a teacher to make real changes in their teaching?”

For example, to seeing if there is a relationship between the co-efficients of a quadratic equation of the form:

$$ax^2 + bx + c = 0 \text{ and } cx^2 + bx + a = 0$$

that is, reversing the co - efficients. If so, could this relationship be extended to higher order polynomials. Without a graphical calculator, this would be a daunting task for the teacher and the student, as much experimentation would be required and numerous quadratic equations would be needed to test theories posed by the students.

Consider, $x^2 - 3x + 2 = 0$ and $2x^2 - 3x + 1 = 0$



[Note: That there is a relationship the solutions to the equations they are reciprocals of each other]

Figure 18: $ax^2 + bx + c = 0$ and $cx^2 + bx + a = 0$, reversing the co - efficients.

These kinds of learning tasks can alter the class dynamics, as activities become more student centered and move away from dependence on set text book exercise. The graphical calculator reduces the time needed to solve the quadratics chosen by the student and allows more time for the student to explore, experiment and discover.

National recommendations produce a new philosophical direction that can drive pedagogic changes. Teachers can either adopt a 'wait-and-see' attitude and follow recommendations when these become more popular or

they can become attracted to instructional aids that have the potential to affect real changes in the classroom.

At the Annual National Council of Teachers of Mathematics (N.C.T.M.) Conference in Boston in 1995, forty-one of the presentation titles specifically referred to the graphical calculator (Slavitt, 1996). The graphical calculator can affect the nature of the instructional environment and the avenues of context delivery, but also perhaps more importantly, it can affect the nature of the mathematics also being discussed.

What are we gaining or losing?

Just what do we gain and what do we lose by using a graphical calculator? This study highlights some of the effects of introducing a novel pedagogical aid in the mathematics classroom.

Three specific uses of the graphical calculator available to the classroom teacher and students are:

- * A means of providing alternative problem solving strategies.
- * A means of transmitting information in support of numerical, algebraic and geometrical explanations.
- * An investigating tool and development of alternative approaches to the learning and teaching of mathematics.

Examples in graphing with the graphical calculator can utilise the use of the:
- ZOOM and the TRACE features. This was evident in the student practices.

Early use of the graphical calculator in studies seemed to have focus on providing alternative strategies to the usual problems posed in mathematics. Studies showed that even when other topics were covered, the teacher spent more time with the graphical calculator to illustrate the concepts. The pedagogical power of the graphical calculator was being utilised, because the topics were being introduced using graphical images supplied by the graphical calculator. One of the problems encountered in such studies was that initially, students had to be able to find the right buttons.

The graphical calculator can introduce numerical inaccuracies and inexactness to problems encountered. For example, here is a picture of a cubic and a so called 'tangent line'. By zooming in on the point of intersection it can be shown that the line is actually not a tangent to the curve. The screen pixel definitions can at time lead to visual discrepancies that the teacher and the student need to be aware of when looking at the 'finer detail' of the mathematics that they see on the graphical calculator screen.

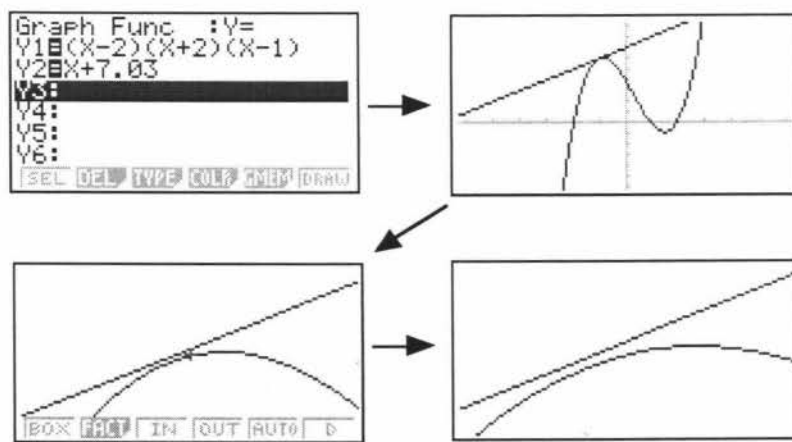


Figure 19: Pixel definition on the screen and graphing discrepancies.

Generally, the most desirable result in mathematics teaching is a blend of traditional and 'alternative' teachings approaches using the graphical calculator. Therefore the teacher is also faced with the task of fitting technology that specialises in collectively connecting the 'rule of three', namely the algebraic, numeric and graphic forms, collectively into the curriculum. In order to do this it was realised that a modification in teacher attitudes and teaching experiences needs to be made, to accommodate the presence of the graphical calculator. The use of the graphical calculator was associated with higher levels of disclosure in the classroom, and included higher levels of questioning by the instructor, as well as more active learning behaviour by the students.

Dwyer (1994) identified changes that tended to occur with electronic learning technology introduction in the classroom:

From	To
Class activity teacher centred	Learner centred
Teacher role Expert fact teller	Partner, sometimes learner
Student role Listener	Partner, sometimes teacher
Teaching emphasis Facts, skills, memorisation	Relationships, investigations
Accumulation of facts	Transformation of facts
Quantity of problems done	Quality of understanding
Assessment Norm-referenced	Criterion referenced
Drill and practice	Investigation, expression, collaboration, communication

Table 1: Changes with electronic learning technology introduction in the classroom.

Dwyer (1994) reported that:

“ . . .the greatest student advances occurred in classes where the teachers were beginning to achieve a balance between the appropriate use of direct instruction strategies and collaborative, inquiry - driven, knowledge construction strategies.”

The graphical calculator can allow for investigations and explorations from a graphical and numerical perspective together, while relating these to their algebraic form. We may then ask, can graphical calculators make investigations broader and more flexible, when more traditional methods are used? For example, if students are given a collection of graphs, the students are able to experiment with the graphical calculator to find their equations by matching what is on their screens, to that on the worksheet.

It seems that, even in conjunction with traditional curriculum delivery, the use of the graphical calculator can stimulate student involvement in classroom discourse, particularly with regard to the inclusion of the student in the problem solving aspects of the instruction.

Comparisons have been made with regard to student performance using pencil and paper methods and calculator / computer systems. In particular, calculus problems solved using computer software that employs symbolic manipulations, show there is a de-emphasis on computational skills and an increasing emphasis on the concepts of calculus (Brody & Erlwange, 1987). The graphical calculator can compute virtually all limits, derivatives and integrals posed in most calculus texts and can handle the problem, symbolically, numerically and graphically.

Three studies on students using computer algebra systems in calculus performed on conceptual examinations, scored as well as, or better than, students in traditional versions of the course (Hawker 1986, Heid 1988, Judson 1988). One could argue that this too, could result from graphical calculator use.

Graphical calculators offer possibilities of performing in seconds and by means of a few key presses, many tasks that have formerly required a range of mathematical skills, including: long division, solving quadratics and drawing graphs. For many teachers, this is unsettling and disquiet is expressed overtly in the concerns about the loss of, or devaluation of, the “basic” skills and rules. There may also be a more deeply rooted concern in that the graphical calculator challenges present traditional pedagogies.

One of the most important tasks in mathematics education today is the inclusion into the curriculum, of teaching methods that take advantage of technology, (MinNZC, pp 14). Developments with graphical calculators during the last decade or so have presented us with an inexpensive and powerful tool which challenges the traditional assumptions previously held about what we should teach, how we should teach and what students should and can learn.

There are now a profusion of technological innovations available for classroom use whether it is a computer or a graphical calculator and there are now many mathematics educators actively exploring these technological aids with a view to include them into mathematics courses.

Adjustments need to be made by teachers in their delivery of secondary school mathematics courses, due mainly to the changing face technology in

education, industry and commerce. Fey (1989) suggested some of the changes we need to include are:

1. Context / Process goals - Trends show a decreasing attention to aspects of mathematics work readily done now by machines, but also an increasing emphasis on conceptual thinking and metacognition. This change focusses on enhancing the curriculum and using applications of greater complexity, rather than using problems which were accessible to students via more traditional teaching methods.
2. Teaching / Learning styles - There has been a shift from teacher role of expositor and drillmaster to that of task setter, counsellor, information resourcer, manager and explainer. Teachers also must have an empathy with students themselves whilst, the students are encouraged to be actively engaged in more self directed learning activities.

These developing initiatives should impact on the mathematical content delivered in classrooms, namely by:

1. Numerical computation.
2. Graphic computation.
3. Symbolic and algebraic computation.
4. Multiple representation of the mathematical information.
5. Programming languages and
6. Connections being made in transferring knowledge to computer, science and other curriculum areas.

Graphical calculators could cause a shift in emphasis from traditional computational procedures to the requisites of higher order thinking involved in problem solving. They can also help to enhance student conceptual understanding and foster positive attitudes toward mathematics, Hembree and Dessart (1986), Brolin (1987), Wynands (1984). Research by Demana and Leitzel, (1988) determined that the search for patterns in tables of values related to numerical variables, is more effective due to a “computation rich” transition from numerical to the abstractions of algebra. Similarly, numerical approaches to calculus are proving effective also. Trends therefore appear to dictate the use of graphical calculators in learning situations, after students have acquired some measure of personal skills. In an algorithmic procedure for instance, learned procedures can be reinforced or maintained by using the graphical calculator to practice this skill.

Research suggests that students who use graphing technology, whether it be computer applications or graphical calculators research they are better able to relate graphs to their equations (Ruthven, 1990), read and interpret graphical information better, are better at finding an algebraic representation for a graph (Ruthven, 1990), have a better understanding for global features of functions (Beckmann, 1989), and have better understanding about the connections between graphical, numerical and algebraic representations (Beckmann, 1989 & Browning, 1990).

Graphical calculators can provide a useful alternative method in the illustration and representation of co-ordinate graphs as pictures of algebra ought to have very powerful effects on a student's understanding. By entering rules for one or more relations, choosing domain and range values, focusing attention on smaller or larger ‘view-windows’ when graphing, re-

drawing various changes of scale, graphs redrawn and transforming the graph or making it animated, or 'dynamic', by varying coefficients, the student can see on the graph the effects of these varying coefficients.

Most students perceive mathematics as a 'game' whereby they seek to figure out the secrets possessed by the teacher, however, when the students are able to see graphs drawn for them on the screen, and can manipulate these graphs themselves, the classroom becomes a setting for student collaboration, where they can try to make more sense out of the mathematics being investigated or displayed. The teacher's role then shifts from demonstrating, "how to" produce a graph, to "explaining what is the graph saying", hence, the student task can now shift from plotting points and drawing curves to writing explanations on key graphical points or global features illustrated by the graph that is under scrutiny.

Classroom dynamics can change with graphical calculators being used suggested Dunham et al (1994). He noted that students became more active in classrooms when using graphing technology, with more group work, investigations and problem solving being observed. Solutions to equations and algebraic expressions, indefinite and definite integrals, power series expansions, summing series, and solving differential equations can be easily calculated and examined, hence classroom climate moves to more cognitive approaches to mathematical learning. Applying mathematics to real world problems and investigating their solutions with the graphical calculator should encourage a rich, positive mathematical learning environment.

Thompson (1985) in his study, found that many students, who never stopped

being “answer” orientated, became extremely frustrated with open ended problems. If getting the right answers was not the name of the game, then they did not know what was the game being played, with little or no understanding that “the answer” was most typically a method or a generalisation of a method, and not merely a number. If this is the case, then the implementation of MinNZC will certainly take a long period of time, not only for the teacher, but also for the students, in order to feel comfortable with it’s contents and aims.

Knight & Meyer (1996) highlighted ‘Critical Factors in the Implementation of the New Mathematics Curriculum’, noting that factors such as classroom preparation, intrusion into teachers’ leisure time, too much curriculum change in recent years, insufficient support from the Ministry of Education with these changes, dual assessment systems in place for senior students and the time frame for the introduction of MinNZC document, all contributed to negativity, stress and uncertainty for the teachers of mathematics.

Conclusions born out from this study were:

1. *Curriculum change is very stressful for teachers.*
 2. *The problems associated with curriculum change are long term. If they are not addressed they will not go away, and the curriculum will not be implemented.*
 3. *A new curriculum cannot be implemented without the input of significant financial resources.*
 4. *For any particular curriculum change, research is needed to determine how these financial resources might best be used.*
- (Knight & Meyer, 1996, pp 38.)

Thus, we need to be aware that these same issues may arise out of the implementation of technology into the classroom, in particular, graphical calculators. Lack of teaching resources, worksheets, teaching ideas and guidance to the use in the mathematics curriculum hamper the integrated use of technological tools in the delivery of the mathematics curriculum.

A questionnaire about the use of, and attitudes to, calculators and computers in mathematics teaching was conducted in 1995 and every school in New Zealand was given the opportunity to respond. Findings from this study (Thomas, Tyrrell & Bullock, 1995) concluded that teacher training was necessary for teachers to feel confident about integrating mathematical technology into their teaching practices. Teacher beliefs and attitudes were major elements and factors that affected technology incorporation into their teaching.

As the use of instructional technology in education expands, it becomes important that all students have the opportunity to acquire competence with that technology, more so in relation to gender issues, technology anxiety, technology confidence and technology liking. There must be a balanced link between relationships in mathematics performance and student / teacher attitudes to technology and it is logical to conclude that, the greater the technology confidence, the higher the mathematics scores posted. It was interesting to note that Munger & Lloyd (1989) found no gender related differences were reflected in their study.

Analysis of examination scripts, Boers and Jones (1993) of thirty seven first semester students studying calculus, showed that the graphical calculator was under-utilised by students, both in amount of time it was used and the

way in which it was used. The capacity of the students to deal simultaneously with graphical and algebraic information from two independent sources seemed to be the major obstacle for effective use. This suggests that students do not feel comfortable with the use of multiple representations in solving equations.

While most teachers feel comfortable in moving between an algebraic and graphical representation, this is not necessarily the case for students. (Dunham & Osborne 1991). The ability to integrate algebraic and graphical information is indeed a learnt skill and just because the student has the graphical calculator to make it operational, confidence and competence does not necessarily manifest itself, demonstrating that the skills need to be nurtured.

Traditional secondary school mathematics places a heavy emphasis on algebra over numerical and geometrical or graphical solutions to mathematics problems. The shift in emphasis on algebra to more number, table and graphical patterning work, brings numerical and graphical approaches to problem solving into the mathematical content. In particular, this is shown by statements contained in MinNZC:

“ . . .guessing and checking, drawing a diagram, making lists, looking for patterns, classifying, substituting, rearranging, . . . ”

(MinNZC, 1992, pp 11)

The analysis of teaching styles.

The Technology in Mathematics Education (T.I.M.E.), Teachers and Calculators at Kaitaia (T.A.C.K.), and Texas Instruments Research at Glenfield (T.I.R.A.G.) studies (Barton, Bullock, Buzeika, Ellis, Regan, Thomas, & Tyrell, 1993) provided evidence that effective use of the graphical calculator goes hand in hand with passing control of the learning situation from the teacher to the students. In T.I.R.A.G., the class which was most under teacher control was mostly locked into questions about the calculator, rather than questions about the mathematics. In these series of studies the effects on learning, combined with the graphical calculator use, became evident as an increase in student motivation was noted across all student levels of involvement, whether male or female, senior or junior, top stream or bottom stream classes.

Student imagery, in their setting out of problems, became a question of being 'Left-justified' and the answer 'Right-justified', just like what was appearing on the screen before them. The projects also suggested that there was some evidence to show that students with calculators perform better than their peers (Hembree et al, 1996), however this was not considered statistically significant.

The importance of teacher input is vital in the positive implementation of the graphical calculator, including teacher attitude, the skills that they bring with them into the classroom, and classroom resources that they have produced or are currently using. The T.I.M.E. study concluded that the implementation of technology had effects in learning and that there was a necessity for teacher development. Teaching and learning styles were adjudged to be

more student centred and encouraged problem solving activities.

Positive aspects that were drawn out by the T.I.M.E. study included comments such as:

“All students considered equal.”

“Less financial family members appreciative of being able to use the technology.”

“More ‘what if’ questions were asked.”

“Maths concepts were being explored rather than the students becoming bogged down with the arithmetic.”

“The students appreciated being able to change axis scales easily.”

Descriptions: class use of the graphical calculator.

In the T.I.M.E. project, the experimental groups were aware of their participation in an ‘experiment’, hence the Hawthorne Effect cannot be discarded when analysing the evidence that was collected from the teachers and students alike.

As we have discussed earlier, the graphical approach adds new light to known algebraic concepts, allowing students to gain depth while maintaining their interest and motivation. “Ability to check answers” can be interpreted not only meaning to check numerically, but also to graphically check answers obtained, enabling the student to think graphically about problems before trying to solve them algebraically. Because the graphical calculator is

more interactive and visual in presenting topics, better visualisation skills are developed, as well as providing virtually immediate feedback (Ainley, 1995).

By using a graphical calculator, students can come to associate a visual image with an algebraic expression. Both linear and non linear relationships can be explored when the capability exists for the use of only a few key strokes (Shumway, 1992). Students can come to know about mathematical equations when they can watch the graph being drawn right in front of their eyes and can easily repeat the drawing process if necessary.

Teaching strategies for graphical calculator in classroom could be grouped as:

1. Graphical calculator under the control of teacher and interaction initiated by the teacher.
2. The graphical calculator is used as a common resource in the classroom, with interaction between students and graphical calculator being direct and not controlled by the teacher.
3. The graphical calculator is used by individual students or by groups and the interaction between them is direct.
4. Graphical calculator is used without restriction (free access).

Because technology is changing the face of mathematics and its uses, technology can be considered as an "equaliser", diminishing differences between males and females, poor and good mathematics students.

At present, mathematics is in a state of change, its role becoming a tool for problem solving, communication and reasoning, requiring students to write

about the mathematical concepts that they are using, and further developing their understanding of the knowledge that is being learnt.

Collis (1987) concluded that a student's attitude to mathematics is related to their personal participation and also achievement. Armstrong and Price (1982) suggested that strategies incorporated into classroom practices need to be developed in conjunction with positive attitudes of the students to the technology. Even though the study was on secondary school students' attitudes to computers, the researchers concluded that computer use in the mathematics class revealed negative attitudes with some female students. Because attitude, participation and achievement are frequently inter-related for secondary school students, the repercussions of the transfer of attitudes toward mathematics to that of attitudes towards graphical calculators, would seem to suggest it could be detrimental to female students.

However, the graphical calculator alone will not bring a change in the classroom as, the beliefs and attitudes of teachers themselves will be a major influence in successful acceptance and implementation. Maddux (1994) believes information technology will remain relatively unimportant until teachers incorporate it successfully into their teaching practices, while Veen (1993) makes the observation that change requires teachers to alter their beliefs and practices and this will come very slowly.

Some possible suggestions to facilitate technology implementation into classroom practices are:

- * teacher training, improved teacher confidence in the use of the graphical calculator.

- * better classroom management with the graphical calculator.
- * good ideas and resources to be developed for classroom use.
- * adequate funding in resourcing and teacher professional development.

The rationale for the study was that the use of graphical calculators would have a strong, positive impact on student learning, (Quesada & Maxwell, 1994). Studies by Demana and Waits (1988, 1992), Methven (1990) and Dion (1990) indicated that graphical calculator use reflects the potential of students' to increase their understanding of mathematical concepts, particularly by presenting a mathematical problem algebraically, numerically and graphically which enables student understanding. We want students to understand things, and not just be able to do them, even though most students are content with simply being able to 'do' things. The graphical calculator allows students to 'do' more mathematics, more easily, and to benefit by increased confidence and decreased anxiety, in itself a positive influence. The use of graphical calculators does have a positive impact on student learning, but would it be fanciful to expect dramatic increases in conceptual understanding? If we want more understanding than perhaps what we need to do is teach less material! We must ask ourselves if the graphical calculator makes a difference in the students' enjoyment, motivation, confidence and understanding.

"What have we learnt?" was a question posed to the teachers by these researchers and responses were wide - ranging and informative. Comments included:

"Introduction to algebraic substitution was easy."

"The 'True Algebraic Logic' was an excellent new feature that reinforced the mathematical argument."

"Some students were able to acquire advanced mathematical modelling skills at an earlier level."

"Complexities of keystrokes, hindered progress at times."

"Good for slow students, as they were an equaliser."

"A good way to introduce technology into the classroom setting."

"Interesting incidence with the way student's setting out of work."

"Examination issues were often discussed in relation to how much working is needed to be shown."

"What constitutes working when graphical calculators are used."

Therefore, the comments revealed that the graphical calculator appeared to make mathematical investigation simpler and more interesting, however, feelings about assessment if graphical calculators were to be used in the classroom, was an important issue for both the teacher and the student alike.

Factors Affecting Implementation.

Fear that calculators will undermine mathematics learning is widespread, in spite of considerable research that suggests that calculators can play an important role in student learning and understanding of mathematics (Hembree and Desert, 1986). When there are calculators available then the mathematics instruction could have emphasis on the development of meaningful concept development and consolidation of the concept by practising and applying to real life problems.

There is also the risk that new methods may not succeed right away which is especially important in secondary schools where they contribute to the efficiency of the system and the competition between students for limited places in tertiary institutions. Thornberg (1994) gives an analogy for what happened with computers in secondary schools:

“ An entity will preserve itself and resist infection of foreign organisms at all costs. To preserve the status quo, a body first tries to reject the foreign organism by putting a shield around it . . . computers ended up in many cases inside labs.”

Lietzel (1993) noted the “explosive growth in the use of graphing calculators in secondary schools”. There have been a number of projects and related studies into the effects that the graphical calculator can have on student understanding in mathematics (Britton, 1994, Barton et al, 1993), as it has generally been accepted that certain computer packages can enhance the learning of mathematics (Britton, 1994). Therefore, it is logical to ask the question, “What are the effects of the graphical calculator?”

The integration of the graphical calculator into an undergraduate mathematics programme at the University of Sydney was primarily introduced in order to encourage the use of the calculator as an exploratory tool, enabling the study of practical problems arising in the teaching, rather than contrived ones. The previous year the graphical calculator was used for demonstration purposes only during lectures, and in 1994, this was extended to having students using them during tutorials.

One of the important aspects of the introduction of the graphical calculator

into teaching programmes is the familiarisation of it to both the teaching staff and the students. As discussed previously, students and teachers can and will at times feel threatened by, or anxious, about mathematics, however there has been evidence that students with the graphical calculator can gain benefits from the illustrations of mathematical concepts shown by the calculator.

Being able to use the graphical calculator is in itself a new skill to be learnt but once this has been accomplished, then the focus can be on the mathematical concepts, rather than the mechanical processes. In this study it was found after the first year of the project that student performance (based primarily on their examination results) was not significantly different from student results of previous years, although the students were able to solve more practical problems rather than contrived ones. One of the factors that could have influenced the results was infrequent graphical calculator use by the students during tutorial time. Those students who invested the time, (some students purchased their own calculator) and who became familiar with operating the calculator, claimed that their understanding of many mathematical concepts improved.

Student questionnaires were also another way of gauging the success of the project with those involved in the programme. A questionnaire subsequently given to those involved in the project, revealed criticism of criteria preventing the use of graphical calculators in the final examination, although its use was permitted in the term tests. A significant number of students therefore, felt that it was a pointless exercise since they were not allowed to use them in the final examination. Comments made included the following opinion, *"We should be learning to do maths under the conditions we will have in the*

exam.” This is, of course, an example of the perennial problem of students being concerned solely with passing examinations. Students also revealed that they took a ‘black box’ approach to the use of the graphical calculator, and learnt to press buttons without applying insight and meaning to the mathematics involved in the problem. Other responses were positive, such as:

“Not having studied maths for ten years or so, my computational and algebraic skills are not very good, but I can think logically, and the TI-85 has allowed me to concentrate on the logic underlying the mathematical ideas.” Britton comments, *“For the students who will be users of mathematics in their future studies and careers, this is precisely the sort of use of this technology that I would like to encourage.”* , (Britton, 1994, pp 126).

The T.I.M.E. project (Barton et al, 1993) covered the spectrum of technologies used in primary, intermediate and secondary schools, with emphasis on teacher participation in the project. In the past, when technology was introduced into schools some teachers ‘ran’ with it while others continued with the status quo. This shift in ideology included what they called self-induction of teachers, in order to facilitate the effective use of technology and its potential to promote change in teaching practices.

Student views on the use of calculators in their classroom studies show that two emerging aspects of attitude and beliefs emerged (Ruthven, 1994). Firstly, there is the students’ degree of enjoyment and the gaining of confidence in alternative approaches to calculations and secondly, an emerging perception of scepticism about the calculator’s legitimacy. Also, there appears to be marked differences in gender and school attitudes to

calculator use in 13 and 14 year olds, with results suggesting that ultimately it is the students who in essence decide for themselves if they do not require the calculator or want to be dependent on them, either by decreasing the challenge of the work or making the work achievable.

"Nevertheless, I believe that a certain amount of understanding seeps in while students are using graphics calculators. With the added benefits of increasing confidence and decreasing anxiety for many students, I can see no good reason for not using these calculators. We just shouldn't expect miracles."

(Britton, 1994, pp 127.)

Pedagogical implications.

When we consider the opportunities and challenges of learning and teaching mathematics in this informative and technological age, we also must address changes in context and forms of mathematical activity that are used in society. As this change is presented, it must be acknowledged by all mathematics teachers and educators at every level of the teaching process.

New forms of mathematical knowledge, brought about by new forms of human activity using new tools and changes, acknowledges the way that mathematics is used in a technologically based society, and this presents a new challenge to mathematics educators. In the past, mathematics has generally been an activity enjoyed by the 'academic elite' however nowadays, mathematics is becoming a basic necessity in the organisation, creativity and opportunity that arises in modern society.

“Mathematics education tools such as pen and paper, paints, and computers according to Vygotsky, are cultural artifacts.”

(Crawford, 1995, pp 47)

Advanced technological tools make it possible for new forms of mathematical activity, thus changing the nature of knowledge and expertise of both teachers and students, and pressuring teachers to update their skills and to rethink old conceptions of knowing, teaching and learning so that students can assimilate the necessary expertise required in modelling the teacher and the mathematics.

From a Vygotskian perspective, mathematical activity in the reproducing of a lesson, using a text book as a source as well as highly automated teaching strategies, involves little conscious intellectual activity on the part of the teacher. On the other hand, creating new kinds of learning activities in the classroom that represents mathematics in new ways, requires personal commitment by the teacher.

“Now that there are machines to carry out the forms of mathematics technique that have dominated the curriculum at all levels, it is time to reconsider the narrow operational focus of students’ mathematical activities.” (Crawford, 1995, pp 60)

The use of graphical calculators could mean that there is to be a different approach to the learning of mathematics, with an emphasis on learning how the symbolic language of mathematics is used for graphs and their interpretation, and on mathematical concepts that will not become obsolete when the next generation of graphical calculators arrive. There are a number

of important questions to be asked and then resolved, in order to implement the graphical calculator successfully. We need to consider whether the graphical calculators improve learning, and if so, then problems have to be designed that can reflect the students' comprehension and understanding. For instance, if we ask ourselves, "Does the graphical calculator improve learning and if we use graphical calculators, how do we design problems to reflect the student's comprehension?" A decision must also be made to answer the question, "What do you want the students to learn?" as opposed to "What the calculator can do without the students learning anything." The teacher must be able to identify when the student is actually learning and understanding the mathematical concepts taught and not merely being adept at manipulating the calculator to arrive at a result. The graphical calculator is a valuable conceptual tool assisting students to understand the following:

- * How graphs relate to their algebraic expressions,
- * How graphs relate to numerical relationships they express,
- * How graphs relate to equations that need to be solved, and to expressions that need to be maximised or minimised,
- * How the appearance of graphs depends on the window selected,
- * How to recognise and graphically interpret common types of expressions and equations.

Most students learn mathematics by 'doing' mathematics, however teachers know that while many students do not retain methods that they practice well, the students who illustrate, explain, and symbolise methods and facts have another way to remember what they study. The researcher feels this different approach to learning mathematics is more appropriate to today's students' needs in the mathematics classroom.

Influencing factors - A teacher's perspective.

For many teachers, faced with the impending arrival of technology such as the graphical calculator into their classroom, there is a period of anxiety, and in some cases, a resistance to its intrusion. They realise that this piece of technology will increase their workload, due to the familiarisation process that is necessary, adding one more thing to think about, however new learning is a necessity. Hope (1987) described, "*To some the introduction of new technology is a threat, . . . prescription for this is a generous dose of hand-holding.*"

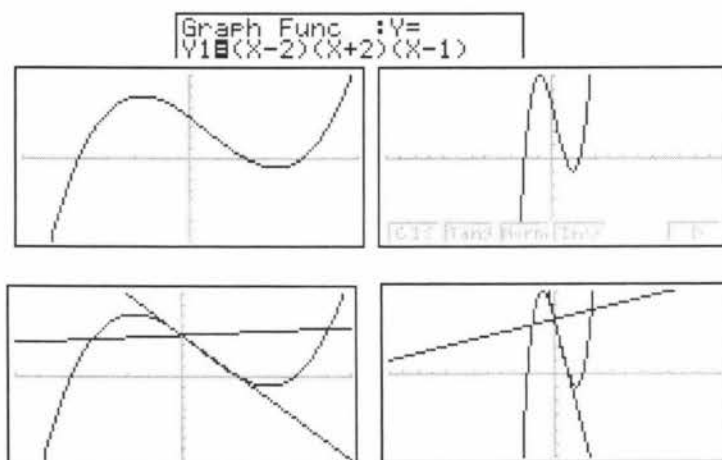
Concerns very evident at the start of the project, particularly for novice users, were that students who use the technology will have inadequate algebraic manipulative skills. With the MinNZC having the focus on real-world problem solving, some teachers felt that students entering year 11-13 mathematics courses would enter with lower skills in solving equations, factorising and the manipulation of algebraic expressions than in previous years. It is undeniable that the graphical calculators will reduce the amount of symbolic and algebraic manipulations that the students need to perform and some stated that the de-emphasis of the manipulative skills in algebra is perhaps long overdue.

Several issues arise in the graphical calculator use and this is evident for example, in the drawing of perpendicular lines. Due to the rectangular screen. If the width of the screen is say 95 pixels and the height is say 63 pixels, things perpendicular do not appear perpendicular and this apparent distortion gives rise to the student believing that the on screen results are incorrect. Can the students then believe what they see on the screen is true?

It would seem that the curve and line either appear tangential or intersecting, however the 'zooming in' facility will reduce this error, or the manipulation of the domain and range values, will show that there is in fact no intersection at all! This demonstrates that in order to arrive at a correct conclusion, the student has to be able to use the calculator with some competence.

Considerations about the screen size is a definitely concern. To see such things as perpendicular line to be drawn non - perpendicular. For instance, consider the 'normal' and 'tangent' lines drawn on the curve:

$$y = (x - 2)(x + 2)(x - 1) \quad \text{at } x = 0, y = 4$$



[The illustration shows that clearly the 'normal' and 'tangent' lines do not appear perpendicular on the original screen setup.]

Figure 20: 'normal' and 'tangent' lines drawn to a curve illustrating the need to set domain and range values.

Students need to acquire an intelligent partnership with the calculator, and not blindly accept what appears on the screen to always be correct. To overcome difficulties students need to have mathematical understanding of the representations being made on the graphical calculator (Salomon, Perkins, & Globerson, 1991).

Influencing factors - A student's perspective.

Initially, some students thought that the graphical calculator was going to solve all of their mathematical worries, such as passing examinations. Student resistance was encountered when it was perceived that new skills had to be learnt in order to operate the calculator, and these attitudes had to be addressed and corrected.

The view that mathematics learning is a pleasurable activity is often balked at by the majority. Cockroft (1982) said that mathematics was a subject to be endured, not enjoyed, and those students with feelings of anxiety and helplessness with the subject would support his claim.

Summary.

It is evident that the learning process involves the co-ordination and execution of strategy uses. Metacognition requires the ability to plan, monitor and regulate, and cognition requires control processes to guide it. Pressley, Borkowski & Schneider (1987, pp 93), who developed the 'Good Strategy User' model, stated:

"When children actively monitor strategic implementation and performance, they can increase understanding and how to use the strategy and the benefits gained from it."

A plan is then formed for executing these strategies and progress during the use of the strategies is monitored, if the teaching of metacognitive strategies has been effective. In the face of difficulties, ineffective strategies

will be abandoned for others, hopefully more favourable.

It is also important to remember that strategic behaviour requires an awareness on the part of the learner to realise the need for personal goals, and how to view learning. These factors can influence the variety of strategies that the learner has in their armoury and the ability to use them effectively and flexibly.

Students' learning goals influence their choice of learning strategies and their beliefs about themselves and this also influences their learning strategy use. As strategies are goal orientated, the students' metacognitive knowledge will influence the choice of learning strategies employed. The student's motivation to learn is an important aspect in the learning situation. A positive attitude to learning either can affect themselves making personal gains in their understanding in a particular subject area or their overall achievement.

To achieve successful learning, both the teacher and the student need to work in partnership to develop metacognitive knowledge and behaviours useful to them. Ideally, the learning environment needs to provide opportunities that require higher order thinking and strategy use, that will enhance positive learning outcomes. Successful students plan their work, are able to self instruct, self access and correct their own work as well as being able to modify their learning tasks and to know when it is appropriate to seek help. By controlling and monitoring their understanding in the learning process, these students are demonstrating that they are engaging in metacognitive behaviours.

The importance and relevance of metacognition in the implementation of

successful student learning is, one of the key factors in gauging teaching and learning effectiveness.

It would seem that the graphical calculator has a role in the instructing and learning of mathematics in the classroom, however, it must be made clear to the student that the graphical calculator is not a crutch to lean on, but an accessory to help them to learn the mathematics. Factors which are able to be controlled can either influence a positive or a negative impact that the graphical calculator can have in classroom use.

The advantages quite possibly outweigh the disadvantages for both students and teachers. Introducing the calculator cannot be piecemeal, and needs to be orchestrated with a clear purpose and with endeavour, for there to be an appreciation of what it truly has to offer to those who use it. This study attempts to begin the examination of the introduction of graphical calculators into schools.

These studies have shown that there is presently certain aspects of teacher and student resistance to the implementation of graphical calculators into mathematics learning and instruction. Students felt that they had to learn something new on top of the mathematics, as well as being doubtful that this new tool may not be a means to substitute the familiar and tried pencil and paper algorithms. Therefore, how does the student incorporate this new tool into their current learning practices so that it does not become a hindrance in their own learning of mathematics?

For teachers, the graphical calculator presents another skill to be mastered and then included into their teaching practices. Problems may occur in the assessment of the topic when graphical calculators have been used.

Questions arise such as, "What areas of the mathematics curriculum can be taught 'better' or in a different manner using the graphical calculator." and "What mathematical concepts remain important and what areas of mathematics lose it's relevance when the graphical calculator is used?"

These implications of pedagogical strategies can have a dramatic effect on students and teachers alike in the positive (and negative) response from the use of the graphical calculator. If the graphical calculator is to play a part in the teaching of mathematics, we must be aware of all the sentiments in order to facilitate acceptance of the calculator's role. There may also be a transitional period, during which teachers and student alike become familiar and comfortable with this new mathematical 'tool'.

Chapter 5

Research Method.

“The integration of the calculator into the curriculum where it plays a central role in the learning process is a worthy goal for the research in the 1990s.” (Hembree & Dessert, 1992, pp31.)

Introduction.

Dunham & Dick (1994) published an overview of research that had to date been made on the effects of graphical calculators in the United States, and discussed its importance on mathematics teaching. They noted:

“The graphing calculator phenomenon is new enough that relatively little research has found its way into the journal literature.” (Dunham & Dick, 1994, pp 440)

They examined the use of the graphing calculator and drew the following conclusions:

“The early reports from research indicate that graphing calculators have the potential to dramatically affect teaching and learning of mathematics particularly in the fundamental areas of functions and graphs. Graphing calculators can empower students to be better problem solvers. Graphing

calculators can facilitate changes in students and teachers' classroom roles, resulting in more interactive and exploratory learning environments . . . The evidence supporting the use of graphics calculators . . . certainly suggests that this technology can be a catalyst for, and not an obstacle to, mathematics learning." (Dunham & Dick, 1994, pp 444)

The general purpose of this study is to examine the use of graphical calculator technology in mathematics classrooms, especially as technology has dramatically changed the way mathematics is used and valued in the world today. Eventually all of us, teachers, students, parents and employers will need to decide what to use and when to use it.

To encourage the use of graphical calculators, the pedagogy in the classroom needs to be adapted for its use. For example, investigations with the graphical calculator in pre-calculus and calculus courses, provide 'hands-on' experimentation, as the students discover the mathematics in question that follow with graphical calculator use. With the use of a graphical calculator, the student can grasp new ideas and concepts that are being explored and then to focus more on the mathematical implications, rather than the labour associated with pencil and paper algorithms. By their own directed discoveries, the student is able to make the mathematical connections and hence construct their own mathematical knowledge effectively.

The research setting.

In order to undertake the project on graphical calculators in the classroom, it was necessary to have a representative group of schools that were willing to be included in all phases of the study. As it is the teachers' who are working at the 'coal face' of student learning, teacher input into this study was a necessary and valuable commodity, drawing on the teacher's experiences and use of the graphical calculator in their classroom.

The time period of three school terms was appropriate and using terms one, two and three in 1997 meant that the data collection from the schools was finished prior to term four, which was a short term and teachers could focus on their senior examination preparations.

A pre - calculator use questionnaire was submitted to those schools who expressed an interest in being involved in the study and the researcher was able to form a group of eight schools from the lower part of the North Island. To gain an overview of the participating schools and the teachers who were to be involved in the project, the questionnaire gave statistics on gender, age, individual teaching experience and a general description of the school type and current technology presently available to them in their respective schools.

This questionnaire was followed up with a sheet that was to be completed by the teacher when the graphical calculators were used in the classroom. Entitled, "A slice in time", this sheet gave an overview of the lesson and teacher perception of the students' reception, acceptance, use, motivational affects and feelings towards the graphical calculators as they were used in

the lesson.

Nearing the conclusion of the study, a post - use questionnaire was given to the teachers involved for a personal evaluation of the graphical calculator use in their classrooms. This questionnaire helped to describe how teachers found the integration of the graphical calculator progressed in their classroom and teaching practices and the learning experiences the calculators had on themselves and the students.

Finally, the study ended with an interview with some of the participating teachers to enable them to expand further on their experiences, and to identify any significant trends in teacher use of the graphical calculator.

In the beginning . . .

In 1995, a teacher development training programme was initiated by the researcher to give teachers the opportunity to gain skills and to have access to graphical calculators. This programme recognised that advancements in this type of technology was making considerable progress in it's functionality and it's pricing was becoming realistic for the consumer, namely teachers and students of mathematics.

The research was to be conducted primarily by a mixture of qualitative and quantitative methods and interviews (Triangulation). The researcher set up a database of contact schools, and with the help of Monaco Corporation and Casio calculators, was able to approach secondary school Mathematics Departments and their staff for inclusion in this research. An introductory

programme to graphical calculators was drawn up and included participants:

- (i) Attending two workshops, each of four hours duration, that introduce the teacher to the graphical calculator and its functions.
- (ii) Completing a questionnaire at the beginning of the first workshop [pre - graphic calculator use]
- (iii) Attending the second workshop to finish explanations and to troubleshoot if there were problems with the teacher familiarity process. The teachers also had some time to themselves to become more familiar with the calculator and the following term a class set of graphical calculators was provided for each school to use with their classes.
- (iv) Completing a follow up questionnaire [post - graphic calculator use] to see how the graphical calculator was used and to collate reactions to their use by teachers and students.
- (v) A 'slice in time' check sheet also facilitated the collection of classroom activity, providing information for the use of the graphical calculator in the classroom programmes.

Piloting the study.

During the year of 1996, a pilot project was made by the researcher with his own classes. Graphical calculators were made available to his year 9, year 11 (School Certificate), year 12 (Mathematics with Applications) and year 13 Mathematics with Statistics classes. The calculators were available to the students at all times if they wanted to use them in class time, and the students were able to take a calculator home if they so requested. The

students at particular times through the year had instruction on using the calculators for specific activities. For instance, the year 9 class were plotting co - ordinate points and drawing lines, with the aim of being able to produce a flag of a country or a product logo on the calculator using point plotting, saving it as a written programme and printing it.

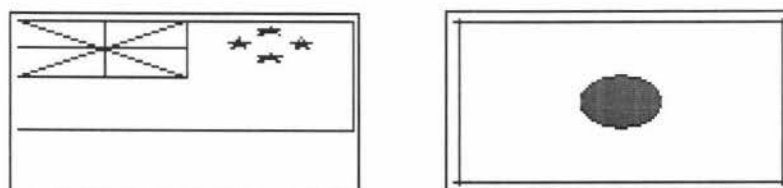


Figure 21: Student illustrations of N.Z. and Japanese flags.

Another activity was to reinforce the BEDMAS concept and it's connections to algebra. These activities were well received and the students worked very collaboratively together.

The year 11 class were not so receptive, and could be described as an accelerant group of students who were very formal in their approach to learning mathematics as a whole. Most particularly, the girls preferred pencil and paper methods whereas the the boys would use the calculators, more as a check on their solutions, rather than working with them as a primary source to begin their 'attack' on solutions.

The year 12 and 13 classes began the year hesitantly, using the calculators gradually, until they saw the relevance for using them. A requirement of the year 12 class was for them to be able to use the calculators in assessments. The year 13 class began to see the relevance of the calculators when they could see connections between the theory, the pencil and paper work and what the calculator offered in terms of reducing the the workload, once the

theory had began to make sense to them.

By the middle of the year, the students who had been using the graphical calculators regularly were becoming proficient in operating them and at times it was 'second nature' for them to automatically use the graphical calculator, rather than their own scientific calculator.

Data collection.

The main role of teachers involved in the study was to use the graphical calculators as an effective teaching device. The workshops for the teachers provided a mechanism for them to embrace the graphical calculator, rather than give it a cursory glance.

The workshops offered a necessary platform for novices and experienced users of graphical calculators alike, to share their own experiences with the use of the technology within the classroom setting and to evaluate it's relevance as the project took it's course.

This was the basis for the development of the teacher workshops, deemed necessary for those teachers who were to be involved in the study. Those who were not familiar with the graphical calculator had the opportunity to learn new skills in a non-threatening environment, supported by their peers and the researcher. Problems that each participant experienced were addressed and appropriate action taken to remedy the problem, or if there was a limitation in the use of the calculator, this was discussed and conclusions were made.

An aim of the study was to view the classroom dynamics and understanding mathematical concepts taught to the students while integrating graphical calculators into the classroom setting. This was supplemented by following up some of the participant teachers and interviewing them, collecting classroom data and attempting to compare knowledge formation by the students. The use of conventional and non-conventional teaching modes, as described by the teacher participants, when using the graphical calculators as a learning tool, was also discussed.

Classroom co-operation would be for the teacher to incorporate the following into their teaching programme:

1. A more practical and investigative approach to mathematical learning using the graphical calculators.
2. The graphical calculators to be available and accessible to the students and their use will be examined when looking at student strategies, rather than the teachers' perspectives.
3. The stimulus of the concept that mathematical ideas are the focus of classroom activity and not merely routine mechanical algorithms.

The Project Design.

The research project was divided into four phases:

Phase 1 - the training of teachers to become familiar with the graphical calculator.

Phase 2 - the provision of support in the schools with school visits and

regular contact to be made with each school.

Phase 3 - the period in which the graphical calculators were used in the classrooms, whereby students become familiar with the graphical calculator.

Phase 4 - interviews with teachers to obtain an appreciation of the effects that the graphical calculators had in their classroom practices.

Each teacher was provided with a checklist to '√' and 'x' for each lesson where the calculators were used. This list represented '**a slice in time**', as the research progressed throughout the project time frame and it was hoped that this aspect of self - appraisal would also act as an impetus for change. The 'slice in time' provided periodic classroom observations throughout the period when teachers used the graphical calculators

All participants received a comprehensive workbook and associated instructional booklets to help in the familiarisation process, and the researcher would provide 'follow up' on a regular basis, by personally visiting the schools involved in the study. This was expected to help in providing continued motivation with the study by the participating schools and also give an opportunity to reflect on what had happened in the classroom, as well as offering suggestions to the teachers for continued use. Some examples of trialled worksheets and 'starters' were distributed if requested, but as each school had their own schema for each year level, it was left to individual teachers to assess when it was appropriate to use the graphical calculators.

A focus of the study was the liaison of the researcher and the teacher and provision of support for the teacher. Informal discussions held, showed how

the calculator could complement or supplement the activities that could be used in the classroom.

Consent was sought from the Principals and the Schools' Board of Trustees, by way of a letter of introduction of the researcher and an explanation of the study that was to be carried out. It included a request for the participating schools' assistance and co-operation in the study and was followed by information being sent to the staff involved. A Staff Consent Form was also developed for the teachers.

Ethical considerations.

All teachers involved in the study were informed of the purpose of the research project and their role in it, prior to the commencement of the research project. Anonymity was paramount throughout and a policy of cooperativeness, guidance and tuition were a priority throughout when required.

The data base.

A representative sample of schools from throughout New Zealand was sought, to participate in the study. During every school term, schools in particular areas of New Zealand were asked to attend the workshops and some were selected on a "first in, first served" basis. They, in essence, formed a 'cluster group'. The selection of the participants from the schools was be dependent on the classroom teacher's decision to be involved in the

programme.

A questionnaire (Appendix 3) provided preliminary information on the teachers covering gender, age, ethnicity, years in the teaching profession, years in teaching of mathematics classes, their qualifications background, the classes that they taught this year and a sharing of their ideas about the graphical calculator being used in the classroom. A description of the schools appears in Appendix 1.

Summary.

Eight secondary schools availed themselves of the study and consisted of:

- (a) 1 city co-educational school (year 9 - 13) using Sharp calculators,
- (b) 2 female integrated city schools (year 7 - 13) using Texas calculators,
- (c) 4 provincial town co-educational schools (year 9 - 13) using Casio calculators and
- (d) 1 male city state school (year 9 - 13) using Casio calculators

covering the lower half of the North Island, with the total number of teachers entering the research being approximately 30 in number. Included in the study was one school that had not been involved in the teacher workshops but wanted to be included that the graphical calculators owned by them would begin to be used more by the staff and the students.

Each school was able to decide the areas in which the calculators were to be used and which level of student classes was to be involved. The teachers were encouraged to report on their own and also the students' response, to the use of graphical calculators in their classrooms. Most of the schools

elected to restrict the graphical calculator use to the senior school students, with reasons ranging from the graphical calculator functions being more suited to the senior students to the concerns of management and the security of the calculators.

The total time involved in the collection of data was during the first three terms of the academic year of 1997. The researcher was 'free' to offer the support to teachers during the latter two terms as he was not teaching classes himself.

The classroom teachers elected to use the graphical calculators according to their own personal style of teaching. Options ranged from using it to introduce a topic, or for maintenance of a learned skill, or for checking or clarifying pencil and paper methods that had been done previously. Even though classroom practices were 'usual' to the students and teachers alike, the teaching and learning styles could alter as the calculator became integrated with the learning process.

Chapter 6

Research results.

The 22 teachers involved in the study, ranged in ages from 25 to 50 +, the average age being 38 years. All of the teachers had a diploma of teaching and their qualifications in mathematics ranged from none to a master's degree in statistics. The majority of the teachers involved had at least 200 level mathematics papers in their degree.

The number of years involved in teaching ranged from two years (teacher recruitment from the United Kingdom) to over ten years, the majority having taught for eight years or more. Most worked in their teaching position in a full time capacity.

The school numbers indicated that there were three distinct 'types' of school, those which had:

- (i) student numbers < 500,
- (ii) 500 < student numbers < 1000 and
- (iii) 1000 < student numbers < 1500

These figures included secondary schools and secondary schools with an attached intermediate school, co - educational, and single sexed schools.

Mathematics courses were compulsory to year 12 (Form 6) level and the majority of the participating schools were involved in some aspect of Unit Standards assessment.

The majority of the teachers owned their own graphical calculator and there

was a 50 / 50 split between student non use and use of a graphical calculator, prior to the study. Approximately 66% of the teachers reported that some students in their class owned their own graphical calculator, but found this was confined to their senior students (year 11 student or higher).

All of the teachers were aware that graphical calculators were allowed in the NZQA examinations with which they were involved, namely School Certificate, and Bursary Examinations. All of the schools had computer access, in particular, spreadsheet software, such as Claris Works and / or Microsoft Office. There were only a few schools who had programmes such as Omnigraph, Algeblasta, Stat Lab, Math Helper, Australian Stat Pac, New Zealand Stat Pac, Greenglobs and PC Info indicating clearly, that only a limited array of software was available in the schools involved in this study.

Some comments from the Pre-graphical calculator use questionnaire.

What were some of the teachers expectations from the teacher workshops?

The following are some of their comments:

“Opportunities for greater exploration of graphs and their comparisons.”

“Enough expertise and confidence to be able to use the graphical calculator for myself and to teach its use in the classroom.”

“Heaps! With more confidence with technology, students will not have to do so many tedious calculations and the subject should become more meaningful and interesting.”

"More skills and knowledge about technology available."

"Better preparation for post school study and the work place."

"To learn to use the graphical calculator so that we can assist students in their use."

"Confidence and incentive to use graphical calculator in the classroom."

"To show the ease of use of the graphical calculator for teachers and students alike."

"A sense of realism for this subject called maths. Fun, kinesthetic, . . . "

"Increased confidence to use graphical calculator and ability to competently introduce them to students."

"A chance to upskill myself in the use of this technology. Some ideas and resources for direct implementation into the classroom."

"Increased power to analyse data. Speed of calculation, display etc, implying more practice. More motivation as they get to use the technology in the classroom. Better examination results."

"A chance to deepen use and confidence with the graphical calculator."

"The chance to solve problems related to 'real life' situations. Using data, patterns and graphs to assist both conceptualisation of the problem and avenues for solving the problem."

"Ability to introduce graphical calculator to the students."

"Practical applications and simulation problems."

"Make me more competent to help students with their graphical calculator and assimilate them into the classroom (hopefully)."

"For the brighter students it has a lot to offer, for the poorer students, not much."

"To introduce graphical calculator to the students. To update myself

with the latest tools!"

Evaluation of the 'slice in time' recordings.

235 'slice in time' sheets (Appendix 5) were returned and showed there were a diverse array of uses for the calculators. The majority of teachers used the calculators themselves prior to the students using them in the lesson, mainly to increase teacher confidence. Attitudes to its use, such as increased enjoyment, motivation, understanding and student confidence in their own personal abilities, was duly noted in the 'slice in time' recordings.

The length of time during which the calculators were used ranged from a ten minute revision sequence, that required differentiating at a point, to three days of investigations on graphing, differentiating, examining, and integrating functions.

The 'slice in time' indicated that the graphical calculator was associated with higher levels of classroom interaction between teacher to student and student to student. Higher levels of questioning were being asked and students were exhibiting more 'active' learning behaviours. There was a mix of traditional instruction and student centered learning with the graphical calculator.

The graphical calculator appears to have the potential to modify learning behaviours of students and to stimulate student involvement. The students were on task and were using the calculators appropriately, although some teachers did find that some students got sidetracked by trying to test the

calculators robustness. This will be discussed later in the interview sequence.

The use of the calculators was mainly confined to senior classes, but at the interviews all but one of the teachers said that in the future they would use the calculators with junior classes. The main reason given indicated that teachers felt students were still in the key stroking phase and worksheets or class work had to be supplemented with the teacher either writing the key stroking sequence on the black board or on the worksheets. This method appeared to work well and teachers found that there was a reduction in individual students 'getting lost' For those teachers that did not do this or something similar, such as use the calculator handbooks, it was found that some students gave up and felt frustrated.

Some teachers commented in the questionnaire sheets that because they are not using the calculators regularly, the key stroking sequences was still a major hurdle at this stage. Regular users felt that this could be addressed by constant exposure of the calculators to the students.

Some of the recorded student comments recorded on the 'slice in time' sheets were:

"It's not the mathematics that we have been doing in the past."

"I don't understand how the calculator does it, so I won't use it."

"To pass the test, I have to show the working, how do I show the working that I have done on the calculator."

"I use the graphical calculator to check what I have done by hand, if I get it right then that's great."

"If the calculator answer disagrees with my pencil and paper method,

then I check it by placing the calculator along side my work and do it again slowly, checking each step as I go."

It was obvious that the responses by the students as a whole, welcomed the calculators into the classroom, as they added a new and fresh dimension into their mathematics learning.

Evaluation of teacher post - use questionnaire responses. (Appendix 4)

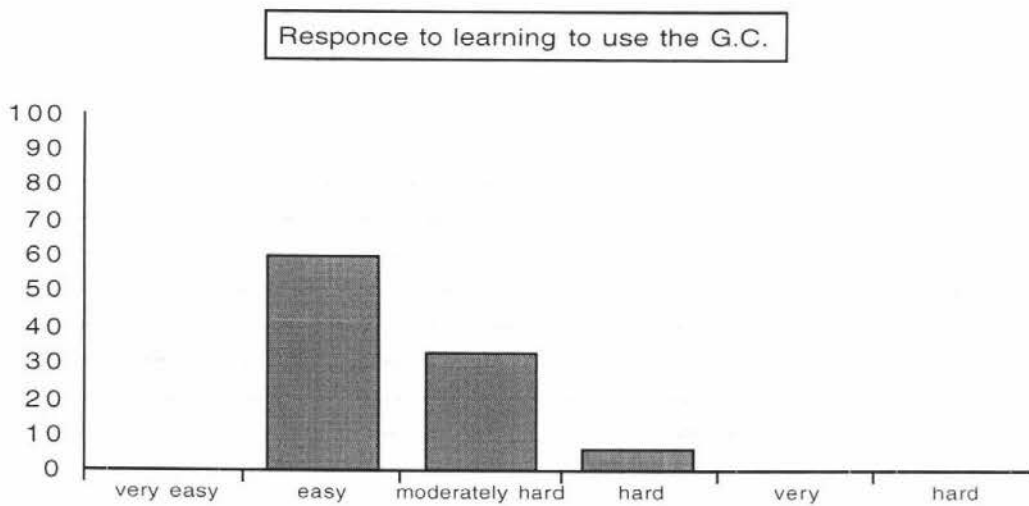


Figure 22: Teacher response to learning to use the G.C.

Most teachers said that they either found it easy / moderately hard to learn to use the graphical calculator, with two teachers finding it very difficult.

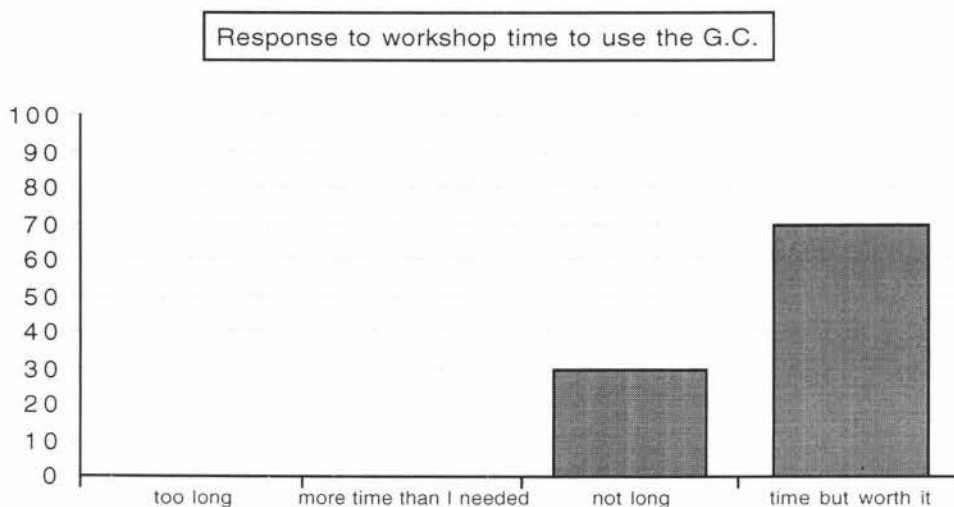


Figure 23: Workshop time taken to become familiar with the G.C.

The majority found that the workshop time was useful, but that it was not long enough for some of the teachers to really come to grips with the graphical calculator.

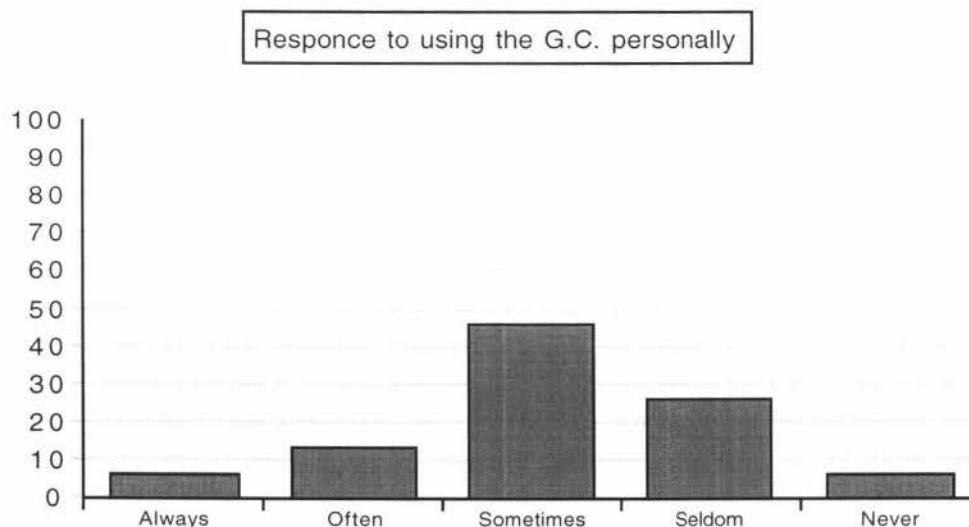


Figure 24: Teacher's personal response to using the G.C.

Personal teacher use of the graphical calculator was recorded as sporadic or

seldom. This highlighted the fact that constant use of the calculators is needed, in order to overcome the hardships involved in using and becoming familiar with calculator operation.

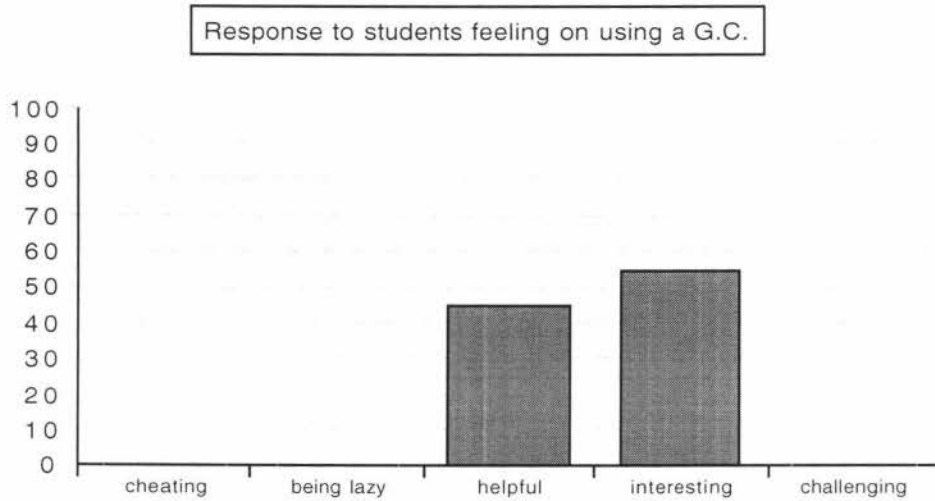


Figure 25: Student response about about using the G.C.

The teachers recorded that the students felt that 'doing' mathematics with the graphical calculator was helpful and interesting. The teachers' feelings on the use of graphical calculators by the students varied between lazy, helpful and interesting.

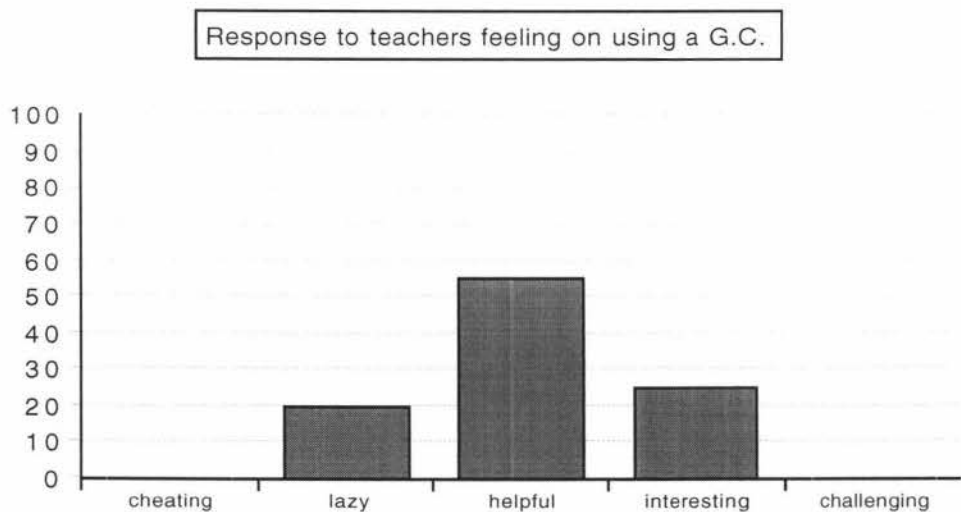


Figure 26: Teacher feelings about using the G.C.

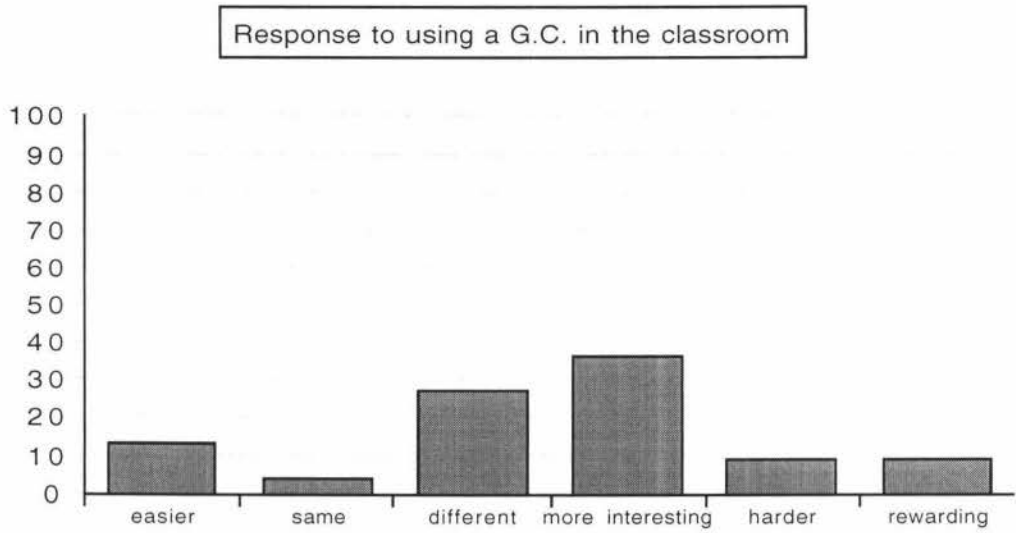


Figure 27: Graphical calculator use in the classroom.

When using graphical calculators in the classroom, teachers felt that individual students reacted differently, but with a majority felt mathematics became more interesting.

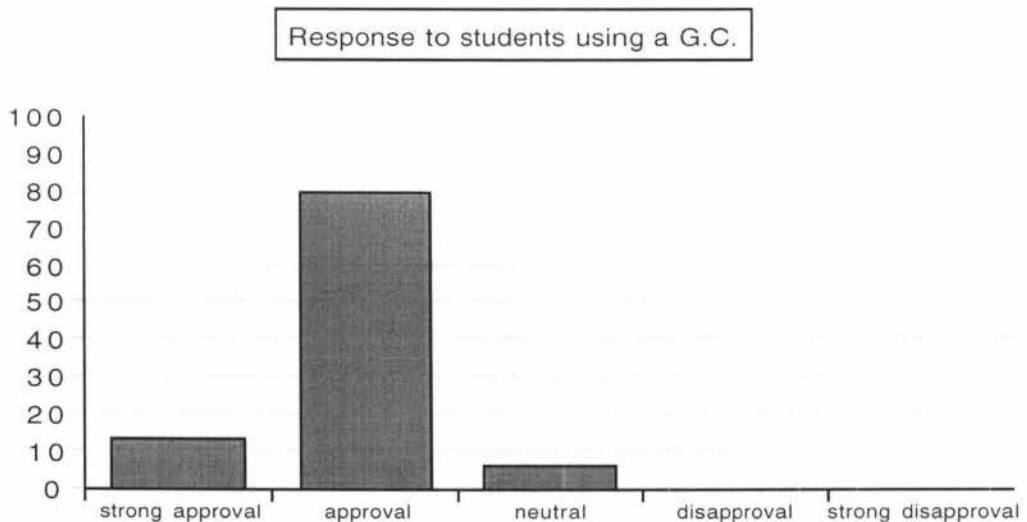


Figure 28: Students feelings on using the G.C.

There was a noticeable, positive response when asked if the calculators

helped. The teachers also felt that co - operation during the lessons became easier.

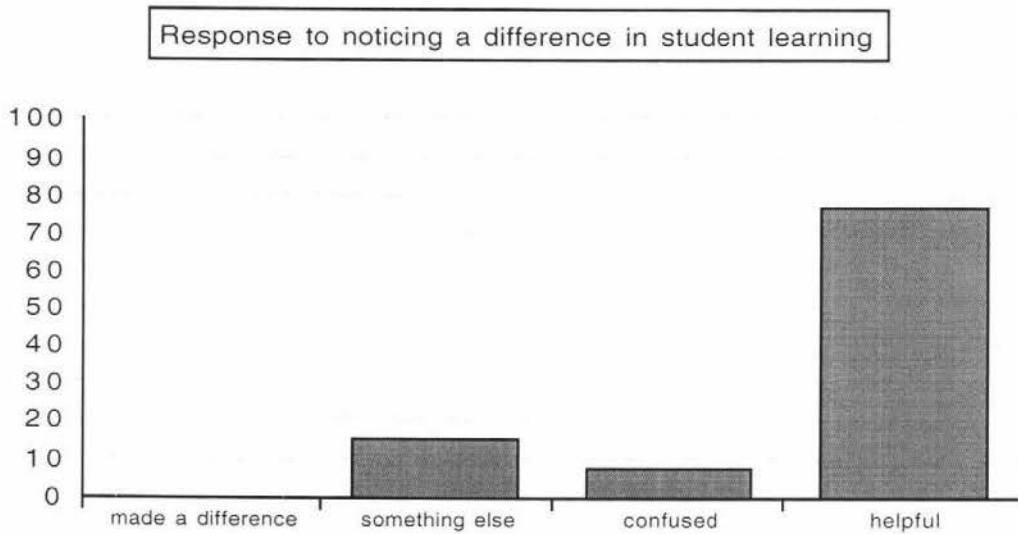


Figure 29: Differences noted in student learning.

On - task behaviour was divided, with no change to a greater level of on - task behaviour, and some teachers reported that students were more investigative in their personal approach to the mathematics being taught.

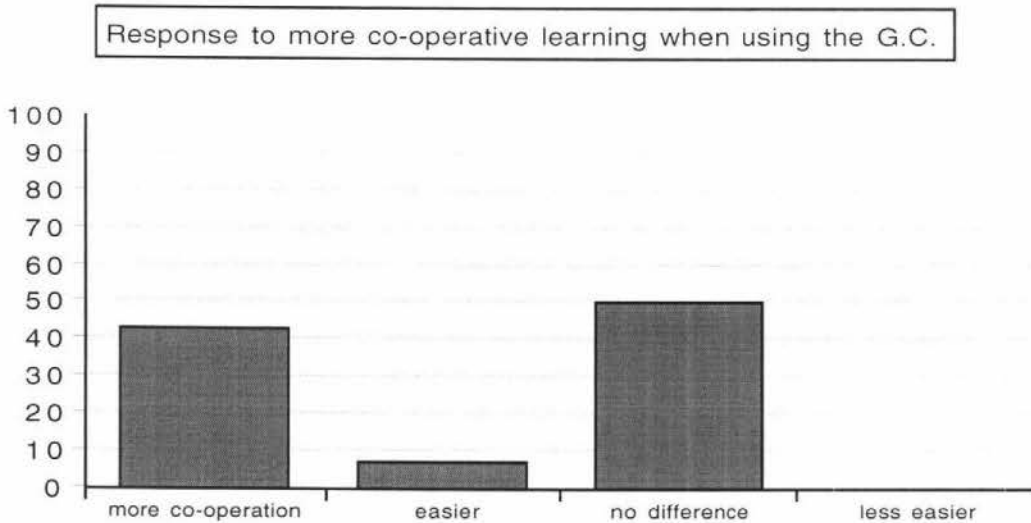


Figure 30: Response to co-operative learning.

The graphical calculator fitted into a role that shifted from teacher directed to that of being student centered.

The 'slice in time' record of what the teachers used the graphical calculators for in the classrooms. (Appendix 5)

Uses of the graphical calculators stated by the teachers in the 'slice in time' sheets were:

- * Transformations of families of functions,
 $f(x) = (x + a)^2 + k$, $y = f(x + a)$, $y = k f(x + a)$ etc.,.
- * Finding intercepts and gradients of various functions.
- * Identifying shapes of various functions.
- * Solving Quadratic equations,
(i) in the equation editor,
(ii) by tracing the functions,
(iii) with complex solutions.
- * Graphing trigonometric functions and reciprocal trigonometric functions.
- * Definite integration and areas under curves.
- * Complex number calculations.
- * Normal distribution probability calculations.
- * Plotting points, joining points, graphing lines and checking equations of lines.
- * Matching equations of various types of functions and being able to extract information about intercepts etc.,.
- * Finding turning points and matching these with the pencil and paper answers.
- * Finding limits and what significance asymptotes have.

- * Simultaneous equations.
- * Conic sections.
- * Drawing polynomials.
- * Verifying the theory with calculator calculations, BEDMAS, integration and differentiation for example.
- * Designing of appropriate involvement that the graphical calculator can play in Bursary examinations.
- * Plotting sequence and looking for the patterns.
- * Quadratic investigations. The calculator is programmed to draw parabolas and the students enter the co - efficients to attempt to match the calculators parabola.
- * Drawing $f(x)$, $f'(x)$, $f''(x)$ to see the relationships between, turning points, maximums and minimums, points of inflexions.
- * Graphing lines and curves to find points of intersections and checking examples worked algebraically.
- * Exploration of transformations of graphs.
- * Statistical simulations.
- * Identifying various functions to investigate shape and properties.
- * Investigating gradient and y axis intercept of straight lines.
- * Experimenting with Domain and Range values to see effects of the graphs drawn.
- * Appreciating what a solution means graphically and to find a starting interval for the bisection method.
- * Checking graphical solutions by 'zooming in'.
- * Seeing the relationships between graphs of functions and graphs of their gradient functions.
- * Running a programme to illustrate binary search methods.
- * Evaluating limits to sequences and series and comparing the relationship to integration.

The teacher interviews. (Appendix 5)

The interviews with the teachers spanned three weeks in time and were held in the participating schools. All interviews were conducted individually, except for two sessions due to timetable constraints and a group interview took place instead. The teachers have been coded, for example **A2** is teacher 2 from school A (see Appendix 1).

It was interesting to note in the teacher interviews, that applying the graphical calculator to mathematical problems had varying results in the classroom. F2 felt that, *"The form 4 students experiment far more easily than the teacher. The students will explore more and 'figure things out', then try something else."*

However, A2 had a different approach and felt that his students mastered the necessary skills with a somewhat casual and simplistic approach. *"I just chucked the calculators at them and said this is what they can do, here's the book, here's an example, buttons to push are on page 200 and something. We had being doing it by hand and found that the calculator came up with the same answer. So away they went, using it as an alternative, simpler method. I told them to figure out how it does this, as I didn't know. So it was more student centred and discovery orientated."*

It appears that this teacher gave some responsibility for the learning to his students and felt that the graphical calculator was an appropriate and successful tool for doing this. B1 agreed with A2. He also felt that, *"The graphical calculator has enabled me to develop a more student centered, activity based teaching style which I prefer."*

F1 also supported comments by F2, and said, *“Graphical calculators fit in with my personal teaching style, that of being interactive.”*, and *“Small groups made it easier to get around because of students getting lost for instance. There needs to be a balance of traditional teacher up the front and ‘free learning’ for instruction, as at the time there was more emphasis on the key stroking aspect of the calculator use.”*

G2 explained his classroom trials with the calculators more fully than F3. He considered that, *“The graphical calculator encourages you to move away from the transmission model of teaching. There are constraints on investigation type activity, due to having to get them through the syllabus in a set time frame. Using the graphical calculator as a tool to reinforce ideas rather than having the students re-discover these rules or mathematical concepts and ideas you are trying to get across. Students are not up to speed with using the calculators yet in general, for example with my form five students the calculators come out periodically, so students were on the wrong screen at times, we are not using them enough. There is a definite need for a balance between chalk and talk, discovery and practising skills and using the graphical calculators in the classroom.”*

While some teachers supported small group work, others favoured a large group approach, and it is important for teachers to experiment personally with the calculators, and to select a mode of content delivery with which they feel comfortable.

C1 suggests, *“Introductions to new areas needs to be slow, and step by step. Then the students are encouraged to look up procedures in the manual to improve and extend their own methods. Small classes make it possible to*

give a lot of individual instruction to those who struggle or are not very confident either with the mathematics or using the calculators.”

G1 concurred with C1 saying that, *“I prefer one calculator between two students. I’m personally not keen on group work and with the graphical calculators that’s the same. A one to one ratio would be ideal.”*

A1 however, had concerns about this technology in her classroom, and felt that it tended to dominate the teaching aspect in the classroom. She stated, *“I don’t want the technology to get between me and the teaching. We are spending far more time on working out how to press the buttons so we lost sight of where we are going and that was because I was not sure how to do it. For example when I’m doing limits and zooming in, zooming in and zooming in, it took far longer to zoom in that it should have and instead of saying here is a quick way of showing limits, it wasn’t. I want to use it as a tool that is quicker and at this stage it is not.”*

In summarising the first question in the interview sessions, it is fair to say that ten out of the fourteen interviewed supported graphical calculators in a working role in their classrooms, however class grouping and sizes, as well as personal teaching styles must be considered, so that teachers feel comfortable and in control of the classes progress.

H1 summed up in a very positive way - he felt, *“Because I’ve got into using the graphical calculators this year, I think it will influence my thinking and planning in the future. This year if I’ve been doing a topic and found it appropriate then I would use the graphical calculators. I would like to see all staff using the graphical calculators and be directed to have a go and if*

necessary those who feel more confident can help others.”

Five out of the fourteen teachers showed that when answering the second interview question, they felt unsure of their own skills and abilities when using the calculators. F3 said, *“I’m still learning after six months and still not confident. Teachers don’t get time, it was good having two workshop sessions. Resolving difficulties in using the calculators becomes a shared problem with your colleagues.”* Whereas, D2 felt, *“It didn’t take me long at all, once you could do one thing it could lead on to something else. If anything goes wrong all is not lost, the students will often find a way. I can troubleshoot really well, what have they done wrong sort of thing. “*

G1, although was not very confident of her own ability, had worked out her own way of introducing the calculators to the students, while at times only remaining one step ahead of the students. She observed that, *“I prefer to go through something myself before the students do it and I write down what I think the students should be doing. Whatever we have done with the calculators, we have produced worksheets to use in conjunction with the calculator activity. I am not threatened by the students knowing more. I like it to be a useful exercise, rather than students just fiddling, the lessons have definite structure. Having been to graphical calculator workshops at conferences and got thoroughly lost, I don’t want that happening to the students because I know exactly how they feel.”*

A1 felt that she was not confident because of time constraints that had limited her own time in learning the skills necessary for using the graphical calculator in class, *“I’m not yet confident because I haven’t spend very much time, because I haven’t got the time to spend.”*

C1 said that, *"I felt more confident by the end of the second course, but it was also helped by the extra visits you made to the College."* F1 supported C1's comment with, *"I feel confident, with the support from others, I am very willing to use others for help too."*

B1 reported that, *"With the calculators, once I had worked through the MinNZC I felt more confident in using them and meet curriculum objectives."*

G2 said that he had used graphical calculators at Training College and felt comfortable with them, and it was interesting to note that he felt personal ownership was a benefit. He said, *"I played around with it for quite a while and become interested in its abilities."* He also provided a day inservice for all the staff at the college, to begin using graphical calculators more in the classrooms. This highlights that there needs to be a staff member who is interested and enthusiastic as calculators are implemented into classroom practices.

From the results of interview question two, it appears necessary for teachers who are using the calculators in the classroom situation, to feel personally capable and comfortable with the calculator. Support and personal time must be provided or set aside, so that teachers are able to come to terms with the calculator's abilities, and of course, their own.

Question three, concerning anxiety about the use of graphical calculator, saw G2's comments being fairly detailed and informative. He observed that, *"If you're not proficient and you move into a mode that you're not familiar with then it could be a nightmare, for example degrees → radians and*

differentiation. Both teachers and students need to invest a little time to become familiar with it and it would pay off particularly in lesson planning and the continuity of their use. The graphical calculator is definite advantage for SFC students as there is no restriction in their use in assessments at this college and a number of students have purchased their own, you certainly don't want to 'flog' the calculators benefits but the advantage is quite considerable. It's not going to be the saviour to some students who are perhaps struggling, but if you use it with understanding then it's amazing, its graphical, you can see what's going on. I don't think that when I was at secondary school I was able to picture a function and make connections between the algebraic and graphical representations."

"The students that use them, their understanding has greatly increased and they are beginning to see the applications. For example, one of the students (Form 6) programmed the calculator for two players to play noughts's and crosses's. To programme a calculator to play noughts's and crosses's is tremendous. The programming side is good because if you can programme something in and understand what you are doing, then you are understanding what formulas and such like are doing for you. That goes hand in hand with understanding what's going on. That following something through in programming and the logic involved to when it ends up is great, to follow it through to its conclusion is what mathematics is all about.", commented G2.

F3 noted that, "Students asked questions and you couldn't remember the procedure for setting up the calculator. The purple book was very helpful rather than the manual." Concerns were raised that students needed to refer to the instructional text at times, whereas B1 had concerns that tasks needed

to be practised regularly if procedures were not to be forgotten. He also felt that mixed abilities classes could be covered with student centered workbooks based on MinNZC.

F1 had some concerns initially, but later felt able to direct students in using techniques and to answer questions more confidently as exposure to calculator use increased. It was again apparent that that necessary support structures for teachers must be in place if the graphical calculator is to be included in lesson planning and initiated readily as a valuable learning tool. With any concept or new tool, it would be reasonable to expect a period of exploration, practice and experimentation, in order to become familiar with the territory in which this new technology can take us. However, the researcher feels that we may compliment and enhance confidence and the ability of teachers and students alike, by providing support, either by personal visits (by experienced users) or written support, in the form of lesson activities that can be used in the class, specifically designed for every graphical calculator model.

Eleven of the fourteen teachers affirmed that graphical calculators aided student understanding. G2 felt that, "*Their understanding is far better when we practice with the calculator. Does it water down the algebra? I think it makes it more accessible to the average student.*" F2 said, "*You can see what is happening on the screen, not just an answer. You can easily edit if they have punched in any incorrect entry. The seven or eight lines of screen is excellent. If a student has got answer wrong then checking to see what they've put in is very easy. For example, viewing the screen can show that 9 has been entered twice or they've added rather than multiplied for instance.*"

F1 noted that by using the graphical calculator, students were able to cover work in one lesson that would otherwise have taken three or four days, saying, *"They have potential to give more aid, and that will come as the students themselves can use the graphical calculator with greater familiarity. A whole chapter that would take 3 or 4 days was done in 1 lesson."* H1 observed, *"Using the calculator to reinforce pen and paper is noteworthy. To show them that this is true, by doing something by pen and paper and hoping that it is right and checking on the calculator confirms it. The students who have their own calculator are now using it to try and make full benefit of it."*

C1 used the graphical calculator as a replacement for Omnigraph, revising the graphing section for his senior classes and said that he was pleased with the test results that followed. He felt that using the calculators enhanced student understanding and would make a good revision tool for other sections of work in this area of mathematical study.

B1 said, *"Worksheets he developed covered aspects of mathematics that could be taught using the graphical calculator, for example, transformations of functions."* A1 had doubts and noted that, *"Sometimes it has, because we've used them for graphing. They were easy, but for differentiation you could only find values of point derivatives and not $d/dx (f(x), x)$."*

The responses from question four indicated that the graphical calculator had a positive impact on student revision and understanding of pre-learnt ideas and practices. The teachers also liked the independence the calculators gave students in checking results, particularly in graph drawing and the quick visual response that the calculator offers, either confirmed their results

or queried student answers.

Question five in the interview concerned the reinforcement of ideas that had been developed in the classroom and the way in which teachers felt that the calculator helped. The visual aspects were felt by all teachers to aid checking and reinforcing mathematical ideas. F3 noted, *"When introducing power, e^x functions it usually it takes ages, but I was able to do all in one lesson, as the students could see the different shapes and patterns of each family of these functions."* F1 also commented, *"To see pattern in, as a check, a good way to look at particular features of functions that they would not focus on when drawing them by hand. Laborious drawing reduces the student to extract significant features in the graph."* C1 said, *"The graphing section of the SFC course was covered before the work was done with the calculators, therefore its main purpose was reinforcement and revision."*

B1 used the graphical calculators to check Box and Whisker plots and said the calculator used the same methods that the students had been taught previously by hand and on the computer. A2 used the calculator to, *"check solutions."* The calculator complimented the visual learning, revision and checking solutions.

In order to discover how the teachers felt about using the calculators to introduce ideas, the next question in the interview asked how they felt concerning this. As previously commented, F1 felt the calculators shortened the time taken in covering some topics, particularly graphing. In question six she noted that, *"Patterning, looking at formulae later and how substitution works. Will do that more with F6 and calculus work."* G2 felt that he had positive results in this area, saying, *"Yes, to introduce trigonometric*

equations, as I cannot draw well on the blackboard. I would love O.H.P. projector unit. I would use it all the time to draw the graphs. It's just so much quicker, here it is, as my diagrams are not all that brilliant, so it would aid myself and hence the students."

A2 also observed positive results using the graphical calculator and felt that they were particularly useful in checking results. Although B1 to date had not used the calculator to introduce ideas, he stated that he would do so in the future. He saw that the patterns could be quickly picked up by the students where the calculator had 'list' space. *"I have not used the graphical calculator to introduce ideas, but will so in the future. Patterns can be quickly picked up by students using graphical calculators and generalisations were made."*

C1 said, *"None of the topics in his classroom were introduced by using the calculator"*, and A1 gave an emphatic, *"No"* to the question. D2 had, *"I used the graphical calculators mainly with functions. With my form 5 class for example, 2^x , circles, hyperbola, parabola. My Mathematics with Calculus class, as many aspects as possible, using the graphical calculator to introduce (using the overhead protector graphical calculator). Then, go back to the theory of say numerical methods, for example, with a complicated function, just a 10 minute little look and then they can zoom in on the solution, go back and do it by the bi section method. They found it useful to use as a starter or as an introduction. The O.H.P. unit is ideally the best way to get all to see the steps and stages along the way. The students are getting very good at using them and there is someone there to help, a lot of collaboration exists between the students."* Seven of the teachers believed that the calculators were of value in introducing new mathematical ideas and

then developing the idea with theory The calculator had a role in this area, especially when teachers became more familiar with identifying real life and practical ideas for introducing topics to the class.

Question seven asked teachers to note whether students were investigating and experimenting more with the graphical calculator and also to observe if it was used to test and check ideas.

Six of those interviewed felt that the calculator helped the students in this area. B1 said, *"The graphical calculators speed and ease of use enables students to try out ideas and test more theories using such techniques as trial and error and looking for patterns."* A2 noticed that students used the calculator to check solutions while G2 commented that, *"The Quadratic programme that I wrote the students found very easy to do, by imputing numbers (co-efficients of a quadratic), most of the graphs attempted were eventually drawn by all students. It still relied on the students to use pen and paper to check, expand and try out before they entered the co-efficients."* G1 said, *"It encouraged investigation. It is easy to draw graphs, it is easy to go a little further and its like having a toy, like the tamagotchi."* H1 felt that the calculator had indeed been useful, saying, *"We were drawing straight line graphs $y = mx + c$ and some were checking their thoughts to see where it cuts the y axis and so on. The calculator certainly reduced the time required to look at family of curves due to the fact that the students can reproduce more graphs, more quickly and efficiently and even look at more advanced ideas and types of graphs."*

A1 however, did not feel her students used the calculator for investigating or experimenting, as they were more involved in trying to come to terms with it,

as well as learning the ideas being taught. She felt that their focus was on key stroking rather than on the mathematics. Not all of C1's students were comfortable with using the calculators and noted, *"Not all of the students were comfortable with the use of the calculator, but a number of the students did a lot of their own experimenting. As the calculators become more common, hesitant students should also become more investigative."*

D3 was not confident in using all aspects of the calculator (Did not attend workshop), but when she began using the graphical calculators in her classes she noted that, *"The students loved using them - both of the groups, (year 10 and 11 classes). I used them to introduce the students to exploring graphs, trying out families of linear functions, $y = mx + c$. I have not done this type of approach before, the students enjoyed it, we couldn't have done it without the graphical calculator. The students found it easy to use although I'm not that confident myself with them. I was anxious when using graphical calculator to start with because of the fact I was not familiar."*

F3 said "Yes", to the question, but F1 felt her students didn't experiment or use the calculators investigatively. She replied, *"No, it hasn't happened yet, partly due to nature of the class I have used them with. They like to be lead rather than to find out for themselves. But I can see that when students become familiar, this will happen, presently the from three students I have I could see them getting into investigative type scenarios."*

E1 reported, *"I used the calculators a lot and one positive for the calculators is that initially some girls did not want anything to do with them, but now they are quite happy to use them. The boys tend to risk take more than the girls, but the girls tend to stay on task better."* D3 also commented, *"The students*

were certainly on task more, interacted more with each other and with what they were doing. The conversations reflected what was happening on the calculators and the more able students picked up the functions and key pressing of the graphical calculator more easily. They appeared to have more confidence in what they were doing. The poor students didn't give up there was a lot of collaboration, peer support."

Therefore, half of the teachers interviewed felt that the graphical calculator was a useful tool for investigative approaches by the students. As the teachers had found themselves, when the students were comfortable with the calculators' operation, then the focus shifted to the learning. The calculator then becomes an adjunct, in the same way other mathematical tools have done in the past, such as logarithm tables.

In question eight, the researcher asked teachers if graphical calculators would be used as an assessment tool or made available in assessments in their schools. Seven teachers felt that they would be used, however A2 had reservations, saying, *"They would be good for that, as long as the student's understand the process and used it as a tool. As soon as it starts becoming the method then it's time to worry or you're not laying down the basis. Use it as a tool not the method to solve problems. For example, finding the area bounded between two curves, not testing the finding the co-ordinates, it is testing the finding of the area defined."* B1 noted that it was possible, *"A class set of 30 calculators would speed up the process. The classroom dynamics would discourage teachers to try this unless they had one graphical calculator for each student."*

D1 had this to say, *"Assessment with graphical calculator is something we*

need to consider. We used to use the computers, we probably should. We need to promote them to the students. In the next 2 or 3 years, especially for the younger ones who have had the exposure, they see it as quite a good learning tool. Much easier than going over to the Computer Suite, the portability of the graphical calculator is a real plus. Assessability to the computers is difficult here. We are going to have to invest in more graphical calculator soon. "

A typical response from the teachers who were in favour of this was G1, who replied, *"It reinforces their knowledge. For my assignments a more motivated student, I can always see him with his graphic calculator, when we are doing stuff he's checking it on the calculator, he's always looking at how he can use it."*

Question nine referred to the type of support that was warranted if graphical calculators were to be implemented into teaching programmes. Teachers responded to this question well and came up with a variety of suggestions, which indicated that they had given this some thought. G2 in particular, had a variety of ideas. He said, *"Some people will run with it, others are still on the start line, others feel it is going to ruin arithmetic forever. There is not much support, or resources, not yet. A good idea if someone took the initiative to write a heap of programmes that could be put on a 'chip' and you just plugged it in to the calculators, like a floppy disk. It needs someone in each Mathematics department to engender enthusiasm."* A1 appeared to have concerns saying, *"If graphic calculators are going to be implemented into the curriculum, then we (teachers) need time to learn how to use them and that is personal time to sit down and be dedicated to it. This is how I would like to learn, in pairs, sharing sessions as long as they started from where*

everybody was, hands on - in specific areas such as how are you teaching introductory calculus, for example. This is the scheme we have at our school, this is the requirements, this is how you can use the calculator to fit into your schools scheme.” D3 stated, “I always put the instructions on the board for keystroking, mainly because they could focus on the mathematics and not the keystroking. The students were doing a lot of exploring, lots of “What if’s” for various graphs. They were interpreting the graph on the screen. If the graph wasn’t on the screen, they interacted with others about them to rectify the problem, such as changing the domain and range values.” This indicated that although there is a need for resources, simple explanations and examples can ease the problems that may surface when students do not know what they are supposed to do.

D2 felt that there was a definite need for teacher support, and stated, *“Support is definitely needed. At present you need to make up your own worksheets and that takes time. Any resources for the classroom need to be tried and tested and they need to be specific to the school’s type of calculator. , that is, calculator specific. More in-house and teacher support training, such as applying how to use the graphical calculator in other ways. For example, programming skills, statistical functions. “*

A2 felt there was a definite need for resources if calculators were to be implemented, saying, *“We need resources to assist in the classroom. We need the time to become familiar with the calculator. Administration of them - security / batteries / keeping clean, the management issues, but most importantly, we need time - just sitting down and planning your classes to use them. A question of prioritising and some would prioritise it lower, just like computer use.”* F1 felt, *“Areas where they are particularly useful, not just*

general use. Resources that focus on really enhancing a lesson. Workshops that focus on specific aspects, for instance mathematics with statistics or year 9 Algebra. Sharing Sessions need to be interactive and not just swapping notes / worksheets as these just get filed away. Would have to be hands on. Everything has to be hands one, so that all participants are benefiting."

D2 had some suggestions to offer, *"Support to classroom teacher is needed, time to plan, and prepare. For example, in department meetings, we are going to teach this with the graphical calculator so everyone is familiar with what's happening in all classes A lot of teachers are cautious, they won't get in there without the support. Worksheets and lesson planning with O.H.P. masters would be great."*

These responses reveal that there is a genuine need for classroom resources, lesson planning and professional development courses involving learning and focusing on specific needs of the teachers if calculators are to be integrated into mathematics programmes. It is also necessary to be aware that some teachers will not want to 'run with it' - they all may have particular reasons for putting the graphical calculator aside. It would be preferable to have a unified stance, particularly in an individual school setting, and this needs to be addressed in each school as the department members begin to acknowledge the importance of this new technology. There are no easy answers, but the researcher believes that as more graphical calculators surface in a school, either by teacher or student choice, then that is the time when these concerns should be resolved. This may occur when new and energetic teachers move into the teaching profession.

To the question, Do you think that the graphical calculator could be a catalyst

for change in the teaching and learning of mathematics in secondary schools?, eleven of the fourteen interviewed agreed that it could. F1 said, *“Sure, even the scientific calculator, whilst is a catalyst for change, has not changed the teaching methods and content that is totally unnecessary with the use of graphic calculators. Teachers seem to have a phobia of students having to know how to do it. The pencil and paper algorithms are still a major focus in today’s classroom.”* B1 referred to the potential of the graphical calculator when he said, *“. . . the latest model, Casio cfx 9850, has a potential for a wide application and move students away from key skills to application and real life problem solving.”* H1 felt all students, especially senior students, should have one and be encouraged to use them, rather than expecting the mathematics departments to provide them. A1 had doubts, stating, *“I don’t think it has been a catalyst. They would be if you were, for instance, assessing with specific objectives to be met by the student using the calculator. It is a very useful tool if I could use it properly. When I first saw a scientific calculator that came to being in the early ‘70s, at a maths day, it was \$360, when your rent was \$14 and milk cost 4c a pint. We thought that would be really wonderful, we could see the uses but not as that cost and I think its the same for the graphic calculator. The uses are there, but the price needs to be more accessible, when enough students can afford to buy one so that the majority of the class had one.”*

These comments showed that the teachers felt the calculators should be available to students, preferably by personal ownership, however, there were significant concerns over their financing. Students also needed to be encouraged to use them as a supplement to pencil and paper method and the other tools already at their disposal.

Question eleven, which asked if teachers felt that using graphical calculators

made the quality and higher levels of mathematics more accessible, only two of the teachers felt this to be true. C1 said, *"Patterns of work can be investigated more easily because the calculations are done quickly and can be seen on the screen."* B1 stated, *"Yes, 'high levels' of mathematics becomes more accessible to students using the graphical calculator, also it can reinforce and encourage the learning of higher level mathematics."* There were several opinions as to the reasons why teachers disagreed. A1 said, *"It make higher level mathematics more accessible, but I don't think it aids their understanding necessarily. Use it to understand the level that they are supposed to be at."* F1 noted, *"No, but I can see the potential for it to happen. I try to go as far as I can take the class at the time, for example a Form 3 class could move into Form 6 ideas in a particular area of mathematics."*

Question twelve dealt with the issue of access to better quality mathematics if calculators were used. F1's reply was comprehensive ,saying, *"The graphic calculators made them focus on particular features of the graphs. They had a better understanding of looking for and considered that their overall ability is mediocre but they were excited and starting to make sorts of comments that you always hope students make but don't when they are drawing them."* C1 observed that, *"I do not foresee any major change in the teaching, but the graphical calculator will allow another approach (tool) to get the information across to the students. The graphic calculator will allow for some aspects of the curriculum that normally take a lot of time with calculations to be done quickly, but there will still be a need for work by the students manually in these areas."* D3 reported that, *"Some areas of the mathematics curriculum are antiquated. MinNZC has changed its focus onto application rather than learning algorithms. We still need a balance between the theory and the*

graphical calculator for practising the theory, the graphical calculator can help here."

Question thirteen asked if teachers noted any evidence, from observing the students, of a better understanding of the mathematics that was being examined while using the calculators. If so, they were asked to note also if this occurred, and also if a particular gender was a factor to be considered. F3 noted that there had not been enough time given to evaluate these types of considerations and felt that some students gave up, deterred by too much key pressing. Others in their classes, already anxious about their mathematical ability and the demands of technology, did not like using the calculator. Some students, they noted, preferred to write in their books, while those less able preferred to share the calculators and follow others' lead. From the research it appeared girls tended to show less favourable attitudes, towards graphical calculators, one reason being that girls tend to prefer a more structured classroom setting compared to that of boys. Girls perhaps, have had less experience and access time with technology.

C1 commented, *"The idea of the domain and range needing to be set brought home the idea of the size or rate of change of x and y values for different graphs that are drawn. In statistics, the realisation of the restrictions put on solving for the normal distribution were mainly due to the tables we use in the logarithm books to calculate the values of probabilities from the z -scores, the students that had a better understanding of this work were usually the students who were able to experiment more because they recognised the patterns and made the connections that were overlooked by the others."* B1 also stated, *"Form 3 are working on Form 4 work, all in the accelerant class. The students picked up the ideas very quickly."* A2

observed, "For $\int f(x) dx$ -ve answer means below x axis, numerical $\int f(x) dx$ got an answer, and the area $\int f(x) dx$ got a different answer, than abs $\int f(x) dx$ to get numerical answer for the area and it worked very well. Poorer students picked up that idea as they could see it. Reinforcing with a sketch of the possible solution was possible so the students could get a 'feel' for what is being asked." H1 felt that the average to above average student appeared to be getting the most out of the calculators, while students who did not enjoy mathematics, "it was not their thing" - gained very little.

The interviews revealed a mixed response for question fourteen. G2 said, "The 'brighter' kids take to it better than 'poorer' students. I could train them to use the graphic calculator, but I don't know whether their understanding would be better." C1 said, "The less able student did not get as much out of the applications, because they were still struggling to make the connections, (algebra and graphing). They are also more prone to make mistakes in their working, therefore the use of the calculator for helping the student to pick up on these mistakes did not help the less able student in their corrections."

A1 felt that the poor students got lost, however B1 observed that, "Students with 'poor' mathematical skills often feel empowered when using the graphical calculator, provided simple and clear instructions are given to perform the desired tasks. The graphical calculator enable students with 'poor' mathematical ability to complete tasks that they may have got bogged down and discouraged with and not completed if it was to be completed by pencil and paper methods." This was echoed by D3 saying, "They find them very helpful, I can just do this on the graphical calculator. They are also a good tool for the less able and perhaps a means for them to continue with their mathematics studies to higher levels whereas in the past, they would

not have been able to carry on."

All but one of the teachers felt that more time was needed to give a definite response to question fifteen. G1 commented, *"I think the second year of using them is going to be more telling, we will feel more confident with the calculators, using them a lot more and the students also becoming more familiar also."* "Students starting to see links between numeric, graphs, algebra when using them.", was the response from H1. A1 felt that her own competence with the calculators was still a problem, saying, *"I myself are still getting to terms with the technology. I think that it would come for example, for trigonometry you aren't using the tables to get values your using the graph and the plotting, much more easier. Using the manuals, we got very tangled up, following the instructions we had trouble clearing screens. We would follow what we thought were the steps in the manual and it wouldn't come out correctly and we gave up and taught the way I've always done it. It's not because I don't like new things and I don't like taking risks because I'll waltz them up to the computer room to work on that because I'm confident and I know what I'm doing and I can show them what to do. It's a competence thing really, if I'm not competent then I'll say to the students 'I'm still learning' and it means I can't help them when they need help. And although we can work it out together, I find that it is taking too long."* C1 said that more time was warranted to identify shifts, while B1 was positive about the calculators impact, *"The graphical calculator when used effectively enable the emphasis from calculating answers to thinking more critically about mathematics. they can also be used to enrich and speed up the learning process."*

The last question in the interview asked the teachers to review the year with the calculators and to reflect on particular aspects of the calculator's impact

in their classes. F1 and B1 reported that the students accepted the graphical calculators and delighted in using them. F1 replied that, *"The students are blown away, the fact that the calculator can help them to find things that are really difficult to do with pencil and paper and that they are allowed to use them in the examination. They feel they have a tremendously unfair advantage over others without one."* B1 answered with, *"They loved it and enjoyed the challenge."* C1 noted, *"The calculator gave the thinking student a chance to shine, and the what if approach allowed for this type of student to gain confidence in their ability."* D3 concluded that, *"We need to introduce graphical calculators to lower forms, I have used them with them and they think it's great. Like computers they've been brought up with them and they are confident with using technology as opposed to us (teachers and parents)."*

D2 looked at the impact of graphical calculators on different aspects of topics being taught and felt that this technology made some views redundant, stating, *"With these calculators is there a necessity now for log / log, log / linear now. So the curriculum needs to change somewhere, so that the technology can be used constructively. For example, the bisection method, is it needed now that we can zoom in on solutions to graphical questions. No one in "real life" would use this method nowadays, they would all be done on a computer. We are still teaching theory that in some ways does it need to be understood. Or is it just for the benefit of history? Will we ever let go of those links to history. For example logarithms are basically redundant. We need them to a certain extent? It's only those who are going on that there is specific relevance for this knowledge. But it's important history, its fascinating to use (teachers, mathematicians), but to the students to a certain degree, its good project ideas only."*

D1 had this to say, *“A number of juniors have purchased their own and want to use it for their senior year. The 4th Form students were investigated for example trig ...Some of the students want to do that “Duke of Edinburgh” type programme (started from Britain), this will also get us (teachers) involve too.”* D1 referred to a programme being run by The University of Auckland, under the auspices of the Mathematics Education Unit, which promotes a scheme for New Zealand secondary school students as an ‘Open Calculator Challenge’, where students can explore mathematics with a graphical calculator and complete set tasks involving programming, graphing, statistics and “performance” at bronze, silver and gold levels. The scheme is modelled on one that has been set up at the Open University, UK.

Summary.

As with any new idea or tool, there are those who accept, experiment and embrace it, while others take longer to adjust and to assimilate changes in prior learning techniques and familiar procedures. It appears that both the teacher and student groups have a diversity of feelings towards using graphical calculators in these early days of its emergence into New Zealand classrooms. However, it seems that both teacher and student feedback indicates calculator use is favoured, especially as familiarity with its abilities increases. It is important that both students and teachers allow themselves time to become familiar with the graphical calculator and for teachers to plan and incorporate it into their learning activities. The teachers that were interviewed indicated that support is definitely needed, as it is apparent that some feel threatened by the calculators’ impact on the more traditional methods of learning and doing mathematics.

Chapter 7

Discussion and Conclusions.

Although the phenomenon of the graphical calculator is still relatively new, this study identified and examined the different view points that arose when calculators were used in the classroom.

The study examined:

- (1) Graphical calculators and student achievement.
- (2) Graphical calculators and student conceptual understanding.
- (3) Graphical calculators and problem-solving.
- (4) Graphical calculators and classroom dynamics.

We may then ask if the graphical calculator can act as a catalyst for, and not as an obstacle to, mathematics learning and the research sought answers to questions such as:

- * "What aspect of the graphical calculator brings about improved understanding?",
- * "What paper and pen skills retain their importance?",
- * "Can technology use impede understanding?",
- * "What accounts for the success or failure in implementing the use of graphical calculators?",
- * "Will a particular gender favour the use of the graphic

calculator?"

These questions originally formed the basis of the project and aimed at furnishing evidence that teachers who used graphical calculators in their classroom practices had:

1. provided another mathematical tool that could enhance the level of understanding of mathematics by the students in the classroom.
2. provided alternative approaches to the introduction, maintenance and investigative methods to the delivery of the curriculum.
3. provided evidence that the use of technology in mathematics classrooms opened up the area of secondary mathematics education to scrutiny, in terms of how mathematics teachers are empowering themselves and how the students to come to terms with technology use in mathematics instruction.

The project also aimed at bringing about evidence that generally students who used graphical calculators in their classroom learning had:

1. benefited from its use.
2. gained more insight into the mathematics and the concepts that were being taught by the use of / or with the graphical calculator in their learning.

Significant factors in implementation.

The project examined the teaching practices in current use in the classroom

setting and the research indicated that irrespective of teaching style, the graphical calculator could be used in the classroom as part of instruction practices.

However, to achieve a balance between the graphical calculator use and pencil and paper algorithms in the classroom, one needs to be aware that teaching practices with which teachers are comfortable, as well as considering their historical links, may be perceived as being under threat by new technology. In order for a positive introduction and assimilation into classroom practices, it is a necessary requirement for teachers and students to be familiar with the graphical calculator and its functions.

How often, when we introduce and practice a new technique or skill do we hear the students inevitable response - "Why?" If the students have an idea about the processes involved, then the explanation from the teacher or the text book would be enhanced by graphical calculator use. The research indicated that students involved in the research was no different, and it was clear that students needed reassurance in order to appreciate the benefits of mastering the calculator.

In maintenance of a particular technique, for instance, the finding of possible factors of a polynomial, the guess and check method of substitution in x -values into the equation for $f(x)$ can be done quickly and efficiently by using as in the method outlined below:

An alternative approach to the factor theorem and solving $f(x) = 0$ was investigated by some of the students in the classes.

$$\text{For } f(x) = x^3 - x^2 - x - 15$$

(a) Calculate $f(3)$ and $f(-5)$



Figure 31: ‘guess and check’ method for the factor theorem using the G.C.

That is, $f(3) = 0$ and $f(-5) = -160$

Therefore by the factor theorem $(x - 3)$ is a factor.

By using the ‘?’ sets up for a ‘guess and check’ method, utilising the factor or remainder theorem. As the students are familiar with the technique of substitution, instead of the laborious tasks of continual substituting x-values like, $(-5)^3 - (-5)^2 - (-5) - 15 =$ and then having to edit the equation for each x-value entry, they are able to arrive at a solution more easily and can see along the way, the relevance to mathematical ideas that they have experienced in the past. The project indicated that the teachers that used this method or something similar, the students found it was more easily dealt with using the graphical calculator with the students finding it easier to link with their prior learning of algebraic substitution methods.

It is also worthwhile noting that for various reasons, not all teachers or students are ready for the change to graphical calculator use in the classroom. The criteria for the use of graphical calculators in the classroom and examination settings needs to be established and in particular, guidelines need to be set in place, to determine the acceptable amount of information used from graphical calculators and the writing and recording that is necessary, in order to be awarded full marks. In reference to the Chief

Examiners report for School Certificate, 1996, alternative approaches to questions using the graphical calculator were given in the report and in it marks were not deducted for appropriate graphical calculator answers. If calculators are not allowed in internal examinations, students will often question the validity of having to learn the operating skills necessary, seeing it as having to learn another 'thing' in mathematics.

As the response to the teacher interviews indicated, some teachers resisted the introduction of graphical calculators because they too, would have to master new skills. Teachers are often under pressure for all sorts of reasons, throughout the school year, and it is difficult for some teachers to find the 'extra' time necessary for learning calculator skills and then to incorporate it into their lesson plans is a big ask of some. Many changes have been placed before teachers over the last few years and the graphical calculator is another change that they may have to face, understand, master and possibly implement.

Those teachers involved in the project using calculators found that it was necessary to provide their own 'worksheets' and other teaching materials, as there is little material or text books available at present to reinforce the graphical calculator use in the classroom. This is mainly due to the fact that until now, manufacturers have not given this area of their invention a priority. The small number of educationalists writing resources and the variety of graphical calculators presently on the New Zealand market, have not assisted developments in this area. However, a solution must be given priority, if teachers are to support the graphical calculators introduction into classrooms.

It is necessary for consistency in the selection of the models of calculators if schools are to include them in their school's resources. One solution could be that a school purchases class sets, however monetary constraints could limit the number of sets available for school and individual use. Other alternatives could include students being encouraged to purchase their own, or to use some form of hireage system, with the hire fee offsetting future purchases of upgrades and replacements. Schools need to set guidelines so that uniformity is achieved, as each model of graphical calculator has its own idiosyncrasies, and a wide range of them operating within a school would be counter productive. As the teacher becomes more familiar with one type of calculator, they should feel more comfortable if another brand is brought in, by either the department in future purchases, or by individual students.

When teachers introduce the students to the calculator, they should be aware that this needs to be done with a minimum of 'key stroking'. This practice will shorten the complexities of the calculator instructions so that the focus is on the mathematics and not with trying to 'get around' the technology. Small groups of students should be instructed at a time, progressively culminating in the whole class participation, once the whole picture is in place! An example of this could be:

Long division of polynomials.

- | | |
|--------|---|
| Step 1 | Guess and check methods to find a solution to $f(x) = 0$. |
| Step 2 | Draw the graph of $f(x)$ to confirm when $f(x) = 0$ i.e. x intercepts. |
| Step 3 | Practice long division by pencil and paper methods if $(x-a)$ is a factor to find the quotient. |
| Step 4 | Confirm by drawing $f(x) / (x-a)$ and that the quotient function that they draw are the same as the pencil and paper result. |
| Step 5 | Relating the connections between algebra result of long division and the graphical result. |
| Step 6 | Find the other solutions if there are any by either factorising the quotient function and solving e.g. using the quadratic formula or finding the x intercepts of the quotient function by tracing the functions. |

Table 2: Long division of polynomials on the G.C.

By following a procedure similar to this, the connections between the algebra, the numerical and the graphical aspects are being continually reinforced, with 'key stroking' on the graphical calculator kept to a minimum.

Assessment issues.

As the graphical calculator is a relatively new phenomenon to most secondary school mathematics teachers and students, a question arises, "How do we assess the students if graphical calculators are to be incorporated into our teaching schemes?"

This was one of the concerns that surfaced during the project, however, an answer to this is easy. If the mathematics department already uses computers in their assessment, this can be extended into other areas and levels to involve the graphical calculator. For departments that do not generally use technology, the transition needs to be more slowly introduced, as each department should decide at which year level and what topic area will come under scrutiny for technology use. Teachers should develop assessments with which they feel comfortable and should be aware that there is support provided for those teachers who are to be involved. These include provision of basic worksheets, assistance through teacher workshops and ensuring that the teachers concerned have ready access to, and practice with, a graphical calculator.

The study has indicated that computer access in schools appears to be an important issue with most schools and in some it is critical. One particular concern was that computer room(s) were not as free as they have been in the past. This is due to the introduction of compulsory keyboarding classes for the junior school and senior typing classes, and also senior computer studies classes taking the lions' share of this school resource. Other departments also use the computer suites, resulting in not enough time for either complete block bookings of the computers, or the time available being either ill-timed or not at the appropriate time, due to timetabling constraints.

If graphical calculators are to be used in the classroom, then there must definitely be a keen, informed staff member, or someone who is readily accessible and available to the school, to offer encouragement and support to the teaching staff involving themselves in graphical calculator use. This person is perhaps the 'guru' and cajoles the staff by offering teacher and

student sessions on the calculator use, whether it be key stroking work or applications of particular aspects of instruction to specific uses, to quicken the instruction time that is available to the classroom teacher.

Results from this project indicated that some students are prepared to accept increased responsibility for their own learning, and to nurture and develop personal responsibility. Some helpful initiatives could be:

1. Learning logs: Students are encouraged to keep a learning log which records the knowledge worth crystallising from the learning. Examples could include, a series of keystroking sequences for a specific task, a record of work related to the graphical calculator, and particular situations in which to use a graphical calculator. By keeping a log, students are turning their own thoughts into writing about the lesson with the graphical calculator. This log then becomes a planned part of the lesson. Students are able to monitor their progress with the calculator, and show each task and goal reached as they progress. By resetting goals and adding others, the students are also re-evaluating their progress and procedures and this can be compared to journal writing in an English lesson.

2. Student responsibility to learning: Students are able to take a greater responsibility for their learning, firstly, by giving them information about what is to be learnt using the graphical calculator and reinforcing this as the lessons progress. They are expected to record not snippets, but relevant information to the learning tasks in which they are engaged. Prior to the introduction of any technology, students should be told that the more active you are in your learning, the better it is likely to be and that it is important to give that new knowledge time and practice, for example, to memorise, to

understand when working through problems. The students should be clear in their minds that their work should be reinforced by learning strategies, such as rehearsal, note taking, modelling, elaborating, discussing and trialing, to help in their understanding of the mathematics and procedures in their working of problems.

3. Cognition / Metacognition: Messages that students receive from teachers need to be consistent with the learning goals and thus are able to engage in appropriate learning strategies using the graphical calculator. It is also important to establish guidelines of graphical calculator use. If it is appropriate to introduce a mathematical tool into classroom practices, but not in an assessment or examination setting, whether it be internal or external in origin from a students perspective, there may be little incentive for the students to acquire the extra necessary skills in order to operate a new mathematical tool that has no recognised validity outside the classroom setting.

Teaching strategies, such as modelling, can be augmented with the use of the graphical calculator, whether it be to follow a mathematical argument in class, and making notes about what needs to be done, or debating results and then returning to each part of the problem to specify which components of the solution can be carried out, using the calculator. Moving through these stages will encourage students to reflect inductively on the information that is to be modelled.

The teaching variable.

It would seem clear from the results that within the teaching of the mathematics curriculum in each of the schools involved in this study, an explanation of differences in graphical calculator use reflects the ability and readiness for both teachers and students, to the graphical calculator phenomenon.

The difference in 'quality' of graphical calculator teaching may lie in the different goals that teachers set in their implementation into their classroom practices. Whether it be individual teaching, small group or whole class teaching or other variations, it is not easy for teachers to bring about gains in student knowledge and skills merely by aspiration. Gains in learning are reflected by gains in student performance.

Relevance to schools.

This research was undertaken with realistic expectations of a practical result corresponding to the conditions found in most New Zealand secondary schools, in an average classroom setting. If the average student is able to achieve under normal conditions, then effective results are likely to be promising for the rest of the student and teacher body. If the result could be obtained in some normal classes with average students by a practising teacher, then there should be a high probability that successful outcomes could be achieved by other students in other schools, in other mathematics classrooms, and with other teachers.

Transfer of knowledge, skills and motivation.

The matter of student motivation towards learning how to operate the graphical calculator can bring, to some, another critical factor in their learning and understanding of secondary school mathematics. Observations from the interviews are worth noting. Firstly, secondary school students do not necessarily welcome work on improving their learning and there is something of an initial reluctance to change learning habits that have developed over a long time, namely pencil and paper algorithms and previously learnt methods of tackling mathematical problems. As mentioned earlier, students can be put off by too much work on learning a new skill. We should also be aware of the paradox that teachers do most of the work to help students become autonomous learners, however the best motivation for work on learning new skills is the demonstrable fact that improvement will occur, is sustainable and can develop further.

“Graphics calculators are a force for change in mathematics learning and teaching. If all students in a class have graphics calculators then the teaching will be different. In addition, both what is learnt and the ways in which it is learned will change.”

(Technology In Mathematics Education, 1993, pp 82.)

Implications.

The implications of this piece of research may be briefly stated:

1. It is possible to help secondary school students to improve their

- mathematical skills and learning with the graphical calculator.
2. It is practicable to accomplish this objective in terms of normal classroom constraints and resources.
 3. The gains are sufficient and worthwhile in terms of improved knowledge and skill.
 4. It is probable that the majority of secondary school students could gain from using the graphical calculator, in particular, the average student.
 5. Benefits can be achieved through a style of teaching that facilitates self-directed learning.
 6. Graphical calculators appear to develop student ability to reflect upon their learning and to develop planning skills and self - monitoring abilities.
 7. All secondary school students should experience using the graphical calculator.
 8. Prospective and current teachers need training in the use of graphical calculators.
 9. Enhanced learning shows benefits in student performance.

“Effective use of graphics calculators is dependent on teacher understanding the changes in mathematics and in teaching which are possible. our experience is that this understanding takes a considerable time to develop. . .”

(Technology In Mathematics Education, 1993, pp 76.)

The graphical calculator poses some interesting consequences from the beginning of its journey from its packaged box to the hands of the user. The lack of familiarity that some teachers experienced was mainly due to not fully

utilising their own personal skills with the calculator. Knowing that the calculator can do a particular function does not mean that as soon as you have picked it up for the first time you suddenly can solve mathematical problems with a push of a button. Competence takes time, an investment of personal time but does not automatically ensure success when a few buttons are pushed. However, if time is taken to use and practice the calculators functions, then the individual mathematician becomes aware of the diverseness of functionality available to them.

Just like any new learning and teaching skill picked up from an inservice course or from a discussion with a fellow teacher, previous personal practice and exploration with the calculator is advisable. Even the 'bare bones' of knowledge of the calculator will prevent the transference of new skills to students becoming daunting and even threatening. Failure to feel able, competent and comfortable with the calculator and its capabilities, will probably result in the teacher regressing into familiar methods of instruction and learning for their classes, as it is sometimes easier to withdraw from a potentially threatening situation.

Noticeable changes when using graphical calculators.

Many of the teachers involved in the study noted that students using graphical calculators were able to see the connections between numerical, algebraical and graphical forms of mathematics. The graphical calculator has the potential to extend student understanding, rather than simply 'doing' the work for them and can improve classroom efficiencies by removing

tedious calculations. It also provides a way to recall patterns, equations and the like by scrolling through past work. In graph drawing, the graphical calculator provides a vehicle that can make connections with table values, algebraic relationships and their graphs, under investigation, allowing teachers and students to move away from more traditional approaches to mathematics learning. The graphical calculators 'hands - on' approach gives the students a feeling of doing 'real' mathematics.

Teachers were generally in control of class sets of graphical calculators, and they were usually provided for a specific activity only, limiting access, however individual ownership would encourage students to explore further the options available to them more. As time and access constraints are lifted, students can then develop a sense of proficiency in operating the graphical calculator.

It was evident from the research that as students became familiar with the graphical calculator they became less dependent on the teacher, and became more confident and investigative and to try their own ideas about the mathematics in which they were involved. In some instances, some students gained confidence in the mathematics being studied at the time and began to attempt more, taking risks in a way they previously never would have attempted.

Influencing the curriculum.

The graphical calculator may influence the curriculum, by providing new opportunities to the teaching and learning of mathematics and new ways for

students and teachers to explore alternative mathematical algorithms. It also brings exciting and innovative approaches to 'traditional problems'. Because the graphical calculator has a number of different 'icons', it can support a number of different mathematical activities from graphing, statistical simulations and normal distribution analysis to stochastic modelling and linear programming.

The MinNZC document actively encourages the use of technology in the mathematics classrooms of New Zealand. At present, there are minimal resources available that encourage graphical calculator use, either in their implementation or in support for the classroom teacher. Workshops are a start in helping teachers to become 'friendly' with this new technology, and the research showed that using the learning strategy of 'scaffolding' helped to accomplish this. Teachers need support as they learn how to use the graphical calculator successfully, to incorporate them into their lesson plans and to make a transition from 'pencil and paper' algorithms. It is not enough to accept it simply as a powerful learning tool that can be picked up and immediately mastered. Some teachers are still cautious in adopting the graphical calculator into their own teaching practices because they feel the 'pen and paper' algorithms provide substance and offer a proven set of logical steps for students to recognise and follow. But do they? Is it possible that some are reluctant to face change?

The research indicated that the graphical calculator can offer a richer and more accessible mathematical experience from which the student can learn and it also provides a more meaningful approach to mathematics education.

Conclusions and recommendations.

This research has shown that it is possible and worthwhile to teach students to learn to use the graphical calculator, in order to facilitate learning and to enable them to then apply their mathematical knowledge to technology use. As is often the case, a multi-faceted approach holds the promise for teachers and students alike, however the current administrative changes in education make recommendations for action on a national scene more difficult.

The following recommendations are indicated from the research:

1. **At a school level.**
 - 1.1 Schools should investigate for themselves the extent to which they develop student skills in the implementation of the graphical calculator into their mathematics programmes.
 - 1.2 Mathematics departments should formulate a policy that includes graphical calculators, into their current teaching and assessment practices.
 - 1.3 That such policies and strategies include:
 - 1.3.1 Goals.
 - 1.3.2 Objectives.
 - 1.3.3 The effects of current research on learning and teaching with graphical calculators.
 - 1.3.4 Means of monitoring and assessing achievement of the goals.
 - 1.3.5 How progress in learning the skills with the graphical calculator are reported to parents and students in the reporting process.
 - 1.4 Facilitating graphical calculator use at all student levels.
 - 1.4.1 Providing department members and colleagues in other

departments, for example, science department, means of facilitating knowledge transfer.

- 1.4.2 Forging common links to improve interdepartmental knowledge in developing student learning skills in technology.

2. **At the teacher level.**

Teachers should:

- 2.1 Investigate their own classroom practices continuously in order to gain a better understanding of learning with technology.
- 2.2 Reflect upon their own teaching, in order to make the processes involved overt to themselves.
- 2.3 Help students to set their own learning goals with this technology.
- 2.4 Help students put together general or specific learning needs with the use of the graphical calculator.
- 2.5 Use the graphical calculator to increase student involvement in classroom learning.
- 2.6 Develop methods that facilitate conscious transfer of learning strategies and knowledge across tasks and subjects.
- 2.7 Help students to check their own progress and to access their own achievement when using the graphical calculator.
- 2.8 Allow sufficient time for this to occur.

3. **At the system level.**

- 3.1 The Implementation Division of the Ministry of Education should promote a project that helps schools develop teacher knowledge and practice of graphical calculators in the classroom.
- 3.2 New Zealand Association of Mathematics Teachers should examine the contribution made by graphical calculators and their implication to

mathematics education in the future and continue to investigate the potential for further development in this area.

- 3.3 Conduct a survey to collect information about the needs of secondary schools in relation to graphical calculator use and support.

Education aspires to the empowerment of individuals to make the most of their lives, and to equip them with skills that will maximise choices available to them as we all move into the twenty first century, is an aim that is surely a priority.

“Mathematics is not about getting answers, it is about gaining insight into the methods whereby those answers are to be obtained”

(Gardiner, 1992, pp 10)

There appears from the research, to be three phases, or transition periods, for the use or implication of the graphical calculator:

- (i) Using the graphical calculator as a mathematical tool that sits along side the exercise book and used primarily for checking pencil and paper algorithms,
- (ii) Using the graphical calculators as a teaching and learning tool which allows the student and the teacher to make connections between numeric, graphic and algebraic modes of thinking and solving and being a vehicle to make these connections and to supplement the theory,
- (iii) Using the graphic calculator as an exploratory tool, which introduces the students to the mathematical content being taught. It may be used to discover the mathematical concepts for themselves or for concepts

the student has previously studied. Teacher guidance helps to fit these 'discoveries' into the 'community of mathematical practices' - a constructivist learning approach.

The graphical calculator can be used to focus on the mathematical content and / or its application to real world scenarios and is not a tool that should 'bog' students down with algebraic manipulation.

We need to encourage H.O.D's. and teachers to integrate graphical calculators into classroom practices and assessment procedures, thereby encouraging teachers to use and update their own personal skills and providing students with the chance to utilise the calculators potential. Information from the teacher workshops showed that there was a strong feeling from teachers that schools should be integrating the graphical calculators into classrooms if a move to higher order thinking is to occur

Teacher workshops are required to provide school use of graphical calculators by both students and teachers, to move to that higher order thinking (metacognition).

Ideas that the graphical calculator presents have surfaced in the survey results, and require discussion here. Some teacher resistance emerged, the cause being the requirement for teachers to rethink their teaching strategies, and move from the transition model to a more constructivist model of classroom practice.

The question of ownership arises - is it the responsibility of the student or of the school? Student ownership of graphical calculators usually results in

them making good use of its functions and to utilise it to its full potential. If the school owns class sets, however, it is the responsibility of the department to integrate the graphical calculator into all classroom practices, including assessment. As previously discussed, student resistance to their introduction could be a consideration, if assessment practices in schools are not revised.

Clarification of assessment practices needs to be addressed also, in regard to the inclusion of graphical calculators into mathematical programmes. These practices will keep within the spirit the new mathematics curriculum demands.

Teacher education and inservice, either internally or by an external provider, is necessary. There appear to be three types of 'teacher moulds', namely:

- (i) the teacher who 'runs' with the graphical calculator,
- (ii) the teacher who is cautious to start with and then realises the graphical calculators potential,
- (iii) the teacher who will never consider looking at it.

Integration of the graphical calculator into teaching practices should be done at all levels, so that as the student moves up through the school levels, the focus on the graphical calculator use changes from a focus on 'key stroking' to concentrating on the mathematics in question. Presently in both New Zealand and Australian schools, resources are becoming available that focus either solely on the 'key stroking' aspect or on the mathematics that can be done with a graphical calculator.

Assessment strategies.

Innovative and imaginative developments in these areas could embrace the graphical calculator into assessment and examination areas. If we can visualise an examination room where the students use graphical calculators as part of the legitimate tools in use, then we can also foresee departures from the more historic methods of assessment. In examination room in the future the students may work with their graphical calculator and arrive at a situation where they are required to 'screen dump' information obtained and shown on the graphical calculator screen. They then move off to a computer, and using the necessary computer software, 'screen dump' this information, print their results and then continue with their examination.

Modelling of a mathematics learning cycle when using the graphical calculator.

A proposed model for the establishing of an on-going evaluation of the graphical calculators utilise teacher and student input into classroom practices.

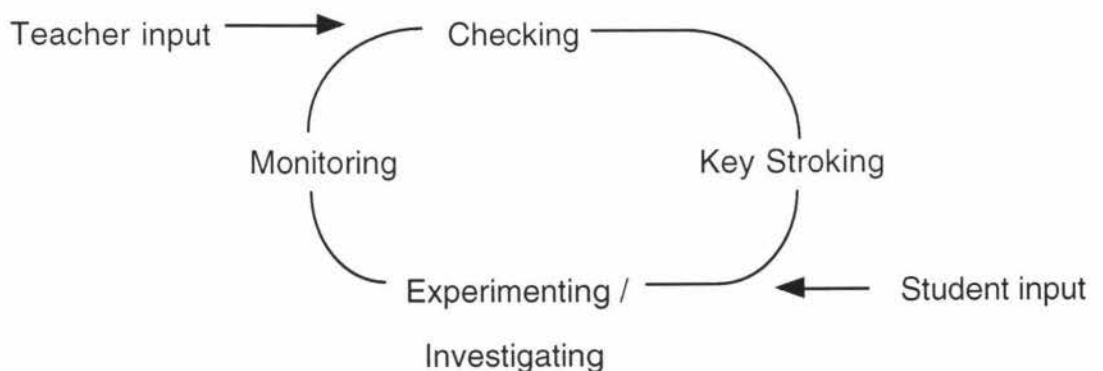


Figure 32: Modelling of a mathematics cycle when using the G.C.

Summary.

To use the graphical calculator successfully in the classroom requires teachers to exhibit personal technological capability and competence in using this technology. It is important that there is a period of teacher training provided prior to calculator use in the classroom, with a period of student training prior to using calculators in mathematics instruction and learning. Sound planning of graphical calculator use in the classroom and a selected use of graphical calculator functions in each lesson sequence should provide:

- (i) confidence building,
- (ii) maintenance of learnt 'key stroking' skills and
- (iii) to give direction in the calculators' purpose in the mathematical knowledge under investigation.

The earlier the graphical calculators are introduced to secondary school students, the quicker the students will consider it to be another useful tool in their personal armoury of tools, at their disposal, assisting in developing their mathematical understanding. Academically low achieving students with reading difficulties will find it essential that written instructions are simplistic in nature.

The issue of absenteeism was not addressed in the project, but this could pose a potential problem for the students who are absent from class.

Teacher intervention should be an ongoing activity, to ensure students are kept on track, and to provide leading questions that help students to

understand the activities in which they are involved.

Students should be encouraged to write up findings and results displayed on the calculator screen, to provide a record of their activities. When they return to this activity or something similar, this log can become a valuable reference source, that reinforces the calculator's operation and the skills that they gain as learning takes place. This would reinforce not only the skills in operating the graphical calculator, but also the mathematical skills involved in the problem and by encouraging students to do this, teachers are promoting more student responsibility for developing their own learning techniques. Log - keeping is a good way of promoting and fostering student responsibility for personal learning and subsequent independence.

We live in an age where technology plays a major role, and it is important that tomorrow's scientists, technicians and citizens are familiar and comfortable with as many facets of this technology as possible.

Once the graphical calculator becomes an accepted and integral part of classroom learning, it will become an invaluable aid to both the teacher and student alike, just like it's predecessors were in the past. Classroom dynamics can change with graphical calculator use but teachers need to be motivated to move with change. The graphical calculator could be a potential vehicle to change and enhance progress with technology in classrooms as we move towards the next century.

There is little doubt that in some time in the future the graphical calculator will be relegated to the archives of history, gathering dust in some display cabinet along with the abacus, the slide rule and the other tools of the past.

But for now, the graphical calculator takes its place at the leading edge of technology used in today's classroom, and it is possible that the students who at present are learning to master its complexities, will be using it to design the technology of the future.

Enter the graphical calculator!

Appendix 1

Participating Schools' Description Table.

	Description	School Roll	Ethnic Composition		Gender	
A.	State coeducational secondary school. Situated in a rural provincial town.	450	Pakeha	88%	Female	51%
			Maori	10%	Male	49%
			P.Is	2%		
B.	Form 3 - 7 state educational boys' school. Situated in a large city.	500	Pakeha	70%	Male	100%
			Maori	30%		
C.	Form 3 - 7 state educational co- school. Situated in a rural town.	400	Pakeha	65%	Female	53%
			Maori	35%	Male	47%
D.	An integrated secondary with attached Intermediate Situated in a city of a rural province.	450	Pakeha	88%	Female	100%
			Maori	10%		
			P.Is	2%		
E.	Form 3 - 7 state coeducational school. Situated in a rural provincial town.	600	Pakeha	81%	Female	52%
			Maori	19%	Male	48%
F.	An integrated form 1 - 7 girls' school. Situated in a urban city.	250	Pakeha	84%	Female	100%
			Maori	8%		
			P.Is	2%		
			Asian	5%		
			Other	1%		
G.	State coeducational secondary school. Situated in a urban city.	1000	Pakeha	79%	Female	46%
			Maori	8%	Male	54%
			P.Is	6%		
			Asian	7%		
H.	State coeducational secondary school. Situated in a rural provincial town.	550	Pakeha	75%	Female	51%
			Maori	22%	Male	49%
			P.Is	2%		
			Other	1%		

Appendix 2

I would like to take this opportunity to introduce myself to you as Principal of _____ College. My name is Derek Smith and I am assistant H.O.D of Mathematics at _____ College, _____.

I am presently an extramural student at Massey University completing a Masters Degree in Mathematics Education. I am currently completing my thesis which is entitled 'The Effects that Graphical Calculators have in the Enhancement of Mathematical Learning in the Classroom'. My supervisor for this research is Dr Gordon Knight.

I have made contact with members of your Mathematics Department at recent workshops that I direct. I would appreciate the assistance of the members in the Mathematics Department in my planned research. There is no direct involvement of the students at your college, hence I will not be interviewing the students or requesting them to complete questionnaires etc.

Your school would not be identified in the writings of the thesis, and the individual responses that the mathematics staff make will not be identified personally either. If you would like any further information regarding the research, I would be happy to make myself available for an interview.

Staff that are involved in the research project would be free to withdraw from the study at any time. Can I reiterate that the information provided to me as the researcher is confidential to the study and that they have the right to refuse to answer any particular questions.

Thanking you for your time.

Yours faithfully,

Derek Smith

Appendix 3

Pre Use of Graphical Calculators Questionnaire.

Preamble:

Thank you for agreeing to take part in this study on 'The Effects that Graphical Calculators have in the Enhancement of Mathematical Learning in the Classroom.' Your contribution will go towards helping in the analysis and understanding of the effects that this new technology has in the learning of mathematics.

All your responses and personal details will be held in strictest confidence. The data collected from the teacher involvement will be presented in aggregated form, and your individual details will not be passed on to anyone.

If you have any questions about the study, or are unsure about any of the details, please do not hesitate to contact the researcher.

1. What is your gender?

Male

Female

2. What is your age category?

< 25 years

26 - 30

31 - 35

36 - 40

41 - 45

46 - 50

50 +

3. What teaching qualifications do you have? (Please state)

University Degree

College of Education Diploma

Polytechnic Diploma

If you have a University Degree, at what level of Mathematics did you study to?

Level 1

Level 2

Level 3

Level 4+

4. Number of years in the teaching service?

0 - 2 2 - 4 4 - 6 6 - 8 8 - 10 10 +

5. Are you teaching part- time or full time?

Part - time Full - time

State hours

6. How many years have you been teaching mathematics?

0 - 1 2 - 3 4 - 5 6 - 7 8 +

6. What is your school type that you are presently teaching in?

Area School J 1 - F 7 (Y 0 - Y 13)

Secondary School with an attached Intermediate F 1 - F 7 (Y 7 - Y 13)

Secondary School F 3 - F 7 (Y 9 - Y 13)

7. What is the size of your student population at your school.

X < 500 500 < X < 1000 1000 < X < 1500 1500 < X < 2000 X > 2000

8. Is mathematics compulsory at your school?

Yes No

If you answered yes to the previous question then, at what year level does mathematics become voluntary? Explain.

9. Is your school using or contemplating the implementation of Mathematics Unit Standards?

Unit Standards 1996 ?

At what level ?

Unit Standard in the future?

Year 11 Year 12 Year 13
F5 F6 F7

10. Do you own, or have access to a graphic calculator?

Yes

No

If you answered yes to the previous question, What type of graphic calculator is it?

State Type(s):

11. Does your school have graphic calculators for

staff use?

Yes

No

student use?

Yes

No

12. Have some of the students that you teach own a graphic calculator?

Yes

No

If so, what % (approx):

<Y7

Y7

Y8

Y9

Y10

Y11

Y12

Y13

<F1

F1

F2

F3

F4

F5

F6

F7

13. Can students use graphic calculators in

your school internal examinations?

Yes

No

Do not know

in N.Z.Q.A. examinations?

Yes

No

Do not know

in other external examinations?

Yes

No

Do not know

14. Are there mathematical software on your school's computers that students have access to?

Spreadsheet / Database

For example: Excel, Claris, MS Works

Mathematica

Maple

Math II plus

Appendix 4

Post Use of Graphical Calculators Questionnaire.

Preamble:

Thank you for agreeing to take part in this study on 'The Effects that Graphical Calculators have in the Enhancement of Mathematical Learning in the Classroom.' Your contribution will go towards helping in the analysis and understanding of the effects that this new technology has in the learning of mathematics.

All your individual responses and personal details will be held in strictest confidence. The data collected from the teacher involvement will be presented in aggregated form, and your individual details will not be passed on to anyone.

If you have any questions about the study, or are unsure about any of the details, please do not hesitate to contact the researcher.

1. When learning to use the graphic calculator did you find it

very easy easy moderately hard hard very hard

2. The workshop time taken in learning how to use the graphic calculator was

too long more time than I needed not long time, but it was worth it I am familiar with the G C

3. Do you use a graphic calculator personally

always often sometimes seldom never

4. Do you think that the students feel that doing the mathematics with the graphical calculator was

cheating being lazy helpful interesting challenging

5. Do you think that using the graphic calculator is

cheating lazy helpful interesting challenging

6. The general student response to the use of the graphic calculator was

strong approval neutral disapproval strong disapproval

7. When you used the graphic calculator in the classroom, the mathematics to the students was [multiple responses if required]

easier the same different more interesting harder rewarding

8. Using the graphic calculator in the classroom have you noticed a difference in the student learning and understanding?

made a difference been something else to learn confused the students helped the students understand

9. In your opinion were the students more co-operative with their learning of the content using the graphic calculator ?

more co-operation easier no difference less easier less co-operative

10. (a) Was there a greater level of on-task behaviour

yes no no change

(b) Was there a greater level discussion among the students regarding the mathematics being learnt?

yes no no change

11. Did the students show more investigative approaches when using the graphic calculators?

yes no no change

Appendix 5

Slice in Time Sheet.

A Slice in Time....

Please complete as soon as possible after the lesson that is to be evaluated.

Date: _____ Time: _____ Period: _____
Class: _____ Year Level: _____ Number of students present: _____

Activity:
(Explain briefly and / or attach the worksheet / lesson plan.)

Student participation and evaluation:

On task activity:

< 25% 25 - 50% 50 - 75% 75% +

Calculator use:

Appropriate

Inappropriate

The activity was

well
received

not well
received

by the students.

Further investigation on this topic with the graphic calculator is appropriate:

Yes

No

Were the students communicating mathematically, when on task with the graphical calculator ?

Yes

No

Attitudes of the students towards the use of the graphic calculator in class as seen in:

Enjoyment

Motivation

Confidence

Understanding

Increase Decrease

Increase Decrease

Increase Decrease

Increase Decrease

The students generally found using the graphic calculator in this activity

very
easy

easy

neutral

hard

very
hard

Using the graphic calculator encouraged the students to

risk take

make critical decisions

experiment

With the use of the graphic calculator, did the noise level

Increase

Decrease

Did the graphic calculator activity encourage

information processing

analysing information

accuracy

student confidence

checking outcomes

thinking critically

testing ideas and solutions

Thank you for your assistance.

Appendix 6

Teacher Interview Schedule and Questions.

1. Have you become more aware of your preferred teaching style since using the graphical calculators?
2. How long did / has it taken you to feel confident in using the graphical calculator ?
3. Were you anxious about using the graphical calculators in your classroom ? Why ? How did you over come this ?
4. Do you think that the graphical calculator has aided student understanding when they were used in the classroom ?
5. Did you use the graphical calculator to reinforce ideas that were being developed in your lessons ? How ?
6. Did you use the graphical calculator to introduce mathematical ideas that you wanted to develop ? How ?
7. Were the students investigating more, experimenting more with the graphical calculators to test ideas and theories ?
8. Would you use graphical calculators in assessments at your school in the future ?
9. What sort of support do you think is warranted if graphical calculators were to be integrated fully into your mathematics programmes ? (For example, resources, ongoing workshops, sharing sessions.)
10. Do you think that the graphical calculator was / has been / could be a catalyst for change in the teaching and learning of mathematics in secondary schools?
11. Does the quality and 'higher levels' of mathematics become more accessible to you and the students when using the graphical calculators ?
12. Does the graphical calculator give you, and your students, better access to:
 - (a) better quality mathematics?
 - (b) higher level mathematics?
13. Have you seen any evidence from the students of better understanding of the mathematics being examined when the

graphical calculators have been used ? What mathematics class level ? What gender ? What level of the student's ability ?

14. Have students with a 'poor' mathematical ability applied themselves better to the tasks when using a graphical calculator ?
15. Has the emphasis shifted from calculating answers to thinking more critically about the mathematics when students are using the graphical calculator ?
16. Could you share with me some of your experiences that you may have experienced either yourself or with a student or the class as a whole and the graphical calculator.

Appendix 7

Supporting the numerical, graphical and algebraic approaches to problem solving.

You are offered two jobs starting on the 1st July 1997. Firm A offers you \$40000 per year to start and promises an annual rise of 4% every 1st July. At Firm B you start at \$30000, but with an annual rise of 6% per year. On July 1st of which year will the job at Firm B pay more than the job at Firm A?

We know that in any particular year n say, your salary will be:

$$A = 40000(1.04)^n$$

$$B = 30000(1.06)^n$$

Numerical Approach
plan.

Create a table of values showing the annual salary under each

With the Graphic Calculator define the functions, set up and display a table of values for each function

```

Table Func  iY=
Y1=40000(1.04)^X
Y2=30000(1.06)^X
Y3:
Y4:
Y5:
Y6:
[SEL] [DEL] [TYPE] [CLR] [RANG] [TABL]
    
```

```

Table Range
X
Start:0
End :20
Pitch:1
    
```

X	Y1	Y2
0	40000	30000
1	41600	31800
2	43264	33708
3	44994	35730

X	Y1	Y2
4	46794	37874
5	48666	40146
6	50612	42555
7	52637	45108

X	Y1	Y2
8	54742	47815
9	56932	50684
10	59209	53725
11	61578	56948

X	Y1	Y2
12	64041	60365
13	66602	63987
14	69267	67827
15	72037	71896

X	Y1	Y2
16	74919	76210
17	77916	80783
18	81032	85630
19	84273	90767

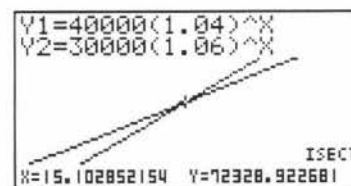
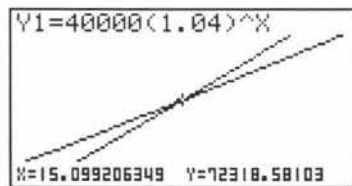
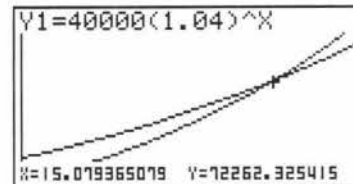
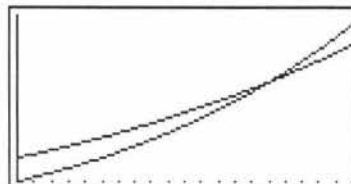
The table of values shows that after 15 years (2012) the B plan salary (\$71896) has nearly overtaken the corresponding A plan salary (\$72037), and we can conclude that the B plan overtakes the A plan by the 16th year (1997 + 16 = 2013). So by July 1st, 2013

you will be earning more under the B plan.

Graphical Approach Enter $Y1 = 4000(1.04)^X$, $Y2 = 30000(1.06)^X$ and set the view window to $[0,20]_x$ and $[30000,100000]_y$. Press F6 [DRAW].

```
Graph Func :Y=
Y1=40000(1.04)^X
Y2=30000(1.06)^X
Y3:
Y4:
Y5:
Y6:
[SEL] [DEL] [TYPE] [CLR] [MEM] [DRAW]
```

```
View Window
Xmin : 0
max : 20
scale : 1
Ymin : 30000
max : 100000
scale : 0
[INIT] [TRIG] [STD] [STO] [RCL]
```



Tracing and Zooming or employing the [Intersect] option, we can estimate that at year 15.10 years the salary will be equal from both Firms. So by July 1st, 2013 you will be earning more under the B plan.

Algebraic Approach

$$\text{Solve } 40000(1.04)^X = 30000(1.06)^X$$

$$4(1.04)^X = 3(1.06)^X$$

$$\log \frac{4}{3} = \log \left(\frac{1.06}{1.04} \right)^X$$

$$\log \frac{4}{3} = X \log \left(\frac{1.06}{1.04} \right)$$

$$\frac{\log 4}{3} = x$$

$$\log\left(\frac{1.06}{1.04}\right)$$

$$15.10 = x$$

Therefore, in $1997 + 16 = 2013$, plan B would produce the larger salary.

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