TEACHERS DEVELOPING COMMUNITIES OF MATHEMATICAL INQUIRY

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ABSTRACT

This study explores how teachers develop communities of mathematical inquiry which facilitate student access to, and use of, proficient mathematical practices as reasoned collective activity. Under consideration are the pathways teachers take to change classroom communication and participation patterns and the mathematical practices which emerge and evolve, as a result.

Sociocultural theories of learning underpin the focus of the study. A synthesis of the literature reveals the importance of considering the social and cultural nature of students' learning and doing mathematics in intellectual learning communities—communities in which shared intellectual space creates many potential learning situations.

A collaborative classroom-based qualitative approach—design research—falls naturally from the sociocultural frame taken in the study. The design approach supported construction of a communication and participation framework used to map out pathways to constitute inquiry communities. Study group meetings, participant and video observations, interviews, and teacher recorded reflections in three phases over one year supported data collection. Retrospective data analysis used a grounded approach and sociocultural activity theory to present the results as two teacher case studies.

Managing the complexities and challenges inherent in constituting communication and participation patterns each teacher in this study successfully developed communities of mathematical inquiry within their own classrooms. Important tools that the teachers used to mediate gradual transformation of classroom communication and participation patterns from those of conventional learning situations included the communication and participation framework and the questions and prompts framework.

Significant changes were revealed as the teachers enacted progressive shifts in the sociocultural and mathematical norms which validated collective inquiry and argumentation as learning tools. Higher levels of student involvement in mathematical dialogue resulted in increased intellectual agency and verbalised reasoning. Mathematical practices were shown to be interrelated social practices which evolved within reasoned discourse.

The research findings provide insights into ways teachers can be assisted to develop a range of pedagogical practices which support the constitution of inquiry communities. For New Zealand teachers, in particular, models for ways teachers can draw on and use their Maori and Pasifika students' ethnic socialisation to constitute mathematical inquiry communities are represented in the case study exemplars.

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E rima te'arapaki, te aro'a, te ko'uko'u te utuutu, 'iaku nei.

Under the protection of caring hands there's feeling of love and affection.

TABLE OF CONTENTS

ABSTRACT ii

ACKNOWLEDGEMENTS iii

TABLE OF CONTENTS iv

LIST OF TABLES x

CHAPTER 1: INTRODUCTION 1

1.1 Introduction 1

1.2 Research aim 2

1.3 Background context of the study 2

1.4 Rationale for the study 5

1.5 Overview of the thesis 7

CHAPTER 2: THE BACKGROUND TO THE STUDY 8

2.1 Introduction 8

2.2 Mathematical practices 9

2.2.1 Mathematical practices as reasoned collective activity 11
2.2.2 The discourse of communities of mathematical inquiry 12

2.3 Sociocultural learning perspectives and activity theory 14

2.3.1 Communicative interaction and the mediation of mathematical practices 14
2.3.2 Zones of proximal development 17

2.4 Communities of mathematical inquiry 21

2.4.1 The communication and participation patterns of communities of mathematical inquiry 23

2.4.2 Socio-cultural and mathematical norms 25

2.5 Summary 26

CHAPTER 3: THE BACKGROUND RESEARCH ON THE TEACHING AND LEARNING OF MATHEMATICAL PRACTICES IN COMMUNITIES OF MATHEMATICAL INQUIRY 28

3.1 Introduction 28

3.2 Structuring communities of mathematical inquiry 29

3.2.1 Models of teachers mediating mathematical inquiry cultures 30
3.2.2 Variations in practices of classroom inquiry communities 34
3.2.3 The socio-cultural and mathematical norms of communities of mathematical inquiry 37
3.2.4 Intellectual partnerships in the mathematical discourse of inquiry 39

3.3 Forms of discourse used in mathematics classrooms 42

3.3.1 Univocal and dialogic discourse 42
3.3.2 Inquiry and argumentation 44
3.3.3 Interactional strategies used by teachers to engage students in the discourse 47
3.3.4 The mathematical discourse and the development of situated identities 48
3.3.5 Examples of frameworks used to structure collective reasoning during inquiry and argumentation 50

3.4 Summary 54

CHAPTER 4: THE MATHEMATICAL PRACTICES OF COMMUNITIES OF MATHEMATICAL INQUIRY 56

4.1 Mathematical practices 56

4.2 Mathematical explanations 57

4.3 Mathematical justification 59

4.4 Mathematical generalisations 65

4.5 Mathematical representations and inscriptions 71

4.6 Using mathematical language and definitions 72

4.7 Summary 74

CHAPTER 5: METHODOLOGY 76

5.1 Introduction 76

5.2 Research question 77

5.3 The qualitative research paradigm 77

5.4 Design research 78

5.4.1 Testing conjectures: Communication and participation framework 80

5.5 Ethical considerations 86

5.5.1 Informed consent 86
5.5.2 Anonymity and confidentiality 87

5.6 The research setting 88

5.6.1 Description of the school 88
5.6.2 The participants and the beginning of the research 89
5.6.3 Participants in the study groups 91
5.6.4 The case study teachers and their students 92
5.6.5 Study group meetings 92

5.7 Data collection 94

5.7.1 Data collection in the classrooms 95
5.7.2 Participant observation 95
5.7.3 Video-recorded observations 96
5.7.4 Documents 97
5.7.5 Interviews with teachers 98
5.7.6 Exit from the field 99

5.8 Data analysis 99

5.8.1 Data analysis in the field 100
5.8.2 Data analysis out of the field 100
5.8.3 Sociocultural activity theory data analysis 103

5.9 Data presentation 104

5.9.1 Trustworthiness, generalisability and ecological validity 105

5.10 Summary 106

CHAPTER 6: LEARNING AND USING MATHEMATICAL PRACTICES IN A COMMUNITY OF MATHEMATICAL INQUIRY: AVA 108

6.1 Introduction 108

6.2 Teacher case study: Ava 109

6.3 Establishing mathematical practices in a community of mathematical inquiry 110

6.3.1 Constituting shared ownership of the mathematical talk 110
6.3.2 Constituting a safe learning environment 112
6.3.3 Collaborative construction of mathematical explanations in small groups 114
6.3.4 Making mathematical explanations to the large group 116
6.3.5 Learning how to agree and disagree to justify reasoning 119
6.3.6 Generalising mathematical reasoning 121
6.3.7 Using and clarifying mathematical language 123
6.3.8 Summary of the first phase of the study 125

6.4 Extending the mathematical practices in a mathematical inquiry community 125

6.4.1 Providing an environment for further intellectual growth 126
6.4.2 Collectively constructing and making mathematical explanations 130
6.4.3 Engaging in explanatory justification and mathematical argumentation 132
6.4.4 Problem solving and inscribing mathematical reasoning 136
6.4.5 Justifying and generalising mathematical reasoning 138
6.4.6 Using mathematical language 141
6.4.7 Summary of the second phase of the study 142

6.5 Owning the mathematical practices in a community of mathematical inquiry 142

6.5.1 Maintaining intellectual partnerships in collective inquiry and argumentation 143

6.5.2 Transforming informal inscriptions to formal notation schemes 148
6.5.3 Increasing the press for generalising reasoning 149
6.5.4 Summary of the third phase of the study 152

6.6 Summary 152

CHAPTER 7: LEARNING AND USING MATHEMATICAL PRACTICES IN A COMMUNITY OF MATHEMATICAL INQUIRY: MOANA 154

7.1 Introduction 154

7.2 Teacher case study two: Moana 155

7.3 Changing the interaction norms towards a community of mathematical inquiry 157

7.3.1 The initial start to change the communication and participation patterns 157
7.3.2 Constituting shared mathematical talk 159
7.3.3 Constructing more inclusive sharing of the talk in the community 162
7.3.4 Learning to make mathematical explanations 165
7.3.5 Learning how to question to make sense of mathematical explanations 168
7.3.6 Summary of the first phase of the study 170

7.4 Further developing the communication and participation patterns of a community of mathematical inquiry 171

7.4.1 Collectively constructing and making mathematical explanations 172
7.4.2 Providing a safe risk-taking environment to support intellectual growth 175
7.4.3 Positioning students to participate in the classroom community 178
7.4.4 Providing explanatory justification for mathematical reasoning 181
7.4.5 Exploring relationships and pattern seeking 185
7.4.6 Using mathematical language 187
7.4.7 Summary of the second phase of the study 188

7.4 Taking ownership of mathematical practices on a community of mathematical inquiry 189

7.5.1 Using models of cultural contexts to scaffold student engagement in inquiry and argumentation 189
7.5.2 Further developing student agency of the mathematical discourse 192
7.5.3 Problem solving and a shift towards generalising 194

7.5.4 Justifying explanatory reasoning through inscriptions 196
7.5.5 Summary of the third phase of the study 198

7.6 Summary 198

CHAPTER 8: CONCLUSIONS AND IMPLICATIONS 200

8.1 Introduction 200

8.2 The pathways to developing communities of mathematical inquiry 201

8.2.1 Scaffolding student communication and participation in mathematical practices 203

8.3 Supporting students to become members of communities of mathematical inquiry 207

8.4 Supporting teachers to construct communities of mathematical inquiry 209

8.5 Limitations 212

8.6 Implications and further research 213

8.7 Concluding words 215

REFERENCES 217

APPENDICES 244

Appendix A: Teacher information sheet and consent form. 244

Appendix B: Student information sheet and consent form. 247

Appendix C: Board of Trustees Information sheet and consent form. 249

Appendix D: Parent and care-giver information sheet and consent form. 252

Appendix E: The framework of questions and prompts. 254

Appendix F: Sample transcript with teacher annotations. 256

Appendix G: Problem example developed in the study group to support early algebraic reasoning. 257

Appendix H: Problem examples developed in the study group which required multiple ways to validate the reasoning. 258

Appendix I: Problem examples developed in the study group which supported exploration of partial understandings. 259

Appendix J: Moana's chart for the ground rules for talk. 260

Appendix K: Examples of expansions of the communication and participation framework. 261

Appendix L: A section of the table of data for the activity setting in the classroom. 262

List of Tables

Table 1: Assumptions about doing and learning mathematics implicit in teacher-student interactions. 33

Table 2: The communication and participation framework 84

Table 3: A time-line of data collection 94

Table 4: Example of the codes 102

Table 5: Examples of the themes for teacher and student actions 102

Table 6: Examples of the evidence for one theme 103